Enhancement of single-lap joint strength under tension using piezoelectric actuation

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ENHANCEMENT OF SINGLE-LAP JOINT STRENGTH UNDER TENSION USING PIEZOELECTRIC ACTUATION

A Thesis

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for degree of Master of Science in Mechanical Engineering in The Department of Mechanical Engineering

by

Ryan Michael Meyer
B.S. Louisiana State University, 2009
May, 2010
This thesis is dedicated
to my parents
Milton Clayton Meyer, Jr.
and
Terry Conaway Meyer
and
my fiancée
Sarah Lyn Rogers
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ABSTRACT

Composite adhesively bonded joint structures have seen increased use over conventional riveted due to their ease of manufacturing and quality strength-to-weight ratios. However, adhesively bonded joints lack the rigidity and failure strength of riveted assemblies, due to high stress concentrations which develop in the adhesive layer, which makes them prone to undesirable deformation and failure. Piezoelectric materials feature the ability to produce mechanical strain when subjected to an applied voltage and vice-versa. Therefore, it is proposed to use piezoelectric actuators to transmit counterbalancing forces to increase the rigidity and strength of composite joints. Existing studies have been limited in that they have focused on the effect of piezoelectric actuators bonded to or embedded in a single beam. This work studies the use of surface bonded piezoelectric linear strain actuators, and their effect on induced static deflection of composite joint structures. Euler-Bernoulli beam theory is utilized to derive the elastic curves of the proposed structures due to actuation forces and external loading, while accounting for physical parameters of the structures. By decreasing the deflection within the adhesively bonded region, especially at the critical joint locations where high stress concentrations are developed in the adhesive layer, the overall strength of the joint can be improved. The effects of straight beam actuation and small beam curvature actuation are discussed. A numerical study is performed at evaluating the effectiveness of piezoelectric actuation based on straight beam and small curvature beam actuation, as well as bond location. It is shown that small beam curvature actuation increases the amount of deflection which can be produced by the piezoelectric actuator. The effects of four possible piezoelectric bond regions are analyzed. It is shown that bonding the piezoelectric actuator in Region II most effectively reduces detrimental deflections caused by tensile loading at the critical joint locations.
CHAPTER 1: INTRODUCTION TO PIEZOELECTRIC MATERIALS

1.1 HISTORY

In 1880, the brothers Jacques and Pierre Curie discovered the phenomenon now known as the piezoelectric effect. They discovered that certain crystalline materials (Quartz was the material which lead to the discovery), when subjected to a mechanical strain, would become polarized. They found that if these materials were subjected to forces in tension or compression, they would generate voltages of opposite polarity which were proportional to the applied forces. This phenomenon is called the direct piezoelectric effect; derived from the Greek word “piezein” which in English translates as “to press or to squeeze” (Curie et al., 1880, Schwartz, 2009). Years later, the direct effect would be implemented in applications such as strain gauges and energy harvesters. They later went on to find that the converse of the relationship was true. They found that by applying a voltage to these crystalline materials, it would cause them to expand or contract in proportion to the strength of the applied electric field. This is known as the inverse or converse piezoelectric effect. The converse effect is what allows piezoelectric materials to be used as mechanical actuators by subjecting them to applied electric fields (APC International, Ltd., 2009, History of Piezoelectricity, 2009). By the 1950s, the development of Lead zirconate titanate (PZT) based materials lead to widespread use of piezoelectric ceramics in industrial applications due to their excellent electromechanical properties (Schwartz, 2009).

1.2 THE PIEZOELECTRIC EFFECT

The direct piezoelectric effect refers to the situation when an applied mechanical force results in an electric charge being generated by the piezoelectric material. This is the basis for which piezoelectric materials are implemented as sensors in structural monitoring applications. Figure 1-1 illustrates the direct piezoelectric effect for both tension and compression loading.
Notice the opposite polarity for the generated voltage due to tensile and compressive loading. Piezoelectric materials are an excellent design choice for strain gauges due to the fact that they are the only type of strain gauge that is self-powered. Commercial-bought piezoelectric materials are generally manufactured with electrodes attached to their surface to collect the generated electric charge. By connecting the electrodes to an oscilloscope, voltmeter, or computer data acquisition unit, real time structural behavior data can be collected and analyzed.

The converse piezoelectric effect is the exact opposite of the direct effect. An electric field can be applied to the electrodes of a piezoelectric material to induce strain in the material, as seen in Figure 1-2. The corresponding mechanical strain can be used to transmit forces to an otherwise passive structure. This ability of piezoelectric materials is the foundation for their implementation as mechanical actuators.

1.3 PIEZOELECTRIC COEFFICIENTS

The piezoelectric strain, “d”, and voltage, “g”, coefficients are important material characteristics to consider when selecting a piezoelectric material for a certain application. The
"g" coefficients are relevant to the direct piezoelectric effect while the "d" coefficients are applicable to the converse piezoelectric effect. These coefficients are useful in determining the ability of a piezoelectric material to act as either a sensor or an actuator. Piezoelectric materials which have favorable strain and voltage coefficients can act as both a sensor and an actuator, making them effective transducers (Piezo Systems, Inc., 2010, Schwartz, 2009).

1.3.1 “d$_{ij}$” CONSTANTS

The "d" coefficients, or strain constants, are the piezoelectric constants which relate the mechanical strain produced by an applied electric field. The strain constants are important parameters when determining a piezoelectric material’s ability as an actuator. Specifically, d$_{ij}$ is defined as the ratio of the strain produced in the j-direction to the electric field applied in the i-direction. The units of the “d” constants are therefore meters per meter, per volts per meter, or reduced to meters per volt (Piezo Systems, Inc., 2010, Schwartz, 2009).
1.3.2 “$g_{ij}$” CONSTANTS

The “$g$” coefficients, or voltage constants, are the piezoelectric constants which relate the electric field produced by an applied mechanical stress. The voltage constants play an important role when selecting a piezoelectric material to be a sensor. The constants $g_{ij}$ are defined as the ratio of the open circuit electric field in the i-direction caused by an applied mechanical stress in the j-direction. The units of the “$g$” constants are therefore volts per meter, per N per meter squared (Piezo Systems, Inc., 2010, Schwartz, 2009).

1.4 CONSTITUTIVE EQUATIONS

The following are the constitutive equations which describe the coupling effect between the mechanical and electrical properties of piezoelectric materials for actuator applications. Equation 1.1 represents the total strain of the piezoelectric material caused by mechanical stress and applied electric field. Equation 1.2 represents the total electric displacement due to mechanical stress and applied electric field (Schwartz, 2009).

$$
\varepsilon_p = S_{pq}^e \sigma_q + d_{ip} E_i \quad p, q = 1,2, ..., 6 \quad 1.1
$$
$$
D_i = d_{iq} \sigma_q + \varepsilon_{ik}^e E_k \quad i, k = 1,2,3 \quad 1.2
$$

$\varepsilon_p$ = strain vector (m/m)

$\sigma_q$ = stress vector (N/m$^2$)

$E_k$ = applied electric field vector (V/m)

$D_i$ = electric displacement vector (C/m$^2$)

$S_{pq}^e$ = elastic compliance matrix (m$^2$/N)

$d_{ip}$ = piezoelectric strain/charge coefficient matrix (m/V)

$\varepsilon_{ik}^e$ = electric permittivity matrix (F/m)
1.5 TYPES OF PIEZOELECTRIC MATERIALS

Piezoelectric materials can come in many different forms, ranging from simple, single crystals to composite piezoelectric materials which combine the material characteristics of both piezoelectric ceramics and piezoelectric polymers. The different material types can exhibit diverse electromechanical properties which can make one type of material more suitable for particular applications. The following sections briefly introduce the most popular types of piezoelectric materials.

1.5.1 SINGLE CRYSTALS

There have been numerous ferroelectric and nonferroelectric single crystals which have been demonstrated to possess piezoelectric characteristics. The most common, and arguably most important, single crystal piezoelectric material is quartz (SiO$_2$). Quartz features small but extremely stable piezoelectric properties which makes their performance very reliable. The most common application of quartz is wrist watches where the piezoelectric properties of the crystal are used to accurately keep time. Other common single crystal types include ferroelectric lithium niobate (LiNbO$_3$) and lithium tantalite (LiTaO$_3$) which are primarily used in surface acoustic wave devices (Schwartz, 2009).

1.5.2 CERAMICS

Piezoelectric ceramics are one of the most widely used types of piezoelectric material. The first piezoelectric ceramic to be developed was Barium Titanate (BaTiO$_3$) in the 1940s. Barium Titanate is a widely used capacitor material due to its high dielectric constant. The most technologically important piezoelectric ceramics feature a perovskite crystalline structure, an example is illustrated in Figure 1-3, with a general formula of ABO$_3$, where A=Na, K, Rb, Ca, Sr, Ba, Pb, etc. and B=Ti, Sn, Zr, Nb, or W. Piezoelectric ceramics typically feature high
Figure 1-3 Perovskite Crystalline Structure (PbTiO₃) (Schwartz, 2009)

electromechanical coupling characteristics which makes them excellent induced strain actuators. However, ceramics are, in general, very brittle which can lead to undesirable cracking if subjected to large deformations (Schwartz, 2009).

1.5.3 POLYMERS

Unlike piezoelectric ceramics, piezoelectric polymers are very flexible and are capable of sustaining higher deformations before failure. However, they also exhibit less electromechanical coupling than ceramics, making them undesirable choices as actuators, but are more often used as sensors in devices like microphones, biomedical devices, and strain gauges. The most common types of piezoelectric polymers include polypropylene, polystyrene, and polyvinylidene fluoride (PVDF). PVDF features the greatest piezoelectric electromechanical coupling of the piezoelectric polymers (Schwartz, 2009).
1.5.4 COMPOSITES

Recently, researchers have been working on methods to combine the excellent electromechanical effects of piezoelectric ceramics with the flexibility of polymers to overcome the previously mentioned limitations. The method of combining piezoelectric ceramics with polymers to create a piezoelectric composite allows the piezoelectric properties to be tailored for specific applications. The resulting properties are strongly dependent upon the properties of the ceramic and polymer, as well as the manner in which they are connected. The goal of manufacturing a piezoelectric composite is to create a material which has high electromechanical properties while maintaining adequate flexibility (Schwartz, 2009). Figure 1-4 is a picture of a piezoelectric fiber composite manufactured by Advanced Cerametrics. The composite features ceramic fibers embedded in a polymer resin matrix (Advanced Cerametrics, 2009).

![Piezoelectric Fiber Composite](Image)

**Figure 1-4** Piezoelectric Fiber Composite (Ryan Meyer, 2009)

1.6 COMMON DIRECT EFFECT APPLICATIONS

As previously mentioned, the direct piezoelectric effect allows piezoelectric materials to convert mechanical strain into electric potential. However, the strain of a piezoelectric material will not produce an electric potential forever. The mechanical movement of the piezoelectric
crystals generates a charge, which, when collected on electrodes, produces electric potential, but that charge will dissipate over time. Therefore, piezoelectric measuring systems act as active electrical systems. The piezoelectric crystals will only produce an electrical potential when they experience a change in loading. This makes piezoelectric materials ineffective for static measurements. However, they can still act as excellent strain gauges for loads which change over time. This ability makes piezoelectric materials suitable accelerometers and dynamic strain gauges. Piezoelectric materials can function as accelerometers by being attached to the body whose motion is in question. The piezoelectric can then convert the G-force experienced by the body in motion to an electric potential. This potential can be quantified to determine the rate of acceleration experienced by the object (The Principles of Piezoelectric Accelerometers, 2004). Piezoelectric accelerometers are often used as vibration sensors that can be attached to rotating machinery to detect any unusual vibratory disturbances. Recently, they have been implemented inside the sole of a runner’s shoe which can detect the impact of the runner’s foot with the ground, as seen in Figure 1-5. The data is collected by a transmitter unit to give the wearer real-time feedback about their speed and distance traveled.

![Image of a piezoelectric accelerometer](http://static.howstuffworks.com)

Figure 1-5  Shoe Sole Accelerometer (http://static.howstuffworks.com)
Another common application of the direct piezoelectric effect is energy harvesting systems. The concept of piezoelectric energy harvesters is to collect the charge created by dynamic strain of the piezoelectric materials and store the charge using a capacitor. Researchers at Cornell University are currently developing a synthetic tree which uses piezoelectric materials as leaves. The idea is that when the wind blows and bends the piezoelectric leaves, the charge generated can be stored and used later. A conceptual design of the Tree Energy Harvester is presented in Figure 1-6. The direct effect can also be utilized in acoustical sensors, such as guitar pickups, shown in Figure 1-7, which detect pressure waves in the air due to the vibrating strings.

Figure 1-6 Tree Energy Harvester Concept (Flapping Piezo-Leaf Generator for Wind Energy Harvesting)
1.7 COMMON CONVERSE EFFECT APPLICATIONS

The converse effect allows an electric field to be applied to a piezoelectric material which will in turn induce strain within the material. In this sense, the converse effect allows piezoelectric materials to behave as actuators. Piezoelectric materials can work as both static and dynamic actuators, provided that a power supply is present to apply an electric field. NASA has developed small scale aircraft wing models with attached piezoelectric actuators, seen in Figure 1-8, to try and control wing deflection occurring during flight. There have been numerous studies done using piezoelectric actuators to damp out free and forced vibrations on cantilever beams. Laser displacement sensors or piezoelectric sensors are used to monitor the dynamic state of the vibrating beam. The sensor signal is then sent through a data acquisition
Figure 1-8  Aircraft Wing Model with Piezoelectric Actuators (http://nasaimages.org)

unit and a feedback controller is utilized to send a control signal to the piezoelectric actuator. The result is improved damping characteristics of the passive structure. A picture of one such experimental setup is shown in Figure 1-9 (Slovak Technical University). Another application are piezoelectric motors, shown in Figure 1-10, which take advantage of the converse effect by producing surface acoustic waves (SAW) to produce rotary and/or linear motion to drive the motor (Physik Instrumente, 2010). Piezoelectric materials are also used in many alarm systems where a piezoelectric layer is used to excite a speaker driver to create a buzzing sound, illustrated in Figure 1-11.
Figure 1-9 Piezoelectric Vibration Damping Actuator Experiment (Slovak Technical University)

Figure 1-10 Piezoelectric Motor (http://www.micromechatronicsinc.com/)
1.8 EXISTING STUDIES IN MODELING OF PIEZOELECTRIC ACTUATION

Although the phenomenon of piezoelectricity was first discovered in 1880, analytical actuation models did not emerge until the 1980s. The first analytical study of the mechanical coupling of segmented piezoelectric actuators to a structural member was developed by Edward F. Crawley and Javier de Luis in 1987. The model was able to predict the motion of the structure as a result of subjecting a piezoelectric actuator to a specified excitation voltage. A scaling analysis was also performed to demonstrate how the effectiveness of actuation changes with different piezoelectric materials and structural scale. The modeling accounted for shear lag due to the thickness of the adhesive layer material used to bond the piezoelectric actuator to the structure. The following equation was developed to obtain an explicit expression for the shear stress applied by the piezoelectric actuator to the substructure accounting for shear lag effects.

\[
\frac{\tau}{E_B} = \frac{-\bar{G}}{t_3} \left( \frac{\sinh \Gamma x}{\cosh \Gamma} \right) \Lambda
\]

\[\tau = \text{shear stress applied to the substructure}\]
\[ E_B = \text{substructure elasticity modulus} \]
\[ \tilde{G} = \text{modulus ratio of shear layer to piezoelectric} \]
\[ \tilde{E} = \text{modulus ratio of beam to piezoelectric} \]
\[ \tilde{t}_s = \text{nondimensional bonding layer thickness} \]
\[ \Gamma = \text{nondimensional shear transfer parameter} \]
\[ \tilde{x} = \text{nondimensional piezoelectric centered coordinate} \]
\[ \Lambda = \text{piezoelectric strain} \]

The formula was later reduced to the ideal case of no shear lag when the entire shear force is effectively transferred at a concentrated point at the ends of the piezoelectric.

\[
\frac{F}{E_B t_B b} = \frac{1}{(\psi + \alpha)} \left[ \frac{\epsilon^{\delta+}_B + \epsilon^{\delta-}_B}{2} + \frac{\epsilon^{\delta+}_B - \epsilon^{\delta-}_B}{2} \tilde{x} \right] - \left[ \frac{1}{\psi + \alpha} \right] \Lambda \tag{1.2}
\]

\[ F = \text{concentrated shear force applied to the substructure at the piezoelectric ends} \]
\[ t_B = \text{beam thickness} \]
\[ b = \text{beam width} \]
\[ \psi = \text{effective stiffness ratio} \]
\[ \alpha = \text{structure equilibrium parameter} \]
\[ \epsilon^{\delta-}_B = \text{known substructure strain at the left (−) end of the piezoelectric segment} \]
\[ \epsilon^{\delta+}_B = \text{known substructure strain at the right (+) end of the piezoelectric segment} \]

The concept was later extended to embedding the actuators within the substructure, and experimental work was performed to validate the modeling (Crawley and Luis, 1987).

In 1989, Jan G. Smits and Susan I. Dalke formulated the constituent equations of piezoelectric bimorphs with the upper and lower elements having opposite polarization, causing
bending when excited by an electric field. Their model was a result of their development of an integrated microbot with legs that were moved piezoelectrically. The model was derived using the strain-energy method and was capable of predicting the free-end slope and deflection as a function of applied electric field or external loading, while subjected to various boundary conditions (Smits and Dalke, 1989). Smits later extended the study to a piezoelectric heterogeneous bimorph. The heterogeneous bimorph consists of a piezoelectric element bonded to the top of a nonpiezoelectric element. A similar model was derived using the strain-energy method to predict the free-end slope and deflection for an applied electric field and the structure subjected to given boundary conditions (Smits and Choi, 1991).

The use of piezoelectric materials as actuators in active vibration control has been thoroughly established. The use of sensors and actuators in a feedback control loop is said to make an otherwise passive structure, a “smart structure” instead. T. Bailey and J. Hubbard designed an active vibration damper for a cantilever beam using distributed-parameter actuator and distributed-parameter control theory. A piezoelectric polymer bonded to one side of the cantilever beam was utilized as the actuator. The control algorithm for the active damper was designed using Lyapunov's second method for distributed-parameter systems. The angular velocity of the cantilever beam free end was used to determine all vibration modes of the beam. Vibrations were successfully controlled using both a linear constant-gain controller and a nonlinear constant-amplitude controller (Bailey and Hubbard, 1985). Lightweight composite structures have become increasingly popular in the aerospace industry for their excellent strength-to-weight ratios. However, they are still prone to undesirable vibrations. J. Han et al developed an analytical active vibration control model for composite structures using laminated beam theory in conjunction with the Ritz method. A linear quadratic Gaussian (LQG) control
algorithm was utilized to design the feedback control system. A Kalman filter was used to estimate the state value from the measured feedback system. The experimental setup can be seen in Figure 1-12. The feedback control system was implemented experimentally with very good results. Figure 1-13 shows the sensor output voltage for both the controlled and uncontrolled vibration tests (Han et al, 1997).

Figure 1-12  Active Vibration Control Equipment Setup (Han et al, 1997)
Existing research has also been done in the area of deflection analysis and shape control of structures using piezoelectric actuators. B. N. Agrawal and K. Treanor have studied the effectiveness of using piezoceramic actuators to control the shape of smart structures. They utilized Euler-Bernoulli beam theory to develop deflection models for a cantilever beam with multiple surface bonded actuators. The multiple actuator model can be seen in Figure 1-14. The differences between desired shape and actual shape were studied as a function piezoceramic actuator location and applied voltage (Agrawal and Treanor, 1999).

Figure 1-13 Transient Vibration Control Results for First Bending Mode (Han et al, 1997)
P. Gaudenzi and R. Barboni derived simple solutions for Euler-Bernoulli beams actuated by pairs of piezoelectric patches while subjected to several static loading scenarios and boundary conditions. The solutions presented the transverse deflection and rotation of the beam with respect to beam position and piezoelectric actuator properties, length, and bond position.

**Cantilever Boundary Conditions**

\[
\begin{align*}
w_1(x, a, h, m) &= 0 \quad 1.3 \\
w_2(x, a, h, m) &= \frac{-m(2x + h - 2a)^2}{8EI} \quad 1.4 \\
w_3(x, a, h, m) &= \frac{hm(a - x)}{EI} \quad 1.5
\end{align*}
\]

**Simply Supported Boundary Conditions**

\[
\begin{align*}
w_1(x, a, h, m) &= -\frac{hm(a - L)}{EIL}x \quad 1.6 \\
w_2(x, a, h, m) &= -\frac{m}{2EI}x^2 - \frac{m[2a(h - L) - hL]}{2EIL} - \frac{m(2a - h)^2}{8EI} \quad 1.7 \\
w_3(x, a, h, m) &= \frac{ahm}{EI} - \frac{ahm}{EIL}x \quad 1.8
\end{align*}
\]

\[x\quad = \quad \text{position along the beam}\]

\[a\quad = \quad \text{location to the center of the piezoelectric patch}\]
\( h \) = piezoelectric patch length

\( m \) = piezoelectric induced bending moment

\( EI \) = bending stiffness of the beam

\( L \) = length of the beam

Subscripts 1,2,3 denote the different domains along the beam

\[
0 \leq x \leq \frac{h}{2}, \quad a - \frac{h}{2} \leq x \leq a + \frac{h}{2}, \quad a + \frac{h}{2} \leq x \leq L
\]

The fundamental solutions illustrate the ability of piezoelectric actuators to induce deflection in passive structures and for simple design purposes. For example, it was shown that for the cantilever beam subjected to a concentrated load, \( P \), at its free-end, the following solutions can be obtained for the optimal piezoelectric length and position required to return the free-end position to equilibrium conditions (Gaudenzi and Barboni, 1999).

\[
h = \frac{PL^2}{2m} \quad 1.9
\]

\[
a = \frac{L}{3} \quad 1.10
\]

More complicated beam geometries have also been analyzed with respect to induced static deflection. Y. Kuang et al derived the static responses of circular curved beams excited by piezoelectric actuators. The governing equations were derived under the assumptions of small curvature and one-dimensional beam theory. The results were verified experimentally and with Finite Element Modeling software ANSYS (Kuang et al, 2007).

However, while there have been extensive studies regarding the induced static deflection of beams with piezoelectric actuators, the work has limitations. First, the studies often neglect the effect of the piezoelectric actuators on the bending stiffness of the beam. This renders the
results of the works impractical for soft and thin structures where the effect of the piezoelectric layers can be quite large on the rigidity of the overall structure. Also, previous studies limited themselves to single beam structures. There is limited work on the effects of piezoelectric actuation of joint structures featuring multiple substructures adhesively bonded to create a more complicated configuration (i.e., lap joints, strap joints, etc.). The aim of this thesis is to develop an analytical model which can predict the ability of piezoelectric materials to reduce deflection within an adhesively bonded single lap joint caused by tensile loading.
CHAPTER 2: THE SMART JOINT

2.1 SMART STRUCTURES

A structure can be defined as a “smart structure” if it is capable of three functions: sensing, processing and control, and actuating. The structure must have a way of monitoring or sensing its current state. The structure must be able to process the information provided by the sensing component to determine how the structure should react. Finally, once the structure has determined how to react to the sensed state of the system, it must be able to actuate itself to return it to its desired equilibrium state (Chee et al., 1998). “Smart” structures are said to be a subset of “intelligent” structures in that “intelligent structures also feature the ability to learn and react based on past experiences” (Gandhi and Thompson, 1992). Smart structures are increasing in popularity because they are capable of reacting to unforeseen loading conditions, such as vibration and impact loading. Piezoelectric materials are seeing increased use in smart structures, especially in the aerospace industry, where there is a constant search for lightweight materials. They are being used more frequently because they are lightweight, do not drastically alter the passive structure, and are capable of acting as both the sensing and actuating elements of the smart structure (Chee et al., 1998).

2.2 SMART COMPOSITE JOINT

This research is in contribution to fulfilling the goals and objectives of the NASA/EPSCoR funded project to design a durable, reliable, and intelligent adhesively bonded composite joint. A schematic diagram of the proposed smart joint can be seen in Figure 2-1. The smart composite joint design problem is being approached from two different perspectives; using shape memory polymer for the adhesive bond layer and piezoelectric materials to counter balance external loading. The ultimate goal of the smart joint is to reduce the interfacial shear stress which arises
in the adhesive bondline, as seen in Figure 2-2 ("Smart Adhesively Bonded High Performance Joint for Composite Structures" NASA/EPSCoR Project Proposal, 2007).

**Figure 2-1** Smart Joint Configuration ("Smart Adhesively Bonded High Performance Joint for Composite Structures" NASA/EPSCoR Project Proposal)

**Figure 2-2** Adhesive Bondline Stress Distribution ("Smart Adhesively Bonded High Performance Joint for Composite Structures" NASA/EPSCoR Project Proposal)
The high shear stress concentrations in Figure 2-2 can result in a tensile force being applied to a single lap joint, presented in Figure 2-3. The eccentricity of the applied force results in an induced moment throughout the beam and causes high stress concentrations at the ends of the adhesive bondline. The goal of this study is to reduce the deflection within the adhesively bonded region of the single lap joint in order to reduce the high shear stress concentrations that result from tensile loading. However, this is a preliminary study and further work should be carried out utilizing Laminated Plate Theory to determine the local strain within the adhesive layer of the joint.

![Figure 2-3 Single Lap Joint under Tension](image)

2.2.1 SHAPE MEMORY POLYMER

In recent years, a new type of smart polymer has been developed called a shape memory polymer. Shape memory polymers feature the ability to “remember” their original shape in the event of deformation or cracking. The shape memory polymer can be “programmed” to remember their initial shape by using an appropriate curing cycle. By using shape memory polymer in the adhesive layer of composite joints, if the adhesive layer becomes damaged, the shape memory polymer can be thermally reactivated to recover its original shape. An illustration of this phenomenon can be seen in Figure 2-4 (NASA/EPSCoR “Smart Adhesively Bonded High Performance Joint for Composite Structures” Project Proposal, 2007).
2.2.2 PIEZOELECTRIC MATERIALS

Piezoelectric composite materials have been selected to be used as actuators within the smart composite joint system. Piezoelectric composite materials were chosen over piezo-ceramics or piezo-polymers because they combine the excellent electromechanical properties of piezo-ceramics with the flexibility of piezo-polymers. A schematic of a piezoelectric composite is presented in Figure 2-5. A high voltage power supply is used to excite the piezoelectric layers to induce counterbalancing forces on the smart joint. The ultimate goal of using the piezoelectric materials within the smart joint is to increase the strength and durability of the joint by inducing counterbalancing forces which will in turn decrease the stress concentrations within the adhesive bondline (“Smart Adhesively Bonded High Performance Joint for Composite Structures” NASA/EPSCoR Project Proposal, 2007). Analytical models have been derived for a smart
composite pipe joint with piezoelectric layers subjected to bending (Cheng et al, 2006), as well as under tensile loading (Cheng et al, 2006).

Figure 2-5  Piezoelectric Fiber Composite Exploded View (“Smart Adhesively Bonded High Performance Joint for Composite Structures” NASA/EPSCoR Project Proposal)
CHAPTER 3: STATIC DEFLECTION MODELING

When considering the use of piezoelectric materials as actuators in smart joint applications, it is desired to be able to predict some measure of the performance of the piezoelectric actuation. The purpose of using actuators as active elements of the smart joint is to improve the overall strength of the adhesively bonded joint structure. Since static deflection and failure strength are directly related, it is desirable to use piezoelectric actuators to reduce undesirable deflections caused by external loading, in particular, deflections which occur at the ends of the adhesively bonded joint because that is where the largest stress concentrations are developed due to applied loading. In order to predict the behavior of the smart joint when subjected to external loads and piezoelectric actuation, static deflection models have been developed through the use of Euler-Bernoulli beam theory. Euler-Bernoulli beam theory requires the internal bending moment and flexural rigidity of the structure to be known. Therefore, an actuation force model is developed, as well as a method for determining the equivalent flexural rigidity of different regions of the joint structures.

Euler-Bernoulli beam theory is utilized to determine the elastic curves of the proposed structures. Euler-Bernoulli beam theory can be easily implemented provided that the internal bending moment and flexural rigidity of the structure is known. Integrating Equation 3.1 twice and applying the appropriate boundary conditions will yield the elastic curve of a given structure (Beer et al, 2006, Budynas and Nisbett, 2008).

$$E I y'' = M(x) \text{  \ \ \ 3.1}$$

$M(x)$ = internal bending moment function

$E I$ = flexural rigidity

$y''$ = 2nd derivative of the elastic curve
The flexural rigidity for joint structures does not remain constant throughout the structure due to the use of different material elements and a non-uniform cross-section. The internal bending moment is caused by external loading and the piezoelectric actuator force mechanism.

The piezoelectric actuator is able to induce deflection in structures by converting an electric potential to a mechanical force. Applying an electric field is assumed to produce linear strain in the piezoelectric actuator. Equation 3.2 formulates the induced mechanical strain as a function of applied electric field and piezoelectric actuator properties. An equivalent force, which acts on the cross-section centroid of the piezoelectric actuator, can then be determined from the magnitude of the induced strain. Equation 3.3 utilizes Hooke’s law to derive the equivalent force produced by the induced strain.

\[
\Lambda = \frac{d_{31}V}{t_A} \quad 3.2
\]

\[
F = t_A b_A E_A \Lambda \quad 3.3
\]

\( \Lambda \) = induced piezoelectric strain
\( d_{31} \) = piezoelectric strain constant (strain/electric field)
\( V \) = applied voltage
\( E_A \) = modulus of elasticity of the piezoelectric actuator
\( t_A \) = piezoelectric actuator thickness
\( b_A \) = piezoelectric actuator width
\( F \) = equivalent force generated by piezoelectric actuation

In order to utilize Euler-Bernoulli beam theory, the flexural rigidity of the structure being analyzed must be known. For a simple homogenous structure, the flexural rigidity will remain constant for the entire span of the beam. However, when analyzing the induced deflection of
structural joints with surface bonded piezoelectric actuators, the flexural rigidity will vary due to the non-uniform nature of the structure geometry and the use of different material elements. The equivalent flexural rigidity, Equation 3.4, is determined by summing the flexural rigidity contributed by each unique structural element.

\[ EI = \sum_{i=1}^{n} E_i I_i \]  \hspace{1cm} 3.4

From Equation 3.4, it is obvious that the modulus of elasticity and second area moment of inertia must be known for each structural element.

- \( EI \) = equivalent flexural rigidity
- \( E \) = modulus of elasticity
- \( I \) = second area moment of inertia

### 3.1 CANTILEVER SINGLE LAP JOINT FIBER COMPOSITE BEAM

Adhesively bonded joint structures have seen their usage in the aerospace industry increase recently due to their ease to manufacture and ability to possess excellent strength-to-weight ratios. However, although they may be strong in comparison to their weight, they still suffer from large stress concentrations that form within the adhesive layer during external loading, leaving them susceptible to undesirable deformations and premature failure. The aim of this study is to illustrate the ability of surface bonded piezoelectric actuators to reduce the undesirable deformations in the adhesive bondline caused by tensile loading on a cantilever single lap joint.

The cantilever single lap joint beam’s physical model with relevant dimensions can be seen in Figure 3-1. The coordinate system defined in Figure 3-1 will be used in the following
sections detailing the elastic curves induced by pre-load and post-load piezoelectric actuation. A single lap joint composite beam presents a few unique problems when analyzing the effects of piezoelectric actuation on the static deflection of the structure. First, when using composites joints in a structure, the modulus of elasticity can vary for different material types used within the joint. It is also important to note that a single lap joint with a surface bonded piezoelectric actuator is not symmetric about the x-coordinate axis, as defined in Figure 3-1. Therefore, the neutral axis position will vary along the beam due to the changing cross-section and differences in modulus of elasticity between the piezoelectric actuator and beam substrate. The location of
the neutral axis, weighted by the elasticity modulus, cross-sectional area, and location of the
centroid with respect to the x-axis for each material, can be defined by Equation 3.5

\[ d_E = \frac{\sum_{i=1}^{n} E_i A_i c_i}{\sum_{i=1}^{n} E_i A_i} \]  

\( d_E \) = distance from the x-axis to the neutral axis  
\( A_i \) = cross-sectional area of a material constituent for a given section of the beam  
\( E_i \) = modulus of elasticity of a material constituent  
\( c_i \) = distance from the x-axis to the cross-sectional centroid of a material constituent  
\( n \) = total number of material constituents for a given beam region

It is important to know the location of the neutral axis when determining the second-area
moment of inertia for each individual material constituent. As mentioned before, the structure is
not symmetric about the x-axis; therefore the parallel axis theorem, Equation 3.6, must be used to
determine the second area moment of inertia for each unique material in a given section of the
beam. Once the second area moment of inertia is known for each beam element, Equation 3.4
can be applied to determine the equivalent flexural rigidity for each region of the beam.

\[ I_x = \overline{I_{x'}} + A(c - d_E)^2 \]  

\( I_x \) = second area moment of inertia with respect to the neutral axis  
\( \overline{I_{x'}} \) = second area moment of inertia with respect to the cross-sectional centroid

Figure 3-2 shows the four possible regions of the lap joint in which the piezoelectric
actuator can be surface bonded. The following sections will derive the elastic curves of the
composite lap joint for each possible bond region due to straight beam and small beam curvature
actuation.
3.1.1 STRAIGHT BEAM ACTUATION

Straight beam actuation produces a pair of equal and opposite forces located at the left and right ends of the piezoelectric actuator, seen in Figure 3-3, with a magnitude defined by Equation 3.3. The forces produce equivalent concentrated moments due to the forces acting parallel to the neutral axis as defined by Equation 3.5. The resulting moments have a magnitude defined by Equation 3.7. As a result, an internal bending moment only exists between the ends of the actuator. The straight beam actuation model is applied if the piezoelectric is actuated prior to an external load applied to the beam. Since the beam has no undergone any deflection, it will be straight. The piezoelectric actuator can be actuated in advance to induce deflection prior to an applied external load to minimize the overall deflection cause by external loading.

\[ m = F(c_p - d_E) \]  \hspace{1cm} 3.7

- \( m \) = induced piezoelectric moment
- \( F \) = piezoelectric force
- \( c_p \) = distance from the x-axis to the piezoelectric centroid
- \( d_E \) = distance from the x-axis to the neutral axis
Bond Region I

Figure 3-4  Bond Region I Straight Beam Actuation Model

Domain 1: \[ 0 \leq x \leq a - \frac{l}{2} \]
\[ X_1 = x \quad 3.8 \]
\[ y_1(x) = 0 \quad 3.9 \]

Domain 2: \[ a - \frac{l}{2} \leq x \leq a + \frac{l}{2} \]
\[ X_2 = x - \left( a - \frac{l}{2} \right) \quad 3.10 \]
\[ y_2(x) = \frac{mX_2^2}{2EI_2} \quad 3.11 \]

Domain 3: \[ a + \frac{l}{2} \leq x \leq 2L - L_f \]
\[ X_3 = x - \left( a + \frac{l}{2} \right) \quad 3.12 \]
\[ y_3(x) = X_3 \left. \frac{dy_2}{dX_2} \right|_{x=a+\frac{l}{2}} + y_2 \big|_{x=a+\frac{l}{2}} \quad 3.13 \]
Bond Region II

When the piezoelectric actuator is bonded in Region II, with the left end in Region I and the right end in Region III, the induced moment at the left and right ends are no longer equal in magnitude due to the location of the neutral axis being different in Region I and Region III because of the lap joint. Therefore, the magnitude of the induced moments will not be equal at the left and right ends of the piezoelectric. Additional moments of equal and opposite direction are assumed to act at the lap joint interface to account for the change in neutral axis location.

![Figure 3-5 Bond Region II Straight Beam Actuation Model](image)

Domain 1:

\[ 0 \leq x \leq a - \frac{L}{2} \]

\[ X_1 = x \]

\[ y_1(x) = 0 \]

Domain 2:

\[ a - \frac{L}{2} \leq x \leq L - L_f \]

\[ X_2 = x - \left( a - \frac{L}{2} \right) \]

\[ y_2(x) = \frac{m^- X_2^2}{2E I_2} \]

Domain 3:

\[ L - L_f \leq x \leq a + \frac{L}{2} \]
\[
X_3 = x - (L - L_f)
\]

\[
y_3(x) = \frac{m^*X_3^2}{2EI_3} + X_3 \frac{dy_2}{dx} \bigg|_{x=L-L_f} + y_2 \bigg|_{x=L-L_f}
\]

Domain 4:
\[
a + \frac{l}{2} \leq x \leq 2L - L_f
\]

\[
X_4 = x - \left(a + \frac{l}{2}\right)
\]

\[
y_4(x) = X_4 \frac{dy_3}{dx} \bigg|_{x=a+\frac{l}{2}} + y_3 \bigg|_{x=a+\frac{l}{2}}
\]

Bond Region III

![Diagram of Bond Region III Straight Beam Actuation Model]

Figure 3-6  Bond Region III Straight Beam Actuation Model

Domain 1:
\[
0 \leq x \leq a - \frac{l}{2}
\]

\[
X_1 = x
\]

\[
y_1(x) = 0
\]

Domain 2:
\[
a - \frac{l}{2} \leq x \leq a + \frac{l}{2}
\]

\[
X_2 = x - \left(a - \frac{l}{2}\right)
\]
\[ y_2(x) = \frac{mX_2^2}{2EI_2} \quad 3.25 \]

Domain 3:

\[ a + \frac{l}{2} \leq x \leq 2L - L_1L \]

\[ X_3 = x - \left(a + \frac{l}{2}\right) \quad 3.26 \]

\[ y_3(x) = X_3 \frac{dy_2}{dX_2} \bigg|_{x=a+\frac{l}{2}} + y_2\bigg|_{x=a+\frac{l}{2}} \quad 3.27 \]

**Bond Region IV**

![Figure 3-7 Bond Region IV Straight Beam Actuation Model](image)

Domain 1:

\[ 0 \leq x \leq a - \frac{l}{2} \]

\[ X_1 = x \quad 3.28 \]

\[ y_1(x) = 0 \quad 3.29 \]

Domain 2:

\[ a - \frac{l}{2} \leq x \leq a + \frac{l}{2} \]

\[ X_2 = x - \left(a - \frac{l}{2}\right) \quad 3.30 \]

\[ y_2(x) = \frac{mX_2^2}{2EI_2} \quad 3.31 \]
Domain 3: 
\[ \frac{a + \frac{l}{2}}{2} \leq x \leq 2L - L_f \]

\[ X_3 = x - \left( a + \frac{l}{2} \right) \]

\[ y_3(x) = X_3 \frac{dy_2}{dX_2} \bigg|_{x=a+\frac{l}{2}}^{x=a+\frac{l}{2}} + y_2 \bigg|_{x=a+\frac{l}{2}} \]

3.1.2 SMALL BEAM CURVATURE ACTUATION

The small beam curvature actuation model assumes that the beam has already undergone small deflections prior to exciting the piezoelectric actuator. The deflection may be a product of external loading. As a result of differences in slope along the beam, the piezoelectric actuation forces no longer act in a purely axial direction, illustrated in Figure 3-8. The beam curvature affects the forces produced by the piezoelectric actuator. There will now be forces acting in both the x and y directions, Equation 3.34 and Equation 3.35 respectively, which are functions of the slope of the beam at the left and right ends of the piezoelectric actuator. The concentrated moments will also vary with respect to the slope of the beam, Equation 3.36.

Figure 3-8  Small Beam Curvature Piezoelectric Actuation Model
If the beam deflections are assumed to be the result of an applied external load, the elastic curve of the beam due to external loading must be derived prior to determining the elastic curve due to piezoelectric actuation in order to determine the angles at which the piezoelectric forces act. The elastic curves due to external loading are not presented within the body of this paper. However, they have been derived and used in the ensuing numerical analysis performed with the aid of MATLAB. The resulting code can be referenced in the Appendix.

\[ F_x = F \cos \theta \]  \hspace{2cm} (3.34)

\[ F_y = F \sin \theta \]  \hspace{2cm} (3.35)

\[ m = F_x (c_A - d_E) = F \cos \theta (c_A - d_E) \]  \hspace{2cm} (3.36)

**Figure 3-9  Bond Region I Small Beam Curvature Actuation Model**

Domain 1:

\[ 0 \leq x \leq a - \frac{l}{2} \]

\[ X_1 = x \]  \hspace{2cm} (3.37)

\[ y_1(x) = \frac{1}{EI_1} \left( \left( \frac{m^+ - m^- + F_y^+ (a + \frac{l}{2}) - F_y^- (a - \frac{l}{2})}{2} \right) X_1^2 + \frac{(F_y^- - F_y^+) X_1^3}{6} \right) \]  \hspace{2cm} (3.38)
Domain 2:

\[ a - \frac{l}{2} \leq x \leq a + \frac{l}{2} \]

\[ X_2 = x - (a - \frac{l}{2}) \]

\[ y_2(x) = \frac{1}{EI_2} \left( \frac{m^+ + F_y^+ l}{2} X_2^2 - \frac{F_y^+ X_2^3}{6} \right) + X_2 \frac{dy_1}{dX_1} \bigg|_{x=a-\frac{l}{2}} + y_1 \bigg|_{x=a-\frac{l}{2}} \]

\[ 3.39 \]

Domain 3:

\[ a + \frac{l}{2} \leq x \leq 2L - L_f \]

\[ X_3 = x - (a + \frac{l}{2}) \]

\[ y_3(x) = X_3 \frac{dy_2}{dX_2} \bigg|_{x=a+\frac{l}{2}} + y_2 \bigg|_{x=a+\frac{l}{2}} \]

\[ 3.41 \]

\[ 3.42 \]

**Bond Region II**

The piezoelectric post-load actuation model for Bond Region II is illustrated in Figure 3-10. As discussed in the straight beam actuation section, when the piezoelectric actuator is bonded in a location where it overlaps the lap joint interface, the change in neutral axis location must be accounted for. In the straight beam case, equal and opposite moments were assumed to act at the lap joint interface.

![Figure 3-10  Bond Region II Small Beam Curvature Actuation Model](image)

38
However, for the small beam curvature actuation, the moments will no longer be equal in magnitude to the moments acting at the left and right ends of the actuator, due to the slope of the beam affecting the magnitude of the piezoelectric force in the x-direction, and consequently, the induced moment. The forces in the y-direction at the joint interface are not shown in Figure 3-10 because they are equal and opposite in direction and cancel each other out.

Domain 1: \[ 0 \leq x \leq a - \frac{l}{2} \]
\[ X_1 = x \]
\[ y_1(x) = \frac{1}{EI_1} \left( \frac{\left( m^+ - m^- + m_j^+ + F_y^+ \left( a + \frac{l}{2} \right) - F_y^- \left( a - \frac{l}{2} \right) \right) X_1^2}{2} + \frac{(F_y^- - F_y^+) X_1^3}{6} \right) \]

\[ \text{Domain 2:} \quad a - \frac{l}{2} \leq x \leq L - L_j \]
\[ X_2 = x - \left( a - \frac{l}{2} \right) \]
\[ y_2(x) = \frac{1}{EI_2} \left( \frac{\left( m^+ + m_j^- + m_j^+ + F_y^+ l \right) X_2^2}{2} - \frac{F_y^+ X_2^3}{6} \right) + X_2 \frac{dy_1|_{x=a-l/2}}{dX_1} + y_1|_{x=a-l/2} \]

\[ \text{Domain 3:} \quad L - L_j \leq x \leq a + \frac{l}{2} \]
\[ X_3 = x - \left( L - L_j \right) \]
\[ y_3(x) = \frac{1}{EI_3} \left( \frac{\left( m^+ + F_y^+ \left( l - (L - L_j) + \left( a - \frac{l}{2} \right) \right) \right) X_3^2}{2} - \frac{F_y^+ X_3^3}{6} \right) \]
Domain 4:

\[ a + \frac{l}{2} \leq x \leq 2L - L_j \]

\[ X_4 = x - \left( a + \frac{l}{2} \right) \]

\[ y_4(x) = X_4 \left. \frac{dy_3}{dx} \right|_{x=a+\frac{l}{2}} + y_3 \left|_{x=a+\frac{l}{2}} \right. \]

Bond Region III

\[ m^- \quad \text{Small Beam Curvature Actuation Model} \]

Domain 1:

\[ 0 \leq x \leq L - L_j \]

\[ X_1 = x \]

\[ y_1(x) = \frac{1}{EJ_1} \left( \frac{m^+ - m^- + F_y^+(a + \frac{l}{2}) - F_y^-(a - \frac{l}{2})}{2} \right) X_1^2 + \frac{(F_y^- - F_y^+)X_1^3}{6} \]

Domain 2:

\[ L - L_j \leq x \leq a - \frac{l}{2} \]

\[ X_2 = x - (L - L_j) \]
\[ y_2(x) = \frac{1}{EI_2} \left( \frac{(m^+ - m^- + F_Y^+ \left( (a + \frac{l}{2}) - (L - L_j) \right)) - F_Y^- \left( (a - \frac{l}{2}) - (L - L_j) \right)}{2} \right. \]

\[ \left. + \frac{(F_Y^- - F_Y^+)X_2^3}{6} \right) + X_2 \frac{dy_1}{dx} \bigg|_{x=L-L_j} + y_1 \bigg|_{x=L-L_j} 3.54 \]

Domain 3:
\[ a - \frac{l}{2} \leq x \leq a + \frac{l}{2} \]
\[ X_3 = x - \left( a - \frac{l}{2} \right) \]
\[ y_3(x) = \frac{1}{EI_3} \left( \frac{(m^+ + F_Y^+ l)X_3^2}{2} - \frac{F_Y^+ X_3^3}{6} \right) + X_3 \frac{dy_2}{dx} \bigg|_{x=a-\frac{l}{2}} + y_2 \bigg|_{x=a-\frac{l}{2}} 3.56 \]

Domain 4:
\[ a + \frac{l}{2} \leq x \leq 2L - L_j L \]
\[ X_4 = x - \left( a + \frac{l}{2} \right) \]
\[ y_4(x) = X_4 \frac{dy_3}{dx} \bigg|_{x=a+\frac{l}{2}} + y_3 \bigg|_{x=a+\frac{l}{2}} 3.58 \]

Bond Region IV

![Diagram of Bond Region IV Small Beam Curvature Actuation Model]

**Figure 3-12** Bond Region IV Small Beam Curvature Actuation Model
\[ X_1 = x \quad (3.59) \]

\[
y_1(x) = \frac{1}{EI_1} \left( \frac{\left( m^+ - m^- + F_y^+ \left( a + \frac{l}{2} \right) - F_y^- \left( a - \frac{l}{2} \right) \right) X_1^2}{2} + \frac{(F_y^- - F_y^+)X_1^3}{6} \right) \quad (3.60)
\]

**Domain 2:** 
\[ L - L_j \leq x \leq L \]
\[ X_2 = x - (L - L_j) \quad (3.61) \]

\[
y_2(x) = \frac{1}{EI_2} \left( \frac{\left( m^+ - m^- + F_y^+ \left( a + \frac{l}{2} \right) - F_y^- \left( a - \frac{l}{2} \right) \right) X_2^2}{2} \right. \\
\left. + \frac{(F_y^- - F_y^+)X_2^3}{6} \right) + X_2 \frac{dy_1}{dX_1} \bigg|_{x=L-L_j} + y_1 |_{x=L-L_j} \quad (3.62)
\]

**Domain 3:** 
\[ L \leq x \leq a - \frac{l}{2} \]
\[ X_3 = x - L \quad (3.63) \]

\[
y_3(x) = \frac{1}{EI_3} \left( \frac{\left( m^+ - m^- + F_y^+ \left( a + \frac{l}{2} \right) - F_y^- \left( a - \frac{l}{2} \right) - L \right) X_3^2}{2} \right. \\
\left. + \frac{(F_y^- - F_y^+)X_3^3}{6} \right) + X_3 \frac{dy_2}{dX_2} \bigg|_{x=L} + y_2 |_{x=L} \quad (3.64)
\]

**Domain 4:** 
\[ a - \frac{l}{2} \leq x \leq a + \frac{l}{2} \]
\[ X_4 = x - \left( a - \frac{l}{2} \right) \quad (3.65) \]
\[ y_4(x) = \frac{1}{EI_4} \left( \frac{(m^+ + F_y^+ l)X_4^2}{2} - \frac{F_y^+ X_4^3}{6} \right) + X_4 \frac{dy_3}{dX_3} \bigg|_{x=a-\frac{l}{2}} + y_3 \bigg|_{x=a-\frac{l}{2}} \quad 3.66 \]

Domain 5:

\[ a + \frac{l}{2} \leq x \leq 2L - l_f \]

\[ X_5 = x - \left( a + \frac{l}{2} \right) \quad 3.67 \]

\[ y_5(x) = X_5 \frac{dy_4}{dX_4} \bigg|_{x=a+\frac{l}{2}} + y_4 \bigg|_{x=a+\frac{l}{2}} \quad 3.68 \]
CHAPTER 4: NUMERICAL STUDY AND DISCUSSION

In order to evaluate the effectiveness of piezoelectric actuation, a numerical study is performed on the cantilever single lap joint beam under piezoelectric actuation and tensile loading. Table 4-1 lists the beam and actuator parameters chosen for the numerical study. The single lap joint beam substrate material was chosen to be made of polyvinyl chloride (PVC) to decrease the overall rigidity of the beam, increasing the effect of piezoelectric actuation. The single lap joint beam was also chosen to have a high length-to-thickness ratio to amplify the effect of piezoelectric actuation. The piezoelectric actuator was chosen to be a ceramic-type based on the high electromechanical coupling properties afforded. The piezoelectric actuator properties and dimensions were chosen based on available commercially produced actuators [Piezo Systems, Inc., 2010]. A piezoelectric actuator is assumed to be bonded at the midpoint of each bond region, as presented in Chapter 3. The aim of this numerical study is to highlight the ability of piezoelectric materials to reduce the deflections, caused by applied tensile loading, within the adhesively bonded region of the single lap joint while also illustrating the differences between straight beam and small beam curvature actuation for the four possible bond regions.

Table 4-1 Single Lap Joint Beam and Piezoelectric Actuator Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Single Lap Joint Beam</th>
<th>Piezoelectric Actuator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus (GPa)</td>
<td>4</td>
<td>70</td>
</tr>
<tr>
<td>Width (mm)</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>6</td>
<td>0.6</td>
</tr>
<tr>
<td>Length (mm)</td>
<td>200</td>
<td>50</td>
</tr>
<tr>
<td>Joint Length (mm)</td>
<td>100</td>
<td>-</td>
</tr>
<tr>
<td>d31 (strain/electric field)</td>
<td>-</td>
<td>320 x 10^{-12}</td>
</tr>
</tbody>
</table>
The tensile loading model is presented above in Figure 4-1. The deflection of points labeled A and B in Figure 4-1 are of most importance in this analysis due to the large stress concentrations which are developed at the ends of the adhesive bonding layer during tensile loading.

Due to eccentricity of the applied tensile load, a constant internal bending moment is created throughout the beam. The induced bending moment causes bending and the beam will deflect. The elastic curves for the four piezoelectric bond regions due to a 1 N applied tensile load are shown in Figure 4-2. The aim of using piezoelectric actuation is to reduce the deflection in the, at points A and B, caused by the applied tensile load. Points A and B correspond to x-locations of 100 mm and 200 mm respectively in this numerical study. In order to highlight the difference in effect of straight beam and small curvature beam actuation, an actuation of 50 Volts is applied across the piezoelectric actuator in both actuation scenarios. Figure 4-3 shows the elastic curves induced by straight beam actuation. During straight beam actuation, only a pair of induced moments exists as there is no y-component of the actuation force. As a result, the piezoelectric layer only induces a bending moment in the region between the left and right ends of the piezoelectric layer. No deflection will occur to the left of the left end of the piezoelectric layer.
Figure 4-2  Tensile Free-End Load Induced Displacement: Tensile Free End Load (1 N)

Figure 4-3  Piezoelectric Induced Displacement Curves: Straight Beam Actuation (50 V)
due to there being no reaction at the support. It is obvious from inspection of Figure 4-3, Bond Region I produces the most deflection throughout the span of the lap joint beam. Less deflection is induced as the piezoelectric actuator is bonded further away from the cantilever support. This is a result because when the actuator is bonded closer to the cantilever support, there is greater length of the beam for the bending effects to propagate.

For the small beam curvature actuation scenario, the actuation voltage is applied after the beam has deflected due to the tensile load. The elastic curves for small beam curvature actuation are presented in Figure 4-4. It is seen that the small beam curvature piezoelectric actuation is more effective at inducing deflection throughout the entire span of the beam when compared to straight beam actuation. This occurs due to the y-component of the induced piezoelectric force. The y-component of the force creates a reaction at the cantilever support, allowing for an

![Figure 4-4 Piezoelectric Induced Displacement Curves: Small Beam Curvature Actuation (50 V)](image-url)
increased influence over a greater portion of the beam. In all four possible bond regions, the deflection induced at critical points A and B are increased, with deflection values listed in Table 4-2.

**Table 4-2 Piezoelectric Induced Deflection at Critical Adhesive Joint Locations A and B**

<table>
<thead>
<tr>
<th></th>
<th>Straight Beam</th>
<th>Small Beam Curvature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( y_A ) (mm)</td>
<td>( y_B ) (mm)</td>
</tr>
<tr>
<td>Region I</td>
<td>-0.0140</td>
<td>-0.0421</td>
</tr>
<tr>
<td>Region II</td>
<td>-0.0018</td>
<td>-0.0214</td>
</tr>
<tr>
<td>Region III</td>
<td>0.0000</td>
<td>-0.0064</td>
</tr>
<tr>
<td>Region IV</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

As previously mentioned, the purpose of using piezoelectric actuators on the single lap joint is to reduce the deflections at the ends of the adhesively bonded joint, locations A and B, which will benefit the joint integrity by decreasing stress concentrations that exist due to tensile loading. In order to evaluate the effectiveness of the piezoelectric actuators to reduce tensile loading deflection at the critical joint locations A and B, the elastic curves due to actuation and tensile loading are superimposed for all four possible bond regions. The resulting elastic curves illustrate the combined displacement of the single lap joint subjected to both tensile loading and piezoelectric actuation. Figure 4-5 shows the combined loading elastic curves due to straight beam actuation and tensile loading. Figure 4-6 shows the combined loading elastic curves due to small beam curvature actuation and tensile loading. The effectiveness of piezoelectric actuation can be gauged by how closely the actuation can return the critical joint locations A and B to equilibrium, i.e. zero deflection. Table 4-3 lists the deflection at the critical joint locations A and B before and after straight beam piezoelectric actuation. Table 4-4 lists the deflection at locations A and B before and after small beam curvature actuation.
Figure 4-5  Straight Beam Actuation – Combined Loading Displacement Curves

Figure 4-6  Small Beam Curvature Actuation – Combined Loading Displacement Curves
From inspection of Table 4-3 and Table 4-4, it is clear that bonding the piezoelectric actuator in Region II is capable of returning the critical joint locations A and B closer to equilibrium than any other bond region for both straight beam and small beam curvature actuation. It can be concluded that bonding the piezoelectric actuator in Region II, with the actuator partially bonded overlapping joint location A, will be more beneficial in reducing the deflection of both critical joint locations, which will in turn enhance the overall strength of the single lap joint.

The difference in displacement between location A and location B is another important factor to consider when analyzing which bond region will have the greatest benefit to the
strength of the joint. Minimizing the differences in displacement between the two critical joint locations will ensure that both critical locations are affected equally by actuation. Region II is shown to have the smallest difference in displacement between location A and location B for both straight beam and small beam curvature actuation. The small differences in displacement between location A and location B indicate that the piezoelectric actuation mechanism for Region II bond location produces displacement nearly opposite that of the applied tensile loading, which ensures that actuation will affect both critical joint locations, A and B, equally. Table 4-5 lists the differences in combined displacements for each proposed bond region at the critical joint locations A and B for both straight beam actuation and small beam curvature actuation.

Table 4-5  Location A and B Combined Displacement Differences

<table>
<thead>
<tr>
<th>Region</th>
<th>$\Delta y_{AB}$ (mm) Straight Beam Actuation</th>
<th>$\Delta y_{AB}$ (mm) Small Beam Curvature Actuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region  I</td>
<td>0.0132</td>
<td>0.0135</td>
</tr>
<tr>
<td>Region  II</td>
<td>0.0015</td>
<td>0.0027</td>
</tr>
<tr>
<td>Region  III</td>
<td>0.0153</td>
<td>0.0135</td>
</tr>
<tr>
<td>Region  IV</td>
<td>0.0222</td>
<td>0.0186</td>
</tr>
</tbody>
</table>
CHAPTER 5: CONCLUSIONS AND RECOMMENDATIONS

5.1 CONCLUSIONS

The elastic curve equations for straight beam and small beam curvature actuation of a single lap joint with a surface bonded piezoelectric actuator have been derived. By applying the appropriate polarity to the piezoelectric layer will induce a force and equivalent moment to counterbalance the effects of tensile loading. By inducing counterbalancing deflection in the single lap joint, the high stress concentrations which exist at the left and right extremes of the adhesive bonding layer can be reduced.

For the single lap joint, it was shown that straight beam and small beam curvature actuation will have a different effect on induced displacement as demonstrated in the model. Small beam curvature actuation was shown to be capable of inducing greater displacements than straight beam actuation due to the nature of the actuation mechanism. Straight beam actuation produces a pair of concentrated moments and will induce an internal bending moment only between the two ends of the piezoelectric layer. The increase in induced displacement for small beam curvature actuation is a result of the actuation mechanism producing vertical forces as well as concentrated moments which create a reaction at the cantilever support. The reaction is capable of producing an internal bending moment along a greater portion of the beam, from the cantilever support to the right end of the piezoelectric layer, resulting in increased displacements. It was also shown that the piezoelectric bond location has a great effect on the effectiveness of piezoelectric actuation to reduce deflections at the critical joint locations due to tensile loading. Bond Region II was shown to be most effective reducing deflection at the critical joint locations. Bond Region II was also shown to have an equal effect in reducing deflections in both critical joint locations.
Adhesively bonded joints have become more prevalent in recent years because of the many advantages offered over conventional riveted assemblies using conventional design materials such as steel and aluminum. Advantages include reduced cost, reduced weight, and reparability. Adhesively bonded joints are especially prevalent in the aircraft and aerospace industry where weight reduction is a high priority. However, the advantages often come at the expense of rigidity, making the composite joint structures susceptible to large deformations and premature failure due to the large stress concentrations which develop at the extreme ends of the adhesive layer. The presented modeling indicates that piezoelectric actuation can be used to induce beam deflection to reduce the detrimental effects of tensile loading on a single lap joint, without significantly altering the passive behavior of the joint structure. Deflection and failure strength are directly related, therefore, reducing deflection at the critical joint locations will increase the strength of the composite joint structure. Using piezoelectric actuators to reduce deflection and thus enhance the strength of the structure seems feasible and may offer improved joint performance without significant changes to the original structure. However, an on-site power supply must be present to excite the piezoelectric actuator.

5.2 LIMITATIONS AND RECOMMENDATIONS

The deflection analysis presented is preliminary work to determine the feasibility in using piezoelectric actuation to enhance the strength of a single lap joint under tensile loading. There are still a number of limitations of the proposed modeling. One limitation of using deflection as an indication of joint integrity is that it does not account for the local stress / strain within the adhesive layer. Another limitation is the force which can be generated by the piezoelectric actuator. The magnitude of the induced force is not large, limiting the applications in which piezoelectric actuators can be used. For example, large structures and structures made of
substrate materials which are very stiff, such as steel and aluminum, will not benefit from piezoelectric due to the high rigidity of the beam. As a result, the piezoelectric actuator will not be able to produce significant deflections within the adhesively bonded joint. Finally, an on-site external power supply must be used to excite the piezoelectric actuators, which may be very expensive.

Suggestions for future work include an experimental validation of the proposed modeling should be conducted to better understand the actuation mechanism for real world applications. The modeling should be verified by measuring the induced displacement of the critical adhesively bonded joint locations. The modeling should be expanded further using Laminated Plate Theory to determine the local stress and strain within the adhesive layer due to tensile and piezoelectric loading. Additional experimental studies should also be performed to test the effect of piezoelectric actuation on the failure strength of joint structures.
REFERENCES


APPENDIX A: MATLAB – STRAIGHT BEAM ACTUATION

The following MATLAB code, and slightly modified versions, was used to generate the graphs for the elastic curves of the composite lap joint beam due to piezoelectric actuation and tensile loading.

%% Straight Beam - Elastic Curves for a Cantilever Single Lap Joint
clear all
% close all
% % GLOBAL PARAMETERS
% Beam Geometry and Physical Constants
EB=4e9; % Young’s Modulus of Beam (N/m²) was 55
LB=200e-3; % Beam length (m)
bB=20e-3; % Beam width (m)
h=6e-3; % Beam thickness (m)
Lj=0.5*LB; % Joint Length (m)
EBpsi=6.25*10^6; % From Prepreg Data
Echeck=EBpsi*6894.75729;

% Beam Joint Configuration
xA=0;
xB=LB-Lj;
xC=LB;
xD=2*LB-Lj;
N=4;

% Piezo Geometry and Physical Constants
EP=70e9; % Young’s Modulus of Piezo(N/m²) - SI Units
g31=25e-3; % Piezo Electric Constant - SI Units (V*m/N)
d31=320e-12; % (strain/field)
LP=50e-3; % Piezo length (m)
bP=bB; % Piezo width (m)
t=0.6e-3; % Piezo thickness (m)

% External Load
P=-1;

% Piezo Actuation ==> Applied Voltage ==> Induced Strain and Force
v=-50;
e=d31*(v/t);
% e=(v/t)/(g31*EP); % Piezo strain
f=t*bP*EP*e; % Induced Piezo Force (N)
fpounds=(f/9.81)*2.20462262;
thetaL=0;
thetaR=0;
thetaA=0;

%% Region I
%% Beam Thickness INPUT Arrays
HB=[h h h 2*h h];
HP=[0 t 0 0 0];

% Piezo Location
a=LB/4;
% Piezo Location (m)
oxL=a-LP/2;
% L Piezo Extreme (m)
oxR=a+LP/2;
% R Piezo Extreme (m)

%x Vectors
x1=linspace(0,xL,N);
x2=linspace(xL,xR,N);
x3=linspace(xR,xB,N);
x4=linspace(xB,xC,N);
x5=linspace(xC,xD,N);
XI=[x1 x2 x3 x4 x5];
XImm=XI*1000;
X=[xA xL xR xB xC xD];

% Array storing Area of beam in each region
AB=bB*HB;
% Array storing centroid of beam wrt Y-axis
cB=[h/2 h/2 h/2 0 -h/2];
% Array storing Area of piezo in each region
AP=bP*HP;
% Array storing centroid of piezo wrt Y-axis
cP=[0 h+t/2 0 0 0];
% Array storing the location of neutral axis in each region
dE=(EB*AB.*cB+EP*AP.*cP)./(EB*AB+EP*AP);
% Arrays storing Moment of Inertia for Beam and Piezo
IB=(bB*HB.^3)/12+AB.*(cB-dE).^2;
IP=(bP*HP.^3)/12+AP.*(cP-dE).^2;
% Array storing Equivalent EI
EI=EB*IB+EP*IP;

% External Loading
MA=P*h/2;
% [xA xL xR xB xC xD]
% M and V give reaction at A. The rest of the values are determined in the
% for loop.
M=[-MA 0 0 0 0];
V=[0 0 0 0 0];
% dY and Y give slope and displacement at boundary A. The rest of the
% values are determined in the for loop.
dY=[0 0 0 0 0];
Y=[0 0 0 0 0];
% Initialize B vector. B1-B10 that result as constants of integration.
A=zeros(1,5);
B=zeros(1,5);
x(1,:)=x1;x(2,:)=x2;x(3,:)=x3;x(4,:)=x4;x(5,:)=x5;
dyI_load=zeros(5,N);
yI_load=zeros(5,N);
for i=1:5
% Integration Constants
A(i)=dY(i);
B(i)=Y(i);
% yI_load(x) and yI_load'(x)
dyI_load(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^2/2+M(i)*(x(i,:)-X(i)))+A(i);
yI_load(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^3/6+M(i)*(x(i,:)-X(i)).^2/2)+A(i)*(x(i,:)-X(i))+B(i);
% Slope and Deflection at right end of each region (used for
% integration constants in next region eqn)
dY(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^2/2+M(i)*(X(i+1)-X(i)))+A(i);
Y(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^3/6+M(i)*(X(i+1)-X(i)).^2/2)+A(i)*(X(i+1)-X(i))+B(i);
% Moment and Shear at right end of each region
M(i+1)=V(i)*(X(i+1)-X(i))+M(i)+M(i+1);
V(i+1)=V(i)+V(i+1);
end
YI_load=[yI_load(1,:) yI_load(2,:) yI_load(3,:) yI_load(4,:) yI_load(5,:)];
YI_loadmm=YI_load*1000;
YAloadI=Y(4);
YBloadI=Y(5);
% thetaL=atan(abs(dY(2)));
% thetaR=atan(abs(dY(3)));

% Piezo Response
cL=cP(2)-dE(2);
FxL=f*cos(thetaL);
FyL=f*sin(thetaL);
ML=FxL*cL;

cR=cL;
FxR=f*cos(thetaR);
FyR=f*sin(thetaR);
MR=FxR*cR;
Ax=FxR-FxL;  % Reaction Force - X Component (lb)
Ay=FyL-FyR;  % Reaction Force - Y Component (lb)
MA=ML-MR+FyL*xL-FyR*xR; % Reaction Moment (lb*in)

% [A  L  R  B  C  D]
V=[Ay -FyL FyR 0 0 0];
M=[-MA ML -MR 0 0 0];
A=zeros(1,5);
B=zeros(1,5);
dyI=zeros(5,N);
yI=zeros(5,N);
dY=[0 0 0 0 0 0];
Y=[0 0 0 0 0];
% Elastic Curves
for i=1:5
    % Integration Constants
    A(i)=dY(i);
    B(i)=Y(i);
    % yI(x) and yI'(x)
    dyI(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^2/2+M(i)*(x(i,:)-X(i)))+A(i);
    yI(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^3/6+M(i)*(x(i,:)-X(i)).^2/2)+A(i)*(x(i,:)-X(i))+B(i);
    % Slope and Deflection at right end of each region (used for
    % integration constants in next region eqn)
    dY(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^2/2+M(i)*(X(i+1)-X(i)))+A(i);
    Y(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^3/6+M(i)*(X(i+1)-X(i)).^2/2)+A(i)*(X(i+1)-
    X(i))+B(i);
    % Moment and Shear at right end of each region
    M(i+1)=V(i)*(X(i+1)-X(i))+M(i)+M(i+1);
    V(i+1)=V(i)+V(i+1);
end
YI=[yI(1,:); yI(2,:); yI(3,:); yI(4,:); yI(5,:)];
YImm=YI*1000;
YImm=YIbothmm=(YI+YI_load)*1000;

YApiezoI=Y(4);
YBpiezoI=Y(5);
% Region II
% Beam Thickness INPUT Arrays
HB=[h h 2*h 2*h h];
HP=[0 t t 0 0];

% Piezo Location
a=LB/2;  % Piezo Location (m)
xL=a-LP/2;  % L Piezo Extreme (m)
xR=a+LP/2; % R Piezo Extreme (m)

% NEUTRAL AXIS LOCATION, 2ND AREA MOMENT OF INERTIA, EQUIVALENT EI

x1=linspace(0,xL,N);
x2=linspace(xL,xB,N);
x3=linspace(xB,xR,N);
x4=linspace(xR,xC,N);
x5=linspace(xC,xD,N);
XII=[x1 x2 x3 x4 x5];
XIImm=XII*1000;
X=[xA xL xB xR xC xD];
% Array storing Area of beam in each region
AB=bB*HB;
% Array storing centroid of beam wrt Y-axis
cB=[h/2 h/2 0 0 -h/2];
% Array storing Area of piezo in each region
AP=bP*HP;
% Array storing centroid of piezo wrt Y-axis
cP=[0 h+t/2 h+t/2 0 0];
% Array storing the location of neutral axis in each region
dE=(EB*AB.*cB+EP*AP.*cP)./(EB*AB+EP*AP);
% Arrays storing Moment of Inertia for Beam and Piezo
IB=(bB*HB.^3)/12+AB.*(cB-dE).^2;
IP=(bP*HP.^3)/12+AP.*(cP-dE).^2;
% Array storing Equivalent EI
EI=EB*IB+EP*IP;

% External Loading
MA=P*h/2;
% [xA xL xR xB xC xD]
% M and V give reaction at A. The rest of the values are determined in the
% for loop.
M=[-MA 0 0 0 0 0];
V=[0 0 0 0 0 0];
% dY and Y give slope and displacement at boundary A. The rest of the
% values are determined in the for loop.
dY=[0 0 0 0 0 0];
Y=[0 0 0 0 0 0];
% Initialize B vector. B1-B10 that result as constants of integration.
A=zeros(1,5);
B=zeros(1,5);
x(1:)=x1;x(2:)=x2;x(3:)=x3;x(4:)=x4;x(5:)=x5;
dyII_load=zeros(5,N);
yII_load=zeros(5,N);
for i=1:5

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% Integration Constants
A(i)=dY(i);
B(i)=Y(i);

% yII_load(x) and yII_load'(x)
dyII_load(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^2/2+M(i)*(x(i,:)-X(i)))+A(i);
yII_load(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^3/6+M(i)*(x(i,:)-X(i)).^2/2)+A(i)*(x(i,:)-X(i))+B(i);

% Slope and Deflection at right end of each region (used for
% integration constants in next region eqn)
dY(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^2/2+M(i)*(X(i+1)-X(i)))+A(i);
Y(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^3/6+M(i)*(X(i+1)-X(i)).^2/2)+A(i)*(X(i+1)-X(i))+B(i);

% Moment and Shear at right end of each region
M(i+1)=V(i)*(X(i+1)-X(i))+M(i)+M(i+1);
V(i+1)=V(i)+V(i+1);
end
YII_load=[yII_load(1,:) yII_load(2,:) yII_load(3,:) yII_load(4,:) yII_load(5,:)]
YII_loadmm=YII_load*1000;

YAloadII=Y(3);
YBloadII=Y(5);

% thetaL=atan(abs(dY(2)))
% thetaR=atan(abs(dY(4)))
% thetaA=atan(abs(dY(3)))

% Piezo Response
cL=cP(2)-dE(2);
FxL=f*cos(thetaL);
FyL=f*sin(thetaL);
ML=FxL*cL;
FxLA=f*cos(thetaA);
FyLA=f*sin(thetaA);
MLA=FxLA*cL;

cR=cP(3)-dE(3);
FxR=f*cos(thetaR);
FyR=f*sin(thetaR);
MR=FxR*cR;
FxRA=f*cos(thetaA);
FyRA=f*sin(thetaA);
MRA=FxRA*cR;

Ax=FxR-FxL-FxRA+FxLA; % Reaction Force - X Component (lb)
Ay=FyL-FyR-FyLA+FyRA; % Reaction Force - Y Component (lb)
MA=ML-MR-MLA+MRA+FyL*xL-FyR*xR-FyLA*xB+FyRA*xB; % Reaction Moment (lb*in)
% [A L B R C D]
V=[Ay -FyL FyLA-FyRA FyR 0 0];
M=[-MA ML MRA-MLA -MR 0 0];
A=zeros(1,5);
B=zeros(1,5);
dyII=zeros(5,N);
yII=zeros(5,N);
dY=[0 0 0 0 0];
Y=[0 0 0 0 0];
% Elastic Curves
for i=1:5
% Integration Constants
A(i)=dY(i);
B(i)=Y(i);
% yII(x) and yII'(x)
dyII(i,:)=((1/EI(i))*(V(i)*(x(i,:)-X(i)).^2/2+M(i)*(x(i,:)-X(i)))+A(i);
yII(i,:)=((1/EI(i))*(V(i)*(x(i,:)-X(i)).^3/6+M(i)*(x(i,:)-X(i)).^2/2)+A(i)*(x(i,:)-X(i))+B(i);
% Slope and Deflection at right end of each region (used for integration constants in next region eqn)
dY(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^2/2+M(i)*(X(i+1)-X(i)))+A(i);
Y(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^3/6+M(i)*(X(i+1)-X(i)).^2/2)+A(i)*(X(i+1)-X(i))+B(i);
% Moment and Shear at right end of each region
M(i+1)=V(i)*(X(i+1)-X(i))+M(i)+M(i+1);
V(i+1)=V(i)+V(i+1);
end
YII=[yII(1,:) yII(2,:) yII(3,:) yII(4,:) yII(5,:)];
YIImm=YII*1000;
YIIbothmm=(YII+YII_load)*1000;

YApiezoII=Y(3);
YBpiezoII=Y(5);

%% Region III
% Beam Thickness INPUT Arrays
HB=[h 2*h 2*h 2*h h];
HP=[0 0 t 0 0];

% Piezo Location
a=(3/4)*LB; % Piezo Location (m)
xL=a-LP/2; % L Piezo Extreme (m)
xR=a+LP/2; % R Piezo Extreme (m)

% NEUTRAL AXIS LOCATION, 2ND AREA MOMENT OF INERTIA, EQUIVALENT EI
x1=linspace(0,xB,N);
x2=linspace(xB,xL,N);
x3=linspace(xL,xR,N);
x4=linspace(xR,xC,N);
x5=linspace(xC,xD,N);
XIII=[x1 x2 x3 x4 x5];
XIIImm=XIII*1000;
X=[xA xB xL xR xC xD];

% Array storing Area of beam in each region
AB=bB*HB;
% Array storing centroid of beam wrt Y-axis
cB=[h/2 0 0 0 -h/2];
% Array storing Area of piezo in each region
AP=bP*HP;
% Array storing centroid of piezo wrt Y-axis
cP=[0 0 h+t/2 0 0];
% Array storing the location of neutral axis in each region
dE=(EB*AB.*cB+EP*AP.*cP)./(EB*AB+EP*AP);
% Arrays storing Moment of Inertia for Beam and Piezo
IB=(bB*HB.^3)/12+AB.*(cB-dE).^2;
IP=(bP*HP.^3)/12+AP.*(cP-dE).^2;
% Array storing Equivalent EI
EI=EB*IB+EP*IP;

% External Loading
MA=P*h/2;
% [xA xB xL xR xC xD]
% M and V give reaction at A. The rest of the values are determined in the
% for loop.
M=[-MA 0 0 0 0 0];
V=[0 0 0 0 0 0];
% dY and Y give slope and displacement at boundary A. The rest of the
% values are determined in the for loop.
dY=[0 0 0 0 0 0];
Y=[0 0 0 0 0 0];
% Initialize B vector. B1-B10 that result as constants of integration.
A=zeros(1,5);
B=zeros(1,5);
x(1,:)=x1;x(2,:)=x2;x(3,:)=x3;x(4,:)=x4;x(5,:)=x5;
dyIII_load=zeros(5,N);
yIII_load=zeros(5,N);
for i=1:5
    % Integration Constants
    A(i)=dY(i);
    B(i)=Y(i);
% yIII_load(x) and yIII_load'(x)
dyIII_load(i,:)=((1/EI(i))*(V(i)*(x(i,:)-X(i))).^2/2+M(i)*(x(i,:)-X(i)))+A(i);
yIII_load(i,:)=((1/EI(i))*(V(i)*(x(i,:)-X(i))).^3/6+M(i)*(x(i,:)-X(i)).^2/2)+A(i)*(x(i,:)-X(i))+B(i);
% Slope and Deflection at right end of each region (used for
% integration constants in next region eqn)
dY(i+1)=((1/EI(i))*(V(i)*(X(i+1)-X(i))).^2/2+M(i)*(X(i+1)-X(i)))+A(i);
Y(i+1)=((1/EI(i))*(V(i)*(X(i+1)-X(i))).^3/6+M(i)*(X(i+1)-X(i)).^2/2)+A(i)*(X(i+1)-X(i))+B(i);
% Moment and Shear at right end of each region
M(i+1)=V(i)*(X(i+1)-X(i))+M(i)+M(i+1);
V(i+1)=V(i)+V(i+1);
end
YIII_load=[yIII_load(1,:); yIII_load(2,:); yIII_load(3,:); yIII_load(4,:); yIII_load(5,:)];
YIII_loadmm=YIII_load*1000;

YAloadIII=Y(2);
YBloadIII=Y(5);
% thetaL=atan(abs(dY(3))); % thetaR=atan(abs(dY(4)));
% Piezo Response
cL=cP(3)-dE(3);
FxL=f*cos(thetaL);
FyL=f*sin(thetaL);
ML=FxL*cL;
cR=cL;
FxR=f*cos(thetaR);
FyR=f*sin(thetaR);
MR=FxR*cR;

Ax=FxR-FxL; % Reaction Force - X Component (lb)
Ay=FyL-FyR; % Reaction Force - Y Component (lb)
MA=ML-MR+FyL*xL-FyR*xR; % Reaction Moment (lb*in)

% [A B L R C D]
V=[Ay 0 -FyL FyR 0 0];
M=[-MA 0 ML-MR 0 0];
A=zeros(1,5);
B=zeros(1,5);
dyIII=zeros(5,N);
yIII=zeros(5,N);
dY=[0 0 0 0 0];
Y=[0 0 0 0 0];
% Elastic Curves
for i=1:5
% Integration Constants
A(i)=dY(i);
B(i)=Y(i);
% yIII(x) and yIII'(x)
dyIII(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^2/2+M(i)*(x(i,:)-X(i)))+A(i);
yIII(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^3/6+M(i)*(x(i,:)-X(i)).^2/2)+A(i)*(x(i,:)-X(i))+B(i);
% Slope and Deflection at right end of each region (used for % integration constants in next region eqn)
dY(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^2/2+M(i)*(X(i+1)-X(i)))+A(i);
Y(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^3/6+M(i)*(X(i+1)-X(i)).^2/2)+A(i)*(X(i+1)-X(i))+B(i);
% Moment and Shear at right end of each region
M(i+1)=V(i)*(X(i+1)-X(i))+M(i)+M(i+1);
V(i+1)=V(i)+V(i+1);
end
YIII=[yIII(1,:) yIII(2,:) yIII(3,:) yIII(4,:) yIII(5,:)];
YIIImm=YIII*1000;
YIIIbothmm=(YIII+YIII_load)*1000;

YApiezoIII=Y(2);
YBpiezoIII=Y(5);

%% Region IV
% Beam Thickness INPUT Arrays
HB=[h 2*h h h h];
HP=[0 0 0 t 0];

% Piezo Location
a=LB+LB/4; % Piezo Location (m)
xL=a-LP/2; % L Piezo Extreme (m)
xR=a+LP/2; % R Piezo Extreme (m)

% NEUTRAL AXIS LOCATION, 2ND AREA MOMENT OF INERTIA, EQUIVALENT EI
x1=linspace(0,xB,N);
x2=linspace(xB,xC,N);
x3=linspace(xC,xL,N);
x4=linspace(xL,xR,N);
x5=linspace(xR,xD,N);
XIV=[x1 x2 x3 x4 x5];
XIVmm=XIV*1000;
X=[xA xB xC xL xR xD];
% Array storing Area of beam in each region
AB=bB*HB;
% Array storing centroid of beam wrt Y-axis
cB=[h/2 0 -h/2 -h/2 -h/2];
% Array storing Area of piezo in each region
AP=bP*HP;
% Array storing centroid of piezo wrt Y-axis
cP=[0 0 0 t/2 0];
% Array storing the location of neutral axis in each region
dE=(EB*AB.*cB+EP*AP.*cP)./(EB*AB+EP*AP);
% Arrays storing Moment of Inertia for Beam and Piezo
IB=(bB*HB.^3)/12+AB.*(cB-dE).^2;
IP=(bP*HP.^3)/12+AP.*(cP-dE).^2;
% Array storing Equivalent EI
EI=EB*IB+EP*IP;

% External Loading
MA=P*h/2;
% [xA xB xC xL xR xD]
% M and V give reaction at A. The rest of the values are determined in the
% for loop.
M=[-MA 0 0 0 0 0];
V=[0 0 0 0 0 0];
% dY and Y give slope and displacement at boundary A. The rest of the
% values are determined in the for loop.
dY=[0 0 0 0 0 0];
Y=[0 0 0 0 0 0];
% Initialize B vector. B1-B10 that result as constants of integration.
A=zeros(1,5);
B=zeros(1,5);
x(1,:)=x1;x(2,:)=x2;x(3,:)=x3;x(4,:)=x4;x(5,:)=x5;
dYIV_load=zeros(5,N);
yIV_load=zeros(5,N);
% External Load Curve
for i=1:5
    % Integration Constants
    A(i)=dY(i);
    B(i)=Y(i);
    % yIV_load(x) and yIV_load'(x)
    dYIV_load(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^2/2+M(i)*(x(i,:)-X(i)))+A(i);
    yIV_load(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^3/6+M(i)*(x(i,:)-X(i)).^2/2)+A(i)*(x(i,:)-X(i))+B(i);
    % Slope and Deflection at right end of each region (used for
    % integration constants in next region eqn)
    dY(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^2/2+M(i)*(X(i+1)-X(i)))+A(i);
    Y(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^3/6+M(i)*(X(i+1)-X(i)).^2/2)+A(i)*(X(i+1)-
    X(i))+B(i);
    % Moment and Shear at right end of each region
    M(i+1)=V(i)*(X(i+1)-X(i))+M(i)+M(i+1);
    V(i+1)=V(i)+V(i+1);
end
YIV_load=[yIV_load(1,:) yIV_load(2,:) yIV_load(3,:) yIV_load(4,:) yIV_load(5,:)];
YIV_loadmm=YIV_load*1000;

YAloadIV=Y(2);
YBloadIV=Y(3);
% thetaL=atan(abs(dY(4)));
% thetaR=atan(abs(dY(5)));

% Piezo Response

cL=cP(4)-dE(4);
FxL=f*cos(thetaL);
FyL=f*sin(thetaL);
ML=FxL*cL;

[cR=cL; FxR=f*cos(thetaR); FyR=f*sin(thetaR); MR=FxR*cR;]

Ax=FxR-FxL;  % Reaction Force - X Component (lb)
Ay=FyL-FyR;  % Reaction Force - Y Component (lb)
MA=ML-MR+FyL*xL-FyR*xR;  % Reaction Moment (lb*in)

% [A  B  C  L  R  D]
V=[Ay 0 0 -FyL FyR 0];
M=[-MA 0 0 ML -MR 0];
A=zeros(1,5);
B=zeros(1,5);
dyIV=zeros(5,N);
yIV=zeros(5,N);
dY=[0 0 0 0 0];
Y=[0 0 0 0 0];

% Elastic Curves
for i=1:5

% Integration Constants
A(i)=dY(i);
B(i)=Y(i);
% yIV(x) and yIV'(x)
dyIV(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^2/2+M(i)*(x(i,:)-X(i)))+A(i);
yIV(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^3/6+M(i)*(x(i,:)-X(i)).^2/2)+A(i)*(x(i,:)-X(i))+B(i);
% Slope and Deflection at right end of each region (used for
% integration constants in next region eqn)
dY(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^2/2+M(i)*(X(i+1)-X(i)))+A(i);
Y(i+1)=(1/Ei(i))*V(i)*(X(i+1)-X(i))^3/6+M(i)*(X(i+1)-X(i))^2/2+A(i)*(X(i+1)-X(i))+B(i);

% Moment and Shear at right end of each region
M(i+1)=V(i)*(X(i+1)-X(i))+M(i)+M(i+1);
V(i+1)=V(i)+V(i+1);

end

YIV=[yIV(1,:) yIV(2,:) yIV(3,:) yIV(4,:) yIV(5,:)];
YIVmm=YIV*1000;

YIVbothmm=(YIV+YIV_load)*1000;

YApiezoIV=Y(2);
YBpiezoIV=Y(3);

%% PLOTS
set(0,'DefaultFigurePosition',[400 200 2000 1100]);
figure(1);set(gcf,'color','w');
ExternalLoadComparison=plot(XImm,YI_loadmm,'o-',XIImm,YII_loadmm,'x-',XIIImm,YIII_loadmm,'+',XIVmm,YIV_loadmm,'*');
set(ExternalLoadComparison,'LineWidth',2);set(gca,'xlim',[0 300],'ylim',[0 0.1]);set(gca,'FontSize',30,'FontName','Times');
title('External Load Displacement - Straight Beam Actuation','FontSize',36,'FontName','Times','FontWeight','b');
xlabel('x (mm)','FontSize',36,'FontName','Times','FontWeight','b');
ylabel('Displacement (mm)','FontSize',36,'FontName','Times','FontWeight','b');
legend('Region I','Region II','Region III','Region IV','Location','NorthWest');

set(0,'DefaultFigurePosition',[400 200 2000 1100]);
figure(2);set(gcf,'color','w');
PreLoadComparison=plot(XImm,YI_bothmm,'o-',XIIImm,YII_bothmm,'x-',XIIIImm,YIII_bothmm,'+',XIVmm,YIV_bothmm,'*');
set(PreLoadComparison,'LineWidth',2);set(gca,'xlim',[0 300],'ylim',[-0.1 0.1]);set(gca,'FontSize',30,'FontName','Times');
title('Piezoelectric Induced Displacement - Straight Beam Actuation','FontSize',36,'FontName','Times','FontWeight','b');
xlabel('x (mm)','FontSize',36,'FontName','Times','FontWeight','b');
ylabel('Displacement (mm)','FontSize',36,'FontName','Times','FontWeight','b');
legend('Region I','Region II','Region III','Region IV','Location','SouthWest');

set(0,'DefaultFigurePosition',[400 200 2000 1100]);
figure(3);set(gcf,'color','w');
PreLoadTotalComparison=plot(XImm,YIbothmm,'o-',XIIImm,YIIbothmm,'x-',XIIIImm,YIIIbothmm,'+',XIVmm,YIVbothmm,'*');
set(PreLoadTotalComparison,'LineWidth',2);set(gca,'xlim',[0 300],'ylim',[-0.05 0.05]);set(gca,'FontSize',30,'FontName','Times');
title('Total Displacement','FontSize',36,'FontName','Times','FontWeight','b');
xlabel('x (mm)','FontSize',36,'FontName','Times','FontWeight','b');
ylabel('Displacement (mm)','FontSize',36,'FontName','Times','FontWeight','b');
legend('Region I','Region II','Region III','Region IV','Location','SouthWest')

YAload=1000*[YAloadI YAloadII YAloadIII YAloadIV];
YBload=1000*[YBloadI YBloadII YBloadIII YBloadIV];
Yloadstraight=[YAload YBload]

YApiezo=1000*[YApiezoI YApiezoII YApiezoIII YApiezoIV];
YBpiezo=1000*[YBpiezoI YBpiezoII YBpiezoIII YBpiezoIV];
Ypiezostraight=[YApiezo YBpiezo]

YtotalAstraight=YAload+YApiezo;
YtotalBstraight=YBload+YBpiezo;

Ytotalstraight=[YtotalAstraight YtotalBstraight]
APPENDIX B: MATLAB – SMALL BEAM CURVATURE ACTUATION

The following MATLAB code, and slightly modified versions, was used to generate the graphs for the elastic curves of the composite lap joint beam due to piezoelectric actuation and tensile loading.

%% Small Beam Curvature - Elastic Curves for a Cantilever Single Lap Joint
clear all
close all

%% GLOBAL PARAMETERS
% Beam Geometry and Physical Constants
EB=4e9; % Young’s Modulus of Beam (N/m^2) was 55
LB=200e-3; % Beam length (m)
bB=20e-3; % Beam width (m)
h=6e-3; % Beam thickness (m)
Lj=0.5*LB; % Joint Length (m)
EBpsi=6.25*10^6; % From Prepreg Data
Echeck=EBpsi*6894.75729;

% Beam Joint Configuration
xA=0;
xB=LB-Lj;
xC=LB;
xD=2*LB-Lj;
N=4;

%% Piezo Geometry and Physical Constants
EP=70e9; % Young’s Modulus of Piezo(N/m^2) - SI Units
g31=25e-3; % Piezo Electric Constant - SI Units (V*m/N)
d31=320e-12; % (strain/field)
LP=50e-3; % Piezo length (m)
bP=bB; % Piezo width (m)
t=0.6e-3; % Piezo thickness (m)

% External Load
P=-1; % External Load (N)

% Piezo Actuation ==> Applied Voltage ==> Induced Strain and Force
v=-50; % Applied Voltage (V)
e=d31*(v/t);
%f=fbp*Ep*e; % Induced Piezo Force (N)
fpounds=(f/9.81)*2.20462262;
%% Region I
% Beam Thickness INPUT Arrays
HB=[h h h 2*h h];
HP=[0 t 0 0 0];

% Piezo Location
a=LB/4; % Piezo Location (m)
xL=a-LP/2; % L Piezo Extreme (m)
xR=a+LP/2; % R Piezo Extreme (m)

% x Vectors
x1=linspace(0,xL,N);
x2=linspace(xL,xR,N);
x3=linspace(xR,xB,N);
x4=linspace(xB,xC,N);
x5=linspace(xC,xD,N);
XI=[x1 x2 x3 x4 x5];
XImm=XI*1000;
X=[xA xL xR xB xC xD];

% Array storing Area of beam in each region
AB=bB*HB;
% Array storing centroid of beam wrt Y-axis
cB=[h/2 h/2 h/2 0 -h/2];
% Array storing Area of piezo in each region
AP=bP*HP;
% Array storing centroid of piezo wrt Y-axis
cP=[0 h+t/2 0 0 0];
% Array storing the location of neutral axis in each region
dE=(EB*AB.*cB+EP*AP.*cP)./(EB*AB+EP*AP);
% Arrays storing Moment of Inertia for Beam and Piezo
IB=(bB*HB.^3)/12+AB.*(cB-dE).^2;
IP=(bP*HP.^3)/12+AP.*(cP-dE).^2;
% Array storing Equivalent EI
EI=EB*IB+EP*IP;

% External Loading
MA=P*h/2;
% [xA xL xR xB xC xD]
% M and V give reaction at A. The rest of the values are determined in the
% for loop.
M=[-MA 0 0 0 0 0];
V=[0 0 0 0 0];
% dY and Y give slope and displacement at boundary A. The rest of the
% values are determined in the for loop.
dY = [0 0 0 0 0];
Y = [0 0 0 0 0];

% Initialize B vector. B1-B10 that result as constants of integration.
A = zeros(1,5);
B = zeros(1,5);
x(1,:) = x1; x(2,:) = x2; x(3,:) = x3; x(4,:) = x4; x(5,:) = x5;
dyI_load = zeros(5,N);
yI_load = zeros(5,N);

for i = 1:5
    % Integration Constants
    A(i) = dY(i);
    B(i) = Y(i);

    % yI_load(x) and yI_load'(x)
    dyI_load(i,:) = (1/EI(i))*(V(i)*(x(i,:)-X(i)).^2/2+M(i)*(x(i,:)-X(i)))+A(i);
    yI_load(i,:) = (1/EI(i))*(V(i)*(x(i,:)-X(i)).^3/6+M(i)*(x(i,:)-X(i)).^2/2)+A(i)*(x(i,:)-X(i))+B(i);

    % Slope and Deflection at right end of each region (used for
    % integration constants in next region eqn)
    dY(i+1) = (1/EI(i))*(V(i)*(X(i+1)-X(i)).^2/2+M(i)*(X(i+1)-X(i)))+A(i);
    Y(i+1) = (1/EI(i))*(V(i)*(X(i+1)-X(i)).^3/6+M(i)*(X(i+1)-X(i)).^2/2)+A(i)*(X(i+1)-X(i))+B(i);

    % Moment and Shear at right end of each region
    M(i+1) = V(i)*(X(i+1)-X(i))+M(i)+M(i+1);
    V(i+1) = V(i)+V(i+1);
end

YI_load = [yI_load(1,:) yI_load(2,:) yI_load(3,:) yI_load(4,:) yI_load(5,:)];
YI_loadmm = YI_load*1000;

YAloadI = Y(4);
YBloadI = Y(5);

thetaL = atan(abs(dY(2)));
thetaR = atan(abs(dY(3)));

% Piezo Response
cl = cP(2)-dE(2);
FxL = f*cos(thetaL);
FyL = f*sin(thetaL);
ML = FxL*cl;

CR = cl;
FxR = f*cos(thetaR);
FyR = f*sin(thetaR);
MR = FxR*CR;

Ax = FxR-FxL; % Reaction Force - X Component (lb)
Ay = FyL-FyR; % Reaction Force - Y Component (lb)
MA=ML-MR+FyL*xL-FyR*xR; % Reaction Moment (lb*in)

% [A    L    R    B    C    D]
V=[Ay -FyL FyR 0 0 0];
M=[-MA ML -MR 0 0 0];
A=zeros(1,5);
B=zeros(1,5);
dyI=zeros(5,N);
yI=zeros(5,N);
dY=[0 0 0 0 0 0];
Y=[0 0 0 0 0 0];
% Elastic Curves
for i=1:5
    % Integration Constants
    A(i)=dY(i);
    B(i)=Y(i);
    % yI(x) and yI'(x)
    dyI(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^2/2+M(i)*(x(i,:)-X(i)))+A(i);
    yI(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^3/6+M(i)*(x(i,:)-X(i)).^2/2)+A(i)*(x(i,:)-X(i))+B(i);
    % Slope and Deflection at right end of each region (used for
    % integration constants in next region eqn)
    dY(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^2/2+M(i)*(X(i+1)-X(i)))+A(i);
    Y(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^3/6+M(i)*(X(i+1)-X(i)).^2/2)+A(i)*(X(i+1)-X(i))+B(i);
    % Moment and Shear at right end of each region
    M(i+1)=V(i)*(X(i+1)-X(i))+M(i)+M(i+1);
    V(i+1)=V(i)+V(i+1);
end
YI=[yI(1,:) yI(2,:) yI(3,:) yI(4,:) yI(5,:)];
YImm=YI*1000;
YIbothmm=(YI+YI_load)*1000;
YApiezoI=Y(4);
YBpiezoI=Y(5);
% Region II
% Beam Thickness INPUT Arrays
HB=[h h 2*h 2*h h];
HP=[0 t t 0 0];

% Piezo Location
a=LB/2; % Piezo Location (m)
xL=a-LP/2; % L Piezo Extreme (m)
xR=a+LP/2; % R Piezo Extreme (m)

% NEUTRAL AXIS LOCATION, 2ND AREA MOMENT OF INERTIA, EQUIVALENT EI
x1=linspace(0,xL,N);
x2=linspace(xL,xB,N);
x3=linspace(xB,xR,N);
x4=linspace(xR,xC,N);
x5=linspace(xC,xD,N);
XII=[x1 x2 x3 x4 x5];
XIImm=XII*1000;
X=[xA xL xB xR xC xD];
% Array storing Area of beam in each region
AB=bB*HB;
% Array storing centroid of beam wrt Y-axis
cB=[h/2 h/2 0 0 -h/2];
% Array storing Area of piezo in each region
AP=bP*HP;
% Array storing centroid of piezo wrt Y-axis
cP=[0 h+t/2 h+t/2 0 0];
% Array storing the location of neutral axis in each region
dE=(EB*AB.*cB+EP*AP.*cP)./(EB*AB+EP*AP);
% Arrays storing Moment of Inertia for Beam and Piezo
IB=(bB*HB.^3)/12+AB.*(cB-dE).^2;
IP=(bP*HP.^3)/12+AP.*(cP-dE).^2;
% Array storing Equivalent EI
EI=EB*IB+EP*IP;
% External Loading
MA=P*h/2;
% [xA xL xR xB xC xD]
% M and V give reaction at A. The rest of the values are determined in the
% for loop.
M=[-MA 0 0 0 0 0];
V=[0 0 0 0 0 0];
% dy and Y give slope and displacement at boundary A. The rest of the
% values are determined in the for loop.
dY=[0 0 0 0 0 0];
Y=[0 0 0 0 0 0];
% Initialize B vector. B1-B10 that result as constants of integration.
A=zeros(1,5);
B=zeros(1,5);
x(1,1)=x1;x(2,1)=x2;x(3,1)=x3;x(4,1)=x4;x(5,1)=x5;
dyII_load=zeros(5,N);
yII_load=zeros(5,N);
for i=1:5
  % Integration Constants
  A(i)=dY(i);
  B(i)=Y(i);
% yII_load(x) and yII_load'(x)
  dyII_load(i,:)=((1/El(i))*(V(i)*(X(i,:)-X(i)).^2/2+M(i)*(X(i,:)-X(i)))+A(i));
  yII_load(i,:)=((1/El(i))*(V(i)*(X(i,:)-X(i)).^3/6+M(i)*(X(i,:)-X(i)).^2/2)+A(i)*(X(i,:)-X(i)))+B(i);

% Slope and Deflection at right end of each region (used for integration constants in next region eqn)
  dY(i+1)=((1/El(i))*(V(i)*(X(i+1)-X(i)).^2/2+M(i)*(X(i+1)-X(i)))+A(i));
  Y(i+1)=((1/El(i))*(V(i)*(X(i+1)-X(i)).^3/6+M(i)*(X(i+1)-X(i)).^2/2)+A(i)*(X(i+1)-X(i)))+B(i);

% Moment and Shear at right end of each region
  M(i+1)=V(i)*(X(i+1)-X(i))+M(i)+M(i+1);
  V(i+1)=V(i)+V(i+1);
end

YII_load=[yII_load(1,:) yII_load(2,:) yII_load(3,:) yII_load(4,:) yII_load(5,:)];
YII_loadmm=YII_load*1000;

YAloadII=Y(3);
YBloadII=Y(5);

thetaL=atan(abs(dY(2)));
thetaR=atan(abs(dY(4)));
thetaA=atan(abs(dY(3)));

% Piezo Response
  cL=cP(2)-dE(2);
  FxL=f*cos(thetaL);
  FyL=f*sin(thetaL);
  ML=FxL*cL;
  FxLA=f*cos(thetaA);
  FyLA=f*sin(thetaA);
  MLA=FxLA*cL;

  cR=cP(3)-dE(3);
  FxR=f*cos(thetaR);
  FyR=f*sin(thetaR);
  MR=FxR*cR;
  FxRA=f*cos(thetaA);
  FyRA=f*sin(thetaA);
  MRA=FxRA*cR;

  Ax=FxR-FxL-FxRA+FxLA; % Reaction Force - X Component (lb)
  Ay=FyL-FyR-FyLA+FyRA; % Reaction Force - Y Component (lb)
  MA=ML-MR-MLA+MRA+FyL*xL-FyR*xR-FyLA*xB+FyRA*xB; % Reaction Moment (lb*in)

% [A L B R C D]
% Elastic Curves
for i=1:5
    % Integration Constants
    A(i)=dY(i);
    B(i)=Y(i);
    % yII(x) and yII'(x)
    dyII(i,:)=((1/EI(i))*(V(i)*(x(i,:)-X(i)).^2/2+M(i)*(x(i,:)-X(i)))+A(i));
    yII(i,:)=((1/EI(i))*(V(i)*(x(i,:)-X(i)).^3/6+M(i)*(x(i,:)-X(i)).^2/2)+A(i)*(x(i,:)-X(i))+B(i));
    % Slope and Deflection at right end of each region (used for
    % integration constants in next region eqn)
    dY(i+1)=((1/EI(i))*(V(i)*(X(i+1)-X(i)).^2/2+M(i)*(X(i+1)-X(i)))+A(i));
    Y(i+1)=((1/EI(i))*(V(i)*(X(i+1)-X(i)).^3/6+M(i)*(X(i+1)-X(i)).^2/2)+A(i)*(X(i+1)-X(i))+B(i));
    % Moment and Shear at right end of each region
    M(i+1)=V(i)*(X(i+1)-X(i))+M(i)+M(i+1);
    V(i+1)=V(i)+V(i+1);
end
YII=[yII(1,:) yII(2,:) yII(3,:) yII(4,:) yII(5,:)];
YIImm=YII*1000;
YIIbothmm=(YII+YII_load)*1000;

YApiezoII=YII(3);
YBpiezoII=YII(5);

%% Region III
% Beam Thickness INPUT Arrays
HB=[h 2*h 2*h h h];
HP=[0 0 t 0 0];

% Piezo Location
a=(3/4)*LB; % Piezo Location (m)
xL=a-LP/2; % L Piezo Extreme (m)
xR=a+LP/2; % R Piezo Extreme (m)

% NEUTRAL AXIS LOCATION, 2ND AREA MOMENT OF INERTIA, EQUIVALENT EI
x1=linspace(0,xB,N);
x2=linspace(xB,xL,N);
x3=linspace(xL,xR,N);
x4=linspace(xR,xC,N);
x5=linspace(xC,xD,N);
XIII=[x1 x2 x3 x4 x5];
XIIImm=XIII*1000;
X=[xA xB xL xR xC xD];
% Array storing Area of beam in each region
AB=bB*HB;
% Array storing centroid of beam wrt Y-axis
cB=[h/2 0 0 0 -h/2];
% Array storing Area of piezo in each region
AP=bP*HP;
% Array storing centroid of piezo wrt Y-axis
cP=[0 0 h+t/2 0 0];
% Array storing the location of neutral axis in each region
dE=(EB*AB.*cB+EP*AP.*cP)./(EB*AB+EP*AP);
% Arrays storing Moment of Inertia for Beam and Piezo
IB=(bB*HB.^3)/12+AB.*(cB-dE).^2;
IP=(bP*HP.^3)/12+AP.*(cP-dE).^2;
% Array storing Equivalent EI
EI=EB*IB+EP*IP;

% External Loading
MA=P*h/2;
% [xA xL xR xB xC xD]
% M and V give reaction at A. The rest of the values are determined in the
% for loop.
M=[-MA 0 0 0 0 0];
V=[0 0 0 0 0 0];
% dY and Y give slope and displacement at boundary A. The rest of the
% values are determined in the for loop.
dY=[0 0 0 0 0 0];
Y=[0 0 0 0 0 0];
% Initialize B vector. B1-B10 that result as constants of integration.
A=zeros(1,5);
B=zeros(1,5);
x(1,:)=x1;x(2,:)=x2;x(3,:)=x3;x(4,:)=x4;x(5,:)=x5;
dyIII_load=zeros(5,N);
yIII_load=zeros(5,N);
for i=1:5
% Integration Constants
A(i)=dY(i);
B(i)=Y(i);
% yIII_load(x) and yIII_load'(x)
dyIII_load(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^2/2+M(i)*(x(i,:)-X(i)))+A(i);
yIII_load(i,:)= (1/EI(i))*(V(i)*(x(i,:)-X(i)).^3/6+M(i)*(x(i,:)-X(i)).^2/2)+A(i)*(x(i,:)-X(i))+B(i);
% Slope and Deflection at right end of each region (used for
% integration constants in next region eqn)
dY(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^2/2+M(i)*(X(i+1)-X(i)))+A(i);
Y(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^3/6+M(i)*(X(i+1)-X(i)).^2/2)+A(i)*(X(i+1)-X(i))+B(i);
% Moment and Shear at right end of each region
M(i)=V(i)*(X(i+1)-X(i))+M(i)+M(i+1);
V(i+1)=V(i)+V(i+1);
end
YIII_load=[yIII_load(1,:) yIII_load(2,:) yIII_load(3,:) yIII_load(4,:) yIII_load(5,:)];
YIII_loadmm=YIII_load*1000;
YAloadIII=Y(2);
YBloadIII=Y(5);
thetaL=atan(abs(dY(3)));
thetaR=atan(abs(dY(4)));

% Piezo Response
cL=cP(3)-dE(3);
FxL=f*cos(thetaL);
FyL=f*sin(thetaL);
ML=FxL*cL;

cR=cL;
FxR=f*cos(thetaR);
FyR=f*sin(thetaR);
MR=FxR*cR;

Ax=FxR-FxL; % Reaction Force - X Component (lb)
Ay=FyL-FyR; % Reaction Force - Y Component (lb)
MA=ML-MR+FyL*xL-FyR*xR; % Reaction Moment (lb*in)

% [A  B  L  R  C D]
V=[Ay 0 -FyL FyR 0 0];
M=[-MA 0 ML -MR 0 0];
A=zeros(1,5);
B=zeros(1,5);
dyIII=zeros(5,N);
yIII=zeros(5,N);
dY=[0 0 0 0 0];
Y=[0 0 0 0 0];
% Elastic Curves
for i=1:5
    % Integration Constants
    A(i)=dY(i);
    B(i)=Y(i);
    % yIII(x) and yIII'(x)
    dyIII(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^2/2+M(i)*(x(i,:)-X(i)))+A(i);
    yIII(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^3/6+M(i)*(x(i,:)-X(i)).^2/2)+A(i)*(x(i,:)-X(i))+B(i);
    % Slope and Deflection at right end of each region (used for int constants in next region eqn)
    dY(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^2/2+M(i)*(X(i+1)-X(i)))+A(i);
    Y(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^3/6+M(i)*(X(i+1)-X(i)).^2/2)+A(i)*(X(i+1)-X(i))+B(i);
    % Moment and Shear at right end of each region
    M(i+1)=V(i)*(X(i+1)-X(i))+M(i)+M(i+1);
    V(i+1)=V(i)+V(i+1);
end
YIII=[yIII(1,:); yIII(2,:); yIII(3,:); yIII(4,:); yIII(5,:)];
YIIImm=YIII*1000;

YIIIbothmm=(YIII+YIII_load)*1000;

YApiezoIII=Y(2);
YBpiezoIII=Y(5);

%% Region IV
% Beam Thickness INPUT Arrays
HB=[h 2*h h h h];
HP=[0 0 0 t 0];

% Piezo Location
a=LB+LB/4; % Piezo Location (m)
xL=a-LP/2; % L Piezo Extreme (m)
xR=a+LP/2; % R Piezo Extreme (m)

% NEUTRAL AXIS LOCATION, 2ND AREA MOMENT OF INERTIA, EQUIVALENT EI
x1=linspace(0,xB,N);
x2=linspace(xB,xC,N);
x3=linspace(xC,xL,N);
x4=linspace(xL,xR,N);
x5=linspace(xR,xD,N);
XIV=[x1 x2 x3 x4 x5];
XIVmm=XIV*1000;
X=[xA xB xC xL xR xD];

% Array storing Area of beam in each region
AB=bB*HB;

% Array storing centroid of beam wrt Y-axis
cB=[h/2 0 -h/2 -h/2 -h/2];
% Array storing Area of piezo in each region
AP=bP*HP;
% Array storing centroid of piezo wrt Y-axis
cP=[0 0 0 t/2 0];
% Array storing the location of neutral axis in each region
dE=(EB*AB.*cB+EP*AP.*cP)./(EB*AB+EP*AP);
% Arrays storing Moment of Inertia for Beam and Piezo
IB=(bB*HB.^3)/12+AB.*(cB-dE).^2;
IP=(bP*HP.^3)/12+AP.*(cP-dE).^2;
% Array storing Equivalent EI
EI=EB*IB+EP*IP;

% External Loading
MA=P*h/2;
% [xA xL xR xB xC xD]
% M and V give reaction at A. The rest of the values are determined in the
% for loop.
M=[-MA 0 0 0 0 0];
V=[0 0 0 0 0 0];
% dY and Y give slope and displacement at boundary A. The rest of the
% values are determined in the for loop.
dY=[0 0 0 0 0 0];
Y=[0 0 0 0 0 0];
% Initialize B vector. B1-B10 that result as constants of integration.
A=zeros(1,5);
B=zeros(1,5);
x(1,:)=x1;x(2,:)=x2;x(3,:)=x3;x(4,:)=x4;x(5,:)=x5;
dyIV_load=zeros(5,N);
yIV_load=zeros(5,N);

% External Load Curve
for i=1:5

% Integration Constants
A(i)=dY(i);
B(i)=Y(i);
% yIV_load(x) and yIV_load'(x)
dyIV_load(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^2/2+M(i)*(x(i,:)-X(i)))+A(i);
yIV_load(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^3/6+M(i)*(x(i,:)-X(i)).^2/2)+A(i)*(x(i,:)-X(i))+B(i);
% Slope and Deflection at right end of each region (used for
% integration constants in next region eqn)
dY(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^2/2+M(i)*(X(i+1)-X(i)))+A(i);
Y(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^3/6+M(i)*(X(i+1)-X(i)).^2/2)+A(i)*(X(i+1)-X(i))+B(i);
% Moment and Shear at right end of each region
M(i+1)=V(i)*(X(i+1)-X(i))+M(i)+M(i+1);
V(i+1)=V(i)+V(i+1);
end  
  YIV_load=[yIV_load(1,:); yIV_load(2,:); yIV_load(3,:); yIV_load(4,:); yIV_load(5,:)];
  YIV_loadmm=YIV_load*1000;

  YAloadIV=Y(2);
  YBloadIV=Y(3);
  thetaL=atan(abs(dY(4)));
  thetaR=atan(abs(dY(5)));

  % Piezo Response
  cL=cP(4)-dE(4);
  FxL=f*cos(thetaL);
  FyL=f*sin(thetaL);
  ML=FxL*cL;

  cR=cL;
  FxR=f*cos(thetaR);
  FyR=f*sin(thetaR);
  MR=FxR*cR;

  Ax=FxR-FxL;
  % Reaction Force - X Component (lb)
  Ay=FyL-FyR;
  % Reaction Force - Y Component (lb)
  MA=ML-MR+FyL*xL-FyR*xR;
  % Reaction Moment (lb*in)

  % [A  B  C  L  R  D]
  V=[Ay 0 0 -FyL FyR 0];
  M=[-MA 0 0 ML -MR 0];
  A=zeros(1,5);
  B=zeros(1,5);
  dyIV=zeros(5,N);
  yIV=zeros(5,N);
  dY=[0 0 0 0 0];
  Y=[0 0 0 0 0];

  % Elastic Curves
  for i=1:5
    % Integration Constants
    A(i)=dY(i);
    B(i)=Y(i);
    % yIV(x) and yIV'(x)
    dyIV(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^2/2+M(i)*(x(i,:)-X(i)))+A(i);
    yIV(i,:)=(1/EI(i))*(V(i)*(x(i,:)-X(i)).^3/6+M(i)*(x(i,:)-X(i)).^2/2)+A(i)*(x(i,:)-X(i))+B(i);
    % Slope and Deflection at right end of each region (used for
    % integration constants in next region eqn)
    dY(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^2/2+M(i)*(X(i+1)-X(i)))+A(i);
Y(i+1)=(1/EI(i))*(V(i)*(X(i+1)-X(i)).^3/6+M(i)*(X(i+1)-X(i)).^2/2)+A(i)*(X(i+1)-X(i))+B(i);

% Moment and Shear at right end of each region
M(i+1)=V(i)*(X(i+1)-X(i))+M(i)+M(i+1);
V(i+1)=V(i)+V(i+1);

end

YIV=[yIV(1,:); yIV(2,:); yIV(3,:); yIV(4,:); yIV(5,:)];
YIVmm=YIV*1000;
YIVbothmm=(YIV+YIV_load)*1000;

YApiezoIV=Y(2);
YPpiezoIV=Y(3);

%% PLOTS
set(0,'DefaultFigurePosition',[400 200 2000 1100]);
figure(1);set(gcf,'color','w');
ExternalLoadComparison=plot(XImm,YI_loadmm,'o-',XIIImm,YII_loadmm,'x-',XIIIImm,YIII_loadmm,'+-',XIVmm,YIV_loadmm,'*');
set(ExternalLoadComparison,'LineWidth',2);set(gca,'xlim',[0 300],'ylim',[0 0.1]);set(gca,'FontSize',30,'FontName','Times');
title('External Load Displacement','FontSize',36,'FontName','Times','FontWeight','b');
xlabel('x (mm)', 'FontSize',36,'FontName','Times','FontWeight','b');
ylabel('Displacement (mm)', 'FontSize',36,'FontName','Times','FontWeight','b');
legend('Region I','Region II','Region III','Region IV','Location','NorthWest');

set(0,'DefaultFigurePosition',[400 200 2000 1100]);
figure(2);set(gcf,'color','w');
PreLoadComparison=plot(XImm,YImm,'o-',XIIImm,YIIImm,'x-',XIIIImm,YIIIImm,'+-',XIVmm,YIVmm,'*');
set(PreLoadComparison,'LineWidth',2);set(gca,'xlim',[0 300],'ylim',[0 0.1]);set(gca,'FontSize',30,'FontName','Times');
title('Piezoelectric Induced Displacement - Small Beam Curvature Actuation','FontSize',36,'FontName','Times','FontWeight','b');
xlabel('x (mm)', 'FontSize',36,'FontName','Times','FontWeight','b');
ylabel('Displacement (mm)', 'FontSize',36,'FontName','Times','FontWeight','b');
legend('Region I','Region II','Region III','Region IV','Location','SouthWest');

set(0,'DefaultFigurePosition',[400 200 2000 1100]);
figure(3);set(gcf,'color','w');
PreLoadTotalDisplacementComparison=plot(XImm,YIbothmm,'o-',XIIImm,YIIbothmm,'x-',XIIIImm,YIIIbothmm,'+-',XIVmm,YIVbothmm,'*');
set(PreLoadTotalDisplacementComparison,'LineWidth',2);set(gca,'xlim',[0 300],'ylim',[-0.05 0.05]);set(gca,'FontSize',30,'FontName','Times');
title('Total Displacement','FontSize',36,'FontName','Times','FontWeight','b');
xlabel('x (mm)','FontSize',36,'FontName','Times','FontWeight','b');
ylabel('Displacement (mm)','FontSize',36,'FontName','Times','FontWeight','b');
legend('Region I','Region II','Region III','Region IV','Location','SouthWest')

YAload=1000*[YAloadI YAloadII YAloadIII YAloadIV];
YBload=1000*[YBloadI YBloadII YBloadIII YBloadIV];
Yloadcurve=[YAload YBload]

YApiezo=1000*[YApiezoI YApiezoII YApiezoIII YApiezoIV];
YBpiezo=1000*[YBpiezoI YBpiezoII YBpiezoIII YBpiezoIV];
Ypiezocurve=[YApiezo YBpiezo]

YtotalAcurve=YAload+YApiezo;
YtotalBcurve=YBload+YBpiezo;

Ytotalcurve=[YtotalAcurve YtotalBcurve]
VITA

Ryan Meyer is a Master’s candidate in the Department of Mechanical Engineering at Louisiana State University. He has been a graduate research assistant since beginning the accelerated master’s program under the guidance of Dr. Su-Seng Pang. He received his Bachelor of Science in Mechanical Engineering from Louisiana State University in May 2009. He grew up in Mandeville, Louisiana, and currently resides in Baton Rouge, Louisiana. He is the oldest of five children. He will be marrying his long-time friend and current fiancée, Sarah Lyn Rogers, on November 27, 2010.