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Numerical Modeling of Wetland Hydrodynamics

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NUMERICAL MODELING OF WETLAND HYDRODYNAMICS

A Dissertation
Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in
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by
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Dedicated to my wife, Priyadarshini
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# TABLE OF CONTENTS

ACKNOWLEDGMENTS .................................................................................................................. iii

ABSTRACT ..................................................................................................................................... v

CHAPTER 1: INTRODUCTION ..................................................................................................... 1

CHAPTER 2: LARGE EDDY SIMULATION MODELING OF UNIDIRECTIONAL AND WAVE FLOWS THROUGH VEGETATION .............................................................. 6
  Introduction .............................................................................................................................. 6
  Numerical Methods ................................................................................................................ 9
  Numerical Domain and Boundary Conditions .................................................................. 11
  Results .................................................................................................................................. 15
  Conclusions .......................................................................................................................... 37

CHAPTER 3: LES MODELING OF WAVE OVERTOPPING INDUCED VERTICAL CIRCULATION BETWEEN A BREAKWATER AND A MARSH EDGE ........................................ 41
  Introduction .......................................................................................................................... 41
  Numerical Model Description ............................................................................................. 43
  Model Verification ............................................................................................................... 49
  Numerical Experiment Results ........................................................................................... 53
  Conclusions .......................................................................................................................... 67

CHAPTER 4: MODEL DRIVEN PARAMETRIC DESIGN OF BREAKWATER LAYOUT FOR MARSH SHORELINE PROTECTION ................................................................. 70
  Introduction .......................................................................................................................... 70
  Numerical Model Description ............................................................................................. 72
  Numerical Experiment Results ........................................................................................... 76
  Conclusions .......................................................................................................................... 97

CHAPTER 5: BOUSSINESQ MODELING OF WAVE INDUCED HYDRODYNAMICS IN COASTAL WETLANDS DURING HURRICANE ISAAC .................................................. 99
  Introduction .......................................................................................................................... 99
  Numerical Model Description ............................................................................................. 103
  Model Verification ............................................................................................................... 106
  Numerical Experiment Results ........................................................................................... 113
  Conclusions .......................................................................................................................... 129
  Appendix ................................................................................................................................ 132

CHAPTER 6: CONCLUSIONS .................................................................................................. 133

REFERENCES ......................................................................................................................... 137

APPENDIX: PERMISSIONS ..................................................................................................... 150

VITA ........................................................................................................................................... 152
ABSTRACT

Wave induced erosion accounts for as much as 26% of landloss in coastal Louisiana. This dissertation work, focuses on answering research questions relevant to the design of two shoreline protection methods (a) vegetated wetlands and (b) nearshore breakwaters. Two types of numerical models are used - the three-dimensional (3D) Navier Stokes Equation for small to medium scale experiments and a depth integrated, fully non-linear Boussinesq model for dispersive waves for larger, field scale studies. The former model provides insights into the 3D hydrodynamics of wave interaction with vegetation canopies and breakwaters, while the latter model focuses on the horizontal two dimensional wave induced hydrodynamics. For 3D modeling, flow around vegetation stems, considered as a rigid array of cylinders are explicitly resolved and was used to investigate the influence of stem sheltering and non-linear free surface interaction with emergent stems under wave flow. The model can also simulate the wave breaking on mudflats and transmission over breakwaters and is used to conduct a parametric study of the functional design of a breakwater, placed close to a marsh edge. It was found that the crest elevation with respect to the marsh platform plays a critical role in regulating the wave damage on the platform. A vertical circulation was also found to be existent, driven by over-topped waves, between the marsh edge and the breakwater, which was otherwise absent without the structure. The depth integrated Boussinesq model was used to develop model driven empirical relations for wave breaking induced rip currents, which can flush out sediments through the breakwater gaps. A similar relation was obtained for the wave height immediately behind the gap, which can also cause edge erosion. The results are combined to lay new design guidelines for the design of marsh edge protection structures. Finally, an improved depth integrated vegetation drag formulation, applicable for the fully non-linear Boussinesq equations, using the higher order expansion of the horizontal velocity, is introduced. The new model is validated against laboratory experiments and is extended to simulate the effects of wave induced currents on a natural wetland during peak storm conditions of Hurricane Isaac (2012).
CHAPTER 1
INTRODUCTION

The State of Louisiana, possessing more than 3 million acres of wetlands, a 37% share of the nations total, has suffered more than 1833 sq miles (or 90% of the nations total) coastal land loss between 1932 and 2010 (Couvillion et al., 2011), with recent trends from 2004-2009 (Dahl and Stedman, 2013) showing an almost doubling of the erosion rate compared to the previous period (1988-2004). This continuous loss, apart from affecting the fragile ecosystem which houses a diverse species of flora and fauna, has far reaching commercial impacts. By 2050, the fishing industry is slated to lose a staggering $37 billion (LADNR, 1999) as a direct result of coastal land loss.

Both natural (Day et al., 2007) and anthropogenic (Kesel et al., 1992) factors are responsible for the wetland loss in this region. Powerful hurricanes like Katrina and Rita, regularly erode marshes and bring excess saltwater into the wetlands. Increase in rates of global sea level rise, together with the natural avulsion of the Mississippi River Delta system have only compounded the degradation. Subsidence, a phenomenon which causes wetlands to sink due to geological movement of deposits along fault lines and the compaction of loosely deposited sediments, is another major natural cause of wetland loss. Among anthropogenic factors, the presence of flood-control structures like dams and levees (Kesel et al., 1992; Barras et al., 1994) along the Mississippi River, built since the 1920s, have resulted in a massive reduction of the natural sedimentary load which used to nourish the wetlands and form new land at the river mouth. Fresh water, which previously helped to reduce marsh salinity and provide nutrients was also no longer available and resulted in the breakup and dispersal of large amounts of nutrient-starved wetlands. As subsidence increases, the sea reaches inland and wave erosion occurs at the exposed wetland fringes leading to further degradation. Land loss trends since the 1950s indicate that over 20,000 square miles of land in coastal Louisiana will have been lost to the sea between 1956-2050 if no action is taken.

Erosion of the marsh edge by high frequency, moderate to high energy wind waves is a major cause of wetland loss in coastal Louisiana accounting for almost 26% of the total land loss (Penland et al., 2003). To combat the devastating wetland loss, rock dikes, near-shore breakwaters, artificial oyster reefs, among other structures have been built in south Louisiana (CPRA, 2012). However due to soft soils and fine sediments in Louisianas estuaries and bays, innovative engineering design of shoreline protection systems that are different from those applied to sandy coasts is needed. The use of natural vegetation canopies and
low crested breakwaters with light weight aggregate cores, which are placed close to the shoreline, have found increased application as effective wave barriers.

In this dissertation work we focus on addressing three specific research aspects relevant to the design of these two most common forms of shoreline protection alternatives. The dissertation is composed of six chapters, the first and the last being a general introduction to the problem and the conclusions inferred from this research. Chapter two to five comprise of the body of the work and are each written in the form of a journal paper, discussing a literature review relevant to the topic, details of specific numerical experiments conducted, the results from the experiments and the conclusions inferred from them.

The second chapter addresses the aspect of quantification of wave induced stem drag in vegetation canopies and its relation to wave properties, which have tremendous implication in estimating wave reduction by wetland canopies. The open-source computational fluid dynamics (CFD) model OpenFOAM (Weller and Tabor, 1998) is used to solve the three dimensional (3D) Navier Stokes equations with Large Eddy Simulation (Vreman et al., 1994) turbulence closure, in order to simulate unidirectional and wave flow around vegetation canopies idealized as rigid cylinders. Massively parallel Large Eddy Simulation (LES) experiments were conducted to study the flow fields developed by uni-directional flow over submerged vegetation and wave flow through emergent vegetation. For the submerged vegetation, vertical profiles of mean and turbulent horizontal and vertical velocities were found to be in good agreement with laboratory experiments. Canopy-averaged bulk drag coefficient calculated from the depth-integrated forces on the cylinders compares well with empirical measurements. For the emergent vegetation, wave induced drag forces were calculated and the inertia and pressure-drag coefficients were compared with laboratory experiments which were found to be in good agreement. Vertical variation of forces and moments about the stem base are presented and compared with a single stem case under a variety of wave conditions and Keulegan Carpenter (KC) numbers. It is seen that at low KC numbers effect of the inertia force is significant. The vertical variation of the velocity field near the free surface, within 2 to 3 diameters from the cylinder center was found to be considerably influenced by the free surface flux between the wave trough and the crest, over a time period, which creates a predominant recirculation zone behind the cylinder in the direction of wave propagation.

The third chapter extends the application of the 3D Navier Stokes model in OpenFOAM to field scale and simulates the wave induced hydrodynamics in the vertical plane between a breakwater and a marsh edge. Low-crested breakwater structures are often used to protect
marsh shorelines in sheltered waters. This chapter deals with the wave overtopping and
subsequent evolution of the vertical flow field in between the breakwater and the marsh
edge under winter storm conditions. The goal was to determine the critical crest elevation
and distance from the marsh edge that is likely to cause minimum damage to the marsh
platform and induce sediment deposition under different, realistic wave conditions. A three-
dimensional Navier Stokes based model with Volume of Fluid method for the free surface
and Large Eddy Turbulence closure is used to simulate the flow field using OpenFOAM.
Model results are validated against laboratory experiments for wave breaking and empirically
derived equations for wave transmission. Free-surface envelopes, wave heights and mean
velocity profiles are presented and compared for different wave heights, breakwater crest
height, submergence level and distances from the shoreline, with and without a breakwater,
placed on a realistic bathymetry. In absence of a structure, the wave dynamics on the
mudflat are dominated entirely by the submergence level, with the maximum reflected wave
height at the marsh edge occurring when water levels are between half of the marsh scarp
to the marsh surface. Increasing the water level decreases the reflection effect but may
cause increased wave heights to reach the platform and propagate with non-linear triad
interactions. A breakwater works best when the crest elevation is maintained at or above
the platform height it is protecting. It is observed that when the intervening structure is
placed within a distance, equal to about twice the incident wavelength from the marsh edge
a strong vertical circulation is set up due to the overtopped waves, the strength of which
increases with decreasing distance between the structure and marsh edge. It is hypothesized
that if sufficient dredged sediments are available in between the marsh edge and breakwater,
the vertical circulation has the potential to lift the suspended sediment and deposit it
on the platform due to the onshore flux, thereby re-nourishing the wetland. Placing the
breakwater too close to the shore (≤0.75 times the wavelength) seems to offset any potential
advantage from this circulation. This is the first study employing LES to understand the
hydrodynamics around submerged breakwaters near a marsh edge in field scale.

The fourth chapter documents the model driven parametric study of the functional
design of a continuous breakwater system with a fish dip in between. A well validated, fully
non-linear depth integrated Boussinesq model, for dispersive waves, FUNWAVE-TVD (Shi
et al., 2012) is used for this purpose. Fish-dips are regular openings in continuous breakwater
systems designed to facilitate the exchange of water as well as to allow fish passage. During a
storm when the breakwater is submerged, wave breaking induces ‘rip currents’ through these
fish dips and the resulting off-shore directed current can flush away fine sediments already
suspended by the transmitted waves and long-shore currents. Two hundred and fifty six (256) field scale simulations were run for varying offshore wave characteristics, typical of cold front conditions, distances of the structure from the edge, crest elevations and gap width ratios to develop empirical non-dimensional relationships for both the mean current and the wave height. The resultant data set is used to obtain a non-dimensional empirical equation for use by practicing engineers, relating the mean flushing current through the fish dip with five important design parameters - the distance from marsh edge, relative crest elevation, gap ratio, wave height, time period and water depth. The wave height near the marsh edge, just behind the dip, is also of concern due to the increased wave height behind a fish dip, caused by a combination of wave current interaction and wave diffraction through the dip shoulders. This wave height is empirically related to the same functional design parameters. It was found that wave non-linearity at the fish dip, aspect ratio of the space between the breakwater and the marsh edge, the gap ratio of the fish dip with respect to the continuous expanse of the breakwater and the ratio of the relative crest elevation to the incident wave height had strong correlations with the two design variables. In general, the mean cross shore velocity decreases exponentially for increasing gap ratio up to about 0.3, after which it becomes almost zero, while wave height shows the opposite trend of increasing up to 0.3 and becoming nearly constant after that. Both the mean current and wave height behind the fish dip increases with decreasing ratio of relative crest elevation to the incident wave height on account of stronger breaking on the adjacent breakwater crests. Mean velocity increases with increasing distance from the marsh edge till about 2.5 wavelengths, after which it becomes steady, while wave heights show the reverse trend. The horizontal extent along the marsh edge, that experiences high energy wave action behind the fish dip was also quantified. It was found that combined wave current interaction along the middle of the fish dip and refraction, diffraction from the sides of the dip can cause potential higher incident wave heights to reach the the marsh edge even at distances up to 1.5 times the size of the fish dip on either side of the dip centerline. Additional shore protection measures are recommended if the fish dip gap ratio exceeds 0.05. The results from Chapter 2 and 3 are combined to illustrate most critical design parameters and lay new design guidelines in the design of marsh shoreline protection systems in the conclusions section.

The fifth chapter presents the development and application of a Boussinesq wave model, based on the CACTUS (Goodale et al., 2003) framework, called CaFUNWAVE, which is similar to FUNWAVE-TVD but has a superior treatment of the vegetation drag force term as it uses the higher order expansion of the horizontal velocity and is also consistent with
the fully non-linear Boussinesq governing equations. The model is validated against laboratory experiments on a vegetated sloping beach and then extended to simulate the wave induced circulation on a Louisiana wetland during Hurricane Isaac. The wave attenuation results using the improved formulation were compared with those using the first order or the reference velocity as well as with analytical solutions using linear wave theory. The analytical solution using the depth varying velocity, predicted by the linear wave theory, was shown to match the model results with the depth varying, fully expanded velocity approach very well for all wave cases, except under high $kh$ conditions ($k$ being the wave number and $h$ the water depth) when the wave amplitude was equal or greater than the distance between the still water level and the canopy top. The extension of the model to simulate peak storm waves showed that vegetation is very effective in reducing setup on platforms while also effectively reducing the wave energy within the first few hundred meters. The adaptive meshing capabilities of CaFUNWAVE are exploited to simulate wave impact along highly resolved and irregular marsh edges. The sensitivity of circulation patterns on the mudflat and the platform to wave directions are identified as well as the importance of vegetation in reducing wave breaking induced setup on the platform, during hurricanes revealed. Wave generated circulation patterns showed that for vegetated platforms, the circulation on the mudflat is more profound with coherent large circulation eddies located near the marsh edges. For non-vegetated platform, the wave induced currents, generated by strong overtopping of the waves at the edge are strongest on headland-type marsh lobes that are exposed to wave attack from multiple directions, while sheltered shorelines with a constant wave attack direction, are somewhat better protected.
1. Introduction

Flows through riverine and coastal vegetation have been an area of active research among civil engineers for the past half a century. Submerged riverine vegetation exists largely in unidirectional flow environments and induces vegetation drag thereby reducing the mean flow velocities within the canopy with respect to the free upper layer while emergent vegetation canopies in coastal wetlands are most effective in attenuating the wave height by dissipating the wave energy. In both the cases the forces exerted by the plant stems on the surrounding flow strongly influence the mean and instantaneous flow field, species and sediment transport processes. Experimental work in unidirectional flow over vegetation (Kouwen et al. (1969); Tsujimoto et al. (1991); Dunn et al. (1996); Fairbanks and Diplas (1998)) have successfully established that the mean horizontal velocity profile within a submerged vegetated layer shows significant departure from the universal logarithmic law and the flow over is more akin to a mixing layer flow analogous to what is seen in atmospheric canopies rather than a perturbed boundary layer flow (Ghisalberti and Nepf (2002); Ghisalberti and Nepf (2004); Ghisalberti and Nepf (2006); Nepf and Ghisalberti (2009)). It was also found that apart from the variation in the mean quantities large spatial variation in the vertical profiles of kinetic energy and Reynolds stresses exist within the canopy López and García (2001).

Experimental studies looking into wave attenuation through emergent and submerged artificial as well as live vegetation (Asano et al. (1988); Kortlever (1994); Lowe et al. (2005a); Lowe et al. (2005b); Augustin et al. (2009); Chakrabarti and Smith (2011); Jadhav and Chen (2013); Jadhav et al. (2013)) found that the wave height decay and resulting turbulence is closely related to population, diameter of the stems, degree of submergence, rigidity of the stems as well as on the wave conditions. Recently Hu et al. (2014) shows that the ratio of the imposed current velocity to the amplitude of the orbital velocity determines whether a following current increases or decreases the vegetation drag. They used the Morrison’s

equation to calculate a period averaged and spatially averaged drag coefficient from known force data, the first observation of its kind.

The first analytical model, based on the wave period averaged energy conservation was conceptualized by Dalrymple et al. (1984) in which the local flow velocity was assumed to follow linear wave theory. The drag force was represented by the quadratic law using the wave orbital velocity, while the inertia component was assumed to vanish when averaged over a wave period. Subsequent improvement in modeling efforts (Kobayashi et al. (1993); Mendez and Losada (1999); Mendez and Losada (2002); Maza et al. (2013); Chen and Zhao (2012); Ma et al. (2013); Wu et al. (2013); Zhao and Chen (2014); Marsooli and Wu (2014); Zhu and Chen (2015)) introduce a drag force based sink term in the momentum equation to represent vegetation drag. Although this class of models are relatively easier to implement in the governing equations and are able to give reasonable results on mean flow quantities, they have several limitations. Firstly, since the drag coefficient is unknown in the governing equations, they require calibration against known wave height decay curves from wave flume experiments and the calibrated coefficients thus become a function of the fidelity of the correlation of the modeled and experimental data. Secondly, the momentum loss due to turbulent interaction of the flow between the stems, and bed friction are all lumped into a drag force term, which may be one of the reasons for the large scatter commonly seen between studies of different researchers. Lastly, these models do not give any insights into the nature of eddy shedding from vegetation stems and resultant turbulent flow structure within canopies.

In view of the limitations of the above models, a more direct approach to solving the wave vegetation interaction problem is proposed in form of high resolution Large Eddy Simulation (LES) Sagaut (1998) based turbulence modeling using three dimensional (3D) Navier Stokes equation where the flow structure within the canopy is resolved up to a reasonable order of accuracy verified by first order (mean) and second order (turbulent quantities) from known experiments. The only assumption in our model is that the ‘large eddy’ flow features which develop around the cylinders are the principal energy extractors from the overall flow. This is the first study simulating and investigating free surface non-linear wave flow around an array of rigid vertical cylinders by resolving the flow field in the canopy. The drag force is calculated by integrating the time varying instantaneous pressure field around each cylinder in the vegetation array and no calibration is required.

LES is an intermediate approach between Reynolds Average Navier Stokes (RANS) or Unsteady RANS (URANS) and Direct Numerical Simulation (DNS). RANS based models
(Shimizu and Tsujimoto (1994); Naot et al. (1996); López and García (1997); Fischer et al. (2001); Neary (2003)) used earlier required explicit specification of momentum sinks to account for the drag force. On the other hand DNS is still restricted to small scale domains \(O(10^{-3})\) to \(O(10^{-2})m\) and low Reynolds number environments (Liu et al. (2011); Liu et al. (2014)). The LES method has been applied before to study the uni-directional flow around arrays of emergent and submerged vegetation stems (Cui and Neary (2002); Stoesser et al. (2009); Palau-Salavador et al. (2010)). These LES studies offered a higher degree of detail into the flow and an opportunity to answer hitherto unanswered research questions albeit at the cost of greater computational resources when compared to URANS based models. However LES studies in vegetated flows are still limited to relatively moderate Reynolds numbers \(O(10^2 - 10^3)\) due to the need for very fine grids near the vegetation elements and corresponding very small time steps as well as the sheer size of the problem. Luckily the waves in the estuarine zone where vegetated coastal wetlands exist exhibit orbital velocity Reynolds number in that range. LES studies of this nature can only be run at present in dedicated High Performance Computing (HPC) clusters with large number of nodes, fast networks and large storage. The simulations in this paper were run in a 146 TFlops peak performance cluster with 440 compute nodes, each node having two 8-Core Sandy Bridge Xeon 64-bit processors operating at 2.6GHz with 32 GB of RAM and sharing a 40Gb/s Infiniband network interface between them. Typically 64 to 160 nodes were allocated for each run based upon scalability curves and depending upon the problem size.

In this paper our principal aim is to validate the 3D LES model based on OpenFOAM in application to vegetated flow and apply the model to study the wave interaction with an array of cylinders with an emphasis on understanding how the drag force on the cylinder varies vertically and the influence of the free surface hydrodynamics in modifying the velocities around the cylinder. The first part of the paper deals with the comparison of mean and turbulent velocities for the uni-directional flow over an array of submerged cylinders against the experimental results of Liu et al. (2008). The distribution of depth-integrated drag force in the canopy has been investigated and the bulk drag coefficient calculated from the modeled forces has been compared with the empirically derived relations by Ghisalberti and Nepf (2004), relating the submerged vegetation canopy drag coefficient to that in emergent vegetation. The agreement was found to be within 5\% and further validates the empirical equation. The second part shows the application of the model to study the wave interaction with an array of cylinders as well as a single cylinder. The drag and inertial coefficients induced by the wave forces acting on a cylinder are obtained by fitting Morrison’s equation
to the LES model computed force data and these are then compared with those obtained in the experiments of Hayashi and Chaplin (2012). Insights into the vertical variation of the wave forces, vertical variation of the amplitude of wave velocity as well as the complex vortex shedding mechanism from a single cylinder are presented for a variety of wave heights and time periods. An attempt has been made to explain the complex hydrodynamics at the free surface due to non-linear wave interaction with a cylinder, either isolated or in an array. This work is expected to be the beginning of several future studies looking at wave and vegetation interaction using LES. With the development of HPC infrastructure this approach has the advantage of revealing several important canopy scale phenomena like the role of the spacing between stems, characteristic vortex features in canopy associated with wave conditions, relative importance of the orbital components in the energy reduction as well as unique flow features due to wave-current-vegetation interactions.

2. Numerical Methods

All the experiments in this paper were conducted using OpenFOAM version 2.1.1 Weller and Tabor (1998). For the uni-directional flow experiments over submerged vegetation the three-dimensional incompressible LES filtered continuity and the Navier Stokes equation were used to resolve the flow field. The equations can be expressed as,

\[ \nabla \cdot \mathbf{u} = 0 \]  \hspace{1cm} (1)

\[ \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{uu}) = -\frac{1}{\rho} \nabla p + \nabla \cdot \nu (\nabla \mathbf{u} + \nabla \mathbf{u}^T) + \nabla \cdot \tau_{\text{sgs}} \]  \hspace{1cm} (2)

where vector notations are represented as bold faced characters with \( \mathbf{u} = (u, v, w) \) and \( u, v \) and \( w \) being the magnitude of velocities in the X, Y and Z directions respectively, \( t \) is the time variable, \( \rho \) density of water, \( p \) the pressure, \( \nu \) the kinematic viscosity and \( \tau_{\text{sgs}} \) being the sub-grid scale modeled shear stress. The LES spatial filter operator, written generically is

\[ \overline{\phi}(x) = \frac{1}{V} \int_V \phi(x')G(x, x')dx' \]  \hspace{1cm} (3)

where \( \Omega \) is the fluid domain and \( G \) the filter function that specifies the scales of the resolved eddies. In OpenFOAM, LES filtering is provided implicitly by the finite-volume discretization itself and the filtering operator in Eqs. (1) and (2), represented by the overbar, becomes

\[ \overline{\phi}(x) = \frac{1}{V} \int_V \phi(x')dx' \]  \hspace{1cm} (4)
for all \((x' \in V)\), \(V\) being the volume of a computational cell and the filter function is the simple box-hat filter,

\[
G(x, x') = \begin{cases} 
\frac{1}{V} & \text{if } x' \in V \\
0 & \text{otherwise}
\end{cases} \quad (5)
\]

For the free surface wave flow experiments over emergent vegetation, in addition to solving Eq. (1), the LES filtered combined equation for the air and water flows are solved,

\[
\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p^* - g \mathbf{x} \nabla \rho + \nabla \cdot [\mu \nabla \mathbf{u} + \rho \tau_{sgs}] \quad (6)
\]

In both the Eqs. (2) and (3) the sub-grid scale stress term \(\tau_{sgs}\) was modeled by the dynamic Smagorinsky model Germano et al. (1991) with local averaging of the Smagorinsky constant. The effective viscosity was clipped such that it is never allowed to be less than zero in order to keep back-scatter effects resulting from negative sub-grid scale viscosities physically consistent. This is the same LES model that was also used by Stoesser et al. (2009) and their results have been also presented later as a comparison.

In the Volume of Fluid (VOF) Hirt and Nichols (1981) method since air and water are treated in a single phase calculation throughout the domain the physical properties of the mixture \((\rho \text{ and } \mu \text{ in Eq. (3)})\) are calculated as weighted average of their respective properties with the weighting done by the phase fraction, that is,

\[
\rho = \alpha \rho_w + (1 - \alpha) \rho_a \quad (7)
\]

\[
\mu_{eff} = \alpha \mu_{eff,w} + (1 - \alpha) \mu_{eff,a} \quad (8)
\]

where the subscripts represent water \((w)\) or air phase \((a)\) values and the effective viscosity is \(\mu_{eff} = \mu + \mu_{sgs}\). The phase fraction \(\alpha\) can have values within \([0, 1]\) with \(\alpha = 0\) signifying fully air phase and \(\alpha = 1\) fully water phase with cells having intermediate values being representative of partially air and water filled zones. OpenFOAM uses a modified version of the two-fluid Eulerian model approach typically used in two-phase flows to describe the transport of the phase fraction. In the Multidimensional Universal Limiter for Explicit Solution or MULES Berberović et al. (2009), the transport of the phase fraction can be represented by a single combined evolution equation which is written as,

\[
\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{u}) + \nabla \cdot [\mathbf{u} \alpha (1 - \alpha)] = 0 \quad (9)
\]
where \( \mathbf{u} = \alpha \mathbf{u}_w + (1 - \alpha) \mathbf{u}_a \) and \( \mathbf{u}_r = \mathbf{u}_w - \mathbf{u}_a \) represents the relative velocity vector termed as ‘compression velocity’ which in turn introduces an additional convective term referred as ‘compression term’ to the governing equation. This term can be properly tuned to achieve a very high interface resolution and avoiding the need for special interface treatments used in classical VOF method and makes this VOF technique very effective and unique though requiring significantly small time steps \( O(10^{-3} - 10^{-4}) \)s for practical wave flow problems.

The numerical domain was discretised using the finite volume method with both spatial and temporal discretisations being second order accurate. The Pressure Implicit with Splitting of Operators (PISO) Ferziger and Peric (2001) algorithm with two pressure corrector steps and two non-orthogonal correctors were used for the pressure-velocity solver. For the solution of the linear algebraic equations the Pre-conditioned Conjugate Gradient method (PCG) with Diagonal Incomplete Cholesky (DIC) (for symmetric matrices) and Preconditioned Bi-Conjugate gradient (PBiCG) with Diagonal Incomplete LU (for asymmetric matrices) were used as the preconditioner.

3. Numerical Domain and Boundary Conditions

For the uniform flow over submerged vegetation experiment the domain size was similar to the experiments of Liu et al. (2008) and numerical simulations of Stoesser et al. (2009). Length, width and height of the domain were respectively \( 40D \), \( 20D \) and \( 18D \) where \( D = 6.35mm \) is the diameter of each cylinder. Sixteen cylinders were arranged in a staggered pattern with distance between them being \( 10D \) (Fig. 1(a) and 1(b)). For the results presented in this paper measurements were taken at the six locations marked 1 to 6 as in Fig. 1(b). At the inlet, outlet and sides of the domain the cyclic boundary condition for velocity and pressure were imposed while the top was given a full slip boundary condition treating it as a rigid lid. No slip boundary condition was imposed for the bottom of the domain as well as on the cylinder walls. In order to correctly simulate the near wall turbulence the Spalding’s smooth wall function Spalding (1961) was used to calculate the near wall turbulent eddy viscosity. Flow was driven by specifying the pressure gradient at every time step such that the bulk velocity in the domain \( (U_b) \) was kept constant. The domain had a total of 12,764,590 finite volume cells with the near wall resolution of 0.45mm corresponding to \( y^+ = 46 \). The mesh was refined in the XY plane near the cylinder walls (Fig. 2(a)) while in the Z direction refinement was done to capture the bottom boundary layer using a geometric expansion. The near wall aspect ratio was maintained at \( \Delta x : \Delta y : \Delta z = 1 : 1 : 1 \). The channel, cylinder and canopy Reynolds numbers were respectively \( Re_{channel} = 38,000 \),
Figure 1: Numerical domain and sampling locations.

\(Re_D = 2100\) and \(Re_{canopy} = 1340\).

For the wave flow experiments over emergent vegetation, the waveFOAM toolbox Jacobsen et al. (2012) was used. The domain was 0.25\(m\) wide, 0.75\(m\) high and \((1.8 + 3L)m\) long, where \(L\) was the wavelength of the incident wave, with the central 1.8\(m\) in the flume occupying the vegetation zone as in the original laboratory experiment (Fig. 1(c)). The free lengths (before and after the vegetation zone) are considerably shorter than in the original experiment in order to reduce computational cost. Also since we are mostly concerned with the hydrodynamics inside the vegetation zone it was not important to have such a long domain. Also the width of the domain is 0.25\(m\) versus 0.8\(m\) in the original experiment as sensitivity analysis with larger width did not show any appreciable differences to the force
and velocity measurements at the central cylinder. Fifty three cylinders were placed in a staggered arrangement such that the nearest neighbor distance between them was $10D$ (Fig. 1(d)). Velocity measurements were taken from locations marked by ‘X’ as in Fig. 1(d) and averaged to yield a representative velocity for the evaluation of drag coefficients from force measurements of the central cylinder (53). Surface elevation readings taken from these locations, on the same X co-ordinate as the central cylinder on either side, were also averaged to represent surface elevation reading at the cylinder location. For the isolated cylinder experiments only cylinder 53 was placed in the domain. Vertical profile of three-dimensional velocity data from locations 1 to 4 in Fig. 1(d) have been used to compare maximum wave velocities in the canopy and the effect of the cylinders on the wave flow. At the inlet the surface elevation of waves were set by specifying the phase fraction ($\alpha$) using Stokes’ second order wave theory. In order to damp the waves generated in the numerical flume a numerical sponge layer or relaxation zone is needed. Relaxation zones, for both isolated cylinder and array cases extended up to one wavelength into the domain from the inlet and outlet. For the relaxation zone a function similar to that used by Engsig-Karup (2006) and shown
in Eq. (10) was used where exponent $p$ was chosen as 3.5 based on sensitivity analysis of the reflection coefficients with $w$ and $\sigma$ being the weighting function and the local distance function respectively Jacobsen et al. (2012).

$$w = 1 - \sigma^p$$

(10)

The local Courant number correction method of Seng et al. (2012) was applied as an additional temporal correction within the relaxation zones. The mesh was refined in the XY plane similar to Fig. 2(a) and also in the Z direction near the bottom boundary layer as well as near the fluid cylinder interface (Fig. 2(b)). Minimum cell sizes outside the central 2.0m zone were double the size of the largest cell within the vegetation zone and were expanded near the inlet and outlet boundaries. Local refinement was done at the Still Water Level (SWL) so that the maximum oscillation of the free surface remained within the refined zone. Aspect ratio of cells at the free surface were kept 1:1:1, except within the relaxation zones where the cells were expanded in the horizontal direction.

The sides of the domain had full slip walls while the top was given an atmospheric pressure condition. The bottom of the domain and cylinder walls had the no slip condition along with the Spalding’s wall function Spalding (1961) for the turbulent viscosity term. The wave conditions and mesh resolutions are given in Table 1. Linear wave theory was used to calculate the maximum velocity at free surface which is used to calculate the non-dimensional parameters in the table. The near wall aspect ratio was maintained at $\Delta x : \Delta y : \Delta z = 1 : 1 : 2.5$. Sensitivity analysis of the near wall mesh requirements indicated that a minimum of 64 points along the cylinder circumference was required to accurately simulate the wake structures and account for the pressure fluctuations near each cylinder. A near wall resolution of $D/20 = 0.5mm$ was maintained along the circumference with the minimum cell size in the wall normal direction being $D/50 = 0.2mm$, the cells being stretched away from the wall. For both the uni-directional and wave flow simulations, time stepping was done adaptively such that the maximum Courant number ($max(C_r)$) in the domain does not exceed 0.2. The Courant number ($C_r$) for a given cell with volume $V$ is defined as $C_r = \frac{\sum |U_i A_i| \Delta t}{0.5 V}$, where the velocity and area vector of the $i^{th}$ face are $U_i$ and $A_i$. The factor 0.5 ensures that the mean of the velocities are taken for opposite faces when calculating the sum. The numerical time stepping interval ($\Delta t$) varied between $10^{-6}$ to $10^{-5}$ seconds for the unidirectional flow runs and between $10^{-6}$ to $0.5 \times 10^{-3}$ seconds for the wave flow runs.
Table 1: Free Surface Wave Flow: Experimental Conditions and Problem Size

<table>
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<th>H (cm)</th>
<th>T (s)</th>
<th>KC</th>
<th>Re</th>
<th>β</th>
<th>kH/2</th>
<th>h/L</th>
<th>Ur</th>
<th>Array (# cells)</th>
<th>Isolated (# cells)</th>
<th>Flume (# cells)</th>
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<td>732</td>
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<td>0.066</td>
<td>0.42</td>
<td>0.28</td>
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<td>2,735,400</td>
<td>203,500</td>
</tr>
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<td>0.96</td>
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<td>0.42</td>
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<td>203,500</td>
</tr>
<tr>
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<td>22.8</td>
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<td>80</td>
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<td>0.42</td>
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<td>2,735,400</td>
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<td>0.10</td>
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<td>414,500</td>
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<td>71.5</td>
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<tr>
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<td>15.73</td>
<td>16,579,100</td>
<td>4,915,300</td>
<td>414,500</td>
</tr>
</tbody>
</table>

4. Results

The goal of using 3D LES modeling is to better understand the hydrodynamics within vegetation canopies and quantify the variation of forces acting on the stems. However due to the scarcity of available laboratory experiments in the literature which simultaneously document mean and turbulence quantities as well as drag forces, it was decided to conduct separate validations for uni-directional flow where the mean and turbulence quantities could be verified and free surface wave flows where the force coefficients could be validated.

4.1. Uni-directional Flow

The experimental conditions were chosen similar to the laboratory experiments of Liu et al. (2008) where the bulk velocity \( U_b \) in the channel was specified as 0.33 m/s. An initial ramp up time of 18 flow through periods \( t_f = \frac{l}{U_b} \) were allowed before data collection started. The choice of this period was based on the fact that the mean and turbulence velocity profiles were devoid of any initial effects after 18 flow through cycles. The mean velocity results became independent of the averaging period after 45 flow through cycles beyond the initial ramp time while the mean turbulence quantities became invariant of the averaging period after 50 flow through periods beyond the initial ramp time. The independence of the results on the averaging period was quantified by comparing the difference in the root-mean square deviation (RMSD) with respect to the experimental data for the vertical profiles at the six chosen locations for each incremental averaging period with a step size of 5. When the difference in the RMSD between two profiles obtained from two successive averaging periods became less than 1%, the latest averaging period was chosen as being sufficient for data analysis purposes. Thus the data presented in this paper was obtained by averaging over 55 flow through periods after the initial 18 flow through periods.
Figure 3: Uni-directional flow validation: Comparison of non-dimensionalised mean horizontal velocity ($U$), turbulent horizontal velocity ($U_t$), mean vertical velocity ($W$) and turbulent vertical velocity ($W_t$) for the six locations (V1, V2, V3, V4, V5 and V6) as marked in Fig. 1(b) with the experimental data adapted from Liu et al. (2008) and the high-resolution LES simulation of Stoesser et al. (2009). The black horizontal line signifies the top of the vegetation canopy. $U_b$ and $U_*$ are respectively the bulk and shear velocities in the channel for flow ramping. Temporal data was collected at 333Hz which was sufficient to trap the tail of the Kolmogorov $-5/3$ spectrum.

In Fig. 3 the first and third row of panels from the top show the vertical profile of the variation of mean horizontal ($U$) and vertical ($W$) velocities, non-dimensionalised by the bulk velocity ($U_b$) at each of the six locations shown previously in Fig. 1(b). The second and fourth row of panels show the vertical profiles of the turbulence component.
of the velocities in the two directions non-dimensionalised by the shear stress defined as $U_* = \sqrt{ghS}$, where $h = 0.114m$ was the height of the domain as well as the depth of water and $S = 0.003$ the slope of the channel in the original laboratory experiment. The turbulent velocities were calculated as root mean square of the observed velocities, i.e,

$$U_t = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (U_i - U)} \tag{11a}$$

$$W_t = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (W_i - W)} \tag{11b}$$

where, $U_i$ and $W_i$ are resolved instantaneous horizontal and vertical velocity values.

In general it is observed that the mean horizontal velocities at all the six locations match well with the experimental data and also compare favorably with the numerical results of Stoesser et al. (2009), considering the fact that the latter used a mesh with about twice the resolution as the present simulation. The stream-wise velocities within the canopy layer is considerably lower than that in the fluid layer above the canopy and follows the general trend observed in flow over submerged vegetated canopies. The fluid on the top of the canopy accelerates and produces the classical hyperbolic tangent profile with the inflection point close to the top of the canopy. The fact that all the six positions show the same global hyperbolic type profile suggests the spanwise homogeneity of this phenomenon, with the locations just behind ($V1$) and before ($V2$) the stem showing large gradients of horizontal velocity near the top of the canopy indicating the effect of the recirculation zone, which can be defined as the sheltered region within the wake of a cylinder, in the dominant flow direction, within which the centerline mean streamwise velocities are negative. It is worthwhile to note that the mean horizontal velocity within the recirculation zone is actually better predicted in our model than in Stoesser et al (2010) who have noted that the accuracy of prediction of this region is strongly dependent upon the choice of Sub-grid Scale (SGS) model, LES data averaging period or measurement period in the original experiment. In our initial sensitivity runs we found that the accurate prediction of the mean and turbulent quantities in the recirculation zone is much more sensitive to the near wall mesh than the averaging period or the choice of the SGS model, assuming of course the averaging period is large enough that statistically steady state turbulence profiles have been reached.
within 1-2% error. Hence our model consistently under predicts the gradients of mean and turbulence quantities in the velocity profiles within the recirculation zone and even more so at vertical \( V1 \). An important goal of this research is to use LES simulations, with mesh sizes and temporal resolutions, which can successfully simulate experiments conducted in typical wave flumes found in engineering laboratories, within a reasonable level of accuracy. These experiments may either be in field scales or prototype scales of larger coastal wetland canopies mimicking the height, density and diameter of the rigid portion of the stems. As a secondary condition, the total size of the problem in these simulations should be such that they can be run in conventional High Performance Computing (HPC) clusters with relatively moderate computational effort. Hence the near wall resolution, which yields acceptable results for mean quantities in the majority of the locations is considered satisfactory.

The stream-wise turbulence velocity profiles (second row from the top in Fig. 3), agree very well with the experimental data except at location \( V1 \) due to the same reason as explained above. The peak of the intensities occur at or near the stem tops with the peak lying a little below the tops at the free locations (\( V2 \) and \( V5 \)) suggesting that stream-wise vortices advected from preceding stems are pushed down when they reach the sideways free locations. It is also noted that the LES model slightly under predicts the peaks at those two locations as was observed by Stoesser et al. (2009). The present model predictions are found to be better than those by Stoesser et al. (2009) in trapping the vertical gradients of the turbulent horizontal velocities at the free locations (\( V2 \) and \( V5 \)). Also in the above canopy region the turbulence stream wise velocities are better predicted consistently, particularly near the full slip wall boundary at the top. The difference, in our opinion is due to the full slip boundary condition set at the top versus the symmetry condition as imposed by Stoesser et al. (2009) who instead explained their overprediction of turbulence intensities near the rigid lid to coarse resolution and to large, roughness scale turbulence elements originating at the actual free surface which is treated as a rigid lid for our experiment. Simulations run with symmetry condition in our case in fact produced similar overshoots due to reflection of turbulent vortices back into the domain and it is recommended that a full slip top wall is a much better boundary condition than symmetry for setups of this type.

The vertical profiles for the vertical velocity (third row from top in Fig. 3) agree reasonably well with the experiment except within the recirculation regions (\( V1 \) and \( V6 \)) where the model does not predict the peaks very well. The vertical velocities also seem to be a little overpredicted both within and above the canopy. The LES model captures the peaks of the profile very well for the sheltered zones just outside the recirculation region (\( V3 \) and \( V8 \)).
while it seems to miss those in the free zones (V2 and V5) and is attributed to lower resolution at these locations.

Velocity profiles for the turbulent component of the vertical velocity (fourth row from top Fig. 3) are found to agree well consistently for all the locations except for the recirculation zones (V1 and V6). The underprediction of the turbulent quantities within the recirculation zones in our opinion is due to the lower grid resolution than that used by Stoesser et al. (2009). The larger cell sizes within this zone is unable to trap the eddies with the peak turbulent energy and thus cause and underprediction of the turbulent velocities. The peak of the profiles occur just below the canopy top for the free zones (V2, V5) and at the canopy top for the sheltered zones (V1, V3, V4 and V6) and compares well with the results of Stoesser et al. (2009).

In addition to sampling velocity profiles, the depth-integrated forces on each cylinder was computed as a sum of pressure ($F_p$) and viscous ($F_v$) forces as $F_{tot} = F_p + F_v$, where,

$$F_p = \int_{-h}^{h} \int_{0}^{2\pi} p\cos(\phi)rd\phi dz,$$  \hspace{1cm} (12a) \\

$$F_v = \int_{-h}^{h} \int_{0}^{2\pi} \tau_{eff}\sin(\phi)rd\phi dz$$  \hspace{1cm} (12b)

while $\tau_{eff} = (\tau + \tau_{sgs})dU/dn|_{cyl-wall}$ being the effective shear stress on the cylinder wall with the gradient calculated with the wall parallel velocity. Here $r$ and $\phi$ are the radius of the cylinder and the angle made by the infinitesimally small element located along the cylinder surface on which the forces act, with the axis of the cylinder in the XY plane. The force data were sampled at 50Hz and averaged over 55 flow through periods to yield a representative steady state force. The time-averaged and root mean square (RMS) forces for each of the cylinders is shown in Table 2. The mean forces showed little deviation from cylinder to cylinder with the percentage coefficient of variation ($\frac{\sigma}{\mu} \times 100$), where $\sigma$ and $\mu$ are respectively the standard deviation and mean of the forces on all cylinders, being about 0.6%, while the same for the RMS values was about 10%.

The canopy-averaged mean force, $\overline{F}_{tot} = \frac{1}{N} \sum_{i=1}^{N} (\overline{F}_{pi} + \overline{F}_{vi})$ was used along with the mean velocity of the cells immediately at the top of the canopy ($\overline{U_{hv}}$) to compute the canopy representative drag coefficient Tanino et al. (2005) given by $C_d = \frac{F_{tot}}{0.5p\overline{U_{hv}}^2h_vD}$, where $h_v = 12D$. The calculated value for $\overline{F}_{tot}$, $\overline{U_{hv}}$ and $C_d$ from the model results were
Table 2: Uni-directional Flow: Force Distribution among Cylinders in the Channel

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<tr>
<th>Cyl #</th>
<th>$F_{pi}$</th>
<th>$F_{vi}$</th>
<th>$F_{pi, rms}$</th>
<th>$F_{vi, rms}$</th>
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</thead>
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<tr>
<td></td>
<td>x10⁻³N</td>
<td>x10⁻³N</td>
<td>x10⁻³N</td>
<td>x10⁻³N</td>
</tr>
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</tr>
<tr>
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</tr>
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</table>

16.77x10⁻³ N, 43.2 cm/s and 0.371 respectively. This value of $C_d$ has been compared with the theoretically derived coefficient from Ghisalberti and Nepf (2004). The vertical variation of the ratio $\zeta = \frac{C_d(z)}{C_{dA}}$ is given in Ghisalberti and Nepf (2004) as

$$\zeta = \begin{cases} 
1.4(\frac{\zeta}{\pi})^{2.5} + 0.45, & \text{if } 0 \leq x \leq 0.76 \\
-4.8(\frac{\zeta}{\pi}) + 4.8, & \text{if } 0.76 < x \leq 1 
\end{cases}$$

(13)

where $C_{dA} = \frac{(1+10Re_{h,v,D}^{-2/3})}{11.6}$ \left(1.16 - 9.31(ad) + 38.6(ad)^2 - 59.8(ad)^3\right) = 0.609 is the empirical expression for bulk drag coefficient of a random, emergent array of cylinders as obtained by Nepf (1999) and $Re_{h,v,D} = \frac{U_{h,v}D}{\nu} = 2743$ is the Reynolds number using the velocity at the canopy top ($U_{h,v}$). The value of $ad$ for the present case is 0.071.

In order to derive the value of the depth averaged drag coefficient, the expression in Eq. (8) is integrated as follows,
\[ \zeta = \frac{1}{0.76} \int_0^{0.76} \left[ 1.4 \left(\frac{z}{h}\right)^{2.5} + 0.45 \right] d\left(\frac{z}{h}\right) + \frac{1}{1 - 0.76} \int_{0.76}^{1} \left[ -4.8 \left(\frac{z}{h}\right) + 4.8 \right] d\left(\frac{z}{h}\right) = 0.633 \]  

(14)

which gives a depth averaged drag coefficient of \( C_{d,Nepf} = 0.386 \). This was found to be in close agreement with the model predicted value of \( C_d = 0.371 \), with a percentage difference of 3.9%. This analysis in addition to providing an estimate of range of bulk drag forces also validates numerically the method of Ghisalberti and Nepf (2004) for drag force estimation in submerged canopy flows and lends confidence to the present model which can be used to supplement drag coefficient laboratory data-sets of both submerged and emergent vegetation canopies in uni-directional flow in the future.

4.2. Free surface Wave Flow

Six different wave conditions were simulated as shown in Table 1, with each case being run for an array of cylinders, an isolated cylinder and a flume with no cylinders. The linear dimensions of the flume was the same (0.25m wide, 0.75m high and 1.8+3L m long, where \( L \) is the wavelength of the particular case) for a given case for the three setups, with the same refinement regions (near the free surface and the bottom boundary) as described before. Table 2 also shows the problem sizes in terms of the number of cells for each experiment. The degree of wave non-linearity is represented by the Stokes parameter \( (\epsilon = kH/2) \) and Ursell number \( (Ur = H/h \left( \frac{h}{L} \right)^2 = \frac{HL^2}{h^3}) \), where \( H \) is the wave height at the wave-maker, \( h \) uniform water depth, and \( L \) the wavelength calculated by the dispersion relationship. For a given \( h/L \) the degree of non-linearity increases with increase in \( Ur \), while \( \epsilon \) gives an absolute estimate of the degree of non-linearity among different \( h/L \) cases, with higher values indicating greater non-linearity. Velocity data was sampled at the four locations marked in Fig. 1(d) for both the array and isolated cylinder experiments. In addition to the isolated cylinder runs, velocity data were also sampled in the central vertical plane stretching from -25D to 25D with the cylinder placed at the origin. This was done to quantify and compare the relative influence of the presence of the cylinder in modifying the wave field in either direction and compare with the array cylinder velocity fields. Pressure (\( p \)) and velocities were collected for all the cells around the cylinders. Waves were started with a still water condition in the flume and it took approximately 18 wave periods from the start of the wave-maker to eliminate the effect of the initial conditions. The data were sampled at a frequency of 50/T Hz (\( T \) being the wave period of the wave conditions simulated) over a period of 35 complete wave periods after the initial spin-up time of 20 wave periods. The
choice of the averaging period was led by the fact that about 28 – 32 wave cycles were required for averaging before the phase-averaged force coefficients for the central cylinder showed less than 1% variation for subsequent increase in number of averaging cycles. Thus a total of 55 waves were introduced into the domain and averaging was done over the last 35 wave periods. Since the goal in this paper is to understand the hydrodynamics and resultant forces associated with the mean wave energy, the data were analyzed and frequencies larger than the second order peak of the surface elevation spectrum were filtered out using a fast Fourier Transform algorithm in Matlab.

In order to calculate the drag \( C_d \) and inertia \( C_m \) coefficients, the Morrison’s equation Sumer and Fredsøe (1997) as shown below was fitted to the sampled numerical phase-averaged total force \( F_{\text{tot}} \), calculated in the same way as before in Eq. (12), over one time period, using a non-linear least square fitting technique Isaacson et al. (1991).

\[
F_{\text{tot}}(t) = F_{\text{drag}}(t) + F_{\text{inertia}}(t) = \frac{1}{2} C_d \rho l D U(t) |U(t)| + \rho C_m \pi D^2 \frac{\partial U(t)}{\partial t}
\]  

(15)

where \( U(t) \) is the phase-averaged velocity measured at the free surface, averaged at the sampling locations in Fig. 1. The least-square method minimizes the mean square error \( \epsilon^2 \) between the measured and predicted forces to determine \( C_d \) and \( C_m \) and is expressed as,

\[
\epsilon^2 = \frac{1}{N} \sum_{i=1}^{N} (F_{n_i} - F_{p_i})^2
\]  

(16)

where subscripts \( n \) and \( p \) respectively represent the sampled (numerical) and predicted (using Morrison’s equation) values of the total force as in Eq. (15), and \( N \) is the total number of discrete data points. Using \( \frac{\partial \epsilon^2}{\partial C_d} = 0 \) and \( \frac{\partial \epsilon^2}{\partial C_m} = 0 \), the following two equations with two unknowns \( C_d \) and \( C_m \) are obtained, which were then solved using Matlab,

\[
f_d \sum U^4(t) + f_m \sum U(t)|U(t)|\dot{U}(t) = \sum U(t)|U(t)|F_{n}(t)
\]  

(17)

\[
f_d \sum U(t)|U(t)|\dot{U}(t) + f_m \sum \dot{U}^2(t) = \sum \dot{U}(t).F_{n}(t)
\]  

(18)

where, \( \dot{U}(t) \) is the first derivative of the phase-averaged velocity at the free surface and
\[ f_d = \frac{1}{2} \rho C_d D \quad \text{and} \quad f_m = \frac{\pi}{4} \rho C_m D^2. \] The summation limits have been omitted for ease of reading and span from 1 to N, where N is the total number of discrete intervals within a time period. \( N = 300 \) was selected for all runs.

Figure 4: Validation of force coefficients from wave flow experiments: Comparison of drag \( (C_d) \) and inertia \( (C_m) \) predicted by the model for the central cylinder in the array and for the isolated cylinder tests with the corresponding experimental values adapted from Hayashi and Chaplin (2012) at two different \( \beta \) values and for a range of KC numbers.

Figure 4 shows the comparison of the variation of simulated drag and inertia coefficients with Keulegan Carpenter number \( (KC = U_{max}T/D, \text{where } U_{max} \text{ is the maximum orbital velocity from linear wave theory at the SWL}) \) for the array and isolated cylinder cases with the experimental values along with the \( \beta = Re/KC \) numbers for each case. The agreement with the experimental data is good and shows that the model is able to simulate the fitted force coefficients accurately. Both \( C_d \) and \( C_m \) decreases with \( KC \) number for the range of values tested for the \( \beta = 80 \) case while for the smaller \( \beta = 30 \) we find the coefficients remain almost constant with \( KC \) showing only a reduction at \( KC = 143 \). The isolated cylinder shows a marginally lower force coefficients than the corresponding array cylinder while there is little overall difference between the array and isolated cylinder results in the simulations and supports the observations of Hayashi and Chaplin (2012) who concluded that the spacing between the arrays \( (10D) \) was likely large for any significant vortex interactions. This is an interesting departure from the uni-directional flow results where we found that at similar spacings vortex interactions are quite active and influence the shedding dynamics of the cylinders.
Fig. 5(a) shows the time histories (about 4 wave cycles) of forces acting on the cylinders located along the center line (see Fig. 5(b) for exact cylinder locations) in the direction of wave propagation for cases 1, 2, 5, and 6. Results for case 3 are similar in trend to case 1 and 2 while those of 4 are similar in trend to 5 and 6 and are not shown here. While the RMS force magnitudes show little overall variation down the flume, the higher $KC$ number cases show a greater variation, the cylinders in the front of the flume (3 and 23) experiencing higher forces than those towards the back (83 and 103). There is a marginal increase in
the force experienced by the central cylinder (53) and may be due to a weak reflection from the back of the channel causing a partial standing wave anti-node at this point. The isolated cylinder consistently experiences higher forces than the array cylinder at the same location. The column of panels on the right shows the force histories averaged over a single phase and time lagged over a single time window and clearly shows the variability in the force histories as well as the effect of the non-linearity of the wave condition particularly at higher $KC$ numbers where a clear inflection point is visible below the SWL line. The crest is more peaked than the trough at large $KC$ numbers with the force direction showing an asymmetry with the wave phase.

Figure 6: Comparison of forces and moments at low and high $KC$ numbers. Moments are measured about the cylinder base. The dark dotted line signifies the end of each wave period.

The top row of panels in Fig. 6 shows the time series of pressure ($F_p$), viscous horizontal ($F_{v,x}$), viscous vertical ($F_{v,z}$) and the total force ($F_{tot} = F_p + F_{v,x}$) over about 4 wave cycles for case 1 and case 6. The bottom row of panels shows the time series of the corresponding moments about the stem base. The surface elevation ($\eta$) is also plotted on a second y axis on the right. The viscous forces are almost in phase with the surface elevation and consequently leads the pressure force by almost 90° at the lower $KC$ number. This phase difference is however not so pronounced at the higher $KC$ number. In terms of force magnitudes we find that at low $KC$ numbers the viscous horizontal force contributes significantly to the total force while at higher $KC$ the contribution is negligible. The viscous vertical force is also higher than the viscous horizontal force at $KC = 9.1$. The viscous forces appear to be positive mostly with sharp peaks and shallow troughs. The phase difference between the pressure force and the surface elevation is near 90° at $KC = 9.1$ suggesting the contribution
of the inertia effect while the forces are almost in phase with the surface elevation at the higher $KC$. The addition of the viscous force to the pressure force decreases the phase difference somewhat between the total force and the surface elevation. The moments follow a similar trend to the forces except a change in scale as expected.

![Figure 7: Effect of time lagging fitted force curves at low and high $KC$ numbers](image)

The upper row of panels in Fig. 7 show the fitted drag and inertia forces according to the Morrison’s equation at two representative $KC$ numbers. The drag force and inertia force are $90^\circ$ out of phase with each other due to the nature of the Morrison’s equation with the inertia force leading the drag force. It is found that at lower $KC = 9.1$, the inertia force is significant in magnitude and is even greater than the drag force, while at the higher $KC$ the drag force is much larger. This causes a temporal spreading of the total force for the lower $KC$ number and results in a phase difference with the surface elevation. At the higher $KC$ number where the drag force is dominating and is almost in phase with the surface elevation the total force follows a similar trend. The lower row of panels in Fig. 7 shows the time lagged forces. The time lagging was done by calculating the correlation between the surface elevation time series and the forces time series and finding out the the lead or lag and shifting the forces time series accordingly. It is seen that the effect of time lagging causes the predicted total forces to be almost double that of the measured total forces for the lower $KC$ case while the total force increases only slightly for the higher $KC$ case. In most large scale models it is a convenient practice to only consider time and depth averaged
drag forces based on some apriori assumed drag coefficient value and to neglect the effect of the inertia forces for calculating the total forces acting on model plants, on account of inertia forces being conservative (for linear wave assumption only). Based on the findings here it appears that while it may be fine to neglect the inertia effect at high $KC$ numbers on account of them being too small, the same cannot be said at low $KC$ numbers where an over-estimation of total forces might result by assuming drag contribution only.

In Fig. 8, the vertical variation of forces (Fig. 8(a)) and vertical variation of corresponding moments (Fig. 8(b)) acting on the central cylinder (53) in Fig. 5(b) for four representative cases (Case 1, 2, 5 and 6) are shown. The crest or high water level (HWL), still water level (SWL) and trough or low water level (LWL) are marked by black dashed lines as well as the central zero line passing through the zero mark on the X axis. For the force calculations, equations similar to Eqs. (12) were used except the depth-integrating operation. Calculation of forces around the air water interface can be difficult in VOF models due to absence of a well defined interface, thus in order to only consider fluid filled regions, pressure ($p^*$) and effective viscosity ($\tau_{eff}$) values from cells having void fraction equal to...
or greater than 0.5 were used in the equations. Maximum and minimum forces and corresponding moments about the base of the cylinder on the flume bed, signify the maximum positive and maximum negative forces acting on the cylinder over the entire wave record, while the RMS and mean quantities have their usual meaning. For all the cases the peaks of the maximum (positive) force and moment occur just below the HWL but above the SWL. A similar phenomenon was observed by Torum (1989) who explained it by the analogy of unidirectional flow with free surface where the pile up and increase of water level on the upstream side and the additional draw-down on the downstream face causes the pressure difference. As will be explained later in this section a similar gradient in the mean velocity (and hence pressure) exists between the SWL and the HWL, on either side of the cylinder in the wave direction, due to the net positive velocity associated with the shoreward motion of the wave crest. The rising limb of the wave causes a recirculation in the horizontal plane, which spans vertically throughout between the wave crest and trough while the falling limb of the wave has a recirculation primarily only below the SWL. This is also confirmed by the evidence that the minimum force (maximum of negative forces) almost vanish between the SWL and HWL suggesting that the recirculation region on the rising limb of the wave is stronger than the one developed during the falling limb. The peak of the RMS and minimum (maximum negative) forces and moments value occur just below the LWL. The mean values show a slight negative bias below the LWL but are entirely positive between the LWL and HWL with a peak lying at almost the same level as the maximum force. A local peak in the RMS value at that level is also noticed. A clear distinction can be seen between the higher $K_C$ and lower $K_C$ number cases. The lower $K_C$ number cases show a pronounced hyperbolic profile while for the higher $K_C$ numbers the profiles are flatter in nature. Also interesting is the distinct boundary layer effect on the force profiles. The force profiles show a decrease within the wave bottom boundary layer and this decrease is seen within a distance of $5 - 6cm$ from the bed for the lower $K_C$ numbers to $2 - 3cm$ for the higher $K_C$ numbers.

In order to analyze the hydrodynamics in the water column and investigate the spatial influence of the cylinder in the wave flow in the flume, the vertical and horizontal velocities at several locations in the central plane were compared for the array and isolated cases. Fig. 9 and Fig. 10 shows the profiles of horizontal ($U_{rms}$) and vertical ($W_{rms}$) RMS of the wave velocities at 10 locations (5 on each side of the cylinder, where distances are made non-dimensionalised by the cylinder diameter) including the 4 locations as marked in Fig. 1(d) for the array case for wave cases 5 ($kH/2 = 0.027$) and 6 ($kH/2 = 0.054$), where for
Figure 9: Variation of root mean square horizontal wave orbital velocities ($U_{RMS}$), along the flume center-line, at various distances from the cylinder center. The Stokes’ second order profile uses the free surface elevation at the flume center (no cylinder case).

The temporal RMS operation considers velocity values from points below the water surface only. The results from the other cases show similar conclusions and are not shown here. The dotted dark black line shows the theoretical velocity profile at the center of the flume from the wave flume only case (no cylinder) calculated using the inviscid Stokes’ second order theory Svendsen (2006). In order to calculate the Stokes’ theoretical profiles the observed water surface elevation time series for each wave case was
fitted for some unknown $x$ and $H$ against the Stokes second order equation for surface elevation as,

$$\eta_{\text{stokes}}(t) = \frac{H}{2} \cos(kx - \omega t) + \frac{1}{16}kH^2(3coth^3kh - cotkh)cos(2\phi)$$

(19)

where $\phi = kx - \omega t$ is the wave phase. The best fit $x$ and $H$ were substituted in the equations.
for horizontal ($u_{stokes}$) and vertical ($w_{stokes}$) orbital velocities as,

\begin{align}
    u_{stokes}(t) &= \frac{\omega H \cosh k(z + h)}{2 \sinh kh} \cos(kx - \omega t) + \frac{3}{16} \frac{c(kH)^2 \cosh 2k(z + h)}{\sinh kh} \cos(2\phi) \\
    w_{stokes}(t) &= \frac{\omega H \sinh k(z + h)}{2 \sinh kh} \sin(kx - \omega t) + \frac{3}{16} \frac{c(kH)^2 \sinh 2k(z + h)}{\sinh kh} \sin(2\phi)
\end{align} 

(20a)
(20b)

It was found that the horizontal and vertical velocity profiles at far field locations $|X| = 10D$ $|X| = 8.67D$ for the isolated cylinder case almost matches the numerical results for the no cylinder case, meaning that there is very little variation of the global wave flow field at these distances on either side of the cylinder. The array cylinder results (shown by green line) show that the velocities are slightly less than the isolated and no cylinder cases due to the small damping effect of the cylinders in the front half of the canopy. Within $|X| \leq 2D$ we see significant departure of the wave orbital horizontal RMS velocities (Fig. 9). Below the wave trough or low water level (LWL) the recirculation and turbulence action on either sides of the cylinder decreases the wave excursions significantly compared to the no-cylinder case. The higher wave height case (Case 6) shows larger deviation due to larger turbulence loss and thus the greater wave orbital energy damping by the stem. Velocity profiles for both array and isolated cylinders in the region between the wave crest and the wave trough differ significantly on either side of the cylinder within the recirculation zone ($|X| \leq 2D$).

This apparent anomaly is explained further in Fig. 11 which shows the mean horizontal velocities at the same locations as in Fig. 9 for the two cases, with the temporal mean velocity calculated between the trough and the crest considering points below the water surface only. Below the wave trough the mean flow is almost zero for the far field locations ($|X| = 10D$ and $|X| = 8.67D$) while within $X| \leq 2D$, below the wave trough, the profiles show a net positive current and behind the cylinder a net negative current due to the recirculation region existing on either sides of the cylinder. Between the wave trough and the crest, the far-field profiles show a net positive current due to the forward motion of the wave crest being entirely positive as is expected for progressive waves. Near the cylinder, within the recirculation zone, we observe a decrease of this positive current behind the cylinder in the wave direction due to the effect of the surface vortex (and hence negative velocities) created during the forward thrust of the crest behind the cylinder in the wave direction. This also explains the existence of a free surface velocity gradient on either side of the cylinder between the trough and the crest as was mentioned before. The RMS horizontal velocity profiles (Fig. 9) likewise show a reduction behind the cylinder between
the trough and the crest compared to the points in front of the cylinder in the wave direction. The reduction in the free surface horizontal wave velocity seems to increase with wave non-linearity (Case 5 vs Case 6) as greater the non-linearity greater is the Stokes drift associated with the wave orbital excursion within the trough and the crest. In other words, since the time averaged horizontal flow between the trough and the crest behaves as an uniform flow, with a vertical variation, it creates a mean damping effect similar to that observed in uniform flow around a cylinder while the flow below the trough, having predominantly wave orbital signatures is damped similar to wave damping in submerged vegetation. This separation of the hydrodynamic regime for an emergent vegetation problem has potential important influence on simplified quadratic law based formulations commonly used in large scale models and can be a topic of future research.

Vertical wave velocity RMS profiles (Fig. 10) show similar trends at far field locations ($X = 10D$ and $X = 8.67D$) for array, isolated and no cylinder cases. Within the recirculation zone ($X \leq 2D$) the profiles are similar except near the free surface. In front of the cylinder, the obstruction by the cylinder causes run up on the sea-ward face during the forward motion of the wave crest followed by a localized re-circulation vortex created below the SWL during the flow reversal phase and impinging into the SWL. This causes a local increase in vertical velocity excursions. Behind the cylinder, since there is no impinging below the SWL of the surface vortex, as the recirculation exists only when the crest is in shoreward motion causing lesser vertical velocity excursion amplitudes compared to that on the front face.

The above free surface phenomenon is visualized in Fig. 12(a), where the left panel is for wave case 5 and right panel for wave case 6. The figure shows from top to bottom respectively the mean horizontal, mean vertical, RMS horizontal and RMS vertical velocities over 20 wave periods, presented with the colorbar on the right. The crest, mean and trough locations are also indicated in the same figure. As explained in the line diagrams (Figs. 9 - 11), we observe a net positive horizontal velocity flux above the SWL (top panels) which increases with increasing non-linearity. Fig. 12(b) (bottom two rows) show conceptually the the wave height integrated horizontal recirculation zones around the cylinder below and above the SWL during the rising limb (shoreward motion of wave crest) and falling limb (seaward motion of wave crest) of the free surface (locations shown relative to the cylinder in the top row). Thus over a complete cycle, the excess flux above the SWL causes an uniform-flow type phenomenon with a pronounced recirculation zone behind the cylinder in the direction of wave propagation, particularly between the crest and the SWL, which makes
the depth-integrated recirculation zone between the trough and the crest, stronger in the shore-ward direction of the cylinder. The shear effect of the flux extends downward below the SWL too. The mean vertical velocities in Fig. 12(a) show an asymmetrical signature with a net negative zone between the crest and the trough behind the cylinder and one between the trough and the SWL in front of it. This is due to the impinging action of the surface vortex into the SWL during the flow reversal in the wave crest. The RMS horizontal velocities (\( \bar{U} \)) along the flume centerline, at various distances from the cylinder center. The array case results are almost similar to the corresponding isolated cases and is omitted here.
Vertical distribution of mean ($U$ and $W$) and root mean square ($U_{RMS}$ and $W_{RMS}$) velocities between the trough ($\eta_{\text{min}}$) and the crest ($\eta_{\text{max}}$) for Case 5 and Case 6 near the emergent cylinder (isolated). Conceptual sketch of horizontal recirculation zones below and above SWL at different wave phases ($L_R$ is the length of the recirculation zone).

Figure 12: Hydrodynamics between the crest and the trough.

Velocity plots indicate bulk of the free surface energy is damped between the SWL and wave crest, while below the trough the damping occurs on both sides with similar magnitudes just like wave damping around submerged bluff bodies.

Vortex growth, shedding and subsequent force variations near the free surface from an emergent surface piercing cylinder has been a topic of active research (Kjeldsen et al. (1986); Torum (1989); Quetey et al. (2004); Barlas (2012)). However detailed visualization of the vortex shedding near the surface for an emergent cylinder has not been reported before in the literature. In Fig. 13 we present the three-dimensional vortex visualizations from the isolated cylinder run corresponding to Case 5 as in Table 2 as a representative of the free surface damping phenomenon explained before. Vortex cores are represented by the Q-criterion Hunt et al. (1988) and iso-surfaces are drawn at constant $Q = 10$ with
Figure 13: Vortex shedding from an isolated cylinder (Wave Case 5) at various stages of a wave period: Vortex iso-surfaces are drawn according to the Q criterion and colored by vorticity ($\omega$) about the z-axis.

coloring done by vorticity (Red-Green-Blue scale) about the z-axis with red representing a clockwise rotating vortex (positive) when viewed in the z-direction from the origin and blue a counter-clockwise (negative) vortex. The Q-criterion is mathematically represented as,

$$Q = \frac{1}{2} (||\Omega||^2 - ||S||^2)$$  \hspace{1cm} (21)

where $||\Omega|| = tr[\Omega \Omega^T]^{1/2}$ and $||S|| = tr[SS^T]^{1/2}$ are respectively the asymmetric and symmetric part of $\nabla \upsilon$ defined as,
\[ S = \frac{1}{2}(\nabla \mathbf{u} + (\nabla \mathbf{u})^t), \quad (22a) \]

\[ \Omega = \frac{1}{2}(\nabla \mathbf{u} - (\nabla \mathbf{u})^t) \quad (22b) \]

A vortex region is said to exist in a flow field if \( Q > 0 \) in the cells comprising that region and may be thought as local regions having greater rotation rate than strain rate. Fig. 13(a) to 13(h) shows the instantaneous vortex iso-surfaces generated near the free surface from the cylinder at eight equal intervals of time along a representative wave period \( (T) \). The figures have been magnified to show the region near the vicinity of the free surface with black lines marking the crest elevation or High Water Level (HWL), the mean water level or Still Water Level (SWL) and trough elevation or Low Water Level (LWL). At \( t = T/8 \) (Fig. 13(a)) the free surface is accelerating and descending downwards as shown by the arrow direction. Surface vortex \( A \), having its axis almost parallel to the SWL and forming a circular arc around the cylinder is the remnant of the vortex shed in the preceding cycle while two counter-rotating vortex pairs (B-B’) also shed in the preceding cycle is seen. A violent free surface also exists near the cylinder-water interface as well as strong isolated vortex streaks formed just below the free surface. At \( t = T/4 \) (Fig. 13(b)) as the downward acceleration of the free surface has reached a peak, we find the vortex \( A \) has formed almost a helical shape, is impinging below the free surface and appears to be dissipating. This vortex aligns itself mostly between the SWL and somewhat below the LWL and is responsible for the increase in vertical velocity excursions in this zone. The counter-rotating pair has weakened and is ready to be flipped over the cylinder. The rotation within the boundary layer also has changed direction with a weak vorticity structure developing. At \( t = 3T/8 \) (Fig. 13(c)), as the downward motion of the free surface decelerates the vortex \( A \) is seen to be dissipating further and moving downwards in a helical pattern. B-B’ vortices have flipped sides at this point and has been also been pushed down with the top part almost dissipating under the strong vertical velocities. At \( t = T/2 \) (Fig. 13(d)), as the wave is about to change direction, the vortex \( A \) has almost dissipated with only weak helical signatures surrounding the cylinder. B-B’ has completely dissipated from the window at this stage. At \( t = 5T/8 \) (Fig. 13(e)) as the free surface accelerates upwards, we find the counter-rotating vortex pair (C-C’) that was developing within the boundary layer since \( t = T/4 \) has now
shed completely. It is worth noticing that vortex is almost gone except for a few streaks near the free surface. The rise of the free surface is actually causing a stretching of the counter-rotating pair and they reach very near to the free surface. No prominent surface vortices are visible unlike what we saw at $t = T/8$. At $t = 3T/2$ (Fig. 13(f)), the free surface has reached the highest peak of upward acceleration and causes significant stretching and thinning of the C-C’ pair. Surface Vortex A has started shedding and is present only as few streaks. The recirculation of this zone associated with the rising limb is what is giving rise to the negative velocity zone behind the cylinder in the wave direction and will cause dampening of the surface energy in the subsequent two phase instants. C-C’ is about to flip over the cylinder and we can see new counter-rotating vortex signatures developing within the boundary layer which will eventually become the B-B’ pair in the subsequent cycle. At $t = 7T/8$ (Fig. 13(g)), the free surface rise has decelerated and we observe the detachment of the C-C’ vortex pair and formation of a new A vortex. It is seen that stronger vortices are formed under the free surface on the lee-side of the cylinder during the rising limb of the wave than during the falling limb. This causes stronger recirculation region in the wave direction and consequently higher positive forces on the cylinder under the SWL during the rising cycle of the wave than during the falling limb and explains the asymmetrical surface velocities observed before on either side of the cylinder. Finally at $t = T$ (Fig. 13(h)), we observe C-C’ is completely dissipated while vortex A aligns itself in a circular arc around the cylinder near the free surface.

5. Conclusions

In this paper the uni-directional flow over submerged vegetation and free surface wave flow over emergent vegetation were simulated using the dynamic Smagorinsky LES turbulence closure scheme in order to validate and demonstrate the capabilities of the model in application to vegetated flows. The vegetation was idealized as an array of rigid cylinders. The main conclusions from this study can be summarized as:

1) Low-resolution LES, with adaptive mesh refining around the cylindrical stems is able to produce fairly accurate flow descriptions of the first order and second order velocity quantities, comparable to results of other high-resolution studies. The present simulation results matched the mean and turbulent velocities within and above the canopy from experiments of Liu et al. (2008) with reasonable accuracy. Further, bulk drag coefficient calculated from the depth-integrated, direct force measurements around the cylinders, match the value predicted by the empirical equations of Ghisalberti and
Nepf (2004) and Nepf (1999), indicating that optimal-resolution LES simulations yielding acceptable velocity distribution within the canopy can be used to evaluate drag coefficients within a margin of error. In terms of computational effort, the present simulations run approximately 25 times faster than the high resolution LES experiments of Stoesser et al. (2009) in comparable hardware. It was found that a minimum of 60 grid points were required on the cylinder circumference to accurately simulate the drag and inertia forces yet maintain optimal resolution away from the cylinder in the free zones within the canopy where the grid was coarsened, while the vertical aspect ratio ($\Delta x : \Delta z$) can be increased up to 2.5 without compromising the model results.

2) Force coefficients calculated by fitting the Morrison’s equation to the observed forces in the wave flow experiments matched well with the experiments of Hayashi and Chaplin (2012). No significant differences in force measurements were found between the isolated and array cylinder, which is an indication that the hydrodynamics around the central array cylinder is similar to that of an isolated cylinder for the spacing chosen.

3) Vertical variation of mean and root mean square wave velocity profiles at several locations along the central vertical plane on either side of the cylinder at the array center were compared with those at the same locations around the isolated cylinder and that in the wave flume without a cylinder. Notable differences were seen in the velocities on either side of the cylinder in the zone between the wave trough and wave crest and this difference seems to increase with wave non-linearity. The shoreward motion of the wave energy (during the rising limb of the water surface) associated with the wave crest impacting the cylinder, creates a localized recirculation zone spanning throughout between the wave trough and the wave crest on the shoreward side of the cylinder, while the seaward motion of the wave crest (during the falling limb of the water surface) creates a recirculation zone located only between the the wave trough and the still water level on the seaward side of the cylinder. Also the seaward recirculation zone between the trough and the still water level is weaker compared to the shoreward one (during the shoreward motion of the crest) for waves with greater non-linearity due to Stokes’ drift. Thus over one representative period, a net reduction of mean and root mean square velocities is seen shoreward of the cylinder in the vicinity of the SWL with respect to an equidistant point in front of the cylinder. For the cases investigated, this effect was most prominent within 2 diameters from the cylinder center. This also verifies the fact that the array density was too sparse for notable hydrodynamic interaction effects to be observed for the array cylinders and explains
the agreement of the drag coefficient values with the isolated cylinder case.

4) Vortex shedding was found to be largely localized around one to two diameter distance on either side of the cylinder and no appreciable vortex interaction with the neighboring cylinders were noted. The vortex evolution pattern was found to be different between the zone above the wave trough from the rest of the cylinder. This is due to the fact that the net horizontal wave orbital velocities, between the SWL and the crest, always point in the direction of wave propagation during the forward motion resulting in a stronger recirculation shoreward of the cylinder between the trough and the crest over a wave period. The vertical orbital velocity amplitudes show little deviation from Stokes’ velocities except near the SWL, where the runup effect on the seaward face of the cylinder causes a local increase of the vertical velocity amplitude. This also indicates that the horizontal velocity components are damped more than the vertical component for the majority of the cylinder height.

5) Vertical profile of maximum force and moment on the cylinder suggests the peak force occurs somewhere between the SWL and the HWL and agrees with experimental evidence in literature.

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The following symbols are used in this paper:

\[ t = \text{time} \ (s); \]
\[ g = \text{acceleration due to gravity} \ (9.81 \text{m/s}^2); \]
\[ l, B = \text{length and width of domain respectively} \ (m); \]
\[ h = \text{water depth or submerged height of the domain} \ (m); \]
\[ D = \text{cylinder diameter} \ (m); \]
\[ U_b = \text{bulk velocity in the channel} \ (m/s); \]
\[ U_c = \text{averaged velocity in the canopy layer} \ (m/s); \]
\[ U_s = \text{shear velocity} \ (m/s); \]
\[ \text{Re}_D = \frac{U_b D}{\nu} = \text{cylinder Reynolds number}; \]
\[ \text{Re}_{\text{channel}} = \frac{U_b B}{\nu} = \text{channel Reynolds number}; \]
\[ \text{Re}_{\text{canopy}} = \frac{U_c D}{\nu} = \text{canopy Reynolds number}; \]
\[ \rho_w = \text{density of water} \ (10^3 \text{kg/m}^3); \]
\[ \rho_a = \text{density of air} \ (1 \text{kg/m}^3); \]
\[ \nu_w = \text{molecular kinematic viscosity of water} \ (1 \times 10^{-6} \text{m}^2/\text{s}); \]
\[ \nu_a = \text{molecular kinematic viscosity of air} \ (0.148 \times 10^{-6} \text{m}^2/\text{s}); \]
\[ \mu_w = \rho_w \nu_w = \text{dynamic molecular viscosity of water}; \]
\[ \mu_a = \rho_a \nu_a = \text{dynamic molecular viscosity of water} \ (10^3 \text{kg/m}^3); \]
\[ y^+ = \frac{U_* y}{\nu_{eff}} = \text{non-dimensional wall units, } y \text{ is measured along local normal to the wall}; \]
\[ \Delta x, \Delta y, \Delta z = \text{cell sizes in X, Y and Z directions} \ (m); \]
\[ L = L_0 \tanh(kh) = \text{wavelength of wave solved iteratively using the dispersion relationship} \ (m); \]
\[ L_0 = \frac{gT^2}{2\pi} = \text{deep-water wavelength} \ (m); \]
\[ H = \text{mean wave height} \ (m); \]
\[ T = \text{wave period} \ (s); \]
\[ \omega = \frac{2\pi}{T} = \text{wave angular velocity} \ (\text{rad/s}); \]
\[ k = \frac{2\pi}{L} = \text{wave number} \ (1/m); \]
\[ x, y, z = \text{generic x, y and z co-ordinates}; \]
CHAPTER 3
LES MODELING OF WAVE OVERTOPPING INDUCED VERTICAL CIRCULATION BETWEEN A BREAKWATER AND A MARSH EDGE

1. Introduction

One of the effective shoreline protection methods frequently employed to protect marsh edges in erosive wave forcing regimes in cohesive, soft bed environments is the low-crested breakwater structure, placed at some distance from the marsh edge (Geesey et al., 2011; Douglass et al., 2012). Most of these rubble-mound structures are relatively impermeable having a Light Weight Aggregate Core (LWAC) (USACE, 2015) and run approximately parallel to the shore with few gaps. During a typical winter storm (i.e. a cold-front passage), these structures may become completely submerged and waves can overtop and reach the marsh platform. While a lot of experimental research has been done in the past (Seelig, 1980; van der Meer, 1988; dAngremond et al., 1996; Seabrook and Hall, 1998; van der Meer et al., 2005; Carevic et al., 2013) to understand two-dimensional (2D) wave transmission over submerged breakwaters, they have mostly concentrated on understanding the wave overtopping and free-surface variations in front and behind individual rubble mound breakwaters. Studies that have investigated the nature of the vertical velocity field around these structures have only presented the instantaneous vortex dynamics under progressive waves (Petti et al., 1994; Chang et al., 2005; Ou et al., 2010; Wu and Hsiao, 2013) and solitary waves (Chang et al., 2001), while mean velocity induced circulations have not been investigated in detail. This phenomenon is indeed important in the context of marsh edge hydrodynamics where the mean circulation field, particularly the excess momentum induced onshore flux near the top of the water column, can transport sediment on the marsh platform and may have implications in wetland restoration and management practice. The instantaneous overtopped turbulent jet formed as a result of wave breaking on the breakwater crest is found to create an onshore scour hole, the location and dimensions of which are independent of the shape of the breakwater and depend upon the Keulegan Carpenter (KC) number, calculated using the crest width, as well as the incident wave height (Young and Testik, 2009). Thus if sufficient dredged sediment is available behind the breakwater, the turbulent jet can scour the sediment while the mean circulation eddy moves the sediment to the top of the water column from where the onshore flux can transport this sediment to the marsh
platform. This phenomenon can thus be used as an important nature-based solution for re-nourishment of the wetland without added dredging costs and is an inspiration for the present work. Together with the reduced wave energy at the marsh edge and the presence of a vertical circulation, a properly designed and maintained breakwater system can thus be also exploited for coastal wetland nourishment and keeping up with sea level rise.

In numerical modeling studies, past attempts at simulating the wave transmission over submerged structures have included a variety of techniques, from simple analytical methods Lamb (1932); Ogilvie (1960) to the Boundary Element Method (BEM) solving the Laplace’s equation (Christou et al., 2008) and Boussinesq based shallow water equations (Beji et al., 1992) as well as parabolic mild slope equations (Johnson et al., 2005). More recently, with the advent of increased computing capabilities Reynolds Averaged Navier Stokes (RANS) based models together with the Volume of Fluid (VOF) method (Hirt and Nichols, 1981) for the free surface handling, have found increased application in this problem (Shen et al., 2004; Garcia et al., 2004). One of the advantages of RANS-VOF methods is that these do not require explicit wave breaking conditions like those in shallow water based techniques and are able to handle complicated wave breaking and transmission problems accurately with calibration required for the optimum grid size and turbulence model only. While the focus of most of these previous studies were on the wave field evolution behind the breakwater and instantaneous velocity fields (Huang and Dong, 1999; Chang et al., 2001; Hsu et al., 2004; Chang et al., 2005; Yang et al., 2007; Ou et al., 2010; Bozorgnia et al., 2014) only Garcia et al. (2004) shows the mean velocity field formed in the vertical plane behind the breakwater as a result of wave overtopping. However they did not consider the modification of this flow field by any secondary intervening structure behind the breakwater, similar to what would be expected if the breakwater were placed in front of a marsh scarp. More research is needed to understand how the circulation changes with factors like breakwater relative submergence, incident wave height, relative crest height of the structure with respect to the marsh edge as well as the distance from the marsh edge. Hence, the functional design of these LWAC structures placed in front of a marsh edge is still a challenge as no guideline exists as to how these structures modify the mean flow field and likely influence the resulting sediment transport between the breakwater and the marsh platform. Our present study is an attempt towards understanding these dynamics using Large Eddy Simulation (LES) based turbulence closures, which have been found to be superior to RANS approaches when dealing with wave breaking (Zhao et al., 2004), representation of eddy structures and wave transmission through partially immersed breakwaters (Gotoh et al., 2004). This is the first study applying
LES to understand hydrodynamics around submerged breakwaters in field scale. This work also validates the modeling capabilities of the numerical model OpenFOAM (Weller and Tabor, 1998) which has been used here to simulate wave breaking, overtopping and the mean vertical circulation created due to the waves overtopping a trapezoidal impermeable breakwater placed at some distance from the marsh edge. A realistic equilibrium profile (Wilson and Allison, 2008) is selected for the bathymetry, while breakwater geometry, wave characteristics, submergence levels and breakwater distance from marsh edge were chosen to mimic field scale conditions. In addition, no-structure conditions, with the equilibrium bathymetry only, at different submergence levels have been investigated to illustrate the choice of optimum crest elevation of the structure. The results show that the strength, shape and distribution of the mean recirculation cells between the breakwater and the marsh edge are a function of the relative rate of submergence, wave height as well as distance from the edge. Further, comparison with experiments conducted without a structure show that such a recirculation is absent in front of the marsh edge.

The paper is arranged as follows: The numerical model description section presents the governing equations, including turbulence closure schemes, grid and numerical settings and an overview of the design parameters investigated. The verification section presents results from the simulations comparing numerical results to laboratory experiments for wave breaking and empirical equations for wave transmission. The results section presents and discusses free-surface envelopes, wave height evolution for with-structure and no-structure cases, mean vertical velocity profiles or undertows and finally circulation patterns represented by streamlines and vector plots for a variety of design conditions. Most important design considerations include the relative submergence levels, relative crest elevation with respect to marsh platform elevation, incident wave height and distance from the marsh edge. Seven conclusions are provided based on the findings in the final section.

2. Numerical Model Description

2.1. Governing Equations

All the experiments in this paper were conducted using OpenFOAM version 2.2.x (Weller and Tabor, 1998). The three-dimensional RANS or LES filtered (filtering operation denoted by overbar) Navier Stokes and the incompressible continuity equations can be expressed as,

\[
\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p - \mathbf{g} \times \nabla \rho + \nabla \cdot [\mu \nabla \mathbf{u} + \rho \mathbf{t}] \tag{1}
\]

\[
\nabla \cdot \mathbf{u} = 0 \tag{2}
\]
where vector notations are represented as bold faced characters with $\mathbf{u} = (u, v, w)$ and $u$, $v$ and $w$ being the magnitude of velocities in the X, Y and Z directions respectively, $t$ is the time variable, $\rho$ density, $p$ the dynamic pressure and $\mu$ the dynamic viscosity. Here, $\mathbf{r} = 2\mu_{turb}\mathbf{S} - \frac{2}{3}k\mathbf{I}$ represents the Reynolds stress tensor for RANS (where $\mu_{turb} = \mu_t$) or sub-grid scale stress tensor for LES where (where $\mu_{turb} = \mu_{sgs}$).

The temporal filter has the form $\bar{\phi}(t) = \frac{1}{\Delta t} \int_0^{\Delta t} \phi'(t) dt$ for RANS models, with the accent representing fluctuating component in time. In addition to solving the above equations, the model also solves the transport equations for the kinetic energy and dissipation. A number of RANS models such as $k-\epsilon$ (Launder and Sharma, 1974), $k-\omega$ (Wilcox, 2002) and $k-\omega$ SST (Menter, 1993, 1994) were tested for their accuracy in the verification studies and the $k-\omega$ SST model was found to perform best for the nearshore wave breaking case. Hence we describe governing transport equations relevant to $k-\omega$ SST here only. For the remaining models the reader is referred to the cited references. The transport equations for kinetic energy ($k$) and specific turbulent dissipation rate ($\omega$) for $k-\omega$ SST are,

$$\frac{\partial \rho k}{\partial t} + \nabla \cdot [\rho \mathbf{u} k] = \min(P, 10\beta \rho k \omega) - \beta^* \rho \omega k$$

$$+ \nabla \cdot [(\mu + \sigma_k \mu_t) \nabla k]$$

$$\frac{\partial \rho \omega}{\partial t} + \nabla \cdot [\rho \mathbf{u} \omega] = \frac{\gamma}{\nu_t} P - \beta \rho \omega^2 + \nabla \cdot [(\mu + \sigma_\omega \mu_t) \nabla \omega]$$

$$+ 2(1 - F_1) \frac{\rho \sigma_\omega^2}{\omega} \nabla k \cdot \nabla \omega$$

(3)

(4)

where, $P = \mu_t (\nabla \times \mathbf{u}) \cdot (\nabla \times \mathbf{u})^T$ according to Mayer and Madsen (2000), $\mu_t = \rho k / \omega$ and coefficients $\sigma_k$, $\sigma_\omega$, $\beta$ and $\gamma$ are blended by an inner (1) and outer (2) coefficients ($\phi_1$ and $\phi_2$) as $\phi = F_1 \phi_1 + (1 - F_1) \phi_2$, with the full form of $F_1$ as in Menter et al. (2003). Values of constants used for the blending are $\beta_1 = 0.075$, $\beta_2 = 0.083$, $\beta^* = 0.09$, $\sigma_\omega = 0.5$, $\sigma_k1 = 0.5$, $\sigma_k2 = 1$.

For LES models, spatial filtering is provided implicitly by the finite-volume discretization itself by a simple box-hat filter where filtering operator in Eqs. (1) and (2), becomes $\bar{\phi}(x) = \frac{1}{V} \int_V \phi'(x) dx$ (for all $x \in V$), $V$ being the volume of a computational cell, the accent denoting spatially varying component and the filter function. The subgrid scale stress $\tau_{sgs}$ can be modeled by different techniques which leads to different formulations of the LES model constant. From the authors’ previous experience (Chakrabarti et al., 2016) the Dynamic Mixed Smagorinsky Model (DMM henceforth) (DMM2 in Vreman et al., 1994) was found to perform best and has been used in this study. The subgrid scale viscosity is given by $\mu_{sgs} = \rho C_{DMM} \Delta^2 S$, where $C_{DMM}$ is the DMM model constant as derived in Zang.
et al. (1993), $\Delta = (\Delta x \Delta y \Delta z)^{1/3}$ being the grid-filter width.

For the free-surface representation, the Volume of Fluid (VOF) (Hirt and Nichols, 1981) method is used where air and water are treated in a single phase calculation throughout the domain and the physical properties of the mixture ($\rho$ and $\mu$ in Eq. (1)) are calculated as weighted average of their respective properties with the weighting done by the phase fraction, i.e., $\rho = \alpha \rho_w + (1-\alpha)\rho_a$ and $\mu_{\text{eff}} = \alpha \mu_{\text{eff},w} + (1-\alpha)\mu_{\text{eff},a}$, where the subscripts represent water ($w$) or air phase ($a$) values and the effective viscosity is $\mu_{\text{eff}} = \mu + \mu_{\text{turb}}$. The phase fraction ($\alpha$) can lie within $[0, 1]$ with $\alpha = 0$ signifying full air phase and $\alpha = 1$ full water phase and cells having intermediate values being representative of partially air and water filled zones. OpenFOAM uses a modified version of the two-fluid Eulerian model approach called Multidimensional Universal Limiter for Explicit Solution or MULES (Berberović et al., 2009) to describe the transport of the phase fraction by a single combined evolution equation, written as,

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{u}) + \nabla \cdot [\mathbf{u}_r \alpha (1-\alpha)] = 0$$

where $\mathbf{u} = \alpha \mathbf{u}_w + (1-\alpha)\mathbf{u}_a$ and $\mathbf{u}_r = \mathbf{u}_w - \mathbf{u}_a$ represents the relative velocity vector termed as ‘compression velocity’ which in turn introduces an additional convective term referred as ‘compression term’ to the governing equation. This term can be properly tuned to achieve a very high interface resolution and avoiding the need for special interface treatments used in classical VOF method.

The numerical domain was discretised using the finite volume method with both spatial and temporal discretisations being second order accurate. The Pressure Implicit with Splitting of Operators (PISO) (Ferziger and Peric, 2001) algorithm with two pressure corrector steps and two non-orthogonal correctors were used for the pressure-velocity solver. For the solution of the linear algebraic equations the Pre-conditioned Conjugate Gradient method (PCG) with Diagonal Incomplete Cholesky (DIC) (for symmetric matrices) and Preconditioned Bi-Conjugate gradient (PBiCG) with Diagonal Incomplete LU (for asymmetric matrices) were used as the preconditioner.

2.2. Model Setup

For the first verification study of wave breaking on a 1:35 slope (Ting and Kirby, 1994) the numerical wave tank was 28 m long (instead of 40 m long in the laboratory experiment) and 0.8 m high. The wavemaker was placed at the left boundary at a water depth of 0.4m and a 4m relaxation zone similar to Jacobsen et al. (2012) was provided at the beginning of the domain, while at the end a 3.7m (about one wavelength) sponge layer absorbed any outgoing waves. Regular waves of height 0.125m and time period 2s were generated by the
A sensitivity analysis conducted for both 3D LES and RANS models showed a minimum of 15 cells were required in the spanwise (width) direction to successfully simulate the breaker and obtain accurate results. The results from the two cases are shown later in the verification section. A grid sensitivity analysis ($\Delta x = \Delta z = 10, 5, 2.5$ and $1.25$ mm) near the breaker zone found that a grid size of $5$ mm was optimum for obtaining accurate free-surface envelopes within the surf zone and the location of the breaker. A grid size of $2.5$ mm improved the undertow results slightly (8% difference over the whole profile) than the $5$ mm case. Based on the fact that a 3D grid for the $5$ mm case consisted of about 4.3 million cells while the $2.5$ mm grid had about 8.3 million cells without significant benefit in accuracy, it was decided to use the $5$ mm grid as appropriate for capturing the breaker turbulence in production runs to make optimum use of computational resources. The aspect ratio was maintained as $\Delta x: \Delta z: \Delta y = 1:1:1.5$ for points near the free-surface except within the inlet relaxation zone where the grid was stretched by a factor of 1.2 in the x direction towards the wavemaker. The grid resolution within $+8$ cm to $-7$ cm above and below the SWL respectively was maintained constant in the vertical direction and stretched by a factor of 1.1 away from this zone. The near bed resolution required to adequately resolve the bottom portion of the undertow profile was found to be $1.5$ mm, hence a further vertical stretching was done away from the bed within the bottom $6$ mm of the domain.

The second set of verification experiments of wave propagation over a submerged trapezoidal breakwater (van der Meer et al., 2005) consisted of a domain $50$ m long, $20$ cm wide and variable depth of $(1 - R_c)$ m, where $R_c$ is the difference in elevation between the SWL and the breakwater crest measured as negative when the crest is under water. The height of the breakwater crest from the bed was constant $1$ m. Sponge layers extended inwards into the domain up to $1.5$ wavelengths from either end of the flume. Regular waves of height $H=0.4$ m and period $T=2.5$ s, representative of target experimental conditions of production runs were simulated by the stream function theory. A trapezoidal breakwater with crest width (B) $1$ m, and variable seaward slopes (1:2 and 1:3) depending upon the configuration was placed at the center of the flume. A finer grid resolution with half the grid size ($\Delta x = \Delta z = 1$ cm) from the rest of the domain, was provided within the -1.5L to 1.5L (L being the wavelength) on either side of the breakwater center. It was found that at least 25 cells in the vertical direction across the wave height over the crest was required to get acceptable agreement of transmission coefficients with empirical values. Near the bed a boundary layer zone of 6 mm was provided with smallest cell of 2 mm. The incident and transmitted wave
heights were calculated at points (L+B/2) away from the center on either side of the breakwater. The three-gauge technique of Mansard and Funke (1980) was used to filter out the reflected wave signal from the seaward wave gauge and only incident wave height was used to calculate $H_i$. The breakwaters were assumed smooth with the no-slip condition imposed with the smooth wall function (Spalding, 1961) applied for the near wall turbulent viscosity.

For the production runs, simulating the wave overtopping and resulting hydrodynamics behind a breakwater placed in front of a marsh edge, the numerical domain setup is as shown in Fig. 1. Table 1 shows the values of different parameters for cases presented that illustrate the most important results relevant to this study. In the setup $L_{slope} = 23m$ and was held constant and all distances were measured with respect to the local origin placed at the SWL on the marsh edge. For the nearshore bathymetry from the edge to a distance $L_{slope}$ offshore, an equilibrium bathymetry profile equation valid for Louisiana coasts (Wilson and Allison, 2008) was assumed. The scarp height ($h_{sc} = 0.5m$), marsh platform slope ($1:200$), breakwater side slopes ($1:3$ seaward and $1:2$ shoreward) and crest width ($B = 1m$) were chosen representative of field conditions and held constant. Regular waves at the inlet were specified using stream-function theories on a flat bottom which extended for $12m$ from the wavemaker to the start of the slope with a relaxation zone length of $L_{wm}$. Shoreward, the platform was extended upto $(23+2L_{wm})$ meters from the marsh edge with a $2L_{wm}$ length sponge layer at the end. This was done based on a sensitivity analysis to ensure that the outlet sponge was sufficiently long to absorb the outgoing waves and was far away from the zone of interest ($-20m \leq x \leq 15m$) to influence the results. The domain was $10cm$ wide in the spanwise direction.

![Figure 1: Definition of functional design parameters.](image)

For all simulations, the front and back sides of the domain had cyclic walls while the top was given an atmospheric pressure condition. The bottom of the domain and all structure walls had a no slip condition together with the Spalding’s smooth wall function (Spalding, 1961) for the turbulent viscosity term for LES models. For the $k-\omega$ SST model, $k$ and
Table 1: Values of design parameters (Fig. 1) for different cases:

<table>
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<th>Case Type</th>
<th>Name</th>
<th>$h_{mp}$</th>
<th>$H$</th>
<th>$L_{wm}$</th>
<th>$h_{wm}$</th>
<th>$h_{tbw}$</th>
<th>$R_c$</th>
<th>$L_d$</th>
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<td>-</td>
<td>-</td>
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<tr>
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<td>1.86</td>
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<td>1.26</td>
<td>-</td>
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<td>1.26</td>
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$\omega$ were defined at the walls using a technique similar to Jacobsen et al. (2012) and Nichols and Nelson (2004).

Fig. 2 shows the grid with every alternate point shown for visual convenience. Cells were stretched within the sponge layers towards the wavemaker and the outlet from within the domain by a factor of 1.1 in the X direction. Most of the remaining part of the domain had uniform resolution with adaptive gridding performed near the free surface (Zone 1) ($-H/2 \leq z \leq 0.75H$) throughout the domain and near the breakwater (Zone 2) and one
further step of finer resolution (Zone 3 and Zone 4) between $L_d/2 \leq x \leq 15m$. The later was done to enhance the resolution of the circulation features which mostly hug the face of the marsh edge. The grid sizes within the fine meshed free surface (Zone 1) and around the breakwater (Zone 2) were $\Delta x: \Delta z: \Delta y = H/40:H/40:1.5*(H/40)$ while near the marsh edge in (Zone 3 and Zone 4) these were $\Delta x: \Delta z: \Delta y = H/80:H/80:1.5*(H/80)$. The total number of cells in the domain varied between about 6.8 million to 15.5 million depending upon the wave and water depth conditions, with about 88% of the cells lying within adaptively refined areas.

Figure 2: The numerical mesh is shown here. Adaptive mesh refinement done near the free-surface, around the breakwater, bottom boundary layer of the bed and near the marsh edge.

3. Model Verification

3.1. Wave Breaking on a 1:35 Slope

Fig. 3 shows the comparison of the water surface envelopes averaged over 20 wave periods with the experimental data (Ting and Kirby, 1994). Among the RANS models tested, the $k-\omega$ SST performed the best and has been compared here with the LES DMM. The 2D and 3D results refer to setups with one grid and 20 grid points in the spanwise (Y) direction. The location of the breaker was found to be more dependent upon the order of the wave theory used at the wavemaker, while the breaker height and the envelopes within the surf-zone were found to be more dependent on the type of turbulence model, resolution and whether it is
2D or 3D. The stream-function theory together with the 3D LES DMM was found best for estimating the breaker location \((X_{b,\text{num}} = 6.52m \text{ against } X_{b,\text{expt}} = 6.4m)\), breaker height \((H_{b,\text{num}} = 0.157m \text{ against } H_{b,\text{expt}} = 0.1625mm)\), the free-surface variation and wave setup within the surf-zone. Both RANS and LES 2D models tend to predict the breaker location somewhat before the experimental value. Also RANS models underestimate the breaker height and is found to be unsuitable for the present purpose while 2D LES seems to show a buildup of wave height before breaking. The difference in the 2D and 3D LES results for the same grid sizes in our opinion is due to the difference in the subgrid scale viscosity for the two cases, while for the RANS this is likely due to artificial diffusion from the spanwise direction. Also assuming no numerical diffusion it has been shown that back-scatter effects are likely to occur in 2D (Kraichnan, 1967), where the turbulent energy is transferred back to the resolved scales. Wave breaking is essentially a 3D process and the eddy filter scales in the LES DMM being dependent on the grid size, should always be run in the 3D mode. It is seen that to simulate wave breaking LES DMM requires a minimum number of cells (15 for acceptable results) in the spanwise direction to correctly extract the energy as the waves break.

![Graph](image-url)

Figure 3: Model verification for wave breaking on a 1:35 beach. Comparison of free-surface envelopes with experiments of Ting and Kirby (1994).

A series of snapshots in Fig. 4 show the 3D breaking process in detail. Different camera angles capture the complete spilling breaker process with vortex cores represented by the Q-criterion (Hunt et al., 1988) with iso-surfaces drawn at constant \(Q = 10\). Vortex iso-
Figure 4: The 3D wave breaking process is shown here. Panels left to right show before, at and after breaking vortex iso-surfaces colored by vorticity. Surface coloring is done by vorticity (Red-Green-Blue scale) ($\omega_y$) about the Y axis. For the simulated spilling breaker case it is seen that the model is able to successfully simulate the spiller as it hits the water surface and shoots up. Post breaking the LES simulations show a strong turbulent region in the surf zone, which is an important capability for our current problem where wave breaking on the breakwater is likely to induce similar turbulent zone between the marsh edge and the structure.

Fig. 5 shows the comparison of undertow profiles with experimental values just after the breaking point and within the surf zone. Here the RANS models are tested with 3D LES DMM with and without a rough wall function. It was found that in order to better match the experimental undertow profile near the bed immediately after the breaker point a rough wall function was needed together with a near bed resolution of 1.5mm. The
Figure 5: Modeled results of wave breaking induced undertow compared with experiments of Ting and Kirby (1994)

‘nutURoughWallFunction’ in OpenFOAM was used for this purpose, which is based on the rough wall formulation of Cebeci and Bradshaw (1977). The undertow profiles show improved agreement of the 3D LES with wall function results over RANS and is thus adopted for subsequent production runs.

3.2. Wave Transmission over Submerged Breakwater

Figure 6: Verification of wave transmission over submerged structures. Transmission coefficient ($K_t = H_t/H_i$) compared with empirical equations of Seelig (1980) and van der Meer et al. (2005).
In order to test the adequacy of the numerical resolution and the ability of the LES model to handle the violent breaking process over a low crested submerged breakwater, wave transmission coefficients from a set of 3D numerical experiments with regular waves ($H = 0.4m$, $T = 2.5s$) propagating over a trapezoidal breakwater of constant height $h_{t,bw} = 1m$ were obtained. Fig. 6 shows the transmission coefficient ($K_t = H_t/H_i$, where $H_t$ and $H_i$ are transmitted and incident waves respectively) plotted against the ratio $Rc/H_i$. These are compared with the empirical equations of van der Meer et al. (2005) and Seelig (1980) for two different sets of breakwater front slopes. The transmission coefficient ($K_t = H_t/H_i$, where $H_t$ and $H_i$ are transmitted and incident waves respectively) was calculated and plotted against the ratio $Rc/H_i$ for all the cases. The error-bars represent the standard deviation in the original laboratory experiments. Comparisons with the equations by Seelig (1980) and van der Meer et al. (2005) show good agreement with the Seelig (1980) regression equation between $-1.5 :\leq Rc/H_i \leq -0.5$ for slope 1:2.

The van der Meer et al. (2005) experiments used the significant wave height in the original expression, while the Seelig (1980) used the mean wave height as the latter experiments were on regular waves exclusively. Consequently we believe the Seelig (1980) equation is a better fit for our case here as our experiments were with regular waves only. The van der Meer et al. (2005) equation seemed to be better in agreement when $H_i = 0.63Hs$ (assuming Rayleigh waves, where $H_s$ is the incident mean wave height) was used with the agreement being even slightly better for the 1:3 slope problem. However none of the equations show the apparent flattening of the transmission curve for the numerical results as they approach the $Rc/H_i = 0$ mark. It is possible that the free-surface turbulent interactions from the broken wave has a different behavior as the crest free-board decreases to below half of the incident wave height, leading to some scatter in the data which may not be captured well by the model. Nevertheless our results indicate that the model is capable of representing laboratory scale tests for these problems within the acceptable error of the original measurements and can be used for our work.

4. Numerical Experiment Results

4.1. No Structure Condition

Wave propagation and transmission over a typical mudflat with a marsh edge, under realistic field conditions were investigated by a series of numerical experiments as in Table 1. Fig. 7 shows the free-surface envelopes under natural (no-breakwater) conditions where offshore incident wave height and time periods were held constant at $H=20cm$ and $T=2s$,
respectively, and the still water level is increased from -50cm to +60cm (bottom to top panels) with respected to the platform elevation at the marsh edge.

It is seen that when the water level is at the toe (NS20-1) the waves break before reaching the marsh toe and the local wave height at the toe is only 6cm. When the water level reaches half of the scarp height (NS20-2), the local wave height increases to 38cm due to reflection from the scarp while when the water level is at the scarp height (NS20-3), the wave height is seen to be 35cm, indicating an almost full reflection under those submergence conditions.
Both these conditions are likely to cause considerable pounding forces at the marsh scarp but less scour at the toe due to presence of an anti-node there. When the water level increases to 30cm above the marsh platform (NS20-4), only partial reflection from the edge is seen, however the wave breaks on the marsh platform itself. This scenario is likely to cause damage to the marsh scarp due to pounding force action and also some damage to the platform as a result of the shear stress due to breaking. Also the reduction of the reflection will cause a shear stress increase at the toe. Finally at 60cm above the platform (NS20-5), the waves almost show no reflection patterns but instead display modulation patterns on the platform due to the triad interaction of the non-linear waves generated as they move over the marsh edge. This scenario is likely to cause much less damage due to the pounding action of the forces but more due to the shear action on both the platform and the toe.

4.2. With Structure Condition

A series of experiments, (denoted by prefix WS-) as shown in Table 1, with a trapezoidal breakwater placed at varying distances from the marsh edge and different crest elevations, were conducted. Water levels for the three no-structure cases (NS20-3, NS20-4 and NS20-5) were selected for comparisons as for the other two cases, the wave will not overtop the breakwater to reach the marsh edge and is not of interest here. Fig. 8 shows the comparison of wave heights at two submergence levels (lower panel $h_{mp} = 0cm$, upper panel $h_{mp} = 30cm$) with and without a structure. Transmission coefficients are included in parentheses in the legend. It is seen that the breakwater is very effective in attenuating the wave energy when the water level is the same as that of the crest elevation (WS20-3)
and successfully protects the scarp both from the pounding action of the reflected waves as well as the shear stress of transmitted broken waves. However, once the water level on the platform increases beyond 30cm (free-board $|Re| > 1.5H$, to be more accurate), the breakwater is almost inefficient (cases WS20-4-LC1 and WS20-4LC2) while the reflection effect also increases if the breakwater crest elevation becomes less than the marsh platform elevation. It is therefore recommended that the breakwater crest be always maintained higher than the platform elevation, in order protect the scarp from the pounding action of the reflected waves and to maintain the intended function of the breakwater. Note that the breakwater becomes completely inefficient, if it sinks at or below half of the marsh scarp height.

4.3. Comparison of Undertow under No-Structure and With-Structure Conditions

![Figure 9: Comparison of undertow profiles for no-structure (NS) conditions at two different submergence levels ($h_{mp} = -0.5$, i.e., the foot of the toe and $h_{mp} = 0$, i.e., at the marsh platform level) with the corresponding wave case for submergence level at $h_{mp} = 0$ with a structure (WS).](image)

In addition to comparing the free-surface envelopes and wave heights, the mean velocity or undertow profiles between the breakwater and the marsh edge under no-structure and with-structure conditions were also compared at several vertical transects in the vicinity of the marsh edge and the breakwater structure. The significance of the undertows (which are calculated by averaging velocities for all submerged points) lie in their role in suspending sediments from the zone between the marsh edge and the structure and moving them back on to the platform. If sufficiently strong undertow is present, then it is possible to lift sediments up and carry them shoreward by the near surface current. The presence of the breakwater is seen to drive a strong vertical circulation under submerged conditions, which is absent without the structure. The excess momentum flux that overtops the structure carries with it the bulk of the wave energy at the free surface as a result of blockage action of the structure and causes periodic passage of momentum flux along the free surface which on reaching
the marsh edge drives a current. Fig. 9 shows the undertow profiles under no-structure condition, with the distance from the marsh edge indicated in the figures (negative values indicate distances offshore from the marsh edge while positive values are distances onshore from the edge). It is observed that the breaking of the waves (breaking point $X_b=-6.1m$) in the NS20-1 case (water level at marsh toe) (Fig. 7) causes a strong undertow to develop whose strength decreases steadily towards the marsh edge. As the water level increases further waves no longer break, through there is a pronounced reflection from the edge. Consequently no significant undertow profile can be observed for NS20-3 case. Interestingly, when a structure is present, we see that an undertow forms soon after the overtopped waves cross the shoreward structure toe ($X=-2m$) and grows stronger upto $X=-0.5m$, then decreases quickly to the marsh edge as the waves break considerably as they approach the edge.

Figs. 10(a) and 10(b) show the variation of the undertow for points offshore (on mudflat)
and onshore (on platform) of the marsh edge, respectively. The submergence level and wave height were $h_{mp} = 0.3m$ and $H = 0.2m$. For points offshore of the marsh edge, when no-structure is present we do not observe any undertow signatures. However when a structure is present, the strength of the undertow depends upon the relative crest height ($R_c$) and incident wave height ($H_i$). Under the tested condition for $H_i = 20cm$ and $R_c = -30cm$, we find that the waves break and produce a strong undertow from $-2m \leq X \leq 0m$. Thus for the breakwater to be both effective in terms of producing a circulation as well as protect the edge from wave damage, it is important to consider the $R_c/H_i$ ratio. For points on the platform (Fig. 10(b)), it is seen that the secondary breaking of the waves at the marsh edge under both with and without structure conditions (NS20-4 and WS20-4), when $h_{mp} = +0.3m$, we see undertows developing, though they are of different natures. The undertow strength for the WS20-4 case is maximum near the marsh edge and decreases shoreward and vanishes almost...
after \( X = 1.5m \), representative of a strong secondary breaking event near the edge, while under no structure condition (NS20-4), the undertow profile has an 'S' shape near the marsh edge, decreases in strength shoreward up to about \( X = 1.5m \) and again increases in strength showing a typical undertow signature from \( X = 3m \) onward. As will be explained in the following section in the circulation streamlines, the formation of the 'S' shaped undertow profile is due to a combination of reflection effect from the marsh edge, surging of the overtopped flux as well as the excess momentum flux generated from the local breaking at the edge, while the typical undertow profile further into the platform is exclusively due to secondary breaking of the waves.

Figs. 11(a) and 11(b) show the variation of undertow profiles under different submergence levels as well as varying incident wave heights (\( H_i = 0.2m \) and \( H_i = 0.4m \)) under with-structure condition for offshore and onshore points of the marsh edge, respectively. On the mudflat, it is seen that when the submergence level is lower (\( h_{mp} = +0.3m \)) the higher wave height (WS40-4) tends to drive a stronger recirculation between \(-3m \leq X \leq 0m\)
and the onset of the undertow is closer to the breakwater onshore toe. Another interesting fact observed is the shifting of the negative peak in both the cases towards the bed as one approaches the marsh edge. This is because of the unique circulation cell shape set up under this condition and will be explored further in the following section. When the water level rises ($h_{mp} = +0.6m$) the structure no longer offers a significant breaking barrier for the $H_i = 0.2m$ waves and subsequently no undertow forms while the larger wave height still forms an undertow, albeit the onset is further away from the breakwater onshore toe than for the lower submergence level for the same wave height. The WS40-5 case also shows a 'S' profile on the mudflat at the marsh edge, which is attributed to a circulation cell being located only in the top half of the water column, the lower half being blocked by the presence of the marsh edge. Thus it can be said that a higher submergence level with a lower wave height causes a weaker circulation compared to a lower submergence with a higher wave height on the mudflat. On the platform (Fig. 11(b)) at the lower submergence level the difference in the undertow profiles between the two wave heights is not so evident due to the fact that the secondary breaking induced by the marsh edge is almost the same for both the cases. However, for the greater submergence level, the difference becomes more noticeable as for the smaller wave height we have no secondary breaking on the marsh edge, while the larger wave height shows pronounced breaking and development of a strong undertow. Notice in particular that no 'S' shaped undertow profile forms on the marsh platform in the presence of the structure.

Figs. 12(a) and 12(b) show the variation of undertow profiles on the mudflat and the platform under different relative submergence levels, where the crest height is varied with respect to the marsh edge while keeping the submergence on the platform ($h_{mp} = +0.3m$) and incident wave height ($H_i = 0.2m$) constant. The distance from the marsh edge varies between 5.08 to 8.08m due to the different crest elevations with same forward and backward slopes as well as crest width, thereby having slightly different locations where the shoreward end of the toe meets. On the mudflat, it is observed that the WS20-4 and WS20-4-HC1 cases both induce similar undertow profiles with the undertow onset for WS20-4-HC1 being slightly further away from the edge due to the reduced relative crest elevation ($R_c$) and the fact that the $X = -4.5m$ point is at the shoreward slope of the structure, consequently this setup shows stronger breaking. Both the WS20-4-LC1 and WS20-4-LC2 (the latter not shown due to similar results) cases do not show any undertows which is in agreement with the wave height evolution results in Fig. 8 which showed that waves do not break due to the large $|R_c/H_i|$ for either of these cases. Consequently allowing the breakwater to
sink below the platform elevation is not recommended as no circulation induced sediment
transport is going to happen in these cases. As before in previous figures we also observe the
shifting of the the negative peak in the undertows towards the bed as one approaches the
marsh edge. Similarly, on the platform (Fig. 12(b)) both the WS20-4-LC1 and WS20-4-LC2
show similar profiles with the 'S' curve towards the marsh edge with increasing undertow
signature as a result of wave breaking as one moves into the platform, much like the case
with no-structure (NS20-4, Fig. 10(a)). When the crest elevation is either at the platform
elevation or above it, we see a difference in the mean velocity profiles between the two cases.
This is because the relative crest height $|R_c/H_i|$ for WS20-4 is greater than that for WS20-4-
HC1, which means the former setup results in greater overtopping and wave heights. These
waves break on the platform for a second time and give rise to the undertow for WS20-4
while for the WS20-4-HC1 case the wave height at the marsh platform is already small due
to the primary breaking on the breakwater is not high enough to break at this submergence
level and likewise does not show any undertow signature. The influence of this combined
dynamics of wave height variation and circulation gives rise to unique circulation patterns
which will be further discussed in the next section.

Figs. 13(a) and 13(b) show the variation of the undertow profiles on the mudflat and the
platform for different distances of the breakwater centerline from the marsh edge with labels
WS20-4-Ld1, WS20-4 and WS20-4-Ld3 being results from increasing distances respectively
(See Table 1 for exact distances). WS20-4-Ld2 results were similar to WS20-4 ones and
is therefore not shown. On the mudflat, WS20-4 and WS20-4-Ld2 show strong undertow
patterns between $-1.5m \leq X \leq 0m$, indicating that the dominant circulation cell resides
within 1.5m from the edge. The reason WS20-4-Ld1 shows an undertow only within 1m
from the edge is because the distance between the marsh edge and the breakwater is too
short (3.09m) in this case for the undertow circulation cell to develop and most of the
recirculation zone is concentrated near the edge. Consequently, having the breakwater too
close to the edge may not be favorable as the circulation net effective zone is small and may
not suspend adequate sediment. Also the turbulent jet from the primary breaking on the
structure crest can reach the marsh edge and cause scour at the marsh face. On the other
hand placing the structure too far away from the edge (15.73m for the WS20-4-Ld3 case)
reduces the strength of the undertow and is evident in the almost constant profile for this
case. For distances greater than 20m it was found that the undertow profiles almost vanish
and are almost similar to cases without a structure. There is a likely relationship between
the ratio of the wavelength to the distance of the breakwater in order for the undertow
Figure 13: Comparison of undertow profiles for with-structure conditions for varying distances of the structure from the marsh edge

to be the strongest, but is not explored here as only one offshore wavelength was tested here. On the platform, we find for locations within the first 1m from the edge, closer the breakwater to the edge stronger is the undertow. All the three cases WS20-4-Ld1, WS-20-4 and WS20-4-Ld3 show high near bed velocities. The similarity of the profiles WS20-4-Ld1 and WS20-4 are due to similar nature of the marsh edge vortex on the mudflat as will be shown later, while for WS20-Ld-3 the profiles show similar behavior to the NS20-4 case (no-structure in Fig. 10(b)) with the 'S' shaped curve near the edge and transforming to a more conventional undertow as one goes away from the edge into the platform. Note, unlike the no-structure case no strong breaking is observed around the 3m mark because of the lower wave heights reaching the platform compared to the no-structure case. Thus in conclusion we may say that in order to exploit the benefits of the circulation adequately, such that we have high enough circulation velocities on the mudflat to suspend and move sediments onshore while minimum breaking and undertow, the WS20-4-Ld2 with a distance...
of 11.1m seems to be ideal.

4.4. Comparison of Vertical Circulation Patterns

The vertical circulation effect observed in presence of the breakwater, driving the undertow profiles in the previous section is analyzed further in Fig. 14, which shows the span-wise averaged mean circulation patterns using streamlines (top panel of each subfigure) and corresponding vector plots (bottom panel of each subfigure) for the selected cases from Table 1. The vectors are plotted at every 200 points for clarity and velocity results are averaged over a total of 25 waves. Fig. 15 shows the circulation pattern from the no-structure case with similar wave conditions.

4.4.1. Effect of the Structure Distance from the Marsh Edge

Comparing WS20-4-Ld1 (14(a)), WS20-4 (14(b)), WS20-4-Ld2 (14(c)) and WS20-4-Ld3 (14(d)) we find that the position of the breakwater with respect to the edge strongly influences the size of clockwise rotating (driven by the on-shore flux) dominant recirculation cell immediately adjacent to the marsh edge. As the breakwater is moved closer to the edge, the size of the circulation cell decreases, with the length varying between about 1.4m (smallest, WS20-4-Ld1) to about 2.1m (largest, WS20-4-Ld2).

For WS20-4-Ld1, WS20-4 and WS20-4-Ld2 cases, apart from the clockwise rotation for the dominant eddy, it is also found that the eddy rotation just outside the surf zone on the platform is also clockwise, with this being the strongest for the WS20-4-Ld1 case followed by WS20-4. This is because as seen in Fig. 8 the wave for WS20-4 does not undergo breaking at the marsh edge, unlike the NS20-4 case. Instead the wave height remains almost unchanged and in fact slightly increases and breaks only at or after 2m. This deferment of the breaking event causes not only the stretching of the dominant mudflat eddy into the platform but also shifts the clockwise (seen between $0.6 \leq X \leq 1.5m$ for the WS20-4 case) further into platform when the breakwater is closer to the marsh edge (WS20-4-Ld1). On the other hand, WS204-Ld2 seems to be the ideal case, where the mudflat eddy is strong enough to carry the sediment on the platform, where it can be deposited.
Figure 14: Vertical circulation patterns for with-structure cases. Top panel for each subfigure represents streamlines while bottom panel shows the vector plots.
In real life probably an equilibrium distance similar to this distance will be established if the breakwater is placed too close to the edge. In addition to these circulation cells another pair of anticlockwise rotating cells can be observed, one close to the shoreward corner of the breakwater crest and the other at the toe of the breakwater in the WS20-4-Ld1 case. The former eddy can be of possible significance for the stability of the armor layer on that corner while the latter may cause scour at the breakwater shoreward toe. Comparing with WS20-4 and WS20-4-Ld2 cases it seems these eddies exist as companion eddies of the large clockwise marsh edge eddy for smaller breakwater distances from the marsh edge. In addition, moving the breakwater too close to the shore also causes the generation of very small but strong anti clockwise eddies at the marsh face (WS20-4-Ld1).

Interestingly, WS20-4-Ld3 does not show a coherent circulation pattern near the marsh edge even though it seems to have a weak undertow on the platform. This is similar to the no-structure case, NS20-4 (15) which does not show any circulation obviously, which means as the breakwater moves away from the edge, it behaves more like a no-structure case in terms of circulation patterns. Another similarity between the two cases is observed in that the weak eddy that forms at the marsh edge with an anti-clockwise rotation just outside the surf zone on the platform, for the NS20-4 case is also observed in the WS20-4-Ld3 case. This is because the obstruction from the marsh edge causes flow separation at the marsh edge face which is somewhat stronger during the offshore cycle of the wave phase. This is because vertical velocity excursions are higher at the edge (being a partial anti-node location) than horizontal excursions, subsequently the edge being at a steep downward angle experiences a stronger downward velocity than the horizontal velocity on the platform, which also gives this eddy the anticlockwise rotation and potentially moves away any scoured sediment from the marsh.

Figure 15: Vertical circulation pattern for no structure (NS-20-4) case. Top panel represents streamlines while bottom panel shows the vector plot.
A second anticlockwise eddy on the marsh platform stretching from $0.2 \leq X \leq 0.4$ m is observed which can be attributed to a surging breaker formation at this location. Most of the wave energy moves above the still water level at this location, while near the bed the detached jet from the marsh edge forms an on-shore motion. A third companion weak eddy rotating in clockwise direction is located approximately midway between these two eddies near the free-surface. A few apparent free-surface eddies located near the free-surface on the mudflat are actually very weak, as apparent from the vector plot and are of not much interest.

4.4.2. Effect of the Structure Crest Height with respect to Platform Elevation

Figs. 14(b) and 14(e) show the circulation patterns when the structure crest elevation is raised above the platform while keeping the water depth unchanged (WS20-4 and WS20-4-HC1 respectively). Raising the structure shortens the main eddy size and moves the eddy center downward. This is because the smaller opening causes lesser wave power to enter the sheltered area, while the mean overtopped flux remains relatively constant. This delays or almost eliminates any possibility of secondary breaking on the platform while keeping the onshore flux more or less unchanged.

Comparing Figs. 14(e) and 14(f) it is seen that increasing the water level ($h_{mp} = 0.6$ m from $h_{mp} = 0.3$ m) as well as the crest of the structure (to double the marsh edge scarp height) but keeping the wave height constant (WS20-4-HC2 and WS20-4-HC1) breaks up the main clockwise eddy into two parts, a clockwise rotating top part and another anticlockwise rotating bottom part. A strong shear layer can be seen bordering the counter rotating eddies. On the platform the secondary breaking is visible again as the relative crest elevation to wave height ($|R_c/H_i|$) increases. This scenario is not desirable as the not only the counter clockwise eddy at the toe of the marsh is likely to cause erosion and net movement of sediment, the advantage from the onshore flux is also somewhat lost as whatever sediment is suspended from the mudflat a major part of it will be redeposited on the mudflat by this second lower eddy. In conclusion we may say a high crest structure with respect to the marsh edge is good only if it can successfully block off the wave energy. If sufficient wave overtopping is still present, it will be detrimental to the survivability of the marsh edge scarp.

5. Conclusions

In this work, we have simulated wave overtopping, transmission and resultant hydrodynamics over a submerged breakwater placed close to a marsh edge using Large Eddy
Simulation filtered Navier Stokes Equations in OpenFOAM. The following conclusions can be drawn from this study:

1. The OpenFOAM based RANS and LES filtered Navier Stokes equations were verified against laboratory measurements for free surface envelopes and undertow profiles. Agreement with the Dynamic Mixed Model (DMM) based LES model was found to be superior to RANS models and was henceforth chosen for this study.

2. The model’s capability to simulate wave transmission over impermeable trapezoidal breakwater structures in near field scale conditions were tested for two different seaward slopes and the agreement was found to be good when compared with the empirical equations of van der Meer et al. (2005) and Seelig (1980). The van der Meer et al. (2005) seemed to match the results better when the wave height was represented as the mean wave height instead of significant wave height in the original equation.

3. The test conditions for production runs were chosen specifically to mimic field conditions in terms of wave characteristics, equilibrium bathymetry and breakwater geometry. Conditions where no structure was present were compared with those without structure cases. When no structure is present, the wave envelope within the first 2-3 wavelengths from the marsh edge on the mudflat is in general dominated by the submergence level with respect to the marsh scarp. When the submergence is at the marsh toe, waves break before reaching the edge, while when the submergence is at or below the marsh scarp height, waves are reflected back from the edge and slamming effects of the incident waves would dominate the failure mode. When the submergence exceeds the marsh platform elevation, the reflection effect decreases and the waves may or may not break over the platform depending upon the incident wave height on the marsh edge. When the waves don’t break on the platform, triad interaction is found to dominate the evolution of waves on the platform.

4. An intervening structure is most efficient in attenuating the waves if the height of the structure is at or above the marsh scarp elevation. For situations where the still water level is above the platform, the design crest elevation should be chosen taking into consideration the relative crest height to wave height ratio ($R_c/H_i$) based on transmission coefficient curves.

5. When waves are allowed to overtop a structure, the excess momentum flux creates a circulation pattern in the vertical plane, the size and shape of which depends upon
several factors like distance of the breakwater from the marsh edge, incident wave height, relative crest elevation, and wavelength of the incident wave. It was found that for the constant wavelength tested, the breakwater works optimally when the centerline is located around 11.1m (about twice the wavelength in this case) from the marsh edge. If sufficient sediment supply is available the vertical circulation may pickup the sediment and deposit on the platform. Increasing the distance causes a decrease in the circulation strength and seems to vanish (therefore approach no-structure condition) when the distance is over 20m. Placing the breakwater too close ($\leq 4m$ or about 0.75 times the wavelength in this case) causes significant undertow both on the mudflat and the platform.

6. Increasing the incident wave height in general causes greater circulation strength and thus stronger undertow both on the mudflat as well as on the platform due to secondary breaking. Increasing the submergence level keeping the wave height constant may cause no circulation either on the mudflat or the platform if the ratio $R_c/H_i$ is too small as most of the wave energy will go over the structure and there will be no breaking on the platform due to deeper depth.

7. A higher crest elevation over the marsh platform is not necessarily suited if it does not attenuate the wave height sufficiently. In this case the overtopped wave energy onshore flux gives rise to a counter rotating pair of circulation cells close to the marsh edge with the top one being clockwise and the bottom one at the marsh edge toe anticlockwise. This anticlockwise eddy cell can be strong enough to not only scour the marsh edge toe but rob away sediment from the onshore flux and deposit back to the mudflat.

As future work it is recommended that laboratory and field experiments be conducted to validate the findings of this numerical study.

6. Acknowledgements

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1. Introduction

Low-crested breakwater structures, placed at some distance from the marsh edge are frequently adopted as shore protection solutions to protect fragile marsh edges in erosive wave forcing regimes (Geesey et al., 2011; Douglass et al., 2012). Particularly in cohesive, soft bed environments, like Louisiana, the use of Light Weight Aggregate Core (LWAC) breakwaters, which have an impermeable, light geotechnical fabric wrapped core and a thin, heavier armor layer has seen increased application in recent years (USACE, 2015). While mostly continuous, these breakwater systems have regular gaps, called fish dips, generally about 20-50 feet (6-15 m) wide, placed at every 1000 feet (305 m) of the breakwater length. Sometimes gaps may be wider and placed much closer, at arbitrary distances from each other, to accommodate highly irregular marsh shoreline breaks or outflowing wetland streams. Fish dips facilitate the exchange of water as well as allow fish passage. The breakwater itself is an effective wave barrier when emergent, particularly towards the beginning of its design life and successfully retains the fine, wave suspended sediment behind it. However, as the breakwater settles in course of its design life, it tends to remain submerged for a significant amount of time during cold fronts and tropical storms, with higher incident breaking wave heights, on account of the larger water depth in front of the structure. Raising the breakwater to the design level is often too costly and though submerged structures can be made effective again by widening the crest (Capietti, 2011), gaps within the structures can still cause ‘rip current’ type flow, induced by wave breaking on the breakwater crests. This phenomenon drives a return current that can move sediment from behind breakwater out into the open sea. Longshore feeder currents driven by the rip instability also cause the longshore motion of the sediment towards the openings, with the resultant system being reduced from accreting to an eroding regime. Thus the optimization of the functional design of a submerged breakwater system, which not only is expected to perform its primary objective of reducing the wave energy behind the structure, but also conserve the sediment between the breakwater and the marsh edge, becomes an important secondary objective.

Rip currents, frequently observed in barred sandy beaches, are narrow (10 – 20m in
longshore direction) offshore directed currents, spanning the entire water column within the surf zone and are the primary drivers of seaward transport of water and sediment (Shepard et al., 1941). Experimental investigations into rip currents dynamics in the laboratory (Haller and Dalrymple, 2001; Dronen et al., 2002; Haas and Svendsen, 2002) and in the field (Smith and Largier, 1995; MacMahan et al., 2005) have identified vortical instability mechanisms responsible for rip current fluctuations, a phenomenon that is the primary life risk for swimmers. Influence of bar gap widths and bar lengths under different wave conditions was studied experimentally by Kennedy et al. (2008), who noted that while the rip neck width increased slowly with increasing channel width, maximum offshore and longshore velocities showed little dependence on bar or gap lengths, and instead scaled better with the rate of generation of circulation. Also volumetric offshore flow rates through the rip channel showed no proportionality with the bar length calculated from simple mass transport hypothesis. Numerical simulations too have confirmed the weak correlation of maximum velocities with rip spacing (Svendsen et al., 2001; Yu and Slinn, 2003). Thus the primary focus of this paper is on the mean rip current flow (typically averaged over 60-180 wave periods) which is devoid of the relative uncertainty of the fluctuations and can be better measures of suspended sediment transport. Experimental studies on rip currents through gaps between detached, permeable, low crested breakwater structures have been performed under the EU DELOS project by Zanuttigh and Lamberti (2006); Vicinanza et al. (2009), but is not the primary focus here as LWAC structure is mostly impermeable.

In the field of numerical modeling of rip currents, depth-integrated models, either phase averaged (Haas et al., 2003) or phase resolving (Chen et al., 1999a; Yang et al., 2015; Ketabdari et al., 2015), offer the best compromise between accuracy of the results and computational costs, though a limited number of studies employing three-dimensional Navier Stokes Equations (Hur et al., 2012) or Smoothed Particle Hydrodynamics (SPH) (Haas et al., 2012) have been performed. Also two dimensional depth integrated models can be run for a large number of design conditions, with moderate computational costs on existing HPC systems and provide a platform to develop model driven parametric relationships between design conditions and flow variables that are of interest to engineers. The phase averaged models have however been found to overpredict the current magnitudes (Johnson et al., 2005; Zanuttigh and Lamberti, 2006), through the breakwater gaps as well as setup behind it. In particular the capability of the FUNWAVE-TVD (Shi et al., 2012), based on the fully non-linear Boussinesq (Chen, 2006) equations and using the Total Variable Diminishing (TVD) based hybrid approach to handle wave breaking, is impressive (Fang et al., 2014) and is...
the model of choice for our case. The FUNWAVE model has been validated extensively in the past with laboratory experiments and has been shown to be very effective in simulating highly asymmetric waves in shallow water (Kennedy et al., 2001), wave-breaking induced currents (Chen et al., 1999a), long-shore currents (Chen et al., 2003), wave setup close to the shoreline (Chen et al., 2000; Kennedy et al., 2000), wave-current interaction (Chen et al., 1998, 1999b), wave transformation in inlets and harbors (Shi et al., 2001) and wind effects on wave propagation (Chen et al., 2004; Liu et al., 2015). The model is here used to simulate highly non-linear wave transformation over a realistic mudflat and wave breaking over low crested breakwaters with gaps, placed close to a vegetated marsh platform, in field scale.

The main objective of this paper is to develop model driven empirical relations based on a parameter space investigation, connecting the mean offshore directed current through the fish dip as well as the wave height behind the fish dip to the main functional design parameters of the breakwater. This is the first study of this type employing hundreds of simulation runs to develop empirical equations that can be applied to practical field scale problems. This paper is divided into three following subsections, numerical model description (including governing equations and numerical model setup), numerical results (where the development of the parametric relations from the model runs is discussed) and finally the conclusions. It is expected that this study will help practicing engineers to better design low crested structures applicable for shoreline protection of marsh edges.

2. Numerical Model Description

2.1. Governing Equations

The governing equations FUNWAVE-TVD are the 2DH depth-integrated conservation and horizontal momentum equations (Chen, 2006; Shi et al., 2012),

$$\eta_t + \nabla \cdot [(h + \eta)(u_\alpha + \tilde{u}_2)] = 0$$ (1)

$$u_{\alpha,t} + (u_{\alpha} \cdot \nabla)u_{\alpha} + g\nabla\eta + V_1 + V_2 + V_3 + R = 0$$ (2)

where, boldface notations denote vector quantities with $\eta =$free surface elevation at time $t$, $h =$still water depth, $u_\alpha = u_\alpha i^x + v_\alpha i^y$ the reference velocity at reference elevation $z_\alpha = -0.53h + 0.47\eta$ defined by Kennedy et al. (2001), $u_{\alpha,t} =$differential of $u_\alpha$ with respect to time. At any elevation $z$, the velocity is

$$u_2(z) = (z_\alpha - z)\nabla A + \frac{1}{2}(z_\alpha^2 - z^2)\nabla B$$ (3)
and \( \tilde{u}_2 \) = depth averaged \( O(\mu^2) \) contribution to the velocity field, given by

\[
\tilde{u}_2 = \frac{1}{(h + \eta)} \int_0^{h+\eta} u_2(z)dz
\]

\[
= \left( \frac{z_\alpha^2}{2} - \frac{1}{6}(h^2 - h\eta + \eta^2) \right) \nabla B + \left( z_\alpha + \frac{1}{2}(h - \eta) \right) \nabla A \tag{4}
\]

where \( A = \nabla.(h\mathbf{u}_a) \), \( B = \nabla.(\mathbf{u}_a) \) and \( \mathbf{V}_1 \) and \( \mathbf{V}_2 \) are dispersive Boussinesq terms given by,

\[
\mathbf{V}_1 = \left( \frac{z_\alpha^2}{2} \nabla B + z_\alpha \nabla A \right) - \nabla \left( \frac{\eta^2}{2} B_t + \eta A_t \right) \tag{5}
\]

\[
\mathbf{V}_2 = \nabla \left[ (z_\alpha - \eta)(\mathbf{u}_a.\nabla)A + \frac{1}{2}(z_\alpha^2 - \eta^2)(\mathbf{u}_a.\nabla)B + \frac{1}{2} \nabla(A + \eta B)^2 \right] \tag{6}
\]

\( \mathbf{V}_3 \) represents the \( O(\mu^2) \) contribution (\( \mu = h/L \) or dispersivity where \( L \) is the wavelength, is the perturbation parameter for all Boussinesq higher order expansions) to the expression for \( \omega \times \mathbf{u} = \omega i^z \times \mathbf{u} \), \( i^z \) being the unit vector in \( z \) (vertical) direction and is written as,

\[
\mathbf{V}_3 = \omega_0 i^z \times \tilde{u}_2 + \omega_2 i^z \times \mathbf{u}_a \tag{7}
\]

where

\[
\omega_0 = (\nabla \times \mathbf{u}_a).i^z = v_{a,x} - u_{a,y} \quad \text{and}
\]

\[
\omega_2 = (\nabla \times \mathbf{u}_2).i^z = z_{a,x}(A_y + z_{a,By}) - z_{a,y}(A_x + z_{a,Bx})
\]

\( \mathbf{R} \) here represents the collection of all physical diffusion and dissipation terms. For our present model we consider bottom friction, vegetation drag and subgrid lateral mixing,

\[
\mathbf{R} = \mathbf{R}_f - \mathbf{R}_s + \mathbf{R}_v \tag{8}
\]

Following Chen et al. (1999a), the bottom friction sink term is modeled by a quadratic law, but using the Manning’s friction factor \( n \) as,

\[
\mathbf{R}_f = \frac{gn^2}{(h + \eta)^{1/3}} \mathbf{u}_a |\mathbf{u}_a| \tag{9}
\]

The subgrid turbulent mixing sink term is given as,

\[
\mathbf{R}_s = R_{s,x} i^x + R_{s,y} i^y
\]

\[
= \frac{1}{h + \eta} \left[ \left( \nu_s[(h + \eta)u_a]_x + \frac{1}{2} \nu_s[(h + \eta)u_a]_y + \nu_s[(h + \eta)v_a]_x \right) i^x \right. \tag{10}
\]

\[
\left. + \left( \frac{1}{2} \nu_s[(h + \eta)v_a]_x + \nu_s[(h + \eta)u_a]_y \right) i^y \right]
\]

where \( \nu_s \) is the subgrid eddy viscosity represented by a Smagorinsky (1963) Large Eddy Simulation (LES) type eddy viscosity model,

\[
\nu_s = C_s \Delta X \Delta Y \left[ \tilde{v}_a^2 + \tilde{v}_\alpha^2 + \frac{1}{2} \left( \tilde{u}_a + \tilde{v}_\alpha \right)^2 \right]^{\frac{1}{2}} \tag{11}
\]
with Smagorinsky constant $C_s = 0.25$ and $\bar{u}_\alpha$ and $\bar{v}_\alpha$ being the time averaged velocities in X and Y directions, with the averaging done over 3 wave periods.

A value of $n = 0.03$ for the Manning’s coefficient (corresponding to barren wetland in (Dietrich et al., 2008)) was used for the mudflat and on the platform for non-vegetated cases, and $n = 0.033$ corresponding to dry rubble or riprap for all breakwater surfaces. For cases where the marsh platform was vegetated a value of $n = 0$ was used, meaning that the wave decay on the vegetated platform was attributed to vegetation only. Turbulent mixing term was calculated using Eq. 10.

The vegetation drag source term, used to represent wave damping by vegetation on the marsh platform, was represented by a quadratic drag law, using the reference velocity $u_\alpha$, to obtain the depth integrated vegetation drag force,

$$R_v = \frac{1}{2(h + \eta)}C_dNa_u_\alpha \left| u_\alpha \right|$$  \hspace{1cm} (12)

where, $C_d =$bulk drag coefficient of the canopy, $A = d_v min(h_v, h)$ =the plant frontal area, $N =$density of vegetation expressed as stems per unit plan area of vegetation canopy, $d_v =$the stem diameter and $h_v =$the stem height. The use of the first order reference velocity approach to estimate the drag force is considered sufficient as the $kh$ ($k$ being the wave number and $h$ the water depth) value on the vegetated platform is low enough due to smaller water depth than the mudflat, to neglect higher order effects. In Chapter 4 of this dissertation, Chakrabarti et al. (2016) discusses in detail the need for using a higher order expansion of the depth varying velocity for larger $kh$ problems.

For temporal discretization (Shi et al., 2012), third-order Strong Stability-Preserving (SSP) RungeKutta scheme was adopted and an adaptive time stepping based on Courant Friedrichs Lewy (CFL) criteria was employed such that the maximum Courant ($Co$) number did not exceed 0.5 in any part of the domain. Since a hybrid finite volume and finite difference scheme was used for spatial discretization, a fourth order MUSCL-TVD scheme with a van-Leer limiter was used to discretize the first order derivatives in the flux terms followed by an HLL approximate Riemann solver that was used to get the fluxes at cell interfaces. Higher order derivatives in the dispersive and source terms were discretized using a central difference scheme at the cell centroids (Shi et al., 2012).

2.2. Numerical Setup

Fig. 1 shows a representative numerical domain. The domain was 200m long and 150+$L_{fd}$ m wide, where $L_{fd}$ was the fish dip width that varied from 7.5 – 67.5m from case to case. A constant side slope of 1:4 (V:H) was provided for the side walls of the
fish dip and slopes of 1:3 and 1:2 for the seaward and shoreward slopes of the breakwater, consistent with existing designs for these types of breakwaters. Uniform grid resolution of $dx = 0.1m$ and $dy = 0.2m$ was maintained which allowed between 60-100 cells per wavelength within the surf zone. The number of cells varied between $1.5 - 2.2$ million for each run. A realistic equilibrium bathymetry, based on the empirical equation of Wilson and Allison (2008) applicable for Louisiana estuaries, was assumed between the marsh edge and the offshore depth. The breakwater dimensions and fish dip sizes were similar to field conditions. For the demonstration runs and to test the relative sensitivity of the longshore domain size, three such breakwater gaps were simulated with periodic conditions at the north and south ends. For the parametric study simulations, in order to save computational cost, only one gap was considered after it was found that multiple gaps had little effect on the time time averaged currents. Sponge layers $20m$ in length, were provided on east and west ends.

Random waves were generated using TMA shallow water spectrum, at 5-5.6m water depth by a source type wave maker and were shoaled up to the equilibrium mudflat profile, which began at 50m from the marsh edge. The vegetation on the platform had constant $C_d = 1.0$, $N = 500$ stems/sqm, and stem length $h_v = 25cm$. The simulations were run till quasi-steady state wave heights were reached, which was about 60-120 wave periods after the first wave hit the end of the domain and the mean circulation results averaged over the

![Figure 1: Numerical Domain. Colorbar signifies depth in meters from SWL.](image)
next 120 wave periods for the parametric study. For the vorticity results, three wave periods was used.

3. Numerical Experiment Results

<table>
<thead>
<tr>
<th>#</th>
<th>Parameter</th>
<th>Values</th>
<th># of Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$H_{mo}$</td>
<td>0.2, 0.4, 0.6, 0.8 m</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>$T_p$</td>
<td>2, 3 s</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>$GR = L_{fd}/L_b$</td>
<td>0.05, 0.10, 0.15, 0.30</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>$L_{dist}$</td>
<td>10, 20, 30, 40 m</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>$R_c$</td>
<td>-0.3, -0.6 m</td>
<td>2</td>
</tr>
</tbody>
</table>

Total number of cases 256

![Figure 2: Breakwater Layout](image)

In this study the main objectives was to develop model driven empirical relations for the offshore directed rip current and the wave height behind the fish dip with the functional
design parameters of a breakwater system. These relations can enable practicing engineers to better estimate sediment budgets as well as to address the need for further protection behind the gap when designing such structures. The five design parameters that were found to have the most effect on these two flow variables are the wave height offshore of the structure \((H_{mo})\), typically measured or obtained from hindcasting at a few hundred feet offshore of the shoreline), peak wave period \((T_{p0})\), gap ratio \((GR)\) expressed as the ratio of the fish dip width \((L_{fd})\) to the continuous length of the breakwater \((L_b)\), the distance of the structure centerline from the marsh edge \((L_{dist})\) and relative crest elevation \((R_c)\) of the breakwater from the design still water level. Fig. 2 shows the definition of the parameters and the model layout. The values of these parameters that were tested in this study are given in Table 1. These values were chosen within ranges that represent moderate wave energy environments, typical of cold front and tropical storms. During hurricanes, though larger waves are expected, the water depth in front of the structure and also on the platform will be higher and cause significantly less breaking at the structure and hence is not the primary concern here. Further hurricanes have much higher return periods and are not the primary functional design criteria for marsh shore protection structures, which are mainly meant to protect the edge and the platform from more frequently occurring, moderate energy waves during cold fronts and tropical storms.

3.1. Wave Overtopping Induced 2D Horizontal Circulation through Breakwater Gaps

As waves overtop the breakwater during a storm event, the broken waves create unique vortex patterns and the excess overtopped flux raises the water level behind the breakwater. In order to balance this imbalance in the momentum, a circulation pattern is set up between the lee side of the breakwater and the open area seaward of the breakwater. Long-shore currents are setup and circulation jets emanate as rip-currents through the fish dips. The left panels of Fig. 3(a) (for random waves) and Fig. 3(b) (for regular waves) show free-surface elevations and the right panels corresponding vortex patterns as waves break and overtop the structure. Note the increased instability in the irregular wave vorticity versus that for regular waves.

Fig. 4 shows the resultant circulation pattern under random waves, averaged over 60 wave periods after quasi steady state conditions were reached in the domain. The left panel is for a fish dip opening of 5% and right panel for an opening of 45% of the continuous length of the breakwater. Note the stronger velocities for reduced openings. This is qualitatively similar to rip current patterns typically seen during wave breaking over a barred beach. Fig. 5 shows the variation of cross-shore (top row of panels) and long-shore (bottom row of
panels) velocities for the two gap opening ratios (0.05 for red and 0.45 for blue) at several distances from the marsh edge. The breakwater is at 30m (about 3 wavelengths) (negative values indicate offshore). It is observed that a larger opening causes smaller cross-shore strength of the jet but stronger longshore velocities. On the other hand a smaller fish dip size will cause an increased strength of the jet through the fish dip with smaller long-shore currents between the breakwater and the edge.

3.2. Wave Amplitude Variation

Fig. 6 shows the variation of modeled wave amplitude (1st and 2nd harmonics) along two transects, the top panel at the centerline of the fish dip and the bottom along the centerline of the breakwater continuous length. The dashed line signifies the location of the breakwater. It is seen that the 1st harmonic increases as the wave approaches the breakwater due to the waves traveling against the circulation jet exiting from the fish dip. Behind the breakwater the jet velocity decreases and the amplitude decreases too. At the centerline of the breakwater section, we do not observe any increase in wave amplitude as no circulation jet is present. Instead the waves simply undergo a steep change in amplitude and increase in second order harmonics after the breaking event. These results also are qualitatively
Figure 4: Circulation Patterns averaged over 12 mins
Figure 5: Variation of Long-shore ($V_{\text{mean}}$) and Cross-shore ($U_{\text{mean}}$) currents with Gap Opening Sizes. Red is for $R=0.05$ (5%) and blue for $R=0.45$ (45%).
similar to those of Chen et al. (1999a) and prove that the wave transformation over the breakwater is similar to that over a barred beach. A number of interesting observations are noted at the marsh edge, where wave breaking causes the second harmonic (or non-linearity) to increase. On the platform the vegetation helps in damping the wave, while a platform with no vegetation shows a more gradual wave height reduction due to bottom roughness and breaking. Secondary breaking events on the platform are also noted for the non-vegetated case. It is also seen that the marsh edge, behind a fish dip experiences larger wave amplitudes than that behind that behind the continuous section of the breakwater and is a potential cause for concern. This can mean that the marsh edge will scour faster at the fish dip location. Later in this paper this wave height at the marsh edge behind the fish dip is modeled empirically and related to the design parameters.

3.3. Parametric Modeling of Cross-shore Mean Velocity ($U_{cs,mean}$)

The existence of the cross-shore current ($U_{cs,mean}$) through the fish dip was shown conceptually in Fig. 4 and quantitatively in Fig. 5 for a few specific cases earlier. In this section we develop a regression based empirical model for the mean current by performing a multivariate linear regression on the data from 256 cases. The fish dip current at every
point along the fish dip opening width was averaged over 120 wave periods after a quasi-equilibrium wave and current state was established. The spatially varying current was then averaged along the entire width of the fish dip to obtain the average current magnitude for each of the 256 cases. The current was always found to point offshore (-ve) and the absolute value \( U_{cs,mean} \) is therefore of interest here and is modeled parametrically.

Fig. 7 plots numerically simulated mean current magnitudes against the parametric model predicted values for all the 256 cases. The \( R^2 \) of the fit is 0.93 with a root mean square error of \( RMSE = 0.006 \text{m/s} \). It is also seen that the regression model slightly underpredicts the velocities for values greater than 30cm/s.

The non-dimensional parameters, formed as a combination of the 5 variables of interest (Table 1), were carefully selected to relate physical aspects of the flow in the fish dip, incident wave characteristics and the breakwater layout. Care was also taken to choose the right algebraic functions for each term that will provide the best fit. The resulting non-dimensional relationship for the mean current magnitude can be given as,

\[
\frac{U_{cs,mean}}{\sqrt{gh}} = 10^{-2} \times \left| -2.7 + 1.63 \frac{H_{mo}L_{p0}}{h^2} + 16.1 \tanh \left( \frac{L_d}{L_b + L_{fd}} \right) - 0.55 \ln(GR) - 0.1 \left( \frac{R_c}{H_{mo}} \right)^2 \right|
\]  \hspace{1cm} (13)

Here, the symbols are as in Fig. 2 and are defined along with applicable ranges as follows,

\[
\begin{align*}
U_{cs,mean} & = \text{Best Fit Predicted \( U_{cs,mean} \) (m/s)} \\
\text{Numerical Model Data (U_{cs,mean}) (m/s)} & = \text{Best Fit Predicted (U_{cs,mean}) (m/s)} \\
R^2 & = 0.93; \text{RMSE} = 0.006 \text{m/s}
\end{align*}
\]

Figure 7: Best Fit Regression Model for Simulated Cross-shore Mean Velocity
Table 2: Symbols and their Applicable Ranges for use in Eqn. 13:

<table>
<thead>
<tr>
<th>#</th>
<th>Symbol</th>
<th>Description</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$U_{cs,mean}$</td>
<td>Cross-shore Mean Velocity Magnitude</td>
<td>NA</td>
</tr>
<tr>
<td>2</td>
<td>$g$</td>
<td>Gravity Constant</td>
<td>$9.81 \text{m/s}^2$</td>
</tr>
<tr>
<td>3</td>
<td>$k_{p0} = 2\pi/(gT_{p0}^2/2\pi)$</td>
<td>Deep Water Wave Number corresponding to $T_{p0}$</td>
<td>$0.04 - 0.2 \text{m}^{-1}$</td>
</tr>
<tr>
<td>4</td>
<td>$H_{mo}$</td>
<td>Incident Wave Height, $(0.50 \text{m or 2 wavelengths from BW})$</td>
<td>$0.2 - 1.0 \text{m}, \text{Max } 0.63 \text{h}$</td>
</tr>
<tr>
<td>5</td>
<td>$L_{p0} = gT_{p0}^2/2\pi$</td>
<td>Deep Water Wavelength corresponding to $T_{p0}$</td>
<td>$5 - 25 \text{m } (T_{p0} = 2 - 4 \text{s})$</td>
</tr>
<tr>
<td>6</td>
<td>$h$</td>
<td>Fish dip centerline depth</td>
<td>$1.2 - 3 \text{m}$</td>
</tr>
<tr>
<td>7</td>
<td>$L_d$</td>
<td>BW centerline distance from Marsh Edge</td>
<td>$10 - 40 \text{m}$</td>
</tr>
<tr>
<td>8</td>
<td>$L_b$</td>
<td>Continuous length of BW</td>
<td>$150 \text{m}$</td>
</tr>
<tr>
<td>9</td>
<td>$L_{fd}$</td>
<td>Fish Dip Width</td>
<td>$7.5 - 45 \text{m}$</td>
</tr>
<tr>
<td>10</td>
<td>$GR = L_{fd}/L_b$</td>
<td>Gap Ratio</td>
<td>$0.05 - 0.40$</td>
</tr>
<tr>
<td>11</td>
<td>$R_c$</td>
<td>Relative crest height wrt SWL*</td>
<td>$-0.60 \text{to } -0.15 \text{m}^2$</td>
</tr>
</tbody>
</table>

*--ve indicates submerged crest

The significance of each non-dimensional term is explained below:

- **Term 1**: This is the non-dimensional cross-shore mean velocity based Froude number at the fish dip, non-dimensionalised by the shallow water celerity ($\sqrt{gh}$) at the fish dip water depth.

- **Term 2**: This term (alternatively written as $H_{mo}/h$) is similar to the Ursell number ($H_{mo}/(h/L_{p0})^2$) for waves, with the only difference being the power in the denominator and gives an estimate of how non-linear the wave is at the fish-dip location, assuming the same wavelength as the deep water. It is to be noted that replacing this term with the Ursell number expression gave a lower regression coefficient ($R^2 = 0.83$) and hence the expression in its present form is preferred. Greater the value of this term greater is the mean fish dip velocity, indicating that the more non-linear the waves, the higher are the velocities.

- **Term 3**: This ratio is a measure of the aspect ratio of the rectangular region behind the breakwater and the marsh edge. Greater this term, greater is the magnitude of the fish dip mean velocity because of two reasons mainly. First, as we move away from the marsh edge (increasing $L_d$), the water depth at the fish dip location increases and greater is the wave height at breaking on the adjacent breakwater sections resulting in stronger fish dip currents. Second, as the distance increases the size of the eddy associated with the fish dip current increases in size too (Fig. 4) and causes stronger currents, though there seems to be a limit to this growth size.
• **Term 4**: This is the gap ratio \( \left( \frac{L_{fd}}{L_b} \right) \) of the fish dip, expressed as a ratio of the fish dip width to the continuous length of the breakwater section. The fish dip mean current decreases with increasing this ratio logarithmically as will be shown later in this section. Based on the results it seems that for gap ratios greater than 0.3 the mean current varies little, and at ratio greater than or equal to 1 we have no mean current at all.

• **Term 5**: This term is the ratio of the relative crest elevation \( \left( \frac{R_c}{H_{mo}} \right) \) to the wave height \( (H_{mo}) \) (maximum permitted value of \( H_{mo} \) is 0.63h) and is a measure of the strength of the breaking at the breakwater. Greater this ratio, lower is the current magnitude due to deeper depth with respect to the incident wave height over the breakwater and consequently weaker breaking.

In the next few subsections we investigate in detail the variation of the cross-shore mean fish dip current with each of the five design parameters one by one, to provide a detailed picture of these variations with each parameter.

3.3.1. **Variation of Cross-shore Mean Velocity with Distance of Breakwater From Shoreline**

Fig. 8 and Fig. 9 show the variation of the mean fish dip cross-shore velocity magnitude with distance of the breakwater from the shoreline for \( T_{p0} = 2s \) and \( T_{p0} = 3s \) respectively. Both model results and those predicted by the regression equation are shown. It is seen that the mean velocity increases with increasing distance with a flattening curve above 30m (about 3 wavelengths) for wave heights upto 40cm, for greater wave heights the increase continues above 40m (about 4 wavelengths). Also the mean current increases with increasing wave heights, with other factors remaining constant. The regression model values match well for most of the numerical results except for waves greater than or equal to 80cm, probably due to secondary non-linear effects that cause significant rip current jet instability within the averaging period. Comparing the two figures, we also see that longer peak wave periods cause greater fish dip currents with almost a doubling of the magnitudes for wave heights greater than equal to 40cm.

3.3.2. **Variation of Cross-shore Mean Velocity with Fish Dip Gap Ratio**

Fig. 10 shows the variation of the mean fish dip current with the gap ratio of fish dip opening to the continuous breakwater length. With increasing value of this ratio the mean current decreases and theoretically, the ratio has no effect on the current after a value of 1. In reality the variation of the current seems to become negligible after a value of 0.3,
Figure 8: Cross-shore Mean Velocity Variation with Distance of Breakwater from Shoreline for $T_p = 2s$

Figure 9: Cross-shore Mean Velocity Variation with Distance of Breakwater from Shoreline for $T_p = 3s$
suggesting that from a sediment conservation perspective a fish dip gap ratio greater or equal to 0.3 will cause only a linear increase of the outflow flux as the gap size is increased. However it must be noted that even though mean current may be less for a larger gap the flux may not be due to the larger width and can still cause sediment loss. On the other hand in reality a smaller ratio may be required from erosion perspective, as will be discussed later when effect of this ratio on the wave heights at the marsh edge behind the fish dip are explained.

3.3.3. Variation of Cross-shore Mean Velocity with $R_c/H_{mo}$

Fig. 11 shows the variation of the mean fish dip velocity with the ratio of relative crest elevation to the incident wave height. It is seen that increasing $|R_c/H_i|$ (i.e, the crest becomes progressively submerged) increases the mean current. This is due to stronger wave breaking on the adjacent breakwater shoulders which results in greater excess momentum flux reaching behind the structure, which in turn drives a stronger longshore feeder eddy immediately behind the gap that ultimately causes an even stronger offshore current. This also implies that during high storm surge conditions ($h_{mp} > 1m$), waves upto 1m will not cause significant mean current while on the other hand typical water levels, during a combination of high tides and cold front conditions ($h_{mp} < 0.5m$), with moderate energy waves ($H_{mo} < 1m$), will result in significant fish dip current magnitudes.
3.4. Parametric Modeling of Wave Height at Marsh Edge behind the Fish Dip

The wave height near the marsh edge immediately behind a fish dip is a cause for concern as was described in section 3.2 due to combined wave current effect as well as the diffraction of the unbroken waves from the breakwater edges as was shown numerically (free-surface elevation on left panels) in Fig. 3. Field evidence of this type of high energy wave conditions and subsequent erosion effects can be seen in the satellite images (courtesy google map) and in field photos (Fig. 12), taken during a survey conducted by Louisiana State University researchers in a shore protection project (LA CPRA Project number PO-72) in Lake Borgne, LA, USA designed to protect the Biloxi State Wildlife Management Area. The high energy zone is termed the Effective Wave Energy Zone \((W_{eff})\) in this paper and measured linearly along the shoreline. Note that this effect is also present even when the breakwater is not submerged, though no wave-current induced wave height amplification exists and the wave height at the edge may be taken as minimum of the unbroken wave height and the maximum possible wave height at 1m away from the edge \((0.63h_{1m})\). The wave height at the marsh edge has spatial variation that depends not only on the gap size, but wave conditions and the breakwater layout.

Fig. 13 shows the plot of the numerically simulated and parametric equation predicted significant wave height at the shore \((H_{mo,shr-fd})\), behind the fish dip, for all the 256 cases. \(H_{mo,shr-fd}\) as shown in Fig. 2 was calculated 1m away from the marsh edge (to eliminate possible breaking effects due to the platform and includes reflection from the edge) and denotes the mean of the top two maximum wave heights within the effective wave energy.
zone. This was done because, as will be seen later the significant wave height has long-shore spatial variations along the extent of $W_{eff}$, picking the mean of the top two maximum values is conservative for design purposes. The regression has an $R^2$ of 0.93 with a $RMSE = 0.1m$. A safety factor of 10cm is therefore recommended, particularly for design incident wave

![Figure 12: Satellite and survey photos of typical erosional features due to high energy wave action behind a fish dip.](image)

(a) Increased wave height behind a fish dip at PO-72 site. Courtesy Google Maps.

(b) Fish dip gap

(c) Erosional scarp behind gap

Figure 12: Satellite and survey photos of typical erosional features due to high energy wave action behind a fish dip.
heights greater than 60cm. The non-dimensional parameters were carefully selected as a combination of the 5 variables of interest in Table 1 based on their physical significance and how well the combination could best fit the model. The resulting non-dimensional empirical relationship for the significant wave height at the shore, during submerged breakwater conditions is,

$$\frac{H_{mo,shr-fd}}{H_{mo}} = -0.09 + 0.71 \frac{H_{mo}}{h} + 0.14 \frac{L_{p0}}{h} + 0.96 \tanh\left(\frac{L_d}{L_b + L_{fd}}\right) + 0.13 \log(\text{GR}) + 0.37 \log\left(\frac{R_c}{H_{mo}}\right)$$  \hspace{0.5cm} (14) \]

and simpler form for the emergent breakwater conditions,

$$\frac{H_{mo,shr-fd}}{H_{mo}} = \min\left(H_{mo}, 0.63h_{1m}\right)$$  \hspace{0.5cm} (15) \]

Here, the symbols are as in Fig. 2 and are defined along with appropriate ranges as follows,

The significance of each non-dimensional term is as follows:

- **Term 1**: This is the non-dimensional maximum significant wave height at the marsh edge behind the fish dip, non-dimensionalised by the incident wave height and is a
Table 3: Symbols and their Ranges for use in Eqn. 14:

<table>
<thead>
<tr>
<th>#</th>
<th>Symbol Description</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$H_{mo,shr-fd}$ Maximum significant wave height at 1m from marsh edge</td>
<td>NA</td>
</tr>
<tr>
<td>2</td>
<td>$H_{mo}$ Incident Wave Height, (850m or 2 wavelengths from BW)</td>
<td>0.2 – 1.0m, max 0.63h</td>
</tr>
<tr>
<td>3</td>
<td>$h$ Fish dip centerline depth</td>
<td>1.2 – 3m</td>
</tr>
<tr>
<td>4</td>
<td>$L_{p0} = gT_{p0}^2/2\pi$ Deep Water Wavelength corresponding to $T_{p0}$</td>
<td>5 – 25m ($T_{p0} = 2 – 4$s)</td>
</tr>
<tr>
<td>7</td>
<td>$L_d$ BW centerline distance from Marsh Edge</td>
<td>10 – 40m</td>
</tr>
<tr>
<td>8</td>
<td>$L_b$ Continuous length of BW</td>
<td>150m</td>
</tr>
<tr>
<td>9</td>
<td>$L_{fd}$ Fish Dip Width</td>
<td>7.5 – 45m</td>
</tr>
<tr>
<td>10</td>
<td>$GR = L_{fd}/L_b$ Gap Ratio</td>
<td>0.05 – 0.40</td>
</tr>
<tr>
<td>11</td>
<td>$R_c$ Relative crest height wrt SWL*</td>
<td>-0.60 to -0.15m*</td>
</tr>
<tr>
<td>12</td>
<td>$h_{1m}$ Water Depth at 1m away from edge</td>
<td>NA</td>
</tr>
</tbody>
</table>

* – * indicates submerged crest

- Term 2: This term is the ratio of wave height with respect to the water depth and is measure of non-linearity of the wave. Greater the non-linearity, for a given wave length, greater is the wave height at the edge.

- Term 3: This term indicates the dispersion parameter of the wave. Larger the value, greater is the wave height due to increasing wave length of the incident waves.

- Term 4: This ratio is a measure of the aspect ratio of the rectangular region behind the breakwater and the marsh edge. Greater this term, greater is the wave height and actually shows a similar trend to the current case. As we move away from the marsh edge (increasing $L_d$), the wave current interaction effect at the edge decreases thus reducing the wave height. Secondly, as the distance increases the secondary breaking induced by shoaling between the breakwater and the edge further decreases the final wave height at the edge.

- Term 5: This is the gap ratio ($L_{fd}/L_b$) of the fish dip. The wave height increases with increasing gap ratio logarithmically and has an opposite behavior to that in current case, where the current decreases with increasing gap ratio.

- Term 6: This term is the ratio of the relative crest elevation ($R_c$) to the wave height ($H_{mo}$) (maximum permitted value of $H_{mo}$ is 0.63$h$). Greater this ratio, weaker is the measure of how much the wave energy at the shoreline decreases (or increases) with respect to the incident wave energy.
breaking and lower the wave height and the trend is similar to that in the current case except that the relation is a logarithmic one in this case than a parabolic one for the current.

In the following sections we will look at the variations of the fish dip shoreline wave height with respect to each of the five main design parameters.

3.4.1. Variation of Wave Height behind Fish Dip with Breakwater Distance from Shore

Fig. 14 and Fig. 15 shows the variation of the wave height at the shoreline behind the fish dip ($H_{mo,shr-fd}$) with distance of the breakwater from the shoreline, for $T_{p0} = 2s$ and $T_{p0} = 3s$ respectively. Model results as well as those predicted by the regression equation are shown. It is seen that the wave height behind the fish dip decreases with increasing distance with a flattening of the curve towards the incident wave height, beyond 30m for all the cases. In particular larger incident wave heights seem to undergo a greater decrease with distance than smaller wave heights. It also appears that the wave height slightly decreases then increases to a constant value between 20m and 30m for the 80cm case for $T_{p0} = 3s$. 

Figure 14: Maximum Wave Height Variation with Distance of Breakwater from Shoreline, measured at 1m offshore from the Marsh Edge behind the Fish Dip for $T_{p0} = 2s$. 

91
Comparing the two figures, we also see that longer peak wave periods cause greater wave heights. For a typical design it may be inferred that increasing the distance beyond 30m will not have any effect in changing the wave height at the shore behind the fish dip.

3.4.2. Variation of Wave Height near the Marsh Edge behind Fish Dip with Fish Dip Gap Ratio

Fig. 16 shows the variation of the fish dip shoreline wave height with the gap ratio of fish
dip. It is seen that increasing the gap ratio increases the wave height up to about 0.3 after which the curve flattens. This is the same value at which the cross-shore current becomes constant and indicates that opening larger than this will not significantly decrease the wave height. Even though the wave height becomes constant at 0.3, gap ratios greater than 0.1 is not recommended to keep the wave height at the marsh edge below 0.3m.

3.4.3. Variation of Wave Height near the Marsh Edge behind Fish Dip with $R_c/H_{mo}$

![Figure 17: Maximum Wave Height Variation with $R_c/H_{mo}$, measured at 1m offshore from the Marsh Edge behind the Fish Dip](image)

Fig. 17 shows the variation of fish dip shoreline wave height with the ratio of relative crest elevation to the incident wave height. It is seen that increasing $|R_c/H_i|$ (i.e., the crest becomes progressively submerged) decreases the wave height, due to stronger breaking on the adjacent breakwater shoulders, similar to the current case. From the model results we may conclude that most of the wave damage at the shoreline behind the fish dip is likely to occur when water depth on the breakwater is less than or equal to 0.75 times the incident wave height. From a design perspective the breakwater will be submerged during hurricanes and such damage will be inevitable unless special shore protection structures are build (e.g., revetments, additional wave barrier, etc.) in front of the marsh edge behind the fish dip. In the following section, the effective zone of the high energy wave action is quantified with respect to the fish dip gap size and guidelines for design of the extent of additional shore protection structures provided.
3.5. Extent of the Effective Zone of Wave Action ($W_{eff}$) near Marsh Edge behind the Fish Dip

Figure 18: Effective Zone of Wave Action behind Fish Dip for varying Gap Ratios for $T_{p0} = 2$ s (a-c) and $T_{p0} = 3$ s (d-f). Here $H_{mo}$ varies from 20-60cm.
Figure 19: Effective Zone of Wave Action behind Fish Dip for varying Gap Ratios for $T_{p0} = 2\text{s}$ (a-c) and $T_{p0} = 3\text{s}$ (d-f). Here $H_{mo}$ varies from 20-60cm.

Fig. 18 shows the variation of wave height near the marsh edge behind the fish dip for two different wave conditions (left column $T_{p0} = 2\text{s}$, right column $T_{p0} = 3\text{s}$) for increasing wave
heights (top to bottom), for a constant submergence level \( h_{mp} = 0.3m \) and distance of breakwater \( L_d = 20m \). As seen from the figure the wave height near the marsh edge shows spatial variation in the long-shore direction. The data in each panel shows the variation of \( H_{mo-shr-fd} \) with distance measured on either side of the fish dip center line (located at 0m) along the X axis. Data from only one side (the one with greater maximum wave height) of the fish dip centerline is plotted here and mirrored on the opposite side in order to make the extent of \( W_{eff} \) consistent for design and also to use the more conservative value of the wave height from two sides of the fish dip centerline. Different gap ratios \( (GR = 0.05, 0.1, 0.15, 0.3) \) are shown with vertical solid lines representing the extent of the fish dip for each case. For both the wave periods studied, it is seen that the wave heights at the marsh edge between the fish dip increases with increasing wave height. Increasing gap ratio causes increased wave heights in this zone with the \( GR = 0.3 \) an amplification of the incident wave height for the \( T_{p0} = 3s \) case. It is seen that even a 60cm wave with \( T_{p0} = 3s \), which is a typical condition during storm, can cause wave heights greater than 0.33m (or 1ft) for \( GR > 0.05 \).

Fig. 19 shows the same conditions as in Fig. 18, except that the submergence has increased to \( h_{mp} = 0.6m \). It is interesting to note that increasing the water depth causes the wave height at the fish dip shoreline to become more or less constant for the \( T_{p0} = 2s \) case for almost all the gap ratios for a given wave height. However the increased depth causes a slightly greater wave height at the edge also. The extent of the \( W_{eff} \) appears to be also the same for the different gap ratios for the smaller waves. As far as longer waves are concerned, for \( T_{p0} = 3s \) (right panels), under increased submergence levels, the cases with \( GR > 0.05 \) in fact show an increase of the shoreline wave height beyond the incident wave height and is therefore a \( GR > 0.05 \) is not recommended.

In conclusion we recommend the following guidelines towards the design of the gap ratio based on fish dip shoreline erosion perspectives,

- **Gap Ratio:** It is recommended that the gap ratio be maintained at 0.05 if no additional shore protection structures at the shoreline behind the marsh edge is provided. If additional shore protection is provided then the gap ratio may be increased upto a maximum of 0.3, however the designer should in that case follow guidelines for \( W_{eff} \) (below) to adequately protect the exposed shoreline behind the fish dip.

- **\( W_{eff} \) and Length of Additional Shore Protection:** The following sizes of \( W_{eff} \) and
consequent minimum extent of shore protection behind the fish dip is recommended:

\[
W_{eff} = \begin{cases} 
2L_{fd} & \text{if } GR \leq 0.05, \text{ no additional protection needed;} \\
3L_{fd} & \text{if } 0.05 < GR \leq 0.2, \text{ additional protection needed;} \\
L_{fd} & \text{if } 0.2 < GR \leq 0.3, \text{ additional protection needed;} 
\end{cases}
\]

\( L_{fd} \) is the fish dip opening size and except for the case with \( GR \leq 0.05 \), all other gap ratios need additional protection equal to \( W_{eff} \).

4. Conclusions

In this paper the FUNWAVE-TVD Boussinesq model was applied to simulate in field scale, the wave breaking and resulting breaking generated rip currents through gaps in a submerged breakwater, placed close to a vegetated marsh platform and on a realistic equilibrium mudflat bathymetry, under typical cold front wave conditions. The specific conclusions that were obtained from this study are as follows:

1. An empirical equation relating five design parameters - incident offshore wave height, wave period, gap ratio, distance of structure from marsh edge and relative crest elevation, to the mean current through the fish dip was developed by simulating 256 total cases for realistic range of these parameters. A non-dimensional relationship (Eq. 13) relating the local Froude number within the fish dip to the non-linearity factor of the waves, the aspect ratio factor of the region behind the breakwater, the gap ratio and the free-board (breaking strength) factor was established by a multiple linear regression.

2. The wave height at the shoreline behind the fish dip was found to be higher than the wave heights behind the protected portions of the breakwater due to two reasons 1) During emergent conditions local diffraction and the fact that unbroken waves reach the marsh edge cause increased wave heights, while 2) During submerged conditions, together with local diffraction, wave-current interaction as a result of the rip current increases the resultant wave height behind the fish dip. Similar to the mean current this wave height was also modeled empirically and a non-dimensional relationship (Eq. 14) relating the non-linearity parameter, the dispersion parameter, the aspect ratio factor, the gap ratio and the free-board (breaking strength) factor was obtained for submerged and emergent conditions (Eq. 15).
3. The breaking-generated mean current in the fish dip was found to increase with increasing distance up to 30 m (2.5-4 times the wavelength), at which the current becomes the maximum and no appreciable increase is seen. Since this value range of 2.5-4 times the wavelength is greater than the 1.5-2.5 times the wavelength range (found from vertical circulation considerations as in Chapter 3), it is therefore recommended that distance be kept about 2-2.5 times the wavelength for the final design.

4. The breaking-generated mean current decreases with increasing gap ratio till about 0.3 after which it will decrease very slowly to the theoretical limit of zero mean current. On the other hand, from wave energy perspective, a maximum gap ratio of 0.05 can be allowed beyond which the waves can cause damage to the shoreline behind the fish dip. It is therefore recommended that to keep the discharge through the fish dip small as well as to protect the shoreline behind the marsh edge the fish dip gap ratio be kept at 0.05 or less. Where this is not possible, due to construction or other difficulties a maximum ratio up to 0.3 can be used. However, adequate shoreline protection (revetments) is recommended spanning up to a distance of 2 times the fish dip for the zone behind the fish dip in those cases.

5. The empirical equations and design guidance presented in this paper are based on numerical experiments using a well tested Boussinesq model. Laboratory or field experiments are highly recommended to validate the findings.

5. Acknowledgements

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1. Introduction

Coastal wetlands help in maintaining water quality, provide natural habitat for a variety of flora and fauna (Shutes, 2001) as well as control soil erosion (Gaciaa and Duarteb, 2001). Apart from the numerous ecological benefits (Barbier, 2007), vegetated wetlands can be used effectively to dissipate wave energy and storm surge through form drag of the stems and turbulent dissipation within the canopies (Danielsen et al., 2005; Kathiresan and Rajendran, 2005; Barbier et al., 2008; Costanza et al., 2008). A large number of experimental and numerical modeling studies, extending from laboratory scale to field scale have been performed over the past two decades, to improve the knowledge base of wave and surge interaction with vegetation.

Knutson et al. (1982) was one of the first to investigate wave damping properties of *Spartina alterniflora* marshes in Chesapeake Bay, VA, USA. While wave attenuation, in the field, during low to moderate wave energy conditions, have been well reported in the literature (Möller et al., 1999; Tschirky et al., 2000; Bradley and Houser, 2009), studies looking at higher wave energy environments, such as those during tropical storms or hurricanes are relatively scarce (Jadhav and Chen, 2013; Jadhav et al., 2013), primarily due to difficulty of deployment and measurement during a storm. On the other hand, laboratory experiments, performed under well controlled environments with submerged artificial vegetation stems, either rigid (Lowe et al., 2007) or flexible (Asano et al., 1988; Kobayashi et al., 1993; Schuten et al., 2004; Blackmar et al., 2014) and mimicking real vegetation properties in terms of diameter, density and rigidity, have been used to quantify wave attenuation and deduce analytical solutions based on the wave energy balance approach. Studies into emergent vegetation using both rigid and flexible dowels (Augustin et al., 2009) have related the drag coefficient to the Reynolds number and Keulegan Carpenter number of the flow. In addition to artificial vegetation in the laboratory, some researchers have also conducted experiments with a variety of natural vegetation species, either in typical flume scales (Fonseca, 1992; Wu et al., 2011, 2012) or in near field scales (Chakrabarti and Smith, 2011; Chakrabarti, 2011; Blackmar, 2013), all of which have contributed immensely to the existing database of
bulk drag coefficients of different vegetated canopies under waves, against which numerical experiments can be validated. In particular, Wu et al. (2012) reports a wide range of regular and random wave experiment results for wave breaking on a vegetated beach and presents an opportunity to validate numerical models that simulate wave breaking on wetlands. This paper is the first to validate this experiment numerically using a Boussinesq model.

In the field of numerical modeling, preliminary work, before the advancement of super computers, involved developing analytical solutions, assuming linear wave theory for the horizontal velocity, to the wave energy conservation equation or by solving simplified forms of the momentum equation (Kobayashi et al., 1993; Mendez and Losada, 1999) and assuming loss due to vegetation as another energy dissipation term like bottom friction (Knutson et al., 1982; Camfield, 1983) or a quadratic drag force term (Dalrymple et al., 1984). Mendez and Losada (2004) extended the theory of Dalrymple et al. (1984) to an empirical model for wave damping and wave breaking over variable depth vegetation fields and provided calibrated drag coefficients for artificial flexible plant canopies. Studies into random wave attenuation by vegetation has been extended by Chen and Zhao (2012) to gain insights into the spectral distribution of the energy dissipation.

With the advent of High Performance Computing (HPC) since the last two decades, the research effort has shifted to numerically solving either the 2D (Maza et al., 2013; Wu et al., 2013; Ma et al., 2013) or 3D (Li and Yan, 2007; Zhan et al., 2014; Marsooli and Wu, 2014) Navier Stokes Equations with Volume of Fluid (Hirt and Nichols, 1981) approach for the free surface, or the simplified Euler equations with non-hydrostatic approach (Ma et al., 2013; Zhu and Chen, 2015) for the free surface and considering vegetation resistance as a time and spatially dependent sink term in the momentum equation. Multi layer approaches (Lynett and Liu, 2004b,a; Zhu et al., ????) that solve the Euler equations and match the independent quadratic polynomial expansions for each layer at the interfaces, have been successfully applied to investigate tsunami generated landslides (Lynett and Liu, 2005). A large eddy simulation model, running on a massively parallel computer, that resolves the flow around individual cylinders has also been reported recently (Chakrabarti et al., 2016). However, the above models require at least several vertical layers for accuracy and can only be applied to laboratory scales with the currently available computational power. For entire basin or ocean scales, wave action balance type models are used (Zhao and Chen, 2014; Hu et al., 2015), which effectively eliminate the vertical coordinate from the calculations. These models, however, do not resolve the wave phase and hence fail to explain effects such as wave diffraction and reflection, nor do they provide any knowledge of the horizontal velocity.
that drive wave generated circulation patterns, all of which are very important physical phenomena within a few hundred meters of the marsh edges that move sediment. It is here that the merit of phase resolving long-wave models, that solve the depth averaged (shallow water) equations, lie.

The application of exclusively shallow water models are however limited to non-dispersive or weakly dispersive ‘long-waves’ only, eg. tsunamis (Thuy, 2010) and tidal waves (Wu et al., 2001). Frequency dispersion of water waves is a phenomenon by which waves of different wavelengths travel at different phase speeds and hence fully dispersive wave models are required to simulate typical wind driven waves, propagating from deep water to shallow water. One way of incorporating the dispersive behavior is by solving the Green-Naghdi equations (Green and Naghdi, 1976) and has been developed (Zhang et al., 2013, 2014; Panda et al., 2014) and applied to investigate several nearshore phenomena (Zhang et al., 2015). In our present work we, follow our discussion on Boussinesq models, instead, from the extended equations of Peregrine (1967). Madsen et al. (1991) introduced a third-order term to the classical equations by Peregrine (1967), with a free parameter, into the momentum equation and improved the model performance. Nwogu (1993) derived a new set of governing equations, from the three-dimensional Euler equations using a concept of reference horizontal velocity evaluated at a reference depth, that was applicable from deep to shallow water, with phase speed errors less than 2% between $kh = 0$ to $\pi$, where $k$ is the wavenumber $h$ the water depth. Wei and Kirby (1995) derived a fully nonlinear form of the equations, applicable for nonlinear waves, which were shown to be very effective in simulating highly asymmetric waves in shallow water (Kennedy et al., 2001), wave-breaking induced currents (Chen et al., 1999a), particularly when coupled with a one-equation turbulence model for simulating turbulent kinetic energy produced by wave breaking (Nwogu and Demirbilek, 2001), long-shore currents (Chen et al., 2003), wave setup close to the shoreline (Chen et al., 2000; Kennedy et al., 2000), wave-current interaction (Chen et al., 1998, 1999b), wave transformation in inlets and harbors (Shi et al., 2001), wind effects on wave propagation (Chen et al., 2004; Liu et al., 2015) and surf-zone tracer dispersion (Feddersen, 2007; Spydell and Feddersen, 2009). A slightly different set of equations from Wei and Kirby (1995) was developed in Lynett et al. (2002) and Lynett and Liu (2002), the latter even allowing for the time variation of the dispersive terms, which extended the model for simulation of landslides generated tsunamis (Lynett, 2006) and tsunami runup effects (Korycansky and Lynett, 2007; Geist et al., 2009). The most complete set of equations, correcting deficiencies in the representation of the higher order advection terms was proposed in Chen (2006), who
eliminated the \( z \) dependency by double-integrating the Boussinesq equations, resulting in a model that included the vertical vorticity. This approach forms the basis of the present state of art Boussinesq model FUNWAVE or ‘FUlly Nonlinear (Boussinesq) WAVE’ model. This work also extended the model for application for flow through porous media by considering a friction force term consisting of a linear (laminar) friction force, a turbulent (nonlinear) drag force and an inertia force. A similar model to FUNWAVE, developed by Kim et al. (2009), has been applied to investigate hurricane wave overtopping (Lynett et al., 2010) and dispersive effects at a surge bore front (Kim and Lynett, 2011) and recently extended to handle multiple layers (Kim and Lynett, 2012).

In spite of the massive advances in Boussinesq modeling in FUNWAVE, important issues still remained (Zhen, 2004) which was preventing its expansion to more practical problems. The model showed weak instability at high wave numbers near the grid Nyquist limit, instability in the eddy viscosity model for wave breaking and also in ‘beach slot’ technique, which was being used to handle the moving water line in the swash zone. In order to address these deficiencies Shi et al. (2012) proposed a new version, FUNWAVE-TVD, employing a hybrid method (Tonelli and Petti, 2009, 2010), by which the advective part of the nonlinear shallow water equations are handled using the TVD finite volume method while dispersive and source terms (existing in the fully non-linear Boussinesq equations) are handled using conventional finite differencing. This method allows wave breaking to be handled implicitly by the treatment of weak solutions in the shock-capturing TVD scheme, making an explicit formulation for breaking wave dissipation unnecessary and thus ridding the model of one source of instability. In addition, shoreline movement may be handled quite naturally as part of the Reiman solver underlying the finite volume scheme and the slot technique associated instability can be eliminated too.

The application of Boussinesq wave models in vegetated flows is relatively new, where the vegetation resistance is handled either as a Darcy type bottom resistance (Yang et al., 2015), by solving a separate canopy flow model (Karambas et al., 2016) or by applying a quadratic drag source term to the the depth averaged Boussinesq equations (Kuiry and Ding, 2016). In fact, Kuiry and Ding (2016) used the depth averaged velocity without any expansion to higher orders, in quantifying the vegetated drag force by the quadratic law, raising questions as to the accuracy of the model to correctly handle highly non-linear waves. In absence of any existing studies on wave vegetation interactions using the fully non-linear Boussinesq equation (Chen, 2006), this paper is aimed at developing and testing a consistently derived vegetation model for use with the widely used Boussinesq equations.
In this paper we introduce an extended version of FUNWAVE-TVD model, which was developed within the CACTUS framework (Goodale et al., 2003) using the same equations as in FUNWAVE-TVD. Though CACTUS has been used widely in the field of astrophysics (Löffler et al., 2014), its application in coastal engineering is limited (Tao et al., 2010; Oler et al., 2015). The two main improvements over the existing FUNWAVE-TVD are the use of the Adaptive Mesh Refinement (AMR) (Schnetter et al., 2004) capability of CACTUS, which now enables the user to simulate highly irregular shorelines with greater resolution within the surf zone at reasonable computational costs and an improved higher order representation of the vegetation drag force. The three main objectives of this paper are thus as follows: 1) To validate the vegetation model, using the higher order expansion of the depth varying horizontal velocity in the quadratic drag term, against laboratory experiments; 2) compare the wave attenuation, for different wave conditions, using the two approaches: (a) by using the expanded depth varying velocity and (b) the reference velocity, in the stem depth integrated vegetation drag force term. They will also be compared with the linear wave theory based analytical solutions derived from the wave energy balance equation; and 3) extend the validated vegetation model to a realistic field problem and investigate hurricane wave breaking induced setup and circulation under different storm scenarios with and without vegetation cover in a Louisiana wetland.

2. Numerical Model Description

2.1. Governing Equations

The main governing equations for CaFUNWAVE, similar to FUNWAVE-TVD are the 2DH depth-integrated conservation and horizontal momentum equations (Chen, 2006; Shi et al., 2012),

\[ \eta_t + \nabla \cdot [(h + \eta)(u_\alpha + \tilde{u}_2)] = 0 \]  
\[ u_{\alpha,t} + (u_\alpha \cdot \nabla)u_\alpha + g \nabla \eta + \mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3 + \mathbf{R} = 0 \]

where, boldface notations denote vector quantities with \( \eta = \) free surface elevation at time \( t \), \( h \) = still water depth, \( u_\alpha = u_\alpha i_x + v_\alpha i_y \) the reference velocity at reference elevation \( z_\alpha = -0.53h + 0.47\eta \) defined by Kennedy et al. (2001) and \( u_{\alpha,t} = \) differential of \( u_\alpha \) with respect to time. At any elevation \( z \), the velocity is

\[ u_2(z) = (z_\alpha - z) \nabla A + \frac{1}{2}(z_\alpha^2 - z^2) \nabla B \]
and the depth averaged $O(\mu^2)$ contribution to the velocity field, $\tilde{u}_2$ is given by

$$\tilde{u}_2 = \frac{1}{(h + \eta)} \int_0^{h + \eta} u_2(z)dz$$

where $A = \nabla.(hu_\alpha)$, $B = \nabla.(u_\alpha)$ and $V_1$ and $V_2$ are dispersive Boussinesq terms given by,

$$V_1 = \left(\frac{\eta^2}{2}\right)B + \frac{1}{2}\left(\eta^2 + \eta B\right)$$

$$V_2 = \nabla \left[ (z_\alpha - \eta)(u_\alpha \nabla)A + \frac{1}{2}(\eta^2 - \eta^2)(u_\alpha \nabla)B + \frac{1}{2}\nabla(A + \eta B) \right]$$

$V_3$ represents the $O(\mu^2)$ contribution ($\mu = h/L$ or dispersivity where $L$ is the wavelength, is the perturbation parameter for all Boussinesq higher order expansions) to the expression for $\omega \times u = \omega i^z \times u$, $i^z$ being the unit vector in z (vertical) direction and is written as,

$$V_3 = \omega_0 i^z \times \tilde{u}_2 + \omega_2 i^z \times u_\alpha$$

where

$$\omega_0 = (\nabla \times u_\alpha).i^z = v_{\alpha,x} - u_{\alpha,y}$$

$$\omega_2 = (\nabla \times u_2).i^z = z_{\alpha,x}(A_y + z_\alpha B_y) - z_{\alpha,y}(A_x + z_\alpha B_x)$$

$R$ here represents the collection of all physical diffusion and dissipation terms. For our present model, we consider bottom friction, vegetation drag and subgrid lateral mixing,

$$R = R_f + R_v - R_s$$

Following Chen et al. (1999a), the bottom friction sink term is modeled by a quadratic law, but using the Manning’s friction factor $n$ as,

$$R_f = \frac{g n^2}{(h + \eta)^{1/3}} |u_\alpha|$$

The subgrid turbulent mixing sink term is given as,

$$R_s = R_{s,x} i^x + R_{s,y} i^y$$

$$= \frac{1}{h + \eta} \left[ \left( \nu_s [(h + \eta)u_\alpha]_x + \frac{1}{2} \nu_s [(h + \eta)u_\alpha]_y + \nu_s [(h + \eta)v_\alpha]_x \right) i^x \right.$$ \hspace{1cm} (10)

$$+ \left. \left( \nu_s [(h + \eta)v_\alpha]_x + \nu_s [(h + \eta)u_\alpha]_y \right) i^y \right]$$

where $\nu_s$ is the subgrid eddy viscosity represented by a Smagorinsky (1963) Large Eddy Simulation (LES) type eddy viscosity model,

$$\nu_s = C_s \Delta X \Delta Y \left[ \tilde{u}_\alpha^2 + \tilde{v}_\alpha^2 + \frac{1}{2}(\tilde{u}_\alpha + \tilde{v}_\alpha)^2 \right]^{1/2}$$
with Smagorinsky constant $C_s = 0.25$ and $\bar{u}_\alpha$ and $\bar{v}_\alpha$ being the time averaged velocities in X and Y directions, with the averaging done over 3 wave periods.

$R_f = 0$ and $R_s = 0$ was assumed for the 1D vegetated beach, in the verification of Wu et al. (2012) experiments presented in this paper, thus the momentum source term and hence wave attenuation was entirely due to vegetation effects. For the 2D field cases, a value of $n = 0.03$ for the Manning’s coefficient (corresponding to barren wetland in (Dietrich et al., 2008)) was used for the mudflat and $n = 0$ for the vegetated platform, meaning that wave decay on the platform was due to vegetation only in this case too. Also in the 2D case, turbulent mixing term was active and calculated using Eq. 10.

The vegetation drag source term was represented by two approaches. In the first approach only the reference velocity $u_\alpha$ was used to obtain the depth integrated vegetation drag force source term and will be referred to as the **first order approach** henceforth,

$$R_v = \frac{1}{2(h + \eta)} C_d N A u_\alpha |u_\alpha|$$  \hspace{1cm} (12)

where, $C_d =$bulk drag coefficient of the canopy, $A = d_v \min(h_v, h) =$the plant frontal area, $N =$density of vegetation expressed as stems per unit plan area of vegetation canopy, $d_v =$the stem diameter and $h_v =$the stem height.

The second approach, henceforth referred as the **full integration approach** involves using the depth varying velocity, $u(z)$, obtained by expanding the horizontal velocity by the perturbation approach of Hsiao and Liu (2002) as,

$$u(z) = u_\alpha - \mu^2 \left[ \frac{1}{2}(z^2 - z_\alpha^2) \nabla B + (z - z_\alpha) \nabla A \right] + H.O.T$$  \hspace{1cm} (13)

where $H.O.T$ contains higher order terms of $O(\mu^4)$ and above and using it to find the depth integrated stem drag force term as,

$$R_v = \frac{1}{2(h + \eta)} \int_0^{\min(h_v, h+\eta)} C_d N d_v u(z) |u(z)| dz$$  \hspace{1cm} (14)

In CaFUNWAVE when implementing the full integration approach the higher order terms up to fourth order were preserved. MATHEMATICA was used to simplify the source term above and to get a form that could be easily written in C++ code format in CACTUS. Eqns. 26 and Eqns. 27 , shown in the appendix, represent the contribution of terms below $O(\mu^4)$ to the final drag force. The resulting integrand was integrated using numerical integration within CaFUNWAVE.

A very similar approach to the first order approach, but using the depth averaged velocity ($\bar{u}$), instead of the reference velocity, was used by Kuiry and Ding (2016) in their 1D depth averaged Boussinesq equations. While Kuiry and Ding (2016) showed validations of their
model for a number of different laboratory experiments of Wu et al. (2011) with vegetation, none of the cases were for large $kh$, which is the expected condition for hurricane waves at field scale depths. Further, while the experimental data were matched by calibrating $C_d$, the $C_d$ values were considerably larger than those found in the original experiments using analytical solutions of simple wave energy balance equation and linear wave theory for the depth variation of the velocity. The full integration approach is therefore introduced here with two primary goals in mind, first, to develop a consistent vegetation drag model that can be used for the present type of Boussinesq models that use the reference velocity and second to show the need of the physically realistic full integration approach, that retains the higher order terms arising out of the velocity expansion in the drag force which is a more realistic description of the wave drag across large number of wave and submergence conditions. It must be pointed out that the effect of vegetation is still reflected as as a momentum sink, hence, within the canopy, the vertical variation of horizontal the velocity profile still maintains its wave signature, only with a lower amplitude due to the attenuation and does not undergo any fundamental change in shape due to the presence of vegetation.

Temporal and spatial discretization schemes were the same as in FUNWAVE-TVD (Shi et al., 2012). For temporal discretization, third-order Strong Stability-Preserving (SSP) RungeKutta scheme was adopted and an adaptive time stepping based on Courant Friedrichs Lewy (CFL) criteria was employed such that the maximum Courant ($Co$) number did not exceed 0.5 in any part of the domain. For the runs employing AMR, constant time stepping was used, with the time step size decided such that $Co < 0.5$ based on the smallest grid size. Since a hybrid finite volume and finite difference scheme was used for spatial discretization, a fourth order MUSCL-TVD scheme with a van-Leer limiter was used to discretize the first order derivatives in the flux terms followed by an HLL approximate Riemann solver that was used to get the fluxes at cell interfaces. Higher order derivatives in the dispersive and source terms were discretized using a central difference scheme at the cell centroids.

3. Model Verification

3.1. 1D Wave Breaking and Setup on a Vegetated Beach (Wu et al., 2012)

The CaFUNWAVE model was compared with laboratory flume experiments conducted at the University of Mississippi by Wu et al. (2012), who studied the wave breaking and setup through an array of rigid vegetation on a sloping beach. Fig. 1 shows the numerical domain used to simulate the laboratory experiments. The vegetation properties were same
as in the original experiments with density ($N$) 3182 stems/m$^2$, uniform stem diameter ($d_v$) 3.2mm and uniform stem height ($h_v$) of 20cm.

Figure 1: Numerical Domain for 1D Boussinesq Modeling

The CaFUNWAVE model uses a linear wavemaker and in order to create linear wave conditions, the numerical wavemaker had to be shifted to deeper depth than what was originally in the laboratory experiment. The domain was extended using a slope leftward from where the wavemaker was located in the original experiment. In order to ensure that this change of the inlet domain did not alter the accuracy of the results, the wave height generated at the wavemaker was calibrated to match the wave height at the entrance of the vegetation canopy ($H_i$) (or in case of no vegetation, the point where the vegetation canopy would have been located). In addition, careful attention was paid to match the simulated overall wave height profile, from the origin (the toe of the 1:7 beach) upto the beginning of the canopy with the experimental data. For all calibration results reported in this study a Mean Average Percentage Error (MAPE) of less than 5% and correlation coefficient $R^2 > 0.95$ (for cases where profiles had to be matched) was adopted as reliable metrics. The waves were generated at 1m water depth and the water depth in the zone of interest (from origin upto the end of the canopy) was maintained constant at 0.4m as in the original experiment. Sponge layers of 7.5m length were provided at the inlet and outlet of the flume to absorb the generated waves. The main goal of this verification study was to identify the optimum grid resolution requirement, the accuracy of the AMR method and ability of the improved vegetation treatment to handle wave height and wave setup evolution on a vegetated slope, similar to what is expected in the field. Numerical modeling results
from five experiments selected from the original data-set and spanning a range of wave conditions are presented here. The wave conditions, namely the wave period (T), incident wave height at canopy entrance from experiment (H_{i-Expt}), the corresponding calibrated incident wave height in the numerical model (H_{i-Num}), the wavelength (L) at 0.4m depth, the \( kh \) value (\( k \) being the wave number and \( h = 0.4m \) the water depth) and the calibrated vegetation drag coefficient (\( Cd_{veg} \)) that was used to match the experimental values are given in Table 1.

<table>
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<th>( H_{i-Num} ) (cm)</th>
<th>L (m)</th>
<th>( kh )</th>
<th>( Cd_{veg} )</th>
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</tbody>
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3.1.1. Grid Sensitivity Analysis

A grid sensitivity analysis is important when simulating wave breaking for this type of Boussinesq model which uses the TVD (Total Variable Diminishing) technique in conjunction with a non-linear shallow water equation to handle wave breaking implicitly. The threshold for switching off the Boussinesq terms was determined when wave height to water depth ratio \( (H/h) \) exceeded 0.8. This means that optimal grid resolution is important to introduce just the right amount of diffusion at the breaking point to correctly mimic the physics of the wave breaking. Wave case 400142160 was selected for the grid sensitivity test, which was the most non-linear among the two shortest wavelength cases (40028160 and 400142160). The grid was refined by AMR, with the inlet AMR boundary being placed at origin \( (X = 0m) \) and the outlet at the point where the beach ends \( (X = 17.5m) \) (Fig. 1). Separate tests with regular waves on a flat bed showed that if the AMR boundary was placed at a point where \( kh > 0.8\pi \), dispersion error propagated within the AMR region, hence it was carefully ensured that the AMR boundary be placed at \( kh < 0.8\pi \) for all the simulations in this paper. In the present simulations none of the \( kh \) values at the AMR boundaries exceeded 0.3\pi. The grid size outside of the AMR region was fixed at 0.08m with successive refinement levels placed within \( 0 \leq X \leq 17.5 \) to obtain the target grid size on the beach. The grid sizes within this zone were varied as 0.04m(L/71), 0.02m(L/142),
0.01\(m/(L/284)\) and 0.005\(m/(L/568)\) for sensitivity analysis, where \(L = 2.84m\) was the wavelength. The grid sensitivity results are shown in Fig. 2. The drag coefficient for the vegetated cases was calibrated with respect to the 0.01\(m\) grid and maintained constant for all the resolutions. It is seen that both the 0.04\(m\) and the 0.02\(m\) grids cause the breaking location to shift shoreward, though the wave heights before the breaking point are better correlated. These two resolutions also show greater wave heights post breaking. The finer grid sizes (0.005\(m\) and 0.01\(m\)) on the other hand have better agreement with experimental results for the breaker location as well as wave height post breaking. However the 0.005\(m\) grid slightly underpredicts the wave height before the breaker location indicating that the 0.01\(m\) resolution is optimal for this case.

![Graph showing grid sensitivity test for Wave Case 400142160](image)

For the case with vegetation, it is seen that the results are not very sensitive to the grid resolution, neither at the breaker point, nor in the canopy, except for the coarsest grid (0.04\(m\)) which shows a premature breaker just before the canopy. Before the breaking point, the 0.02\(m\) grid has the best agreement for the wave height with the experimental data,
the 0.005m and 0.01m both show slightly larger wave heights. A clear reflection pattern is observed, similar to the experiments, in the region before the canopy for the vegetated cases and likely increases the incident wave height at the canopy. Based on these results the 0.01m grid is chosen for the remaining simulations. In general 275 points per wavelength was found to be an optimal grid resolution for non-vegetated cases with breaking, while for cases with breaking in the vegetation canopy, this may be relaxed to 85-100 points per wavelength. At least 25 points per wavelength was required to simulate and shoal the waves accurately, without diffusion, in the non-breaking zone.

3.1.2. Comparison of Wave Height ($H$) and Wave Setup with Experiments

Figs. 3 and 4, show the comparison of the numerical results with laboratory experiments of Wu et al. (2012) for wave height and wave breaking induced setup on a vegetated beach for the five test cases. As seen from the figures the agreement is very good, especially for the vegetated cases. The breaker location and the breaking wave height are well predicted by the model for both vegetated and non-vegetated cases. For the non-vegetated cases the only issue was the overprediction of the broken wave height in the surf-zone post breaking. This was a problem which could not be fixed, neither by changing grid resolution (Fig. 2), nor by altering the $H/h$ threshold for breaking. It is believed that this could be a deficiency of the TVD Boussinesq modeling approach, which fails to simulate the violent free surface breaking cascade within the surf zone. In Chakrabarti and Chen (2016), it was shown that a Navier Stokes model with an LES turbulence closure can better model the surf zone free surface dynamics. Nevertheless, for our field application since the platform is vegetated, this deficiency is not of concern for the results presented later, except the non-vegetated case, in which case the repercussions are stated later.

The over prediction of the wave height is also the reason why the setup (Fig. 4) is also underpredicted. A higher wave height in the surf zone means lesser secondary breaking and hence lesser excess momentum flux is transferred to the water column, consequently the smaller setup. Regarding the vegetated cases, the setup (and also set-down) are considerably lower than the non-vegetated cases, proving the ability of the model to predict the notion that vegetation on a slope can reduce breaking induced wave setup. While the model is now verified for laboratory scale experiments, before going to field scale application, the applicable range (particularly model sensitivity to $kh$ and $h_v/h$ or the submergence ratio) when the full depth integrated velocity approach may be superior to the first order approach will be investigated in the following section.
Figure 3: Comparison of simulated Significant Wave Height ($H_s$) with Wu et al (2012)
Figure 4: Comparison of simulated Wave Setup with Wu et al (2012)
4. Numerical Experiment Results

4.1. Comparison of Wave Attenuation through Vegetation using the two Approaches: Reference Elevation Velocity ($U_a$) and Depth Integrated fully expanded Velocity ($U_v$)

Existing Boussinesq models like FUNWAVE-TVD uses the reference velocity ($U_a$) for calculating the drag force by the vegetation while CaFUNWAVE uses the vertical profile of the horizontal velocity to compute the depth integrated force. Additionally, all the higher order Boussinesq terms, from the expansion of the horizontal velocity, are retained in CaFUNWAVE, when formulating the quadratic drag law equation. In order to compare the difference in the wave attenuation for these two approaches, CaFUNWAVE model runs were set up to run separate simulations in the first order (using the velocity at the reference elevation) mode as well as in the full depth integration mode (using the depth integrated velocity). Simulations were run for each mode, with varying incident wave heights ($H = 10 - 110 cm$), water depths ($h = 1 - 8 m$) and submergence ratios ($h_v/h = 0.25 - 1.0$) but with constant time period ($T = 3.5s$) and drag coefficient ($C_d = 1.0$) over a vegetation canopy (60m length, $d_v = 10mm$, density $N = 1000$ stems/m$^2$) placed on a flat bottom wave flume. $kh$ ranged from 0.19$\pi$ to 0.84$\pi$ and covers the complete applicable range for field scale wave conditions. AMR was not used for these cases and uniform grid resolution of 0.1m was maintained ($L/104 - L/190$) with maximum and minimum wavelengths tested being $L = 10.4m$ and $L = 18.9m$. In addition to the numerical results, wave attenuation predicted by two analytical solutions have been compared. The first one, derived below and is similar to Dalrymple et al (1984), but uses the velocity at the reference elevation. The second one is the analytical solution of Mendez and Losada (2004), which uses the depth integrated velocity, assuming linear wave theory for the full depth. These solutions are meant to test the accuracy of the code and investigate under what wave conditions, they may or may not be applicable.

In order to derive the analytical solution which uses the velocity at the reference elevation, following Dalrymple et al (1984), we start from the energy balance equation for waves through vegetation,

$$\frac{dE C_g}{dx} = -\epsilon_d$$  \hspace{1cm} (15)

where, $E$ = wave energy density = $1/2 \rho a^2$, $\rho$ = fluid density, $g$ = gravity, $a$ = wave amplitude = $H/2$, $H$ = wave height at location $x$, $C_g$ = wave group velocity = $nC$, $n = 1/2(1 + 2kh/sinh(2kh))$, $C$ = wave celerity = $\sqrt{(g/k)\tanh(kh)}$, $k = 2\pi/L$ = wave number, $L$ = wave...
length, \( h \) = water depth and \( \epsilon_D \) = energy dissipation per unit planform area of the vegetated patch, which may be written as

\[
\epsilon_D = F_D U_a = \frac{1}{2} \rho C_d N A U_a |U_a| U_a
\]  

(16)

\( F_D \) = drag force, written as quadratic drag law and \( C_d \), \( A \), \( N \), \( d_v \) and \( h_v \) are the same as was defined in 12. Using linear wave theory, \( U_a \) can be expressed as,

\[
U_a = \frac{\pi H \cosh(k(h + z_a))}{\sinh(kh)}
\]  

(17)

Substituting 16 in 15 and using 17, we may write for a flat bed,

\[
\frac{dEC_g}{dx} = \frac{1}{2} \rho C_g \frac{d^2 a}{dx^2} = \frac{4\pi^3 \rho C_d A \cosh^3(k(h + z_a))}{T^3 \sinh^3(kh)} \left[ a^3 \right] B
\]  

(18)

Solving for the above equation yields,

\[
\frac{a}{a_0} = \frac{H}{H_0} = \frac{1}{1 + \frac{H_0}{\rho g C_g H_0}} = \frac{1}{1 + \alpha x}
\]  

(19)

where \( a_0 \) is the incident wave amplitude at the beginning of canopy \( (x = 0) \) and

\[
\alpha = \frac{4\pi^3 C_d A \cosh^3(k(h + z_a))}{T^3 \rho g C_g \sinh^3(kh)} H_0
\]  

(20)

By knowing \( H_0 \), which is the wave height at the beginning of the canopy the complete distribution of wave height \( H(x) \) can thus be obtained within the vegetation length.

The second analytical solution is the one by Mendez and Losada (2004), who extended Dalrymple et al (1984) formulation for flat bottom and found the solution for the depth integrated force case, as

\[
\frac{a}{a_0} = \frac{H}{H_0} = \frac{1}{1 + \beta x}
\]  

(21)

where

\[
\beta = \frac{4}{9\pi} C_d N k \left( \frac{\sinh^3(kh_v) + 3\sinh(kh_v)}{\sinh(2kh) + 2kh \sinh(kh)} \right)
\]  

(22)

Similar to Eq. 19, by knowing \( H_0 \) at the beginning of the canopy, we can estimate the wave height variation within the total vegetated length. Wave height distributions within the canopy, computed using Eqs. 19 and 21 will be henceforth respectively termed as \( H_Ua \) and \( H_Uv \) and corresponding first order and full integrated velocity approach numerical results as \( H_1 \) and \( H_F \) respectively.

Fig. 5 shows the wave height (normalized by the wave height at the canopy entrance) variation within the vegetated zone for a lower \( kh \) value (left panel) and a higher \( kh \) value (right panel) and for increasing incident wave heights from top to bottom. In general, the
Figure 5: Comparison of First Order and Full Integration Approaches: Varying Wave Heights
attenuation using the full integration approach is lower than the reference velocity approach. The water depth was \( h = 2\) m for \( kh = 0.29\pi \) with wavelength \( L = 13.8m \) and \( h = 8m \) for \( kh = 0.84\pi \) with wavelength \( L = 19.05m \). The vegetation height was half the water depth \( (h_v/h = 0.5) \). The description texts also show the mean percentage difference over the total vegetated length, of first order simulated wave heights with the full integration approach wave heights \( ([H_F - H_1]/H_F \times 100\%) \) as well as the mean percentage difference of the Mendez and Losada (2004) analytical solution with the simulated full integration values \( ([H_F - H_{UV}]/H_{UV} \times 100\%) \).

The former difference gives a quantitative idea of the departure of the first order computed wave heights with respect to the fully integrated velocity approach, while the second difference provides an idea of how much difference is in the prediction using the linear wave theory from the non-linear (simulated) case. It is seen that with increasing wave height the difference in attenuation using the first order velocity and the fully integrated velocity increases for both lower and higher \( kh \). One key difference is for lower \( kh \) most of the deviation is within the first wavelength or so while for the higher \( kh \) the deviation increases linearly along the flume. This is because, for the lower \( kh \), the depth being smaller for the same wave height, the ratio \( H/h \) and the ratio \( h/L \) are smaller than the corresponding large \( kh \) case. This means that the Ursell number \( (Ur = (H/h)/(h/L)^2) \) is larger and hence the waves are more non-linear for the lower \( kh \) cases. Thus the difference in the velocity \( (U_a) \) at the reference elevation \( (z_a = -0.531h + 0.235(H)) \), from the depth integrated one \( (U_v) \) will be greater for the larger \( kh \) cases due to the steeper hyperbolic nature of the wave velocity profile. The increased deviation within the first wavelength or so, for the lower \( kh \) case, at higher wave heights, is due to the shift of the reference elevation upwards \( (H/h \) being higher). The difference becomes negligible as the wave is damped \( (H/h \) becomes lower). This comparison signifies the need for full integration of the velocity profile for the larger \( kh \) problems when computing the vegetation drag. Fig. 5 also shows that the wave decay \( (H_{Ua}) \), using linear wave theory to predict the velocity at the reference elevation, is predicted increasingly higher for higher wave heights than the first order wave decay \( (H_1) \), due to the increasing non-linearity of the test cases at low \( kh \). Interestingly for this case of submerged vegetation, the Mendez and Losada (2004) analytical solution \( (H_{UV}) \) predicts very well the attenuation from the full integration approach \( (H_F) \) for both lower and higher \( kh \), implying that the non-linearity of the waves may not be a major factor when depth integrated drag force is estimated by considering the vertical profile of the velocity even if linear wave theory is assumed. For the higher \( kh \), \( H_{UV} \) actually underpredicts the decay.
slightly, compared to $H_F$. Overall the Mendez and Losada (2004) analytical solution can be taken as a reliable estimate of the wave decay for $kh$ ranges tested here and for submerged vegetation with $h_v/h = 0.5$.

Fig. 6 shows the comparison of the wave decay curves for increasing vegetation emergence ratio $h_v/h = 0.25-1.0$. Results are shown for a low $kh = 0.29\pi$ and a high $kh = 0.84\pi$. Wave height, wave period and drag coefficient were held constant at $H = 90cm$, $T = 3.5s$ and $Cd = 1$. Water depth was $h = 2m$ for $kh = 0.29\pi$ and $h = 8m$ for the $kh = 0.84\pi$. Thus for
a given $kh$ case, the wave non-linearity is constant as $Ur$ is constant. It is seen that for both low and high $kh$ cases, the full depth integrated velocity approach shows less attenuation than the first order approach for $h_v/h < 0.75$, while above this limit, $h_v/h > 0.75$, the reverse is observed. The difference is more evident for the larger $kh$ case than the lower one. This is due to the fact that as the stem height increases, the effective vegetation drag increases too, a fact that the first order approach, where the reference elevation remains relatively unchanged, is unable to explain. In fact for the large $kh$ case it is seen that the average deviation can be greater than 100% when $h_v/h > 0.75$ (near emergent to emergent case). Thus the full integration approach is an important addition when extending the model to field scale conditions with large $kh$, large wave height and a heterogeneous environment consisting of both emergent and submerged vegetation. The model is seen to under predict the attenuation for emergent cases at higher $kh$ compared to the Mendez and Losada (2004) analytical solution ($H_{lv}$). This is possibly due to the inaccuracy of the $O(\mu^2)$ model to reproduce the full depth varying velocity profile for the large $kh$ case as has been shown before by Lynett (2006) by a two layer model which can simulate higher $kh$ cases with greater accuracy. Also when amplitude of the wave is comparable to the distance between the canopy top and the still water level, the phase averaged drag force, simulated by CaFUNWAVE will likely be lower than linear wave theory, on account of taking average of depth integrated forces using different integration limits under the trough and the crest.

4.2. Wave Induced Hydrodynamics in Wetlands during Hurricane Isaac

The CaFUNWAVE model was extended to simulate field scale waves on a vegetated wetland during Hurricane Isaac, which struck Louisiana on August 2012. Isaac was a Category I hurricane at the time of its landfall which it made twice along the coast of Louisiana, the first at Southwest Pass on the mouth of the Mississippi River around 00:00 UTC 29 August 2012 and the second one at just west of Port Fourchon around 08:00 UTC 29 August 2012. Isaac then gradually weakened and dissipated inland. Field data from the storm was collected by coastal researchers at Louisiana State University (LSU) (Hu et al., 2015) using pressure type wave gauges deployed before the hurricane. The field site for the present case was located near Bohemia, about 35 miles south east of New Orleans (Fig. 7).

Our current study area includes the vicinity of the region where the wave gauges 1 and 2 (henceforth referred as WG1 and WG2) were deployed and which experienced the maximum wave heights ($>1m$) during the period 02:00 UTC to 09:00 UTC of 29 August 2012. Three separate wave conditions (Table 2), representative of storm conditions just before the local peak wave height was reached, at the peak wave height and immediately after the passage of
the peak storm was simulated, to gain insights into the wave hydrodynamics on the mudflat and the platform. In addition, a separate hypothetical case was run, with no vegetation cover on the wetland, using Case 2 (peak storm) conditions, in order to understand the role of the vegetation in modifying the wave hydrodynamics.

Table 2: Numerical Setup: Wave and Water Depths Before (Case 1), During (Case 2) and After (Case 3) the Peak Storm passed the Study Site

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<th>$h_{WG1}$ (m)</th>
<th>$h_{WG2}$ (m)</th>
<th>$H_{wm}$ (m)</th>
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<td>4.70</td>
<td>3.50</td>
<td>1.62</td>
<td>5.00</td>
<td>-2.65</td>
</tr>
<tr>
<td>Case 3: Veg</td>
<td>09:00 08/29/2012</td>
<td>13.09</td>
<td>4.04</td>
<td>2.84</td>
<td>1.52</td>
<td>4.52</td>
<td>15.70</td>
</tr>
<tr>
<td>Case 2: No Veg</td>
<td>06:00 08/29/2012</td>
<td>13.75</td>
<td>4.70</td>
<td>3.50</td>
<td>1.62</td>
<td>5.00</td>
<td>-2.65</td>
</tr>
</tbody>
</table>

4.2.1. Numerical Setup

Fig. 8(a) shows the complete domain including the bathymetry of the wetland and on the mudflat (under water). Lidar data (collected between January and March of 2011 by USGS) was used to generate the wetland bathymetry (Fig. 8(b)). The locations of the two

Figure 7: Field Site Location (photo courtesy Google Maps)
wave gauges are marked by X (cross marks) and the internal wavemaker, with a bottom elevation $-10m$ NAVD88 ($h_{wm}$), placed at $X = 1300m$, is denoted by the black dotted line. Waves propagated from east to west. A $1:40$ slope, meeting the mudflat at $X = 1050m$ was set to shoal the waves up to the mudflat from the wavemaker. The total size of the domain was $1.5km \times 1.7km$, with a $100m$ sponge layer to the west and $50m$ in other three directions. The wavelength at the wave maker varied from $L = 31.3 - 38.1m$ for the three cases. Grid sizes of $\Delta x = 0.6m$ and $\Delta y = 1.0m$ were selected for the base grid, which assured at least 50 cells per wavelength for the waves to approach the mudflat along the dominant ($X$) direction. One AMR region ($800m \times 1600m$) was defined, denoted by the black box. Thus the grid size within the innermost block was $\Delta x = 0.3m$ and $\Delta y = 0.5m$ which ensured $80 - 120$ cells per wavelength over the vegetated wetland and at the marsh edge where the breaking was most intense.

For the far field in the east, at the wave maker, the waves were specified by a TMA shallow water spectrum with an angular spreading value of $15^\circ$. The wave height, wave period and wave direction were taken from hourly 1D wave spectrum data, extracted at the wave maker location, from a SWAN model (Hu et al., 2015) run over the whole Gulf of Mexico domain. Using the exact water level from SWAN at the wavemaker, however, did
not yield good agreements with observed data, likely because the water level at the wave maker from the SWAN model included large scale wind driven storm surge effects that is beyond the scope of the present model. We are instead modeling the wave breaking induced effects on the wetland and the priority is to recreate the wave conditions near the gauge sites accurately so that the correct forcing can be provided for the secondary effects. It was decided to use the mean of the observed water levels at the WG1 and WG2 locations as the water level at the wave maker for each case. This amounted to a difference in about $10 - 12\%$ of the water level from SWAN value at the wavemaker. The only other change that had to be made to the SWAN data was the wave height for Case 3, where we set a wave height of $1.52m$ instead of the one from SWAN of $1.45m$, to match the gauge data.

In order to set the vegetation properties, data from vegetation survey, conducted a month after the storm (September 2012) were used. In the field, the length for the rigid stems varied from $64 - 82cm$ among different sampling sites on the platform, with average uniform stem diameter between $6.2 - 7.0mm$. A uniform value of $h_v = 73cm$ for the stem height and $d_v = 6.4mm$ for the vegetation was set for the numerical runs, representative of the mean vegetation characteristics. The vegetation canopies were thus completely submerged during the storm passage. The vegetation density over the platform was maintained at $N = 350stems/m^2$, close to that used in Hu et al. (2015) for saline marsh.

In order to create the underwater bathymetry an equilibrium profile for the mudflat was assumed similar to Wilson and Allison (2008), who proposed an equation of the following type to describe the one dimensional equilibrium mudflat profile for Louisiana wetlands,

$$y = a + be^{cx}$$  \hspace{1cm} (23)

where $a$, $b$ and $c$ are constants, $y$ the difference of elevation between the mudflat point at $x$ from the elevation of the local marsh edge. In Wilson and Allison (2008), the values of the coefficients obtained from the best fit equation from their field data were $a = -1.5$, $b = 1.2$ and $c = -0.05$. Using Wilson and Allison (2008)'s exact coefficients did not yield good agreement of the model results with the observed data. It was found that Wilson and Allison (2008)'s equilibrium profile equation was predicting a deeper depth at the WG1 location on the mudflat. It was therefore decided to use actual survey data, collected in the month following Isaac (on 09/11/2012) by the team at LSU, to create a new equilibrium profile equation for the present case. Fig. 9 shows the comparison of data obtained from the field survey transect passing through the wave gauges (WG1 and WG2) with the lidar data (on the marsh platform) and the data extracted from the equilibrium profile (on the mudflat) used for the present study. The black dashed line indicates the marsh edge.
While Wilson and Allison (2008) fitted a single equation to a number of survey profiles, in this case due to the absence of further survey profiles, it was decided to calibrate the complete 3D bathymetry using an equation similar to Wilson and Allison (2008), until a transect profile matching the survey at the study site was obtained. The calibration operation involved three main steps. First, the land water boundary was extracted from the lidar data using a technique similar to binary image extraction and land masses separated from water bodies by closed polygons. Then the land points were given the corresponding lidar data elevations while the water points were kept unassigned. Next the distance of all water points from all land-water boundaries obtained and sorted to find the least distance for a given water point. Since the Wilson and Allison (2008) type equation is a 1D equation, now that the distance of each point from the nearest marsh edge was known, different combinations of coefficients were used to create different 3D underwater bathymetries. The
coefficients that resulted in the bathymetry, whose transect through the survey locations provided the closest match to the observed survey profile, was chosen. The coefficients used were $a = -3$, $b = 3$ and $c = 0.01$. The whole operation was conducted in MATLAB v 15.0 using Image Processing and Mapping Toolbox. The process was parallelized to use the most of the available computational power and can process bathymetries consisting of several hundred million 3D data points within an hour in regular desktop workstations. The final elevations of WG1 and WG2 in the domain were $-0.95m$ NAVD88 and $0.25m$ NAVD88, compared to the actual survey elevations of $-0.84m$ and $0.248m$ NAVD88 respectively. The difference in the elevations at WG1 was due to the concave shape of the profile (Fig. 9) which cannot be represented by the equilibrium profile. It is likely that this concavity is due to sediment being moved during the hurricane or before (during the period between the lidar and survey profiles) due to erosion at the marsh edge. Indeed, the marsh platform edge is seen to have receded by about 6-7m over the 21 months between the two readings. The difference in this bed elevation for WG1 had little effect however on the wave height at this location for the test cases conducted here, likely due to significantly larger depths during the peak storm, compared to this difference. The dotted magenta line, passing through the mean location of the wave gauges and perpendicular to the local mean marsh edge orientation represents the transect along which wave height and wave breaking induced setup were analyzed and presented in the following section.

Each numerical simulation was run for 35mins of storm duration and wave data averaged over the last 20mins. For the circulation patterns, 60 waves were used as the averaging period. It took about 150-200 secs for a single wave to cross the domain and about 15 mins to reach quasi-steady state after which wave data averaging was started. The total number of cells in the domain was 8.09 million. If AMR was not used the number of cells for the simulations with the smallest resolution for the entire domain would have been 17 million and likely prohibitively expensive to run. The simulations were run in parallel using 5 nodes with 20 cores in Supermike HPC platform, provided by LSU High Performance Computing.

4.2.2. Wave Height and Wave Setup

Table 3 shows the observed water depth and wave characteristics at the two gauge locations (denoted by subscripts WG1 and WG2), where $h$, $H_s$ and $T_p$ represent water depth, significant wave height and peak wave period respectively. $K_v$ is the ratio of wave heights at WG1 and WG2, and provide an estimate of effective wave energy reaching the marsh platform. Table 4 shows the corresponding simulated wave characteristics at the same locations, including the wave breaking induced setup ($S$, +ve means increase in water
Table 3: **Observed** Wave Characteristics at the two Wave Gauges

<table>
<thead>
<tr>
<th>Run #</th>
<th>$h_{WG1}$</th>
<th>$h_{WG2}$</th>
<th>$H_{sWG1}$</th>
<th>$H_{sWG2}$</th>
<th>$K_v = \frac{H_{sWG2}}{H_{sWG1}}$</th>
<th>$T_{pWG1}$</th>
<th>$T_{pWG2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: Veg</td>
<td>4.2</td>
<td>3.1</td>
<td>1.24</td>
<td>1.0</td>
<td>0.81</td>
<td>4.54</td>
<td>4.64</td>
</tr>
<tr>
<td>Case 2: Veg</td>
<td>4.7</td>
<td>3.5</td>
<td>1.40</td>
<td>1.30</td>
<td>0.93</td>
<td>4.95</td>
<td>5.83</td>
</tr>
<tr>
<td>Case 3: Veg</td>
<td>4.0</td>
<td>2.9</td>
<td>1.20</td>
<td>1.05</td>
<td>0.88</td>
<td>4.53</td>
<td>4.57</td>
</tr>
<tr>
<td>Case 2: No Veg</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4: **Numerically Simulated** Wave Characteristics at the two Wave Gauges

<table>
<thead>
<tr>
<th>Run #</th>
<th>$h_{WG1}$</th>
<th>$h_{WG2}$</th>
<th>$S_{WG1}$</th>
<th>$S_{WG2}$</th>
<th>$H_{sWG1}$</th>
<th>$H_{sWG2}$</th>
<th>$K_s = \frac{H_{sWG2}}{H_{sWG1}}$</th>
<th>$T_{pWG1}$</th>
<th>$T_{pWG2}$</th>
<th>$Cd_{veg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: Veg</td>
<td>4.03</td>
<td>2.96</td>
<td>-21.4</td>
<td>-6.0</td>
<td>1.20</td>
<td>1.02</td>
<td>0.85</td>
<td>4.40</td>
<td>4.51</td>
<td>0.9</td>
</tr>
<tr>
<td>Case 2: Veg</td>
<td>4.67</td>
<td>3.46</td>
<td>-4.3</td>
<td>-2.6</td>
<td>1.35</td>
<td>1.17</td>
<td>0.87</td>
<td>5.10</td>
<td>5.20</td>
<td>1.0</td>
</tr>
<tr>
<td>Case 3: Veg</td>
<td>3.93</td>
<td>2.75</td>
<td>-13.4</td>
<td>-9.0</td>
<td>1.18</td>
<td>0.93</td>
<td>0.79</td>
<td>4.30</td>
<td>4.45</td>
<td>0.75</td>
</tr>
<tr>
<td>Case 2: No Veg</td>
<td>4.60</td>
<td>3.57</td>
<td>-9.8</td>
<td>+6.5</td>
<td>1.49</td>
<td>1.36</td>
<td>0.92</td>
<td>4.92</td>
<td>4.95</td>
<td>-</td>
</tr>
</tbody>
</table>

level and -ve means a decrease) and bulk drag coefficient ($Cd_{veg}$) for the vegetation that was used to calibrate the wave heights at the two gauges. It is seen that overall the results are in good agreement.

The water depths are slightly under predicted at both locations because this model is only simulating wave breaking induced setup, wind induced water level setup is not accounted for. The wave heights at the WG1 are also well predicted, while at WG2 the wave heights at the peak hurricane and immediately after the peak (Case 2 and 3) are underpredicted. Modifying the bulk drag coefficient or the stem length did not cure this problem (it changed the wave heights at WG1 too when vegetation properties were altered). It is likely that as the peak storm passed over the platform, the vegetation suffered catastrophic damage and altered the moderately uniform bulk drag behavior that was before the storm. The higher observed wave heights at WG2 (on the vegetated platform) is a complex effect of the heterogeneous nature of the vegetation damage on the wetland. The non-vegetated run for Case 2 shows higher wave heights at both the gauge locations compared to the vegetated run. Also, the setup ($S$) for the non-vegetated case at the WG2 location is higher than that for the vegetated case.

Figs. 10 and 11 show the variation of the wave breaking induced setup and the significant wave height across the marsh edge from the mudflat up to the platform. It is seen that the setup for the vegetated cases is significantly lower than the non-vegetated case (average setup on platform 0.28m), because of the stronger wave breaking at the edge and the also on the platform for the non-vegetated case. This further shows that the setup on the platform for
the vegetated cases are not very sensitive to wave direction during the peak of the storm. On the other hand, we see different set-downs (lowering of the water level) on the mudflat with different wave directions before and after the peak storm. This is due to complex 2D
circulation effects, as will be shown in the next section, that cause the lowering of the water level on the mudflat by a momentum induced pressure gradient driven by wave breaking at the surrounding asymmetrically oriented marsh lobes. It, however, must be mentioned that, as was pointed out before the TVD scheme tends to underestimate the wave setup slightly due to higher wave heights within the surf zone for non-vegetated cases, hence, in reality, the actual setup is probably somewhat higher than the model predicted value.

The wave heights (Fig. 11) also show similar influence of the breaking generated current. There is a local increase of wave height between $X = 400m$ and $X = 600m$ due to wave current interaction arising out of circulation patterns to be discussed later. At the marsh edge all the cases show significant wave breaking. The vegetated cases show a typical exponential decay of the wave height on the platform which is not present for the non-vegetated case. The latter undergoes a series of wave breakings and likely triad interaction effects on the platform. The wave height variations seem to be much more correlated to wave direction, with the shore normal waves showing a higher wave height at the gauge locations compared to the angled wave cases.

The relative asymmetry of the wetland lobes in the north and south of the study transect changes the wave decay curves due to complex 2D attenuation and wave diffraction, refraction effects. Overall the model is able to recreate the three storm states with good accuracy and is therefore used to explain the breaking generated circulation patterns in the next section, which can have important influence on the net sediment transport during such an event.

4.2.3. Wave Breaking Generated Circulation on the Wetland

Figs. 12(a) to 12(c) represent the circulation patterns, during the three stages of the storm, generated by averaging the velocity field over 60 waves after steady wave characteristics were established and therefore represent pseudo-stationary circulation patterns representative of the hourly storm conditions. In addition, Fig. 12(d) shows the non-vegetated condition corresponding to Case 2 (Table 2).

The figures contain the reference vector at the top right corner and the black arrow pointing east to west indicate the dominant wave direction for the particular case. The wave induced current values show a wide range from $0.05 - 1.02 m/s$ on the mudflat while on the platform they are much smaller (about $0.01 - 0.08 m/s$) indicating that circulations on the mudflat are much stronger than on vegetated wetland platforms. For storm conditions just immediately prior and at the peak, (Case 1 and Case 2), two circulation cells within the central mudflat and hugging the north and south lobes. These are generated due to
Figure 12: Wave Induced Circulations at different stages of the storm (Table 2). The black arrow signifies the dominant wave direction.
wave breaking along the inner edges of the C shaped landmass and drive contra rotating currents which in turn cause an angular jet-stream from the shore to the outer open water. It is this jet that causes local wave height increase on the mudflat between 450 to 600 m off the marsh edge. Stronger longshore currents are visible along the inner edges when the wave direction is negative (Case 1).

As the wave direction becomes almost shore normal (Case 2, Fig. 12(b)), we see a decrease of these longshore currents as majority of the waves no longer break at an angle along the edges. On the other hand longshore breaking induced currents can now be seen along the continuous marsh edges towards the north and south of the domain and will likely cause increased longshore transport of the sediment. The central jet is also weaker with the two feeder eddies becoming weak and more separated. Nonetheless, both these conditions represent scenarios when the sediment, suspended by the high energy waves can be driven out of the central land mass by the angular jet.

![Figure 13: Quasi-equilibrium wave field at peak storm, corresponding to Case 2 (Table 2): vegetated and non-vegetated scenarios.](image)

When the wave direction is positive (Case 3, Fig. 12(c)) it is seen that the eddy hugging the northern lobe has lost its circular structure while the southern eddy has become stronger and bigger. An important difference during this scenario is the strong clockwise rotating longshore circulations along the northern edges of the open marsh edge which have produced circulation features of their own now. The asymmetry of the northern and southern expanse...
of the natural marsh edge means that though the wave direction is nearly the opposite of that in Case 1 (12(a), the circulation patterns are different. A dominant return flow can be seen flowing into the open water fed by the long shore breaking induced circulations along this angled marsh edge, as well as by the southern lobe hugging clockwise rotating larger eddy.

Fig. 12(d) shows the Case 2 conditions with no vegetation and shows a marked difference in the circulation patterns on the mudflat and the platform. Without vegetation currents are much stronger (upto 1.2 m/s) within 100m from the marsh edge, due to stronger breaking and overtopping of the broken waves on the platform, particularly for the southern and northern triangular marsh lobes whose edges are exposed to wave attack along multiple directions. Fig. 13 shows a snapshot of the wave field once the wave height reached an equilibrium and explains the phenomenon better. Vegetation (Fig. 13(a)) seems to be quite effective in protecting exposed marsh platforms, that are subjected to wave attack from multiple directions, by dissipating the hurricane wave energy quickly within the first few hundred meters from the edge, while non-vegetated platforms are subjected to increased breaking and likely higher shear stresses due to the overtopped waves. The overtopping induced currents are lower over sheltered marsh edges which are exposed to waves in one direction only. Also vegetated wetlands seem to break up long crested waves and reduce the energy from focusing of diffracted waves, due to the irregular wetland shapes.

5. Conclusions

In this paper, an improved way of handling wave attenuation by vegetation, applicable for the fully non-linear Boussinesq equations was implemented in the CACTUS framework. Adaptive Mesh Refinement (AMR) was used to generate high resolution grids for areas of main interest in the domains. The model was validated against 1D laboratory experiments for wave breaking on a vegetated beach and extended for 2D application to field scale, in order to explain breaking wave generated complex hydrodynamics in a wetland under hurricane waves. The specific conclusions from the present study can be summarized as follows:

1. This paper documents an improvement of the formulation of the vegetation drag force under waves, over existing depth averaged based Boussinesq models. A full depth varying velocity profile, based on higher order expansion of the reference velocity, is used in the depth integrated quadratic drag law formulation. This new approach, for application in fully non-linear Boussinesq equations, has been validated for wave height
and setup results from a laboratory experiment for regular waves on a 1D vegetated beach. The results are in very good agreement. Wave setup on vegetated beach slopes was found to be considerably reduced or almost zero compared to non-vegetated beach canopies.

2. The capabilities of the improved drag formulation over the first order reference velocity based formulation is illustrated for a variety of regular wave heights, different water depths and vegetation submergence levels. The attenuation curves are compared with analytical solutions based on linear wave theory. The linear wave theory based analytical solution, considering the depth varying velocity, appears to match very well with the higher order expanded velocity formulation, in predicting wave attenuation by Boussinesq models for submerged vegetation stems. The model results show some deviation from linear wave theory solution only at high $kh$ ($k$ wavenumber, $h$ water depth) for near emergent or emergent conditions, likely due to the inaccuracy of the $O(\mu^2)$ models prediction of the vertical velocity profile.

3. Adaptive Mesh Refinement (AMR) was used for the first time within the Boussinesq framework and makes field scale, hurricane wave simulations over highly resolved, natural, irregular marsh shorelines and nearshore mudflat bathymetries a possibility at reasonable computational effort. The use of AMR reduced the domain size by over 50% over traditional uniform grid domain.

4. The model was extended to simulate waves during Hurricane Isaac over a wetland in south eastern Louisiana. In order to create the bathymetry for the field scale domain, Lidar data was used to create the wetland bathymetry while an equilibrium mudflat profile, generated using an innovative 3D calibration of the mudflat profile with field survey data, were used to describe the mudflat bathymetry.

5. The main goal was to simulate peak wave conditions during a typical hurricane and gain new insights into the wave breaking induced hydrodynamics. Three wave cases, representative of hourly averaged Hurricane Isaac storm conditions at the study site, immediately before the peak, at the peak and immediately after the peak were simulated. Far field wave conditions were implemented by the TMA spectrum with wave characteristics extracted from 1D averaged spectrum of a larger scale SWAN model run over the whole Gulf of Mexico. Vegetation drag coefficients were calibrated to match wave heights observed at two wave gauges placed in the field.
6. The effect of the vegetation on wave height, setup and set-down were investigated for the first time using a phase resolving model for hurricane waves on a wetland. Cross-shore variation of wave height and wave setup were presented to show the effect of wave angle. Results from a non-vegetated hypothetical case were also presented to show how vegetation can reduce setup during a storm. It was found that for vegetated platforms the setup was not very sensitive to wave directions. On the other hand, set-down on the mudflat depended on 2D circulation effects that drove a momentum induced pressure gradient on the mudflat. The circulation on the mudflat leads to a larger resultant velocity, causing a further reduction of mean water level in addition to breaking generated set-down. Setup for non-vegetated case, with an average value of $0.28 m$ was observed to be much higher on the platform than the vegetated case, during the peak storm. Wave heights were also larger compared to the vegetated case. Wave energy attenuated exponentially on the mudflat for the vegetated cases irrespective of wave direction, while for the non-vegetated case, wave breaking at the edge on the platform was the primary cause of energy reduction.

7. The role of a vegetated platform in modifying the wave breaking induced circulation during hurricanes was also illustrated. Two dimensional circulation patterns representative of hourly wave conditions during the three stages of the storm were presented along with the results from the non-vegetated case for the peak storm. For vegetated cases, it was found that complex circulation patterns change based on dominant wave direction and the current magnitudes show a wide range ($0.05 - 1.02 m/s$) on the mudflat with the strongest currents along the marsh edges. On the platform the current values rarely exceed $0.1 m/s$ indicating that breaking induced currents on vegetated platforms are not significant. Specific circulation patterns, that are likely to move the already suspended sediment due to wave action, away from the wetland, were illustrated. While coherent circulation cells were visible on the mudflat for vegetated platforms, for non-vegetated platforms, the largest magnitude currents (upto $1.2 m/s$) was observed on the marsh lobes that are exposed to wave attack from multiple directions. Sheltered, non-vegetated continuous marsh edges, which are only exposed to waves from a single direction, however seem to be better protected from this onshore directed overtopping flux. Thus for non-vegetated areas, sediment is likely to be eroded from exposed lobes and driven onshore, during a hurricane, while vegetation can effectively protect the first few hundred meters from such harsh wave conditions and prevent sediment from being stripped off the platform.
6. Acknowledgments

The study was supported in part by the National Science Foundation (NSF Grant SEES-1427389, and CCF-1539567) and the Louisiana Sea Grant. Arash Karimpour, Ranjit Jadhav and Kyle Parker assisted in the field experiments in Louisiana. Frank Loffler and Jian Tao assisted in Cactus implementation. Any opinions, findings, conclusions and recommendations expressed in this paper are those of the authors and do not necessarily reflect the views of the NSF or NOAA.

7. Appendix

Eqn. 14 can be written as follows, after taking the depth averaged constants out of the integral,

\[
R_v = \frac{1}{2(h + \eta)} C_d N d_v \int_0^{\min(h_v, h + \eta)} I d\eta
\]  

(24)

where the integrand \( I \) can be written, using Eqn. 13, as follows, neglecting terms of \( O(\mu^4) \) and above,

\[
I = \left( u_\alpha + \mu^2 \left[ \frac{1}{2} (z_\alpha^2 - z^2) \nabla B + (z_\alpha - z) \nabla A \right] \right) u_\alpha + \mu^2 \left[ \frac{1}{2} (z_\alpha^2 - z^2) \nabla B + (z_\alpha - z) \nabla A \right] \]  

(25)

\( I = (I_x, I_y) \) can be split in to \( X \) and \( Y \) components for ease of programming and the final forms of \( I_x \) and \( I_y \) after simplification and substitution of \( A = \nabla \cdot (h u_\alpha) \), \( B = \nabla \cdot (u_\alpha) \), \( u_\alpha = (u_\alpha, v_\alpha) \) become,

\[
I_x = \frac{1}{2 \sqrt{u_\alpha^2 + v_\alpha^2}} \left[ 2u_\alpha (u_\alpha^2 + v_\alpha^2) + \mu^2 (z_\alpha - z) \left( 4u_\alpha^3 h_{xx} + u_\alpha v_\alpha |2u_\alpha, y| h_{xy} \right. \right. \\
+ 2h_y (2v_\alpha, y + u_\alpha, x) + (z + 2h + z_\alpha)(v_\alpha, yy + u_\alpha, xy) + 2v_\alpha (h_{yy} + h_{xx}) \right. \\
\left. + v_\alpha^2 (2h_x (v_\alpha, x + u_\alpha, x) + 2h_y v_\alpha, x + 2v_\alpha h_{xy} + (z + 2h + z_\alpha)(v_\alpha, xy + u_\alpha, xy) \right) \right] \\
\]  

(26)

\[
I_y = \frac{1}{2 \sqrt{u_\alpha^2 + v_\alpha^2}} \left[ 2v_\alpha (u_\alpha^2 + v_\alpha^2) + \mu^2 (z_\alpha - z) \left( 2u_\alpha^3 h_{xy} \right. \right. \\
+ 2v_\alpha^2 (2v_\alpha, y + 2u_\alpha, x) + 2h_y (2v_\alpha, y + u_\alpha, x) + (z + 2h + z_\alpha)(v_\alpha, yy + u_\alpha, xy) \right. \\
\left. + v_\alpha^2 (2h_x u_\alpha, y + 2h_y (2v_\alpha, y + u_\alpha, x) + (z + 2h + z_\alpha)(v_\alpha, yy + u_\alpha, xy) + 2v_\alpha (h_{yy} + h_{xx}) \right) \right] \\
\]  

(27)
CHAPTER 6
CONCLUSIONS

In this dissertation, numerical modeling of wave interaction with two types of shore protection solutions - vegetated wetlands and nearshore breakwaters was undertaken and the results discussed. Studies were performed from laboratory scales up to field scales and attempts were made to answer specific questions relevant to the design of each shore protection alternative. In particular the drag coefficients of emergent wetland canopies under wave flow were explicitly verified with laboratory experiments and new insights into wave interaction with free surface interaction with emergent stems obtained. Also the effectiveness of vegetation canopies in reducing breaking induced setup and protecting marsh platforms from wave induced currents during a typical hurricane was demonstrated. Numerical results obtained from a parametric study of various breakwater layouts were used to generate empirical relations for mean current through fish dips as well as lay new design guidelines.

The specific conclusions that were obtained from this study are as follows:

1. In the 3D numerical experiments with regular waves through an emergent vegetation canopy, it was found that the forces experienced by the central cylinder in the array were similar to that on the isolated cylinder, implying no significant hydrodynamic interaction from stem to stem at the spacing chosen (about 200 stems/sq m). Vortex shedding was also found to be largely localized around one to two diameter distance on either side of the cylinder and no appreciable vortex interaction with the neighboring cylinders were noted. The vortex evolution pattern was different between the zone above the wave trough from the rest of the cylinder, because the net horizontal wave orbital velocities, between the SWL and the crest, always point in the direction of wave propagation during the forward motion resulting in a stronger recirculation shoreward of the cylinder between the trough and the crest over a wave period.

2. The vertical orbital velocity amplitudes show little deviation from Stokes' velocities except near the SWL, where the runup effect on the seaward face of the emergent stem causes a local increase of the vertical velocity amplitude. This also indicates that the horizontal velocity components are damped more than the vertical component for the majority of the cylinder height.

3. The difference in velocities on either side of the emergent cylindrical stem, in the zone between the wave trough and wave crest increases with increasing wave non-linearity.
The shoreward motion of the wave energy (during the rising limb of the water surface) associated with the wave crest impacting the cylinder, creates a localized recirculation zone spanning between the wave trough and the wave crest on the shoreward side of the cylinder. The seaward motion of the wave crest (during the falling limb of the water surface) creates a recirculation zone located only between the the wave trough and the still water level on the seaward side of the cylinder. Also the seaward recirculation zone between the trough and the still water level is weaker compared to the shoreward one (during the shoreward motion of the crest) for waves with greater non-linearity due to Stokes’ drift. Thus over one representative period, a net reduction of mean and root mean square velocities is seen shoreward of the cylinder in the vicinity of the SWL with respect to an equidistant point in front of the cylinder.

4. Vertical profile of maximum force and moment on an emergent stem in waves, suggests the peak force occurs somewhere between the SWL and the HWL.

5. The use of a LES Dynamic Mixed Model in the 3D Navier Stokes equations was found to be a better turbulence closure choice to simulate wave breaking than RANS models. Tests on wave transformation on a mudflat found that when no structure is present, the wave envelope on the mudflat, within the first 2-3 wavelengths from the marsh edge is dominated by the submergence level with respect to the marsh scarp.

In particular, when the water level is at or below the marsh scarp height, waves are reflected back from the edge and slamming effects of the incident waves would dominate the failure. When the submergence exceeds the marsh platform elevation, the reflection effect decreases and the waves may or may not break over the platform depending upon the incident wave height on the marsh edge. When the waves don’t break on the platform, triad interaction is found to dominate the evolution of waves on the platform mode.

6. An intervening structure is most efficient in attenuating the waves if the height of the structure is at or above the marsh scarp elevation. For situations where the still water level is above the platform, the design crest elevation should be chosen taking into consideration the relative crest height to wave height ratio, based on wave transmission coefficient curves.

7. When waves are allowed to overtop an impermeable structure, the excess momentum flux creates a circulation pattern in the vertical plane, the size and shape of which depends upon several factors like distance of the breakwater from the marsh edge,
incident wave height, relative crest elevation, and wavelength of the incident wave. For the experiments conduct edit was found that breakwater works optimally when it is placed at a distance, of about twice the incident wavelength, from the marsh edge. Increasing the distance causes a decrease in the circulation strength and seems to vanish (and therefore approach no-structure condition) when the distance is over 20m. Placing the breakwater too close (4m or about 0.75 times the wavelength in this case) causes significant undertow both on the mudflat and the platform.

8. An empirical non-dimensional relationship relating the mean cross-shore current through a fish dip, with five functional design parameters: incident wave height, wave period, gap ratio, distance of structure from marsh edge and relative crest elevation, was established based on model results from a large number of runs using the depth integrated fully non-linear Boussinesq equations. Non-linearity of the wave, aspect ratio of the gap behind the breakwater, the gap ratio and the ratio of the relative crest elevation to the incident wave height were the physical parameters of interest that determined the strength of the current. The mean current was found to increase with increasing distance for about 30m, while it decreased with increasing gap ratio till about 0.3.

9. The wave height behind a fish dip can cause erosion at the marsh edge. A similar empirical non-dimensional relationship as for the current was also obtained for this wave height. The wave height decreased with increasing distance upto 30m and gap ratio upto 0.3, showing an opposite trend to the current.

10. The zone of high wave action extended from 2 to 3 times the fish dip width along the marsh edge, behind the gap for gap ratios less than about 0.2. It is recommended that gap ratio be not more than 0.05 if no additional shore protection is used and never should exceed 0.3.

11. Based on the 2DH and 2DV results it can be concluded that the optimal distance of the breakwater from the marsh edge should be 2-2.5 times the incident wavelength in order to simultaneously exploit the benefits of the vertical circulation as well as to keep offshore rip current low. The crest elevation of the structure should not be allowed to fall below the platform elevation during the course of its design life.

12. An improved formulation for the vegetation induced drag force, incorporating the depth varying, higher order expansion of the horizontal velocity was implemented in a fully non-linear Boussinesq model, named CaFUNWAVE within the CACTUS framework. The model was shown to match very well experimental results for wave
height and setup for wave transformation on a vegetated laboratory scale beach. The new treatment of the drag force was found to predict the attenuation through near emergent or emergent canopies much better for large wave height and \(kh\) cases (\(k\) being the wave number and \(h\) the water depth) and is an important addition for application for hurricane conditions on natural wetlands.

13. The CaFUNWAVE model was extended to simulate waves during peak storm conditions on a natural wetland during Hurricane Isaac. It was found that for vegetated platforms, the setup was not very sensitive to wave directions while set down on the mudflat was, primarily due to 2D circulation effects on the mudflat. Setup for non-vegetated case, with an average value of 0.28m, was observed to be much higher on the platform than the vegetated case, during the peak storm. Wave heights were also larger compared to the vegetated case. Wave energy attenuated exponentially on the platform for the vegetated cases irrespective of wave direction, while for the non-vegetated case, wave breaking at the edge on the platform was the primary cause of energy reduction.

14. Two dimensional circulation patterns representative of hourly wave conditions during the three stages of the storm were presented along with the results from the non-vegetated case for the peak storm. For vegetated cases, it was found that complex circulation patterns change based on dominant wave direction and the current magnitudes show a wide range (0.05-1.02m/s ) on the mudflat with the strongest currents along the marsh edges. On the platform the current values rarely exceed 0.1m/s indicating that breaking induced currents on vegetated platforms are not significant. While coherent circulation cells were visible on the mudflat for vegetated platforms, for non-vegetated platforms, the largest magnitude currents (upto 1.2m/s ) was observed on the marsh lobes that are exposed to wave attack from multiple directions. Sheltered, non-vegetated continuous marsh edges, which are only exposed to waves from a single direction, however seem to be better protected from this onshore directed over-topping flux. Thus for non-vegetated areas, sediment is likely to be eroded from exposed lobes and driven onshore, during a hurricane, while vegetation can effectively protect the first few hundred meters from such harsh wave conditions and prevent sediment from being stripped off the platform.
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