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Working active power, reflected active power, and detrimental active power in the power system

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WORKING ACTIVE POWER, REFLECTED ACTIVE POWER, AND DETRIMENTAL ACTIVE POWER IN THE POWER SYSTEM

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ABSTRACT

Study of decomposition of the active power into components with deeper economical meanings, namely working active power, reflected active power, and detrimental active power is the subject of the thesis. The decomposition of active power will be based on the Current's Physical Component (CPC) Theory.

Working active power is equivalent to useful power that is the rate of energy used to do work, such as mechanical power. Reflected active power is the rate of energy transfer that the load sends back to the supply which dissipates off the supply resistance. This reflected active power is not taken into account on traditional power meters and the utility is not compensated for the power sent to the load to generate the reflected active power. Detrimental active power is the power that the supply sends to the load that is not considered useful power. The traditional power meter takes into account detrimental active power and the customer is paying for power that does not convert to useful work and potentially harming equipment.

Working active power shows that the standard definition for active power does not fully take into account all economical responsibilities. With active power decomposed into economical components, the economic responsibilities can be accounted to the correct party.

CHAPTER 1. INTRODUCTION

1.1 Energy Provider and Consumers

Since the turn of the 19th century, electric energy has played a critical role in industrialization of society. Just as creating fire by cavemen revolutionized human kind, utilizing electrical energy is no doubt on par. Without this energy many objects for today would not exist. Even the most basic light bulb would not be possible.

In 1831, Michael Farady took the first step in the revolution and discovered electromagnetic induction which paved the way for machines that would produce electricity. Almost half a century later, Thomas Edison finally commercialized electricity to a commodity in direct current form. Soon Nikola Telsa showed prospects of using alternating current as a more efficient transfer of energy. After several years of debate, the United States decided to use alternating current as their standards for developing what we know today as the power grid across the United States.

In today's society, nearly every household is connected to the electric grid. The electric grid is a system of the power lines that connect all the households or more generally, consumers to the energy provider. There are several energy providers in the United States that continuously provide electric energy for the consumers such as an industrial complex or a house. The energy provider is responsible for the generation and delivery of electric energy to the meter on the consumer. The consumer is then charged a set rate for energy used. Typical rates would be 12 cents per kilowatt-hour or KWh for short.

In the United States, there is roughly 4000 billion KWh produced a year in 2008. The average residential consumer uses 920 KWh a month in 2008 [1]. The usual appliances in the

house that uses majority of electricity are the air conditioning systems, refrigerators, and electric stoves. Could one imagine a world without these appliances with today's standards of living?

1.2 Power Theory

The development of a power theory has always traced its origin back to accurately account energy usage, which in turn drives the power economy. Since the discovery of electric energy, many individuals have strived to develop an accurate model for energy consumption. Several terms such as active power, reactive power, apparent power, and power factor try to name the physical phenomenon at hand.

Active power denotes the physical or real power. In other words, active power is what creates torque on the shaft of a machine or pushes an object in the ideal situation. These ties into the economy model as the rate of energy usage for the consumer and should pay the utility company accordingly. Reactive power is the rate of energy that is the oscillating in the system caused by the phase shift between voltage and current. This is considered wasted energy as it cannot be harnessed to do any work. Apparent power was then developed as the multiplication of rms voltage and rms current. This apparent power is mainly used by the utility company as a means to rate equipment on the power grid. Finally, power factor is the ratio of the active power to the apparent power.

1.3 Thesis Subject and Objective

The thesis subject is the study on decomposition of the active power into components with deeper economic meanings, namely “working active power”, “detrimental active power”, and “reflected active power.”

The thesis objective is to define the working active power, reflected active power, and detrimental active power in the CPC based power theory and to create a mathematical model and illustrate it with Matlab. Then the algorithm for the Matlab based Fast Fourier Transform will be developed to use in conjecture with the working, reflected, and detrimental active powers.

The working, reflected, and detrimental active powers will first be described in an abstract way and it will be shown how it ties into the economic mode as well how the utility company and consumers can benefit from accounts based on working energy. Next a mathematical model will be built that will enable calculation of these powers under all circumstances while a Matlab model will be coded to verify the results of a numerical example that demonstrates the viability of the thesis model.

Secondly, a Fast Fourier Transform or FFT algorithm will be used to sample phase voltage and current readings real time from the power system. This will allow us to use the thesis theory and calculate the working, reflected, and detrimental active powers in the power system. This FFT algorithm will have to be very efficient because the values must be acquired and processed in real time.

1.4 Approach of Thesis Objective

First the economic driving force behind the idea of the thesis subject will be covered. This will include the utility and consumer relationship and technological background that will help integrate the thesis subject. Also, the background of the CPC power theory will be discussed along with the basic meanings of traditional powers and complex root mean square values.

Second, the active power term will be broken down into several more specific powers. This will be the working active power, reflected active power, and detrimental active power.

Mathematical and economical reasoning will be given and definitions will be developed to support the idea of the decomposition. After a basis for understanding the main concept is created, supporting illustrations will be given along with simulations to verify the results. After the results are verified for single phase, the same concept will be extended to three phase systems. In addition, the algorithm for calculating the crms values will be derived.

CHAPTER 2. ENTITIES OF POWER SYSTEMS

2.1 Utilities and Supply Quality

The utility company is a service for the people to provide electric energy to the general public at the most affordable rate. Customers are charged a fair rate for the energy they use which is directly proportional to the cost of running the utility company. The utility companies are to keep a certain level of supply quality for the consumer's satisfaction. This supply quality is defined by convention to keep the power system reliable and efficient.

The utility company has been, via history, the sole provider of electric energy to society. They have been considered a service to the community. Their duties are to provide reliable and affordable electric energy to everyone. Reliability is keeping the power grid online all year long for all hours of the day. Finally, affordable electric energy is making sure costs are low enough that the average citizen can afford electric energy as a common commodity rather than a luxury. The government has closely mandated the industry as to utility companies have regional monopolies to keep this affordability to minimum.

Electric energy is delivered to every home and business. This energy is produced by the utility company. The utility company is responsible for both the production and delivery of the electric energy. This chain starts off as a fuel source rather it be fossil fuels, renewable energy, or nuclear energy. This natural energy is the converted to electric energy via a generator at a power plant. The electric energy is then transferred across power lines via the transmission grid and then through the distribution grid to the meter outside the consumer's home or industrial plant.

The consumer uses this electric energy delivered by the utility company to perform some sort of real work such as heating elements, rotary machines, or powering computers. The quality of supply is defined by the utility company. A good supply quality would mean electric energy

being delivered at a constant voltage with little to no noise or interference. These are vital as rotary machines with low supply quality would suffer from increased heating which lowers life span of the equipment and reduces torque on the output shaft thus increasing electric energy usage. Power supplies for computers would suffer from noise and interference and reduce the life span of the inverter.

The utility company and the consumer adhere to standards on what defines good supply quality. These standards would include stable rms value of voltage, constant frequency, symmetrical voltage, sinusoidal signal, the absence of transient waveforms in the voltage, and no high frequency noise present. In realistic terms, stable rms voltage and constant frequency are usually maintained by the utility company fairly well. Symmetrical and sinusoidal voltage waveforms are of lesser importance in large power grids as most utility companies do not see this as a vital aspect to keep to an absolute minimum. Lastly, transients and high frequency noises are almost impossible to completely eliminate and are passed on as unavoidable.

2.2 Consumers and Loading Quality

The consumers buy electric energy from the utility company. These clients can be from residential houses to industrial complexes, all the way to huge military complexes. The consumer is what drives the market with power demand. Consequently, most if not all of the income that the utility company generates is from the consumers themselves.

A few examples of consumers and their variety would be: individual residential houses, small commercial businesses, larger industrial complexes such as oil refineries or chemical plants, and lastly huge complexes such as military bases or NASA space centers. The specific loads these consumers have can range from individual three ton air conditioning systems,

elevator motors, massive water pumps, to welding factories and steel mills. As one can see, the demand can vary greatly. Even in more detail, one can divide the demand into varying ranges of the day such as noon when everyone has their air conditioning system running versus at night when very few have their systems running.

The utility company is obliged to provide electric energy to all consumers regardless of their energy consumption or location. These conditions affect how much the consumer pays for their energy consumption. The utility companies do have to look out for their own wellbeing. If a consumer has such a “bad” load that it would affect the utility companies’ grid, then the utility company has the rights to charge extra penalties to cover the monetary losses that the utility company suffers directly from adding the consumer to the power grid. What then is a “bad” load?

Loading quality can be defined as specific attributes the consumer’s equipment possesses that would harm the power grid. To be defined as a “good” load the loading equipment would have these attributes: balanced, resistive, linear, and time-invariant. A balanced load is when all three phases in the system have equal impedances. This would create a scenario where all three phase currents are balanced which minimizes power losses. An ideal resistive load would not have any inductance or capacitance, meaning a purely resistive circuit. Linearity of the load means that the load does not cause waveform distortion. Time-invariant loads means that the load has constant parameters therefore, the current rms does not change.

Realistically, some of the above loading qualities are very hard to acquire to perfection. Having a balanced load is the easiest criteria to meet, as most motors are naturally balanced loads. Most commonly, when the power grid intentionally splits the phases into single loads there will be unbalanced loads. Resistive loads are virtually impossible to perfect as naturally all

devices have some natural occurring inductances and capacitances, these are very small though. More so, some equipment such as motors, have inductances in the windings needed for producing magnetic flux necessary for their operation. Linear loads are loads that do not create harmonics in the load current; this would exclude harmonic generating loads such as AC/DC converters, arc welders, and fluorescent lamps. Time-invariant loads would consist of equipment that has a constant current draw and would not deviate which would create a constant current rms. Of course these conditions cannot be fulfilled to perfection, but having a load closer to the ideal qualities is what defines a “good” loading quality.

2.3 Billing and Revenues

The utility company is privileged for billing the customer to cover their overhead costs of operations and the delivery of energy to the consumer. The consumer is then responsible for paying the charged payments at the end of a monthly billing cycle. The monthly bill includes several key components: overhead charges, energy usage, fuel surcharge, and power factor penalty. The commissioner decides a fair rate to charge for these components based on administrative costs, energy creation and delivery, additional fuel costs, and extra losses incurred on the utility company based on degraded loading quality of a specific customer.

Overhead charges are considered charges that are charged to the customer to keep the utility company’s infrastructure operational. These are usually in the form of a flat fee that is imposed on the consumer before energy consumption is accounted for. It is worthy to note that the customer charge is not the entire overhead cost as other charges usually mask more overhead charges.

Energy usage charge is the actual consumption of energy in Kilo-Watt-Hours or kWh. At the end of the month a metering service comes to check the meter by the customer. In the simplest case of individual residential homes for example; in Louisiana, a direct rate is applied such as 12 cents per kWh. In bigger consumer such as industrial plants, a more complex charging rate is used such as sliding block rate.

Fuel surcharge is the charge per kWh of additional cost of fuel that is required to provide the electric energy to be delivered. Such an example is a petrol generator. Since the price of oil tends to fluctuate constantly, the fuel surcharge is used to compensate the utility for the increased cost of oil to produce energy for the consumer.

Power factor penalty is a surcharge on specific consumers when their loading quality is so degraded that there is a significant energy loss upon delivery to the consumer and the increased cost of equipment. The utility company would then have to create more energy to overcome the energy loss that is a result of the degraded loading quality that the consumer is responsible for. Therefore, the consumer should have to pay for the extra energy loss rather than the utility company.

Each utility company has their own incentive to create profits for their company. A natural monopoly occurs due to government regulation to keep charged rates affordable, or in a deregulated market, competition naturally keeps charges down to an affordable rate. At the end of a fiscal year the utility company will look at their own profits versus losses. In reality the utility company will then look at the differences and if they are insufficient funds, then they will charge their customers more money to cover the losses from the previous fiscal year. In the end of the day, the customers are the ones who are affected by all variables in the economic model. This gives a very low incentive for the utility company to upgrade their equipment or improve

their power grid to the benefit of the customers unless forced by an outside force such as governmental regulations.

The customers for the most part have little to no power to compel the utility company to improve the performance of the power grid due to the monopolistic nature of the business. The average customer does not know the difference from loading quality and supply quality so the customer will have to suffer from lower supply quality without knowledge of its existence. Furthermore, the utility company can strong arm the customer to improving the loading quality by installing compensators that do not give economic benefit to the customer.

Additionally, since the utility company is also unknown of the idea of supply quality and loading quality, there are specific customers that have such low loading quality that their specific load affects the rest of the system as supply quality due to the Thevenin's equivalent model of the source as seen from the customer's side. The utility company is unaware of this degradation and suffers monetary losses from it, but at the end of the year the company will spread the losses of the specific customers across all other customers to cover their "unknown" loss they reported in the previous fiscal year.

CHAPTER 3. POWER THEORY

3. 1 Various Approaches to Power Theory

Power Theory is a century old issue with an objective to create a system of conceptual, physical, technical, and economic meanings to reveal and describe the power phenomena in the electrical system. Another aspect of power theory would be the more practical such as compensation. Engineers are most familiar with the active power defined as mean value of the instantaneous power, namely

$$P \triangleq \frac{1}{T}\int_{0}^{T} u(t)i(t)dt$$

which in sinusoidal systems can be calculated as,

$$P \triangleq \frac{1}{T}\int_{0}^{T} u(t)i(t)dt = U \cdot I \cdot \cos\varphi$$

$$\varphi = \varphi_u - \varphi_i$$

Another important power quantity is a reactive power. Most engineers believe that reactive power is related to energy storage by electric or magnetic field of capacitors and inductors, but this opinion is not entirely correct as reactive power occurs because of a phase shift between voltage and current. Assuming a linear RLC circuit, the reactive power in sinusoidal conditions is defined as,

$$Q \triangleq \frac{1}{T}\int_{0}^{T} u(t)i\left(t - \frac{T}{4}\right)dt = U \cdot I \cdot \sin\theta$$

Apparent power is defined as the product of voltage rms and current rms of the system,

$$S = \|u\|\|i\|$$

where the rms or root mean square values are defined as,

$$\|x\| = \sqrt{\frac{1}{T}\int_0^T [x(t)]^2 \, dt}$$

The voltage and current rms values affect dimension of transmission equipment, meaning they are related to the investment cost needed for delivery of the voltage and current of $u(t)$, $i(t)$ and the energy to its consumer. Therefore, the apparent power is closely related to the cost of energy and the cost of delivering it.

The power factor is defined as the ratio of the active and apparent power, i.e.,

$$\lambda = \frac{P}{S}$$

and within sinusoidal system simplified to $\lambda = |\cos\varphi|$. Power factor is a term used to describe the utilization of the power system grid to supply energy to the load. Loads draw current for energy delivery. Due to the presence of reactive power, additional current flows through the system. Thus the apparent power is higher than the active power. Utilities build their power system based on the apparent power of transmission equipment, but the customer is charged for active power. Therefore, the utility has an unnecessary burden upon them to support customers with higher reactive current demands. Power utilities use the value of power factor to recover monetary losses by financial fines or an increase in the price of energy delivered to customers with low power factor.

In sinusoidal conditions, apparent power, active power, and reactive power in a single phase system satisfy the relationship,

$$S^2 = P^2 + Q^2 \tag{1}$$

In 1892, Steinmetz performed a famous experiment with a circuit that had an electrical arc as a load.

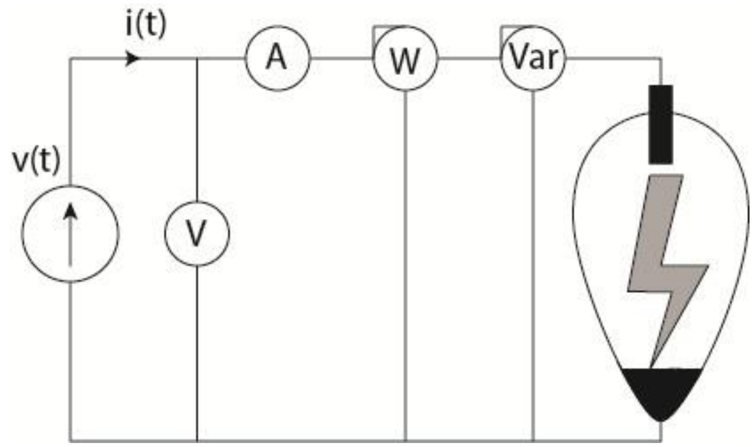


Fig. 1.1 Circuit in Steinmetz experiment

This experiment showed that the above relation between the apparent, the active and the reactive powers is not correct, but

$$P^2 + Q^2 > S^2$$

Stienmentz's experiment demonstrated that the power equation is not valid when the current is nonsinusoidal. The experiment then blew apart the engineering community and then there were many attempts at clarifying the above observation.

In an attempt to explain this experiment, several schools of power theory of systems with non-sinusoidal voltage and current were established. Out of the many schools of power theory early on there were two schools that stood out, Budeanu's and Fryze's power theory.

In 1927, Budeanu developed a new definition of the reactive power and consequently, and a new power equation. Four years later, Fryze came to light with his own school of power theory, which introduced the idea of current orthogonal decomposition into the active and reactive currents. Consequently, he suggested a new definition of the reactive power, different

from the Budeanu's definition. Budeanu applied concept of harmonics for his definition of the reactive power, while Fryze did it directly in the time domain.

For defining the active and reactive powers, Budeanu's power theory breaks down the voltage and current into their respective harmonic components and expresses them in the form

$$Q_{\mathrm{B}} \triangleq \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n$$

At such a definition of Budeanu's reactive power, the power equation $P^2 + Q^2 = S^2$ is not fulfilled. Therefore, Budeanu suggested that there is another power in the circuit. Because it only occurs in the presence of the voltage and current distortion, Budeanu introduced the name of the ***distortion power*** for this power. It was defined as

$$D \triangleq \sqrt{S^2 - (P^2 + {Q_{\mathrm{B}}}^2)}$$

Fryze created a power theory based on load current orthogonal decomposition. First consider the basic LTI load, shown in Fig. 1.2, which supplied with voltage $u(t)$ has the active power P. Such a load draws current $i(t)$.

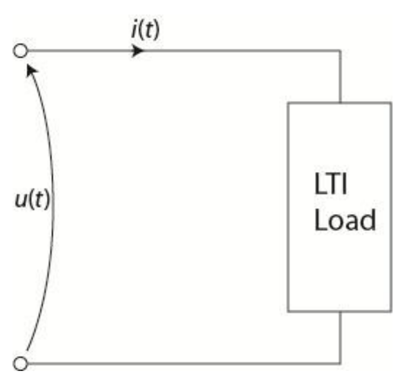


Fig. 1.2 LTI system

With respect to the active power P at voltage $u(t)$, such a load is equivalent to a purely resistive load of conductance G, such that

$$G\|u\|^2 = P \quad , \quad G = \frac{P}{\|u\|^2} \triangleq G_\mathrm{e}$$

Such conductance was referred to by Fryze as equivalent conductance. The current of such resistive load was named ***active current*** $i_\mathrm{a}(t)$. It is equal to

$$i(t) = G_\mathrm{e}\, u(t) \triangleq i_\mathrm{aF}(t)$$

The remaining current,

$$i(t) - i_\mathrm{aF}(t) = i_\mathrm{rF}(t)$$

is the reactive current according to Fryze's definition. Active power can be expressed as,

$$P = \frac{1}{T}\int_0^T u(t)\cdot i(t)\cdot dt = \frac{1}{T}\int_0^T u(t)\cdot (i_\mathrm{aF}(t) + i_\mathrm{rF}(t))\cdot dt = \frac{1}{T}\int_0^T u(t)\cdot i_\mathrm{aF}(t)\cdot dt$$

3.2 Discussion of Fryze and Budeanu Power Theory

Budeanu's power theory uses harmonic decomposition for defining the reactive power. Fryze frowned upon the idea of Budeanu's reactive power theory because of the use of harmonics, which Fryze does not regard as physical quantities. Measurement of Budeanu's reactive power and distorted power proved very difficult to measure with equipment at that time period showed no way to prove the theory incorrect with measurements. Instead, turning to the fundamental issues of Budeanu's power theory shows that there is no correlation of Budeanu's reactive power and distorted power to the power phenomenon of a circuit and that distortion power is not even related to waveform distortion. [2]

Let us assume that a periodic voltage u with a frequency of ω_1 contains only one harmonic of order $n\omega_1$,

$$u = u_n \triangleq \sqrt{2}U_n \cos(n\omega_1 t)$$

With a linear time invariant load of Y_n, the load current has the form,

$$i = i_n \triangleq \sqrt{2} I_n \cos(n\omega_1 t + \varphi_n), \quad I_n = U_n Y_n$$

Instantaneous power can be defined in the following way,

$$p_n(t) = \frac{dW_n}{dt} = u_n i_n(t) = U_n I_n \cos\varphi_n \ [1 + \cos 2n\omega_1 t] + U_n I_n \sin\varphi_n \ \sin 2n\omega_1 t$$

$$p_n(t) = P_n \ [1 + \cos 2n\omega_1 t] + Q_n \ \sin 2n\omega_1 t$$

The first, $P_n \ [1 + \cos 2n\omega_1 t]$, term cannot fall below zero, due to the limits of cosine function therefore is considered permanent energy flow. The latter shows that energy flow can be bi-directional, thus meaning an alternating component of instantaneous power between the load and source. In the Budeanu's reactive power definition,

$$Q_{\mathrm{B}} = \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n = \sum_{n=1}^{\infty} Q_n$$

amplitudes Q_n of alternating components of the instantaneous power are added harmonic by harmonic. Each component has a different frequency and phase angle which is not taken into account with Budeanu's reactive power. Even though each harmonic's Q_n has physical meaning, it is lost with the generalization to Budeanu's reactive power. Non-zero Q_n means that there is energy oscillation in the system of *n* order harmonic, but Q_n can be positive or negative. Thus they can mutually cancel to zero but oscillation still exists because these oscillations have different frequencies. Therefore, oscillation of energy that do exist, do not contribute to the apparent power *S* increase, as it is in sinusoidal situation. Cancelation of amplitudes Q_n means that Q_{B} is not a measurement of energy oscillation.

The distortion power, *D* was introduced by Budeanu. Its name does not explicitly state its meaning. It implies mutual distortion of the voltage and current waveform in the circuit. The

distortion power is equal to zero if the voltage waveform is sinusoidal, but the distortion power is also equal to zero when a non-sinusoidal voltage is applied to a resistive load or in other words, the current waveform is not distorted with respects to the voltage waveform. Distortion power could be non-zero when current waveform is different with respects to voltage waveform.

The original Budeanu's definition of distortion power can be expressed in the form,

$$D \triangleq \sqrt{S^2 - P^2 - {Q_\mathrm{B}}^2} = \sqrt{\frac{1}{2}\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{nm}} \;, \tag{2}$$

with terms A_{nm} equal to,

$$A_{nm} \triangleq (U_n I_m - U_m I_n)^2 + 2U_n U_m I_n I_m[1 - \cos(\varphi_n - \varphi_m)] \geq 0$$

Since terms A_{nm} are non-negative, the distortion power is equal to zero on the condition that all terms A_{nm} are equal to zero which requires that,

$$\frac{I_n}{U_n} = \frac{I_m}{U_m} \quad and \quad \varphi_n = \varphi_m = \varphi$$

this requires that the impedances for each harmonic frequency be equal to each other for all order harmonics in the supply voltage, i.e.,

$$\boldsymbol{Y}_n = \boldsymbol{Y}_m, \tag{3}$$

On the other hand, when there is no current distortion with respects to the voltage, i.e., it can be related to voltage as follows,

$$i(t) = k \cdot u(t - \tau),$$

then the crms values of the voltage and current harmonics has to fulfill the relationship,

$$\boldsymbol{I}_n = ke^{-jn\omega_1\tau}\boldsymbol{U}_n = \boldsymbol{Y}_n\boldsymbol{U}_n \;.$$

It means that if there is no mutual distortion of the voltage and current, then the admittance cannot be constant, rather changes by harmonic order as follows,

$$\boldsymbol{Y}_n = ke^{-jn\omega_1\tau} \neq \boldsymbol{Y}_m,$$

and this contradicts the condition (3) for the zero distortion power *D*.

Fyrze objected to the Budeanu's idea of power theory and the using harmonics in power theory. Instead he used current decomposition defined in the time domain. This approach was very simple in definition and easy for instrumentation to calculate at that time period.

Despite all the attractiveness of Fryze's power definition, there are major deficiencies that hinder the validity of Fryze's power concept. First, there is no clear physical interpretation for the Fryze's reactive current $i_{\mathrm{rF}}(t)$. All that is know is that $i_{\mathrm{rF}}(t)$ is the remaining current that is not being used for work. Second, there is no explicit relationship between $i_{\mathrm{rF}}(t)$ and the load parameters. Therefore, it does not help in evaluating load parameters to minimize the Fryze's reactive current. Lastly, there are no fundamentals associated with improving power factor and loading quality. It is not to say that Fryze's power theory is not ground breaking. The concept of current decomposition is such a powerful idea that even today it is used as the basis for modern power equations.

3.3 Current's Physical Components for LTI Load

The development of Currents' Physical Components (CPC) power theory started in 1984 in the paper [3]. The CPC power theory starts with the derivation from a LTI system supplied with nonsinusoidal voltage such that the voltage has the form with added notation of $\mathbf{U}_n$ describing a complex RMS value or magnitude and phase angle:

$$u(t) = U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{U}_n e^{jn\omega_1 t} \quad , \qquad \boldsymbol{U}_n = Ue^{j\propto}$$

An LTI load can be described by its admittance in the form:

$$\boldsymbol{Y}_n \triangleq G_n + jB_n$$

Therefore this gives us the current's form of:

$$i(t) = Y_0 U_0 + \sqrt{2}\, Re \sum_{n \in N} \boldsymbol{Y}_n \boldsymbol{U}_n \cdot e^{jn\omega_1 t}$$

Now the current can be broken down into fictitious currents. First being the active current component $i_a(t)$ which is associated with the permanent energy flow P with G_e being the equivalent conductance of the load with respect to the active power.

$$i_a(t) \triangleq G_e U_0 + \sqrt{2}\, \mathrm{Re} \sum_{n \in N} G_e \boldsymbol{U}_n\, e^{jn\omega_1 t}, \quad G_e = \frac{P}{\|u\|^2}$$

Then the active current is subtracted from the load current with the remaining current components:

$$i(t) - i_\mathrm{a}(t) = (Y_0 - G_e) U_0 + \sqrt{2}\, \mathrm{Re} \sum_{n \in N} (\boldsymbol{Y}_n - G_e) \boldsymbol{U}_n\, e^{jn\omega_1 t}$$

$$i(t) - i_\mathrm{a}(t) = (Y_0 - G_e) U_0 + \sqrt{2}\, \mathrm{Re} \sum_{n \in N} (jB_n + G_n - G_e) \boldsymbol{U}_n\, e^{jn\omega_1 t}$$

Now we can see there are two components of the current that is remaining. First one being the reactive current which is the current component within the load current that has a phase shift of 90° in the form of:

$$i_\mathrm{r}(t) = \sqrt{2}\, \mathrm{Re} \sum_{n \in N} jB_n \boldsymbol{U}_n\, e^{jn\omega_1 t}$$

The other part of the remaining current is the ***scattered current***. This arises from the conductance changing from the equivalent conductance per harmonic.

$$i_\mathrm{s}(t) = (G_0 - G_\mathrm{e}) U_0 + \sqrt{2}\, \mathrm{Re} \sum_{n \in N} (G_n - G_e) \boldsymbol{U}_n\, e^{jn\omega_1 t}$$

The current in the CPC power theory can be broken down into several components in the form of:

$$i(t) = i_{\mathrm{a}}(t) + i_{\mathrm{r}}(t) + i_{\mathrm{s}}(t)$$

Due to the mutual orthogonality of all components [3], rms value of this current components satisfy the relationship,

$$\|i\|^2 = \|\mathrm{i_a}\|^2 + \|i_{\mathrm{r}}\|^2 + \|i_{\mathrm{s}}\|^2$$

3.4 Currents' Physical Components for HGL

When looking at a circuit with harmonic generating load (HGL), such as a fluorescent lamp or an AC/DC converter, the load acts like a current source of higher order harmonics. Additionally, each harmonic could have a voltage source on the supply side representing supply voltage distortion or the harmonic current will create a voltage difference across the supply impedance if no voltage source is present. This current and voltage at load terminals could be expressed as sum of harmonics, i.e.,

$$u(t) = \sum_{n \in N} u_n\,(t) \qquad\qquad i(t) = \sum_{n \in N} i_n\,(t)$$

with the active power per harmonic being:

$$P_n = \frac{1}{T}\int_0^T u_n(t) \cdot i_n(t) \cdot dt$$

From the load terminal, samples of the voltage u_k and current i_k can be acquired. With the discrete Fourier transform (DFT), the crms quantities can be calculated and the phase angle between the voltage and current, i.e.,

$$\boldsymbol{U}_n = Ue^{j\propto_n} \quad , \quad \boldsymbol{I}_n = Ie^{j\beta_n} \quad , \quad \varphi_n = \alpha_n - \beta_n$$

With the crms quantities and phase angle, the power per harmonic can be described in the form,

$$P_n = \mathrm{Re}\{\boldsymbol{U}_n \boldsymbol{I}_n{}^*\} = U_n I_n \cos\varphi_n$$

The phase angle can then be observed to see if the active power is positive or negative by the condition:

$$|\varphi_n| \le \frac{\pi}{2}, \qquad positive\ power$$

$$|\varphi_n| > \frac{\pi}{2}, \qquad negative\ power$$

Therefore a subset of N can be formed with the notation N_C which represents the positive orientation of flow and N_L which represents the negative orientation. Using the above conditions we can write the subsets in the form:

$$if\ |\theta_n| \le \frac{\pi}{2}, then\ n \in N_C$$

$$if\ |\theta_n| > \frac{\pi}{2}, then\ n \in N_L$$

With the above subsets, the voltage and current can be decomposed into harmonic specific components such as:

$$i(t) = \sum_{n\in N} i_n(t) = \sum_{n\in N_C} i_n(t) + \sum_{n\in N_L} i_n(t) = i_C(t) + i_L(t)$$

$$u(t) = \sum_{n\in N} u_n(t) = \sum_{n\in N_C} u_n(t) + \sum_{n\in N_L} u_n(t) = u_C(t) - u_L(t)$$

Negative sign of the voltage u_L means that it occurs as the effect of the voltage drop of the load originated current on the supply source internal impedance.

With super position principle, the system can be broken down into two groups of sources. One is composed of voltage harmonics located in the distribution system resulting in energy

flowing from the supply to the load. The second group is composed of current harmonics originated in load resulting in energy flowing from the load back to the source. System decomposition according to the superposition principle is shown in Figs. 3.1 and 3,2.

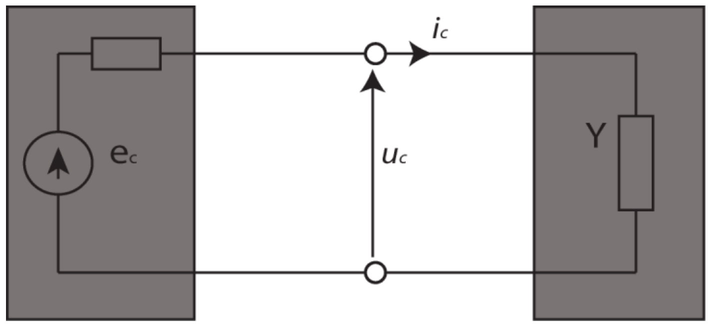


Fig. 3.1 Figure for $n \in N_C$

The voltage $u_C(t)$ composed of harmonics of the order n from the subset N_C, shown in the Fig. 3.1, is the voltage at terminals of a passive load of admittance $\boldsymbol{Y}_n$ for harmonic frequencies.

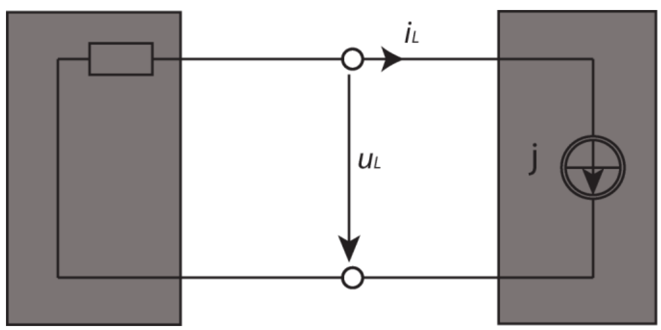


Fig. 3.2 Figure for $n \in N_L$

The voltage $u_L(t)$ composed of harmonics of the order n from the subset N_L, shown in the Fig. 3.2, is the voltage caused by load originating current harmonics at terminals of distribution system regarded as a passive device.

The load in Fig. 3.1 is LTI which we can then use traditional CPC theory and decompose current i_C into active, reactive, and scattered current as so:

$$i_C(t) = i_{aC}(t) + i_r(t) + i_s(t)$$

where

$$i_{\mathrm{aC}}(t) \triangleq G_{\mathrm{eC}} U_0 + \sqrt{2}\,\mathrm{Re} \sum_{n \in N} G_{eC} \boldsymbol{U}_n \, e^{jn\omega_1 t} \quad , \qquad G_{\mathrm{eC}} = \frac{P_{\mathrm{C}}}{\|u_{\mathrm{C}}\|^2}$$

which then, can be transposed to the load current in the form:

$$i(t) = i_{\mathrm{C}}(t) + i_{\mathrm{L}}(t) = i_{\mathrm{aC}}(t) + i_{\mathrm{r}}(t) + i_{\mathrm{s}}(t) + i_{\mathrm{L}}(t)$$

Note that since $i_{\mathrm{C}}(t)$and $i_{\mathrm{L}}(t)$ are orthogonal, the rms values of current components fulfill the relationship,

$$\|i\|^2 = \|i_{\mathrm{aC}}\|^2 + \|i_{\mathrm{r}}\|^2 + \|i_{\mathrm{s}}\|^2 + \|i_{\mathrm{L}}\|^2$$

CHAPTER 4. ACTIVE POWER

4.1 Useful and Useless Active Power

The active power (real power) is regarded in electrical engineering community as useful power, but in present power systems this common opinion cannot be true. This is because of the voltage and current distortion and asymmetry. Harmful effects of asymmetry and distortion are most visible in three phase induction motors.

A majority of the energy in the power grid is used to convey mechanical energy via electric motors. The motor's rotor is pulled by a rotating magnetic field created by the positive sequence voltage of the first harmonic. Negative sequence voltage components and harmonics of the 2nd and 5th order create a field in the opposite direction resulting in counter torque on the shaft. Harmonics of the 7th order would rotate 7 times faster than the fundamental voltage component which acts similar to the situation of a locked rotor and does not contribute to useful power but is only converted to heat instead. Therefore, active power of negative sequence and higher order harmonics do not contribute to useful power on the shaft.

Power electronic equipment is becoming more common in today's industry. Voltage asymmetry and harmonics degrade the performance of such equipment. AC/DC converters suffer from voltage asymmetry and harmonics by having their output waveform ripple increased and the supply current contains asymmetry and non-characteristic harmonics. Additionally, higher order harmonics will often contribute to extra energy loss and increased temperature, consequently shortening the life span of sensitive computer equipment.

Only in purely resistive LTI load such as incandescent light bulbs or industrial heating units is the active power a synonym for useful power. Useful energy is the electric energy that is converted into work. In the example of a three phase motor, the positive sequence component of

the voltage and current convey energy to the shaft of the motor which is used for work such as operating an elevator. Energy that is delivered by negative sequence components and higher order harmonics do not convert to energy used for work resulting in useless energy.

The part of active power that conveys useful energy for the operation of equipment will be defined as ***working active power***, P_{w}. A rectifier produces current harmonics at normal operation; the energy needed to produce the current harmonics is delivered by the fundamental voltage and current harmonic. The working power is then regarded as the output power, power losses, and the power needed for harmonic generation. Therefore the working active power P_{w} is higher than the active power P. The consumer should then be charged for working active power P_{w} rather than the active power P. Thus there is a need to differentiate working active power P_{w} from active power P.

4.2 Working Active Power and Current

Working active power is defined as the component of active power needed for operating equipment. The most basic circuit suffices to differentiate active power P and working active power P_{w}. The system is assumed to have sinusoidal supply voltage with HGL with current j composed of harmonic order n = 2,3,4…, i.e.,

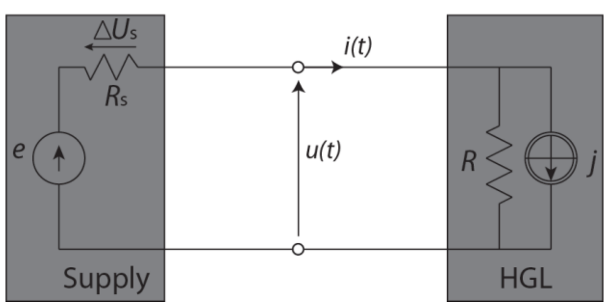


Fig. 4.1 Sinusoidal supply with resistive HGL

Assuming that the load generated current *j* is composed of higher order harmonics; the load current of a resistive load has the form of,

$$i(t) = \sum_{n \in N} i_n = i_1 + i_{\mathrm{h}} = i_{1\mathrm{aC}} + i_{\mathrm{h}}$$

where $i_{1\mathrm{aC}}$ corresponds to the active current of the first harmonic order. This current is in phase with the voltage fundamental harmonic and shall be coined ***working current***,

$$i_{\mathrm{w}} \triangleq i_{1\mathrm{aC}}$$

Therefore, the load current can be expressed in the form,

$$i = i_{\mathrm{w}} + i_{\mathrm{h}}$$

The load voltage, $u(t)$, is distorted by the distorted load current *i*. It has a form,

$$u(t) = \sum_{n \in N} u_n = u_1 + u_{\mathrm{h}}$$

The fundamental harmonic of the load voltage, u_1, is coined as the ***working voltage***, i.e.,

$$u_{\mathrm{w}} \triangleq u_1$$

The active power of the load is defined as:

$$P = \frac{1}{T}\int_0^T u(t)i(t)dt = \sum_{n \in N} P_n = P_1 + P_2 + P_3 + \cdots + P_n$$

The working active power is the product of the fundamental harmonic components of the working current and the working voltage meaning,

$$P_{\mathrm{w}} \triangleq U_1 I_1$$

The rest of the higher order harmonic powers associated with load generated current *j* makes up the remaining active power, i.e.,

$$P_n = U_n I_n = (-R_{\mathrm{s}} I_n) I_n = -R_{\mathrm{s}} {I_n}^2$$

This part of active power that originates from the load side is dissipated in the supply resistance. Thus being written as,

$$P_{\mathrm{h}} \triangleq (P_2 + P_3 + \cdots + P_n) < 0$$

Therefore, the active power has the form of,

$$P = P_{\mathrm{w}} + P_{\mathrm{h}}$$

4.3 Reflected Active Power and Currents

Active power of higher order harmonics P_{h} is the average rate of energy flow at higher order frequencies from the load back to the source over a period T. This harmonic active energy is dissipated over the supply and therefore referred to as ***reflected active power***, P_{r}, which has a negative value as compared to P_{h}, i.e.,

$$P_{\mathrm{r}} \triangleq -P_{\mathrm{h}}$$

The reflected active power is the rate of energy dissipation off the supply resistance by the harmonic current generated from the load, i.e.,

$$P_{\mathrm{r}} = R_{\mathrm{s}} \|i_{\mathrm{h}}\|^2 = -P_{\mathrm{h}}$$

The harmonic active power is negative to denote that the energy flows from the load back to the source for harmonics $n > 1$. The energy that flows from the load back to the source via harmonic current is delivered from the fundamental harmonic of the supply source. Thus, the rms current of the fundamental harmonics of a resistive load are,

$$I_1 = \frac{P_1}{U_1} \triangleq I_{\mathrm{w}} = \frac{P_{\mathrm{w}}}{U_{\mathrm{w}}} = \frac{P + P_{\mathrm{r}}}{U_{\mathrm{w}}} > \frac{P}{U_{\mathrm{w}}} \triangleq I_{10}$$

where I_{10} denotes the rms current of a non-HGL. The derivation shows that the generating current harmonics in the load increases the fundamental current, I_1, or the rms value of the working current.

The active power of the supply resistance contains two components. Is the reflected active power associated with harmonic current,

$$\Delta P_{\mathrm{sh}} = R_{\mathrm{s}} \|i_{\mathrm{h}}\|^2 = P_{\mathrm{r}}$$

and secondly, the active power on the supply source impedance associated with the fundamental current harmonic,

$$\Delta P_{\mathrm{s1}} = R_{\mathrm{s}} {I_1}^2$$

These two components increases the overall active power of the system in the form of,

$$\Delta P_{\mathrm{s}} = \Delta P_{\mathrm{s1}} + \Delta P_{\mathrm{sh}} = R_{\mathrm{s}} {I_1}^2 + P_{\mathrm{r}} = R_{\mathrm{s}} {I_1}^2 + P_{\mathrm{w}} - P$$

Notice that it is possible to calculate the supply resistance R_s from measurements of voltage and currents from the load terminals so that,

$$R_{\mathrm{s}} = \frac{P_{\mathrm{r}}}{\|i_{\mathrm{h}}\|^2} = \frac{P_{\mathrm{w}} - P}{\|i\|^2 - {I_1}^2}$$

This means that active power on the supply source impedance can be expressed in terms of known parameters, i.e., reflected active power in the form,

$$\Delta P_{\mathrm{s}} = R_{\mathrm{s}} {I_1}^2 + (P_{\mathrm{w}} - P) = (P_{\mathrm{w}} - P) \frac{\|i\|^2}{\|i\|^2 - {I_1}^2}$$

$$\Delta P_{\mathrm{s}} = P_{\mathrm{r}} \frac{\|i\|^2}{\|i\|^2 - {I_1}^2}$$

Therefore if the measurement of working and active powers along with the rms values of the load current and the fundamental current can be calculated, then it is possible to calculate the active power loss of the supply system.

Illustration 1. Calculate the working and reflected active powers of the load with a rectifier supplying a DC motor from an AC source. Assume that the rectifier is lossless and neglect the inductance of the motor armature (the motor has only purely resistive losses) and the AC voltage *e* is sinusoidal with purely resistive supply source impedance. (see Appendix A for detailed calculation)

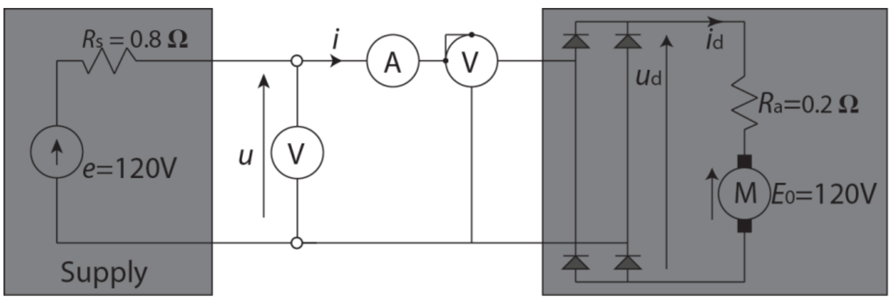


Fig. 4.2 Supply for DC motor from AC voltage source

With the load voltage *u* and the supply current *i* in the form,

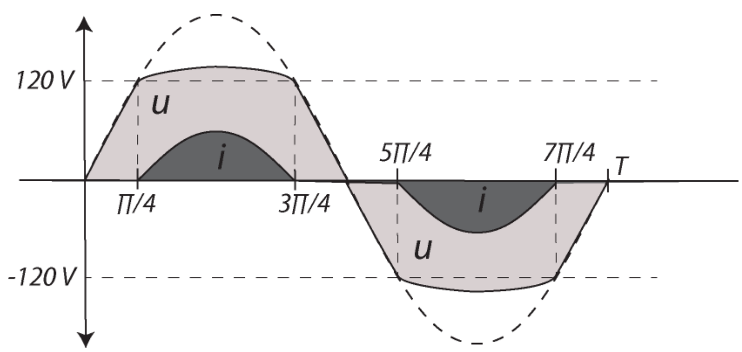


Fig. 4.3 Voltage and current waveform for illustration 1

First the fundamental current i_1 must be calculated from the supply voltage *e* and supply current *i* i.e.,

$$e = \sqrt{2}E\cos\omega_1 t = 120\sqrt{2}\cos\omega_1 t$$

$$i(t) = \frac{\sqrt{2}E\cos\omega_1 t - E_0}{R_\mathrm{s} + R_\mathrm{a}} = \frac{120\sqrt{2}\cos\omega_1 t - 120}{0.8 + 0.2}$$

with the crms $\boldsymbol{I}_1$ calculated with,

$$\boldsymbol{I}_1 = \frac{\sqrt{2}}{T}\int_0^T i(t)e^{-j\omega_1 t}dt = 21.8e^{-j\frac{\pi}{2}} \text{ [A]}$$

Then the fundamental voltage u_1 must be calculated from the current waveform i that has the form of,

$$\boldsymbol{U}_1 = \frac{\sqrt{2}}{T}\int_0^T u(t)e^{-j\omega_1 t}dt = 102.56e^{-j\frac{\pi}{2}} \text{ [V]}$$

Next the rms current, $\|i_\mathrm{d}\|$ is calculated from the load current i_d waveform,

$$\|i_\mathrm{d}\| = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} = 25.48 \text{ [A]}$$

Afterwards, the DC current $\bar{i_\mathrm{d}}$ is calculated from the load current i_d waveform,

$$\bar{i_\mathrm{d}} = \frac{1}{T}\int_0^T i_\mathrm{d}(t)dt = 16.39 \text{ [A]}$$

Finally the working active power can be calculated as a product of the voltage and current magnitudes,

$$P_1 = U_1 I_1 \cos\varphi_1 = 102.56 \cdot 21.8 = 2235.8 \text{ [W]}$$

and the active power on the load,

$$P = \bar{i_\mathrm{d}}E_0 + \|i_\mathrm{d}\|^2 R_a = 16.38(120) + 25.48^2\,(0.2) = 2097 \text{ [W]}$$

Therefore reflected active power being,

$$P_{\mathrm{r}} = P_{\mathrm{w}} - P = 2235.8 - 2097 = 138.8 \text{ [W]}$$

The main conclusion from the resistive system is that the working active power P_{w} is higher than active power P by the difference of reflected active power P_{r}. Since the working active power P_{w} is the power needed to run equipment that generate harmonics, the customer should be charged the energy needed to run the device, referred as ***working energy***, i.e.,

$$W_{\mathrm{w}} \triangleq \int_{0}^{\mathrm{month}} P_{\mathrm{w}}\, dt$$

With working active power in a single phase system, similarly the idea of working active power can be adapted to the three phase system. First consider the model below which is a three wire circuit with a three phase AC/DC converter and symmetrical supply sources.

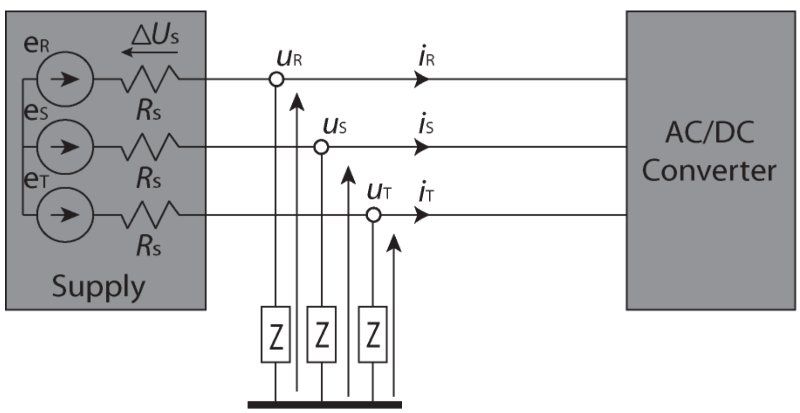


Fig. 4.4 Circuit with three phase AC/DC converter

The three currents and three voltages can be represented in vector notation such as,

$$\begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix} \triangleq \boldsymbol{i} \qquad\qquad \begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix} \triangleq \boldsymbol{u}$$

The load current of the three phase rectifier will have the form of,

$$\boldsymbol{i} = \sum_{n \in N} \boldsymbol{i}_n = \boldsymbol{i}_1 + \boldsymbol{i}_{\mathrm{h}} = \boldsymbol{i}_{1\mathrm{aC}} + \boldsymbol{i}_{\mathrm{h}}$$

The current in phase with the voltage fundamental harmonic is working current,

$$\boldsymbol{i}_{\mathrm{w}} \triangleq \boldsymbol{i}_{1\mathrm{aC}}$$

Therefore, the load current can be expressed in the form,

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{w}} + \boldsymbol{i}_{\mathrm{h}}$$

The load voltage $\boldsymbol{u}$ is distorted by to the distorted load current $\boldsymbol{i}$, thus it has a form,

$$\boldsymbol{u} = \sum_{n \in N} \boldsymbol{u}_n = \boldsymbol{u}_1 + \boldsymbol{u}_{\mathrm{h}}$$

The fundamental harmonic of the load voltage $\boldsymbol{u}_1$ is the working voltage, i.e.,

$$\boldsymbol{u}_{\mathrm{w}} \triangleq \boldsymbol{u}_1$$

The active power of the load is defined as:

$$\mathrm{P} = (\boldsymbol{u}, \boldsymbol{i}) = \sum_{n \in N} P_n = P_1 + P_2 + P_3 + \cdots + P_n$$

The working active power is the scalar product of the fundamental harmonic components of the working current and the working voltage meaning,

$$P_{\mathrm{w}} \triangleq (\boldsymbol{u}_1, \boldsymbol{i}_{1\mathrm{aC}})$$

The rest of the higher order harmonic powers associated with load generated current makes up the remaining active power, i.e.,

$$P_n = (\boldsymbol{u}_n, \boldsymbol{i}_n) = (-R_{\mathrm{s}}\|\boldsymbol{i}_n\|)\|\boldsymbol{i}_n\| = -R_{\mathrm{s}}\|\boldsymbol{i}_n\|^2$$

Similarly to the single phase system,

$$P_{\mathrm{h}} \triangleq (P_2 + P_3 + \cdots + P_n) < 0$$

Therefore the active power will have the form of,

$$P = P_{\mathrm{w}} + P_{\mathrm{h}}$$

Next consider the model below which is a three wire circuit with LTI, resistive and unbalanced loads and symmetrical supply sources.

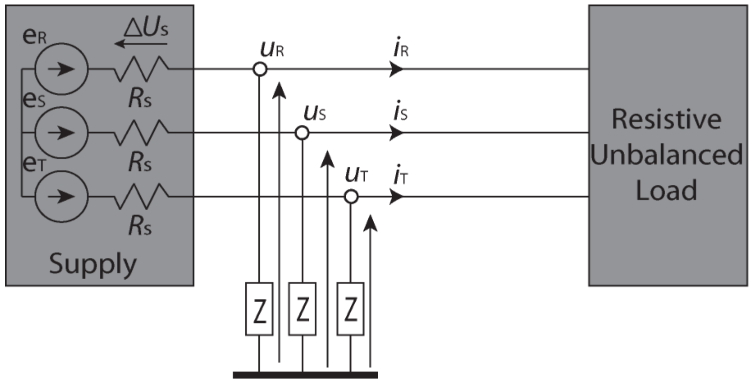


Fig. 4.5 Three phase system unbalanced load

The three phase system can then be represented in symmetrical components of positive and negative sequence with the relationship below,

$$\begin{bmatrix} \boldsymbol{u}^{\mathrm{p}} \\ \boldsymbol{u}^{\mathrm{n}} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^{*} \\ 1 & \alpha^{*} & \alpha \end{bmatrix}\begin{bmatrix} u_r \\ u_s \\ u_t \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{i}^{\mathrm{p}} \\ \boldsymbol{i}^{\mathrm{n}} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^{*} \\ 1 & \alpha^{*} & \alpha \end{bmatrix}\begin{bmatrix} i_r \\ i_s \\ i_t \end{bmatrix}$$

$$\alpha = 1e^{j120^{\circ}}$$

Such that the load voltage and currents will have the form,

$$\boldsymbol{u} \triangleq \boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}}$$

$$\boldsymbol{i} \triangleq \boldsymbol{i}^{\mathrm{p}} + \boldsymbol{i}^{\mathrm{n}}$$

The working current and voltage is the positive sequence components of the current and voltage,

$$\boldsymbol{u}_{\mathrm{w}} \triangleq \boldsymbol{u}^{\mathrm{p}}$$

$$\boldsymbol{i}_{\mathrm{w}} \triangleq \frac{P^{\mathrm{p}}}{\|\boldsymbol{u}^{\mathrm{p}}\|}\boldsymbol{u}^{\mathrm{p}}$$

Active power is calculated as the scalar product of the positive sequence and negative sequence components of voltage and current. The scalar product of components of different sequences are orthogonal, thus having a zero scalar product (see Appendix B for details) Therefore, the active power at the load terminal is,

$$P = (\boldsymbol{u}, \boldsymbol{i}) = (\boldsymbol{u}^{\mathrm{p}}, \boldsymbol{i}^{\mathrm{p}}) + (\boldsymbol{u}^{\mathrm{n}}, \boldsymbol{i}^{\mathrm{n}}) = P^{\mathrm{p}} + P^{\mathrm{n}}$$

The positive sequence components will contribute to active power P^{p}. Assuming sinusoidal symmetrical voltage source $\boldsymbol{e}$, the negative sequence voltage occurs due to the current passing through the supply resistance R_s, i.e.,

$$P^{\mathrm{n}} \triangleq (\boldsymbol{u}^{\mathrm{n}}, \boldsymbol{i}^{\mathrm{n}}) = (-R_s \boldsymbol{i}^{\mathrm{n}}, \boldsymbol{i}^{\mathrm{n}}) = -R_s \|\boldsymbol{i}^{\mathrm{n}}\|^{2} < 0$$

The active power by the difference of P^n is higher than the loaded active current P in such a form,

$$P^{\mathrm{p}} = P - P^{\mathrm{n}} > P$$

The working active power is the active power of the positive sequence of the three phase system.

$$P_{\mathrm{w}} \triangleq P^{\mathrm{p}}$$

By definition, the negative sequence component is the active enregy that travels from the load back to the source and dissipates on the supply resistance and should therefore be considered reflected active power, i.e.,

$$P_{\mathrm{r}} \triangleq -P^{\mathrm{n}} = -(\boldsymbol{u}^{\mathrm{n}}, \boldsymbol{i}^{\mathrm{n}})$$

With the reflected active power, the supply resistance can be calculated by the difference of working active power and load active power divided by the rms negative sequence current,

$$R_s = \frac{P_{\mathrm{w}} - P}{\|\boldsymbol{i}^{\mathrm{n}}\|^2}$$

Similarly to the single phase system, the rms value of the working active current must be higher than the rms value of load active current in such a way that,

$$||\mathbf{i}^{\mathrm{p}}|| = ||\boldsymbol{i}_{\mathrm{w}}|| = \frac{P_{\mathrm{w}}}{||\boldsymbol{u}_{\mathrm{w}}||} > \frac{P}{||\boldsymbol{u}^{\mathrm{p}}||}$$

With the additional rms current, there will be additional losses in the supply resistance along with the reflected active power in the form,

$$\Delta P_s = R_s||\boldsymbol{i}^{\mathrm{p}}||^2 - P^{\mathrm{n}} = R_s||\boldsymbol{i}^{\mathrm{p}}||^2 + R_s||\boldsymbol{i}^{\mathrm{n}}||^2 = R_s||\boldsymbol{i}||^2$$

Illustration 2. A three phase purely resistive system with a balanced load is assumed. The distribution voltage is sinusoidal and symmetrical with line to ground voltage rms *E*= 240V. The load parameters are selected so the load active power P = 100kW and the power loss in the supply is ΔP_s = 5kW. The resulting system is shown below in Fig. 4.6

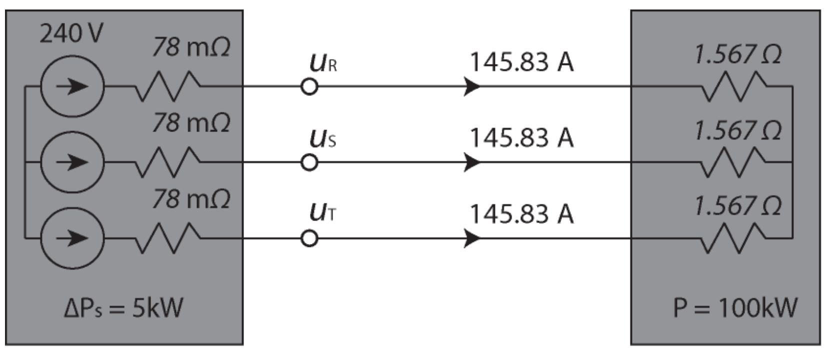


Fig. 4.6 Three phase resistive system with balanced load

With respects to the load active power *P*, the above balanced system can be rewritten as an unbalanced system with the same supply source, i.e.,

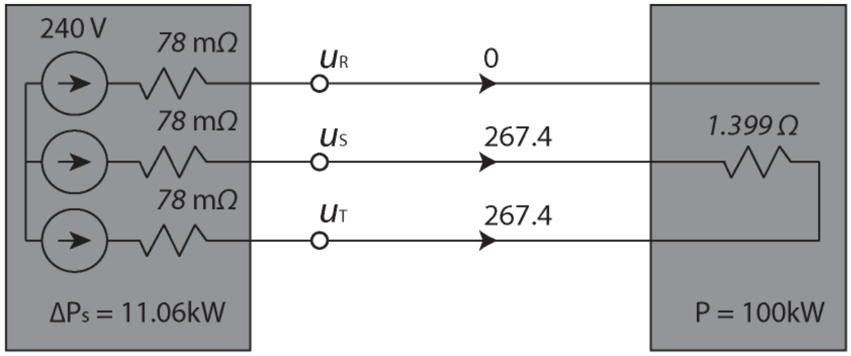


Fig. 4.7 Three phase resistive system with unbalanced load

With line voltage $\boldsymbol{E}_{\mathrm{R}} = 240e^{j0}$[V], the line currents crms value of the system are,

$$\boldsymbol{I}_{\mathrm{S}} = \frac{\boldsymbol{E}_{\mathrm{S}} - \boldsymbol{I}_{\mathrm{T}}}{2R_{\mathrm{S}} + R} = \frac{240e^{-j120} - 240e^{j120}}{2(0.078) + 1.399} = 267.4e^{-j90}\text{ [A]}$$

$$\boldsymbol{I}_{\mathrm{T}} = -\boldsymbol{I}_{\mathrm{S}} = 267.4e^{j90}\text{[A]}$$

Then the load voltage's crms value are,

$$\boldsymbol{U}_{\mathrm{S}} = \boldsymbol{E}_{\mathrm{S}} - R_{\mathrm{S}}\boldsymbol{I}_{\mathrm{S}} = 240e^{-j120} - 0.078 \times 267.4e^{-j90} = 222.2e^{-j122.6}$$

$$\boldsymbol{U}_{\mathrm{T}} = \boldsymbol{E}_{\mathrm{T}} - R_{\mathrm{S}}\boldsymbol{I}_{\mathrm{T}} = 240e^{j120} - 0.078 \times 267.4e^{j90} = 222.2e^{j122.6}$$

The load current can then be decomposed into symmetrical components, i.e.,

$$\begin{bmatrix}\boldsymbol{I}^{\mathrm{p}}\\ \boldsymbol{I}^{\mathrm{n}}\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & \alpha & \alpha^*\\ 1 & \alpha^* & \alpha\end{bmatrix}\begin{bmatrix}\boldsymbol{I}_{\mathrm{R}}\\ \boldsymbol{I}_{\mathrm{S}}\\ \boldsymbol{I}_{\mathrm{T}}\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & \alpha & \alpha^*\\ 1 & \alpha^* & \alpha\end{bmatrix}\begin{bmatrix}0\\ 267.4e^{-j90}\\ 267.4e^{j90}\end{bmatrix} = \begin{bmatrix}154.4e^{j0}\\ -154.4e^{j0}\end{bmatrix}\text{[A]}$$

Similarly, the load voltage can be decomposed into symmetrical components, i.e.,

$$\begin{bmatrix}\boldsymbol{U}^{\mathrm{p}}\\ \boldsymbol{U}^{\mathrm{n}}\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & \alpha & \alpha^*\\ 1 & \alpha^* & \alpha\end{bmatrix}\begin{bmatrix}\boldsymbol{U}_{\mathrm{R}}\\ \boldsymbol{U}_{\mathrm{S}}\\ \boldsymbol{U}_{\mathrm{T}}\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & \alpha & \alpha^*\\ 1 & \alpha^* & \alpha\end{bmatrix}\begin{bmatrix}240e^{j0}\\ 222.2e^{-j122.6}\\ 222.2e^{j122.6}\end{bmatrix} = \begin{bmatrix}228e^{j0}\\ 11.83e^{j0}\end{bmatrix}\text{[A]}$$

Therefore, the working active power of the load equates to,

$$P_{\mathrm{W}} = P^{\mathrm{p}} = (\boldsymbol{u}^{\mathrm{p}}, \boldsymbol{i}^{\mathrm{p}}) = 3U^{\mathrm{p}}I^{\mathrm{p}} = 3 \times 11.83 \times 154.4 = 3(35.2\mathrm{k}) = 105.6\mathrm{k[W]}$$

and the reflected active power equals to,

$$P_{\mathrm{r}} = P^{\mathrm{n}} = (\boldsymbol{u}^{\mathrm{n}}, \boldsymbol{i}^{\mathrm{n}}) = 3U^{\mathrm{n}}I^{\mathrm{n}} = 3 \times 11.83 \times 154.4 = 3(1.83\mathrm{k}) = 5.48\mathrm{k[W]}$$

The working active current rms value is equal to,

$$||\boldsymbol{i}_{\mathrm{w}}|| = \frac{P_{\mathrm{w}}}{||\boldsymbol{u}_{\mathrm{w}}||} = \frac{105.6\mathrm{k}}{\sqrt{3} \times 228} = 267.4\ [\mathrm{A}]$$

Thus the active power loss on the supply is elevated to,

$$\Delta P_{\mathrm{S}} = R_{\mathrm{S}}||\boldsymbol{i}_{\mathrm{w}}||^2 + P_{\mathrm{r}} = 0.078 \times 267.4^2 + 5.48\mathrm{k} = 11.06\mathrm{k[W]}$$

4.4 Detrimental Active Power and Currents

Let us assume that the supply voltage at terminals of an induction motor is asymmetrical because of distribution internal voltage $\boldsymbol{e}$(t) asymmetry. The equivalent circuit of such a supply is shown in Fig. 4.8.

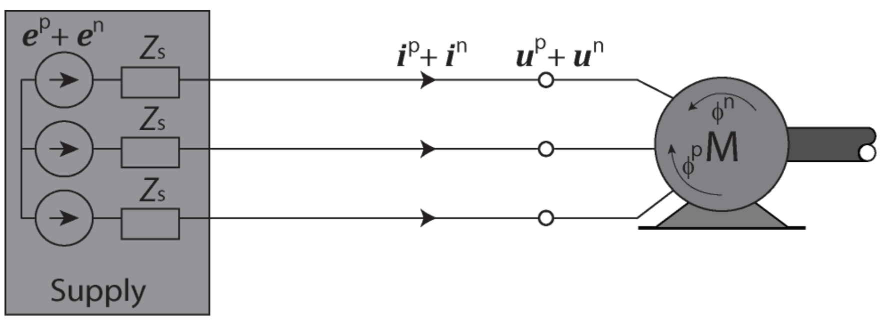


Fig. 4.8 Three phase motor with asymmetrical supply voltage

Therefore, the load voltage and current contains positive and negative sequence components, i.e.,

$$\boldsymbol{u} \triangleq \boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}} \tag{4}$$

$$\boldsymbol{i} \triangleq \boldsymbol{i}^{\mathrm{p}} + \boldsymbol{i}^{\mathrm{n}} \tag{5}$$

Active power is the scalar product of the load voltage and current $P = (\boldsymbol{u}, \boldsymbol{i})$. According to (4) and (5), active power is composed of the positive and negative sequence components which can be expressed as follows,

$$\mathrm{P} = (\boldsymbol{u}, \boldsymbol{i}) = (\boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}}, \boldsymbol{i}^{\mathrm{p}} + \boldsymbol{i}^{\mathrm{n}}) = (\boldsymbol{u}^{\mathrm{p}}, \boldsymbol{i}^{\mathrm{p}}) + (\boldsymbol{u}^{\mathrm{n}}, \boldsymbol{i}^{\mathrm{n}}) = P^{\mathrm{p}} + P^{\mathrm{n}}$$

because vectors of different sequences are orthogonal (see Appendix B), i.e., $(\boldsymbol{u}^{\mathrm{p}}.\boldsymbol{i}^{\mathrm{n}}) = (\boldsymbol{u}^{\mathrm{n}}.\boldsymbol{i}^{\mathrm{p}})=0$. Thus, the voltage and current of the positive and negative sequences convey the energy to the motor separately.

The supply current $\boldsymbol{i}$(t), which in fact is the stator current of the motor, creates a rotating magnetic field in the motor. However, fields created by the positive and negative sequence components $\boldsymbol{i}^{\mathrm{p}}$ and $\boldsymbol{i}^{\mathrm{n}}$ rotate in the opposite directions. The dominating field is created by the positive sequence component so that the motor rotates according to the rotation of the positive sequence component. Hence, the field created by the negative sequence components rotates in the opposite direction to the rotor rotation. Thus, only active power P^{p} can be converted to mechanical power on the motor shaft and can be regarded as the working active power.

The negative sequence component of the current in the induction motor stator creates a magnetic field that rotates in the direction opposite to the direction of the rotor rotation. This magnetic field creates a negative torque which reduces the resulting torque on the motor shaft. Moreover, the negative sequence magnetic field rotates with respect to the rotor at nearly twice the synchronous speed. Since the voltage induced in the rotor is proportional to the relative rotational speed of the magnetic field, this voltage could be relatively high even at low value of this negative sequence field. The rotor current caused by the induced voltage results in energy dissipation and consequently, in temperature increase. This reduces the life span of the motor.

Thus negative sequence active power cannot be regarded as useful active power, but instead as ***detrimental active power*** , denoted P_{d}, i.e.,

$$P_{\mathrm{d}} \triangleq P^{\mathrm{n}}$$

Consequently, the active power is composed of two components of different usefulness for the induction motor owner,

$$P = P_{\mathrm{w}} + P_{\mathrm{d}}$$

This means that the working active power P_{w} is lower than the active power P by the detrimental active power P_{d}. Customers who are charged for the working energy W_{w} would not have to pay for energy that causes harmful effects on their own equipment. Therefore, charging for working energy is clearly justified when the distribution system is the source of voltage asymmetry even though the customer's loads are linear, balanced, and time-invariant.

When the supply voltage is not only asymmetrical, but also distorted by harmonics, the load voltage and current vectors can be decomposed as follows,

$$\boldsymbol{i} \triangleq \boldsymbol{i}_1^{\mathrm{p}} + \boldsymbol{i}_1^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{h}} \tag{6}$$

$$\boldsymbol{u} \triangleq \boldsymbol{u}_1^{\mathrm{p}} + \boldsymbol{u}_1^{\mathrm{n}} + \boldsymbol{u}_{\mathrm{h}} \tag{7}$$

Active power is the scalar product of the load voltage and current $P = (\boldsymbol{u}, \boldsymbol{i})$. Taking into account (6) and (7), it can be expressed as follows,

$$P = (\boldsymbol{u}, \boldsymbol{i}) = \left(\boldsymbol{u}_1^{\mathrm{p}} + \boldsymbol{u}_1^{\mathrm{n}} + \boldsymbol{u}_{\mathrm{h}}, \boldsymbol{i}_1^{\mathrm{p}} + \boldsymbol{i}_1^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{h}}\right)$$

Because of the voltage and current of different sequences are orthogonal (see Appendix B) and harmonics of different orders are mutually orthogonal, we obtain,

$$P = \left(\boldsymbol{u}_1^{\mathrm{p}}, \boldsymbol{i}_1^{\mathrm{p}}\right) + \left(\boldsymbol{u}_1^{\mathrm{n}}, \boldsymbol{i}_1^{\mathrm{n}}\right) + (\boldsymbol{u}_{\mathrm{h}}, \boldsymbol{i}_{\mathrm{h}}) = P_1{}^{\mathrm{p}} + P_1{}^{\mathrm{n}} + P_{\mathrm{h}}$$

Thus, the active power is composed of active powers of the positive and negative sequence components of the fundamental harmonic and active power of higher order harmonic components. Therefore the voltage and current of the positive and negative sequences and harmonic components convey energy to the motor separately.

The current harmonics of order $n = 3k-1$ are harmonics of the negative sequence, therefore, they create magnetic fields that rotate in the direction opposite to the direction of the rotor rotation. These current harmonic components in the induction motor stator create a magnetic field that rotates in the direction opposite to the direction of the rotor rotation. This magnetic field creates a negative torque which reduces the resulting torque on the motor shaft. Moreover, the magnetic field rotates with respect to the rotor at nearly $n = 3k$ the synchronous speed. For example, the 5th order current harmonic creates a negative sequence magnetic field six times faster than synchronous speed. Since the voltage induced in the rotor is proportional to the relative rotational speed of the magnetic field, this voltage could be extremely high even at low value of this magnetic field created by the 5th order current harmonic. The rotor current caused by the induced voltage results in energy dissipation and consequently, in temperature increase. This reduces the life span of the motor.

The current harmonics of order $n = 3k+1$ are harmonics of the positive sequence, therefore, they create magnetic fields that rotate in the positive direction to the direction of rotor rotation. These current harmonic components in the induction motor stator create a magnetic field rotates with respect to the rotor at nearly $n = 3k$ the synchronous speed. For example, the 7th order current harmonic creates a magnetic field six times faster than synchronous speed. Since the voltage induced in the rotor is proportional to the relative rotational speed of the magnetic field, this voltage could be high even at low value of this magnetic field created by the 7th order current harmonic. This magnetic field creates a situation similar to a locked rotor in the induction motor. The rotor current caused by the induced voltage results in energy dissipation and consequently, in temperature increase. This ultimately reduces the life span of the motor.

Therefore, harmonic active power and negative sequence power cannot be regarded as useful power and therefore considered detrimental active power, i.e.,

$$P_{\mathrm{d}} \triangleq P_{\mathrm{h}} + P_1^{\mathrm{n}}$$

For calculating particular components of the active power, meaning the working, P_{w}, and detrimental active power, P_{d}, complex rms (crms) values of voltages and currents fundamental and higher order harmonics have to be known. Digital signal processing which calculates Discrete Fourier Transform (DFT) on the voltage and current samples is needed for that.

To calculate the active power P, samples of the voltage and current must be taken from the three phase system. The three phase system is sampled via Digital Signal Processing (DSP) equipment in the form of phase voltages $u_{\mathrm{R}}(k)$, $u_{\mathrm{S}}(k)$, $u_{\mathrm{T}}(k)$ and phase currents $i_{\mathrm{R}}(k)$, $i_{\mathrm{S}}(k)$, $i_{\mathrm{T}}(k)$. With the sampled voltages and current, the active power can then be calculated by the sum of each phase's instantaneous active power averaged over the sampling points N.

To calculate the working active power P_{w}, the sampled voltages and currents are then transformed by the Discrete Fourier Transform (DFT) to extract the crms voltages for the fundamental, $\boldsymbol{U}_{\mathrm{R1}}$, $\boldsymbol{U}_{\mathrm{S1}}$, $\boldsymbol{U}_{\mathrm{T1}}$, and crms currents for the fundamental, $\boldsymbol{I}_{\mathrm{R1}}$, $\boldsymbol{I}_{\mathrm{S1}}$, $\boldsymbol{I}_{\mathrm{T1}}$. Then the crms voltages and currents of the fundamental are used to calculate the positive sequence components of the fundamental voltage, $\boldsymbol{U}_1^{\mathrm{P}}$ and the positive sequence components of the fundamental current, $\boldsymbol{I}_1^{\mathrm{P}}$. Finally, the positive sequence components $\boldsymbol{U}_1^{\mathrm{P}}$ and $\boldsymbol{I}_1^{\mathrm{P}}$ are used to calculate the working active power P_{w}. Therefore the detrimental active power can be calculated as the difference of active power P and working active power P_{w}, i.e.,

$$P_{\mathrm{d}} = P - P_{\mathrm{w}}$$

Detrimental power can also be associated with harmonic active power in a three phase rectifier shown below. Assume that the supply voltage of a six pulse rectifier, as shown in Fig. 4.9, is symmetrical but contains harmonics.

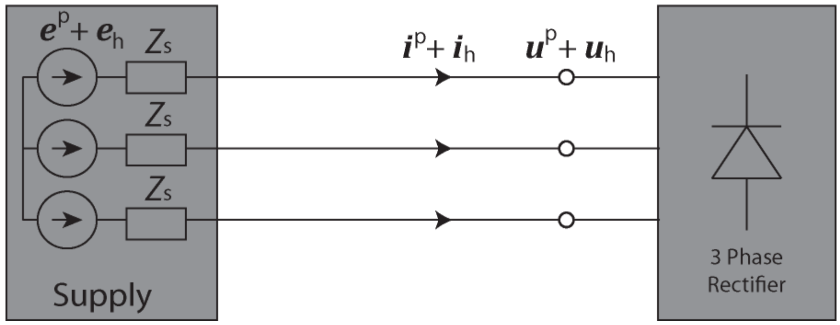


Fig. 4.9 Rectifier in three phase system

In such a case, the vectors of the supply voltage and current can be decomposed as follows:

$$\boldsymbol{i} \triangleq \boldsymbol{i}_1{}^{\mathrm{p}} + \boldsymbol{i}_{\mathrm{h}}$$

$$\boldsymbol{u} \triangleq \boldsymbol{u}_1^{\mathrm{p}} + \boldsymbol{u}_{\mathrm{h}}$$

The current decomposition has an instance when both the harmonic generation originates from the supply and the load side. Because both the supply and load harmonic generation are random by nature and statistically two random events are mutually exclusive, the harmonic generated current would be considered orthogonal. Because the DFT is used to calculate the crms value that results in only the magnitude and phase angle of the load current, only the sum of the currents can be obtained as an approximation of the actual harmonic generated current.

With the load current, the active power at load terminals consist of,

$$P = (\boldsymbol{u}, \boldsymbol{i}) = \left(\boldsymbol{u}_1^{\mathrm{p}}, \boldsymbol{i}_1^{\mathrm{p}}\right) + \left(\boldsymbol{u}_{\mathrm{h}}, \boldsymbol{i}_{\mathrm{h}}\right) = P_1^{\mathrm{p}} + P_{\mathrm{h}}$$

Working active power is denoted as the positive sequence active power of the fundamental, i.e.,

$$P_{\mathrm{w}} = P_1^{\mathrm{p}}$$

while the harmonic active power P_h is separated by positive and negative polarities. The first case would be if the harmonic generation is on the supply side and the second case is when the harmonic generation is on the load side. The active power would be calculated as such,

$$P_\mathrm{h} = (\boldsymbol{u}_\mathrm{h}, \boldsymbol{i}_\mathrm{h}) = \sum_{n \in N_\mathrm{C}} (\boldsymbol{u}_\mathrm{n}, \boldsymbol{i}_\mathrm{n}) + \sum_{n \in N_\mathrm{L}} (-R_\mathrm{S} \|\boldsymbol{i}_\mathrm{n}\|^2, \boldsymbol{i}_\mathrm{n})$$

$$P_\mathrm{h} = \sum_{n \in N_\mathrm{C}} P_n - \sum_{n \in N_\mathrm{L}} P_n$$

With three phase rectifier, higher order harmonics disturb the performance of the rectifier. Nonsinusoidal voltage contributes to commutation of the diodes slightly off from the normal commutation angle, thus creating noncharacteristic harmonics in the current and increasing the output ripples on the DC voltage side. With increase in current rms due to increased current harmonics, there will be additional losses as well as temperature increases that would ultimately shorten the lifespan of the equipment. Therefore harmonic power P_h originating from the supply side cannot be regarded as useful power, and thus is considered detrimental active power P_d, i.e.,

$$P_\mathrm{d} = \sum_{n \in N_\mathrm{C}} P_n$$

Similar to the single phase rectifier, the harmonic power P_h originating from the load side is regarded as reflected active power P_r, i.e.,

$$P_\mathrm{r} = \sum_{n \in N_\mathrm{L}} P_n$$

In such a system, the active power P contains both reflected active power P_r and detrimental active power P_d, i.e.,

$$P = P_\mathrm{w} + P_\mathrm{d} - P_\mathrm{r}$$

With Fourier analysis of the load voltage and current, the sign of the harmonic active power P_n can be obtained to separate these powers. Although it is impossible to single out reflected active power P_r and detrimental active power P_d because each individual component within the same frequency level cannot be decomposed. Only the difference can be shown, i.e.,

$$P_\mathrm{w} - P = P_\mathrm{r} - P_\mathrm{d}$$

Similar to the scenario of detrimental active power of the three phase system, the single phase system can be regarded as a decomposition of a three phase system. Take for example the case of a single phase rectifier shown below,

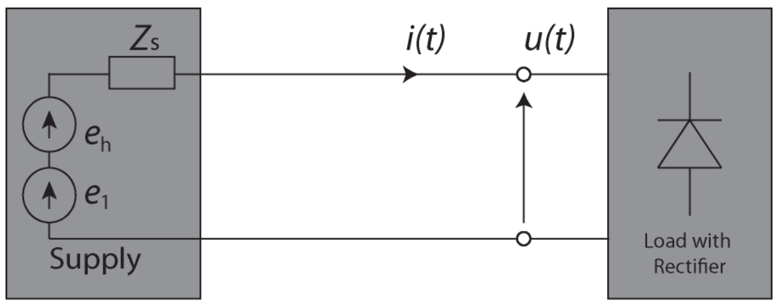


Fig. 4.10 Single phase rectifier

In this case, the load voltage and current contains harmonics supplied by the source, i.e.,

$$i(t) = \sum_{n \in N} i_n = i_1 + i_\mathrm{h} = i_{1\mathrm{aC}} + i_\mathrm{h}$$

$$u(t) = \sum_{n \in N} u_n = u_1 + u_\mathrm{h}$$

With the active power of the load is calculated as,

$$P = \sum_{n \in N} P_n = P_1 + P_2 + P_3 + \cdots + P_n$$

Where active power of harmonic order *n* is,

$$P_n = \frac{1}{T}\int_0^T u_n(t) i_n(t) dt$$

With harmonic active power P_{h} is the active power of higher order harmonics, i.e.,

$$P_{\mathrm{h}} = P_2 + P_3 + P_4 + \cdots + P_n$$

Because the harmonic sources are on both the supply side and load side, harmonic active power P_{h} is split into two subsets i.e.,

$$P_{\mathrm{h}} = \sum_{n \in N_{\mathrm{C}}} P_n - \sum_{n \in N_{\mathrm{L}}} P_n$$

Thus similar to the three phase rectifier, the harmonic power cannot be regarded as useful active power and therefore is regarded as detrimental active power and reflected active power, i.e.,

$$P_{\mathrm{d}} = \sum_{n \in N_{\mathrm{C}}} P_n \quad , \quad P_{\mathrm{r}} = \sum_{n \in N_{\mathrm{L}}} P_n$$

With working active power P_{w} similar to the three phase rectifier, i.e.,

$$P_{\mathrm{w}} = P + P_{\mathrm{r}} - P_{\mathrm{d}}$$

When a system consist of loads in addition to HGLs that are reactive and time-varying such as RLC in Fig. 4.11, the voltages and currents still remain periodic. Therefore the previous analysis does not change drastically.

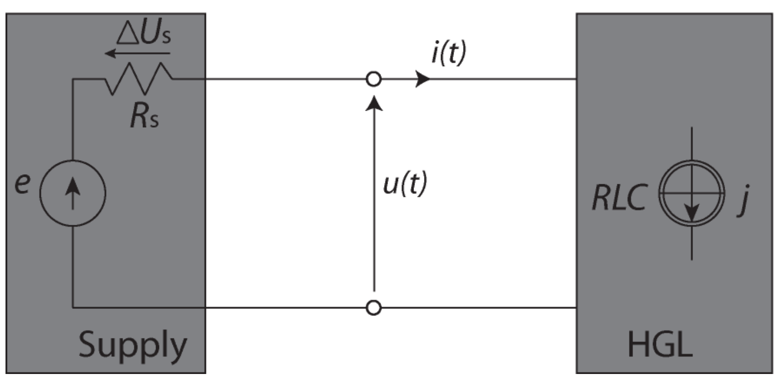


Fig. 4.11 System with HGL and RLC loads with sinusoidal voltage

The difference would be the addition of the fundamental reactive component, $i_{1\mathrm{r}}$, in the load current. Therefore, the current can be decomposed into several components, i.e.,

$$i(t) = \sum_{n \in N} i_n = i_1 + i_{\mathrm{h}} = i_{1\mathrm{aC}} + i_{1\mathrm{r}} + i_{\mathrm{h}}$$

with the active current component composed of the fundamental active current, i.e.,

$$i_w \triangleq i_{1\mathrm{aC}} \, ,$$

Thus the decomposition of the load current is expressed as,

$$i(t) = i_{\mathrm{w}} + i_{1\mathrm{r}} + i_{\mathrm{h}} \, .$$

The working active power is then expressed as,

$$P_{\mathrm{w}} \triangleq P_1 = \mathrm{Re}\,\{\boldsymbol{U}_1 \boldsymbol{I}_1{}^{*}\} = U_1 I_1 \cos\varphi_1 = U_{\mathrm{w}} I_{\mathrm{w}} \, .$$

With the addition of reactive devices, scattered current will have to be taken into account. This will interact with the working active power and create a non-orthogonal relationship. The load current has the form,

$$i = i_{\mathrm{aC}} + i_{\mathrm{s}} + i_{\mathrm{r}}$$

with active current defined as,

$$i_{\mathrm{aC}} = G_{\mathrm{eC}} u$$

working active current can then be specified as,

$$i_{\mathrm{w}} = i_{\mathrm{a1C}} = G_1 u_1$$

In the crms form, working active power fulfills the relationship,

$$\boldsymbol{I}_{\mathrm{w}} = G_1 \boldsymbol{U}_1$$

with scattered current as,

$$\boldsymbol{I}_{\mathrm{s}} = (G_1 - G_{\mathrm{eC}}) \boldsymbol{U}_1$$

When these cmrs currents are added together,

$$\boldsymbol{I}_{\mathrm{w}} = \boldsymbol{I}_{\mathrm{a1}} + \boldsymbol{I}_{\mathrm{s1}} = G_1\boldsymbol{U}_1$$

Therefore the active working current is composed of the first fundamental active current and scattered current.

$$i_{\mathrm{w}} = i_{\mathrm{a1}} + i_{\mathrm{s1}}$$

While the detrimental active current is the higher order harmonics of the active and scattered current,

$$i_{\mathrm{d}} = \sum_{n=2}^{N} i_{an} + i_{sn}$$

Because the working active current and the detrimental active current is associated with two different sets namely active current and scattered current, the current decomposition of the load current,

$$i = i_{\mathrm{w}} + i_{\mathrm{d}} + i_{\mathrm{r}}$$

is not orthogonal yet in the real world there is still economical meaning to working active current.

4.5 Measurement of Power Components

To calculate the active power P, samples of the voltage and current must be taken from the power system. The power system is sampled from data acquisition and processed by Digital Signal Processing (DSP) equipment in the form of load voltage $u(k)$ and load current $i(k)$. With the sampled voltage and current, the active power can then be calculated by the sum of instantaneous active power averaged over the sampling points N, i.e.,

$$P = \frac{1}{N}\sum_{n=0}^{N-1}[u(k)i(k)]$$

To calculate the working active power P_{w}, the sampled voltage and current is then transformed by the Discrete Fourier Transform (DFT) to extract the crms voltage for the fundamental, $\boldsymbol{U}_1$ and crms current for the fundamental, $\boldsymbol{I}_1$, i.e.,

$$\boldsymbol{U}_1 = \frac{\sqrt{2}}{N}\sum_{k=0}^{N-1} u(k)\, e^{-j2\frac{\pi}{N}k}$$

According to Nyquist's sampling criterion, a sequence of uniformly spaced samples of the order greater than $2f_1$ is needed to recover the signal's original form. In such a case to find the fundamental signal at uniform sampling, N = 3. Therefore, the Discrete Fourier Transform (DFT) can be expressed as,

$$\boldsymbol{U}_1 = \frac{\sqrt{2}}{3}\sum_{k=0}^{2} u(k)\, e^{-j2\frac{\pi}{3}k}$$

and similar for the crms currents for the fundamental, $\boldsymbol{I}_1$.

Therefore, the positive sequence components $\boldsymbol{U}_1^{\mathrm{P}}$ and $\boldsymbol{I}_1^{\mathrm{P}}$ are used to calculate the working active power P_{w}, i.e.,

$$P_{\mathrm{w}} = Re\{\boldsymbol{U}_1\, \boldsymbol{I}_1{}^*\}$$

To calculate the active power P, samples of the voltage and current must be taken from the three phase system. The three phase system is sampled via Digital Signal Processing (DSP) equipment in the form of phase voltages $u_{\mathrm{R}}(k)$, $u_{\mathrm{S}}(k)$, $u_{\mathrm{T}}(k)$ and phase currents $i_{\mathrm{R}}(k)$, $i_{\mathrm{S}}(k)$, $i_{\mathrm{T}}(k)$. With the sampled voltages and currents, the active power can then be calculated by the sum of each phase's instantaneous active power averaged over the sampling points N, i.e.,

$$P = \frac{1}{N}\sum_{n=0}^{N-1}[u_R(k)i_R(k) + u_S(k)i_S(k) + u_T(k)i_T(k)]$$

To calculate the working active power P_w, the sampled voltages and currents are then transformed by the Discrete Fourier Transform (DFT) to extract the crms voltages for the fundamental $\boldsymbol{U}_{R1}$, $\boldsymbol{U}_{S1}$, $\boldsymbol{U}_{T1}$, and crms currents for the fundamental, $\boldsymbol{I}_{R1}$, $\boldsymbol{I}_{S1}$, $\boldsymbol{I}_{T1}$, i.e.,

$$\boldsymbol{U}_{\mathrm{R}} = \frac{\sqrt{2}}{N}\sum_{k=0}^{N-1} u_R(k)\, e^{-j2\frac{\pi}{N}k}$$

According to Nyquist's sampling criterion, a sequence of uniformly spaced samples of the order greater than $2f_1$ is needed to recover the signal's original form. In such a case to find the fundamental signal at uniform sampling, N = 3. Therefore, the Discrete Fourier Transform (DFT) can be expressed as,

$$\boldsymbol{U}_{\mathrm{R1}} = \frac{\sqrt{2}}{3}\sum_{k=0}^{2} u_R(k)\, e^{-j2\frac{\pi}{3}k}$$

and similar for the crms voltages for the fundamental $\boldsymbol{U}_{S1}$, $\boldsymbol{U}_{T1}$, and crms currents for the fundamental, $\boldsymbol{I}_{R1}$, $\boldsymbol{I}_{S1}$, $\boldsymbol{I}_{T1}$.

Then the crms voltages for the fundamental $\boldsymbol{U}_{R1}$, $\boldsymbol{U}_{S1}$, $\boldsymbol{U}_{T1}$, and crms currents for the fundamental, $\boldsymbol{I}_{R1}$, $\boldsymbol{I}_{S1}$, $\boldsymbol{I}_{T1}$ are used to calculate the positive sequence components of the fundamental voltage, $\boldsymbol{U}_1^{\mathrm{P}}$ and the positive sequence components of the fundamental current, $\boldsymbol{I}_1^{\mathrm{P}}$,

$$\boldsymbol{U}_1^{\mathrm{P}} = \frac{1}{3}[\boldsymbol{U}_{R1} + \alpha \boldsymbol{U}_{\mathrm{S1}} + \alpha^* \boldsymbol{U}_{\mathrm{T1}}]$$

$$\boldsymbol{I}_1^{\mathrm{P}} = \frac{1}{3}[\boldsymbol{I}_{R1} + \alpha \boldsymbol{I}_{\mathrm{S1}} + \alpha^* \boldsymbol{I}_{\mathrm{T1}}]$$

Finally, the positive sequence components $\boldsymbol{U}_1^{\mathrm{P}}$ and $\boldsymbol{I}_1^{\mathrm{P}}$ are used to calculate the working active power P_w, i.e.,

$$P_{\mathrm{w}} = Re\left\{\boldsymbol{U}_1^{\mathrm{P}}\, \boldsymbol{I}_1^{\mathrm{P}^*}\right\}$$

CHAPTER 5. FUNDAMENTAL CRMS ALGORITHM

5.1 DFT Calculations for CRMS Values

The rms values of voltage and current with the phase shift are invaluable resources of information on the power system performance. The use of the rms values and phase shift are for the utilization of power system stability, protection, and energy accounting. By combining the two variables, complex RMS form can be expressed by a rms magnitude with a phase shift denoted,

$$\boldsymbol{U}_1 \triangleq U_1 e^{j\propto_1} \quad , \quad \boldsymbol{I}_1 \triangleq I_1 e^{j\beta_1}$$

which can be defined more generally as,

$$\boldsymbol{X}_1 \triangleq X_1 e^{j\propto_1} = \mathrm{Re}\{\boldsymbol{X}_1\} + j\mathrm{Im}\{\boldsymbol{X}_1\}$$

The fundamental harmonic x(t) can be related to the crms with the following,

$$x_1(t) \triangleq \sqrt{2} Re\left\{\boldsymbol{X}_1 e^{j\omega_1 t}\right\} \quad , \quad \omega_1 \triangleq 2\pi f_1$$

When a system has strong distorted waveforms, the crms values enable the calculation of reactive power of the first harmonic, Q_1 which then enables the calculation for power factor improvements.

The quantity x within a sinusoidal system of frequency f_1 can be used to calculate the crms value. According to Nyquist's sampling criterion, a sequence of uniformly spaced samples denoted x_k of the order greater than $2f_1$ are needed to recover the signal's original form. In such a case to find the fundamental signal at uniform sampling, N = 3. Therefore, the Discrete Fourier Transform (DFT) will be in the general form of,

$$\boldsymbol{X}_n = \frac{\sqrt{2}}{N} \sum_{k=0}^{N-1} x_k \, e^{-j2\frac{\pi}{N}nk}$$

and more specifically,

$$\boldsymbol{X}_1 = \frac{\sqrt{2}}{3}\sum_{k=0}^{2} x_k\, e^{-j2\frac{\pi}{3}k}$$

Therefore, the crms values of quantity x can be determined by three samples, x_0, x_1, x_2 at points $t_k \triangleq 2\frac{\pi}{3}k$ such that,

$$\mathrm{Re}\{\boldsymbol{X}_1\} = \frac{1}{3\sqrt{2}}(2x_0 - x_1 - x_2)$$

$$\mathrm{Im}\{\boldsymbol{X}_1\} = \frac{-1}{\sqrt{6}}(x_1 - x_2)$$

Such is the case that only three samples of the voltage, u_0, u_1, u_2 and three samples of the current, i_0, i_1, i_2 are needed to determine the crms values $\mathbf{U}_1$ and $\mathbf{I}_1$. Determining the crms values for the fundamental harmonic, $\mathbf{X}_1$ from a non-sinusoidal quantity such as,

$$x_0(t) \;\triangleq\; X_0 + \sqrt{2}\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{X}_n\, e^{jn\omega_1 t}$$

requires a few issues to be addressed.

First, $x_0(t)$ must be approximated with an *M*-order trigonometric polynomial, i.e.,

$$x_0(t) \;\triangleq\; X_0 + \sqrt{2}\mathrm{Re}\sum_{n=1}^{M-1} \boldsymbol{X}_n\, e^{jn\omega_1 t}$$

If the calculation of $\boldsymbol{X}_1$ is to applied with a digital method, the value of *M* affects the number of samples *N* of x_k with sampling rate of the A/D converter affected as well. Consequently, the amount of calculations needed and the hardware used will also be affected. Reduction of the amount of calculations as much as possible will reduce the hardware requirements for real-time application.

DFT calculations usually use the Fast Fourier Transform (FFT) method for calculation of periodic sequences to provide X_0 and the rest of the crms values $\boldsymbol{X}_n$ for 1 to M-1 at the same time. If only $\boldsymbol{X}_1$ is needed, then all the other calculations are considered a waste and burden on the hardware. Therefore, a more efficient method than FFT algorithm is required for real-time applications.

5.2 Direct DFT Calculation Algorithm

The frequency spectrum of quantity x with frequency f_1 be limited by Mf_1 frequency. In other words, $\boldsymbol{X}_n = 0$ for $n \geq M$ while M is an even integer. Using the Nyquist's criterion, the frequency spectrum of the sampled sequence x_k is not aliased under the condition that N number samples which are taken uniformly over the period T_1 is greater than 2M. If the sampling number N is chosen as a multiplicity of four, i.e., $N = 4K$, and $2K = M$, then the crms value of fundamental harmonic $\boldsymbol{X}_1$ is equal to,

$$\boldsymbol{X}_1 = \frac{\sqrt{2}}{N}\sum_{k=0}^{N-1} x_k\, e^{-j2\frac{\pi}{N}k} = \frac{\sqrt{2}}{N}\sum_{k=0}^{4K-1} x_k\, \boldsymbol{V}^k \ , \quad \boldsymbol{V} \triangleq 1e^{-j\frac{\pi}{2K}}$$

Where $\boldsymbol{V}^{k+M} = -\boldsymbol{V}^k$ and $\boldsymbol{V}^0 = 1$, the DFT formula above can be expressed as,

$$\boldsymbol{X}_1 = \frac{\sqrt{2}}{N}\left[(x_0 - x_M) + \sum_{k=1}^{2K-1}(x_k - x_{k+M})\,\boldsymbol{V}^k\right]$$

Next the value inside the summation are denoted as $x_k - x_{k+M} \triangleq s_k$ and with $\boldsymbol{V}^k = -\mathrm{j}1$, the above formula can be further rearranged as such,

$$\boldsymbol{X}_1 = \frac{\sqrt{2}}{N}\left[s_0 - \mathrm{j}s_k + \sum_{k=1}^{K-1}(s_k\boldsymbol{V}^k + s_{M-k}\boldsymbol{V}^{M-k})\right]$$

The complex coefficients $\boldsymbol{V}^k$ benefits from having quadrant symmetry, i.e.,

$$\text{Re}\,\{\boldsymbol{V}^{M-k}\} = -\text{Re}\,\{\boldsymbol{V}^k\}\,, \quad \text{Im}\,\{\boldsymbol{V}^{M-k}\} = \text{Im}\,\{\boldsymbol{V}^k\}$$

Therefore, the crms formula can be broken down into real and imaginary parts, i.e.,

$$\text{Re}\,\{\boldsymbol{X}_1\} = \frac{\sqrt{2}}{N}\left[s_0 + \sum_{k=1}^{K-1}(s_k - s_{M-k})\,\text{Re}\,\{\boldsymbol{V}^k\}\right]$$

$$\text{Im}\,\{\boldsymbol{X}_1\} = \frac{\sqrt{2}}{N}\left[-s_k + \sum_{k=1}^{K-1}(s_k + s_{M-k})\,\text{Im}\,\{\boldsymbol{V}^k\}\right]$$

The above formula for the direct DFT calculation assumes that N is a multiplicity of 4 which provides us with an efficient algorithm for calculating the crms value of the fundamental harmonic. Additionally, if the values for Re $\{\boldsymbol{V}^k\}$, Im $\{\boldsymbol{V}^k\}$, and $\sqrt{2}/N$ are stored in the computer's memory, then the algorithm only requires $3M - 4$ additions and M multiplications.

For example, assume that the quantity x has harmonics of order $n = 0,1,2,3,4,5$. First assume that $N = 12$ thus $M = 6$ and $K = 3$. Therefore the values for Re$\{\boldsymbol{V}^k\}$ and Im $\{\boldsymbol{V}^k\}$ can be stored as,

$$\text{Re}\,\{\boldsymbol{V}^1\} = \cos(-\pi/6) = \sqrt{3}/2$$

$$\text{Im}\,\{\boldsymbol{V}^1\} = \sin(-\pi/6) = -1/2$$

$$\text{Re}\,\{\boldsymbol{V}^2\} = \cos(-\pi/3) = 1/2$$

$$\text{Im}\,\{\boldsymbol{V}^2\} = \sin(-\pi/3) = -\sqrt{3}/2$$

Therefore, the crms value can be calculated as such,

$$\text{Re}\,\{\boldsymbol{X}_1\} = \frac{\sqrt{2}}{12}\left[s_0 + \frac{\sqrt{3}}{2}(s_1 - s_5) + \frac{1}{2}(s_2 - s_4)\right]$$

$$\text{Im}\,\{\boldsymbol{X}_1\} = -\frac{\sqrt{2}}{12}\left[s_3 + \frac{1}{2}(s_1 + s_5)\ + \frac{\sqrt{3}}{2}(s_2 + s_4)\right]$$

The calculation of the crms value $\boldsymbol{X}_1$ in the example requires 14 additions and 6 multiplications. The A/D converter needs to only provide N = 12 samples for each period T_1 to fulfill the Nyquist' criterion.

The algorithm to calculate the crms value $\boldsymbol{X}_1$ of the fundamental harmonic discussed is very efficient computationally. This direct DFT calculation requires much lower processing power than conventional FFT algorithm along with low sampling rate. These benefits enable the calculation of crms value $\boldsymbol{X}_1$ in real time. Such calculation can be used to calculate the crms values of the load voltage $\boldsymbol{U}_1$ and current $\boldsymbol{I}_1$ to then calculate the working active power P_{w}.

CHAPTER 6. SOFTWARE VERIFICATION

6.1 Software Overview

The software used for numerical calculations and verifications is MATLAB. MATLAB is a numerical computing program developed by MathWorks and stands for MATrix LABoratory. Various operations include matrix manipulation, plotting of data and functions, and interfacing with other computer languages such as C++. The version used in the thesis is MATLAB 2010a.

The MATLAB environment includes a GUI that will let the user input commands, declare variables, and run simulations. Secondary files called “m-files” are created and used similar to a function in programming language. These files will be used to create the programming code in MATLAB to simulate and verify the numerical examples given in the thesis as well as testing the DFT algorithm. The input of the user is similar to a typical computer programming language with a syntax input line shown in Fig. 6.1.

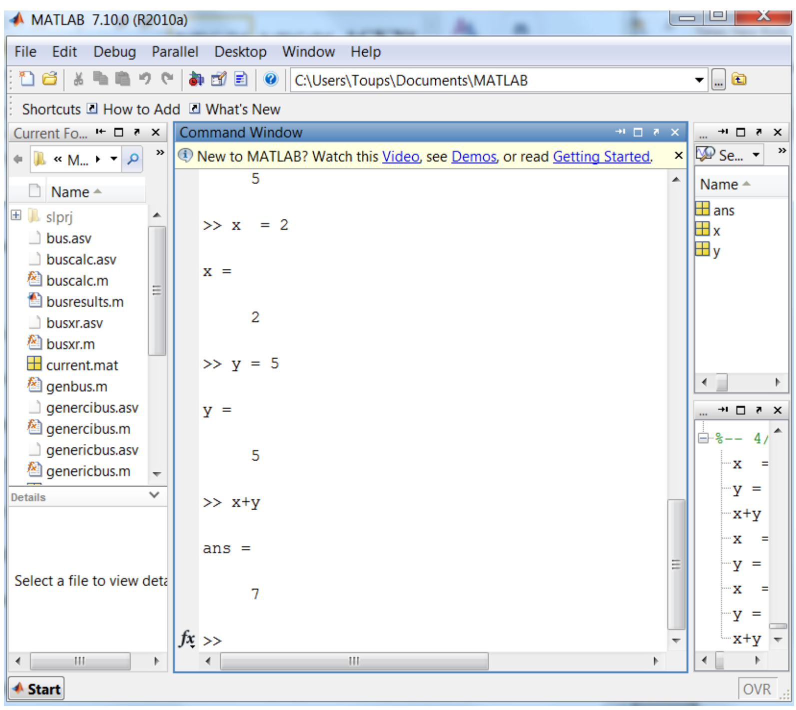


Fig. 6.1. MATLAB software

In addition to the standard MATLAB program, a sub-program called SIMULINK is available to use. SIMULINK is a gui that uses block diagrams to simulate control systems. With SIMULINK there is a program called “powergui” which is a circuit builder and simulator specifically designed for power applications. With the powergui simulator, the examples can be further evaluated on a circuitry level. The input for the “powergui” is similar to SIMULINK where the user would place blocks and parts and connect metering devices or scopes with wires such as in Fig. 6.2.

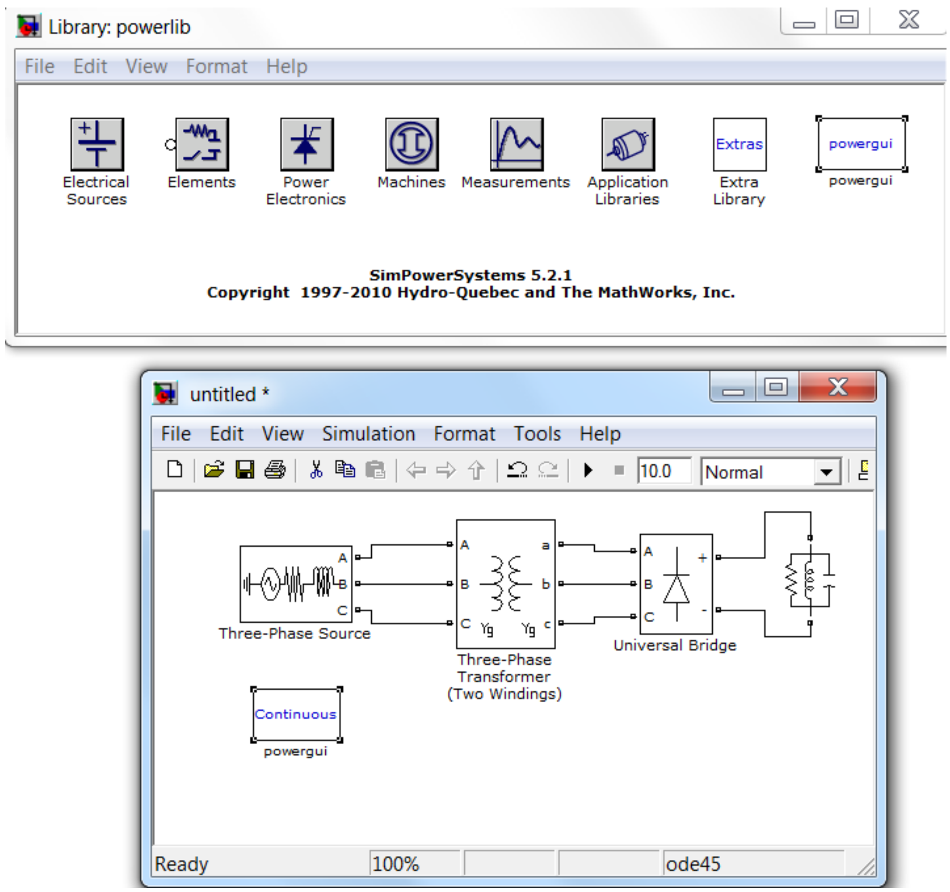


Fig. 6.2 Powergui software

6.2 MATLAB Simulation

The first numerical example was given with a single phase rectifier with sinusoidal voltage supply from illustration 1. Recalling the illustration, calculate the working and reflected active powers of the load with a rectifier supplying a DC motor from an AC source. Assume that the rectifier is lossless and neglect the inductance of the motor armature (the motor has only purely resistive losses) and the AC voltage *e* is sinusoidal with purely resistive supply source impedance. (see appendix for detailed calculation)

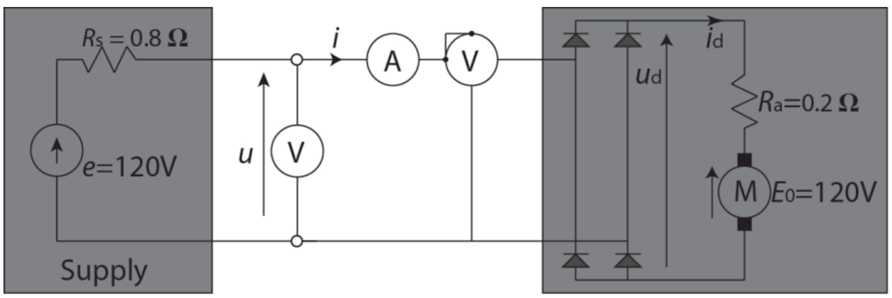


Fig. 6.3 Supply for DC motor from AC voltage source

With the load voltage *u* and the supply current *i* in the form,

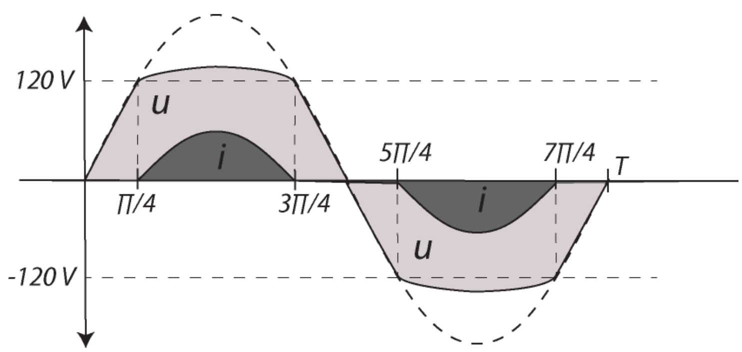


Fig. 6.4 Voltage and current waveform for illustration 1

A m-file named "1phaserectifier.m" is created to verify the numerical example using the following algorithm in Table 6.1. After the initial setup of variables and constants, the program

follows a similar logic based on the numerical example. The m-file is listed under the Appendix C for reference.

Step 1	Initial setup and variable decleration
Step 2	Fundamental current integration
Step 3	Fundamental integration of part A
Step 4	Fundamental integration of part B
Step 5	Fundamental rebuilding and shifting of original voltage waveform
Step 6	Rms current integration
Step 7	Dc current integration
Step 8	Power calculations

Table 6.1 Algorithm for verification of illustration 1

The resulting output shows the verification of the illustration with a slight difference due to rounding of numerical example versus rounding of a computer simulation in table 6.2.

Variable	Numerical Example	Simulated Results
Active Power (P)	2097 [W]	2097.1 [W]
Working Active Power (P_1)	2235.9 [W]	2236 [W]
Reflected Active Power (P_r)	138.8 [W]	138.92 [W]

Table 6.2 Output of the simulation versus numerical calculations for illustration 1

After the simulated results verified the numerical example, the illustration is then built in the powergui interface. The circuit diagram is built according to Fig. 6.3 with added

measurement blocks to verify the results. The circuit diagram itself can be found in the Appendix C. The circuit diagram of the model is shown in Fig. 6.5.

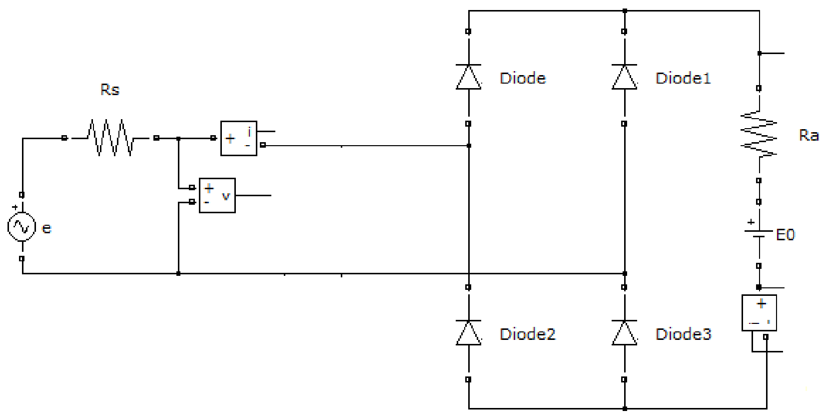


Fig. 6.5. Powergui circuit of illustration 1

The output measurement devices are shown with the verified data in Fig. 6.6.

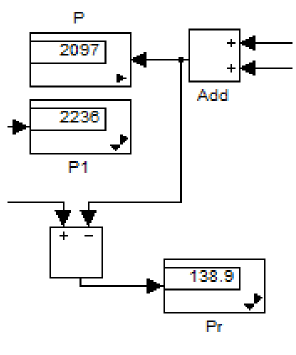


Fig.6.6 Output of powergui modeling of illustration 1

The output of the simulated circuit indeed matches with the results from the previous verification and the numerical example.

Next, illustration 2 was verified using the powergui interface. Recall illustration 2, a three phase purely resistive system with a balanced load is assumed. The distribution voltage is sinusoidal and symmetrical with line to ground voltage rms E= 240V. The load parameters are

selected so the load active power P = 100kW and the power loss in the supply is ΔP_s = 5kW. The resulting system is shown below in Fig. 6.7.

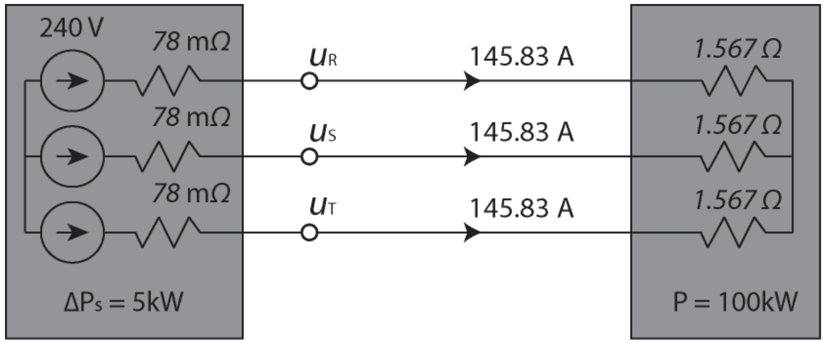


Fig. 6.7 Three phase resistive system with balanced load

With respects to the load active power *P*, the above balanced system can be rewritten as an unbalanced system with the same supply source, i.e.,

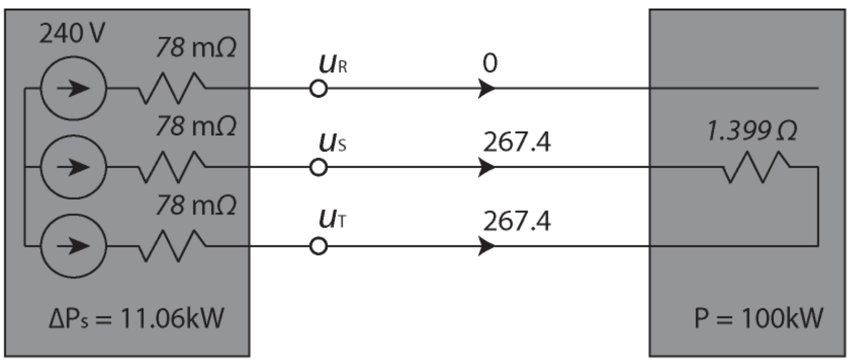


Fig. 6.8 Three phase resistive system with unbalanced load

The illustration is constructed using the powergui interface. The circuit diagram is built according to Fig. 6.8 with added measurement blocks and sequence component subsystems. The circuit diagram and subsystems can be found in the Appendix C. The circuit diagram of the model is shown in Fig. 6.9.

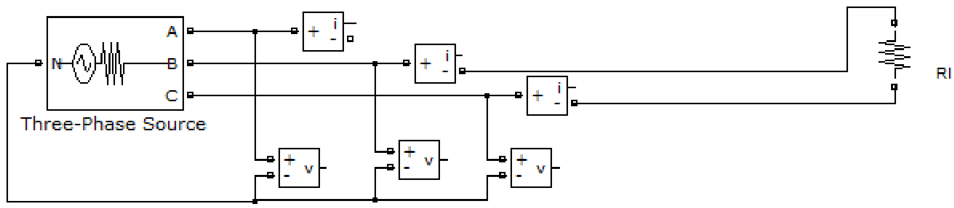


Fig. 6.9 Powergui circuit for illustration 2

The output measurement devices are shown with the verified data in Fig. 6.10.

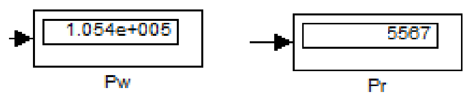


Fig. 6.10 Output of powergui modeling for illustration 2

The output of the simulated circuit approximately matches with the results from the numerical example. The results are shown in Table 6.3.

Variable	Numerical Example	Modeling Verification
Working Active Power (P_1)	105.6 k [W]	105.4 k [W]
Reflected Active Power (P_r)	5.48 k [W]	5.567 k [W]

Table 6.3 Output of the simulation versus numerical calculations for illustration 2

The differences in values come from the rounding of numerical example versus the more accuracy of computer simulated rounding.

Lastly, a m-file is created to test and verify the DFT algorithm explained in the previous section. The m-file can be referenced in the Appendix C. The test waveform consists of the fundamental and summation of all odd ordered harmonics to the 49th harmonic, i.e.,

$$x(t) = 100\sqrt{2}\sin(\omega_1 t) + \sum_{n=2}^{49} 5\sqrt{2}\sin(n\omega_1 t)$$

The DFT algorithm follows the following steps outlined in Table 6.4.

Step 1	Setup the test waveform and sample the waveform for N = 100.
Step 2	Declare constants and parameter values.
Step 3	Calculate the values for Re$\{\boldsymbol{V}^k\}$ and Im $\{\boldsymbol{V}^k\}$ into vector form.
Step 4	Calculate the values for the $x_k - x_{k+M} \triangleq s_k$ values into vector form.
Step 5	Setup scaling value $\frac{\sqrt{2}}{N}$ and add initial values s_0 and $-s_k$ to Re $\{\boldsymbol{X}_1\}$, Im $\{\boldsymbol{X}_1\}$
Step 6	Add up the rest of values for $\sum_{k=1}^{K-1}(s_k - s_{M-k})$ Re $\{\boldsymbol{V}^k\}$, $\sum_{k=1}^{K-1}(s_k + s_{M-k})$ Im $\{\boldsymbol{V}^k\}$ To Re $\{\boldsymbol{X}_1\}$, Im $\{\boldsymbol{X}_1\}$
Step 7	Multiply Re $\{\boldsymbol{X}_1\}$, Im $\{\boldsymbol{X}_1\}$ by scaling factor $\frac{\sqrt{2}}{N}$ and generate error percentage.

Table 6.4 DFT Algorithm

The results of the DFT algorithm m-file is as follows,

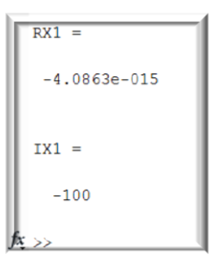


Fig. 6.11 DFT algorithm output

Using MATLAB the illustrations are checked via a computer simulated environment and have proven to be correct. The DFT algorithm written using the theory in chapter 6 is also tested and with the output being satisfactory.

CHAPTER 7. CONCLUSION

7.1 Conclusion

The thesis demonstrates that active power decomposition into working, reflected, and detrimental active power enriches the CPC power theory which gives a more detailed approach that can be used to accurately dictate financial losses by both parties. Working active power is the component of active power needed to run equipment and produce mechanical work. Reflected active power is the component of active power that the energy provider sends to the load so that the HGL or unbalanced loads to create the harmonic generating current or negative sequence current. Detrimental active power is the component of active power that the utility sends to the customer and is charged for energy that does not convey useful energy and possibly harming the customer's equipment.

With the decomposition of working, reflected, and active power, both the utility company and customer can benefit economically. The party that is the cause of the increased energy usage would be accurately billed for the energy usage. Overall, with accurate readings of power, this will give incentive for both customers and utilities to raise their supply quality or load quality which will reduce the overall energy usage in the power system.

7.2 Future Progress

With the working, reflected, and detrimental active power theory defined and simulated, the next step would be to start creating software to accurately read the current and voltages from the load terminals and calculate the working active power. After the software is complete, the

hardware will be assembled to create a physical meter to calculate the working active power in both single phase and three phase systems and test the device in real time.

REFERENCES

[1] U.S. Energy Information and Administration Frequently Asked Questions March 17, 2011 <http://www.eia.doe.gov/tools/faqs/>

[2] Leszek S. Czarencki, "What is Wrong with Budeanu Concept of Reative and Distortion Power and Why It Should be Abandoned" IEEE Trans on Instr. and Meas., Vol. IM-31, No. 3,1987, pp. 834-837

[3] Leszek S. Czarencki "Currents' Physical Components (CPC) Power Theory: Part 1-Single-Phase-Circutis" Electric Power Quality and Utilization Journal, vol. XI, No. 2, 2005

APPENDIX A: CRMS CALCULATION FOR ILLUSTRATION 1

Fundamental current crms calculation

$$i(t) = \frac{120\sqrt{2}\cos(\omega_1 t) - 120}{0.8 + 0.2}$$

$$\boldsymbol{I}_1 = \boldsymbol{X}_1 e^{-j\frac{\pi}{2}} - \boldsymbol{X}_1 e^{-j\frac{3\pi}{2}}$$

$$\boldsymbol{X}_1 = \frac{1}{\sqrt{2}\pi}\int_{-\pi/4}^{\pi/4}(I_{\max}\cos\theta - 120)e^{-j\theta}d\theta = \frac{I_{\max}}{\sqrt{2}\pi}\int_{-\pi/4}^{\pi/4}\cos\theta\, e^{-j\theta}d\theta - \frac{1}{\sqrt{2}\pi}\int_{-\pi/4}^{\pi/4}120e^{-j\theta}d\theta$$

$$= \frac{I_{\max}}{\sqrt{2}\pi}\int_{-\pi/4}^{\pi/4}\frac{e^{j\theta} + e^{-j\theta}}{2}e^{-j\theta}d\theta - \frac{120}{\sqrt{2}\pi}\int_{-\pi/4}^{\pi/4}e^{-j\theta}d\theta = \frac{I_{\max}}{2\sqrt{2}\pi}\int_{-\pi/4}^{\pi/4}1 + e^{-2j\theta}d\theta - \frac{120}{\sqrt{2}\pi}\int_{-\pi/4}^{\pi/4}e^{-j\theta}d\theta$$

$$= \frac{I_{\max}}{2\sqrt{2}\pi}\theta\Big|_{-\pi/4}^{\pi/4} + \frac{I_{\max}}{2\sqrt{2}\pi}\frac{e^{-2j\theta}}{-2j}\Bigg|_{-\pi/4}^{\pi/4} - \frac{120}{\sqrt{2}\pi}\frac{e^{-j\theta}}{-j}\Bigg|_{-\pi/4}^{\pi/4}$$

$$= \frac{I_{\max}}{2\sqrt{2}\pi}\left[\frac{e^{-j\frac{\pi}{2}} - e^{j\frac{\pi}{2}}}{-2j}\right] - \frac{120}{\sqrt{2}\pi}\left[\frac{e^{-j\frac{\pi}{4}} - e^{j\frac{\pi}{4}}}{-j}\right] + \frac{I_{\max}}{2\sqrt{2}\pi}\left[\frac{\pi}{4} - \frac{-\pi}{4}\right]$$

$$= \frac{I_{\max}}{2\sqrt{2}\pi}\left[\frac{-j - j}{-2j}\right] - \frac{120}{\sqrt{2}\pi}\left[\frac{\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)}{-j}\right] + \frac{I_{\max}}{2\sqrt{2}\pi}\frac{\pi}{2}$$

$$= \frac{I_{\max}}{2\sqrt{2}\pi}\left[\frac{2j}{2j}\right] - \frac{120}{\sqrt{2}\pi}\left[\frac{\sqrt{2}j}{j}\right] + \frac{I_{\max}}{4\sqrt{2}} = \frac{I_{\max}}{2\sqrt{2}\pi} - \frac{120}{\pi} + \frac{I_{\max}}{4\sqrt{2}}$$

$$= \frac{120\sqrt{2}}{2\sqrt{2}\pi} - \frac{120}{\pi} + \frac{120\sqrt{2}}{4\sqrt{2}} = \frac{60}{\pi} - \frac{120}{\pi} + 30 = 10.9e^{j0°}$$

$$\boldsymbol{I}_1 = \boldsymbol{X}_1 e^{-j\frac{\pi}{2}} - \boldsymbol{X}_1 e^{-j\frac{3\pi}{2}} = 10.9e^{-j\frac{\pi}{2}} + 10.9e^{-j\frac{\pi}{2}} = 21.8e^{-j\frac{\pi}{2}}$$

Fundamental voltage crms calculation

$$u(t) = 120\sqrt{2}\cos(w_1 t) - 0.8i(t)$$

$$\boldsymbol{U}_1 = \boldsymbol{A}_1 + \boldsymbol{B}_1 e^{-j\frac{\pi}{2}} - \boldsymbol{A}_1 e^{-j\pi} - \boldsymbol{B}_1 e^{j\frac{\pi}{2}}$$

$$\boldsymbol{X}_1 = \frac{1}{\sqrt{2}\pi}\int_{-\pi/4}^{\pi/4} E_{\max}\cos\theta\, e^{-j\theta}d\theta = \frac{E_{\max}}{\sqrt{2}\pi}\int_{-\pi/4}^{\pi/4}\cos\theta\, e^{-j\theta}d\theta = \frac{E_{\max}}{\sqrt{2}\pi}\int_{-\pi/4}^{\pi/4}\frac{e^{j\theta}+e^{-j\theta}}{2}e^{-j\theta}d\theta$$

$$= \frac{E_{\max}}{2\sqrt{2}\pi}\int_{-\pi/4}^{\pi/4} 1 + e^{-2j\theta}\theta = \frac{E_{\max}}{2\sqrt{2}\pi}\left[\theta\Big|_{-\pi/4}^{\pi/4} + \frac{e^{-2j\theta}}{-2j}\Bigg|_{-\pi/4}^{\pi/4}\right] = \frac{E_{\max}}{2\sqrt{2}\pi}\left\{\left[\frac{\pi}{4} - \frac{-\pi}{4}\right] + \left[\frac{e^{-j\frac{\pi}{2}} - e^{j\frac{\pi}{2}}}{-2j}\right]\right\}$$

$$= \frac{E_{\max}}{2\sqrt{2}\pi}\left[\frac{\pi}{2} + \frac{-j-j}{-2j}\right] = \frac{E_{\max}}{2\sqrt{2}\pi}\left[\frac{\pi}{2}+1\right] = \frac{120\sqrt{2}\left[\frac{\pi}{2}+1\right]}{2\sqrt{2}\pi} = 49.1e^{j0^\circ}$$

$$\boldsymbol{B}_1 = \boldsymbol{X}_1 - 0.8\boldsymbol{I}_1 = 49.1e^{j0^\circ} - 0.8\left(10.9e^{j0^\circ}\right) = 49.1e^{j0^\circ} - 8.72e^{j0^\circ} = 40.38e^{j0^\circ}$$

$$\boldsymbol{A}_1 = \frac{1}{\sqrt{2}\pi}\int_{-\pi/4}^{\pi/4} E_{\max}\sin\theta\, e^{-j\theta}d\theta = \frac{E_{\max}}{\sqrt{2}\pi}\int_{-\pi/4}^{\pi/4}\sin\theta\, e^{-j\theta}d\theta = \frac{E_{\max}}{\sqrt{2}\pi}\int_{-\pi/4}^{\pi/4}\frac{e^{j\theta}-e^{-j\theta}}{2j}e^{-j\theta}d\theta$$

$$= \frac{E_{\max}}{\sqrt{2}\pi}\int_{-\pi/4}^{\pi/4}\frac{1-e^{-2j\theta}}{2j}d\theta = \frac{E_{\max}}{\sqrt{2}\pi}\left[\frac{\theta}{2j}\Bigg|_{-\pi/4}^{\pi/4} + \frac{e^{-2j\theta}}{-2j(2j)}\Bigg|_{-\pi/4}^{\pi/4}\right] = \frac{E_{\max}}{\sqrt{2}\pi}\left\{\left[\frac{\frac{\pi}{4} - \frac{-\pi}{4}}{2j}\right] + \left[\frac{-e^{-j\frac{\pi}{2}} + e^{j\frac{\pi}{2}}}{4}\right]\right\}$$

$$= \frac{E_{\max}}{\sqrt{2}\pi}\left[\frac{-(-j)+j}{4} + \frac{\pi}{4j}\right] = \frac{E_{\max}}{\sqrt{2}\pi}\left[\frac{j}{2} + \frac{\pi}{j4}\right] = \frac{120\sqrt{2}\left[\frac{j}{2} - j\frac{\pi}{4}\right]}{\sqrt{2}\pi} = -10.9e^{j\frac{\pi}{2}}$$

$$\boldsymbol{U}_1 = \boldsymbol{A}_1 + \boldsymbol{B}_1 e^{-j\frac{\pi}{2}} - \boldsymbol{A}_1 e^{-j\pi} - \boldsymbol{B}_1 e^{j\frac{\pi}{2}} = -10.9e^{j\frac{\pi}{2}} + 40.38e^{-j\frac{\pi}{2}} + 10.9e^{-j\frac{\pi}{2}} - 40.38e^{j\frac{\pi}{2}}$$

$$\boldsymbol{U}_1 = 102.56e^{-j\frac{\pi}{2}}$$

Current rms calculation

$$i(t) = \frac{120\sqrt{2}\sin(\omega_1 t) - 120}{0.8 + 0.2}$$

$$\|i\| = \sqrt{\|x\|^2 + (-\|x\|)^2}$$

$$\|x\|^2 = \frac{1}{2\pi}\int_{\pi/4}^{3\pi/4} i^2\,(t)\,dt = \frac{1}{2\pi}\int_{\pi/4}^{3\pi/4} [120\sqrt{2}\sin(\omega_1 t) - 120]^2\,dt$$

$$= \frac{120^2}{2\pi}\int_{\pi/4}^{3\pi/4} [2\sin^2(\omega_1 t) - 2\sqrt{2}\sin(\omega_1 t) + 1]\,dt$$

$$= \frac{120^2}{2\pi}\left[\left(t - \frac{\sin(2\omega_1 t)}{2}\right)\Bigg|_{\pi/4}^{3\pi/4} + 2\sqrt{2}\cos(\omega_1 t)\big|_{\pi/4}^{3\pi/4} + t\big|_{\pi/4}^{3\pi/4}\right]$$

$$= \frac{120^2}{2\pi}\left[2\left(\frac{3\pi}{4} - \frac{\pi}{4} - \left(\frac{1}{2} - \frac{1}{2}\right)\right) + 2\sqrt{2}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) + \frac{\pi}{2}\right] = \frac{120^2}{2\pi}\left(\frac{\pi}{2} + 1 - 4 + \frac{\pi}{2}\right)$$

$$\|x\| = 18.0$$

$$\|i\| = \sqrt{\|x\|^2 + (-\|x\|)^2} = \sqrt{18.0^2 + (-18.0)^2} = 25.48$$

DC current calculation

$$i(t) = \frac{120\sqrt{2}\cos(\omega_1 t) - 120}{0.8 + 0.2}$$

$$\overline{|\imath|} = \frac{1}{\pi/2}\int_0^{\pi/4} 120\sqrt{2}\cos(\omega_1 t) - 120\ \,dt = \frac{240}{\pi}\left[\sqrt{2}\sin(\omega_1 t)\big|_0^{\pi/4} - t\big|_0^{\pi/4}\right]$$

$$= \frac{240}{\pi}\left[\sqrt{2}\left(\frac{1}{\sqrt{2}} - 0\right) - \frac{\pi}{4}\right] = \frac{240}{\pi}\left(1 - \frac{\pi}{4}\right)$$

$$\overline{|\imath|} = 16.39$$

APPENDIX B: ORTHOGONALITY PROOFS

Proof of orthogonality of harmonics of different order

$$x(t) = X_r \sin(r\omega_1 t) \quad , \quad y(t) = Y_s \sin(s\omega_1 t) \qquad s \neq r$$

$$(x_r, y_s) = \frac{1}{T}\int_o^T X_r \sin(r\omega_1 t)\, Y_s \sin(s\omega_1 t)\, dt = \frac{X_r Y_s}{T}\int_o^T \sin(r\omega_1 t)\sin(s\omega_1 t)\, dt$$

$$= \frac{X_r Y_s}{T}\int_o^T \frac{1}{2}\cos[(r-s)\omega_1 t]\cos[(r+s)\omega_1 t]\, dt$$

Integration of $\cos\theta = 0$ over the period, but $\cos(0) = 1$ As long as $s \neq r$ then

$\cos[(r-s)\omega_1 t] = 0.$

Therefore, $(x_r, y_s) = 0$ such that harmonics of different order are orthogonal to each other.

$$(x_r, y_s) = 0$$

Proof of orthogonality of positive sequence components and negative sequence components

$$\alpha = 1e^{j120^\circ}$$

$$\begin{bmatrix} \boldsymbol{x}^{\mathrm{p}} \\ \boldsymbol{x}^{\mathrm{n}} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^* \\ 1 & \alpha^* & \alpha \end{bmatrix}\begin{bmatrix} X_r \\ X_s \\ X_t \end{bmatrix}$$

$$(\boldsymbol{x}^{\mathrm{p}}, \boldsymbol{y}^{\mathrm{n}}) = \frac{1}{T}\int_o^T \boldsymbol{x}^{\mathrm{p}\,\mathrm{T}} \boldsymbol{y}^{\mathrm{n}} dt = \frac{1}{T}\int_o^T [X_{\mathrm{r}} \quad \alpha X_{\mathrm{s}} \quad \alpha^* X_{\mathrm{t}}]\begin{bmatrix} Y_{\mathrm{r}} \\ \alpha^* Y_{\mathrm{s}} \\ \alpha Y_{\mathrm{t}} \end{bmatrix} dt$$

$$= \frac{1}{T}\int_o^T X_{\mathrm{r}} Y_{\mathrm{r}} + \alpha X_{\mathrm{s}} \alpha^* Y_{\mathrm{s}} + \alpha^* X_{\mathrm{t}} \alpha Y_{\mathrm{t}} dt = \frac{1}{T}\int_o^T X_{\mathrm{r}} Y_{\mathrm{r}} + X_{\mathrm{s}} Y_{\mathrm{s}} + X_{\mathrm{t}} Y_{\mathrm{t}} dt$$

$$= \frac{1}{T}\int_o^T X_{\mathrm{r}} Y_{\mathrm{r}} + \alpha X_{\mathrm{r}} \alpha Y_{\mathrm{r}} + \alpha^* X_{\mathrm{r}} \alpha^* Y_{\mathrm{r}} dt = \frac{X_{\mathrm{r}} Y_{\mathrm{r}}}{T}\int_o^T (1 + \alpha^* + \alpha) dt = 0$$

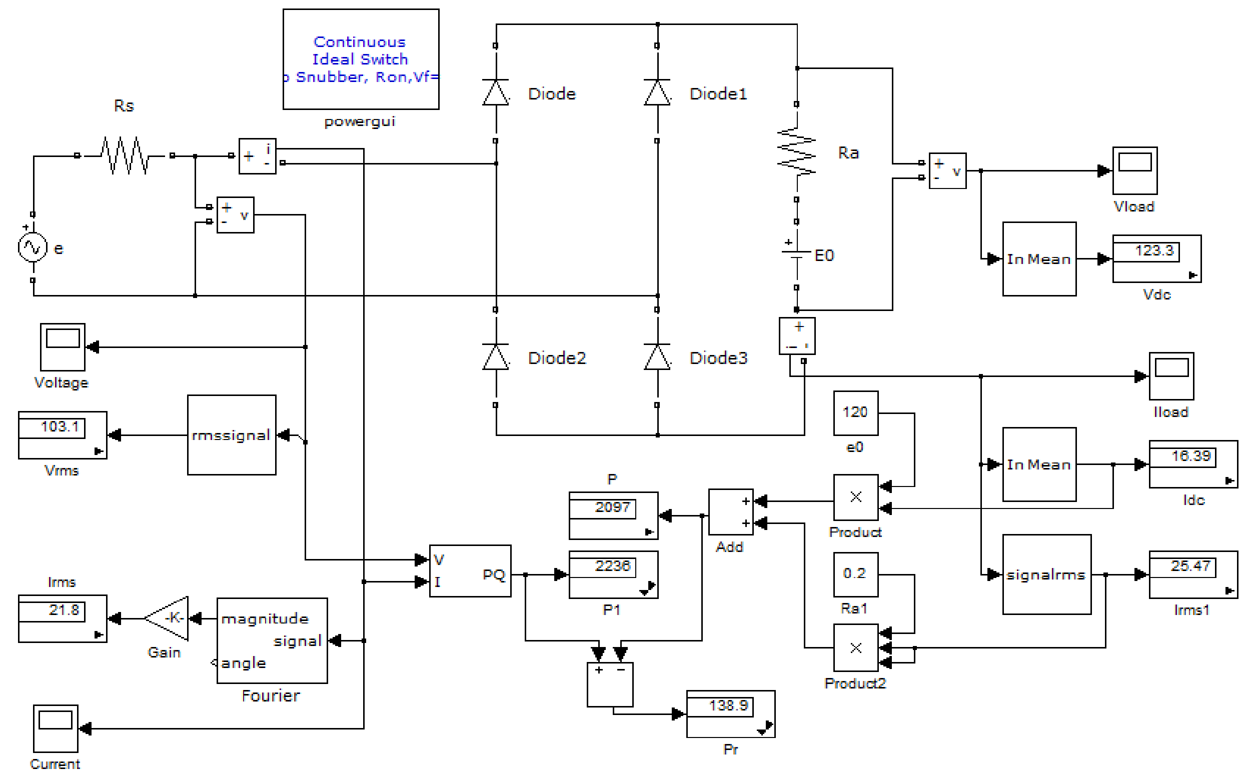
APPENDIX C: MATLAB FILES

Matlab code for verification of illustration 1

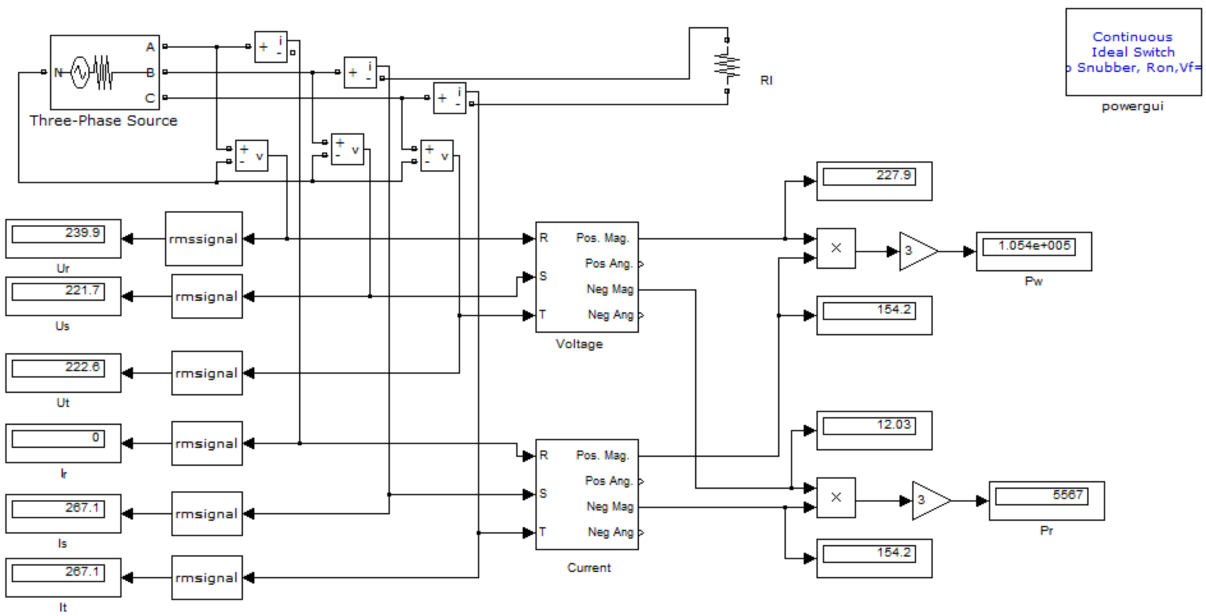
```
%Initial setup and variable decleration
Vmax = 120*sqrt(2);
Imax = 120*sqrt(2);

syms x;
Rs = 0.8;
Rl = 0.2;


%Fundamental Current Integration
d = (Imax*cos(x)-120).*exp(-j*x);
up = pi/4;
low = -pi/4;
D = double(1/(2^0.5*pi)*int(d,x,low,up));
I = D*exp(-j*pi/2) - D*exp(-j*3*pi/2);
Imag = abs(I);
Iangle = angle(I)*180/pi;


%Fundamental Integration of part A
f = sin(x).*exp(-j*x);
up = pi/4;
low = -pi/4;
A = double(Vmax/(2^0.5*pi)*int(f,x,low,up));


%Fundamental Integration of part B
g = cos(x).*exp(-j*x);
up = pi/4;
low = -pi/4;
X = double(Vmax/(2^0.5*pi)*int(g,x,low,up));
B = X - D*Rs;


%Fundamental Shifting and Rebuilding of Original Voltage Waveform
U = A + B*(-j) + (-A)*(-1) - B*(j);
Umag = abs(U);
Uangle = angle(U)*180/pi;
P1 = Umag * Imag;

%RMS Current Integration
i = sqrt(2)*120*sin(x)-120;
up = 3*pi/4;
low =pi/4;
AC = sqrt(double((1/(2*pi))*int((i^2),x,low,up)));
RMS = (2*AC^2)^0.5;

%DC Current
```

```
i = sqrt(2)*120*cos(x)-120;
up = pi/4;
low = 0;
DC = double((2/pi)*int(i,x,low,up));

%Power calculations
P1 = Umag * Imag
P = DC*120+RMS^2*0.2
Pr = P1-P
```

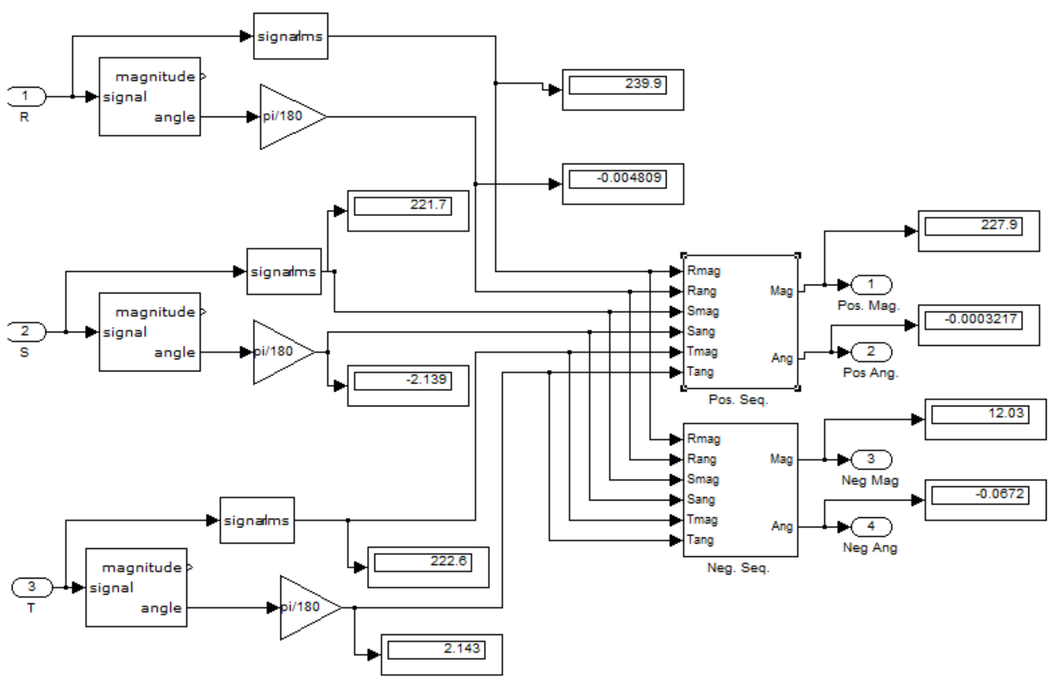
Matlab powergui circuit for verification of illustration 1



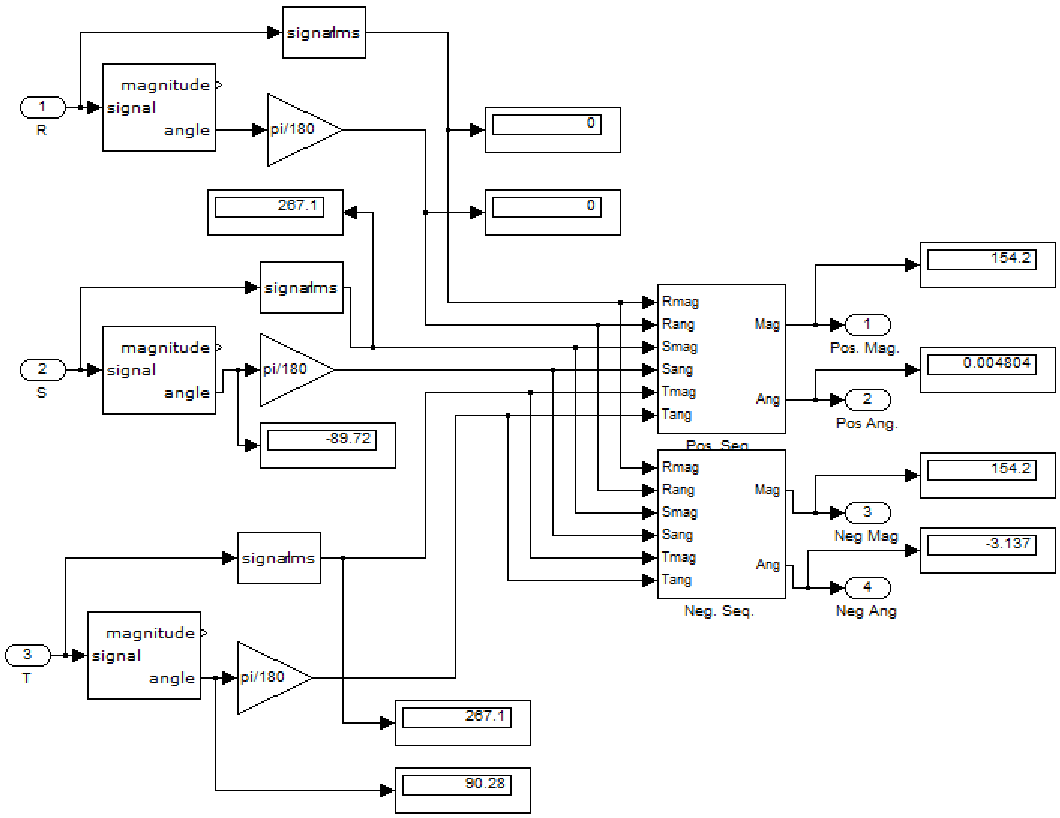
Matlab code for verification of illustration 2



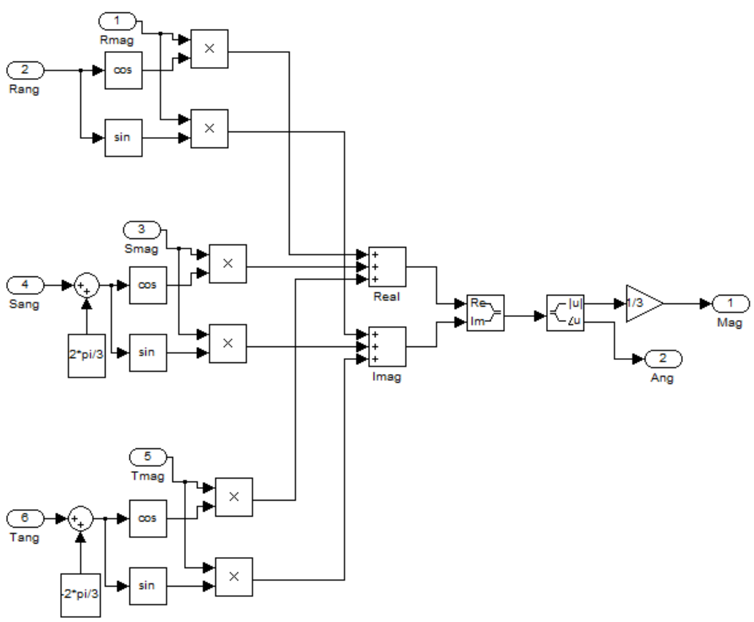
Voltage Subsystem



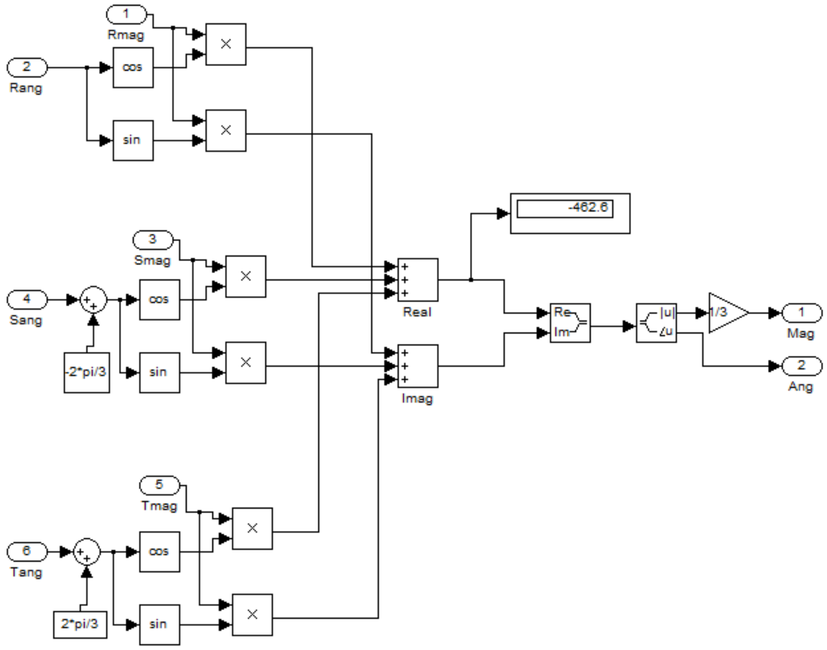
Current Subsystem



Pos. Seq Subsystem



Neg Seq subsystem



Matlab code for fundamental crms algorithm

```
syms x
%Setup of the problem with test waveform x

Amp = 100;

for c = 0:1:99
    x(c+1) = (sqrt(2)*Amp* sin(c/100 * 2*pi) + sqrt(2)*50* sin(c/100 *
5*2*pi));
    x(c+1) = (x(c+1) + sqrt(2)*5* sin(c/100 * 7*2*pi)+ sqrt(2)*5* sin(c/100 *
11*2*pi));
    x(c+1) = (x(c+1) + sqrt(2)*5* sin(c/100 * 13*2*pi)+ sqrt(2)*5* sin(c/100
* 17*2*pi));
    x(c+1) = (x(c+1) + sqrt(2)*5* sin(c/100 * 19*2*pi)+ sqrt(2)*5* sin(c/100
* 23*2*pi));
    x(c+1) = (x(c+1) + sqrt(2)*5* sin(c/100 * 25*2*pi)+ sqrt(2)*5* sin(c/100
* 29*2*pi));
    x(c+1) = (x(c+1) + sqrt(2)*5* sin(c/100 * 31*2*pi)+ sqrt(2)*5* sin(c/100
* 35*2*pi));
    x(c+1) = (x(c+1) + sqrt(2)*5* sin(c/100 * 37*2*pi)+ sqrt(2)*5* sin(c/100
* 41*2*pi));
    x(c+1) = (x(c+1) + sqrt(2)*5* sin(c/100 * 43*2*pi)+ sqrt(2)*5* sin(c/100
* 47*2*pi));
    x(c+1) = (x(c+1) + sqrt(2)*5* sin(c/100 * 49*2*pi));
end
```

```
x = double(x);


%Basic Arithemtic and parameters setup
N = 100;
K = 25;
M = 50;
twoK = K*2;

%Calculate real and imaginary parts for V
for c = 1:1:(K-1)
    RV(c) = cos(-pi*c / (twoK));
    IV(c) = sin(-pi*c / (twoK));
end

%Calculate the values for s(k) into vector form
top = 2*M;
for c = 1:1:M
    s(c) = double(x(c) - x(c+M));
end

%Setup Initial values and the first terms of Real(X1) and Imag(X1)
C = 2^0.5/N;
RX1 = s(1);
IX1 = -s(K+1);

%summation of all the values of s(k) with the RV and IV scalars.
%NOTE: In the FT algorithm theory, s starts s(0), in MATLAB the index must
start at 1.
%so s(1)represents the s(0) in DFT algorithm theory. hence why there is the
"+1" in s(c+1) indexing.

for c = 1:1:(K-1)
    RX1 = RX1 + RV(c)*(s(c+1) - s(M+1-c));
    IX1 = IX1 + IV(c)*(s(c+1) + s(M+1-c));
end

%Multiply by sqrt(2)/N.
%Output
RX1 = RX1 * C
IX1 = IX1 * C
```

VITA

Tracy Toups was born in 1984 in Thibodaux, Louisiana. He graduated high school from Baton Rouge Magnet High in Baton Rouge, LA. After attending Louisiana State University, he acquired a Bachelor of Science in Electrical Engineering in 2007. After a short break, he enrolled at Louisiana State University as a graduate student in the Electrical and Computer Engineering Department where he is currently pursuing a doctoral degree in electrical engineering.