

2006

Essays on the Bayesian estimation of stochastic cost frontier

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**ESSAYS ON THE BAYESIAN ESTIMATION OF STOCHASTIC COST
FRONTIER**

A Dissertation

Submitted to the Graduate Faculty of the Louisiana
State University and
Agricultural and Mechanical College
In partial fulfillment of the
Requirements for the degree of
Doctor of Philosophy

in

The Department of Economics

By

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B.S., Xiangtan University, 2000
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December 2006

To my dear parents Shenglin Zhao and Guilin Guo, to my dear husband Hui Wang , and to my dear uncle John Guo, who continue to love, support, pride, encourage and inspire me through every stage of my life. Thank You and I Love You!

ACKNOWLEDGMENTS

I would like to express my most sincere gratitude, appreciation, admiration and respect to my committee chair, Dr. Dek M. Terrell, for his inspiration, guidance, encouragement and support throughout my Ph.D. study in LSU. Without help from Dr. Dek M. Terrell, this dissertation would have been impossible to complete. Thank You, Dr. Terrell, for everything!

I am also deeply indebted to my committee members, Dr. R. Carter Hill, Dr. Eric T. Hillebrand and Dr. David E. Dismukes for their valuable comments and helpful suggestions. Special thanks also go to Dr. Sudipta Sarangi, Dr. David Bransington, Dr. Robert J. Newman, Dr. Chris E. Papageorgiou, all other faculty members, staff and my colleagues in the Department of Economics at LSU for their continuous encouragement and support.

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ABSTRACT

This dissertation consists of three essays that focus on a Bayesian estimation of stochastic cost frontiers for electric generation plants. This research gives insight into the changing development of the electric generation market and could serve to inform both private investment and public policy decisions.

The main contributions to the growing literature on stochastic cost frontier analysis are to

1. Empirically estimate the possible efficiency gain of power plants due to deregulation.
2. Estimate the cost of electric power generating plants using coal as a fuel taking into account both regularity restrictions and sulfur dioxide emissions.
3. Compare costs of plants using coal to those who use natural gas.
4. Apply the Bayesian stochastic frontier model to estimate a single cost frontier and allow firm type to vary across regulated and deregulated plants. The average group efficiency for two different types of plants is estimated.
5. Use a fixed effects and random effects model on an unbalanced panel to estimate group efficiency for regulated and deregulated plants.

The first essay focuses on the possible efficiency gain of 136 U.S. electric power generation coal-fired plants in 1996. Results favor the constrained model over the unconstrained model. SO_2 is also included in the model to provide more accurate estimates of plant efficiency and returns to scale.

The second essay focuses on the predicted costs and returns to scale of coal generation to natural gas generation at plants where the cost of both fuels could be obtained. It is found that, for power plants switching fuel from natural gas to coal in 1996, on average, the expected fuel cost would fall and returns to scale would increase.

The third essay first uses pooled unbalanced panel data to analyze the differences in plant efficiency across plant types – regulated and deregulated. The application of a Bayesian stochastic frontier model enables us to apply different mean plant inefficiency terms by plant type on a single stochastic frontier. The fixed effect panel estimation technique is then applied to the same unbalanced panel data. The results provide evidence that deregulated power plants are more cost-efficient than regulated plants.

CHAPTER 1. INTRODUCTION

This study focuses on a Bayesian estimation of stochastic cost frontiers for U.S. electric generation plants. The dissertation consists of three essays.

The first essay, “Measuring Potential Efficiency Gains in the Presence of Undesirable Outputs of Electricity Generation,” examines the efficiency of 136 coal-fired U.S. electric power generation plants in 1996 using a Bayesian stochastic frontier model that imposes monotonicity and concavity restrictions on frontier. The results confirm that this constrained model yields more accurate results than an unconstrained model. In particular, the shares and elasticities are well behaved, and the standard deviations are reduced substantially. More important, when Sulfur Dioxide (SO_2) is treated as a “bad” output, include SO_2 into the model, and impose monotonicity and concavity restrictions on both input prices and SO_2 , measures of plant efficiencies rise by 1%. Power plants are not as inefficient as we believed, because the pollution abatements were treated as inefficiency. Imposing monotonicity and concavity restrictions only generates minor differences for individual firms’ returns to scale. The results also show that once we include SO_2 in the model, average returns to scale for the constrained model rises from 1.17 to 1.194, and 132 out of 136 plants in the sample exhibit increasing returns to scale.

The current literature has been relatively sparse on empirical studies of inter-fuel change in the electric generation market. The second essay, “Comparing the Cost of Coal and Natural Gas Electricity Generation,” compares the predicted costs and returns to scale of coal generation to natural gas generation at plants where the cost of both fuels could be obtained. Higher natural gas prices have raised the costs of power generation in plants using that fuel. Using a Bayesian stochastic frontier model that imposes monotonicity and concavity, I show that, for power plants switching fuel from natural gas to coal in 1996, on average, the expected fuel cost would fall and

returns to scale would increase. The results provide insight into how the optimal fuel choice for electricity generation varies with the relative prices of those fuels.

Assuming the same technology across different types of power plants, the third essay, “Measurement of Efficiency in Panel Data: A Stochastic Cost Frontier Analysis of the U.S. Electricity Generation,” applies two different estimation methods to show that deregulated power plants are more efficient than those regulated plants. One method is the application of the Bayesian stochastic frontier model to a pooled panel dataset. Different mean plant inefficiency terms by plant type on a single stochastic frontier are allowed for. The finite sample approximation of posterior means and highest density regions for model and inefficiency parameters are another advantage of the Bayesian technique. The probability that deregulated firms are more efficient than regulated firms is also calculated. The other model is the classical fixed effects method applied on an unbalanced panel of power plants. The results in this chapter are consistent with the previous conclusion that deregulated power plants are more efficient than regulated power plants.

CHAPTER 2. THE STOCHASTIC FRONTIER MODEL AND RESTRICTIONS ON MONOTONICITY AND CONCAVITY

2.1 The Stochastic Frontier Model

Microeconomic theory states that the objective of firms is to produce the maximum output utilizing given inputs, to minimize costs at given outputs, or to efficiently allocate input and output in order to maximize profits. Based on the objective of measuring how close firms are to achieving these objectives, the frontier approach to measuring the productive efficiency of firms is becoming increasingly widespread. Productive efficiency is measured as the distance to a particular frontier, such as the production frontier, cost frontier, revenue frontier, and profit frontier.

The basis for frontier analysis was developed from the theoretical literature on productive efficiency, with the work of Koopmans (1951), Debreu (1951), and Shephard (1953). Koopmans laid out a mathematical definition of technical efficiency: “A producer is technically efficient if, and only if, it is impossible to produce more of any output without producing less of some other output or using more of some input.” Debreu and Shephard introduced a distance function to model multiple-output technology as an output-oriented measure of the distance of a producer from a frontier (Debreu) or of an input-oriented measure of technical efficiency (Shephard). Farrell (1957) provided a measurement application on U.S. agriculture and was the first to measure productive efficiency empirically. He showed the definition and decomposition of cost efficiency into its technical and allocative components and used linear programming techniques to estimate efficiency.

The literature on the frontiers model consists of two competing estimation techniques categorized as the mathematical programming approach and the econometric approach. The mathematical programming approach, also known as “Data Envelopment Analysis” (DEA) approach, was introduced by Charnes, Cooper, and Rhodes (1978). They proposed a ratio definition of efficiency, which reduces multiple outputs and multiple inputs, for each decision making unit to be evaluated, into a single-input–single-output form, and produces an efficiency

measure. The production functions reveal the technically efficient input-output relationships for a firm, and therefore constitute a frontier. In cost applications, it seeks the minimum cost associated with the highest output. DEA imposes no explicit distribution assumptions, but involves the construction of a nonparametric frontier or a piecewise linear surface obtained from the observed data set. The mathematical programming model is applied to an observed sample to construct a production frontier as well as to compute efficiency scores relative to the constructed frontiers. Each decision making unit's performance is evaluated relative to the best practice frontier by solving linear programming problems. One of the chief shortcomings of the DEA models is their non-stochastic nature. Therefore, the calculated frontier and consequently the obtained efficiency scores can be contaminated by any statistical noise in the data. The lack of a stochastic element also makes traditional DEA models quite sensitive to outliers.

The stochastic frontier approach (SFA) allows the possibility that the relative performance of a firm may vary due to a stochastic shock (such as weather, machinery performance, demographic factors and even luck) outside its control. Therefore, SFA can separate impact on a firm's output due to an external stochastic error from the variation due to technical inefficiency.

The econometric approach does a good job in handling statistical noise, but it requires an explicit ad-hoc assumption about the distribution of the efficiency component. The econometric approach was first originated almost simultaneously by Aigner, Lovell, and Schmidt (1977), Meeusen and van den Broeck (1977), and Battese and Corra (1977). The original Stochastic Frontier Model used a composed error structure, where one part of the composed error structure represents the statistical noise, which is generally assumed to be normally distributed, and the other part of the composed error term¹ indicate inefficiency, which is generally assumed to follow a particular one-sided distribution.

¹ The two sided error model was initially proposed by Aigner, Amemiya, and Poirier (1976) where in this model errors were allowed to be both positive and negative, but in which positive and negative errors could be assigned in different weights.

Aigner, Lovell, and Schmidt (1977) specified the stochastic frontier production function as

$$y_i = f(x_i, \beta) + \varepsilon_i, \quad (2.1)$$

where y_i is observed output, x_i is a vector of inputs, and β is a vector of unknown parameters.

The composed error term ε_i is specified as

$$\varepsilon_i = u_i - v_i, \quad v_i \geq 0. \quad (2.2)$$

The first error component u_i is independently and identically distributed as $u_i \sim N(0, \sigma_u^2)$ and captures the effects of statistical noise such as random effects of measurement error and external shocks out of control, while v_i is intended to capture technical inefficiency, which can be measured as the deficiency in output away from the maximum possible output given by the stochastic production frontier $y_i = f(x_i, \beta) + u_i$. The property that $v_i \geq 0$ ensures all the observed outputs should lie below or on the stochastic frontier.

2.1.1 The Distributions of the Inefficiency Error Term

With respect to the distribution of inefficiency term v_i , Meeusen and van den Broeck used an exponential distribution, Battese and Corra applied a half normal distribution, and Aigner, Lovell, and Schmidt considered both. Later, Stevenson (1980) proposed a Gamma and truncated normal distribution for v_i , and Greene (1990) assigned a two-parameter Gamma distribution. For the stochastic frontier production function $y_i = f(x_i, \beta) + \varepsilon_i$, if we assume that two error terms u_i and v_i are independent of input variables x_i and also independent of each other, then we can apply one of the above distributions, define the likelihood function, and compute the maximum likelihood estimates. Various techniques can be applied to test the appropriateness of the distribution assumption and to find which distribution fits the data best. For example, Lee (1983) and Schmidt and Lin (1984) proposed using Lagrange multiplier techniques to test the appropriateness of various

distributions of v_i . Sengupta (1993) also proposed entropy measures to determine the underlying distribution empirically.

2.1.2 Stochastic Cost Frontier

Estimation of cost efficiency differs from estimation of technical efficiency in several aspects, such as data requirements, number of outputs, quasi-fixity of some inputs and decomposition of efficiency itself. Unlike the output-oriented approaches to the estimation of technical efficiency, the estimation of cost efficiency requires us to apply an input-oriented approach on the cost frontier. For the purpose of illustration, consider the single-equation stochastic cost function model

$$\ln(C_i) = \ln C(p_i, q_i; \beta) + u_i + v_i, \quad (2.3)$$

where C_i is the observed cost, p_i is a vector of input prices, q_i is a vector of output prices, β is a vector of technology parameters to be estimated, v_i is a non-negative stochastic error capturing the effects of inefficiency and u_i is a symmetric error component reflecting the statistical noise.

$C(p_i, q_i)$ is the deterministic kernel of the stochastic cost frontier $C(p_i, q_i) \exp(v_i)$. The measure of cost efficiency is then

$$CE_i = \frac{C(p_i, q_i; \beta) \cdot \exp\{v_i\}}{C_i}, \quad (2.4)$$

where CE_i reflects the ratio of the minimum possible cost, given inefficiency v_i , to actual total cost.

If $C_i = c(p_i, q_i; \beta) \cdot \exp\{v_i\}$, then $CE_i = 1$ and we can say that firm i is fully efficient. Otherwise actual cost for firm i exceeds the minimum cost so that $0 \leq CE_i < 1$.

To illustrate, the following Figure 2.1² shows the idea behind the composed error model. For the purpose of illustration, we assume no fixed costs. Expressed using the terminology defined before, this line is the cost frontier - $c(p_i, q_i)$. The points A and B on the graph represent the

² This figure and illustration are adapted from Dek Terrell's lecture notes.

reported cost and input price for two different plants. Remember, the deviation of plant i 's cost from the cost frontier includes two stochastic error terms, inefficiency (v_i) and measurement error (u_i). Note that both plants in this example report higher costs than would a fully efficient plant to produce one unit of electricity.

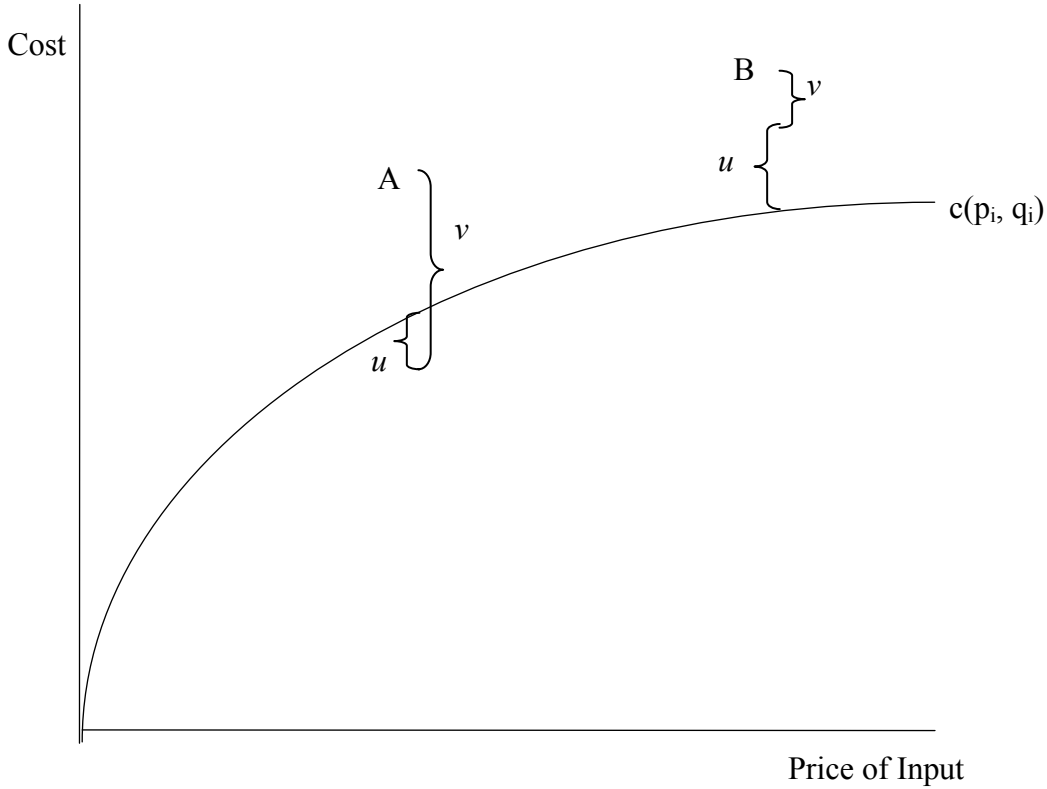


Figure 2.1 - Inefficiency and Measurement Plots

Aigner, Lovell, and Schmidt (1977) assumed $u_i \sim N(0, \sigma_u^2)$ and $v_i \sim |N(0, \sigma_v^2)|$. The log likelihood function for the exponential distribution is written as

$$\ln L = \frac{N}{2} \ln(2/\pi) - N \ln \sigma + \sum_{i=1}^N \ln \left[1 - \Phi \left(-\frac{\varepsilon_i \lambda}{\sigma} \right) \right] - \frac{1}{2\sigma^2} \sum_{i=1}^N \varepsilon_i^2, \quad (2.5)$$

where $\varepsilon_i = u_i + v_i$ is a vector of composed error term, N refers to the number of observations,

$\sigma^2 = \sigma_u^2 + \sigma_v^2$, $\lambda = \frac{\sigma_u}{\sigma_v}$, and $\Phi(\cdot)$ refers to the standard normal distribution function.

To obtain the consistent and asymptotically efficient maximum likelihood estimates, the log-likelihood function shown in equation (2.5) can be numerically maximized with respect to the parameters λ , β and σ . An alternative approach to estimate the model is corrected ordinary least squares (COLS), which was first applied to stochastic frontiers by Olson, Schmidt, and Waldman (1980). A production (or cost) function is first estimated using ordinary least squares (OLS). The OLS intercept estimate is then shifted up by the value of the largest positive residual to represent the production frontier, or shifted down the value of the smallest positive residual to represent the cost frontier. They observed that Ordinary Least Squares (OLS) provide a biased estimate for the constant term but the best linear unbiased estimates for all other parameters. Olsen, Schmidt and Waldman (1980) show that the biased OLS estimates of the constant can be corrected by adding mean μ of ε_i , because it equals the estimated bias. For example, if $\mu = -\sqrt{2/\pi\sigma_u}$, we can add $\mu = \sqrt{2/\pi\sigma_u}$ in the constant term to correct the bias. Olsen, Schmidt, and Waldman (1980) used a Monte Carlo approach to examine both COLS and MLE. They found that COLS performs as well as MLE in the stochastic frontier model for normal and half normal distributions. They also found that MLE tended to outperform COLS in sample sizes larger than 400. Take the cost frontier for example, another disadvantage of COLS is that it tends to penalize the efficiency estimates of units that have relatively large OLS residuals, since there is a restricted parallel shift on cost frontier. This problem becomes more severe in a heteroskedastic sample.

2.1.3 Stochastic Frontier Functional Forms

Stochastic frontier analysis begins with the selection of a functional form. In previous literature, there are several different functional forms that have been applied with the stochastic frontiers model, such as Cobb-Douglas, Constant Elasticity of Substitution (CES), Generalized Leontief, translog and Fourier. Among all of these functional forms, the Cobb-Douglas is one of the most commonly used. The single output log-linear Cobb-Douglas cost frontier is written as

$$\begin{aligned}
\ln E_i &\geq \beta_0 + \beta_q \ln q_i + \sum_n \beta_n \ln p_{ni} + v_i \\
&= \beta_0 + \beta_q \ln q_i + \sum_n \beta_n \ln p_{ni} + v_i + u_i
\end{aligned}
\tag{2.6}$$

where $E_i = \sum_n p_{ni} q_{ni}$ is the cost expenditure incurred by firm i , q_i is vector of produced outputs produced by firm i . The two components u_i and v_i of composed error term ε_i are the same as defined before. By construction, the Cobb-Douglas frontier is homogenous of degree one.

Schmidt (1985-1986) observed that flexible functional forms envelope the data more closely and proposed using them in frontier analysis. The single output Cobb-Douglas cost frontier can be easily extended to the multiple-output Cobb-Douglas cost frontier model. However, Hasenkamp (1976) noted that a function having the Cobb-Douglas form cannot accommodate multiple outputs without violating the requisite curvature properties in output space. In addition, the simple functional form of Cobb-Douglas can hardly capture the true complexity of the production technology, therefore leaving the unmodeled complexity in the error term and biasing estimates of cost inefficiency. In this regard, the translog cost function becomes the most popular functional form in the current literature.

The translog cost frontier was originally provided by Christensen, Jorgenson, and Lau (1971). The translog cost function takes a second-order Taylor series expansion about mean of the data and can approximate any well-behaved cost frontier. Therefore, contrary to a Cobb-Douglas cost function, flexible translog cost function can accommodate multiple outputs without violating the requisite curvature properties in output space. If we assume the cost frontier takes the log-quadratic translog functional form, then the single-output translog variable stochastic cost frontier model can be written as

$$\begin{aligned}
\ln VE_i &= \beta_0 + \beta_q \ln q_i + \sum_n \alpha_n \ln p_{ni} + \sum_j \beta_j \ln z_{ji} + \frac{1}{2} \beta_{qq} (\ln q_i)^2 \\
&+ \frac{1}{2} \sum_n \sum_k \alpha_{nk} \ln p_{ni} \ln p_{ki} + \frac{1}{2} \sum_j \sum_r \beta_{jr} \ln z_{ji} \ln p_{ri} \\
&+ \sum_n \sum_j \gamma_{nj} \ln p_{ni} \ln z_{ji} + \sum_n \alpha_{qn} \ln q_i \ln p_{ni} \\
&+ \sum_j \beta_{qj} \ln q_{ji} + v_i + u_i
\end{aligned} \tag{2.7}$$

where $i = 1, \dots, I$ refers to the i^{th} producer, $x_i = (x_{1i}, \dots, x_{Ni}) > 0$ refers to a vector of variable inputs, $p_i = (p_{1i}, \dots, p_{Ni})$ is input price, $z_i = (z_{1i}, \dots, z_{Ji}) > 0$ is a vector of quasi-fixed inputs, and q_i is output for producer i . $VE_i = \sum_n p_{ni} x_{ni}$ refers to variable expenses in production and the stochastic variable cost frontier is $vc(q_i, p_i, z_i; \beta) \cdot \exp(v_i + u_i)$. $vc(q_i, p_i, z_i; \beta)$ is nonincreasing in quasi-fixed inputs z_i under strong monotonicity property. It is a convex function of (q_i, z_i) and also it is linearly homogeneous in (q_i, z_i) .

Assuming no quasi-fixed input, we can then build a multiple-output stochastic total cost frontier using translog functional forms. The deterministic form of the translog cost function can be written as

$$\begin{aligned}
\ln E_i &= \beta_0 + \sum_m \alpha_m \ln q_{mi} + \sum_n \beta_n \ln p_{ni} + \frac{1}{2} \sum_m \sum_j \alpha_{mj} \ln q_{ji} \\
&+ \frac{1}{2} \sum_n \sum_k \beta_{nk} \ln p_{ni} \ln p_{ki} + \frac{1}{2} \sum_n \sum_m \gamma_{nm} \ln p_{ni} \ln p_{mi}
\end{aligned} \tag{2.8}$$

From Shephard's lemma, the share equation of associated input cost can be developed as $S_{ni} = \partial \ln E_i / \partial \ln p_{ni} = p_{ni} x_{ni} / E_i$, which leads to

$$S_{ni} = \beta_n + \sum_k \beta_{nk} \ln p_{ki} + \sum_m \gamma_{nm} \ln q_{mi}, \quad n = 1, \dots, N, \tag{2.9}$$

where Young's theorem requires imposing symmetry restriction

$$\begin{aligned}
\alpha_{mj} &= \alpha_{jm}, \\
\beta_{nk} &= \beta_{kn}
\end{aligned} \tag{2.10}$$

Also the restriction of homogeneity of degree one in input prices requires

$$\begin{aligned}\sum_n \beta_n &= 1, \\ \sum_n \beta_{nk} &= 0 \quad \forall k, \\ \sum_n \gamma_{nm} &= 0 \quad \forall m\end{aligned}\tag{2.11}$$

In this dissertation, the translog functional form is used as the cost frontier. Discussion of application in detail will be explored in Chapter 5.

2.1.4 Observation-Specific Estimates of Inefficiency

Jondrow, Lovell, Materov, and Schmidt (1982) and Kalirajan and Flinn (1983) suggest a method to decompose the composed error ε_i , where $\varepsilon_i = u_i + v_i$, from the cost frontier to obtain the firm-specific estimates of inefficiency v_i . This method uses the expected value or mode of v_i conditional on ε_i as

$$E(v|\varepsilon) = \frac{\sigma_u^2 \sigma_v^2}{\sigma^2} \left[\frac{\phi(\varepsilon\lambda/\sigma)}{1 - \Phi(\varepsilon\phi/\sigma)} - \left(\frac{\varepsilon\lambda}{\sigma} \right) \right]\tag{2.12}$$

and

$$\begin{aligned}M(v|\varepsilon) &= \varepsilon \left(\sigma_v^2 / \sigma^2 \right) && \text{if } \varepsilon \geq 0, \\ &= 0 && \text{if } \varepsilon < 0\end{aligned}\tag{2.13}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal density and cumulative distribution function of the standard normal distribution, respectively. $\sigma^2 = \sigma_u^2 + \sigma_v^2$ and $\lambda = \frac{\sigma_v}{\sigma_u}$ are as defined before as in equation (2.5). The disadvantage of the above approach is that the specific distributional assumptions must be made of uncertainty on both inefficiency v and statistical noise u .

2.2 The Bayesian Stochastic Frontier

The major advantages of the Bayesian stochastic frontier include accounting for parameter uncertainty by assigning probability distributions and easily imposing economic regularity (such as

monotonicity and concavity) restrictions on functional form. Using the translog functional form and imposing monotonicity and concavity restrictions leads to a smooth frontier.

This method also enables us to obtain exact small sample results, derive full posterior distribution of any observation-specific efficiency or functions of efficiencies. Thus we can compute the exact standard deviations and make inferences to about whether the efficiency of one firm is statistically different from that of another. It is very difficult to obtain small sample estimates and make reliable inferences regarding questions of individual firm efficiency based on results from non-Bayesian statistical methods. For example, the standard errors of firm efficiency are difficult for frequentists to compute.

The goal of the stochastic frontier model is to estimate the cost function of an efficient firm in an industry, which is called the cost frontier. The inefficiency is measured by the deviations from the frontier. From the stochastic frontier model, inferences can be drawn about the efficiency of the industry or even each firm, and the cost function of the efficient firm. This dissertation estimates elasticities of substitution, returns to scale, and efficiency of electric power generation plants using coal as source.

The Bayesian analysis in the estimation of the cross-sectional stochastic cost frontier was first introduced by Van Den Broeck, J. Gary Koop, J. Osiewalski, and M. F. Steel (1994). They use posterior model probabilities as weights and mix several different inefficiency distributions to treat uncertainty regarding the sampling models. To illustrate, the basic Bayesian principles are introduced. Bayes law indicates that

$$p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)}, \quad (2.14)$$

where $p(\theta)$ refers to the prior density of a vector of unobservable parameters θ , which seeks to explain Y , Y is a vector of observable data, $p(\theta|Y)$ is the posterior density of θ . For simplicity,

the posterior density $p(\theta|Y)$ is written as proportional to the product of prior density $p(\theta)$ and prior observation of data $p(Y|\theta)$.

$$p(\theta|Y) \propto p(\theta)p(Y|\theta) \quad (2.15)$$

Inferences are made on the parameters from the posterior distribution $p(\theta|Y)$. Theoretically, the marginal posterior density of θ_i could be computed by integrating the joint posterior density of θ with respect to all elements of θ except for θ_i itself. However, the integral generally cannot be computed analytically. Van Den Broeck, Koop, Osiewalski, and Steel (1994) show that an alternative is to make Monte Carlo sampling draws from an importance sampling density and use them to reveal features of the posterior distribution of $p(\theta|Y)$ distribution. However, Koop, Steel, and Osiewalski (1993) argue that this approach is computationally difficult and therefore recommend the use of Gibbs sampling. The Gibbs sampler is directly based on conditional densities. It is a strategy of sequentially drawing from the full conditional posterior distributions. We sample n values of θ_i for the posterior, then we can evaluate moments numerically. Let $g(\theta)$ represent these functions of interest, such as returns to scale and firm efficiency. The posterior mean for a function of interest can be computed as $E[g(\theta)] = \frac{1}{n} \sum_{i=1}^n g(\theta_i)$.

In this dissertation, a linear translog frontier was chosen to generate a linear composed error model which is showed as follows:

$$\begin{cases} y_i = X_i\beta + u_i + v_i, \\ u_i \sim N(0, \sigma^2), \\ v_i \sim EXP(\lambda) \end{cases} \quad (2.16)$$

where y_i refers to the log cost for plant i , X_i refers to a row vector of independent variables used to create the translog frontier, and β refers to a column vector of coefficients of the translog. For

the residual terms, u_i denotes the two-sided error term accounting for measurement error, and v_i is the non-negative one-sided error term reflecting plant inefficiency.

A flat prior for β and a gamma prior for λ^{-1} and σ^2 have been chosen:

$$\begin{aligned}\pi(\beta) &\propto 1, \\ \pi(\lambda^{-1}) &= f_G\left(\lambda^{-1} \mid 1, -\ln(r^*)\right), \\ \pi(\sigma^{-2}) &= f_G\left(\sigma^{-2} \mid \frac{\tau}{2}, \frac{s_p^2}{2}\right)\end{aligned}$$

where $f_G(\cdot \mid \nu_1, \nu_2)$ is a gamma density with mean ν_1/ν_2 and variance ν_1/ν_2^2 . $r_i = \exp(-v_i)$ measures the efficiency of the i 'th plant, and r^* is the prior median for efficiency.

Koop, Steel, and Osiewalski (1993) derived the conditional densities for the stochastic frontier model used in this dissertation. The conditional density for the model parameters and the variance of v_i is

$$\begin{aligned}p(\beta, \sigma^{-2} \mid y, \lambda^{-1}, \nu) &= f_G\left(\sigma^{-2} \mid \frac{N-2}{2}, \frac{1}{2}(y-\nu-x\hat{\beta})'(y-\nu-x\hat{\beta})\right) \\ &\quad \times f_N\left(\beta \mid \hat{\beta}, \sigma^2(x'x)^{-1}\right)\end{aligned}\tag{2.17}$$

where $\hat{\beta} = (x'x)^{-1}x'y^*$, $y^* = y - \nu$, $f_G(\cdot \mid \nu_1, \nu_2)$ is a gamma density with mean ν_1/ν_2 and variance ν_1/ν_2^2 , and $f_N(\cdot \mid d, D)$ refers to normal density with mean d and covariance matrix D . The conditional distribution of λ given ν , β and σ^{-2} is

$$p(\lambda^{-1} \mid \beta, \sigma^{-2}, \nu, y) = p(\lambda^{-1} \mid \nu) = f_G(\lambda^{-1} \mid n+1, \nu'\iota - \ln(r^*)),\tag{2.18}$$

where ι is a $n \times 1$ vector of ones. Given σ^{-2} , λ^{-1} and data y , the conditional posterior for ν is

$$p(\nu \mid y, \beta, \lambda^{-1}, \sigma^{-2}) = f_N\left(\nu \mid y - x'\beta - \frac{\sigma^2}{\lambda}\iota, I_n\sigma^2\right) \prod_{i=1}^n I(\nu_i \geq 0)\tag{2.19}$$

This is the normal distribution truncated below at 0, where $I(\cdot)$ is the indicator function and I_n is the $n \times n$ matrix.

Later, Koop, Osiewalski, and Steel (1997) and Fernandez, Osiewalski and Steel (1997) extend the application of Bayesian techniques to the measurement of economic efficiency to panel data.

2.3 Restrictions on Monotonicity and Concavity

There are two criteria for judging a cost function: regularity and flexibility. If a cost function is regular, it satisfies the restrictions implied by economic theory; if it is flexible, it includes a wide variety of functional forms. According to Diewert (1974), a cost function is flexible if the level of cost and all of its first and second derivatives coincide with those of an arbitrary cost function that satisfies the linear homogeneity in prices properly at any point in an admissible domain. Empirical applications of flexible functional forms often fail to satisfy economic theoretical restrictions. For example, economic theory states that cost function must be monotone and concave to input price. Monotonicity refers to the condition that the cost should be non-decreasing with respect to inputs. A necessary and sufficient condition for concavity is that a twice continuously differentiable cost has negative semi-definite matrix of second order partial derivatives of the cost function with respect to inputs prices. From an empirical perspective, there are both costs and benefits to imposing these theoretical restrictions. Imposing true monotonicity and concavity properties on a true function form unambiguously increases the estimation efficiency and makes resulting inferences more precise. Cost functions that violate economic theoretical properties may cause biased estimation and may lead to inferences that violate basic economic theory. If the functional form for the cost frontier is mis-specified, the argument for imposing monotonicity and concavity may be less compelling. For example, Wales (1977) believes that

parameterization of the translog violating concavity may approximate more complex technologies better than a restricted version of the translog.

Some previous studies restrict parameters to impose concavity and/or monotonicity globally on functions. For example, Jorgensen and Fraumeni (1981) impose global concavity of the sectoral price functions. The concavity constraints contribute to the precision of their estimates but require that the share of each input be nonincreasing in the price of input itself. Tombazos (1998) combines a global imposition of concavity with a symmetric normalized quadratic representation of the unit cost function to estimate the Allen-Uzawa effect of various categories of imports on U.S. primary factors. However, these global restrictions may lead to a significant loss of functional flexibility. For example, Diewert and Wales (1987) find that imposing global restrictions on a translog cost function tends to bias own price elasticities upward (in absolute value). They also find that imposing concavity on the generalized Leontief cost function forces all inputs to be substitutes. On the other hand, global restrictions also proved restrictive for other functional forms. For example, Wales (1977) analyzes empirically the ability of the Translog (TL) and Generalized Leontief (GL) functional forms to approximate constant elasticity of utility functions. He finds that the flexible forms provide a good local approximation but do not always provide a good approximation over a range of observations. Hence the finding in practice of observations for which the regularity conditions required by economic theory are not satisfied need not imply the absence of an underlying utility-maximizing process, but may simply reflect the inability of the flexible form to approximate the true utility function over the range of the data. The same argument applies to violations of monotonicity and concavity. Terrell (1995) uses simulations to examine the ability of a series expansion such as the Asymptotically Ideal Production Model (AIM) to approximate arbitrary cost functions. He finds that without imposing concavity on AIM, there are relatively few violations of concavity in the region of price space considered. The simulation also provides strong evidence against the non-negativity constraints used by Barnett, Geweke and

Wolfe (1991) to impose concavity on AIM. The paper indicates that the constraints reduce the flexibility of AIM enough to render it incapable of approximating most of the CES and translog technologies considered. Gagne and Ouellette (1998) investigate the behavior of three flexible functional forms: the translog, the symmetric McFadden (SMF), and the symmetric generalized Barnett (SGB), and argue that imposing curvature properties globally is too stringent.

Terrell (1996) assesses the benefits of imposing monotonicity and concavity on cost functions over a range of prices through an appropriate choice of prior distribution. Using Berndt-Wood data, results reveal that imposing monotonicity and concavity over large ranges of prices restricts the functional forms in a manner similar to global restrictions, but that restrictions over smaller ranges of prices may be imposed without a significant loss in flexibility.

Later a sequence of papers also follows proposing alternative methods to impose regularity locally. Ryan and Wales (2000) propose and illustrate a method for imposing concavity at a chosen single reference point, which may result in concavity at many points, in Translog and Generalized Leontief cost functions, while at the same time maintaining flexibility of the forms. Applying the procedure in the consumer demand context, Ryan and Wales (1998) find that the local imposition results in concavity being satisfied at all data points for both forms. They claim that at least for the data set they considered, there is no need to resort to more complex functional forms that allow concavity to be imposed globally, while at the same time maintaining flexibility.

Salvanes and Tjotta (1998) suggest a procedure to calculate the region where an estimated translog cost function meets the required regularity conditions (positive cost, positive marginal cost, homogeneous, monotonicity and concavity in input prices). They calculate this region for the U.S. Bell cost function as reported by Evans and Heckman (1984, 1986), and show that the estimated cost function had negative marginal cost in most of the test region.

Koebel, Falk and Laisney (2003) present a new method for imposing and testing concavity of cost functions using asymptotic least squares, which can be easily implemented even for

nonlinear cost functions. By analyzing a generalized Box-Cox cost function, they present a parametric concavity test and compare price elasticities when curvature conditions are imposed versus when they are not. They find that although concavity is statistically rejected, estimates were not very sensitive to its imposition in their application.

As we have discussed above, economic theory requires that cost functions satisfy certain properties such as monotonicity and concavity, which involve restricting the sign of the regression's partial derivatives. A cost function such as the translog, which involves taking a second-order Taylor series expansion about a point, is locally flexible but may not be regular. However, the cost function can be made regular at a particular data point by imposing restrictions.

The cost frontier provides the amount it would cost an efficient plant facing a given set of prices p_i to produce a given level of output q_i . Let $c(p, q)$ denote the cost frontier, expressed as a frontier of input price vector p and output vector q . Although the underlying cost function is unknown, microeconomic theory requires several properties of cost function. First, cost must be monotone to both input prices and output, which requires that $\frac{\partial c(p, q)}{\partial p} > 0$ and $\frac{\partial c(p, q)}{\partial q} > 0$. Since input and output levels are positive, monotonicity requires that the elasticity of output is positive at all data points. Second, a necessary and sufficient condition for a twice differentiable cost frontier to be concave in input prices requires that the Hessian matrix $\frac{\partial^2 c(p, q)}{\partial p \partial p'}$ be negative semi-definite and it rules out an upward sloping input demand. Last, theory also requires that the cost frontier should satisfy homogeneity of degree one in the input prices, that is $c(tp) = tc(p)$, for $t > 0$.

In this dissertation, a translog function is used in its restricted and unrestricted forms to discuss the importance of using flexible functional forms and its implication on the efficiency measures. The translog function has a flexible functional form where output elasticity and input

levels can vary across observations. Following the method proposed by Terrell (1996), monotonicity and concavity restrictions are imposed locally on the cost frontier. Parametric restrictions are imposed over the relevant range of data ψ ³, where the inferences are made. The indicator function is defined as

$$\begin{cases} h(\beta) = 1 & \text{stochastic frontier satisfies monotonicity and concavity} \\ h(\beta) = 0 & \text{otherwise} \end{cases} \quad (2.20)$$

The above indicator function is then included into the informative prior to impose regularity conditions and implement Gibbs sampling technique. As the number of iterations goes to infinity, the Gibbs Sampling method obtains the samples that converge to a random sample from the posterior density. Please see appendix B for a detailed description of the procedure for Gibbs sampling.

³ See Figure 5.3 to understand how I chose ψ .

CHAPTER 3. LITERATURE REVIEW ON ELECTRICITY GENERATION AND POLLUTION AS BYPRODUCT

3.1 Electricity Deregulation and Restructuring

Electricity restructuring and deregulation refers to the current trend in which many states are reorganizing traditional U.S. monopoly electric service to allow operations and charges to be separated or "unbundled" into generation, transmission, and distribution and retail services. This will allow customers in some states to buy retail electric service from competing providers. With enactment of the Energy Policy Act in 1992 (EPACT), the competition in wholesale power sales received a boost. This allowed U.S. non-utility power producers access to the transmission grid to sell power in an open market. In 1996 Federal Energy Regulatory Commission (FERC) enacted another Orders 888, which reduced monopoly power over the transmission of electricity and encouraged competition. State initiatives to promote retail competition began in 1996 with Rhode Island. Since 1996, 49 states and the District of Columbia have addressed some aspect of electric restructuring. All state initiatives attempt to incorporate some form of stranded cost recovery for the incumbent utility and separate electric generation from distribution.

Later in December 1999, FERC issued Order 2000 which focused on the organization and governance of the transmission grid. FERC encouraged the creation of regional transmission organizations to control and operate the transmission grid free of any discriminatory practices by the end of 2001.

As of February 2003, 24 states⁴ and the District of Columbia have implemented retail competition by mandating customer choice for electricity providers, either through their legislatures or by regulatory orders. The Energy Information Administration (EIA) states that "Alaska and South Carolina had legislation or regulatory orders pending. Sixteen States still had ongoing

⁴ 24 states include Arizona, Arkansas, California, Connecticut, Delaware, Illinois, Maine, Maryland, Massachusetts, Michigan, Montana, Nevada, New Hampshire, New Jersey, New Mexico, New York, Ohio, Oklahoma, Oregon, Pennsylvania, Rhode Island, Texas, Virginia, and West Virginia.

legislative or regulatory investigations, and there were 8 States where no restructuring activities had taken place)” (“The Changing Structure of the Electric Power Industry 2000: An Update,” Energy Information Administration, 2000, p. 182).

Electricity restructuring and deregulation are expected to affect the structure and operation of utilities, either individually or as an industry. Restructuring and deregulation in the U.S. electric market in the past decade has brought competition into generator sector. The competitive electricity generation market rewards the minimization of generation costs. Since the telecommunications and banking industries in the U.S. have been made more competitive by the deregulation and introduction of competition, we expect that the electric power industry should have the potential for similar efficiency gains. In the traditional cost-of-service regulation systems, electricity companies receive a risk-free fixed rate of profit for their generation of electricity, and profit is a function of capital investments. Also, wholesale and retail electricity power prices are calculated based on a utility's costs. Thus, regulated companies can still add the costs resulting from any inefficiency to the price they charge for electricity and pass the costs to customers. Therefore the regulated companies may have an incentive to increase, rather than decrease, their costs. In contrast, deregulation should theoretically give firms the incentives to lower costs to be technologically efficient in order to maximize their profits. However, little is known empirically about the magnitude of such effects except for some studies.

To study the potential benefit of competition introduced by deregulation and restructuring, in 1999, the U.S. Department of Energy (DOE) released “Supporting Analysis for the Administration’s Comprehensive Electricity Competition Act”. DOE claims that by year 2010, competition would reduce electric prices nationally by 14% on average. They found that all regions of the country will benefit, but customers in high cost areas will benefit more through larger price reductions. However, there have also been studies that predicted price increases where the DOE had forecasted price declines and savings benefits.

Kleit and Terrell (2001) estimated potential efficiency gains by examining 78 steam plants using natural gas as primary fuel. Results indicate that plants on average could reduce cost by up to 13% by eliminating production inefficiency.

In addition, Hiebert (2002) analyzed the generation plants operating cost efficiency over the period 1988-1997 and claimed evidence that the average operating efficiency of coal plants increased in states where the transition to retail competition had begun.

To add some value to current literature and to answer some questions related to the debate regarding deregulation, Chapter 5 examines the efficiency and returns to scale of 136 U.S. electric power generation coal-fired plants in 1996. That application uses a Bayesian stochastic frontier model that imposes monotonicity and concavity. Using a single stochastic frontier in Chapter 7, I also estimate the mean posterior inefficiency for regulated and deregulated power plants.

3.2 Effects of Pollution Control on Efficiency

One of the major sources of air pollution in the United States is electricity generation. Electricity generators that burn fossil fuels contribute a large part of emission pollutions such as sulfur dioxide (SO_2) and nitrogen dioxide (NO_2). The Clean Air Act Amendments implemented in 1990 greatly influence power plants' technical efficiency, especially for coal fueled power plants. Consequently, the emissions of SO_2 from U.S. coal-burning electric plants were reduced from around 19 million tons in 1980 to 8.95 million tons by the year 2000.

Historically, most papers have ignored undesired outputs (“bads”) in modeling production functions or cost functions for regulated industries. For example, Nelson (1984), Baltagi and Griffin (1988), and Callan (1991) estimate productive change in electric utility industry without including pollution as a bad output, but their models include inputs used to control pollution.

However, a growing literature has studied productivity change and efficiency in industries when an undesirable output (a bad) is a by-product in the production process. For example, Pasurka

(2003) calculated the association between changes in SO₂ emissions and changes in technical efficiency, changes in the output mix and input growth. He found that changes in the output mix were strongly associated with changes in SO₂ emissions. Another study from Dorfman and Atkinson (2005) measured productivity change and efficiency when an undesirable output is a by-product. They also found that failing to credit the utilities for their efforts aimed at reducing SO₂ emissions would have underestimated their true technological progress by 35%.

The cost frontier includes a combination of costs: costs resulted from reducing pollution emission and costs associated with regular inputs such as labor, capital and fuel. Without a doubt, a reduction of SO₂ emission requires power plants to substitute other inputs in electric generation. Controlling the effects of bad will make our estimates more precise and resulting inferences more reliable, and also shed some insights into the tradeoff between electricity and pollution control. In the application considered in this paper, electric plants produce not only good output, which is electricity, but also undesired outputs (“bads”), such as pollution, specifically sulfur dioxide. Accounting for bad outputs when we estimate plant efficiency and make subsequent inferences is a crucial point.

3.3 Alternate Fuel Sources in Electric Generation Utilities

With the ongoing restructuring and deregulation in the U.S. electricity market, power plants are expected to be more flexible in switching fuels from one to another due to economic incentive. As competition among different electric suppliers becomes more severe, electric generators tend to minimize cost and take advantage of movements among different fuels. Today, coal is the most common fuel used to generate electricity. According to the EIA⁵, U.S. electric power industry net generation in 2004 was 3941 billion KWH and 49.8% was generated using coal. In the past several years, higher natural gas prices have increased the costs of power generation in plants using that

⁵ Source: Energy Information Administration, Form EIA-906, "Power Plant Report."

fuel, especially in the days following hurricanes Katrina and Rita. However, coal prices are relatively stable. Given this situation, it is possible that electric utilities, which are able to use a variety of fuels and switch between them, may be very responsive to changes in relative fuel prices and then inter-fuel substitution in existing power plants will be substantial, especially that between natural gas and coal. Additionally, some conversions of electric plants are relatively straightforward and inexpensive, such as a conversion from gas/oil to coal.

Some prior literature has also found the substitution power between gas and coal. For example, McDonnell (1991) used a translog cost model with six fuels: natural gas, coal, petroleum, nuclear, hydro on a cross-sectional data for 82 privately owned electric utilities during 1987. He found that oil is the most price elastic and coal is the least. His results showed that coal and gas have the highest cross price elasticities suggesting they are the best substitutes.

Chapter 6 of this dissertation compares the predicted costs and returns to scale of coal generation to natural gas generation at plants where the cost of both fuels could be obtained. The results will provide insight into how the optimal fuel choice for electricity generation varies with the relative prices of those fuels.

CHAPTER 4. DATA DESCRIPTION

The first two papers use the Utility Data Institute (UDI) Production Costs Database (1998), which provides plant level information including total cost, plant location, fuel prices⁶ (both average price of natural gas and average price of coal burned at each plant), and two measures of output in electricity generating plants in year 1996. Output in megawatt hours and peak output in megawatts are used to account for the fact that some power plants (especially for power plants using natural gas as sources) exist mainly to provide output during periods of peak demand.

The second data source, the Bureau of Labor Statistics (BLS), supplies county level data on wages by SIC code on U.S. manufacturing wages. For the data set applied in cross-sectional regression in Chapter 5 and Chapter 6, I compute the average annual manufacturing wage for worker in the county where the power plant is located. I particularly use the average wage of SIC 4900 (Standard Industrial Classification), which includes the establishments involved in the generation, transmission, and/or distribution of electricity or gas or steam. To avoid too much missing data, the computed wages in the panel used in Chapter 7 focus on all the manufacturing workers. Therefore, the average wage in the panel from 1994 to 2000 is much lower than the 1996 average wage⁷.

The third data source is the emissions data and compliance reports (1996) from U.S. Environmental Protection Agency (EPA). The Summary Emissions Report provides tons of sulfur dioxide (SO₂) emitted by each electricity generation plant since the beginning of the year.

For the fourth data source, Hilt (1996) provides plant level measures of the capital stock, taxes, overhead, depreciation, and operating and management expenses. Allocating firm level data

⁶ Price of coal is measured by average cost of coal burned per million BTU, and price of natural gas is measured by average cost of natural gas burned per million BTU.

⁷ See the average wage in table 4.1 and table 4.2.

to each plant provides these variables. In this data set, Hall and Jorgenson's (1971) method is used to compute the price of capital.

In addition, we want to estimate the efficiency change of electricity generation plants if the plants used alternative fuel sources, for example, switch fuels between coal and natural gas. Therefore, an alternative fuel price is needed. The last data source is Monthly Cost and Quality of Fuels for Electric Plants Data (1996)⁸, provided by Energy Information Administration (EIA). This data set includes information on type of fuel purchase, fuel price, fuel cost, fuel type, fuel origin, fuel quantity and fuel quality. The source for this data is FERC Form No. 423: "Monthly Report of Cost and Quality of Fuels for Electric Plants."

The measured variables are as follows:

Total Cost	= annual total production expenses (sum of operations [including fuel] and maintenance expenses). (\$) The variable employed as dependent variable is cost_i , which is measured as the natural log of total cost minus the natural log of price of capital for plant i .
Annual Output	= net generation produced in reporting year. (MW: measured in megawatt hours)
Peak Output	= net peak demand on plant during reporting year. (MW)
Wage (labor price)	= average annual manufacturing wage for workers in the county where the power plant is located,
Price of Fuel	= average cost of coal or gas burned per million BTU,
Log Relative Wage	= natural log of wage minus natural log of capital price,
Log Relative Fuel Price	= natural log of fuel price minus natural log of capital price,
Log (SO ₂)	= natural log of sulfur dioxide emission.

⁸ See <http://www.eia.doe.gov/cneaf/electricity/page/ferc423.html> for the data set.

$$\text{Price of Capital}^9 = \frac{(1 - \text{taxrate} \times z) \times (r + \text{deprate} + \text{omrate} + \text{ohrate})}{(1 - \text{taxrate})}$$

where *taxrate* is the statutory corporate profits tax rate, *z* is the present value of depreciation deductions on one dollar's worth of capital investment, *r* is the discount rate, *omrate* refers to the other operating and management expenses and *ohrate* refers to the overhead rate.

Table 4.1 contains summary statistics of data sets used in Chapter 5 and Chapter 6 within this dissertation. Our first sample includes 139 coal-fired U.S. power plants in 1996 (shown as column 2 and column 3 in Table 4.1). With sulfur dioxide emission included, three plants were dropped due to data constraints and therefore the second sample includes 136 coal-fired plants (shown as column 4 and column 5 in Table 4.1). The standard error of $\log(\text{SO}_2)$ is about 1.22, which indicates substantial variation in the sulfur dioxide emission across plants.

The third sample consists of 78 electric power generation plants using natural gas as the primary fuel in 1996 (shown as column 6 and column 7 in Table 4.1). This data set is constructed for the purpose of comparing estimated cost changes resulting from the expected fuel source change. In the third sample, we find that only one firm also used coal as fuel in year 1996. Therefore, due to this data constraint, we choose utility-level prices as the alternative fuel price¹⁰ in the fourth sample (shown as column 8 and column 9 in Table 4.1). We assume that electric generation natural gas-fired power plants would choose to use alternative fuel – coal. By doing so, we are interested in the effects on fuel cost and returns to scale for power plants switching fuel in the generation of electricity. The sample size in the fourth data set dropped to 51 due to lack of data on alternative fuel prices at some plants. In the 1996 data set, there is a substantial variation in the price of natural gas. The standard deviation of gas price is \$0.4603 relative to the mean of \$2.7052 per million

⁹ Hazilla and Kopp (1986) provided the detail methods of construction of price of capital.

¹⁰ See appendix table A1 for utility level price of alternative fuel.

BTU. If those plants change to use coal as alternative fuel, we find that they would face an average coal price at \$1.3203 per million BTU with standard deviation at \$0.2554.

Table 4.2 contains summary statistics for the sample employed in Chapter 7 in this dissertation. This sample consists of an unbalanced panel of 287 power plants over a 7-year period from 1994 to 2000. The overall sample includes 1341 observations with a minimum two observations per plant (shown as column 2 and column 3 in Table 4.2). Six¹¹ out of forty states in our data set have restructuring undergoing. Therefore, the data sets include 74 plants in the deregulated sample and 213 plants in the regulated sample.

Descriptive statistics for deregulated plants and regulated plants are shown as column 4, 5 and column 6, 7 respectively in Table 4.2. Utility Data Institute (UDI) Production Costs Database (updated in 2001) provides plant level information including total cost, plant location, fuel prices¹² (average price of coal burned at each plant), and output in electricity generating plants from year 1981 to 2000.

The price of capital is computed based on data extracted from UDI and FERC (the Federal Energy Regulatory Commission) Form 1 "Annual Report of Major Electric Utilities, Licensees and Others" (filed by investor-owned utilities). Although our sample is restricted to the investor-owned utilities, they are the largest component of the electric industry in terms of power generation, value of assets, and total revenues.

Utility level costs are allocated to plant level using electric output weights. The data consists of an unbalanced panel of 287 steam power plants in 40 states over the 1994 through 2000 period. Small plants are excluded and are defined as those for which net output of electricity is less than 10,000 megawatt hours in any of our sample years.

¹¹ Six states: IL, MD, MI, MT, NY, PA.

¹² Price of coal is measured by average cost of coal burned per million BTU.

Table 4.1: Summary Statistics for U.S. Power Plants in Year 1996

Variable	Coal fueled, no SO2		Coal fueled, with SO2		Gas fueled		Alternative Coal	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std Dev.
Total Cost (in millions)	90.12	77.79	91.86	77.74	50.26	51.04	46.99	49.28
Annual Output (in millions)	4.61	4.28	4.71	4.28	1.53	1.73	1.47	1.76
Peak Output	853.0014	673.0422	869.5382	671.0055	643	547.8342	625.8077	524.713
Wage	44,268.44	6,632.71	44,229.45	6,622.17	45,251.40	7,195.68	43,425.79	6,318.89
Price of Fuel	1.452	0.4117	1.4518	0.4142	2.7052	0.4603	1.3203	0.2554
Price of Capital	0.9282	0.3545	0.9187	0.3474	1.0182	0.3902	0.9535	0.353
Log Relative Wage	3.9005	0.3321	3.9083	0.3276	3.8439	0.3656	3.8636	0.3473
Log Relative Fuel Price	0.4573	0.3864	0.4654	0.3866	1.0264	0.397	0.3619	0.3191
Log (SO ₂) (in tons)			9.8312	1.2177				
Sample Counts	139		136		78		51	

Table 4.2: Summary Statistics for a Panel of U.S. Power Plants in Year 1994 to 2000

Variable	All Plants		Deregulated Plants		Regulated Plants	
	Mean	Std Deviation	Mean	Std Deviation	Mean	Std Deviation
Total Cost (in millions)	84.45	77.5	77.25	66.36	86.49	80.29
Annual Output (in millions)	4.614	4.47	3.81	3.73	4.84	4.63
Wage	32,179.1	8501.7	35,106.45	8,455.37	31,349.92	8,333.80
Price of Coal	1.371	0.358	1.491	0.369	1.336	0.348
Price of Capital	1	0.446	1.021	0.473	0.994	0.437
Log Relative Wage	3.53	0.52	3.61	0.47	3.51	0.53
Log Relative Coal Price	0.375	0.512	0.45	0.536	0.354	0.503
Sample Counts	1341		296		1045	

To compute plant-level price of capital, we need values of other variables, such as the plant-level tax rate, plant-level maintenance rate, plant-level depreciation rate, plant-level overhead and other operating and management expenses. However, from FERC Form-1, we can only observe values at utility-level of the above variables. The next several steps allocate each value at plant level.

First, plant-level annual electricity output q_{it} and utility-level annual electricity output Q_{jt} are obtained, where i refers to different plants, j refers to different utilities and $t = 1994, \dots, 2000$ refers to different years. Then, to each plant, output weight is computed as $\frac{q_{it}}{Q_{jt}}$ and assign this weight to obtain above plant-level values.

As discussed earlier in this chapter, different approaches are used to compute the average wage in Table 4.1 and Table 4.2. Therefore, the average wage falls from around \$40,000 in Table 4.1 to around \$30,000 in Table 4.2.

CHAPTER 5. MEASURING POTENTIAL EFFICIENCY GAINS IN THE PRESENCE OF UNDESIRABLE OUTPUTS OF ELECTRICITY GENERATION

5.1 Introduction

Electricity restructuring and deregulation are expected to affect the structure and operation of the utilities, bring competition, and therefore increase efficiency of individual plants. This chapter examines the efficiency of 136 coal-fired U.S. electric power generation plants in 1996 using a Bayesian stochastic frontier model that imposes monotonicity and concavity. Results confirm that this constrained model yields more accurate and favorable results than an unconstrained model: Shares and elasticities are well behaved, and the standard deviations are largely reduced. How much will controlling for Sulfur Dioxide (SO₂) affect plants' efficiency? When SO₂ is treated as "bad" output and included into model, and monotonicity and concavity restrictions are imposed on both input prices and SO₂, measures of plant efficiencies rise by around 1%. Therefore the costs for pollution abatement are treated as part of inefficiency if SO₂ is not included in the model. However, imposing monotonicity and concavity restrictions only generates minor differences for individual firms' returns to scale. Results also show that once SO₂ is included in the model, average returns to scale for the constrained model rises from 1.17 to 1.194, and 132 out of 136 plants in the sample exhibit increasing returns to scale.

5.2 The Bayesian Stochastic Cost Frontier

This study uses the flexible translog cost frontier, which allows the above properties to be imposed. An earlier study by Kleit and Terrell (2001) includes both peak and off-peak output of electricity generation. Here we do not include peak output, because peakers (plants specializing in generating electricity during periods of peak demand) tend to rely on gas rather than coal as a fuel. Unlike Kleit and Terrell (2001), we add sulfur dioxide (SO₂) as "bad" output and include SO₂ in our model. Therefore, we list two models for the translog frontier with three inputs (labor, capital and coal). The first model (one-output model) includes non-peak electricity output:

$$f(p, q) = a_0 + \sum_{i=1}^3 a_i \ln p_i + \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 a_{ij} \ln p_i \ln p_j + b_1 \ln q + \gamma_1 (\ln q)^2. \quad (5.1)$$

While another model (two-outputs model) includes two outputs - both normal time electricity output (measured in megawatt hours) and SO₂ (measured as tons of sulfur dioxide emitted by each electricity generation plant):

$$f(p, q) = a_0 + \sum_{i=1}^3 a_i \ln p_i + \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 a_{ij} \ln p_i \ln p_j + b_1 \ln q_1 + \gamma_1 (\ln q_1)^2 + c_1 \ln q_2 + c_{11} (\ln q_2)^2 + d_1 \ln q_1 \times \ln q_2 \quad (5.2)$$

where

$$a_{ij} = a_{ji}, \quad \forall i, j = 1, 2, 3,$$

$$\text{and } \sum_{i=1}^3 a_i = 1, \quad \sum_{j=1}^3 a_{ij} = 0, \quad \forall i, j = 1, 2, 3.$$

Note p_1 refers to the price of labor, p_2 prices of fuel, and p_3 price of capital. q_1 is electricity output and q_2 is SO₂. Concavity and monotonicity restrictions depend on only relative prices – individual input price relative to capital price. The homogeneity of degree one is imposed on the translog here and the models are presented in the way of relative prices. Equation (5.1) becomes

$$f(p, q) - \ln p_3 = a_0 + a_1 (\ln p_1 - \ln p_3) + a_2 (\ln p_2 - \ln p_3) + \frac{1}{2} a_{11} (\ln p_1 - \ln p_3)^2 + \frac{1}{2} a_{22} (\ln p_2 - \ln p_3)^2 + a_{12} (\ln p_1 - \ln p_3) (\ln p_2 - \ln p_3) + b_1 \ln q + \gamma_1 (\ln q)^2 \quad (5.3)$$

and equation (5.2) reduces to

$$f(p, q) - \ln p_3 = a_0 + a_1 (\ln p_1 - \ln p_3) + a_2 (\ln p_2 - \ln p_3) + \frac{1}{2} a_{11} (\ln p_1 - \ln p_3)^2 + \frac{1}{2} a_{22} (\ln p_2 - \ln p_3)^2 + a_{12} (\ln p_1 - \ln p_3) (\ln p_2 - \ln p_3) + b_1 \ln q_1 + \gamma_1 (\ln q_1)^2 + c_1 \ln q_1 + c_{11} (\ln q_2)^2 + d_1 \ln q_1 \times \ln q_2 \quad (5.4)$$

Also note that the relative prices of labor ($\ln p_1 - \ln p_3$) and fuel ($\ln p_2 - \ln p_3$) tend to be positively correlated. The translog function imposes homogeneity of degree one in factor prices under using parameter restrictions. As a second order approximation to an arbitrary cost frontier, the translog also fulfills Diewert's minimum flexibility requirement for flexible function forms. We compute the share equations associated with a respective input by taking the first derivative of the log cost frontier with respect to the log input price:

$$s_i(p, q) = a_i + \sum_{j=1}^3 a_{ij} \ln p_j \quad (5.5)$$

At any given price, the monotonicity and concavity conditions can be easily verified based on restrictions derived from the translog cost frontier. For the translog, monotonicity in input prices ($\partial c / \partial p_i > 0$) requires that nonnegative values for all shares in equation (5.5). The negative semidefinite values for the Hessian matrix $\nabla^2 c$ (where c is the cost frontier) ensures the concavity restriction. The concavity can also be verified by using translog shares. Let A represent the $n \times n$ symmetric matrix of a_{ij} , s the vector of n equations, and \hat{s} an $n \times n$ diagonal matrix with the shares on the main diagonal. Diewert and Wales (1987) show that the translog cost frontier satisfies concavity if and only if $A - \hat{s} + s s^T$ is a negative semi-definite matrix.

Moreover, given the translog frontier, monotonicity in output from one-output model requires $\partial c / \partial q > 0$, which simply implies:

$$b_1 + 2\gamma_1 \ln q > 0 \quad (5.6)$$

and monotonicity in output from two-output model requires both $\partial c / \partial q_1 > 0$ and $\partial c / \partial q_2 < 0$, which imply:

$$\begin{cases} b_1 + 2\gamma_1 \ln q_1 > 0 \\ c_1 + 2c_{11} \ln q_2 < 0 \end{cases} \quad (5.7)$$

It is important to emphasize again the distinction between global and local concavity restrictions. Global concavity restricts all positive input prices to obey the law of demand. By contrast, the imposition of local concavity is defined as constraining a single input price, multiple input price combinations, or a range of input prices, to obey economic theory.

To solve the problem of concavity violations, Jorgenson and Fraumeni (1981) imposed concavity on the translog at all positive input prices by forcing the symmetric matrix A to be negative semi-definite. However, their method causes the translog to overestimate own-price elasticities and biases cross-price elasticities. Alternatively, this chapter follows Terrell (1996) method, which uses a prior to impose monotonicity and concavity over a certain range of input prices where inferences are to be drawn. This method treats the flexible function form as local rather than global approximation to impose restrictions without greatly reducing the flexibility of the translog. His results showed little bias over small ranges, but significantly increased bias while imposing constraints over a wide range of input prices, hence loss in flexibility of function forms.

5.3 Methodology

Monotonicity and concavity restrictions are imposed through the prior. Bayesian analysis allows us to combine a subjective probability distribution summarizing what one knows about a parameter, with any available sample information, to obtain a more reflective posterior distribution. The prior distribution used is the same as that analyzed by Kleit and Terrell (2001).

The cost frontier $f(p_i, q_i)$ represents the costs that an efficient plant faces given a set of prices (p_i) for inputs used to produce a given level of output (q_i). If a plant's observed costs exceed the cost frontier $f(p_i, q_i)$, then that deviation is partly attributed to inefficiency. Therefore, plant inefficiency can be measured by deviations from the cost frontier.

Following Kleit and Terrell's earlier work, I implement the method for multiple inputs and express the log of total cost of the plant as:

$$\ln(c_i) = f(p_i, q_i) + u_i + v_i \quad (5.8)$$

where $f(p_i, q_i)$ is the cost frontier. The deviation of plant i 's cost from the cost frontier includes two stochastic error terms: inefficiency (v_i) and measurement error (u_i). (u_i) represents statistic noise and is generally assumed that $u_i \sim IIDN(0, \sigma^2)$ as common in this literature. The inefficiency error term (v_i) is nonnegative and always serves to increase cost. Therefore, (v_i) follows a particular one-sided distribution and here we assume (v_i) follows an exponential distribution with scale parameter λ . Exponential distribution is chosen because van den Broek, Koop, Osiewalski, and Steel (1994) argue that this distribution for inefficiency (v_i) is more robust to prior assumptions about parameters than other distributions.¹³ Through Bayesian estimation, we are able to assign the distribution to u_i and v_i , though it is hard to identify the above two error terms using other estimation technique. The translog cost is applied to the specific function form $f(p_i, q_i)$, since stochastic frontier models typically choose a linear function to generate a linear composed error model which is showed as following:

$$\begin{cases} y_i = X_i\beta + u_i + v_i, \\ u_i \sim N(0, \sigma^2), \\ v_i \sim EXP(\lambda) \end{cases} \quad (5.9)$$

where y_i refers to the log cost for plant i , X_i refers to a row vector of independent variables used to create the translog frontier, and β refers to a column vector of coefficients of the translog. For the residual terms, u_i denotes two-sided error terms accounting for measurement error, and v_i is the non-negative one-sided error term reflecting plant inefficiency. The linear error model that combines the cost function with the translog functional form is consistent with the prior information in Kleit and Terrell (2001).

¹³ Also see Kleit and Terrell (2001).

Bayesian models require choosing a prior parameter that can summarize our best initial guess of the efficiency of a median power plant. Therefore, we choose a flat prior for β and a gamma prior for λ^{-1} and σ^2 ,

$$\begin{aligned}\pi(\beta) &\propto 1, \\ \pi(\lambda^{-1}) &= f_G\left(\lambda^{-1} \mid 1, -\ln(r^*)\right), \\ \pi(\sigma^{-2}) &= f_G\left(\sigma^{-2} \mid \frac{\tau}{2}, \frac{S_p^2}{2}\right)\end{aligned}\tag{5.10}$$

where $f_G(\bullet \mid \nu_1, \nu_2)$ is a gamma density with mean ν_1/ν_2 and variance ν_1/ν_2^2 . $r_i = \exp(-\nu_i)$ measures the efficiency of the i 'th plant, and r^* is the prior median for efficiency. Following van den Broek et al. (1994) and Koop et al. (1994), we set the same value $r^* = 0.875$ ¹⁴. With the number of observations in our sample, this implies a weak prior on λ and the results should not be sensitive to the choice of r^* . Note that in equation 2.18, it was discussed that the conditional distribution of λ given ν , β and σ^{-2} is

$$p(\lambda^{-1} \mid \beta, \sigma^{-2}, \nu, y) = p(\lambda^{-1} \mid \nu) = f_G(\lambda^{-1} \mid n+1, \nu' \iota - \ln(r^*))$$

Since ι is a $n \times 1$ vector of ones, the expected value of λ^{-1} is basically

$$E(\lambda^{-1}) = \frac{-\ln(r^*) + \sum_{i=1}^n \nu_i}{n+1} \approx n\bar{\lambda} + 0.133 \text{ for } r^* = 0.875. \text{ When } n \text{ is large enough (} n = 10,000 \text{ in this}$$

dissertation) and $\bar{\lambda}$ is not too small, the result is not sensitive to the choice of r^* .

Figure 5.1 and Figure 5.2 show the density plots for priors $\pi(\lambda^{-1})$ and $\pi(\sigma^{-2})$.

¹⁴ They tried different values for prior median efficiency, and find this value provides the highest accordance between prior and the sample information. $\nu_1 = 1$ is the most conservative model, and out-of-sample efficiency is not affected much by $\nu_1 = 1$.

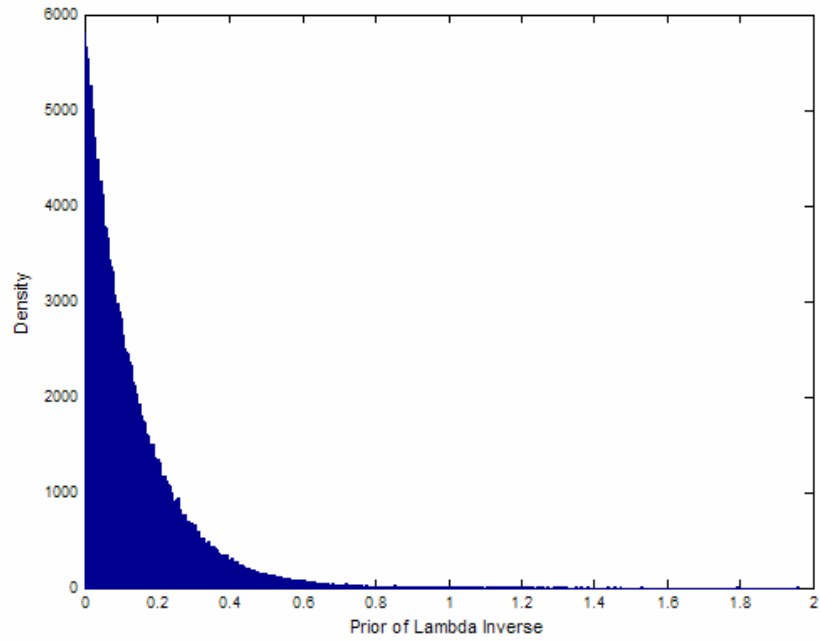


Figure 5.1 - Density Plot of Prior $\pi(\lambda^{-1})$

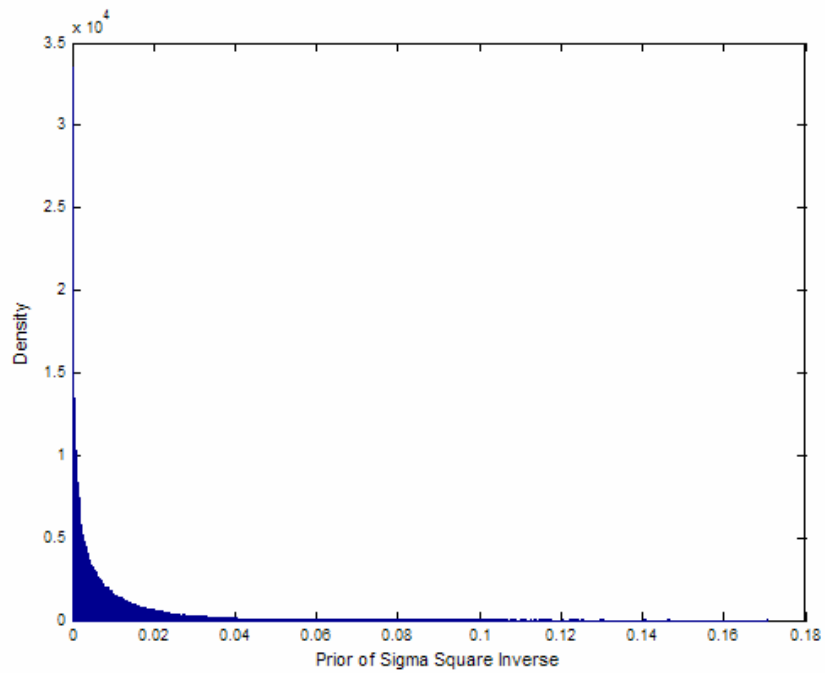


Figure 5.2 - Density Plot of Prior $\pi(\sigma^{-2})$

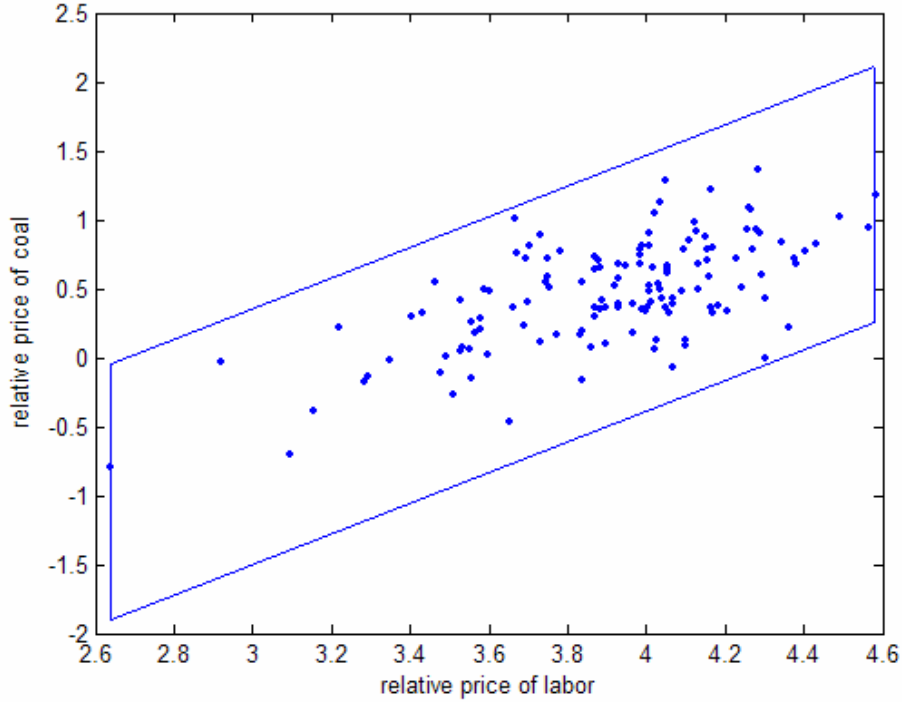


Figure 5.3 - The Region Ψ versus Relative Prices in the Dataset

Fernandez, Osiewalski, and Steel show that an uninformative prior for σ^2 leads to an improper prior in a cross-sectional application such as this one, so we choose a gamma prior for σ^2 . Following Kleit and Terrell (2001) and based on previous studies, we set $\tau = 1$ and $s_p^2 = 0.03$, which implies a weak prior on σ^2 as well.

As Terrell (1996) does, we also use the prior to incorporate monotonicity and concavity restrictions. We define $h(\beta)$ as the indicator function. Let $h(\beta) = 1$ if the stochastic frontier satisfies monotonicity and concavity for all price combinations in a region of prices and output Ψ , $h(\beta) = 0$ otherwise. Furthermore, this full prior allows us to slice away the portion of the posterior density that violates economics theory, specifically the properties of cost function. Since concavity and monotonicity depend only on relative price, we define the region in terms of relative prices¹⁵.

¹⁵ Relative prices are defined as p_L/p_K - price of labor relative to price of capital, and p_F/p_K - price of fuel relative to price of capital.

The prior density is presented by this indicator function.

$$\pi(\beta, \sigma^2, \lambda^{-1}) \propto \pi(\beta) \pi(\lambda^{-1}) \pi(\sigma^{-1}) h(\beta). \quad (5.11)$$

The prior imposes monotonicity and concavity over the price and output combinations Ψ where the inferences will be drawn. It means that the prior sets the probability of parameter values that violates the microeconomic theory at relevant prices to zero. Figure 5.3 graphs the parallelogram Ψ . It is defined in terms of the relative price of labor and relative price of coal because monotonicity and concavity restrictions depend on only relative prices. This region also includes a positive correlation between the relative prices of coal and labor which are typically observed in data sets similar to this one.

Combining the prior and the likelihood generates the posterior density $p(\theta)$, where $\theta = (\beta, \sigma^2, \lambda, \nu)$. Also, the efficiency measures, elasticities, and returns to scale are all functions of θ . We denote the ratio of plant i 's cost to the cost of an efficient firm as the measure of efficiency, equivalently,

$$r_i = \frac{\text{cost of an efficient firm}}{\text{cost for firm } i} = \frac{\exp(f(p_i, y_i))}{\exp(f(p_i, y_i) + v_i)} = \exp(-v_i). \quad (5.12)$$

As discussed earlier, peak output is not included in regression. When we compute the returns to scale, we list two models: one model with only normal time electricity output, another one with two outputs - both normal time electricity output and SO₂. Similar to Caves, Christensen, Swanson (1981), the returns to scale in the above two models are

$$RTS_1 = \left(\frac{\partial \ln C}{\partial \ln q} \right)^{-1} = \frac{1}{b_1 + 2\gamma_1 \ln q} \quad (5.13)$$

$$RTS_2 = \left(\sum_{i=1}^2 \frac{\partial \ln C}{\partial \ln q_i} \right)^{-1} = \frac{1}{b_1 + 2\gamma_1 \ln q_1 + c_1 + 2c_{11} \ln q_2}^{16}. \quad (5.14)$$

Note the efficiency measures, elasticities, and returns to scale are the function of parameters in this paper. Let $g(\theta)$ represents these functions of interest. In theory, the moments of $g(\theta)$ could be obtained from the posterior density through integration. For example, the posterior mean for these functions of interest is

$$E[g(\theta)] = \int g(\theta) p(\theta) d\theta. \quad (5.15)$$

However, it is generally impossible to compute the integral analytically as in most Bayesian applications. Therefore, we use Gibbs Sampler¹⁷ to provide the necessary sample for the posterior. If we sample n values of θ_i for the posterior, then we can evaluate moments numerically. Van Koop, Osiewalski, and Steel (1995) first introduce the Gibbs sampler for the stochastic frontier model. The posterior mean for a function of interest is computed as $E[g(\theta)] = \frac{1}{n} \sum_{i=1}^n g(\theta_i)$. I draw 11,000 Gibbs Sampler and dropped the first 1,000 points to avoid the sensitivity of starting values. The density plots were drawn and showed that the functions of interest normally converge around 2,000 Gibbs draws. This dissertation uses the algorithm employed by Kleit and Terrell (2001) that includes an appropriate prior for σ^2 and an accept-reject element¹⁸ to impose monotonicity and concavity restrictions.

¹⁶ Note that q_1 refers to the normal time electricity output and q_2 refers to the sulfur dioxide emission-the “bad” output.

¹⁷ Please see appendix B for the procedure of Gibbs Sampler in detail.

¹⁸ In practice, at each draw from the Gibbs Sampler, we check whether all of the regularity conditions hold at each point inside Ψ . If any of the regularity conditions are violated at any data point, then the Gibbs draw is dropped from the sample used for numerical integration.

5.4 Results

This chapter estimates two models. The first does not control for the effects of sulfur dioxide (SO₂) emissions. The second model including SO₂ emissions is then presented and the differences are compared.

For results, we choose to report summary statistics from the posterior densities of all parameters and to graph posterior densities for several quantities of interest, such as inputs shares and elasticities. Note again that the compelling advantages of Bayesian inference, including being able to draw finite-sample inferences on various functions of parameters: firm efficiencies, relative efficiency, efficiency score rankings, the returns to scale function, and the probability that one firm is more efficient than another.

5.4.1 No Control for SO₂ Emissions

Using the 139 coal fueled power plant in 1996 data without sulfur dioxide included, we estimate the parameter posterior moments, inputs shares and elasticities, plant-level efficiencies and returns to scale in both the unconstrained and constrained model. Our results show large improvements of the constrained model over the unconstrained model.

Table 5.1 (A) and (B) present the posterior moments for the unconstrained and constrained model parameters estimated using the 1996 coal sample. This table also presents the posterior moments for the frontier parameters λ and σ^2 . The results produce very similar estimates for mean inefficiency (λ), 12.49% for the unconstrained model and 12.14% for the constrained model. In other words, 1996 electric generation plants using coal as resource exhibited average inefficiency of roughly 12%, or were 88% efficient. From the posterior standard deviations, we can see a slight improvement in precision for estimates of efficiency (the standard deviation of λ decreases from 0.0188 to 0.0186) and more dramatic increases in precision for other model parameters.

Table 5.1: Posterior Moments for Model Parameters, n=139**(A) Unconstrained**

Parameter	Mean	Std. Dev.	5th percentile	95th percentile
a_0	10.0971	2.1030	6.6216	13.5262
a_1	0.6058	1.0147	-1.0468	2.2688
a_2	0.7341	0.6400	-0.3331	1.7549
a_{11}	-0.1129	0.2772	-0.5685	0.3383
a_{22}	0.1114	0.1846	-0.2095	0.4054
a_{12}	-0.0041	0.1780	-0.2891	0.2959
b_1	-0.0694	0.1117	-0.2449	0.1205
γ_1	0.0316	0.0040	0.0248	0.0378
σ^2	0.0079	0.0022	0.0048	0.0119
λ	0.1249	0.0188	0.0948	0.1562

(B) Constrained

Parameter	Mean	Std. Dev.	5th percentile	95th percentile
a_0	10.7843	0.8104	9.4116	12.0676
a_1	0.2424	0.1551	0.0163	0.5177
a_2	0.7242	0.1456	0.4650	0.9396
a_{11}	-0.0226	0.0417	-0.0941	0.0413
a_{22}	-0.0127	0.0488	-0.0968	0.0628
a_{12}	0.0082	0.0401	-0.0533	0.0768
b_1	-0.0573	0.1066	-0.2320	0.1210
γ_1	0.0313	0.0038	0.0248	0.0373
σ^2	0.0082	0.0023	0.0050	0.0125
λ	0.1214	0.0186	0.0915	0.1531

Note: Posterior moments are computed based on 10,000 points generated from the Gibbs sampling algorithm. The first 1000 points are dropped to avoid sensitivity to starting values. The endpoints of the 90% confidence region are the 5th and 95th percentiles of the marginal densities.

Table 5.2: Shares and Elasticities, n=139
(A) Unconstrained

Shares	Posterior Mean	Posterior Std. Dev.	5th percentile	95th percentile
s_L	0.1638	0.0463	0.0882	0.2389
s_F	0.7692	0.0372	0.7075	0.8299
s_K	0.0670	0.0391	0.0023	0.1312
Elasticities	Posterior Mean	Posterior Std. Dev.	5th percentile	95th percentile
ϵ_{LL}	-1.6570	2.3373	-5.1005	1.3341
ϵ_{FF}	-0.0870	0.2465	-0.5188	0.2971
ϵ_{KK}	-3.1816	198.2453	-6.1157	6.5459
ϵ_{LF}	0.7196	1.4367	-1.3200	2.7255
ϵ_{LK}	0.9374	1.5935	-0.9258	3.3517
ϵ_{FL}	0.1598	0.2407	-0.2192	0.5686
ϵ_{FK}	-0.0728	0.1785	-0.3623	0.2279
ϵ_{KL}	8.1528	489.7012	-4.9313	9.3162
ϵ_{KF}	-4.9712	304.6732	-9.1219	4.1662

(Table 5.2 continued)

(B) Constrained

Shares	Posterior Mean	Posterior Std. Dev.	5th percentile	95th percentile
s_L	0.1582	0.0396	0.0935	0.2236
s_F	0.7502	0.0357	0.6914	0.8080
s_K	0.0916	0.0290	0.0459	0.1417
Elasticities	Posterior Mean	Posterior Std. Dev.	5th percentile	95th percentile
ε_{LL}	-0.9931	0.2790	-1.4632	-0.5792
ε_{FF}	-0.2670	0.0783	-0.4087	-0.1545
ε_{KK}	-1.0985	0.3475	-1.6327	-0.5028
ε_{LF}	0.7930	0.2527	0.3923	1.1950
ε_{LK}	0.2001	0.2346	-0.0935	0.6399
ε_{FL}	0.1693	0.0729	0.0645	0.3027
ε_{FK}	0.0977	0.0484	0.0333	0.1885
ε_{KL}	0.3007	0.2870	-0.1740	0.7636
ε_{KF}	0.7978	0.2878	0.3651	1.3125

Note: This table presents the posterior mean for shares and elasticities calculated at the mean value of all prices and output. Posterior moments are computed based on 10,000 points generated from the Gibbs sampling algorithm. The first 1000 points are dropped to avoid sensitivity to starting values. The endpoints of the 90% confidence region are the 5th and 95th percentiles of the marginal densities. s_L : share of labor, s_F : share of coal, s_K : share of capital. ε_{LL} : own price elasticity of labor, ε_{FF} : own price elasticity of coal, ε_{KK} : own price elasticity of capital. ε_{LF} : cross price elasticity of labor given coal, ε_{LK} : cross price elasticity of labor given capital, ε_{FL} : cross price elasticity of fuel given labor, ε_{FK} : cross price elasticity of coal given capital, ε_{KL} : cross price elasticity of capital given labor, ε_{KF} : cross price elasticity of capital given coal.

Table 5.2 (A) and (B) present the posterior means for shares and price elasticities for the unconstrained and constrained model, respectively, evaluated at the means for prices and output in the 1996 data set. First, focus on the posterior moments for the shares of inputs. In the unconstrained model, it is obvious that the largest predicted expenditure share is that of fuel (coal) with 76.92% of total expenditures, while 16.38% expenditures go to labor and 6.7% go to capital. By imposing monotonicity and concavity, we found similar estimates for shares. Panel (B) shows the constrained model. The largest predicted expenditure share is also that of fuel (coal) with 75.02% of total expenditures, while 15.82% expenditure goes to labor and 9.16% goes to capital. The constrained model exhibits a slight increase of capital share and slight decreases of shares of labor and fuel. Generally speaking, for the posterior moments for the shares of inputs, the constrained model has smaller posterior standard deviations and narrower confidence regions. The 90% highest density regions for shares estimates from both the constrained and unconstrained model are plausible, containing no negative values and no values greater than one. But the 90% highest density regions for shares estimates from the constrained model are narrower than those in the unconstrained model.

The large improvements of the constrained model over the unconstrained model are best shown from estimates of price elasticities. In both the constrained and unconstrained model, the posterior means for own price elasticities are all negative. This is plausible because according to the law of demand, we expect that electricity generation plants will decrease their demand for inputs in response to an increase in input prices. A closer look at the posterior standard deviation and 90% highest density regions show the drawbacks of the unconstrained model. The own price elasticity of labor (ε_{LL}) posterior standard deviation shrinks from 2.3373 in the unconstrained model to 0.279 in the constrained model; the own price elasticity of fuel (ε_{FF}) posterior standard deviation shrinks from 0.2465 in the unconstrained model to 0.0783 in the constrained model; and

the own price elasticity of capital (ε_{KK}) posterior standard deviation shrinks largely from 198.3453 in the unconstrained model to 0.3475 in the constrained model.

Moreover, the confidence regions for own price elasticities in the unconstrained model are very wide and all 95th percentiles are positive numbers, which contradicts the law of demand. By contrast, after imposing monotonicity and concavity restrictions, all own price elasticities' confidence intervals are much narrower and in the negative range. For example, the 90% highest density region for ε_{KK} shrinks from [-6.1157, 6.5459] in the unconstrained model to [-1.6327, -0.5028] in the constrained model. Obviously, the constrained model produces point estimates and highest density regions more consistent with economics theory.

To explain the intuition behind the improvement in precision of imposing regularity conditions, we also generate the marginal density plots for the input shares and own-price elasticities from both the unconstrained and the constrained models, again evaluated at mean price. Figure 5.4 graphs the density plot for the unconstrained model. The plot for input shares shows that labor share and capital share may be negative, which is economically implausible. Regarding the own price elasticity, the histograms for three inputs show that own price elasticity for labor, fuel (coal), and capital can be positive, suggesting that firms may increase the investment of inputs after the rise of input prices. This contradicts economic theory, in particular, law of demand. The histograms are graphed for own price elasticities to avoid smoothing away extreme values.

The marginal density plots for the input shares and own-price elasticities from the constrained model are presented in Figure 5.5. In contrast to the unconstrained model, the posterior densities place no mass on economically implausible frontiers. Shares for labor, fuel and capital are all constrained to be positive and less than one. These histograms show the effect of the constraints. All own-price elasticities are now negative, implying that the firm will respond to higher prices by reducing the usage of inputs.

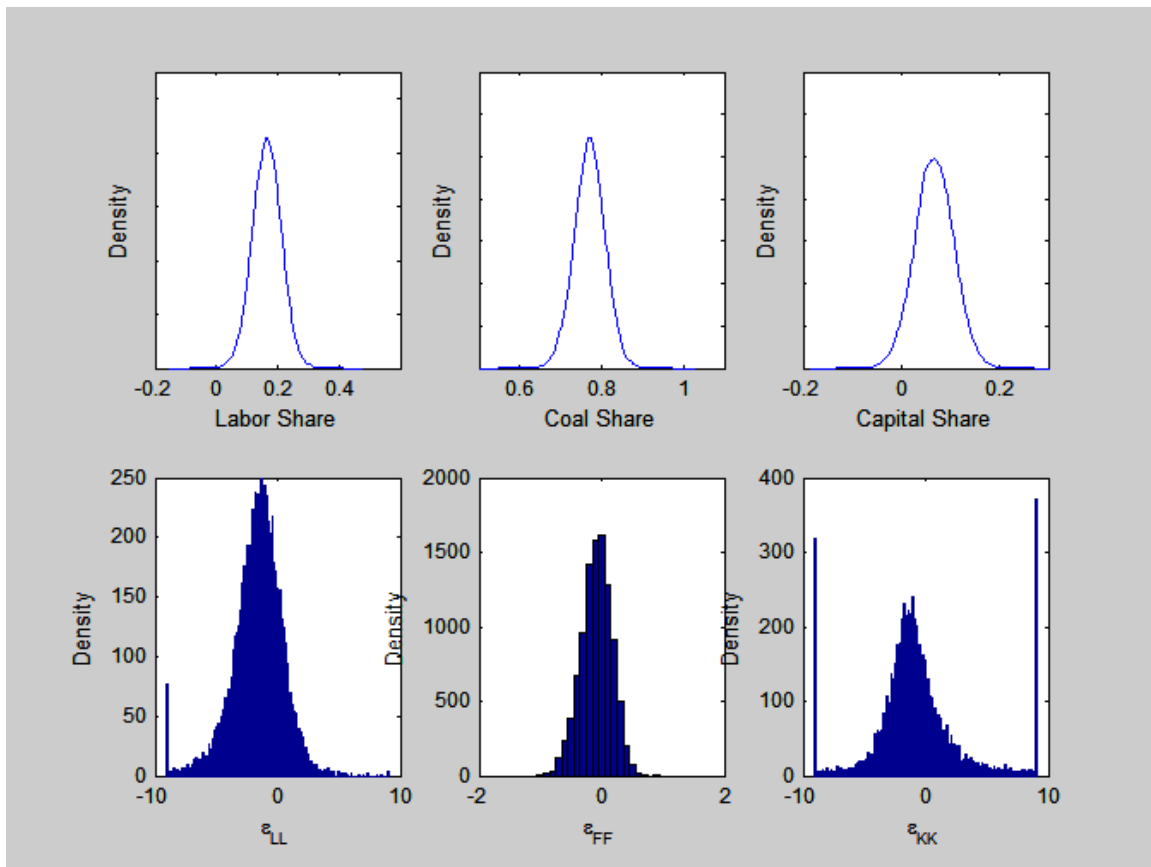


Figure 5.4 - Marginal Density Plots for Shares and Elasticities- Unconstrained model, with Sulfur Dioxide (SO₂) Excluded

In addition, Figure 5.5 shows that some densities are asymmetric, for example, ε_{FF} and ε_{LL} . Kleit and Terrell (2001) found similar results and they suggested, “perhaps reflecting that fact that the constrained posterior density ‘slices away’ the portion of the unconstrained posterior density that violates monotonicity and concavity” (p. 528). This slicing away of mass is especially clear in a comparison of ε_{LL} and ε_{KK} , where substantial mass associated with positive own price elasticities from Figure 5.4 is eliminated in Figure 5.5 by imposing concavity. The asymmetries further imply that we should not estimate this model and make any inference based on normal distribution.

Figure 5.4 also explains the very large posterior standard deviations¹⁹ for elasticities in the unconstrained model. The marginal density plots for the capital share and labor share in Figure 5.4

¹⁹ In Table 5.2 (A) unconstrained model, standard deviation for ε_{KK} is 198.2453, and standard deviation for ε_{LL} is 2.3373.

show a significant mass near zero. The mass near zero turns into very extreme values of elasticities, because the own price elasticity computation involves division by the shares. For example, own price elasticity of labor is computed as $\varepsilon_{LL} = \frac{a_{11}}{s_L} + s_L - 1$. These extreme values are all grouped into the last single bar as shown in the histogram for own-price elasticities. For example, for both ε_{KK} and ε_{LL} , the histograms show significant posterior mass associated with extreme values, which leads to large posterior standard deviations.

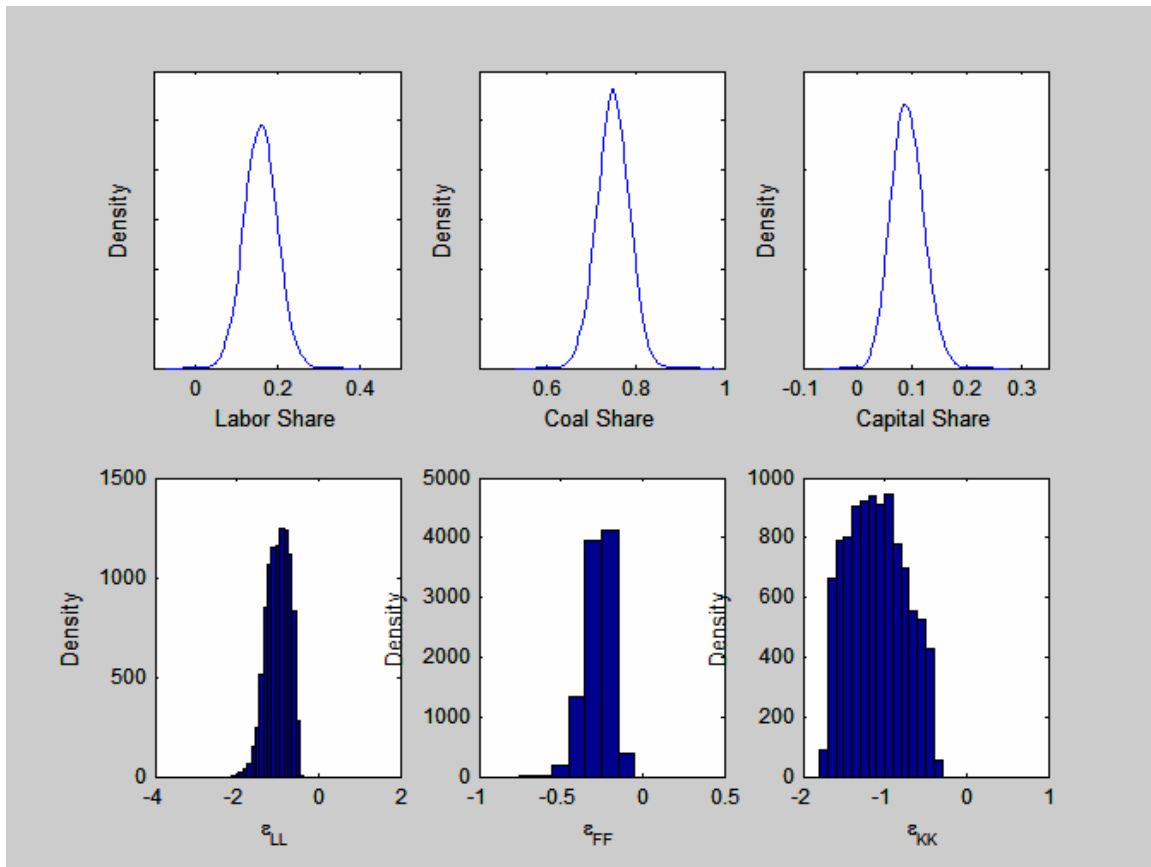


Figure 5.5 - Marginal Density Plots for Shares and Elasticities- Constrained model, with Sulfur Dioxide (SO₂) Excluded

Table 5.3 presents the returns to scale results in unconstrained model for 139 firms in our sample. All plants in the entire data set of 139 exhibit increasing returns to scale and the confidence region for 3 of 139 contain returns to scale equal to one.

Table 5.3: Plant Returns to Scale–Unconstrained, No Sulfur Dioxide, 139 Firms Using Coal

Plant	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
RS NELSON	1.1119	0.0035	1.0992	1.1495
ALBRIGHT	1.2534	0.0171	1.2423	1.2864
ALLEN	1.1038	0.0045	1.0908	1.1424
AM WILLIAMS	1.1223	0.0021	1.11	1.1587
ARAPAHOE	1.2223	0.0122	1.2119	1.2536
ARKWRIGH	1.408	0.0447	1.3866	1.4713
ARMSTRONG	1.1782	0.0056	1.1677	1.2107
ASHTABULA	1.2	0.0088	1.1896	1.2313
AVON LAKE	1.1207	0.0023	1.1083	1.1573
BAILLY	1.1563	0.0025	1.1453	1.1899
BARRY	1.0479	0.0113	1.0324	1.0946
BC COBB	1.1841	0.0065	1.1736	1.2162
BEEBEE	1.3423	0.0323	1.3265	1.3889
BELLE RIVER	1.0648	0.0093	1.0501	1.1092
BLACK DOG	1.225	0.0126	1.2145	1.2564
BOARDMAN (OR)	1.1937	0.0079	1.1833	1.2255
BRANDON SHORES	1.0633	0.0095	1.0486	1.1078
BRAYTON POINT	1.0663	0.0092	1.0517	1.1104
BREMO BLUFF	1.2405	0.0151	1.2297	1.2725
BRUCE MANSFIELD	1.0291	0.0134	1.0126	1.0787
BRUNNER ISLAND	1.0745	0.0082	1.0603	1.1174
CAMEO	1.3254	0.0293	1.3107	1.369
CANE RUN	1.157	0.0026	1.146	1.1905
CHALK POINT	1.1149	0.0031	1.1023	1.1522
CHEROKEE (CO)	1.1125	0.0034	1.0999	1.1501
CHESAPEAKE	1.1358	0.0003	1.1241	1.1705
CHESTERFIELD	1.0734	0.0083	1.0592	1.1164
CHOLLA	1.1053	0.0043	1.0924	1.1437
CLIFFSIDE	1.1331	0.0007	1.1213	1.1682
COMANCHE (CO)	1.117	0.0028	1.1045	1.1542
CONEMAUGH	1.0457	0.0116	1.03	1.0927
COUNCIL BLUFFS	1.1084	0.0039	1.0956	1.1464
CP CRANE	1.181	0.006	1.1704	1.2133
CR HUNTLEY	1.1312	0.0009	1.1194	1.1665

(Table 5.3 continued)

CRAWFORD	1.1812	0.006	1.1707	1.2136
CRIST	1.1349	0.0004	1.1232	1.1698
CROMBY	1.228	0.0131	1.2175	1.2595
CRYSTAL RIVER 1&2	1.1014	0.0048	1.0883	1.1403
CRYSTAL RIVER 4&5	1.1014	0.0048	1.0883	1.1403
DAN RIVER	1.2582	0.0179	1.2469	1.2918
DH MITCHELL	1.2036	0.0094	1.1933	1.2349
DICKERSON	1.1399	0.0002	1.1283	1.1743
DUNKIRK	1.1341	0.0006	1.1224	1.1691
EDDYSTONE	1.1168	0.0028	1.1044	1.154
EDWARDSPORT	1.395	0.0422	1.3748	1.4547
EW STOUT	1.1474	0.0013	1.1361	1.1813
FISK	1.2555	0.0175	1.2443	1.2888
FLINT CREEK (AR)	1.1431	0.0007	1.1316	1.1773
FOUR CORNERS	1.0351	0.0128	1.019	1.0837
GADSDEN NEW	1.3281	0.0298	1.3132	1.3721
GIBSON	1.0199	0.0144	1.003	1.0705
GORGAS TWO	1.0622	0.0096	1.0474	1.107
GREEN RIVER	1.28	0.0215	1.268	1.3161
HA WAGNER	1.1424	0.0006	1.131	1.1766
HAMMOND	1.1589	0.0029	1.148	1.1923
HARRINGTON	1.0705	0.0087	1.0561	1.114
HARRISON	1.0339	0.0129	1.0176	1.0826
HATFIELDS FERRY	1.061	0.0098	1.0461	1.1059
HAWTHORN	1.1661	0.0039	1.1553	1.199
HICKLING	1.4369	0.0504	1.4124	1.508
HIGH BRIDGE	1.239	0.0148	1.2283	1.271
HOLTWOOD	1.3171	0.0278	1.303	1.3591
HOMER CITY	1.0396	0.0123	1.0236	1.0876
HT PRITCHARD	1.2768	0.0209	1.265	1.3127
HUNTINGTON	1.0868	0.0067	1.0731	1.1278
IATAN	1.1084	0.0039	1.0956	1.1464
JACK WATSON	1.1077	0.004	1.0948	1.1459
JH CAMPBELL	1.0656	0.0092	1.051	1.1099
JIM BRIDGER	1.0329	0.013	1.0165	1.0817

(Table 5.3 continued)

JM STUART	1.0323	0.0131	1.016	1.0812
JOLIET	1.0909	0.0062	1.0773	1.1312
JR WHITING	1.1905	0.0074	1.1801	1.2225
KANAWHA RIVER	1.1845	0.0065	1.174	1.2165
KEYSTONE (PA)	1.0387	0.0124	1.0226	1.0867
KILLEN	1.1173	0.0028	1.1048	1.1545
KINCAID	1.1385	0	1.1269	1.173
KINTIGH	1.1145	0.0032	1.102	1.1518
KRAFT	1.3162	0.0277	1.3022	1.358
LANSING SMITH	1.2542	0.0172	1.243	1.2874
LEE (SC)	1.2347	0.0141	1.2241	1.2664
LIMESTON	1.0483	0.0113	1.0327	1.0949
LOUISA	1.1206	0.0024	1.1083	1.1573
MARTIN LAKE	1.0216	0.0143	1.0048	1.072
MARYSVILLE	1.6267	0.0926	1.5795	1.7663
MAYO	1.141	0.0004	1.1295	1.1754
MCMEEKIN	1.2071	0.0099	1.1967	1.2383
MERAMEC	1.2019	0.0091	1.1915	1.2331
MERRIMACK	1.1569	0.0026	1.1459	1.1904
MIAMI FORT	1.0753	0.0081	1.0611	1.1181
MICHIGAN CITY	1.1595	0.0029	1.1486	1.1929
MILL CREEK (KY)	1.0638	0.0095	1.0491	1.1083
MILLER	1.0095	0.0156	0.992	1.0619
MINNESOTA VALLEY	1.9636	0.1856	1.8606	2.2716
MOHAVE	1.0569	0.0103	1.0419	1.1023
MONTROSE	1.163	0.0034	1.1521	1.1961
MOUNT TOM	1.2497	0.0165	1.2386	1.2824
MUSKOGEE	1.0518	0.0109	1.0365	1.098
NAUGHTON	1.1044	0.0045	1.0914	1.1428
NAVAJO	1.032	0.0131	1.0157	1.081
NILES (OH)	1.2276	0.013	1.2171	1.2591
NOBLESVILLE	1.4331	0.0497	1.4091	1.5033
NORTHEASTERN 3&4	1.478	0.0589	1.4491	1.5621
OTTUMWA	1.1245	0.0018	1.1123	1.1607
PAWNEE	1.1483	0.0014	1.137	1.1821

(Table 5.3 continued)

PICWAY	1.3416	0.0322	1.3258	1.3881
PIRKEY	1.1061	0.0042	1.0932	1.1445
PORTLAND (PA)	1.1961	0.0082	1.1857	1.2278
POSSUM POINT	1.1956	0.0082	1.1853	1.2274
POWERTON	1.0813	0.0073	1.0674	1.1231
RE BURGER	1.1774	0.0055	1.1669	1.2099
RIVER ROUGE	1.1372	0.0001	1.1255	1.1717
RIVERBEND (NC)	1.1936	0.0079	1.1832	1.2254
RIVESVILLE	1.4997	0.0636	1.4684	1.5914
ROXBORO	1.0364	0.0126	1.0203	1.0848
RP SMITH	1.3934	0.0419	1.3733	1.4526
SALEM HARBOR	1.1478	0.0013	1.1365	1.1816
SAN JUAN (NM)	1.0472	0.0114	1.0317	1.094
SCHILLER	1.2819	0.0218	1.2698	1.3183
SEWARD	1.2257	0.0127	1.2152	1.2571
SHAWVILLE	1.133	0.0007	1.1213	1.1682
SIOUX	1.1256	0.0017	1.1134	1.1617
ST JOHNS RIVER	1.0565	0.0103	1.0415	1.102
STATE LINE	1.1878	0.007	1.1774	1.2199
TANNERS CREEK	1.1064	0.0042	1.0935	1.1448
TECUMSEH (KS)	1.2448	0.0157	1.234	1.2772
TITUS	1.228	0.0131	1.2174	1.2595
TOLK	1.0799	0.0075	1.066	1.1221
TRENTON CHANNEL	1.1225	0.0021	1.1101	1.1588
TYRONE (KY)	1.5126	0.0663	1.4797	1.6085
VALMONT	1.2328	0.0138	1.2221	1.2644
WABASH RIVER	1.1423	0.0005	1.1308	1.1765
WAUKEGAN	1.141	0.0004	1.1295	1.1754
WC BECKJORD	1.089	0.0064	1.0755	1.1296
WH SAMMIS	1.0324	0.0131	1.016	1.0813
WHITE BLUFF	1.0523	0.0108	1.037	1.0984
WILL COUNTY	1.117	0.0028	1.1045	1.1542
WOOD RIVER (IL)	1.1697	0.0044	1.159	1.2027
WYODAK	1.1508	0.0017	1.1396	1.1845
YATES	1.1378	0.0001	1.1262	1.1723

Table 5.4: Plant Returns to Scale – Constrained, No Sulfur Dioxide, 139 Firms Using Coal

Plant	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
RS NELSON	1.1138	0.044	1.1013	1.1504
ALBRIGHT	1.2544	0.0088	1.2436	1.2865
ALLEN	1.1058	0.0458	1.0929	1.1434
AM WILLIAMS	1.1242	0.0416	1.112	1.1597
ARAPAHOE	1.2236	0.0171	1.2132	1.2537
ARKWRIGH	1.4076	0.0372	1.3871	1.4706
ARMSTRONG	1.1797	0.0284	1.1691	1.2105
ASHTABULA	1.2014	0.0229	1.191	1.2312
AVON LAKE	1.1226	0.042	1.1104	1.1583
BAILLY	1.158	0.0337	1.147	1.1904
BARRY	1.0502	0.0575	1.0348	1.0956
BC COBB	1.1856	0.0269	1.1749	1.2159
BEEBEE	1.3425	0.0167	1.3271	1.3883
BELLE RIVER	1.067	0.0541	1.0525	1.1101
BLACK DOG	1.2262	0.0164	1.2158	1.2565
BOARDMAN (OR)	1.1952	0.0245	1.1847	1.2252
BRANDON SHORES	1.0656	0.0544	1.0509	1.1089
BRAYTON POINT	1.0685	0.0538	1.054	1.1114
BREMO BLUFF	1.2416	0.0123	1.2311	1.2727
BRUCE MANSFIELD	1.0315	0.0612	1.0154	1.0793
BRUNNER ISLAND	1.0767	0.0521	1.0626	1.1185
CAMEO	1.3257	0.0116	1.3114	1.3682
CANE RUN	1.1587	0.0335	1.1477	1.1909
CHALK POINT	1.1168	0.0433	1.1044	1.153
CHEROKEE (CO)	1.1145	0.0438	1.102	1.1509
CHESAPEAKE	1.1376	0.0385	1.1259	1.1718
CHESTERFIELD	1.0756	0.0523	1.0614	1.1176
CHOLLA	1.1073	0.0454	1.0945	1.1446
CLIFFSIDE	1.1349	0.0392	1.1231	1.1693
COMANCHE (CO)	1.1189	0.0428	1.1066	1.1549
CONEMAUGH	1.048	0.058	1.0325	1.0937
COUNCIL BLUFFS	1.1104	0.0448	1.0977	1.1473
CP CRANE	1.1825	0.0277	1.1718	1.213
CR HUNTLEY	1.1331	0.0396	1.1212	1.1676

(Table 5.4 continued)

CRAWFORD	1.1827	0.0276	1.1721	1.2132
CRIST	1.1367	0.0387	1.125	1.171
CROMBY	1.2292	0.0156	1.2188	1.2596
CRYSTAL RIVER 1&2	1.1034	0.0463	1.0905	1.1414
CRYSTAL RIVER 4&5	1.1034	0.0463	1.0905	1.1414
DAN RIVER	1.2592	0.0075	1.2481	1.2917
DH MITCHELL	1.205	0.022	1.1946	1.2347
DICKERSON	1.1417	0.0376	1.1301	1.1752
DUNKIRK	1.1359	0.0389	1.1242	1.1703
EDDYSTONE	1.1188	0.0429	1.1064	1.1547
EDWARDSPORT	1.3947	0.033	1.3754	1.4543
EW STOUT	1.1492	0.0358	1.1378	1.182
FISK	1.2565	0.0082	1.2455	1.2887
FLINT CREEK (AR)	1.1448	0.0368	1.1334	1.1782
FOUR CORNERS	1.0375	0.06	1.0216	1.0844
GADSDEN NEW	1.3284	0.0124	1.3139	1.3713
GIBSON	1.0223	0.063	1.0057	1.0713
GORGAS TWO	1.0644	0.0546	1.0497	1.1079
GREEN RIVER	1.2808	0.0014	1.269	1.3156
HA WAGNER	1.1441	0.037	1.1327	1.1776
HAMMOND	1.1606	0.0331	1.1496	1.1926
HARRINGTON	1.0727	0.0529	1.0584	1.1151
HARRISON	1.0362	0.0603	1.0203	1.0833
HATFIELDS FERRY	1.0632	0.0549	1.0484	1.1068
HAWTHORN	1.1677	0.0313	1.1568	1.1992
HICKLING	1.4361	0.0466	1.413	1.5074
HIGH BRIDGE	1.2401	0.0127	1.2296	1.2711
HOLTWOOD	1.3175	0.0092	1.3038	1.3584
HOMER CITY	1.0419	0.0592	1.0262	1.0883
HT PRITCHARD	1.2776	0.0024	1.2659	1.312
HUNTINGTON	1.0889	0.0495	1.0754	1.1289
IATAN	1.1104	0.0448	1.0977	1.1473
JACK WATSON	1.1096	0.0449	1.0969	1.1467
JH CAMPBELL	1.0678	0.0539	1.0533	1.1108
JIM BRIDGER	1.0352	0.0605	1.0193	1.0825

(Table 5.4 continued)

JM STUART	1.0347	0.0606	1.0187	1.082
JOLIET	1.0929	0.0486	1.0796	1.1323
JR WHITING	1.1919	0.0253	1.1814	1.2222
KANAWHA RIVER	1.186	0.0268	1.1753	1.2162
KEYSTONE (PA)	1.041	0.0594	1.0253	1.0875
KILLEN	1.1192	0.0428	1.1069	1.1552
KINCAID	1.1403	0.0379	1.1287	1.1741
KINTIGH	1.1165	0.0434	1.1041	1.1527
KRAFT	1.3166	0.0089	1.3029	1.3573
LANSING SMITH	1.2552	0.0086	1.2443	1.2874
LEE (SC)	1.2359	0.0138	1.2254	1.2668
LIMESTON	1.0506	0.0574	1.0352	1.096
LOUISA	1.1225	0.042	1.1103	1.1582
MARTIN LAKE	1.024	0.0626	1.0075	1.0727
MARYSVILLE	1.6237	0.1144	1.5791	1.7616
MAYO	1.1428	0.0373	1.1313	1.1762
MCMEEKIN	1.2084	0.0211	1.198	1.2383
MERAMEC	1.2032	0.0224	1.1929	1.233
MERRIMACK	1.1585	0.0336	1.1475	1.1908
MIAMI FORT	1.0774	0.0519	1.0634	1.1191
MICHIGAN CITY	1.1611	0.0329	1.1502	1.1933
MILL CREEK (KY)	1.066	0.0543	1.0514	1.1092
MILLER	1.0119	0.0649	0.9949	1.0619
MINNESOTA VALLEY	1.9548	0.2573	1.858	2.2595
MOHAVE	1.0592	0.0557	1.0443	1.1035
MONTROSE	1.1647	0.0321	1.1538	1.1965
MOUNT TOM	1.2507	0.0098	1.24	1.2825
MUSKOGEE	1.0541	0.0567	1.0389	1.0991
NAUGHTON	1.1063	0.0457	1.0935	1.1438
NAVAJO	1.0344	0.0606	1.0184	1.0817
NILES (OH)	1.2288	0.0157	1.2184	1.2592
NOBLESVILLE	1.4324	0.0453	1.4096	1.5023
NORTHEASTERN 3&4	1.4768	0.0604	1.4497	1.5613
OTTUMWA	1.1264	0.0411	1.1143	1.1616
PAWNEE	1.15	0.0356	1.1387	1.1828

(Table 5.4 continued)

PICWAY	1.3418	0.0165	1.3265	1.3874
PIRKEY	1.1081	0.0453	1.0953	1.1453
PORTLAND (PA)	1.1975	0.0239	1.187	1.2274
POSSUM POINT	1.1971	0.024	1.1866	1.227
POWERTON	1.0835	0.0506	1.0697	1.1243
RE BURGER	1.1789	0.0286	1.1683	1.2098
RIVER ROUGE	1.139	0.0382	1.1273	1.173
RIVERBEND (NC)	1.195	0.0245	1.1846	1.2251
RIVESVILLE	1.4983	0.0679	1.469	1.5894
ROXBORO	1.0387	0.0598	1.0229	1.0856
RP SMITH	1.3931	0.0325	1.374	1.4522
SALEM HARBOR	1.1495	0.0357	1.1382	1.1823
SAN JUAN (NM)	1.0495	0.0577	1.0341	1.095
SCHILLER	1.2827	0.0009	1.2708	1.3178
SEWARD	1.2269	0.0162	1.2165	1.2572
SHAWVILLE	1.1349	0.0392	1.1231	1.1693
SIOUX	1.1274	0.0409	1.1154	1.1625
ST JOHNS RIVER	1.0588	0.0558	1.0438	1.1031
STATE LINE	1.1893	0.026	1.1787	1.2196
TANNERS CREEK	1.1084	0.0452	1.0956	1.1456
TECUMSEH (KS)	1.2459	0.0111	1.2352	1.2773
TITUS	1.2291	0.0156	1.2187	1.2596
TOLK	1.0821	0.0509	1.0682	1.123
TRENTON CHANNEL	1.1243	0.0416	1.1122	1.1598
TYRONE (KY)	1.511	0.0724	1.4803	1.6061
VALMONT	1.2339	0.0144	1.2235	1.2647
WABASH RIVER	1.144	0.037	1.1326	1.1774
WAUKEGAN	1.1428	0.0373	1.1313	1.1762
WC BECKJORD	1.0911	0.049	1.0777	1.1307
WH SAMMIS	1.0347	0.0606	1.0188	1.082
WHITE BLUFF	1.0546	0.0566	1.0394	1.0995
WILL COUNTY	1.1189	0.0428	1.1066	1.1549
WOOD RIVER (IL)	1.1713	0.0305	1.1606	1.2025
WYODAK	1.1525	0.035	1.1413	1.1852
YATES	1.1396	0.0381	1.128	1.1735

To compare with the results from unconstrained model, Table 5.4 presents the returns to scale results in the constrained model. All power plants in the entire data set of 139 exhibit increasing returns to scale and only one plant's confidence region contains returns to scale equal to one. For 139 firms in the entire sample, the average returns to scale is 1.1688 for the unconstrained model and 1.17 for the constrained model. These results demonstrate that imposing monotonicity and concavity restrictions only generate minor differences for individual firm's returns to scale.

5.4.2 Models Including SO₂ Emissions

As discussed earlier, a growing literature has studied productivity change and efficiency in industries when an undesirable output (a "bad") is a by-product in the production process. We apply our methodology to the 136 U.S. electric utilities in the year 1996. The number of plants in our sample dropped from 139 to 136 due to sulfur dioxide (SO₂) data restriction. The good output is the quantity of electric power generated, and the bad is SO₂ emissions. Three inputs measured are labor, capital, and coal. According to Atkinson and Dorfman (2005), "Title IV of the 1990 Clean Air Act Amendments reduced emissions of SO₂ from U.S. coal-burning electric utilities from about 19 million tons in 1980 to 8.95 million tons by the year 2000" (p. 457). The increased reduction of SO₂ emissions over time has likely had an important impact on the levels of technical efficiency for those power plants. Controlling the effects of bad will make our model's efficiency estimates more robust and efficient.

Atkinson and Dorfman (2005) utilize the Kim limited-information likelihood derived by minimizing the entropy distance subject to the moment conditions from the Generalized Method of Moments estimator. One of the chief shortcomings of the distance functions is their non-stochastic nature. Therefore, the calculated frontier and consequently the obtained efficiency scores can be contaminated due to the extent of the statistical noise in the data. Our methods are different from theirs. We impose both monotonicity and concavity on all inputs, output and the SO₂ emissions, while they only impose monotonicity restriction. They treat the bad as technology shifter of an

input distance function and model a system of nonlinear equations subject to endogeneity. But we treat the bad as one byproduct and apply Bayesian stochastic frontier model to estimate cost frontier and measure the inefficiency as the deviations from the frontier.

Table 5.5 (A) and (B) present the posterior moments for the unconstrained and constrained model parameters estimated. The model estimates power plants using coal as a fuel source in year 1996 with sulfur dioxide included as bad output.

Table 5.5 also presents the posterior moments for the frontier parameters λ and σ^2 . Similar to estimates in model excluding SO₂, these results produce very close estimates for mean inefficiency (λ), 11.58% for the unconstrained model, and 11.53% for the constrained model. Therefore in our sample, electric generation coal-fired plants exhibited an average inefficiency of roughly 11.5%, or were 88.5% efficient in 1996. If we do not control for SO₂ emissions, the mean inefficiency (λ) is 12.49% for the unconstrained model and 12.14% for the constrained model. Now, the electric generation plants exhibited average inefficiency of about 12%. Intuitively, in order to reduce bad by-product, power plants need to decrease production of desirable output – electricity, or increase usage of inputs. Therefore, by controlling the SO₂ emission in our model, we realized that power plants are not as inefficient as we believed, because initially we treat the costs to control pollution as inefficiency. The posterior mean inefficiency falls by about 1% after controlling the effect of SO₂. Moreover, the improvement of controlling SO₂ emission model can be shown from the standard deviation of λ , which decreases from 0.0185 to 0.0173 in constrained model.

We also estimate parameters by OLS. The results by OLS are similar to those from the Bayesian unconstrained model but with relatively larger standard errors and wider confidence intervals.

**Table 5.5: Posterior Moments for Model Parameters (1996 coal data)
- Sulfur Dioxide (SO₂) Included
(A) Unconstrained**

Parameter	Mean	Std. Dev.	5 th percentile	95 th percentile
a ₀	13.6401	2.2752	9.8773	17.3270
a ₁	-0.1900	1.0150	-1.8360	1.5020
a ₂	0.8148	0.6381	-0.2515	1.8508
a ₁₁	0.0837	0.2772	-0.3796	0.5324
a ₂₂	-0.0580	0.1972	-0.3899	0.2585
a ₁₂	-0.0044	0.1786	-0.2934	0.2953
b ₁	-0.1255	0.2030	-0.4566	0.2159
γ ₁	0.0281	0.0086	0.0137	0.0420
c ₁	-0.2993	0.1543	-0.5471	-0.0423
c ₁₁	0.0009	0.0050	-0.0076	0.0089
d ₁	0.0171	0.00946	0.0015	0.0327
σ ²	0.0077	0.0023	0.0046	0.0118
λ	0.1158	0.0185	0.0863	0.1469

(B) Constrained

Parameter	Mean	Std. Dev.	5 th percentile	95 th percentile
a ₀	12.9453	1.2862	10.7686	14.9795
a ₁	0.2159	0.1437	0.0062	0.4695
a ₂	0.7248	0.1306	0.4976	0.9258
a ₁₁	-0.0221	0.0377	-0.0869	0.0363
a ₂₂	-0.0234	0.0508	-0.1082	0.0576
a ₁₂	0.0102	0.0358	-0.0478	0.0693
b ₁	-0.1365	0.2005	-0.4604	0.2002
γ ₁	0.0284	0.0084	0.0143	0.0422
c ₁	-0.2987	0.1494	-0.5424	-0.0507
c ₁₁	0.0007	0.0050	-0.0080	0.0084
d ₁	0.0174	0.0092	0.0020	0.0325
σ ²	0.0074	0.0020	0.0046	0.0110
λ	0.1153	0.0173	0.0889	0.1453

Note: Posterior moments are computed based on 10,000 points generated from the Gibbs sampling algorithm. The endpoints of the 90% confidence region are the 5th and 95th percentiles of the marginal densities.

The parameters of translog frontier do not reveal much information and are difficult to interpret. Therefore, we look at the posterior mean for shares and elasticities to better understand the efficiency gains.

Table 5.6 (A) and (B) show the posterior mean for shares and price elasticities for unconstrained and constrained models, evaluated at the means for prices and output in the 1996 data set with sulfur dioxide included as bad output.

We focus on the posterior moments for the shares of inputs first. As shown in unconstrained model panel (A), the largest predicted expenditure share is that of fuel (coal) with 77.06% of total expenditures, then of labor with 13.53% and the least of capital with 9.41%. Table 5.6 (B) presents the constrained model. After imposing monotonicity and concavity, the largest predicted expenditure share is also that of fuel (coal) with 75.4% of total expenditures, then of labor with 13.44% and the least of capital with 11.16%. Comparing the constrained model to the unconstrained model, the share of labor is quite similar, the share of fuel falls by 1.66%, and the share of capital rises by 1.75%. Generally speaking, for the posterior moments for the shares of inputs, constrained models have smaller posterior standard deviation and narrower confidence interval. The 90% density regions for shares estimates from both constrained and unconstrained model are plausible, containing no negative values and no values greater than one.

Next, we compare the estimates of inputs shares between SO₂ controlled (Table 5.6 B) and no SO₂ controlled constrained model (Table 5.2 B). The largest predicted expenditure share is that of fuel (coal) with around 75% of total expenditures for both models. Also, the share of labor falls by 2.38% while the share of capital rises by 2% as we move from no SO₂ controlled constrained model to an SO₂ controlled constrained model. The increase of capital share may be explained by the fact that power plants need more capital investment once they were required to reduce the quantity of SO₂ emissions. Therefore, properly accounting for the reduction of this bad will lead to more precise predicted expenditure shares.

**Table 5.6: Shares and Elasticities (1996 coal data)-Sulfur Dioxide (SO₂) Included
(A) Unconstrained**

Shares	Posterior Mean	Posterior Std. Dev.	5th percentile	95th percentile
s_L	0.1353	0.0455	0.0608	0.2080
s_F	0.7706	0.0371	0.7101	0.8315
s_K	0.0941	0.0386	0.0313	0.1586
Elasticities	Posterior Mean	Posterior Std. Dev.	5th percentile	95th percentile
ϵ_{LL}	-0.3214	11.3461	-4.3297	3.7793
ϵ_{FF}	-0.3063	0.2649	-0.7587	0.1131
ϵ_{KK}	-0.0805	25.9276	-2.8844	3.6497
ϵ_{LF}	0.8536	13.5263	-1.8526	3.3961
ϵ_{LK}	-0.5322	4.7693	-2.9540	2.0816
ϵ_{FL}	0.1304	0.2383	-0.2553	0.5281
ϵ_{FK}	0.1759	0.1920	-0.1347	0.4950
ϵ_{KL}	-1.6332	31.4438	-6.5300	2.3826
ϵ_{KF}	1.7137	11.6880	-1.6063	4.8575

(Table 5.6 continued)

(B) Constrained

Shares	Posterior Mean	Posterior Std. Dev.	5th percentile	95th percentile
s_L	0.1344	0.0380	0.0723	0.1974
s_F	0.7540	0.0341	0.6969	0.8085
s_K	0.1116	0.0302	0.0636	0.1623
Elasticities	Posterior Mean	Posterior Std. Dev.	5th percentile	95th percentile
ε_{LL}	-1.0422	0.3005	-1.5567	-0.5960
ε_{FF}	-0.2775	0.0807	-0.4206	-0.1576
ε_{KK}	-1.0930	0.3433	-1.6100	-0.5025
ε_{LF}	0.8205	0.2667	0.3715	1.2272
ε_{LK}	0.2217	0.2940	-0.1551	0.7843
ε_{FL}	0.1482	0.0673	0.0491	0.2703
ε_{FK}	0.1293	0.0610	0.0470	0.2428
ε_{KL}	0.2301	0.2726	-0.1934	0.6984
ε_{KF}	0.8629	0.2963	0.3964	1.3662

Note: Table 5.6 (A) and (B) present the posterior mean for shares and elasticities calculated at the mean value of all prices and output. Posterior moments are computed based on 10,000 points generated from the Gibbs sampling algorithm. The first 1000 points are dropped to avoid sensitivity to starting values. The endpoints of the 90% confidence region are the 5th and 95th percentiles of the marginal densities.

s_L : share of labor, s_F : share of coal, s_K : share of capital. ε_{LL} : own price elasticity of labor, ε_{FF} : own price elasticity of coal, ε_{KK} : own price elasticity of capital. ε_{LF} : cross price elasticity of labor given coal, ε_{LK} : cross price elasticity of labor given capital, ε_{FL} : cross price elasticity of fuel given labor, ε_{FK} : cross price elasticity of coal given capital, ε_{KL} : cross price elasticity of capital given labor, ε_{KF} : cross price elasticity of capital given coal.

(Table 5.6 continued)

(C) Using OLS and Delta Method

Shares	Mean	Std. Dev.	5th percentile	95th percentile
S_L	0.1399	0.0527	0.0532	0.2267
S_F	0.7741	0.0473	0.6963	0.8519
S_K	0.0860	0.0449	0.0122	0.1598
Elasticities	Mean	Std. Dev.	5th percentile	95th percentile
ϵ_{LL}	-0.7300	1.9574	-3.9503	2.4903
ϵ_{FF}	-0.3807	0.2759	-0.8345	0.0732
ϵ_{KK}	-0.9698	14.0682	-24.1148	22.1751
ϵ_{LF}	1.1200	6.6489	-9.8188	12.0588
ϵ_{LK}	-0.3900	1.3815	-2.6628	1.8828
ϵ_{FL}	0.2024	0.2818	-0.2612	0.6661
ϵ_{FK}	0.1782	0.1978	-0.1472	0.5036
ϵ_{KL}	-0.6347	14.3001	-24.1611	22.8918
ϵ_{KF}	1.6045	14.0620	-21.5303	24.7394

From the results of the SO₂ controlled model, we also notice that the constrained model has a smaller posterior standard deviation and a narrower confidence region in terms of posterior moments for the shares of inputs. There are no negative values and no values greater than one contained in the 90% density regions for shares estimates from both the constrained and unconstrained model.

The estimates of price elasticities reveal the large improvements of the constrained model over the unconstrained model. According to the law of demand, we expect that electric generation plants will decrease their demand for inputs in response to an increase in input prices. Therefore, the negative point estimates of own price elasticities²⁰ in both the constrained and unconstrained model are all plausible. A closer look at the posterior standard deviation and 90% highest density regions shows the drawbacks of the unconstrained model. The own price elasticity of labor (ε_{LL}) posterior standard deviation falls from 11.3461 in the unconstrained model to 0.3 in the constrained model; the own price elasticity of fuel (ε_{FF}) posterior standard deviation falls from 0.2649 in the unconstrained model to 0.0807 in the constrained model; and the own price elasticity of capital (ε_{KK}) posterior standard deviation falls largely from 25.9276 in the unconstrained model to 0.3433 in the constrained model.

Moreover, similar to what we observed in the previous model when SO₂ was excluded, the confidence regions for own price elasticities in the unconstrained model are very wide and all 95th percentiles are positive numbers, which contradicts the law of demand, while all own price elasticities' confidence regions are much narrower and in the negative range, after imposing monotonicity and concavity restrictions. For example, the 90% highest density region for ε_{LL} falls from [-4.3297, 3.7793] in the unconstrained model to [-1.5567, -0.596] in the constrained model and the 90% highest density region for ε_{KK} falls from [-2.8844, 3.6497] in the unconstrained model

²⁰ See ε_{LL} , ε_{FF} and ε_{KK} in Table 5.6.

to $[-1.61, -0.5025]$ in the constrained model. We conclude that the constrained model produces point estimates and highest density regions more plausible to economics theory.

Table 5.6 (C) represents shares and elasticities estimated using the OLS and Delta methods. The results are also similar to results estimated by the Bayesian unconstrained model.

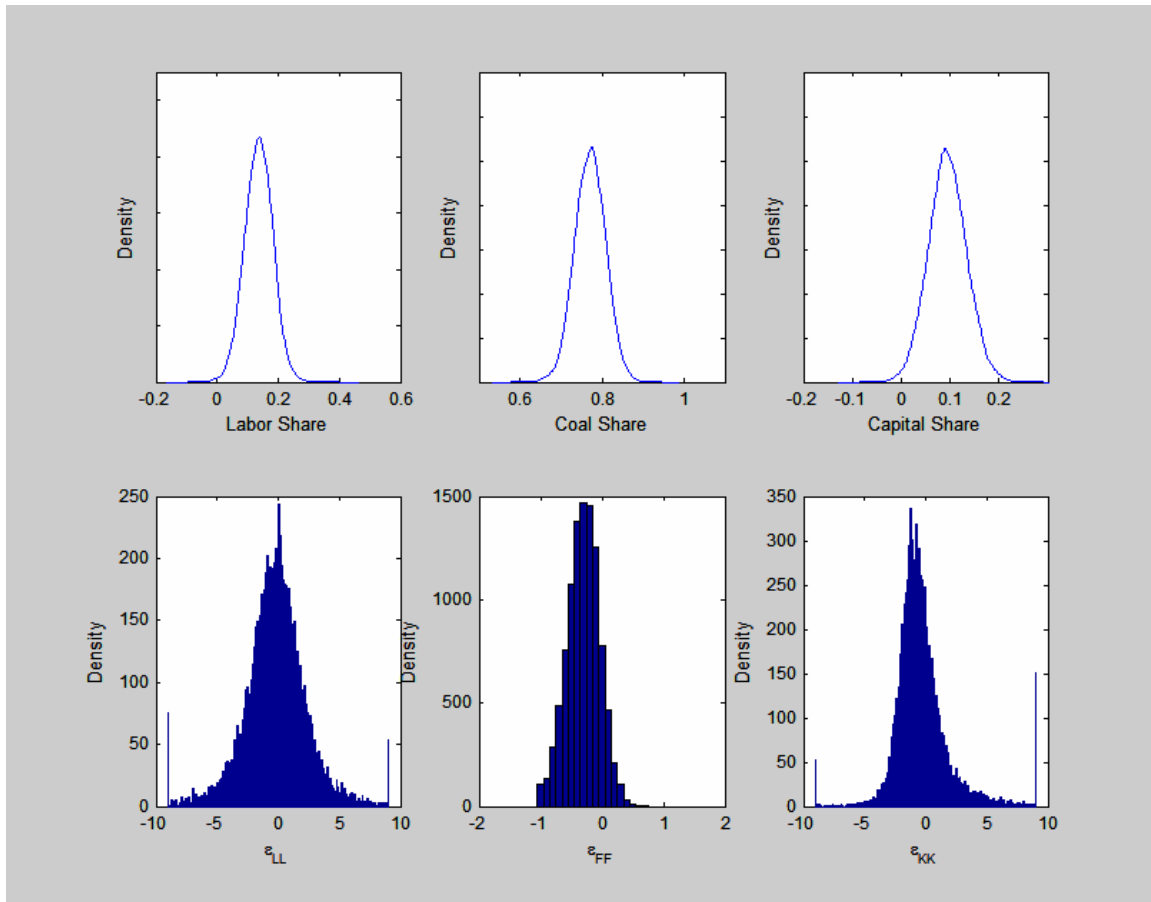


Figure 5.6 - Marginal Density Plots for Shares and Elasticities- Unconstrained model, Sulfur Dioxide (SO₂) Included

To show the improvement in precision of imposing regularity conditions again, in the model including SO₂, we generate the marginal density plots for the input shares and own-price elasticities from both the unconstrained and the constrained models, evaluated at mean price. Figure 5.6 graphs the density plot for the unconstrained model. The plot for input shares shows that labor share and capital share may be negative, which is economically implausible. Another drawback is

shown from the own price elasticity. Regarding the own price elasticity, the histograms for three inputs show that own price elasticity for labor, fuel (coal) and capital can be positive, suggesting that firms may increase the investment of inputs after the rise of input prices. This contradicts economic theory and, in particular, law of demand.

Figure 5.7 presents the marginal density plots for the input shares and own-price elasticities from the constrained model. Just as for the previous model with SO₂ excluded, the constrained posterior densities place no mass on economically implausible frontiers. Notice that the shares for labor, fuel, and capital are all constrained to be positive and less than one. These histograms also show that all own-price elasticities are now negative, suggesting that firms will reduce their usage of inputs in response to a rise in the input's price.

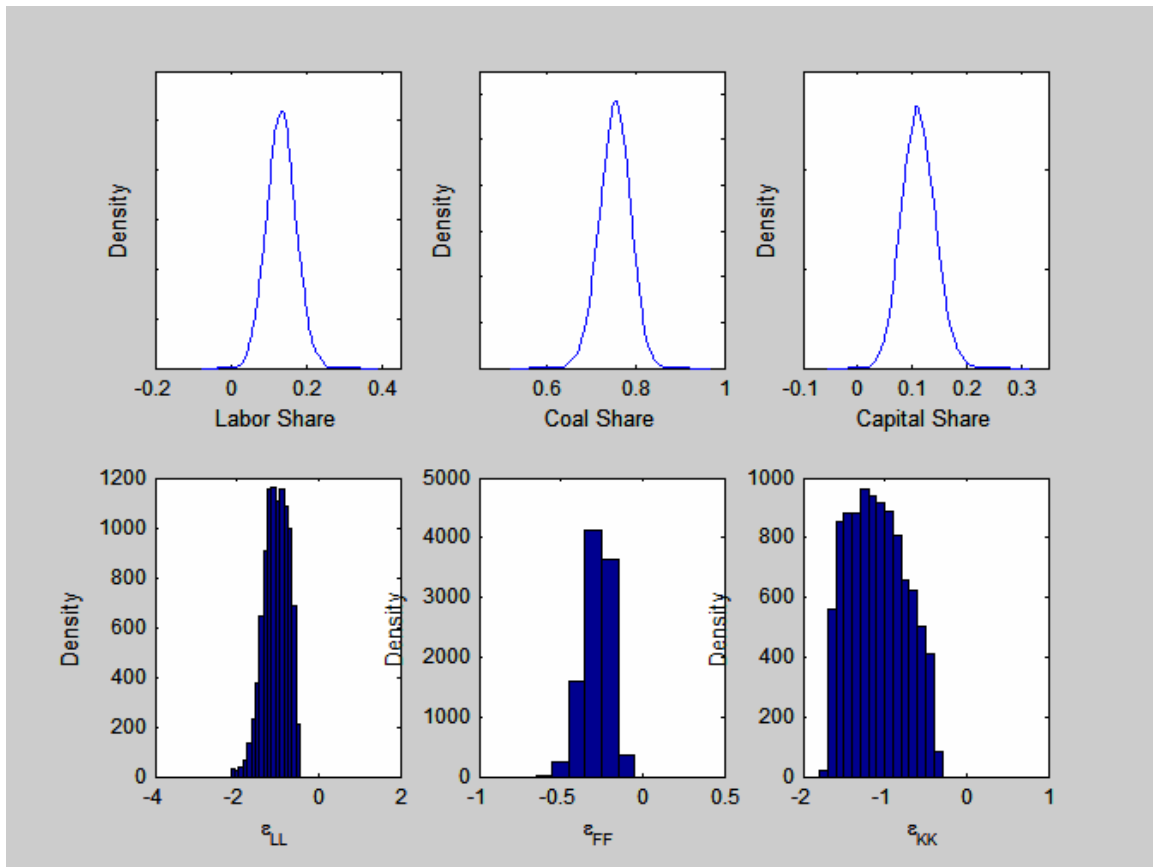


Figure 5.7 - Marginal Density Plots for Shares and Elasticities - Constrained model, Sulfur Dioxide (SO₂) Included

The densities of ε_{FF} and ε_{LL} are shown to be asymmetric as what we observed in previous model excluding SO₂. In addition, comparison of densities of ε_{LL} and ε_{KK} across two figures indicates that the substantial mass associated with positive own price elasticities from Figure 5.6 is eliminated in Figure 5.7 by imposing concavity.

Also Figure 5.6 explains the very large posterior standard deviations²¹ for elasticities in the unconstrained model.

Table 5.7 contains posterior moments for efficiency score that summarize how efficiently a firm produces given its current level of output for both unconstrained and constrained model. The plant average of posterior efficiency is 0.898 for the unconstrained model and 0.899 for the constrained model. For example, the posterior mean for the Allen facility in the constrained model suggests that it is 90.98% efficiency, or equivalently, its costs could be reduced by 9.02% by eliminating all inefficiency. For 124 of the 136 plants, the posterior means in the constrained model indicate that they are more than 80% efficiency and 88 of the 136 plants are more than 90% efficient. Also, Figure 5.8 shows the histograms for the posterior of λ^{-1} (the inverse of power plants' inefficiency). We can also compare the posterior density plots with the prior density plots shown as Figure 5.1.

Table 5.8 summarizes the returns to scale results with control for pollution - tons of sulfur dioxide (SO₂) emissions for each power plant in year 1996. No monotonicity or concavity restriction is imposed in the results of this table. Compared the one-output model (no SO₂ included), the difference is once SO₂ emissions are included in the model, 4 out of all 136 plants in our entire sample exhibit decreasing returns to scale and the confidence region for 12 of 136 contains returns to scale equal to one. On the other hand, in the one-output model, all plants in the entire data set of 139 exhibit increasing returns to scale and the confidence region for 3 of 139 contain returns to

²¹ In Table 5.6 (A) unconstrained model, standard deviation for ε_{KK} is 25.9276, and standard deviation for ε_{LL} is 11.3461.

scale equal to one. Results of returns to scale in the constrained model with SO₂ emission included are shown in Table 5.9. Here we impose monotonicity and concavity on not only all inputs, but also the “bad” output. Imposing monotonicity and concavity restrictions generate similar results as in the unconstrained model.

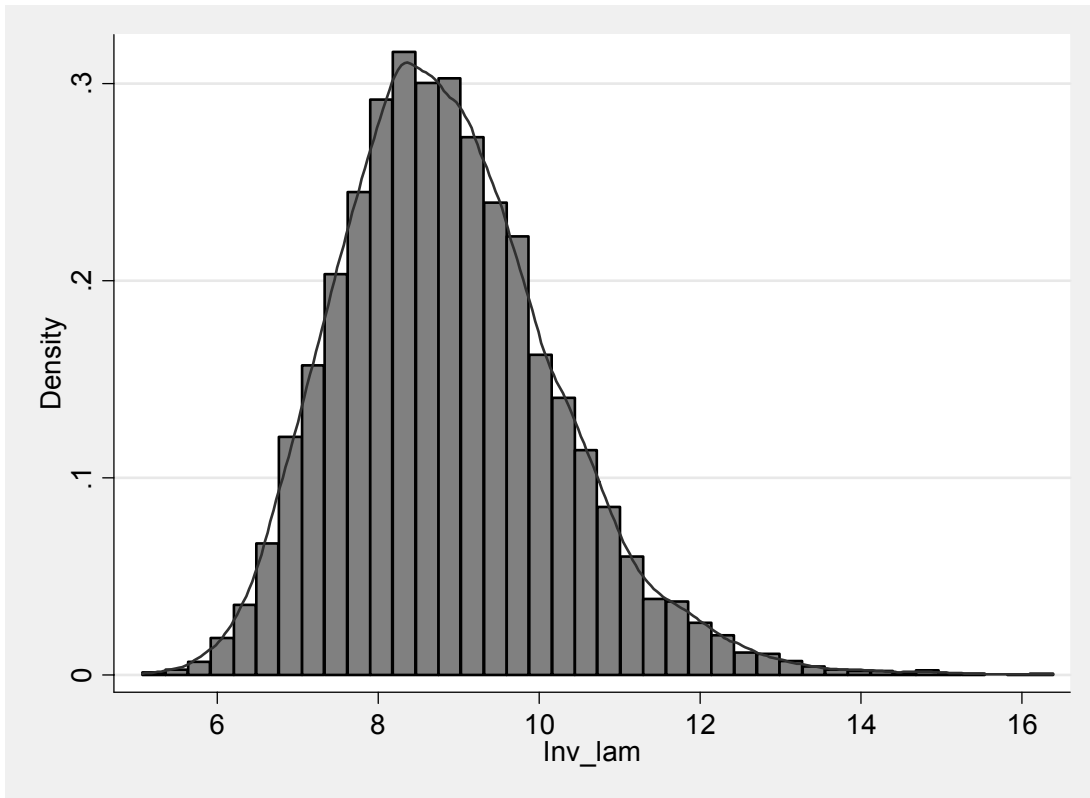


Figure 5.8 - Posterior Density of λ^{-1}

Table 5.7: Plant Efficiency (Coal), Sulfur Dioxide (SO₂) Included, 136 firms

Plant	Unconstrained				Constrained			
	Posterior Mean	Posterior S.D.	5th percentile	95th percentile	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
ALBRIGHT	0.7388	0.0742	0.6867	0.9032	0.7445	0.0717	0.6198	0.9058
ALLEN	0.9076	0.0584	0.8677	0.9943	0.9098	0.0571	0.7833	0.9948
AM WILLIAMS	0.9689	0.0277	0.9558	0.9991	0.9689	0.0273	0.901	0.9991
ARAPAHOE	0.9416	0.0444	0.9151	0.9981	0.9428	0.044	0.8362	0.9979
ARKWRIGH	0.6602	0.074	0.6078	0.8292	0.659	0.0696	0.5423	0.8144
ARMSTRONG	0.9533	0.0381	0.9327	0.9985	0.9547	0.0374	0.861	0.9984
ASHTABULA	0.7689	0.0772	0.7141	0.9385	0.7671	0.074	0.6354	0.9325
AVON LAKE	0.8933	0.0622	0.8501	0.9933	0.8935	0.0613	0.7653	0.9922
BAILLY	0.7936	0.0777	0.7379	0.9552	0.7962	0.0739	0.6636	0.9528
BARRY	0.9487	0.041	0.9255	0.9984	0.9507	0.0395	0.8529	0.9983
BC COBB	0.9567	0.0359	0.9371	0.9986	0.9592	0.0343	0.8727	0.9987
BELLE RIVER	0.9367	0.0481	0.9085	0.9977	0.9312	0.0504	0.8131	0.9972
BLACK DOG	0.8407	0.0745	0.7873	0.9832	0.8454	0.0708	0.714	0.9815
BOARDMAN (OR)	0.9037	0.0602	0.8638	0.9948	0.9023	0.0605	0.7725	0.9946
BRANDON SHORES	0.9482	0.0414	0.925	0.9983	0.9471	0.0417	0.8462	0.9981
BRAYTON POINT	0.9274	0.051	0.8943	0.9966	0.9287	0.0504	0.8137	0.9965
BREMO BLUFF	0.9105	0.0585	0.8706	0.9955	0.9077	0.0583	0.7818	0.9954
BRUCE MANSFIELD	0.8733	0.0682	0.8255	0.9899	0.8711	0.0675	0.7356	0.9891
BRUNNER ISLAND	0.9153	0.0562	0.8789	0.9958	0.9141	0.056	0.7904	0.9955
CAMEO	0.9663	0.0304	0.9523	0.9991	0.969	0.0281	0.8941	0.9991
CANE RUN	0.8096	0.0741	0.757	0.9651	0.8153	0.0721	0.6826	0.9651
CHALK POINT	0.7538	0.0722	0.7022	0.9145	0.7529	0.0704	0.6262	0.9073
CHEROKEE (CO)	0.9155	0.0561	0.8778	0.9962	0.9213	0.0532	0.8007	0.9964
CHESAPEAKE	0.9497	0.0402	0.9281	0.9981	0.9502	0.0396	0.8536	0.9984
CHESTERFIELD	0.9498	0.0401	0.9281	0.9982	0.9498	0.0405	0.8486	0.9983
CHOLLA	0.9332	0.0567	0.9039	0.9979	0.9356	0.0491	0.8169	0.9976
CLIFFSIDE	0.9429	0.045	0.917	0.9979	0.9415	0.0441	0.8373	0.9979
COMANCHE (CO)	0.9523	0.0395	0.931	0.9985	0.955	0.0369	0.8639	0.9985
CONEMAUGH	0.8977	0.0649	0.8532	0.9943	0.9015	0.0621	0.7726	0.9945

(Table 5.7. continued)

Plant	Unconstrained				Constrained			
	Posterior Mean	Posterior S.D.	5th percentile	95th percentile	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
CR HUNTLEY	0.9274	0.0514	0.8952	0.9972	0.9239	0.0527	0.8061	0.9963
COUNCIL BLUFFS	0.9178	0.0557	0.8803	0.9963	0.9238	0.0525	0.8032	0.9967
CRAWFORD	0.863	0.0717	0.8122	0.989	0.8616	0.0693	0.727	0.9859
CRIST	0.8626	0.0694	0.8129	0.9872	0.8587	0.069	0.7266	0.9851
CROMBY	0.8231	0.0754	0.7683	0.9738	0.8225	0.0722	0.6882	0.9688
CRYSTAL RIVER 1&2	0.95	0.0406	0.9272	0.9984	0.9491	0.0404	0.852	0.9983
CRYSTAL RIVER 4&5	0.9439	0.0436	0.9183	0.998	0.9429	0.0436	0.8381	0.9981
DAN RIVER	0.8966	0.0623	0.8536	0.9938	0.8954	0.0613	0.766	0.9927
DH MITCHELL	0.9111	0.058	0.872	0.9956	0.9127	0.0566	0.7894	0.9959
DICKERSON	0.9348	0.0477	0.9056	0.9974	0.9345	0.0473	0.8225	0.9972
DUNKIRK	0.947	0.0423	0.9241	0.9982	0.9441	0.0438	0.8372	0.998
EDDYSTONE	0.7031	0.0726	0.6529	0.8701	0.702	0.067	0.5852	0.8518
EDWARDSPORT	0.8626	0.0724	0.811	0.9875	0.8635	0.0712	0.7215	0.9873
EW STOUT	0.9425	0.0442	0.9172	0.9976	0.94	0.0458	0.8294	0.9979
FISK	0.9196	0.0563	0.8833	0.9967	0.9175	0.0561	0.791	0.9965
FLINT CREEK (AR)	0.9462	0.0421	0.9222	0.998	0.9457	0.0426	0.8429	0.9981
FOUR CORNERS	0.895	0.0625	0.852	0.9933	0.8975	0.0613	0.7673	0.9935
GADSDEN NEW	0.8663	0.0716	0.8149	0.9891	0.8632	0.0706	0.723	0.9872
GIBSON	0.9271	0.0513	0.8948	0.9967	0.9293	0.0507	0.8117	0.9971
GORGAS TWO	0.9279	0.0529	0.8958	0.9969	0.93	0.0521	0.8084	0.9976
GREEN RIVER	0.916	0.0574	0.8785	0.9962	0.925	0.0527	0.8044	0.9966
HA WAGNER	0.8811	0.0648	0.8353	0.9908	0.8819	0.0646	0.7516	0.9908
HAMMOND	0.7228	0.0721	0.6731	0.8829	0.7224	0.0694	0.6012	0.8741
HARRINGTON	0.9528	0.039	0.9324	0.9984	0.9539	0.0377	0.8599	0.9986
HARRISON	0.9521	0.0399	0.931	0.9984	0.9549	0.038	0.8591	0.9986
HATFIELDS FERRY	0.9375	0.0468	0.9093	0.9977	0.9396	0.046	0.8312	0.9977
HAWTHORN	0.8989	0.0658	0.8548	0.9948	0.9173	0.0558	0.7913	0.996
HIGH BRIDGE	0.9402	0.0489	0.9133	0.9981	0.9495	0.0408	0.8492	0.9983
HOMER CITY	0.9319	0.0492	0.9007	0.9972	0.9345	0.0483	0.8216	0.9973
HOLTWOOD	0.8103	0.0765	0.755	0.969	0.8104	0.0738	0.6771	0.9641
HT PRITCHARD	0.8411	0.073	0.7878	0.9809	0.8375	0.0716	0.7029	0.9787

(Table 5.7. continued)

Plant	Unconstrained				Constrained			
	Posterior Mean	Posterior S.D.	5th percentile	95th percentile	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
HUNTINGTON	0.9343	0.0526	0.905	0.9976	0.9378	0.0473	0.8249	0.9974
IATAN	0.9595	0.0347	0.9419	0.9988	0.9616	0.0332	0.8772	0.9988
JACK WATSON	0.8893	0.0643	0.8452	0.9922	0.8885	0.0643	0.754	0.9913
JH CAMPBELL	0.9652	0.031	0.9508	0.999	0.9678	0.0287	0.8942	0.9991
JIM BRIDGER	0.8919	0.0654	0.8467	0.9937	0.89	0.0632	0.7563	0.9924
JM STUART	0.9368	0.0478	0.9091	0.9975	0.9373	0.0471	0.8292	0.9976
JOLIET	0.8603	0.0695	0.8116	0.985	0.8582	0.0692	0.7226	0.9855
JR WHITING	0.9557	0.0368	0.9362	0.9985	0.954	0.0378	0.8605	0.9985
KANAWHA RIVER	0.9697	0.0271	0.9567	0.9991	0.9703	0.0267	0.9016	0.9991
KEYSTONE (PA)	0.9487	0.0413	0.9256	0.9984	0.9506	0.0405	0.8504	0.9982
KILLEN	0.9477	0.0413	0.9239	0.9981	0.9496	0.0405	0.8504	0.9983
KINCAID	0.8957	0.0615	0.8534	0.9931	0.8973	0.0606	0.7707	0.9933
KINTIGH	0.9503	0.0398	0.9282	0.9983	0.9495	0.041	0.8493	0.9984
KRAFT	0.8134	0.0757	0.7594	0.9695	0.8107	0.0737	0.679	0.964
LANSING SMITH	0.9508	0.0415	0.9292	0.9985	0.9507	0.0402	0.8501	0.9983
LEE (SC)	0.9448	0.0424	0.9199	0.9978	0.9449	0.0427	0.8413	0.998
LIMESTON	0.8457	0.0725	0.7941	0.9817	0.847	0.0701	0.7128	0.9801
LOUISA	0.9699	0.0276	0.9569	0.9992	0.9702	0.0266	0.9009	0.9991
MARTIN LAKE	0.8985	0.0628	0.8555	0.9944	0.9053	0.0591	0.7753	0.9946
MARYSVILLE	0.8756	0.0839	0.822	0.994	0.8644	0.0858	0.6752	0.993
MAYO	0.9246	0.0531	0.8916	0.997	0.9223	0.053	0.8016	0.9961
MCMEEKIN	0.9745	0.0238	0.9637	0.9993	0.975	0.0237	0.912	0.9993
MERAMEC	0.5649	0.0593	0.5236	0.6991	0.5657	0.0547	0.4725	0.6908
MERRIMACK	0.6219	0.0789	0.5665	0.7963	0.6015	0.0603	0.4978	0.7349
MIAMI FORT	0.9438	0.0435	0.9183	0.9979	0.9469	0.0422	0.8438	0.9983
MICHIGAN CITY	0.9272	0.0508	0.8943	0.9973	0.927	0.0509	0.811	0.9966
MILL CREEK	0.8891	0.0629	0.8461	0.9926	0.8925	0.0619	0.764	0.9921
MILLER	0.9513	0.0402	0.9298	0.9983	0.9494	0.0415	0.8461	0.9984
MOHAVE	0.8936	0.0631	0.8495	0.9938	0.8984	0.061	0.7678	0.9935
MONTROSE	0.859	0.0706	0.808	0.986	0.8577	0.0682	0.7274	0.9844
NOBLESVILLE	0.9577	0.0362	0.9391	0.9987	0.9597	0.0347	0.8715	0.9987
NORTHEASTERN 3&4	0.9659	0.0318	0.952	0.999	0.9651	0.0327	0.8814	0.9991
OTTUMWA	0.9535	0.0381	0.9327	0.9984	0.9553	0.0368	0.8639	0.9983

(Table 5.7. continued)

Plant	Unconstrained				Constrained			
	Posterior Mean	Posterior S.D.	5th percentile	95th percentile	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
PAWNEE	0.9343	0.0485	0.9047	0.9975	0.9379	0.0467	0.8278	0.9976
PICWAY	0.9303	0.0508	0.899	0.9973	0.932	0.0494	0.8166	0.9973
MUSKOGEE	0.9433	0.0442	0.9172	0.9978	0.9476	0.0413	0.8471	0.9981
NAUGHTON	0.908	0.0579	0.8692	0.9949	0.9099	0.057	0.7883	0.995
NAVAJO	0.8838	0.0647	0.8386	0.9916	0.8876	0.0636	0.7551	0.9919
NILES (OH)	0.9189	0.0554	0.8824	0.9963	0.9263	0.0516	0.8062	0.9968
PIRKEY	0.9315	0.0491	0.9008	0.997	0.9335	0.0483	0.8203	0.9972
PORTLAND (PA)	0.927	0.0537	0.8941	0.9971	0.9243	0.053	0.8048	0.9967
POSSUM POINT	0.9491	0.0404	0.9269	0.998	0.9489	0.0405	0.8502	0.9983
POWERTON	0.8929	0.0862	0.8445	0.9963	0.89	0.0895	0.6646	0.9966
RE BURGER	0.9147	0.0594	0.8767	0.9963	0.923	0.0541	0.8008	0.9963
RIVER ROUGE	0.9328	0.0491	0.9031	0.9973	0.9336	0.0482	0.8207	0.9973
RIVERBEND (NC)	0.9318	0.0495	0.9015	0.997	0.9317	0.0492	0.8177	0.9971
RIVESVILLE	0.8869	0.0706	0.8385	0.9939	0.8871	0.0697	0.738	0.9937
ROXBORO	0.9433	0.0441	0.9175	0.9982	0.9447	0.0429	0.8392	0.9979
RP SMITH	0.9616	0.0335	0.9446	0.9988	0.9613	0.0342	0.872	0.999
RS NELSON	0.7489	0.0736	0.6965	0.9136	0.7469	0.0711	0.6228	0.9027
SALEM HARBOR	0.8831	0.0657	0.837	0.992	0.8817	0.0648	0.751	0.9905
SAN JUAN (NM)	0.8294	0.072	0.7778	0.9745	0.8276	0.0717	0.692	0.9713
SCHILLER	0.6024	0.064	0.5582	0.7489	0.6058	0.0607	0.5032	0.7423
SEWARD	0.9304	0.0512	0.898	0.9973	0.934	0.048	0.8229	0.9971
SHAWVILLE	0.8899	0.0642	0.846	0.9928	0.8985	0.0611	0.7687	0.9935
SIOUX	0.876	0.0669	0.8276	0.9908	0.8764	0.0666	0.7428	0.9889
ST JOHNS RIVER	0.8949	0.0629	0.852	0.993	0.8934	0.0626	0.761	0.9935
STATE LINE	0.902	0.0608	0.8602	0.9949	0.901	0.0608	0.769	0.9943
TANNERS CREEK	0.9458	0.0425	0.9207	0.9979	0.9474	0.0415	0.8467	0.998
TECUMSEH (KS)	0.9414	0.0452	0.9153	0.9979	0.9385	0.0465	0.8276	0.9975
TITUS	0.9411	0.0452	0.9143	0.998	0.942	0.0437	0.8391	0.9977
TOLK	0.9346	0.0498	0.905	0.9974	0.9359	0.0474	0.8241	0.9976
WC BECKJORD	0.9416	0.049	0.9162	0.998	0.9452	0.0458	0.8302	0.9981
WH SAMMIS	0.9434	0.0443	0.9189	0.9979	0.946	0.0426	0.8426	0.9981
WHITE BLUFF	0.9271	0.051	0.895	0.9968	0.928	0.0506	0.8105	0.9968
WILL COUNTY	0.8875	0.0683	0.8406	0.9932	0.8876	0.0645	0.7526	0.992
TYRONE (KY)	0.8639	0.077	0.8107	0.9897	0.8656	0.0761	0.7054	0.9898

(Table 5.7. continued)

Plant	Unconstrained				Constrained			
	Posterior Mean	Posterior S.D.	5th percentile	95th percentile	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
WOOD RIVER (IL)	0.9666	0.0294	0.9527	0.9989	0.9676	0.0287	0.893	0.9991
TRENTON CHANNEL	0.9096	0.0587	0.8706	0.9952	0.9036	0.0596	0.7748	0.9942
VALMONT	0.9722	0.0254	0.9605	0.9992	0.972	0.0259	0.9042	0.9993
WABASH RIVER	0.852	0.0715	0.8015	0.9846	0.8571	0.0687	0.7223	0.9837
WAUKEGAN	0.922	0.0531	0.8874	0.9961	0.923	0.0523	0.804	0.9965
WYODAK	0.865	0.0725	0.8133	0.9895	0.8602	0.0698	0.7255	0.9853
YATES	0.6907	0.07	0.6422	0.85	0.6899	0.0661	0.5772	0.8361
Median	0.926				0.925			
Mean	0.898				0.899			

Note: posterior moments are computed based on 10,000 points generated from the Gibbs sampling algorithm. The endpoints of the 90% confidence region are the 5th and 95th percentiles of the marginal densities.

**Table 5.8: Plant Returns to Scale-Unconstrained Coal Data
- Sulfur Dioxide (SO₂) Included, 136 firms**

Plant	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
RS NELSON	1.132	0.0038	1.0915	1.1761
ALBRIGHT	1.3128	0.0085	1.2636	1.3601
ALLEN	1.1156	0.0055	1.076	1.1587
AM WILLIAMS	1.1331	0.0148	1.0974	1.1713
ARAPAHOE	1.3007	0.0401	1.2433	1.3543
ARKWRIGH	1.5327	0.0161	1.404	1.6646
ARMSTRONG	1.1924	0.0352	1.1556	1.2329
ASHTABULA	1.1984	0.0674	1.141	1.2635
AVON LAKE	1.124	0.025	1.0878	1.1636
BAILLY	1.2249	0.0611	1.1579	1.2946
BARRY	1.013	0.0513	0.959	1.0763
BC COBB	1.2265	0.0039	1.1917	1.2618
BELLE RIVER	1.0627	0.0109	1.0163	1.1137
BLACK DOG	1.3239	0.0734	1.2469	1.4
BOARDMAN (OR)	1.2624	0.0408	1.2086	1.3159
BRANDON SHORES	1.0479	0.0297	1.001	1.0987
BRAYTON POINT	1.0547	0.0253	1.0086	1.1037
BREMO BLUFF	1.3091	0.0134	1.2596	1.3543
BRUCE MANSFIELD	1.0105	0.0223	0.9583	1.068
BRUNNER ISLAND	1.0486	0.0483	0.9996	1.1052
CAMEO	1.4826	0.1046	1.3603	1.5976
CANE RUN	1.191	0.0043	1.1559	1.2276
CHALK POINT	1.111	0.033	1.072	1.1536
CHEROKEE (CO)	1.131	0.001	1.0914	1.1739
CHESAPEAKE	1.1433	0.0257	1.1083	1.1811
CHESTERFIELD	1.0531	0.0402	1.0056	1.1059
CHOLLA	1.1298	0.0135	1.0843	1.1803
CLIFFSIDE	1.1416	0.0228	1.107	1.1791
COMANCHE (CO)	1.1425	0.0099	1.1002	1.1882
CONEMAUGH	1.0704	0.0386	1.0019	1.1485
COUNCIL BLUFFS	1.1207	0.0066	1.0822	1.1627
CP CRANE	1.1987	0.0314	1.1632	1.237
CR HUNTLEY	1.1237	0.0449	1.0814	1.1704
CRAWFORD	1.2352	0.0237	1.1918	1.2785
CRIST	1.1404	0.0281	1.1049	1.1789
CROMBY	1.3067	0.0374	1.2496	1.3593
CRYSTAL RIVER 1&2	1.0964	0.0292	1.0564	1.14
CRYSTAL RIVER 4&5	1.0889	0.0399	1.0459	1.1364
DAN RIVER	1.3191	0.009	1.2683	1.368
DH MITCHELL	1.2666	0.0258	1.22	1.3109
DICKERSON	1.1467	0.0283	1.1115	1.1849
DUNKIRK	1.1279	0.0444	1.0859	1.1741
EDDYSTONE	1.1535	0.0277	1.1028	1.208

(Table 5.8 continued)

Plant	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
EW STOUT	1.1515	0.0355	1.1147	1.1923
FISK	1.3637	0.0695	1.2836	1.4391
FLINT CREEK (AR)	1.1783	0.0131	1.1386	1.2206
FOUR CORNERS	1.0232	0.0146	0.9717	1.0809
GADSDEN NEW	1.4183	0.0091	1.3346	1.4995
GIBSON	0.9852	0.0419	0.9299	1.049
GORGAS TWO	1.0139	0.0735	0.9507	1.0893
GREEN RIVER	1.3343	0.0319	1.2731	1.3969
HA WAGNER	1.1624	0.0099	1.1294	1.1984
HAMMOND	1.1723	0.0274	1.1386	1.2093
HARRINGTON	1.0651	0.0178	1.0202	1.1138
HARRISON	1.0424	0.0169	0.9818	1.1116
HATFIELDS FERRY	1.0236	0.0588	0.9684	1.089
HAWTHORN	1.2147	0.023	1.1715	1.2588
HICKLING	1.6531	0.1099	1.474	1.8289
HIGH BRIDGE	1.3348	0.059	1.2638	1.4013
HOLTWOOD	1.3948	0.0207	1.3179	1.4714
HOMER CITY	1.0031	0.0507	0.9482	1.0671
HT PRITCHARD	1.3549	0.0062	1.2938	1.4107
HUNTINGTON	1.1058	0.0127	1.0573	1.16
IATAN	1.123	0.0031	1.0839	1.1655
JACK WATSON	1.0786	0.0646	1.0243	1.1416
JH CAMPBELL	1.0453	0.0374	0.9973	1.0983
JIM BRIDGER	1.0288	0.0021	0.9749	1.0903
JM STUART	1.0009	0.0414	0.9476	1.0618
JOLIET	1.0998	0.0045	1.0575	1.1461
JR WHITING	1.2344	0.0029	1.1994	1.2692
KANAWHA RIVER	1.2241	0.0006	1.1907	1.2584
KEYSTONE (PA)	0.9978	0.0561	0.9409	1.0653
KILLEN	1.1218	0.022	1.0855	1.1613
KINCAID	1.1573	0.0101	1.1238	1.1939
KINTIGH	1.1375	0.0071	1.0963	1.1825
KRAFT	1.4046	0.0038	1.3271	1.4785
LANSING	1.344	0.0393	1.279	1.4021
LEE (SC)	1.3002	0.0119	1.2527	1.3438
LIMESTON	1.0384	0.0162	0.9892	1.0925
LOUISA	1.1396	0.0018	1.1021	1.1802
MARTIN LAKE	0.9876	0.0417	0.9326	1.0509
MARYSVILLE	2.0654	0.296	1.6769	2.5074
MERAMEC	1.2385	0.0141	1.205	1.2736
MERRIMACK	1.1652	0.0338	1.1294	1.2048
MIAMI FORT	1.0554	0.0402	1.0082	1.108
MICHIGAN CITY	1.1906	0.0012	1.1575	1.2256
MAYO	1.1616	0.0086	1.1283	1.1979
MCMEEKIN	1.2488	0.0093	1.2141	1.2841

(Table 5.8 continued)

Plant	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
MILL CREEK (KY)	1.0487	0.0294	1.0019	1.0993
MILLER	0.98	0.0314	0.9241	1.0424
MOHAVE	1.0459	0.021	0.9986	1.0972
MONTROSE	1.2111	0.0236	1.1672	1.2558
MUSKOGEE	1.0422	0.0172	0.9937	1.0952
NAUGHTON	1.1154	0.0069	1.0761	1.1581
NAVAJO	1.0089	0.0297	0.9569	1.0661
NILES (OH)	1.2624	0.0305	1.2195	1.3078
NOBLESVILLE	1.5873	0.0088	1.4392	1.7363
NORTHEASTERN 3&4	1.5943	0.0848	1.415	1.8003
OTTUMWA	1.1436	0.0034	1.107	1.1829
PAWNEE	1.1834	0.0104	1.145	1.2239
PICWAY	1.4209	0.0347	1.3317	1.5152
PIRKEY	1.1073	0.0221	1.069	1.1486
PORTLAND (PA)	1.2199	0.0301	1.1836	1.2586
POSSUM POINT	1.2414	0.003	1.206	1.2767
POWERTON	1.2338	0.2774	1.0383	1.48
RE BURGER	1.174	0.0593	1.124	1.2308
RIVER ROUGE	1.1633	0.0017	1.1275	1.202
RIVERBEND (NC)	1.2108	0.0384	1.1713	1.2541
RIVESVILLE	1.7728	0.1459	1.5382	2.0131
ROXBORO	1.0114	0.0339	0.9597	1.0687
RP SMITH	1.5754	0.0904	1.4291	1.7151
SALEM HARBOR	1.164	0.0181	1.1313	1.1995
SAN JUAN (NM)	1.0352	0.019	0.986	1.0891
SCHILLER	1.3728	0.0231	1.306	1.4336
SEWARD	1.2659	0.0216	1.2258	1.3071
SHAWVILLE	1.1265	0.0443	1.0846	1.1728
SIOUX	1.1169	0.0442	1.0746	1.1635
ST JOHNS RIVER	1.0574	0.0032	1.008	1.1128
STATE LINE	1.2562	0.0437	1.2006	1.312
TANNERS CREEK	1.0898	0.0474	1.0445	1.1408
TECUMSEH (KS)	1.3369	0.0491	1.2697	1.3982
TITUS	1.2732	0.0152	1.2327	1.3137
TOLK	1.0829	0.0091	1.0391	1.1302
TRENTON CHANNEL	1.131	0.0183	1.0956	1.1695
WH SAMMIS	0.9925	0.0528	0.936	1.0594
WHITE BLUFF	1.0343	0.0294	0.9857	1.0873
WILL COUNTY	1.1409	0.0074	1.0999	1.1859
WOOD RIVER (IL)	1.2051	0.0003	1.1715	1.2398
WYODAK	1.1906	0.0165	1.1497	1.2336
YATES	1.1431	0.0297	1.1073	1.182
TYRONE (KY)	1.8044	0.1658	1.554	2.0645
VALMONT	1.3183	0.0457	1.2555	1.376

(Table 5.8 continued)

Plant	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
WABASH RIVER	1.1365	0.0471	1.0936	1.1839
WAUKEGAN	1.1747	0.0116	1.1353	1.2168
WC BECKJORD	1.1849	0.1484	1.0585	1.3313
Median	1.15			
Mean	1.196			

**Table 5.9: Plant Returns to Scale-Constrained Coal Data
- Sulfur Dioxide (SO₂) Included, 136 firms**

Plant	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
ALBRIGHT	1.3122	0.0288	1.2634	1.3581
ALLEN	1.1133	0.0174	1.0764	1.155
AM WILLIAMS	1.131	0.0149	1.0983	1.168
ARAPAHOE	1.2993	0.0463	1.244	1.3522
ARKWRIGH	1.5347	0.0417	1.4062	1.6676
ARMSTRONG	1.1911	0.0104	1.1562	1.2285
ASHTABULA	1.1976	0.0026	1.1419	1.2594
AVON LAKE	1.122	0.0103	1.0885	1.1594
BAILLY	1.2224	0.0485	1.1593	1.2904
BARRY	1.0106	0.0063	0.9593	1.0693
BC COBB	1.2249	0.0278	1.1913	1.2585
BELLE RIVER	1.06	0.0123	1.0159	1.1096
BLACK DOG	1.3222	0.0599	1.2471	1.3971
BOARDMAN (OR)	1.2606	0.0439	1.2083	1.3122
BRANDON SHORES	1.0455	0.0042	1.0022	1.095
BRAYTON POINT	1.0522	0.0063	1.0099	1.1007
BREMO BLUFF	1.3082	0.037	1.2589	1.352
BRUCE MANSFIELD	1.0078	0.0051	0.9586	1.0635
BRUNNER ISLAND	1.0464	0.0032	0.9995	1.0992
CAMEO	1.4823	0.0832	1.3635	1.5977
CANE RUN	1.1891	0.0257	1.1562	1.2244
CHALK POINT	1.109	0.0064	1.0729	1.1492
CHEROKEE (CO)	1.1287	0.0207	1.0919	1.1702
CHESAPEAKE	1.1414	0.0112	1.1096	1.1767
CHESTERFIELD	1.0508	0.0004	1.0069	1.1006
CHOLLA	1.1273	0.0253	1.0842	1.175
CLIFFSIDE	1.1397	0.0122	1.1079	1.1746
COMANCHE (CO)	1.1401	0.0248	1.1009	1.184
CONEMAUGH	1.067	0.0307	1.0012	1.1422
COUNCIL BLUFFS	1.1184	0.0173	1.0825	1.1589
CP CRANE	1.1974	0.0123	1.1637	1.2334
CR HUNTLEY	1.122	0.0023	1.083	1.1651
CRAWFORD	1.2334	0.0358	1.1918	1.2757
CRIST	1.1385	0.0101	1.1059	1.1749

(Table 5.9 continued)

Plant	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
CROMBY	1.3054	0.0457	1.2502	1.3574
CRYSTAL RIVER 1&2	1.0943	0.0071	1.0572	1.136
CRYSTAL RIVER 4&5	1.0868	0.0024	1.0467	1.1313
DAN RIVER	1.3186	0.029	1.268	1.3663
DH MITCHELL	1.2651	0.0387	1.2201	1.3084
DICKERSON	1.1449	0.0104	1.1127	1.1808
DUNKIRK	1.1262	0.0028	1.0875	1.1687
EDDYSTONE	1.1509	0.0319	1.1029	1.2039
EDWARDSPORT	1.519	0.0432	1.3977	1.6417
EW STOUT	1.1499	0.0078	1.1155	1.1875
FISK	1.3625	0.0615	1.2839	1.4381
FLINT CREEK (AR)	1.1761	0.0282	1.1391	1.2171
FOUR CORNERS	1.0204	0.0087	0.9715	1.0757
GADSDEN NEW	1.4189	0.036	1.3361	1.5001
GIBSON	0.9826	0.0039	0.9303	1.043
GORGAS TWO	1.0119	0.0155	0.9515	1.0825
GREEN RIVER	1.3342	0.0209	1.2729	1.3958
HA WAGNER	1.1605	0.0185	1.1301	1.1944
HAMMOND	1.1707	0.0123	1.1393	1.2049
HARRINGTON	1.0626	0.0098	1.0205	1.1099
HARRISON	1.0392	0.0215	0.9816	1.1056
HATFIELDS FERRY	1.0215	0.0088	0.9686	1.0818
HAWTHORN	1.2127	0.0342	1.1714	1.2558
HICKLING	1.6551	0.0996	1.4776	1.8327
HIGH BRIDGE	1.3334	0.0556	1.2641	1.3999
HOLTWOOD	1.3952	0.0296	1.3186	1.4725
HOMER CITY	1.0007	0.0065	0.9487	1.0605
HT PRITCHARD	1.3545	0.0374	1.2939	1.4093
HUNTINGTON	1.1031	0.0236	1.0572	1.1548
IATAN	1.1207	0.0187	1.0841	1.1619
JACK WATSON	1.0769	0.0083	1.0251	1.136
JIM BRIDGER	1.0259	0.0138	0.974	1.0852
JM STUART	0.9983	0.0029	0.9476	1.0563
JH CAMPBELL	1.043	0.0011	0.9986	1.0936
JOLIET	1.0973	0.0168	1.0573	1.1423

(Table 5.9 continued)

Plant	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
JR WHITING	1.2329	0.0279	1.199	1.2661
KANAWHA RIVER	1.2225	0.0259	1.1901	1.2549
KEYSTONE (PA)	0.9955	0.009	0.9413	1.0578
KILLEN	1.1197	0.0114	1.0862	1.1578
KINCAID	1.1553	0.0181	1.1247	1.1899
KINTIGH	1.1351	0.0234	1.0966	1.1785
KRAFT	1.4049	0.0371	1.3276	1.4785
LANSING SMITH	1.3431	0.0491	1.2791	1.401
LEE (SC)	1.2991	0.0358	1.2521	1.341
LIMESTON	1.0357	0.0089	0.9891	1.0879
LOUISA	1.1373	0.0202	1.1027	1.1766
MARTIN LAKE	0.9849	0.0036	0.9329	1.0449
MARYSVILLE	2.0724	0.2034	1.6896	2.5257
MAYO	1.1596	0.019	1.1288	1.1939
MCMEEKIN	1.2475	0.0242	1.2127	1.2809
MERAMEC	1.2373	0.0216	1.2039	1.2706
MERRIMACK	1.1636	0.0093	1.1302	1.2001
MIAMI FORT	1.0532	0.0005	1.0094	1.1026
MICHIGAN CITY	1.1887	0.0236	1.1578	1.2226
MILL CREEK (KY)	1.0462	0.0044	1.0031	1.0958
MILLER	0.9772	0	0.9248	1.0385
MOHAVE	1.0434	0.0075	0.9992	1.0935
MONTROSE	1.209	0.0342	1.1676	1.2525
MUSKOGEE	1.0395	0.0088	0.9938	1.0909
NAUGHTON	1.113	0.0168	1.0765	1.1543
NAVAJO	1.0063	0.0022	0.9578	1.0622
NILES (OH)	1.2616	0.0167	1.2193	1.3048
NOBLESVILLE	1.5898	0.0559	1.4413	1.7413
NORTHEASTERN 3&4	1.5981	0.0169	1.4198	1.7996
PIRKEY	1.1051	0.0105	1.0697	1.1451
PORTLAND (PA)	1.2187	0.0141	1.1838	1.256
POSSUM POINT	1.2399	0.0284	1.2052	1.2734
PAWNEE	1.1813	0.0275	1.1459	1.22
PICWAY	1.4218	0.0258	1.3312	1.5144
OTTUMWA	1.1414	0.0198	1.1076	1.1795

(Table 5.9 continued)

Plant	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
POWERTON	1.2283	0.1127	1.0397	1.4699
RE BURGER	1.1729	0.0006	1.1248	1.2262
RIVER ROUGE	1.1612	0.023	1.1277	1.1986
RIVERBEND (NC)	1.2096	0.0102	1.1716	1.2503
RIVESVILLE	1.7762	0.1232	1.5453	2.0192
ROXBORO	1.0088	0.0007	0.9607	1.0645
RP SMITH	1.5765	0.0859	1.4316	1.7181
RS NELSON	1.1296	0.0219	1.0916	1.1725
SALEM HARBOR	1.1622	0.0154	1.1322	1.1948
SAN JUAN (NM)	1.0325	0.0077	0.9863	1.0844
SCHILLER	1.3724	0.0452	1.3049	1.4329
SEWARD	1.265	0.0205	1.2249	1.3049
SHAWVILLE	1.1248	0.0028	1.086	1.1675
SIOUX	1.1151	0.0023	1.0759	1.1583
ST JOHNS RIVER	1.0546	0.0149	1.0073	1.1081
STATE LINE	1.2543	0.0445	1.2009	1.308
TANNERS CREEK	1.0879	0.0006	1.0456	1.1355
TECUMSEH (KS)	1.3357	0.0522	1.2702	1.3972
TITUS	1.2723	0.0235	1.2323	1.3108
TOLK	1.0804	0.0141	1.0389	1.1267
TRENTON CHANNEL	1.1289	0.0134	1.0963	1.1655
TYRONE (KY)	1.8081	0.1332	1.5611	2.0722
VALMONT	1.3169	0.0496	1.2564	1.3743
WABASH RIVER	1.1349	0.0022	1.0951	1.179
WAUKEGAN	1.1726	0.0274	1.1359	1.2129
WC BECKJORD	1.1808	0.0736	1.0581	1.3239
WH SAMMIS	0.9901	0.0079	0.9366	1.0523
WHITE BLUFF	1.0318	0.0036	0.9867	1.0835
WILL COUNTY	1.1386	0.0237	1.1003	1.1815
WOOD RIVER (IL)	1.2033	0.0251	1.1718	1.2368
WYODAK	1.1885	0.0302	1.1503	1.2297
YATES	1.1413	0.0096	1.1086	1.1776
Median	1.1504			
Mean	1.194			

Moreover, we compare the constrained model with and without SO₂ included. Our results show that once one considers plants' costs on SO₂ emission control, all plants' returns to scale rise by 2.4% on average. This result is also consistent with the previous literature.

5.5 Conclusion

This essay examines the possible efficiency gain of 136 U.S. electric power generation coal-fired plants in 1996 using a Bayesian stochastic frontier model that imposes monotonicity and concavity restrictions on the frontier. My results confirm that this constrained model yields more accurate and favorable results than an unconstrained model; shares and elasticities are well behaved, and the standard deviations are largely reduced. I must emphasize that when I treat Sulfur Dioxide (SO₂) as "bad" output in the model, and impose monotonicity and concavity restrictions on both input prices and SO₂, our measures of plant efficiencies rise by 1%. Imposing monotonicity and concavity restrictions only generate minor differences for individual firms' returns to scale. The results also show that once I include SO₂ in the model, average returns to scale for the constrained model rises from 1.17 to 1.194, and 132 out of 136 plants in our sample exhibit increasing returns to scale.

CHAPTER 6. COMPARING THE COST OF COAL AND NATURAL GAS ELECTRICITY GENERATION

6.1 Introduction

In Chapter 5, using a Bayesian stochastic frontier model that imposes monotonicity and concavity, I analyzed the potential efficiency gain of 136 U.S. electric power generation coal-fired plants in 1996. This chapter provides a comparison of expected costs for U.S. power plants switching fuel from natural gas to coal in 1996.

Mark Ballard (2005)²² stated that, Entergy, as one of the largest power companies in Louisiana, claims that, from October 2004 to October 2005, the total bill for a residential customer using 1000 kilowatts-hours of electricity increased nearly 30%, solely due to the cost of natural gas. The once-low price of natural gas has skyrocketed since the last decade. According to this news, it costs Entergy about \$100 to produce a megawatt hour of electricity using natural gas, while only \$17 using coal. Given this situation, power plants may take steps to help reduce their reliance on gas-fired generators.

With the ongoing restructuring and deregulation in the U.S. electricity market, along with the skyrocketed price of natural gas, power plants are expected to be more flexible in switching from one source of fuel to another. How much would power plants save by switching from natural gas to coal? And how would switching to coal affect the point on the long run average cost curve at which firms in the industry operate? This dissertation is the first study to investigate these issues. Using cross-sectional data and applying a Bayesian stochastic frontier model that imposes monotonicity and concavity, I show that, for power plants switching fuel from natural gas to coal in 1996, the expected fuel cost per megawatt hour would fall by an average 3.23% and returns to scale would increase by an average 7.2%.

²² Mark Ballard (2005), "Electric shock on utility bills-Entergy warns users to brace for record highs," *The Advocate*, October 11, 2005.

6.2 Methodology

To fulfill the objective of this chapter, the Bayesian statistic is applied to impose monotonicity and concavity restrictions on stochastic translog cost frontier. From the estimates of plant-level inefficiency, one can easily derive and compare the expected total cost using coal and using natural gas. It is assumed that there are no changes of each firm's price of labor and price of capital after the fuel switch. Therefore, the only change is fuel prices and a move from the natural gas frontier to the coal frontier. Also the total relative cost $\ln(c_i / p_K)$ used to estimate the coal frontier is also taken from the relative cost of each plant using gas. Equation (6.1) computes the expected total cost of electricity generation for natural gas fired power plants that switch to use coal instead.

$$\begin{aligned} E(\ln c_i^{gas}) &= E(X_i^{gas} \beta^{gas}) + E(v_i^{gas}) \\ E(\ln c_i^{coal}) &= E(X_i^{coal} \beta^{coal}) + E(v_i^{coal}) \end{aligned} \quad (6.1)$$

Note that we have discussed how to combine the prior and the likelihood to generate the posterior density $p(\theta)$, where $\theta = (\beta, \sigma^2, \lambda, \nu)$. Estimates of β and ν_i can be easily derived from posterior density through Gibbs Sampling, which relies on taking successive draws from conditional densities to generate a sample of estimation. The joint density of the Gibbs sampler converges to true posterior density. I generate 11,000 parameter vectors and drop the first 1,000 to avoid the sensitivity to initial starting values. All numerical integration results are based on the remaining 10,000 parameter vectors from the posterior.

I am interested in a comparison of average costs per megawatt hour between using natural gas and coal as major sources. Equation (6.2) and (6.3) below show the expected per unit cost of electricity generation using natural gas and coal, respectively.

$$c_i^{gas} / mwhr = \exp(\ln c_i^{gas} - \ln q_{1i} - \ln q_{2i}), \quad (6.2)$$

where q_{1i} is normal electricity output for firm i and q_{2i} refers to peak output of electricity for firm i .

$$c_i^{coal} / mwhr = \exp(\ln c_i^{coal} - \ln q_{1i}^{coal}). \quad (6.3)$$

I do not consider peak output for plants using coal, because electricity in peak hours tends to be generated through natural gas instead of coal.

6.3 Results

6.3.1 Switching Fuel from Natural Gas to Coal

The movement of electric restructuring, the sharp increased price of natural gas, and more competitive markets may lead to changes of fuels used in electricity generation for some power plants. For simplicity, it is assumed that plants incur no extra cost in facilities or structures from switching fuel source. Labor, fuel and capital are the three inputs inside the model. Therefore, the only change in the model is the switch from natural gas price to coal price. Given the current increase in natural gas prices (especially following hurricanes Katrina and Rita) and a relatively stable coal price, we would expect a fall in estimated total cost for plants switching from gas to coal, under the previous assumption.

Therefore, I am interested in examining the potential impacts on plant-level expected total costs and returns to scale if power plants switch fuel source from natural gas to coal using 1996 data. Comparisons of plant-level expected total costs and returns to scale, assuming plants switching fuel from natural gas to coal are demonstrated as follows.

Table 6.1 shows the predicted costs, posterior standard deviation and their 90% confidence region. The per-unit cost on average decreases by \$1.5 using the price of coal and natural gas in 1996. However, if more recent data are applied on coal and gas prices, the expected coal costs should be much lower than the expected natural gas costs. With more updated data available, this dissertation can easily be extended to a more interesting policy-oriented aspect.

Table 6.1: Comparison of Predicted Costs-Constrained, n=51

Plant	COST/MWHR USING GAS				COST/MWHR USING COAL			
	Posterior Mean	Posterior S.D.	5th percentile	95th percentile	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
MORGAN CREEK (LA)	27.903	0.9208	26.059	29.68	25.579	1.2852	22.958	28.007
WILLOW GLEN (LA)	29.831	1.1859	27.461	32.095	26.794	1.3644	24.106	29.453
ALAMITOS	33.197	1.6449	29.908	36.386	31.32	1.4339	28.458	34.091
ALBANY	35.905	1.1364	33.715	38.144	36.529	2.0563	32.402	40.501
ATKINSON	83.588	4.4525	74.6	92.208	88.557	6.3373	76.938	101.94
BOWLINE POINT	39.705	1.7186	36.328	43.122	40.33	1.7394	36.925	43.749
CEDAR BAYOU	23.62	1.2068	21.245	25.976	24.987	1.8057	21.434	28.599
COOL WATER	29.614	1.3786	26.915	32.353	39.82	2.1627	35.864	44.288
CUNNINGHAM	26.136	0.8562	24.425	27.805	28.254	1.2168	25.994	30.765
DECORDOVA	26.261	1.128	24.025	28.485	24.96	1.7149	21.561	28.212
EAGLE MOUNTAIN	33.63	1.099	31.4	35.723	28.27	1.4331	25.391	31.027
EATON	53.557	2.9582	47.769	59.399	54.963	3.024	49.204	61.128
EDGEWATER (OH)	150.56	11.981	125.94	173.84	157.83	16.002	127.97	192.09
EL SEGUNDO	40.444	1.3359	37.81	43.09	36.044	1.5185	33.149	39.131
ETIWANDA	46.005	1.9463	42.171	49.897	37.563	2.0072	33.91	41.716
GADSBY	44.662	1.5289	41.69	47.65	47.608	2.7731	42.163	53.007
GRAHAM	27.435	0.8739	25.646	29.123	24.181	1.1143	21.929	26.293
GREENWOOD (MI)	94.21	6.5773	81.67	107.36	68.238	4.9979	58.394	77.87
HANDLEY	27.555	1.0954	25.333	29.685	23.875	1.2555	21.329	26.262
HUTCHINSON	51.577	1.9899	47.727	55.51	61.723	3.3723	55.196	68.428
JONES	24.038	0.8041	22.424	25.621	24.209	1.0946	22.184	26.504
KNOX LEE	31.682	0.9403	29.751	33.494	27.632	1.0965	25.512	29.841
LAKE CATHERINE	31.687	0.7669	30.165	33.176	28.59	1.1679	26.385	30.951
LAKE CREEK (TX)	30.456	0.9765	28.508	32.353	25.553	1.0753	23.467	27.668
LAKE HUBBARD	29.548	0.8846	27.751	31.257	27.271	1.5769	24.032	30.211
LEWIS CREEK	28.034	1.018	26.116	30.121	28.265	1.7529	25.206	32.018
LON HILL	28.149	0.6678	26.82	29.448	28.231	1.0424	26.17	30.249
MADDOX	23.736	1.2475	21.309	26.214	31.751	1.5664	28.839	35.005
MANDALAY	33.056	0.9678	31.168	34.97	34.072	1.4753	31.315	37.066
MUSTANG	37.95	2.0105	33.788	41.713	22.611	1.2039	20.389	25.081
NICHOLS	27.556	0.7705	26.019	29.078	29.032	1.3685	26.391	31.769
NORTH LAKE	31.475	0.708	30.081	32.847	29.732	1.4482	26.696	32.394
NUECES BAY	25.167	0.7992	23.613	26.759	26.267	1.033	24.195	28.246
OCOTILLO	56.708	1.7674	53.268	60.213	51.849	2.7875	46.455	57.319
ORMOND BEACH	38.277	1.9268	34.456	42.114	33.04	1.279	30.565	35.582

(Table 6.1 continued)

Plant	COST/MWHR USING GAS				COST/MWHR USING COAL			
	Posterior Mean	Posterior S.D.	5th percentile	95th percentile	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
PERMIAN BASIN	27.405	0.8442	25.675	29.02	23.898	1.0506	21.782	25.898
PLANT X	28.769	0.7042	27.368	30.133	29.405	1.2632	27.032	32.036
RE RITCHIE	30.735	1.0428	28.742	32.854	29.438	1.7202	26.384	33.109
REDONDO BEACH	35.012	0.9755	33.065	36.882	33.079	1.5648	30.031	36.196
REEVES	59.529	2.3021	54.903	64.012	66.363	4.3748	58.391	75.635
RIO PECOS	23.688	1.0342	21.642	25.733	29.871	1.3025	27.441	32.548
RIVERSIDE (GA)	99.395	4.9871	89.479	109.14	86.291	5.7093	75.45	98.157
RIVERSIDE (OK)	37.42	1.856	33.665	40.977	24.701	1.0775	22.536	26.743
SABINE	25.926	1.2784	23.447	28.422	25.902	2.1276	21.757	30.167
SAGUARO	71.244	2.69	65.828	76.495	60.355	3.3479	53.934	67.108
STRYKER CREEK	26.29	0.8851	24.455	27.967	20.646	0.8144	19.027	22.212
SWEATT	53.549	2.5471	48.548	58.439	46.522	2.6549	41.648	51.941
TRADINGHOUSE CREEK	25.248	1.0554	23.156	27.317	20.643	1.2914	18.029	23.158
TULSA	41.166	1.4203	38.461	44.001	44.194	2.5881	39.097	49.258
VALLEY (TX)	132.01	10.344	111.44	152.38	117.74	8.7661	100.65	135.83
ZUNI	150.44	12.672	124.71	175.12	182.86	18.397	148.25	222.29
MEAN	44.52431				43.08702			
MEDIAN	31.687				29.732			

Table 6.2: Comparison of Plant Returns to Scale-Constrained, n=51

Plant	Natural Gas				Alternative Coal			
	Posterior Mean	Posterior S.D.	5th percentile	95th percentile	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
MORGAN CREEK	1.0378	0.0557	0.9779	1.1023	1.0665	0.023	0.9797	1.1759
WILLOW GLEN	1.1378	0.0517	1.0131	1.2937	1.0443	0.0241	0.955	1.1577
ALAMITOS	1.1839	0.0481	1.0343	1.3753	1.0654	0.0231	0.9784	1.1749
ALBANY	1.1462	0.0389	1.0962	1.1992	1.3211	0.0051	1.2618	1.3894
ATKINSON	1.3508	0.0014	1.2274	1.4942	1.9335	0.0839	1.6718	2.2435
BOWLINE POINT	1.2583	0.0305	1.1293	1.4134	1.2381	0.0121	1.1719	1.3163
CEDAR BAYOU	1.1174	0.0565	0.9817	1.2903	0.9928	0.0262	0.8985	1.1142
COOL WATER	1.021	0.053	0.9318	1.1237	1.3183	0.0054	1.2591	1.3867
CUNNINGHAM	1.0233	0.0536	0.962	1.0901	1.2038	0.0147	1.134	1.2868
DECORDOVA	1.0349	0.0567	0.9698	1.1062	1.0441	0.0241	0.9547	1.1575
EAGLE MOUNTAIN	1.19	0.0362	1.1136	1.2777	1.2548	0.0108	1.1906	1.3305
EATON	1.1227	0.0403	0.9842	1.29	1.7293	0.0468	1.5679	1.913
EDGEWATER (OH)	1.4489	0.0247	1.2498	1.7046	2.6018	0.2599	1.945	3.5657
EL SEGUNDO	1.1948	0.0387	1.0935	1.3155	1.1913	0.0156	1.1199	1.2765
ETIWANDA	1.2485	0.0295	1.1392	1.3785	1.2722	0.0094	1.2095	1.3459
GADSBY	1.2108	0.0267	1.1362	1.2909	1.5491	0.0202	1.4555	1.6526
GRAHAM	1.0526	0.0532	0.9986	1.1101	1.1104	0.0207	1.0296	1.2105
GREENWOOD (MI)	1.5796	0.0405	1.3395	1.8911	1.7014	0.0423	1.5512	1.8706
HANDLEY	1.1633	0.0471	1.042	1.3128	1.0916	0.0217	1.0078	1.1952
HUTCHINSON	1.2312	0.0225	1.1386	1.3345	1.6655	0.0367	1.5303	1.8172
JONES	1.0241	0.0552	0.9732	1.0784	1.1127	0.0206	1.0321	1.2124
KNOX LEE	1.0782	0.0484	1.0299	1.1278	1.2092	0.0143	1.1404	1.2915
LAKE CATHERINE	1.1234	0.045	1.0636	1.1907	1.1881	0.0158	1.1163	1.2739
LAKE CREEK (TX)	1.0827	0.0468	1.0298	1.1381	1.2687	0.0097	1.2056	1.3425
LAKE HUBBARD	1.1273	0.0468	1.0483	1.2176	1.1356	0.0192	1.0577	1.2314
LEWIS CREEK	1.0584	0.0532	1	1.1212	1.0992	0.0213	1.0167	1.2016
LON HILL	1.0849	0.0487	1.0354	1.138	1.1748	0.0167	1.1015	1.2625
MADDOX	0.9865	0.0566	0.8853	1.1044	1.3041	0.0067	1.2444	1.3736
MANDALAY	1.0625	0.0505	1.014	1.1133	1.1799	0.0164	1.1072	1.2669
MUSTANG	1.1617	0.0366	1.11	1.2166	1.3466	0.0027	1.2878	1.4133
NICHOLS	1.0811	0.0485	1.0332	1.131	1.1969	0.0152	1.1263	1.2814
NORTH LAKE	1.1158	0.047	1.0488	1.1905	1.1525	0.0182	1.0767	1.2453
NUECES BAY	1.0337	0.0547	0.9819	1.0887	1.1053	0.021	1.0237	1.2066
OCOTILLO	1.1861	0.0307	1.1112	1.2675	1.5266	0.0173	1.4392	1.6211
ORMOND BEACH	1.2607	0.0342	1.1065	1.455	1.1817	0.0162	1.1091	1.2683

(Table 6.2 continued)

Plant	Natural Gas				Alternative Coal			
	Posterior Mean	Posterior S.D.	5th percentile	95th percentile	Posterior Mean	Posterior S.D.	5th percentile	95th percentile
PERMIAN BASIN	1.0498	0.0539	0.9931	1.1107	1.0942	0.0216	1.0109	1.1974
PLANT X	1.0923	0.0467	1.0445	1.1415	1.2259	0.0131	1.1584	1.3059
RE RITCHIE	1.1521	0.0433	1.0707	1.2462	1.1703	0.017	1.0964	1.2587
REDONDO BEACH	1.1072	0.051	1.0188	1.2115	1.08	0.0223	0.9949	1.1867
REEVES	1.2465	0.0195	1.1415	1.3654	1.746	0.0495	1.5769	1.9382
RIO PECOS	0.9962	0.0556	0.9077	1.0981	1.272	0.0094	1.2093	1.3457
RIVERSIDE (GA)	1.2526	0.0184	1.106	1.4299	1.9902	0.0956	1.6992	2.3409
RIVERSIDE (OK)	1.0881	0.0516	1.0151	1.1709	1.0914	0.0217	1.0076	1.195
SABINE	1.0623	0.0596	0.9535	1.1954	0.9711	0.027	0.8747	1.0957
SAGUARO	1.2752	0.0146	1.1819	1.3809	1.71	0.0436	1.556	1.8834
STRYKER CREEK	1.0641	0.0527	1.0043	1.129	1.101	0.0212	1.0188	1.203
SWEATT	1.1429	0.037	1.0163	1.2932	1.704	0.0427	1.5527	1.8748
TRADINGHOUSE CREEK	1.0704	0.0565	0.9761	1.1814	1.0146	0.0254	0.9225	1.132
TULSA	1.12	0.0404	1.0362	1.2132	1.4701	0.0104	1.3967	1.5504
VALLEY (TX)	1.5962	0.0601	1.3889	1.8493	2.151	0.132	1.7738	2.6307
ZUNI	1.4451	0.0238	1.2483	1.6974	2.5787	0.2524	1.9372	3.5112
Median	1.123				1.204			
Mean	1.157				1.352			

Note: posterior moments are computed based on 10,000 points generated from the Gibbs sampling algorithm. The endpoints of the 90% confidence region are the 5th and 95th percentiles of the marginal densities.

In addition, Table 6.2 presents the effects on plant-level returns to scale for firms who would switch fuel from natural gas to coal in the year 1996. Returns of scale are computed as shown in equation 5.2. Posterior standard deviation and a 90% confidence interval are presented along with each point estimation. The results show that, on average, the plant returns to scale increase from 1.157 to 1.352 if power plants switch fuel from natural gas to coal. In other words, switching fuel in 1996 can be more effective in lowering costs for plants by increasing output to take advantage of economies of scale.

6.4 Conclusion

The current literature has been relatively lacking in terms of empirical studies of inter-fuel change in the electric generation market. Chapter 6 compares the predicted costs and returns to scale of coal generation to natural gas generation at plants where the cost of both fuels could be obtained. Higher natural gas prices have raised the costs of power generation in plants using that fuel. Using a Bayesian stochastic frontier model that imposes monotonicity and concavity, I show that, for power plants switching fuel from natural gas to coal in 1996, on average, the expected fuel cost would fall by an average 3.23% and returns to scale would increase by an average 7.2%. My results provide insight into how the optimal fuel choice for electricity generation varies with the relative prices of those fuels. From the standpoint of firms, the key question is whether the expected cost savings based on expectations of future gas and coal prices would exceed the cost of conversion.

CHAPTER 7. MEASUREMENT OF EFFICIENCY IN PANEL DATA: STOCHASTIC COST FRONTIER ANALYSIS OF U.S. ELECTRICITY GENERATION

7.1 Introduction

Since 1996, twenty-four U.S. states and the District of Columbia have either enacted enabling legislation or issued a regulatory order to implement retail access²³ for electric industry restructuring and deregulation. We expect that electricity restructuring and deregulation in the U.S. electric market will affect the structure and operation of the utilities, and thus bring competition into the generator sector. Economists have long argued that competition has brought important efficiency benefits. Measuring the efficiency of power generation plants is an important issue. While many studies have examined the efficiency of the U.S. electric market, less effort has been devoted to quantifying the efficiency implications of deregulation within the electric industry. Therefore previous literature applied techniques that tend to bring bias into efficiency estimates.

With this background, the current chapter tends to examine the “real” efficiency by plant types. In another words, the power plants’ efficiency is computed conditional on whether the plant is a deregulated firm or a regulated one. In a Bayesian context, we can calculate group efficiency for regulated and deregulated power plants using a single stochastic frontier. The key contribution of this paper is to distinguish the types of power plants while computing inefficiency.

I also apply the frequentist classical panel data estimation technique – fixed effects and random effects methods on the stochastic frontier model, though the Hausman test rejects random effects and favors the fixed effects technique. The fixed effects results show that deregulated plants are more efficient than regulated plants and further confirmed that correctly analyzing the influence of deregulation will lower the bias of firm efficiency computation. As such, this chapter contributes

²³ Information including state restructuring legislation and status of state destructing activity is collected from Energy Information Administration report: http://www.eia.doe.gov/cneaf/electricity/chg_str/restructure.pdf.

to the current broad debate on the role of deregulation in the efficiency gain and therefore is a closely policy relevant research.

The remainder of this chapter is organized as follows. Section 7.2 describes the two empirical models, and analyzes the general formulation of estimation methods such as the fixed and random effects models. Section 7.3 reports the results of the empirical analysis. Section 7.4 concludes and provides further research. The data are described in Chapter 4 in this dissertation.

7.2 Model

7.2.1 Single Bayesian Stochastic Frontier Model

$$f(p_{it}, q_{it}) = a_0 + \sum_{i=1}^3 a_i \ln p_{it} + \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 a_{ij} \ln p_{it} \ln p_{jt} + b_1 \ln q_{it} + \gamma_1 (\ln q_{it})^2 + \beta_1 t + \beta_2 t^2 + \beta_3 (\ln q_{it} \times t) \quad (7.1)$$

$$\ln(c_i) = f(p_i, q_i) + u_i + v_i, \quad (7.2)$$

$$\begin{aligned} \ln(C_{it} / P_{Kit}) = & \alpha_0 + \alpha_1 \ln(P_{Lit} / P_{Kit}) + \alpha_2 \ln(P_{Fit} / P_{Kit}) + \frac{1}{2} a_{11} \ln(P_{Lit} / P_{Kit})^2 \\ & + \frac{1}{2} a_{22} \ln(P_{Fit} / P_{Kit})^2 + a_{12} \ln(P_{Lit} / P_{Kit}) \ln(P_{Fit} / P_{Kit}) \quad , \quad (7.3) \\ & + b_1 \ln q_{it} + \gamma_1 (\ln q_{it})^2 + \beta_1 t + \beta_2 t^2 + \beta_3 \ln q_{it} \times t + u_i + v_i \end{aligned}$$

where $f(p_{it}, q_{it})$ refers to the cost frontier, C_{it} denotes the total cost for firm i , which depends on input prices p_{it} ($i = L$ (labor), F (fuel) and K (capital)) and outputs q_{it} (total generation in megawatt hours). t and t^2 refers to time trend indicator for year and square of year, where $t = 1, 2, 3, \dots, 7$ for year 1994 to 2000. $(\ln q_{it} \times t)$ refers to the interaction term of log output and year trend indicator. I impose homogeneity of degree one and symmetry on translog cost function and enforce the restrictions $a_{ij} = a_{ji}$, $\sum_{i=1}^3 a_i = 1$ and $\sum_{j=1}^3 a_{ij} = 0$ for $i, j = 1, 2, 3$. Residual terms are defined the same way as before, where u_i denotes two-sided error terms that capture measurement error, and v_i is the non-negative one-sided stochastic error term reflecting plant inefficiency. The

statistical noise u_i is generally assumed as $u_i \sim IIDN(0, \sigma^2)$ and the nonnegative, one-sided inefficiency error term v_i is assumed to follow an exponential distribution with scale parameter λ . Here I allow the inefficiency to differ across two types of firms, with λ_1 serving as the mean of v_i for regulated power plants and λ_2 for deregulated power plants. The single frontier $f(p_i, q_i)$ is the average of $f(p_{it}, q_{it})$ across years.

Following the model employed in Lewis and Anderson (1999), I extend the model developed in van den Broek et al. (1994) and Koop, Osiewalski, and Steel's (1994) by allowing the mean inefficiency to differ across plant type. Intuitively, this implies that all firms face the same cost frontier but allows for different inefficiency estimates for regulated and deregulated plants in the sample.

To choose a prior parameter that can summarize the best initial guess of the efficiency of median power plant, we follow van den Broek et al. (1994) and Koop, Osiewalski, and Steel's (1994) and choose a flat prior for β where $\pi(\beta) \propto 1$, and a gamma prior for λ_j^{-1} and σ^2 ,

$$\begin{aligned}\pi(\lambda_j^{-1}) &= f_G(\lambda_j^{-1} | 1, -\ln(r_j^*)) \\ \pi(\sigma^{-2}) &= f_G\left(\sigma^{-2} | \frac{\tau}{2}, \frac{s_p^2}{2}\right).\end{aligned}\tag{7.4}$$

Recall that λ_j is the scale parameter that fixes the mean and variance of the exponential density function, where j is the plant type: $j=1$ refers to regulated plants and $j=2$ refers to deregulated power plants. $f_G(\bullet | \nu_1, \nu_2)$ denotes a gamma density with mean ν_1/ν_2 and variance ν_1/ν_2^2 .

The complete prior is defined as

$$\pi(\beta, \sigma^2, \lambda_j^{-1}) \propto \sigma^{-2} f_G(\lambda_j^{-1} | 1, -\ln r_j^*)\tag{7.5}$$

where $r_{ij} = \exp(-v_{ij})$ measures the efficiency of the i^{th} plant in type j , and r_j^* is the prior median for the plant type's efficiency. Following van den Broek et al. (1994), Koop et al. (1994) and previous work, we set the same value $r_1^* = r_2^* = 0.875^{24}$, which implies no prior about the efficiency comparison between deregulated plants and regulated plants.

The conditional density functions for λ_1^{-1} , λ_2^{-1} and v_{ij} are shown as following:

$$\pi(\lambda_1^{-1} | \text{data}, \nu, \beta, \sigma^{-2}) = \pi(\lambda_1^{-1} | \nu) = f_G(\lambda_1^{-1} | j_1 + 1, \nu' i_{j_1} - \ln r_1^*), \quad (7.6)$$

$$\pi(\lambda_2^{-1} | \text{data}, \nu, \beta, \sigma^{-2}) = \pi(\lambda_2^{-1} | \nu) = f_G(\lambda_2^{-1} | n - j_1 + 1, \nu' i_{n-j_1} - \ln r_2^*), \quad (7.7)$$

$$\pi(v_{ij} | \text{data}, \beta, \lambda_1^{-1}, \lambda_2^{-1}, \sigma^{-2}) \propto f_N(\nu | y - X\beta - \frac{\sigma^2}{\lambda_j}, I_n \sigma^2) \prod_{i=1}^n I(v_i \geq 0), \quad (7.8)$$

where $\hat{\beta} = (X'X)^{-1} X'Y - \nu$, $f_N(\bullet | u, \Sigma)$, i is a $n \times 1$ vector of ones, f_G is a gamma density, I_n refers to the $n \times n$ identity matrix, n represents the total number of power plants in the sample, $I(\bullet)$ denotes the indicator function, j_1 is the number of regulated plants, and $n - j_1$ is the number of deregulated plants. Note that the mean of plant efficiency error term is $y - X\beta - \frac{\sigma^2}{\lambda_j}$ in equation

(7.8) instead of $y - X\beta - \frac{\sigma^2}{\lambda}$.

This approach differs from others that evaluated the efficiency effect in previous literature by following Lewis and Anderson (1999) to allow firm efficiency to vary across firm types. Previous studies assume single plant efficiency for different plant types in the same frontier. But plant type itself may cause a difference in plant efficiency. Allowing only one λ will likely generate an estimated inefficiency that is a weighted average of the efficiency across two different types of firms and thus bias the efficiency estimates. Lewis and Anderson (1999) also argue that

²⁴ I assigned different values ranging from 0.6 to 0.99 to the prior median and found that the results are not sensitive to the value of prior median.

Tobit results and simple t-test results that attempt to distinguish the efficiency between two different types of firms will also be biased. Therefore, if we allow two different efficiencies across plant types and assign two different λ on the same frontier, then on average, deregulated plants would have larger mean efficiency than that of regulated firms, thus yielding more accurate estimated efficiency effects of deregulation and restructuring.

7.2.2 Regular Fixed Effects Model

Schmidt and Sickles (1984) pointed out several limitations of cross-sectional stochastic frontier models²⁵. First, maximum likelihood estimation of stochastic frontier models assumes that the inefficiency term and other regressors such as input prices, are independent. But no such strong assumption is needed in panel data estimation. Second, by adding repeated observations for each producer, the panel data stochastic frontier models provide consistent estimates. Last, Schmidt and Sickles (1984) stated that when panel data are available, we can obtain all the relevant frontier parameters by using the traditional fixed effects and random effects model for panel data.

The basic stochastic frontier cost function model is written as

$$C_{it} = \alpha + x_{it}'\beta + u_{it} + v_i \quad v_i \geq 0, \quad (7.9)$$

where α is a scalar intercept, $u_{it} \sim N(0, \sigma_u^2)$ is the statistical noise that vary across time and units, and v_i is the positive inefficiency error term, which varies across units but is constant over time. In natural logs form, v_i is the percentage deviation of plant specific observed costs from the minimum costs required as the cost frontier. $x_{it}'\beta$ contains the terms in cost function which are functions of input prices, output. I also include functions of time trend into $x_{it}'\beta$ to account for technique change.

If I define $\alpha_i = \alpha + v_i$, then equation (7.9) is shown as the standard panel data model:

²⁵ See Kumbhakar and Lovell (2000, p. 95-96) for reference.

$$C_{it} = \alpha_i + x_{it}'\beta + u_{it}. \quad (7.10)$$

We can follow the following steps to estimated firm specific inefficiency.

First, using the “within-groups” transformation, I can estimate transformed equation (7.11) through OLS.

$$(C_{it} - \bar{C}_i) = \beta'(X_{it} - \bar{X}_i) + u_{it}. \quad (7.11)$$

Then the measure of efficiency for plant i is the ratio of plant i 's cost to that of the efficient plant shown as

$$\gamma_i = \frac{\text{cost for an efficient firm}}{\text{cost for firm } i} = \exp(-\nu_i). \quad (7.12)$$

Therefore the cost inefficiency is $1 - \gamma_i$. Sometimes we can use ν_i itself to measure cost inefficiency, because $\nu_i \approx 1 - \exp(-\nu_i) = 1 - \gamma_i$ for small values of ν_i . To compute the individual estimated inefficiency, we can rank the intercept α_i as $\alpha_{(1)} \geq \alpha_{(2)} \dots \geq \alpha_{(N)}$ where N refers to the index of the plant with the smallest value of α_i or smallest value of ν_i among all N firms²⁶. Therefore, the plant's relative inefficiency rather than absolute efficiency is

$$\begin{aligned} \hat{\nu}_i &= \alpha_{(i)} - \alpha_{(N)} \quad \text{and} \\ \hat{\gamma}_i &= \exp(-\hat{\nu}_i), \quad i = 1, 2, \dots, N-1. \end{aligned} \quad (7.13)$$

The fixed effects model provides the consistent estimates for cost inefficiency, but it excludes time invariant regressors. In addition, if the inefficiency term is independent of other regressors, then the random effects model produce more efficient estimates and therefore should be preferable to the fixed effect model. However, I used the Hausman test and the results reject the hypothesis that the errors terms are independent of regressors at 1% significance level for all three segments: pooled plants, regulated and deregulated plants. Therefore, no further analysis of the random effects model will be pursued.

²⁶ I followed the method proposed by Schmidt and Sickles (1984) to estimates the technical inefficiency, based on the fixed effects estimates.

7.3 Results

7.3.1 Single Bayesian Stochastic Frontier Results

Under a single stochastic frontier methodology, I allow λ to vary with firm types using Bayesian statistics. This makes firm efficiency computation more accurate by allowing mean efficiency to differ for regulated and deregulated plants.

Table 7.1: Posterior Moments for Model Parameters (Unconstrained)

Parameter	Mean	Std. Dev.	5 th percentile	95 th percentile
a_0	10.3885	0.4821	9.5920	11.1777
a_1	0.1799	0.0108	0.1625	0.1974
a_2	0.8802	0.1113	0.8616	0.8985
a_{11}	0.1042	0.0454	0.0291	0.1788
a_{22}	0.0622	0.0493	-0.0194	0.1439
a_{12}	-0.0466	0.0412	-0.1138	0.022
b_1	0.1073	0.0664	-0.0014	0.2168
γ_1	0.0261	0.0023	0.0223	0.0229
β_1	-0.0079	0.0272	-0.0521	0.0365
β_2	0.0027	0.0011	0.0008	0.0045
β_3	-0.0009	0.0018	-0.0038	-0.0021
σ^2	0.0091	0.0008	0.0078	0.0103
λ_1	0.1229	0.0069	0.112	0.135
λ_2	0.1182	0.0096	0.1033	0.1348
λ_1/λ_2	1.0451	0.0821	0.915	1.18
Prob($\lambda_1/\lambda \geq 1$)	70.1%			
Odds Ratio	2.34:1			

Note: posterior moments are computed based on 10,000 points generated from the Gibbs sampling algorithm. The endpoints of the 90% confidence region are the 5th and 95th percentiles of the marginal densities.

Table 7.1 presents the posterior moments for the translog cost frontier parameters, σ^2 , two efficiency parameters and their ratio. Figure 7.1 also shows the posterior density of σ^2 . Ninety percent confidence intervals for all parameters are constructed though Bayesian statistics and Gibbs

sampling. Figure 7.2 presents the change of $\ln(C_i / P_{Ki})$ over time, where $t = 1, 2, \dots, 7$ refers to year indicator from 1994 to 2000. Notice that $\ln(C_i / P_{Ki})$ first decreases then starts to rise at an increasing rate later.

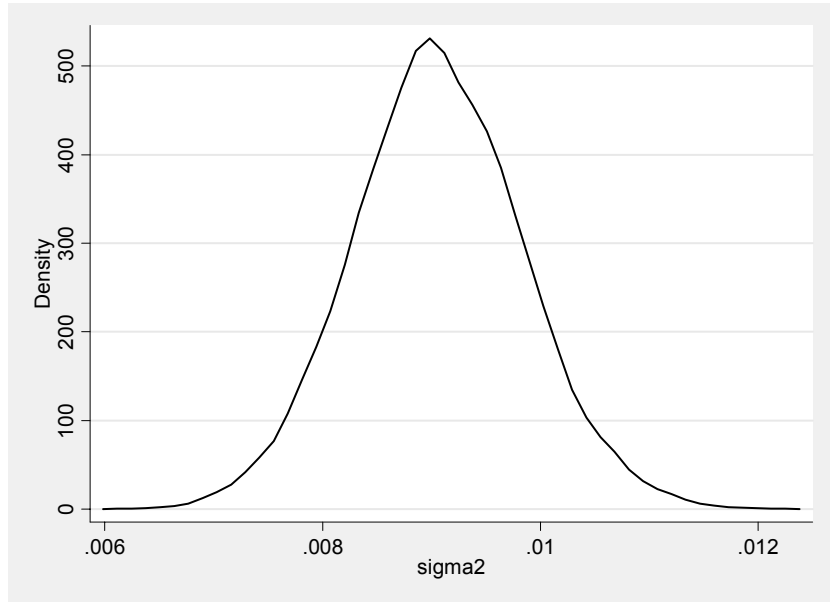


Figure 7.1 - Posterior Density of σ^2

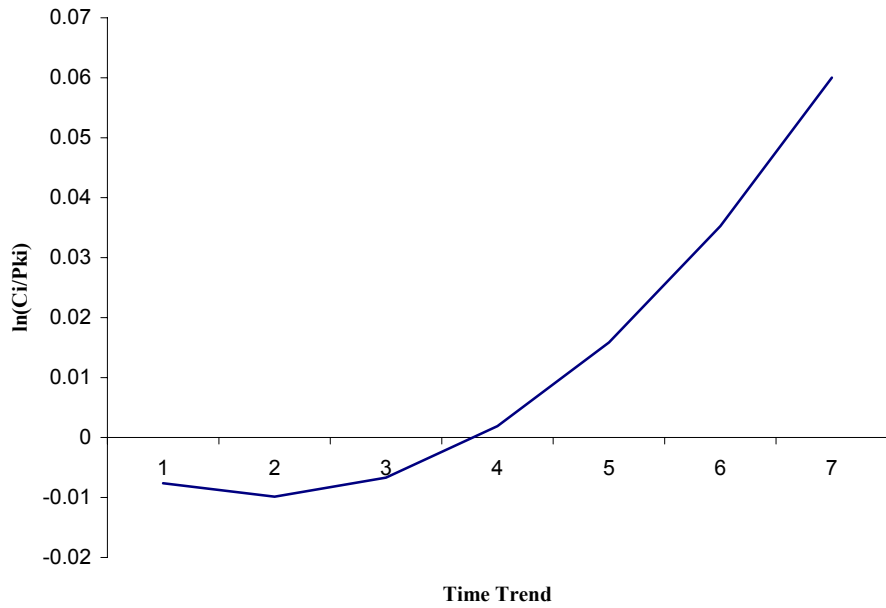


Figure 7.2 - Change of $\ln(C_i / P_{Ki})$ over time

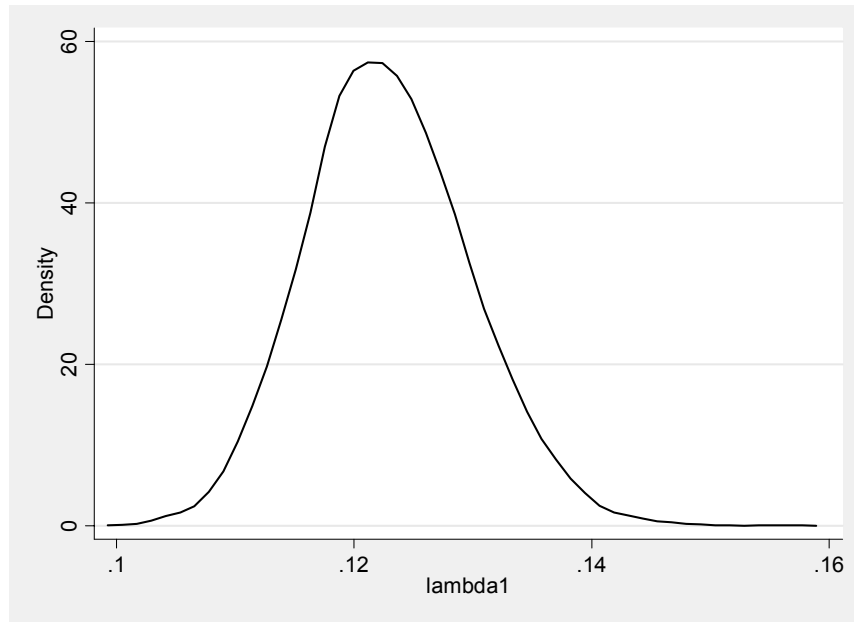


Figure 7.3 - Posterior Density of λ_1 (posterior mean inefficiency of regulated plants)

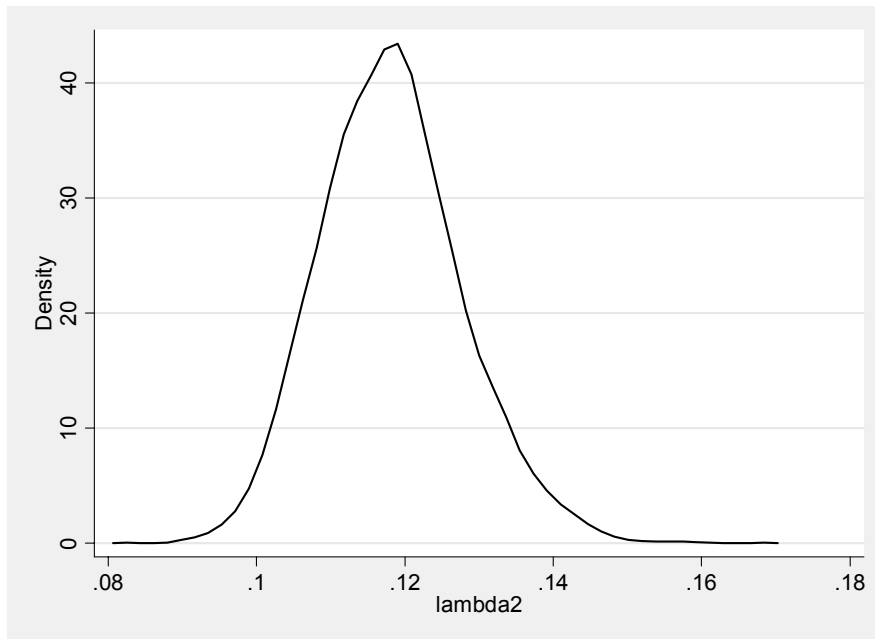


Figure 7.4 - Posterior Density of λ_2 (posterior mean inefficiency of deregulated plants)

Figure 7.3 and Figure 7.4 depict the posterior marginal density plots of λ_1 and λ_2 . The posterior mean parameter for the regulated plants λ_1 is 0.1229, implying that on average regulated power plants are about 87.7% efficient. In other words, regulated plants can further reduce their

costs about 12.29% by increasing efficiency without changes in inputs. Also posterior mean parameters λ_2 is 0.1182 for deregulated plants, which suggests that, on average, deregulated power firms are about 88.2% efficient. Therefore, costs of regulated plants can be further reduced about 11.82% by increasing efficiency. In addition, Figure 7.5 shows the posterior marginal plot of λ_1/λ_2 . Figure 7.6 and Figure 7.7 show the histograms for the posterior of inverse of regulated plants' inefficiency and the posterior of inverse of deregulated plants' inefficiency respectively. We can also compare the posterior density plots with the prior density plots shown as Figure 5.1.

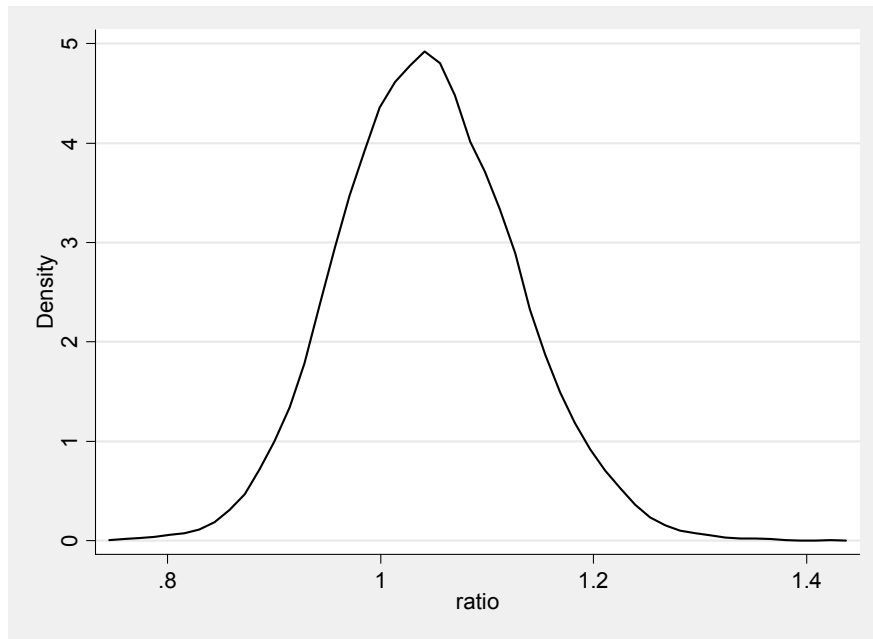


Figure 7.5 - Posterior Density of λ_1 / λ_2

Applying the 10,000 Gibb Sampler points, I also compute the probability of $(\lambda_1 / \lambda_2) \geq 1$ as 70.1%. The results provide only weak evidence that deregulated plants are more efficient than regulated plants. Posterior odds ratio can be used as another alternative to conduct inferences on efficiency measures across regulated and deregulated firms. An odds ratio provides the probability of occurrence of an event. Symbolically, the posterior odds ratio is computed

as $P(\lambda_1 > \lambda_2) / [1 - P(\lambda_1 > \lambda_2)]$. The posterior odds that deregulated plants are more efficient than regulated plant are 2.34 to 1.

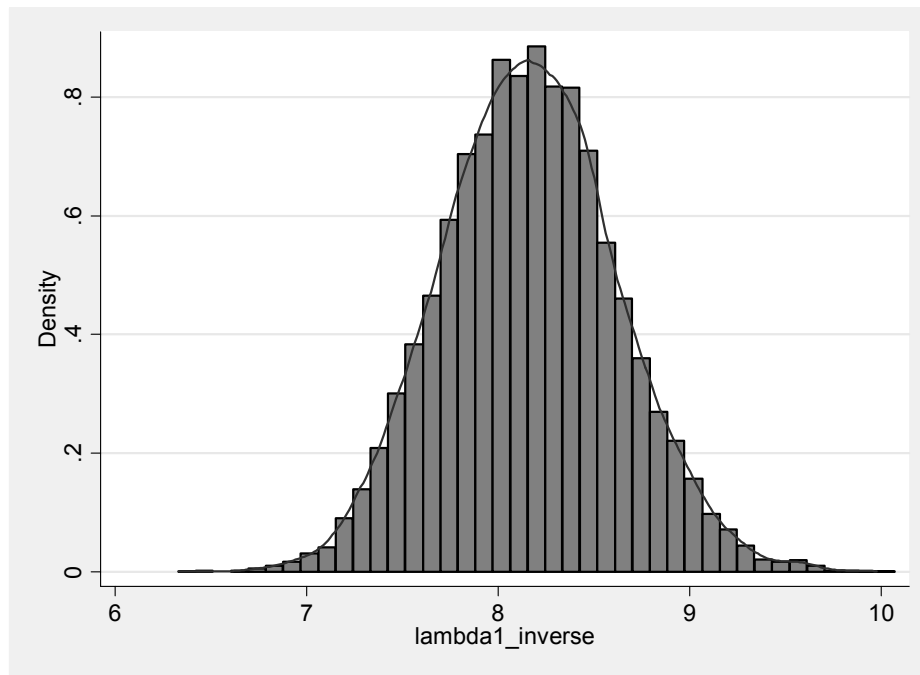


Figure 7.6 - Posterior Density of λ_1^{-1}

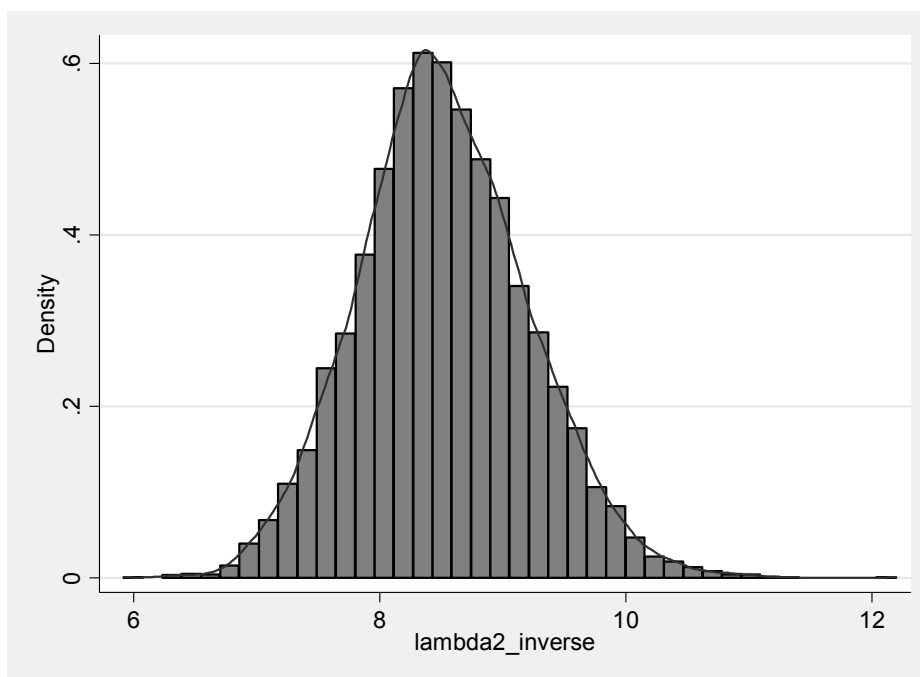


Figure 7.7 - Posterior Density of λ_2^{-1}

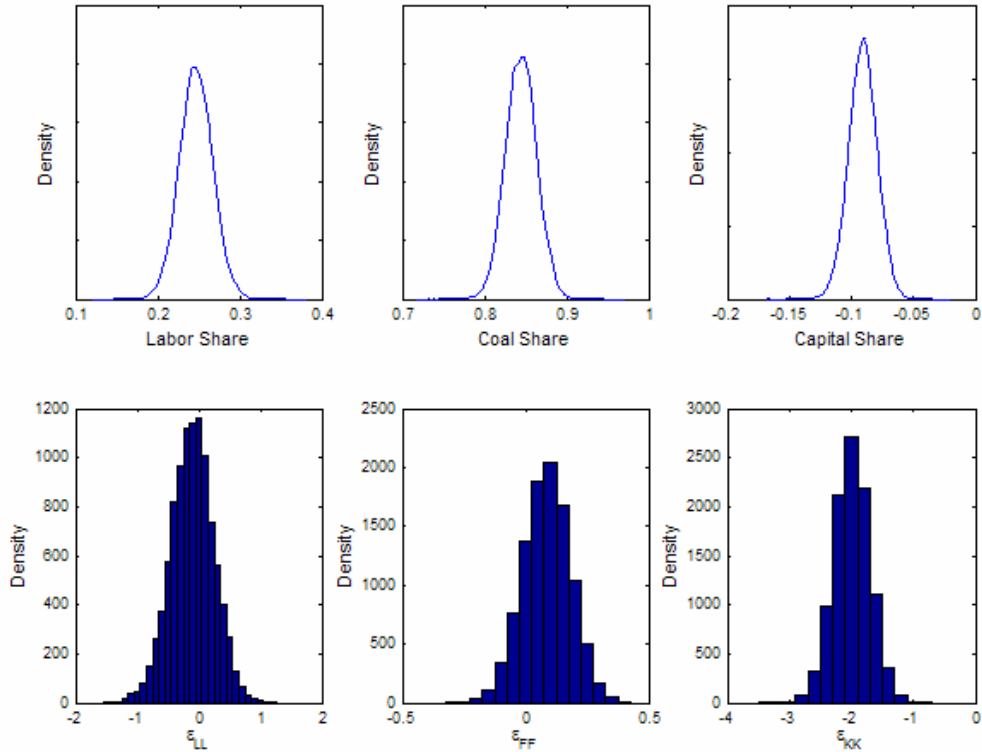


Figure 7.8 - Marginal Density Plots for Shares and Elasticities-Panel Unconstrained model

Figure 7.8 graphs the marginal density plots for inputs shares and inputs own price elasticities. Recall that Chapter 5 discussed how the prior can be used to incorporate monotonicity and concavity restrictions on the stochastic frontier model. I define $h(\beta)$ as the indicator function in the prior. For example, $h(\beta) = 1$ if the stochastic frontier satisfies monotonicity and concavity for all price combinations in a region of prices and output Ψ . I then slice away the portion of the posterior density that violates economics theory. However, the above method does not apply to this data set because all computed price of capital are negative and therefore all distributions of capital share are negative from Figure 7.8. Because there are no restrictions of monotonicity and concavity on the frontier model, in Figure 7.8, we can see labor share and capital share may be negative. In addition, the histograms for three inputs show that own price elasticity for labor, fuel and capital can be positive, which serves as a disadvantage here. Unfortunately, attempts to impose

monotonicity and concavity failed because the Gibbs sampler failed to provide parameter vectors that satisfied the constraints²⁷.

I also show the convergence of functions of interests in Figure 7.9. With 10,000 Gibbs Sampler draws, it is very clear from the graphs that the functions of interests generally converge with 2,000 Gibbs points.

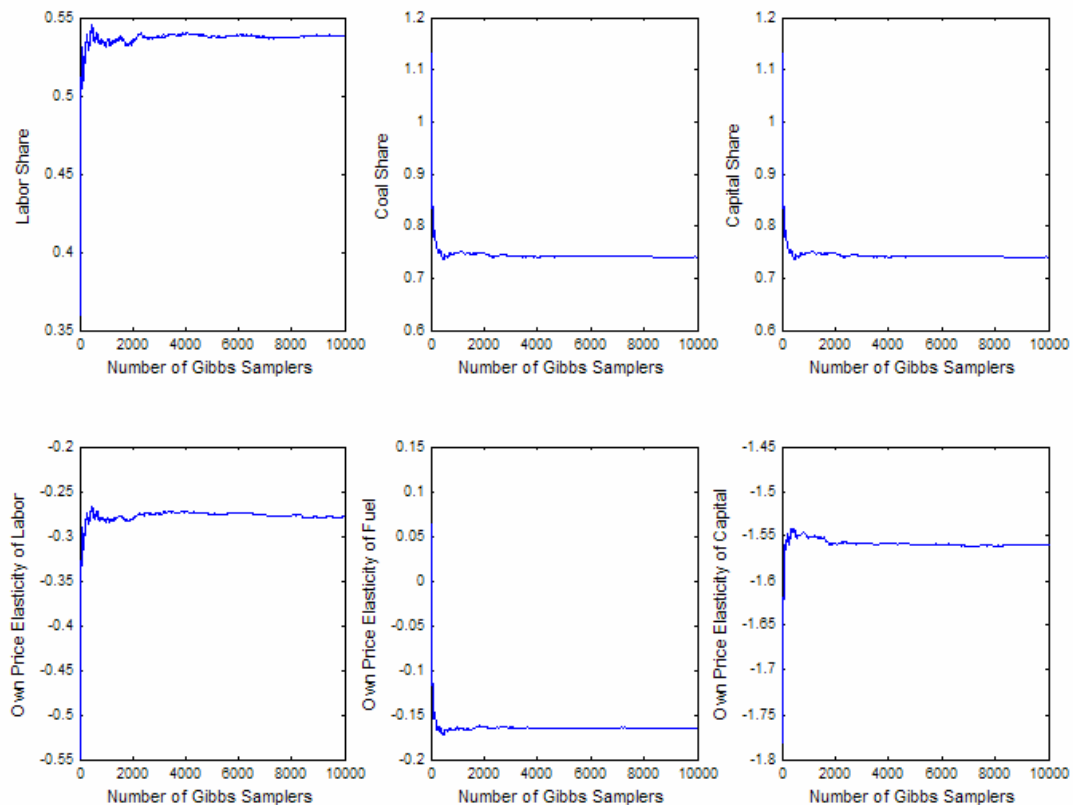


Figure 7.9 - Convergence of Functions of Interests

7.3.2 The Classical Fixed Effects Stochastic Frontier Results

I employed Hausman tests for fixed and random effects regression for all three segments: pooled, regulated and deregulated plants respectively. The results show p-values close to 0 based

²⁷ Because the mass associated with the share of capital is all negative (see Figure 7.8), the Gibbs Sampler fails to provide parameter factors that satisfy the monotonicity to input price condition. This may due to data inconsistency to concavity condition and may also be due to multicollinearity in the model.

on χ^2_9 distribution and thus I reject the exogeneity of error terms for all three segments.

Table 7.2 provides point estimates and 90% confidence intervals for the relative efficiency measures γ_i^* for the classical fixed effects models. The usual point estimates are based on the fixed effects within estimates.

Moreover, estimates of the Fixed Effects model on pooled plants, regulated and deregulated plants are presented in Table 7.2. Few comments can be made on these estimates except to note that the variance of inefficiency term σ_v^2 is large relative to the variance of noise σ_u^2 in all three segments. Table 7.3 provides the descriptive statistics for the estimated inefficiency distributions. The results confirmed my conclusion in section 7.3.1 that, on average, deregulated plants appear to perform more efficiently than regulated plants do.

**Table 7.2: Estimated Stochastic Frontiers, Fixed Effects (within) Parameters
(Estimated standard errors in parentheses)**

Parameter	Fixed Effects Pooled Plants (n=1,341)	Fixed Effects Regulated Plants (n=1,045)	Fixed Effects Deregulated Plants (n=296)
a_0	11.99838 (1.286197)	11.7638 (1.293991)	16.76347 (4.280534)
a_1	.2605614 (.0263896)	.2579327 (.0265007)	.1985803 (.0883732)
a_2	.6439428 (.0265571)	.6021828 (.0266522)	.8389107 (.0901075)
a_{11}	.1330127 (.0580547)	.1139068 (.0566059)	.2249215 (.2405868)
a_{22}	-.0253781 (.0603316)	-.0154117 (.0608661)	-.0395475 (.2018976)
a_{12}	-.0375941 (.0533172)	-.0335942 (.0528109)	-.0943405 (.1943892)
b_1	.234374 (.1814992)	.2948796 (.1820654)	-.4960522 (.606084)
γ_1	.0108395 (.0065006)	.0079096 (.0065068)	.0380995 (.0217486)
β_1	-.0111802 (.0196498)	.0060311 (.0203237)	-.1184108 (.0627742)
β_2	.0006397 (.0007236)	.0004911 (.0007396)	.0018637 (.0024698)
β_3	.0003122 (.001331)	-.0007823 (.0013662)	.0071885 (.0041817)
σ_u	.08073891	.07493731	.09669781
σ_v	.37381077	.41125152	.28248969

Kim and Schmidt (2000) states that the “fixed effects models (either classical or Bayesian) yield much lower efficiency levels than random effects models” and “from a classical point of view, the fixed effects estimates of efficiency levels are biased downward.” This partially explains the fact that our classical fixed effect inefficiency estimates are lower than the estimates using the single Bayesian stochastic frontier as shown in Table 7.1.

Table 7.3: Estimated Inefficiencies for Fixed Effects Stochastic Frontier Models

	Mean	Standard Deviation	Minimum	Maximum
Pooled Plants	0.2950	0.1827	0.0194	1.0198
Regulated Plants	0.3175	0.2013	0.0022	1.0811
Deregulated Plants	0.2098	0.1483	0.0018	0.7870

7.4 Conclusion

Assuming the same technology across different types of power plants, this study applies two different estimation methods to show that deregulated power plants are more efficient than those regulated plants. One method is the application of the Bayesian stochastic frontier model on a pooled panel data. We are able to apply different mean plant inefficiency terms by plant type on a single stochastic frontier. The precise parameter, inefficiency estimates and confidence intervals in finite samples are another advantage of the Bayesian technique. We are also able to calculate the probability that deregulated firms are more efficient than regulated firms. The posterior odds that deregulation is more efficient are 2.43 to 1, using this probability.

The other is the classical fixed effects method applied on an unbalanced panel of power plants. The finding is consistent to our previous results that deregulated power plants are more efficient than regulated power plants.

The fixed effects approach analyzed in the dissertation so far has considered the cost inefficiency term to be time-invariant. If the time dimension is large, we can also extend the fixed effect model by letting the inefficiency error term vary across both plants and year. That is, extend

equation (7.9) to $C_{it} = \alpha + x_{it}'\beta + u_{it} + v_{it}, v_{it} \geq 0$. The model can be estimated by creating dummy variables when N - the observation in the sample is not too large.

The current paper can also be extended to the Bayesian fixed effects or the Bayesian random effects model. In addition, the use of a Metropolis-Hastings sampling algorithm could potential solve the problems that prevented from the imposing of monotonicity and concavity restrictions on cost frontier as in previous chapters.

CHAPTER 8. SUMMARY AND CONCLUSIONS

This dissertation consists of three essays that focus on Bayesian estimation of stochastic cost frontiers for electric generation plants. Applying Bayesian methods to impose monotonicity and concavity restrictions on a cost frontier, we are able to develop a precise perspective regarding specific issues such as plant-level production costs, efficiency, and returns to scale. Essay one uses cross sectional data to empirically examine the possible efficiency gain of power plants due to the deregulation. I also estimate the cost of electric power generating plants using coal as a fuel taking into account both regularity restrictions and sulfur dioxide (SO_2) emissions. Essay two observes the higher price of natural gas and compares the predicted costs and returns to scale of coal generation to natural gas generation at plants where the cost of both fuels could be obtained. Essay three estimates average group efficiency for two different types of plants by applying the Bayesian stochastic frontier model on a single cost frontier and allowing firm type to vary across regulated and deregulated plants. Additionally, essay three uses the classical fixed effects model and random effects model on an unbalanced panel to estimate group efficiency for regulated and deregulated plants.

Results lend support to previous literature and confirm that a constrained Bayesian stochastic frontier model yields more precise estimates than an unconstrained one. For example, all shares and own price elasticities of inputs are well behaved in the constrained model. The results also indicate that failure to account for pollution reduction in the model will underestimate plants' returns to scale and overestimate plant inefficiency. Using a Bayesian stochastic frontier model that imposes monotonicity and concavity, I find that, for power plants switching fuel from natural gas to coal in 1996, on average, the expected fuel cost would fall and returns to scale would increase. The results also suggest that for power plants switching fuel from natural gas to coal in 1996, on average,

the expected fuel cost would fall and returns to scale would increase. Furthermore, the findings also provide evidence that deregulated power plants are more cost-efficient than regulated plants.

This dissertation can be further extended in two ways. First, it is anticipated that expected costs will be even further lower for gas-fired plants that change to use coal if techniques on more recent data are applied.

Second, the fixed effects approach analyzed in the dissertation so far has considered the cost inefficiency term to be time-invariant. If the time dimension is large, we can also extend the fixed effect model by letting the inefficiency error term vary across both plants and year. The model can be estimated by creating dummy variables when N - the observation in sample is not too large.

Last, the final chapter could also be extended to apply a Bayesian fixed effects or Bayesian random effects model. In addition, the use of a Metropolis-Hastings sampling algorithm could potentially solve the problem that prevented the imposing of the monotonicity and concavity restrictions on the cost frontier as in previous chapters.

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APPENDIX A: SUPPLEMENTARY DATA

Table A1 below provides the price of alternative source for the 1996 data set. This data set is used when we focus on the cost change when power plants switch fuel source from natural gas to coal in 1996.

Table A.1: Price of Coal as Alternative Source for Firms using Natural Gas in 1996, n=56

Plant	Utility Level Price of Coal	Plant Level Price of Coal
ALAMITOS	1.5247	
ALBANY	1.2813	
ATKINSON	1.4935	1.3203
BOWLINE POINT	1.9199	
CEDAR BAYOU	1.5466	
COOL WATER	1.5247	
CUNNINGHAM	1.6706	
DECORDOVA	1.049	
EAGLE MOUNTAIN	1.049	
EATON	1.3171	
EDGEWATER (OH)	1.1084	
EL SEGUNDO	1.5247	
ETIWANDA	1.5247	
GADSBY	0.9234	
GRAHAM	1.049	
GREENS BAYOU	1.5466	
GREENWOOD (MI)	1.3116	
HANDLEY	1.049	
HUTCHINSON	1.1776	
JONES	1.6706	
KNOX LEE	1.4288	
LAKE CATHERINE	1.4481	
LAKE CREEK (TX)	1.049	
LAKE HUBBARD	1.049	
LEWIS CREEK	1.421	
LON HILL	1.3541	
MADDOX	1.6706	
MANDALAY	1.5247	

(Table A.1 continued)

Plant	Utility Level Price of Coal	Plant Level Price of Coal
MORGAN CREEK	1.049	
MUSTANG	0.8043	
NICHOLS	1.6706	
NORTH LAKE	1.049	
NUECES BAY	1.3541	
OCOTILLO	1.3775	
ORMOND BEACH	1.5247	
PERMIAN BASIN	1.049	
PH ROBINSON	1.5466	
PLANT X	1.6706	
RE RITCHIE	1.4481	
REDONDO BEACH	1.5247	
REEVES	1.642	
RIO PECOS	1.3522	
RIVERSIDE (MD)	1.4306	
RIVERSIDE (GA)	1.469	
RIVERSIDE (OK)	1.2013	
SABINE	1.421	
SAGUARO	1.3775	
SAM BERTRON	1.5466	
STRYKER CREEK	1.049	
SWEATT	0.8284	
TH WHARTON	1.5466	
TRADINGHOUSE CREEK	1.049	
TULSA	1.2013	
VALLEY (TX)	1.049	
WILLOW GLEN	1.421	
ZUNI	0.984	

APPENDIX B: PROCEDURE OF GIBBS SAMPLER

Conditional on ν , the model simplifies to the normal linear regression model $y - \nu = x\beta + u$.

Treating $y - \nu$ as y^* , the conditional densities are defined as:

$$\beta \mid \sigma^2, \nu, \lambda, y \sim N(\hat{\beta}, (x'x)^{-1} \sigma_u^2), \text{ where } \hat{\beta} = (x'x)^{-1} x'y^* \text{ and } y^* = y - \nu$$

and (A.1)

$$\frac{1}{\sigma_u^2} \mid \beta, \nu, \lambda, y \sim \Gamma\left(\frac{T-2}{2}, \frac{(y^* - x\beta)'(y^* - x\beta)}{2}\right),$$

with ν known, β and σ^2 provide no additional information about the mean of the exponential distribution. The conditional distribution of λ is then:

$$\lambda^{-1} \mid \beta, \sigma^2, \nu, y \sim \Gamma(n+1, [v'i - \ln(r^*)]^{-1}). \quad (\text{A.2})$$

where i represents an $n \times 1$ vector of ones.

To understand this conditional distribution, recall that the mean of a $\Gamma(\alpha, \beta)$ is $\alpha\beta$, so

$$E[\lambda^{-1}] = \frac{n+1}{\sum_{i=1}^n v_i - \ln(r^*)}$$

If the sample is large the mean of λ^{-1} is roughly $1/\text{mean}(\nu)$, which is the inverse of the maximum likelihood estimate of λ given ν . The prior simply increases or decreases the mean slightly to reflect the prior estimate of efficiency. For more than 10,000 observations and average sample efficiency greater than 0.01, the prior does little to affect results.

The final conditional density applies to the vector ν , containing the inefficiency error for each firm. Jondrow et. al. (1983) show that this conditional distribution is

$$\nu \mid \beta, \sigma^2, \lambda, y \sim TN\left(y - x\beta - \frac{\sigma_u^2}{\lambda}, \sigma_u^2 I\right) \quad (\text{A.3})$$

where TN is normal distribution truncated below at 0.

The Gibbs Sampler for this problem is directly based on these conditional densities and is implemented as follow.

Step 1: Choose initial starting values $\lambda^{[0]}, \nu^{[0]}$.

Step 2: Sample $\beta^{[1]}$ and $\sigma^{[1]}$ conditional on $\lambda^{[0]}, \nu^{[0]}$ from equation (A.1).

Step 3: Sample $\lambda^{[1]}$ given $\nu^{[0]}, \beta^{[1]}$ and $\sigma^{[1]}$ based on equation (A.2).

Step 4: Sample $\nu^{[1]}$ given $\lambda^{[1]}, \beta^{[1]}$ and $\sigma^{[1]}$ using equation (A.3).

Step 5: Iterate to complete the sample using for integration.

The Gibbs sampler converges to actual joint density as the iterations approach infinity. In this dissertation, I generate 11,000 parameter vectors and drop the first 1000 to avoid sensitivity to starting values. Monotonicity and concavity conditions are checked for each parameter vector generated from the Gibbs Sampler. If conditions are violated, the parameter vector is dropped from the sample used for numerical integration.

VITA

Xia Zhao obtained her Bachelor of Science degree in accounting in 2000 from Xiangtan University in China. She then obtained her Master of Science degree in economics from Louisiana State University in 2003. During her graduate study at Louisiana State University, Xia Zhao taught "Principle of Microeconomics" at the undergraduate level for three years. Her working paper, "Comparing the Cost of Coal and Natural Gas Electricity Generation: A Bayesian Study," a joint work with Professor Dek M. Terrell and Professor Andrew Kleit, was presented at the Southern Economic Association 74th Annual Meeting in 2004 and the Fifteenth Annual Meeting of the Midwest Econometric Group in 2005. Currently she is a candidate for the degree of Doctor of Philosophy in economics at Louisiana State University, which will be awarded at the December 2006 commencement.