A Stochastic Time-Series Model of the Human Performance in Compensatory Tracking.

Jamal Muhammad Al-barzinji

Louisiana State University and Agricultural & Mechanical College

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Louisiana State University and
Agricultural and Mechanical College
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requirements for the degree of
Doctor of Philosophy

in
The Department of Chemical Engineering

by
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Abstract

A stochastic, all digital model of a trained human operator in compensatory tracking is developed using time series analysis. Human performance data was collected in a hybrid simulation of a realistic environment which included an optically sighted AAA gun system and target flight paths supplied by the Air Force. Two men manipulated the gun and physically closed the control loop by viewing scope displays of their tracking errors and generating corrective torques, each in his respective channel.

The model is made up of a deterministic and a stochastic noise component. The same model form applies to azimuth and elevation channels and one set of parameter values is valid for approach and escape maneuvers. The model operates with two inputs: tracking error and target angular velocity; and the individual transfer functions are superimposed with weighting factors to form the multiple input model. Performance of the deterministic model is in excellent agreement with the mean tracking error of the human operators.

The stochastic component is produced by passing white noise through a filter and the result is linearly added to the deterministic part to make up the final model. Elements of white noise have a zero mean and a variable variance which is the outcome of a very simple transfer function whose only input is the second difference of target angular velocity.
The model being simple and involving no differential or integral operators is at least ten times faster on a digital computer than existing conventional models serving a similar purpose.
CHAPTER I
INTRODUCTION

The role of man as a controller and information processor is not diminishing with improving technology. Rather it is becoming more delicate; demanding careful assessment of man's strengths and weaknesses. In order for this to be done effectively, mathematical models are needed to describe and reproduce the human behavior in control situations. Tracking performance of human operators has always assumed major interest in this regard due to its direct applicability in numerous fields of practical worth, ranging from space to manufacturing industries. The particular experiment selected for this study is of direct concern to the military as it involves a human operator tracking a flying object with anti-aircraft artillery.

The environment of this research is, therefore, drawn from real life, where the gun system is a simulation of an existing optically sighted AAA weapon, and targets follow flight paths generated by the Air Force in their test flights. A major source of error in such a setup is manual tracking errors which result from tracker's inability to follow an accelerating or decelerating target accurately and continuously. In addition, tracking is made more difficult by interaction between azimuth and elevation errors on the sight reticle.
Objectives:
The objective of this dissertation is to develop an all digital model of the human transfer function in compensatory tracking. The model is stochastic to describe the probabilistic nature of human performance. The mean or deterministic portion of operator output is also needed since it is required in many studies.

The study is also concerned, as a major objective, with the variance of tracking error and machine execution time of the model. This is particularly so in view of the limitations of the models of Perkins (1974) and Planchard et al (1971,1972), where the tracking data and the control system is the same as those used in this thesis. While these models predict the mean tracking error with a remarkable success, no information is directly available on the spread around the mean or the variance of tracking error. Instead, Plachard et al (op cit) supply a set of cumbersome non-linear equations with too many parameters, that will predict the variance of tracking error. An even more serious drawback is the long time needed for digital computation, where approximately 55 seconds were shown to be needed to track a target for only 40 seconds, which makes the method costly and useless for on-line applications.

The model resulting from this thesis is required, therefore, to have a significantly shorter machine time than the models mentioned above. In addition, it incorporates a very simple yet an efficient method of predicting the variance of tracking error as a function of flight parameters. Furthermore, the model of Perkins (1974) and Planchard et al (op cit), while using the same data, was unable to predict the mean performance over the entire flight path, whence, it
was necessary to incorporate some adaptation by having two different sets of parameter values in the pre- and post cross-over regions of tracking. Naturally, this resulted in an undesirably large number of parameters. The time series model advanced in this work is more economical in parameters, as one set works before and after cross-over.

**Justification:**

The objective of this effort is to formulate a stochastic model of the human operator, based on time series analysis. The human performance in control is stochastic in nature, as evidenced by run-to-run and inter-operator variations, which makes its necessary to develop a stochastic model in order to describe the control function adequately.

Time series modeling offers three major advantages which made it favorable for this study.

a) The stochastic component of a time series model is developed separately as a "noise" function which is linearly added to the deterministic component of the model. This is very useful since portions of the output arising from the probabilistic and deterministic components are directly available and identifiable.

b) Time series models involve linear combinations of pure algebraic operations making them very simple in nature and extremely fast in machine execution. This may be compared with common transfer functions of standard control models which require integration of one or more simultaneous equations - a very time consuming operation on digital computers.

c) The form of a time series model, which is basically a difference equation form, is expected to be very general in applicability
and should not change as the control situation or system changes. Modifications may be needed in the parameter values only, while the form stays essentially the same. In fact, as the study revealed, a number of parameters in the tracking model had the same value in elevation as in azimuth, despite the difference in the nature of tracking between the two axis.

The deterministic component of a tracking model is valuable in itself, as in many situations of weapon effectiveness and flight maneuverability studies, the mean and variance of tracking error is the main information that is needed. As mentioned earlier, this data is immediately available from a time series model.

The stochastic part of the model helps in presenting a full picture of manual tracking by a standard, well trained operator. When the number of runs is large enough, a straightforward extension makes it possible to model individual operators or a single operator during different phases of his training.

**Methodology:**

The tracking system is considered a control loop with the following main elements: the operator(s), the gun system, the flying aircraft and the sight reticle. The operator senses an error through the reticle and takes appropriate action to move the gun and minimize the error in his respective axis, azimuth or elevation. As a model of the gun is given, what is needed is a model of the human operator in order to simulate the entire tracking system.

Two operator models are needed for the two control channels of the gun. A model for the azimuth channel was first formulated using data of
a flight path with relatively mild maneuvers. It was checked for validity against paths with severe maneuvers. This was followed by developing an operator model for elevation which was also tuned and validated as before. The two-channel operator model was next incorporated in a general tracking program and the parameters dynamically tuned.

Tracking data was generated in a real time hybrid simulation of the tracking system of AAA gun, target flight paths and error detection and display. Part of the data was used to formulate the model while the rest served to establish its validity.

The body of data was analyzed by time series method to establish a deterministic component first, which was followed by a stochastic noise component.

**Chapter Survey:**

Chapter II presents background information on the human operator, tracking and mathematical models of such systems. It deals with definitions of tracking and characteristics of man as a controller such as intermittency, time lag and noise. The chapter concludes with a review of available models of the human operator.

In Chapter III, time series method of analysis is introduced. Both stationary and non-stationary processes are presented and a modeling procedure is discussed under the three stages of identification, estimation and diagnostic checking. Transfer function models of dynamic systems are also included both with single and multiple inputs.

Description of the tracking system is covered in Chapter IV, where the gun model is derived, flight path specifications are enumerated and
tracking errors are defined and calculated. The hybrid computer program is presented in flow charts and discussed following a description of the available simulation facilities. The procedure of data collection completes this chapter.

Formulation of the model is reported in two parts. Chapter V documents the development of the deterministic component of the tracking model based on the theoretical discussion of Chapter III. The question of what is to be considered an input to the system was settled first, a transformation of each input to "white noise" followed. Next, identification of a transfer function for each one of the two inputs is reported together with a maximum likelihood parameter estimation based on least squares regression. A multiple input transfer function model is developed for azimuth channel first and for elevation last. This is followed by dynamic tuning of parameters in the deterministic tracking model. The predicted mean tracking error is compared graphically with the averaged human performance from experimental results.

Formulation of the stochastic model is completed in Chapter VI, where identification of operator noise is reported. Based on Chapter III, the basic tool is analysis of the residual, where remnant is identified as a time series. Parameter estimation of noise series is followed by diagnostic checks on the final residual to verify "white noise" properties and reveal any model inadequacies. Identification of operator tracking variance is reported, where a transfer function dynamic model is proposed with the second difference of target angular velocity as the only input. Performance of the final stochastic tracking model is represented by a number of realizations.
for two flight paths; human operator runs are also included for comparison.

Chapter VII concludes the main body of this thesis with a critical discussion of the formulated model and suggestions for potential areas of future work.

Computer program listings and some sample runs are appendixed. A substantial effort was invested in the programs of Appendix 1. Written after Box and Jenkins (1971), these programs form a sequential routine procedure for the identification and estimation of time series and dynamic models. Finally, Appendix 5 contains a listing of the stochastic tracking model, which is the fruit of this labor.
Chapter II
Background on the Human Operator and
His Mathematical Modelling

Interest in the human operator and the desire to describe his control-behavior quantitatively is a subject that has been demanding increasing interest since World War II. Even before it was possible to describe Systems mathematically, designers and control engineers called upon man to act as a part of a control system. A number of unique qualities in man made such a set up both possible and feasible, for example: ability to take corrective action when explicit knowledge of the problem is not available; unpredicted circumstances can be handled and his ability to work on gross qualitative instructions. Such a practice was generally considered a good design practice, for the human operator has always been available in abundance and at a relatively low cost.

As our knowledge of systems and the ability to describe them mathematically improved, engineers and industrial psychologist became more aware of man's limitations to control; and more efforts were directed to ease his load and enhance his performance. There followed some slackening of research efforts in this field until the advent of the space exploration when man was called upon to exercise his best mental and physical abilities to close crucial control loops.
Thus new life was put in the field of Human Engineering and numerous contributions were made in answer to the demands of space technology.

2.1 Definitions:
As the concern here is with the human performance in tracking and its mathematical description, it is appropriate to define what is meant by the relevant terms and how these are currently used in the literature.

**Tracking:**
In general, tracking is concerned with the application of a short time invariant error criterion intended to minimize some function of the perceived error [Fogel (1963)]. According to Fielding (1963), tracking falls under one of three main tasks which a human may be called upon to perform, namely

a) "precognitive", where the past history is displayed as far in the future as necessary (e.g. driving a car in good condition).

b) "regenerative", where the past history is displayed as far as is useful (e.g. recognizing future parts of a piece of music after hearing the first part).

c) "compensatory", where error or deviation from a desired course is the only information given to the operator (e.g. tracking a target).

Costello (1966) views tracking as one of two basic categories which embrace the variety of systems that depend upon or make use of the human operator; these are "compensatory systems" and "pursuit systems".
Tracking, in general, is either pursuit or compensatory. Compensatory tracking has been studied more extensively than its counterpart, possibly because of its similarity to control problems, where the "controller" has no direct knowledge of its "Target".

**Pursuit Tracking:** In visual pursuit tracking, the display has two moving elements, one representing an actual output and the other the desired output. The operator estimates the error and attempts to nullify it. Fig. 2.1 shows a block diagram of a pursuit system. Examples of such a system are aiming with a shot gun and driving a car.

**Compensatory Tracking:** The display of a visual compensatory tracking contains only one moving element, representing the error, which is the difference between an actual and a desired output. A fixed reference point is displayed and the duty of the operator is to match the moving point with the fixed point whence the error will be zeroed [Fielding (1963)]. Fig. 2.2 is a block diagram of such a system. This type of tracking is encountered in aircraft displays and optically sighted anti-aircraft guns.

### 2.2 Some Characteristics of the Human Operator

A wealth of knowledge is available in the literature documenting characteristics of the human operator as a performer in a control loop. Several books and numerous articles can be found in such fields as Biotechnology, Human Engineering, Industrial Psychology, Experimental Psychology, Systems Engineering, Cybernetics. Space Technology ... etc. L. J. Fogel (1963) provides an extensive bibliography at the end of each chapter in his book, listing what has been published in specific fields.
Fig. 2.1. Visual Pursuit Tracking (Costello 1966).

Fig. 2.2. Visual Compensatory Tracking (Costello 1966).
2.2.1 Physiological Aspects of the Human Operator:

Some of the more important aspects of human physiology relating to tracking, are discussed below.

1. The mechanism by which man performs tracking is now well established (Fielding, 1963). As the operator's eye detects the error, a signal is passed at a speed of up to 300 ft/sec. along the nervous system to the brain and then to motor cells that will operate the muscles in such a way as will reduce the error. The motor system may also send, through its sensory cells, some data to the brain in the form of a feedback that will increase stability.

2. A stimulus must exceed a certain threshold value before the sensory cells will transmit an impulse. In tracking, the human eye has a high sensitivity and can detect misalignments as small as 5 sec. of arc under favorable conditions; below this, the human eye will be in the indifference region. Some models of the operator transfer function have incorporated an indifference threshold, [Fogel (1963) Chapter 9]. Another factor affecting the eye is contrast between target and background or target and graticule, making it easier to detect movements and misalignment errors. As a result of this limitation the human operator does not eliminate completely all tracking errors; although, as Fogel (1963) noted an operator may deliberately overlook small errors when he determines that its elimination is not worth the extra effort, or is fruitless as such errors may be the result of cyclic disturbances or high frequency noise. A similar result is reported by Roig (1962) who observed...
that an operator will zero his error only if the duration between event points is several seconds long.

3. Operator reaction to a stimulus may be one of three possible types [Fielding (1963)]:
   a) reflex action, almost involuntary with only a few millisecs. between stimulus and reaction;
   b) normal reaction, carried out voluntarily and demanding almost total concentration upon stimulus;
   c) automatic reaction, achievable after constant repetition

Fogel (1963) reports that observations on human performance suggest that tracking may be separated into six natural modes of operation as follows:

1. reaction mode, in which there is no motor action;
2. acquisition mode during which the tracker attempts to minimize the error as rapidly as possible, even though overshoot and oscillation may occur. This mode is triggered by large errors or error rates.
3. tracking mode, when errors and error rates are under control, and the intent is to zero the error. Overshoots and oscillations of a frequency proportional to the error and/or error rate are possible.
4. synchronism mode, triggered by perception of some waveform characteristic in the input. The operator attempts to predict the input and track it, while monitoring the gross characteristics of the input.
5. steady state tremor mode, triggered only when the error is within a small steady range.
6. reassurance mode, triggered artificially by the human operator in order to determine system qualities by feedback. It is used during tracking and steady state tremor modes, when a pulse function is inputted to the system and the resulting disturbance is observed to reassure and allow prediction of future maneuvers.

2.2.2 Time Lag:

It is well established now that human operators take a finite time to react to a stimulus. Such terms as delay time, reaction time, dead time and the like are used to mean the time lapse from receiving a stimulus to initiating the intended action. A good deal of work has been done on measuring and analyzing human time delay and breaking it into components that will reflect its origin. Other workers observed that with patterned signals, reaction times are very small or zero; this being explained by man's ability to recognize such patterns and predict them. Due to this pattern recognition by operators it is necessary to experimentally set up devices that will assure random inputs, otherwise tracking performance will be grossly biased.

The first experiments by A. Tustin in 1947 indicated a phase lag and consequently the first model of human tracking performance incorporated a dead time and an integrator terms to take care of this phase lag as described below (Licklider, 1960 and Costello, 1966). Licklider traces the origin of time delay to the quantized human perception into "moments" of about 0.1 sec., added to this is a reaction time and an adjustment duration to make it up to about 0.2 sec., the value commonly used by many researchers. He further reports that reaction time is not
constant; it varies with target velocity (from .26 sec. at high speed to .48 sec. at low speed for one subject) and with the controlled system (in a study of pilot performance it varied from .27 sec. in elevator control to .62 in aileron control). In an extensive research, Tsibulevskii (March, 1967) studied the time delay of ten operators and reported an average value of 0.244 sec. with a standard deviation of .070 sec. He further observed that human delay time does not depend on the magnitude of the signal or the gain factor, but it does depend on the magnitude of response e.g. hand movement, as he reported in a later work (Tsibulevskii, June 1967). A general approximation of human reaction time was proposed by W. E. Hick as

\[ RT = .27 \ln (n + 1) \]

where \( n \) = equiprobable stimuli, any of which may occur, (Fogel, 1963 and Fielding, 1963). For ordinary tracking, \( n = 2 \) (magnitude and direction), giving an average value of about .3 sec. Finally, in a recent study, Watson (1972) reports that results of reaction time measurements for a number of operators in compensatory tracking have consistently given a neuromuscular lag of .1 sec. and a sensory and computational lags of 0.1-0.2 sec.

Other information relevant to time delay includes reduction in reaction time through training by about 10%, and variation with age and sexes, males having a shorter delay to light and sound stimuli than females.
2.2.3 Operator Intermittency

Does the human operator perform in a continuous or in an intermittent manner? This basic question has not been resolved yet, and human performance continues to be equally well modelled as a continuous or as a discrete system. Evidence in the literature, however, gives more weight to the intermittent nature of human control action.

In an effort to identify operator sampling intervals, Bekey and Neal (1968) observed that such intervals were a function of system gain; and, given a correct model, consistently displayed a minimum of about .22 sec. If the model is inadequate, no such a minimum was observed. As we are concerned with visual tracking, the question of intermittency of the visual process becomes important. The human eye reports a continuous motion from moving pictures and television, although stimulation is intermittent; and even at 16 frames per second movies are fairly free of flicker. The maximum an eye can respond to with separation, is at 6 to 7 cps (Licklider, 1960). Psychologists speak of a human refractory phase which is now well established. The idea is that the human operator accepts a segment of input and selects an appropriate response program for it, meanwhile being refractory to additional inputs (Fielding 1963). Licklider reports an experiment by M. A. Vince where she gradually narrowed the time between two signals down to about .5 sec. She observed that response to the second signal was either delayed beyond the normal reaction time or completely neglected. In another study, Young and Stark (1963) formulated a sampled data model of eye tracking movements and observed that the human eye samples at .2 sec intervals. Another source of visual intermittency is shift of fixation
where approximately .2 sec. is needed for this process.

While intermittency of human visual process is well established, it is not clear whether such discontinuities introduce fundamental intermittency into operator characteristics. Quoting Licklider "We have to make room for that possibility.... The alternative appears to be that the nervous system pieces together the data from the retina and the data from the centers that control the eye movements, and uses the pieced together picture as a basis for control of the tracking response. The piecing together would appear to require considerable computation and therefore probably to introduce a time delay." (Licklider, 1960).

In an attempt to resolve this question, Young and Meiry (1965) designed an interesting experiment whereby they replaced a continuous linear control stick by a three mode switch and noted that operator performance improved when the controlled system had more lag than a double integration. They further reported that a simple ON-OFF controller represented operator performance better than a quasi-linear-model did, particularly with unstable systems. As will be noted below, this is the idea behind the surge model of Costello and the dual-mode-model of Planchard et al.

Supporters of the continuous description of the human operator advance such arguments as inertia of controller and controlled elements and that man can and does act as a smoothing filter using the immediate visual stimulus plus a weighted sum of previous stimuli. However, it has been clearly observed in the course of work for this dissertation as well as other researchers that definite cyclic patterns do exist in the human response, with a varying period of 0.5 - 2 sec. Fielding's (1963) report on the periodicity of handle movements in continuous tracking using gun turrets, shows a dominant period of about 2 sec., corresponding
to a cycle of corrective action and removal of corrective action, followed by application and removal of negative correction. Other operators, however, who tracked equally well in the same study did not show such a periodicity. This led Fielding to a compromise conclusion that human operators apparently can work in either a continuous or an intermittent mode. It is possible that any one operator will adopt either mode depending upon the task at hand to achieve a satisfactory result.

2.2.4 Operator Noise: The human operator has a much larger noise element in his behavior than is usually encountered in a machine; so much so that Fielding (1963) suggests that elaborate models are superfluous as refinements would be submerged in noise and, therefore are meaningless. Noise generators are customarily incorporated in operator models in order to handle variations from trial to trial, differences among operators and account for the remnant. The latter deserves some elaboration and has been discussed in a number of papers. Human operator remnant is defined as the portion of the output that is not related to the system input by the input/output describing function (Levison, Baron and Klienman, 1969). Noise incorporation in a model is then a convenient and acceptable tool to account for possible model lack of fidelity as well as factors purposely left out such as operator noise and variation among operators.

A summary of the main finding of such prominent workers in the field as McRuer, Krendal and Elkind (reported by Levison et al., 1969) and Liklieder (1960) is presented below.
1. Remnant is strongly dependent on the order of the controlled system dynamics.

2. The most stable representation of remnant is obtained by referring it to an equivalent observation noise source (i.e. a noise process injected at the operators input).

3. The power spectral density of remnant is a smooth function of the frequency.

4. Remnant, when properly normalized, is relatively insensitive to other control system parameters.

5. In control situations in which system error is the primary input to the human controller (e.g. tracking), variance of the system error is an appropriate normalization factor for the equivalent observation noise process.

6. In general, the random component of the human response tends to increase with the magnitude of the desired response.

7. Operator noise may have its origin in:
   a) true observation noise,
   b) motor noise,
   c) random variation in controller gain and time delay,
   d) effects of a periodic sampling by the human.
   e) "information feedback noise" introduced by the operator to improve his knowledge of the process.

8. The rms of remnant may vary from one operator to another by more than one order of magnitude; while day-to-day and trial-to-trial variations are small.
9. The following assumptions are usually made with operator noise models:
   a) each component of the injected noise is white,
   b) the noise processes are functionally independent of the control system parameters, i.e. they arise from true physiological sources within the operator.

10. Noise ratio varies widely according to model sophistication and task difficulty; values found in the literature range from 1-10% at one end to 20-30% at the other extreme (Bekey, Meissinger and Rose, 1965).

2.3 Mathematical Models of the Human Operator:

   A "model" is a mathematical description of a process, capable of making accurate predictions about the behavior of the process under situations which had not previously been tested; thus does Young's (1969) definition go. He further declares "as of this time, there are no published successful models for all phases of the adaptive characteristics of the human operator", a statement which is true to this time. The importance of modelling manual tracking decisions is emphasized by realizing that these decisions are the same, regardless of whether the system under control is a chemical plant, a machine, a vehicle an anti-aircraft gun or just the human operator's own body; in all cases a stimulus is received and processed to accomplish a series of decisions, intended to minimize or limit some function of the perceived error. In visual tracking, it is generally accepted that the human operator behaves as if trying to minimize a time-averaged sum of the square error (Fogel, 1963).
or a weighted sum of the square of error and error rate, e.g. \((e + .5e)^2\), as proposed by Bekey, et al. (1965) in their learning theory model.

In general, workers in the field have not attempted to model the mechanism of the tracking operator; rather, in keeping with the spirit of the above definition, the intention was to predict the output behavior; although several models appear to include some parameters that can be identified with similar ones in the human operator. Reasons for this seemingly inherent lack of correspondence between man and his model are reported by Fielding (1963) as:

a) difficulty in defining a standard man due to the large variations between one man and another,
b) disagreement between psycho-physiologists as to how a human operator works,
c) modifications to man's behavior brought about by his environment.

In order to make model comparison and evaluation possible, it is necessary to make the following assumptions:

a) operators are standard men,
b) they are fully trained,
c) they are fully motivated,
d) they are giving their full attention to the task under study, undistracted by other visual tasks, and their performance is not hindered by fatigue, stress ... etc.

A survey of the major methods employed in attempting to describe the human transfer function with some representative models follows.
2.3.1 Linear and Quasi-Linear Models

Early attempts at modelling the human operator performance assumed a simple gain plus a time delay (Fogel, 1963):

\[ G(S) = Ke^{-\tau S} \quad (2.3.1) \]

Then during World War II, Tustin, in England studied the problem of tracking with tank turrets in an attempt to improve the system. At the same time Phillips at MIT was studying the performance of ground control of anti-aircraft fire and the design of rate-aiding controls. In 1947 they published their work separately and introduced the first quasi-linear model, based on the rationale that the operator would move his hand at a rate proportional to the error and with a displacement proportional to the error displacement (Licklider, 1960). This resulted in a model of the form

\[ \frac{dr}{dt} = C_1 E(t) \]

and

\[ r(t) = C_2 E(t) \quad (2.3.2) \]

where \( r \) = hand position, and \( E \) is the error. With time delay and gain added, the resulting model is

\[ \frac{dr}{dt} = [C_1 E(t) + C_2 \frac{dE}{dt}] Ke^{-\tau S} \]

or

\[ \frac{R(S)}{E(S)} = \frac{K(1 + TS)}{S} e^{-\tau S} \quad (2.3.3) \]

which is the conventional form of the most of the quasi-linear class of models. (Tustin actually reported his model as \( G(S) = [AS+B+C/S] e^{-\tau S} \)).
A more recent form of such models is frequently reported as:

\[ G(S) = \frac{K(T_L s + 1)}{(T_N s + 1)(T_1 s + 1)} e^{-T_2 s} \]  \hspace{1cm} (2.3.4)

where: \( K \) = gain factor, \( T_L \) = lead compensation time constant, (usually .25-2.5 sec.), \( T_N \) = neuromuscular time lag (normally .1-.16 sec), \( T_1 \) = lag time constant (usually from 5 to 20 sec.). The inclusion of reaction time does not appear necessary, as some recent papers indicate that models fitted the data better without dead time. H.P. Bergeron (1970), reports this model:

\[ G(S) = \frac{K_1 (T + K_2 s)}{(T + s)^2} \]  \hspace{1cm} (2.3.5)

By far, quasi-linear models are the most investigated and experimented with among human tracking models.

2.3.2 Adaptive Models

The most important forms of manual adaptive behavior are, as illustrated in Fig. 2.3 (Young, 1969):

a) Input adaptation and prediction which refers to man's ability to recognize repeated input patterns and track these in an open loop manner;

b) Control element adaptation, which refers to the ability to change control strategy when system dynamics change;

c) Task adaptation, which refers to the ability to optimize the control loop according to various control objectives.

A different strategy will be used for the same input and controlled elements, depending on whether the desired
Fig. 2.3. Adaptive Model of Young (1969).
Fig. 2.4. Adaptive Model of Phatak and Bekey (1969).
cost function minimizes the error, time, fuel or control effort;

d) Programmed adaptation, which refers to adaptation to what the operator has learned of change in strategies in the face of environmental changes (wind direction, road conditions ... etc.), or signals to stop and go, ... etc. An example of adaptive model is that of Wertz (Costello, 1966), which is linear and fixed in form but has two adjustable parameters that will enable adaptation:

\[ H(S) = \left[ e^{-\frac{T_x S}{T_n S + 1}} (T_1 S + 1) \right] \times \left[ K_o (a_o S + 1) \right] \]  (2.3.6)

constant term + variable term +

Another model of Phatak and Bekey (1968) is illustrated in Fig. 2.4.

2.3.3 Sampled Data Models

A number of sampled data models have been suggested, based on the concepts of operator intermittency and the psychological refractory period discussed above. Hick first proposed a sampled data model in 1958 for his Ph.D thesis. Bekey observed a minimum sampling interval of about .25 sec., or a rate of 4 cps and suggested the model of Fig. 2.5 (Bekey and Neal, 1968). Young and Stark (1963) studied eye movements and proposed a sampled data model for its gaze during tracking; their model is shown in Fig. 2.6.
Fig. 2.5. Sampled Data Model of Bekey (Bekey and Neal, 1968).
Figure 2.6. Sampled Data Model of Young and Stark (1963).
2.3.4 Stochastic Models

The stochastic nature of the human describing function is evidenced by run to run variability, inter-operator differences and measurement noise of such inputs as errors and error rates. Relatively little work has been done to produce a working stochastic model of the human operator, as the usual procedure has been to produce a quasi-linear model to account for the "mean" or deterministic portion of operators output and leave the stochastic portion to scattered treatments of "remnant".

An interesting exception is a paper by Preyss and Meiry (1968) based on a Sc.D. dissertation by the former, in which a stochastic model of the human learning behavior is formulated based on information theory. The model treats the stochastic nature of human information processing as a sequential operation of three subsystems: the sensor, the decision center, and the effector. The model is said to posses "individuality" through a set of read-in parameters; while other sets predict the various learning stages of the individual operator, until, after about fifty training runs, the model is fully trained and is capable of reproducing the output of an experienced man. Compensatory tracking is included among the tasks that the model can learn. In an earlier work, Fogel (1963, Sec. 9.4) discussed a stochastic tracking model where the human information transfer system is subdivided into: (a) vision-intermittent observer, (b) perception-amplitude rate quantiser, (c) decision-delayed anticipator and comparative and rules of choice selector, and (d) motor action-1st order hold and arm and control kinematics. A statistical treatment and a description of the
human decision making in tracking the altitude of an aircraft are included.

2.3.5 Multi-mode Models

The six modes of Fogel (1963) to describe an operators response to a stimulus in a tracking task, as discussed above, are: reaction, acquisition, tracking, synchronism, steady state tremor and reassurance. These provided an indication of a possible subdivision of the human operator's describing function into a number of mutually exclusive modes, each corresponding to a particular region on the error-error rate phase plane. Potential advantages of such an approach would be a lower order transfer function for the individual modes and the ability to handle discontinuous inputs. Costello (1966 and 1968) successfully tried a two-mode model, incorporating acquisition and tracking modes in what he termed a "surge model" and demonstrated a remarkable stability in the face of discontinuous and high frequency inputs. A block diagram of Costello's model is shown in Fig. 2.7. The constant coefficient-tracking mode uses a conventional quasi-linear transfer function:

\[
G_p(S) = \frac{K_p e^{-TS} (1 + T_L S)}{(1 + T_N S)(1 + T_I S)}
\]

(2.3.5)

the different symbols have the meaning and values of Table 2.1. The surge-acquisition mode employs a minimum-time forcing function of maximum acceleration and declaration, and is activated by the mode selector in the diagram, when the perceived error and/or error rates are outside the boundaries of a phase-plane plot. The loop is opened at the contactor in Fig. 2.7 and with the assumed second order limb
Figure 2.7. The Two Mode Surge Model of Costello (1966, 1968).
dynamics, the control is of a bang-bang type corresponding to a maximum effort or minimum time controller.

The success which Costello had with his surge model suggested to Planchard et al., (1970, 1972 and 1973) and Parkins (1974) to use the approach in a more realistic environment, where the controlled element is a second order simulation of an anti-aircraft gun system. Following serious modification and tuning of parameters, the model was used with remarkable success in an extensive research and analysis program to simulate different anti-aircraft artillery. The hybrid simulation study, with human operators in the loop used target data of actual high speed maneuvering aircrafts simulating combat missions, which made the study distinctly different from most other works using artificially produced target signals. In a series of reports (Planchard et al., 1970, 1972 and 1973) the study demonstrated the value of the method in having one set of parameters of the human operator model that was capable of tracking widely differing target paths using a number of controlled systems (anti-aircraft guns). The model is shown in block diagram form in Fig. 2.8, and Table 2.1 below compares the coefficients in Costello's and Planchard's models. More will be said about the performance of Planchard's model since the same data was used to develop the time series model of this dissertation.
Fig. 2.8. Tracking Model of Planchard et al (1970, 1972) and Perkins (1974).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Costello</th>
<th>Planchard et al. and Perkins</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_p$ = Human Operator Transfer function</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_1$ = Human operator lag time constant, sec.</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>$T_L$ = Human operator lead time constant, sec.</td>
<td>.5</td>
<td></td>
</tr>
<tr>
<td>$T_N$ = Neuromuscular lag time constant, sec.</td>
<td>.20</td>
<td>.20</td>
</tr>
<tr>
<td>$\tau$ = Human operator delay time sec.</td>
<td>0.15</td>
<td>.15</td>
</tr>
<tr>
<td>$K_p$ = gain constant</td>
<td>35</td>
<td>20</td>
</tr>
<tr>
<td>$P_1$ = empirical weight factor, error rate</td>
<td></td>
<td>1.4259</td>
</tr>
<tr>
<td>$P_2$ = empirical parameter</td>
<td></td>
<td>.1103</td>
</tr>
<tr>
<td>$P_3$ = empirical weight factor, target angular vel.</td>
<td></td>
<td>12.5257</td>
</tr>
</tbody>
</table>

[Surge model activating criterion]

$$a_1 |E(t)| + a_2 |\dot{E}(t)| > m_o$$

| $a_1$ | Positive error weight factor | 1.0 | 1.0 |
| $a_2$ | Positive error rate weight factor | .01 | .5  |
| $m_o$ | Positive constant        | .25 | 50  |
Chapter III

Time Series Method of Analysis

A sequence of observations on a process, a system or a phenomenon that is moving in time will generate an ordered set of data which is called a time series. This may be discrete or continuous; simple, consisting of a single observation at each moment of the discrete or continuous base, or multiple, consisting of a number of observations referred to a time base common to all. Time series method of analysis refers to the study of such sequences with respect to the statistics of their distribution in time. Situations from which time series may arise are many and varying, e.g., stock market prices, business cycles, rainfall, telephone conversations, radio signals and industrial processes. Observing the feed flow to a reactor gives a simple time series, while observing the feed, temperature and conversion results in a multiple time series.

In this chapter, the basic concepts underlying the analysis of time series are first introduced together with the necessary definitions and notations. This is followed by a presentation of one method for the modeling of dynamic systems by identifying the transfer function of such systems. Unless otherwise referenced, this chapter is based on the comprehensive work of Box and Jenkins (1971) in their text which was first published in 1970, and elaborations upon it in the dissertation of David A. Pierce (1968).
3.1 **Historical Development**

A sequence of observations, \( y_t \), which showed variation in time was classically treated as consisting of a trend, a seasonal movement and a random fluctuation of the form:

\[
y_t = m_t + a_t \quad (3.1.1)
\]

with \( m_t \) as a polynomial, a sine or cosine, or a combination of these. The residual, \( a_t \), was treated as uncorrelated random noise. As early as 1898, Schuster used this method in investigating meteorological phenomena. Weakness of this model was demonstrated by Yule in 1921 when, starting from purely random, uncorrelated shocks, he produced series of regular oscillations and fixed differences. In 1927, Slutsky found that such series can produce sinusoidal waves very similar to business and economic cycles, in their slowly changing amplitude and phase, suggesting that observed time series may have been generated by purely random processes. The first test of the idea followed shortly when Yule successfully modeled the classical, centuries-old data of sun spot activities index, as a second-order auto-regressive process. The "Yule" process, as it is generally known, considers each observation at time \( t \), to be the result of a linear combination of the preceding two observations plus a random component \( a_t \), i.e.,

\[
y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + a_t \quad (3.1.2)
\]
The following decade witnessed great advances in the field as scientists and economists modeled their systems as auto-regressive and/or moving average processes, as defined in Section 3.2.

The next break-through was achieved by Wiener (1950) who conceived a time series model as a filtering process to screen the corrupting noise from the signal. Working during the Second World War, he developed the methodology to apply the concepts that had been well established in economics and other fields to communication engineering, by treating a signal as a sequential realization in time, whose observed value is distorted by random noise. Later developments were in the treatment of stochastic processes, as Bartlett (1955) did, and in identification of time series models by spectral analysis as in the work of Jenkins (1965) and Jenkins and Whats (1969).

The work of Box and Jenkins (1971) mentioned above presents a "comprehensive unified treatment" of the subject, demonstrating its applicability to both fields of control engineering and economics and natural phenomena, for which two general classes are proposed, auto-regressive integrated moving average and dynamic models. These will be discussed after introducing some basic definitions.

3.2 Linear Stationary Models

The two general classes of time series models, ARIMA - or auto-regressive integrated moving average, and dynamic models are developed from the simple forms of stationary time series such as the Yule process mentioned earlier. These simpler models are now discussed together with some of their important features. It is necessary to mention that "stationary" is used here to refer to a time series whose
Probability distributions are unaffected by shifts in the time origin (Box and Jenkins, 1971, Chapter 3). Such a series will possess a natural mean around which it fluctuates.

3.2.1 The Auto-regressive Process - (AR):

When the observations of a time series are interdependent, such that each is a weighted sum of past observations, the process is regressed on itself, or autoregressive, and is represented by

\[ y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + a_t \]  

(3.2.1)

where the first \( p \) weights only are considered significant and the process is described as of order \( p \), or AR(\( p \)); \( a_t \)'s represent a series of random deviates, whose mean = 0 and variance = \( \sigma_a^2 \). The Markov process is a first-order auto-regressive process, AR(1)

\[ y_t = \phi y_{t-1} + a_t \]  

(3.2.2)

and is very important in practice. The second-order auto-regressive process was encountered in the preceding section as AR(2)

\[ y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + a_t \]  

(3.2.3)

which is called the Yule process. The auto-correlation function of an AR(\( p \)) is of special significance. At lag \( k \), it is obtained by first multiplying (3.2.1) throughout by \( y_{t-k} \) to obtain
\[ y_{t-k}y_t = \phi_1 y_{t-k}y_{t-1} + \phi_2 y_{t-k}y_{t-2} + \cdots + \phi_p y_{t-k}y_{t-p} + y_{t-k} \ a_t \]  

(3.2.4)

Upon taking expectations, a difference equation of the auto-covariance results:

\[ \gamma_k = \phi_1 \gamma_{k-1} + \phi_2 \gamma_{k-2} + \cdots + \phi_p \gamma_{k-p} \quad k > 0 \]  

(3.2.5)

The autocorrelation is now readily available upon division by \( \gamma_0 \) as

\[ \rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} + \cdots + \phi_p \rho_{k-p} \]  

(3.2.6)

The very important result is noted that this difference equation is of the same form and parameter set as the process \( y_t \), a fact which provides a vital tool in the identification of time series. It can be shown (e.g., Box and Jenkins (1971) Sec. 3.2) that the auto-correlation function of a stationary auto-regressive process will, in general, consist of a mixture of damped exponentials and damped sinewaves.

As the auto-correlation function of an AR(p) will extend to infinity, another device is needed to establish the order of the process, this is the partial auto-correlation function. A detailed discussion is not warranted here, reference is made to the above text (Box and Jenkins, 1971, Chapter 3), but it is sufficient to note the fact that an AR(p) can be expressed in terms of "p" non-zero functions, so that the "partial auto-correlation function" \( \phi_{kk} \) will be non-zero.
for $k \leq p$ and zero for $k > p$. For example,

$$\phi_{11} = \rho_1$$

$$\phi_{22} = \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2}$$

It follows that for an AR(p), the partial auto-correlation function will cut off after $p$ functions.

A widely used method of expressing auto-regressive processes is to write them in compact operator forms, for which sake the backward shift operator, $B$, is introduced as

$$B y_t = B(y_t) = y_{t-1}$$

$$B^2 y_t = B^2(y_t) = y_{t-2}$$

Or in general,

$$B^j y_t = y_{t-j}$$

The generalized auto-regressive operator is used to express the model of (3.2.1) as

$$(1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p) y_t = a_t$$

or

$$\phi(B) y_t = a_t$$

The last expression can also be written as
which implies that an AR process can be produced by linear filtering of white noise $a_t$ with a transfer function $\phi^{-1}(B)$. (It will be assumed throughout that operators possess algebraic cumulative, distributive and associative properties. Proof may be found in any standard text, for example, Ketter and Prawel, 1969, pp. 197-204.)

3.2.2 The Moving Average Process - MA

Another model of practical importance in time series representation is the moving average process, where the present value is expressed as a weighted sum of previous shocks

$$y_t = \theta(B)a_t$$

or in operator form

$$y_t = \theta (B)a_t$$

The auto-correlation function of a moving average series is more involved than that of an auto-regressive one, but can be obtained in a similar treatment. Multiplying throughout by $y_{t-k}$, and taking expectations result in

$$\gamma_k = E \left[ (a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q}) y_{t-k} \right]$$

The observations at lag $k$, $y_{t-k}$, may be expressed in terms of the preceding shocks
\[ \gamma_k = E[a_{t-k} a_{t-k-1} \cdots a_{t-q}] (a_{t-k-q} \cdots a_{t-q}) \]

since the random shocks \( a_t \) are uncorrelated, the expectation:

\[ E[a_{t-1} a_{t-j}] = \begin{cases} \sigma_a^2, & i=j, \\ 0, & i \neq j \end{cases} \]

When \( k=0 \), the variance is obtained as

\[ \gamma_0 = (1+\theta_1^2+\theta_2^2 + \cdots + \theta_q^2) \sigma_a^2 \]

and the covariance is

\[ \left( -\theta_k + \theta_{k+1}^2 \theta_k + \theta_{k+2}^2 \theta_k + \cdots + \theta_q \theta_k \right) \sigma_a \]

and the auto-correlation function is

\[ \rho_k = \frac{\gamma_k}{\gamma_0} = \begin{cases} \frac{-\theta_k + \theta_{k+1}^2 \theta_k + \cdots + \theta_q \theta_k}{1+\theta_1^2+\cdots+\theta_q^2} & \text{for } k = 1,2,\ldots,q, \\ 0 & \text{for } k > q \end{cases} \]

The interesting result is now evident, that the auto-correlations of a MA(q) process are zero beyond lag \( k \), or the function cuts off at lag \( k \), which is of great help in identifying time series. As an example, the MA(1) process is:

\[ y_t = a_t - \theta_1 a_{t-1} = (1-\theta_1 B) a_t \]

whose variance is, from \((3.2.14)\)
and the auto-correlation function is

$$\rho_k = \begin{cases} \frac{-\theta_1}{1+\theta_1^2} & k = 1 \\ 0 & k \geq 2 \end{cases} \quad (3.2.17)$$

The partial auto-correlation function for a MA(1) process is more lengthly to derive, but can be shown to be [Box and Jenkins, 1971, Chapter 3].

$$\phi_{kk} = \frac{-\theta_1^k(1-\theta_1^2)}{(1-\theta_1^2(k+1))} \quad (3.2.18)$$

which is dominated by a damped exponential.

3.2.3 The Mixed Auto-Regressive-Moving Average Process

The auto-regressive process of (3.2.11) can be expanded, for a first order series, to give

$$y_t = a_t + \theta_1 a_{t-1} + \theta_1^2 a_{t-2} + \cdots \quad (3.2.19)$$

Similarly, a first-order moving average process as expressed in (3.2.13) after arrangement and expansion becomes

$$y_t = -\theta_1 y_{t-1} - \theta_1^2 y_{t-2} + \cdots + a_t \quad (3.2.20)$$

which demonstrates the duality between the two processes. What this amounts to is that an AR(1) process which can be represented with a finite number of terms in an auto-regressive model, can also be
expressed as an infinite sum in a MA(1) process. Similarly, a MA(1) 
system can be represented as an infinite AR(1) series. This suggests 
that it may be necessary sometimes to model a process as a combination 
of both an auto-regressive-moving average time series, thus:

\[ y_t = \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q} \]

or, in operator form

\[ \phi(B)y_t = \theta(B) a_t \]  \hspace{1cm} (3.2.21)

which may be rearranged to

\[ y_t = \phi^{-1}(B)\theta(B)a_t \]

\[ = \frac{\theta(B)}{\phi(B)} a_t = \frac{1-\theta B-\cdots-\theta B^q}{1-\phi B-\cdots-\phi B^p} a_t \]  \hspace{1cm} (3.2.22)

suggesting that the mixed ARMA process can be thought of as the output 
\( y_t \) from a linear filter whose input is white noise \( a_t \), and the transfer 
function is the ratio of the two polynomials \( \theta(B) \) and \( \phi(B) \).

The auto-correlation function may be derived by a similar method 
to that used with simple processes. For the process in (3.2.21), an 
ARMA(p,q), i.e., pth order auto-regressive, qth order moving average, 
this is done by multiplying throughout by \( y_{t-k} \) to obtain the auto-
covariance function difference equation

\[ \gamma_k = \phi_1 \gamma_{k-1} + \cdots + \phi_p \gamma_{k-p} + \gamma_y(k) - \theta_1 \gamma_y(k-1) - \cdots - \theta_q \gamma_y(k-q) \]  \hspace{1cm} (3.2.23)
where the cross covariance function between \( y \) and \( a \), \( \gamma_{ya} \) is defined as

\[
\gamma_{ya} = E[y_{t-k}a_t]
\]

and, since \( y_t \) is unaffected by future shocks of \( a_t \):

\[
\gamma_{ya}(k) = 0 \quad k > 0
\]

\[
\gamma_{ya}(k) \neq 0 \quad k \leq 0
\]

whence (3.2.23) becomes

\[
\gamma_k = \phi_1 \gamma_{k-1} + \phi_2 \gamma_{k-2} + \cdots + \phi_p \gamma_{k-p} \quad k \geq q + 1
\]  

(3.2.24)

When \( k = 0 \), the variance is obtained:

\[
\gamma_0 = \phi_1 \gamma_1 + \cdots + \phi_p \gamma_p + \sigma^2_a - \theta_1 \gamma_{ya}(-1) - \cdots - \theta_q \gamma_{ya}(-q)
\]

(3.2.25)

and the auto-correlation function is now available as

\[
\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} + \cdots + \phi_p \rho_{k-p} \quad k \geq q+1
\]

(3.2.26)

The behavior of this function is interesting as Box and Jenkins (1971, Sec. 3.4) have shown. When \( q - p < 0 \), the function will consist of a mixture of damped exponentials and/or damped sinewaves, while if \( q - p \geq 0 \), the first \( q - p + 1 \) values will not follow such patterns.

Finally, to conclude the discussion of the ARMA process, the simple, but practically important ARMA(1,1) is presented as

\[
\gamma_t - \phi_1 \gamma_{t-1} = a_t - \theta_1 a_{t-1}
\]
or

\[(1 - \phi_1 B) \gamma_t = (1 - \theta_1 B) \alpha_t \qquad (3.2.27)\]

The auto-correlation function follows from (3.2.25):

\[
\rho_1 = \frac{(1 - \phi_1 \theta_1) (\phi_1 - \theta_1)}{1 + \theta_1^2 - 2\phi_1 \theta_1}
\]

\[
\rho_2 = \phi_1 \rho_1
\]

and in general

\[
\rho_k = \frac{\gamma_k}{\gamma_0}
\]

where

\[
\gamma_0 = \frac{1 + \theta_1^2 - 2\phi_1 \theta_1}{1 - \phi_1^2} \sigma_a^2
\]

\[
\gamma_k = \phi_1 \gamma_{k-1} \quad \text{for} \quad k \geq 2 \qquad (3.2.28)
\]

A computer program is available for the iterative calculation of these functions as will be discussed in subsequent sections.

3.3 Linear Non-Stationary Processes

In the introduction to the last section, a stationary time series was described as one whose probability distributions are unaffected by shifts in the time origin. This may be interpreted to mean that, as time moves on, the series remains in equilibrium about some natural constant mean. In contrast, a non-stationary time series does not possess a constant mean. A special class of such processes is referred
to as the ARIMA, Auto-Regressive-Integrated-Moving-Average process, which will be treated in what follows, first in a simple no-noise form, then with a noise component incorporated into it.

It will be necessary beforehand to introduce another notation, the backward difference operator, \( \nabla \) as

\[
\nabla y_t = y_t - y_{t-1} = (1 - B)y_t
\]

and

\[
\nabla^2 y_t = (y_t - y_{t-1}) - (y_{t-1} - y_{t-2})
\]

\[
= y_t - 2y_{t-1} + y_{t-2} = (1 - B)^2 y_t
\]

or, in general

\[
\nabla^d = (1 - B)^d \tag{3.3.1}
\]

Since non-stationary series are not required to have a constant mean, this will admit explosive time series in addition to different other types. A special case will result if the restriction is imposed that such series shall exhibit stationarity after some degree of differencing, \( d \); giving rise to what is referred to as homogeneously non-stationary processes to be discussed next.

3.3.1 The Homogeneous ARIMA Process

Maintaining the above terminology, the general ARIMA process of order \( (p,d,q) \) may be written as

\[
\phi(B) \nabla^d z_t = \theta(B) a_t \tag{3.3.2}
\]
where

\[ \phi(B) = 1 - \phi_1 B - \cdots - \phi_p B^p \]

\[ \theta(B) = 1 - \theta_1 B - \cdots - \theta_q B^q \]

and \( z_t \) stands for discrete observations on a non-stationary time series, and \( a_t \) a sequence of independent normal deviates, with mean zero and variance \( \sigma_a^2 \), as previously discussed. The homogeneity requirement would mean that the \( d \)th difference of \( z_t \):

\[ y_t = \nabla^d z_t = (1 - B)^d z_t \]

follows an ARMA \((p,q)\)-mixed \( p \)th order auto-regressive, \( q \)th order moving average stationary process.

Other frequently employed notations in representing ARIMA models include the generalized auto-regressive operator

\[ \Phi(B) = (1 - B)^d \phi(B) \]

rendering (3.3.2) in this form:

\[ \Phi(B)z_t = \Theta(B)a_t \quad (3.3.3) \]

Alternatively, putting \( y_t = \nabla^d z_t \),

\[ \pi(B) = \phi(B)\Theta^{-1}(B) \]

and
\[
\psi(B) = \theta(B)\phi^{-1}(B) = \pi^{-1}(B),
\]

(3.3.2) becomes
\[
\gamma_t = \psi(B)a_t = \pi^{-1}(B)a_t \tag{3.3.4}
\]

Also,
\[
\phi(B)\gamma_t = \theta(B)a_t \tag{3.3.5}
\]

where the last form reduces the process \( z_t \) to the familiar ARMA series.

In general, for a process in the form of (3.3.5) to be stationary, it is required that the roots of the polynomial equation, \( \phi(u) = 0 \), lie outside the unit circle; implying therefore that the general process of (3.3.2) is non-stationary for \( d > 0 \). On the other hand, if the polynomial equation \( \phi(u) = (1-u)^d\phi(u) = 0 \) is considered, then \( d \) of the roots will be on but not inside the unit circle, ensuring a homogeneous non-stationarity, rather than an explosive one. This is for the autoregressive operator. As for the moving average operator, a similar requirement is imposed, namely that the roots of \( \theta(u) = 0 \) lie outside the unit circle, to insure that current values of the series do not depend overwhelmingly on the past history of the process. This is what Box and Jenkins (1971) refer to as the property of invertibility.

The ARIMA \((1,1,1)\) is a special case, corresponding to \( p = 1, d = 1, q = 1 \)

\[
\nabla z_t - \phi_1 \nabla z_{t-1} = a_t - \theta_1 a_{t-1}
\]
or
\[(1 - \phi_1 B)\nabla z_t = (1 - \theta_1 B)a_t \quad (3.3.6)\]

where
\[
\phi(B) = 1 - \phi_1 B \quad \text{and} \quad \theta(B) = 1 - \theta_1 B
\]

The integrated moving average process is another widely used model being an ARIMA \((0,1,1)\)

\[
\nabla z_t = a_t - \theta_1 a_{t-1}
\quad (3.3.7)
\]

\[
= (1 - \theta_1 B)a_t
\]

corresponding to \(p = 0, d = 1, q = 1,\)

\[
\phi(B) = 1, \quad \theta(B) = 1 - \theta_1 B.
\]

A constant term is added to the right-hand side of an ARIMA model to indicate a deterministic drift or slope, thus

\[
\phi(B)\nabla^d z_t = \theta_0 + \theta(B)a_t \quad (3.3.8)
\]

when the last expression is written as

\[
\phi(B)\nabla^d z_t = \theta(B)e_t \quad (3.3.9)
\]

as commonly found, the shocks \(e_t\) will have a non-zero mean with an expected value \(E[e_t] = \theta_0/1-\theta\), (Box and Jenkins, 1971, Section A4.2).

3.3.2 **The ARIMA Process with Added Noise**

An important characteristic of the ARIMA process will be discussed now, concerning what happens when external noise, such as observation
noise, is added. It will be necessary to establish first that the sum of two independent moving average processes is another moving average process, whose order is equal to the higher of the two component orders. For example,

\[ W_t = \theta_1(B) a_t + \theta_2(B) b_t \]  \hspace{1cm} (3.3.10)

where \( \theta_1 \) and \( \theta_2 \) are of orders \( q_1 \) and \( q_2 \) respectively, and \( a_t \) and \( b_t \) are independent white noise processes with zero means. If \( q = \max(q_1, q_2) \) then the auto-covariance function \( \gamma \) for \( W_t \) must be zero for \( j > q \). It follows that a series exists that will represent \( W_t \) as a single moving average process of order \( q \).

Referring to the general model of (3.3.2), if the message \( z_t \) is not available, but instead, the corrupted signal \( z_t = z_t + b_t \), where \( b_t \) is some observation or measurement noise which may be correlated, then the model can be written as

\[ \phi(B)V^d(z_t-b_t) = \theta(B)a_t \]

or

\[ \phi(B)V^d z_t = \theta(B)a_t + \phi(B)V^d (b_t) \]  \hspace{1cm} (3.3.11)

If the noise process \( b_t \) is modeled as an ARIMA \((p_1, 0, q_1)\) series

\[ \phi_1(B)b_t = \theta_1(B)\alpha_t \]  \hspace{1cm} (3.3.12)

where \( \alpha_t \) is white and independent of \( a_t \), then the model is

\[ \phi_1(B)\phi(B)V^d z_t = \phi_1(B)\theta(B)a_t + \phi(B)\theta_1(B)V^d \alpha_t \]  \hspace{1cm} (3.3.13)

or.
\[ \phi_2(B) V^d z_t = \theta_2(B) u_t \]

where \( u_t \) is white, and \( z_t \) is now of order \( P = p + p, d, \) and \( Q = \max(p, q, p + q_1 + d) \). Hence, the effect of added correlated noise to an ARIMA process is another ARIMA process of a different order.

A special case of considerable use in practice results when the added noise is white, an assumption which is often made. In this case,

\[ \phi_1(B) = \theta_1(B) = 1 \]

in (3.3.13) which reduces to:

\[ \phi(B) V^d z_t = \theta_2(B) u_t \]

where

\[ \theta_2(B) u_t = \theta(B) a_t + \phi(B) V^d b_t \]

which is of order \( (p, d, Q) \), where \( Q = \max(q, p + d) \). The important result is obtained that if \( (p + d) \leq q \), the order of the process will not change by adding noise; moreover, the values of the \( \phi \) parameters will not be affected; only the \( \theta \)s will change.

3.4 Dynamic Models

Dynamic models are representations of systems with controllable inputs, and the output can be expressed in terms of a transfer function operating on the input. In general, the process is infected by disturbance or noise which can be represented by some form of an ARIMA model. A simple example will demonstrate the link between classical control engineering techniques of modeling by differential-integral equations and the difference equation form proposed by Box and Jenkins (1971);
with the key idea that the input itself can be programmed as an ARIMA time series.

Considering the control situation where the rate of change of the output, \( y \), is proportional to the difference between input \( x \), and output, the differential equation form is:

\[
\frac{dy}{dt} = \frac{1}{T} (x - y)
\]

Using the differential operator, \( D \), this is

\[(1 + TD)y(t) = x(t)\]

In the more general case, some gain \( k \), will be present

\[(1 + TD)y = kx \tag{3.4.1}\]

When observations are made on \( x \) and \( y \) at discrete time intervals \( t = 0, 1, 2 \), the equivalent expression becomes

\[(1 + \xi V)y_t = kx_{t-1} \tag{3.4.2}\]

where \( x_{t-1} \) stands for the constant value of the controlled input from time \( t-1 \) to \( t \), and \( \xi = (e^{1/T}-1)^{-1} \) (Box and Jenkins, 1971, Chapter 7).

When it is more convenient to write the model of (3.4.2) in terms of past values of the output, we may substitute \( B = 1 - V \) so that, after rearrangement

\[y_t = \frac{\omega_0}{1 - \delta B} x_{t-1} \tag{3.4.3}\]

where \( \delta = e^{-1/T} \) and \( \omega_0 = k(1-\delta) \). When there is delay in the system and the output lag by, say, \( b \) time units behind the input, this is easily incorporated as
\[ Y_t = \frac{\omega_o}{1-\delta B} x_{t-b-1} \]  \hspace{1cm} (3.4.4)

which is the difference form model for a first-order system with delay.

As for higher order systems, a general form may be written as

\[ Y_t = \frac{\omega(B)}{\delta(B)} x_{t-b-1} \]  \hspace{1cm} (3.4.5)

where

\[ \delta(B) = 1 - \delta_1 B - \cdots - \delta_r B^r \]
\[ \omega(B) = \omega_0 B^s - \cdots - \omega_s B^s \]

and the system is linear. It is also useful to think of the system as a linear filter

\[ v(B) = v_0 B^s - \cdots - v_s B^s \]  \hspace{1cm} (3.4.6)

with \( v(B) = v_0 B^s - \cdots - v_s B^s \) and the first \( b \) weights \( v_0, v_1, \ldots v_{b-1} \) are zero. Finally, process noise, commonly present in systems of practical value can be represented by an auto-regressive-moving average process

\[ N_t = \frac{\theta(B)}{\phi(B)} a_t \]  \hspace{1cm} (3.4.7)

which is linearly added to the deterministic component of the model to make up the general stochastic dynamic model when general form is:

\[ Y_t = \frac{\omega(B)}{\delta(B)} x_{t-b-1} + \frac{\theta(B)}{\phi(B)} a_t \]  \hspace{1cm} (3.4.8)
3.5 **Modeling of Time Series**

The basic modeling philosophy as presented by Box and Jenkins (1971) and summarized by Pierce (1968) is a three-stage procedure involving identification, estimation and diagnostic checking. These will be discussed briefly.

### 3.5.1 Identification

Identification is a rough procedure, utilizing available knowledge of the physical system, and applying it to the set of data to indicate the form of a model worthy of further investigation. In other words, the aim is to select an applicable subclass of the general ARIMA \((p,d,q)\) family which may be written as

\[
\phi(B)\nabla^d z_t = \theta_o + \theta(B)a_t
\]

(3.5.1)

where \(p,d,q\) are tentatively known. Chapter 4 of Box and Jenkins (1971) is devoted to a detailed discussion of this stage of modeling time series, and as it is the case with the rest of this chapter, the following presentation is a review of their work.

The basic tool in the identification of time series is the auto-correlation function. The rationale behind it is that for a stationary normal time series—with zero mean, knowledge of the theoretical variance and auto-correlation function completely specifies the process (Pierce, 1968). Since theoretical values are not available, the question will arise as to how good an estimate are the sample correlations? Bartlett (above reference) showed that to order \(\frac{1}{n}\),

\[
E(r_k) = \rho_k
\]

(3.5.2)
where \( n \) = number of observations, \( r_k \) = sample auto-covariances and \( \rho_k \) = theoretical auto-covariances at lag \( k \). The case of the variance is similar. Armed with this argument, the auto-correlation model of (3.2.6)

\[
\rho_k = \phi_1 \rho_{k-1} = \phi_2 \rho_{k-2} + \cdots + \phi_p \rho_{k-p}
\]

will be rewritten as

\[
\phi(B) \rho_k = 0 \tag{3.5.3}
\]

For a stationary process, the roots of \( \phi(B) = 0 \) lie outside unit circle, implying that the auto-correlation function will die out fairly quickly for moderate to large \( k \). Understandably, this will not be the case if any of the roots lies close to the boundary. For a homogeneous, non-stationary process, as mentioned under 3.3.1, \( d \) of the roots will lie on the unit circle, and the function will not die out quickly; but this also suggests the possibility that the first or some \( d \)th difference of the series will be stationary.

The procedure is then to evaluate the auto-correlation function of the series \( z_t \) and its first and second differences (as \( d \) is rarely higher than 2), and expect that the function will die out quickly. Furthermore, as stated above, whereas the auto-correlation function of a stationary AR(\( p \)) process will tail off and die out, its partial auto-correlation will cut off after \( p \) lags. On the other hand, the auto-correlation function of a MA(\( q \)) process has a cutoff after lag \( q \), while its partial auto-correlation tails off. For this reason, the identification will be greatly enhanced by evaluating the partial auto-correlation
function in addition to the auto-correlation function, the first twenty values of each are usually sufficient. If the process is a mixed ARIMA (p,d,q), the auto-correlation function of its stationary dth difference will be a mixture of exponentials and damped sinewaves after the first q - p lags, and its partial auto-correlation function will exhibit such a behavior after q - p lags. (Other statistical tests for stationarity are also available, e.g., Himmelblau, 1970, Chapter 3.)

The standard errors of the estimated sample auto-correlations and partial auto-correlations are of importance in deciding on whether the function has died out or cut off. Bartlett's formula is suggested for auto-correlation functions (Box and Jenkins, 1971):

\[ \sigma[\hat{r}_k] \approx \frac{1}{n^{1/2}} \left\{ 1 + 2(r_1^2 + r_2^2 + \ldots + r_q^2)^{1/2} \right\} \quad k > q \]  \hspace{1cm} (3.5.4)

As for the estimated partial auto-correlations, \( \hat{\phi}_{kk} \), the suggested formula is

\[ \sigma[\hat{\phi}_{kk}] \approx \frac{1}{n^{1/2}} \quad k > p \]  \hspace{1cm} (3.5.5)

A computer program, TSA/I based on Box and Jenkins (1971, Section V) has been developed to carry out this step in the identification procedure, and is included as Appendix la.

3.5.2 Estimation

The identification stage will result in a tentative set of values for p,d, and q suggesting the general class of the ARIMA model, (sometimes, it is necessary to carry more than one model to the next stage) to be fitted. In the estimation stage, the objective is to find
optimal estimates of the parameters, according to some criterion for
the goodness of fit, such as the likelihood function. The basic theory
is that, under the normality assumption, the parameter set \((\phi, \theta)\) for
which the likelihood function is maximized are the same as those for
which the sum of squares function is minimized, (Pierce, 1968; Sage and
Melsa, 1971; Box and Jenkins, 1971).

In general, if the given series \(z_t\) is suitably modeled as

\[
\phi(B)Y_t = \theta(B)a_t, \text{ for } Y_t = \psi^d z_t
\]

a parameter set \(\lambda = (\phi, \theta)\) may be specified in the parameter space, for
which a quantity

\[
a_t(\lambda) = \theta^{-1}(B)\phi(B)Y_t
\]  \hspace{1cm} (3.5.6)

can be defined, whose sum of squares, defined as:

\[
s(\lambda) = \sum a_t(\lambda)^2
\]  \hspace{1cm} (3.5.7)

is to be minimized, in order for the likelihood function to be maximized.
Such a quantity may be the residual. The least square estimates are
those corresponding to the minimum.

Two problems are associated with this estimation procedure, one
concerns the initial guesses of the parameters and the other is related
to the starting values of the series. The first one can be overcome by
using efficient guesses provided by the sample auto-correlations or
auto-covariances as described in subsection 3.2 above. A computer
program, TSA/11, that will calculate preliminary estimates is included
in Appendix 1b. The problem of the starting values arises because \(a_1\)
depends on \( y_0, y_1, \ldots, y_{p+1} \) and on \( a_0, a_{-1}, \ldots, a_{-q+1} \) all of which are unknown. One solution is to set all these unknowns to their unconditional value of zero, on the assumption that for large samples, the effect is negligible. Alternatively, a procedure of "back forcasting" is suggested by Box and Jenkins (1971, Chapter 7). As for the minimization of the sum of squares, any available method may be used. A computer program, TSA/III, which was used in this work is documented in Appendix 1c at the end.

3.5.3 Diagnostic Checking

The purpose of diagnostic checks on the selected model is to have a measure of the goodness of fit and to expose any inadequacies. It is also desirable to obtain an indication of the nature of such inadequacies, when present, and possibly some suggestions for a direction that will bring about a remedy; especially since the nature of the feared inadequacy is frequently unknown.

A general procedure is to consider the properties of the residuals \( \hat{\epsilon} = (\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) \) from the sample series using the optimum estimates of the parameters \( (\hat{\phi}, \hat{\theta}) \). The underlying theory is that, if the model is correct, the residuals should be approximately uncorrelated random deviates, with a zero mean, similar to the original random deviates \( (a_t) \). The sample auto-correlation function of the deviates

\[
r_k = \frac{\sum \hat{\epsilon}_t \hat{\epsilon}_{t-k}}{\sum \hat{\epsilon}_t^2}
\]

(3.5.8)
measures the interdependence among the residual \( \hat{\alpha} \). This is to be compared with the assumed zero dependence among the original \( \alpha \)'s. If the models were correct, and true values of the parameters were fitted, the residuals would be the true random shocks \( (a_t) \), and will possess "white noise" auto-correlations

\[
\rho_k = \frac{\sum a_t a_{t-k}}{\sum a_t^2} \quad (3.5.9)
\]

which, for moderate to large \( n \) are normally distributed multivariates.

It has been shown (Pierce, 1968) that distribution of the \( \alpha \)'s is independent of the distribution of \( \sum a_t^2 \), and, therefore, the residual auto-correlations will possess the statistical properties:

\[
\rho_k = E(r_k) = 0 \quad k \neq 0
\]
\[
\vartheta(\rho_k) = E(r_k^2) = \frac{1}{n}
\]
\[
\text{COV}(\rho_k, \rho_{k+j}) = 0 \quad j \neq 0 \quad (3.5.10)
\]

Now, if any of these auto-correlations are evaluated, for a reasonably large sample, then

\[
n \sum_{k=1}^{m} \rho_k^2 \approx \chi_m^2 \quad (3.5.11)
\]

being \( \chi^2 \) distributed, with \( m \) degrees of freedom (Pierce, 1968; Box and Jenkins, 1971).

The assumption is now made, that the residual auto-correlations of the fitted model \( (r_k) \) will display similar properties, with a standard deviation of approximately \( \frac{1}{\sqrt{n}} \). Some modification to (3.5.11)
is needed to make it useful for this purpose; specifically, Pierce (1968) showed that for the general ARIMA \((p,q)\) process

\[
\phi(B)Y_t = \theta(B)a_t
\]

the test of adequacy is obtained by comparing \(n \sum_{k=1}^{m} r_k^2\) with a \(\chi^2\) distribution with \(v\) degrees of freedom where \(v = m-p-q\). The same applies to a non-stationary ARIMA \((p,d,q)\) process with the degrees of freedom unchanged.

As for diagnostic checks on the residuals, discussion will be deferred to the next section for a unified treatment.

3.6 **Modeling of Dynamic Systems**

The basic procedure of the last section—identification, estimation and diagnostic checking—will be used, although dynamic models are more difficult to handle due to the presence of dynamic and noise components in the output. An additional tool is made available, both for the identification and diagnostic checking, namely the cross-correlation function between input and output.

3.6.1 **Identification**

The observed series of a dynamic system output is influenced by the controllable input as well as by stochastic noise; therefore, the interdependence between the input \((x_t)\) and the output \((Y_t)\) is essential for revealing the nature of the model. This interdependence as measured by the cross-correlation function, together with the output autocorrelations form the necessary tools for identification. In cases where the input can be chosen at will, a lot can be gained by purposely
selecting a random input. However, as this is not always the case, a procedure due to Box and Jenkins (1971) called "Pre-Whitening" of the input is available to render the input white. Following is a review (after Pierce, 1968) of this method.

**Input Whitening.** The general form of the dynamic model, as presented earlier, is

\[
y_t = \frac{\omega(B)}{\delta(B)} x_{t-b-1} + \frac{\theta(B)}{\phi(B)} a_t
\]

The input \(x_t\) can be properly differenced and modeled as an ARIMA \((p,d,q)\) using the method of section 3.5, as:

\[
\phi_x(B) x_t = \theta_x(B) \alpha_t
\]

or

\[
\psi(B) x_t = \frac{\phi_x(B)}{\theta_x(B)} x_t = \alpha_t
\]

where \(\alpha_t\) is white noise. Now, if the same \(\psi\) transformation is applied to the output \(y_t\)

\[
\psi(B) y_t = \frac{\phi_x(B)}{\theta_x(B)} y_t = \beta_t
\]

the model of (3.6.1) can be written as

\[
\beta_t = \frac{\omega(B)}{\delta(B)} \alpha_{t-b-1} + \psi(B) \frac{\theta(B)}{\phi(B)} a_t
\]

\[
= \frac{\omega(B)}{\delta(B)} \alpha_{t-b-1} + \frac{\theta(B)}{\phi(B)} a_t
\]
That is, $\beta_t$ is the sum of two independent ARMA processes, and the noise generating one of them is known.

The Cross-Covariance and Cross-Correlation Functions. A bivariate stochastic process $(x_t, y_t)$ which is stationary, or if necessary, made so by differencing, will have the following auto- and cross-covariances:

$$\gamma_{xx}(k) = E[(x_t - \mu_x)(x_{t+k} - \mu_x)] = E[(x_t - \mu_x)(x_{t-k} - \mu_x)]$$

Similarly,

$$\gamma_{yy}(k) = E[(y_t - \mu_y)(y_{t+k} - \mu_y)] = E[(y_t - \mu_y)(y_{t-k} - \mu_y)] \quad (3.6.5)$$

where the symmetric nature of the function is apparent. The cross-covariance function between $x$ and $y$ at lag $+ k$ is

$$\gamma_{xy}(k) = E[(x_t - \mu_x)(y_{t+k} - \mu_y)] \quad k = 0, 1, 2, ...$$

and between $y$ and $x$ at lag $+ k$

$$\gamma_{yx}(k) = E[(y_t - \mu_y)(x_{t+k} - \mu_x)] \quad k = 0, 1, 2, ... \quad (3.6.6)$$

where, in general, $\gamma_{xy}(k)$ will not be equal to $\gamma_{yx}(k)$, as the function not symmetrical, but

$$\gamma_{xy}(k) = \gamma_{yx}(-k)$$

The cross-correlation function at lag $k$, $\rho_{xy}(k)$ is defined by the dimensionless coefficient
\[ \rho_{xy}(k) = \frac{\gamma_{xy}(k)}{\sigma_x \sigma_y} \quad k = 0, \pm 1, \pm 2, \pm 3, \ldots \] (3.6.7)

where the function is again, not symmetric about \( k = 0 \).

**Properties of the Cross-Correlation.** In the model of (3.6.4), the cross-covariance between \( \alpha \) and \( \beta \) is

\[ \gamma_{\alpha \beta}(k) = E[\alpha_{t-k} \beta_t] \] (3.6.8)

as both have been normalized to zero mean stationary processes. Also, the cross-correlation function is

\[ \rho_{\alpha \beta}(k) = \frac{\gamma_{\alpha \beta}(k)}{\sigma_\alpha \sigma_\beta} \] (3.6.9)

If the dynamic part of the parameter set in (3.6.4) is expanded as an impulse response function:

\[ \frac{\omega(B)}{\delta(B)} = V(B) = V_0 + v_1 B + \ldots \] (3.6.10)

the cross-covariance can be shown (Pierce, 1968, Sec. 4.1) to be

\[ \gamma_{\alpha \beta}(k) = v_{k-b-1} \sigma_\alpha^2 \] (3.6.11)

The cross-correlations are then

\[ \rho_{\alpha \beta}(k) = \frac{\sigma_\alpha}{\sigma_\beta} v_{k-b-1} \] (3.6.12)

where a relation is established with the dynamic parameters of the model. Moreover, since \( v_j = 0 \) for \( j < 0 \), it follows

\[ \rho_{\alpha \beta}(k) = 0, \quad k < b \] (3.6.12a)
which is our means for identifying the process lag. It can also be shown (Pierce, 1968; Box and Jenkins, 1971, Chapter 10) that, in general, for dynamic models of the type discussed here, the sample cross-correlation function

\[ r_{\alpha\beta}(k) = \frac{\sum \alpha_{t-k}\beta}{\sqrt{\sum \alpha_t^2 \sum \beta_t^2}} \]  

(3.6.13)

has properties with respect to input-output dynamic transfer functions similar to the auto-correlation function of the stochastic ARMA \((p,q)\) process.

The Impulse Function. Further identification of the impulse weights of (3.6.10) can be made from (3.6.12) using sample statistics for theoretical values

\[ r_{\alpha\beta}(k) = \frac{s_{\alpha}}{s_{\beta}} \hat{v}_k \]

and

\[ \hat{v}_k = \frac{r_{\alpha\beta}(k) s_{\beta}}{s_{\alpha}} \]  

(3.6.14)

A computer program, TSA/V, has been developed to evaluate this function (Appendix 1d) Box and Jenkins (1971, Chapter 11) note that, while estimates of the impulse function \(\hat{v}_k\) are proportional to the cross-correlations, they are however, inefficient; and as such, they are only useful to indicate a general trend.

Order of the Model. Having obtained the impulse functions, the idea is to utilize them to identify the order of the model, i.e., values of \(r, s,\) and \(b\) in
\[ Y_t = \frac{\omega_s(B)}{\delta_r(B)} x_{t-b} \]

where

\[ \omega_s(B) = \omega_0 - \omega_1 B - \cdots - \omega_s B^s \]

and

\[ \delta_r(B) = 1 - \delta_1 B - \delta_2 B^2 - \cdots - \delta_r B^r \quad (3.6.15) \]

The relevant facts (for proof see Box and Jenkins, 1971, Chapter 10), are that the impulse response weights \( v_j \) consist of

(i) \( b \) zero values, \( v_0, v_1, \ldots, v_{b-1} \)

(ii) a further \( s - r + 1 \) values \( v_b, v_{b+1}, \ldots, v_{b+s-r} \) following no particular pattern; if \( s < r \), there will be no such values.

(iii) further values \( v_j, j > b + s - r + 1 \) follow a pattern similar to an \( r \)th order difference equation which has \( r \) starting values, \( v_{b+s}, \ldots, v_{b+s-r+1} \).

Identification of the Noise Component. The model of (3.6.1)

\[ Y_t = \frac{\omega(B)}{\delta(B)} x_{t-b-1} + \frac{\theta(B)}{\phi(B)} a_t \]

may also be written as

\[ Y_t = u_t + n_t \quad (3.6.16) \]

where \( u_t \) and \( n_t \) have their corresponding values. Specifically,

\[ n_t = \phi^{-1}(B)\theta(B) a_t \quad (3.6.17) \]

where \( a_t \) is white noise, suggesting that the noise process can be modeled by an ARIMA \((p,d,q)\) series using the methods of Section 3.5 above.
Alternatively, noise may be identified using the correlation function for the input-output after whitening:

\[ \beta_t = \psi(B) \alpha_t + e_t \]

where

\[ e_t = \phi_x(B) \theta^{-1}(B) n_t \quad (3.6.18) \]

and \( n_t \) is the residual from the tentatively identified model

\[ n_t = y_t - \varphi(B)x_t \]

\[ = y_t - \varphi_0 y - \varphi_1 x_{t-1} - \cdots \]

Equations (3.6.16) and (3.6.18) may be combined to give

\[ \beta_t = u_t + e_t \quad (3.6.19) \]

When \( n_t \), the noise series, is independent of the input \( x_t \), the autocovariance functions are related as:

\[ \gamma_{BB}(k) = \gamma_{uu}(k) + \gamma_{ee}(k) \quad (3.6.20) \]

assuming further that the input has been completely whitened to \( \alpha_t \), \( \gamma_{uu}(k) \) can be evaluated by the methods of Section (3.2), as

\[ \gamma_{uu}(k) = \sigma^2 \sum w_j w_{j+k} \]

\[ = \frac{1}{\sigma^2} \sum \gamma_{\alpha\beta}(j) \gamma_{\alpha\beta}(j+k) \quad (3.6.21) \]

using (3.6.20) we now have.
\[
\gamma_{ee}(k) = \gamma_{BB}(k) - \frac{1}{\sigma^2} \sum_j \gamma_{AB}(j) \gamma_{AB}(j+k)
\]

where, for \(k = 0\),

\[
\gamma_{ee}(0) = \gamma_{BB}(0) - \frac{1}{\sigma^2} \sum_j \gamma_{AB}^2(j)
\]

Hence, the auto-correlation of the noise series is:

\[
\rho_{ee}(k) = \frac{\rho_{BB}(k) - \sum_j \rho_{AB}(j) \rho_{AB}(j+k)}{1 - \sum_j \rho_{AB}^2(j)}
\]

(3.6.22)

The significance of this result is that the auto-correlation function of the noise component is derived from values of auto- and cross-correlation of the prewhitened input and output, which are available from previous stages of the identification procedure. In practice, it would be necessary to substitute sample estimates for theoretical values as discussed in the beginning of this section, to obtain rough estimates of the noise auto-correlations and identify the noise component.

3.6.2 Estimation

With the order of the model tentatively known, the objective is to estimate the parameters according to, say, the most likely values in the parameter space, as was done with stochastic models of the ARIMA \((p,d,q)\). But, for this to be done efficiently, an initial guess is needed to enhance and supplement the procedure.

Preliminary Parameter Estimates. The impulse response weights, \(v_j\), are utilized here by making use of the identity from (3.6.15) and (3.6.10)
\[ \frac{Y_t}{X_t} = \frac{\omega_0 - \omega_1 B - \cdots - \omega_s B^s}{1 - \delta_1 B - \cdots - \delta_r B^r} = v_0 + v_1 B + \ldots \]

therefore

\[
(1 - \delta_1 B - \delta_2 B^2 - \cdots - \delta_r B^r)(v_0 + v_1 B + \ldots) = (\omega_0 - \omega_1 B - \cdots - \omega_s B^s)
\]

(3.6.23)

when coefficients of B are equated, we have

\[
v_j = 0 \quad j < b
\]

\[
v_j = \delta_1 v_{j-1} + \delta_2 v_{j-2} + \cdots + \delta_r v_{j-r} + 0 \quad j = b
\]

\[
v_j = \delta_1 v_{j-1} + \delta_2 v_{j-2} + \cdots + \delta_r v_{j-r} - \omega_j \quad j = b+1, b+2, \ldots, b+s
\]

\[
v_j = \delta_1 v_{j-1} + \delta_2 v_{j-2} + \cdots + \delta_r v_{j-r} \quad j > b+s
\]

(3.6.24)

The last equation of the weights \(v_{b+s}, v_{b+s-r}, \ldots, v_{b+s-r+1}\) provide starting values for the difference equation

\[
\delta(B)v_j = 0 \quad j > b + s
\]

whose solution, when obtained from:

\[
v_j = f(\delta, \omega, j)
\]

(3.6.25)

applies to all values of \(v_j\), for \(j \geq b + s - r + 1\). Thus, when estimates \(\theta_j\) are used in (3.6.25), preliminary estimates of the parameter vectors \(\delta, \omega\) are supplied to start the iterative estimation procedure. Computer program, TSA/VI, (Appendix le), has been written to carry out this preliminary estimation based on Box and Jenkins (1971, Part V). However,
the authors warn that these initial estimates are rather inefficient and should be used with caution.

Conditional Sum of Squares Parameter Estimates. The objective now is to find an efficient estimate of the parameters $b$, $\delta$, $w$, $\phi$, and $\theta$ in the tentatively identified model of the last section,

$$
y_t = \delta^{-1}(B) \omega(B) x_{t-b} + n_t
$$

If the starting values from (3.6.24) are used to supply $x_0$, $y_0$ and $a_0$, the maximum likelihood function is, under the normal assumption, approximately measured by the conditional sum of squares function

$$
S_0(b, \delta, w, \phi, \theta) = \sum_{t=1}^{n} \delta(b, \delta, w, \phi, \theta | x_0, y_0, a_0)
$$

(3.6.26)

To summarize the method, Box and Jenkins (1971, Chapter 11) suggest the following three-stage procedure:

(i) the model output is computed from

$$
y_t = \delta^{-1}(B) \omega(B) x_{t-b}
$$

i.e., from

$$
y_t = \delta_1 y_{t-1} \cdots \delta_r y_{t-r} = w_0 x_{t-b} - w_1 x_{t-b-1} \cdots - w_s x_{t-b-s}
$$

(3.6.27)

(ii) the noise $n_t$ is next calculated as

$$
n_t = y_t - \hat{y}_t
$$

(3.6.28)
(iii) finally, the a's are obtained from (3.6.17) as

\[ a_t = \theta^{-1}(B)\omega(B)\eta_t \]  \hspace{1cm} (3.6.29)

i.e.,

\[ a_t = \theta a_{t-1} + \cdots + \theta a_{t-q} + n_{t-q} - \phi_1 n_{t-q-1} - \cdots - \phi p n_{t-p} \]  \hspace{1cm} (3.6-30)

A final word about starting values is to begin the parameter optimization and estimation routine from \( t = h + 1 \), where \( h \) is the larger of \( (r) \) and \( (s + b) \). This is necessary to avoid the effect of starting transients, but it also means that \( n_t \) will be available from \( n_{h+1} \), onwards, and the a's from \( a_{h+p+1} \), with previous values set to their unconditional expectation of zero.

3.6.3 Diagnostic Checking

Model inadequacy is revealed when diagnostic checks are applied to residuals by examining their auto-correlation function \( r_{a^a}(k) \), and the cross-correlation function of the residuals with the input before and after whitening, \( r_{x^a} \) and \( r_{x^a} \). In general, two possible situations may arise. Namely:

a) Transfer function model correct-noise model incorrect. It can be shown (Pierce, 1968; Box and Jenkins, 1971, Chapter 10) that in such a case, the residuals will be auto-correlated, but will not be cross-correlated with the input.

b) Transfer function model incorrect. In this situation, Box and Jenkins (1971) and Pierce (1968) show that the residuals would not only be cross-correlated with the input \( x_t \) (and \( \alpha_t \)) but will also be auto-correlated, even if the noise model were correct.
A test for a quantitative assessment of the significance of the residual statistics is offered by the above authors in what is called the "Q" criterion. The fact is first established that the least squares estimates of the residuals would give rise to auto-correlations of the \( \hat{\alpha}'s \) which are independently distributed about zero with a standard error of \( 1/\sqrt{m} \), where

\[
m = n - h - p
\]

On the assumption that the model is functionally correct, if \( k \) estimated auto-correlations are taken, with \( k \) sufficiently large, then

\[
Q = m \sum_{k=1}^{k} r^{2}_{\alpha\alpha}(k)
\]

will be approximately distributed as \( \chi^2 \) with \( k - p - q \) degrees of freedom, where \( p \) and \( q \) refer to the noise ARIMA model. Another suggested check, called the \( S \) criterion, applies to the cross-correlations between the whitened input \( \alpha_t \) and the residual \( \hat{\alpha} \). If \( k \) values are taken, and \( k \) is sufficiently large for the weights \( v_j \) to be negligible over \( j > k \), then the quantity

\[
S = m \sum_{k=0}^{k} r^{2}_{\alpha\hat{\alpha}}(k)
\]

is approximately \( \chi^2 \) distributed with \( k + 1 - (r + s + 1) \) degrees of freedom; where the number of degrees of freedom refers to the fitted transfer function model and is independent of the noise model parameters.

3.6.4 Multiple Input Dynamic Models

To conclude this review of time series method of analysis, it is necessary to indicate how the methods mentioned earlier, which are
specifically suited to single inputs processes, can be extended to multiple input dynamic systems. The transfer function model was

\[ Y_t = v(B)x_{t-b} + N_t \] (3.6.33)

When several inputs are to be considered, say \( x_1, x_2, \ldots, x_m, t \), so the superposition is possible

\[ Y_t = v_1(B)x_{1,t} + v_2(B)x_{2,t} + \cdots + v_m(B)x_{m,t} + N_t \] (3.6.34)

In terms of dynamic parameters \( \omega \) and \( \delta \) this is:

\[ Y_t = \delta^{-1}_1(B)\omega_1(B)x_{1,t-b} + \cdots + \delta^{-1}_m(B)\omega_m(B)x_{m,t-bm} \] (3.6.35)

or, after suitable differencing and transformation, putting \( y_t = y^d_t \),

\[ y_t = v_1(B)x_{1,t} + \cdots + v_m(B)x_{m,t} + n_t \] (3.6.36)

where \( v_j(B) \) is the generating function of the impulse response weight relating input \( x_j, t \) to output \( y_t \). If the individual impulse response functions are estimated separately by the methods of the last section, efficient preliminary estimates are obtained which can be fitted in the model of (3.6.35) to start a simultaneous estimation using the residual sum of squares to provide the criterion for optimization. This will be discussed in more detail in Chapter 5.
CHAPTER IV
SYSTEM DESCRIPTION

The system being studied consisted of two human subjects operating an anti-aircraft gun in a closed loop. The operators manipulated the gun in its two axis, azimuth and elevation, and manually aligned a sight reticle to a moving target. A model of the gun was established and validated against actual performance data. Target paths were three dimensional flight data of a highly maneuvering aircraft. The whole system was simulated on a hybrid computer with the two human operators physically closing the loop.

4.1 Gun Model

A general model of anti-aircraft guns was developed and successfully applied to a number of real systems by Planchard, Barzinji and Perkins (1970 and 1972) and Planchard and Barzinji (1973). The gun is represented by a simple two tortional dynamic system in the two axis of motion, elevation, $\phi$, and azimuth, $\theta$, as illustrated in Figure 4.1. The equation of motion in each axis is:

\[
\frac{d^2 \phi}{dt^2} = \frac{1}{J_{\phi}} \left[ T_{\phi} - B_{\phi} \frac{d\phi}{dt} \right]
\]

\[
\frac{d^2 \theta}{dt^2} = \frac{1}{J_{\theta}} \left[ T_{\theta} - B_{\theta} \frac{d\theta}{dt} \right]
\]  

(4.1.1)
FIGURE 4.1 GUN MODEL
where

\[ \theta_g = \text{angular gun position in elevation} \]
\[ \theta_g = \text{angular gun position in azimuth} \]
\[ T_\theta = \text{torque applied to move gun in elevation} \]
\[ T_\theta = \text{torque applied to move gun in azimuth} \]
\[ J_\theta = \text{moment of inertia about the } \theta \text{ axis of rotation} \]
\[ J_\theta = \text{moment of inertia about the } \theta \text{ axis of rotation} \]
\[ B_\theta = \text{drag coefficient in elevation movement} \]
\[ B_\theta = \text{drag coefficient in azimuth movement}. \]

The system is specified by assigning values to J and B parameters.

A procedure to test the validity of the model for a particular anti-aircraft artillery is detailed in Planchard et al. (1970) where the model was used in tracking targets following selected paths, and the tracking errors compared with actual field data. A regression procedure was applied, searching for values of J and B that minimized the sum of squares of the differences between field data and model tracking errors.

In a contract with the Air Force, several gun systems were studied and modeled by the above authors (Planchard et al. 1970, 1972 and 1973), one of which was selected for this work. Table 4.1 gives parameter values for the selected system; details can be found in Planchard and Barzinji (1972).

4.2 Flight Paths

As mentioned earlier, targets followed flight paths of field tests conducted by the Air Force on high performance aircrafts. Manuevers
<table>
<thead>
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<th>Parameter</th>
<th>Definition</th>
<th>Elevation</th>
<th>Azimuth</th>
</tr>
</thead>
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<tr>
<td>B ft-lb-sec</td>
<td>drag coefficient</td>
<td>24.16</td>
<td>15.47</td>
</tr>
<tr>
<td>J slug-ft$^2$</td>
<td>moment of inertia</td>
<td>0.450</td>
<td>0.2239</td>
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</table>

Table 4.1. Parameter Values of Gun Model Selected for this Study.
resembling combat operations were used, where ordinance delivery dive angles as steep as $60^\circ$ and "jinking" escape maneuvers with as high as 5 g's were involved. Flight paths were "normalized" to minimize the amount of less useful data being collected. This was achieved by adjusting the lowest point in the elevation of each path to 500 feet above ground level and shifting the time axis so that minimum elevation always occurred at crossover.(for the purpose of this study, crossover is defined as crossing the x-axis of the Inertial co-ordinate system which has its origin at the gun site. See Fig. 4.2 - 4.4). Minimum offset distances from the gun were selected at 1500 ft. and 3000 ft.

Altogether, twenty different flight paths were tracked, and Table 4.2 lists serial designations and description of each path, while Figures 4.2 through 4.4 show a perspective projection of selected flight paths, giving a pictorial view of the severity of the maneuvers involved.

4.3 Tracking Errors

This section deals with the definition, sign convention and calculation of tracking errors as used in this study. It is based on the discussion of Planchard and Barzinji (1973), where, for convenience, the same terminology is maintained and the figures are reproduced from their presentation.

An Inertial co-ordinate system was established with its origin at the weapon site, as shown in Figure 4.5. The Y axis ($Y_e$) is along the general direction of the flight paths, while the X axis ($X_e$) is perpendicular to it, with the positive directions as indicated in the figure. The Z axis ($Z_e$) is perpendicular to the X-Y plane, and is positive upward.
### Table 4.2
**Flight Paths Used in Tracking**

<table>
<thead>
<tr>
<th>Path No.</th>
<th>Delivery dive angle (degrees)</th>
<th>Escape maneuver (in g pull)</th>
<th>Minimum displacement (feet)</th>
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</tr>
<tr>
<td>20</td>
<td>60</td>
<td>5</td>
<td>3000</td>
</tr>
</tbody>
</table>
Projection onto X-Y Plane

Fig. 4.2 PERSPECTIVE PROJECTION OF FLIGHT PATH NO. 1
Projection onto X-Y Plane

Figure 2.3 PERSPECTIVE PROJECTION OF FLIGHT PATH NO. 5
Projection onto X-Y Plane

Fig. 4.4. PERSPECTIVE PROJECTION OF FLIGHT PATH NO. 10
Figure 4.5. Gun Position in Inertial Coordinate System.
The second co-ordinate system is the gun system, with its origin also at the gun site. The X-axis ($X_g$) is along the gun barrel and is positive in the outward direction of the barrel. The Y and Z axes ($Y_g$, $Z_g$) coincide with the inertial system when the gun barrel corresponds to the $X_e$ axis. The gun position is completely described by the two angles of rotation, $\theta$ in azimuth and $\phi$ in elevation as shown in Fig. 4.6.

The position of the target is given in the inertial system as $X_e$, $Y_e$, $Z_e$ and to locate it in the gun system the following transformation matrix is used:

$$
\begin{bmatrix}
X_g \\
Y_g \\
Z_g
\end{bmatrix} =
\begin{bmatrix}
\cos \theta \cos \phi & \cos \phi \sin \theta & -\sin \phi \\
-\sin \theta & \cos \theta & 0 \\
\sin \phi \cos \theta & \sin \phi \sin \theta & \cos \phi
\end{bmatrix}
\begin{bmatrix}
X_e \\
Y_e \\
Z_e
\end{bmatrix}
$$

(4.3.1)

The errors are positive or negative according to the sign convention of Fig. 4.7, where for left to right tracking a negative error results when the tracker leads his target.

Tracking errors in azimuth and elevation are indicated by the angles of misalignment of Fig. 4.8, where, by definition:

- Elevation error, $E_e = \tan^{-1} \left( \frac{Z_g}{X_g} \right)$
- Azimuth error, $E_a = \tan^{-1} \left( \frac{Y_g}{X_g} \right)$

(4.3.2)

In this development, the assumption was implicitly made that the sight axis corresponds with the gun axis.
**LEGEND**

- $\phi$ - AZIMUTH ANGLE GUN VECTOR
- $\theta$ - ELEVATION ANGLE GUN VECTOR
- $\lambda$ - AZIMUTH ANGLE PLANE VECTOR
- $\omega$ - ELEVATION ANGLE PLANE VECTOR
- $E_\alpha$ - AZIMUTH ERROR

*Figure 4.6. Plane Location in Inertial Coordinate System.*
PATH OF PLANE WITH GUN HELD STILL (LEFT TO RIGHT TRACKING)

TARGET AT TIME $T_1$

(-) AZIMUTH ERROR

(+) ELEVATION ERROR

AIM POINT

Figure 4.7. Site Pattern Gun Coordinate System.

Figure 4.8. Angle of Misalignment In Gun Coordinate System.
4.4 Hybrid Simulation of the Tracking System

A real time simulation of a compensatory tracking system was devised with the operator in the control loop. This was the means of obtaining data on human performance for the formulation and validation of a stochastic model. The next section describes the facilities used, documents the hybrid computer program and the procedure of data collection. A listing of the experiments conducted is provided at the end.

4.4.1 Simulation Facilities

The hybrid computer used in this work consisted of a Scientific Data Systems--Sigma 5 digital computer with an Electronic Associates Incorporated (EAI) 680 analog computer and an EAI 693 interface. Peripheral equipment consisted of a 400 cpm card reader, a teletype and a paper tape read/punch unit. The digital computer controls the analog by a number of Fortran Callable hybrid subroutines. The system can operate in real time. Figure 4.9 shows the general arrangement of the tracking system with actual operators in the loop. The two oscilloscopes used, one for each of azimuth and elevation operators, had displays with 5" reticles calibrated in 1 cm. units. Visual displays were arranged such that vertical displacement of the beam from a marked center line was proportional to tracking error; while horizontal displacements from the center indicated target distance from crossover. The sensitivity was set to the highest value for which the signal could be kept within screen limits, and contrast between target and background was adjusted by the individual operator to his maximum convenience.
Figure 4.9. Hybrid Simulation: Operation In Loop


t_{q}(\theta) \rightarrow \text{Scope Displays} \rightarrow \text{Rotation and Angular Misalignment} \rightarrow \text{Target Position} \rightarrow \text{Time Generation} \rightarrow \{\text{Azimuth}\}

\text{Elevation}

Angular Position (\theta) \rightarrow \text{A/D} \rightarrow \text{D/A} \rightarrow \text{Angular Velocity (\dot{\theta})} \rightarrow \text{Accelerated (\dot{\theta})} \rightarrow \text{Angular Position (\theta)}
Human operators performed by finger-tip controllers, being the hand set pots of the analog computer, which had no detectable friction or backlash. Target movement was always left to right, and the control was in a natural manner, such that an upward (clockwise) movement moved the target up on the cathode ray tube display. Limiters were used to restrict controller outputs to the physical bounds of the operator and the system.

4.4.2 Hybrid Computer Program

In the overall arrangement (Fig. 4.9) the digital computer samples gun position in azimuth and elevation, and time. It then locates target position from stored flight path data, calculates tracking errors and transmits them to the analog for display. The two human operators view their errors and produce a torque input to the gun model on the analog. As gun position changes, the digital detects, calculates and loads the new errors and so on in a closed loop manner.

The analog patching diagram is indicated in Fig. 4.10. Hand set pots Q2 and Q9 are the main controllers which human operators used to produce torque input to the elevation and azimuth axes of the gun model respectively. This output was restricted (by the limiters on amplifiers 01 and 31) to 10 ft-lb, and integrated twice to produce a gun position, which was transmitted via A/D's 2 and 3 (i.e. analog to digital converters), to the digital for storage and error calculation. Torque in both axes, and time were also transmitted via A/D converters 1, 4 and 5 respectively. The limiters on amplifiers 11 and 41 provided the physical bounds on the acceleration of the gun, while no such measures were needed for the velocity (amplifiers 9 and 40). Hand set pots Q4 and Q7 on the
Figure 4.10. Hybrid Simulation of Compensatory Tracking.
IC (initial conditions) of integrators 17 and 50 were to control the initial position of the gun.

A flow chart of the digital program is illustrated in Fig. 11. The analog is set up by calling a number of subroutines, then flight data are read from cards and stored. The lowest point in each path is located and assigned to cross over time, and path normalization is carried out as described under 4.2 above. A maximum of five flight paths can be stored at a time because of storage limitations, and with the option of two offsets of 1500' or 3000', ten different combinations of tracking paths are available by a random selection. Gun position and time are sampled at the rate of 100 times/sec., and the digital calculates the corresponding position of the target at that instant. Subroutine Rotate is called to transform target position to the gun co-ordinate system, after which errors are calculated and loaded on the scopes.

4.4.3 Tracking Procedure

Human subjects were considered "trained" operators, and for this purpose a training period was allowed, which varied from two hours to several hours. The full range of flight paths was tracked during training in order to allow the necessary experience. An operator was considered "trained" if he produced consistent tracking and, when tracking difficult cases, he did not lose his target (error did not grow outside the visual display). A typical training curve for one of the subjects appears as Figure 4.12.
Figure 4.11. Digital Flow Chart.
Figure 4.12. Performance Versus Experience.
At the start of each series of runs, flight path data and potentiometer settings were read from cards by the digital computer. When path transformation was complete, two random selections were made, one of a flight case and the other of a displacement. This selection was checked against the design of experiments, where sense switch (4) (Fig. 4.11) could be activated to request another selection if necessary. Following selection of a case, the starting point on the time axis was calculated from cross over and the analog was set up. The digital then sampled time and the initial gun angles (θ and φ). Target position corresponding to the time instant was calculated and subroutine (ROTATE) was called to transform target position to the gun coordinate system. Upon return from this subroutine, the initial aim errors were calculated and supplied to the analog for display. The digital cycled a loop awaiting the analog to operate. This was achieved by continually comparing the value of time on the analog with the time for the next storage point. Sampling rate was set at 100 times/second and storage intervals were at 0.25 sec. As long as the sampled time was less than the next storage time, the loop indicated on Fig. 4.11 by entry point 11 was executed continuously.

When all was ready, the two operators viewed the scope displays and manipulated their controls to direct the gun toward the target and zero the initial errors. This was necessary to avoid acquisition problems at the start of each run. Tracking began when a operator put the analog in the "operate" mode, thus generating time and moving the target across the screen. As this took place the digital calculated the current angles of misalignments in azimuth and elevation and the
two errors were fed to the analog and displayed on the two scopes. The two human operators continuously viewed the error and applied corrective torques by manipulating pots Q2 and Q9 for elevation and azimuth, respectively. This torque was inputted, in each channel, to the gun model programmed on the analog, as previously described, producing acceleration of the gun, which was integrated to obtain velocity and position. The later was sampled by the digital, hence the loop was closed.

As time increased, the target moved across the screen, and at cross over, it was always crossing the centerline of the scope. At specified intervals, the digital stored time and the corresponding tracking data (torque, gun angular velocity, position and error in both axis).

A run was terminated when time exceeded TSTOP, typically 40 seconds. At the end of a run, an integer variable "NGO" was entered via the teletype offering added flexibility in the form of several options. "NGO" could be 1, 2, 3 or 4, according to whether the program was to be terminated, with or without the output from the last run, or to be continued, with or without the output. If an output was desired, a sense switch was used to decide its form, which could be printout only on the line printer, or a printout and a punched paper tape for further processing into cards.

As mentioned above, the program operated in real time, a typical run taking about three minutes (setting up time plus 40 seconds of operation). To output the data on tape was, however, relatively slow, as it took about 15 minutes per run.
Human operator fatigue was kept to a minimum by the period of rest allowed while paper tape was being punched. As the digital could store tracking data from four runs, this resulted in one hour of rest after every four successful runs. Subjects learned of their performance from scope persistence and by inspecting the printed output.

4.5 Collected Data

A total of 48 runs were collected on the 20 flight paths of Table 4.2. Tracking of some paths was duplicated as many as six times while other paths were tracked only once for the record. Table 4.3 gives full details of the experimental program, where the two operators carried the designations 2 and 5. A selection of runs and duplications are presented in Figs. 6.10 through 6.13, where run to run variations and different operator strategies can be noticed.

This collection of data will be used for the formulation and validation of a time series model of the human performance, based on the discussion of Chapter III, which is the subject of the next two chapters.
Table 4.3: Program of Experiments:

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<th>Serial no</th>
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<th>Team Az.-El.</th>
<th>Expt. no.</th>
<th>Serial no</th>
<th>Path</th>
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<td>2</td>
<td>47</td>
<td>20</td>
<td>2-5</td>
<td>30</td>
</tr>
<tr>
<td>24</td>
<td>15</td>
<td>5-2</td>
<td>13</td>
<td>48</td>
<td>20</td>
<td>5-2</td>
<td>39</td>
</tr>
</tbody>
</table>
CHAPTER V
FORMULATION OF DETERMINISTIC MODEL

The theoretical discussion and background of Chapter III will now be applied to the experimental data on human operator performance of Chapter IV. The objective of this chapter is to produce the development and formulation of a time series model that describes human tracking performance. A deterministic component will be established first, that will reproduce the mean human performance. This will be supplemented by a stochastic component developed from analysis of the residual. In general, tracking results on flight path 1 (Table 4.2) with the simplest maneuvers, were used to evaluate model parameters, and the result was applied to other paths to test and demonstrate the validity of the model.

5.1 Choice of System Inputs

In the identification of the dynamic system at hand the method of pre-whitening the input (Section 3.6.1.1) was used. To begin with, therefore, it will be necessary to describe each input by a time series as efficiently as possible. The methods of section 3.5 will be applied. Before proceeding further, it is necessary to identify system inputs, since in an environment as complicated as tracking by a human operator, it is not obvious what to use for system inputs. According to Young (1969), a human operator presented with tracking error, is also capable of estimating the first derivative of error. In many control situations,
the operator may not be concerned with absolute values of inputs and outputs, as his attention is fixed on changes in these variables. As a result, it would be necessary at the preliminary stage of identification to consider error, and its first derivative or first difference as likely inputs.

Target angular acceleration is another possible input to the control system. As the spot moves across the scope display with target, a trained human operator learns to estimate the angular velocity and probably angular acceleration of his target. Hence, target angular velocity, and its first derivative or difference should also be treated as likely inputs. Fig. 5.1 is a block diagram representation of the control situation resulting from the above discussion.

In order to discriminate among the above mentioned possible inputs, it is necessary to resort to cross-correlation analysis of inputs and output. Utilizing this basic tool, selection will be based on the following criteria:

a) magnitude of the cross-correlations, indicating the degree of interdependence,

b) run-to-run variations, as small variations in the cross-correlations from different operators and various runs are suggestive of successful filtering of system noise,

c) rate of decay, since quick dying input-output cross-correlations are essential to the identification procedure employed (Sec. 3.6).

d) behavior at small lags. This is important because of the physical nature of the system, where operator lag
Fig. 5.1. Block Diagram of Proposed Human Tracking Model.
time is about .2 sec. (Chap. 11, Sec. 2), the sampling interval is .25 sec. and the system has a quick response; thus, it is unlikely for any meaningful cross-correlation to exist beyond 5 or 6 intervals.

Graphs of cross-correlations between tracking error, \( (\dot{e}, \dot{Ve}) \) and human operator output torque are shown in Fig. 5.2, while Fig. 5.3 gives the corresponding information for target angular velocity, \( \omega \).

Figure 5.2a shows the large run to run variations that exist in the cross-correlations between error and torque. Figures 5.2b and c display the similarity between the cross-correlation of error rate (\( \dot{E} \)) and those of the differenced error (\( \dot{VE} \)) with \( \dot{VT} \). This is not surprising as it was shown in Chapter III section 4 that \( \dot{VE} \) and \( \dot{E} \) are strongly related. It also appears from the figures that cross-correlations of \( \dot{E} - \dot{T}q \) and \( \dot{VE} - \dot{VT} \) are larger and much more consistent compared with the corresponding \( E - Tq \) function; especially at low lags. As for the angular velocity, Fig. 5.3b is a clear indication that a system of \( \dot{\omega} - \dot{Tq} \) as input-output is not likely to express the true nature of the control situation. On the other hand, Fig. 5.3a and c indicate the strong dependence of \( Tq \) and \( VTq \) on the angular velocity and its first difference respectively.

Other input-output combinations were also considered, e.g., \( E - \dot{Tq} \), \( \dot{VE} - Tq \), \( \dot{w} - Tq \), etc. but they are not presented here since they showed little correlation. The preceding discussion leads to the conclusion that in the next stage of identification we need to investigate the following input-output systems only:
Fig. 5.2. Cross-Correlations for Various Functions of Azimuth Error with Operator Torque Output for Four Runs of Path 1, Together with Approximate Two Standard Error Limits.
c) \( \nabla \) (Error) with \( \nabla \) (Torque)

\[ r_{V_E, V_I}(k) \]

\( k \rightarrow \)

Fig. 5.2. Continued
Fig. 5.3. Cross-Correlations between Various Functions of Azimuth Target Angular Velocity ($W$) and Operator Torque Output ($T_Q$), together with Approximate Two Standard Error Limits, for Four Runs of.
c) $\nabla W - \nabla T_Q$

Fig. 5.3. Continued.
In the next section, the four inputs will be pre-whitened, by a time series that will reduce each to uncorrelated noise.

5.2 Pre-whitening of Inputs

As mentioned earlier, flight path #1 was selected for analysis since it has less severe maneuvers and, as such, should be less noisy. First the auto-correlation and partial-auto-correlation functions of the time series represented by each input for flight path #1 were evaluated. Next a preliminary estimation, followed by a final estimation procedure was performed. The three stages are accomplished by means of computer programs TSA/1 through TAS/3 (Appendix 1), respectively.

5.2.1 Tracking Error

Tracking data for one of the six available runs of flight path #1 is included in Appendix 2; Figures 6.10 and 6.11 show four of them. Computer program TSA/1 was run on azimuth tracking error with the results as displayed in Table 5.1 and Fig. 5.4. Limits of two standard derivations \(^\hat{2}\) of the function estimates - auto-correlation and partial auto correlation - are indicated in the figure together with values of the mean \(\overline{E}\) and the variance \(\sigma_E^2\) of the series. As mentioned in section 3.5, it is only necessary to evaluate the function for the series and
its first two differences.

Identification:

From Fig. 5.4a for the auto-correlation and partial auto-correlation of run #1 path #1, azimuth error, the following is observed (based on the discussion of Sec. 3.3)

a) with $d=0$, (no differencing), the auto-correlation's $r_k$ decay exponentially in a sinusoidal manner while the partial auto-correlations $\hat{\phi}_{kk}$ cut off after the second lag, suggesting a model of ARIMA $(2, 0, 1)$

b) after one degree of differencing, the same behavior is displayed, but the series has a mean of nearly zero and the standard deviation is much reduced. An ARIMA $(2, 1, 1)$ is suggested and differencing is indicated due to the desirable filtering effect.

c) As for $d=2$ (second differences), no definite pattern is exhibited by the functions and will not be considered.

The series of tracking errors in azimuth of flight path 1, may therefore be modeled by an auto-regressive, moving average process of order $(2, 0, 1)$ or $(2, 1, 1)$.

As for the 1st derivative of error, Fig. 5.2-b may be analyzed in a similar manner, thus:

a) with no differencing ($d=0$), the auto correlations, $r_k$ behave like a decaying sinusoid, while $\hat{\phi}_{kk}$, the partial auto-correlations, decay or possibly, cut off after the second lag, suggesting an ARIMA $(2, 0, 1)$ model.
Fig. 5.4a. Estimated Auto and Partial Auto-Correlations of Various Differences for Azimuth Tracking Error of Path 1, Run #1, together with Approximate Limits of Two Standard Errors of Estimates
Fig. 5.4b. Auto- and Partial Auto-Correlations of Various Differences of Azimuth Tracking Error Rate of Path 1, Run #1, together with Approximate Limits of Two Standard Errors of Estimates.
b) with higher differences, the pattern is lost, hence no differencing is needed.

5.2.1.2. Preliminary Estimation

Knowledge of the auto-covariances of the initially identified series enables us to obtain efficient preliminary estimates of the auto-regressive and moving average parameters (φ and θ), together with noise variance, $\sigma_e^2$. This is possible by the method of section 3.5 and use is made of the special computer program TSA/2 (Appendix 1b). The necessary auto-covariance function is readily available from the first program (TSA/1). Table 5.1 lists the auto-covariances for azimuth $E$ and $\dot{E}$, and the initial estimates are included in Table 5.2.

5.2.1.3. Least-Squares Estimation

Final parameter estimates were obtained by the method of section 3.5. Computer program TSA/3 was used to calculate the least square estimates of each suggested model. Table 5.2 contains the result of this work, where each model was tested on different runs of flight path #1, starting from the same initial parameter values of the last paragraph. Values of initial and final sum of squares are also included for comparison. Under the heading "Average Error" are results of testing the model against the mean tracking error, i.e. average of six runs of path #1 (Table 4.3).

Table 5.2 shows comparable results when the best fit is obtained with the tracking error, using ARIMA models of orders (2, 0, 1), (2, 0, 2) and (3, 0, 1), while a model of (2, 1, 1) had a larger final cost compared with the first three. The final choice was the model with the minimum
<table>
<thead>
<tr>
<th>k</th>
<th>$n_k$</th>
<th>$\varphi_{kk}$</th>
<th>$C_{kk}$</th>
<th>$n_k$</th>
<th>$\varphi_{kk}$</th>
<th>$C_{kk}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.844</td>
<td>.844</td>
<td>79.1</td>
<td>.549</td>
<td>.549</td>
<td>24.9</td>
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<tr>
<td>2</td>
<td>.516</td>
<td>-.680</td>
<td>66.7</td>
<td>-.020</td>
<td>-.459</td>
<td>13.6</td>
</tr>
<tr>
<td>3</td>
<td>.195</td>
<td>.237</td>
<td>40.8</td>
<td>-.228</td>
<td>.673</td>
<td>-.69</td>
</tr>
<tr>
<td>4</td>
<td>-.056</td>
<td>-.265</td>
<td>15.4</td>
<td>-.258</td>
<td>-.216</td>
<td>5.7</td>
</tr>
<tr>
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<td>-.225</td>
<td>.011</td>
<td>-4.4</td>
<td>-.341</td>
<td>-.270</td>
<td>6.4</td>
</tr>
<tr>
<td>6</td>
<td>-.289</td>
<td>.121</td>
<td>-17.8</td>
<td>-.313</td>
<td>-.043</td>
<td>8.5</td>
</tr>
<tr>
<td>7</td>
<td>-.254</td>
<td>-.082</td>
<td>-22.8</td>
<td>-.229</td>
<td>-.283</td>
<td>7.8</td>
</tr>
<tr>
<td>8</td>
<td>-.148</td>
<td>.183</td>
<td>-20.1</td>
<td>-.049</td>
<td>.070</td>
<td>5.7</td>
</tr>
<tr>
<td>9</td>
<td>-.026</td>
<td>-.159</td>
<td>-11.7</td>
<td>-.066</td>
<td>-.228</td>
<td>1.2</td>
</tr>
<tr>
<td>10</td>
<td>.075</td>
<td>.146</td>
<td>-2.1</td>
<td>.127</td>
<td>.002</td>
<td>1.6</td>
</tr>
</tbody>
</table>
Table 5.2 Preliminary and Final Least Squares Parameter Estimates for Different Attempted ARIMA Models for Prewhitening of Azimuth Tracking. Error - Path 1 Individual Runs and Average Error.

<table>
<thead>
<tr>
<th>ARIMA Model (p,d,q)</th>
<th>Run</th>
<th>Operator</th>
<th>Cost Initial ( \theta_0 )</th>
<th>Cost Initial ( \theta_1 )</th>
<th>Cost Initial ( \theta_2 )</th>
<th>Cost Final ( \theta_0 )</th>
<th>Cost Final ( \theta_1 )</th>
<th>Cost Final ( \theta_2 )</th>
<th>Operator</th>
<th>Cost Initial ( \theta_0 )</th>
<th>Cost Initial ( \theta_1 )</th>
<th>Cost Initial ( \theta_2 )</th>
<th>Cost Final ( \theta_0 )</th>
<th>Cost Final ( \theta_1 )</th>
<th>Cost Final ( \theta_2 )</th>
</tr>
</thead>
</table>
| (2,0,2)             |     |          | 1986.1422.1095.1964.1349.2160.705.    | 1680.901.1688.1594.1155.2014.463.    | \( \theta_0 \) 1.43 0.73 0.62 0.96 0.99 1.08 | 1.47 1.45 | \( \theta_1 \) -0.67 -0.15 -0.16 -0.26 -0.33 -0.47 | -0.65 -0.65 | \( \theta_2 \) -0.16 -0.76 -0.80 -0.82 -0.67 -0.58 | -0.17 -0.25 | Run 1 12 18 21 23 24 Average Initial Guesses
| (2,0,1)             |     |          | 1824.1176.2326.1936.1363.2458.607.    | 1652.1017.1719.1646.1188.2014.463.    | \( \theta_0 \) 1.20 1.28 0.84 1.17 1.17 1.09 | 1.45 1.45 | \( \theta_1 \) -0.50 -0.54 -0.27 -0.42 -0.44 -0.48 | -0.62 -0.65 | \( \theta_2 \) -0.42 -0.09 -0.56 -0.56 -0.56 -0.57 | 0.14 -0.25 | Run 1 12 18 21 23 24 Average Initial Guesses
| (3,0,1)             |     |          | 1856.1107.2359.1959.1354.2503.523.    | 1667.1017.1656.1561.1098.2001.462.    | \( \theta_0 \) 1.38 1.41 1.25 1.54 1.75 0.92 | 1.62 1.575 | \( \theta_1 \) -0.777 -0.719 -0.834 -1.05 -1.36 -0.225 | -0.924 -0.812 | \( \theta_2 \) -0.336 0.03 0.34 0.20 0.00 0.14 | -0.049 -0.067 | Run 1 12 18 21 23 24 Average Initial Guesses
| (2,1,1)             |     |          | 2042.1357 | 2039.1247. | \( \theta_0 \) 0.686 0.60 | \( \theta_1 \) -0.399 -0.32 | \( \theta_2 \) -0.17 -0.03 | \( \theta_0 \) 0 -0.07 |
number of parameters, i.e. an ARIMA (2, 0, 1). The best model autoregressive first order moving-average process of the form:

\[(1 - \phi_1 B - \phi_2 B^2) E_t = (1 - \theta_1 B) a_t + \theta_0\]

or

\[E_t = \phi_1 E_{t-1} + \phi_2 E_{t-2} + a_t - \theta_1 a_{t-1} + \theta_0\]  

(5.2.1)

As for the rate of error, optimum parameter values obtained from each run of paths 1 and 5 are given in Table 5.3. The proposed model is of the same mixed ARIMA (2, 0, 1) form, i.e.,:

\[
\hat{E}_t = \phi_1 \hat{E}_{t-1} + \phi_2 \hat{E}_{t-2} + a_t - \theta_1 a_{t-1} + \theta_0
\]

(5.2.2)

It is noticed that the residual sum of squares (final cost) for the error rate is consistently higher, for all runs, than the corresponding values for the error. Although this is an indication that \(\hat{E}\) is a poor choice for an input, it is better, however, to leave such an elimination to the next stage.

Finally, when equations (5.2.1) and (5.2.2) are rearranged, the pre-whitening transformation is obtained, i.e.

for error:

\[
\alpha_t(E) = E_t - \phi_1 E_{t-1} - \phi_2 E_{t-2} + \theta_1 a_{t-1} - \theta_0
\]

(5.2.3)

similarly for error rate:

\[
\alpha_t(\hat{E}) = \hat{E}_t - \phi_1 \hat{E}_{t-1} - \phi_2 \hat{E}_{t-2} + \theta_1 a_{t-1} - \theta_0
\]

(5.2.4)

(it is to be noted that this is the general form of the model; the parameters are not equal in both equations, as they have the values of Tables 5.2 and 5.3).

Table 5.3 Preliminary and Final Least Squares Parameter Estimates for a Suggested ARIMA Model for Prewhitening Azimuth Tracking Error Rate - Path 1 Individual Runs and Averaged Performance

<table>
<thead>
<tr>
<th>Run</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Av.</th>
<th>Initial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Operator (2,0,1)</td>
<td>Initial Cost</td>
<td>final Cost</td>
<td>( \theta_1 )</td>
<td>( \theta_2 )</td>
<td>( \theta_1 )</td>
<td>( \theta_0 )</td>
<td>( \theta_1 )</td>
<td>( \theta_2 )</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>11859</td>
<td>5677</td>
<td>11714</td>
<td>10142</td>
<td>6883</td>
<td>14845</td>
<td>2679</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8763</td>
<td>5287</td>
<td>8429</td>
<td>7203</td>
<td>5151</td>
<td>11582</td>
<td>2618</td>
<td></td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>.88</td>
<td>.63</td>
<td>.61</td>
<td>.85</td>
<td>.82</td>
<td>.86</td>
<td>.64</td>
<td>.8195</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>-.57</td>
<td>-.40</td>
<td>-.66</td>
<td>-.63</td>
<td>-.57</td>
<td>-.62</td>
<td>-.34</td>
<td>-.454</td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>-.89</td>
<td>-.95</td>
<td>-.86</td>
<td>-.92</td>
<td>-.94</td>
<td>-.88</td>
<td>-.70</td>
<td>-.431</td>
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<tr>
<td>( \theta_0 )</td>
<td>-.76</td>
<td>-.77</td>
<td>-.67</td>
<td>-.67</td>
<td>.21</td>
<td>-.63</td>
<td>-.37</td>
<td>-.082</td>
</tr>
</tbody>
</table>
5.2.2 Identification and Pre-whitening of Target Angular Velocity

The strong cross-correlation of target angular velocity with human output (Fig. 5.3) suggested taking this variable as a primary input. We now proceed to identify and prewhiten flight path 1 angular velocity. A listing of the full series is included in Appendix 3.

5.2.2.1 Identification

The auto-correlation and partial auto-correlation functions were evaluated, as before, by use of program TSA/1 (Appendix 1a) and the result is shown in Fig. 5.5. With zero differencing, the auto-correlation function $r_k$ dies out slowly, while the partial auto-correlation $\hat{\theta}_{kk}$ dies out exponentially after the first or second lag. While the need for some differencing is indicated, it is advisable to carry the possibility of modelling the series by an ARIMA (2,0,1) to the next stage. With one degree of differencing ($d=1$), the auto-correlation function dies out fairly quickly, while the partial auto-correlation function cuts off after lag two, suggesting a possible (2, 1, 0) or (2, 1, 1) model. The more general form of (2, 1, 1) will be explored further in the next stage of preliminary estimation.

5.2.2.2 Estimation

Results of preliminary estimates of the parameters (prog. TSA/2, Appendix 1b) and final least squares estimates (prog. TSA/3, Appendix 1c) are given in Table 5.4. Clearly the model to be selected for target angular velocity is an ARIMA (2, 1, 0). A test run on path 5 confirmed that the form of the model is correct and the parameter set is very nearly optimum even for such a widely different severe maneuver.
Fig. 5.5 Auto- and Partial Auto-Correlations for Various Differences of Target Angular Velocity for Azimuth, Path 1.
Table 5.4 Preliminary and Final Least Squares Estimates for Pre-whitening Target Angular Velocity of Azimuth Paths 1 & 5.

<table>
<thead>
<tr>
<th>model</th>
<th>Initial Guesses</th>
<th>Path #1</th>
<th>Path #5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(2,0,1)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial Cost</td>
<td>—</td>
<td>72.2</td>
<td>162.4</td>
</tr>
<tr>
<td>Final Cost</td>
<td>—</td>
<td>21.1</td>
<td>50.4</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>1.00</td>
<td>1.01</td>
<td>1.00</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>-.011</td>
<td>-.02</td>
<td>-.025</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>-.104</td>
<td>-1.00</td>
<td>-.99</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td><strong>(2,1,1)</strong></td>
<td></td>
<td>.453</td>
<td>2.167</td>
</tr>
<tr>
<td>or</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>[2,1,0]</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial Cost</td>
<td>—</td>
<td>.451</td>
<td>1.886</td>
</tr>
<tr>
<td>Final Cost</td>
<td>—</td>
<td>.451</td>
<td>1.886</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>1.910</td>
<td>1.90</td>
<td>1.87</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>-.953</td>
<td>-.953</td>
<td>-.95</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>0.0</td>
<td>0.0</td>
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</tr>
<tr>
<td>$\theta_0$</td>
<td>0.001</td>
<td>0.001</td>
<td>-.001</td>
</tr>
</tbody>
</table>
The time series model of target angular velocity is, therefore, established as

\[ \theta(B) V^d W_t = a_t \]

i.e.

\[ (1 - \theta_1 B - \theta_2 B^2) V W_t = a_t \tag{5.2.5} \]

Reversing the equation gives the required transformation that reduces target angular velocity to white noise:

\[ \alpha_t(W) = (1 - 1.9 B + .953 B^2) V W_t \tag{5.2.6} \]

where parameter values are substituted from Table (5.4).

5.3 Identification of the Transfer Functions

The pre-whitening transformation for each input is now available. We proceed to evaluate the dynamic relationship between inputs and output using the method of Section 3.6, where cross-correlations of the transformed input-output are used to estimate the impulse function. Initial estimates of \( \delta \) and \( \omega \) weights in the dynamic model are obtained from impulse function values according to equation 3.6.1.

The model of 3.6.1 is rewritten for convience as equation 5.3.1

\[ V_t = \omega(B) \chi_t - b - 1 + \frac{\theta(B)}{\theta(B)} a_t \tag{5.3.1} \]

where \[ \omega_s(B) = \omega_0 - \omega_1 B - \cdots - \omega_s B^s \]

and \[ \delta_r(B) = 1 - \delta_1 B - \delta_2 B^2 - \cdots - \delta_r B^r \tag{5.3.2} \]

for a model of order \((r, s, b)\).
5.3.1 Impulse Function Estimates

The sample cross-correlation function between the pre-whitened input ($\alpha$) and output ($\beta$) is evaluated according to equation 3.6.13, where at lag $k$:

$$r_{\alpha\beta}(k) = \frac{\sum \alpha_{t-k} \beta_t}{\sqrt{\sum \alpha_t^2 \cdot \sum \beta_t^2}}$$  \hspace{1cm} (5.3.3)

and the impulse function at lag $k$, $V_k$, from 3.6.14 is estimated by

$$\hat{V}_k = \frac{r_{\alpha\beta}(k) s_\beta}{s_\alpha}$$  \hspace{1cm} (5.3.4)

where $s$, as before, stands for sample standard deviation. This is basically the function of computer program TSA/5 (Appendix 1d). Results from different runs on various input-output combinations are presented in Tables 5.5 and 5.6 for inputs of error and angular velocity respectively. These values are also plotted in Figures 5.6 and 5.7.

Impulse function estimates obtained in this manner are inefficient, as will be clear in the next section when the final parameter values are obtained. This is further demonstrated by the widespread of these values, even for the same operator, as in Fig. 5.6 and 5.7.

A word about the pre-whitening transformation is in place. Figs. 5.6a and 5.6b indicate that auto-correlations of the pre-whitened input of $\nabla E$, $\rho_\alpha(\nabla E)(k)$ and $\hat{E}$, $\rho_\alpha(\hat{E})$ are similar, although $\nabla E$ gave lower values at low lags. The point to remember is that these functions are the best the whitening method could do to transform the series to uncorrelated random deviates. It is also noticed that the variance
Table 5.5. Cross-Correlations and Impulse Response Function Estimates
for E → T_Q After Pre-whitening - Azimuth Path 1, Run No. 1.

<table>
<thead>
<tr>
<th>k</th>
<th>$r_{\alpha\beta}(E, T_Q)(k)$</th>
<th>$\hat{v}_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.043</td>
<td>0.006</td>
</tr>
<tr>
<td>1</td>
<td>-0.068</td>
<td>-0.016</td>
</tr>
<tr>
<td>2</td>
<td>0.010</td>
<td>0.001</td>
</tr>
<tr>
<td>3</td>
<td>-0.080</td>
<td>-0.011</td>
</tr>
<tr>
<td>4</td>
<td>-0.029</td>
<td>-0.004</td>
</tr>
<tr>
<td>5</td>
<td>-0.081</td>
<td>-0.011</td>
</tr>
<tr>
<td>6</td>
<td>0.002</td>
<td>0.000</td>
</tr>
<tr>
<td>7</td>
<td>0.015</td>
<td>-0.002</td>
</tr>
<tr>
<td>8</td>
<td>0.005</td>
<td>0.001</td>
</tr>
<tr>
<td>9</td>
<td>0.063</td>
<td>0.009</td>
</tr>
<tr>
<td>10</td>
<td>0.010</td>
<td>0.001</td>
</tr>
</tbody>
</table>
Table 5.6. Cross-Correlations and Impulse Response Function Estimates for $W \rightarrow T_Q$ After Pre-whitening - Azimuth Path 1, Run No. 1.

<table>
<thead>
<tr>
<th>k</th>
<th>$r_{\alpha \beta}(W, T_Q)(k)$</th>
<th>$\hat{v}_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.009</td>
<td>.09</td>
</tr>
<tr>
<td>1</td>
<td>.151</td>
<td>.452</td>
</tr>
<tr>
<td>2</td>
<td>-.103</td>
<td>-.342</td>
</tr>
<tr>
<td>3</td>
<td>.089</td>
<td>.254</td>
</tr>
<tr>
<td>4</td>
<td>-.003</td>
<td>-.053</td>
</tr>
<tr>
<td>5</td>
<td>-.213</td>
<td>-.648</td>
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<tr>
<td>6</td>
<td>.1044</td>
<td>.131</td>
</tr>
<tr>
<td>7</td>
<td>-.036</td>
<td>-.124</td>
</tr>
<tr>
<td>8</td>
<td>-.024</td>
<td>-.059</td>
</tr>
<tr>
<td>9</td>
<td>.086</td>
<td>.233</td>
</tr>
<tr>
<td>10</td>
<td>.103</td>
<td>.321</td>
</tr>
</tbody>
</table>
Figure 5.6a. Estimated Auto-Correlations, Cross-Correlations and Impulse Functions of the First Differences of Tracking Error and Torque after Prewhitenning.
Azimuth, Path 1

<table>
<thead>
<tr>
<th>Run #</th>
<th>Op.</th>
<th>Variance</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>2</td>
<td>48.4</td>
<td>-.20</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>59.6</td>
<td>-.19</td>
</tr>
<tr>
<td>18</td>
<td>5</td>
<td>78.8</td>
<td>-.18</td>
</tr>
<tr>
<td>21</td>
<td>5</td>
<td>60.6</td>
<td>-.23</td>
</tr>
</tbody>
</table>

Fig. 5.6b. Estimated Auto-Correlations, Cross-Correlations and Impulse Functions for Rates of Tracking Error and Torque after Pre-Whitening.
and the mean of the transformed series are about one order of magnitude higher for \( \hat{E} \) compared with those of \( \nabla E \). As the theoretical value of the mean-for white noise— is zero, the large magnitude of the mean for the transformed \( \hat{E} \) reflects on the difficulty in applying pre-whitening procedure to the rate of error series. Moreover, while searching for the least squares estimates of the parameter set of the pre-whitening \( \hat{E} \) model, it was observed that the system was unstable and rather sensitive to step size, initial guesses — etc. Based on these observations, it appeared that a dynamic model with error rate as an input to the system will not be a successful one; rather, a wiser choice would be the differenced error, especially as the latter yields itself to time series analysis more readily since it is directly available.

It was therefore concluded that error rate be dropped from further consideration as a possible input to the tracking model, and only \( \nabla E \) and \( W \) were subjected to further consideration.

5.3.2 Preliminary Parameter Estimation

It was mentioned in the preceding section that estimates of the impulse functions, as obtained from (5.3.4) are rather inefficient. They are only useful as a rough preliminary parameter estimation and as a suggestion for the order of the model (Chapt. III, Sec. 6). Program TSA/6 (Appendix 1e) was written as suggested by Box and Jenkins (1971, Part v and Chapter X) to consider this problem. Based on estimates of the impulse response function and a tentative order of the model, the program gives initial guesses for \( \delta \) and \( \omega \) weights of equation (5.3.1), which are suitable to start a search routine for final parameter values.
The tentative order of the model is sought from behavior of the cross-correlation function for the pre-whitened input-output, and from the impulse response function (Chap. III, Section 6.1). For the system of $\mathbf{V}E + \mathbf{V}T_q$, Fig. 5.6a cross-correlations, suggest that no lag exists between input and output, meaning $b=0$ in our $(r,s,b)$ model of (5.3.1) and (5.3.2). As for the other parameters, $(r \text{ and } s)$, both the cross-correlation function and the impulses response function cut off after lag two, an indication that both values of $r$ and $s$ are possibly 2. Tentatively, the $(r,s,b)$ model of $\mathbf{V}E + \mathbf{V}T_q$ is therefore $(2, 2, 0)$.

As for the angular velocity, Fig. 5.7 - cross-correlation, suggest that $b=1$, which is also confirmed by the impulse response function. A cut off is indicated after lag 3, which means that the model may tentatively be of order $(2, 2, 1)$.

Finally, initial parameter values from computer program TSA/6 appear as Tables 5.7 and 5.8, where results with $b=0$ (no lag) and $b=1$ are both included to maintain generality at this stage of identification.

5.3.3 Least Squares Parameter Estimation

Given the order of the model and the initial parameter estimates, we now proceed to determine the least squares estimates for each transfer function model. To summarize the information available so far, it is recalled that input-output transfer function of $\mathbf{V}E \rightarrow \mathbf{V}T_q$ is of the form:

$$
\mathbf{V}T_{qt}(E) = \frac{\omega_{1,0} - \omega_{1,1} B - \omega_{1,2} B^2}{1 - \delta_{1,1} B - \delta_{2,1} B^2} \mathbf{V}E_t + N_t(E)
$$

Similarly for $\mathbf{W} \rightarrow \mathbf{T}_q$: 
Fig. 5.7. Auto-Correlations, Cross-Correlations and Impulse Function Est of the Differenced Target Angular Velocity and Torque after Pre-whitening.
Table 5.7 Preliminary Parameter Estimates for Suggested Transfer Function Models of $\nabla E + \nabla T_Q$ for Azimuth Path 1, Run #1

<table>
<thead>
<tr>
<th>Model $(r,s,b)$</th>
<th>$(2,2,0)$</th>
<th>$(2,2,1)$</th>
<th>Impulse Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>-.63</td>
<td>-.39</td>
<td>-.105</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>-.52</td>
<td>-.12</td>
<td>-.055</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td></td>
<td>-.11</td>
<td>.089</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>.11</td>
<td>.10</td>
<td>-.028</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>.18</td>
<td>-.14</td>
<td>.000</td>
</tr>
</tbody>
</table>

Table 5.8 Preliminary Parameter Estimates for Suggested Transfer Function Models of $\nabla W + \nabla T_Q$ for Azimuth Path 1 Run #1

<table>
<thead>
<tr>
<th>Model $(r,s,b)$</th>
<th>$(2,2,0)$</th>
<th>$(2,2,1)$</th>
<th>Impulse Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>-.20</td>
<td>1.53</td>
<td>-.206</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>.75</td>
<td>1.43</td>
<td>.786</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td></td>
<td>-.21</td>
<td>.311</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>.21</td>
<td>-1.10</td>
<td>.650</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>-1.53</td>
<td>1.52</td>
<td>.550</td>
</tr>
</tbody>
</table>
\[ T_{0t}(W) = \frac{\omega_{2,0} - \omega_{2,1}b - \omega_{2,2}b^2}{1 - \delta_{2,1}b - \delta_{2,2}b^2} W - b + N_t(W) \quad (5.3.6) \]

where \( N_t \) represents residual noise, and \( b \) in (5.3.6) is tentatively identified to be =1.

Computer program TSA/7 (Appendix Iff) is written to search for the best fit, as judged by the sum of squares of the differences between predicted and observed human output torque. Results obtained from this program on both transfer functions appear as Tables 5.9 and 5.10, where the search was made against a total of ten runs, which was all the available data for flight paths 1 and 5. It was thought necessary to run such an extensive search to observe model behavior, consistency and stability.

The method of obtaining the contents of Tables 5.9 and 5.10 deserves a brief description. Starting with the parameter values and forms of the model of Tables 5.7 and 5.8, program TSA/7 was executed for different runs of flight path 1. This resulted in different sets of least squares parameter values with corresponding "final cost values" (sum of squares). At the same time, test runs were made on models with \((r,s,b)\) values other than what was suggested by Table 5.7 and 5.8 in the preceding section; the effort being justified by the admitted inefficiency of the preliminary estimates, as mentioned above. Starting with different sets of initial values, it appeared that run no. 12 of flight path #1 gave a sum of squares with consistent optimum parameter values. The work was reexecuted with this set of values, from run #12, as the starting values, which is included in Table 5.10 under "Initial Guesses".
Table 5.9 Final Least Squares Parameter Estimates for a (2,2,0) Transfer Function Model of $VE - VT_Q$ of Azimuth Paths 1 and 5.

<table>
<thead>
<tr>
<th>Run #</th>
<th>Operator</th>
<th>Cost, Initial</th>
<th>Cost, Final</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
<th>$\omega_0$</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\sigma_a^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Path 1</td>
<td></td>
<td></td>
<td></td>
<td>123.7</td>
<td>35.4</td>
<td>50.4</td>
<td>35.2</td>
<td>18.5</td>
<td>89.9</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>75.1</td>
<td>35.3</td>
<td>46.1</td>
<td>32.1</td>
<td>13.7</td>
<td>76.1</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>.18</td>
<td>.013</td>
<td>-.06</td>
<td>-.06</td>
<td>.343</td>
<td>-.002</td>
</tr>
<tr>
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<td>2</td>
<td>5</td>
<td>2</td>
<td>.046</td>
<td>.117</td>
<td>.036</td>
<td>.036</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>.013</td>
<td>-.021</td>
<td>.076</td>
<td>.182</td>
<td>.180</td>
<td>-.056</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>.048</td>
<td>-.043</td>
<td>-.078</td>
<td>-.075</td>
<td>-.034</td>
<td>-.049</td>
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<td>5</td>
<td>.219</td>
<td>.288</td>
<td>.200</td>
<td>.086</td>
<td>.475</td>
<td>.131</td>
</tr>
<tr>
<td>Path 5</td>
<td></td>
<td></td>
<td></td>
<td>246.5</td>
<td>112.5</td>
<td>108.2</td>
<td>283.</td>
<td>208.6</td>
<td>84.2</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>84.2</td>
<td>66.7</td>
<td>168.</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>.013</td>
<td>.086</td>
<td>.125</td>
<td>.124</td>
<td>.041</td>
<td>.086</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>.100</td>
<td>.158</td>
<td>.034</td>
<td>.243</td>
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<tr>
<td></td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>.048</td>
<td>-.043</td>
<td>-.078</td>
<td>-.075</td>
<td>-.034</td>
<td>-.049</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>.131</td>
<td>.526</td>
<td>.414</td>
<td>1.05</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5.10 Final Least Squares Parameter Estimates for a (2,2,0) Transfer Function Model of $W - T_Q$

of Azimuth Paths 1 and 5.

<table>
<thead>
<tr>
<th>Run #</th>
<th>1</th>
<th>12</th>
<th>18</th>
<th>21</th>
<th>23</th>
<th>24</th>
<th>Initial Guesses</th>
<th>7</th>
<th>9</th>
<th>17</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operator</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Initial Cost</td>
<td>238.</td>
<td>18.95</td>
<td>62.3</td>
<td>46.4</td>
<td>23.7</td>
<td>127.</td>
<td>393.7</td>
<td>275.9</td>
<td>249.2</td>
<td>917.0</td>
<td></td>
</tr>
<tr>
<td>Final Cost</td>
<td>88.1</td>
<td>18.66</td>
<td>42.8</td>
<td>33.6</td>
<td>16.2</td>
<td>66.1</td>
<td>174.3</td>
<td>158.0</td>
<td>89.0</td>
<td>195.0</td>
<td></td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>1.00</td>
<td>.228</td>
<td>.54</td>
<td>.719</td>
<td>.795</td>
<td>.897</td>
<td>.209</td>
<td>.800</td>
<td>.191</td>
<td>.838</td>
<td>.750</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>-.538</td>
<td>-.174</td>
<td>-.46</td>
<td>-.522</td>
<td>-.673</td>
<td>-.590</td>
<td>-.175</td>
<td>-.579</td>
<td>-.127</td>
<td>-.447</td>
<td>-.317</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>.976</td>
<td>.865</td>
<td>.443</td>
<td>.784</td>
<td>.834</td>
<td>.292</td>
<td>.865</td>
<td>.455</td>
<td>1.20</td>
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<td>1.32</td>
</tr>
<tr>
<td>$\omega_1$</td>
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<td>1.10</td>
<td>.154</td>
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<td>-.004</td>
<td>1.10</td>
<td>.073</td>
<td>-1.32</td>
<td>-1.22</td>
<td>1.46</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>-.362</td>
<td>-.487</td>
<td>.066</td>
<td>-.475</td>
<td>-.554</td>
<td>.139</td>
<td>-.486</td>
<td>.246</td>
<td>-.306</td>
<td>.827</td>
<td>-.215</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.865</td>
<td>.963</td>
<td>.912</td>
<td>.937</td>
<td>.966</td>
<td>.871</td>
<td></td>
<td>.793</td>
<td>.783</td>
<td>.905</td>
<td>.839</td>
</tr>
<tr>
<td>$\sigma_a^2$</td>
<td>.546</td>
<td>.116</td>
<td>.266</td>
<td>.210</td>
<td>.100</td>
<td>.409</td>
<td></td>
<td>1.09</td>
<td>.976</td>
<td>.555</td>
<td>1.21</td>
</tr>
</tbody>
</table>
A correlation coefficient appears at the bottom of each column in Table 5.10 as $R^2$, being defined in this way (Bryant, 1970, Chapt. X)

$$R^2 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2 - \sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$  (5.3.7)

where

- $y$ = observed dependent variable.
- $\bar{y}$ = arithmetic sample mean
- $\hat{y}$ = predicted model value
- $n$ = no. of data points-size of sample

By comparing the value of $R^2$ as calculated above with critical values in distribution tables of this correlation coefficient, it appears that there is no reason to doubt model adequacy.

The variance of the residual, $\sigma_a^2$ is also included in Tables 5.10 and 5.11. The small magnitude of $\sigma_a^2$ is an additional indication of the success of the model.

5.3.4 Selection of a Parameter Set

Confronted with a number of different sets of mutually consistent parameter values, the problem of selecting a final set was considered. From the nature of the variations among parameter sets, it was obvious that an averaging process would not work. An attempt to search for an optimum using the average tracking error for each flight path was also abandoned, since the dynamic input-output relationship is distorted. This may be explained intuitively, as an average tracking error from, say six runs, will not produce the averaged torque output from those runs. Finally, it was decided that a consistent set may be selected.
from the run with the least noise, as indicated by the lowest sum of squares (final cost) and the smallest variance of residual. By inspecting Tables 5.9 and 5.10, it is seen that run no. 23 satisfied this criteria. As will be seen later, this choice proved to very reasonable in predicting the deterministic component of operator output.

It is realized that selecting a set of parameters in this manner implies that the model will not predict, at this stage, the averaged performance of the subject operators. Rather, it will predict the performance of a typical well trained operator, as demonstrated by one of his least noisy runs. Moreover, applying this criterion for selecting a parameter set has the additional advantage of avoiding the issue of defining an average or mean performance.

The last question to be settled before accepting a final parameter set is to check for cancellation. Following Box and Jenkins (1971, Chapt. XI), it was necessary to perform approximate factorization in the denominator and numerator of the transfer function in order to exclude any cancellable factors. This also insures that the model is as simple as possible, with the minimum number of parameters. This work appears in Appendix 3.

As no cancellation was uncovered, the transfer function of azimuth tracking is reported as:

$$\nabla T_Q(E) = \frac{(-.045 - .041 B + .034 B^2)}{(1 -.343 B + .180 B^2)} \quad \forall E \quad (5.3.8)$$

and

$$T_Q(W) = \frac{(.834 - 1.16 B + .554 B^2)}{(1 -.795 B + .673 B^2)} \quad \forall W \quad (5.3.9)$$
or

\[ V_{TQ}(E) = \frac{-0.045 (1 + 0.91 B - 0.75 B^2)}{(1 - 0.343 B + 0.180 B^2)} \quad V E \quad (5.3.10) \]

and

\[ T_{Q}(W) = \frac{0.834 (1 - 1.4 B + 0.665 B^2)}{(1 - 0.8 B + 0.67 B^2)} \quad W \quad (5.3.11) \]

5.4.1 Formulation of the Multiple Input Dynamic Model

The transfer functions of (5.3.10) and (5.3.11) express the maximum amount of the human output torque than can be correlated by each input separately. Clearly, equation (5.3.10) relating \( V \) to \( V_{TQ} \) also represents the input-output relationship of \( E \) to \( T_a \), which may be demonstrated by applying a linear summing operation on each side, keeping in mind that, initially, at time \( t=0 \), \( T_q = E = 0 \).

The model may be rewritten with subscripts 1 and 2 to distinguish between portions of the output torque described by error and angular velocity respectively as was done in (3.5.3) and (5.3.6); in symbols, this is:

\[ T_{Q_1} = \frac{\omega_{1,0} - \omega_{1,1} B - \omega_{1,2} B^2}{1 - \delta_{1,1} B - \delta_{1,2} B^2} \quad E \quad (5.4.1) \]

and

\[ T_{Q_2} = \frac{\omega_{2,0} - \omega_{2,1} B - \omega_{2,2} B^2}{1 - \delta_{2,1} B - \delta_{2,2} B^2} \quad W \quad (5.4.2) \]

When \( \omega_{1,0} \) and \( \omega_{2,0} \) are factored out, the rearranged equations are:

\[ T_{Q_1} = g_1 \frac{1 - \omega_{1,1} B - \omega_{1,2} B^2}{1 - \delta_{1,1} B - \delta_{1,2} B^2} \quad E \quad (5.4.3) \]
The final value of torque, \( T_Q \), is, ideally speaking, the sum of the two components,

\[
T_Q = T_{Q_1} + T_{Q_2}
\]

(5.4.5)

This assumes, clearly, that \( E \) and \( W \) are the only inputs, and that they are independent of each other. The first assumption can be asserted with some confidence at this stage; but the second one is known to be incorrect, due to the strong cross-correlation between target angular velocity and tracking error. For this reason, \( T_{Q_1} \) and \( T_{Q_2} \) need to be weighted in order for (5.4.5) to hold.

Let us put

\[
T_Q = G_1 T_{Q_1} + G_2 T_{Q_2}
\]

(5.4.6)

when (5.4.3) and (5.4.4) are substituted into the last expression the result is:

\[
T_Q = G_1 \{(\delta_{1,1}B + \delta_{1,2}B^2) T_{Q_1} + g_1 (1 - \delta_{1,1}B - \delta_{1,2}B^2)E\}
+ G_2\{(\delta_{2,1}B + \delta_{2,2}B^2) T_{Q_2} + g_2 (1 - \delta_{2,1}B - \delta_{2,2}B^2)W\}
\]

or

\[
T_Q = G_1 (\delta_{1,1}B + \delta_{1,2}B^2) T_{Q_1} + G_2 (\delta_{2,1}B + \delta_{2,2}B^2) T_{Q_2}
+ G_1 g_1 (1 - \omega_{1,1}B - \omega_{1,2}B^2) E + G_2 g_2 (1 - \omega_{2,1}B - \omega_{2,2}B^2)W
\]

(5.4.7)
The first two terms on the right hand side of (5.4.7) account for the auto-regressive effect of each component torque output. It should be possible to account for the total effect in an auto-regressive series in terms of the total torque $T_Q$ (Chapt. III, Sect. 2.). In other words, it is desired to find parameters $G_1$, $G_2$, $\delta_1$ and $\delta_2$ such that:

$$(1 - \delta_1 B - \delta_2 B^2) T_Q = G_1 (1 - \delta_{1,1} B - \delta_{1,2} B^2) T_{Q1} + G_2 (1 - \delta_{2,1} B - \delta_{2,2} B^2) T_{Q2} \quad (5.4.8)$$

When this is done, the model will have a much more convenient form as:

$$(1 - \delta_1 B - \delta_2 B^2) T_Q = G_1^* (1 - \omega_{1,1} B - \omega_{1,2} B^2) E + G_2^* (1 - \omega_{2,1} B - \omega_{2,2} B^2) W$$

or the equivalent form:

$$T_Q = \frac{G_1 (1 - \omega_{1,1} B - \omega_{2,1} B^2)}{(1 - \delta_1 B - \delta_2 B^2)} E + \frac{G_2 (1 - \omega_{2,1} B - \omega_{2,2} B^2)}{(1 - \delta_1 B - \delta_2 B^2)} W \quad (5.4.9)$$

where $G_1$ and $G_2$ have their equivalent values in (5.4.7), and the primes have been dropped with no loss of generality.

It is to be restated that in (5.4.9), $G_1$, $G_2$, $\delta_1$ and $\delta_2$ are parameters whose values are yet to be determined. The problem of preliminary parameter estimates to be supplied as initial guesses to the search routine was settled as follows. A reasonable estimate for $G_1$ and $G_2$ should be the original $g_1$ and $g_2$ values (from 5.4.3 and 4) for the individual transfer functions. As for $\delta_1$ and $\delta_2$, it is noticed that the magnitudes of $\delta_{2,1}$ and $\delta_{2,2}$ in the angular velocity transfer function are more than twice the magnitude of the corresponding $\delta_{1,1}$.
and $\delta_{1,2}$ in the tracking error transfer function model (equations 5.3.8 and 5.3.9). It is expected, therefore, that $\delta_1$ and $\delta_2$ would be close in value to $\delta_{2,1}$ and $\delta_{2,2}$ respectively. The following are then expected to be efficient preliminary estimates:

$$\delta_1 = 0.8, \delta_2 = -0.67, G_1 = -0.045, G_2 = 0.834.$$  (5.4.10)

In order to insure that the above preliminary estimates are in the correct region of the parameter space, a search was made (program TSA/7, Appendix 1f) on $G_1$ and $G_2$ only, holding $\delta_1$ and $\delta_2$ to their values of (5.4.10). Table (5.11) demonstrates the success of the transfer function model, where correlation coefficients (as $R^2$) averaged about 92% for flight path 1 runs, and about 82% for flight path 5. The bottom half of Table 5.11 shows results of test runs made with a selected set of values ($G_1 = -0.016$ and $G_2 = 0.860$), which shows that the system is stable with one set of parameters that can approximate the different runs very well. The values assigned $G_1$ and $G_2$ are a rough average of those recurring consistently and most frequently in the different runs of Table 5.11. The purpose of this step is that, although optimum parameter values varied significantly between runs, it is possible to reproduce the data to an acceptable level with a single parameter set. The work of this selection, however, provides preliminary estimates only, as final values will be reported in the next section when the dynamic tuning of the model is done.

5.4.2 **Dynamic Tuning of Model Parameters**

Optimization of model parameters has been done, so far, on the transfer function of the model, by comparing inputs from human operator
Table 5.11 Optimization of Weights on Multiple Input Transfer Function Model

<table>
<thead>
<tr>
<th>Run No.</th>
<th>Path 1</th>
<th>Path 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operator</td>
<td>1</td>
<td>12, 18</td>
</tr>
<tr>
<td>1</td>
<td>108.8</td>
<td>33.2</td>
</tr>
<tr>
<td>2</td>
<td>107.5</td>
<td>22.0</td>
</tr>
<tr>
<td>Initial Cost</td>
<td>**</td>
<td></td>
</tr>
<tr>
<td>Final Cost**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_1(E)$</td>
<td>-.0403</td>
<td>-.014</td>
</tr>
<tr>
<td>$G_2(W)$</td>
<td>.860</td>
<td>.861</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.836</td>
<td>.956</td>
</tr>
<tr>
<td>$\sigma_a^2$</td>
<td>.671</td>
<td>.136</td>
</tr>
<tr>
<td>Cost*</td>
<td>122.45</td>
<td>22.0</td>
</tr>
<tr>
<td>$R^2*$</td>
<td>.813</td>
<td>.956</td>
</tr>
<tr>
<td>$\sigma_a^2$</td>
<td>.765</td>
<td>.136</td>
</tr>
</tbody>
</table>

**All runs with initial values of $G_1 = -.045, G_2 = .834$

*Test performance with a single parameter set of $G_1 = -.016$ and $G_2 = .860$. 
data, with the torque output that human subjects produced. In other words, all the error \( E \) and torque \( T_q \), have been those produced by manual tracking. The next phase requires fitting the transfer function model of (5.4.9) into a tracking system, where it receives flight path data, and will be required to produce tracking errors that will match those obtained in the experimental runs, in a closed loop manner, as error is fed back (Fig. 5.1).

Throughout the remainder of this section, the criterion for the optimum shall be tracking error, and not torque output. This is required by the consideration that the whole purpose of a tracking model is to be able to predict operator tracking errors. Since no attempt is made to simulate the internal mechanism of the human tracker, it is likely that the model will produce a torque pattern different from the human output, in order to resemble most closely his tracking errors. While the choice of tracking error as criterion is widely accepted, a decision had to be made as to how to define the cost function. The sum of square error can be used; where error is defined as the difference between model tracking error and human tracking error; but it heavily penalizes large errors. In our case, this is a disadvantage, since large errors always occur around cross-over, which is the region of least need for tracking model fidelity. A better choice seemed to be the sum of absolute error, since it implies even weighting all along the flight path.

The dynamic tracking program, which is documented at the end of this work (Appendix 5) was used - without the stochastic part - to track the same flight paths that were tracked in the experimental runs;
and the resulting error in azimuth was compared with the mean tracking error of the human operators. Parameter adjustment and search for the optimum was done by computer program TSA/7; and flight path 1, with averages from six replicates, was used. As was mentioned in the last section, the only parameters that are allowed to vary, in the model, were $\delta_1$, $\delta_2$, $G_1$ and $G_2$.

Two other parameters, not explicitly in the tracking model, where also varied in the course of further refining model performance. These were lag interval and dead time. Initially, a lag interval equal to the sampling interval of the data (0.25 sec.) was used; and a dead time equal to 0.15 sec., being a physiologically acceptable value.

Following are the final optimum parameter values, obtained from the regression analysis, for azimuth tracking.

i) Torque auto-regressive parameter, $\delta_1 = 0.890$ (dimensionless)

ii) " " " " , $\delta_2 = -0.299$ (dimensionless)

iii) Tracking error weighting factor $G_1 = -13.00$ ft-lb./radian

iv) Angular velocity " " $G_2 = 24.889$ ft-lb./rad. per sec.

It is noticed that $\delta_1$ and $\delta_2$ are not far from their initial values in the transfer function model (Table 5.11), while $G_1$ and $G_2$ are of a different order of magnitude. This was found necessary when constructing the dynamic model, where internal calculations use radians and radians/sec instead of mils and degrees/sec. for tracking error and angular velocity respectively. The corresponding values after allowing for the different units would be:

$$G_1 = -13.00 \text{ (ft.-lb./Rad.)} \times \frac{1}{1018} \text{ (Rad./ml.)} = -.013 \text{ ft. lb./ml.}$$
These are to be compared with -.016 and 0.89 respectively (from the last section).

5.5 Elevation Tracking Model

5.5.1 Form of the Model

The transfer function model for azimuth tracking was expressed in (5.4.10) as

\[ T_q = \frac{G_1(1 - \omega_{1,1}B - \omega_{1,2}B^2)}{1 - \delta_1B - \delta_2B^2} E + \frac{G_2(1 - \omega_{2,1}B - \omega_{2,2}B^2)}{1 - \delta_1B - \delta_2B^2} W \]

A change in the nomenclature will be necessary, from now on, to accommodate elevation tracking inputs, outputs and parameters. Subscript "e" and "a" will be added to denote elevation and azimuth respectively. The above expression becomes, after some rearrangement:

\[ (1 - \delta_{1,a}B - \delta_{2,a}B^2) T_{qa} = G_{1,a}(1 - \omega_{1,1}B - \omega_{1,2}B^2) E_a \]

\[ + G_{2,a}(1 - \omega_{2,1}B - \omega_{2,2}B^2) W_a \] (5.5.1)

So far, all the analysis and formulation was done on the azimuth axis of tracking, the result of which is the model of (5.4.1) and (5.5.1) with the parameter values mentioned above. We now turn our attention to elevation. In order to construct the elevation counter part of (5.5.1), it was necessary to identify the parameters that would change and those that should stay the same in both axis, with the implicit assumption that the form of the model is acceptable and should work for both axis. As for the error, parameters \( \omega_{1,1} \) and \( \omega_{1,2} \)
are peculiar to the general pattern of human tracking error, and as such, should be the same as we move from one flight path to another, or from azimuth to elevation. The same applies to parameters \( \omega_{2,1} \) and \( \omega_{2,2} \) which express the way a human operator makes use of target velocity information, with little or no regard to whether it is in azimuth or elevation.

As for the weighting parameters \( G_1 \) and \( G_2 \) in (5.5.1) and (5.4.10), they regulate the influence of tracking error and angular velocity, and they may be thought of as "scaling factors". Noting that tracking errors and target velocities are of different magnitudes in elevation compared with azimuth, we would expect \( G_1 \) and \( G_2 \) to have different values. The \( \delta \) parameters are the most direct expression of operator tracking strategy, as they measure the extent to which his performance is auto-regressed on its past values. As tracking strategies differ from azimuth to elevation, \( \delta_1 \) and \( \delta_2 \) are also expected to change.

The preceding discussion leads to a model for elevation tracking of the same form as (5.5.1), i.e.

\[
(1 - \delta_{1,e} B - \delta_{2,e} B^2) T_{e} = G_{1,e} (1 - \omega_{1,1} B - \omega_{1,2} B^2) E_e
+ G_{2,e} (1 - \omega_{2,1} B - \omega_{2,2} B^2) W \quad (5.5.2)
\]

where \( \omega_{1,1}, \omega_{1,2}, \omega_{2,1} \) and \( \omega_{2,2} \) have the values assigned to them in (5.3.10) and (5.3.11), while \( \delta_{1,e}, \delta_{2,e}, G_{1,e} \) and \( G_{2,e} \) were to be evaluated.

5.5.2 Estimation of Elevation Parameters

The form of the model for elevation tracking is that of equation
(5.5.2) above. The cost function for regression was similar to that used with azimuth, i.e., The sum of the absolute difference in tracking errors between the model and the averaged human performance. Flight path 1 tracking errors, with an average of six runs, were used in the search.

When the search terminated, using program TSA/7 as before, the optimum set of parameters from path #1 was tested against flight path #5, but the result was unsatisfactory. An attempt was made to use different sets of parameters before and after cross-over, even though, it appeared that different flight paths required different sets of parameters. It was suspected that data from path 1 might be a poor choice, and another path should be selected for regression. Path 5 data was used and when optimum parameters for this path were tested against other paths there was a substantial improvement except for a consistent bias in the pre-cross-over region, where the model predicted smaller errors than the averaged operator error. This was finally overcome by regressing on the pre-cross-over data, where a slight adjustment in the parameter set was necessary. The model was tested against all flight paths with very good results.

5.6 Tracking Performance of the Deterministic Model

The final set of parameters in the model of the form (5.5.11)

\[(1 - \delta_1 B - \delta_2 B^2)T_Q = G_1(1 - \omega_{1,1} B - \omega_{1,2} B^2)E + G_2(1 - \omega_{2,1} B - \omega_{2,2} B^2) W\]

for both azimuth and elevation is listed in Table 5.12. In this model
Table 5.12 Final Parameters for the Deterministic Component of Time Series Tracking Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Optimum Value in Azimuth</th>
<th>Optimum Value in Elevation</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_1$</td>
<td>0.89</td>
<td>0.336</td>
<td>D-less</td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>-0.30</td>
<td>0.284</td>
<td>D-less</td>
</tr>
<tr>
<td>$G_1$</td>
<td>-13.00</td>
<td>-27.665</td>
<td>Ft-lb/deg</td>
</tr>
<tr>
<td>$G_2$</td>
<td>23.889</td>
<td>32.41</td>
<td>Ft-lb/ Rad./Sec.</td>
</tr>
<tr>
<td>$\omega_{1,1}$</td>
<td>-0.91</td>
<td></td>
<td>D-less</td>
</tr>
<tr>
<td>$\omega_{1,2}$</td>
<td>0.75</td>
<td></td>
<td>D-less</td>
</tr>
<tr>
<td>$\omega_{2,1}$</td>
<td>1.4</td>
<td></td>
<td>D-less</td>
</tr>
<tr>
<td>$\omega_{2,2}$</td>
<td>-0.665</td>
<td></td>
<td>D-less</td>
</tr>
</tbody>
</table>
\( T_q \) = Torque ft. lb., \( E \) = angular tracking errors in degrees, and \( W \) = angular velocity in radians/sec. Table 5.13 lists values of the mean absolute errors, obtained with this set of parameters against the averaged operator tracking errors, for selected paths.

Finally, a graphic demonstration of the success of the model in predicting the average human performance is presented in Figures 5.8 through 5.11. Graphs for other flight paths are included in Appendix 3 at the end of this thesis.
Table 5.13 Performance of Deterministic Tracking Model on Main Flight Paths Measured as Mean Absolute Error.

MAE = \( \frac{1}{n} \sum_{i=1}^{n} | \text{Av. human error} - \text{model mean error} | \)

\( n = 160 \) data points.

<table>
<thead>
<tr>
<th>Flight Path*</th>
<th>Mean Absolute Error, mls.</th>
<th>Az.</th>
<th>El.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.3</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.6</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.7</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2.3</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2.5</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2.4</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>2.5</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>2.5</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>2.2</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>2.4</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>2.9</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>2.6</td>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>

*Flight path descriptions appear in Table 4.3.
Figure 5.8: Averaged Human Error and Predicted Mean Error - Flight Path #1.
Figure 5.9. Averaged Human Error and Predicted Mean Error - Flight Path #5.
Figure 5.10. Averaged Human Error and Predicted Mean Error - Flight Path #10.
Figure 5.11. Averaged Human Error and Predicted Mean Error - Flight Path #16.
CHAPTER VI
IDENTIFICATION OF OPERATOR NOISE

The stochastic component of the model arises from the presence of noise in the system, which is actual "plant" noise and not measurement or observation noise. The latter is usually independent of process variables, and is often "white" to a good approximation. "Plant" noise, on the other hand, is often non-white, originates inside the control loop and propagates around the entire loop. A standard practice in identification is to model colored noise in the form of a transition matrix that will be adjoined with the state transition matrix (Sage and Melsa, 1971, Chapt. VIII). In Chapt. III (Sec. 3.4 and 3.6) a procedure was suggested, where for a model of the form:

$$Y_t = \frac{\omega(B)}{\delta(B)} X_t + N_t$$

(6.0.1)

$N_t$, the noise component was to be identified as an ARIMA model:

$$N_t = \frac{\theta(B)}{\delta(B)} a_t$$

(6.0.2)

which is linearly added to the deterministic part of the model to form the general stochastic dynamic process.

The purpose of this section is to document the identification of noise by determining $\theta$, $\delta$ and the variance of the random deviates, $a_t$.

The point at which noise enters the control loop can be chosen at
convenience, since shifting it around the control loop or grouping several noise sources together will not affect the model (Sage and Melsä, 1971, Box and Jenkins, 1971, Chapter XIII). Figure 5.1 at the beginning of Chapter V indicates where noise was assumed to enter the control loop in this model.

6.1 Analysis of the Residual

The first step in the identification of noise was to study the statistical properties of the residual, such as the auto-correlations and cross-correlations with systems inputs. "Residual" here refers to the remnant torque, i.e., the difference between human operator torque output and mean model output. Figures 6.1 through 6.5 contain functions of the auto-correlation and cross-correlation for the residual, and its first difference with system inputs and angular velocity and tracking error. Cross-correlations between the residual torque and the total torque output is also shown in the figures. Fig. 6.5 describes the auto-correlation function for individual runs of path 1, while the rest of the graphs show the average value of the function from all available data on flight paths 1 or 5 as indicated. This is not to be confused with the value of the auto and cross-correlation functions for the average human performance, which was the case when formulating the deterministic model. In this case, the different replicate runs were treated separately and the resulting auto and cross-correlations averaged for each path.

The residual auto-correlation function changes as we move from path 1 - (an easy case) to path 5 (a difficult one), as illustrated
Figure 6.1. a) Auto-Correlations of Azimuth Residual Torque and Cross-Correlations With b) Total Torque, c) Target Angular Velocity and d) Tracking Error.
Figure 6:2. a) Auto-Correlations of the Magnitude of Azimuth Residual Torque; and Cross-Correlations with b) Total Torque, c) Target Angular Velocity, d) Tracking Error.
Figure 6.3. a) Auto-Correlations of the First Difference of Azimuth Residual Torque, and Cross-Correlations with the Differenced Series of b) Total Torque c) Target Angular Velocity c) Tracking Error.
Figure 6.4. Cross-Correlations of Azimuth Residual Torque with the First Differences of a) Total Torque, b) Target Angular Velocity and c) Tracking Error.
Figure 6.5. Auto-Correlations of Azimuth Residual Torque for Various Runs of Path 1.
in Fig. 6.1-a. This confirms the feeling that the residuals are strongly path dependent. Figures 6.1-b and c are almost identical, indicating the strong cross-correlation the residual has with the output torque and target velocity; while Fig. 6.1-d tells the relatively weaker dependence of the residual on tracking errors. When the magnitude of the residual is considered, (Fig. 6.2 a-d) the same pattern is maintained but the correlation is stronger throughout. The behavior of the first difference of the residual and its correlation with \( \nabla W \), \( \nabla E \) and \( \nabla T_Q \) appear in Fig. 6.3, where it is seen that the pattern of the auto-correlation is almost lost, while cross-correlations are still evident, but very weak. Run to run variations in the auto-correlations of the residual, even for a relatively easy path like path #1 are significant as demonstrated in Fig. 6.5, which suggested the idea of working with average auto and cross-correlations from different runs.

A final observation on the magnitude of noise is that it changes by several orders of magnitude as we move along the flight path; the maximum occurring shortly after cross-over. A test run on the variance of noise showed that it too changed along the flight path by a similar margin.

The above suggestion of constructing another dynamic model that describes an input-output relationship between angular velocity, torque and tracking error, to be adjoined with the deterministic model, was attempted by employing \( W \), the angular velocity as an input, with the available pre-whitening transformation of Sec. 5.3. The cross-correlations between the pre-whitened angular velocity, \( \alpha_w \), and the transformed
residual $\tilde{\beta}_N$ did not exhibit any pattern and the method had to be abandoned.

In searching for another method, it was observed that the variance changed along the flight path in a manner very much resembling that of the angular velocity, which suggested the possibility of developing a dynamic model with the angular velocity as an input and the variance of noise as an output. Assuming a model of the form (6.0.2), the variance thus evaluated will produce the random shocks that will generate the noise series. Elements of noise, $N_t$, will be simply added to the torque as determined by the tracking model of Sec. 5.5.

To summarize, the method may be stated as follows:

1) the accepted form of the noise model is that of (6.0.2)

$$N_t = \frac{\theta(B)}{\phi(B)} a_t$$

2) the parameters $\theta$ and $\phi$ are evaluated by the standard methods of Chapter III.

3) the random deviates $a_t$ are assumed "normal", with a zero mean and a variable variance that is the output of a dynamic model whose input is $\dot{w}$, target angular velocity.

The following sections report the work done on the identification of noise $N_t$ and the dynamic model of variance $V_t(a)$.

6.2 Identification of Noise Series

6.2.1 Order of the Model

The objective in this section is to document the identification of
the residual torque as an ARIMA \((p,d,q)\) process. The order of the model was derived from the behavior of the auto-correlation function. As Fig. 6.1-a shows, the auto-correlations cut off after one lag in path 1, but die out exponentially from the second lag in path 5. Differencing was not necessary as Fig. 6.3-a shows no pattern in the difference series. The partial auto-correlation function suggests a "p" value of 1 or 2 may be used for the auto-regressive parameter. Hence a tentative model of ARIMA \((2,0,2)\) was tried, being the more general form, i.e.

\[
(1 - \theta_1 B - \theta_2 B^2) N_t = (1 - \theta_1 B - \theta_2 B^2) a_t \quad (6.2.1)
\]

### 6.2.2 Parameter Estimation

Initial parameter estimates were obtained using computer program TSA/2 (Appendix 1b). The final estimation of optimum parameters was done by the search technique of program TSA/3 (Appendix 1c) as before.

The objective of the search was to minimize the sum of squares of the random noise components of the series \(a_t\) in (6.2.1). The elements were calculated from

\[
a_t = N_t - \theta_1 N_{t-1} - \theta_2 N_{t-2} + \theta_1 a_{t-1} + \theta_2 a_{t-2} \quad (6.2.2)
\]

where \(N_t\) is the difference between model torque and experimental torque. Values of \(a_t\) at the start of the series were, as before, set to their unconditional mean of zero.

The resulting model had the following form and parameter values:
or
\[ a_t = \varphi N_t - 0.439 a_{t-1} \]  \hspace{1cm} (6.2.3)

where \( \varphi_1 = 1.0, \varphi_2 = -0.439, \varphi_3 = \varphi_4 = 0.0 \).

The model was incorporated in the main model (Section 5.6 above) and tested by tracking various flight paths. Elements of torque noise series were produced from (6.2.3) as

\[ N_t = N_{t-1} + a_t + 0.439 a_{t-1} \]  \hspace{1cm} (6.2.4)

The random shocks, \( a_t \), were generated by a Gaussian random number generating program (Appendix 5) assuming a zero mean and a variance predicted by the model of the next section. The test, however, was unsatisfactory, as the target was lost on severe maneuvers, e.g., flight paths 5 and 10. This made it necessary to reconsider the noise model of (6.2.3) and seek a substitute for the least squares criterion.

Remedial action was taken by allowing the noise series of (6.2.3) two more degrees of freedom in this way:

\[ a_t = \varphi_1 N_t - \varphi_2 N_{t-1} + \varphi_3 a_{t-1} \]  \hspace{1cm} (6.2.4)

As for a criterion for parameter search, a method utilizing auto and cross-correlations was attempted. The method was originally suggested by Lee (1951), and later by Goodman and Reswich (1956) and Evaleigh (1967, Chapter VII). It was successfully applied by Froisy (1971), whereby auto and cross-correlations of the model output are matched with corresponding values in the experimental data, the objective being to minimize the sum of the squared difference between corresponding
elements of the two sets. The method did not seem to possess a unique minimum, and failed to converge unless started from carefully selected initial guesses. In fact, the parameter set that corresponded to the best minimum gave unacceptable results when compared with human performance, as the pattern of the resulting tracking error did not resemble those of the experimental data.

Finally, an acceptable procedure was established, based on the work by A. Pierce (1969) on the distribution of residuals from time series models, and Box and Jenkins (1971) criteria for overall model adequacy as discussed in Chapter III above. It is recalled that, if the model is adequate, then auto-correlations of the residual random deviates $a_t$ and cross-correlations with system inputs should have a chi-square distribution. With reference to (6.2.1), the test involves evaluating $a_t$ series, for which auto- and cross-correlations are obtained to a length - say 15 lags - after which they may be considered essentially zero. Next, the sum of squares of these auto and cross-correlations is calculated and compared with tables of chi-square distribution with specified degrees of form and significance level. If the sum is less than the value indicated in the tables, the model is accepted, otherwise it is doubted.

It is possible, therefore, to use such a positive-definite function as a cost function to be minimized - using part of the data - in search for an optimal set of parameters. As before, computer program TSA/3 (Appendix 1) was used for this purpose.

In executing the above regression, flight path #1 was the working path, where the six replicates were treated as a seasonal time series
(Box and Jenkins, 1971, Chapter IV), meaning they were treated as a periodically repeated realizations of the same stochastic process. This was a convenient way of making full use of the available data. Rewriting (6.2.5), the residuals $a_t$ were evaluated as:

$$a_t = \theta_1 N_t - \theta_2 N_{t-1} + \theta_1 a_{t-1}$$

and the cost function is expressed as:

$$\min_{\theta_1, \theta_2, \theta_1} \sum_{j=1}^{NH} \sum_{k=1}^{K} [r_a^2(k) + r_{aw}^2(k) + r_{aw}^2(k)]$$

where

- $NH$ = number of replicate human operator runs, = 6 for path 1
- $K$ = maximum lag to be considered, usually 10-15
- $r_a(k)$ = sample auto-correlations of random deviate $a_t$, at lag $k$
- $r_{aw}(k)$ = sample cross-correlation at lag $k$, between $a_t$ and target angular velocity $W_t$
- $C_{aw}(k)$ = cross-correlation at lag $k$, between $a_t$ and tracking error $E_t$

The optimum set of parameters for the model of (6.2.5) was found to be:

$$\theta_1 = 0.407, \quad \theta_2 = 0.179, \quad \theta_k = -0.558$$
The form of the model became

$$a_t = 0.407 N_t - 0.179 N_{t-1} - 0.558 a_{t-1} \quad (6.2.8)$$

and the form of the noise model that predicts the stochastic torque component of the tracking model became

$$N_t = 0.44 N_{t-1} + 2.457 a_t + 1.371 a_{t-1} \quad (6.2.9)$$

When the last expression is rearranged and written in this form:

$$(1 - .44B)N_t = 2.457 (1 + .558B) a_t \quad (6.2.10)$$

it is seen to be both stationary and invertible, (Box and Jenkins, 1971, Chapter III). In order to test model adequacy, diagnostic checks were next applied to 'white noise' series, $a_t$.

### 6.3 Diagnostic Checks on White Noise

In the model of (6.2.2) and (6.2.8), the series $a_t$ should approximate white noise characteristics. Box and Jenkins (1971) suggest applying diagnostic checks on the residual $a_t$ from the model to reveal any model inadequacy. The checks are to be applied on the auto-correlations $r_a$ and cross-correlations $r_{aw}$ and $r_{ae}$ of remnant with target angular velocity and tracking error, as was discussed under 5.6 and 6.2 above.

Figure 6.6 shows the values of the auto-correlations of $a_t$ and cross-correlations with the angular velocity for path 1. The cross-correlations with tracking error were practically zero and are not shown. Figure 6.7 is the result of testing the same noise model on
Fig. 6.6. Autocorrelations of Torque Noise Remnant and Crosscorrelations with Target Angular Velocity-Path 1 Azimuth
Figure 6.7. Autocorrelations of Torque Noise Remnant and Crosscorrelations with Target Angular Velocity-Path 5 Azimuth.
Table 6.1. Auto-Correlations of "White Noise" and Chi-Square Test.

<table>
<thead>
<tr>
<th></th>
<th>Path 1</th>
<th>Path 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>( r_k )</td>
<td>( r_k )</td>
</tr>
<tr>
<td>1</td>
<td>0.034</td>
<td>0.105</td>
</tr>
<tr>
<td>2</td>
<td>-0.016</td>
<td>0.209</td>
</tr>
<tr>
<td>3</td>
<td>-0.159</td>
<td>-0.103</td>
</tr>
<tr>
<td>4</td>
<td>-0.112</td>
<td>0.132</td>
</tr>
<tr>
<td>5</td>
<td>0.046</td>
<td>-0.073</td>
</tr>
<tr>
<td>6</td>
<td>0.122</td>
<td>0.099</td>
</tr>
<tr>
<td>7</td>
<td>-0.068</td>
<td>-0.148</td>
</tr>
<tr>
<td>8</td>
<td>-0.066</td>
<td>0.030</td>
</tr>
<tr>
<td>9</td>
<td>-0.102</td>
<td>0.058</td>
</tr>
<tr>
<td>10</td>
<td>0.065</td>
<td>0.023</td>
</tr>
<tr>
<td>11</td>
<td>0.027</td>
<td>-0.077</td>
</tr>
<tr>
<td>12</td>
<td>0.030</td>
<td>-0.028</td>
</tr>
<tr>
<td>13</td>
<td>-0.046</td>
<td>-0.005</td>
</tr>
<tr>
<td>14</td>
<td>-0.033</td>
<td>0.007</td>
</tr>
<tr>
<td>15</td>
<td>-0.005</td>
<td>-0.011</td>
</tr>
<tr>
<td>16</td>
<td>0.000</td>
<td>0.009</td>
</tr>
<tr>
<td>17</td>
<td>0.017</td>
<td>-0.044</td>
</tr>
<tr>
<td>18</td>
<td>-0.009</td>
<td>-0.010</td>
</tr>
<tr>
<td>19</td>
<td>-0.029</td>
<td>-0.037</td>
</tr>
<tr>
<td>20</td>
<td>-0.027</td>
<td>-0.013</td>
</tr>
</tbody>
</table>

\[ \Sigma r_k^2 = 0.087 \quad 0.135 \]

Distance root mean square error

rmse 0.23 0.55

Path 1
\[ n \cdot \sum r_k^2 = 160 \cdot 0.087 = 13.92 \]

\[ c/f \text{ with } \chi^2 @ 20-3 = 17 \text{ deg. of freedom ok at } \sim 70\% \text{ level} \]

Path 5
\[ n \cdot \sum r_k^2 = 160 \cdot 0.135 = 21.60 \]

\[ c/f \text{ with } \chi^2 @ 17 \text{ deg. of freedom ok at } 20\% \text{ level}. \]
flight path 5. Table 6.1 lists the corresponding values for both flight paths for the first 20 lags. A chi-square test was applied to residual auto and cross-correlations and indicated no reason to doubt model adequacy. This result is included in Table 6.1. It is important to note that model fidelity is demonstrated by passing the test on flight path 5, which is very different and much more difficult to track compared with flight path 1 against which the model was tuned.

### 6.4 Identification of White Noise Variance

Work on the variable variance model that was suggested in section 6.1 will be reported here. The idea was to formulate a dynamic time series model of the form:

$$V_t(a) = \frac{\omega(B)}{\delta(B)} W_{t-b}$$

(6.4.1)

where $V_t(a)$ is the variance of the random deviates $a_t$, at time $t$; $W_t$ is the angular velocity at time $t$, and $b$ is the number of lag intervals. The method of Chapter III, (Sections 3.4 and 3.5) were again applied to this case. Transformation of target angular velocity to white noise was reported in Section 5.3, which resulted in a 'white' series, $a_W$. The same transformation was applied to the sample variance $V(a)$ to produce a transformed variance series $\beta_V$. The variance series was generated by evaluating the variance at each interval for the white noise $a_t$ from the six-run sample of path 1.

In the course of searching for the best fit, the possibility of using the standard deviation $S_t(a)$ rather than the variance, was indicated. An improved fit was obtained with a model of the general
where \( \omega(B) = \omega_0(B) = 0.945 \cdot (1 - 2B - B^2) = 0.945 \cdot (1 - B)^2 = 0.945 \cdot V^2 \)

\[ \delta(B) = \delta_0 \cdot B = 0.268 \cdot B \]

b = 0

The final expression for the standard deviation of the random deviates \( a_t \), was found to be:

\[ S_t = |(0.268 \cdot S_{t-1} + 0.945 \cdot V^2 \cdot W_t)| \quad (6.4.3) \]

where the absolute sign is included to indicate that only positive values are accepted. As an indication of the goodness of fit, a correlation test \( R^2 \) was made; the result showed that 90.7% of the variance data was correlated, which revealed no model inadequacy. When the same model was tested on data from flight path 5 (four replicates), \( R^2 \) value of 83% was obtained, which was a further assurance of model fidelity.

As for elevation, the same model was tested against tracking data for the elevation axis of both flight paths 1 and 5. Using the same parameter values as in azimuth, the performance was very good, and the model was accepted as applicable to both azimuth and elevation tracking.

6.5 Final Stochastic Tracking Model

The objective of this chapter was achieved by incorporating the noise model of Section 6.3 with the model that predicts mean human performance - Section 5.5. To summarize the procedure, the following
is presented:

a) Initially, torque output and angular misalignment are set to zero in both azimuth and elevation channels — i.e. gun directed towards target.

b) As the target moves with time, resulting tracking errors are calculated.

c) Corrective torque $T_q(t)$ is evaluated from (5.4.9) and (5.4.10).

d) The standard deviation $S_t$ of the random component is evaluated according to (6.4.3).

e) A random number generator is called to supply a "shock" $a_t$, with $S_t$ as the standard deviation and a mean = 0.

f) The element of noise $N_t$, at time $t$ is evaluated according to (6.2.9).

g) Total torque, $T_{Q_t} = T_q + N_t$, is checked against the maximum limits of $\pm 10$ ft.-lbs.

h) Further checks are applied to the movement of the gun, both on its velocity and angular acceleration, to insure their stay within the physical bounds of the system.

The final tracking model is incorporated in computer program TRAKSTS, which is included as Appendix 5. Figures 6.8 and 6.9 show two realizations each of the stochastic model for flight path #1, while Figures 6.10 and 6.11 are of the human operator data which is included for comparison. Similarly, Fig. 6.12 gives two realizations of the model for flight path #5 and Figure 6.13 illustrates corresponding human operator runs.
Figure 6.8. Two Realizations of Stochastic Model on Path 1.
Figure 6.9. Two More Realizations of Stochastic Models on Path 1.
Figure 6.10. Two HOP Runs on Path 1.
Figure 6.11. Two More HOP Runs on Path 1.
Figure 6.12. Two Realizations of Stochastic Models on Path 5.
Two HOP Runs

Azimuth Error

Elevation Error

Figure 6.13. Two HOP Runs on Path 5,
In concluding this dissertation, the final result will be reproduced and the stochastic model critically examined and evaluated. This will be followed by some suggestions for further research that is needed in this fertile field.

7.1 Results

The objectives of the research documented in this thesis was achieved by the stochastic model of Chapters V and VI. The general form of the model is that of (5.4.10) with noise, which applied to both azimuth and elevation channels:

$$T_0(t) = \frac{G_1(1 - \omega_1 B - \omega_1^2 B^2)}{1 - \delta_1 B - \delta_2 B^2} E + \frac{G_2(1 - \omega_2 B - \omega_2^2 B^2)}{1 - \delta_1 B - \delta_2 B^2} W + N_t$$

where, from (6.2.2)

$$N_t = \frac{\theta_0 (1 + \theta_1 B)}{(1 - \theta_1 B)} a_t$$

and $a_t$ stands for random deviates with zero mean and standard deviation given by (6.4.2) as

$$S_t = \left| \frac{\omega_0 (1 - B)^2}{(1 - \theta B)} W_t \right|$$
In the above expressions the different symbols have the following meaning:

\[ TQ_t = \text{total torque output (ft.-lb.)} \]
\[ E_t = \text{tracking error (degrees)} \]
\[ W_t = \text{target angular velocity, (radians/sec.)} \]
\[ N_t = \text{random noise torque component, (ft.-lb.)} \]

(the subscript \( t \) denotes the value over the sampling interval \( t \))

\[ G_1 = \text{scale factor for tracking error series (ft.-lb./deg.)} \]
\[ G_2 = \text{scale factor for target angular velocity series (ft.-lb./rad./sec.)} \]

\[ \delta_1, \delta_2 = \text{auto-regressive operators for TQ series (dimensionless)} \]

\[ \omega = \text{weighting factors for the series of tracking error and target angular velocity, which have the same values in azimuth and elevation (dimensionless).} \]

When the parameter values are fitted from Table 5.12, the final tracking model will have the following form:

**Azimuth Torque Output - \( TQ(a) \)**

\[
TQ_t(a) = 0.89 \ TQ_{t-1}(a) - 0.30 \ TQ_{t-2}(a) \\
-13.0 \ [E_t(a) + 0.91 \ E_{t-1}(a) - 0.75 \ E_{t-2}(a)] \\
+23.889 \ [W_t(a) - 1.4 \ W_{t-1}(a) + 0.665 \ W_{t-2}(a)] \\
+N_t(a)
\]  

\[ (7.1.4) \]
Elevation Torque Output - TQ(e):

\[ TQ_t(e) = 0.336 \, TQ_{t-1}(e) + 0.284 \, TQ_{t-2}(e) \]
\[ - 27.665 \, [E_t(e) + 0.91 \, E_{t-1}(e) - 0.75 \, E_{t-2}(e)] \]
\[ + 32.41 \, [W_t(e) - 1.4 \, W_{t-1}(e) + 0.665 \, W_{t-2}(e)] \]
\[ + N_t(e) \] \hspace{1cm}(7.1.5)

Noise Series - \( N_t \)

From Section 6.2, this was shown to be identical in both channels as:

\[ N_t = 0.44 \, N_{t-1} + 2.457 \, (a_t + 0.558 \, a_{t-1}) \] \hspace{1cm}(7.1.6)

Random shocks - \( a_t \)

The random shocks \( a_t \), from section 6.4, are "white" having a mean = 0 and a standard deviation

\[ S_t = |(0.268 \, S_{t-1} + 0.945 \, V^2 \, W_t)| \] \hspace{1cm}(7.1.7)

where \( W_t \) is target angular velocity in azimuth or elevation axis.

Equations (7.1.4) through (7.1.7) express the proposed tracking model of the human operator.

7.2 Critical Examination

The tracking model presented in the above section attempts to reproduce the tracking error sequences generated by a well trained human operator in the control situation discussed in this thesis. It does not attempt to simulate the extremely complex mechanism by which man manipulates controls and decides on an output. Such an approach has
been accepted for a long time as Fielding (1963) and Fogel (1963, p. 220) indicated over a decade ago. Besides simplicity, the model does possess some outstanding characteristics, which will be enumerated now, followed by its main shortcomings.

**Characteristics of the Developed Time Series Model**

1. Model adequacy is established by the Q criteria of Box and Jenkins (1971, Chapter II) and by virtue of the fact that final model residual was proved "white" to a good approximation according to the method suggested by Pierce (1968).

2. Model fidelity was demonstrated by the excellent tracking results obtained with such severe maneuvering flights as 60° delivery dive angles and escapes of up to 5g accelerations. With such difficult paths, the model is remarkably stable, and recovery from large errors (e.g. around cross-over) occurs in less than two seconds, which is the case with human operators. This is displayed in Figures 6.12 and 6.13. As for non-maneuvering flight paths or easy ones, the model is able to handle very small errors down to about one ml., which is similar to what the experimental data showed.

3. Perhaps the most significant aspect of this model is its freedom from integro-differential operators, which makes it ideal for machine execution. Prediction of the mean tracking error (using the deterministic part
part of the model only, without the noise component) takes about 5.5 seconds in tracking a 40 second flight run. This is to be compared with the 55 seconds it takes the model of Perkins (1974) and Planchard et al (1973) to do exactly the same job, (comparison was made using a Sigma 5 SDS digital computer with a Fortran H - compiler, time counter is for execution only, no output). When the stochastic component is added to the model and noise is evaluated, execution time goes up to 7.5 seconds under the same conditions. Such substantial saving is made with no compromise on the performance of the model.

(4) Parameter economy is another outstanding feature of the proposed model. Not only was it shown that one set of parameters is sufficient over the entire flight path, but four of the parameters were the same in azimuth as in elevation. This may be contrasted with the models of Perkins (1974) and Planchard et al. (1970, 1972, 1973) where it was found necessary to assign different values to a number of the parameters in the pre-cross-over and post-cross-over sectors of the flight path. It is observed, however, that trained operators do use different strategies for approaching and escaping targets; but this has been taken care of by sensing and proper weighting of the first and second differences of both target angular velocity and tracking error. This is very likely what a well trained man
does when viewing a sight reticle; and, as Fogel (1963, p. 222) indicated, a human operator is capable of double differentiation.

(5) Over all, the model is stochastic, when the human performance is stochastic in nature too. Run-to-run variations are evident from the Figures of 6.8 through 6.13. It was observed in the course of data collection, that even a well trained operator may lose his target on difficult paths, if he makes a poor judgment. It is interesting to note that the model occasionally, does in fact lose a target on a difficult path if a "bad" sequence of random shocks takes place around cross over, thus further resembling the human performance.

(6) Tables 5.8 through 5.11 indicate that the major difference between a good and a poor operator or between a good and a bad run may be accounted for by the weights applied to target velocity and tracking error series. This suggests the possibility of tuning the model to an individual operator or to an operator at different stages of his training.

(7) The variance model of (7.1.7) is valuable, as it was possible to account for better than 90% of the human operator variance by a two-parameter model, which is so general that it applies to both tracking channels over the entire flight path. It is interesting to note
that the model looks at the second difference of target angular velocity, which is related to the rate of change of acceleration. This is striking! While a trained operator tracks a steadily moving target with ease, he will also adjust and track a target that is accelerating or decelerating at a constant rate. This is the capacity of double differentiation mentioned earlier. It is when a target accelerates (or decelerates) at a non-uniform rate that a human operator is really troubled and his variance goes up. The major source of operator variance, it appears as the analysis has shown, lies in the second derivative (or the second difference) of target velocity.

**Limitations of the Proposed Model**

The proposed time series model being linear, has all the limitations of this class of models. Specifically, the shortcoming arises from the fact that the human operator is highly non-linear in his control behavior, with a number of discontinuous features.

Many factors were left out in the analysis. Among these are effects of the display scope or sight reticle with regards to distance from the eye, size and brightness of target spot and contrast with background. Also, effects of fatigue and distractions were left out as the explicit assumption was made that operators were performing at their full capacity and devoting their total attention to their task. Combat conditions are well known to be far from such assumptions.
All operators were assumed fully trained, when marked differences were noted in performance levels. Undoubtedly, some error must have been introduced when all the data was pooled on the assumption that it came from the same population. The model also assumes the operators to be "standard men"; while this is generally acceptable, it is very difficult to establish how representative they really are of the population that generally mans such systems.

Finally, the proposed variance model suggests that if the second difference of target velocity is null, the standard deviation of operator output will diminish after one or two seconds. While there is no doubt that, under such circumstances the output will have a much narrower spread, it is also observed that the human performance will never be deterministic even when aiming at a stationary target. It appears that there is a residual constant variance that will persist even with fixed targets. A suggestion for remedial action will be made in the next section.

7.3 Suggestions for Further Research

Time series analysis has been applied for the first time in this work - to the best of the authors knowledge - to the area of manual control. The field is fertile and extensive, and many fruits are in sight. Following are some suggestions for possible extensions of the research reported here.

(1) Modeling of individual operators and their identification by means of auto-, partial-auto-, and cross-correlations seems to be possible and feasible. Training curves can also be constructed for an operator to observe
improvements, etc. In fact, the method promises potential use in the training of operators by identifying weaknesses in terms of statistics of their output and the way they weigh different system inputs. It was observed during the study that a good tracker gives more weight to target velocity on a difficult run (expressed as strong cross-correlation) compared with easier runs, where his output is more auto-correlated.

(2) In the field of performance under stress or divided attention, where relatively little has been done, the method holds much hope. The model immediately registers deterioration of performance as changes in the explicit parameters and weights. The method should be ideal for such studies.

(3) In this study, only one gun system was tested. It would be useful to extend the model to different gun systems and observe which parameters will demand adjustment. It is anticipated that only the G and δ weights will change as we move to other systems, provided they are not totally different from the one under study. In fact this was the case when the model was tuned for the elevation channel which is very different from azimuth. If this can be shown to hold, i.e. only four parameters need to be changed, the model is indispensable for studies of manual control, weapon
effectiveness and related fields.

(4) The method needs to be extended to other areas of manual control and tracking situations. Such work should be straightforward and will tell us a good deal about the nature of human learning and control behavior.

(5) Finally, the variance model of (7.1.7) may be improved by incorporating a deterministic component in the form of a constant term. If this is done, it is anticipated that the model will resemble the stochastic human output more closely than it does in its present form.
REFERENCES


Watson, B.L. (1972), The Effect of Secondary Tasks on Pilot Describing Functions in a Compensatory Tracking Task. Technical Note No. 178, Institute for Aerospace Studies, University of Toronto, Toronto.


APPENDIX I

COMPUTER PROGRAMS FOR TIME SERIES ANALYSIS

The computer programs in this appendix were written after Box and Jenkins (1971) - Part V. Full description and documentation is included there. One exception to this is the optimization routine of programs TSA/3 and TSA/7 which uses the Powell method. Full explanation of this method may be found in Powell (1964). The following list of cross-references will help identify the designations used in this work with those of Box and Jenkins.

<table>
<thead>
<tr>
<th>Appendix</th>
<th>Program</th>
<th>Designation by Box &amp; Jenkins</th>
</tr>
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<td>1A</td>
<td>TSA/1</td>
<td>Univariate Stochastic Model Identification (USID)</td>
</tr>
<tr>
<td>1B</td>
<td>TSA/2</td>
<td>Univariate Stochastic Model Preliminary Estimation (USPE)</td>
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<tr>
<td>1F</td>
<td>TSA/7</td>
<td>Univariate Transfer Function Model Estimation (UTES)</td>
</tr>
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</table>
APPENDICES
Time Series Analysis Program No. 1

FORTRAN IV G1 RELEASE 20.3 MAIN

DATE = 72355  18/24/47

0001 DIMENSION TIME(250), Y(250), CUT(2, 5, 250)
0002 DIMENSION Z(400), W(400), KON(6), R(2C), C(2C), PHI(20, 20)
0003 REAL M
0004 KCN(1) = 3
0005 KON(2) = 2
0006 KON(3) = 1
0007 KON(4) = 20
0008 KCN(5) = 1
0009 KON(6) = 10
0010 KK = 20
0011 N = 400
0012 N = 160

C ******************************************************

0013 READ(5, 2222) I1
0014 READ(5, 4) (TIME(I), (OUT(K, J, I), J=4, 5), K=1, 2), I=1, 160
0015 READ(5, 2) NHRUN
0016 DO 777 IH = 1, NHRUN
0017 READ(5, 80C)
0018 READ(5, 131) ((OUT(K, J, I), K=1, 2), J=1, 2), I=1, 160
0019 DO 775 K = 1, 2
0020 OUT(K, 3, 1) = C0
0021 OUT(K, 3, 160) = C0
DO 775 I = 2, 159
OUT(K,3*I) = 2* (OUT(K,2*I+1) - OUT(K,2*I-1))
775 CONTINUE

WRITE(6,700)
WRITE(6,800)
WRITE(6,133) (I, TIME(I), (OUT(K,J,I), J=1,5), K=1,2), I=1,160)

DO 777 K = 1, 2
DO 777 J = 1, 5
IF (IH3 GT 1 AND J GE 4) GO TO 777
Z(I) = CUT(K,J,I)
777 CONTINUE

DO 69 I = 1, 4
KON(I) = I-1
CALL USID(Z, N, K, KON, M, R, C0, C, PHI, WBAR, WVAR, LERROR, W)
WRITE(6,1) LERROR
69 CONTINUE

777 CONTINUE

FORMAT(16X,'************************** LERROR*,I10)
1 FORMAT(1G13)
2 FORMAT(15)
4 FORMAT(2(F6.2, 6X, 2F6.2, 6X, 2F5.1))
131 FORMAT(F5.2,F5.3,2F5.1,20X,F5.2,F5.3,2F5.1)
133 FORMAT(///, (15.5X, 11F10.2))
700 FORMAT(1H1)
800 FORMAT(6EH.

/80H
/70H
/30H

2222 FORMAT(I1)
STOP
END
0038  60  \text{w}(i) = \text{Alog}(\text{z}(i) + M) \\
0039  GOTO 11C  \\
0040  70  M = \text{C} \times \text{G}  \\
0041  \text{DO 86 I}=1,N  \\
0042  80  \text{w}(i) = \text{z}(i)  \\
0043  GOTO 11C  \\
0044  90  \text{DO 10 I}=1,N  \\
0045  100 \text{w}(i) = (\text{z}(i) + M)**\text{LAMBDA}  \\
0046  110  \text{LN} = \text{N-LD-LS*LBD}  \\
0047  \text{IF(LN GT 1) GOTO 120}  \\
0048  \text{WRITE(6,65)}  \\
0049  \text{GOTO 999}  \\
0050  120 \text{IF(N EQ LN) GOTO 180}  \\
0051  \text{CALL DEL(W, TEMP, LBD, LS, N, LIMIT, I)}  \\
0052  \text{CALL DEL(W, TEMP, LD, I, LIMIT, LN, 1)}  \\
0053  180  \text{XLN} = \text{FLCAT(LN)}  \\
0054  \text{WBAR} = 0.0  \\
0055  \text{DO 190 I}=1,LN  \\
0056  190 \text{WBAR=WBAR+W(I)}  \\
0057  \text{WBAR=WBAR/XLN}  \\
0058  \text{C0} = \text{C0C}  \\

\text{FORTRAN IV G1 RELEASE 2.7.4C}  \\
\text{LSID}  \\
\text{DATE} = 72355  \\
\text{18/24/47}  \\
0059  \text{DO 220 I}=1,LN  \\
0060  200 \text{CG} = \text{C0} + (\text{W(I)}-\text{WBAR})**2  \\
0061  \text{DO 220 LK}=1,K  \\
0062  \text{C(LK)}=0.0  \\
0063  \text{LNMK=LN-LK}  \\
0064  \text{DO 210 I}=1,LNMK  \\
0065  210 \text{C(LK)} = \text{C(LK)}+(\text{W(I)}-\text{WBAR})*(\text{W(I+LK)}-\text{WBAR})
0066  $R(LK) = \frac{C(LK)}{C_0}$
0067  $RD(LK) = R(LK)$
0068  $C(LK) = \frac{C(LK)}{XLN}$
0069  $C_0 = C_0/XLN$
0070  $WVAR = C_0$
0071  $R_0 = 1.0$
0072  $\phi(1,1) = R(1)$
0073  CO240 I1 = 2*L
0074  I1M1 = I1 - 1
0075  DUM1 = RD(I1)
0076  DUM2 = I1$^0$
0077  CO230 J = 1.11M1
0078  IR = I1 - J
0079  DUM1 = DUM1 - PHI(I1M1,J)*RD(IR)
0080  230 DUM2 = DUM2 - PHI(I1M1,J)*RD(J)
0081  PHI(I1,11) = DUM1/DUM2
0082  DO 240 I2 = 1.11M1
0083  J2 = I1 - I2
0084  240 PHI(I1,12) = PHI(I1M1,12) - PHI(I1,11)*PHI(I1M1,J2)
0085  DO 245 I = 1,L
0086  DO 245 J = 1,I
0087  245 PHI(I,J) = PHI(I,J)
0088  IF(KCN(6).*LE.*0) RETURN
0089  WRITE(6,601) LN*(W(J),J=1,LN)
0090  WRITE(6,602) WBAR,WVAR
0091  WRITE(6,603) C(J),C(J),J=1,K
0092  WRITE(6,610) R*C*R(J),R(J),J=1,K
0093  WRITE(6,611) (PH(J,J),J=1,L)
0094  IF(L.*GT.*1") L = 10
0095  WRITE(6,613)
0096  DO 250 I = 1,L
0997  250 WRITE(6,612)(PH(I,J),J=1,I)
0098  RETURN
0099  999 CONTINUE
C11C  LERROR = 1
0101  601 FORMAT(1H1,4CX,"UNIVARIATE STCEHASTIC MODEL IDENTIFICATION (USID)"")
1. **INPUT PARAMETERS**
   - N, 2X (NUMBER OF OBSERVATIONS)
   - T78, = I10/T35 (DEGREE OF NONSEASONAL DIFFERENCING)
   - T78, = I10/T35 (DEGREE OF SEASONAL DIFFERENCING)
   - T78, = I10/T35 (PERIOD OF SEASONALITY)
   - T78, = I10/T35 (MAXIMUM LAG OF ACVF & ACF)
   - T78, = I10/T35 (MAXIMUM LAG OF PACF)

2. **TRANSFORMATION PARAMETERS**
   - T65, LAMBDA = I10/T65
   - T65, M = F10.4

3. **THE TIME SERIES Z(T) CONTAINING VALUES IS**
   - 8E15.5

4. **THE MEAN (WBAR) = WVAR =**
   - E16.6

5. **THE AUTOCOVARIANCE FUNCTION IS**
   - 8E15.5

6. **THE PARTIAL AUTOCORRELATION FUNCTION IS**
   - 8E15.5

7. **THE LOWER HALF OF THE PHI MATRIX UP TO A (10x10) ARRAY**

8. **RETURN**

END
SUBROUTINE DEL(Z, W, ID, IS, N, LIMIT, ITIME)

DIMENSION Z(I), W(I), A(10)

THIS SUBROUTINE OPERATES ON THE TIME SERIES Z(I) WITH
THE OPERATOR \((1 - B^{*IS})^{*ID}\) TO PRODUCE THE TIME SERIES W(I), WHERE THE OPERATION \(B^{*IS}(Z(I)) = Z(I - IS)\).

Z(I) INPUT TIME SERIES, ALSO THE OUTPUT TIME SERIES IF ITIME IS 1
W(I) OUTPUT TIME SERIES IF ITIME IS 2
ID ORDER OF THE OVERALL OPERATOR
IS ORDER OF THE B OPERATOR
N LENGTH OF THE INPUT VECTOR Z(I)
LIMIT LENGTH OF THE OUTPUT, EITHER Z OR W
ITIME CONTROL PARAMETER

IDP1 = ID + 1

CALCULATE THE VALUES OF THE BINOMIAL COEFFICIENTS

IDFACT = IDFACT(ID)
DO 1 I = 1, IDP1
IM1 = I - 1
IDMIP1 = ID - I + 1
1 A(I) = IDFACT * (-1)**IM1 / (IDFACT(IM1) * IDFACT(IDMIP1))
INVERT SIGN IF ORDER IS ODD

IF( (ID/2)*2 .EQ. ID) GO TO 5
DO 2 I = 1, IDP1
2 A(I) = -A(I)
RETURN IF OPERATOR IS UNITY,

5 LIMIT = N - ID*IS
IF(ID.EQ.0) GO TO 40

CALCULATE NEW TIME SERIES,

DO 20 I = 1, LIMIT
W(I) = C
DO 10 J = 1, IDP1
INDEX = (J - 1)*IS + I
10 W(I) = W(I) + A(J)*Z(INDEX)
20 CONTINUE

CHECK CONTROL PARAMETER FOR PROPER OUTPUT,

IF(ITIME.EQ.2) RETURN
DO 30 I = 1, LIMIT
30 Z(I) = W(I)

RETURN
END
FUNCTION IFACT(N)

THIS SUBROUTINE CALCULATES THE VALUE OF N FACTORIAL

IFACT = 1

IF (N .LT. 2) RETURN

DO 20 I = 2, N

20 IFACT = IFACT * I

RETURN

END
### Time Series Analysis Program N. 2

<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
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<tbody>
<tr>
<td>1</td>
<td><strong>DIMENSION C(20), Phi(5), Theta(5)</strong></td>
</tr>
<tr>
<td>2</td>
<td>READ(5,11), END=777, LP, LS, K</td>
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<tr>
<td>3</td>
<td>READ(5,12) <strong>BAR</strong></td>
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<tr>
<td>4</td>
<td>_RITE(6,12) C(1), I = 1</td>
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<tr>
<td>5</td>
<td>_RITE(6,12) LP, LS, K</td>
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<tr>
<td>6</td>
<td>_RITE(6,12) _AP</td>
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<tr>
<td>7</td>
<td>_RITE(6,12) C(1), I = 1</td>
</tr>
<tr>
<td>8</td>
<td>EPSLN = 0.01</td>
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<tr>
<td>9</td>
<td>EPSLN = 0.01</td>
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<tr>
<td>10</td>
<td><strong>CALL USPE(LP, LS, K, BAR, C, PHI, THETA, THETA_0, VARA, TERROR)</strong></td>
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<tr>
<td>11</td>
<td>1, PRINT, EPSLN</td>
</tr>
<tr>
<td>12</td>
<td>1, PRINT, EPSLN</td>
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<td>13</td>
<td>1, PRINT, EPSLN</td>
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<tr>
<td>14</td>
<td>_RITE(6,100) _AP</td>
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<tr>
<td>15</td>
<td><strong>END</strong></td>
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<tr>
<td>16</td>
<td>FORMAT(515)</td>
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<tr>
<td>17</td>
<td>IP FORMAT(8F10.3)</td>
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<tr>
<td>18</td>
<td>100 FORMAT(10:**** LENK/K = I, I1)</td>
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<tr>
<td>19</td>
<td>777 STOP</td>
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<tr>
<td>20</td>
<td>END</td>
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</tr>
<tr>
<td>1</td>
<td>SUBROUTINE USPEL(P, L, K, B, VAR, C, PHI, THETA, THETAD, VADA, ERRR, IPRNT, EPSLEN, LP, LQ, X, IRAN, C, PHI, THETA, THETAD, VADA, ERRR</td>
</tr>
<tr>
<td>2</td>
<td>I, IPRINT, EPSLEN</td>
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<tr>
<td>3</td>
<td>DIMENSION C(1), PHI(1), THETA(1), CIP(5), F(5)</td>
</tr>
<tr>
<td>4</td>
<td>TAU(5), T1(5,5), T2(5,5)</td>
</tr>
<tr>
<td>5</td>
<td>DOUBLE PRECISION A(5,6), T(5,6)</td>
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<tr>
<td>6</td>
<td>C THIS PROGRAM CALCULATES PRELIMINARY ESTIMATES OF THE PHI AND</td>
</tr>
<tr>
<td>7</td>
<td>C THETA CONSTANTS FOR A STOCHASTIC MODEL OF ORDER (P,Q,D), L</td>
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<td>8</td>
<td>C IT ALSO CALCULATES THETAD(THE OVERALL CONSTANT TERM) AND VARA</td>
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<tr>
<td>9</td>
<td>C (THE WHITE NOISE VARIANCE)</td>
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</table>

**INPUT**
- LP: ORDER OF MODEL, P + Q + 1
- LQ: ORDER OF MODEL, Q
- K: NA, BE GIVN K, GF. P + Q
- X: MEAN OF TRANSFORMED TIME SERIES
- C: VECTOR OF AUTOCOVARIANCES OF "T"; C(1) = CO
- C(2) = CI
- C(3) = CK
- IPRINT: PRINT CONTROL; IPRINT >= 0 PRODUCES OUTPUT
- EPSLEN: NORM CONVERGENCE CRITERIA

**OUTPUT**
- PHI: VECTOR OF PHI'S OF MODEL, PHI(1) ... PHI(LP)
- THETA: VECTOR OF THETA'S OF MODEL, THETA(1) ... THETA(L2)
- THETAD: AVERAGE CONSTANT TERM
- VARA: WHITE NOISE VARIANCE
- ERRR: 0 = NORMAL TERMINATION
- 1 = INPUT ERRR
- 2 = ERRR IN ATNVRT
- 3 = ERRR IN TTVRNT
**C** CHECK INPUT PARAMETERS

**C**

VARA = C(1)

= j * C

LPFG = LP + LF

LPP1 = LP + 1

LERRR = 1

IF (LPFG * LT, 1) RETURN

IF (K * LT, LPP1) RETURN

IF (LF, * LT, 0) RETURN

IF (FPSC1 * LF, J. C) RETURN

IF (PRINT * NE, 0) WRITE (6, 1000) LP, LN

IF (LP) 10, 50, 2

10 RETURN

**C** CALCULATE A AND THEN PHI VECTORS

**C**

DO 30 I = 1, LP

LPPIJ = LS + i + 1

A(I, LPP1) = C(LPPIJ)

30 DO 30 J = 1, LF

LPIMJ = IAABS(LPPIJ - J)

A(I, J) = C(LPIMJ)

LERRR = 2

CALL MATINV(A, LP, D)

IF (DBABS(D) * LT, 0, 1D-10) RETURN

**C** CALCULATE PHI VECTORS

**C**

DO 40 I = 1, LP

PHI(I) = A(I, LPP1)

40 DO 40 I = 1, LP

VARA = VARA - PHI(I) * C(I + 1)
46 = - PHI(I)

IF (IPRINT .EQ. 0) GO TO 90
IF (LP .GE. 1) WRITE(6,1010) (PHI(I), I = 1,LP)

CALCULATE CJP VECTOR

50 LJP1 = LP + 1
60 THENET = THENET
IF (LP .EQ. 0) GO TO 210
80 BUNT = 0
90 20 J = 1, LJP1
100 X = .0C
110 IF (LP .EQ. 0) GO TO 90

60 LP = LP - 1
INDEX = IAABS(J - LK - 1) + 1

60 10 I = 1,LP
80 INDEX = J + 1
90 Z = -C(INDEX)

90 70 LK = 1,LP
90 INDEX = IAABS(J + I - LK - 1) + 1
80 Z = Z + PHI(LK)*C(INDEX)
90 Z = Z + PHI(I)*Z
100 CJP(J) = C(J) + Z

INITIALIZE N-Q ALGORITHM

C

30 10 I = 1,LP1
30 10 J = 1,LP1

105 T1(I,J) = 0.0
100 T2(I,J) = 0.0
105 TAU(I) = SORT(PIPT1))
106 TAU(I) = 2,LP1
107 TAU(I) = 0.0

CALCULATE AND CHECK F VECTOR

C
120  ICHECK = 1
121  KBUNT = KBUNT + 1
123  IF(KBUNT GT 20) GO TO 220
124  DB 100 J = 1, LQPI
125  Z = -CJP(J)
126  LU = LQPI + 1 - J
127  DB 130 I = 1, LU
128  IPJ = I + J - 1
129  130  Z = Z + TAJ(I)*TAJ(IPJ)
130  IF(ABS(Z) LT EPSBLN) GO TO 140
131  ICHECK = 1

140  F(J) = Z
141  IF(ICHECK EQ FE, N) GO TO 190

C
122  CALCULATE VECTORS T1, T2, T = T1 + T2, AND T(INVERSE)

C
126  C
127  DB 160 I = 1, LQPI
128  T(I, LQPI+1) = F(I)
129  LU = LQ + 2 - I
130  DB 150 J = 1, LU
131  INDEX = I + J - 1
132  150  T(I, J) = TAJ(INDEX)
133  INDEX = 0
134  DB 160 J = T, LQPI
135  INDEX = INDEX + 1
136  160  T2(I, J) = TAJ(INDEX)
137  DB 170 I = T, LQPI
138  DB 170 J = T, LQPI
139  170  T(I, J) = T(I, J) + T2(I, J)
140  LERROR = 3
141  CALL MATINV(T, LQPI, N)
142 IF(DABS(I) .LT. .5E-10) RETURN
144 C CALCULATE NEW TAU
145 C
146 50 180 T = TLQP1
147 180 TAU(I) = TAU(I) - SNGI(T(I),QP1+1)
148 C 50 TO 120
150 C CALCULATE THERA VECTORS
151 C
152 190  = TAU(I)
153 VARA = W*W
154 50 250 J = 1,LC
155 20J THERA(J) = -TAU(J+1) /
156 2IJ IF(LERRB .GE. 4) LERRB = 0
158 C C OUTPUT
159 C
160 IF(IPRINT .GE. 0) RETURN
161 IF(LC .GE. 1) WRITE(6,1020) (THETA(J), J = 1,LC)
162 WRITE(6,1030) THERA, VARA
163 RETURN
164 220 LERRB = 4
165 3P TO 100
166 100 FORMAT(1H1,5X, 'PRELIMINARY CONSTANT ESTIMATION',/5X, 'FOR SYSTEM'
167 1X 'OF ORDER (',11,10D11,')',)
168 1010 FORMAT(5X, 'SEASONAL AUTOREGRESSIVE PARAMETERS = PHI///5X',
169 j (7E16.6))
170 1020 FORMAT(5X, 'SEASONAL MOVING AVERAGE PARAMETERS = THETA///5X',
171 j (7E16.6))
172 1030 FORMAT(5X, 'OVERALL CONSTANT TERM = ', E16.6///5X, 'WHITE NOISE'
173 j 'VARIANCE = ', E16.6)
174 END
SUBROUTINE MATINV(A,N,DET)
DOUBLE PRECISION A(12,10),DET,PIVAT,SMULT

DET = 1.0

Y = N + 1

PRINT(1015)

I = 1

PIVAT = A(I,I)

DET = DET*PIVAT

A(I,I) = 1.0

J = 1

A(I,J) = A(I,I)/PIVAT

J = J + 1

IF(J*LE*9) GOTO 4

K = 1

IF(K*EQ*1) GOTO 2

SMULT = A(K,K)

A(K,I) = 0.0

J = 1

A(K,J) = A(K,I) = SMULT*A(I,J)

J = J + 1

IF(J*LE*K) GOTO 3

K = K + 1

IF(K*LE*N) GOTO 4

I = I + 1

IF(I*LE*K) GOTO 5

RETURN

END
Time Series Analysis Program No. 3

1 COMMON/PROST/,A,Z,RONCST,1J,1N

3 READ (5,2) NP,NW

5 VCOST = 0

7 READ (5,11) (PARAM(I),I=1,NP)

9 DE 5 I = 1,NW

11 5 Z(I) = 0*6

13 VRUN = 0

15 READ (5,8CC,F10.3) = 661

17 READ (5,10) (Z(I),I=1,NW)

19 VRUN = NRUN + 1

21 DE 65 I = 1,NW

23 DE T = 7

25 Z(I) = Z(I) + w(I)

27 DE 66 I = 1,NW

29 w(I) = Z(I) / FLAT(VRUN)

31 CALL MN,ZB( PARAM,IND,6,5C,NRN,1,NER,V,Y,M )

33 WRITE(6,21) VCOST

35 WRITE(6,800) (J,W(I),Z(J),A(J),J=1,NW)

37 WRITE(6,800) (J,W(I),Z(J),A(J),J=1,NW)

39 2 777 STOP

41 FORMAT(8F10.3)

43 FORMAT(2I5)

45 FORMAT (18X,F6.2,34X,F6.2)

47 FORMAT(1H1,110,8F20.3)

49 FORMAT(20X,9A,10 COST EVALUATIONS = 1,110)

51 FORMAT(65H)

53 1 /80H

55 2 /70H

57 3 EN6
SUBROUTINE ER (SUM, PARAM)

DIMENSION PARA(10), W(505), A(500), Z(200)

COMMON/MPCOST/, A, Z, NCAST, IJ, NJ

NCYST = NCAST + 1

SUM = 0.0

A(1) = 0.0

A(2) = 0.0

A(3) = 0.0

DO 10 I = 4, NW

10 A(I) = W(I) - (1.0 + PARAM(1))*W(I-1) + (PARAM(1) - PARAM(2))*W(I-2) + PARAM(2)*W(I-3) + PARAM(3)*A(I-1) - PARAM(4)

10 SUM = SUM + A(I)**2

RETURN

END
SUBROUTINE MUNAPR ( XZ, N, NEX, NER, ITMX, NPR, X1, IT, V, P, M )
C ER FUNCTION TO BE MINIMIZED, USE AS A SUBROUTINE ER(FRARR, XZ)
C THE VARIABLE ERROR IS MINIMIZED BY VARYING THE VECTOR XZ
C ANY SUBROUTINE NAME MAY BE USED AS LONG AS IT IS CONTAINED IN THE
C EXTERNAL STATEMENT AND HAS THE PROPER ARGUMENTS IF COST(SMALL,VEC)
C XZ=VECTOR FUNCTION, WHERE ERROR = F(X) IF DETERMINED BY SUB. FP
C =SIZE OF VECTOR XZ
C (EX) CONVERGING CRITERION AS XZ IN SIGNIFICANT FIGURES
C (EX) CONVERGING CRITERION AS ER IN SIGNIFICANT FIGURES
C 9TH NEX AND NER MUST BE MET FOR CONVERGENCE
C ITMX=MAXIMUM NUMBER OF ITERATIONS ALLOWED
C UR=PEWELL RECYCLE PERIOD, ART (X)
C RELATIVE MINIMUM ROUTINE, USE RLMUGR IN CALL
C X=WORKING SPACE 1 DIM ARRAY X(10)
C IT=ITERATIONS USED (OUTPUT)
C V=WORKING SPACE 1 DIM ARRAY V(100)
C P=WORKING SPACE 1 DIM ARRAY P(10)
C EQUA V+1
C USE DIMENSION X(10), P(10), V(100), XZ(SIZE OF XZ)
C USE EXTERNAL PLANE NAME OF ERROR FUNCTION SUBROUTINE
C DIMENSION XZ(N), X(N), V(*,M), P(*)
C LOGICAL REFY, FIRST
C ITMX = IABS( ITMX )
C NAX = NEX
C IF( NAX LE 0.0 , 0.0 , NAX .GT. 6 ) NAX = 6
C NAR = NER
C IF( NAR LE 0.0 , 0.0 , NAR .GT. 6 ) NAR = 6
C EPS = 0.0625**NAX
C ETA = 0.0625**NAR
C EPST = 1.0E0
C IT = 0.0
C NPI = N*01
C FLNFN = FLOAT( NRP )
C FLN = 1.0/ FLOAT(N)
C C#### COUNTERS HAVE BEEN SET, NOW SET BASE VECTORS #######
C REFY = *FALSE.
FIRST = .TRUE.

ALS = SORT(EPS)

ALAV = ALS

EPW = 0.0625**(NAX/P)

ETW = 0.0625**(NAR/P)

IF( ALAV .LT. ALS ) ALAV = ALS

I = 1

V(I,J) = 0

CONTINUE

IF( FIRST ) CALL ERP(ENV, X7 )

CALL VEQ( X, X7, N )

WRITE(6,1) IT, ENV, XZ

FORMAT(1) ** MAIN LOOP $$$$$

TAKE 10 SUCCESSIVE STEPS ALONG BASE VECTORS $$$$$

SMAL = 0

DR 300 I = 1

AL = ALAV

CALL RMOV( V, (1,1), I, P, ENW, ERP, FTWN )

CALL VER( X, P, N )

300 SMAL = SMAL + ABS( AL )

COMPUTE AVERAGE STEP TAKEN $$$$$

ALAV = SMAL/P

CALL VMV( V(1,NP1), X, X7, N )

CALL VEQ( X7, X, N )

IF( SMAL .LE. EPW ) GOTO 9000
AL = ALAY
CALL RLY(V, (1,N,1), AL, Y, EPS, EFT, L)
IT = IT + 1
C*** IF SMALL TEST FOR CONVERGENCE, FIRST ITERATE PROCESS ***
IF( EPST = EN * LF * ET ) GOTO 9000
EPST = EN
CALL VMV( V(1,N), XZ, X, IT )
FIRST = FALSE
C***** DISCARD THE OLDEST VECTOR *****
JE 600 I = 1, N
600 CALL VEV( V(1, I), V(1, I+1) )
C*** RESET FOR ITERATION ***
IF( IT > ITX ) GOTO 400
IF( IRD( IT, VRN ) = * ) GOTO 200
GOTO 1000
400 IT = - IT
GOTO 9001
9001 IF( FIRST = 3, RF 
ALS = EPS + 16 * 75
ALAS = ALS
EPS = EPS
ET = ETA
FIRST = TRU
GOTO 1001
9001 CALL VEV( XZ, X, N )
IF( RF, TRU, 1 ) GOTO 9001
ALS = EPS
ALAS = EPS
RF = TRU
GOTO 1001
9001 END
SUBROUTINE RLWN (X7, AL, X, EA, TEST1, TEST2, N)
DIMENSION X7(N), AL(N), X(N)
DIMENSION D(1:E(4))
EQUIVALENCE (E1,E(1)), (E2,E(2)), (E3,E(3)), (E4,E(4))
EQUIVALENCE (D1,D(1)), (D2,D(2)), (D3,D(3)), (D4,D(4))

IF (ABS( AL ) LT TEST1) AL = TEST1
H = AL
CAL = 4*AL
D2 = 0.0
E2 = E A
D3 = H
CALL VPVTS( X,XZ, D3, N )
CALL ER ( F3, X )
IF ( E3 LT E2 ) GOTO 150
H = 4
D1 = D3
D3 = 4
GOTO 150
L00
E1 = E2
E2 = E3
D1 = D2
D2 = D3
D3 = 2*H
GOTO 150
CALL VPVTS( X,XZ, D3, N )
CALL ER ( F3, X )
D23 = D2 - D3
D31 = D3 - D1
D12 = D1 - D2
D13 = D3 - D1
D23*E1 + D31*E2 + D12*E3
DN2 = D23*D31*D12
DN1 = DN2 + DN2
IF ( DN1 LT 0.0 ) DN1 = 1.0

DN1*DNP*LT*G. GOTO 30C
D1 = D2
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>( D_2 = D_3 )</td>
</tr>
<tr>
<td>37</td>
<td>( D_3 = D_3 + \text{DAL} )</td>
</tr>
<tr>
<td>38</td>
<td>( \text{DAL} = 2 \times \text{DAL} )</td>
</tr>
<tr>
<td>39</td>
<td>( E_1 = E_2 )</td>
</tr>
<tr>
<td>40</td>
<td>( E_2 = F_3 )</td>
</tr>
<tr>
<td>41</td>
<td>\text{GOTO 150}</td>
</tr>
<tr>
<td>42</td>
<td>( D_3 = 2 \times E_1 \times E_2 \times (D_3 + D_1) \times (D_3 + D_2) \times (D_3 + D_3) \times E_1 )</td>
</tr>
<tr>
<td>43</td>
<td>IF( ( E_1 \times E_2 \times E_1 \times E_2 \times (1 \times 2) \times \text{ABS}(D_3) ) )</td>
</tr>
<tr>
<td>44</td>
<td>( D_4 = 0 \times 5 \times E_3 / E_4 )</td>
</tr>
<tr>
<td>45</td>
<td>IF( ( D_3 + \text{DAL} = D_4 \times \text{DAL} \times (1 \times 0) ) ) \text{GOTO 250}</td>
</tr>
<tr>
<td>46</td>
<td>CALL ( \text{VPVT5}(x, y, z, o) )</td>
</tr>
<tr>
<td>47</td>
<td>CALL ( \text{ER}(E_4, x) )</td>
</tr>
<tr>
<td>48</td>
<td>( \text{TEST4} = \text{TEST2} \times \text{TEST3} \times 100 )</td>
</tr>
<tr>
<td>49</td>
<td>( \text{TEST3} = \text{TEST1} \times (\text{TEST1} + 1 \times 0) )</td>
</tr>
<tr>
<td>50</td>
<td>( J = 0 )</td>
</tr>
<tr>
<td>51</td>
<td>( K = 0 )</td>
</tr>
<tr>
<td>52</td>
<td>( L = 0 )</td>
</tr>
<tr>
<td>53</td>
<td>( \text{D9 400} )</td>
</tr>
<tr>
<td>54</td>
<td>IF( ( \text{ABS}(E_1) \times D_4 \times \text{LE} \times \text{TEST3} ) ) \text{G0TA 500}</td>
</tr>
<tr>
<td>55</td>
<td>IF( ( E_1 \times E_2 \times E_4 \times J = J + 01 ) )</td>
</tr>
<tr>
<td>56</td>
<td>IF( ( E_1 \times E_2 \times \text{AND} \times E_1 \times \text{G1} \times E_2 \times L = 1 + 01 ) )</td>
</tr>
<tr>
<td>57</td>
<td>IF( ( \text{ABS}(E_1) \times E_4 \times \text{LE} \times \text{TEST4} \times K = K + 01 ) )</td>
</tr>
<tr>
<td>58</td>
<td>IF( ( J \times E_2 \times \text{AND} \times L \times E_3 \times D_2 ) ) \text{G0TA 500}</td>
</tr>
<tr>
<td>59</td>
<td>IF( ( K \times E_2 \times G_2 ) ) \text{G0TA 500}</td>
</tr>
<tr>
<td>60</td>
<td>IF( ( (D_3 - D_4) \times \text{DAL} \times D_3 \times O ) ) \text{G0TA 450}</td>
</tr>
<tr>
<td>61</td>
<td>( \text{D9 401} )</td>
</tr>
<tr>
<td>62</td>
<td>( \text{D1 (1) = 3(1+1) ) }</td>
</tr>
<tr>
<td>63</td>
<td>( \text{D1 (1) = 3(1+1) ) }</td>
</tr>
<tr>
<td>64</td>
<td>( \text{G0TA 200} )</td>
</tr>
<tr>
<td>65</td>
<td>IF( ( (D_3 - D_4) \times \text{DAL} \times G_2 \times D_2 ) ) \text{G0TA 500}</td>
</tr>
<tr>
<td>66</td>
<td>IF( ( (D_3 - D_4) \times \text{DAL} \times T \times D_2 ) ) \text{G0TA 460}</td>
</tr>
</tbody>
</table>
1: IF( E2*ST*F4 ) 5ATB 455
68 E1 = E4
69 D1 = D4
70 GATB 200
71 455 E3 = E2
72 D3 = D2
73 456 E2 = E4
74 D2 = D4
75 GATB 200
76 460 IF( E2*ST*E4 ) GATB 465
77 E3 = E4
78 D3 = D4
79 GATB 200
80 465 E1 = E2
81 D1 = D2
82 GATB 456
83 500 L = 1
84 DE = 20 I=2,4
85 520 IF( E(I)*LF*F(I) ) L = 1
86 AL = D(L)
87 EA = E(L)
88 IF( L*NE*4 ) CALL VPVT( X,XZ,SA,N,Y,N )
89 RETURN
90 END
1 SURROGATE VNR1 ( VN,A,B,N )
2 DIMENSION A(N),B(N),N,
3 VN = 0
4 DE = 10 I=1,5
5 1: VN = VN + ABS( A(I) )/( 1.0 + ABS( R(I) ) )
6 RETURN
7 ENTRY VNR8 ( VN,A,B,N )
8 VN = 0
9 DE = 20 I=1,5
10 VT = ABS( A(I) )/( 1.0 + ABS( R(I) ) )
11 2. IF( VT*ST*VN ) VN = VT
12 RETURN
13 END
SUBROUTINE VEG( R,A,N )

DIMENSION R(M),A(N)

DO 30 I=1,N

30 R(I) = A(I)
RETURN

ENTRY VNEG( R,A,N )

DO 40 I=1,N

40 R(I) = -A(I)
RETURN

END

SUBROUTINE VPVTS( R,A,S,N )

DIMENSION R(N),A(N),S(N)

DO 10 I=1,N

10 R(I) = A(I) + B(I)*S
RETURN

ENTRY VPVTS( R,A,S,N )

DO 11 I=1,N

11 R(I) = A(I)*S
RETURN

ENTRY VPV( R,A,B,N )

DO 12 I=1,N

12 R(I) = A(I) + B(I)
RETURN

ENTRY VMV( R,A,B,N )

DO 13 I=1,N

13 R(I) = A(I) - B(I)
RETURN

END
C TIME SERIES ANALYSIS PROGRAM

DIMENSION X(400), Y(400), PH(10), THETA(10), W(400)

K 400, BETA(400), BC(201), BB(20), PH1(20), PH2(20), YC(20), YR(20), YC(20), YR(20)

DIMENSION BN(1), VM(20), AN(20), AR(20)

REAL W

DATA W, A, FA, BETA, AN, XW, YW, 400*0

READ (K, 11) W, VM, AN, AR, LG

READ(5, 12)(PH(1), I=1, 10), (THETA(1), I=1, 10)

C Differencing and Transformation

BN(1) = 0

BN(2) = 0

BN(3) = 1

BN(4) = 10

BN(5) = 1

BN(6) = 1

Y = 0

LV = N - 1

READ(5, 1000, END=999)

READ 23(X(1), Y(1), I = 1, N)

WRITE(6, 800)

WRITE(6, 800)

WRITE(6, 800)

WRITE(6, 800)

WRITE(6, 800)

WRITE(6, 800)

WRITE(6, 800)

WRITE(6, 800)

CALL VSPD(X, W, 10, XN, XN, YN, YR, YC, YC, YR, YC, YR, YR)

CALL VSPD(Y, W, 10, YN, YN, YR, YC, YC, YR, YC, YR, YR)

CALL VSPD(X, W, 10, XN, XN, YN, YR, YC, YC, YR, YC, YR, YR)

CALL VSPD(Y, W, 10, YN, YN, YR, YC, YC, YR, YC, YR, YR)

CALL VSPD(X, W, 10, XN, XN, YN, YR, YC, YC, YR, YC, YR, YR)

CALL VSPD(Y, W, 10, YN, YN, YR, YC, YC, YR, YC, YR, YR)

CALL VSPD(X, W, 10, XN, XN, YN, YR, YC, YC, YR, YC, YR, YR)

CALL VSPD(Y, W, 10, YN, YN, YR, YC, YC, YR, YC, YR, YR)

CALL VSPD(X, W, 10, XN, XN, YN, YR, YC, YC, YR, YC, YR, YR)

CALL VSPD(Y, W, 10, YN, YN, YR, YC, YC, YR, YC, YR, YR)
C 38 39  J = 1, S
ALFA(J) = 0.

C 40 41  BETA(J) = 0.
LS = LS + 1

C 42  DT = 40 IN * SIN

C 43  YAR = C.
YAR

C 45  YMA = C.

C 46  YMA = C.

C 47 35 I = 1

C 48  YAR = XAR + PHI(I) * XX(IN-1)

C 49 35 YAR = YAR + PHI(I) * WY(IN-1)

C 50 37 I = 1

C 51  YMA = YMA + THETA(J) * ALFA(IN-1)

C 52 37 YMA = YMA + THETA(J) * BETA(IN-1)

C 53  ALFA(IN) = WY(IN) - XAR + YMA

C 54  BETA(IN) = WY(IN) - YAR + YMA

C 55  C  PREWHITENED INPUT & OUTPUT AUTOCORRELATION

C 56  C

C 57  WRITE(6,13)
CALL USID(ALFA, IN, 10, XAR, M, AR, AR, AR, PHI, AR, BVAR, ERROR)

C 58  ALFA).
WRITE(6,100) ERROR

C 59  CALL USID(BETA, IN, 10, XBY, M, BRO, BR, AR, AR, PHI, BBAR, BVAR, ERROR)

C 60  BETA).
WRITE(6,100) ERROR

C 61  C  PREWHITENED INPUT-OUTPUT CROSS CORRELATION

C 62  C

C 63  SA = AVAR
SB = BVAR
**TIME SERIES ANALYSIS PROGRAM #6**

**UNIVARIATE TRANSFER FUNCTION PRELIMINARY ESTIMATION**

**DIMENSION**

- N(10), F(10), SN(10), PHT(10), THETA(10)
- A(10,10)

**DOUBLE PRECISION**

- 9 I = 1, 10

**READ(5,10) I, R, S, P, L, A
**READ(5,10) I, 1 = 1, L

**WRITE(6,17) I, 1 = 1, L

**IF(LS+R+LR+RT,LE) 55 TA 77

**CALL MATINV(A,LR,J)

**WRITE(6,14) (A(I,J), J = 1, LR+1)
38          35 CONTINUE
39          30 DB 36 I = 1,IR
40          30 DEL(I) = A(I,IR)
41          C
42          C ESTIMATE OF RIGHT HAND PARAMETERS
43          C
44          IF(LB*LF*0)69 TA 37
45          H0 = Y1(F3)
46          37 IBS = LR
47          IF(LS*ST*FR)1RG = LS
48          40 DB 45 J = 1,IRG
49          H(J) = 0.
50          09 45 I=1,J
51          IF(J*GT*LR)69 TA 45
52          45 CONTINUE
53          WRITE(6,16)P,0,LF,LR,TR,LS
54          WRITE(6,21)DEL(I),I=1,IR
55          WRITE(6,22)H(J),I=1,IRG
56          77 CONTINUE
57          11 FORMAT(1615)
58          C
59
60          15 FORMAT(8F10.3)
61          16 FORMAT(T45,'INPUT PARAMETERS',/T50,'ID = ''13.,T50,'IR = ''13.,T50,'I = ''13.,T50,'R = ''13.,T50,'S = ''13.)
62          15 = ''13.,T50,'I = ''13.,T50,'R = ''13.,T50,'S = ''13.)
63          17 FORMAT(T35,'INPUT SE RESPONSE FUNCTION = &/160X,F10.9))
64          21 FORMAT(T35,'INITIAL ESTIMATES OF PARAMETERS',/T40,'LEFT HAND SIDE =
65          DELTA = &/160X,F10.21)
66          22 FORMAT(T40,'RIGHT HAND SIDE -V = &/160X,F10.21)
67          100 FORMAT('***PARAMETERS = ',I1)
68          STOP
```fortran
C TIME SERIES ANALYSIS PROGRAM #7

C

DIMENSION PARAM(10), D(10), VY(10), V(100), W(400), PX(400), PXN(400)

DIMENSION WW(400), A(400), P(400), PXN(400)

DIMENSION PHI(20), PR(20), XR(20), XR(20), YR(20), YR(20), AC(20), AR(20)

COMMON/MPPBST/WX, WY, A, Z, N, NCPST, L, R, S, T

REAL M

DATA WX, WY, A, Z /400*0, 400*0, 400*0, 400*0, 400*0 /

DATA KSN /0, 0, 1, 0, 1, 1 /

M = 0

READ 2, N, LR, LS, L, LD

NP = LR + LS + 1

READ (5, 110) (PARAM(I), I = 1, NP)

READ (5, 110) (X/I, I = 1, N)

KSN(1) = LD

CALL USID(I, V, 10, KSN, M, XR, XR, XG, PXH, XR, XBAR, XVAR, LERRAR, WY)

WRITE(6, 100) LERRAR

READ(5, 800) FND = 777

READ(5, 110) (Y/I, WY(I), I = 1, N)

NCST = 0

NM = NP + 1

KRN = 2*NP

C DIFFERENCING & TRANSFORMATION

C

KSN(1) = 0

CALL USID(Y, 10, KSN, M, YR, YR, YG, PH1H, YB0R, YVAR, LERRR, WY)

WRITE(6, 100) LERRR

CALL YNP2R(PARAM, NP, 6, 6, 50, KRN, M, NNP, V, WY, NM)
```
31      WRITE(6,21) NCNST
32      WRITE(6,800)
33      WRITE(6,20) (J, WY(J), 7(J), A(J), J = 1, N )
34      CALL USID(A, N, 10, KON, M, ARO, AR, ACO, AC, PTH, ABAR, AVAR, TERROR, A)
35      WRITE(6,100) LERROR
36      38    TE 7
37      777   STOP
39      1    FORMAT(8F10.3)
40      2    FORMAT(16I5)
41      10   FORMAT(2(F5.2,5x,F5.1,25X))
42      20   FORMAT(1H1,(110,3F20.3))
43      21   FORMAT(POX, IN8. B COST EVALUATIONS = '1,110)
44      100  FORMAT('0**** 1 ERROR = 111)
45      110  FORMAT(12X,F6.2,34X,F6.2)
46      800  FORMAT(66H)
47      1    /ROH
48      2    /ROH
49      3    )
50      END
SUBROUTINE FR (SUM, P)

DIMENSION P(10), WX(400), WY(400), A(400), I, L /400/

COMMON/MPCOST/ W, WY, A, Z, N, NCOST, LR, I, S, R

NCOST = NCOST + 1
SUM = 0.0
LR1 = LR + 1
LR2 = LR + 2
LRS1 = LR + LS + 1
DB 10 I = LSB1, N
ZLH = 0.
ZRH = 0.
DB 8 IR = 1, LR
8  DLH = ZLH + P(IR) * WX(1 IR)
ZRH = -P(LR1) * WX(1 LB)
DB 9 IS = 1, IR, LRS1
18  DX = 1 LB, IS, LBP + 1
9  ZRH = ZRH + P(IS) * WX(DX)
Z(I) = 7LH - 7RH
A(I) = WY(I) = 7T(I)
SUM = SUM + A(I) * 2
FORMAT(20X, 1NE, 3F COST EVALUATIONS = , 5120)
RETURN
WRITE(6, 21) NCOST
END
APPENDIX 2

A SAMPLE OF COLLECTED HUMAN OPERATOR DATA - ONE RUN OF FLIGHT PATH NO. 1
## Appendix 2: Sample Human Operator Data

### Tracking Run No. 23, Flight Path No. 11, Initial 2500FT

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>U1</th>
<th>V1</th>
<th>W1</th>
<th>U2</th>
<th>V2</th>
<th>W2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.04</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
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</tr>
<tr>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
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<tr>
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<td>0.20</td>
<td>0.20</td>
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<td>0.20</td>
</tr>
<tr>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
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</tr>
<tr>
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<td>0.28</td>
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<tr>
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<tr>
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<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Note: The data represents the displacement in various directions (U, V, W) over time.
APPENDIX 3

ADDITIONAL GRAPHS COMPARING AVERAGED HUMAN OPERATOR DATA WITH PREDICTED MEAN TRACKING ERROR OF DETERMINISTIC COMPONENT OF MODEL
Figure A.3.1. Averaged Human Error and Predicted Mean Error - Flight Path #3.
Figure A.3.2. Averaged Human Error and Predicted Mean Error - Flight Path #6.
Figure A.3.3. Averaged Human Error and Predicted Mean Error - Flight Path #8.
Figure A.3.4. Averaged Human Error and Predicted Mean Error - Flight Path #11.
APPENDIX 4.

PARSIMONY AND REDUCTION OF TRANSFER FUNCTIONS

The tentative transfer function models of equations (5.3.8) through (5.3.11) need to be inspected to see if any parameters can be cancelled resulting in a reduced model. This will assure us that the model has no redundant terms or parameters and is as economical as can be. "Parsimony" is another word used by Box and Jenkins (1971, Chapter 3) for such a property. The model is "parsimoneous" when it has the minimum possible number of parameters.

A cancellation test is done by approximate factorization of the numerator and denominator, and identical or similar terms are cancelled. The two transfer functions of tracking error and target angular velocity will now be subjected to such tests.

\[ \omega \rightarrow T Q \]

\[ y_t = \delta^{-1}(B) \omega(B) x_{t-b} + \vartheta^{-1}(B) \Theta(B) a_t \]

\[ \delta(B) y_t = \omega(B) x_{t-b} + ... \]

or \[ y_t = \delta_1 y_{t-1} + \omega_2 y_{t-2} + \omega_0 x_{t-b} - \omega_1 x_{t-b-1} - \omega_2 x_{t-b-2} + ... \]

\[ (1 - \delta_1 B - \delta_2 B^2) y_t = (\omega_0 - \omega_1 B - \omega_2 B^2) x_{t-b} + ... \]

for \( \omega \rightarrow T Q \)
\[ b = 0, \quad \delta_1 = .795, \quad \delta_2 = -.673 \]
\[ \omega_0 = .834, \quad \omega_1 = 1.16, \quad \omega_2 = -.554 \]

\[ y_t = \frac{(.834 - 1.16B + .554B^2)}{(1 - .795B + .673B^2)} W_t + N_t \]

\[ = \frac{.834 \ (1 - 1.4B + .665B^2)}{(1 - .8B + .67B^2)} W_t + N_t \]

Consider

\[ g \omega (1 - \omega_1 B - \omega_2 B^2) \]
\[ (1 - \delta_1 B - \delta_2 B^2) \]

factor out to get

\[ \frac{(B - 1.1 - .6j) \ (B - 1.1 + .6j)}{(B - .6 - 1.1j) \ (B - .6 + 1.1j)} \times \frac{.834 \times .665}{.67} \]

\[ \approx .834 \ \frac{(B - 1.1 - .6j) \ (B - 1.1 + .6j)}{(B - .6 - 1.1j) \ (B - .6 + 1.1j)} \]

\[ \therefore \text{No cancellation is possible.} \]

\[ \Delta E + \Delta T_Q \]

Parameters for Path 1, Run #23, operator 2, with minimum variance:

\[ b = 0, \quad \delta_1 = .343, \quad \delta_2 = .180 \]
\[ \omega_0 = -.045, \quad \omega_1 = .041, \quad \omega_2 = -.034 \]

for a model of

\[ (1 - \delta_1 B - \delta_2 B^2) y_t = (\omega_0 - \omega_1 B - \omega_2 B^2) x_t + N_t \]
\[ \Delta T_Q = y_t = \frac{(-.045 - .041B + .034B^2)(1 - .343B - .180B^2)}{\Delta E + N_t} \]

Consider

\[ = \frac{- .045 (1 + .91B - .75B^2)}{(1 - .343B - .180B^2)} \]

\[ = \frac{- .045 (B + .7)(B - 1.9)(-.75)}{.18 (B - 1.68)(B + 3.58)} \]

\[ = \frac{+.033 (B + .7)(B - 1.9)}{-.18 (B - 1.68)(B + 3.58)} \]

\[ = -.183 \frac{(B + .7)(B - 1.9)}{(B + 3.58)(B - 1.68)} \]

\[ \therefore \text{No cancellation is uncovered.} \]
APPENDIX 5
DOCUMENTATION AND LISTING OF STOCHASTIC TRACKING PROGRAM

The MAIN program initiates IX, the seed number for the Gaussian random number generator, GAUSS. It also reads in NR, the number of different flight paths to be tracked. NTIME is the number of stochastic realizations for each path. The main program then calls the time series tracking subroutine TSTRAK, which returns full output tracking data both in Azimuth and elevation as array BOUT. "MAIN" also prints and punches this data as an output.

TSTRAK begins by calling "DELAY" which initiates a working storage array of all the discrete data that is for tracking. NPT is the number of discrete intervals to be updated. Subroutine TARGET which is called next reads in flight data and defines cross-over in target state vector, after path normalization is done (Chapter IV). Subroutine WHERE is called to locate the target at any time by interpolation of the stored target data. The gun is initially directed towards target and the initial aim error is zero. This is done after statement 10 in TSTRAK, which is followed by an analytical solution of the gun model. IPRINT and KONT are integers to control the frequency of data storage or output.

The main tracking loop is indicated by statement 77 in TSTRAK, which begins by setting initial values of all series to their
unconditional mean of zero. Aim error calculation and evaluation of the angular position and velocity of target is done in subroutine EVAL which is next called and upon return the deterministic component of torque is evaluated.

The stochastic component of torque is determined by first calculating the standard deviation of the random deviates in azimuth and elevation SP and ST, by means of internal function subroutine SIGMA. Subroutine GAUSS is then called to provide the value of the random deviates with mean = 0. Stochastic noise components ANP and ANT are then evaluated in each axis and total torque is calculated as TRPHI and TRTHDA in azimuth and elevation respectively.

The total value of torque is applied to the gun model in each axis and a check is made on the different limits. Storage and updating of the different series - torque, tracking error, target angular velocity, standard deviation of white noise, element of white noise series and stochastic component of torque - is done by calling DLYIN. Subroutine IODATA is called at predetermined intervals to store the desired information.

BLK DATA is the data block which specifies the gun system and supplies parameter values of the tracking model. Subroutine ROTATE has the function of transforming target location from inertial to gun co-ordinate system.
STOCHASTIC TIME SERIES TRACKING MODEL

FORTRAN IV GI RELEASE 2.0

COMMON /EXX/ IX , NTIME
0002 COMMON /BARZ/ BOUT(200,7), JMB
0003 IX = 7014681
0004 READ 2,NR
0005 DO 1 II = 1,NR
0006 DO 1 NTIME = 1,6
0007 JMB = 0
0008 PRINT 211
0009 PRINT 21,IX
0010 PRINT 216
0011 CALL TSTRAK
0012 WRITE(6,216)((BOUT(I,K), K=1,6),(BOUT(I+1,K), K=1,6),I,I=1,JMB,2)
0013 WRITE(7,217)((BOUT(I,K), K=1,6),(BOUT(I+1,K), K=1,6),I,I=1,JMB,2)
0014 1 CONTINUE
0015 STOP
0016 2 FORMAT(I5)
0017 21 FORMAT(20X,'IX = ',I10)
0018 211 FORMAT(IHI,20X,'TIME SERIES STOCHASTIC TRACKING MODEL *)
0019 215 FORMAT(I10,F10.2,10X,3F10.2,10X,3F10.2)
0020 216 FORMAT(4(F9.2),I6)
0021 217 FORMAT(4(F6.2,F6.1,F6.2),I8)
0022 WRITE(6,215)((I,BOUT(I,7),(BOUT(I,K), K=1,6),I=I,JMB)
0023 END
SUBROUTINE TSTRAK

COMMON /EXX/ IX , NTIME

DIMENSION ZZ(6), Z(8), ZI(8), PLANE(6)

COMMON /ALL/STORE(13,51), IS

COMMON IPRINT,KOUNT,DELT,T

COMMON /AREA1/ TI, TL, TN, TD, KP, SPJET, THILIM, TH2LIM,

IPH1LIM, PH2LIM, JTHDA, JPHI, BTHDA, BPHI, RADEG, CMILRD

REAL JTHDA, JPHI, KP

COMMON /WTS/ D1, D2, D1E, D2E, W1, W12, W21, W22, GG1, GG2, G1E, G2E

COMMON /TQCOMP/ ZEP, ZWP, ZTOP, TRPHI, ZET, ZWT, ZQT, TRTHDA, ERP, ERT, WP, WT

SIGMA(S,W) = ABS(0.268*S+0.945*W)

IF (NTIME*GT.1) GO TO 10

DELT = 0.125

NPT = 10

ISTP = 1000

CALL DELAY(NPT)

READ IN FLIGHT DATA AND RETURN TCROSS AND ZZ AT TIME EQUAL 0.000

CALL TARGET(TCROSS,ZZ)

CONTINUE

T = 0.000

CALL WHERE(T, ZZ)

Z(2) = 0.00

Z(4) = 0.00

ZZX = ZZ(1)

ZZY = ZZ(2)

ZZZ = ZZ(3)

DIA = SQRT(ZZ(1)*ZZ(1) + ZZ(2)*ZZ(2))

Z(1) = ATAN2(ZZY, ZZX)

Z(3) = -ATAN2(ZZZ, DIA)
### Nonlinear Gun Model

<table>
<thead>
<tr>
<th>CCCCCC</th>
<th>Nonlinear Gun Model</th>
<th>CCCCCC</th>
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</thead>
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<tr>
<td>C</td>
<td>Z(1) or PHI</td>
<td>C</td>
</tr>
<tr>
<td>Z(1)</td>
<td>Azimuth Angle of Gun</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Z(2)</td>
<td>C</td>
</tr>
<tr>
<td>Z(2)</td>
<td>Angular Velocity of Gun Azimuth</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Z(3)</td>
<td>C</td>
</tr>
<tr>
<td>Z(3)</td>
<td>Elevation Angular Position</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Z(4)</td>
<td>C</td>
</tr>
<tr>
<td>Z(4)</td>
<td>Angular Velocity Elevation</td>
<td></td>
</tr>
</tbody>
</table>

```
0028 PJOB = JPHI / BPHI
0029 TJOB = JTHDA / BTHDA
0030 PJ0BB = PJOB / BPHI
0031 TJ0BB = TJOB / BTHDA
0032 C11 = EXP(( - BPHI / JPHI ) * DELT)
0033 C21 = EXP(( - BTHDA / JTHDA ) * DELT)
0034 C12 = (DELT - PJOB) / BPHI
0035 C22 = (DELT - TJOB) / BTHDA
0036 IPRINT = 0
0037 KOUNT = 0
0038 TSTP = 40.
0039 77 CONTINUE
0040 PHI0 = Z(1)
0041 DPHI0 = Z(2)
0042 THEO = Z(3)
0043 DTHEO = Z(4)
0044 CALL EVAL (T, Z, ERP, ERT, PLANE)
0045 WP = PLANE(2)
0046 WT = PLANE(5)
0047 IF (T .LE. DELT) GO TO 81
0048 TQP1 = STORE(2, IS)
0049 TOT1 = STORE(3, IS)
0050 ERP1 = STORE(4, IS)
```
ERT1=STORE(5,IS)
WP1=STORE(6,IS)
WT1=STORE(7,IS)
TQP2=STORE(2,IS-1)
TQT2=STORE(3,IS-1)
ERP2=STORE(4,IS-1)
ERT2=STORE(5,IS-1)
WP2=STORE(6,IS-1)
WT2=STORE(7,IS-1)
GO TO 85
TRPHI = 0.
TRTHDA = 0.
GO TO 88
CONTINUE
ZEP =GG1 * ( ERP - W11*ERP1 - W12*ERP2 )
ZET = G1E* ( ERT - W11*ERT1 - W12*ERT2 )
ZWPI = GG2 * ( KP - W21*WP1 - W22*WP2 )
ZW2T = G2E * ( WT - W21*WT1 - W22*WT2 )
ZTQP = D1*TQP1 + D2*TQP2
ZTQT = D1E*TQT1 + D2E*TQT2

C *****************************************
C STOCHASTIC COMPONENT EVALUATION
C *****************************************

WWP = (WP - 2.*STORE(6,IS) + STORE(6,IS-1))/RADEG
WWT = (WT - 2.*STORE(7,IS) + STORE(7,IS-1))/RADEG
SP = SIGMA(STORE(8,IS),WWP)
ST = SIGMA(STORE(9,IS),WWT)
CALL GAUSS(IX,SP,0.,AP)
CALL GAUSS(IX,ST,0.,AT)
ANP = 0.44*STORE(12,IS) + 2.457*AP + 1.371*STORE(10,IS)
ANT = 0.44*STORE(13,IS) + 2.457*AT + 1.371*STORE(11,IS)
TRPHI = ZTOP + ZEP + ZWP + ANP
TRTHDA = ZTOT + ZET + ZWT + ANT

IF(ABS(TRPHI).

GT.10.) TRPHI = SIGN(10.*TRPHI)
IF(ABS(TRTHDA).GT.10.) TRTHDA = SIGN(10.*TRTHDA)

CONTINUE

CA1 = PJOB * TRPHI - PJOB * DPHIO
CE1 = TJOB * TRTHDA - TJOB * DTHEO
CA2 = PHIO + PJOB * DPHIO
CE2 = THEO + TJOB * DTHEO

PHI = CA1 * C11 + CE1 * TRTHDA * C21
DPHI = -CA1 * PJOB * C11 + TRPHI / BPHI

IF(ABS(DPHI).GT.PHILIM) GOTO 1
DDPHI = (TRPHI - BPHI * DPHI) / JPHI
IF(ABS(DDPHI).GT.PH2LIM) PHI = PHI0 + (DPHI + 0.5*PH2LIM*DELT)*DELT
GOTO 3

GOTO 1
DPHI = SIGN(PH1LIM,DPHI)
PHI = PHI0 + DPHI * DELT

CONTINUE

THEDA = CE1 * C21 + CE2 + TRTHDA * C22
IF(THEDA.GT.0.0) GOTO 7

DTHEDA = -CE1 * TJOB + C21 + TRTHDA / BTHDA
IF(ABS(DTHEDA).GT.THILIM) GOTO 4

DDTHDA = (TRTHDA - BTHDA * DTHEDA) / JTHDA
IF(ABS(DDTHDA).GT.TH2LIM) THEDA = THE0 + (DTHEDA + 0.5*TH2LIM*DELT)*DELT
GOTO 8

DTHEDA = SIGN(THILIM,DTHEDA)
THEDA = THEO + DTHEDA * DELT
GOTO 8

THEDA = 0.
DTHEDA = 0.

CONTINUE

CALL DLYINC(T,TRPHI,TRTHDA,ERP,ERT,WP,WT,SP,ST,AP,AT,ANP,ANT)
0111  
Z(1) = PHI
0112  
Z(2) = DPHI
0113  
Z(3) = TTHEDA
0114  
Z(4) = DTHEDA
0115  
IF(KOUNT.EQ.1) PRINT 100,2,100,1,AP,ANP,WWT,ST,AT,ANT
0116  
IF(KOUNT.EQ.1) PRINT 222,T,WWP,SP,AP,ANP,WWT,ST,AT,ANT
0117  
T = T + DELL
0118  
IF(T.GE.TS TP) RETURN
0119  
IF(KOUNT.GE.TS TP) RETURN
0120  
KOUNT = KOUNT + 1
0121  
GO TO 77
0122  
222 FORMAT(F10.2,10X,4F10.3,10X,4F10.3)
0123  
223 FORMAT(40X,3F20.2)
0124  
END

FORTRAN IV G1 RELEASE 2.0
BLK DATA
DATE = 73291 08/28/07

0001  
BLOCK DATA
0002  
REAL JTHDA,JPHI,KP
0003  
COMMON /AREA1/ T1,TL,TD,KP,SPJET,TH1LIM,TH2LIM,
       JPH1LIM,PH2LIM,JTHDA,JPHI,BTHDA,BPHI,RADEG,CMLRD
0004  

0005  
 DATA  SPJET  TH1LIM  TH2LIM
0006  
       1 / 450.  ,384  ,3491 /
0007  
       DATA PH1LIM,PH2LIM,JTHDA,JPHI
0008  
       1 /1.0647 ,3491 ,0.45  ,0.2239 /
0009  
       DATA BTHDA,BPHI,RADEG,CMLRD
0010  
       1 /24.1585,15.4585 ,0175 ,1018.6001 /
0011  
       COMMON /WTS/ D1,D2,DI,E,D2E,W11,W12,W21,W22,GG1,GG2,G1E,G2E
0012  
       DATA D1,D2,GG1,GG2 / .891, -.299, -13.0, 24.899 /
0013  
       DATA DI,E,D2E,G1E,G2E / 0.336, 0.284, -27.665, 32.41 /
0014  
       DATA W11,W12,W21,W22 / -.91, 75, 1.4, -665 /
0015  
       END

END
SUBROUTINE IODATA(Z,N,ITIME)

DIMENSION Z(8)
COMMON IPRINT,KOUNT,DELT,T

COMMON /AREA1/ T1,TL,TN,TD,KP,SPJET,TH1LIM,TH2LIM,
PH1LIM,PH2LIM,JTHDA,JPHI,BTHDA,BPHI,RADEG,CMLRD

REAL JTHDA,JPHI,KP
COMMON /BARZ/ BOUT(200,7),JMB

COMMON/TQCOMP/ZEP,ZWP,ZTOP,TRPHI,ZET,ZWT,ZQT,TRTHDA,ERP,ERT,WP,WT

JMB = JMB + 1

K = 2
IPRINT = IPRINT + K

BOUT(JMB,1) = TRPHI
BOUT(JMB,2) = ERP*CMILRD
BOUT(JMB,3) = WP / RADEG
BOUT(JMB,4) = TRTHDA
BOUT(JMB,5) = ERT * CMILRD
BOUT(JMB,6) = WT / RADEG
BOUT(JMB,7) = T

RETURN

PRINT 555,T,ZEP,ZWP,ZTOP,TRPHI,ZET,ZWT,ZQT,TRTHDA

FORMAT(1SX,10F10.3)

END
SUBROUTINE DELAY(NPT)

COMMON /ALL/STORE(13,5),IS

TIME SUBROUTINE CALLED
NUMBER POINTS IN STORAGE ARRAY
LENGTH OF TIME DELAY
NAME OF STORAGE ARRAY
VALUES OF TIME
VALUES OF TORQUE AZIMUTH
VALUES OF TORQUE ELEVATION
AIM ERROR, AZIMUTH
AIM ERROR, ELEVATION
TARGET ANGULAR VELOCITY, AZIMUTH
TARGET ANGULAR VELOCITY, ELEVATION
DELAYED TORQUE AZIMUTH
DELAYED TORQUE ELEVATION
INPUT TORQUE AZIMUTH
INPUT TORQUE ELEVATION
VALUES OF TIME
VALUES OF TORQUE AZIMUTH
VALUES OF TORQUE ELEVATION
AIM ERROR, AZIMUTH
AIM ERROR, ELEVATION
TARGET ANGULAR VELOCITY, AZIMUTH
TARGET ANGULAR VELOCITY, ELEVATION
DELAYED TORQUE AZIMUTH
DELAYED TORQUE ELEVATION
INPUT TORQUE AZIMUTH
INPUT TORQUE ELEVATION

NUM = 3
IS = NUM-1
NPM = NPT - 1
NPP = NPT + 1
DO 11 I = 1,13
STORE(1,2) = 0.0
STORE(1,1) = 0.
STORE(1,NPP) = 10000.0
RETURN
ENTRY DLVIN(TIME, TP, TT, EP, ET, WP, WT, SP, ST, AP, AT, ANP, ANT)
IF(NUM.GT.NPT) GOTO 1
0014 IS = NUM
0015 STORE(1,NUM) = TIME
0016 STORE(2,NUM) = TP
0017 STORE(3,NUM) = TT
0018 STORE(4,NUM) = EP
0019 STORE(5,NUM) = ET
0020 STORE(6,NUM) = WP
0021 STORE(7,NUM) = WT
0022 RETURN
0023 STORE(9,NUM) = ST
0024 STORE(10,NUM) = AP
0025 STORE(11,NUM) = AT
0026 STORE(12,NUM) = ANP
0027 STORE(13,NUM) = ANT
0028 NUM = NUM + 1
0029 RETURN
0030 1 DO 2 J=1,NPM
0031 I = J + 1
0032 DO 2 K = 1,13
0033 2 STORE(K,J) = STORE(K,I)
0034 STORE(1,NPT) = TIME
0035 STORE(2,NPT) = TP.
0036 STORE(3,NPT) = TT
0037 STORE(4,NPT) = EP
0038 STORE(5,NPT) = ET
0039 STORE(6,NPT) = WP
0040 STORE(7,NPT) = WT
0041 STORE(8,NPT) = SP
0042 STORE(9,NPT) = ST
0043 STORE(10,NPT) = AP
0044 STORE(11,NPT) = AT
0045 STORE(12,NPT) = ANP
0046 STORE(13,NPT) = ANP
0047 IS = NPT
0048 RETURN
0049 END
SUBROUTINE TARGET(TCROSS, ZZ)
DIMENSION ZZ(6), FLY(7,81)

NOMENCLATURE

TCROSS TIME TO CROSSOVER
SPJET VELOCITY OF PLANE (IN KNOTS)
(XO, YO, ZO) INITIAL LOCATION OF THE PLANE
ZZ POSITION AND VELOCITY OF PLANE

ZZ(1) X
ZZ(2) Y
ZZ(3) Z
ZZ(4) X VELOCITY
ZZ(5) Y VELOCITY
ZZ(6) Z VELOCITY

FLY ARRAY PLANE STATE VARIABLES VS. T
FLY(1,X) TIME
FLY(2,X) X
FLY(3,X) Y
FLY(4,X) Z
FLY(5,X) X VELOCITY
FLY(6,X) Y VELOCITY
FLY(7,X) Z VELOCITY

READ(5,6) ITYPE, ICASE, NUMBER, DIVE, PULL, DISPLC
GOTO(1,3,50,13), ITYPE
1 XO = 1500.
IF(ICORS.EQ.2) XO = 3000.
YO = 15200.000
ZO = 500.
SPJET = 450.
XV = 0.000
YV = -SPJET*6067./3600.
ZV = 0.000
TCROSS = -Y0/YV
TIME = 0.000
DO 2 I = 1,81
Y = YO + TIME*YV
TIME = TIME + 0.5000
GOTO 13
DO 4 I = 1,NUMBER
READ(5,11)US1,FLY(3,I),FLY(4,I),FLY(2,I),US2,US3,FLY(5,I),FLY(6,I)
CONTINUE
FORMAT(8F10.0)
ADDX = DISPLC - FLY(2,41)
ADDY = -FLY(3,41)
ADDZ = 500.00 - FLY(4,41)
TIME = 0.0000
DO 41 I = 1,NUMBER
FLY(1,I) = TIME
FLY(2,I) = FLY(2,I) + ADDX
FLY(3,I) = FLY(3,I) + ADDY
FLY(4,I) = FLY(4,I) + ADDZ
FLY(6,I) = -FLY(6,I)
TIME = TIME + 0.50000
GOTO 13
DO 51 I = 1,NUMBER
READ(5,11)US1,FLY(3,I),FLY(4,I),FLY(2,I),US2,US3,FLY(5,I),FLY(6,I)
CONTINUE
READ(5,40)(FLY(7,I),I=1,NUMBER)
ADDX = DISPLC - FLY(2,41)
ADDY = -FLY(3,41)
ADDZ = 500.00 - FLY(4,41)
TIME = 0.0000
A = -ATAN(56.25/71.28)
B = COS(A)
C = SIN(A)

DO 52 I = 1, NUMBER

FLY(1, I) = TIME

FLY(2, I) = FLY(2, I) + ADDX

FLY(3, I) = FLY(3, I) + ADDY

FLY(4, I) = FLY(4, I) + ADDZ

DUMMY = FLY(5, I)

FLY(5, I) = FLY(5, I) * B - FLY(6, I) * C

FLY(6, I) = -FLY(6, I) * B - DUMMY * C

TIME = TIME + 0.50000

CONTINUE

TCROSS = 20.0000

WRITE(6, 12) ICASE, DIVE, TCROSS

WRITE(6, 9)

DO 53 I = 1, NUMBER

WRITE(6, 10) (FLY(J, I), J = 1, 7)

RETURN

FORMAT(313, 3F10.3)

FORMAT(F10.5)

FORMAT(59X, 'CASE = ', I3, 2X, 'LEVEL FLYBY', 55X, 'TIME', 'I TO CROSSOVER = ', F10.2, ')


FORMAT(5X, 7F15.2)

FORMAT(8F10.2)

FORMAT(49X, 'CASE = ', I4, 2X, F10.2, 4X, 'DEGREE DIVE', '1*ANGLE/55X, 'TIME TO CROSSOVER = ', F10.2, ')

FORMAT(5X, 'CASE = ', I4, 2X, F10.2, 4X, 'DEGREE DIVE ANGLE')

ENTRY WHERE (T, ZZ)

T3 = T

GOTO(14, 15, 16, 17), ITYPE

ZZ(1) = XO

ZZ(2) = YO + YY * T3

ZZ(3) = ZO
SUBROUTINE EVAL(T,Z,ERROR1,ERROR2,PLANE)
DIMENSION PLANE(6),ZZ(6),ACINER(3),ACGUN(3),DXINER(3),DXGUN(3)
DIMENSION Z(8)

THE PURPOSE OF THIS SUBROUTINE IS TO
CALCULATE THE ANGULAR ALIGNMENT ERRORS
AND ANGULAR POSITION OF THE TARGET

ERROR ARRAY OF ALIGNMENT ERRORS
AND DERIVATIVES

ERROR(1,1) AZIMUTH ERROR
ERROR(2,1) ELEVATION ERROR

GOTO 17

DO 16 I = 1,6

K = I + 1

Z7(I) = (FLY(K,JJ) - FLY(K,J)) * FT + FLY(K,J)

RETURN
<table>
<thead>
<tr>
<th>C PLANE</th>
<th>ARRAY CONTAINING ANGULAR LOCATION OF THE TARGET</th>
</tr>
</thead>
<tbody>
<tr>
<td>C PLANE(1)</td>
<td>AZIMUTH ANGLE</td>
</tr>
<tr>
<td>C PLANE(2)</td>
<td>AZIMUTH VELOCITY</td>
</tr>
<tr>
<td>C PLANE(3)</td>
<td>AZIMUTH ACCELERATION</td>
</tr>
<tr>
<td>C PLANE(4)</td>
<td>ELEVATION ANGLE</td>
</tr>
<tr>
<td>C PLANE(5)</td>
<td>ELEVATION VELOCITY</td>
</tr>
<tr>
<td>C PLANE(6)</td>
<td>ELEVATION ACCELERATION</td>
</tr>
</tbody>
</table>

```plaintext
CALL WHERE(T, ZZ)
DO 1 I = 1, 3
   ACINER(I) = ZZ(I)
1
DO 2 I = 1, 3
   DXINER(I) = ZZ(I + 3)
2
PHI = Z(1)
DPHI = Z(2)
THEDA = Z(3)
DTHEDA = Z(4)
CALL ROTATE(PHI, THEDA, ACINER, ACGUN)
Z1 = ACGUN(1) + ACGUN(1) + ACGUN(2) + ACGUN(2)
Z2 = SQRT(Z1)
Z3 = ACGUN(1) + ACGUN(3) + ACGUN(3) + ACGUN(3)
ERROR1 = -ATAN(ACGU(2)/ACGU(1))
ERROR2 = ATAN(ACGU(3)/ACGU(1))
Z1 = ZZ(1) * ZZ(1) + ZZ(2) * ZZ(2)
Z2 = SQRT(Z1)
Z3 = ZZ(1) * ZZ(4) + ZZ(2) * ZZ(5)
PLANE(1) = ATAN2(ZZ(2), ZZ(1))
PLANE(2) = (ZZ(1) * ZZ(5) - ZZ(2) * ZZ(4)) / Z1
PLANE(3) = 0.0
PLANE(4) = -ATAN2(ZZ(3), Z2)
PLANE(5) = (ZZ(3) * ZZ(6) + ZZ(6) * ZZ(1)) / (ZZ(1) + ZZ(3) * ZZ(3))
PLANE(6) = 0.00
RETURN
END
```
SUBROUTINE ROTATE(A,B,COLD,CNEW)

DIMENSION COLD(3),CNEW(3),C(3,3)

CA=COS(A)
CB=COS(B)
SA=SIN(A)
SB=SIN(B)

C(1,1)=CA*CB
C(1,2)=CB*SA
C(1,3)=-SB
C(2,1)=-SA
C(2,2)=CA
C(2,3)=0
C(3,1)=CA*SB
C(3,2)=SA*SB
C(3,3)=CB

DO 10 I=1,3
SUM=0.
10 CNEW(I)=SUM

DO 20 J=1,3
20 SUM=SUM+C(I,J)*COLD(J)

CNEW(1)=SUM
RETURN
END
SUBROUTINE GAUSS(I,X,S,AM,V)

SUBROUTINE GAUSS

PURPOSE

COMPUTES A NORMALLY DISTRIBUTED RANDOM NUMBER WITH A GIVEN MEAN AND STANDARD DEVIATION

USAGE

CALL GAUSS(I,X,S,AM,V)

DESCRIPTION OF PARAMETERS

I - IX MUST CONTAIN AN ODD INTEGER NUMBER WITH THREE OR LESS DIGITS ON THE FIRST ENTRY TO GAUSS. THEREAFTER IT WILL CONTAIN A UNIFORMLY DISTRIBUTED INTEGER RANDOM NUMBER GENERATED BY THE SUBROUTINE FOR USE ON THE NEXT ENTRY TO THE SUBROUTINE.

S - THE DESIRED STANDARD DEVIATION OF THE NORMAL DISTRIBUTION.

AM - THE DESIRED MEAN OF THE NORMAL DISTRIBUTION.

V - THE VALUE OF THE COMPUTED NORMAL RANDOM VARIABLE

REMARKS

THIS SUBROUTINE USES RANDU WHICH IS MACHINE SPECIFIC

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED

RANDU

METHOD

USES 12 UNIFORM RANDOM NUMBERS TO COMPUTE NORMAL RANDOM NUMBERS BY CENTRAL LIMIT THEOREM. THE RESULT IS THEN ADJUSTED TO MATCH THE GIVEN MEAN AND STANDARD DEVIATION.

THE UNIFORM RANDOM NUMBERS COMPUTED WITHIN THE SUBROUTINE ARE FOUND BY THE POWER RESIDUE METHOD.
SUBROUTINE RANDU(Ix, Iy, Yfl)

PURPOSE

COMPUTES UNIFORMLY DISTRIBUTED RANDOM REAL NUMBERS BETWEEN 0 AND 1.0 AND RANDOM INTEGERS BETWEEN ZERO AND 2**31. EACH ENTRY USES AS INPUT AN INTEGER RANDOM NUMBER AND PRODUCES A NEW INTEGER AND REAL RANDOM NUMBER.

USAGE

CALL RANDU(Ix, Iy, Yfl)

DESCRIPTION OF PARAMETERS

Ix - For the first entry this must contain any odd integer number with nine or less digits. After the first entry, Ix should be the previous value of Iy computed by this subroutine.

Iy - A resultant integer random number required for the next entry to this subroutine. The range of this number is between zero and 2**31.

Yfl - The resultant uniformly distributed, floating point, random number in the range 0 to 1.0.
C REMARKS
C THIS SUBROUTINE IS SPECIFIC TO SYSTEM/360 AND WILL PRODUCE
C 2**29 TERMS BEFORE REPEATING. THE REFERENCE BELOW DISCUSSES
C SEEDS (65539 HERE), RUN PROBLEMS, AND PROBLEMS CONCERNING
C RANDOM DIGITS USING THIS GENERATION SCHEME. MACLAREN AND
C MARSAGLIA, JACM 12, P. 83-89, DISCUSS CONGRUENTIAL
C GENERATION METHODS AND TESTS. THE USE OF TWO GENERATORS OF
C THE RANDU TYPE, ONE FILLING A TABLE AND ONE PICKING FROM THE
C TABLE, IS OF BENEFIT IN SOME CASES. 65549 HAS BEEN
C SUGGESTED AS A SEED WHICH HAS BETTER STATISTICAL PROPERTIES
C FOR HIGH ORDER BITS OF THE GENERATED DEVIATE.
C SEEDS SHOULD BE CHOSEN IN ACCORDANCE WITH THE DISCUSSION
C GIVEN IN THE REFERENCE BELOW. ALSO, IT SHOULD BE NOTED THAT
C IF FLOATING POINT RANDOM NUMBERS ARE DESIRED, AS ARE
C AVAILABLE FROM RANDU, THE RANDOM CHARACTERISTICS OF THE
C FLOATING POINT DEVIATES ARE MODIFIED AND IN FACT THESE
C DEVIATES HAVE HIGH PROBABILITY OF HAVING A TRAILING LOW
C ORDER ZERO BIT IN THEIR FRACTIONAL PART.
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C NONE
C METHOD
C POWER RESIDUE METHOD DISCUSSED IN IBM MANUAL C20-8011.
C RANDOM NUMBER GENERATION AND TESTING
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**Notes:**
- Temperature readings are in °C.
- Time intervals are in minutes.
- Values are approximate and may vary slightly.

**Additional Observations:**
- The data seems to represent temperature changes over time, possibly in a controlled environment such as an experiment or process heating.
- The values indicate a steady increase in temperature, suggesting a consistent heating rate.
- The data points are spread evenly across the range, indicating a uniform observation period.
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VITA

Name: Jamal al-Din Muhammad al-Barzinji

Birth: December 15, 1939, in the City of Mosul, Iraq.

Elementary & Secondary Education:

1946 - 1952 Adnanya Elementary, Mosul, Iraq
1952 - 1957 Central High, Mosul, Iraq

Colleges and Universities:

1957 - 1958 College for Educational Missions, Baghdad, Iraq
1958 - 1959 Doncaster Technical College, Doncaster, England
1969 - 1971 Louisiana State University, M.S. in Chemical Engineering.

Membership in Honorary and Learned Societies:

American Institute of Chemical Engineers - Member
The Honor Society of Phi Kappa Phi
ΦΑτ - Honorary Chemical Society.