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Bridging Gaps: Supplemental Materials for Struggling Students in Common Core Aligned Algebra 1

Reynalin Apinardo Baricuatro

Louisiana State University and Agricultural and Mechanical College

BRIDGING GAPS: SUPPLEMENTAL MATERIALS FOR STRUGGLING STUDENTS IN COMMON CORE ALIGNED ALGEBRA 1

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Reynalin A. Baricuatro
B.S., University of San Carlos, 2004
B.S., St. Theresa's College, 2007
B.S., University of the Visayas, 2008
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ABSTRACT

This thesis emphasizes the need for, and the development of, auxiliary and supplemental materials for students struggling with the Module 1 of the Common Core aligned EngageNY/Eureka Math, Algebra 1 Curriculum, in particular, special education students and regular education students who are missing essential pre-requisite skills. The goal of this thesis is to provide resources which assist both the students and the teachers with the transition from the old Louisiana Comprehensive Curriculum to the intensity, rigor, and mathematical sophistication of the new Common Core aligned curriculum. Clearly, the need for supplementary materials is not restricted to the first of six modules of the EngageNY Algebra 1 Curriculum. However, due to the obvious time constraints, this thesis provides only a "proof of concept" that is restricted to Module 1.

The supplemental materials developed for this thesis focus on recognized foundational mathematics standards which students should have mastered at previous grade levels. The workbook included in this thesis contains exercises to reinforce students' knowledge and proficiency in meeting pre-requisite foundational standards needed for Module 1 of the EngageNY Algebra 1 course. Additionally, it assists in the achievement of "standards-based" or Common Core aligned goals and objectives cited in special education students' Individualized Education Programs (IEPs). The materials are structured systematically so that teachers can continue with their at-grade-level instruction while still addressing the missing mathematical skills.

INTRODUCTION

One of the primary goals of teachers is to convey comprehensible lessons. However, students vary in ability levels, and delivering meaningful lessons that are comprehensible for all students is often not possible. With increasing rigor and mathematical sophistication required by the new Common Core State Standards (CCSS), some students feel overwhelmed, become more and more frustrated, and completely quit. Teachers can prevent this by addressing one of the root causes – students' missing knowledge about how to begin and solve grade-level mathematics problems.

This thesis emphasizes the importance of addressing potential gaps in students' required, pre-requisite mathematical knowledge through the use of "Pre-Lessons." These anticipatory sets (bell-ringers) are designed to systematically follow the first module entitled, "Relationships Between Quantities and Reasoning with Equations and Their Graphs" of the ninth grade EngageNY Algebra 1 curriculum, serving as both intervention and introduction to new lessons. These lessons allow students to review previous skills and retrieve prior knowledge to be applied in the current lessons. The Pre-Lessons are also paced to give teachers the chance to identify struggling students and provide one-on-one intervention before new lessons are introduced.

Pre-Lessons are clustered into topics which start at the foundation phase and address the foundational vocabulary. Each of the 28 Pre-Lessons for Module 1 start at the "Build up! Warm-up!" phase, and proceed through "discovery" phases. The "foundation" phase provides definitions of the foundational vocabulary needed to understand the Pre-Lessons. The "build up" phase provides assignments addressing the

recognized foundational standards for practice and review of the needed skills for students prior to introducing the new lesson. The "discovery" phase bridges the foundational skills to the new lesson.

These Pre-Lessons also cater to the needs of students with disabilities, underachievers, repeaters as well as regular students who are either missing essential skills or need a refresher of skills that they should have mastered in prior math classes.

CHAPTER I
THE PROBLEM AND ITS BACKGROUND

This chapter provides a brief history and a background of the Common Core State Standards (CCSS), the only available fully CCSS-aligned math curriculum (Engage New York/ Eureka Math), and the need for additional resources and teaching tools for the successful adoption of the Common Core State Standards in Mathematics.

Brief Background and History of Common Core

The Common Core State Standards were initiated in 2009 by a group of state leaders that included governors, the National Governors Association (NGA), state commissioners of education, and the Council of Chief State School Officers (CCSSO). The Common Core is a set of high-quality academic standards in mathematics and English language arts/literacy (ELA). These learning goals outline what a student should know and be able to do at the end of each grade (Common Core State Standards Initiative, 2014). Common Core Standards were intended to cultivate a nation-wide set of educational standards that would prepare every graduating high school student for college and a career in an increasingly competitive workforce.

In contrast, state education standards have been around since the early 1990s. By the early 2000s, every state had developed and adopted its own learning standards that specified what tasks students in grades 3-8 and high school should be able to perform successfully. Every state also had its own definition of proficiency, the level at which a student is determined to be sufficiently educated at each grade level. The lack of

standardization of the state standards was one reason why states decided to develop the Common Core State Standards in 2009.

Louisiana's Participation of The Common Core Standards

In 2010, the State of Louisiana decided to adopt the Common Core Standards. The 2013-2014 school year was the year originally intended for initial transition, and implementation was supposed to occur by 2014-2015. However, the Louisiana Department of Education (LDOE) decided that full implementation would be postponed and as of June of 2015, it has become entirely nebulous what will happen with the implementation of the CCSS in our state.

During these years, however, the East Baton Rouge Parish School System did not purchase, prepare, or develop a Common Core aligned curriculum. For the school year 2013-2014, EBRPSS, along with several other school districts in Louisiana, decided to utilize the Engage New York/Eureka Math Curriculum. For the 2014-2015 school year, EBR decided to purchase an Algebra 1 book by McGraw-Hill Education and will use the Engage New York Curriculum as a recommended reference. The Department of Education has rated this book as "Tier III, Not representing quality" (Curricular Resources and Annotated Reviews, 2014). A Tier III rating means that it received a "No" for at least one of the "Non-Negotiable" criteria. The Non-Negotiable criteria areas which received a "No" rating are the following:

- Non-Negotiable 2. Consistent, Coherent Content
- Non-Negotiable 3. Rigor and Balance
- Non-Negotiable 4. Practice-Content Connections

(Curricular Resources and Annotated Reviews, 2014)

Louisiana adopted the Common Core because, according to Louisiana Department of Education (LDOE), the state believes that "Louisiana students are just as capable as any other group of students across our country or in the world," (www.louisianabelieves.com, 2014). Unfortunately, Louisiana students have ranked 44th in English Language Arts and 46th Mathematics proficiency compared to other states. The LDOE thinks that, "We must level the playing field for our kids so they can compete in our ever changing global economy." The Common Core State Standards hold students across the country to the same expectations and allow Louisiana students to compare their performance with other students across the nation through assessments that have been recently developed such as PARCC Assessments and Smart Balanced Assessments (www.louisianabelieves.com, 2014)

Need for Supplementary Resources

During the initial shift to Common Core State Standards (CCSS), a main concern of mathematics teachers was the lack of established guidelines, learning tools, and resources. This resulted in inadequate time for preparation and hindered the chance for teachers to adjust and update materials. School districts had a vision of an easy transition and successful implementation of the Common Core State Standards (CCSS) in Mathematics but failed to provide a curriculum and adequate teaching resources.

For Mathematics, the old Louisiana Comprehensive Curriculum (LCC) is different from the new CCSS-aligned Engage NY Curriculum in regards to pace, rigor, expectations and order of the lessons. There are topics that are assigned in the old LCC for teachers to discuss during Algebra 1 that are now under the eighth grade math strand

in the CCSS. Consequently, gaps in student knowledge frequently exist. Pre-requisite skills that are expected to have been mastered in prior grade levels per CCSS must be covered without compromising the new materials to be introduced so that these gaps do not result in students struggling with the current Algebra 1 lessons. The prevalence of serious gaps in pre-requisite knowledge and skills has left Louisiana Algebra 1 teachers in trying circumstances. They must either devote time to finding resources and reteaching missed common core concepts, or they must ignore these deficiencies and just move on with current lessons. If teachers opt to address deficient skills, they must have resources other than the ones provided by the school district. They must identify the missing skills and search for worksheets and activities to address them. The use of Pre-Lessons presented in this thesis will save some teachers the time they would spend otherwise on identifying their own materials. These Pre-Lessons may also help assure that accommodations and modifications for students with disabilities are provided, along with the achievement of goals and objectives in their CCSS-aligned IEPs..

Brief Background of Engage New York and Eureka Math

The Engage New York Curriculum was developed for New York State to provide its educators with professional learning tools and Common Core-aligned educational resources to support the State in reaching its vision "for a college and career-ready education" for all students. The organization's website, EngageNY.org states the purpose of its development is to provide assistance to educators as they adopt the Common Core State Standards (CCSS):

> In order to assist schools and districts with the implementation of the Common Core, NYSED has provided curricular modules and units in P-12 ELA

> and math that can be adopted or adapted for local purposes. Full years of curricular materials are currently available on EngageNY, for grades Kindergarten through 9th grade in Mathematics and Kindergarten through 8th grade in English Language Arts (ELA). NYSED is working with our partners to deliver high quality curricular materials for all remaining grades in both Mathematics and ELA. (EngageNY.org, 2014)

In preparation for their transition to CCSS, the state of New York has funded professionals to develop a curriculum that is suitable for their students. They were the first to develop a new curriculum that is completely aligned with CCSS. As a result, several school districts, including EBRPSS, have simply adopted the EngageNY Curriculum for the school year 2013-2014.

Eureka Math is a Common Core Aligned Curriculum that is a twin to Engage New York and available for distribution and purchase to teachers outside of New York. It is developed by an organization called Great Minds, Inc. (formerly Common Core, Inc.) which consists of educators, some authors of the CCSS, and state education leaders. The EngageNY/Eureka Mathematics curriculum was developed when the Common Core State standards were finalized after the success of then Common Core, Inc.'s Wheatley Portfolio, the first CCSS aligned ELA curriculum in 2010.

The material upon which Eureka Math is based was originally created through a partnership with the New York State Education Department. Their expert review team, including renowned mathematicians who helped write the new standards, progressions, and the much-touted "Publisher's Criteria," strengthened an already rigorous development process. (GreatMinds, Inc, 2015)

Eureka Math divides the curriculum into modules, the modules into topics, and each topic into lessons. In each module, Eureka Math indicates the foundational standards students need to have mastered before approaching the module and the focus standards for the module. However, while recognizing that there should be remedial standards, and indicating what these are, Eureka Math does not address them individually

nor provided materials to do so. When teachers click on a foundational standard, the program only states the standard with no suggestions for teaching students to accomplish that standard. Teachers have to purchase access to the lower grade level curricula for further information; otherwise, their view of this valuable information is restricted. Eureka Math is a very impressive tool for teachers, but it is expensive. Each online membership costs $150 annually; some discounts are available for teachers in qualified districts. Recently, Eureka Math Workbook has been released which is more reasonably priced at $37.50. Teachers should find the Pre-Lessons available in this thesis as a more economical option. The Pre-Lessons provide a series of materials in ascending stages without the need to purchase several books or access to several curricula.

School Year 2014-2015 – Personal Impetus for this thesis

At the beginning of the school year 2014-2015, our school needed three new Math teachers. They were to hire a teacher for the Algebra I repeaters class at the end of August. However, this teacher resigned after the first few months. I was then tasked to teach this class at the beginning of the second semester. This class of 14 students was composed of students with disabilities (504 or Students with Medical Concerns and Exceptional students), T9 or Transitional 9^{th} grade student, English as Second Language students, and repeater students. Besides the apathy for Mathematics, most of these students had issues that would hinder them from becoming successful in their academics. Some of these difficulties included homelessness, teenage pregnancy, poverty, truancy, violence and behavior concerns.

The result of the midterm benchmark assessment administered in December of 2014, showed that 88.89% of them scored in the Needs Improvement range, 11.11% scored Fair, 0% scored good and 0% scored Excellent. One of the school's grading policies was to allow "Revise/Re-Do". This practice gives students the opportunity to make-up missed assignments, revise low-grade homework and class assignments, and retake failed tests or examinations. With the grading policy, and upon careful evaluation and discussion with our school's Instructional Specialist and fellow Algebra 1 teacher, I agreed that in the next six weeks of instruction would focus on lessons covered in this midterm benchmark assessment. The goal was that students would be equipped with a better understanding of the lessons and prove that understanding in their February re-test. My dilemma was re-teaching an entire module and addressing pre-requisite skills in six weeks. I decided to use the Pre-Lessons designed herein. Upon completion of the six weeks, this same group of students was retested by participating in the same benchmark assessment. Of the 14 students registered in that class, only 7 participated in both examinations. Of the seven students that participated in both examinations, five students advanced a minimum of 10.1 percentage points and as high as 62.6 percentage points. Two students decreased by 3 and 9.6 percentage points respectively. Of the 5 students that advanced in this benchmark, and were in attendance at least 80% of class time, 4 passed the End-of-Course Assessment in Algebra I. Though the sample is limited, the results show a favorable outcome when Pre-Lessons are used to bridge the Module 1 achievement gap.

Globalization and Achievement Gaps

In the mid 1990's, the word "globalization" could be found in headlines of every form of news media in developed and developing countries. Developed countries wanted to find means of importing cheaper materials and products, while developing countries opened export processing zones in the hopes of improving economic growth. Globalization was defined to be a process of international integration, resulting from radically new forms of communication (internet, cellphones, and global media), the global trading and exchange of products, concepts, ideas, and other aspects of culture. Its intention was to break down barriers between societies and conceive an open market economy. However, back then, several critics had already feared a possible negative outcome -- competition in employment. Now, with the world becoming more global, it has become easier for those who are qualified to apply for positions anywhere in the world. I, for one, am a product of globalization. I responded to a US-based job placement agency's website identifying a critical shortage of teachers in the East Baton Rouge Parish School System (EBRPSS). A few weeks later, a group of Principals and Human Resources personnel from EBRPSS arrived in our city, Cebu City, and conducted a job interview. And now, here I am, and I have recently completed my seventh year of teaching for the school district.

Worldwide, jobs have become flexible; outsourcing, virtual assistants, and call centers have already grown. Today, employment depends more and more on qualifications and desired salary. All things being equal, those who offer a lower price for the same credentials will get the job. There are many people in the developing world who are willing to work for a lower salary of what workers in developed countries

demand. With these global truths in mind, American high schools should not produce substandard graduates. Our graduates should undergo internationally competitive training that is at least as good, if not better, compared to what their counterparts in other parts of the world receive.

In a world that has become more globally interconnected than before, the concern for public education and the huge gap in the academic performance of students has become alarming to parents, educators and policy makers. This concern has resulted in an increase in supporters of educational reform, not only in the US but in several countries worldwide.

A recent global study by Julia V. Clark et al. (2014) aimed "to provide a wide array of benefits to the nation in terms of educational achievement gap with particular interest in science, technology, engineering, and mathematics (STEM)." Their study found that several countries have successfully confronted inequities in achievement, proving that any school district, starting with its teachers, can close achievement gaps regardless of their clientele. Their research also found that all students can achieve mastery if they are provided with the right educational tools and opportunities.

The research highlighted three countries: Singapore, Finland and South Korea. These three countries made improvements in their education systems and, after 30 years, they have soared to the top of international rankings in student achievement and attainment, graduating more than 90% of their students from high school and sending the majority of graduates through college. Their strategies, which follow, also have much in common:

• All three nations fund schools adequately and equitably, and add incentives for teaching in high-need schools. All three nations have built their education systems on a strong egalitarian ethos, explicitly confronting and addressing potential sources of inequality. In South Korea, for example, a wide range of incentives are available to induce teachers to serve in rural areas or in urban schools with disadvantaged students. In addition to earning bonus points toward promotion, incentives for equitable distribution of teachers include smaller class sizes, less in-class teaching time, additional stipends, and opportunities to choose later teaching appointments (Kang and Hong 2008). The end result is a highly qualified, experienced, and stable teaching force in all schools, providing a foundation for strong student learning.

• All three nations organize teaching around **national standards and a core curriculum** **“**that focus on higher-order thinking, inquiry, and problem solving through rigorous academic content. Working from lean national curriculum guides that have recommended assessment criteria, teachers collaborate to develop curriculum units and lessons at the school level, and develop school-based” (Clark, 2014)

The researchers have identified several factors contributing to the achievement gap and preventing minority students from achieving success in education in the United States. They are summarized as follows:

• Teachers lack skills to deliver instruction to low-performing students.

• Schools that have a history of low performance, lack rigor in mathematics and science programs. The curriculum is often watered down, and instruction is not designed to challenge students to perform at high levels.

• Inadequate resources are available to deliver challenging instruction in STEM programs.

• Tracking students into classrooms where both teachers and students perform at low levels.

• Racial and linguistic minority students and low-income students historically have not been provided equitable access to resources, instruction, and opportunities to achieve at high levels.

• Schools often place teachers with minimal teaching skills and experiences with high need students. (Clark, 2014)

Clark (2014) believes "that the key factors contributing to the achievement gap can be summed up in two words: equity and access. Overall, minority students have less access to: (1) well-qualified mathematics and science teachers, (2) strong mathematics and science curriculum, (3) resources, (4) classroom opportunities, and (5) information."

Many initiatives in the USA have been implemented to narrow the achievement gap. The study focused on three national reform initiatives: No Child Left Behind Act (NCLB), America COMPETES Act, and Race to the Top. The main objective of these education reforms is to increase overall student achievement, especially increasing the performance of low achievers.

No Child Left Behind Act (NCLB) - The United States set a national goal of ensuring that each student receives an equitable, high-quality education, and that no child is left behind in this goal. This Act focuses on standards and aligns tests to ensure that all students will perform adequately by requiring all public schools receiving federal funding to administer annual standardized tests. It also emphasizes school accountability by requiring schools to show constant improvement, and constant progress monitoring of students.

> "The Act also contains the president's four basic education reform principles: (1) stronger accountability for results, (2) increased flexibility and local control, (3) expanded options for parents, and (4) emphasis on teaching methods that have been shown to work. (Clark, 2014)

America COMPETES Act - The America Creating Opportunities to Meaningfully Promote Excellence in Technology, Education, and Science Act (America COMPETES Act) was signed into law on August 9, 2007. It gained the support of the educational organizations and the business sector. The goals of the COMPETES Act "

focuses on three primary areas of importance to maintaining and improving U.S. innovation in the twenty-first century: (1) increasing research investment; (2) strengthening educational opportunities in STEM, from elementary through graduate school; and (3) developing an innovation infrastructure (Clark, 2014)."

Race to the Top – This initative was a competitive grant funded by the U.S. Department of Education as part of the American Recovery and Reinvestment Act of 2009. This program is a competitive grant program that provides financial assistance to states to produce measurable student gains. The primary goals of the program are improving student achievement, closing achievement gaps, and improving high school graduation rates.

> This program was designed to encourage and reward states creating the conditions for education innovation and reform, achieving improvement in student outcomes, and implementing reform plans in four core areas: (1) adopting standards and assessments that prepare students to succeed in college and the workplace; (2) building data systems that measure student growth and success and inform teachers and principals how to improve instruction; (3) recruiting, developing, rewarding, and retaining effective teachers and principals; and (4) turning around the lowest-performing schools (NSB 2012).

The research by Clark (2014) showed that there are states and school districts that have become successful in narrowing the achievement gaps. They have confronted the inequities in the education system and are providing evidence that improvements are being made in student performance. Clark (2014) focused on these states: District of Columbia, Virginia, Maryland, Louisiana, South Carolina, Mississippi and Texas. These states have raised their test scores and graduation rates by providing resources and making community-wide and long-term investments in disadvantaged children; creating better early-childhood programs; and using clear, ambitious goals for all students and curricula aligned to those goals.

Louisiana has made notable progress in its effort to close the achievement gap between races and socioeconomic groups. Based on the National Assessment of Educational Progress (NAEP) data, Louisiana is one of only two states to narrow the achievement gap between Black and White students in both fourth-grade reading and eighth-grade mathematics from 2003 to 2011. Additionally, since the state implemented its accountability system in 1999, the performance gap between Black and White students on state assessments has narrowed by 11.6 % in English language arts (ELA) and 11.2 % in mathematics. At the same time, from 1999 to 2011, the gap between economically disadvantaged students and their peers also narrowed by 4.4 % in ELA and 5.5 % in mathematics (Louisiana Department of Education 2011).

Louisiana and other states have shown that poor students and minority students can perform well above norms and that the achievement gap can be narrowed if the appropriate instruction, curriculum, and resources are provided. Minority and low income students in these states have made strides in narrowing achievement gaps and attaining the proficiency level that exceeds the averages in their states (Clark, 2014).

Special Education Students

Special Education students are students with disabilities who may need additional support and assistance to reach their maximum potential and become successful in the general curriculum. Disabilities and impairments can be physical, medical, cognitive, emotional, behavioral, social or learning disabilities. Each student has his or her own Individualized Education Program (IEP) wherein learning goals and objectives as well as educational placement and exit documents are documented. Each IEP must have goals

and objectives that are in accordance with the curriculum used. A sample IEP is provided in Appendix B.

An article by C. Samuels (2012) showed the need to link IEPs of students with disabilities to Common Core Standards. “Standards-based” IEPs allow individualized instruction in pursuit of a common goal: helping students with disabilities move toward meeting the same grade-level academic standards that general education students are supposed to meet. In order to write a CCSS-aligned IEP, Special Education teachers must have a good knowledge and familiarity of CCSS. They must be able to identify at which grade level the student is currently functioning and must be able to create a concrete plan for how to move the student toward the grade specific that academic goals of the CCSS.

Some students with disabilities are fully mainstreamed (no special education classes) in the regular education classes and several more are educated in an Inclusion setting. Inclusion is a placement wherein a student with a disability is registered in regular classes and with an opportunity to earn Carnegie Units and exit with a diploma with the services being provided by an Inclusion teacher or a paraprofessional. These students that are on track to receive a diploma would take regular classes and must participate in either the End-of-Course Tests or the Louisiana Alternative Assessment, or both.

A study by Paula Maccini and Joseph Calvin Gagnon on “Perceptions and Application of NCTM Standards by Special and General Education Teachers” showed

that special education teachers have problems following National Council of Teachers of Mathematics (NCTM) standards. NCTM found that

> "A majority of special education teachers indicated they had not heard of the NCTM Standards. Respondents reported teaching mostly basic skills/general math to secondary students with LD and ED, versus higher-level math, such as algebra and geometry. Teachers identified lack of adequate materials as a considerable barrier to successful implementation of activities based on the Standards." (Gagnon & Maccini, 2002)

Potential Gaps Recognized by Louisiana Department of Education and by East Baton Rouge Parish School System.

The Louisiana Department of Education and the East Baton Rouge Parish School System acknowledge several pre-requisite knowledge gaps that may exist for ninth grade Algebra 1 students based on the Grade 8 Common Core Math Standards expectations. It also emphasizes the importance that teachers diagnose their students' needs as part of the planning process. A table (captured from www.louisianabelieves.com, 2014) identifying the State Department of Education's recognized potential gaps is provided as Appendix C.

Every lesson in the EngageNY curriculum is accompanied with pacing guides. If teachers are tasked with diagnosing deficient skills and also re-teaching them, teachers must find the time in between classes to do so. Currently, interventions with struggling students are being provided individually or in groups through pulling them out of their physical education or other elective classes. However, this practice means the student sacrifices that day's lesson in that other subject.

Teachers must find a way to reteach critical skills and pre-requites while linking them to the current lessons within a class time. They must find means to incorporate these pre-requisites in the new topic. The use of Pre-Lessons as a bell ringer activity would allow students to review missing or overlooked skills. Pre-Lessons also afford the teachers the time to identify struggling students and provide immediate one-on-one intervention.

Interventions Used

A main goal of teaching is to deliver lessons in such a manner that the students will be able to retain and recall previous information. Figuratively, math skills can be likened to a building block. It is imperative that one must learn the fundamentals. Previous lessons and skills are a requirement in order for students to become successful on current or future tasks. However, every student learns differently. They learn using different learning styles (Visual, Auditory, and Kinesthetic) and at differing paces. Teachers need to consider these factors and try to accommodate every student. If a student is missing an essential skill or is struggling in the current lesson, teachers need to address this and provide assistance by using different strategies and finding what is appropriate.

Intervention is used to describe activities that provide additional support for students who are struggling to master their critical competencies introduced. In mathematics, several interventions have been used by educators. These are:

Peer-Guided Pause - During large-group math lectures, teachers can help students to retain more instructional content by incorporating brief Peer Guided Pause sessions

into lectures: (1) Students are trained to work in pairs. At one or more appropriate review points in a lecture period, the instructor directs students to pair up to work together for 4 minutes. (2) During each Peer Guided Pause, students are given a worksheet that contains one or more correctly completed word or number problems illustrating the math concept(s) currently being reviewed in the lecture. The sheet also contains several additional, similar problems that pairs of students must work cooperatively to complete, along with an answer key. (3) Student pairs are reminded to (a) monitor their understanding of the lesson concepts; (b) review the correctly solved math model problem; (c) work cooperatively on the additional problems, and (d) check their answers. (4) The teacher can direct student pairs to write their names on the practice sheets and collect the work as a convenient way to monitor student participation and understanding. (Hawkins, J., & Brady, M. P., 1994)

Small Group Instruction - A study by Webb, N., 1991, showed that the experience of students during small group instruction can affect their learning. An ideal small group setting is one where students freely admit whether they do or do not understand the lesson. Immediate feedback is provided frequently, and lessons are conveyed at the level of language of students. (Webb, 1991)

Web-based Learning - Web based learning refers to the use of electronic media, computer software, online tools in education such as MyMathLab and MathXL. It is very useful as an aid in improving student performance. However, a study by Wadsworth, Husman, Mary Duggan and Pennington, 2007 showed that a negative relationship between self-testing and achievement. The results indicated that the more the participants used self-testing strategies, the lower their achievement scores. (Wadsworth,

Husman, Duggan, & Pennington, 2007). This study applies to classes that are fully online and trhe absence of teachers and professors to correct misconceptions showed an undesirable effect. In fact, an inssue in EBRPSS was reported in the news due to the massive cheatingf incidents that occurred wherein students completing e2020 program without reading and instead used web search engines to get the answers.

Assistive Technology **-** It is a legal requirement that if a student has been identified with a disability that he or she must be provided an assistive technology if deemed by the IEP committee to be necessary. It could be range from a low-level technology such as a calculator, to an innovative Augmentative/Alternative Communication. However, assistive technology is not a requirement and is not provided to all regular education students.

Manipulatives and Visual Representation – Many students who struggle with mathematics have difficulty understanding abstract symbols. Providing Math concepts by dramatizing problems using manipulatives allows better visual and discovery of these concepts. (Hecht, Vogi, & Torguezen, 2007)

Most of these interventions are proven effective. But effective implementation for many of these strategies requires either separate class time or substantial costs used for the entire student population. The use of Pre-Lessons allows teachers to integrate an intervention without sacrificing time allocated for elective classes or costing thousands for the school district.

The Pre-Lessons accompanying this thesis can be used alongside other interventions. They can also act as an introduction to any of the above-listed interventions.

CHAPTER II
LESSON DESIGN

The Appendix contains a supplemental resource for Algebra 1 teachers and their students that addresses the ninth grade Algebra 1 foundational standards, thereby assisting in the review and mastery of pre-requisite skills essential to achieving target learning goals. The twenty-eight Pre-Lessons are attached in Appendix A.

During the first year of transition to CCSS, several teachers have encountered a difficulty in conveying coherent lessons without deviating from the rigor and mathematical complexity of the EngageNY curriculum. In order to become successful, they have to allot time to search for materials that would introduce the current lessons without resulting to an immediate confusion among their students; materials that would address essential skills and knowledge gaps. Linda Raush, Curriculum and Instructional Specialist from Roman Catholic Diocese of Baton Rouge stated that,

"A bank of problems addressing prerequisite skills and understandings for identified Common Core State Standards could be helpful to math teachers as they prepare Algebra I lessons."

Before designing the Pre-Lessons, I studied every foundational and focus standard, along with the vocabulary and activities that accompany each lesson in Module 1 of the EngageNY Curriculum. These have been compiled as a workbook composed of twenty-eight (28) Pre-Lessons. The Pre-Lessons are clustered per Topic. Each Topic is preceded by a list of foundational vocabulary and focus vocabulary. Each Pre-Lesson is

systematically structured to begin at the Build-Up! Warm-Up! Phase and carry through to the Discovery phase.

The advantage of using these Pre-Lessons is that they are composed of assignments and exercises based on the foundational standards that the Eureka Math writers themselves have enumerated as well as the potential gaps that the Louisiana State Department of Education has already identified. Furthermore, if the Pre-Lessons are used as described, teachers can use these materials as an anticipatory set or as bell-ringers that allow students to review previous skills while teachers use the time to identify struggling students. Teachers can then provide redirections and corrections to a student's misconceptions as well as provide one-on-one intervention to a struggling student. An Algebra 1 teacher from EBRPSS reviewed that, "The use of Pre-Lessons can prepare my students to handle the rigor of CCSS-aligned lessons."

The use of the Pre-Lessons may also reduce the stress of students intimidated by the rigor and design of the new CCSS-aligned curriculum. Moreover, the special education students could benefit from the Pre-Lessons because they (1) accommodate students and modify the lessons (2) address a Learning Disability in Mathematics Problem Solving (3) address the CCSS-aligned short-term objectives and eventually generalize the current IEP goal. The observation of the chairman of Broadmoor High's Special Education Department was:

> "Working with inclusion students who have specific academic cognitive disabilities is difficult enough. They struggle with word problems. They have difficulties using the information to set up and solve equations. Your worksheets are thoughtful in their design and transmission of information. The

definition, cues, hints, prompts, and graphic organizer definitely would assist the students in solving Algebraic equations. It is good that it includes real life connections and something that students can relate to."

Most of the time, struggling students are identified after a graded assessment. Subsequently, interventions are then provided to these struggling students. But a struggling student who receives a low grade usually becomes a frustrated student. A frustrated student may either realize his mistakes and become challenged or hate the subject and give-up. Pre-Lessons can help teachers avoid this scenario. Pre-Lessons can help teachers identify the struggling student before even beginning the current graded lessons.

Pre-Lessons may not have a revolutionary impact in education. It may not result to scoring mastery or advance in the statewide examinations. Nevertheless, if one, or two or even three struggling students improved their perception in math as well as equipping them to have a fighting chance in understanding, comprehending and passing the lessons in the new Common Core aligned Curriculum, for a teacher, it has resulted to a significant difference. The teaching profession already requires multi-tasking. The use of Pre-Lessons will save teachers the time they would devote to finding materials. It would also assure that accommodations and modifications required for students with disabilities are being provided along with the achievement of goals and objectives in their IEP that are CCSS-aligned.

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APPENDIX A: PRE-LESSONS TO MODULE 1 OF ALGEBRA 1 OF EUREKA MATH

Topic A: Introduction to Functions Studied This Year—Graphing Stories

- Pre-Lesson 1: Graphs of Piecewise Linear Functions
- Pre-Lesson 2: Graphs of Quadratic Functions
- Pre-Lesson 3: Graphs of Exponential Functions
- Pre-Lesson 4: Analyzing Graphs
- Pre-Lesson 5: Two Graphic Stories

Topic B: The Structure of Expressions

- Pre-Lesson 6: Algebraic Expressions—The Distributive Property
- Pre-Lesson 7: Algebraic Expressions—The Commutative and Associative Properties
- Pre-Lesson 8: Adding and Subtracting Polynomials
- Pre-Lesson 9: Multiplying Polynomials

Topic C: Solving Equations and Inequalities

- Pre-Lesson 10: True and False Equations
- Pre-Lesson 11: Solution Sets for Equations and Inequalities
- Pre-Lesson 12: Solving Equations
- Pre-Lesson 13: Some Potential Dangers When Solving Equations
- Pre-Lesson 14: Solving Inequalities
- Pre-Lesson 15: Solution Sets of Two or More Equations (or Inequalities) Joined by "And" or "Or"
- Pre-Lesson 16: Solving and Graphing Inequalities Joined by "And" or "Or"
- Pre-Lesson 17: Equations Involving Factored Expressions
- Pre-Lesson 18: Equations Involving a Variable Expression in the Denominator
- Pre-Lesson 19: Rearranging Formulas
- Pre-Lesson 20: Solution Sets to Equations with Two Variables
- Pre-Lesson 21: Solution Sets to Inequalities with Two Variables
- Pre-Lesson 22: Solution Sets to Simultaneous Equations
- Pre-Lesson 23: Solution Sets to Simultaneous Equations
- Pre-Lesson 24: Applications of Systems of Equations and Inequalities

Topic D: Creating Equations to Solve Problems

- Pre-Lesson 25: Solving Problems in Two Ways—Rules and Algebra
- Pre-Lesson 26: Recursive Challenge Problem—The Double and Add 5 Game
- Pre-Lesson 27: Recursive Challenge Problem—The Double and Add 5 Game
- Pre-Lesson 28: Federal Income Tax

Pre-Lessons for Topic A
Introduction to Functions Studied This Year—Graphing Stories

Topic A introduces to students the main functions that they will work with in Grade 9: piecewise linear, quadratic, and exponential. These are introduced by having them make graphs of a situation in which these functions naturally arise. The lessons of Topic A are:

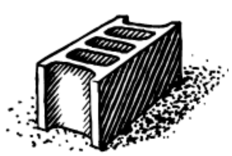
1. Graphs of Piecewise Linear Functions
2. Graphs of Quadratic Functions
3. Graphs of Exponential Functions
4. Analyzing Graphs
5. Two Graphic Stories

The Pre-Lessons to Topic A address necessary skills for students to be able to follow the lessons in Topic A. The Pre-Lessons are preceded with a list of foundational vocabulary and focus vocabulary for Topic A. The individual Pre-Lessons are composed of build up/warm up skills, and individual discovery stages of each lesson.

The Pre-Lessons to Topic A focus on the review of functions. Students should understand that:

(a) A function is a rule that assigns to each input exactly one output;
(b) The graph of a function is the set of ordered pairs consisting of an input and the corresponding output;
(c) Properties of two functions may be represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions);
(d) The functional relationship between two quantities can be described qualitatively by analyzing a graph.

Pre-Lesson 1 reviews the relationship of the table of values and graphs so students would understand "graphing stories" that lead to piecewise linear functions. Pre-Lesson 2 reviews x and y intercepts, and perfect square rational numbers. Pre-Lesson 3 introduces real number exponents and variable exponents. Pre-Lessons 4 and 5 are focused on identifying correlation of two quantities (positive, negative and no correlation), and patterns of association between two quantities.



FOUNDATIONAL STANDARDS

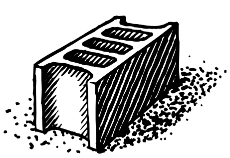
The following skills are necessary for students to be successful in mastering the five Topic A lessons.

- ❖ 8.EE.1 — Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
- ❖ 8.EE.7.a — Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).

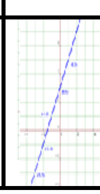
- 8.EE.8 Analyze and solve pairs of simultaneous linear equations.
- 8.EE.8.a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
- 8.EE.8.b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.
- 8.EE.8.c Solve real-world and mathematical problems leading to two linear equations in two variables.

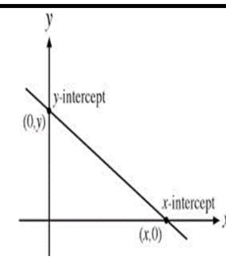
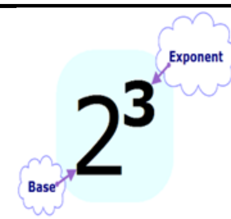
Based on my experience, the following standards are also necessary pre-requisites that need to be addressed by the teacher to prepare students for the Topic A lessons.

- 8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- 8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
- 8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- 8.F.3 Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.
- 8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- 8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.



FOUNDATIONAL VOCABULARY

TERMS	DEFINITION	EXAMPLES (*but not limited to*):
Function	*A function is a rule which assigns to each* ***inputs (independent variable)*** *exactly* ***one output (dependent variables)****. The set of inputs is called the* ***domain*** *and the set of outputs is called the* ***range.***	 

Linear Function	*A function of the form y=mx+b is called linear if m, b are given real numbers and if the domain of the variable x is a subset of the real number line.*	
Equation in One Variable	*An equation is an equality (formula) of the form A=B, where A and B are expressions that may contain one or several variables (unknown) and constants*	$x+1=5-2x$
Linear Equation in One Variable	*An equation is linear if it can be written in the form ax+b=c; where a,b,c are given real numbers and x is a real quantity to be derived.*	$5=3x+2$
Systems of Linear Equation in Two Variables	*A system of two equations is called linear if it can be rewritten in the form ax+by=e, cx+dy=f, where a,b,c,d,e,f are given real numbers and x,y, are unknown numbers to be derived.*	$x + y = 6x + y$ $-3x + y = 2-y$
x-intercept of a linear function	*The value of x for which y=mx+b=0.*	
y-intercept of a linear function	*If y=mx+b, then b is the y intercept of the function; that is the value of y when x=0.*	
Positive Integer Exponents	*The positive integer exponent of a number (base) says how many times to you multiply that number to itself.*	
Non-Linear Function	*A function that cannot be rewritten in the form y=mx+b is called non-linear.*	

FOCUS STANDARDS

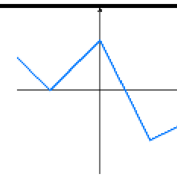
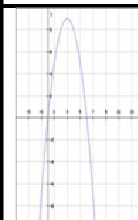
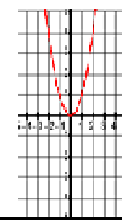
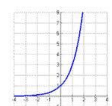
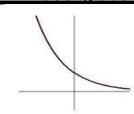
Upon completion of Topic A, students will be able to:

- ❖ N.Q .1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- ❖ N.Q .2 Define appropriate quantities for the purpose of descriptive modeling.

- ❖ N.Q .3 — Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
- ❖ A.CED.2 — Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

FOCUS VOCABULARY

(to be introduced/included in Topic A)

TERMS	**DEFINITION**	**EXAMPLES** ***(but not limited to)***
Piecewise Linear Function	*A piecewise linear function is a function composed of at least two linear functions with non-intersecting domains.*	
Quadratic Functions	*A function of the form $f(x) = ax^2 + bx + c$, is called quadratic if a,b,c are given real numbers and $a \neq 0$. Its graph is called a* ***parabola.***	 
Exponential Functions	*A function of the form $y = ab^x$ is called exponential function if both a and b are greater than 0 and b is not equal to 1.*	 

STUDENT OBJECTIVES

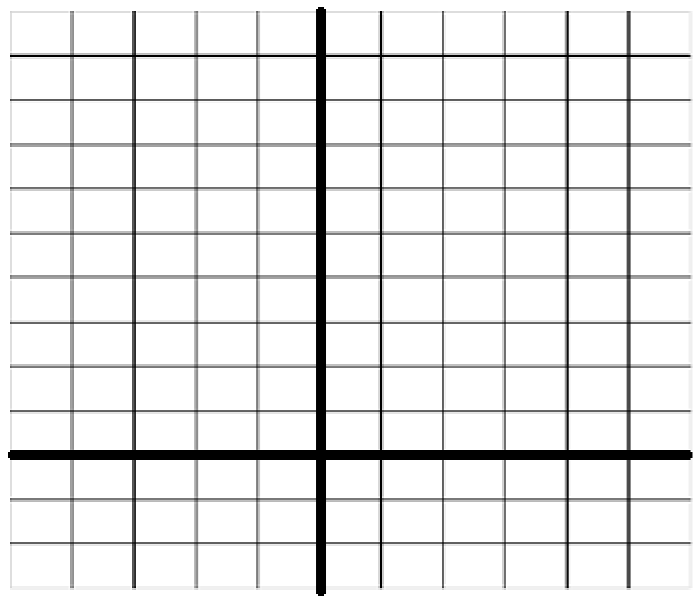
Upon completion of Pre-Lesson 1, I will be able to identify linear functions and its extended forms, tables, graphs, and interval notation.

BUILD UP! WARM UP!

8.F.1 8.F.3 8.F.4

1) (a) Graph the linear function for all real numbers x, given the following sample input-output table.

x	y
-1	3
0	5
1	7

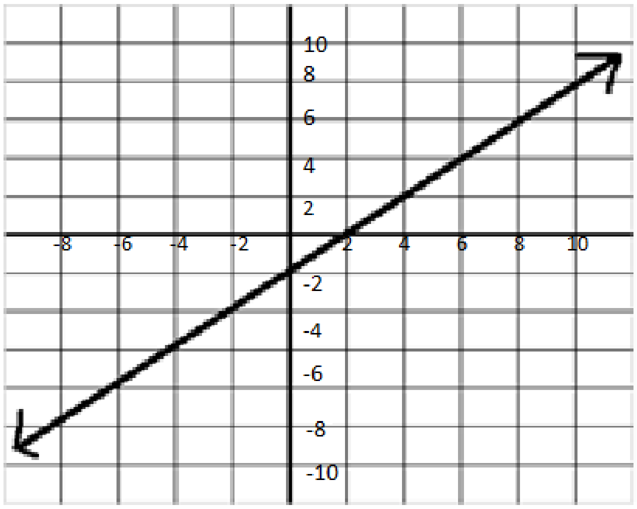


(b) Find the slope, the x intercept and the y-intercept.

(c) What is the linear function? That is, find m and b that y=mx+b.

8.F.1

2) (a) Given the graph of a linear function complete the input-output table.



x	y

HINT: y=mx+b

m = slope

b = y-intercept

(b) What is the linear function it represents?

8.F.4

3) Lucy doesn't like her new short hairstyle and wants to grow it back again. Hair grows at a constant rate of 1.25 cm per month. Write the equation for the function y that represent how many centimeters y her hair grew after x months. What can you say about the domain of the function?

Discovering Graphs of Piecewise Linear Functions

8.F.5 8.F.2

On New Year's Day, it snowed heavily the whole day at a constant rate of 1 cm every hour from 12 midnight to 12 midnight. Construct a graph illustrating the situation.

Graph here!

On a cold Martin Luther King Day, it snowed heavily from 8 am to 11 am. It started out at a constant rate of 1 cm/hour for the first hour, 2 cm/hour for the second hour, and 1 cm/hour for the third hour. It ceased to snow at 11 am. Construct a graph illustrating the situation.

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PREDICT: Would the graph look the same as the graph above?

Graph here!

Now, on a cold Mardi Gras Day, it snowed at a constant rate of 1 cm/hr from midnight until 6:00 am and it stopped snowing for 2 hours. But then it started to snow again, this time at a lighter constant rate of 0.5cm/hr until 6:00 pm. Construct a graph illustrating the situation.

PREDICT: Will the graph look the same as the previous graphs? How do you think the graph would like this time?

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Graph here!

Pre-Lesson 2 GRAPHS OF QUADRATIC FUNCTIONS

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 2, I will be able to (a) solve for intercepts (x and y intercepts), perfect square rational numbers (b) Know and apply the properties of integer exponents to generate equivalent numerical expressions (c) Describe qualitatively the functional relationship between two quantities by analyzing a graph.

BUILD UP! WARM UP

8.EE1 1) Which set of values contain three perfect squares?

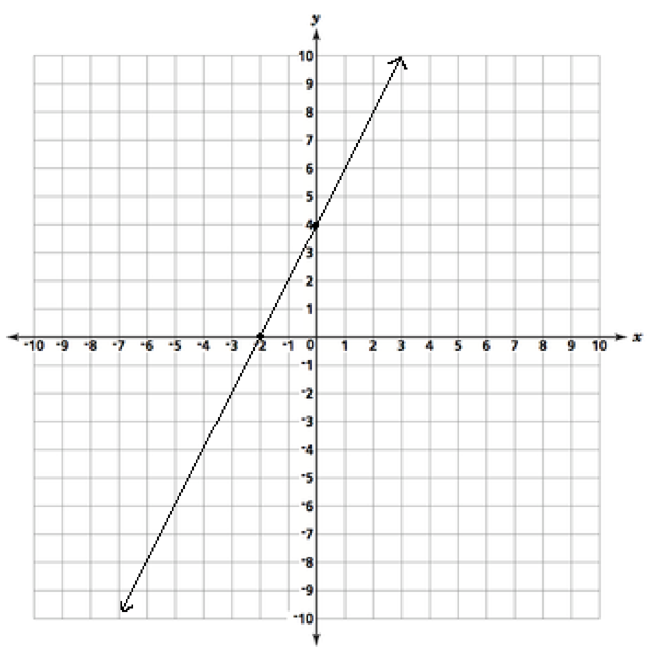
(a) {2, 4, 6} (b) {9, 18, 81} (c) {16, 64, 144} (d) {25, 36, 48}

8.EE.1 2) Square the following:

(a) $\frac{1}{4}$ (b) $\frac{-2}{3}$ (c) $\frac{5}{11}$ (d) 2.5

(e) 1.2 (f) 3.141 (g) $\sqrt{3}$ (h) $3^{2/3}$

3) Given the graph, find the x intercept and the y intercept.



4) Given the linear function y=3x+5, find the x intercept and the y intercept.

5) Given the table, find the x intercept and the y intercept.

x	0	1	2	3	4	5	6
y	6	4	2	0	-2	-4	-6

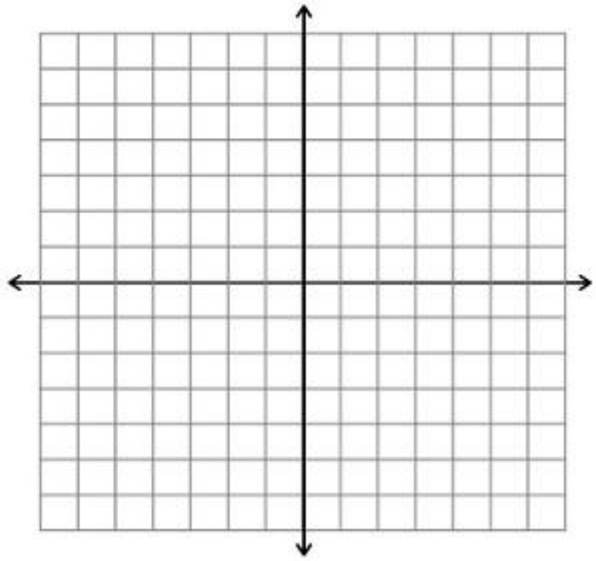
Discovering Graphs of Quadratic Functions

8.F.5

Complete the table

x	$f(x)=x^2$
-2	
-1	
0	
1	
2	

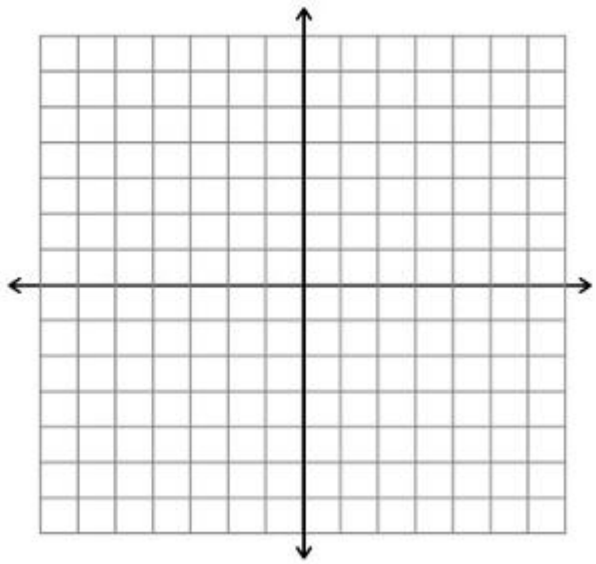
Graph the function $f(x) = x^2$ for all values x.



Now complete this table:

x	$f(x)=-x^2$
-2	
-1	
0	
1	
2	

Graph the function $f(x) = -x^2$ for all values x.



Now compare the two graphs. What is the difference of between the two? Do they look the same?

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Pre-Lesson 3 GRAPHS OF EXPONENTIAL FUNCTIONS

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 3, I will be able to solve real number exponents and variable exponents.

BUILD UP! WARM UP!

8.EE.1 1) Simplify: $\frac{2^4}{2^5}$

8.EE.1 2) Find the missing exponent so that the resulting numerical expressions are equivalent.

(a) $5^x = 125$ (b) $4^x=16$ (c) $(-3)^x=9$ (d) $\frac{1}{3^{-x}} = 9$

8.EE.1 3) Find the value of y given x=2.

(a) $y=3^x$ (b) $y= (\frac{1}{2})^x$ (c) $\frac{-1^x}{3}$ (d) $y=2(1.1)^x$

8.EE.1 4) What is the value of x in the equation $2^{2x+1}=\sqrt{2}$?

8.EE.1 5) Complete the table by filling in the positive values of $y=2^x+5$.

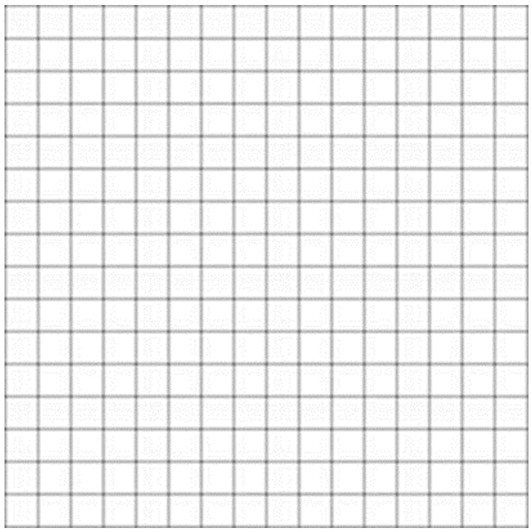
x	-2	-1	0	1	4
y					

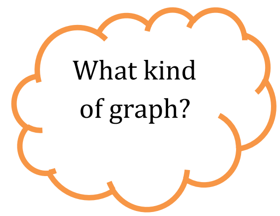
Discovering Exponential Functions

8.F.5 8.F.2

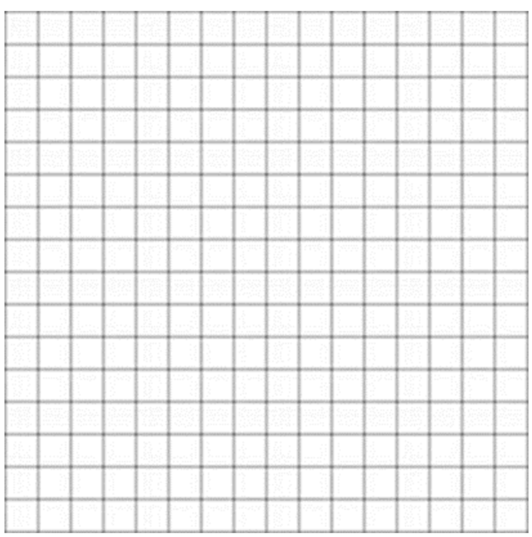
Ruel received an inheritance of $10,000.00 from a rich relative. He wants to place it in a special bank account that would yield the highest interest. He went to two banks and asked for their individual interest rates.

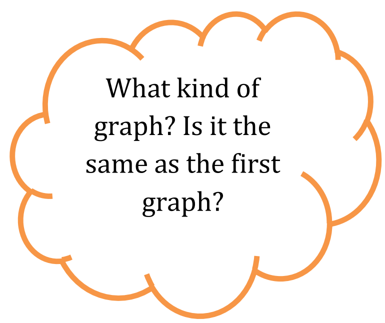
Bank A suggested to put it in a Certificate of Time Deposit and their interest rate is at 5% annually but not compounded. How much total money will Ruel have after 4 years? Solve and illustrate in the graph.





Bank B offered him an interest rate of 4% that is **compounded** annually. How much total money will Ruel have after 4 years? Solve and illustrate in the graph.





Which bank offered him the best deal?

Pre-Lesson 4	ANALYZING GRAPHS

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 4, I will be able to identify correlation of two quantities (positive, negative and no correlation)and patterns of association between two quantities.

BUILD UP! WARM UP!

8.SP.1 1) Christian opened three (3) new restaurants. He collected data for the first 10 days about the number of people visiting and the total sales. The following table shows his results.

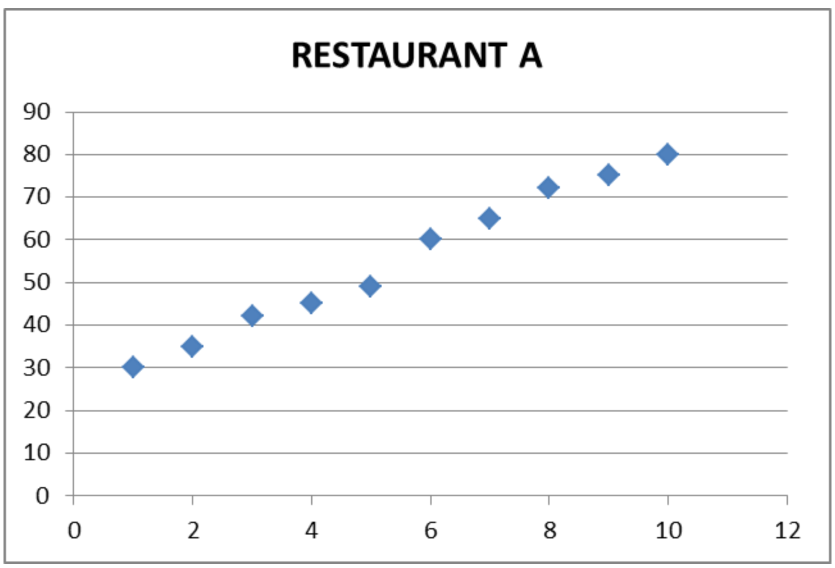
(a) Identify the correlation in each graph (positive, negative or no correlation).

(b) Between Restaurant A and B, which location do you think is better?

(c) What can you conclude on the relationship (increase or decrease) between the number of visitors and the sales in Restaurants A and B?

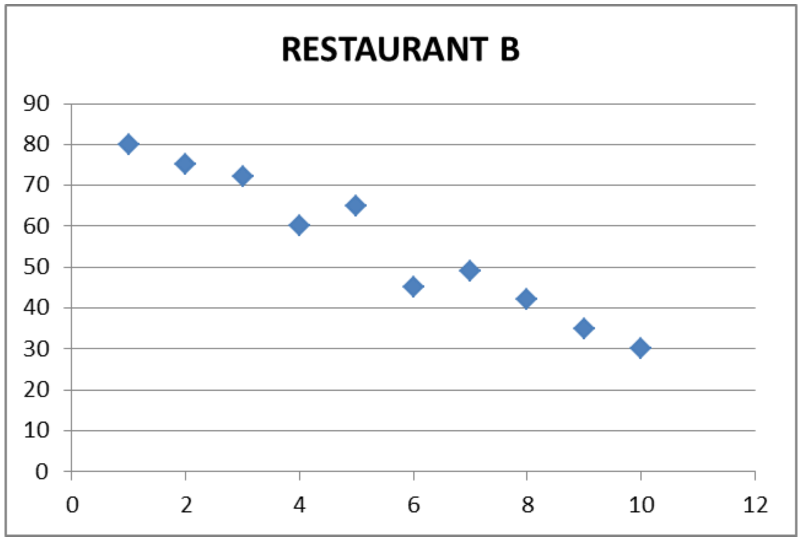
Restaurant A

DAY	# OF VISITORS
1	30
2	35
3	42
4	45
5	49
6	60
7	65
8	72
9	75
10	80



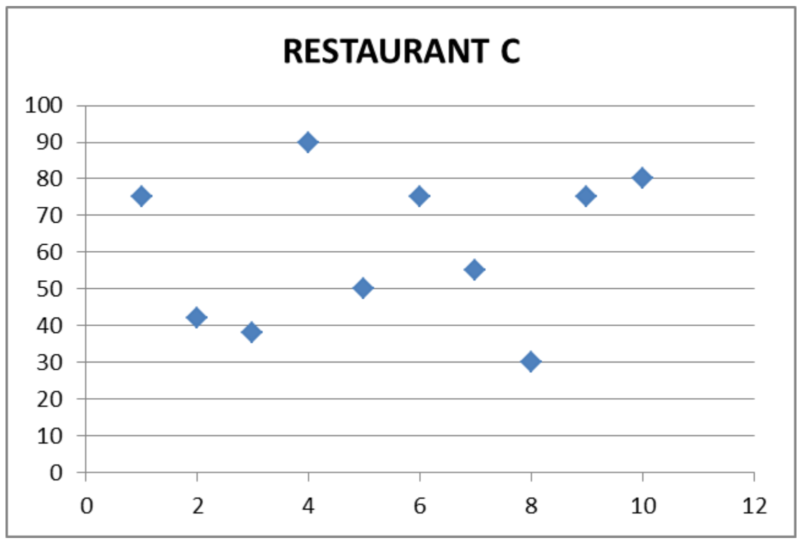
Restaurant B

DAY	# OF VISITORS
1	80
2	75
3	72
4	60
5	65
6	45
7	49
8	42
9	35
10	30



Restaurant C

DAY	# OF VISITORS
1	75
2	42
3	38
4	90
5	50
6	75
7	55
8	30
9	75
10	80



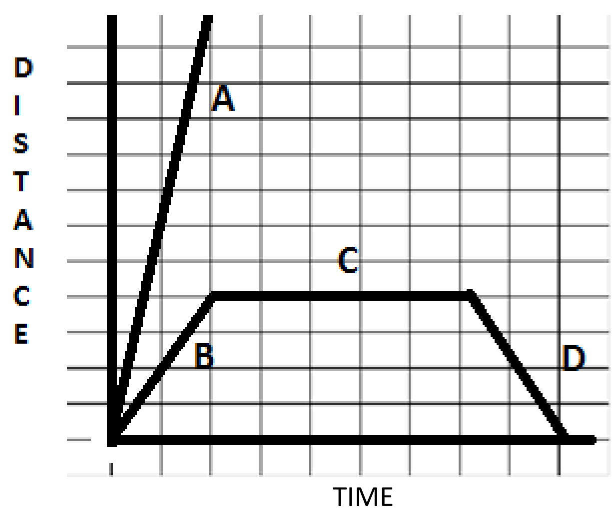
8.F.2 2) Raffy is preparing for a marathon. Match the following preparation activities to the graph:

(a) slow steady speed

(b) stationary

(c) Returning to start

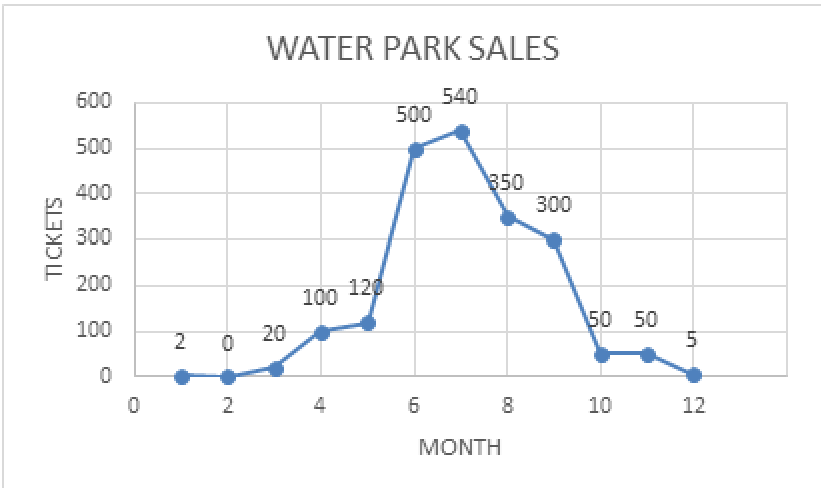
(d) Fast steady speed



Discovering How to Analyze Graphs

8.F.2

The graph below shows the total number of tickets sold at a water park that is open year round. Use the graph to answer the following questions:



What is the **highest** amount of tickets sold? At which three months did it sell the highest?

What is the **lowest** amount of tickets sold? At which three months did it sell the lowest?

What do you think are the factors that could affect the ticket sales?

Pre-Lesson 5 TWO GRAPHIC STORIES

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 5, I will be able to solve systems of equation.

BUILD UP! WARM UP!

8.EE.7.a

1) If the Perimeter of the Figure A is equal to the Perimeter of Figure B, what is the value of x?

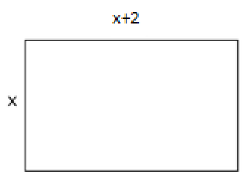


FIGURE A

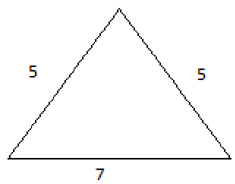
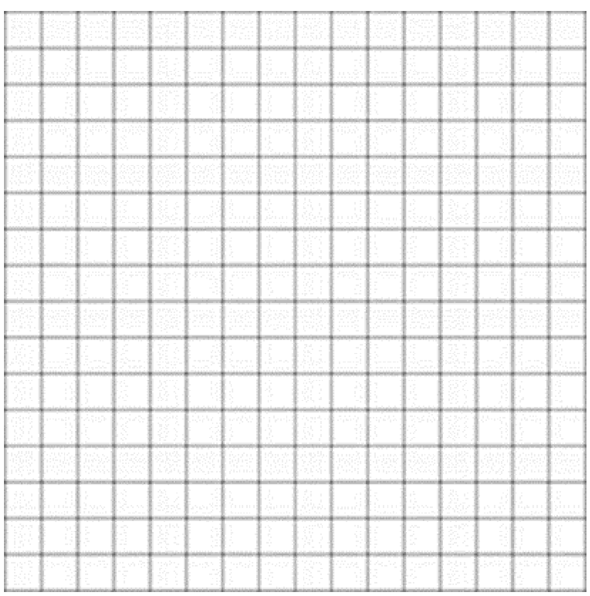


FIGURE B

8.EE.8.a

2) Solve for the solution of the system of equations by graphing. 2y+x=6
y-x=2



8.EE.8 3) How many solutions does the system of equations below have?

$y=-4x+3$ and $8x + 2y = 6$

8.EE.8.c 4) Kristi will be hosting a fashion show. She needed accessories for her models. She ordered purses and shoes online. She bought a total of 27 items. Each purse costs \$16.50 and each shoes costs \$9.25. Kristi spent a total of \$358.50. How many of each did Kristi purchase?

HINT:

Assign one variable for shoes and one for purse.

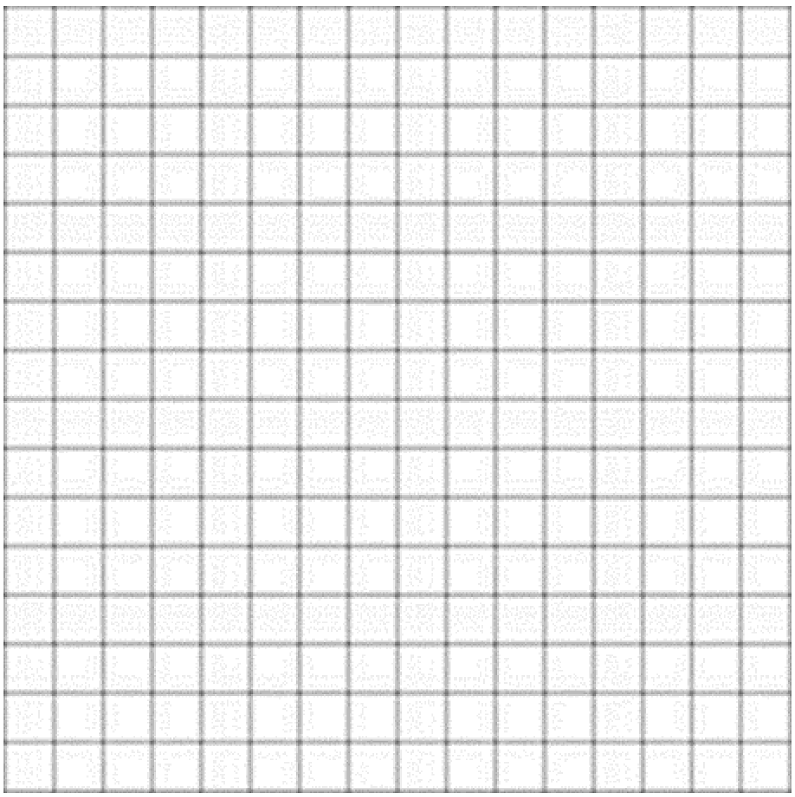
Discovering Two Graphic Stories

8.F.2 8.EE..B

Joseph bought a scented candle. Each minute, the candle burns down the same amount.

x=number of minutes elapsed
y= the height of the candle

(a) Sketch a graph to model the situation given.



HINT: Just estimate!

(b) Libby, bought a soy based scented candle that has the same height as Joseph's candle. It burns twice as fast as the regular scented candle. Now, on the same graph, sketch the situation.

x=number of minutes elapsed
y= the height of the candle

(c) When lit at the same time, will the two candles ever have the same height again?

Pre-Lessons for Topic B
The Structure of Expressions

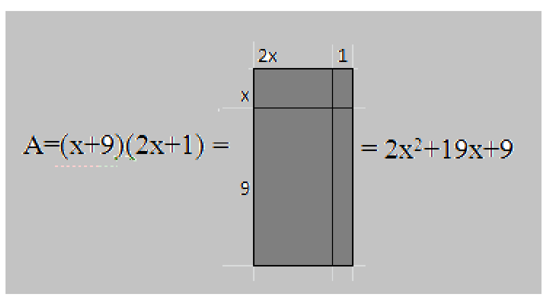
Topic B introduces the fact that arithmetic and algebraic expressions can be written in different but equivalent forms with different structures. It also introduces polynomials and performing operations such as their addition, subtraction and multiplication.

The Pre-Lessons to Topic B address necessary skills for students to become successful in the lessons in Topic B. The Pre-Lessons are preceded with a list of foundational vocabulary and focus vocabulary for Topic B. The individual Pre-Lessons are composed of build up/warm up skills, and individual discovery stages of each lesson. The lessons of Topic B are:

6. Algebraic Expressions—The Distributive Property
7. Algebraic Expressions—The Commutative and Associative Properties
8: Adding and Subtracting Polynomials
9: Multiplying Polynomials

In Lessons 6 and 7, students develop an understanding of what it means for different expressions to be algebraically equivalent. In order for them to do so, Pre-Lessons 6 and 7 are designed to help the students gradually understand how addition, subtraction, multiplication and the distributive property can be used to rewrite algebraic expressions in equivalent forms, emphasizing the difference between purely arithmetic and algebraic equations containing at least one variable. Students also learn to identify whether an equation (equality) is true or false.

In Lessons 8 and 9, students learn to add, subtract, and multiply polynomials and should comprehend that the polynomials form a system that is closed under those operations; That is a polynomial added to, subtracted from, or multiplied by another polynomial always produces another polynomial. Lesson 9 introduces the area model to perform multiplication of polynomials; i.e.



Pre-Lesson 8 focuses on combining like terms as addition and subtraction of polynomials requires students to identify and combine like terms. Pre-Lesson 9 introduces multiplication of polynomials by: (a) translating word problems into algebraic expressions; and (b) completing area problems of a rectangle.

FOUNDATIONAL STANDARDS

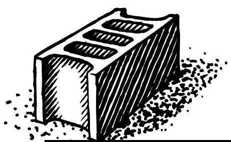
The following skills are necessary for students to be prepared for these lessons:

- **6.EE.3** Apply the properties of operations to generate equivalent expressions.

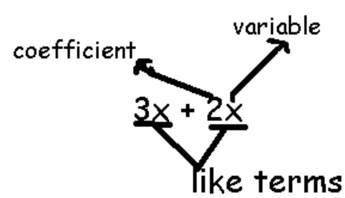
- ❖ **6.EE.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them)
- ❖ **7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Based on experience, the following standard is also a necessary pre-requisite that needs to be addressed by the teacher to prepare students for the lessons that follow:

- ❖ **8.EE.**1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.



FOUNDATIONAL VOCABULARY

TERMS	**DEFINITION**	**EXAMPLES** **(*but not limited to)***
Expression	[1] *An expression is a finite combination of symbols (numbers/constants, variables, operations, functions, punctuation, grouping, and other aspects of logical syntax) that is well-formed according to rules that depend on the context. Mathematical symbols can designate (constants), variables, operations, functions, punctuation, grouping, and other aspects of logical syntax.*	$y(3x + 2)^2$
Numerical Expression	[2]*A numerical expression is a mathematical phrase involving only numbers and one or more operational symbols*	$(4+5)^2 + 2$
Equivalent Numerical Expression	*Two numerical expressions are equivalent if they evaluate to the same number.*	$(4+5)^2 + 2 = (5+2)^2+34$
Term	*A number (constant), a variable or a product of a number and a variable.*	
Like Terms	*Like terms are terms whose variables and their exponents are the same.*	
Coefficient	*A coefficient is a number (constant) used to multiply a variable.*	
Variable	*A variable is a quantity that may assume a set of values.*	

[1] http://en.wikipedia.org/wiki/Expression_(mathematics)
[2] http://www.icoachmath.com/math_dictionary/numerical_expression.html

Polynomial	[3] *A polynomial is an expression consisting of variables and coefficients that involve only the operations of addition, subtraction, multiplication, and non-negative integer exponents.*	x^2+6x+3 not $\frac{3}{x-5}$ or $6x^{-2}$
Monomial	*A polynomial with one term.*	$\mathbf{4x^2}$
Binomial	*A polynomial with two terms.*	$\mathbf{4x^7 + x^3}$
Trinomial	*A polynomial with three terms.*	$\mathbf{2x^2+6x+3}$

FOCUS STANDARDS

"Focus Standards" is a list of standards introduced in a Topic in the EngageNY Curriculum. Topic B presents the following standards to the students:

- A.SSE.2 Use the structure of an expression to identify ways to rewrite it.
- A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

FOCUS VOCABULARY

TERMS	**DEFINITION**	**EXAMPLES** **(*but not limited to)***
Distributive Property	*The distributive property lets you multiply a sum by multiplying each addend individually and then add the products.*	$a(b+c) = ab + ac$
Commutative Law	*The "Commutative Laws" say we can swap numbers over (keeping their signs) and still get the same answer*	$4+5+3 = 3+4+5$ $4\bullet5\bullet(-3) = (-3)\bullet5\bullet4$
Associative law	*The "Associative Laws" says that it does not matter how we group the numbers*	$4+(5+(-3)) = (4+5)-3$ $4\bullet(5\bullet(-3)) = (4\bullet5)\bullet(-3)$

[3] http://en.wikipedia.org/wiki/Polynomial

Pre-Lesson 6 ALGEBRAIC EXPRESSIONS (DISTRIBUTIVE PROPERTY)

STUDENT OBJECTIVES

Upon completion of this Pre-Lesson, I will be able to (a) apply the properties of operations to generate equivalent expressions; (b) identify whether a given equation (equality) is true or false.

BUILD UP! WARM UP!

6.EE.3 7.EE.1 1) Use the order of operations or the distributive property to rewrite and/or evaluate each expression.

a) 5(12+3)

b) 3(6-8)

c) 5(a+9)

d) 4 (4x + 5)

e) -6 + 9y + 5 – 7y

f) 5 – 8x + 9x

g) $\frac{4^3}{4^5} \cdot 2^6$

6.EE.4 2) Verify if the following statements are true.

a) $3(6 + 4) \stackrel{?}{=} 30$

8) $-2(2 - 3) \stackrel{?}{=} -8$

6.EE.4 3) Select all the equations that are equivalent to 2 (9x + 4) = 20 and show your work.

a) 18x+8=20

b) 18x=12

c) x=12/18

d) x=2/3

6.EE.4 4) Select all the equations that is equivalent to 8 (x – 3) = -8 and show your work.

a) 8x-3=-8 b) 8x-24=-8 c) 8x-24=-64 d) x=2

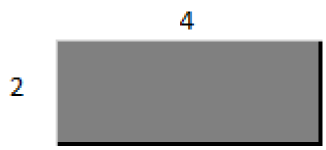
6.EE.4 5) Match equivalent expressions in Column A and B:

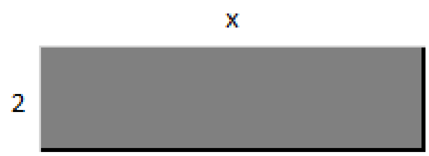
A	B
a) 2 + x	a) 10 +2y
b) 2(5 +y)	b) 2 (8y + 1)
c) 25	c) x + 2
d) 16y + 2	d) 5(3+2)

Discovering Algebraic Expressions (Distributive Property)

Find the area of the given rectangles.

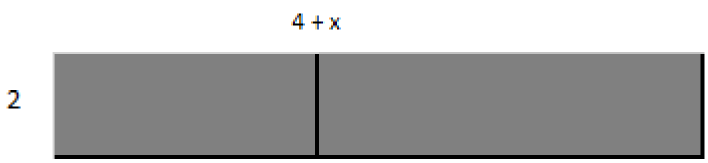
PART A





PART B

Suppose I combine the two figures, what would be the area?



Would the area from Part A and Part B be equal? Why?

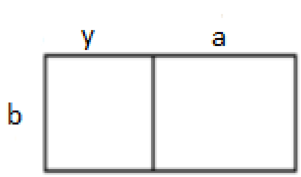
____

____

____

____

Write the area of the whole rectangle in two different ways.



Area as a Product	Area as a Sum

PRE-LESSON 7 ALGEBRAIC EXPRESSIONS (Commutative & Associative Properties)

STUDENT OBJECTIVES

Upon completion of this Pre-Lesson, I will be able to: (a) identify a numerical expression (b) rewrite algebraic expressions in equivalent forms.

BUILD UP! WARM UP!

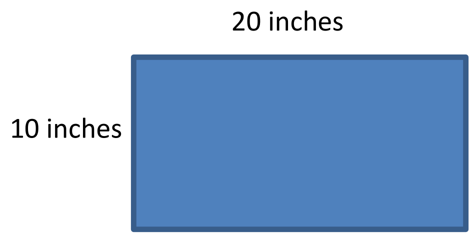
6.EE.4 1) The solution of the equation 4x + 2x= 18 is x=3. Write another equation of the form ax+bx=18 that also has a solution of x=3.

6.EE.4 2) Ms. B asked the class, when can we say that x + 5 = 5 + y? Sophie said that it can happen **only** if x and y are both equal to zero. Ingrid argued that it will happen **<u>every time</u>** both x and y have the same values. Who is correct? Sophie, Ingrid, or both? Explain.

6.EE.4 3) Arya was lifting weights at the gym. On one end of the lifting bar she placed a 5 pound plate then a x pound plate. On the other end, she placed a x pound plate then a 5 pound plate. Does the weight lifting bar have equal weights on both sides? Why or why not?

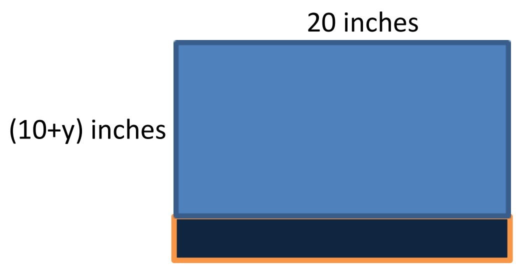
7.EE.1

4) Ryan was wrapping a toy train as birthday gift for his son. The gift wrapper he used for the toy had the following dimensions:



the total area is:
200 sq. inches

Now, Ryan didn't realize the train had a handle. He needs to **extend** the wrapper y inches.



Area of a Rectangle

=

length * width

Select an expression/s that represents the **area of the new** wrapper?

a) (10+y)20 b) 200 + y

b) 200 + 20y d) 200y

Discovering Commutative & Associative Properties

Ray put on his right shoe first and then his left shoe.

Would it be the same if Ray put on his left shoe first and then the right shoe? Why or why not?

Now, Ray wants to brush his teeth. He put toothpaste on his toothbrush and then brushed his teeth.

Would it be the same if Ray brushed his teeth first and then put some toothpaste on his toothbrush?

When adding 4 + 5 + 6, would Ray get the same answer if he adds it as 6 + 5 + 4, or 4+6+5, or 5+4+6?

When multiplying 4*5*6, would Ray get the same answer if he multiplies it as 6*5*4, or 4*6*5, or 5*4*6?

Now Ray drove to Orlando, Florida to go to Disney World. He stopped several times to rest, eat and fill his car with gas. On his first stop, Ray filled his gas tank and it cost him $48.76. He also bought some chicken and paid $5.69. On his second stop, Ray paid 26.83 for gas and $9.99 for a lunch buffet.

How much money did he spend for gas?

How much money did he pay for food?

How much money did he spend total?

When Ray was in Disney World, he met with his cousin, Florence. Ray paid for his ticket as well as his cousin's. The ticket costs $65 each. However, Florence has a coupon and they get a discount of 10% for each ticket.

Write an expression to represent the total expense for tickets?

If Ray would buy his ticket separately, and then Florence's, would he pay the same price? Why?

PRE-LESSON 8 ADDING AND SUBTRACTING POLYNOMIALS

STUDENT OBJECTIVES

Upon completion of this Pre-Lesson, I will be able to (a) combine like terms; (b) apply properties of operations as strategies to add and/or subtract linear expressions with rational coefficients (c) translating given word problems to algebraic expressions.

BUILD UP! WARM UP!

7.EE.1 8.EE.1 1) Simplify the following expressions.

a) $2(4x-5)-7x$

b) $2x+4x^2-6x+2x^2-3$

c) $3^2 \cdot 3^{-5}$

7.EE.1 2) Giselle is making bows for birthday giveaways. She needs to make x bows where each bow needs a 1.5 feet long ribbon and two bows for her dogs where each bow needs 0.6 feet long ribbon. How many yards of ribbon does Giselle need?

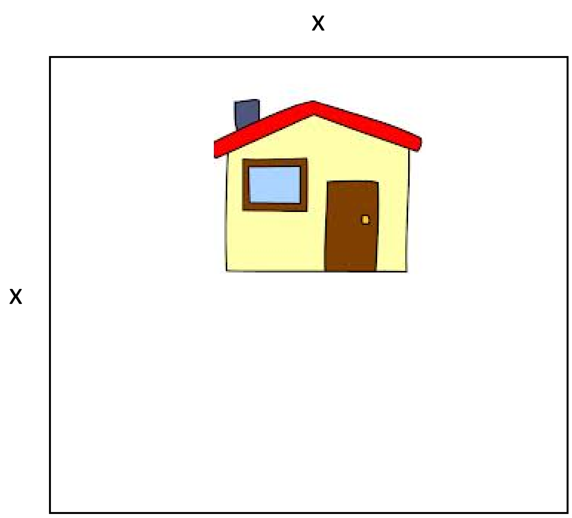
HINT
There are 3 feet in a yard.

7.EE.1 3) Santi drove the same distance for 2 days and 5 miles on the third day. Find the distance Santi drove in the three consecutive days.

DISCOVERING ADDING AND SUBTRACTING POLYNOMIALS

Nicole wants to have a fence around her house. The city records indicate that her land area has a square shape. Her husband wants to do the project himself. He will use a metal material for the side fences, and wooden materials for the front and back. The costs will be the following:

Metal:	\$30 per yard for Metal Fencing Material \$5 fixed cost of decorative materials for metal fence
Wooden:	\$20 per yard for Wooden Fencing Material \$2 fixed cost of decorative materials for wooden fence



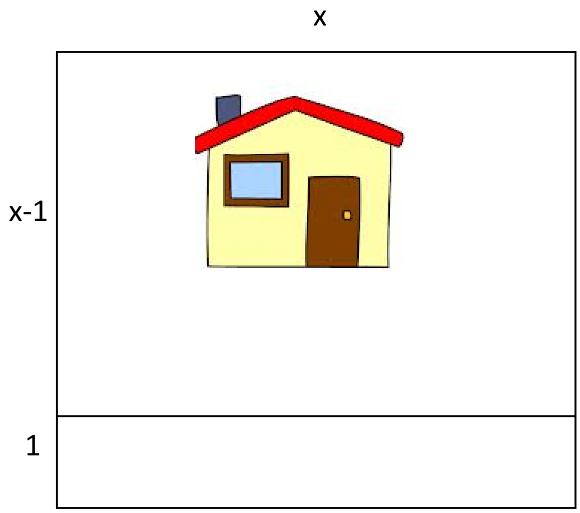
(a) Write an expression representing the cost of metal fencing?

(b) Write an expression representing the cost of wooden fencing?

(c) What is the total cost of fencing (metal and wooden) materials?

(d) Nicole's husband finds out from the Deed of Sale that the land area is 3600 sq. ft. What is the total cost for fencing?

Now, the city government wants to take a part of Nicole's front yard to turn it into a walkway path as illustrated below.



(a) What would be the new perimeter of the fence around Nicole's land?

(b) What would be the new cost of fencing (metal and wooden) materials?

PRE-LESSON 9 MULTIPLYING POLYNOMIALS

STUDENT OBJECTIVES

Upon completion of this Pre-Lesson, I will be able to (a) translating given word problems to algebraic expressions. (b)compute rectangle area (c) apply previous lessons on properties of equality (distributive, commutative and associative) to simplify expressions (d) Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

BUILD UP! WARM UP!

6.EE.3 1) Let an integer be represented by x. Find, in terms of x, the product of three consecutive integers starting with x.

8.EE.1 2) a) Write a variable expression for the area of a square whose sides are x + 8 units long.

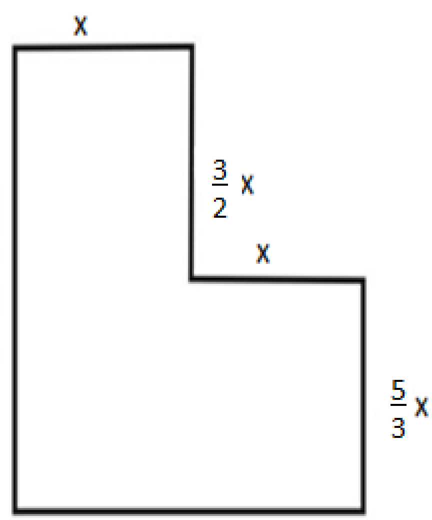
b) Expand by writing it as a trinomial.

6.EE.3 3) The perimeter of a square is 44 inches. What is the area of the square, in square inches?

7.EE.1 4) Given the figure on the right,

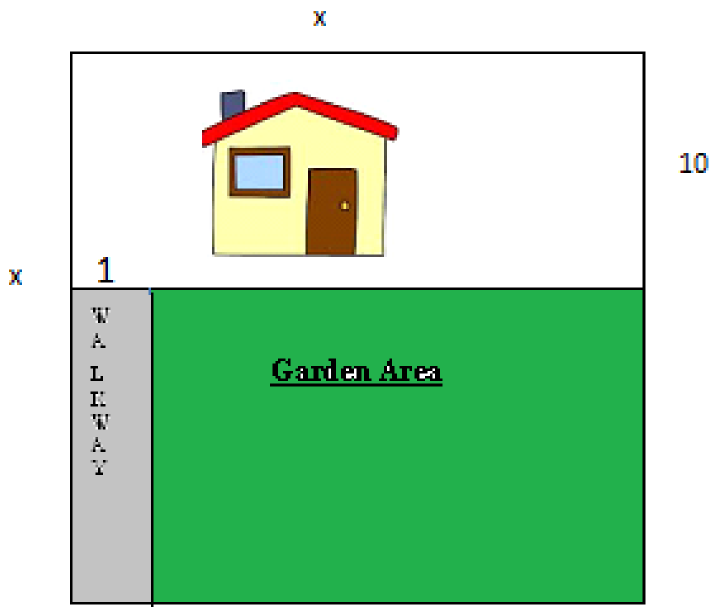
a) find the perimeter of the irregular shape,

b) find the area of the irregular shape.



DISCOVERING MULTIPLICATION OF POLYNOMIALS

Nicole wants to have a landscaping for her garden. A walkway was newly added in her front yard.



(a) What is the area of Nicole's garden?

(b) Express the area of Nicole's garden as a trinomial?

Pre-Lessons for Topic C
Solving Equations and Inequalities

Topic C requires students to apply the theories/concepts that are assumed to have been previously learned in middle school (i.e. variable, solution sets, etc.). They must be able to explain, justify, and evaluate their reasoning as they strategize methods for solving linear and non-linear equations (A-REI.1, A-REI.3, A-CED.4).

The Pre-Lessons to Topic C address necessary skills for students to become successful in the lessons in Topic C. The Pre-Lessons are preceded with a list of foundational vocabulary and focus vocabulary for Topic C. The individual Pre-Lessons are composed of build up/warm up skills, and individual discovery stages of each lesson. The lessons are:

10. True and False Equations
11. Solution Sets for Equations and Inequalities
12. Solving Equations
13. Some Potential Dangers when Solving Equations
14. Solving Inequalities
15. Solution Sets of Two or More Equations (or Inequalities) Joined by “And” or “Or”
16. Solving and Graphing Inequalities Joined by “And” or “Or”

17 Equations Involving Factored Expressions

18. Equations Involving a Variable Expression in the Denominator
19. Rearranging Formulas
20. Solution Sets to Equations with Two Variables
21. Solution Sets to Inequalities with Two Variables
22. Solution Sets to Simultaneous Equations
23. Solution Sets to Simultaneous Equations
24. Applications of Systems of Equations and Inequalities

In Lesson 10, students are taught to understand that an equation is a statement of equality between two expressions. When values are substituted for the variables in an equation, the equation is either true or false. Students find values to assign to the variables in equations that make the equations true statements. Pre-Lesson 10 then focuses on the use of substitution to determine whether a given number in a specified set makes an equation or inequality true.

In Lesson 11, *“students understand that an equation with variables is often viewed as a question asking for the set of values one can assign to the variables of the equation to make the equation a true statement. They see the equation as a “filter” that sifts through all numbers in the domain of the variables, sorting those numbers into two disjoint sets: the Solution Set and the set of numbers for which the equation is false.”* (EngageNY.org, 2014) In this lesson, students are also introduced to solutions to inequalities. Pre-Lesson 11 concentrates on improving student’s skills in:

(a) Substituting values into inequalities,
(b) Translating a given real world problem into inequality statements,
(c) Understanding solving an equation or inequality as a process of answering a question: “which values from a specified set, if any, make the equation or inequality true?”, and
(d) Interpreting statements of inequality as statements about the relative position of two numbers on a number line diagram.

In Lessons 12–14, "*students formalize descriptions of what they learned before (true/false equations, solution sets, identities, properties of equality, etc.) and learn how to explain the steps of solving equations to construct viable arguments to justify their solution methods. They then learn methods for solving inequalities, again by focusing on ways to preserve the (now infinite) solution sets*." (EngageNY.org, 2014) In Pre-Lesson 12, students will solve equations and justify steps using properties of equality. Pre-Lesson 13 addresses the potential misconceptions and common errors/mistakes when solving equations. Pre-Lesson 14 reviews how to:

(a) Translate graphs on the number line into inequality; and

(b) Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem.

In Lessons 15–18 *students investigate solution sets of equations joined by "and" or "or" and investigate ways to change an equation such as squaring both sides, which changes the solution set in a controlled (and often useful) way.*(EngageNY.org, 2014) Pre-Lessons 15-16 focus on the following:

(a) How an inequality can represent different situations;

(b) How to write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Students should learn to recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams, focus on how different inequalities can have the same solutions; and

(c) How to interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.

Pre-Lesson 17 focuses on finding the greatest common factor. Pre-Lesson 18 focuses on the following:

(a) Solving fractions with one denominator as a variable; and

(b) Applying and extending previous understandings of operations with fractions.

In Lesson 19, *"students learn to use these same skills as they rearrange formulas to define one quantity in terms of another".* Pre-Lesson 19 shows students that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Finally, *"in Lessons 20–24, students apply all of these new skills and understandings as they work through solving equations and inequalities with two variables including systems of such equations and inequalities."* (EngageNY.org, 2014) Pre-Lessons 20-21 centers on the students to understand solving an equation or inequality as a process of answering the following question: which values from a specified set, if any, make the equation or inequality true? They learn how to use substitution to determine whether a given number in a specified set makes an equation or inequality true. Pre-Lesson 22 focus on solving word problems leading to equations of the form $px + q = r$, where p, q, and r are specific rational numbers so that students may solve equations of these forms fluently. Pre-Lesson 23 is centered on finding the least common multiple of variable expressions. Pre-Lesson 24 is on the use of variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

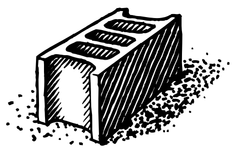
FOUNDATIONAL STANDARDS

The following skills are necessary for students to be successful in mastering the Topic A lessons.

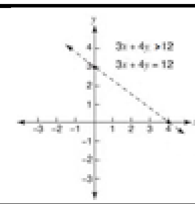
- 6.ee.5 — Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- 6.ee.6 — Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- 6.ee.7 — Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
- 6.ee.8 — Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
- 6.ns.7.a — Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
- 7.ee.2 — Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.
- 7.ee.3 — Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
- 7.ee.4 — Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- 7.ee.4.a — Solve word problems leading to equations of the form $px + q = r$ and $p(x + q)=r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
- 7.ee.4.b — Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.
- 8.EE.7 — Solve linear equations in one variable.
- 8.EE.7.a — Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
- 8.ee.7.b — Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Based on my experiences, the following standards are also necessary pre-requisites that need to be addressed by the teacher to prepare students for the Topic C Lessons.

- 6.ns.4 — Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2)
- 7.ns — Apply and extend previous understandings of operations with fractions.



FOUNDATIONAL VOCABULARY

TERMS	DEFINITION	EXAMPLES (*but not limited to*):
Algebraic Equation	[4]*In mathematics, an algebraic equation or polynomial equation is an equation of the form, P = Q, where P and Q are polynomials.*	$x^2+2x+1=0$
Boundary Line	[5] *A boundary line of an inequality is a line that separates the coordinate plane into half-planes*	
Greatest Common Factor (GCF)	[6]*The* ***Greatest Common Factor*** *(****GCF****) is the largest number/****expression*** *that divides into two or more* ***expressions*** *evenly.*	$3x^2 + 6x$ GCF is $3x$
Identity	*An identity is an equation that is always true.*	Identity Equations 7=7 3+5=5+3 3(2+5x)=6+15x
Inequality Statement	[7]*An Inequality statement is a statement that a mathematical expression is less than or greater than some other expression.*	$3x>5$ $2y\leq16$
Number Sentence	*A statement of equality between two numerical expressions. A number sentence is said to be true if both numerical expressions are equivalent (that is, both evaluate to the same number). It is said to be false otherwise.*	1+2=3 (True) 9+2=4+7 (True) 1=2 (False)

[4] http://en.wikipedia.org/wiki/Algebraic_equation

[5] http://www.mathchamber.com/algebra/docs/general/glossary.htm

[6] http://dmc122011.delmar.edu/math/MLC/Forms/finding_the_greatest_common_factor.pdf

[7] http://encyclopedia2.thefreedictionary.com/Inequality+(mathematics)

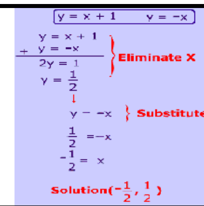
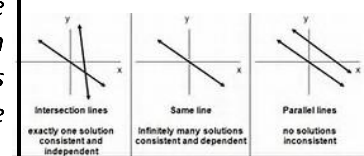
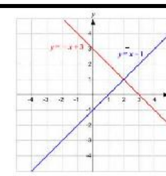
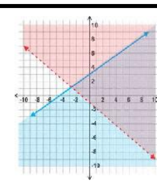
Least Common Multiple (LCM) of Expressions	*The LCM of given expression is the smallest expression that is divisible by each of the given expressions.*	Find the LCM of $4a^2b, 6ab, 8ab^2$. Solution: $4a^2b = 2^2 \times a^2 \times b$ $6ab = 2 \times 3 \times a \times b$ $8ab^2 = 2^3 \times a \times b^2$ $\therefore \text{LCM} = 2^3 \times 3 \times a^2 \times b^2$ $= 24a^2b^2$
Solution Set	*The solution set of an equation written with only one variable is the set of all values one can assign to that variable to make the equation a true statement. Any one of those values is said to be a solution to the equation. To solve an equation means to find the solution set for that equation.*	Solve x-4=0 $x - 4 = 0$ $+4 \quad +4$ $x = 4$ Then the solution is: x=4

FOCUS STANDARDS

Upon completion of Topic C, students will be able to:

- ❖ A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
- ❖ A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.
- ❖ A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- ❖ A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- ❖ A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- ❖ A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- ❖ A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- ❖ A.REI.12 Graph the solutions to a linear inequality in two variables as a half- plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

FOCUS VOCABULARY

TERMS	DEFINITION	EXAMPLES (*but not limited to*)
Compound Inequalities	*A compound inequality is composed of two inequalities joined by the word and (conjuctions) or by the word or (disjunctions).*	*x>5 or x<-1* *x>-3 and x<4*
Elimination Method of Solving Systems of Equations	*[8]The elimination method of solving a system of equations is a method that uses addition or subtraction to eliminate one of the variables to solve for the other variable*	
Solution of Linear System	*[9]A **solution** of a linear system is an assignment of values to the variables x_1, x_2, ..., x_n such that each of the equations is satisfied. A linear system may behave in any one of three possible ways: (a) The system has infinitely many solutions (b)The system has a single unique solution (c)The system has no solution.*	
System of Linear Equation	*[10]A **system of linear equations** (or **linear system**) is a collection of [linear equations] involving the same set of [variables]. The word "system" indicates that the equations are to be considered collectively, rather than individually.*	
System of Linear Inequality	*[11]A System of Equations is a set of two or more equations with the same variables.*	
Zero of a Function	*[12]A **zero**, also sometimes called a **root**, is an input value that produces an output of zero (0).*	*f(x)=(x+3)(x-2)=0* roots of f(x) are x=-3 and x=2

[8] http://www.mathchamber.com/algebra/docs/general/glossary.htm

[9] http://en.wikipedia.org/wiki/System_of_linear_equations

[10] http://en.wikipedia.org/wiki/System_of_linear_equations

[11] http://www.icoachmath.com/math_dictionary/system_of_inequalities.html

[12] http://en.wikipedia.org/wiki/Zero_of_a_function

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 10, I will be able to use substitution to determine whether a given number in a specified set makes an equation or inequality true.

BUILD UP! WARM UP!

6.EE.5

1) Select all the equations or inequalities where x=5 is a solution.

- ○ $2(10-x) > 14$
- ○ $5x-10 \leq 45$
- ○ $50-5x = 25$
- ○ $18+x=27$

6.EE.5

2) Tell whether each algebraic equation is correct. Write True or False on the line next to each number.

____________ a) x-6=4, when x=10

____________ b) 8x =48, when x=7

____________ c) x/3=6, when x =18

______________d) 35-x = 15, when x=12

______________e) x+x+16=20, when x=2

Discovering True and False Equations

Directions: Use what you know to determine if the given number sentences are true or false. Show your work!

1. $2\,(4 + 5) = 2 \bullet 4 + 2 \bullet 5$

This equation is _______________ (true/false) because _____________________________

____

____

2. $10 - (5 + 4) = 10 - 5 + 4$

This equation is _______________ (true/false) because _____________________________

____

____

Pre- LESSON 11 SOLUTION SETS FOR EQUATIONS AND INEQUALITIES

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 11, I will be able to (a) substitute values into inequalities (b) translate a given real world problem into inequality statements (c) understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? (d)interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.

BUILD UP! WARM UP

6.EE.5 1. Select all of the equations or inequalities where x=8 is a solution.

- ○ $3x-11=10$
- ○ $x+9<52$
- ○ $2x+2=18$
- ○ $x-3=5$
- ○ $2x-6\leq 2$

6.EE.6 2. Anne baked 25 cupcakes for their Christmas party. She can pack 7 cupcakes in each container.

Select the inequality that best represents Anne's situation. Let x = the number of cupcake containers needed.

- ○ $25\geq 2.5x$
- ○ $7x\geq 25$
- ○ $25\geq 7x$
- ○ $25\geq x$

6.EE.5 3. Select all the equation or inequality where x=4 is a solution.

- ○ $3x\geq 20$
- ○ $6+x=10$
- ○ $x-2=6$
- ○ $x+4\leq 8$

6.EE.8 4. Bigby's Buffet has a promotion. Children aged less than 7 years old will eat free in the buffet.

a) Write the inequalities that describe all the possible ages, x, of children.

b) Select all of the sets of numbers that represent possible ages of children that would eat free of charge.

- ○ {3, 5, 6}
- ○ {7, 9, 11}
- ○ {2, 5, 6}
- ○ {1, 5, 7}

6.EE.7 5. Will the given equation and inequality have the same solutions? Why or why not?

a) $2x - 6 = 10$

b) $2x - 6 > 10$

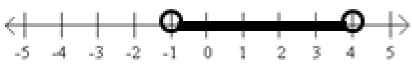
Discovering Solution Sets for Equations and Inequalities

6.NS.7.a

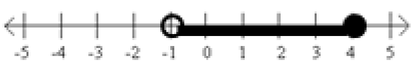
Identify the rule used.

1) 

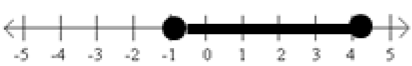
$\{x|-1 \square\, x \,\square\, 4\}$

2) 

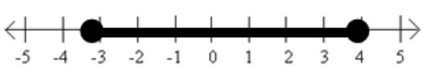
$\{x|-1 \square\, x \,\square\, 4\}$

3) 

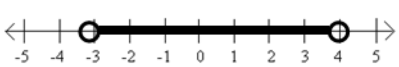
$\{x|-1 \square\, x \,\square\, 4\}$

4) 

$\{x|-1 \square\, x \,\square\, 4\}$

5) 

$\{x|-3 \leq x \leq 4\}$ $[-3, \square]$

6) 

$\{x|-3 < x < 4\}$ $(\square, 4)$

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 12, I will be able to (a) develop a concept on how to initiate and solve an equation (b) solve equations and justify steps using properties of equality.

BUILD UP! WARM UP!

7.EE.3

1. Brandon has a writing assignment that requires him to write 1000 words. He has already written 250 words. If he can write exactly 50 words per minute, how long would it take him to complete his writing assignment?

   Select the strategy that could be used to get the answer.

   ○ Subtract 250 from 1000, and then guess different numbers for the number of minutes. Multiply these guesses by 50, and see which equals 750.

   ○ Add 250 to 1000 and then divide by 50. This will give the number of minutes.

   ○ Divide 1000 by 50 to find the number of minutes.

   ○ Subtract 250 from 1000 then divide by 50.

7.EE.3

2. The boys had a guessing game. Sebastien tried to guess Eric's age and Liam's age. Eric's age plus Liam's age equals 57. Eric is twice as old as Liam. This can be represented by the equation 2x+x=57. Select *all* the statements that are true.

   ○ The variable x represents Liam's age.

   ○ The variable x represents Eric's age.

   ○ The variable x represents Eric's and Liam's combined age.

   ○ To simplify the equation, you can combine like terms to get 3x=57.

   ○ Eric is 38 years old.

7.EE.3

3. Select all of the following scenarios that can be modeled by the equation x+10=50.

   ○ Lachriesha picked 12 fewer oranges than Aaron. Jane picked 50 apples.

   ○ Erin has 10 more hairclips than Ella. Erin has 50 hairclips.

   ○ Zaria paid 10 more dollars for her bus ticket than Joseph. Zaria paid 50 dollars.

   ○ Conner took 10 more seconds to run the race than Alex. Alex finished the race in 50 seconds.

Discovering Solving Equations

7.EE.3 7.EE.2

Addition Property of Equality	If a = b, then a + c = b + c
Subtraction Property of Equality	If a = b, then a - b = b - c
Inverse Property of Addition	a + (-a) = -a + a = 0
Identity Property of Addtion	a + 0 = 0 + a = a
Associative Property of Addition	(a + b) + c = a + (b + c)

	x + 5	=	13	
x + 5 -	______	=	13 - ______	______________ Property of Addition
	x	=	______	______________

The solution set is {   }

Mulitplication Property of Equality	If a = b, then ac = bc
Division Property of Equality	If a = b, and c ≠ 0, then $\frac{a}{c} = \frac{b}{c}$
Inverse Property of Multiplication	If a ≠ 0, then a $\left(\frac{1}{a}\right) = \left(\frac{1}{a}\right) a = 1$
Identity Property of Multiplication	a • 1 = 1 • a = a
Associative Property of Multiplication	a (bc)= (ab)c

Solve the given equation and write the reason for each step, using the properties listed above.

$$\frac{5y}{3} = 20$$

The solution set is {   }

Pre- LESSON 13 SOME POTENTIAL DANGERS WHEN SOLVING EQUATIONS

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 13, I will become familiar with the potential misconceptions and common errors/mistakes when solving equations and address them.

BUILD UP! WARM UP!

8.EE.7.b 1) Select all the equation/s that is/are equivalent to 4(2x + 3) = 20 after performing the distributive property.

- ○ 8x + 3 = 20
- ○ 8x+12 = 80
- ○ 8x + 12 = 20
- ○ 2x + 3 = 5

8.EE.7.b 7.EE.3 2) Mr. Santos gave the equation, 313-312(x+5)=225.

Lorraine's Solution

Step 1	313-312(x+5) =225
Step 2	1(x+5) = 225
Step 3	x+5 = 225
Step 4	-5 = -5
Step 5	x = 220

Mr. Santos said that Lorraine's answer is wrong.

a) At what step did Lorraine make a mistake?

b) Correct Lorraine's mistake.

8.EE.7.b

3) Select all the equation/s that is/are equivalent to 2-4(x+5) = -26 after performing the distributive property.

- ○ -2(x+5) = -26
- ○ 2x + 10 = -26
- ○ -4x - 20 = 26
- ○ -4x - 18 = 26

4) Is the inequality 25 ≥ x the same as x≥25? Why or why not?

8.EE.7 7.EE.3

5) Mrs. Santos asked the class to solve for the value of x in the equation, 13x-21=12x.

Brian's solution:

$$\begin{array}{l} 13x-21=12x \\ \underline{-12x \quad\; -12x} \\ \quad x-21=0 \\ \text{so } x= -21 \end{array}$$

Zara's solution:

$$\begin{array}{l} 13x-21=12x \\ \underline{\quad +21 \;\; +21} \\ \dfrac{13x}{13} = \dfrac{33}{13} \\ \quad x= \dfrac{33}{13} \end{array}$$

Charmaine's solution

$$\begin{array}{l} 13x-21=12x \\ \underline{-13x \quad\; -13x} \\ \quad \dfrac{-21=-x}{-1=-1} \\ \quad 21=x \\ \quad x=21 \end{array}$$

Who has the correct solution? Why? What mistakes did Brian, Zara or Charmaine make?

Discovering Some Potential Dangers When Solving Equations

8.EE.7.b 7.EE.3

Use the table below to answer the questions.

Initial Equation	2(x+3)+5x-x=4(x+4)
Step 1	2(x+3)+4x=4(x+4)
Step 2	2x+6+4x=4x+4
Step 3	6x+6=4x+4
Step 4	6x=4x-2
Step 5	2x=-2
Step 6	x=-1

Alfred solved the equation above. His teacher said that it is wrong because 2(-1+3)+5(-1) is not equal to 4(-1+4).

(a) At what step do you think that Alfred commit a mistake?

(b) Make necessary correction/s to that step.

(c) What is the final answer?

Pre- LESSON 14 SOLVING INEQUALITIES

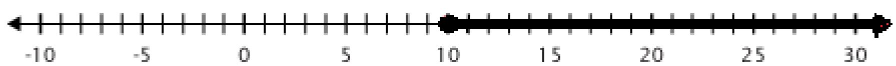
STUDENT OBJECTIVES

Upon completion of Pre-Lesson 14, I will be able to (a) translate graphs on the number line into an inequality statement (b) write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem.

BUILD UP! WARM UP!

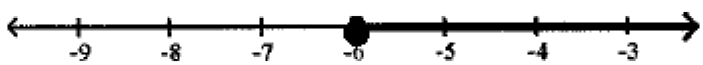
6.NS.7.a 6.EE.8

1) Write the inequality that is represented by the graph shown.



6.NS.7.a 6.EE.8

2) Select all the inequalities or situations that the graph could represent.



- ○ $x \geq -6$
- ○ $-6 \leq x$
- ○ $x < -6$
- ○ $x = -6$

6.EE.8

3) My dog, Oreo, digs a few holes in the sand that are *at least* 10 centimeters deep. Write an inequality that represent all the possible hole depths, x, that Oreo could dig.

6.EE.8

4) The Spanish Club wants to raise $3500 for their field trip. To raise money, they are organizing a school play. The cost to organize the fund raising event is x dollars. Write an inequality to represent this scenario.

Discovering Solving Inequalities

6.EE.5

Given the inequality: $x^2 - 4x \geq 5$

Find two positive and two negative numbers that will make this inequality true.

Verify that your solutions work in the following inequalities. Determine why they should work based on the original inequality.

Given the Inequality: $x^2 - 4x \geq 5$

a) $-4x + x^2 \geq 5$
b) $x(x-4) \geq 5$
c) $x^2 - 4x - 3 \geq 2$
d) $2x^2 - 8x \geq 10$

So far, which inequality came out with a true statement? Why do

____

____

____

So far, which inequality came out with a false statement? Why do you think did it not

____

____

____

____

Pre- LESSON 15 SOLUTION SETS OF TWO OR MORE EQUATIONS (OR INEQUALITIES) JOINED BY "AND" OR "OR"

STUDENT OBJECTIVES

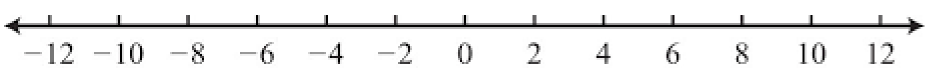
Upon completion of Pre-Lesson 15, I will be able to (a) represent different situations with an inequality (b) write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (c) focus on how different inequalities can have the same solutions. (d) Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.

BUILD UP! WARM UP!

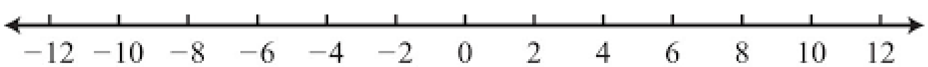
8.EE.7 6.NS.7.a

1. Solve for x and graph on the number line.

(a) $x+5 = 1$ or $x-3=6$
Solution set { }

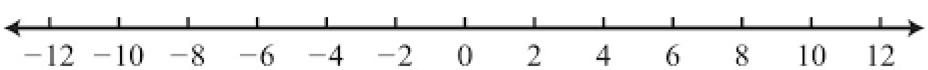


(b) (b) $2x+5=7$ and $2x>0$
Solution set { }



(c) $x+5=2$ or $x+9=6$

Solution set { }



7.EE.4

2. Which situation/s can be modeled by $x \le 25$?

- ○ Mariah Carey has at least 25 songs.
- ○ Tristan weighs more than 25 pounds.
- ○ Raven spends no more than 25 minutes napping.
- ○ The tree is taller than 25 feet.

6.EE.8

3. Students who scored higher than 90% on the spelling bee will participate in the district wide spelling bee. Sofia says that $x \geq 90$ represents the inequality and that she will go to the district wide bee because she scored 90%.

   Which statement best explains why Sofia's statement is wrong?

   - ○ The phrase greater than implies values less than 90 qualify for the district wide spelling bee.
   - ○ The phrase greater than implies values less than or equal to 90 qualify for the district wide spelling bee.
   - ○ The phrase greater than implies values greater than or equal to 90 qualify for the district wide spelling bee.
   - ○ The phrase greater than implies a value equal to 90 does not qualify for the district wide spelling bee.

7.EE.4.b

4. Barbara buys 4 socks that cost the same amount each, and 1 pair of leggings that costs $7.35. Barbara spends less than $13 altogether.

   a) Which inequality can be used to find the cost of each sock?

   - ○ $4s+7.35<13$
   - ○ $4s-7.35<13$
   - ○ $4s+7.35>13$
   - ○ $4s-7.35>13$

   b) What is the maximum cost for each sock that Barbara bought, to the nearest cent?

7.EE.4.b

5. Twice the sum of x and 5 is at most 38.

   a) Write an Inequality that would represent this situation.

   b) What is the greatest value of x that satisfies the inequality in Part a?

Discovering Solution Sets of Two or More Equations (Or Inequalities) Joined By “And” Or “Or”

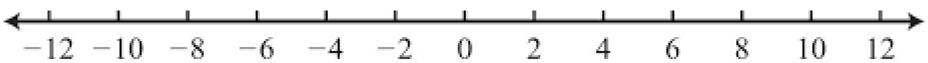
6.NS.7.a 6.EE.5

Given the inequality **$-4<3x+2\leq17$**

List five values that will make this inequality true?

If x=-5, will the inequality still be true?

Graph the solution set on the number line.



Given the inequality **$-4>3x+2\geq17$**

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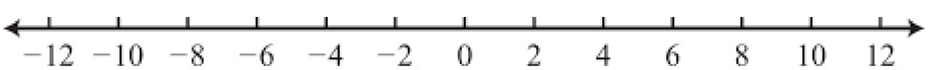


How is this inequality different from the first?

Look at each inequality separately. What values will make each true?

Look at the graph of each inequality. Are they the same or different?

Pre- LESSON 16: SOLVING AND GRAPHING INEQUALITIES JOINED BY "AND" OR "OR"

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 16, I will be able to (a) represent different situations with an inequality (b) write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (c) focus on how different inequalities can have the same solutions. (d) Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.

BUILD UP! WARM UP!

6.NS.7.a

Solve each inequality.

1. Solve the inequality $x + 3 \leq 10$ and graph its solution set the number line.

2) Solve the inequality $x^2 > 36$.

7.EE.4

5) Trey, a realtor, joined an auction for a foreclosed house. His budget is a maximum of $125,000 to bid for the house.

(a) Write an inequality that would represent the above situation where x is the price of the house.

(b) The auction has a minimum bid of $30,000. Write an inequality that would represent the above situation where x is the price of the house.

(c) Write an inequality that would represent both price restrictions.

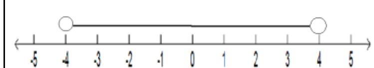
6.EE.5

6) Determine if 2 is a solution to the given equation or inequality.

Equation/Inequality	Yes	No
2x+8=27		
2x+8<27		
9x-3=15		
9x-3≥15		

6.EE.5 6.NS.7.a 6.EE.8

7) Complete the table below.

Verbal Description	Some Possible Solutions	Inequality	Graph
all numbers between 4 and 10	5, 6, 6.5, 7,8.5	x>4 and x<10	
all numbers less than -1 or greater than 6			
all numbers greater than or equal to 2 and less than or equal to 9			
all numbers less than or equal to 5 and greater than -5			
all numbers greater than 5 or less than or equal to -1			

Discover Solving And Graphing Inequalities Joined By "And" Or "Or"

6.NS.7.A

Company A is hiring new trainees who have less than one year experience and supervisors with an experience of at least 5 years.

AGE OF APPLICANTS	NUMBER OF MALE APPLICANTS	NUMBER OF FEMALE APPLICANTS	YEARS OF EXPERIENCE
21	15	8	0-2
22	3	6	0-4
23	5	7	1-5
25	8	9	2-7
30	12	6	0-10

(a) Write an inequality representing the years of experience of 21 year old male applicants and graph on the number line.

(b) Write an inequality representing the years of experience of 25 year old female applicants and graph on the number line.

(c) Write an inequality representing the ages that may have applicants that would qualify for the supervisory position?

Pre- LESSON 17 EQUATIONS INVOLVING FACTORED EXPRESSIONS

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 17, I will be able to find the greatest common factor when given a monomial (b)solve one step or multiple step algebraic equations.

BUILD UP! WARM UP!

Factor the given monomial.

6.NS.4 1) $4y^2 = 2 \bullet$ __ $\bullet\ y \bullet y \bullet y \bullet y$

6.NS.4 2) $8y^2 = 2 \bullet$ __ $\bullet$ __ $\bullet\ y \bullet$ __

Find the Greatest Common Factor of the given monomial

6.NS.4 3) $6x^2y^5$

6.NS.4 4) $9x^3y^3$

6.EE.5 5) Determine a solution that would satisfy the given equation.

a) $x+10=0$ $x–5=0$

b) If joined into one equation, $(x+10)\ (x - 5) = 0$, will the values of x from part a work?

Discovering Equations Involving Factored Expressions

6.NA.4

Given **$x \bullet y = 0$**, what values of x or y would make this equation true?

Now if **$x(x + 2) = 0$**, what values of x would make this equation true?

Now, if **$(x+2)(x-3) = 0$**, what values of x would make this equation true?

In the last equation, how many possible values could x have to make the equation true? Why?

Pre- LESSON 18 EQUATIONS INVOLVING A VARIABLE EXPRESSION IN THE DENOMINATOR

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 18, I will be able to (a) solve fractions with one denominator as a variable (b) apply and extend previous understandings of operations with fractions.

BUILD UP! WARM UP!

7.NS 6.EE.6

1) Evaluate the expression for x = 3.

   a) $\frac{15}{x}$

   b) $\frac{24}{x}$

2) Solve for x.

   a) $\frac{1}{x} + \frac{1}{2} = 1$

   b) $\frac{2}{3} + \frac{1}{x} = 1$

   c) $\frac{3}{x} + \frac{1}{2} = \frac{5}{x}$

   d) $\frac{2}{x} = \frac{8}{28}$

   e) $\frac{3}{4} = \frac{6}{x+1}$

Discovering Equations Involving a Variable Expression in the Denominator

7.NS 6.EE.6

Ralph is planning to renovate his house. He could complete the renovations in the kitchen in 12 hours. His friend, Jet, could finish it in 9 hours.

If they work together, how long would it take them to complete the renovations?

Ralph would also want to renovate the living room. It would take him 5 hours to complete it. His sister, Roxanne, offered to help. She could complete the work in 9 hours.

How long would it take both of them to complete it?

For their bedroom, Ralph thought he could complete it in 9 hours. His wife helped him and they finished it in 4.5 hours. There are two bedrooms left to renovate. Ralph and his wife are trying to make a decision whether to work together or individually assign a bedroom. <u>*If both rooms are the same size, and requires the same amount of work, would it be wiser for them to work together or individually assign a room for each one? What is Ralph's wife's rate of completion?*</u>

Pre- LESSON 19 REARRANGING FORMULAS

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 19, I will be able to (a) comprehend that not all equations would result to a constant answer (b) the same steps/properties of equality would apply to constants and variables (a) understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

BUILD UP! WARM UP!

7.EE.2

Solve for x

1) $2x = 10+2y$

2) $3x - y = 5$

3) $3x = y$

4) $3x - 5 = y$

5) $4x - y + 2 = 5$

6) Given the formula for the volume of a rectangular solid, V=lwh where V=volume, l=length, w=width, and h=height. If the volume of the figure is 165 cubic centimeters, and the length is 8 cm. and width is 2.5 cm, what is the height in cm. of the figure?

Discovering Rearranging Formulas

7.EE.2

1) Solve each equation for x.

$4x+8=16$	$ax+b=c$

What operations did you use to get x alone?

What terms were you able to combine/simplify?

How does the solution appear?

Pre- LESSON 20 SOLUTIONS TO EQUATIONS WITH TWO VARIABLES

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 20, I will be able to solve an equation as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

BUILD UP! WARM UP!

6.EE.5 1) Which ordered pair (x,y) is a solution to 2x+4y=16

(a) (0,4)
(b) (8,0)
(c) (1,4)
(d) (2,4)

6.EE.5 2) Complete the table of values to find solutions of linear equation x+y=5.

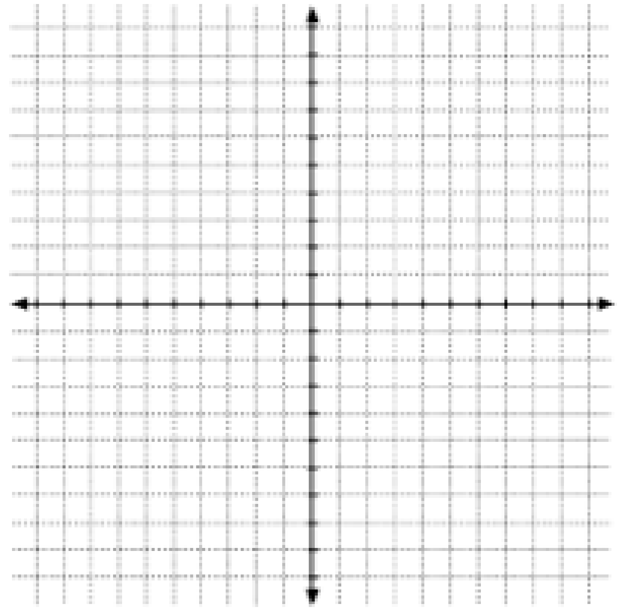
(a)

x	y	(x,y)
-4		
-3		
-2		
-1		
0		
1		
2		
3		
4		
5		

(b) What is the x-intercept?

(c) What is the y-intercept?

(d) Plot in the coordinate grid.

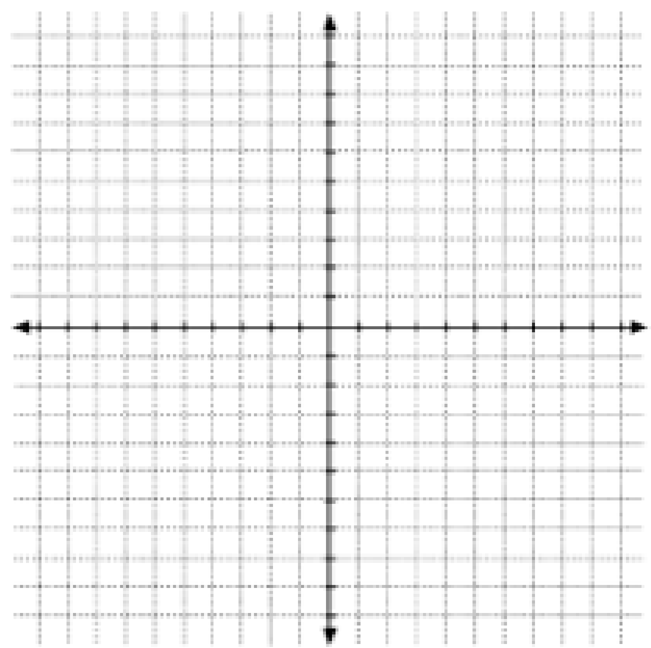


6.EE.5 3) Complete the table of values to find solutions of linear equation x+2y=4

(a)

x	y
0	
	0
1	

(b) Plot in the coordinate grid.



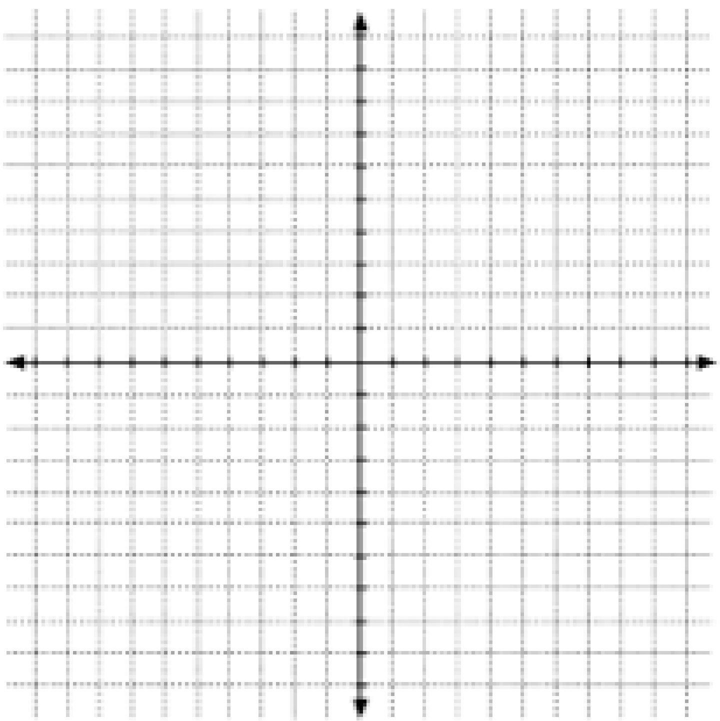
Discovering Solutions to Equations with Two Variables

7.EE.4.a

Aunt Mildred gave you and your brother \$10 each to pay for snacks at the movies. You bought two large size popcorn and two candies and paid \$10 for you and your brother. Your friend bought one large size popcorn and six candies and paid \$10.

a) How much was the price of each candy? Solve algebraically.

b) Solve by graphing.



Pre- LESSON 21 SOLUTION SETS TO INEQUALITIES WITH TWO VARIABLES

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 21, I will be able to solve an inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

BUILD UP! WARM UP!

6.EE.5

1) Is the ordered pair (x, y) a solution of the given inequality?

a) (2,6) ; $y<x+5$

b) (-2,10); $y\geq 2x+5$

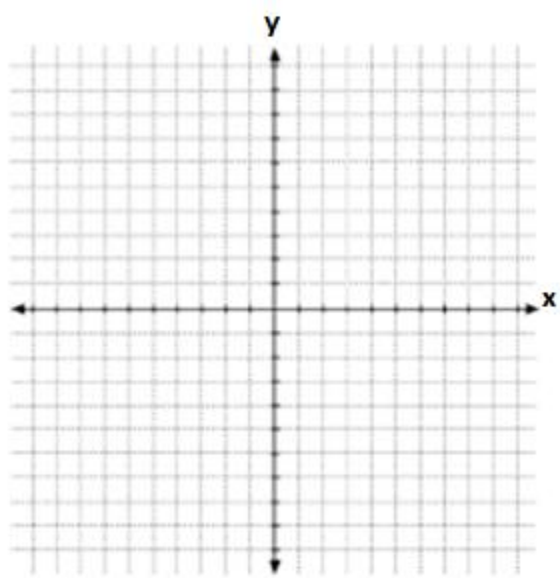
6.EE.5

2) Given the inequality $3x+5y>15$

(a) Complete the table of values for the boundary line $3x+5y=15$. Show your work.

(b) Graph the boundary line $3x+5y=15$ using the table of values.

x	y
0	
	0
-5	



(c) Test several other points in the plane that are not on the boundary line.

Point	Above or Below the Line?	Inequality	True or False
(0, 0)	Below	3(0)+5(0)>15	False
(1,0)			
(0,2)			
(5,2)			
(-2,5)			

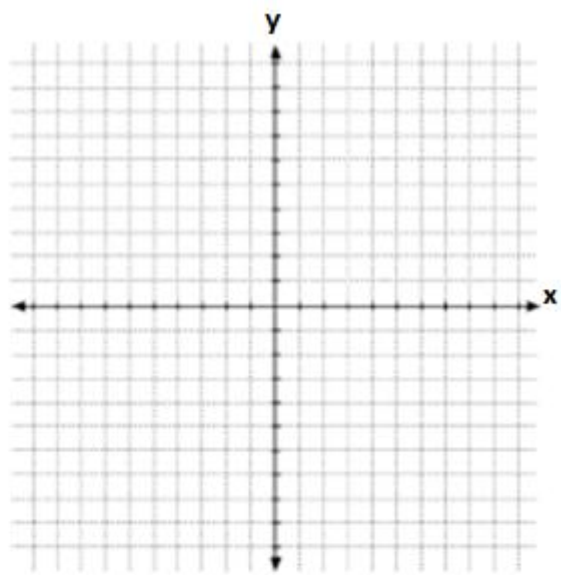
Discovering Solution Sets to Inequalities with Two Variables

6.EE.5

Given the equation 2x-3y=6

a) List 5 possible solutions (x, y) of the inequality.

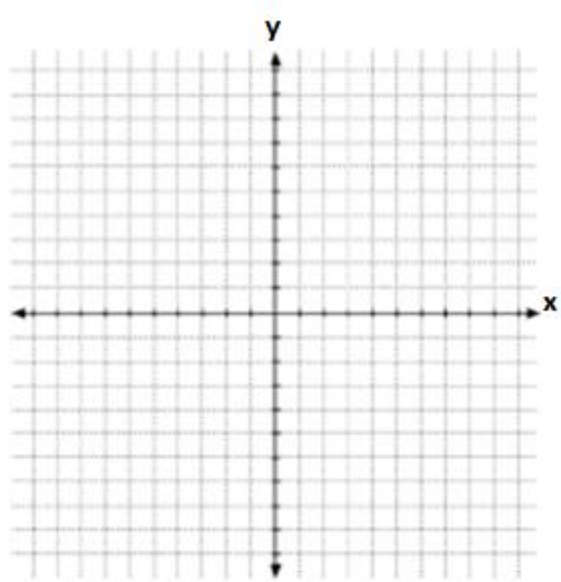
b) Graph the ordered pairs listed in (a).



Given the inequality 2x-y≥6,

a) list 5 possible solutions (x, y) of the inequality, and

b) graph the ordered pairs listed in (a).



Are all solutions (x, y) of

$2x-3y\geq 6$ also solutions to $2x-3y = 6$?

How about all the solutions (x, y) of

$2x-3y = 6$, are these solutions to

$2x-3y\geq 6$ as well?

Pre- LESSON 22 SOLUTION SETS TO SIMULTANEOUS EQUATIONS

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 22, I will be able to solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers so that students may solve equations of these forms fluently.

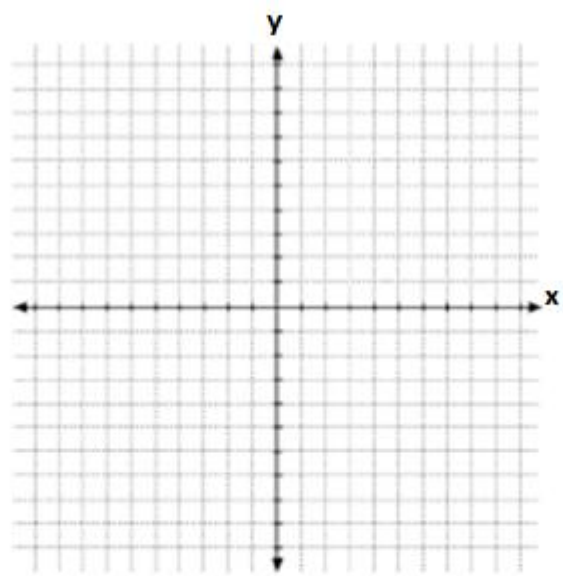
BUILD UP! WARM UP!

7.EE.4.a 1) Given equation, 2x+3y=12.

(a) Complete the table.

x	y
-1	
0	
1	
	0

(b) Graph the points and connect them with a straight line.



7.EE.4.a 2) Given the equation, 2x-3y=12.

(a) Complete the table.

x	y
-1	
0	
1	
	0

(b) On the same grid, graph the points Connect them with a straight line.

3) Did the two the two graphs intersect? What does this mean? Explain.

Discovering Solution Sets to Simultaneous Equations

7.EE.4.a

A restaurant, Yellow Brutus, sells hamburgers and soda. The total number of sodas and hamburgers Yellow Brutus sells today is 175.

(a) Assign and define the variables that would represent the statement above.

(b) Write an equation for the statement above using the variables identified in Part a.

Yellow Brutus charges \$3.50 for hamburgers and \$1.00 per soda and made \$412.50 today.

(c) Write an equation for the statement above using the variables identified in Part a.

4. Complete the table below to guess and check to find the answer to the problem.

Hamburger	Soda	Total Number	Total in \$
25			
	125		
75			
	80		
100			
145			

Pre-Lesson 23 SOLUTION SETS TO SIMULTANEOUS EQUATIONS

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 23, I will be able to (a) find the least common multiple of variable expressions in preparation to solving systems of equation by elimination; (b) understand that equivalent functions may be written in different forms.

BUILD UP! WARM UP!

6.NS.4

1) Find the Least Common Multiple in the following expressions:
   (a) $2x$ and $3x$
   (b) $3y$ and $4y$
   (c) $-2y^2$ and $3y^2$
   (d) $4y$ and $-2y$

7.EE.2 6.NS.4

2) Use the Distributive Property to rewrite each of the following expressions.
   (a) $-5(x^2+1)$
   (b) $4(x+2y)$

6.EE.5

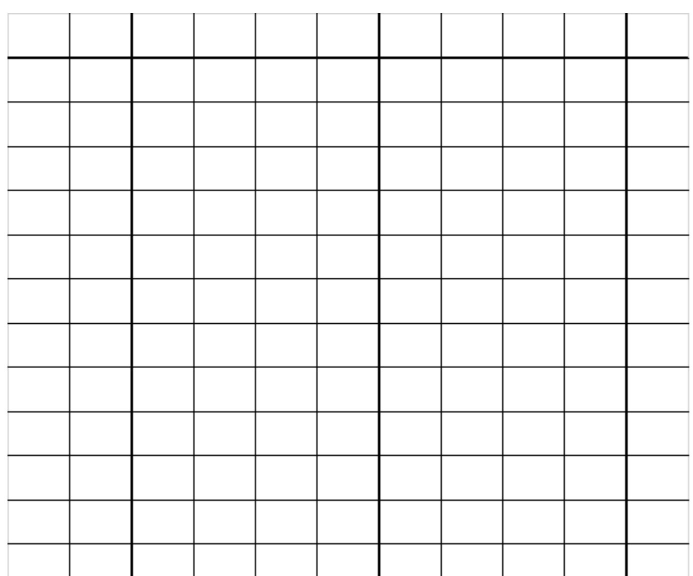
3) a) Given the following equations, complete the table of solutions (x, y).
   b) Graph the given equation for all real numbers x given the sample table.

$x + y=2$

x	y
-1	
0	
1	

$2x+2y=4$

x	y
-1	
0	
1	



Discovering Solution Sets To Simultaneous Equations

8.EE.7.a

Task	System 1	System 2	System 3
	$2x + y = 4$	$y = 3x + 1$	$x + 2y = 5$
	$x + y = 3$	$4y = 12x + 3$	$2x + 4y = 10$
Solve			
How many solution/s?			
Explanation			

Pre- LESSON 24 APPLICATIONS OF SYSTEMS OF EQUATIONS AND INEQUALITIES

STUDENT OBJECTIVES

Upon completion of Pre-Lesson 24, I will be able to (a) use variables to represent numbers and write expressions when solving a real-world or mathematical problem (b) understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set (c) solve real word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

BUILD UP! WARM UP!

6.EE.5 7.EE.4.a

1) a) Complete the table of solutions (x, y) for the given equations:

$x+y=4$

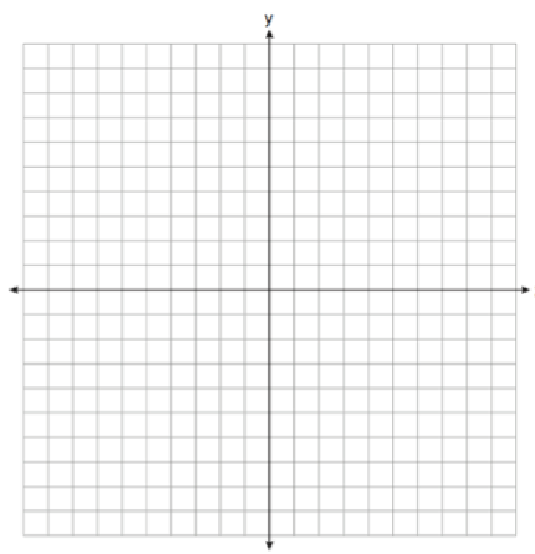
x	y
3	
	0
2	
	-1
0	

$2x+3y=9$

x	y
-1	
	0
0	
	1
6	

b) Look at the two tables above. Can you find the solution to the above system of equations? What is/are the solution/s?

c) Verify your answer by graphing the points.



Discovering APPLICATIONS OF SYSTEMS OF EQUATIONS AND INEQUALITIES

7.EE.4.a   6.EE.6

Our school had a fun run to raise money for the Chess Club. There were a total of 320 tickets sold and the event raised $1810. Tickets were sold for $5 during lunch shift and tickets at the door sold for $7.

(a) Write a system of equation that would represent the scenario above.

(b) How many students bought their tickets during lunch shift?

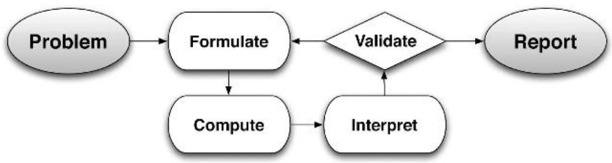
(c) How many students bought their tickets at the door?

The French Club decided that they would organize a similar fundraising event. If the club aims to raise at least $2000, and assuming the same number of students will purchase tickets at the door, how many tickets (minimum) do they have to sell during lunch shift?

Pre-Lessons for Topic D
CREATING EQUATIONS TO SOLVE PROBLEMS

In Topic D, students are introduced to the modeling cycle (see page 61 of the Common Core Learning Standards) through problems that can be solved using equations and inequalities in one variable, systems of equations, and graphing. From the CCLS (page 61):

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making.



The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features; (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables; (3) analyzing and performing operations on these relationships to draw conclusions; (4) interpreting the results of the mathematics in terms of the original situation; (5) validating the conclusions by comparing them with the situation and then either improving the model; (6) or if it is acceptable, reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

SOURCE: https://www.engageny.org/resource/algebra-i-module-1-topic-d-overview

The Pre-Lessons to Topic D address necessary skills for students to become successful in the lessons in Topic D. The Pre-Lessons are preceded with a list of foundational vocabulary and focus vocabulary for Topic B. The individual Pre-Lessons are composed of build up/warm up skills, and individual discovery stages of each lesson. The lessons of Topic B are:

25. Solving Problems in Two Ways—Rules and Algebra (M)
26-27. Recursive Challenge Problem—The Double and Add 5 Game (M, M)s
12. Federal Income Tax (M)

Lesson 25 introduces "*parts of the modeling cycle using problems and situations that students have encountered before: creating linear equations, tape diagrams, rates, systems of linear equations, graphs of systems, etc.*" (EngageNY.org, 2014) Pre-Lesson 25 focuses on how to (a) *solve rate problems, and (b) use unit analysis to write an algebraic expression to represent a situation.*

Lesson 26 and 27 introduce, "*The Double and Add 5 Game, employs the modeling cycle in a mathematical context. In this 2-day lesson, students formulate a model and build an equation to represent the model (in this case, converting a sequence defined recursively to an explicit formula). After they play the game in a specific case, "double and add 5," they have to interpret the results of the mathematics in terms of the original model and validate whether their model is acceptable. Then they use the model to analyze and report on a problem that is too difficult to do "by hand" without the model.*" (EngageNY.org, 2014) Pre-Lesson 26 assists students on how to identify sequences while Pre-Lesson 27 (a) gives students a view on real world applications for sequence, and introduces them to writing a rule for an arithmetic sequence.

Lesson 28 serves as "*a signature lesson on modeling as students take on the very real-life example of understanding federal marginal income tax rates (i.e., the progressive income tax*

brackets). Students are provided the current standard deduction tables per dependent or marital status and the marginal income tax table per marital filing status. For a specific household situation (e.g., married filing jointly with two dependents), students determine equations for the total Federal Income Tax for different income intervals, graph the piecewise-defined equations, and answer specific questions about the total effective rate for different income levels. All elements of the modeling cycle occur as students analyze the information to find, for example, roughly how much their favorite famous performer paid in federal taxes last year" (EngageNY.org, 2014)

FOUNDATIONAL STANDARDS

The following skills are necessary for students to be prepared for these lessons:

- 6.EE.6 — Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- 6.EE.7 — Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.

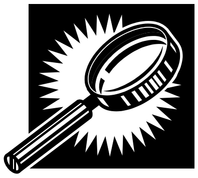
Based on experience, the following standards are also necessary pre-requisites that need to be addressed by the teacher to prepare students for the Lessons that follows.

- 6.RP.A.3.b — Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
- 7.EE.B.4.a — Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

FOUNDATIONAL VOCABULARY

TERMS	**DEFINITION**	**EXAMPLES (*but not limited to*):**
Sequence	[13] *A sequence is an ordered set of mathematical objects in which repetitions are allowed. It contains members (also called elements, or terms).*	The first four odd numbers form the sequence (1,3,5,7)
Term	*Each element in a sequence is called a term.*	

[13] http://en.wikipedia.org/wiki/Sequence



FOCUS STANDARDS

"Focus Standards" is a list of standards introduced in a Topic in the EngageNY/Eureka Math Curriculum. Topic D presents the following standards to the students:

- N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.★
- A-SSE.A.1a Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.A.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.
- A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

FOCUS VOCABULARY

TERMS	DEFINITION	EXAMPLES *(but not limited to)*
Arithmetic Sequence	[14]***Arithmetic sequence*** *is a sequence of numbers such that the difference between the consecutive terms is constant.*	5, 7, 9, 11, 13, 15 … d=2
Common Difference	*Constant difference in an arithmetic sequence*	
Recursive Formula	[15]*Recursive formula is a formula that is used to determine the next term of a sequence using one or more of the preceding terms.*	The recursive formula for 5, 20, 80, 320, … is $a_n = 4a_{n-1}$
Taxable income	[16]*Taxable income is defined as gross income less allowable tax deductions. Taxable income as determined for federal tax purposes may be modified for state tax purposes.*	

[14] http://en.wikipedia.org/wiki/Arithmetic_progression

[15] http://www.icoachmath.com/math_dictionary/recursive_formula.html

[16] http://en.wikipedia.org/wiki/Income_tax_in_the_United_States

Pre-Lesson 25 Solving Problems in Two Ways—Rates and Algebra

STUDENT OBJECTIVES

Upon completion of this Pre-Lesson, I will be able to (a) solve rate problems (b) use unit analysis to write an algebraic expression to represent a situation.

BUILD UP! WARM UP!

6.RP.A.3.B 1) Valerie drove 135 miles in 4.5 hours. What is her average rate in miles per hour?

2) Yummi Café bakes 45 éclairs every 15 minutes. Find the average rate of éclairs made per minute. What is the average rate per hour?

6.EE.6 6.EE.7 3) John ordered tickets online for himself and three friends. There is a \$6 service fee per ticket and a \$5 cost of shipping for the entire order.

(a) Write an algebraic expression to represent the situation.

(b) If he paid a total of \$128, how much is the price per ticket?

6.EE.6 6.EE.7 4) Frank and Alvin are competing on who can perform more push-ups. Frank did 300 push-ups, which was three times the difference of the number of push-ups Alvin did and the number 50. How many push-ups did Alvin do?

6.EE.6 6.EE.7 5) A DVD rental store rents DVDs for \$5.25 each for 3 days. They also charge \$1.25 every day the DVD is late. Ann rented one DVD, and returned it. She was charged a total of \$9.00.

(a) Write an equation that could be used to solve for x, the number of days late Ann turned in the DVD.

(b) Solve your equation.

Discovering Solving Problems in Two Ways—Rates and Algebra

6.EE.6 6.EE.7

Hannah, the cheer captain wants to sell spirit shirts to raise money for the cheerleaders. She bought the Transfer Magic Ink Jet Transfer Paper for $72 total. She bought blank shirts for $1.25 each. Hannah is selling each printed shirt for $5.75.

(a) Complete the table below.

(b) Using the table, how many shirts does Hannah need to sell to break even?

(c) Write an equation that would represent the situation and solve.

NUMBER OFSHIRTS SOLD	HANNAH				MIKAELA			
	SALES	COST PER UNIT	TOTAL COST	TOTAL PROFIT	SALES	COST PER UNIT	TOTAL COST	TOTAL PROFIT
10								
20								
30								
40								
50								
60								
70								
80								
90								
100								

Mikaela wants to sell better spirit shirts than Hannah. She bought the Silhouette Rhinestone kit for a total of \$130. She bought blank shirts for \$5 but sells each printed shirt for \$9.50.

(d) Complete the table above.

(e) How many shirts does Mikaela need to sell to break even?

(f) Write an equation that would represent the situation.

PRE-LESSON 26 Recursive Challenge Problem—The Double and Add 5 Game

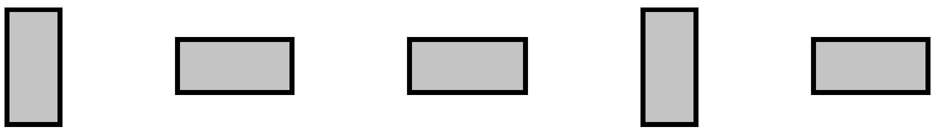
STUDENT OBJECTIVES

Upon completion of this Pre-Lesson, I will be able to identify a sequence.

BUILD UP! WARM UP!

7.EE.B.4.a

1) Consider the pattern below.



____

Based on the pattern, draw the next figure.______________________

2) Consider the pattern. 1, 3, 5, 7, x

Based on the pattern, what would be the value of x?

3) Is there a common difference between each term? If so, what is it?

(a) 3, 6, 9, 12, 15,…

Answer: ___________

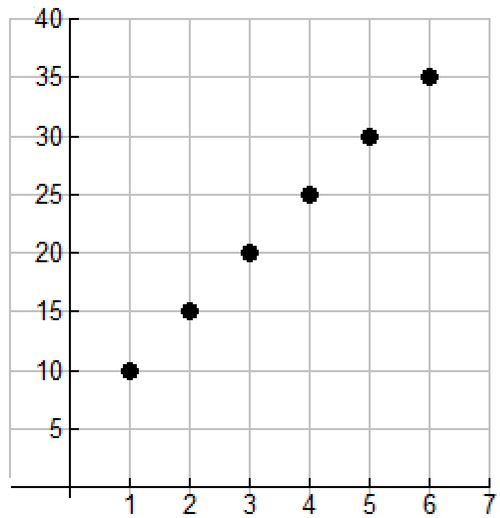
(b) -4,-2, 0, 2, 4,…

Answer: ___________

(b) -5, -2, 1, 4,…

Answer: ____________

4) Given the graph, complete the table.



Term Number	Term	Subscript Notation	Function Notation
1	10	a_1	$f(1)$
2	15		
3			
4			
5			
6			
n			

Discovering Recursive Challenge Problem—
The Double and Add 5 Game

POSITION NUMBER	n	1	2	3	4	5
TERM OF SEQUENCE	f(n)	1	3	5	7	9

(a) Find the common difference by calculating the differences between the consecutive terms.

$3 – 1=$ $5 – 3=$ $7 – 5=$ $9 – 7=$

The common difference, *d,* is _______

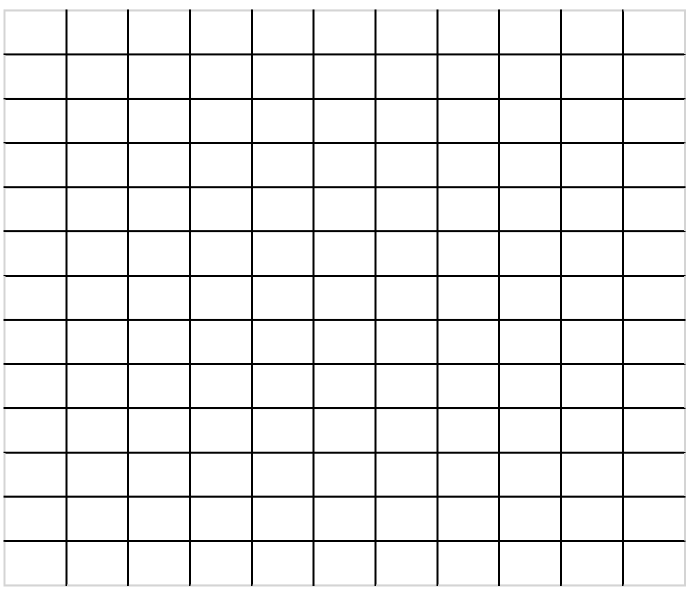
(b) Based on the sequence above, what is *f* (7)?

(c) Based on the sequence above, what will be *f* (60)? Explain your reasoning.

____

____

(d) Graph the sequence below.



(e) Do you thing the relationship between the position numbers and the terms of a sequence is a function? Why or why not?

____

____

PRE-LESSON 27 Recursive Challenge Problem—The Double and Add 5 Game

STUDENT OBJECTIVES

Upon completion of this Pre-Lesson, I will be able to answer real world problems using sequence.

BUILD UP! WARM UP!

1) Match the following formula for the nth term of an arithmetic sequence in Column A to the correct sequence in Column B?

A	B
$A_n=3n+3$	6,7,8,9,10,...
$A_n=2n-1$	6,9,12,15...
$A_n=n+5$	1,3,5,7,...
$A_n=3n-2$	1,4,7,13,...

2) The table below shows the profit, f(n), for the number of cupcakes sold at Reyn's Patisserie:

Cupcakes Sold (n)	Profit (f(n)
5	20
6	24
7	28
8	32
9	36

What is the formula for the Profit *f*(n)**:** _____________________

3) The first 3 terms of an arithmetic sequence are 2, 4, 6.

a) Find the next three terms.

a_4 = _______

a_5 = _______

a_6 = _______

b) What is the formula for the nth term.

c) Use your answer from part (b) to find the 100th term of the sequence.

4) Manny has a job and is saving part of his paycheck each week.

Weeks	Savings (USD)
1	130
2	260
3	390
4	520

(a) How much will Manny have in his bank account at 10 weeks?

(b) At what week will Manny have at least $1500 in his savings?

5) Find the y-differences in the following tables. Considering the y differences, identify whether the table represents a linear function, quadratic function or cubic function.

(a)

x	0	1	2	3	4	5	6
y	0	1	8	27	64	125	216

(b)

x	0	1	2	3	4	5	6
y	1	3	5	7	9	11	13

(c)

x	0	1	2	3	4	5	6
y	1	2	5	10	17	26	37

DISCOVERING RECURSIVE CHALLENGE PROBLEM— THE DOUBLE AND ADD 5 GAME

Alfred borrowed money interest free from his mother to start a business. He is paying the loan in equal monthly payments.

MONTH NUMBER	n	1	2	3	4	5	6	7	8	9	10	11	12
LOAN BALANCE ($)	f(n)	10,000	9,500	9,000	8,500	8,000	7,500	7,000	6,500	6,000	5,500	5,000	4,500

a) Is the sequence of loan balance arithmetic? Explain.

____

____

____

b) Write the recursive rule for the sequence of loan balances.

c) Write the explicit rule.

d) How many months will it take Alfred to pay off the loan?

STUDENT OBJECTIVES

Upon completion of this Pre-Lesson, I will become familiar with solving for the taxable income.

BUILD UP! WARM UP!

Consider the given table below to answer the questions that follows.

Important Tax Tables for this Lesson

Exemption Deductions for Tax Year 2013

Exemption Class	Exemption Deduction
Single	$\$3,900$
Married	$\$7,800$
Married with 1 child	$\$11,700$
Married with 2 children	$\$15,600$
Married with 3 children	$\$19,500$

Standard Deductions Based Upon Filing Status for Tax Year 2013

Filing Status	Standard Deduction
Single	$\$6,100$
Married filing jointly	$\$12,200$

Federal Income Tax for Married Filing Jointly for Tax Year 2013

If taxable income is over--	But not over--	The tax is:	Plus the Marginal Rate	Of the amount over--
$\$0$	$\$17,850$	10%		$\$0$
$\$17,850$	$\$72,500$	$\$1,785.00$	15%	$\$17,850$
$\$72,500$	$\$146,400$	$\$9,982.50$	25%	$\$72,500$
$\$146,400$	$\$223,050$	$\$28,457.50$	28%	$\$146,400$
$\$223,050$	$\$398,350$	$\$49,919.50$	33%	$\$223,050$
$\$398,350$	$\$450,000$	$\$107,768.50$	35%	$\$398,350$
$\$450,000+$		$\$125,846.00$	39.6%	$\$450,000$

Taxable Income: The U.S. government considers the *income* of a family (or individual) to include the sum of any money earned from a husband's or wife's jobs, and money made from their personal businesses or investments. The taxes for a household (i.e., an individual or family) are not computed from the income; rather, they are computed from the household's taxable income. For many families, the household's *taxable income* is simply the household's income minus exemption deductions and minus standard deductions:

(taxable income) $=$ ***(income)*** $-$ ***(exemption deduction)*** $-$ ***(standard deduction)***

All of the problems we will model in this lesson will use this equation to find a family's taxable income. The only exception is if the family's taxable income is less than zero, in which case we will say that the family's taxable income is just $\$0$.

Source: https://www.engageny.org/resource/algebra-i-module-1-topic-d-lesson-28

1) Sandee is single. Her income for the year is $45,482. What is her taxable income?

2) Griselda and Danny is married. Their combined income is $58,000. They have 1 child. What is their taxable income?

3) Bernard and Jennifer is married. Jennifer is a housewife and Bernard is the head of household. Their total income is $65,000. They have 3 children. What is their taxable income?

Use the Table from the "Build-Up! Warm-Up!" to answer the question.

1) Linda and Manny are married taxpayers with no child filing a joint return. Their combined taxable income is $153,900. Use the tax schedule below for married taxpayers filing jointly to calculate Linda and Manny's tax.

APPENDIX B: ALGEBRA I POTENTIAL GAPS IN STUDENT PRE-REQUISITE KNOWLEDGE

Algebra I Potential Gaps in Student Pre-Requisite Knowledge

This document indicates pre-requisite knowledge gaps that may exist for Algebra I students based on what the Grade 8 common core math standards expect. Column four indicates the Algebra I common core standard which could be affected if the Grade 8 gap exists. Other gaps may exist for other reasons; therefore, it important that teachers diagnose their students' needs as part of the planning process.

Domain	Grade 8 CCSS	Wording of Grade 8 CCSS Potential Gap	Algebra I CCSS
The Number System (NS)	**8.NS.A.1**	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	**HSN-RN.B.3**
	8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*	**HSN-RN.B.3**
Equations and Expressions (EE)	**8.EE.A.1**	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.	**HSA-SSE.B.3**
	8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form *x2* = *p* and *x3* = *p*, where *p* is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.	**HSA-REI.B.4**
	8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	**Prerequisite for Linear Equations**
	8.EE.C.8	Analyze and solve pairs of simultaneous linear equations.	**HSA-CED.A.3** **HSA-REI.C.5** **HSA-REI.C.6**

Algebra I Potential Gaps in Student Pre-Requisite Knowledge

Functions (F)	**8.F.A.3**	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*	**Prerequisite for Linear Functions**
	8.F.B.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	**HSF-LE.A.2**
	8.F.B.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	**HSF-LE.B.5**
Statistics and Probability (SP)	**8.SP.A.1**	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	**HSS-ID.B.6**
	8.SP.A.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*	**HSS-ID.C.7**
	8.SP.A.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	**HSS-ID.B.5**

(Algebra I Potential Gaps in Student Pre-Requisite Knowledge, n.d.)

APPENDIX C: SAMPLE INDIVIDUALIZED EDUCATIONAL PLAN (IEP)

INDIVIDUALIZED EDUCATION PROGRAM Student Name: [redacted] DOB: [redacted] Grade: 1st - First CONFIDENTIAL DOCUMENT

LOUISIANA DEPARTMENT OF EDUCATION System: [redacted] Meeting Date: [redacted] State ID: [redacted] Local ID: Page 7 of 13 Revised 2011

Instructional Plan #5

EDUCATIONAL NEED AREA: Social

CONTENT AREA:

☑ ESY Instruction

☐ Targeted for Secondary Transition

Present Level of Academic Achievement and Functional Performance

According to his interim IEP, [redacted] will greet adults when given an indirect cue (expectant look). He will not initiate greetings to peers but he will return greeting with adult prompting. [redacted] independently engages in parallel play with peers. He requires adult support in cooperative activities that require sharing materials and/or taking turns. He benefits from having access to positive peer role models.

Measurable Academic / Functional Goal

[redacted] will increase appropriate interaction with peers by achieving the objectives listed below by the end of the IEP year.

Method of Measurement: Charting

Additional Methods of Measurement:

Date Achieved:

REQUIRED FOR STUDENTS PARTICIPATING IN ALTERNATE ASSESSMENT (LAA1)

MEASURABLE SHORT-TERM OBJECTIVES or BENCHMARKS (Number each objective or benchmark)

#	THE STUDENT WILL	Date Achieved
1	[redacted] will demonstrate cooperative play skills by engaging in a reciprocal activity with a peer for 5 minutes with no more than 2 prompts from an adult 3 out of 4 trials consistently within a nine week period over the IEP year.	
2	Given a social setting, (such as cafeteria/inclusion/recess/free time) respond to peer greeting by waving, making eye-contact or vocalizing with no more than 2 prompts from an adult in 3 of 4 trials consistenly within a nine week period over the IEP period.	
3	[redacted] will independently engage in 3 different appropriate play activities through modeling and prompting (ex. builing a house with legos, making animals out of playdough etc..) in 3 out of 4 trials within a nine week period over the IEP year.	

PERSONNEL RESPONSIBLE FOR IMPLEMENTING GOAL (Check by position)

☑ Special EducationTeacher ☑ Parent ☑ Speech/Language Pathologist ☐Regular EducationTeacher ☑ Student ☑ Adapted Physical Educator

☐ Other Related Service Providers (List)

☐ Other (List)

Captured from: http://www.hdc.lsuhsc.edu/lasard/laqiSocialInteraction.html

VITA

Reynalin Baricuatro was born in Cebu City, Philippines to Manuel Baricuatro and Erlinda Apinardo-Baricuatro. She received her Bachelor's Degree in Management Accounting from the University of San Carlos in March, 2004. She received her Diploma in Secondary Education majoring in Mathematics at St. Theresa's College in March, 2007 and her Diploma in Special Education from the University of the Visayas in March, 2008. She has taught for 7 years at the high school level. She currently teaches Math Inclusion in Algebra and Geometry and Study Skills at Broadmoor Senior High in Baton Rouge. Her passion is to support students with disabilities in obtaining knowledge in Mathematics in the general curriculum. She entered the Graduate School at Louisiana State University Agricultural and Mechanical College in June 2012.