Strength properties of granular materials

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STRENGTH PROPERTIES
OF
GRANULAR MATERIALS

A Thesis
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Louisiana State University and
Agricultural and Mechanical College
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in

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By
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To Sadi.
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ABSTRACT

This thesis presents the results of the experimental work conducted on glass beads in order to investigate the effects of particle size, confining pressure, and surface roughness on the strength properties of the particulate media. Conventional triaxial compression tests were conducted to investigate those effects. Three different sizes of beads were tested: small (diameter = 0.75 – 1.00 mm), medium (diameter = 1.55 – 1.85 mm), and large (diameter = 3.30 – 3.60 mm). The glass beads were subjected to three different confining pressures: 25-, 100-, and 400-kPa. Smooth and etched beads were tested; the etched surface was achieved by submerging the beads in a bath of Hydrofluoric acid. It was found that as the confining pressure increases, the peak stress ratio decreases. Also, it was found that an increase in roughness produces an increase in the peak friction angle. The particle size was found to affect the stress-strain and volumetric strain behavior of the beads; however, a specific trend was not found.
CHAPTER 1
INTRODUCTION

1.1 Problem Statement

A granular material is an assembly of particles; its mechanical behavior depends on the size and shape of the particles, their arrangement, particle-to-particle friction, associated pore spaces, and the degree of saturation. When deformations occur in granular materials, the external forces may cause internal fabric changes, caused by particles sliding, rolling, and interlocking. Those changes will produce a different response of the material behavior. Understanding of such material response is very important in the design of structures such as retaining walls, foundations systems, and dams, because the analyses of these systems are based on the strength and deformation behavior of the material beneath or adjacent to them.

Granular materials are typically investigated using sand. However, there are many questions related to the basic understanding of the friction phenomena such as the effects of particle shape and surface roughness. The literature lacks a systematic experimental investigation that addresses these basic concepts. The experimental work discussed in this thesis was performed utilizing uniform spherical glass beads with known particle shape, size, and surface texture. The objectives were to investigate the effects of particle size, surface texture, and confining pressure in the strength and deformation properties of granular materials.

1.2 Scope of Work

A series of conventional triaxial compression experiments were performed on glass beads to study the effects of particle size, surface texture, and confining pressure on
the strength and deformation properties of granular materials. That type of test was selected because it is the most commonly used to determine the stress-strain properties of soils; therefore, the data acquired can be compared with a large number of other experiments performed with different types of soils. The sizes of beads used were labeled as Small (S, diameter = 0.75 – 1.00 mm); Medium (M, diameter = 1.55 – 1.85 mm); and Large (L, diameter = 3.30 – 3.60 mm). Two different surface textures were tested: smooth and etched. The etched surface was achieved by washing the beads in Hydrofluoric acid. The specimens were tested at three different confining pressures: 25-, 100-, and 400-kPa.

This thesis begins with a literature review (Chapter 2) of the strength properties of granular materials. The description of the glass beads, the technique used to modify their surface texture, and the roughness analyses performed with an optical surface profiler are presented and discussed in Chapter 3. Furthermore, Scanning Electron Microscope images of the beads are presented to show the surface’s roughness at a higher magnification level. In addition, the chapter presents a description of the particle size distribution, specific gravity, and the tests performed to quantify those physical properties of the glass beads.

A description of the equipment used, specimen preparation method, and testing procedure are presented in Chapter 4. Also, a summary of the tests performed is reported. Then, Chapter 5 presents the results obtained from the experimental work. The stress-strain response of the specimens are described, followed by the discussion of the effects of confining pressure, particle size, and surface roughness. A comparison of the
constant volume friction angle calculated from the tests’ data and the ones estimated with Rowe’s solution is also discussed.
CHAPTER 2
STRENGTH PROPERTIES OF GRANULAR MATERIALS

2.1 Friction

The shear resistance between two particles is indirectly quantified through measuring the force required to cause movement between the particles. There are two common approaches to express the frictional resistance, namely to use the friction coefficient or the friction angle. Figure 2.1 presents a sliding block model, if $N$ is the normal force acting on a block, then the maximum shear force, $T_{\text{max}}$, required to slide the block along the surface is $T_{\text{max}} = \mu N$, where $\mu$ is the coefficient of friction between the block and the surface. The second alternative is better explained with the help of Figure 2.2. The friction angle, $\phi_\mu$, can be obtained from the plot of a series of relations of a normal force, $N$, acting on a body versus a shear force, $T$, required to produce the sliding of that body.

![Figure 2.1 Friction coefficient between two surfaces.](image)

$T_{\text{max}} = \mu N$

$\mu = \text{friction coefficient}$

Figure 2.1 Friction coefficient between two surfaces.
There are two basic laws that describe the frictional behavior between two bodies. Those laws were first stated by Leonardo da Vinci in the late 1400’s and then were restated by Amontons in 1699; thus they are known as the Amontons’ laws. The Amontons’ laws establish the following:

- The shear resistance between two bodies is proportional to the normal force between them.
- The shear resistance between two bodies is independent of the size of the bodies.

In granular materials different sources contribute to the frictional resistance, including: sliding and rolling of the particles, resistance to volume change, particle interlocking, and particle crushing. The friction mechanisms of sliding and rolling, resistance to volume change, and particle interlocking can be explained with the help of Figure 2.3. The sliding of the particles in a granular material is not as simple as the sliding of a block over a surface as presented in Figure 2.1; it is more complicated. In a granular material the particles are interlocked as presented in the left side of Figure 2.3;
the particles are in contact one to each other. This interlocking between the particles is directly related to the material density, as denser the material, the greater the interlocking between the particles. If shear stresses are applied to the particles, first a resistance to volume change is developed, followed by the sliding of the particles relative to each other. Then, for a dense specimen the particles will roll up and over each other producing an increase in the volume of the assembly of particles. On the contrary, in a loose specimen the particles will roll down, producing a decrease of volume. Figure 2.3 shows those volume changes which are known as dilatancy effects and will be discussed in more detail in the section 2.5.

![Diagram showing volume changes in granular materials](image)

Figure 2.3 Illustration of volume changes in granular materials subjected to shearing.

Finally, the mechanism of particle crushing occurs at very high stresses. This mechanism increases the frictional resistance between particles because the smaller particles created produce the rearrangement of the assembly of particles. The
rearrangement of the particles produces a denser material, thus increasing the particle interlocking.

2.2 Triaxial Testing

The most common test used to determine the stress-strain properties of a soil is the triaxial test. Figure 2.4 shows a diagram of the triaxial test layout. In this test, a cylindrical specimen, encased in a rubber (latex) membrane, is placed inside a chamber (triaxial cell) that is usually filled with water. At first, the specimen is confined by compressing the water in the cell; then, the specimen is subjected to axial stress until failure.

![Diagram of the triaxial test layout.](image)

Figure 2.4 Diagram of the triaxial test layout.

The application of the axial stress can be performed in one of two ways: by applying dead weights or hydraulic pressure in equal increments until failure (stress-controlled); or, by applying axial deformation at a constant rate by means of a geared or hydraulic loading press.
A triaxial test can be performed under either one of following conditions: drained or undrained. If a drained test is performed, the volume change of the specimen is measured by the amount of water that comes in or out of the cell. If an undrained test is performed, then the changes in pore water pressure inside the specimen are measured.

The common form of triaxial test is the conventional triaxial compression (CTC) test. This test involves loading the specimen in the axial direction while maintaining a constant confining pressure ($\sigma_c$). Based on the assumption that no shear stresses occur at the end platens, $\sigma_c$ and the axial stress: $\sigma_a = \sigma_c + (F_a/A)$ can be taken as the major ($\sigma_1$) and minor ($\sigma_3$) principal stresses, respectively. Figure 2.5 shows the state of stress on a cylindrical specimen.

![Diagram showing the forces and stresses acting on a specimen subjected to CTC.](image)

$\sigma_c + F_a / A = \sigma_1 + \Delta\sigma_a = \sigma_1$

Figure 2.5 Illustration of the forces and stresses acting on a specimen subjected to CTC.

The equations used to analyze the results of a CTC test are:

- **deviator stress**, $q = \sigma_1 - \sigma_3 \quad \text{Equation (2.1)}$

- **axial strain**, $\varepsilon_a = \frac{\Delta h}{h_0} \quad \text{Equation (2.2)}$
volumetric strain, $\varepsilon_v = \frac{\Delta V}{V_0}$  Equation (2.3)

Where: $\Delta h$ and $h_0$ are the change in height and initial height of the specimen, respectively; and, $\Delta V$ and $V_0$ are the change in volume and initial volume of the specimen, respectively.

2.3 Stress-Strain Behavior of Granular Materials During CTC

Figure 2.6 shows the typical behavior of two specimens of granular material subjected to CTC at the same confining pressure. The difference between the specimens is the density, one is dense and the other is loose.

Figure 2.6 Typical behavior of dense and loose granular material specimens subjected to conventional triaxial compression.
The behavior of the dense specimen can be divided into three different stages: initial (before peak), peak, and constant volume (post peak regime). In the initial stage a small decrease in volume is observed. The particles are being pushed together, reaching a denser state. This stage extends to a very low percentage of axial strain. The specimen fails in the peak stage; the deviator stress \( q = \sigma_1 - \sigma_3 \) at the peak is called the shear strength of the material. In this stage an increase in the volume of the specimen occurs because the particles move laterally due to the shear forces. The drop of the strength just after the peak occurs because as the shearing continues, the interlocking between the particles decreases, and consequently, the shear force required to produce that movement also decreases. In the constant volume stage, the particles reached a state of interlocking where shear deformations can occur without any change in volume. From that stage the constant volume friction angle of the material can be calculated; however, when this stage is not reached, it can be estimated as indicated in Figure 2.6 with the dashed line.

The behavior of the loose specimen is different than the behavior of the dense one. For the loose specimen the curve of deviator stress versus axial strain does not show a peak. The deviator stress remains constant once it reaches the maximum shear strength. At the initial stage, the loose specimen also exhibits a volume decrease, but that volume change also remain constant once the maximum shear strength is reached. An explanation to this behavior is that due to the lower density of the specimen, the interlocking between the particles is small; therefore, less shear resistance and no increase in volume is observed.

In general, the stress-strain behavior of granular materials is dependent on their fabric, which includes particle arrangement, size and shape, and surface roughness. The
fabric of a granular material determines the level of interlocking between the particles. The higher the particle interlocking, the higher is the friction resistance between the particles.

2.4 Mohr-Coulomb Theory

The shear strength of soils is usually represented using Mohr-Coulomb theory represented by Mohr circles. Those circles represent the state of stresses of a soil specimen in the plane that contains the major ($\sigma_1$) and minor ($\sigma_3$) principal stresses. If Mohr circles of different specimens of the same material subjected to different confining stresses are drawn together, then the friction angle of the material can be estimated from the slope of the line tangent to the circles, known as failure envelope (Figure 2.7). However, depending on the engineering problem under consideration, either the peak friction angle or the constant volume friction angle is needed. Those angles can be calculated using the Mohr-Coulomb failure criterion:

$$\sigma_1' = \sigma_3' \left( \frac{1 + \sin \phi}{1 - \sin \phi} \right) + 2c' \left( \frac{\cos \phi}{1 - \sin \phi} \right)$$  \hspace{1cm} \text{Equation (2.4)}

For granular materials $c' = 0$, hence the peak friction angle can be calculated as:

$$\phi_p = \sin^{-1} \left( \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} \right)_p$$  \hspace{1cm} \text{Equation (2.5)}

Where the values of $\sigma_1'$ and $\sigma_3'$ are taken at the peak.

Similarly, the constant volume friction angle can be calculated as:

$$\phi_{cv} = \sin^{-1} \left( \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} \right)_{cv}$$  \hspace{1cm} \text{Equation (2.6)}

Where the values of $\sigma_1'$ and $\sigma_3'$ are taken at the constant volume stage.
Figure 2.7 Mohr circles representing the state of stress of three different specimens of the same cohesionless soil (i.e., c=0) subjected to different confining pressures.

2.5 Dilatancy

Dilatancy can be defined as the volume change associated with the application of shear stresses. An increase in volume, or expansion, is known as positive dilation, while a decrease in volume, or contraction, is known as negative dilation. The amount of dilatancy that a granular material can experience is dependent on the particle interlocking, which depends on the fabric of the material. Dilatancy can be estimated from the volumetric strain versus axial strain curve of a material subjected to CTC with the following expression (as stated by Bolton (1986) for plane strain conditions and later derived by Schanz and Vermeer (1996) for triaxial test conditions):

$$\psi = \sin^{-1}\left(\frac{\varepsilon_v}{\gamma}\right) = \sin^{-1}\left[-\left(\frac{\varepsilon_v}{\varepsilon_a}\right)\sqrt{2 + \left(\frac{\varepsilon_v}{\varepsilon_a}\right)}\right]$$  \hspace{1cm} \text{Equation (2.7)}
2.6 Rowe’s Stress-Dilatancy Theory

The stress-dilatancy relation proposed by Rowe (1962) states that the peak friction angle ($\phi'_p$) can be represented as a contribution of three different factors: sliding resistance at particle contacts ($\phi_\mu = \text{angle of interparticle friction}$), particle rearrangement, and dilation. Figure 2.8 shows the contribution of those factors. Rowe proposed an expression that states that the ratio of the work done by the driving stress to the work taken by the driven stress for any strain increment is a constant. That constant, $K$, is related to a soil friction angle ($\phi'_f$):

$$K = \frac{1 + \sin \phi'_f}{1 - \sin \phi'_f} \quad \text{Equation (2.8)}$$

Where $\phi_\mu \leq \phi'_f \leq \phi'_c$, and $\phi_\mu$ and $\phi'_c$ are the interparticle and constant volume friction angles, respectively.

![Figure 2.8 Contributions to shear strength of granular materials (modified from Rowe, 1962; as presented by Mitchell, 1993).](image-url)
For triaxial compression Rowe’s relation states:

\[ K = \frac{\sigma_u \delta e_u}{-2\sigma_r \delta e_r} \quad \text{Equation (2.9)} \]

For plane strain Rowe’s relation states:

\[ K = \frac{\sigma_1 \delta e_1}{\sigma_3 \delta e_3} \quad \text{Equation (2.10)} \]

The constant \( K \) for triaxial compression is supposed to be the same for plane strain.

From the Mohr circle of effective stresses the following expression can be obtained:

\[ \frac{\sigma_1'}{\sigma_3} = \frac{1 + \sin \phi_m'}{1 - \sin \phi_m'} \quad \text{Equation (2.11)} \]

Where \( \phi_m' \) is the mobilized friction angle.

In the same way, from the Mohr circle of strains:

\[ \frac{-\delta e_1}{\delta e_3} = \frac{1 - \sin \psi}{1 + \sin \psi} \quad \text{Equation (2.12)} \]

Where \( \psi \) is the dilatancy angle.

Then, substituting \( \phi_f' \) by \( \phi_{cv}' \) in Equation (2.8) and combining it with Equations 2.10, 2.11, and 2.12, Rowe’s expression turns into:

\[ \sin \phi_m' = \frac{\sin \phi_{cv}' + \sin \psi}{1 + \sin \phi_{cv}' \sin \psi} \quad \text{Equation (2.13)} \]
CHAPTER 3
MATERIALS CHARACTERIZATION

3.1 Introduction

This chapter describes the particulate materials used in the experimental work presented in this thesis. The procedure used to modify the surface texture of the materials; along with a description of the surface roughness analyses that were performed using the Vertical Scanning-Interferometry mode of an optical surface profiler are also reported. Additionally, the chapter presents a description of the physical properties and the tests performed to quantify those physical properties of the particulate material used.

3.2 Materials Description

The materials used in the experimental program were glass beads obtained from Jaygo Incorporated, Union, New Jersey. They are composed of silicon dioxide (72%), sodium dioxide (13%), calcium oxide (9%), magnesium oxide (4%), aluminum oxide (1%), and potassium and ferrum oxide (1%).

Three different sizes of glass beads were used in the investigation. They are labeled as Small (S, diameter = 0.75 – 1.00 mm); Medium (M, diameter = 1.55 – 1.85 mm); and Large (L, diameter = 3.30 – 3.60 mm). Some of the beads were washed in hydrofluoric acid to produce a rough surface texture; therefore, six different types of beads were used in the investigation.

3.3 Surface Roughness Modification

The glass beads come with a shiny smooth surface texture; to achieve a rough texture, they were put in a bath of 15% Hydrofluoric acid for an hour and were stirred
occasionally. Then, the beads were rinsed with distilled water and were dried in the oven. Figure 3.1 shows those steps.

Figure 3.1 Etching procedure steps: (a) submerge the beads in the acid; (b) stir the beads to achieve a uniform etching; (c) first rinse with distilled water; (d) second rinse with distilled water.

3.3.1 Surface Roughness Analysis

To ensure that the surface of the beads was etched with the acid, they were analyzed using a Wyko Optical Interferometer (WOI) manufactured by Veeco Metrology Group which works with the Vision 32 software. In the preparation for the analysis, the beads were glued to a glass slide and then were coated with gold to allow the reflection of the light of the interferometer on the beads’ surface. The WOI has two operating techniques: Phase-Shifting Interferometry (PSI) and Vertical Scanning-Interferometry
(VSI). As stated in the Vision 32 software Manual, the basic interferometric principle is the same for both techniques: “a light beam reflected from a reference mirror combines with a light beam reflected from a specimen to produce interference fringes, where the interference fringes are best seen at focus”. The main difference between the two techniques is that in the VSI mode the light is not filtered and the system measures the degree of fringe modulation (coherence) while in the PSI mode, the light is filtered and the system measures the phase shift of the interference fringes (Wyko Surface Profilers Technical Reference Manual, 1999). The PSI mode is used on smooth, continuous surfaces due to its high lateral resolution while the VSI is used on rougher surfaces where high vertical resolution is needed. For the characterization of the glass beads the VSI mode was used because it gives better vertical resolution to analyze the surface of the beads, which was the main purpose of this analysis.

When the best-contrast fringes are obtained, the surface is scanned while an interference signal for each point in the surface is recorded; this interference signal is demodulated by means of computer algorithms and the vertical position of that point is extracted (Wyko Surface Profilers Technical Reference Manual, 1999). Although the system includes the Vision 32 software, which calculates some roughness indices, the analysis for this investigation was performed using the MathCad software (Alshibli and Alsaleh, 2003). The reason for this is that by using MathCad, the region to be analyzed can be selected while the Vision 32 software analyzes every single point included in the scan. In this investigation only the central part of the surface scanned was analyzed to avoid errors due to the curvature of the beads. Figure 3.2 shows an image obtained from one of the beads; the roughness calculations were performed from the central part of the images.
The roughness indices were calculated relative to a mean reference surface which is the line that runs centrally through the peaks and valleys of the profile (dividing the profile equally above and below the line). The calculated roughness indices were:

- **Average roughness** ($R_a$): overall roughness of the surface.
  \[
  R_a = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} |Z_{ij}| \quad \text{Equation (3.1)}
  \]
  \[
  Z_{ij} = \lambda L_{ij} \quad \text{Equation (3.2)}
  \]

  where: $M$ and $N$ are the number of pixels in the $x$ and $y$ directions, respectively; $Z$ is the surface height at a specific pixel relative to the mean reference surface; $\lambda$ is the wavelength used for the scan; and, $L$ is the wave value for specific coordinates at the particle surface.

- **Root-mean-squared roughness** ($R_q$): standard deviation of the surface heights.
  \[
  R_q = \sqrt{\frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} Z_{ij}^2} \quad \text{Equation (3.3)}
  \]

- **Maximum profile peak height** ($R_p$): height difference between the highest point and the mean reference surface.
- **Maximum profile valley depth** ($R_v$): height difference between the lowest point and mean reference surface.
- **Maximum height of the surface** ($R_t$): height difference between the highest and the lowest points on the surface.
- **Skewness** ($R_{sk}$): represents the symmetry of the surface about the mean reference surface.
- **Kurtosis** ($R_{ku}$): represents the sharpness about the mean reference surface.
Figure 3.2 Example of an image obtained using the WOI.

3.3.2 Statistical Analysis of the Surface Roughness

The surface of approximately 50 beads of each type was analyzed using the WOI. The previously discussed roughness indices were calculated and a statistical analysis of those indices was performed. Tables 3.1 through 3.6 present a summary of the statistical analyses.

Table 3.1 Summary of statistical analysis for small smooth beads.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Roughness Index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_a (\mu m) )</td>
</tr>
<tr>
<td>Mean</td>
<td>0.212</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.014</td>
</tr>
<tr>
<td>Median</td>
<td>0.186</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.101</td>
</tr>
<tr>
<td>Range</td>
<td>0.469</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.095</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.564</td>
</tr>
</tbody>
</table>

Table 3.2 Summary of statistical analysis for small etched beads.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Roughness Index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_a (\mu m) )</td>
</tr>
<tr>
<td>Mean</td>
<td>0.875</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.111</td>
</tr>
<tr>
<td>Median</td>
<td>0.615</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.778</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.184</td>
</tr>
</tbody>
</table>
Table 3.3 Summary of statistical analysis for medium smooth beads.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( R_a (\mu m) )</th>
<th>( R_q (\mu m) )</th>
<th>( R_p (\mu m) )</th>
<th>( R_v (\mu m) )</th>
<th>( R_t (\mu m) )</th>
<th>( R_{sk} )</th>
<th>( R_{ku} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.186</td>
<td>0.223</td>
<td>0.582</td>
<td>0.582</td>
<td>1.165</td>
<td>-0.419</td>
<td>2.648</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.011</td>
<td>0.012</td>
<td>0.029</td>
<td>0.029</td>
<td>0.059</td>
<td>0.120</td>
<td>0.116</td>
</tr>
<tr>
<td>Median</td>
<td>0.173</td>
<td>0.211</td>
<td>0.560</td>
<td>0.560</td>
<td>1.121</td>
<td>-0.835</td>
<td>2.550</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.074</td>
<td>0.083</td>
<td>0.205</td>
<td>0.205</td>
<td>0.410</td>
<td>0.843</td>
<td>0.814</td>
</tr>
<tr>
<td>Range</td>
<td>0.318</td>
<td>0.330</td>
<td>0.892</td>
<td>0.892</td>
<td>1.784</td>
<td>2.868</td>
<td>4.399</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.085</td>
<td>0.104</td>
<td>0.261</td>
<td>0.261</td>
<td>0.523</td>
<td>-1.709</td>
<td>1.463</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.403</td>
<td>0.433</td>
<td>1.153</td>
<td>1.153</td>
<td>2.307</td>
<td>1.158</td>
<td>5.862</td>
</tr>
</tbody>
</table>

Table 3.4 Summary of statistical analysis for medium etched beads.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( R_a (\mu m) )</th>
<th>( R_q (\mu m) )</th>
<th>( R_p (\mu m) )</th>
<th>( R_v (\mu m) )</th>
<th>( R_t (\mu m) )</th>
<th>( R_{sk} )</th>
<th>( R_{ku} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.506</td>
<td>1.734</td>
<td>3.364</td>
<td>3.364</td>
<td>6.729</td>
<td>0.462</td>
<td>1.948</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.120</td>
<td>0.134</td>
<td>0.241</td>
<td>0.241</td>
<td>0.481</td>
<td>0.087</td>
<td>0.061</td>
</tr>
<tr>
<td>Median</td>
<td>1.273</td>
<td>1.444</td>
<td>2.919</td>
<td>2.919</td>
<td>5.837</td>
<td>0.645</td>
<td>1.849</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.828</td>
<td>0.931</td>
<td>1.666</td>
<td>1.666</td>
<td>3.333</td>
<td>0.605</td>
<td>0.419</td>
</tr>
<tr>
<td>Range</td>
<td>3.538</td>
<td>3.997</td>
<td>7.860</td>
<td>7.860</td>
<td>15.720</td>
<td>2.331</td>
<td>1.625</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.354</td>
<td>0.453</td>
<td>0.838</td>
<td>0.838</td>
<td>1.675</td>
<td>-1.124</td>
<td>1.260</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.893</td>
<td>4.450</td>
<td>8.698</td>
<td>8.698</td>
<td>17.395</td>
<td>1.206</td>
<td>2.885</td>
</tr>
</tbody>
</table>

Table 3.5 Summary of statistical analysis for large smooth beads.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( R_a (\mu m) )</th>
<th>( R_q (\mu m) )</th>
<th>( R_p (\mu m) )</th>
<th>( R_v (\mu m) )</th>
<th>( R_t (\mu m) )</th>
<th>( R_{sk} )</th>
<th>( R_{ku} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.240</td>
<td>0.288</td>
<td>0.773</td>
<td>0.773</td>
<td>1.547</td>
<td>-0.026</td>
<td>2.842</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.016</td>
<td>0.018</td>
<td>0.034</td>
<td>0.034</td>
<td>0.067</td>
<td>0.117</td>
<td>0.130</td>
</tr>
<tr>
<td>Median</td>
<td>0.203</td>
<td>0.250</td>
<td>0.749</td>
<td>0.749</td>
<td>1.498</td>
<td>-0.150</td>
<td>2.657</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.116</td>
<td>0.126</td>
<td>0.238</td>
<td>0.238</td>
<td>0.476</td>
<td>0.828</td>
<td>0.916</td>
</tr>
<tr>
<td>Range</td>
<td>0.473</td>
<td>0.556</td>
<td>1.162</td>
<td>1.162</td>
<td>2.324</td>
<td>2.557</td>
<td>4.196</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.095</td>
<td>0.124</td>
<td>0.397</td>
<td>0.397</td>
<td>0.795</td>
<td>-1.315</td>
<td>1.297</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.569</td>
<td>0.680</td>
<td>1.559</td>
<td>1.559</td>
<td>3.119</td>
<td>1.242</td>
<td>5.493</td>
</tr>
</tbody>
</table>
Table 3.6 Summary of statistical analysis for large etched beads.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Roughness Index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_a$ ($\mu$m)</td>
</tr>
<tr>
<td>Mean</td>
<td>0.561</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.038</td>
</tr>
<tr>
<td>Median</td>
<td>0.515</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.266</td>
</tr>
<tr>
<td>Range</td>
<td>1.143</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.135</td>
</tr>
<tr>
<td>Maximum</td>
<td>1.278</td>
</tr>
</tbody>
</table>

The following discussion is related to the values obtained for $R_a$ only. In the case of the small beads, the calculated mean value for the smooth beads is 0.21 $\mu$m, while it is 0.87 $\mu$m for the etched beads. This represents a roughness increase of approximately 314%. The standard deviation calculated for the smooth beads is 0.10 $\mu$m, which is small comparing with the value of 0.78 $\mu$m calculated for the etched beads.

For the medium beads, the mean value is 0.18 $\mu$m for the smooth beads while it is 1.50 $\mu$m for the etched beads; this represents an increase of surface roughness of approximately 733%. The standard deviations calculated vary from 0.07 $\mu$m for the smooth beads to 0.83 $\mu$m for the etched beads.

Finally, for the large beads, the calculated means are 0.24 and 0.56 $\mu$m for the smooth and etched beads, respectively. This represents an approximate 133% of roughness increase. The standard deviations are 0.12 $\mu$m for the smooth beads and 0.27 $\mu$m for the etched beads.

The high values of standard deviation of the etched beads represent a wider range in the surface roughness measured, proving that the surface of the beads was etched with the acid. These observations are confirmed with the images taken with the Scanning Electron Microscope (SEM) at a high magnification level (Figures 3.3 through 3.8).
An important observation from the SEM images is that the surface of the beads was not etched uniformly. There are places rougher than others. That non-uniformity could influence the calculated roughness parameters because only a small part of the surface was analyzed.

Figures 3.9 through 3.14 also show frequency distribution histograms of the roughness indices $R_a$ and $R_q$ for the analyzed beads. From them it can be inferred that all the types of beads show a positively skewed distribution. A positively skewed distribution means that the median is lower than the mean roughness value; hence, more than 50% of the roughness values are lower than the mean roughness value. Additionally, a wider range of roughness is observed for all the etched beads than for the smooth ones, showing again the non-uniformity of the particles’ surface roughness.

Figure 3.3 SEM images of the small smooth beads.
Figure 3.4 SEM images of the small etched beads.

Figure 3.5 SEM images of the medium smooth beads.
Figure 3.6 SEM images of the medium etched beads.

Figure 3.7 SEM images of the large smooth beads.
Figure 3.8 SEM images of the large etched beads.
Figure 3.9 Frequency distribution histograms for the roughness indices (a) $R_a$ and (b) $R_q$ of the small smooth beads.
Figure 3.10 Frequency distribution histograms for the roughness indices (a) $R_a$ and (b) $R_q$ of the small etched beads.
Figure 3.11 Frequency distribution histograms for the roughness indices (a) $R_a$ and (b) $R_q$ of the medium smooth beads.
Figure 3.12 Frequency distribution histograms for the roughness indices (a) $R_a$ and (b) $R_q$ of the medium etched beads.
Figure 3.13 Frequency distribution histograms for the roughness indices (a) $R_a$ and (b) $R_q$ of the large smooth beads.
Figure 3.14 Frequency distribution histograms for the roughness indices (a) $R_a$ and (b) $R_q$ of the large etched beads.
3.4 Materials Properties

3.4.1 Particle Size Analysis

Particle size analyses (ASTM D 422-63) were performed to check the uniformity of the glass beads. The amount of material to be sieved was selected based on the particle size as recommended by Bardet (1997); approximately 100 grams were used for the small and the medium beads, while around 200 grams were used for the large beads.

The particle size distribution curves for the beads are presented in Figures 3.15 and 3.16 for the smooth and etched beads respectively. It can be noticed that the majority of the particles have nearly the same size. The uniformity coefficient ($C_u$) was calculated from the particle size distribution curve of each type of bead (Table 3.7). The values obtained range from 1.10 to 1.32, which indicates a high degree of uniformity. It seems that the etching procedure does not affect the particles uniformity; the $C_u$ values obtained for the etched beads were the same or almost the same as for the smooth beads.

![Particle size distribution curves of the smooth beads.](image-url)

Figure 3.15 Particle size distribution curves of the smooth beads.
Figure 3.16 Particle size distribution curves of the etched beads.

Table 3.7 Coefficient of uniformity (Cu) values for the beads used in the investigation.

<table>
<thead>
<tr>
<th>Type of Bead</th>
<th>Uniformity Coefficient, Cu</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Smooth</td>
<td>1.22</td>
</tr>
<tr>
<td>Small Etched</td>
<td>1.25</td>
</tr>
<tr>
<td>Medium Smooth</td>
<td>1.32</td>
</tr>
<tr>
<td>Medium Etched</td>
<td>1.32</td>
</tr>
<tr>
<td>Large Smooth</td>
<td>1.10</td>
</tr>
<tr>
<td>Large Etched</td>
<td>1.10</td>
</tr>
</tbody>
</table>

3.4.2 Specific Gravity

The specific gravity (Gs) was determined for the six different types of beads for comparison purposes according to ASTM D 854-92. The product information sheet indicates a Gs of 2.5; the values obtained, as presented in Table 3.8, vary from 2.50 to 2.60, which are very close to the specified value. Therefore, the specific gravities measured in the laboratory for each type of beads were used in this investigation.
Table 3.8 Measured specific gravities ($G_s$) for the different types of beads.

<table>
<thead>
<tr>
<th>Type of Bead</th>
<th>$G_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Smooth</td>
<td>2.50</td>
</tr>
<tr>
<td>Small Etched</td>
<td>2.50</td>
</tr>
<tr>
<td>Medium Smooth</td>
<td>2.59</td>
</tr>
<tr>
<td>Medium Etched</td>
<td>2.60</td>
</tr>
<tr>
<td>Large Smooth</td>
<td>2.55</td>
</tr>
<tr>
<td>Large Etched</td>
<td>2.55</td>
</tr>
</tbody>
</table>
CHAPTER 4
EXPERIMENTAL WORK

4.1 Introduction

A series of drained conventional triaxial compression tests were performed on the six types of glass beads. The parameters varied in those tests were particle size, surface texture, and confining pressure. This chapter presents a description of the equipment used to perform those tests, the procedure for specimen preparation and testing, and a summary of the tests performed.

4.2 Equipment Description

The testing equipment utilized for this investigation consists of the following main parts: triaxial cell, loading frame, pressure control panel, and data acquisition system. The triaxial cell used is a conventional axisymmetric one with attached bottom end platen of 71 mm (2.8 in) in diameter. The loading frame is strain-controlled and with a load cell and a Linear Variable Differential Transducer (LVDT) attached to it. Two different load cells were used, one with a maximum load capacity of 1.11-kN (250 lbs), used for the 25- and 100-kPa of confining pressure tests, and other with a 11.12-kN (2500 lbs) load capacity, used for the 400-kPa tests. Also, two different pressure control panels were used for the experimental work. The one presented in Figure 4.1 has a pressure reservoir with the capacity to apply up to 200-kPa of cell pressure; it was used for the 25- and 100-kPa confining pressure tests. It contains the regulators for the cell pressure and the vacuum. In the back, it has two Differential Pressure Transducers (DPT) with capacities of 0.8 and 13.8-kPa. The pressure reservoir consists of two cylinders with deaired water, one inside another. The inner cylinder is the volume change reservoir
while the outer is used to apply the cell pressure. To apply the cell pressure, the top of the reservoirs is pressurized with air by the desired amount. During the test, the DPTs measure the changes in cell pressure caused by changes in the volume of the specimen. When the specimen expands, water comes out of the cell to the inner cylinder and vice versa. These changes in pressure are converted to volume from the water level difference between the inner and outer cylinders. The purpose of using two DPTs is to allow the system to register a wider range of volume changes. For the tests of 400-kPa of confining pressure a control panel with a higher pressure capacity was required. Figure 4.2 shows the control panel used for those tests which has a pressure reservoir with a maximum capacity of 550-kPa. This control panel contains the cell pressure regulator and a DPT; when using this control panel, the vacuum was regulated from the 200-kPa pressure control panel. The pressure reservoir in this control panel consists of two burettes, one next to the other with the same working principle as previously discussed.

Figure 4.1 Pressure control panel with capacity of 200-kPa; used for the 25- and 100-kPa confining pressure tests.
Finally, the data acquisition system consists of a data acquisition unit and a computer with LabView software. The data acquisition unit registers the changes in voltage of the instruments during the test and sends that signal to the computer via a data acquisition card. A LabView application program was custom built to control the experiments, acquire the data, and display primary results as the test proceeds. Figure 4.3 shows the computer with the LabView program acquiring the data for one of the experiments.
4.3 Specimen Preparation

The specimens were prepared in a 71 mm (2.8 in) diameter aluminum split mold. A cylindrical latex membrane is first attached to the bottom end platen using an o-ring; then, the mold is placed around that platen and the membrane is stretched along its inside. Approximately 20-kPa of vacuum needs to be applied from the outside of the mold to keep the membrane aligned to it. Figure 4.4 (a) shows the stretched membrane along the inside of the mold. The mold is then filled with the beads by layers of approximately 2.54 cm (1 in) thick. After a layer is deposited, the beads are plunged with a plastic rod to densify them (Figure 4.4 (b)); this method was selected in order to get more consistency in the void ratio values than with the dry pluviation method. When the mold is completely filled with the beads, the top end platen is attached to it with another o-ring and approximately 25-kPa of vacuum is applied to the inside of the specimen to prevent disturbance. Then, the mold is removed and the cell jacket is put in place along with the top cell plate and the loading ram. Figure 4.5 shows a prepared specimen in the triaxial cell ready to be tested.

![Figure 4.4 (a) Latex membrane stretched along the inside of the mold. (b) Densifying the beads with a plastic rod.](image-url)
4.4 Equipment Preparation

Once the cell is completely assembled, it is placed in the loading frame. The cross beam that has the load cell and the one that has the LVDT are adjusted to the desired height. The cell is then filled with water. Figure 4.6 shows the complete set up of the triaxial cell in the loading frame. Then, the desired cell pressure is applied while reducing the vacuum to avoid confining the specimen to a higher pressure than the desirable test confining pressure. Finally, the vacuum is removed, the specimen is vented, and the test can be started.

4.5 CTC Testing Procedure

All tests were performed under drained conventional triaxial compression (CTC) conditions, where the radial confining pressure ($\sigma_r = \sigma_2 = \sigma_3$) was kept constant while the axial load was increased at a constant displacement rate of 1 mm/min up to approximately 25% of the nominal axial strain.
The axial displacement of the triaxial cell relative to the load frame is measured by the LVDT; the load exerted on the specimen is measured by the load cell; and, the changes in pressure inside the cell produced by the volume changes in the specimen are measured by the DPT. The changes in voltage produced by those instruments are recorded every second by the data acquisition unit which sends the signal to the computer and LabView converts it to the desired engineering units.

4.6 Tests Performed

A series of drained CTC tests were performed on the glass beads with different test parameters of particle size, surface texture, and confining pressure. Table 4.1 presents the tests performed by their designated name along with the void ratio (\(e\)) and relative density (\(D_r\)). The designation of the names is as follows: size (S = small; M = medium; L = large), surface texture (S = smooth; E = etched), and the confining pressure is in kPa.
Table 4.1 Summary of experiments performed.

<table>
<thead>
<tr>
<th>Name</th>
<th>$e_{\text{initial}}$</th>
<th>$D_r^*$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS25-1</td>
<td>0.587</td>
<td>58.42</td>
</tr>
<tr>
<td>SS25-2</td>
<td>0.586</td>
<td>58.60</td>
</tr>
<tr>
<td>SS25-3</td>
<td>0.585</td>
<td>58.77</td>
</tr>
<tr>
<td>SS100-1</td>
<td>0.598</td>
<td>56.49</td>
</tr>
<tr>
<td>SS100-2</td>
<td>0.567</td>
<td>61.93</td>
</tr>
<tr>
<td>SS400-1</td>
<td>0.586</td>
<td>58.60</td>
</tr>
<tr>
<td>SS400-2</td>
<td>0.585</td>
<td>58.77</td>
</tr>
<tr>
<td>SE25-1</td>
<td>0.594</td>
<td>57.19</td>
</tr>
<tr>
<td>SE25-2</td>
<td>0.600</td>
<td>56.14</td>
</tr>
<tr>
<td>SE100-1</td>
<td>0.620</td>
<td>52.63</td>
</tr>
<tr>
<td>SE100-2</td>
<td>0.608</td>
<td>54.74</td>
</tr>
<tr>
<td>SE400-1</td>
<td>0.622</td>
<td>52.28</td>
</tr>
<tr>
<td>SE400-2</td>
<td>0.615</td>
<td>53.51</td>
</tr>
<tr>
<td>MS25-1</td>
<td>0.573</td>
<td>60.88</td>
</tr>
<tr>
<td>MS25-2</td>
<td>0.586</td>
<td>58.60</td>
</tr>
<tr>
<td>MS100-1</td>
<td>0.604</td>
<td>55.44</td>
</tr>
<tr>
<td>MS100-2</td>
<td>0.596</td>
<td>56.84</td>
</tr>
<tr>
<td>MS400-1</td>
<td>0.595</td>
<td>57.02</td>
</tr>
<tr>
<td>MS400-2</td>
<td>0.596</td>
<td>56.84</td>
</tr>
<tr>
<td>ME25-1</td>
<td>0.630</td>
<td>50.88</td>
</tr>
<tr>
<td>ME25-2</td>
<td>0.638</td>
<td>49.47</td>
</tr>
<tr>
<td>ME100-1</td>
<td>0.643</td>
<td>48.60</td>
</tr>
<tr>
<td>ME100-2</td>
<td>0.642</td>
<td>48.77</td>
</tr>
<tr>
<td>ME400-1</td>
<td>0.653</td>
<td>46.84</td>
</tr>
<tr>
<td>ME400-2</td>
<td>0.650</td>
<td>47.37</td>
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<td>LS25-1</td>
<td>0.624</td>
<td>51.93</td>
</tr>
<tr>
<td>LS25-2</td>
<td>0.611</td>
<td>54.21</td>
</tr>
<tr>
<td>LS100-1</td>
<td>0.618</td>
<td>52.98</td>
</tr>
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<tr>
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* Calculated with $e_{\text{max}} = 0.92$ and $e_{\text{min}} = 0.35$ as stated in Soil Mechanics, Design Manual 7.01 of the Naval Facilities Engineering Command (1986); Coduto (1999) after Hough (1969); and, Lambe and Whitman (1969) after Hough (1957).
CHAPTER 5
EXPERIMENTAL RESULTS

5.1 Introduction

This Chapter describes the results of the triaxial compression experiments that were conducted to investigate the effects of surface roughness, confining pressure, and particle size on the strength properties of granular materials (i.e. glass beads). As presented in Chapter 3, at least two conventional triaxial compression (CTC) tests were performed for each combination of test conditions (bead size, surface roughness, and confining pressure) to check repeatability; the results of all tests are presented in the Appendix. They are presented in two forms: the ratio of the principal stresses ($\sigma_1/\sigma_3$) versus the axial strain, and the volumetric strain versus the axial strain. Volume change increase (dilation) is taken as negative and vice versa. Most experiments were run until 25% of the nominal axial strain; however, some experiments were terminated at smaller axial strains due to excessive expansion, which caused damage to the latex membrane that encased the specimen.

5.2 Stress-Strain and Volumetric Strain Behavior

The results of the CTC tests for the six different types of glass beads and at the different confining pressures are shown in Figures 5.1 through 5.6. The stress-strain and volumetric strain versus axial strain results of the small smooth beads are presented in Figure 5.1. In the stress-strain curves of the three different tests, after the peak is reached, a slightly pronounced post peak softening is observed; once the peak stress is reached, the principal stress ratio level out smoothly until the critical state condition is reached. However, only the volumetric strain versus axial strain curve of the 100-kPa
test shows a constant volume at the end, confirming that the critical state condition was reached. The volumetric strain versus axial strain curves of the 25- and 400-kPa tests show a continue increase in the volume change throughout the test. In the 25- and 100-kPa confining pressure tests, load oscillations are observed just before the peak stress ratio and continued until the end of the test.

Figure 5.2 shows the response of the small etched beads. Contrary to the tests with small smooth beads, these exhibit a more pronounced peak stress ratio (more post peak softening) at the different confining pressures; though, those curves do not show the critical state condition. However, the critical state condition is observed in the volumetric strain versus axial strain curves for the 25- and 100-kPa tests. In the 400-kPa test, the specimen exhibited a continuous increase of volume change. In these tests, load oscillations are observed after the peak stress ratio (in the softening and critical state stages) on the 25- and 100-kPa tests, but they have a smaller amplitude than in the small smooth beads (Figure 5.1).

The behavior of the medium smooth beads is shown in Figure 5.3. A pronounced peak stress is not observed in any of the tests; after the maximum principal stress ratio is reached, it decreases very smoothly until the critical state condition is almost reached. The reach of the critical state condition is confirmed by the volumetric strain versus axial strain curve of the 25-kPa test where the volume change became nearly constant at the end. In the other two tests (100- and 400-kPa) a continuing volume increase is observed at the end.

The stress-strain curves of the tests with medium etched beads (Figure 5.4) show a more pronounced post peak softening than the tests performed with the medium smooth
beads (Figure 5.3). Based on those curves, the critical state condition is reached by the 25- and 400-kPa tests. The 100-kPa test had to be stopped at approximately 16% of axial strain due to leaking of water through the membrane, thus it is not possible to observe the critical state condition in that curve. However, the volumetric strain versus axial strain curves for the 25- and 100- kPa indicates that the critical state condition is reached for those tests because a constant volumetric strain is observed at the end. On the other hand, in the test at 400-kPa a continuous volume increase is observed. In these tests, small load oscillations are observed in the softening regime and critical state stages of the 400-kPa experiment (ME400-1).

Figure 5.5 shows the response of the large smooth beads. A pronounced post peak softening is observed for the 100–kPa test, as opposed to the 25- and 400-kPa where stress ratio remains essentially constant after its maximum value is reached. In contrast, in the volumetric strain versus axial strain curves, the only test that confirms that the critical state condition is reached is the 100-kPa where constant volume strain is observed at the end of the test. In the other two tests (25- and 400-kPa) the volume continues increasing until the end of the test. In these tests, load oscillations are observed just before the peak stress ratio and throughout the rest of the test in the 25- and 100-kPa tests.

Finally, the stress-strain and volumetric strain versus axial strain curves of the large etched beads is presented in Figure 5.6. These tests do not show a post peak softening, once the maximum stress ratio is reached, it remains constant or almost constant (100-kPa test) throughout the test, showing that the critical state condition is reached, or close to being reached as in the case of the 100-kPa test. On the other hand,
no constant volumetric strain is observed in any of the volumetric strain versus axial strain curves; the volumetric strains continue to increase throughout the tests. Small load oscillations were observed in the 400-kPa test starting at the peak stress ratio and continuing until the end of the test.

Two phenomena were observed in some of these tests: load oscillations and continued volume increase even at very high strains. The load oscillations were thought to be caused by noise signals; nevertheless, after careful investigation of the oscillation patterns, that hypothesis was eliminated. The oscillations did not follow a specific pattern; they were random (Figure 5.7). Therefore, other possible reason can be the stick-slip phenomenon. That is, the beads stick to each other and then suddenly collapse, that sudden movement produces the drops of the load (Albert et al., 2000). In addition, the long straight vertical lines observed in the oscillations pattern are data points collected at the same percent strain due to the larger sampling resolution in relation with the deformation rate.

The continuous volume increase observed even at high strains appears to be caused by the uniform shape of spherical particles and their uniformity (one size). Due to these two parameters the interlocking among the particles is minimum, thus with further shearing, the particles are going to continue rolling over each other producing the continuous volume increase.
Figure 5.1 Stress-strain and volumetric strain versus axial strain responses of the small smooth beads at the different confining pressures.
Figure 5.2 Stress-strain and volumetric strain versus axial strain responses of the small etched beads at the different confining pressures.
Figure 5.3 Stress-strain and volumetric strain versus axial strain responses of the medium smooth beads at the different confining pressures.
Figure 5.4 Stress-strain and volumetric strain versus axial strain responses of the medium etched beads at the different confining pressures.
Figure 5.5 Stress-strain and volumetric strain versus axial strain responses of the large smooth beads at the different confining pressures.
Figure 5.6 Stress-strain and volumetric strain versus axial strain responses of the large etched beads at the different confining pressures.
5.2.1 Effects of Confining Pressure and Surface Texture

The effects of the confining pressure and surface texture on the stress-strain and volumetric strain versus axial strain behavior of the glass beads can be explained with the help of Figures 5.8 to 5.10. Figure 5.8 shows the behavior of the small smooth and etched beads subjected to the different confining pressures. It can be observed that, for both smooth and etched beads, as the confining pressure increases the peak stress ratio of the beads decreases. Also, the change in volume decreases as the confining pressure increases in the smooth beads; however, in the etched ones, the change in volume is almost the same at the three different confining pressures. The etched beads exhibits higher peak stress ratio than the smooth beads for each one of the three different confining pressures. This proves that the increase in surface roughness produces a greater friction resistance among the beads. The change in volume for the smooth beads
at 25-kPa of confining pressure was the highest, followed by all the etched beads (almost the same change in volume), and then by the 100- and 400-kPa smooth beads.

The behavior of the medium smooth and etched beads at the different confining pressures is shown in Figure 5.9. For the two different surface roughnesses it is observed that the peak stress ratio decreases as the confining pressure increases. Moreover, for both smooth and etched, the volume change decreases as the confining pressure increases from 25-kPa to 100-kPa. However, when the confining pressure increases from 100-kPa to 400-kPa, the volume change is insignificant. Additionally, it can be observed that the peak stress ratio and volume change of the etched beads are higher than the ones for the smooth beads. Again, it was proved that an increase in surface roughness produces higher friction resistance among the particles.

Figure 5.10 shows the behavior of the large smooth and etched beads at the different confining pressures. For both surface textures, the peak stress ratio decreases as the confining pressure increases. Also, the volume change decreases as the confining pressure increases, but only in the etched beads. In the smooth beads, the test subjected to a confining pressure of 100-kPa shows a higher volume change, followed by the 25-kPa and then by the 400-kPa. The peak stress ratio and volume change of the etched beads is higher than that of the smooth ones for the confining pressures of 25- and 400-kPa. Conversely, at a confining pressure of 100-kPa, the smooth and etched beads exhibited the same peak stress ratio, but the smooth beads show a higher volume change than the etched ones.
Figure 5.8 Small smooth and etched beads subjected to different confining pressures (a) 25-kPa, (b) 100-kPa, (c) 400-kPa.
Figure 5.9 Medium smooth and etched beads subjected to different confining pressures (a) 25-kPa, (b) 100-kPa, (c) 400-kPa.
Figure 5.10: Large smooth and etched beads subjected to different confining pressures: (a) 25-kPa, (b) 100-kPa, (c) 400-kPa.
5.3 Peak Friction and Dilatancy Angles

The peak friction angles ($\phi'_p$) and dilatancy angles ($\psi$) were calculated for all the experiments and are presented in Table 5.1 together with the initial and final void ratios, and the average surface roughness ($R_a$). The values of $\phi'_p$ were calculated as follows:

$$\phi'_p = \sin^{-1}\left(\frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3}\right)$$  Equation (2.5)

The average $\phi'_p$ value for all the specimens tested was 27.97°. The dilatancy ($\psi$) angle was calculated from the volumetric strain versus axial strain curves. It was calculated from the steepest portion of the curves (Figure 5.11), using Equation 2.7:

$$\psi = \sin^{-1}\left[-\left(\frac{\varepsilon_v}{\varepsilon_a}\right)\left(2 + \left(\frac{\varepsilon_v}{\varepsilon_a}\right)\right)\right]$$  Equation (2.7)

The calculated average value of $\psi$ for all the specimens was 7.97°.

5.3.1 Effects of Confining Pressure

The variation of the average $\phi'_p$ and $\psi$ angles (average taken from all the tests at the same conditions) with confining pressure is shown in Figures 5.12 and 5.13, respectively. In Figure 5.12 it can be noted that $\phi'_p$ decreases as $\sigma'_3$ increases for the tests with small etched and large smooth and etched beads. This proves that as the confining pressure increases, the dilatancy tendency of the material decreases thus producing the decrease of the shear strength. For the tests with small smooth and medium smooth and etched beads, $\phi'_p$ decreases as $\sigma'_3$ increases from 25- to 100-kPa, but then with further increase of $\sigma'_3$ it remains constant. This indicates that a confining pressure of 100-kPa was enough for the particles to reach their maximum density;
therefore, an increase in confining pressure will not produce further particle arrangement (the peak friction angle will remain constant) although crushing occurs.

In the case of $\psi$, different trends are observed. For the tests with medium etched beads, $\psi$ decreases as $\sigma'_{3}$ increases. In the tests with small smooth beads, $\psi$ decreases as $\sigma'_{3}$ increases from 25- to 100-kPa, but with further increase of $\sigma'_{3}$, it increases to a value close to the one at 25-kPa. On the contrary, for the small etched and large smooth tests, $\psi$ increases as $\sigma'_{3}$ increases from 25- to 100-kPa, but then when $\sigma'_{3}$ increases to 400-kPa, it decreases to a value close to the one at 25-kPa. Moreover, for the tests with medium smooth and large etched beads, $\psi$ remains essentially constant as $\sigma'_{3}$ increases.

![Figure 5.11 Illustration of where the dilatancy angles were calculated.](image)
Table 5.1 Peak friction and dilatancy angles of the experiments.

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* Calculated with $e_{max} = 0.92$ and $e_{min} = 0.35$ as stated in Soil Mechanics, Design Manual 7.01 of the Naval Facilities Engineering Command (1986); Coduto (1999) after Hough (1969); and, Lambe and Whitman (1969) after Hough (1957).
Figure 5.12 Effect of the confining pressure on peak friction angle.

Figure 5.13 Effect of the confining pressure on dilatancy angle.
5.3.2 Effects of Particle Size

Figures 5.14 and 5.15 show the variation of the average \( \phi'_p \) and \( \psi \) with particle size, respectively. Different patterns are observed in Figure 5.14 for the variation of \( \phi'_p \). In the smooth beads tested at 25- and 100-kPa of confining pressure, \( \phi'_p \) decreases as the particle size increases from small to medium, but then it increases again as the size increases from medium to large; the value of \( \phi'_p \) for the large beads is approximately the same as the value for the small (i.e., S>M<L and S≈L). For the smooth beads tested at 400-kPa, it is observed that \( \phi'_p \) decreases as the size increases from small to medium, but then remain essentially constant with further particle size increase (S>M=L).

The tests with the etched beads at the three different confining pressures show a similar behavior as the one with smooth beads at 25- and 100-kPa. The difference is that \( \phi'_p \) increases instead of decreasing as the particle size increases from small to medium. The pattern is: S<M>L where S≈L. A possible reason for this behavior is the amount and type of roughness achieved for each size of beads; this will be discussed in the next section.

In terms of \( \psi \), it decreases as the size increases from small to medium in the smooth beads tested at 25- and 400-kPa; but when the size increases from medium to large, \( \psi \) remains essentially constant. In the tests with smooth beads at 100-kPa, \( \psi \) decreases as the size increases from small to medium; however, when the particle size increases to large, \( \psi \) increases to a value higher than the one for the small beads.

In the tests with the etched beads the same pattern is observed; \( \psi \) increases as the particle size increases from small to medium, but when it increases from medium to small, \( \psi \) decreases to a value lower than the one for the small beads. These trends are
also showed in Figures 5.16 and 5.17 in terms of the stress-strain and volumetric strain versus axial strain results of the three different sizes of beads at each confining pressure.

Figure 5.14 Effect of particle size on peak friction angle.

Figure 5.15 Effect of particle size on dilatancy angle.
Figure 5.16 Small, medium, and large smooth beads subjected to different confining pressures (a) 25-kPa, (b) 100-kPa, (c) 400-kPa.
Figure 5.17 Small, medium, and large etched beads subjected to different confining pressures (a) 25-kPa, (b) 100-kPa, (c) 400-kPa.
5.3.3 Effects of Surface Roughness

The variation of the average $\phi'$ and $\psi$ with mean surface roughness ($R_a$) is shown in Figures 5.18 and 5.19. In those figures it can be observed a pronounced increase of the $\phi'$ and $\psi$ values as $R_a$ increases for the tests with the medium beads at the three different confining pressures. In the tests with small beads, an increase in both angles as $R_a$ increases is observed only in the tests at 100-kPa. In the large beads an increase of $\phi'$ and $\psi$ is observed in the tests at 25- and 400-kPa.

The medium beads attained the highest mean surface roughness after being washed in the Hydrofluoric acid; that is represented in the $\phi'$ and $\psi$ values which are the highest for the medium etched beads at each confining pressure. On the contrary, the small and large etched beads exhibit similar values of $\phi'$ at all the confining pressures although their $R_a$ values differ for more than 50%. A possible reason for this is the difference in the mode of etching that each size of bead attained. For example, as shown in the Scanning Electron Microscope (SEM) images presented in Figure 5.20 and in Chapter 3 (Figures 3.3 to 3.8), the different sizes of beads washed in the hydrofluoric acid attained a different pattern of roughness. In the small beads the acid produced a porous surface, while in the medium ones it created a surface with cleavage; in the large beads, just a little change in the surface roughness was observed and was caused by peeling. Therefore, a possible reason for that behavior of the small and large beads is that independently of the value of $R_a$, each different mode of etching can produce different amounts of friction between the beads. Additionally, as also observed in the SEM images and previously discussed, the roughness of the beads surface was not uniform; some places were more etched than others. That factor could influence the behavior of the
beads because the contact points between particles could be points with different roughnesses.

Moreover, as indicated in Table 5.1, the void ratio values of the small and medium smooth beads are lower than the ones of the etched beads for each respective size. This is not observed for the large beads. The reason for this is that the etched surface creates more friction between the particles and makes it more difficult to densify them. That difference in void ratio is not observed in the large beads because both smooth and etched surfaces have almost the same roughness.

Figure 5.18 Effect of surface roughness on peak friction angle.
5.4 Constant Volume Friction Angle

The constant volume friction angle ($\phi_{cv}$) was calculated from the curves of principal stress ratio versus axial strain using Equation 2.6:

$$\phi_{cv} = \sin^{-1} \left( \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} \right)_{cv}$$  \text{Equation (2.6)}
For the tests were the constant volume stage was not reached, an approximation of $\phi_{cv}$ was calculated from the last portion of the curve, where the slope was becoming horizontal. Additionally, Rowe’s stress-dilatancy expression (Equation 2.13) was used to estimate the constant volume friction angle; deriving Equation 2.13 for $\phi_{cv}$:

$$
(sin \phi_{cv})_R = \left( \frac{\sin \psi_m - \sin \phi'_m}{\sin \psi_m \sin \phi'_m - 1} \right)
$$

Equation 5.1

Where $\psi_m$ and $\phi'_m$ are the mobilized friction and dilatancy angles throughout the tests.

Table 5.2 presents the calculated and estimated (Rowe’s solution) $\phi_{cv}$ for some of the tests. Figure 5.21 shows the variation $\phi_{cv}$ with confining pressure. The calculated $\phi_{cv}$ showed higher values than the estimated with Rowe’s solution. Also, it can be observed that all the calculated $\phi_{cv}$ lie above the particle-to-particle friction angle, $\phi_{\mu} = 17^\circ$ (Rowe, 1962), that is not the same for the $\phi_{cv}$ estimated with Rowe’s solution. Therefore, Rowe’s solution is found to underestimate the $\phi_{cv}$ values.

![Figure 5.21 Constant volume friction angles versus confining pressure.](image-url)
Table 5.2 Calculated and estimated constant volume friction angle.

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<th>$\phi_{cv}^*$ (°)</th>
<th>$\phi_{cv}$ (°) from Rowe's Solution</th>
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*Values calculated from last portion of principal stress ratio versus axial strain curve when constant volume stage was not reached.
CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

6.1 General Conclusions

6.1.1 Surface Roughness Modification

It was found that the purpose of producing a rougher texture in the beads’ surface was achieved by washing them in Hydrofluoric acid. However, the roughness achieved was not uniform and each size of beads attained a different mode of etching.

6.1.2 Triaxial Testing Results

Load oscillations were found in some of the tests maybe due to the stick-slip phenomenon. Also, continuous volume change were observed at the end of some tests; this appears to be caused by the uniform shape of spherical particles and their uniformity that produced a continuous rolling of the beads over each other.

Tests showed that, as the confining pressure increases, the dilatancy tendency of the beads decreases producing a decrease in the peak stress ratio.

The surface roughness was found to affect the behavior of the glass beads. It was found that the etched glass beads exhibited higher peak stress ratio and volume changes than the smooth beads.

The particle size was found to affect the stress-strain and volumetric strain behavior of the beads. However, a specific trend was not found.

6.1.3 Constant Volume Friction Angle

Rowe’s solution was found to underestimate the constant volume friction angles.
6.2 Future Work Recommendations

Based on the findings of the experimental work performed for this thesis, the following recommendations are presented:

- Increase the quality control in the etching procedure in order to obtain a uniform roughness in the surface of the glass beads and the same mode of etching among the different beads’ sizes.

- Further investigation is needed to better understand the load oscillation phenomenon.

- Perform conventional triaxial compression tests with a mixture of different sizes of beads to compare the difference in particle interlocking between the uniform size particles and the mixture.
REFERENCES


APPENDIX
STRESS-STRAIN AND VOLUMETRIC STRAIN
BEHAVIOR OF SPECIMENS TESTED
Figure A.1 Stress-strain behavior of small smooth beads at a confining pressure of 25-kPa.
Figure A.2 Stress-strain behavior of small smooth beads at a confining pressure of 100-kPa.
Figure A.3 Stress-strain behavior of small smooth beads at a confining pressure of 400-kPa.
Figure A.4 Stress-strain behavior of small etched beads at a confining pressure of 25-kPa.
Figure A.5 Stress-strain behavior of small etched beads at a confining pressure of 100-kPa.
Figure A.6 Stress-strain behavior of small etched beads at a confining pressure of 400-kPa.
Figure A.7 Stress-strain behavior of medium smooth beads at a confining pressure of 25-kPa.
Figure A.8 Stress-strain behavior of medium smooth beads at a confining pressure of 100-kPa.
Figure A.9 Stress-strain behavior of medium smooth beads at a confining pressure of 400-kPa.
Figure A.10 Stress-strain behavior of medium etched beads at a confining pressure of 25-kPa.
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Figure A.17 Stress-strain behavior of large etched beads at a confining pressure of 100-kPa.
Figure A.18 Stress-strain behavior of large etched beads at a confining pressure of 400-kPa.
VITA

Brenda Novoa-Martínez was born in Arecibo, Puerto Rico, on April 20, 1975. She attended Lorenzo Coballes-Gandía High School in Hatillo, Puerto Rico, graduating in 1993. From 1993 to 1998 she attended the University of Puerto Rico, Mayagüez Campus, where she obtained the degree of Bachelor of Science in Civil Engineering.

From 1998 to 2001 she worked as a Staff Engineer for the geotechnical consultant firm Luis O. García and Associates, Inc. in Guaynabo, Puerto Rico. In 2000, Brenda married Sadi Torres-Vechini, and in 2001 they moved to Baton Rouge, Louisiana, where Brenda joined the graduate program in Geotechnical Engineering at the Louisiana State University, Baton Rouge, Louisiana. In July 2002, she was awarded the NASA Graduate Students Researchers Program Fellowship.

As a requirement for obtaining the degree of Master of Science in Civil Engineering she will be completing the thesis entitled: “Strength Properties of Granular Materials”.