Three-Dimensional Flow and Sediment Transport at River Mouths.

William Rheuben Waldrop

Louisiana State University and Agricultural & Mechanical College

Follow this and additional works at: https://digitalcommons.lsu.edu/gradschool_disstheses

Recommended Citation

https://digitalcommons.lsu.edu/gradschool_disstheses/2367
INFORMATION TO USERS

This dissertation was produced from a microfilm copy of the original document. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the original submitted.

The following explanation of techniques is provided to help you understand markings or patterns which may appear on this reproduction.

1. The sign or "target" for pages apparently lacking from the document photographed is "Missing Page(s)". If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting thru an image and duplicating adjacent pages to insure you complete continuity.

2. When an image on the film is obliterated with a large round black mark, it is an indication that the photographer suspected that the copy may have moved during exposure and thus cause a blurred image. You will find a good image of the page in the adjacent frame.

3. When a map, drawing or chart, etc., was part of the material being photographed the photographer followed a definite method in "sectioning" the material. It is customary to begin photoing at the upper left hand corner of a large sheet and to continue photoing from left to right in equal sections with a small overlap. If necessary, sectioning is continued again - beginning below the first row and continuing on until complete.

4. The majority of users indicate that the textual content is of greatest value, however, a somewhat higher quality reproduction could be made from "photographs" if essential to the understanding of the dissertation. Silver prints of "photographs" may be ordered at additional charge by writing the Order Department, giving the catalog number, title, author and specific pages you wish reproduced.

University Microfilms
300 North Zeeb Road
Ann Arbor, Michigan 48106
A Xerox Education Company
WALDROP, William Rheuben, 1939-
THREE-DIMENSIONAL FLOW AND SEDIMENT TRANSPORT
AT RIVER MOUTHS.

The Louisiana State University and Agricultural
and Mechanical College, Ph.D., 1972
Engineering, chemical

University Microfilms, A XEROX Company, Ann Arbor, Michigan

THIS DISSERTATION HAS BEEN MICROFILMED EXACTLY AS RECEIVED
Three-Dimensional Flow and Sediment Transport at River Mouths

A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Doctor of Philosophy in

The Department of Chemical Engineering

by

William R. Waldrop
B.S., Auburn University, 1961
M.S., University of Alabama in Huntsville, 1968
Eng., von Karman Institute for Fluid Dynamics, 1969
December, 1972
PLEASE NOTE:

Some pages may have
indistinct print.
Filmed as received.

University Microfilms, A Xerox Education Company
ACKNOWLEDGMENT

My first thanks go to my three girls, Marjorie, Amy and Joan, whose patience and understanding made this degree possible.

Much of the credit for this accomplishment should go to Drs. R. C. Farmer and J. C. Coleman for advice and guidance during this effort and for having the confidence in an aerodynamisist who had the audacity to enter two new fields at the PhD level.

The comments and suggestions of Drs. Bert Wilkins, Ralph Pike, Edward McLaughlin, Philip Bryant and Don Wright, my committee, have been invaluable.

I would like to extend thanks to Mrs. Eleanor S. Thibodeaux for typing and editing and to Mrs. Dunn for preparation of the figures. A Roques named Nancy also deserves thanks for computer programming assistance.

I am grateful for the financial assistance provided by the Coastal Studies Institute through Geography Programs, Office of Naval Research, under Contract N00014-69-A-0211-0003, Project NR 388002.

Finally, I would like to apologize to my dog, Duffy, who became an orphan as a result of my ambition.
ABSTRACT

Three dimensional flow processes resulting from a river emptying into the sea were analyzed. The basic equations which were derived to describe this flow included the effects of buoyancy caused by density differences between the fresh and salt water, inertia from the river and coastal currents, and differences in hydrostatic head throughout the mixing region. Turbulence effects were included through an appropriate eddy viscosity model. Combinations of river stages and tidal currents were represented as systems of steady state flow fields. A numerical procedure was developed and implemented on a digital computer for the solution of the equations. This numerical procedure is classified as an asymptotic time-dependent finite difference technique with certain features of a relaxation technique. Computed flow fields were used to track a distribution of nominal particles representing the suspended load of the river as determined from field data. Deposition of these particles was primarily governed by convective processes, and particle settling velocities which included the effect of a local turbulence level. Deposition rates were used to compute deltaic growth. Results were compared to a delta for which experimental data were available.
# Table of Contents

<table>
<thead>
<tr>
<th>Acknowledgment</th>
<th>..........................................................</th>
<th>i</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Tables</td>
<td>..............................................................................</td>
<td>vi</td>
</tr>
<tr>
<td>List of Figures</td>
<td>..............................................................................</td>
<td>v</td>
</tr>
<tr>
<td>Abstract</td>
<td>..............................................................................</td>
<td>ii</td>
</tr>
<tr>
<td>Chapter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I. Introduction</td>
<td>..............................................................................</td>
<td>1</td>
</tr>
<tr>
<td>II. Delta Models</td>
<td>..........................................................................</td>
<td>8</td>
</tr>
<tr>
<td>A. Current Models</td>
<td>.......................................................................</td>
<td>8</td>
</tr>
<tr>
<td>B. Assessment of Current Delta Models</td>
<td>..............................................</td>
<td>11</td>
</tr>
<tr>
<td>III. Governing Equations for Deltaic Flows</td>
<td>........................................</td>
<td>13</td>
</tr>
<tr>
<td>A. Properties of a Mixture of Salt and Fresh Water</td>
<td>................................</td>
<td>13</td>
</tr>
<tr>
<td>B. Pertinent Conservation Principles</td>
<td>.......................................................</td>
<td>19</td>
</tr>
<tr>
<td>C. Turbulent Mixing Near a River Mouth</td>
<td>..................................................</td>
<td>28</td>
</tr>
<tr>
<td>D. Nondimensionalization of the Governing Equations</td>
<td>................................</td>
<td>33</td>
</tr>
<tr>
<td>IV. Numerical Procedure for the Solution of Deltaic Flows</td>
<td>................................</td>
<td>38</td>
</tr>
<tr>
<td>A. Finite Difference Grid</td>
<td>................................................................</td>
<td>40</td>
</tr>
<tr>
<td>B. Finite Difference Equations</td>
<td>..................................................</td>
<td>45</td>
</tr>
<tr>
<td>C. Initial Conditions</td>
<td>.......................................................................</td>
<td>48</td>
</tr>
<tr>
<td>D. Boundary Conditions</td>
<td>...................................................................</td>
<td>53</td>
</tr>
<tr>
<td>E. Solution Sequence</td>
<td>..................................................................</td>
<td>59</td>
</tr>
<tr>
<td>F. Convergence</td>
<td>.......................................................................</td>
<td>61</td>
</tr>
<tr>
<td>G. Uniqueness</td>
<td>..............................................................................</td>
<td>62</td>
</tr>
<tr>
<td>H. Stability</td>
<td>..............................................................................</td>
<td>65</td>
</tr>
</tbody>
</table>
### CHAPTER

<table>
<thead>
<tr>
<th>V. TRANSPORTATION AND DEPOSITION OF SOIL NEAR THE RIVER MOUTH</th>
<th>page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Transportation of Soil Particles in the Vicinity of a River Delta</td>
<td>71</td>
</tr>
<tr>
<td>B. Numerical Analysis of Deposition Near River Mouths</td>
<td>75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VI. DISCUSSION OF RESULTS</th>
<th>page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Computed Flow Fields in Deltas</td>
<td>85</td>
</tr>
<tr>
<td>B. Deposition of Suspended Material</td>
<td>95</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VII. CONCLUSION</th>
<th>page</th>
</tr>
</thead>
<tbody>
<tr>
<td>-----------------</td>
<td>------</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VIII. RECOMMENDATIONS</th>
<th>page</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>REFERENCES</th>
<th>page</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>NOMENCLATURE</th>
<th>page</th>
</tr>
</thead>
</table>

### APPENDIX

<table>
<thead>
<tr>
<th>A. INCOMPRESSIBLE CONCEPTS FROM A MICROSCOPIC STANDPOINT</th>
<th>page</th>
</tr>
</thead>
<tbody>
<tr>
<td>B. LISTING OF COMPUTER PROGRAM FOR FLUID DYNAMICS</td>
<td>116</td>
</tr>
<tr>
<td>C. LISTING OF COMPUTER PROGRAM FOR CALCULATING THE DEPOSITION OF SUSPENDED PARTICLES</td>
<td>122</td>
</tr>
<tr>
<td>D. RESULTS OF THE FLUID DYNAMICS ANALYSIS PRESENTED AS ISOCONCENTRATION CONTOURS AND VELOCITY VECTORS</td>
<td>163</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VITA</th>
<th>page</th>
</tr>
</thead>
</table>

-iv-
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>South Pass of the Mississippi River</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>Density and Molar Variation with Salt Concentration</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>Density Variation of Fresh Water with Temperature</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>The Effect of Coriolis Effect Upon Fluid Movement</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>Coordinate System for Computation</td>
<td>41</td>
</tr>
<tr>
<td>6</td>
<td>Boundary Conditions for Delta Flow Calculations</td>
<td>55</td>
</tr>
<tr>
<td>7</td>
<td>Description of Actual River Deltas</td>
<td>56</td>
</tr>
<tr>
<td>8</td>
<td>Typical History of Convergence Process</td>
<td>63</td>
</tr>
<tr>
<td>9</td>
<td>Typical Convergence History of Velocity Components</td>
<td>64</td>
</tr>
<tr>
<td>10</td>
<td>Depth Contours on the South Pass Delta</td>
<td>84</td>
</tr>
<tr>
<td>11</td>
<td>Computed Deposition Pattern for a Maximum River Velocity of 2.5 m/sec and a Maximum Coastal Current of -0.5 m/sec</td>
<td>100</td>
</tr>
<tr>
<td>12</td>
<td>Velocity Vectors of Flow Over Deposited Material in a Series of End Views</td>
<td>102</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>General Conservation Laws for Deltaic Flows</td>
<td>27</td>
</tr>
<tr>
<td>2</td>
<td>Nondimensional Terms for a Cartesian Coordinate System</td>
<td>34</td>
</tr>
<tr>
<td>3</td>
<td>Nondimensionalized Basic Equations for Three-Dimensional Isothermal Deltaic Flow</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>Finite Difference Grid Spacing</td>
<td>46</td>
</tr>
<tr>
<td>5</td>
<td>Finite Difference Equations Governing the Fluid Dynamics</td>
<td>49</td>
</tr>
<tr>
<td>6</td>
<td>Run Log for Analysis of Delta Flow Fields</td>
<td>87</td>
</tr>
<tr>
<td>7</td>
<td>Sample Printout of Fluid Dynamics Computer Program</td>
<td>88</td>
</tr>
<tr>
<td>8</td>
<td>Particle Concentration Data for Deposition Calculations</td>
<td>97</td>
</tr>
</tbody>
</table>
CHAPTER I
INTRODUCTION

River deltas have played an important role in men's lives since the beginning of civilization. Here in fact is where most civilizations developed. This is understandable when considering that ships were the primary means of transportation and the delta region represents a bridge between the open ocean and inland waterways.

A river delta is one means by which a continent grows; it is an appendage protruding into the sea. If the river discharges into a sea where large waves or fast currents are common, then a river offing will probably be absorbed and distributed over a wide area at the discretion of the sea. Otherwise, many of the particles transported by the momentum of a river will settle near the river mouth as the river loses its directed momentum as a result of its conflict with the sea. This produces an arm of the continent which is gradually and persistently extended into the sea, until such time as the delta begins to slump down the edge of the continental slope.

In recent times, man has attempted to modify natural delta regions for various reasons. Among the most prominent of these reasons are to provide protection from floods and to make river mouths more amenable to shipping. Extensive knowledge of the natural development of the delta is required to efficiently and effectively perform these modifications. Unanticipated consequences
resulting from poorly planned modifications such as dredging, darning, or jettying are often costly, ineffective, and sometimes catastrophic. Gould (1972) discussed the strong criticism of just such hasty and incomplete planning of the one billion dollar system of dams currently under construction for flood protection in the Netherlands. Sometimes very extensive preliminary investigations are conducted. For example, between 1957 and 1959 the U.S. Army Engineers Waterways Experimental Station conducted seventy-four laboratory scale model tests of the mouth of the Southwest Pass of the Mississippi River as a prelude to jettying and dredging (Patin 1971). Obviously, the cost of such extensive efforts is prohibitive for every proposed modification of river mouths and harbors.

The importance of understanding deltaic development and flow patterns may also be emphasized from a military viewpoint. Quite likely, dredging of the channel will be spasmodic in times of heavy enemy pressure, and knowledge of silting rates of the channel would be desirable. Here also are stratified regions of turbid water, a likely place for submarines to lurk while waiting for ships to enter the river through a narrow channel with treacherous currents and large scale eddies. Knowing the location of such hiding places for a variety of tidal conditions and river stages would thus be desirable from a military standpoint.

It is not surprising that men have exhibited more than a casual interest in comprehending and predicting the dynamics of deltaic phenomena. Unfortunately, these systems include complex,
interacting subsystems with all components casually interrelated. Coleman and Wright (1971) discuss important processes involved in the development of river deltas and review other significant attempts at classifying these processes. The obvious conclusion from reviewing this literature was that there does not exist a typical river delta any more than there exists a typical cloud shape. It is only possible to classify deltas into broad categories for which certain processes are dominant.

The work of Coleman and Wright is part of a current effort by the Louisiana State University Coastal Studies Institute to investigate deltaic regions of the world. This program, which has been in progress for 13 years, is under the auspices of the Office of Naval Research, Geography Program. The study described in this dissertation to develop a mathematical computer model based upon basic physical principles is a portion of this program which was undertaken by the Chemical Engineering Department of Louisiana State University.

The initial stage of the approach presented herein for describing the fluid dynamics and sedimentation process in the delta region are not unlike several previous attempts which will be discussed later. The complexity of the system dictates some idealization as a complete analysis of all influencing parameters will probably never be possible for such a mathematical model. One can easily see that the region of analysis must be restricted to some arbitrary boundary surrounding the delta even though the impact of factors far upstream in the river system and many
kilometers into the open ocean will be felt at the delta. Conditions along these boundaries are considered as known. These boundary conditions can be simplified by specifying them far enough away from the mixing region of the river and sea water so that the interaction of two bodies of water will not influence the boundary conditions.

The equations describing the interaction of the flowing river water and the receiving salt water basin are the basic transport equations. Although these equations are well known, their application to each individual case and the subsequent solution sometimes permit simplifying assumptions about the system to be made. It is here that the analysis described in this report deviates from other approaches. It is customary to assume that the equations cannot be solved in their most general three-dimensional form; consequently, assumptions are made concerning the flow in one or two directions. Many researchers decide a priori that one or two forces or effects are dominant (i.e., buoyancy, viscous forces, inertial, etc.) and neglect the others. These assumptions greatly restrict the applicability of their results.

The approach presented here has more versatility than those of previous attempts as the three-dimensional equations were numerically solved with the aid of a digital computer. All terms deemed significant for controlling the mixing process for most cases of interest were retained in these equations to make the program more versatile. These significant terms include the
effects of buoyancy, inertia, viscosity, diffusion, and pressure. Preliminary investigations indicated that Coriolis effect was insignificant within the region of interest. Wave action from the open ocean was not included directly, but much of its effect can be incorporated into the model used for describing turbulent effects. Although wind shear at the surface was also neglected for these calculations, its future inclusion should not encompass much difficulty.

The philosophy of the solution technique is to represent the mixing region as a system of steady flow patterns. A combination of high and low tide with high and low river stages should provide an envelope for the resulting flow patterns near the mouth of the river. Because the equations are solved in their unsteady form, it would be possible to input periodic boundary conditions and obtain an unsteady solution.

Results of the fluid dynamics analysis are used to trace typical particle trajectories from known locations at the river mouth until they either settle on the bottom or are convected through the boundaries. Inherent in this analysis are the assumptions that interparticular forces are absent and the concentration of particles is sufficiently small that their presence has a negligible influence upon the momentum of the fluid. Effects of variable turbulence levels upon the particle settling velocities were included. The growth rate at specified grid points along the bottom is then computed by summing the particles settled within a given area surrounding that grid point.
The bottom shape is then adjusted to reflect the effect of this growth rate. Slumping and local scour velocities are considered, but subsidence of the bottom caused by the additional weight of the sediment is not. Once the bottom shape has been adjusted to represent a constant settling process over a given increment of time, the fluid dynamics program can be rerun to compute how the new bottom shape will influence the flow patterns. This procedure can be continued almost indefinitely.

Several cases are presented as examples of how the computer program may be used. Most of the comparisons with experimental data are of a qualitative nature partly because the geometry of actual cases is difficult to precisely duplicate, and partly because of a lack of quantitative data.

South Pass of the Mississippi River shown in Figure 1 was chosen as the delta for comparison because a continuous program of investigation of this delta by the Coastal Studies Institute made some quantitative data available. L D. Wright (1970) presents the results of field experiments obtained over a period of years. Aerial surveillance in conjunction with NASA also made shapes of the plume available under different conditions. Unfortunately, logistics problems and the physical size of the delta region limit the data available, even for South Pass. It is hoped that the computation procedure described here can be used in planning optimum regions for sampling in future field experiments.
Figure 1: South Pass of the Mississippi River
CHAPTER II
DELTA MODELS

For centuries man has strived to describe flow from river mouths. The models discussed below by no means represent all such attempts, but represent the current state-of-the-art. After reviewing these models, an assessment of the overall state of delta modeling is presented. The general conclusion from this assessment is that a more comprehensive approach founded upon basic physical principles is needed if computed flows in delta regions are to agree with observed phenomena.

A. CURRENT MODELS

Most attempts at describing the flow of a river into a bay have been based on the approach of neglecting terms of the general flow equations until these equations could be solved in closed form. Among these attempts are those which assume that the momentum loss of the river water is accomplished by turbulent dissipation across free-shear layers as originally developed by Tollmien (1926). Bates (1953) postulated this behavior and Bonham-Carter and Sutherland (1968) formulated a delta growth model using this supposition. In this model experimentally observed vertical profiles of the seaward component of river water were assumed to exist and to be known a priori; therefore, two-dimensional, balanced (i.e. symmetrical) jet profiles in various horizontal planes were used to represent the flow field.
Shear between these two-dimensional planes as well as other
effects in the vertical direction such as vertical turbulence
and buoyancy were neglected.

Fox (1971) presented a slightly different variation of the
same theme. His investigation of a turbulent buoyant jet
operating in a linearly stratified fluid used an integral re­
presentation of two-dimensional governing equations. An
entrainment velocity into the jet was used to vary the density.
A Gaussian velocity distribution within the plume was also
assumed. Dependent variables were computed numerically. This
approach is very similar to a technique used by Hirst (1971).

Tamai (1969) formulated the problem in a more general form
than Fox or Hirst. However, at a crucial point he reverted to
the same logic as far as integrating his equations in the vertical
direction, using an entrainment constant, and assuming a functional
relationship for axial velocity which he honestly admitted did
not match his experimental data. This report does provide
valuable insight and is worth reviewing.

Another treatment of river water flowing into sea water
was presented by Borichansky and Mikhailov (1966). They assumed
that the main forces affecting the flow field were:

1. The friction forces on the lateral surfaces of the
current.

2. The friction forces on the bottom.

3. Inertial terms.
Other influences were neglected. The differential equation upon which their solution is based describes a one-dimensional flow field similar to that of flow through a pipe. Friction forces are represented as coefficients which are to be determined experimentally. Results are in the form of mean velocities which are expressed as a function of distance, geometry, and friction coefficients.

Takano (1954) used the equations for creeping motion to model turbulent, river-plume flow. Although this model is qualitatively reasonable, there is no justification for dropping the inertial terms from the equations of motion. Takano (1955) later modified his solution by including Coriolis effects. Conceptually, only viscous type velocity profiles with very low velocities can be predicted with this model. There is no reason to believe that this is adequate. Variations in the vertical direction are assumed, i.e., three-dimensional equations are not solved, even without inertia terms.

Bondar (1970) hypothesized that hydrostatic pressure differences resulting from buoyancy of the lighter fresh water flowing over the salt water was the primary cause of plume spreading. Wright and Colemen (1971) improved this theory somewhat by including the effect of interfacial waves, and a resulting upward entrainment of salt water into the plume. This led to a reduction of the mean velocity and to an increase in the density of the plume, two factors omitted from Bonders original paper. Theoretical data computed by this method compared
reasonably well with experimental data at South Pass of the Mississippi during low river stage, a time when the effect of buoyancy should be greatest.

B. ASSESSMENT OF CURRENT DELTA MODELS

Several conclusions may be drawn from an analysis of these delta models. Assumptions which simplify the conservation equations are commonly used in order to obtain solutions. This is not necessarily bad, but the choice of which terms can be neglected should be justified on physical bases and not merely for convenience. From the prominent models reviewed, it appears that the dominant forces have not been clearly isolated. Indeed, certain influences are dominant under some conditions, whereas they are negligible under others. Few of the models reviewed specified the assumptions implied by the simplified equations, which were solved; consequently, they gave an impression of far greater applicability than justified.

All of these models fail to be satisfactory because they lack a mechanism for allowing the forces which result from surrounding sea water flows to correctly position the river plume discharge, i.e. they cannot predict simultaneous turbulent dissipation and reaction to pressure forces.

None of the models correctly include the effect of hydraulic head (river stage) on the plume shape. The models involving turbulent dissipation could partially simulate this by varying
the turbulence levels, but this is not an accurate appraisal of the inertial effect due to increasing the surface stage. This effect should increase the spreading of the plume at least near the surface.

The general conclusion from the models reviewed is that all of the models are at best very restricted in their applicability. Many of the models are based upon unjustified and sometimes incorrect premise; the value of these results is questionable. For a model to be practical, then it must include more generality than any of the current models afford and yet not be too expensive to use.
CHAPTER III
GOVERNING EQUATIONS FOR DELTAIC FLOWS

All descriptions of fluid dynamics phenomena are based upon conservation principles. These principles, usually expressed as equations that conserve mass, momentum, and energy, are presented for simple fluids in most standard texts of fluid mechanics; for actual delta flows a more inclusive set of equations is necessary. The flow phenomena of interest here are those involving the relative motion and the mixing of a flowing river of fresh water with a flowing receiving basin of salt water which is not necessarily at the same temperature. Both fluids may be considered incompressible, which means that density is independent of pressure.

Fundamental conservation principles will now be used to develop a rather general set of equations which are applicable for describing deltaic flows. Justifiable assumptions will then be discussed and the governing equations which were solved will be presented.

A. PROPERTIES OF A MIXTURE OF SALT AND FRESH WATER

Because the fluid in the mixing region of the two bodies of water will have two components, sea water and fresh water, some care must be exercised to insure that the most useful form of the pertinent conservation laws is determined. The equations governing the dynamics of both compressible and constant density
fluids are well established, but there appears to be much ambiguity concerning the equations of a two-component fluid which may be considered incompressible. In an effort to clarify these equations, let us look at some properties of an aqueous solution of NaCl (salt), a reasonable facsimile of sea water which contains not only NaCl, but also a weak concentration of a host of other elements. Data in Figure 2 from the Handbook of Chemistry and Physics show that density increases linearly with salt concentration over the range of density of interest. Further analysis of these data indicate that the number of moles of salt and water per unit volume remains constant, even though the salt concentration is increased. This implies a replacement process in which one molecule of water is expelled from the volume for every molecule of salt added. Rearrangement of the molecules within the solution thus has a negligible effect upon the density over this range of salt concentration. Furthermore, by introducing a fictitious molecular weight for salt water, the fluid could be described as a binary mixture of fresh water molecules and salt water molecules.

An important result of the property of constant moles in a given volume is manifested in the statement of the volume dilatation, often used synonomously with a statement of incompressibility. This expression is derived in standard texts of fluid mechanics (e.g., pp. 47-51 of Schlichting 1960) for a one component fluid as

$$\nabla \cdot \mathbf{V} = 0,$$

(3-1)
Figure 2: Density and Molar Variation with Salt Concentration

Figure 3: Density Variation of Fresh Water with Temperature
but how should this velocity vector $\bar{V}$ be defined for multiple component fluids? The definition of the velocity required to specify a constant volume of an incompressible multicomponent isothermal fluid must be based upon the number of moles or molecules of the fluid and independent of the density of each mole, because the total number of moles per unit volume will remain constant even though the density may vary. Stated in a slightly different way, the substantial derivative of the number density $n$ (i.e., the number of molecules per unit volume) will be zero,

$$\frac{\partial n}{\partial t} + \bar{V} \cdot \nabla n = 0, \quad (3-2)$$

only if $\bar{V}$ is defined as a number average velocity. This velocity, often referred to as the molar average velocity, is defined for a binary mixture as

$$\bar{V} = \frac{n_1 q_1 + n_2 q_2}{n_1 + n_2} \quad (3-3)$$

where

$n_1, n_2 =$ number of molecules per unit volume of the mixture, of components 1 and 2 respectively

$q_1, q_2 =$ mean velocity vectors of molecules of components 1 and 2 respectively

The number average velocity differs from the often used mass average velocity defined as
\[
U = \frac{n_1 m_1 q_1 + n_2 m_2 q_2}{n_1 m_1 + n_2 m_2}
\]

\[
= \frac{\rho_1 q_1 + \rho_2 q_2}{\rho_1 + \rho_2}
\]

(3-4)

where:

- \(m_1, m_2\) = mass per molecule of components 1 and 2 respectively
- \(q_1, q_2\) = mean velocity vectors of molecules of components 1 and 2 respectively
- \(\rho_1, \rho_2\) = partial density of components 1 and 2 respectively.

Yih (1960) makes reference to this distinction in defining the volume dilatation, but presents little justification of it, saying that it can be put on a firmer basis from a molecular standpoint. This is done in Appendix A by using a microscopic approach to analyze a binary component, isothermal fluid by a procedure similar to that presented by Williams (1965).

Temperature effects should also be considered because there is no assurance that the river water will be at the same temperature as the ocean. Figure 3 on page 15 presents data from the Handbook of Chemistry and Physics for the density variation of fresh water as a function of temperature. Comparing this figure with the density variation with salt concentration of Figure 2 reveals that if significant salinity differences exist,
these completely overshadow temperature effects upon density for reasonable temperature differences of river and sea water. It was for this reason that the flows in the delta region were considered isothermal, but for the sake of completeness temperature effects will be discussed.

Using the data from Figures 2 and 3, density variation may be approximated as

$$\rho = \rho_f + (\frac{\partial \rho}{\partial S})_T S + (\frac{\partial \rho}{\partial T})_{T, \text{ref}} (T - T_{\text{ref}})$$

(3-5)

where

- \(\rho_f\) = density of fresh water at some reference temperature
- \(S\) = concentration of salt water
- \(T\) = temperature.

Figure 2 indicates that \((\frac{\partial \rho}{\partial S})_T\) is linear and hence constant, and \((\frac{\partial \rho}{\partial T})_S\) can be approximated as linear over a small range of temperature.

As expected, the temperature will also affect the number of moles per unit volume of the fluid. Generally, at higher temperatures the molecules of water will have a greater random velocity; hence, they will demand greater spacing. For non-isothermal fluids, the volume dilatation in Equation (3-1) should be more precisely written as

$$\nabla \cdot \vec{V} = -\frac{1}{\rho} (\frac{\partial \rho}{\partial T})_S \frac{DT}{DT}$$

(3-6)
which is analogous to Equation (46) on page 154 of Yih (1965).

Because the volume expansion due to thermal effects, the right hand side of Equation (3-6), is small for moderate changes in temperature, Equation (3-6) is often approximated by Equation (3-1). For justification of this approximation, the reader may refer to page 154 of Yih (1965) or merely note the slope of the curve in Figure 3, \( \frac{\partial \rho}{\partial T} \).

B. PERTINENT CONSERVATION PRINCIPLES

Now that the proper definition of velocity to correctly define the volume dilatation as zero has been established and the empirical relationship \( \rho = \rho(S,T) \) determined, the conservation equations can now be written. The conservation of mass equation for a multicomponent fluid states that the amount of mass accumulated within a given volume is equal to the net convection and diffusion of mass into the volume. This may either be expressed for the overall density as

\[
\frac{\partial \rho}{\partial t} = - \nabla \cdot (\nabla \rho) - \nabla \cdot J \quad (3-7a)
\]

or, expressed for the mass of an individual species concentration, for this case salt water concentration \( S \), as

\[
\frac{\partial S}{\partial t} = - \nabla \cdot (\nabla S) - \nabla \cdot J \quad (3-7b)
\]

By assuming a Fickian type diffusion process which makes use of a diffusion coefficient \( \hat{D} \) as is customarily done*, the mass flux \( J \)

* The diffusion coefficient will temporarily be denoted \( \hat{D} \).
is defined as

\[ J = - \hat{D} \nabla \rho \]  

(3-8)

or expressed in terms of the salt concentration as

\[ J = - \hat{D} \nabla S \]  

(3-9)

Now for the case of a constant diffusion coefficient \( D \) the two forms of Equation (3-7) become

\[ \frac{\partial \rho}{\partial t} = - \bar{v} \cdot \nabla \rho + \hat{D} \nabla^2 \rho \]  

(3-10a)

\[ \frac{\partial S}{\partial t} = - \bar{v} \cdot \nabla S + \hat{D} \nabla^2 S \]  

(3-10b)

For isothermal fluids, \( \rho = \rho(S) \) by Equation (3-5) and the two forms of Equation (3-10) are redundant.

The conservation of momentum is derived from Newton's second law of motion. Expressed for fluids, this says that the rate of change of momentum of the fluid in a given volume is equal to the summation of the vector forces acting on the fluid. Pertinent forces may consist of pressure forces, shearing forces, and gravity forces.

Unfortunately, for a multicomponent fluid the conservation of momentum equation, and the conservation of energy equation as well, are cumbersome when written in terms of the number average velocity. The skeptic may verify this for himself by using the number average velocity for specifying the momentum of each
component in the Boltzmann equation as presented by Chapman and Cowling (1970) on page 132 of their text. Therefore, the mass average velocity \( \bar{U} \) will be used for presenting the conservation of momentum thus giving

\[
\frac{D(\rho \bar{U})}{Dt} = - \nabla \rho + \mu \nabla^2 \bar{U} - \rho g. \tag{3-11}
\]

In order to solve this equation, it must be made compatible with the conservation of mass equation which is expressed in terms of a number average velocity; therefore, an approximation will be used which was presented by Frank-Kamenetskii (1969) and used, among others, by Daly and Pracht (1968) and Tamai (1969). This approximation states that negligible differences result from using the number average velocity instead of the mass average velocity in the conservation of momentum and energy equations. An idea of the accuracy of this approximation may be obtained by noting that the mass flux vector \( J \) in the volume may be represented as

\[
J = - \hat{D} \nabla \rho
= \rho (\bar{U} - \bar{V}) \tag{3-12}
\]

as derived in Appendix A. Using Equation (3-12), then the substantial derivative of Equation (3-11) can be expressed as

\[
\frac{D(\rho \bar{U})}{Dt} = \frac{D(\rho \bar{V})}{Dt} - \frac{D(\hat{D} \nabla \rho)}{Dt}.
\]

By assuming a constant diffusion coefficient \( \hat{D} \), then
\[
\frac{D(\rho U)}{Dt} = \frac{D(\rho V)}{Dt} - \hat{D}V_2 \rho \frac{\nabla^2}{\rho}
\]

which, with the aid of Equation (3-10a) becomes

\[
\frac{D(\rho U)}{Dt} = \frac{D(\rho V)}{Dt} - \hat{D}^2 V_2 \rho \frac{\nabla^2}{\rho}
\]

(3-13)

For all practical cases, the latter term of Equation (3-13) can be neglected because values of \(\hat{D}\), whether molecular or eddy, are small compared with the dominant terms of the substantial derivative.

By similar arguments, other terms based upon or containing \(U\) in Equation (3-11) can be replaced by \(V\) to a similar order of accuracy, yielding

\[
\frac{D(\rho V)}{Dt} = - \nabla p + \mu \nabla^2 V - \rho \frac{\nabla^2}{\rho}
\]

(3-14)

Combining Equation (3-14) with Equation (3-10a) gives

\[
\frac{D\nabla}{Dt} = - \frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \nabla^2 V - \hat{D} \frac{\nabla^2}{\rho} - \frac{\nabla^2}{\rho}
\]

(3-15)

This is a form of the conservation of momentum equation seldom seen since it includes the effect of diffusion upon the change of momentum within the control volume.

An additional term often appearing in the conservation of momentum equation is derived from the fact that this equation is expressed in a coordinate system which is moving relative to an inertially fixed system. This term, known as the Coriolis force, is actually a misnomer as it is not truly a force, but merely a

* The diffusion coefficient will subsequently be denoted \(D\).
correction because a non-inertial coordinate system is used. The mathematical treatment of this effect will not be presented here as it is eloquently treated in so many texts of physical oceanography, orbital mechanics, meteorology, etc. (e.g., pp. 117-126 of Neumann and Pierson 1966). Instead, a discussion of the origin of this effect is given in an effort to remove some of the confusion associated with it.

Consider a particle of a given mass fixed upon the surface of a spherical earth rotating about an axis extending between the two poles as shown in Figure 4. Associated with this particle is a momentum defined by the mass of the particle and the tangential velocity of that point on the surface. This tangential velocity is in turn a function of the angular velocity of the rotating earth \( \omega \), and a radius of rotation, \( r \), extended normal from the axis of rotation to the particle. If the particle is moving constantly in a southerly direction along the surface of the earth as shown in Figure 4, then it would attain a new momentum because its tangential velocity defined by \( r_2 \) would have increased. In the absence of an additional force to supply this momentum, the particle will assume a velocity opposite the direction of rotation such that its new tangential velocity will precisely balance the previous one. Otherwise, Newton's second law of motion would be violated.

It should also be pointed out that a true inertially fixed coordinate system is not possible; consequently, there exists Coriolis effects due to the angular velocity of the orbit of the
Figure 4: The Effect of Coriolis Effect Upon Fluid Movement
earth around the sun, the movement of the sun in the galaxy, etc. Fortunately, the angular velocity associated with these rotations is so small as to make them completely negligible for problems of interest here. In fact, the Coriolis effect due to a rotating earth is commonly neglected in small scale fluids problems because of the small change in the moment arm from the axis of rotation and the small angular velocity of the earth; \( \omega = 7.29 \times 10^{-5} \) l/sec. Coriolis effect was neglected from this analysis of deltaic flow phenomena because preliminary numerical investigations indicated that the angular velocity of the earth would have to be increased by orders of magnitude before it significantly influenced the flow patterns.

The conservation of energy equation results from applying the first law of thermodynamics to the moving fluid; it states that the rate of increase in total energy is equal to the sum of the rate of work done on the fluid and the rate of heat added from external sources. The total energy of a given volume of fluid consists of three types: internal energy which may be expressed as a function of the temperature, kinetic energy due to the mass velocity of the fluid, and potential energy which is a function of elevation of the particles of fluid. The rate of work done to the fluid results from pressure forces, gravity forces, and viscous and turbulent shearing forces.

Recalling from physics the close relationship between momentum and kinetic energy, it is possible to obtain the kinetic energy of the flow by multiplying Equation (3-15) by the velocity vector.
as demonstrated by Hughes and Brighton (1967). Subtracting the kinetic energy obtained in this way from the conservation of energy equation, Hughes and Brighton obtain the form of the energy equation commonly seen;

\[ \rho C_v \frac{D T}{D t} = \kappa \nabla^2 T - \nabla \cdot q_r + \Phi \]  \hspace{1cm} (3-16)

where, besides the terms previously defined,

- \( C_v \) = specific heat capacity at constant volume
- \( T \) = temperature
- \( \kappa \) = coefficient of thermal conductivity
- \( q_r \) = radiation heat flux vector
- \( \Phi \) = dissipation function resulting from heat generation by viscosity.

The energy equation reduces to a trivial form for isothermal flows when velocities are such that there is little heat generated by viscous dissipation. Thus, the conservation of mass and momentum equations, along with Equation (3-5) which serves as an equation of state, produce a complete and independent set for defining the dependent variables, \( \tilde{V}, \rho, S, p \) and \( T \). These equations, including the general case of non-isothermal effects, are presented in Table 1.

As previously discussed, an isothermal assumption appeared justified for the class of problems under consideration; consequently, Equation (3-16) was not needed and Equation (3-5) was simplified. A cartesian coordinate system was chosen for

Note that there was also a Coriolis effect upon the kinetic energy of the fluid, but it has been subtracted out of the final form of the energy equation.
Table 1: General Conservation Laws for Deltaic Flows

1. Volume Dilatation Equation

\[ \nabla \cdot \mathbf{V} = 0 \quad (3-1) \]

2. Approximate Equation of State

\[ \rho = \rho_f + \left( \frac{\partial \rho}{\partial S} \right)_T S + \left( \frac{\partial \rho}{\partial T} \right)_S (T - T_{ref}) \quad (3-5) \]

3. Species Continuity Equation

\[ \frac{DS}{Dt} = D \nabla^2 S \quad (3-10b) \]

4. Conservation of Momentum Equations

\[ \frac{D\mathbf{V}}{Dt} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{V} - \frac{D}{\rho} \nabla^2 \rho - g \quad (3-15) \]

5. Conservation of Energy Equation

\[ \frac{DT}{Dt} = \frac{\kappa}{\rho C_v} \nabla^2 T - \frac{1}{\rho C_v} \nabla \cdot q + \frac{\phi}{\rho C_v} \quad (3-16) \]
actual calculations, but the equations are equally valid in any coordinate system.

C. TURBULENT MIXING NEAR A RIVER MOUTH

The equations presented in Table 1 were derived by assuming that Stokes' law, Fick's law, and Fourier's law are applicable for defining the viscous shear, mass diffusion, and heat conduction, respectively. Strictly speaking, these laws are applicable for laminar flow only, but flowing rivers are generally turbulent; consequently, flow throughout the interaction region should also logically be turbulent.

The concept of representing turbulent effects in a flowing fluid as eddy viscosity coefficients is probably the most practical and acceptable means currently available. This entails writing the governing equations in terms of mean and fluctuating components to include the transport processes resulting from turbulence. The additional terms occurring in the equations are then combined in each equation as eddy coefficients. Hinze (1959) is among the many authors presenting the details of this procedure.

When the flow is turbulent, these eddy coefficients are several orders of magnitude larger than the molecular coefficients; therefore, the molecular contributions are usually omitted. For such cases the transport coefficients of Table 1 represent only eddy coefficients.

How to specify values of eddy coefficients which adequately describe the microscopic properties of turbulence in macroscopic
equations has been the object of many years of investigation. Excellent summaries of these efforts are provided by Schlichting (1960), Schetz (1969), Hinze (1959), Abramovich (1963), and Farmer and Audeh (1972).

Among the eddy viscosity models proposed, Prandtl's third eddy viscosity model is probably the most popular form currently in use. By assuming that the fluctuating velocities could be determined in terms of a mixing length concept, he was able to express the eddy viscosity in terms of primary characteristics of the mean flow. The mixing length must be determined by evaluation from empirical data for each particular class of problems. Expressed for an incompressible fluid, Prandtl's third model is

\[ \epsilon = k b (V_{\text{max}} - V_{\text{min}}) \]  

(3-17)

where

- \( \epsilon \) = kinematic eddy viscosity coefficient
- \( k \) = empirical constant
- \( b \) = function of the mixing layer thickness
- \( (V_{\text{max}} - V_{\text{min}}) \) = velocity change across the mixing layer.

Both \( k \) and \( b \) have been evaluated experimentally for simple flow fields such as boundary layers, wakes, free shear layers, and two-dimensional and circular jets. Unfortunately, a river emptying into the sea does not precisely fit any of these classifications. The turbulent river should dictate one eddy
viscosity coefficient, the tidal flow of sea water another, and the region of mixing at the intersection of the two streams yet another. Such a complex model for eddy coefficients is not yet available; therefore, it was expedient to use empirically determined eddy viscosity coefficients for a simple flow field which reasonably approximates the emptying of a river into the sea.

The primary region of interest in these computations is near the river mouth. For this reason, eddy viscosity coefficients evaluated from flow of a circular incompressible jet into a quiescent basin were used. It is conceded that river mouths are not circular and the basin is not completely quiescent, but this model should provide reasonable results near the mouth of the river. For this case Prandtl's third model is expressed in many references as

\[ \varepsilon = \frac{\mu_T}{\rho} = 0.0256 \ b \ \tilde{V}_{\text{max}} \]

where

- \( \varepsilon \) = eddy kinematic viscosity coefficient
- \( \mu_T \) = eddy viscosity coefficient
- \( b \) = radius of the jet (used here as 1/2 of the river width)
- \( \tilde{V}_{\text{max}} \) = maximum velocity at the centerline of the jet (i.e., river).

Although we are not dealing with a constant density jet, the error

due to neglecting the density variation when specifying eddy viscosity coefficients will certainly be lost within the range of accuracy of the experimental data.

Tamai (1969) also used Prandtl's third model for the eddy viscosity coefficients of his computations. In his analysis he presented two values of k from experiments of:

a) Hayashi, Shuto, and Yoshida (1969) on a free turbulent jet; \( k = 0.01 \),

b) Albertson, Dai, Jensen, and Rouse (1950) on a warm water jet; \( k = 0.08 \).

The eddy coefficients used by Tamai agree qualitatively with Equation (3-18). He also allowed for spatial variation of \( \kappa \) and \( V_{\text{max}} \) by assuming an analytical form \( * \) of \( \kappa(x) \) and \( V_{\text{max}}(x) \). Incidentally, he reported that this assumption led to results which did not match experimental data.

Eddy transport coefficients of mass, momentum, and energy are related but are not necessarily equal. For example, from field studies of the thermocline, Woods (1970) reports an eddy kinematic viscosity coefficient which was five times larger than the eddy coefficients of mass and heat. According to his explanation, in addition to the mixing caused by the fluctuating terms of the turbulent eddies, there is a momentum exchange between the two fluids because of fluid dynamic drag upon an eddy. The explanation and data of Wood appear plausible; consequently, the

\* Tamai's analysis was applicable only for flows into quiescent basins.
eddy coefficient of mass diffusion $D_T$ may thus be expressed as

$$D_T = \frac{1}{5} \varepsilon.$$  \hspace{1cm} (3-19)

This is, in fact, specifying a Schmidt number of five.

The subject of interfacial waves leading to entrainment of the lower fluid often arises in publications on jet mixing, stratified flow, deltaic mixing, etc. (e.g. Wright and Coleman 1971). Undoubtedly, such waves do occur, but are entirely consistent with the concept of eddy coefficients. The instability of these waves represents the origin of increased turbulence at the free shear layer. As Woods (1970) noted, the turbulence at such a layer appeared to be intermittent and not unlike other investigations of intermittency in turbulent flows (e.g. Yen 1967). This phenomenon was probably due to an instability of these interfacial waves which will break only after a period of growth. If a sufficient number of these waves break for their effect to be averaged between grid points of a finite difference calculation, then a simple form of the eddy coefficients should be adequate to describe their effect upon the flow. If the grid spacing is of the same order as the distances between breaking of interfacial waves, then a more sophisticated form of the eddy coefficients is required to account this intermittency. Probably this phenomenon can best be analyzed by numerical computations such as those of Maslowe and Thompson (1971).

Because different mechanisms such as buoyancy could affect the turbulent exchange of mass, momentum and heat in the different
directions, it is likely that the eddy coefficients should be
different in each direction. Bowden (1965) deduced a functional
relationship between horizontal and vertical transport coefficients,
but his data were based upon mixing along the interface of ocean
currents. Should data become available for specifying such a
relationship for jet mixing, or more specifically for mixing near
river mouths, it could easily be included into the program because
provision was made for use of different eddy coefficients in each
of the three directions. Data presented in the results were based
upon coefficients which were the same in each direction.

D. NONDIMENSIONALIZATION OF THE GOVERNING EQUATIONS

As a matter of preference, the governing isothermal equations
were nondimensionalized before a solution was effected. This
was done by choosing the following basic reference parameters:

a) Density \( \rho_f \) = density of fresh water at a temper­
erature of 20°C.

b) Length \( H \) = maximum depth of the finite
difference grid which is approximately
the maximum depth of the receiving
basin

c) Acceleration of gravity \( g \)

Nondimensional terms were obtained by dividing by combinations of
these reference parameters as shown in Table 2.
Table 2: Nondimensional Terms for a Cartesian Coordinate System

1. Density: \( \tilde{\rho} = \frac{\rho}{\rho_f} \)

2. Concentration of Salt Water: \( \tilde{S} = \frac{S}{\rho_f} \)

3. Length in the \( x_i \) direction: \( \tilde{x_i} = \frac{x_i}{H} ; i=1,2,3 \)

4. Pressure: \( \tilde{P} = \frac{P}{\rho_f H g} \)

5. Time: \( \tilde{t} = \frac{t}{\sqrt{\frac{H}{g}}} \)

6. Velocity: \( \tilde{V} = \frac{V}{\sqrt{Hg}} \)

7. Viscosity Coefficient: \( \tilde{\mu} = \frac{\mu}{\rho_f H g} \)

8. Diffusion Coefficient: \( \tilde{D} = \frac{D}{H g} \)
Using the nondimensional quantities of Table 2 in the previously derived basic equations of isothermal flow, performing the required algebraic manipulations, and subsequently dropping the (\( \sim \)) produced the set of nondimensional equations shown in Table 3. These consist of 5 partial differential equations and 1 algebraic equation which are sufficient to define the 6 dependent variables \((u, v, w, p, S, \rho)\) of a three-dimensional flow field. Note that the viscosity and diffusion coefficients are written in such a manner to permit specifying different coefficients in each of the three directions. This is the general form in which they were solved.

Note that, instead of the dashes (-) used in previous dimensional equations, the equation numbers of Table 3 contain asterisks (*) to denote nondimensionality. This notation will be continued throughout.

Notice that several of the nondimensional terms of Table 3 are actually familiar similarity parameters. For instance, when the true depth of the water is equal to the reference depth \(H\), then the nondimensional velocity is the Froude number. Also, the nondimensional viscosity coefficient \(\tilde{\mu}\) is really the inverse of the Reynolds number. Typical values of the Froude and Reynolds numbers may be computed by assuming

\* The dependent variables \(u, v\) and \(w\) correspond to the scalar components of velocity in the \(x, y,\) and \(z\) direction, respectively.
Table 3: Nondimensionalized Basic Equations for Three-Dimensional Isothermal Deltaic Flow

1. Volume Dilatation

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]  

(3*20)

2. Approximate Equation of State

\[ \rho = 1.0 + KS \]  

(3*21)

3. Conservation of Species Equation

\[ \frac{DS}{Dt} = D_x \frac{\partial^2 S}{\partial x^2} + D_y \frac{\partial^2 S}{\partial y^2} + D_z \frac{\partial^2 S}{\partial z^2} \]  

(3*22)

4. x-Momentum Equation

\[ \rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \left[ \mu_x \frac{\partial^2 u}{\partial x^2} + \mu_y \frac{\partial^2 u}{\partial y^2} + \mu_z \frac{\partial^2 u}{\partial z^2} \right] \]

\[ -u \left[ D_x \frac{\partial^2 \rho}{\partial x^2} + D_y \frac{\partial^2 \rho}{\partial y^2} + D_z \frac{\partial^2 \rho}{\partial z^2} \right] \]  

(3*23)

5. y-Momentum Equation

\[ \rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \left[ \mu_x \frac{\partial^2 v}{\partial x^2} + \mu_y \frac{\partial^2 v}{\partial y^2} + \mu_z \frac{\partial^2 v}{\partial z^2} \right] \]

\[ -v \left[ D_x \frac{\partial^2 \rho}{\partial x^2} + D_y \frac{\partial^2 \rho}{\partial y^2} + D_z \frac{\partial^2 \rho}{\partial z^2} \right] \]  

(3*24)

6. z-Momentum Equation

\[ \rho \frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \left[ \mu_x \frac{\partial^2 w}{\partial x^2} + \mu_y \frac{\partial^2 w}{\partial y^2} + \mu_z \frac{\partial^2 w}{\partial z^2} \right] \]

\[ -w \left[ D_x \frac{\partial^2 \rho}{\partial x^2} + D_y \frac{\partial^2 \rho}{\partial y^2} + D_z \frac{\partial^2 \rho}{\partial z^2} \right] - 1 \]  

(3*25)
\[ H = 15 \text{ m} \]
\[ \bar{V} = 2 \text{ m/sec} \]
\[ b = 100 \text{ m (half width of a river mouth)} \]

\[ \frac{\nu}{\rho_f} = 0.0256 b \bar{V} \text{ (Eddy viscosity model)} \]

which give

Froude number = 0.148

Reynolds number = \( \frac{1}{0.027} \) = 36.

This implies that viscous terms neither dominate nor should be neglected from the conservation of momentum equations.
CHAPTER IV

NUMERICAL PROCEDURE FOR THE SOLUTION OF DELTAIC FLOWS

Although the six governing equations have been expressed as a function of the six dependent equations shown in Table 3, the problem has just begun. Because of their nonlinearity, this set of partial differential equations defy solution in this form. A standard procedure for describing complex equations of this type is to include only the highest order terms; consequently, the equations will reduce to a simpler form. If a proper order-of-magnitude analysis cannot be made, then an often used technique involves hypothesizing which of the terms are dominant in the equations and neglecting the others. However, caution must be used here to avoid losing by over-simplification physical phenomena which are known to be important in deltaic formation.

The philosophy used in this analysis was to avoid making simplifications to the governing equations which would unduly restrict the applicability of the results. Naturally, some simplifying assumptions were made, but only those which have a sound physical basis determined by experiment or observation. This approach makes it unnecessary to hypothesize which of the important parameters such as turbulent interaction, buoyancy, or hydraulic head controls the plume spreading.

It will be conceded that the governing equations should be three-dimensional and nonlinear in type; hence, these equations will be far too complicated to effect an analytical solution.
The alternative is to turn to a numerical solution. For a three-dimensional flow problem, this is a formidable task. Emmons (1970), in his critique of numerical modeling techniques, says "All solutions to date are essentially two-dimensional nonsteady ones. Three-dimensional solutions are of course possible, and would be expected to be equally accurate, but the required machine storage and computing time (and therefore cost) put a limit to one's ambition".

Nevertheless, it was decided that a proper description of deltaic flows could be determined only as a result of a numerical three-dimensional calculation, and that the development of a practical technique should be undertaken, thus providing a choice instead of an echo.

The four classes of numerical solutions considered for solving the equations of Table 3 are the:

1) Method of characteristics
2) Marker in cell technique
3) Asymptotic time dependent techniques
4) Relaxation techniques.

Each method involves solving finite-difference approximations to the differential equations describing the conservation principles. Salient features of each of these techniques along with some of the prominent advantages and disadvantages were reviewed by Waldrop (1972).
The procedure presented in this report can best be described as an asymptotic time-dependent technique with certain prominent features of a relaxation method. In several ways it is similar to a method presented by Callens (1970) for analysis of incompressible boundary layers. Because steady state boundary conditions are imposed, the flow patterns computed should asymptotically converge to a steady state flow field $\frac{\partial (\cdot)}{\partial t} \to 0$, thus deriving the name asymptotic time-dependent. Relaxation techniques are similar in that they treat the time-dependent terms of the conservation equations as an error term of each equation at each grid point. Minimizing these errors also yields a steady state solution. Details of the established procedure are now presented.

A. FINITE DIFFERENCE GRID

The region of computation is covered by a three-dimensional grid system. Grid lines are indexed as $i,j,k$ in the $x,y,z$ directions, respectively. The coordinate system of the region of computation is shown in Figure 5. Values of five of the dependent variables ($p,S,u,v,w$) are indexed by $i,j,k$; thus, they will be known at all intersections of grid lines. The sixth dependent variable, $\rho$, was not subscripted because when needed it was easily computed from Equation (3+21). Four of the dependent variables ($S,u,v,w$) were needed at both the new time ($\tau$) and the previous time ($t$). This necessitated an additional index for time which is shown by example for the dummy dependent variable $Q$: 
Figure 5 : Coordinate System for Computation
where

\[\begin{align*}
T &= \text{value at the new time} \\
i &= \text{index used to designate the x location} \\
j &= \text{index used to designate the y location} \\
k &= \text{index used to designate the z location}.
\end{align*}\]

This FORTRAN type notation will be used throughout to index all dependent variables which were dimensioned in the computer program.

The main region of interest is near the river mouth as may be seen in Figure 5. Resolution can be improved in this region by placing as many grid points as possible there. The region far from the river mouth is of lesser interest, but the calculation must be extended far enough from the mouth such that boundary conditions for the tidal basin can be imposed as independent of the interaction region. This is best accomplished by a grid system with a variable spacing in the horizontal plane. To accomplish this, define

\[\begin{align*}
X &= X[x] \\
Y &= Y[y]
\end{align*}\]

Now, as again demonstrated with a dummy dependent variable \(Q\), the chain rule gives

\[\begin{align*}
\frac{\partial Q}{\partial x} &= \frac{\partial Q}{\partial x} X' \\
\frac{\partial^2 Q}{\partial x^2} &= \frac{\partial^2 Q}{\partial x^2} (X')^2 + \frac{\partial Q}{\partial x} X'' \quad \text{for} \quad x \leq x^* \quad (4.1)
\end{align*}\]

and

\[\begin{align*}
\frac{\partial^2 Q}{\partial x^2} &= \frac{\partial^2 Q}{\partial x^2} (X')^2 + \frac{\partial Q}{\partial x} X'' \quad \text{for} \quad x > x^* \quad (4.2)
\end{align*}\]
where

\[ X' = \frac{dx}{dx}, \]

\[ X'' = \frac{d^2x}{dx^2}. \]

Similarly,

\[ \frac{\partial Q}{\partial y} = \frac{\partial Q}{\partial Y}Y', \]

and

\[ \frac{\partial^2 Q}{\partial y^2} = \frac{\partial^2 Q}{\partial Y^2} (Y')^2 + \frac{\partial Q}{\partial Y} Y''. \]

where

\[ Y' = \frac{dy}{dy}, \]

\[ Y'' = \frac{d^2y}{dy^2}. \]

The basic governing equations of the fluid mechanics shown in Table 3 may now be expressed in the stretched coordinates X and Y with x and y appearing only as independent variables for the derivatives of X and Y.

Consider the following functions of \( X = X(x) \) and \( Y = Y(y) \):

\[ X = \frac{1}{k_1} \tan^{-1}\left(\frac{x}{k_2}\right), \]

\[ Y = \frac{1}{k_3} \tan^{-1}\left(\frac{y}{k_4}\right). \]

Expressed differently,

\[ x = k_2 \tan(k_1 X) \]
and

\[ y = k_4 \tan(k_3 Y) \quad (4*8) \]

If \( X \) and \( Y \) are normalized such that

\[ 0 \leq X \leq 1.0 \]

and

\[ -1.0 \leq Y \leq 1.0, \]

then it is possible to space the grid rows progressively further apart by letting \( x \) and \( y \) vary as \( X \) and \( Y \) are increased in even increments from the zero value at the river mouth. Also, once the functional relationships of \( X \) and \( Y \) are established, then it is sufficient to compute first and second derivatives with respect to \( x \) or \( y \) as required in Equations \((4*1)\) through \((4*4)\). These derivatives are then stored to be recalled when needed for every grid line in the \( x \) and \( y \) direction, respectively.

Note that if

\[ k_1 = \pi/2, \ X = 1.0 : x = \infty \]

and

\[ k_3 = \pi/2, \ Y = \pm 1.0 : y = \pm \infty \]

For such cases, it is possible to extend the grid system from a finite but large value of \( x \) or \( y \) at the next to the last grid row to an infinite distance at the last row. This is merely one small step for a finite difference grid, but one giant step for river deltas.

The grid was not stretched in the vertical direction because
the distances were small compared to the horizontal dimensions of the problem and resolution at all levels were considered equally important. An example of the grid spacing used in actual calculations is presented in Table 4.

B. FINITE DIFFERENCE EQUATIONS

Now that the finite difference grid has been established and appropriate stretching functions defined, the equations can be written in finite difference form. But first, consider the substantial derivative of the momentum equations in an alternative form. Looking only at the x-momentum equation, the substantial form of the x-component of velocity is normally written

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}.$$ 

Recalling for instance that

$$v \frac{\partial u}{\partial y} = \frac{\partial (uv)}{\partial y} - u \frac{\partial v}{\partial y}$$

and using the volume dilatation of Equation (3*20), then the substantial derivative may be written in the alternate form

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + \frac{\partial (u^2)}{\partial x} + \frac{\partial (uv)}{\partial y} + \frac{\partial (uw)}{\partial z} \quad (4*9)$$

This is a preferrable form for finite difference computations according to Emmons (1970).

The finite difference approximations to the partial differential equations can now be written. For partial derivatives
Table 4: Finite Difference Grid Spacing

<table>
<thead>
<tr>
<th>I</th>
<th>x</th>
<th>J</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.40300</td>
<td>1</td>
<td>-833.46973</td>
</tr>
<tr>
<td>2</td>
<td>0.40300</td>
<td>2</td>
<td>-78.05460</td>
</tr>
<tr>
<td>3</td>
<td>1.21425</td>
<td>3</td>
<td>-40.47551</td>
</tr>
<tr>
<td>4</td>
<td>2.04156</td>
<td>4</td>
<td>-26.90610</td>
</tr>
<tr>
<td>5</td>
<td>2.89661</td>
<td>5</td>
<td>-19.79106</td>
</tr>
<tr>
<td>6</td>
<td>3.79259</td>
<td>6</td>
<td>-15.33249</td>
</tr>
<tr>
<td>7</td>
<td>4.74518</td>
<td>7</td>
<td>-12.22017</td>
</tr>
<tr>
<td>8</td>
<td>5.77366</td>
<td>8</td>
<td>-9.88049</td>
</tr>
<tr>
<td>9</td>
<td>6.90271</td>
<td>9</td>
<td>-8.02189</td>
</tr>
<tr>
<td>10</td>
<td>8.16501</td>
<td>10</td>
<td>-6.47968</td>
</tr>
<tr>
<td>11</td>
<td>9.60547</td>
<td>11</td>
<td>-5.15305</td>
</tr>
<tr>
<td>12</td>
<td>11.28811</td>
<td>12</td>
<td>-3.97605</td>
</tr>
<tr>
<td>13</td>
<td>13.30817</td>
<td>13</td>
<td>-2.90281</td>
</tr>
<tr>
<td>14</td>
<td>15.81453</td>
<td>14</td>
<td>-1.89947</td>
</tr>
<tr>
<td>15</td>
<td>19.05457</td>
<td>15</td>
<td>-0.93939</td>
</tr>
<tr>
<td>16</td>
<td>23.47258</td>
<td>16</td>
<td>0.0</td>
</tr>
<tr>
<td>17</td>
<td>29.95651</td>
<td>17</td>
<td>0.93939</td>
</tr>
<tr>
<td>18</td>
<td>40.57684</td>
<td>18</td>
<td>1.89947</td>
</tr>
<tr>
<td>19</td>
<td>61.54504</td>
<td>19</td>
<td>2.90281</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>3.97605</td>
</tr>
<tr>
<td></td>
<td></td>
<td>21</td>
<td>5.15305</td>
</tr>
<tr>
<td></td>
<td></td>
<td>22</td>
<td>6.47968</td>
</tr>
<tr>
<td></td>
<td></td>
<td>23</td>
<td>8.02189</td>
</tr>
<tr>
<td></td>
<td></td>
<td>24</td>
<td>8.02189</td>
</tr>
<tr>
<td></td>
<td></td>
<td>25</td>
<td>9.88049</td>
</tr>
<tr>
<td></td>
<td></td>
<td>26</td>
<td>12.22017</td>
</tr>
<tr>
<td></td>
<td></td>
<td>27</td>
<td>15.33249</td>
</tr>
<tr>
<td></td>
<td></td>
<td>28</td>
<td>19.79106</td>
</tr>
<tr>
<td></td>
<td></td>
<td>29</td>
<td>26.90610</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>40.47551</td>
</tr>
<tr>
<td></td>
<td></td>
<td>31</td>
<td>78.05460</td>
</tr>
<tr>
<td></td>
<td></td>
<td>32</td>
<td>833.46973</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>K</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.10000</td>
</tr>
<tr>
<td>2</td>
<td>0.20000</td>
</tr>
<tr>
<td>3</td>
<td>0.30000</td>
</tr>
<tr>
<td>4</td>
<td>0.40000</td>
</tr>
<tr>
<td>5</td>
<td>0.50000</td>
</tr>
<tr>
<td>6</td>
<td>0.60000</td>
</tr>
<tr>
<td>7</td>
<td>0.70000</td>
</tr>
<tr>
<td>8</td>
<td>0.80000</td>
</tr>
<tr>
<td>9</td>
<td>0.90000</td>
</tr>
<tr>
<td>10</td>
<td>1.00000</td>
</tr>
</tbody>
</table>
with respect to time, a forward difference is used;

\[ \frac{\partial Q}{\partial t} \{\tau,i,j,k\} = \frac{Q\{\tau,i,j,k\}-Q\{\tau,i,j,k\}}{\Delta t} + O(\Delta t), \quad (4.10) \]

where the length of the time step \( \Delta t \) is defined as \( \Delta t = \tau - t \).

Centered differences were used for partial derivatives with respect to spatial dimensions;

\[ \frac{\partial Q}{\partial z} \{t,i,j,k\} = \frac{Q\{t,i,j,k+1\}-Q\{t,i,j,k-1\}}{2\Delta z} + O(\Delta z)^2, \quad (4.11) \]

\[ \frac{\partial^2 Q}{\partial z^2} \{t,i,j,k\} = \frac{Q\{t,i,j,k+1\}-2Q\{t,i,j,k\}+Q\{t,i,j,k-1\}}{(\Delta z)^2} + O(\Delta z)^2 \quad (4.12) \]

Exceptions to the forms of the centered differences of Equations (4.11) and (4.12) were made for the partial derivatives of the salt water concentration \( S \) in the \( y \) direction which is lateral to the river flow. Here, improved resolution was desired; hence

\[ \frac{\partial S}{\partial y} \{t,i,j,k\} = \frac{1}{12 \Delta y} \left[ -8S\{t,i,j+2,k\} + 8S\{t,i,j+1,k\} + 8S\{t,i,j-1,k\} + S\{t,i,j-2,k\} \right] + O(\Delta y)^4 \quad (4.13) \]
\[
\frac{\partial^2 S}{\partial y^2} \{t,i,j,k\} = \frac{1}{(6 \Delta y)^2} \left[ -7 S\{t,i,j+2,k\} + 64 S\{t,i,j+1,k\} - 114 S\{t,i,j,k\} + 64 S\{t,i,j-1,k\} - 7 S\{t,i,j-2,k\} \right] + \mathcal{O}(\Delta y)^4
\]  

(4*14)

Equations (4*10) through (4*14) along with the modified version of the substantial derivative of Equation (4*9) were then used to formulate the finite difference approximations to the basic flow equations of Table 3. These finite difference equations are shown in Table 5 in the form which they were solved. More explanation of this procedure will be presented shortly.

Notice that the z-momentum equation contains neither the time dependent term nor the diffusion component. Both omissions appeared justified from an order of magnitude standpoint as the hydrostatic component of pressure was the overwhelming term of this equation. More discussion of the time dependent term will be presented in the stability section.

C. INITIAL CONDITIONS

Before conditions at a new time step can be computed, all values of the dependent variables must be known for every grid point throughout the region of computation. To begin the computations, it is necessary to initialize all dependent variables at every grid point. For this initialization, a crude guess of the
TABLE 5: FINITE DIFFERENCE EQUATIONS GOVERNING THE FLUID DYNAMICS

1. Approximate Equation of State ($\rho$ computed as required, but not stored)

$$\rho = 1.0 + K S[t,i,j,k], \quad K = \frac{\partial \rho}{\partial S}$$  \hspace{1cm} (4*15)

2. Conservation of Momentum Equation in the x-Direction

$$u[t,i,j,k] = u[t,i,j,k] - X'[i] \left( \left( (u[t,i+1,j,k])^2 - (u[t,i-1,j,k])^2 \right) + \frac{1}{\rho} \left[ p[i+1,j,k] - p[i-1,j,k] \right] \right) \frac{\Delta t}{2 \Delta x}$$

$$- Y'[j] \left( u[t,i,j+1,k] v[t,i,j+1,k] - u[t,i,j-1,k] v[t,i,j-1,k] \right) \frac{\Delta t}{2 \Delta y}$$

$$- \left( u[t,i,j,k+1] w[i,j,k+1] - u[t,i,j,k-1] w[i,j,k-1] \right) \frac{\Delta t}{2 \Delta z}$$

$$+ \left( \mu_x \left[ (X'[i])^2 (u[t,i+1,j,k] - 2 u[t,i,j,k] + u[t,i-1,j,k]) \right] \right) \frac{1}{(\Delta x)^2} + X'[i] (u[t,i+1,j,k] - u[t,i-1,j,k]) \frac{1}{2 \Delta x}$$

$$+ \mu_x \left[ (Y'[j])^2 (u[t,i,j+1,k] - 2 u[t,i,j,k] + u[t,i,j-1,k]) \right] \frac{1}{(\Delta y)^2} + Y'[j] (u[t,i,j+1,k] - u[t,i,j-1,k]) \frac{1}{2 \Delta y}$$

$$+ \mu_z \left[ (u[t,i,j,k+1] - 2 u[t,i,j,k] + u[t,i,j,k]) \right] \frac{1}{(\Delta z)^2} \frac{\Delta t}{\rho} - u[t,i,j,k] \quad \text{[DIFFUSION]} \quad K \Delta t$$  \hspace{1cm} (4*16)
3. Conservation of Momentum Equation in the y-Direction

\[ v_{t,i,j,k} = v_{t,i,j,k} - \frac{At}{2} \left( u_{t,i+1,j,k} v_{t,i+1,j,k} - u_{t,i-1,j,k} v_{t,i-1,j,k} \right) \cdot \frac{Ax}{Ax} \]

\[- Y' \left( \left[ v_{t,i,j+1,k} \right]^2 - \left( v_{t,i,j-1,k} \right)^2 \right) + \frac{1}{\rho} \left[ p_{t,i,j+1,k} - p_{t,i,j-1,k} \right] \cdot \frac{At}{2} \frac{Ay}{Ay} \]

\[- \left( v_{t,i,j,k+1} w_{i,j,k+1} - v_{t,i,j,k-1} w_{i,j,k-1} \right) \cdot \frac{At}{2} \frac{Az}{Az} \]

\[ + \left( \mu_x \left[ \left( Y' \right)^2 \left( v_{t,i+1,j,k} - 2 v_{t,i,j,k} + v_{t,i-1,j,k} \right) \cdot \frac{1}{(Ax)^2} + X' \left( v_{t,i+1,j,k} - v_{t,i-1,j,k} \right) \cdot \frac{1}{(Ax)^2} \right] \right) \cdot \frac{At}{2} \frac{Ax}{Ax} \]

\[ + \left( \mu_y \left[ \left( Y' \right)^2 \left( v_{t,i,j+1,k} - 2 v_{t,i,j,k} + v_{t,i,j-1,k} \right) \cdot \frac{1}{(Ay)^2} + Y' \left( v_{t,i,j+1,k} - v_{t,i,j-1,k} \right) \cdot \frac{1}{(Ay)^2} \right] \right) \cdot \frac{At}{2} \frac{Ay}{Ay} \]

\[ + \left( \mu_z \left[ \left( v_{t,i,j,k+1} - 2 v_{t,i,j,k} + v_{t,i,j,k-1} \right) \cdot \frac{1}{(Az)^2} \right] \right) \cdot \frac{At}{\rho} \cdot \left[ \left( v_{t,i,j,k} \right) \right] \cdot \text{DIFFUSION} \cdot K \cdot \Delta t \]  

(4.17)
Table 5: (Continued)

4. Conservation of Salt Equation

\[ S[t,i,j,k] = S[t,i,j,k] - \left[ u[t,i,j,k] \cdot X'[i] (S[t,i+1,j,k] - S[t,i-1,j,k]) \frac{1}{2 \Delta x} \right. \]

\[ + v[t,i,j,k] \cdot Y'[j] (-S[t,i,j+2,k] + 8 S[t,i,j+1,k] - 8 S[t,i,j-1,k] + S[t,i,j-2,k]) \frac{1}{12 \Delta y} \]

\[ + w[t,i,j,k] \cdot (S[t,i,j,k+1] - S[t,i,j,k-1]) \frac{1}{2 \Delta z} \quad \text{DIFFUSION} \Delta t \]

\[ \text{DIFFUSION} = D_x \left[ (X'[i])^2 (S[t,i+1,j,k] - 2 S[t,i,j,k] + S[t,i-1,j,k]) \frac{1}{(\Delta x)^2} + X'[i] (S[t,i+1,j,k] - S[t,i-1,j,k]) \frac{1}{2 \Delta x} \right. \]

\[ + D_y \left[ (Y'[j])^2 (-7 S[t,i,j+2,k] + 64 S[t,i,j+1,k] - 114 S[t,i,j,k] + 64 S[t,i,j-1,k] - 7 S[t,i,j-2,k]) \frac{1}{(6 \Delta y)^2} \right. \]

\[ + (Y'[j]) (-S[t,i,j+2,k] + 8 S[t,i,j+1,k] - 8 S[t,i,j-1,k] + S[t,i,j-2,k]) \frac{1}{12 \Delta y} \]

\[ + D_z \left[ S[t,i,j,k+1] - 2 S[t,i,j,k] + S[t,i,j,k-1] \right] \frac{1}{(\Delta z)^2} \]
5. Equation of Zero Volume Dilatation
\[
\omega_{i,j,k} = \omega_{i,j,k-1} - 0.25 \left( X'[i] \left[ (u[\tau,i+1,j,k] - u[\tau,i-1,j,k]) + (u[\tau,i+1,k-1] - u[\tau,i-1,k-1]) \right] \frac{1}{\Delta x} 
+ Y'[j] \left[ (v[\tau,i,j+1,k] - v[\tau,i,j-1,k]) + (v[\tau,i,j+1,k-1] - v[\tau,i,j-1,k-1]) \right] \frac{1}{\Delta y} \right) \Delta z
\]

6. Conservation of Momentum Equation in the z-Direction
\[
p_{i,j,k} = p_{i,j,k+1} + 0.5 \left( (w[i,j,k+1])^2 - (w[i,j,k])^2 \right) \rho 
+ \left( \left( 1.0 + X'[i] (u[\tau,i+1,j,k] w[i+1,j,k] - u[\tau,i-1,j,k] w[i-1,j,k]) \frac{1}{2 \Delta x} \right) \rho 
+ Y'[j] (v[\tau,i,j+1,k] w[i,j+1,k] - v[\tau,i,j-1,k] w[i,j-1,k]) \frac{1}{2 \Delta y} \right) 
- \mu_x \left( (X'[i])^2 (w[i+1,j,k] - 2 w[i,j,k] + w[i-1,j,k]) - \frac{1}{(\Delta x)^2} \right) 
+ X'[i] (w[i+1,j,k] - w[i-1,j,k]) \frac{1}{2 \Delta x} \right) 
- \mu_y \left( (Y'[j])^2 (w[i,j+1,k] - 2 w[i,j,k] + w[i,j-1,k]) - \frac{1}{(\Delta y)^2} \right) 
+ Y'[j] (w[i,j+1,k] - w[i,j-1,k]) \frac{1}{2 \Delta y} \right) 
- \mu_z \left( (w[i,j,k+1] - 2 w[i,j,k] + w[i,j,k-1]) \right) \Delta z
\]
flow field will suffice. Initial conditions often used for such starts involve specifying velocities at all grid points extending from the mouth as the velocity of the river within the mouth. Velocities at all other grid points will be specified as having a uniform lateral tidal velocity. This is similar to allowing the river to flow through a cylinder extended to the boundary in the x-direction. Beginning with such crude initial conditions will be designated a "cold start".

Obviously, a close estimate to the steady state variables of the desired solution will expedite the convergence. For this reason, previously computed values for a similar geometry but slightly different boundary conditions are stored on tape and read as initial values. This often decreases computation time by as much as a factor of three.

D. BOUNDARY CONDITIONS

Although the solution technique presented is classified as an initial-value problem with respect to time, it is a boundary-value problem with respect to space. Dependent variables along all six boundaries of the region of computation must be specified for every time step. These boundary conditions are sometimes difficult to define, but they also increase the versatility of the technique. Boundary conditions are what make each problem unique because the solution of the interior points merely involves satisfying the conservation equations which must be done for every
problem. Moretti (1969) discusses many of the pitfalls associated with improperly specified boundary conditions and presents several suggested approaches.

The boundary conditions for deltaic flow are presented in Figure 6. Each set of conditions will be discussed individually.

Conditions within the river mouth are considered known. Fresh water \((S = 0)\) was assumed at this location for the cases computed, but this was arbitrary as any values of \(S\) may be specified. The river mouth used for computations was elliptic in shape. A parabolic velocity distribution in both the lateral and vertical directions was assumed with no net upwelling or tangential flow \((v, w = 0)\). This produced a velocity distribution similar to the isovels on page 155 of Raudkivi (1967). Height of the surface within the mouth was also specified. All boundary conditions within the river mouth are considered known and not allowed to vary with time.

Obviously, the river mouth shown in Figure 6 represents an idealization; Figure 7 from Coleman and Wright (1971) provides examples of real river mouth shapes. The best way to simulate the mixing from the more complicated of these river mouth shapes would be to input experimental data along the upstream plane of the river which could be placed at a convenient location for the experimenter. This would entail modifying the geometry built into the program, including that of the shoreline, but the modification could probably be made with no adverse effects upon stability, run time, etc. For use of such experimental data,
Figure 6: Boundary Conditions for Delta Flow Calculations
Figure 7: Description of Actual River Deltas (from Coleman and Wright (1971))
there is no requirement for independence of the river conditions from the sea, because any set of physically possible steady state data can be input along a boundary conditions. Subsequent versions of this computer program will contain the versatility described above.

The boundary along the beach is vertical as if it were a steep bluff. The coarse spacing of the grid points in the x-direction precludes the resolution required to enforce a no-slip condition for velocity; consequently, reflection principles were used as discussed by Richtmyer and Morton (1967). For reflection, a condition of tangential flow is enforced.

Along the bottom a no-slip condition for velocity is enforced. This says that the current at the bottom is zero. Because the shape of the bottom is irregular, it does not correspond to a grid plane. Consequently, values of the dependent variables for the first grid point above the bottom are approximated. A nonlinear interpolation using values computed above and the no-slip condition on the bottom was used to estimate the velocity components, and extrapolation from above provided S. Computing the pressure at these points presented no problem because of the spatial marching procedure used. This will be discussed shortly.

Pressure at the surface must always be the same as the ambient pressure of the atmosphere taken here to be a gauge pressure of zero. What is not known is the location of the free surface. It must be free to float up or down until all forces balance the
inertia. Because wave action and wind shear have been neglected from this analysis, the surface height must approach a steady state value. For this converged condition, flow at the surface must be tangential.

Basin boundaries to the flanks of the river have been extended to plus and minus infinity for all practical purposes. Here the height of the surface is held constant at ambient sea level, and the water is considered pure sea water ($S = 1.0$). A uniform, parallel boundary layer type velocity profile in the vertical direction (see page 117 of Raudkivi 1967) was input and held constant. Forcing a constant tidal velocity profile on the boundary of the downstream flank of the river caused some concern. To evaluate this effect other boundary conditions were imposed such as linear and nonlinear extrapolation. Results indicated that this boundary was far enough from the region of interest so that no matter how this boundary condition was imposed it had no discernable effect on flow patterns except possibly upon the nearest two grid rows. The upstream boundary was found to influence the flow interaction region implying that its effect was convected downstream.

The boundary downstream of the river mouth was not stretched to infinity because all of the fresh water from the river must flow through this plane. Values of the dependent variables in this plane must be approximated. A linear extrapolation was found to

---

* Convection, often known as advection in physical oceanography and meteorology, is defined as the process by which fluid properties are transported by the mean velocity of the fluid.
be sufficient. This approximation probably somewhat biased the solution at the nearest grid points, but experience indicated that effects are negligible elsewhere. The stretching transformation also serves to minimize these effects.

E. SOLUTION SEQUENCE

The finite difference equations of Table 5 are solved in a particular sequence for each time step. From known conditions at a given time \( t \), new values of \( u \), \( v \), and \( S \) are computed for every grid point at the next time \( \tau \). This means that we are stepping forward in time \( \Delta t \) where \( \Delta t = \tau - t \). The updated dependent variables at \( \tau \) are computed using Equations (4.16), (4.17) and (4.18) along with the algebraic Equation (4.15) as required. Next the vertical component of velocity \( w \) is computed at every grid point by a spatial marching procedure. Beginning at the bottom where \( w \) is always zero, new values of \( w \) are progressively computed in the vertical direction by using the lower value of \( w \), along with the previously computed values of \( u \) and \( v \) at \( \tau \) and the requirement of zero volume dilatation as expressed in Equation (4.19).

This procedure works fine until we reach the surface where we are faced with a dilemma. What should we do with the upwelling (or downwelling) flow at the surface? Only two things can happen; the vertical component of velocity will be redirected in the x or y direction and the surface will move. Experience tells us that the surface is always relatively flat; hence, most of the flow must turn and we have what is commonly known as a boil. In fact,
for a converged steady state solution, the surface must also be steady; therefore, we know that at steady state the vertical component of the velocity at the surface must be zero. Using this logic, the surface height was adjusted at each grid point \((i,j)\) by first writing a Taylor's series

\[
ws\{\tau, i, j\} = ws\{t, i, j\} + \left(\frac{\partial ws}{\partial t}\{t, i, j\}\right)\Delta t + \ldots
\]

where \(ws\) represents the surface velocity. Let \(ws\{t\} = 0\), an assumption certainly valid for \(t = 0\) and \(t \to \infty\). Also, the acceleration of \(w\) at the grid row below the surface \((k = k_{\text{max}} - 1)\) is used to estimate the surface acceleration;

\[
\frac{\partial ws}{\partial t}\{t, i, j\} \approx \frac{0.5}{\Delta t} (w\{\tau, i, j, k_{\text{max}} - 1\} - w\{t, i, j, k_{\text{max}} - 1\})
\]

Now, the vertical component of velocity at the surface becomes

\[
ws\{\tau, i, j\} \approx 0.5(w\{\tau, i, j, k_{\text{max}} - 1\} - w\{t, i, j, k_{\text{max}} - 1\}) \quad (4*21)
\]

A new surface height at each grid interaction \((i,j)\) can now be computed by

\[
h\{\tau, i, j\} = h\{t, i, j\} + ws\{\tau, i, j\}\Delta t \quad (4*22)
\]

The problem of what to do with the flow upwelling (or downwelling) at the surface still remains. Fortunately, \(w\) is always at least an order of magnitude less than \(u\) or \(v\), or both. Boundary conditions at \(x = 0\) must be respected as this is a solid boundary and the constant volume requirement of Equation \((4*19)\) must be satisfied. Because a revelation did not occur, it was decided
to distribute the additional flow from below evenly to the $u$ and $v$ components of velocity at the surface such that Equation (4.19) and boundary conditions were satisfied.

Now that all other dependent variables have been computed at $T$ and the surface height has been adjusted, the pressure $p$ can be computed at all grid points. Once more a spatial marching procedure is used, only now it is begun at the surface where $p = 0$ and proceeds downward as shown by Equation (4.20) of Table 5. The procedure is begun at the next to the top grid plane because the top grid plane is so near the surface (sometimes above and sometimes below) that the pressure here is assumed to be zero. Pressures computed at the next to the top grid plane include the effect of hydrostatic pressure from the true surface location and not the increment contributed from the grid plane above as is done for lower grid planes.

F. CONVERGENCE

The solution should asymptotically approach a steady state solution, but some criteria must be established for determining how fast the solution is proceeding toward convergence, and when it has converged within an acceptable tolerance. To accomplish this, the following terms which are analogous to those used by Prozan (1971) were monitored:

$$\tilde{g}_1 = \sum_{i,j,k} |S[t,i,j,k] - S[t,i,j,k]|$$  \hspace{1cm} (4.23)
\[ g_2 = \sum_{i,j,k} |u(t,i,j,k) - u(t,i,j,k)| \] (4*24)

\[ g_3 = \sum_{i,j,k} |v(t,i,j,k) - v(t,i,j,k)| \] (4*25)

Figure 8 indicates how the solution converges when beginning from a cold start. For this particular computer run, the three components of velocity at one grid point were monitored after each grid step to illuminate the converging process. Results shown in Figure 9 indicate that velocity at this grid point could logically be considered as steady state after only 200 time steps although the program was continued for another 400 time steps.

G. UNIQUENESS

For a given set of boundary conditions, the numerical technique should converge upon one solution regardless of the initial conditions used. This property, known as uniqueness, is often discussed in standard texts on numerical analysis. Unfortunately, for a set of equations as complicated as the ones solved here, a formal derivation of uniqueness is not possible; the alternative is numerical experimentation. To perform this test, two cases were computed, each having the same boundary conditions, but different initial conditions. After sufficient run times to insure convergence, a comparison of the dependent variables revealed negligible differences (typically in the fourth or fifth decimal places) between the two cases.
Figure 8: Typical History of Convergence Process

Grid Size: (17, 31, 11)
Figure 9: Typical Convergence History of Velocity Components
Further proof of uniqueness is provided by results from a pilot program in two-dimensions which was successfully used to compute a problem with a known solution. The problem, similar to one analyzed by Harleman (1966) and Daly and Pracht (1968), was that of a two-dimensional tank with a splitter plate initially separating fresh water on one side and salt water on the other. When the splitter plate was removed, the pressure gradient resulting from the difference in density caused the salt water to flow under the fresh water as a salt wedge. A converged solution was obtained when the salt water lay along the bottom of the tank under a layer of fresh water. Results of the transient response of the fluid and the converged solution may be found in a previous publication by Waldrop (1972).

H. STABILITY

The maximum possible time step for this type of numerical procedure is defined by the Courant-Friedricks-Lewy (CFL) condition which, in the words of Moretti (1969a), says that "The domain of dependence of the partial differential equations must be contained within the domain of dependence of the finite difference equations". Stating this criteria is one thing, but applying it to the three-dimensional set of equations of Table 5 is another. Because the set of time dependent differential equations behave as if they are hyperbolic with respect to time, then presumably characteristic lines exist. The CFL stability criteria says that the maximum time step must be less than that defined by the
intersection of characteristics from the closest grid points in the grid system.

Defining such an intersection of characteristic lines for a set of equations this complicated is not easy. It is no wonder that practitioners turn to approximations to define the stability limit. By analyzing the one-dimensional unsteady incompressible flow equations with a free surface, Stoker (1957) determined that the slope of the characteristics are defined by the speed of a long wave. This says that disturbances are propagated at the wave speed

\[ C \approx \sqrt{gh} \]

Thus, the maximum time step for a finite difference computation of such flows can be determined by letting

\[ C \approx \frac{\Delta x}{\Delta t} \]

producing

\[ \Delta t \leq \frac{1}{\sqrt{gh}} \Delta x, \]

or, in nondimensional form,

\[ \Delta t \leq \Delta x \sqrt{\frac{H}{h}} \tag{4*26} \]

but \( \sqrt{\frac{H}{h}} \approx 1.0 \). Equation (4*26) was used by Harlow and Welch (1965), Laevastu and Robe (1972), and Feigner and Harris (1970) as the incompressible flow analogy of the CFL stability criteria.

Because we are working in three-dimensions, Equation (4*26)

\* Anyone with an abundance of enthusiasm for the mathematics of multi-dimensional characteristics can refer to Courant and Hilbert (1953).
must be modified as

\[ \Delta t \leq \Delta( ) \]  \hspace{1cm} (4*27)

where \( \Delta( ) \) represents the smallest value of \( \Delta x \) or \( \Delta y \) throughout the grid system. As was previously noted, \( \frac{\partial w}{\partial t} \) was omitted from the z-momentum equation of Table 3. This omission apparently eliminates vertical characteristics because the grid spacing of \( \Delta z \) has no effect upon the maximum permissible time step. This is extremely valuable since typical vertical dimensions (and thus grid spacing) are much smaller than those in the horizontal for deltaic flows as may be seen in Table 3.

The possibility of applying this technique to inviscid calculations was investigated by setting the viscosity and diffusion coefficients to zero. This resulted in waviness at sharp gradients which grew into instabilities. From these limited investigations, it was tentatively concluded that this technique is not applicable for the solution of inviscid fluids, but there is very little of that type remaining.

Stability analyses are primarily academic; their results serve only as an initial guess of \( \Delta t \) for the practitioner. Pragmatism dominates in the application of these numerical techniques, as they may be used most efficiently when the largest stable time step is used. Experience indicates that the presence of an instability is easily spotted. Two checks for instability were monitored throughout the calculation:

\[ x \]

Note that \( \Delta x \) and \( \Delta y \) are used and not \( \Delta X \) or \( \Delta Y \).
a) Antimatter criteria - for this case it was manifested as a negative pressure.

b) Rough surf - the free surface at some point had dropped below the next to the top grid plane. When either of these tests indicated an instability, the location of this instability was noted and the computation was terminated.

During the development of this technique, it was noted that the slow convergence of the conservation of species equation retarded convergence of the complete set of equations. This can best be understood by recalling that velocities are free to adjust throughout the region of computation at a rate determined by the surface wave speed, but the density must adjust at a rate governed by the speed of the water and the diffusion. This means that the flow field will rapidly adjust itself to satisfy inertia imbalances, but must wait for the proper density of fluid to be convected or diffuse throughout. The correct buoyancy forces cannot begin to work until this is accomplished.

After this problem was isolated, it was postulated and subsequently verified that the speed of computation of the conservation of species equation had a negligible effect upon the stability of the computation. Therefore, following the philosophy so well put by Tearpair and Seay (1971) that "Moderation in the pursuit of a converged solution is no virtue", the time step used in Equation (4*18) was 5 Δt. Computation times were significantly reduced.
The digital computer program to accomplish this numerical solution is presented as Appendix B. Calculated results from this program will serve as a basis for the analysis of the deposition of suspended material of the river which actually causes the delta to grow.
CHAPTER V

TRANSPORTATION AND DEPOSITION OF SOIL NEAR THE RIVER MOUTH

Predicting the movement of soil particles in or along a boundary of a flowing stream is no trivial task. The motion of the particles is described by Newton's second law of motion, but all forces upon the particle must be known as a function of time and space. Pertinent forces include shear and pressure forces from the flowing fluid, forces due to gravity, and interparticulate forces. Most of the prominent attempts at describing these forces and the resulting movement of soil in flowing streams, along with much relevant experimental data, are reviewed by Raudkivi (1967), Sundborg (1956), Vanoni, Brooks and Kennedy (1961), and Bagnold (1966). Based primarily upon the most plausible ideas of these publications, a brief discussion of factors which influence soil movement and deposition is presented.

As a river empties into the sea, the semi-equilibrium conditions which had existed within the river will be destroyed. Turbulence levels and velocity patterns will be altered. Here certain phenomena heretofore neglected must be considered and empirical relationships for describing flow within a channel become suspect.

This discussion of sediment transport and deposition will be followed by a description of the numerical procedure used for computation of deltaic growth. Basic assumptions of this analysis and the empirical formulae used are stated.
A. TRANSPORTATION OF SOIL PARTICLES IN THE VICINITY OF A RIVER DELTA

Soil particles are involved in a continuous exchange process between the bed and the flowing river. Particles resting upon the bed are freed as a result of shearing forces exerted by the flowing fluid, pressure forces of lift and profile drag, and perhaps gravity forces if the bottom slope is large. Once freed, these particles are transported by the flowing water. Because the density of sand or silt particles is greater than that of the water, these particles will be constantly attempting to settle to the bottom.

Suspended material is transported by the mean local velocity of the flow and by the random fluctuating velocity if the river flow is turbulent which it usually is. For turbulent flows, an intermittent instability of the boundary layer or flow over an uneven bottom may produce a large turbulent vortex which will lift a particle to various heights, depending upon how long the particle remains entrapped or how long the turbulent vortex can maintain its identity.

The transportation of solid particles by the random motion of the turbulence is akin to diffusion. Naturally, prediction of the precise motion of the particles would require a complete knowledge of the motion of the turbulent transporting fluid; this is not yet available. Bagnold (1966) believes that the turbulence must somehow exert an upward stress on the particles in order to maintain a suspension of particles heavier than water.
This led him to hypothesize "--- that the anisotropy of shear turbulence must involve as a second-order effect a small internal dynamic stress directed perpendicularly away from the shear boundary". Unfortunately, as Bagnold conceded, this anistropy is too small to be detected by instruments currently used for such measurements.

The classical approach for explaining the turbulent transporation of particles in a dilute suspension is to consider the flow eddy diffusive, and experimentally determine an eddy diffusivity coefficient of the suspended material by much the same procedure that other eddy coefficients are determined. This coefficient must be a function of the local turbulence level as well as the density and size of the suspended particles. Results of several investigators (e.g. Farmer (1962), Murray (1970) and Ismail (1952)) indicate that the eddy diffusivity coefficients of solid material of the approximate size and density of quartz sand were on the same order of magnitude as the eddy diffusivity coefficients of the fluid. Thus, a mechanism is provided for explaining the vertical transport of suspended material to counterbalance the continuous settling process of the solid particles. This explanation also explains the increased concentration of suspended material near the bottom since the driving force for a diffusive process is a concentration profile.

The settling velocity of sand or silt particles will also be influenced by the local turbulence levels of the fluid. Newton's second law of motion describes the settling of the particles,
but, as discussed by Farmer (1962) and Businger (1965), the drag force on the particle is a nonlinear function of the particle velocity relative to the fluid. Murray (1970) related the drag coefficient to the frequency of turbulence of the fluid. He subsequently estimated these frequencies as a function of the free stream velocity of the fluid and computed the reduction of fall velocity of quartz grains over a range of particle diameters and free stream velocities. His conclusion was that settling velocities of sand could be reduced from the still water values by as much as 40% at velocities typical of those commonly measured in rivers.

Like the turbulent transport of suspended material, the removal of particles from the bottom does not lend itself to rigorous theoretical analysis because the dominant forces on the particles cannot yet be precisely described. According to the classical theory presented by Sundborg (1956), shear and lift forces on the particles lying on the bed are controlled by flow within the laminar sublayer of a large turbulent boundary layer. Because of the long distances of the flow, this turbulent boundary layer often exhibits characteristics of intermittency and its height may reach the free surface. Quite likely, vortices resulting from this intermittancy near the bottom are major contributors to erosion of the bed.

Some material may be transported by movement of the bed itself which behaves as a deformable solid. It reacts to the forces exerted upon it from the fluid flowing above. Movement
of the bed material is difficult to describe because:

a) The shear stresses imparted by the fluid upon the bed material require a definitive description of the fluid velocity.

b) Properties of the bed material must be known to adequately describe movement resulting from shear stresses within the solid.

As a river empties into a receiving basin, the near equilibrium conditions of the sediment exchange with the river bottom are destroyed. The river momentum will be redirected or absorbed, and new turbulence levels and flow patterns will be developed. Wave action, coastal currents and tidal fluctuations are often important in controlling the deposition of suspended material in this region. Primarily from field observations, Coleman and Wright (1971) and Bates (1953) have isolated many of the parameters controlling river delta dynamics.

The mechanics of the erosion, transportation and deposition of soil particles within a river is equally applicable downstream of a river.

Also, where sea and fresh waters mix, a chemical process known as flocculation is a possible influence of sedimentation rates. Flocculation is a process in which the clay particles lose their charge, agglomerate, and settle as larger units. When this occurs these larger units will settle more rapidly than the individual particles. However, in summarizing the reported results of several field investigations Devine (1971) agreed with the
hypothesis of Bates (1953) that flocculation has a negligible effect upon deposition near river mouths. Consequently, flocculation was neglected in the computational procedure presented here.

When considering all of the difficulties involved in describing the movement of solid particles, whether in suspension or as bed material, it is not surprising that most researchers have relied upon empirical techniques. Confidence should be placed upon the results of these empirical techniques only when the predicted case closely corresponds to the data upon which the technique is based. Unfortunately, some of the empirical functions upon which this portion of the computation was forced to rely, are not specifically applicable for flow within the delta region since they are based upon data from flowing rivers or flumes.

B. NUMERICAL ANALYSIS OF DEPOSITION NEAR RIVER MOUTHS

The procedure described here for computing the deposition of suspended material uses the concept of tracking trajectories of nominal particles. These particles represent sediment flux at various locations distributed throughout the river mouth. This approach represents an improvement over that of Bonham-Carter and Sutherland (1968). They also used the concept of tracking nominal particles, but in the technique presented here the fluid mechanics have been described more thoroughly, the geometry of the delta is thought to be more realistic, and the effect of local turbulence levels upon particle settling velocities has been included.
Bed load transport, which becomes even more nebulous as a river empties into the salt water basin, was neglected. Fisk, et al. (1954) report that this form of transport constitutes only ten percent of the total transport of the sediment. The redistribution of settled particles due to wave action was also neglected as was the subsidence of the bottom where material was deposited.

The concentration distribution and size of suspended material at the river mouth was considered known. Samples of suspended material in the lower regions of the Mississippi River, obtained as part of an experimental program by the U.S. Army Corps of Engineers (1939), provide concentrations of several sizes of suspended material for investigations of South Pass of the Mississippi River. Based upon this type of data, a concentration distribution may be expressed in functional form by an empirical relationship presented by Vanoni, et al., and shown here in nondimensional form as

\[
\frac{C}{C_R} = \left[ \frac{(1.0-z)}{(z-d)} \right] \left( \frac{z_R - d}{1.0 - z_R} \right)^k
\]

(5.1)

where

- \(C\) = concentration of suspended material of a given size
- \(R\) = reference level where concentration is known
- \(d\) = maximum river depth
- \(k\) = empirical constant; a function of river velocity, and type and size of suspended material.
Values of $k$ tabulated on page 5-5 of Vanoni, et al. for sand particles are typically of order one, but $k$ can be adjusted to make Equation (5*1) fit experimentally observed profiles at a given river mouth.

Fluid dynamic drag was assumed to be the dominant horizontal force acting upon the suspended material; consequently, the horizontal movement of the particles was controlled by the local mean horizontal velocity of the fluid. In the vertical direction, the particles were assumed to be falling at a constant settling velocity relative to the mean vertical component of velocity of the fluid. This settling velocity (i.e., terminal velocity) is defined as a balance between the fluid dynamic drag and forces resulting from the pull of gravity. The suspended material was idealized as spherical bodies. Gibbs (1971) provides a convenient source of settling velocities in still water for such particles. Because the mean size and settling velocities are data inputs to the computer program, more precise data could easily be used as they become available.

As previously discussed, the settling velocity of particles should reflect the local turbulence level of the fluid. In a broad sense, this local turbulence level may be related to the local velocity gradient $V_q$ when

$$ q = \sqrt{u^2 + v^2 + w^2} $$

$$ \approx \sqrt{u^2 + v^2} $$

(5*2)
Using a nominal decrease in settling velocity of 0.3 from the results of Murray, the settling velocity of each particle was corrected at each time step by

\[ w_s(L) = w_{so}(L) \left[ 1.0 - 0.3 \frac{|v_q|}{v_{q_{\text{ref}}}} \right], \quad (5*3) \]

and

\[ v_{q_{\text{ref}}} \] was arbitrarily specified as

\[ v_{q_{\text{ref}}} = \frac{\text{Maximum river velocity at the mouth}}{\text{Maximum river depth at the mouth}} \]

Once a concentration distribution and settling velocities have been established for suspended material of a given size, nominal particles are distributed and tagged within the river mouth. A volume flux of solid material was then computed and assigned to each nominal particle as

\[ \text{VF} = \frac{C}{\rho_p} (\Delta y)(\Delta z)u \sim \text{cm}^3/\text{sec of solid} \quad (5*4) \]

where

\[ C = \text{concentration of suspended material} \]
\[ \rho_p = \text{density of the suspended particles} \]
\[ u = \text{component of particle velocity in the x-direction.} \]

The trajectory of nominal particle \( L \) was then computed after it left the mouth and moved through the flow field by

\[ x_p \{L,t_1\} = x_p \{L,t_0\} + u \Delta t \quad (5*5) \]
\[ y_p(L_{t_1}) = y_p(L_{t_0}) + v \Delta t \]  \hspace{1cm} (5*6) \\
\[ z_p(L_{t_1}) = z_p(L_{t_0}) + [w + w_s(L)]\Delta t, \]  \hspace{1cm} (5*7)

Where:

- \( x_p, y_p, z_p \) = particle coordinates
- \( L \) = tag number of each nominal particle; \( L=1,2,... \)
- \( u, v, w \) = components of particle velocity (i.e., mean fluid velocity) in the \( x, y, z \) directions, respectively
- \( w_s(L) \) = settling velocity of particle \( L \)
- \( \Delta t \) = time increment; \( \tau-t \).

This procedure was repeated until all nominal particles either settled to the bottom or were convected beyond a region of interest. Because the settling velocity is greater for larger particles, the suspended material from a given location in the river mouth should be sorted with the largest particles settling nearest the river mouth.

Once a particle has settled to the bottom, a test is made to determine the maximum slope of the bottom at that location. If the local bottom slope was greater than the average angle of repose of the particles considered, then the particle was rejected by the bottom by raising the particle a small increment above the bottom and forcing it to be carried with the current until settling elsewhere.
Sundborg (1956) discussed a "deposition velocity" which he defines as a maximum velocity, measured a given height above the bottom, for which suspended particles will settle. He states that experimental observations indicate that this deposition velocity is about two-thirds of the "critical erosion velocity", but the mechanism which prevents particles from settling was not isolated. Sundborg presented the following empirical estimate of "deposition velocity" obtained from flume studies, rewritten in our nondimensional form, as

\[ q_{DV} = 3.8 \sqrt{0.04 \left( \frac{\rho_p}{\rho} - 1.0 \right) D \log_{10} \left( \frac{30.2 \hat{z}}{D} \right)} \]  

(5*8)

where

- \( q_{DV} \) = deposition velocity
- \( \hat{z} \) = height for which \( q_{DV} \) is computed
- \( D \) = diameter of the particles
- \( \rho_p \) = density of the particles
- \( \rho \) = density of the water; \( \rho \approx 1.0 \).

Equation (5*8) was used for a case of:

a) Flow over a parallel bottom

b) Turbulent flow

c) A rough bottom (i.e., sandy soil).

If the magnitude of the fluid velocity vector at a given height above the bottom was greater than \( q_{DV} \) where a particle settles, it was rejected by the bottom and forced to settle elsewhere.
Once all of the nominal particles have settled or have been convected beyond the region of interest, a region surrounding each grid intersection \((i,j)\) was scanned to determine which nominal particles had settled within the domain defined by

\[
x_B < x_p{[}L{]} \leq x_F
\]

and

\[
y_R < y_p{[}L{]} \leq y_L
\]

where

\[
x_B = 0.5[x[i] + x[i-1]]
\]

\[
x_F = 0.5[x[i+1] + x[i]]
\]

\[
y_R = 0.5[y[j] + y[j-1]]
\]

\[
y_L = 0.5[y[j+1] + y[j]]
\]

The growth rate of the bottom at the intersection of grid lines \((i,j)\) was then computed by using only the nominal particles within that domain to compute

\[
GR[i,j] = 1.0 \frac{1.0-P}{L} \sum_{L} \left\{ \frac{VF[L]}{[x_F-x_B][y_L-y_R]} \right\} \quad (5*9)
\]

For loosely packed sand, Sundborg gives a typical value of the porosity of the bottom, \(P\), as 0.4.

Even though the growth rate of the bottom is an instantaneous value, it was assumed that constant conditions persisted sufficiently long to permit an incremental adjustment of the bottom height at
each grid intersection. The new bottom shape may then be used to recalculate the fluid flow field. This procedure could conceivably be continued almost indefinitely.

The computer program used to calculate the transportation and deposition of suspended material in a delta region is presented as Appendix C. Computations performed with the procedure described here are presented in a later section.
Investigations using the previously described computer programs were divided into three areas:

i) Evaluate the effect of systematically modifying the individual parameters which control the flow field.

ii) Compute flow fields spanning the range of coastal currents and river velocities observed near South Pass of the Mississippi River.

iii) Predict delta growth by computing the deposition of suspended material for nominal flow conditions of South Pass.

The presentation and discussion of data calculated to accomplish the first two objectives will be combined because only flow field calculations are involved. The third objective involves sediment transport calculations as well.

As previously discussed, one of the goals of this study was to simulate conditions near South Pass of the Mississippi River for evaluation of results from the numerical model. Actually, the geometry of South Pass was somewhat idealized instead of attempting the more difficult task of precisely duplicating the complex geometry of this region as shown in Figure 10. Future versions of the model will include more realistic details.

The offshore bottom slope used for computations is typical of the bottom slope thought to exist in the initial stages of
Figure 10: Depth Contours on the South Pass Delta
delta development, but it is not typical of the present bottom slope. As a result, flow field comparisons were not as good as they might be, but it was possible to compare deposition trends with current bottom contours to demonstrate that the predicted results are reasonable.

A. COMPUTED FLOW FIELDS IN DELTAS

The influence of individual factors, such as buoyancy, river stage, or turbulent mixing, upon the flow patterns was evaluated by systematically varying input parameters to the computer program and comparing calculated results. The idealized geometry of South Pass of the Mississippi River used in all computations for the flow field analysis is given by:

- Reference Depth, \( H \) --------- 15 meters
- Maximum River Depth--------- 10.5 meters
- Ratio of River Width to Depth---- 13
- Mean Slope of the Offshore Bottom-----0.0026

Surface velocities within the river mouth at the centerline were specified as 1.0 m/sec to reflect low river stage and 4.0 m/sec to simulate flood stage. A nominal value of 2.5 m/sec was used for a baseline computation for comparison with perturbation runs. Surface velocities for coastal currents were varied from 0.0 to 1.0 m/sec to reflect observed currents near the Mississippi River Delta.
The average density of sea water near South Pass is, in nondimensional form, $\rho = 1.022$ or $\sigma_t = 22$ where $\sigma_t$ is defined as

$$\sigma_t = (\rho - 1) \times 10^{-3}.$$  

The difference between the height of the river surface at the mouth and true sea level far from the mouth is difficult to measure, but it is probably never more than 15 or 20 cm (ref. Coleman 1972). For all cases except the run to evaluate this effect, the surface height was set at sea level.

Using these data, the eight cases shown in Table 6 were computed. Tabulations of the dependent variables $(u, v, w, S, p)$ as well as the surface height were outputs of the computer program, but tabulated data were not presented here because of the quantity involved. Table 7 shows a sample of these tabulated data. Comprehension of data in this form is also difficult. Instead, plots of velocity vectors and isoconcentration contours for the eight cases are presented as Appendix D. These plots represent cuts through all three planes of the region - a top view of the surface and of a top view of the six meter depth, a side view of the deepest part of the river extending into the sea, and end views looking toward the river mouth. When viewing these plots, note that all vertical scales have been greatly exaggerated to improve vertical resolution.

All computations were performed with 19 grid rows in the x-direction, 31 in the y-direction, and 10 in the vertical direction.
<table>
<thead>
<tr>
<th>Title</th>
<th>Run</th>
<th>Max. River Velocity (m/sec)</th>
<th>Max. Coastal Current (m/sec)</th>
<th>Elev. of River Surface (cm)</th>
<th>Eddy Visc./Diff. Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>1</td>
<td>2.5</td>
<td>0</td>
<td>22</td>
<td>0</td>
</tr>
<tr>
<td>Buoyancy Analyzed</td>
<td>2</td>
<td>2.5</td>
<td>0</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>Hydraulic Head Effects</td>
<td>3</td>
<td>2.5</td>
<td>0</td>
<td>22</td>
<td>15</td>
</tr>
<tr>
<td>Turbulent Mixing Analyzed</td>
<td>4</td>
<td>2.5</td>
<td>0</td>
<td>22</td>
<td>0</td>
</tr>
<tr>
<td>Low River Stage</td>
<td>5</td>
<td>1.0</td>
<td>0</td>
<td>22</td>
<td>0</td>
</tr>
<tr>
<td>Flood Stage</td>
<td>6</td>
<td>4.0</td>
<td>0</td>
<td>22</td>
<td>0</td>
</tr>
<tr>
<td>Low River Stage Max. Coastal Current</td>
<td>7</td>
<td>1.0</td>
<td>-1.0</td>
<td>22</td>
<td>0</td>
</tr>
<tr>
<td>Flood Stage Max. Coastal Current</td>
<td>8</td>
<td>4.0</td>
<td>-1.0</td>
<td>22</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 7: Sample Printout of Fluid Dynamics Computer Program

<table>
<thead>
<tr>
<th>U</th>
<th>V</th>
<th>W</th>
<th>SALT CONC.</th>
<th>PRESSURE</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>-0.740532E+01</td>
<td>0.00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>1.0</td>
<td>4.0</td>
</tr>
<tr>
<td>0.0</td>
<td>-0.427114E+00</td>
<td>0.00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>1.0</td>
<td>5.0</td>
</tr>
<tr>
<td>0.0</td>
<td>-0.162682E+00</td>
<td>0.00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>1.0</td>
<td>6.0</td>
</tr>
<tr>
<td>0.0</td>
<td>-0.104257E+00</td>
<td>0.00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>1.0</td>
<td>7.0</td>
</tr>
<tr>
<td>0.0</td>
<td>-0.193333E+00</td>
<td>0.00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>1.0</td>
<td>8.0</td>
</tr>
<tr>
<td>0.0</td>
<td>-0.199417E+00</td>
<td>0.00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>1.0</td>
<td>9.0</td>
</tr>
<tr>
<td>0.0</td>
<td>-0.200000E+00</td>
<td>0.00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>1.0</td>
<td>10.0</td>
</tr>
<tr>
<td>0.0</td>
<td>-0.200000E+00</td>
<td>0.00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>1.0</td>
<td>11.0</td>
</tr>
<tr>
<td>0.0</td>
<td>-0.200000E+00</td>
<td>0.00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>1.0</td>
<td>12.0</td>
</tr>
<tr>
<td>0.0</td>
<td>-0.200000E+00</td>
<td>0.00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>1.0</td>
<td>13.0</td>
</tr>
</tbody>
</table>
Because the stretching transformations were used to place some boundary planes far from the river mouth, data at the last few grid planes in the x and y directions were not shown.

The effect of buoyancy of the fresh water upon the flow patterns and mixing process can be determined by comparing data from the Reduced Buoyancy run (Run 2) with that of the Baseline run (Run 1). The density of the sea water for Run 2 was decreased such that it was only 0.5% greater than that of the fresh water ($\sigma_t = 5$). The top and side views of the velocity vectors appear almost identical between these two runs, but mixing occurred much slower for the $\sigma_t = 5$ case as indicated by the concentration contours. Analysis of the velocity vectors in the end views explains this best. Two large symmetrical vortices formed about the centerline of the jet of the Baseline run. These vortices resulted from pressure gradients in the y-direction which were caused by density differences of the two fluids. This is an identical mechanism to the one which induced the movement of the salt wedge and resulting circulation of the tank problem of salt and fresh water described by Waldrop (1972). As a result of these buoyant vortices, salt water is convected into the jet from the bottom as shown in the end views of isoconcentration contours.

The buoyant vortices are weaker for the $\sigma_t = 5$ case because the pressure gradients which serve to drive these vortices are smaller. In the second end view of the decreased buoyancy run, the vortices are barely distinguishable.
The increase in the hydraulic head within the river mouth may be seen by comparing the Higher River Surface run (Run 3) with the Baseline Run. For Run 3, the surface height of the river at the grid row within the river mouth was raised 15 centimeters above the sea level at the far boundary of the region of computation. The height of the surface quickly settled to a variable surface height throughout the plume which never exceeded 4 centimeters. It was typically 1 centimeter higher than the surface height of the jet of the Baseline run. Nevertheless, the increase in pressure at all depths was significant. Comparison with the Baseline run shows that the velocities in the Higher River Surface run were lower in the buoyant vorticies and as a result mixing was delayed. The jet of the Higher River Surface run also appeared to maintain its initial character longer as it was slightly wider at the last grid plane shown in the top view.

Decreasing the eddy coefficients by an order of magnitude provided the most drastic change of all. Velocity vectors in the side view of the Reduced Mixing run had a very pronounced vertical component. Mixing along the centerline was increased near the bottom but decreased near the top. This is confusing until examining the velocity patterns of the end views. Now it is evident that the buoyant vorticies are much more pronounced than those of the Baseline case. Entrainment of salt water from the flanks of the river is more obvious for the Reduced Mixing case. The top views of the velocity profiles also indicate
spreading near the surface and entrainment at the lower depth. Notice that the flow accelerated down the centerline of the jet which was rapidly narrowing.

River velocities also influenced the mixing process of salt and fresh water. This is demonstrated in the plots of the Low River Stage, Baseline, and Flood Stage cases (Runs 5, 1, and 6) which reflect maximum river velocities of 1.0, 2.5, and 4.0 meters per second (m/sec), respectively. A comparison of 2.5 m/sec and 4.0 m/sec data reveals anticipated results. The higher river velocities increased the effect of convection and delayed mixing. Buoyant vortices are less pronounced for the Flood Stage case; consequently, mixing of salt and fresh water was predominately restricted to the horizontal directions.

A comparison of the data for the river velocities of 1.0 m/sec and 2.5 m/sec reveals more subtle differences. Buoyancy was more pronounced for the 1.0 m/sec (Run 5) case and the vortices had induced more mixing near the bottom than for the 2.5 m/sec case (Run 1). Comparing surface velocities in the top views of these two runs reveals a basic difference in the decay of the two jets. The jet of the 2.5 m/sec case slowly spread with an accompanying decrease in velocity, but the 1.0 m/sec jet maintained its initial centerline velocity and decelerated along the borders of the plume. This explains the reduced salinity at the surface and along the centerline of the plume when concentration profiles are compared with Baseline data.
The interaction of the river at low stage (1.0 m/sec) with the maximum observed coastal current (1.0 m/sec) is shown in data from Run 7. Both the river and coastal currents are radically deflected. Velocity vectors in the top views of Run 7 reveal a large scale recirculation region on the leeward side of the river plume. This is a likely place for deposition of suspended material from the edge of the river. Even very fine grain material of very small settling velocities could conceivably become entrapped and settle here.

The velocity patterns of the view nearest to the river mouth closely resemble those of the case with no coastal current, but this symmetrical pattern did not persist very far from the river mouth. The coastal current destroyed the symmetry and apparently attempted to flow under the fresh water. These isoconcentration contours and velocity vectors along with those shown in the top views revealed a three-dimensional flow pattern which would be almost impossible to duplicate or even reasonably approximate in two-dimensions.

The interaction of the river at flood stage (4.0 m/sec) with a coastal current of 1.0 m/sec is shown in the data from Run 8. These data look remarkably similar to those of Run 6, the case of no coastal current. This implies that close to the river mouth a mere coastal current is no match for a river with velocities of up to 4.0 m/sec. Inspection of tabulated data indicated that the plume was increasingly deflected as it moved further from the mouth.
Evidence of this trend can be seen in the velocity vectors of the end view furthest from the mouth and in the isoconcentration contours of the surface.

Data from Figure 12 presented on page 102 of the next section provides a more complete description of the increasing influence of a side current. In this figure a series of velocity vectors from an end view is shown, each progressively further from the river mouth. The ratio of cross currents to maximum river velocity was five for these data; consequently, the velocity vectors shown in Figure 12 should be similar to those of Run 8 where the ratio of cross current to maximum river velocity was four.

Flow conditions of the Low River Stage and Maximum Coastal Current case (Run 7) are comparable to those of published data by Wright and Coleman (1971) from field studies of a flooding-tide condition at the South Pass of the Mississippi River. The numerical model does not contain a provision for the influx of ambient brackish water flowing along the surface between the plume and the sea water, nor does the model contain a true representation of the offshore bottom contours*, but the flow fields and mixing patterns appear remarkably similar. Both sets of data show that the coastal current deflected the river plume shortly downstream of the river mouth. Probably the river plume was somewhat confined on its leeward side by a mud lump and deposited material. A slight deceleration of the flow within the plume is also indicated in both sets of data.

* A future version of the computer model will have these features.
The field data of Wright and Coleman also show salinities within the plume which are greater than those of the ambient fresh water at the periphery. Thermal imagery further indicates that vertical mixing is significant as evidenced by the warm Gulf water which began to emerge at the surface of the plume some two to three channel widths from the mouth. This phenomena has produced much conjecture. Computer calculations showed a similar trend and offer an explanation as well. The buoyancy of the plume developed a set of vortices which entrained the higher density Gulf water near the bottom. Vertical mixing due to turbulence along with vertical convection velocities brought some of the warmer water of higher salinity to the surface. For the case of South Pass, the fresh water of the river was already flowing over a layer of sea water as it left the mouth, but similar mixing patterns would occur had it not been so. The buoyant vortices would have convected the Gulf water underneath the plume anyway.

In summarizing the results of the flow field analysis, it appears that all of the effects investigated have a significant influence upon the mixing and flow patterns in a delta region. Any analysis which neglects or improperly approximates turbulent mixing, hydraulic head, buoyancy, or inertia of either the river or the coastal current will be inadequate. The fact that all of these effects did appear important emphasizes a need for better field data from which more precise boundary conditions could be specified. This is especially true of surface heights and river velocity profiles within the mouth.
Actually, only a very limited amount of data are required to adequately specify the boundary conditions for this computation. With proper planning, a field study should obtain these data with a minimum of effort. On the other hand, extensive field testing throughout the plume provides data for comparison with calculated results, but few of these data are applicable for specifying boundary conditions.

A more generally applicable eddy viscosity model is also desirable. This could be obtained from field studies of river deltas, but would more likely be derived from laboratory investigations.

Regardless of the approximate nature of some of the boundary conditions and the eddy viscosity model, the results of this technique compared remarkably well with field observations, indicating that an effort to obtain better data for boundary conditions is warranted.

B: DEPOSITION OF SUSPENDED MATERIAL

Conditions of the South Pass delta were again used to demonstrate how deposition of suspended material from a river mouth can modify the bottom slope and form sand bars. The initial geometry of this series of calculations was identical to that used in the flow field analysis. A maximum river velocity of 2.5 m/sec was used and the river surface height was fixed at 3cm above mean sea level. A uniform coastal current of -0.5 m/sec at the surface and sea
Concentration levels of suspended material and particle size distributions vary considerably with time within the river, depending mainly upon the river stage. Consequently, representative data were deemed only sufficient to indicate deposition trends. Concentration levels of suspended material were taken from river sampling experiments of the U.S. Army Corps of Engineers (1939) at Mayersville, Miss., some 500 miles upstream of the Mississippi delta. Data used for the computations (shown in Table 8) were obtained on February 1, 1938 when the river was at high stage. The sample shown came from four feet above the bottom and near the center of the river.

Although the sample shown contained some medium and coarse grain sand, Gagliano, Light and Becker (1971) report that only fine sand and smaller ever reach the delta. Studies of the composition of Mississippi River bed material by the U.S. Corps of Engineers (1935) confirm this observation. What happens to large grain particles between Mayersville, Miss., where they are in suspension, and the Mississippi River delta is unclear. Possibly they are collecting in a deep hole somewhere along the route. Nevertheless, the medium and coarse grained sand were not included in the analysis of deposition rates in the delta.

To compute sedimentation rates for the three remaining classes of suspended material, 350 nominal particles of each of the three types of particles * were distributed throughout the

* A total of 1050 nominal particles.
Table 8: Particle Concentration Data for Deposition Calculations

<table>
<thead>
<tr>
<th>Type Material</th>
<th>Mean Dia. (mm)</th>
<th>Empirical Dist. Coeff. k</th>
<th>Particle Density (gm/cm³)</th>
<th>Conc. (ppm by wt.)</th>
<th>Settling Vel. in Still Water (cm/sec)</th>
<th>Ref. Ht. (Depth/River Depth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silt &amp; Clay</td>
<td>0.025</td>
<td>0.14</td>
<td>2.65</td>
<td>705</td>
<td>-0.035</td>
<td>0.90</td>
</tr>
<tr>
<td>Very Fine Sand</td>
<td>0.075</td>
<td>0.450</td>
<td>2.65</td>
<td>522</td>
<td>-0.450</td>
<td>0.90</td>
</tr>
<tr>
<td>Fine Sand</td>
<td>0.175</td>
<td>1.12</td>
<td>2.65</td>
<td>623</td>
<td>-1.800</td>
<td>0.90</td>
</tr>
</tbody>
</table>
river mouth. Their ensuing trajectories were calculated until they either settled permanently on the bottom, or were convected beyond the region of interest.

Results of the first iteration revealed that none of the silt and clay settled within the region of interest. Apparently the small settling velocities associated with these particles was insignificant when compared to the vertical components of velocity, and these particles were convected far from the mouth before settling. This is consistent with reported field observations of the delta. For this reason, silt and clay were omitted from subsequent analyses.

Deposition distributions of the very fine and fine sand particles produced some puzzling results. The empirical function of Sundborg, which was used as criteria for determining if the flow velocity near the bottom was too great to permit particles from settling, never permitted any sand to settle under the plume. According to this function particles of this size should not stick to the bottom if the velocity of the fluid 10 meters above is greater than about 25 cm/sec. In fact, this function would predict considerable erosion for velocities of only 40 cm/sec. This appeared to be a rather severe restriction upon particle settling patterns because we are dealing with velocities of the order of 200 cm/sec in the plume and yet samples of the bottom material show a composition of fine grain sand.
Merely omitting this function which prohibited the particles from sticking to the bottom did not produce entirely satisfactory results either. For this case much of the suspended load settled very near the river mouth, namely in the first grid row. Possibly the particles that settle there are gently moved as bed loads to a bar some distance downstream of the mouth. Unfortunately, the computer model has no provision for moving particles precisely in this manner, but this effect was simulated by multiplying Sundborg's function by a factor of five, thus making it less restrictive. Results then appeared much more reasonable.

The procedure for computing deposition rates was begun by calculating a flow field for the initial geometry used for the flow field analysis. Particles were then allowed to float through the flow field until settling. A summation of where settled particles within a given area surrounding each grid point produced a growth rate for each grid point. Steady flow conditions were assumed to persist sufficiently long for the bottom height to increase by one vertical grid height at the grid point with the greatest growth rate. A new flow field was then computed using the new bottom shape. Results of the third iteration, presented in Figure 11, showed that the maximum deposition occurred at the same grid point all three times; consequently, the bottom was elevated 4.5 meters above its initial value.

Even with the -0.5 m/sec cross current, particles tended to settle on the leeward side of the plume. As expected, more of the
Figure 11: Computed Deposition Pattern for a Maximum River Velocity of 2.5 m/sec; and a Maximum Coastal Current of -0.5 m/sec
fine grain sand settled under the plume and closer to the river mouth than did the very fine grain sand. The patterns of the deposited material indicate the growth of a levee along each side of the plume, but predominately on the leeward side. Particles which settled well outside of the plume on the leeward side were generally of the very fine class, implying that they came from the upper layers of the plume which has a proportionately greater concentration of this type of sand.

Figure 12 shows a series of end views of velocity vectors from the fourth iteration on the flow field. Regions of maximum deposition are obvious from this figure.

Depositional trends predicted agree qualitatively with the bottom shape near South Pass as shown in Figure 10. Sub-aqueous bars are prominent on the leeward side of the plume as the computed growth patterns predict. A bar between 15 and 20 feet below the surface also begins at the river mouth and extends under the plume about two channel widths from the river mouth. Notice in Figure 12 that this is approximately the same point where the maximum deposition occurred. This was due to choosing the coefficient of the empirical function of Sundborg to produce such an effect. Resolving the form or applicability of this function or replacing such a function with a better analysis will be the objective of future improvements to this technique.
Figure 12: Velocity Vectors of Flow Over Deposited Material in a Series of End Views
Figure 12 (Cont'd)
CHAPTER VII
CONCLUSIONS

As a result of this analysis, the following conclusions were reached:

1) The three-dimensional computer model is a practical tool for predicting flow fields and deposition of suspended material on river deltas.

2) Results of both the fluid dynamics and the deposition patterns compared favorably with observed data from the the South Pass of the Mississippi River.

3) An insufficient knowledge of bed movement and the interaction of particles with the bottom limited the generality of the model because the bottom was used as a boundary condition for the deposition calculations.

4) Grid points may be used more efficiently and boundary conditions may be specified more precisely through the use of stretching transformations.

5) Turbulence near the river mouth can be described with an eddy viscosity model of a turbulent jet, but a model which depends upon local properties would be desirable.

6) Results of numerical experimentation showed that buoyancy of the fresh water, inertia of the river and coastal currents, surface height of the river at its mouth, and turbulence all had a considerable influence upon the velocities and mixing patterns of the river plume.
The first four of these refinements can be accomplished with little difficulty, but the fifth will probably entail more effort. To accomplish this, a two-dimensional analysis providing improved resolution near the bottom is envisioned.

Among the many possible applications of this model, the following are suggested:

1) Computations from this model could serve as a preliminary investigation for field studies. This would isolate regions for which extensive testing is justified.

2) Proposed modifications in delta regions could be studied. Specifically, the long and short term effects of artificially creating crevasses from the river could be determined.

3) Systems with time dependent boundary conditions could be studied with minor changes to the program.

4) Mixing of thermal plumes or pollutants could be studied with the program essentially in its current form.
CHAPTER VIII
RECOMMENDATIONS

To date, the majority of the effort expended has been in the development of a practical three-dimensional model. As an assessment of the validity of the model, data computed from idealized geometry were compared with observed conditions near South Pass of the Mississippi River, a delta of rather complex geometry. Because this comparison was generally favorable, a continued effort toward refining the model is warranted.

During the development program and as a result of the comparison with observed data the following refinements to the model are suggested:

1) Several grid rows should be added in the river so that boundary conditions could be specified well upstream of the river mouth.

2) The stretching transformations should be adjusted to optimize the grid spacing and place points further out in the plume.

3) More sophisticated geometry should be added to reflect such features as the influx of brackish water from crevasses, sub-aqueous sand bars, and islands or land extensions which would deflect the flow.

4) The effect of wind shear and wave action should be included.

5) A mechanism should be added in the deposition program to properly move particles along the bottom or to move the bottom itself.
7) Buoyancy develops a pair of large vorticies until cross currents destroy the symmetry of the river plume. These vorticies convect the fresh water toward the surface and away from the centerline. Simultaneously, they also convect the sea water under the plume and ultimately vertically through the center of the jet. This vortex motion along with the accompanying turbulent mixing explains why high salinities characteristic of the bottom water appear in the center of river plumes.

8) The effect of local turbulence upon the settling velocities of suspended material can be adequately related to the local fluid velocity gradient.
REFERENCES


Coleman, J. M., Coastal Studies Institute, LSU, private correspondence (1972).


Gould, Donald, "Holes are Showing in Planned Dutch Defense Against Sea", *Smithsonian*, 3, No. 6, pp. 43-61 (Sept. 1972).


Patin, Thomas, U. S. Army Engineers Waterways Experimental Station, private communication (1971).


Tearpair, P. D. and D. R. Seay, private communication, Desoumette Research Foundation (1971).


NOMENCLATURE

b  function of the mixing layer thickness in eddy viscosity model; radius of the jet
C  wave speed; concentration of suspended material
$C_v$ specific heat capacity at constant volume
D  diffusion coefficient
$D_T$ eddy coefficient of mass diffusion
$D_{x,y,z}$ eddy coefficient of mass diffusion in the $x,y,z$ directions, respectively
$\frac{D}{D_t}$ substantial derivative
d  maximum river depth
f  fresh water
g  acceleration of gravity
GR growth rate of the bottom; Eq. (5*10)
$\bar{g}_1, \bar{g}_2, \bar{g}_3$ functions used to determine convergence; Eqs. (4*23)-(4*25).
H  maximum depth of the finite difference grid
h  vertical distance from the free surface to the bottom
i,j,k  index used to designate the $x,y,z$ location, respectively, in the finite difference grid
J  mass flux vector
K  slope of density versus salt concentration curve; $\frac{\partial \rho}{\partial S}$
k  empirical constant
$k_1,k_2,k_3,k_4$ constants of the stretching transformations
L  tag number of each nominal particle
mass per molecule of species s

number density

number of molecules of species s per unit volume of mixture

porosity of the bottom

pressure

dummy dependent variable used only for demonstration purposes

fluid velocity; $\sqrt{u^2+v^2+w^2}$

deposition velocity; Eq. (5*8)

radiation heat flux vector

mean velocity vectors of molecules of species s

reference level

concentration of salt water

species

temperature

time

mass average velocity

scalar velocities in the x,y,z directions, respectively

number average velocity

volume flux of solid material through the river mouth

settling velocity of a suspended particle

settling velocity of a suspended particle in still water
\( X, Y \)  
- stretched coordinates in the \( x,y \) directions, respectively

\( X', X'' \)  
- first and second derivatives of \( X \) wrt \( x \)

\( x, y, z \)  
- Cartesian coordinates

\( x_B, x_F \)  
- coordinate of the back and front of a region surrounding a grid point; Eq. (5*10).

\( x_p, y_p, z_p \)  
- coordinates of a nominal particle

\( y', y'' \)  
- first and second derivatives of \( Y \) wrt \( y \)

\( y_L, y_R \)  
- coordinates of the left and right of a region surrounding a grid point; Eq. (5*10)

\( \hat{z} \)  
- height for which \( q_{DV} \) is computed; Eq. (5*8)

**Greek**

\( \frac{\partial(\_)}{\partial t} \)  
- partial derivative (shown here wrt time)

\( \epsilon \)  
- kinematic eddy viscosity coefficient

\( k \)  
- coefficient of thermal conductivity

\( \mu \)  
- viscosity coefficient

\( \mu_T \)  
- eddy viscosity coefficient

\( \mu_x, \mu_y, \mu_z \)  
- eddy viscosity coefficients in the \( x,y,\) and \( z \) directions, respectively

\( \rho \)  
- density

\( \rho_p \)  
- density of a suspended particle

\( \rho_s \)  
- partial density of species \( s \)

\( \bar{\phi} \)  
- dissipation function resulting from heat generation by viscosity

\( \bar{\bar{\phi}} \)  
- mean property of a fluid averaged over all species; Eq. (A-1)

\( \tau \)  
- value of a dependent variable after a time step in the finite difference calculation
APPENDIX A
Properties of a constant-molar-density flow such as a mixture of salt and fresh water will be discussed from a microscopic standpoint. The objective of this analysis is to derive statements of zero volume dilatation and the conservation of mass. This can be done either from a continuum approach or from a molecular approach which uses the Boltzmann equation. Williams (1965) discussed each of these approaches in Appendices C and D of his text and Chapman and Cowling (1970) present an eloquent discussion of the molecular approach. Although each approach leads to identical results for the case of interest here, the continuum approach will be used because its application to a mixture of salt and fresh water is more easily comprehended.

Inheret in the continuum approach for analyzing a multicomponent mixture is the assumption that each species exists as a distinct continua within any arbitrary volume with each component obeying the laws of dynamics and thermodynamics. This assumption allows us to define a mean property $\bar{\phi}$ averaged over all species of the mixture as

$$n\bar{\phi} = \sum_{s} n_{s} \phi_{s} \, \quad (A-1)$$

where

$s = \text{species}; \ s = 1,2,3,\ldots$

$\phi_{s} = \text{mean property of species } s$
\( n_s = \) number density of species \( s \)  
\( n = \) number density of the mixture; \( \sum n_s \)

From Equation (A-1), the number average velocity \( \bar{V} \) can be defined as

\[
\bar{V} = \frac{\sum n_s q_s}{\sum n_s} \tag{A-2}
\]

and the mass average velocity \( \bar{U} \) is

\[
\bar{U} = \frac{\sum n_s m_s q_s}{\sum n_s m_s} = \frac{\sum \rho_s q_s}{\rho} \tag{A-3}
\]

where

\( q_s = \) mean velocity of species \( s \)  
\( m_s = \) mass of a molecule of species \( s \)  
\( \rho_s = \) partial density of species \( s \); \( \rho_s = m_s n_s \)  
\( \rho = \) density of the mixture; \( \rho = \sum \rho_s \)

The conservation of molecules of species \( s \) can be expressed as (see Williams' Appendix C)

\[
0 = \frac{\partial n_s}{\partial t} + \nabla \cdot (n_s q_s) \tag{A-4}
\]

By introducing \( \bar{V} \), Equation (A-4) can be rearranged as

\[
0 = \frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \bar{V}) + \nabla \cdot [n_s (q_s - \bar{V})] \tag{A-5}
\]
For the mixture, a summation over all species produces

\[ 0 = \frac{\partial n}{\partial t} + \nabla \cdot (n\vec{V}) + \nabla \cdot \Sigma [n_s(q_s - \vec{v})]. \tag{A-6} \]

With the aid of Equations (A-1) and (A-2),

\[
\sum [n_s(q_s - \vec{v})] = \sum n_s q_s - \vec{v} \cdot \sum n_s = n\vec{V} - \vec{v}_n \]

\[ = 0 \tag{A-7} \]

This reduces Equation (A-6) to

\[ 0 = \frac{\partial n}{\partial t} + \nabla \cdot (n\vec{V}) \tag{A-8} \]

It has already been established from experimental data* that the number density \( n \) varies neither with time nor space for a mixture of salt and fresh water; hence Equation (A-8) further reduces to

\[ \nabla \cdot \vec{V} = 0. \tag{A-9} \]

This is a statement of zero volume dilatation often used synonymously with a statement of incompressibility for constant density flows. A similar derivation in which the mass average velocity was used as the reference velocity does not result in a statement of zero volume dilatation for multicomponent fluids. This is true because the size of the control volume must change to accommodate a volume containing a constant mass and diffusive terms are constantly attempting to change the type of species within the volume.

* See Figure 2 of the main text.
To develop a conservation of mass equation, multiply

Equation (A-5), the statement of the conservation of molecules,

by \( m_s \) which produces

\[
0 = \frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \vec{v}) + \nabla \cdot [\rho_s (q_s - \vec{v})].
\]  
(A-10)

Summing over all species and recalling Equation (A-9) gives

\[
0 = \frac{\partial \rho}{\partial t} + \vec{V} \cdot \nabla \rho + \nabla \cdot \Sigma_s [\rho_s (q_s - \vec{v})].
\]  
(A-11)

Now using Equations (A-1) and (A-3) yields

\[
\Sigma_s [\rho_s (q_s - \vec{v})] = \Sigma_s \rho_s q_s - \vec{V} \Sigma_s \rho_s
\]

\[
= \rho \vec{U} - \rho \vec{V}
\]  
(A-12)

The first two terms of Equation (A-11) are actually the substantial
derivative of the density with \( \vec{V} \) as the reference velocity. Now,

from Equation (A-12),

\[
\frac{D\rho}{Dt} = -[\rho(\vec{U} - \vec{V})]
\]  
(A-13)

By defining

\[
J = \rho(\vec{U} - \vec{V}),
\]  
(A-14)

then \( J \) is a mass flux term into the control volume. This is

actually a diffusion term as discussed in Chapter 3 of the main
text. Combining Equations (A-13) and (A-14) gives a statement of

the conservation of mass;

\[
\frac{D\rho}{Dt} = -\nabla \cdot J.
\]  
(A-15)
If the mass average velocity $\bar{U}$ had been chosen as the reference velocity, then the substantial derivative of Equation (A-15) would be zero, but the diffusion term would have to be included in the statement of volume dilatation, Equation (A-9). It is inconsistent to express both the volume dilatation and the substantial derivative of the density as zero for a multicomponent fluid.
APPENDIX B
LISTING OF COMPUTER PROGRAM FOR FLUID DYNAMICS

This appendix presents the computer program for calculating the fluid dynamics in a delta region. The numerical procedure involves the solution of the finite difference equations described in Chapter IV of the main text. Comment cards were used throughout to indicate which operations were being performed. An effort was made to keep the notation of the program logical, consistent, and compatible with the technical discussion of the report. The only notable difference to the notation of the report is salt concentration which was normalized and denoted C in the computer program. Values of C vary between 0.0 and 1.0 to represent fresh water and sea water respectively.

All data cards to be used in the program are read into SUBROUTINE PRELIM. Proceeding each READ statement is a comment card which defines each term to be read and specifies the appropriate units. These data are nondimensionalized before being used for computations.

At the completion of a run, or at any intermediate time step, data are written on a tape in SUBROUTINE ECRIRE. This provides data for a restart capability, a plot program, or a computation of particle deposition.

When the restart capability is used, data are read from a tape in SUBROUTINE LIRE. The option of calling this subroutine is controlled by a data card as described in SUBROUTINE PRELIM. If
the restart capability is to be used, careful consideration should proceed any changes in the data cards read in SUBROUTINE PRELIM from those used to generate data on the tape.

After the program has completed the desired number of time steps or has detected an instability, it will tabulate data at each grid point as shown in Table 6. Components of velocity are presented in meters per second, but salt concentration and pressure are given in nondimensional form. Following this tabulation of data is a list of the nondimensional surface heights at each grid point throughout the field.

Calculated results of this program are presented in Appendix D as plots of velocity vectors and isoconcentration contours. These plots are generated from a separate program which reads the data from the tape. This program is not presented because it was specifically adapted to the CALCOMP plotter of the IBM-360 digital computer of LSU.

A listing of the computer program is now presented.
COMMON/DIMEN1/U(2,19,31,1C), V(2,19,31,1C), W(19,31,1C)
COMMON/DIMEN2/SURF(19,31), C(2,19,33,10), P(19,31,10)
COMMON/FLOOR/KFLOOR(19,31), ZB(19,31)
COMMON/MOUTH/KBED, JRBNK(11), JLBNK(11), YRBNK(11), YLBANK(11).
1 DEPT(3C)
COMMON/PULL/SX(41), SXX(41), SY(41), SYX(41), X(41), Y(41), Z(16)
COMMON/LIMITS/I MAX, IMAXM1, IMAXM2, JMAX, JMAXM1, JMAXM2, KMAX, KMAXM1, KMAXM2, NMAX, NPRINT, NBAR, JMAXP1, JMAXP1
COMMON/NUMBER/ONE, HALF, FOURTH, PI, SUM, TWO, EIGHT, SIXTH
COMMON/GRID/DX, DX, CX, DT, DTOXSQ, DTOYSQ, DTOZSQ, DTO2DX, DTO2DY, DTO2DZ
1 DZ02DZ, DZ02DY, DZ02DT
COMMON/MISC/ URIVER, VTIDE, DRHOOC, CONST1, LTAPE
COMMON/UNITS/UMAX, VMAX, GRAV, RATIO, HREF, VREF
COMMON/COORD/SX1, SX2, SY1, SY2
COMMON/VELCTY/UNEW, UOLD, VNEW, VOLD, UIP1, UIM1, UJP1, UKP1,
1 UKM1, VIP1, VIM1, VJP1, VM1, VKP1, VM1, WIP1, WIM1, WJP1, WM1, WKP1, KM1
COMMON/STATE/PK, PKP1, PIP1, PI1, PJP1, PJM1, RHO, RHOINV, CIFFUS
COMMON/STEP/MN, MO, K, KBOT
COMMON/CONC/CNEW, COLD, CLP1, CIP1, CIM1, CJP1, CJM1, CKP1, CKM1, CJP2, CJM2
COMMON/FORGOT/XCONST, YCONST, JWAG, DEPTH
C
C
CALL PRELIM
IF (LTAPE .NE. 0) GO TO 7
YOU ARE NOW IN A COLD START REGION
DO 2 I=1, I MAX
DO 2 J=1, J MAX
J J=J+1
SURF(I,J)=DZ
KBOT=KFLOOR(I,J)
ZB1=ZB(I,J)
DO 1 K=KBOT, KMAX
VERT=(Z(K)-ZB1)/(Z(KMAX)-ZB1)
PROFIL=1.0-(-1.0-VERT)**3.0
U(MN, I,J,K)=0.0
U(MO, I,J,K)=0.0
V(MN, I,J,K)=VTIDE*PROFIL
V(MO, l, j, k) = VTIDE*PROFIL
w(I, j, k) = 0
C(MN, I, j, j, k) = 1
C(MO, I, j, j, k) = 1

1 CONTINUE
K = KBOT
KBOTM1 = KBOT - 1
IF(KBOT.EQ.1) GO TO 2
DO 2 K = KBOTM1
U(MN, I, j, k) = 0
U(MO, I, j, k) = 0
V(MN, I, j, k) = 0
V(MO, I, j, k) = 0
W(I, j, k) = 0
C(MN, I, j, j, k) = 1
C(MO, I, j, j, k) = 1
2 CONTINUE
C RIVER VELOCITIES ARE CALCULATED
DO 3 K = KBED, KMAX
3 CALL SHAPE(K)
CALL DEEP
WRITE(6, 1000)
DO 4 K = KBED, KMAX
J1 = JRBANK(K)
J2 = JLEANK(K)
JM1 = J1 - 1
HALF DY = HALF*(Y(J1) - Y(JM1))
RADIUS = Y(J2) + HALF DY
ZFUN = (Z(K) - Z(KBED))/(Z(KMAX) - Z(KBED))
DO 4 J = J1, J2
JJ = J + 1
CALL UDIST(J, K, URIV)
WRITE(6, 1001) J, K, LRIV, DEPTH(J)
DO 4 I = 1, IMAX
U(MN, I, j, k) = URIV
U(MO, I, j, k) = U(MN, I, j, k)
SURF(I,J)=D2*DZ
IF(I*GT*7) GO TO 4
V(MN,I,J,K)=0.0
V(MO,I,J,K)=V(MN,I,J,K)
C(MN,I,J,K)=0.0
C(MO,I,J,K)=0.0
SURF(I,J)=DZ
CONTINUE

C CALCULATE PRESSURE OVER FIELD
DO 5 I=1,I MAX
DO 5 J=1,J MAX
JJ=J+1
RHO=ONE+CONST1*C(MN,I, JJ,KMAX1)
P(I,J,KMAX1)=SURF(I,J)*RHO
P(I,J,KMAX)=0.0
KKMAX=KMAX1-KFLCOR(I,J)
DO 5 KK=1,KKMAX
K=KMAX1-KK
RHO=ONE+CONST1*C(MN,I, KK,K)
DELTA P=DZ*RHO
P(I,J,K)=P(I,J,K+1)+DELTA P
CONTINUE
5 CONTINUE
GO TO 8

C VALUES ARE READ FROM THE TAPE HERE WHEN RESTART IS USED
CALL LIRE
CONTINUE
8 CONTINUE
DO 9 I=1,I MAX
DO 9 K=1,K MAX
C(MN,I,1,K)=1.0
C(MO,I,1,K)=1.0
C(MN,I,JMAX,K)=1.0
C(MO,I,JMAX,K)=1.0
CONTINUE
9 CONTINUE
C INITIAL VALUES ARE NOW COMPLETE
10 N=N+1
NEW SALT CONCS ARE NOW COMPUTED FROM SPECIES CONSERVATION EQs

C CALC NEW VALUES OF U USING X-MOMs EQs AND V USING Y-MOMs EQs

DO 23 I=2,IMAXM1
SX1=SX(I)
SX2=SXX(I)
IP1=I+1
IM1=I-1
DO 21 J=2,JMAXM1
SY1=SY(J)
SY2=SYY(J)
JP1=J+1
JM1=J-1
JJ=J+1
JJP1=JJ+1
JJP2=JJ+2
JJM1=JJ-1
JJM2=JJ-2
KBOT=K FLOOR(I, J)
KBOTP1=KBOT+1
ZB1=ZB(I, J)
ZFUN=(Z(KBOT)-ZB1)/(HALF*DZ+(Z(KBOT)-ZB1))
UOLD=U(MO, I, J, KBOT)
UKP1=U(MO, I, J, KBOTP1)
VOLD=V(MO, I, J, KBOT)
VKP1=V(MO, I, J, KBOTP1)
WNEW=W(I, J, KBOT)
WKP1=W(I, J, KBOTP1)
COLD=C(MO, I, JJ, KBOT)
CKP1=C(MO, I, JJ, KBOTP1)
DO 20 K=KBOTP1, KMAXM1
KP1 = K+1
KM1 = K-1
UIP1 = U(MO, IP1, J, K)
UIM1 = U(MO, IM1, J, K)
UJP1 = U(MO, JP1, K)
UJM1 = U(MO, JM1, K)
UKM1 = UOLD
UOLD = LKP1
UKP1 = U(MO, I, J, KP1)
VIP1 = V(MO, IP1, J, K)
VIM1 = V(MO, IM1, J, K)
VJP1 = V(MO, JP1, K)
VJM1 = V(MO, JM1, K)
VKM1 = VOLD
VOLD = VKP1
VKP1 = V(MO, I, J, KP1)
WKP1 = W(I, J, KP1)
PIP1 = P(IP1, J, K)
PIM1 = P(IM1, J, K)
PJP1 = P(JP1, K)
PJM1 = P(JM1, K)
CIP1 = C(MO, IP1, JJ, K)
CIM1 = C(MO, IM1, JJ, K)
CJP1 = C(MO, JP1, K)
CJP2 = C(MO, JJP1, K)
CJM1 = C(MO, JJM1, K)
CJM2 = C(MO, JJM2, K)
CKM1 = COLD
COLD = CKP1
CKP1 = C(MO, JJ, KP1)
\[ \rho_{inv} = CNE - \text{CONST}1 \times \text{COLD} \]
CALL DIFUSE
CALL SALT
C(MN, I, JJ, K) = CNEW
GBAR = GBAR + ABS(CNEW - COLD)

C

NOW CONVERT FROM DELSQ C TO DELSQ RHO BY MULT. BY CONST1

DIFFUS = CONST1 * DIFFUS

CALL XMOMEQ

U(MN, I, J, K) = UNEW

UBAR = UBAR + ABS(UNEW - UOLD)

CALL YMCMEQ

V(MN, I, J, K) = VNEW

VBAR = VBAR + ABS(VNEW - VOLD)

CONTINUE

CTOP = CNEW + (CNEW - CKM1) * HALF

IF (CTOP < 0.0) CTOP = 0.0

IF (CTOP > ONE) CTOP = ONE

C(MN, I, JJ, KMAX) = CTOP

A NO-SLIP CONDITION HAS BEEN APPLIED ALONG THE BOTTOM

CBOT = TWO * C(MN, I, J, KBOT + 1) - C(MN, I, J, KBOT + 2)

IF (CBOT < ONE) CBOT = ONE

C(MN, I, J, KBOT) = CBOT

U(MN, I, J, KBOT) = ZFUN * U(MN, I, J, KBOT + 1)

V(MN, I, J, KBOT) = ZFUN * V(MN, I, J, KBOT + 1)

CONTINUE

BOUNDARY CONDITIONS FOR JMAX COULD GO HERE

KBOT = KFLOOR(I, JMAX)

DO 22 K = KBOT, KMAX

JJ = JMAX + 1

JJM1 = JMAX

C(MN, I, JJ, K) = C(MN, I, JJM1, K)

C(MN, I, JJMAX, K) = C(MN, I, JJM1, K)

CONTINUE

22 CONTINUE

DO 25 J = 2, 25

JJ = J + 1

KBOT = KFLOOR(IMAX, J)

DO 25 K = KBOT, KMAX

C(MN, IMAX, JJ, K) = TWO * C(MN, IMAXM1, JJ, K) - C(MN, IMAXM2, JJ, K)

U(MN, IMAX, J, K) = TWO * U(MN, IMAXM1, J, K) - U(MN, IMAXM2, J, K)

CONTINUE

25 CONTINUE
IF(U(MN,IMAX,J,K)LT0.0) U(MN,IMAX,J,K)=0.0
V(MN,IMAX,J,K)=TWO*V(MN,IMAXM1,J,K)-V(MN,IMAXM2,J,K)
IF(K,LTBED) GO TO 24
IF(J,LTBANK(K)) GO TO 24
IF(J,GT,JLBANK(K)) GO TO 24
GO TO 25
C(MN,1,JJ,K)=C(MN,2,JJ,K)
U(MN,1,J,K)=-U(MN,2,J,K)
V(MN,1,J,K)= V(MN,2,J,K)
CONTINUE
C
CALC NEW VALUES OF W USING CONTINUITY EQ.
DO 34 I=2,IMAXM1
SX1=SX(I)
IP1=I+1
IM1=I-1
DO 32 J=2,JMAXM1
SY1=SY(J)
JP1=J+1
JM1=J-1
K=KFCOR(I,J)+1
KBOTP2=K+1
COMP1=SX1*DZ02DX*(L(MN,IP1,J,K)-U(MN,IM1,J,K))
COMP2=SY1*DZ02DY*(V(MN,I,JP1,K)-V(MN,I,JM1,K))
ADJ=(Z(K)-ZB(I,J))/DZ
WNEW=-HALF*ADJ*(COMP1+COMP2)
W(I,J,K)=WNEW
WOLD=W(I,J,KMAXM1)
DO 33 K=KBOTP2,KMAXM1
KMI=K-1
WKMI=WNEW
COMP1=SX1*DZ02DX*(U(MN,IP1,J,K)-U(MN,IM1,J,K)+U(MN,IP1,J,KM1)-
1 U(MN,IM1,J,KM1))
COMP2=SY1*DZ02DY*(V(MN,I,JP1,K)-V(MN,I,JM1,K)+V(MN,I,JP1,KM1)-
1 V(MN,I,JM1,KM1))
WNEW=WKMI-HALF*(COMP1+COMP2)
W(I,J,K)=WNEW
CONTINUE
WSURF=WNEW-WOLD
SURFI= SURF(I, J)
WKMAX=(SURFI-DZ)*(((TWO*WNEW/SURF1)-(WKM1/SURF1+DZ)))+
1 TWO*WSURF*DZ/(SURFI+DZ)
(I, J,kMAX)=WKMAX
C
THE SURFACE HEIGHT MAY NOW BE ADJUSTED
SURF2=SURF1+WSURF*DT*Q*5
SURFI= SURF2
IF(SURF2< GT (G) GO TO 31
GO TO 32
31 WRITE(6,1012)N,I,J
GO TO 10
32 CONTINUE
C
BOUNDARY CONDITIONS FOR JMAX COULD GO HERE
34 CONTINUE
CALL SURFAC
DO 38 J=2*JMAXM1
KBOTP1=KFLOOR(1,J)+1
DO 36 K=KBOTP1,KMAX
W(IMAX,J,K)=TWO*W(IMAXM1,J,K)-W(IMAXM2,J,K)
IF(K<LT KBED) GO TO 35
IF(J<LT JRBANK(K)) GO TO 35
IF(J>GT JLBANK(K)) GO TO 35
GO TO 36
35 W(1,J,K)=W(2,J,K)
36 CONTINUE
IF(J<LT JRBANK(KMAX)) GO TO 37
IF(J>GT JLBANK(KMAX)) GO TO 37
GO TO 38
37 U(MN,1,J,KMAX)=-U(MN,2,J,KMAX)
V(MN,1,J,KMAX)= V(MN,2,J,KMAX)
SURF(1,J)=SURF(2,J)
38 CONTINUE
C
CALCULATE NEW PRESSURES FROM Z-MOMENTUM EQ, WITH DW/DT OMITTED
DO 64 I=2,IMAXM1
$\text{SX1} = SX(I)$
$\text{SX2} = SXX(I)$
$\text{IP1} = I + 1$
$\text{IM1} = I - 1$

\begin{verbatim}
DO 63 J = 2 * JMAXM1
    SY1 = SY(J)
    SY2 = SYY(J)
    JP1 = J + 1
    JM1 = J - 1
    JJ = J + 1
    JJP1 = JJ + 1
    JJM1 = JJ - 1
    KBOT = KFLOOR(I, J)
    KMAX = KMAX - KBOT
    WNEW = W(I, J, KMAXM1)
    WK1 = W(I, J, KMAX2)
    DO 63 KK = 1, KMAX
        K = KMAX - KK
        KP1 = K + 1
        KM1 = K - 1
        UIP1 = U(MN, IP1, J, K)
        UIM1 = U(MN, IM1, J, K)
        VJP1 = V(MN, I, JP1, K)
        VJM1 = V(MN, I, JM1, K)
        WIP1 = W(IP1, J, K)
        WIM1 = W(IM1, J, K)
        WJP1 = W(I, JP1, K)
        WJM1 = W(I, JM1, K)
        WK1 = WNEW
    \end{verbatim}

\begin{verbatim}
WNEW = WK1
IF (KEQKBOT) GO TO 60
WK1 = W(I, J, KM1)
GO TO 61
\end{verbatim}

\begin{verbatim}
60 WK1 = -WK1
61 RHO = ONE + CONST1 * C(MN, I, JJ, K)
PKP1 = P(I, J, KP1)
\end{verbatim}
CALL PRESS
IF(KK>N1) GO TO 62
PK=PK*SURF(I,J)/DZ
62 P(I,J,K)=PK
IF(PK>GO TO 63
WRITE(6,1013)N,I,J,K
GO TO 100
63 CONTINUE
C BOUNDARY CONDITIONS FOR JMAX COULD GO HERE
64 CONTINUE
DO 66 J=2,JMAX
KDOT=KFLOOR(IMAX,J)
DO 66 K=KBOT,KMAXM1
P(IMAX,J,K)=2*P(IMAXM1,J,K)-P(IMAXM2,J,K)
C SPECIAL BOUNDARY CONDITIONS IN THE RIVER MOUTH COULD GO HERE
IF(K<KBED) GO TO 65
IF(J<JRANK(K)) GO TO 65
IF(J>LBANK(K)) GO TO 65
GO TO 66
65 CONTINUE
P(I,J,K)=P(I,J,K)
66 CONTINUE
C NEW VALUES OF U,V,W,P AND C HAVE BEEN COMPUTED
I=4
J=JWAG+2
K=KMAXM2
WRITE(6,1012) N,U(MN,I,J,K),V(MN,I,J,K),W(I,J,K)
IF(N>NBAR*)GO TO 69
WRITE(6,1017) N,UBAR,VBAR
NBAR=2*NBAR
69 CONTINUE
IF(N>NPRINT*)GO TO 80
WRITE(6,1010) N
C RESTART CAPABILITY GOES HERE
CALL ECRIRE(MN)
C THIS IS THE END OF THE RESTART SECTION
AN INTERMEDIATE WRITE STATEMENT COULD GO HERE

C CONTINUE

IF(N3GE,NMAX) GO TO 10C

GO TO 1

C IF YOU GET BEYOND THIS POINT, YOU DON'T GET BACK

103 WRITE(6,1C17) N,GBAR,UBAR,VBAR

WRITE(6,1C15)

DO 1C5 I=1,IMAX,2

DO 1C5 J=1,JMAX

JJ=J+1

KBOT=KFLOOR(I,J)

DO 105 K=KBOT,KMAX

C VELOCITIES ARE IN METERS PER SECOND

U1=U(MN,I,J,K)*VREF

V1=V(MN,I,J,K)*VREF

W1=W(I,J,K)*VREF

WRITE(6,1016) U1,V1,W1,C(MN,I,J,K),P(I,J,K),I,J,K

CONTINUE

DO 106 I=1,IMAX

DO 1C6 J=1,JMAX

TOP=100—DZ+SURF(I,J)

WRITE(6,1014) TOP,I,J

CONTINUE

C THIS SECTION IS USED ONLY WHEN PLOTS ARE DESIRED

C THIS IS THE END OF THE PLOT PROGRAM

1020 FORMAT(/,9X,1HJ,9X,1HK,5X,4HURIV,5X,SHDEPT,/,)

1021 FORMAT(8X,12.8X,12.2X,F8.5,2X,F8.5)

1022 FORMAT(5X,2HN=I4.5X,2HU=E13.6,5X,2HV=E13.6,5X,2HW=E13.6)

1023 FORMAT(/,10X,2HN=I4,10X)

1024 FORMAT(5X,8HSURFACE=F8.6,5X,2HI=I3,5X,2HJ=I3)

1025 FORMAT(/,5X,'I AM GOING TO QUIT BECAUSE THE SURF IS TOO ROUGH AT'

1.5X,2HN=I4.5X,2HI=I3,5X,2HJ=I3)

1026 FORMAT(/,5X,'CONGRADULATIONS BILL, YOU HAVE DISCOVERED ANTIMATTER'

1.5X,2HN=I4,5X,2HI=I3,5X,2HJ=I3)

1027 FORMAT(5X,8HSURFACE=F8.6,5X,2HI=I3,5X,2HJ=I3)

1028 FORMAT(/,13X,1HU,19X,1HV,19X,1HW,15X,10HSALT CONC,11X,8HPRESSURE
STOP
END
SUBROUTINE LIRE
COMMON/DIMEN1/V(2,19,31,10),W(19,31,10)
COMMON/DIMEN2/SURF(19,31),C(2,19,33,10),P(19,31,10)
COMMON/FLOOR/KFLOOR(19,31),ZB(19,31)
COMMON/PULL/SX(41),SX(41),SY(41),SY(41),X(41),Y(41),Z(16)
COMMON/MOUTH/KBED,JRBANK(11),JLBANK(11),YRBANK(11),YLBank(11)
COMMON/LIMITS/IMAX,JMAX,KMAX,IMAXM1,JMAXM1,KMAXM1
COMMON/NUMBER/ONE,HALF,FOURTH,PI,SUM,TWO,EIGHT,SIXTH
COMMON/FORGET/XCONST,YCONST,JWAG,DEPTH
COMMON/MISC/URIVER,VTIDE,DRHUOC,CONST,LTape
COMMON/UNITS/UMAX,VMAX,GRAV,RATIO,HREF,VREF

C
C A SUBROUTINE TO READ DATA FROM A TAPE WHEN LTape .NE. C
C
MN=1
MO=2
REWIND 3
READ(3) IMAX,JMAX,KMAX,KBED,JWAG,URIVER,VTIDE,HREF,RATIO
READ(3) (X(I),I=1,IMAX)
READ(3) (Y(J),J=1,JMAX)
READ(3) (Z(K),K=1,KMAX)
READ(3) (YRBANK(K),K=KBED,KMAX)
READ(3) (JRBANK(K),K=KBED,KMAX)
READ(3) (JLBANK(K),K=KBED,KMAX)
READ(3) (((U(MN,I,J,K),I=1,IMAX),J=1,JMAX),K=1,KMAX)
READ(3) (((V(MN,I,J,K),I=1,IMAX),J=1,JMAX),K=1,KMAX)
READ(3) (((W(I,J,K),I=1,IMAX),J=1,JMAX),K=1,KMAX)
JJMAX=JMAX+2
READ(3) (((C(MN,I,JJ,K),I=1,IMAX),JJ=1,JJMAX),K=1,KMAX)
READ(3) (((P(I,J,K),I=1,IMAX),J=1,JMAX),K=1,KMAX)
READ(3) (((SURF(I,J),I=1,IMAX),J=1,JMAX)
READ(3) (((ZB(I,J),I=1,IMAX),J=1,JMAX)
CALL MODIFY
DO 1 I=1,IMAX
1 CONTINUE
DO 1 J=1,JMAX
JJ=J+1
DO 1 K=1,KMAX
U(MO, I,J,K)=U(MN, I,J,K)
V(MO, I,J,K)=V(MN, I,J,K)
C(MO, I,JJ,K)=C(MN, I,JJ,K)
1 CONTINUE
DO 2 I=1,I MAX
DO 2 K=1,KMAX
C(MO, I,1,K)=1.C
2 CONTINUE
RETURN
END
SUBROUTINE MODIFY
COMMON/DIMEN1/U(2,19,31,10),V(2,19,31,10),W(19,31,10)
COMMON/DIMEN2/SURF(19,31),C(2,19,31,10),P(19,31,10)
COMMON/FLOOR/KFLOOR(19,31),ZB(19,31)
COMMON/PULL/SX(41),SXX(41),SY(41),SYY(41),X(41),Y(41),Z(16)
COMMON/MOUTH/KBED,JRBANK(11),JLBANK(11),YRBANK(11),YLBank(11)
COMMON/LIMITS/IMAX,IMAXM1,IMAXM2,JMAX,JMAXM1,JMAXM2,KMAX,KMAXM1,
1 KMAXM2,NMAX,NPRINT,NBAR,JMAXP1,IMAX
COMMON/GRID/DX,DY,DZ,DT,DT0XSQ,DT0XSQ,DT0ZSQ,DT02DX,DT02DY,DT02DZ,
1 DT02DX,DT02DY,DT02DT
COMMON/NUMBER/ONE,HALF,FOURTH,PI,SUM,TWO,EIGHT,SIXTH
COMMON/FORGOT/XCONST,YCONST,JWAG.DEPTCH
COMMON/MISC/VRIVER,VTIDE,DRHOC,CCNST1,LTAPE
COMMON/UNITS/UMAX,VMAX,GRAV,RATIO,HREF,VREF

C
C    THIS SUBROUTINE MAY BE USED TO MODIFY THE BOUNDARY CONDITIONS
C    OR THE GEOMETRY OF THE DATA READ OFF OF THE TAPE
C
MN=1
MO=2
VRIVER=U(MN,1,JWAG,KMAX)
DO 12 I=1,IMAX
  DO 12 J=1,JMAX
  K=0
  K=K+1
  IF(ZK)KLTZB(I,J)) GO TO 10
  KFLOOR(I,J)=K
10 CONTINUE
  I=IMAX
  DO 20 J=2,JMAXM1
  KBOT=KFL0R(I,J)
  DO 20 K=KBOT,KMAX
    IF(JGT25) U(MN,1,J,K)=0.0
    V(MN,1,J,K)=V(MN,2,JMAX,K)
    W(I,J,K)=0.0
20 CONTINUE
SUBROUTINE ECRIRE(MN)
COMMON DIMEN1/U(2,19,31,10),V(2,19,31,10),W(19,31,10)
COMMON DIMEN2/SURF(19,31),C(2,19,33,10),P(19,31,10)
COMMON FLOOR/KFLOOR(19,31),ZB(19,31)
COMMON PULL/SX(41),SX(41),SY(41),SY(41),X(41),Y(41),Z(16)
COMMON MOUTH/KBED,JBANK(11),JLBANK(11),YRBANK(11),YLEANK(11)
COMMON LIMITS/IMAX,IMAXM1,IMAXM2,JMAX,JMAXM1,JMAXM2,KMAX,KMAXM1,
1 KMAXM2,NMAX,NPRINT,NBAR,JMAXP1,JJMAX
COMMON NUMBER/ONE,HALF,FOURTH,PI, SUM, TWO, EIGHT, SIXTH
COMMON FORGOT/XCCNST,YCONST,JWAG,DEPTHG
COMMON MISC/URIVER,VTIDE,DRHOOC,CONST1,LTape
COMMON UNITS/UMAX,VMAX,GRAV,RATIO,HREF,VREF

A SUBROUTINE TO WRITE COMPUTED DATA ON TAPE

REWIND 3
WRITE(3) IMAX, JMAX, KMAX, KBED, JWAG, URIVER, VTIDE, HREF, RATIO
WRITE(3) (X(I), I=1, IMAX)
WRITE(3) (Y(J), J=1, JMAX)
WRITE(3) (Z(K), K=1, KMAX)
WRITE(3) (YRBANK(K), K=KBED, KMAX)
WRITE(3) (YLBANK(K), K=KBED, KMAX)
WRITE(3) (JRBNK(K), K=KBED, KMAX)
WRITE(3) (JLBANK(K), K=KBED, KMAX)
WRITE(3) (((U(MN, I, J, K), I=1, IMAX), J=1, JMAX), K=1, KMAX)
WRITE(3) (((V(MN, I, J, K), I=1, IMAX), J=1, JMAX), K=1, KMAX)
WRITE(3) (((W(I, J, K), I=1, IMAX), J=1, JMAX), K=1, KMAX)
WRITE(3) (((C(MN, I, JJ, K), I=1, IMAX), JJ=1, JJMAX), K=1, KMAX)
WRITE(3) (((P(I, J, K), I=1, IMAX), J=1, JMAX), K=1, KMAX)
WRITE(3) (((SURF(I, J), I=1, IMAX), J=1, JMAX)
WRITE(3) (((ZB(I, J), I=1, IMAX), J=1, JMAX)
WRITE(3) (((KFLOOR(I, J), I=1, IMAX), J=1, JMAX)
RETURN
END
SUBROUTINE PRELIM
COMMON/FLOOR/KFLOOR(19,31),ZB(19,31)
COMMON/PULL/SX(41),SX(41),SY(41),SY(41),X(41),Y(41),Z(16)
COMMON/MOUTH/KBED,JRBANK(11),JLBANK(11),JLBANK(21),YRBANK(11),YLBANK(11)
1 DEPTH(31)
COMMON/GRID/DX,DY,DZ,DT,DTOXSG,DTOYSG,DTOZSG,DTO2DX,DTO2DY,DTO2DZ
1 DZ02CX,DZ02DY,DZ0DT
COMMON/FORGOT/XCONST,YCONST,JWAG,DEPTH0
COMMON/MISC/URIVER,VTIDE,DRHGO,CONST1,LTape
COMMON/UNITs/UMAX,VMAX,GRAV,RATIO,HREF,RAPE
COMMON/LIMITS/IMAX,IMAXM1,IMAXM2,JMAX,JMAXM1,JMAXM2,KMAX,KMAXM1,
1 KMAXM2,NMAX,NPRINT,NBAR,JMAXP1,JMAX
COMMON/VISCITY/COEFVX,COEFVY,COEFVZ,ViSC,VEL,VEL1P1,VELIM1,VELJP1
1 VELJ1P1,VELK1P1,VELKMI
COMMON/DIFF/COEFDX,COEFDY,COEFDZ
COMMON/NUMBER/ONE,HALF,FOURTH,PI,SUM,TWO,EIGHT,SIXTH

PRELIMINARY CALCULATIONS ARE PERFORMED HERE

IF A COLD START IS DESIRED, SET LTAPE=0 IF NOT, LTAPE=1
READ(5,1000) LTAPE
IMAX,JMAX,KMAX ARE THE MAX. NO. OF GRID PTS,
IN X,Y,Z DIRECTIONS
JWAG IS THE NO. OF THE GRID PT ALONG THE CENTERLINE OF THE
RIVER, IT IS USUALLY CONVENIENT TO LET JWAG=(JMAX+1)/2
DT IS THE SIZE OF THE NONDIMENSIONAL TIME STEP
IF UNDECIDED, LET DT=0.1
KBED IS THE K VALUE OF THE GRID PT AT THE DEEPEST PT OF THE
RIVER(J=JWAG)
READ(5,1000) IMAX,JMAX,KMAX,JWAG,KBED
NMAX IS THE MAX. NO. OF TIME STEPS
NPRINT IS THE NO. OF A TIME STEP FOR WHICH AN INTERMEDIATE
WRITE STATEMENT CAN BE USED. VALUES ARE WRITTEN ON TAPE
HERE ALSO
NBAR IS THE TIME STEP NO. WHERE GBAR, UBAR, VBAR ARE PRINTED
LET NBAR=1
READ(5,1000) NMAX,NPRINT,NBAR
C  XCONST,YCONST ARE STRETCHING CONSTANTS SHOWN AS K2 AND K4
C  IN TEXT
READ(5,1001) DT,XCONST,YCONST
C  UMAX=MAX. VELOCITY OF THE RIVER AT THE SURFACE IN THE CENTER
C  M/SEC
C  VMAX=MAX. VELOCITY OF THE TIDE AT THE SURFACE   M/SEC
C  HREF=DEPTH OF DEEPEST GRID POINT TAKEN AS K=0   METERS
C  RATIO=RATIO OF RIVER WIDTH TO DEPTH
C  SIGMAT=MEASURE OF DENSITY OF SEA WATER   SIGMAT=(RHO-1.0)*0.001
READ(5,10C1) UMAX,VMAX,HREF,RATIO,SIGMAT
DX=1.0/FLOAT(IMAX-1)
C  JWAG MUST BE AT LEAST EQUAL TO JMAX/2
DY=1.0/FLOAT(JWAG-1)
DZ=1.0/FLOAT(KMAX)
WRITE(6,1002) IMAX,JMAX,KMAX,NMAX,NPRINT,KBED,JWAG,NBAR
WRITE(6,1003) DX,DY,DZ,DT,SIGMAT,XCONST,YCONST
DTOXSQ=DT/(DX*DX)
DTOYSQ=DT/(DY*DY)
DTOZSQ=DT/(DZ*DZ)
DT02DX=0.5*DT/DX
DT02DY=0.5*DT/DY
DT02DZ=0.5*DT/DZ
DZ02DX=0.5*DZ/DX
DZ02DY=0.5*DZ/DY
DZ02DT=DZ/DT
IMAXM1=IMAX-1
IMAXM2=IMAX-2
JMAXM1=JMAX-1
JMAXM2=JMAX-2
JJMAX=JMAX+1
JJMAX=JMAX+2
KMAXM1=KMAX-1
KMAXM2=KMAX-2
CONST1=CRHO0C*CMAX
WRITE(6,1041)
DO 1 I=1,IMAX
CALL RUBERX(I)
WRITE(6,1041) X(I),SX(I),SXX(I),I
1 CONTINUE
WRITE(6,1042)
DO 2 J=1,JMAX
CALL RUBERY(J)
WRITE(6,1040) Y(J),SY(J),SYY(J),J
2 CONTINUE
WRITE(6,1043)
DO 3 K=1,KMAX
Z(K)=DZ*FLOAT(K)
WRITE(6,1044) Z(K),K
3 CONTINUE
CONST1=CO61*SIGMAT
DEPTHC=I0-Z(KBED)
VREF=SQRT(HREF*GRAV)
URIVER=UMAX/VREF
VTDIE=VMAX/VREF
WRITE(6,1005) UMAX,VMAX,VREF,HREF,RATIC
1005 FORMAT(
          IF YOU DON'T LIKE PRANDTL'S THIRD MODEL FOR A TURBULENT JET,
          THEN YOU HAD BETTER CHANGE THIS
COEFVX=CO0256*(0.5*RATIO*DEPTHC)*URIVER
COEFVY=COEFVX
COEFVZ=COEFVX
          A SCHMIDT NO. OF 5 IS ASSUMED
COEFDX=CO2*COEFVX
COEFDY=CO2*COEFVY
COEFDZ=CO2*COEFVZ
WRITE(6,1006) COEFVX,COEFVY,COEFVZ,COEFDX,COEFDY,COEFCZ
          BOTTOM SHAPE IS COMPUTED HERE
WRITE(6,1007)
DO 12 I=1,IMAX
XFUN=FLOAT(I-1)/FLOAT(IMAXM1)
12 CONTINUE
DO 12 J=1,JMAX
YFLN=FLCAT(J-1)/FLOAT(JMAXM1)
CAUTION. ALWAYS MAKE ZB1 SLIGHTLY GREATER THAN ZERO
ZB1=1605*COS(HALF*PI*XFUN)+5.14
ZB(I,J)=ZB1
K=0
10 K=K+1
IF(Z(K)LT0) GO TO 10
KFLOWR(I,J)=K
ZFLN=(Z(K)-ZB1)/(HALF*DZ+(Z(K)-ZB1))
IF(LTAP=NE=0) GO TO 12
WRITE(6,1008) I,J,K,ZB1,ZFUN,SXFUN,YFUN
12 CONTINUE
1000 FORMAT(A11)
1001 FORMAT(8F10.6)
1002 FORMAT(5X,5HIMAX=I5/,5X,5HJMAX=I5/,5X,5HKMAX=I5/,5X,5HNMAX=I5,
1/5X,7HPRINT=I3/,5X,5HKBED=I5/,5X,5HJWAG=I5/,5X,5SHNBAR=I5)
1003 FORMAT(5X,3HDX=F12.6/,5X,3HDY=F12.6/,5X,3HDZ=F12.6/,5X,3HDY=F12.6/,5X,
1/5HDT=F12.6/,5X,7HSGMAT=F8.4/,5X,7HXCONST=F8.2/,5X,
2/5HYCONST=F8.2/)
1004 FORMAT(5X,6HLTAP=I4)
1005 FORMAT(/,5X,5HUMAX=F7.3,2X,'M/SEC',/,5X,5HVMAX=F7.3,2X,'M/SEC',/,5X,5HVREF=F7.3,2X,'M/SEC',/,5X,5HREF=F7.3,2X,'METERS',/,5X,6HRATIO=F6.3/)
1006 FORMAT(/,5X,32HVISCSITY COEFF. IN X DIRECTION=F10.6/,5X,32HVISCSITY COEFF. IN Y DIRECTION=F10.6/,5X,32HVISCSITY COEFF. IN Z DIRECTION=F10.6/,5X,32HDIFFUSION COEFF. IN X DIRECTION=F10.6/,5X,32HDIFFUSION COEFF. IN Y DIRECTION=F10.6/,5X,32HDIFFUSION COEFF. IN Z DIRECTION=F10.6/)
1037 FORMAT(/,9X,1H1,9X,1H1,6X,6HKFLOWR,6X,2HZB,6X,6HB0TFUN,/)
1042 FORMAT( '13X,1HY,18X,2HSY,18X,3HSYY,13X,1HJ,' )
1043 FORMAT( '13X,1HZ,14X,1HK,' )
1044 FORMAT( '7X,E13,6X,7X,13')
     RETURN
     END
SUBROUTINE RUBEX(I)
COMMON/PULL/SX(41),SXX(41),SY(41),SYY(41),X(41),Y(41),Z(16)
COMMON/NUMBER/ONE,HALF,FOURTH,PI,SUM,TRUE,EIGHT,SIXTH
COMMON/GRID/DX,DY,DZ,DT,DTOXSQ,DTOYSQ,DTOZSQ,DTO2DX,DTO2DY,DTO2DZ
COMMON/FORGOT/XCONST,YCONST,JWAG

A SUBROUTINE FOR CALCULATING STRETCHING FUNCTIONS IN THE X-DIRECTION

XI=I
CAPX=(XI-1.5)*DX
XK=1.45
X(I)=XCONST*TAN(XK*CAPX)
X1=X(I)
CONSQ=XCONST*XCONST
XSQ=X1*X1
DENOM=CONSQ+XSQ
SX(I)=XCONST/(DENOM*XK)
SXX(I)=-TWO*XCONST*X1/(DENOM*DENOM*XK)
RETURN
END
SUBROUTINE RUBERY(J)
COMMON/PULL/SX(41),SXX(41),SY(41),SYY(41),X(41),Y(41),Z(16)
COMMON/NUMBER/ONE,HALF,FOURTH,PI,SUM,TWO,EIGHT,SIXTH
COMMON/GRID/DX,DY,CZ,DT,DTOXSQ,DTOYSQ,DTOZSQ,DTO2DX,DTO2DY,DTO2DZ,
1 DTO2DX,DZO2DY,DZODT
COMMON/FORGOT/XCONST,YCONST,JWAG

C
A SUBROUTINE FOR CALCULATING STRETCHING FUNCTIONS IN THE Y-DIRECTION
C

YK=1.56
CAPY=CY*FLOAT(J-JWAG)
Y1=YCONST*TAN(YK*CAPY)
Y(J)=Y1
CONSQ=YCONST*YCONST
YSQ=Y1*Y1
DENOM=CONSQ+YSQ
SY(J)=(1.0/YK)*YCONST/DENOM
SYY(J)=-(2.0/YK)*YCONST*Y1/(DENOM*DENOM)
RETURN
END
SUBROUTINE SHAPE(K)
COMMON/KMOUTH/KBED, JPBANK(11), JLBANK(11), YREANK(11), YLBANK(11),
1 DEPTH(JC)
COMMON/NUMBER/ONE, HALF, FOURTH, PI, SUM, TWO, EIGHT, SIXTH
COMMON/PULL/SX(41), SXX(41), SY(41), SYY(41), X(41), Y(41), Z(16)
COMMON/LNITS/UMAX, VMAX, GRAV, RATIO, HREF, VREF
COMMON/FORGOT/XCONST, YCONST, JWAG, DEPTHU
C RIVER BOUNDARIES ARE CALC. BASED UPON AN ASSUMED ELLIPTIC SHAPE
C RIGHT AND LEFT IS REF. TO THE RIVER LOOKING DOWNSTREAM
YLBANK( K ) = 0.5 * RATIO * SQRT( DEPTH0**2 - ( ONE - Z(K) )**2 )
YRBANK( K ) = - YLBANK( K )
WRITE(6, 1002) K, YRBANK( K ), YLBANK( K )
C THE FIRST J PT. INSIDE THE MOUTH IS CALC. FOR EACH SIDE
IF( YLBANK( K ) .GT. 0.00001 ) GO TO 1
JRBANK( K ) = JWAG
JLBANK( K ) = JWAG
RETURN
1 J = JWAG - 1
2 J = J + 1
IF( Y(J) .LT. YLBANK( K ) ) GO TO 2
JM1 = J - 1
JLBANK( K ) = JM1
J = JWAG + 1
4 J = J - 1
IF( Y(J) .GT. YRBANK( K ) ) GO TO 4
JP1 = J + 1
JRBANK( K ) = JP1
10)2 FORMAT(5X, 2HK=12, 1CX, 7HYRBANK=F8.5, 10X, 7HYLBANK=F8.5/)
RETURN
END
SUBROUTINE DEEP
COMMON/MOUTH/KBED•JRBANK(11),JLBANK(11),YRBANK(11),YLBank(11),
1 DEPTH(30)
COMMON/LIMITS/IMAX•IMAXM1•IMAXM2•JMAX•JMAXM1•JMAXM2•KMAX•KMAXM1•
1 KMAXM2•NMAX•NPRINT•NBAR•JMAXP1•JJMAX
COMMON/GRID/DX•DY•DZ•DT•DTOXSQ•DTOYSQ•DTOZSQ•DT02DX•DT02DY•DT02DZ•
1 DZ02CX•DZ02DY•DZ02DT
COMMON/PULL/SX(41),SXX(41),SY(41),SYY(41),X(41),Y(41),Z(16)
COMMON/FORGOT/XCONST,YCONST,JWAG,DEPTH0
C
DEPTHS OF THE RIVER AT EACH J GRID POINT
WITHIN THE RIVER MOUTH ARE DETERMINED
C
DEPTH(JWAG)=DEPTH0
JBEGIN=JWAG+1
JEND=JLBANK(KMAX)
DO 10 J=JBEGIN,JEND
K=KMAX+1
8 K=K-1
IF(Y(J)＜LT•YLBANK(K)) GO TO 8
DELTZ=((Y(J)-YLBANK(K))/(YLBank(K+1)-YLBANK(K)))*DZ
DEPTH(J)=10-Z(K)+DELTZ
CONTINUE
JBEGIN=JRBank(KMAX)
JEND=JWAG-1
DO 10 J=JBEGIN,JEND
K=KMAX+1
12 K=K-1
IF(Y(J)＞GT•YRBANK(K)) GO TO 12
DELTZ=((Y(J)-YRBANK(K))/(YRBANK(K+1)-YRBANK(K)))*DZ
DEPTH(J)=10-Z(K)+DELTZ
CONTINUE
RETURN
END
SUBROUTINE UDIST(J,K,URIV)
COMMON/MOUTH/KBED, JRBANK(11), JLEANK(11), YRBANK(11), YLBANK(11), 
1 DEPTH(30)
COMMON/NUMPBER/ONE, HALF, FOURTH, PI, SLIM, TWEM, EIGHT, SIXTH
COMMON/PULL/SX(41), SXX(41), SY(41), SYY(41), X(41), Y(41), Z(16)
COMMON/MICE/ URIVER, VTIDE, DRHOOC, CCNST1, SLAPE
COMMON/FORGOT/XCCNST, YCONST, JWAG, DEPTH0

VELOCITY DISTRIBUTIONS WITHIN THE MOUTH ARE CALC.

RADIUS=3.5*(YLBANK(K)-YRBANK(K))
IF(RADIUS>GEO0001) GO TO 11
URIV=3.5
RETURN
11 CONTINUE
YRIV=Y(J)-YRBANK(K)
COMP1=(ONE-YRIV/RADIUS)*(ONE-YRIV/RADIUS)
COMP2=ONE-((ONE-Z(K))/DEPTH(J))**2.5
URIV=URIVER*(ONE-COMP1)*COMP2
RETURN
END
SUBROUTINE XMOMEQ
COMMON/VELOCITY/UNEW, UOLD, VNEW, VOLD, WNEW, WOLD, UIP1, UIM1, UJP1, UKP1,
1 UKM1, VIP1, VIM1, VJP1, VJM1, VKP1, VKM1, WIP1, WIM1, WJP1, WJM1, WKP1, WKM1
COMMON/STATE/PK, PKP1, PIP1, PIM1, PJP1, PJM1, RHO, RHOINV, DIFFUS
COMMON/GRAID/DX, DY, DZ, DT, DTOXSQ, DTOYSQ, DTOZSQ, DTO2DX, DTO2DY, DTO2DZ,
1 DZ02DX, DZ02DY, DZ02DT
COMMON/COORD/SX1, SX2, SY1, SY2
COMMON/VISCITY/COEFVX, COEFVY, COEFVZ, VISC, VEL, VELIP1, VELIM1, VELJP1,
1 VELJM1, VELKP1, VELKM1

THE X-MOMENTUM EQs IS SOLVED

VEL=UOLD
VELIP1=UIP1
VELIM1=UIM1
VELJP1=UJP1
VELJM1=UJM1
VELKP1=UKP1
VELKM1=UKM1
CALL VISCUS
COMP1=UIP1*UIP1-UIM1*UIM1
COMP2=(PIP1-PIM1)*RHOINV
COMP3=UJP1*VJP1-UJM1*VJM1
COMP4=UKP1*WKP1-UKM1*WKM1
UNEW=UOLD-SX1*DT02DX*(COMP1+COMP2)-SY1*DT02DY*COMP3-DTO2DZ*
1 COMP4+(VISC-UOLD*DIFFUS)*RHOINV
RETURN
END
SUBROUTINE YMQMEQ
COMMON/VELCTY/UNEV, UOLD, VNEW, VOLD, WNEW, UIPI, UIM1, UJP1, UJM1, UKP1,
1 UKM1, VIP1, VIM1, VJP1, VJM1, VKP1, VKM1, WIP1, WIM1, WJP1, WJM1, WKP1, WKM1
COMMON/STATE/PK, PKP1, PIP1, PIM1, PJP1, PJM1, RHO, RHOINV, DIFFUS
COMMON/GRID/DX, DY, DZ, DT, DTOXSQ, DTOYSQ, DTOZSQ, DT02DX, DT02DY, DT02DZ,
1 DT02CX, DT02CY, DT02CZ
COMMON/COORD/SX1, SX2, SY1, SY2
COMMON/VISCCTY/COEFVX, COEFVY, COEFVZ, VISC, VEL, VELIP1, VELIM1, VELJP1,
1 VELJM1, VELKP1, VELKM1

THE Y-MOMENTUM EQ. IS SOLVED

VEL=VOLD
VELIP1=VIP1
VELIM1=VIM1
VELJP1=VJP1
VELJM1=VJM1
VELKP1=VKP1
VELKM1=VKM1
CALL VISCUS
COMP1=UIP1*VIP1-UIM1*VIM1
COMP2=VJP1*VJP1-VJM1*VJM1
COMP3=(PJJP1-PJM1)*RHOINV
COMP4=VKP1*VKP1-VKM1*VKM1
VNEW=VOLD-SX1*DT02DX*COMP1-SY1*DT02DY*(COMP2+COMP3)-DT02DZ*
1 COMP4+(VISC-VOLD*DIFFUS)*RHOINV
RETURN
END
SUBROUTINE DIFUSE
COMMON/CONC/CNEW,COLD,CJP1,CJM1,CKP1,CKM1,CJP2,CJM2
COMMON/GRID/DX, DY, DZ, DT, DTOXSQ, DTOYSQ, DTOZSQ, DTO2DX, DTO2DY, DTO2DZ,
1 DZO2DX, DZO2DY, DZO2DT
COMMON/COORD/SX1, SX2, SY1, SY2
COMMON/DIFF/COEFDX, COEFDY, COEFDZ
COMMON/STATE/PK, PKP1, PIP1, PPM1, PJP1, PJM1, RHO, RHOINV, DIFFUS
COMMON/MISC/URI VER, VTIDE, DRHOC, CONST1, LATAPE
COMMON/PDERIV/PARTX, PARTY, PARTZ
COMMON/NUMBER/ONE, HALF, FOURTH, PI, SUM, TWO, EIGHT, SIXTH

DIFFUSION TERMS ARE CALC.

C

C

TWO C = COLD + COLD
PARTX = DTO2DX*(CJP1 - CIM1)
DTO12Y = SIXTH*DTO2DY
PARTY = DTO12Y*(-CJP2 + EIGHT*(CJP1 - CJM1) + CJM2)
PARTZ = DTO2DZ*(CKP1 - CKM1)
XCOMP1 = SX2*PARTX
XCOMP2 = SX1* SX1* DTOXSQ*(CJP1 - TWO C + CIM1)
YCOMP1 = SY2*PARTY
YCOMP2 = SY1* SY1* DTOYSQ*(-0.19444* (CJP2 + CJM2) + 1.77778*(CJP1 + CJM1)
1 -3.16667*(COLD))
ZCOMP2 = DTO2SQ*(CKP1 - TWO C + CKM1)
DIFFUS = COEFDX*(XCOMP1 + XCOMP2) + COEFDY*(YCOMP1 + YCOMP2) + COEFDZ*
1 ZCOMP2
RETURN
END
SUBROUTINE SALT
COMMON/CONC/CNEW,COLD,CIPL,CIM1,CJPL,CJM1,CKPL,CKM1,CJP2,CJM2
COMMON/GRID/DX,DY,DZ,DT,DTOXSQ,DTOYSQ,DTOZSQ,DTO2DX,DTO2DY,DTO2DZ,
 1 DZO2DX,DZO2DY,DZO2DT
COMMON/COORD/SXI,SX2,SY1,SY2
COMMON/DIFF/COEFDX,COEFDY,COEFDZ
COMMON/VELCTY/UNEW,VOLD,VNEW,VOLD,WNEW,UIP1,UIP1,UIJP1,UIJP1,UKP1,
 1 UKM1,VIPL,VIPL,VJPL,VJPL,VJMI,VJMI,VJPK1,VJPK1,WJPL,WJPL,WJMP1,WJMP1,
COMMON/STATE/PK,PKPI,PPIPI,PMPI,PJP1,PJM1,RHO,RHOINV,DIFFUS
COMMON/PDERIV/PARTX,PARTY,PARTZ

   THE CONSERVATION OF SALT EQ. IS SOLVED
   
   COMP1=SXI*UQLD*PARTX
   COMP2=SY1*VOLD*PARTY
   COMP3=WNEW*PARTZ
   CNEW=COLD+(-COMP1-COMP2-COMP3+DIFFUS)*5.0
   RETURN
END
SUBROUTINE PRESS

COMMON/NUMBER/ONE,HALF,FOURTH,PI,SUM,TWO,EIGHT,SIXTH
COMMON/VELCTY/UNEW,UOLD,VNEW,VCLD,WNEW,UIP1,UM1,UIJP1,UMJP1,UKP1,
1 UKMI,VIM1,VM1,VJMP1,VMJP1,VKM1,WJMP1,WM1,WJM1,WKJP1,WKMI
COMMON/STATE/PKP1,PPI1,PPI1,PJMP1,PJM1,RHO,RHOINV,DFFUS
COMMON/GRID/DX,DY,DZ,DT,DT0XSQ,DT0YSQ,DT0ZSQ,DT02DX,DT02DY,DT02DZ,
1 DT02DX,DZ02DY,DZ02DT
COMMON/COORD/SX1,SX2,SY1,SY2
COMMON/VISCTY/COEFVX,COEFVY,COEFVZ,VISC,VEL,VELIPI,VELIM1,VELJP1,
1 VELJMP1,VELP1,VELKM1

THE Z-MOMENTUM EQ. IS USED TO CALC. PRESSURE

VEL=WNEW
VELIPI=WIP1
VELIM1=WM1
VELJP1=WP1
VELJMP1=WJMP1
VELP1=WP1
VELP1=WKP1
VELKM1=WKMI
CALL VISCUS
COMP1=HALF*(WKP1*WKP1-WKM1*WKMI)*RHC
COMP2=SX1*DZ02DX*(UIP1*WIP1-UM1*WIM1)
COMP3=SY1*DZ02DY*(VJMP1*WP1-VJMP1*WJMP1)
COMP4=VISC*DZ02DT
PK=PKP1+COMP1+((DT+COMP2+COMP3)*RHC-COMP4)
RETURN
END
SUBROUTINE VISCUS
COMMON/GRID/DX, DY, DZ, DT, DTOXSQ, DTOYSQ,DTOZSQ, DTO2DX, DTO2DY, DTO2DZ,
1 DZ02DX, DZ02DY, DZ02DT
COMMON/VISCITY/COEFVX, COEFVY, COEFVZ, VISC, VEL, VELIP1, VELIM1, VELJP1,
1 VELJM1, VELKP1, VELKM1
COMMON/COORD/SX1, SX2, SY1, SY2

VISCOSITY TERMS ARE CALC.

TWOVEL = VEL + VEL
XCOMP1 = SX2 * DTO2DX * (VELIP1 - VELIM1)
XCOMP2 = SX1 * SX1 * DTOXSQ * (VELIP1 - TWOVEL + VELIM1)
YCOMP1 = SY2 * DTO2DY * (VELJP1 - VELJM1)
YCOMP2 = SY1 * SY1 * DTOYSQ * (VELJP1 - TWOVEL + VELJM1)
ZCOMP2 = DTOZSQ * (VELKP1 - TWOVEL + VELKM1)
VISC = COEFVX * (XCOMP1 + XCOMP2) + COEFVY * (YCOMP1 + YCOMP2) + COEFVZ * 
1 ZCOMP2
RETURN
END
SUBROUTINE SURF
COMMON/DIMEN1/U(2,19,31,10),V(2,19,31,10),W(19,31,10)
COMMON/DIMEN2/SURF(19,31),C(2,19,33,16),P(19,31,10)
COMMON/PULL/SX(41),SX(41),SY(41),SY(41),X(41),Y(41),Z(16)
COMMON/MOUTH/KBET, JLBBANK(11), YLBANK(11), YRBANK(11), YLBANK(11),
1 DEPTH(30)
COMMON/LIMITS/IMAX, IMAXM1, IMAXM2, JMAX, JMAXM1, JMAXM2, KMAX, KMAXM1,
1 KMAXM2, NMAX, NPRINT, NBAR, JMAXP1, JMAX
COMMON/GRID/DX, DY, CZ, DT, DTOXSQ, DTOYSQ, DTOZSQ, DTO2DX, DTO2DY, DTO2DZ,
1 DZ02DX, DZ02DY, DZ02DT
COMMON/NUMBER/ONE, HALF, FOURTH, PI, SUM, TWICE, EIGHT, SIXTH
COMMON/STEP/MN, MO, N, KBOT
COMMON/FORGET/XCONST, YCONST, JWAGP1, DEPTH0
COMMON/MISC/URIVER, VTIDE, DRHHOC, CONST1, LTAPR

SURFACE VELOCITIES ARE CALCULATED
JWAGP1=JWAG+1
JWAGM1=JWAG-1
JJJMAX=JWAG-2
DO 10 I=2, IMAXM1
IM1=I-1
IP1=I+1
SX1=SX(I)
V1=V(MN,1, JWAG, KMAXM1)
VWAG=V1
V(MN, I, JWAG, KMAX)=VWAG
VJM1=V(MO, I, JWAGM1, KMAX)
DO 4 J=JWAGP1, JMAXM1
JM1=J-1
SY1=SY(J)
DENOM=SX1+SY1
DWDZ=DZ*EPS*((W(I, J, KMAX)-W(I, J, KMAXM1))
+ (W(IM1, JM1, KMAX)-W(IM1, JM1, KMAXM1)))/DZ
VJM2=VJM1
VJM1=V1
VIN=0.16667*(VJM1+VJM2+V(MN,IP1,JM1,KMAX)+V(MN,IM1,JM1,KMAX)
1 +V(MN,I,JM1,KMAXM1)+V(MO,I,J,KMAX))
V1=VIN-DWDZ*DY/DENOM
V(MN,I,J,KMAX)=V1
CONTINUE
V1=WAG
VJP1=V(MO,I,JWAGP1,KMAX)
DO 6 JJJ=1, JJMAX
J=JWAG-JJJ
JP1=J+1
SY1=SY(J)
DENOM=SY1+SY1
DWDZ=0.5*((W(I,J,KMAX)-W(I,J,KMAXM1))
1 +(W(IM1,JP1,KMAX)-W(IM1,JP1,KMAXM1))/DZ
VJP2=VJP1
VJP1=V1
VIN=0.16667*(VJP1+VJP2+V(MN,IP1,JP1,KMAX)+V(MN,IM1,JP1,KMAX)
1 +V(MN,I,JP1,KMAXM1)+V(MO,I,J,KMAX))
V1=VIN+DWDZ*DY/DENOM
V(MN,I,J,KMAX)=V1
CONTINUE
UJPIM=U(MN,IM1,2,KMAX)
UIM=U(MN,IM1,1,KMAX)
VJP=V(MN,1,2,KMAX)
VJPKM=V(MN,1,2,KMAXM1)
VJP1M=V(MN,1,2,KMAX)
VJP1KM=V(MN,1,2,KMAXM1)
V1=V(MN,1,1,KMAX)
VKM=V(MN,1,1,KMAXM1)
VIN=V(MN,IM1,1,KMAX)
VIKM=V(MN,IM1,1,KMAXM1)
WJP=W(I,2,KMAX)
WJPKM=W(I,2,KMAXM1)
WJP1M=W(IM1,2,KMAX)
WJP1KM=W(IM1,2,KMAXM1)
W1=W(I,1,KMAX)
WKM = W(I, I, KMAXM1)
WIM = W(IM1, I, KMAX)
WIKM = W(IM1, I, KMAXM1)
DO 9 J = 2, JMAXM1
SY1 = SY(J)
JP1 = J + 1
UIJM = UIM
UIM = UJPIM
UJPIM = U(MN, IM1, JP1, KMAX)
VJM = V1
V1 = VJP
VJP = V(MN, I, JP1, KMAX)
VJKM = VKM
VKM = VJPKM
VJPKM = V(MN, I, JP1, KMAXM1)
VIJM = VIM
VIM = VJPIM
VJPIM = V(MN, IM1, JP1, KMAX)
VIKM = VIKM
VIKM = VJPIKM
VJPKM = V(MN, I, JP1, KMAXM1)
D1DY = 0.1250 * (VJP - VJM + VJPKM - VJKM + VJPIKM - VIJM + VJPKM - VIJKM) * (SY1/DY)
WJM = W1
W1 = WJP
WJP = W(I, JP1, KMAX)
WJKM = WKM
WKM = WJPKM
WJPKM = W(I, JP1, KMAXM1)
WIJM = WIM
WIM = WJPIM
WJPIM = W(IM1, JP1, KMAX)
WIJKM = WIKM
WIKM = WJPIKM
WJPIKM = W(IM1, JP1, KMAXM1)
IF (J > GTC) JWAG) GO TO 7
DWDZ = 0.25 * (WJM - WJKM + WIJM + WJKM + W1 - WKM + WIM - WIKM) / DZ
GO TO 8

1 IF(J EQ 0, JAG AND MN EQ 1) GO TO 11
DWDZ = C * 25 * (W1 - WKM + WIM - WIKM + WJP - WJPKM + WJPIM - WJPIM - WJPIM) / CZ

8 CONTINUE
IF(I EQ 2) GO TO 12
IM2 = I - 2
UIN = C * 16667 * (UJPIM + UIM + UIJM + U(MN, IM2, J, KMAX) + U(MN, IM1, J, KMAX1) + U(MN, IM1, J, KMAX1))

10 CONTINUE

12 IF(J GE JRBANK(KMAX) AND J LE JLBANK(KMAX)) GO TO 14
U(MN, I, J, KMAX) = -Q * S * (DXather/SX1) * (DVDF + DWDZ)
GO TO 9

14 U(MN, I, J, KMAX) = UIM - (DX/SX1) * (DVDF + CWDZ)
GO TO 9

END
BLOCK DATA
COMMON/NUMBER/ONE, HALF, FOURTH, PI, SUM, TWO, EIGHT, SIXTH
COMMON/STEP/MN, MO, N, KBOT
COMMON/MISC/ URIVER, VTIDE, DRHOC, CONST1, LTape
COMMON/UNITS/UMAX, VMAX, GRAV, RATIO, HREF, VREF
DATA ONE/1.0/
DATA HALF/0.5/
DATA FOURTH/0.25/
DATA MN/1/
DATA MO/2/
DATA PI/3.14159/
DATA N/C/
DATA DRHOC/0.705/
DATA TWc/2.0/
DATA EIGHT/8.0/
DATA SIXTH/0.166667/
DATA GRAV/9.790/
END
APPENDIX C
This appendix presents the computer program for calculating the deposition of suspended material at a river delta. The numerical procedure used is described in Chapter V. Comment cards have been included to explain the operations being performed.

The geometry of the region and the fluid velocities are read from a tape generated by the fluid dynamics program; therefore, notation between these two programs is consistent.

All data cards are read into this program in SUBROUTINE PRELIM. These data consist primarily of nominal particle spacing, characteristics, and concentration distributions and levels. A comment card is used to define each parameter proceeding a READ card.

At the completion of a specified number of time steps, the program lists the x, y, and z location of each particle and tells whether it has settled to the bottom or not. It then lists the computed growth rate at every grid point as a result of the deposition of each type of material initially in suspension. After searching throughout the entire region for the grid point with the maximum combined growth rate from every kind of particle, the bottom is adjusted upward by a maximum of one grid height. The new bottom location is then listed for each grid point written on tape.

A listing of the program is now presented.
DIMENSION DRIFT(700)
COMMON/VEL/ U(19,32,11), V(19,32,11), W(19,32,11)
COMMON/TOPBOT/ SURF(19,31), ZB(19,31)
COMMON/TURBID/VOLFLO(700), RATE(19,31)
COMMON/LOCATE/XPART(700), YPART(700), ZPART(700), WSETL(700)
COMMON/GRID/II(700), JJ(700), KK(700)
COMMON/SOIL/CONREF(8), EMPEXP(8), WSET0(8), DENSTY(8), DIAM(8)
COMMON/MOUTH/JRBANK(16), JLBANK(16), YRBANK(16), YLBANK(16)
COMMON/LIMITS/IMAX, IMAXM1, IMAXM2, JMAX, JMAXM1, JMAXM2, KMAX, KMAXM1
COMMON/COORD/X(22), Y(41), Z(16)
COMMON/SPEEDS/UPART, VPART, WPART
COMMON/TRANS/NMAX, DELT, RATIO
COMMON/VISC/URIVER, VTIDE, KBED, NNMAX, ZSPACE, DELQRF, KINDS, ZREF, SLUMP
COMMON/FORGOT/XCONST, YCONST, JWAG, DEPTHG, HREF, GRAV, VREF
COMMON/SPACE/DX, DY, DZ
DATA A,B/'SWIM', 'SINK'/
CALL PRELIM
DO 10 KIND=1, KINDS
  WRITE(6,1040) KIND, DIAM(KIND), WSET0(KIND), DENSTY(KIND),
  CONREF(KIND), EMPEXP(KIND)
  WSET0(KIND)=WSET0(KIND)*VREF/0.01
  DIAM(KIND)=DIAM(KIND)*HREF/0.001
  L=3
  N=C
  CALL INITIAL(L, LMAX, KIND)
  WRITE(6,1100)
  DO 3 L=1, LMAX
  DRIFT(L)=A
  DO 50 N=1, NMAX
  DO 45 L=1, LMAX
    IF(DRIFT(L)=EQ B) GO TO 40
    I=II(L)
    J=JJ(L)
    K=KK(L)
    IF(I=GE IMAX) GO TO 40
    IF(J=GE JMAX) GO TO 40
  END
  IF(DRIFT(L)=EQ B) GO TO 40
  IF(J=GE JMAX) GO TO 40
  IF(I=GE IMAX) GO TO 40
  DRIFT(L)=A
  DO 50 N=1, NMAX
  DO 45 L=1, LMAX
    IF(DRIFT(L)=EQ B) GO TO 40
    I=II(L)
    J=JJ(L)
    K=KK(L)
    IF(I=GE IMAX) GO TO 40
    IF(J=GE JMAX) GO TO 40
END
IF(JoLE01) GO TO 40
IF(KoLE0) K=1
IM1=I-1
IPI=I+1
JM1=J-1
JP1=J+1
KM1=K-1
KP1=K+1
XI=X(I)
X1P1=X(IPI)
YJ=Y(J)
YJP1=Y(JP1)
ZK=Z(K)
ZKP1=Z(KP1)
X=P=XPART(L)
YP=YPART(L)
ZP=ZPART(L)
COMPX1=(XP-XIP1)/(XI-XIP1)
COMPX2=(XP-XI)/(XIP1-XI)
ZBOTX1=ZB(I,J)*COMPX1+ZB(IPI,J)*COMPX2
ZBOTX2=ZB(I,JPI)*COMPX1+ZB(IPI,JP1)*COMPX2
ZBOT=(ZBOTX1*(YP-YJP1)-ZBOTX2*(YP-YJ))/(YJ-YJP1)
IF(ZPATZBOT) GO TO 20
KTEST=KMAXM1
ZTEST=Z(KTEST)-ZBOT
UTEST=U(I,J,KTEST)
VTEST=V(I,J,KTEST)
QTEST=SQRT(UTEST*UTEST+VTEST*VTEST)
QDRIFT=4.420*SQRT(DIAM(KIND)*(DENSTY(KIND)-10))*

1 ALOG10(3002*ZTEST/DIAM(KIND))
IF(QTESTGEOQDRIFT) GO TO 14
ZB1=ZB(I,J)
DZBDX=(ZB(IPI,J)-ZB1)/(XIP1-XI)
DZBDY=(ZB(I,JPI)-ZB1)/(YJP1-YJ)
IF(ABS(DZBDX)<1.0E-10) DZBDX=1.0E-10
THETA=ATAN((-DZBDY/DZBDX)
IF(THETA < 0) GO TO 12
DZDX = (ZB(IP1, JP1) - ZB(I, JP1)) / (X1P1 - XI)
THETA = -THETA
12 BETA = ATAN(DZDX * COS(THETA) - DZDY * SIN(THETA))
IF(ABS(BETA) > SLUMP) GO TO 16
GO TO 3C
14 CONTINUE
GO TO 18
16 WRITE(6, 1031) L, I, J
18 ZP = ZBOT + DZ
ZPART(L) = ZP
20 CALL INTERP(I, J, K, L)
CALL GRAD(I, J, K, DELQ)
WSET = WSET0(KIND) * (1.0 - 0.3 * DELQ / DELQF)
XPART(L) = XP + UPART * DELT
IF(XPART(L) < 0.0) XPART(L) = 1.0E-8
YPART(L) = YP + VPART * DELT
ZPART(L) = ZP + (WPART + WSET) * DELT
IF(XPART(L) < X1P1) II(L) = IP1
IF(XPART(L) > X1P1) II(L) = IM1
IF(YPART(L) < YJP1) JJ(L) = JP1
IF(YPART(L) > YJP1) JJ(L) = JM1
IF(ZPART(L) < ZKP1) KK(L) = KP1
IF(ZPART(L) > ZK) KK(L) = KM1
GO TO 40
30 DRIFT(L) = 8
40 CONTINUE
50 CONTINUE
N = NMAX
DO 60 L = 1, LMAX
WRITE(6, 1101) N, L, XPART(L), YPART(L), ZPART(L), II(L), JJ(L), KK(L), DRIFT(L)
60 CONTINUE
WRITE(6, 1020)
DO 88 I = 2, IMAXM1
X1 = X(I)
XIM1 = X(I-1)
XIP1 = X(I+1)
XB = 0.5*(X1 + XIM1)
XF = 0.5*(X1 + XIP1)
DO 86 J = 3, JMAXM2
Y1 = Y(J)
YJM1 = Y(J-1)
YJP1 = Y(J+1)
XR = 0.5*(Y1 + YJM1)
YL = 0.5*(Y1 + YJP1)
AREA = (XF - XB)*(YL - YR)
RATE1 = 0
DO 84 L = 1, LMAX
XP = XPART(L)
IF (XP LT XB) GO TO 84
IF (XP GE XF) GO TO 84
YP = YPART(L)
IF (YP LT YR) GO TO 84
IF (YP GE YL) GO TO 84
IF (DRIFT(L) EQ A) GO TO 84
RATE1 = RATE1 + VOLFLO(L)/AREA
84 CONTINUE
RATE(I, J) = RATE(I, J) + RATE1*1.6667
WRITE (6, 1021) XI, Y1, RATE1
86 CONTINUE
88 CONTINUE
100 CONTINUE
CALL PILEUP
1010 FORMAT (10X, 2HI = 12, 10X, 2HX = F8.4)
1020 FORMAT (1/12X, 1HX = 19, 1HY = 14, 11HGROWTH RATE,/)  
1021 FORMAT (3(7X, 1EI 3, 6))
1030 FORMAT (5X, *PARTICLE NO, 2X, I4, 2X, *WAS REJECTED AT*, 2X, 2HI = 12, 5X  
1 * 2HJ = 12, 5X, *BECAUSE QTEST WAS TOO LARGE*)
1031 FORMAT (5X, *PARTICLE NO, 2X, I4, 2X, *WAS REJECTED AT*, 2X, 2HI = 12, 5X  
1 * 2HJ = 12, 5X, *BECAUSE BETA WAS TOO LARGE*)
1040 FORMAT (1/10X, SHKIND = 19, /10X, 5HDIAM = F9.6, /10X,  
5HDIAM = F9.6, /10X,  
5HDIAM = F9.6,
1  5HSET=F9.6,+/10X,7HDENSTY=F8.6,+/10X, 
2  5HCREF=F9.3,+/10X,7HEMPEXP=F8.3,+/)
110C FORMAT(/,8X,1HN,9X,1HL,10X,10HX PARTICLE,10X,10HY PARTICLE,10X, 
  10HZ PARTICLE,/) 
1101 FORMAT(2(6X,14),3(7X,E13.6),3(7X,I3),5X,A4)
STOP
END
SUBROUTINE PRELIM
COMMON/VEL/U(19,32,11),V(19,32,11),W(19,32,11)
COMMON/TOPOP/SURF(19,31),ZB(19,31)
COMMON/TURBID/VOLFLC(700),RATE(19,31)
COMMON/SOIL/CONREF(8),EMPEXP(8),WSETO(8),DENSTY(8),DIAM(8)
COMMON/MOUTH/JRBANK(16),YRBANK(16),YLBANK(16)
COMMON/TRANS/NMAX,DELTAT,RATIO
COMMON/COORD/X(22),Y(41),Z(16)
COMMON/LIMITS/IMAX,IMAXM1,IMAXM2,JMAD,JMADM1,JMADM2,KNAX,KMAXM1
COMMON/FORGET/XCONST,YCCNST,JWAG,DEPTH0,HREF,GRAV,VREF
COMMON/SPACE/DX,DY,DZ
COMMON/MISC/URIVER,VTIDE,KBED,NNMAX,ZSPACE,DELQRF,KINDS,ZREF,SLUMP

PRELIMINARY CALC's AND DATA READ
NMAX=THE NO. OF TIME STEPS
READ(5,1000) NMAX
DELTAT=THE SIZE OF THE TIME STEP IN SECONDS
READ(5,1001) DELTAT
NNMAX=THE NO. OF NOMINAL PARTICLES TO BE EVENLY DISTRIBUTED
AT EACH LEVEL
KINDS=THE NO. OF KINDS OF PARTICLES
READ(5,1000) NNMAX,KINDS
ZSPACE=VERTICAL SPACING BETWEEN ROWS OF NOMINAL PARTICLES
IN NONDIMENSIONAL FORM. EXAMPLED LET = TO 0.05
ZREF=DEPTH IN METERS BELOW THE SURFACE WHERE
PARTICLE CONC. IS KNOWN WILL BE NONDIMENSIONALIZED
READ(5,1001) ZSPACE,ZREF
DIAM(KIND)=DIAMETER OF THAT KIND OF PARTICLE
READ(5,1001) (DIAM(KIND), KIND=1,KINDS)
CONREF(KIND)=REF. CONC. OF THAT KIND OF PARTICLE AT ZREF
READ(5,1001) (CONREF(KIND), KIND=1,KINDS)
EMPEXP(KIND)=EMPIRICAL EXPONENT FOR PARTICLE CONC. DIST.
READ(5,1001) (EMPEXP(KIND), KIND=1,KINDS)
WSETO(KIND)=STILL WATER SETTLING VELOCITY IN CM/SEC NEGATIVE
READ(5,1001) (WSETO(KIND), KIND=1,KINDS)
DENSTY(KIND)=DENSITY OF A PARTICLE GM/CC
READ(5,1001) (DENSTY(KIND), KIND=1,KINDS)
RE WIND 3
READ(3) IMAX, JMAX, KMAX, KBED, JWAG, URIVER, VTIDE, HREF, RATIO
READ(3) (X(I), I=1, IMAX)
READ(3) (Y(J), J=1, JMAX)
READ(3) (Z(K), K=1, KMAX)
JMAX = JMAX + 2
READ(3) (YRBANK(K), K=KBED, KMAX)
READ(3) (YLBANK(K), K=KBED, KMAX)
READ(3) (JRBANK(K), K=KBED, KMAX)
READ(3) (JLBANK(K), K=KBED, KMAX)
READ(3) (((U(I, J, K), I=1, IMAX), J=1, JMAX), K=1, KMAX)
READ(3) (((V(I, J, K), I=1, IMAX), J=1, JMAX), K=1, KMAX)
READ(3) (((W(I, J, K), I=1, IMAX), J=1, JMAX), K=1, KMAX)
READ(3) (((DUMMY, I=1, IMAX), JJ=1, JMAX), K=1, KMAX)
READ(3) (((DUMMY, I=1, IMAX), J=1, JMAX), K=1, KMAX)
READ(3) (((SURF(I, J), I=1, IMAX), J=1, JMAX)
READ(3) (((ZB(I, J), I=1, IMAX), J=1, JMAX)
DX = 1.0 / FLOAT(IMAX - 1)
DY = 1.0 / FLOAT(JWAG - 1)
DZ = 1.0 / FLOAT(KMAX)
IMAXM1 = IMAX - 1
IMAXM2 = IMAX - 2
JMAXM1 = JMAX - 1
JMAXM2 = JMAX - 2
KMAXM1 = KMAX - 1
KMAXP1 = KMAX + 1
Z(KMAXP1) = 1.0 + DZ
CALL GRIDPT
DEPTH = 1.0 - Z(KBED)
DEGRAD = 3.0 x 14159 / 180.0
GRAV = 9.8179
VREF = SQRT(HREF * GRAV)
DELT = DELTAT / SQRT(HREF / GRAV)
ZREF = (HREF - ZREF) / HREF
C A CONSTANT ANGLE OF REPOSE OF 30 DEGREES WAS ASSUMED
SLUMP = 30.0 x DEGRAD
DO 5 I=1,IMAX
DO 5 J=1,JMAX
U(I,J,KMAXP1)=U(I,J,KMAX)
V(I,J,KMAXP1)=V(I,J,KMAX)
W(I,J,KMAXP1)=-W(I,J,KMAXM1)
5 CONTINUE
URIVER=U(1,JWAG,KMAX)
DELQRF=URIVER/(1.0-Z(KBED))
DO 10 I=1,IMAX
DO 10 J=1,JMAX
RATE(I,J)=0.0
10 CONTINUE
1000 FORMAT(8I10)
1001 FORMAT(8F10.6)
RETURN
END
SUBROUTINE GRIDPT
COMMON/LIMITS/IMAX,IMAXM1,IMAXM2,JMAX,JMAXM1,JMAXM2,KMAX,KMAXM1
COMMON/FORGOT/XCONST,YCONST,JWAG,DEPTH0,HREF,GRAV,VREF
COMMON/COORD/X(22),Y(41),Z(16)
COMMON/SPACE/DX,DY,DZ
C    WRITES OUT GRID LOCATIONS
WRITE(6,273C)
DO 2 I=1,IMAX
  WRITE(6,2001) I,X(I)
WRITE(6,2002)
DO 4 J=1,JMAX
  WRITE(6,2001) J,Y(J)
WRITE(6,2003)
DO 5 K=1,KMAX
  WRITE(6,2001) K,Z(K)
2000 FORMAT(/11X,1HI,15X,1HX,/)  
2001 FORMAT(10X,I2,10X,F10.5)  
2002 FORMAT(/11X,1HJ,15X,1HY,/)  
2003 FORMAT(/11X,1HK,15X,1HZ,/)  
RETURN
END
SUBROUTINE INITIAL(L,LMAX,KIND)

COMMON/VEL/U(19,32,11),V(19,32,11),W(19,32,11)
COMMON/TOPBOT/SURF(19,31),ZB(19,31)
COMMON/TURBID/VOLFLO(700),RATE(19,31)
COMMON/SOIL/CONREF(8),EMPEXP(8),WSETQ(8),DENSTY(8),DIAM(8)
COMMON/MOUTH/JRBANK(16),JLBANK(16),YRBANK(16),YLBank(16)
COMMON/COORD/X(22),Y(41),Z(16)
COMMON/LOCATE/XPART(700),YPART(700),ZPART(700),WSETL(700)
COMMON/GRID/II(700),JJ(700),KK(700)
COMMON/SPACE/DX,DY,DZ
COMMON/LIMITS/IMAX,IMAXM1,IMAXM2,JMAX,JMAXM1,JMAXM2,KMAX,KMAXM1
COMMON/FORGOT/XCONST,YCONST,JWAG,DEPTH0,HREF,GRAV,VREF
COMMON/MISC/URIVER,VTIDE,KBED,NNMAX,ZSPACE,DELQRF,KINDS,ZREF,SLUMP
COMMON/TRANS/NMAX,DELT,RATIO
COMMON/SPEEDS/UPART,VPART,WPART

C PARTICLE PROPERTIES & DISTRIBUTIONS ARE INITIALIZED

ZBED=Z(KBED)
REFFUN=(ZREF-ZBED)/(1.0-ZREF)
CONCRF=CONREF(KIND)
DENSE=DENSTY(KIND)
WSET=WSETQ(KIND)
IF(KIND.GT.1) GO TO 1
WRITE(6,1100)

1 N=0
K=KBED
Z1=Z(KBED)+0.5*ZSPACE
IF(Z1.GE.Z(K+1)) K=K+1
IF(Z1.GE.Z(KMAX)) GO TO 10
KP1=K+1
KM1=K-1
C THIS CASE IS APPLICABLE FOR THE SQUARE SHAPED MOUTH ONLY

J=JRBANK(K)-1
JP1=J+1
JRT=J
JLF=JWAG+(JWAG-JRT)
YL=-0.5*(Y(J)+Y(J+1))
C THIS CASE IS APPLICABLE FOR THE ELLIPTIC SHAPED MOUTH ONLY
Yl=0.5*Ratio*Sqrt(Depth0**2.0-(1.0-Z1)**2.0)
Yr=-Yl
C I WANT TO PLACE NNMAX PARTICLES BETWEEN YR AND YL
YSpace=(YL-YR)/FLOAT(NNMAX)
Area=YSPACE*ZSPACE
Y1=YL-0.5*YSPACE
J=;
3 J=J+1
IF(Y(J)<YL) GO TO 3
JHERBR=J-1
J=JMAX
4 J=J-1
IF(Y(J)>YL) GO TO 4
JHERBL=J
J=JHERBR
JM1=J-1
JP1=J+1
DO 8 NN=1,NNMAX
L=L+1
X1=0.0
Y1=Y1+YSPACE
XPart(L)=X1
YPart(L)=Y1
ZPart(L)=Z1
YJP1=Y(JP1)
IF(YPart(L)<YJP1) J=JP1
J=J1
J=J+1
YJ=Y(J)
YJP1=Y(JP1)
ZK=Z(K)
ZKp1=Z(KP1)
U1=U(1,J,K)
IF(YJ<YT<YRbank(K)) U1=0.0
IF(YJ>YT<YLbank(K)) U1=0.0
UJP1=U(I,JP1,K)
IF(YJP1,LT,YRBANK(K)) UJP1=C0
IF(YJP1,GT,YLBANK(K)) UJP1=C0
UKP1=U(I,K,P1)
UKJP1=U(I,JP1,KP1)
IF(J, EQ,JHERBR) GO TO 50
IF(J, EQ,JHERBL) GO TO 60
COMP1=(Y1-YJP1)/(YJ-YJP1)
COMP2=(Y1-YJ)/(YJP1-YJ)
UBELOW=L1*COMP1+UJP1*COMP2
UABOVE=UKP1*COMP1+UKJP1*COMP2
COMP3=(Z1-ZKP1)/(ZK-ZKP1)
COMP4=(Z1-ZK)/(ZKP1-ZK)
UPART=UBELOW*COMP3+UABOVE*COMP4
GO TO 7
50 COMP5=UJP1*(ZPART(L)-ZKP1)/(ZK-ZKP1)
COMP6=UKJP1*(ZPART(L)-ZK)/(ZK1-ZK)
UPARTZ=COMP5+COMP6
UPART=UPARTZ*(YPART(L)-YR)/(YJP1-YR)
GO TO 7
60 UPARTZ=U1*(ZPART(L)-ZKP1)/(ZK-ZKP1)+UKP1*(ZPART(L)-ZK)/(ZKP1-ZK)
UPART=UPARTZ*(YPART(L)-YL)/(YJ-YL)
7 WSETL(L)=WSE
CONC=CONCREF*{(1+Z1)*REFFUN/(Z1-ZBED)}**EMPEXP(KIND)
VOLFLC(L)=CONC*AREA*UPART/DENSE
II(L)=I
JJ(L)=J
KK(L)=K
IF(KIND,GT,1) GO TO 8
WRITE(6,1101) N,L,X1,Y1,Z1,II(L),JJ(L),KK(L),UPART,AREA
8 CONTINUE
Z1=Z1+ZSPACE
GO TO 2
10 CONTINUE
LMAX=L
L=LMAX-NNMAX
TOP = 1, \text{SURF}(1, \text{JWAG})

\text{CORR} = (\text{TOP} - 2 \text{PART}(\text{LMAX})) / (0.5 \text{ZSPACE})

\text{DO 12} \text{NN} = 1, \text{NNMAX}

\text{L} = \text{L} + 1

\text{VOLFLO}(\text{L}) = \text{VOLFLO}(\text{L}) \times \text{CORR}

\text{12 CONTINUE}

\text{DO 14} \text{L} = 1, \text{LMAX}

\text{WRITE}(6, 1103) \text{L, VOLFLO(L)}

\text{14 CONTINUE}

1103 \text{FORMAT} (5X, 2HL = \text{I4}, 5X, 7HVOLFLO = \text{E13.6})

\text{WRITE}(6, 1102)

1102 \text{FORMAT} (//, 8X, 1HN, 9X, 1HL, 10X, 10HY PARTICLE, 10X, 10H PARTICLE)

\text{WRITE}(6, 1101)

1101 \text{FORMAT} (2(3X, I3), 3(3X, E13.6), 3(3X, I3), 5X, F7.5, 5X, E13.6)

\text{1102 FORMAT} (//, 5X, 27HI HAVE FINISHED WITH INITIAL, //)

\text{RETURN}

\text{END}
SUBROUTINE INTERP(I, J, K, L)
COMMON/VEL/U(19,32,11),V(19,32,11),W(19,32,11)
COMMON/TOPBOT/SURF(19,31),ZB(19,31)
COMMON/TURBID/VOLFLO(700),RATE(19,31)
COMMON/MOUTH/JRBANK(16),JLBANK(16),YRBANK(16),YLBANK(16)
COMMON/LIMITS/IMAX,IMAXM1,JMAX,JMAXM1,KMAX,KMAXM1
COMMON/COORD/X(22),Y(41),Z(16)
COMMON/SPEEDS/UPART,VPART,WPART
COMMON/Locate/XPART(700),YPART(700),ZPART(700),WSETL(700)
COMMON/GRID/II(700),JJ(700),KK(700)

INTERPOLATION SCHEME FOR THE COMPONENTS OF FLUID VELOCITY

IP1=I+1
JP1=J+1
KP1=K+1

COMPX1=(XPART(L)-X(IP1))/(X(I)-X(IP1))
COMPX2=(XPART(L)-X(I))/(X(IP1)-X(I))

COMPY1=(YPART(L)-Y(JP1))/(Y(J)-Y(JP1))
COMPY2=(YPART(L)-Y(J))/(Y(JP1)-Y(J))

COMPZ1=(ZPART(L)-Z(KP1))/(Z(K)-Z(KP1))
COMPZ2=(ZPART(L)-Z(K))/(Z(KP1)-Z(K))

UX1=U(I,J,K)*COMPX1+U(IP1,J,K)*COMPX2
VX1=V(I,J,K)*COMPX1+V(IP1,J,K)*COMPX2
WX1=W(I,J,K)*COMPX1+W(IP1,J,K)*COMPX2

UX2=U(I,J,KPI)*COMPX1+U(IP1,J,KPI)*COMPX2
VX2=V(I,J,KPI)*COMPX1+V(IP1,J,KPI)*COMPX2
WX2=W(I,J,KPI)*COMPX1+W(IP1,J,KPI)*COMPX2

UX3=U(I,JPI,K)*COMPX1+U(IP1,JPI,K)*COMPX2
VX3=V(I,JPI,K)*COMPX1+V(IP1,JPI,K)*COMPX2
WX3=W(I,JPI,K)*COMPX1+W(IP1,JPI,K)*COMPX2

UX4=U(I,JPI,KPI)*COMPX1+U(IP1,JPI,KPI)*COMPX2
VX4=V(I,JPI,KPI)*COMPX1+V(IP1,JPI,KPI)*COMPX2
WX4=W(I,JPI,KPI)*COMPX1+W(IP1,JPI,KPI)*COMPX2

UZ1=UX1*COMPZ1+UX2*COMPZ2
VZ1=VX1*COMPZ1+VX2*COMPZ2
WZ1=WX1*COMPZ1+WX2*COMPZ2
UZ2=UX3*COMPZ1+UX4*COMPZ2
\[ VZ_2 = VX_3 \cdot \text{COMPZ1} + VX_4 \cdot \text{COMPZ2} \]
\[ WZ_2 = WX_3 \cdot \text{COMPZ1} + WX_4 \cdot \text{COMPZ2} \]
\[ UPART = UZ_1 \cdot \text{COMPY1} + UZ_2 \cdot \text{COMPY2} \]
\[ VPART = VZ_1 \cdot \text{COMPY1} + VZ_2 \cdot \text{COMPY2} \]
\[ WPART = WZ_1 \cdot \text{COMPY1} + WZ_2 \cdot \text{COMPY2} \]
RETURN
END
SUBROUTINE GRADI(I,J,K,DELQ)

COMMON/VEL/U(19,32,11),V(19,32,11),W(19,32,11)
COMMON/COORD/X(22),Y(41),Z(16)
COMMON/LIMITS/IMAX,IMAXM1,IMAXM2,JMAX,JMAXM1,JMAXM2,KMAX,KMAXM1
COMMON/MISC/URIVER,VTIME,KBED,NMAX,ZSPACE,DELRDF,INDS,ZREF,SLUMP

C MAGNITUDE OF THE FLUID VELOCITY GRADIENT IS CALC:

IP1=I+1
JP1=J+1
KP1=K+1
XI=X(I)
XIP1=X(IP1)
YJ=Y(J)
YJP1=Y(JP1)
ZK=Z(K)
ZKP1=Z(KP1)
U1=U(I,J,K)
UIP1=U(IP1,J,K)
UJP1=U(I,JP1,K)
UIJP1=U(IP1,JP1,K)
UKP1=U(I,J,KP1)
UJKP1=U(I,JP1,KP1)
V1=V(I,J,K)
VIP1=V(IP1,J,K)
VJP1=V(I,JP1,K)
VIJP1=V(IP1,JP1,K)
VKP1=V(I,J,KP1)
VJKP1=V(I,JP1,KP1)
Q1=SQRT(U1*U1+V1*V1)
QIJP1=SQRT(U1P1*U1P1+VIP1*VIP1)
QJJP1=SQRT(UJP1*UJP1+VJP1*VJP1)
QJKP1=SQRT(UKP1*UKP1+VKP1*VKP1)
QIJJP1=SQRT(UIJP1*UIJP1+VIJP1*VIJP1)
QJKJP1=SQRT(UJKP1*UJKP1+VJKP1*VJKP1)
DQDX=0.5*(QIJP1-Q1+QIJJP1-QJJP1)/(XIP1-XI)
DQDY=(QJP1-Q1)/(YJP1-YJ)
DQDZ = \sqrt{0.5 \times (QKP1 - Q1 + QJKP1 - QJP1) / (ZKP1 - ZK)}
DELT = \sqrt{DQDX^2 + DQDY^2 + DQDZ^2}
RETURN
END
SUBROUTINE PILEUP
DIMENSION BUFFER(3000)
COMMON/TOPBOTT/SURF(19,31),ZB(19,31)
COMMON/TURBID/VOLFL0(700),RATE(19,31)
COMMON/MOUTH/JRBANK(16),JLBANK(16),YRBANK(16),YLBank(16)
COMMON/LIMITS/IMAX,IMAXM1,IMAXM2,JMAX,JMAXM1,JMAXM2,KMAX,KMAXM1
COMMON/COORD/X(22),Y(41),Z(16)
COMMON/MISC/VURIVER,VTIDE,KBED,NNMAX,ZSPACE,DELRGK,KINDS,ZREF,SLUMP
COMMON/SPACE/DX,DY,DZ
COMMON/ORGOT/XCONST,YCONST,JWAG,DEPTO,HREF,GRAV, VREF
C

THIS IS A SUBROUTINE TO ADJUST THE BOTTOM HTs.
C
WRITE THE NEW BOTTOM SHAPE ON TAPE, & PLOT RESULTS
RATMAX=0
DO 10 I=2,IMAXM1
DO 10 J=2,JMAXM1
IF(RATE(I,J).LT.RATMAX) GO TO 10
RATMAX=RATE(I,J)
IGROW=I
JGROW=J
10 CONTINUE
IF(RATMAX.EQ.1.0E-10) WRITE(6,1006)
IF(RATMAX.EQ.1.0E-10) GO TO 100
SCALE=(X(2)-X(1))/DZ
WRITE(6,1006) RATMAX,SCALE,IGROW,JGROW
CALL PLOT(SBUFFER,3000)
CALL FACTOR(0@2)
XMAX=X(IMAXM1)
JBEGIN=5.
JEND=JMAX-4
YMIN=Y(JBEGIN)
YMAX=Y(JEND)
CALL PLOT(0@0,0@0,-3)
K=KMAX
CALL PLOT(X,6,XMAX,3)
YRB=YRBANK(K)-YMIN
CALL PLOT(YRB,XMAX,2)
XUPSTR = XMAX + 1, 0
CALL PLCT(YRB, XUPSTR, 2)
YLB = YLBANK(K) - YMIN
CALL PLCT(YLB, XUPSTR, 3)
CALL PLCT(YLB, XMAX, 2)
YEND = YMAX - YMIN
CALL PLCT(YEND, XMAX, 2)

C
NOW WANT TO INCREASE BOTTOM HTe BY A MAX, INCREMENT OF DZ

TIME = DZ * HREF * 100, 0 * HREF * HREF / RATMAX
WRITE(6, 1005) TIME
WRITE(6, 1007)
DO 30 I = 2, IMAX-1
DO 30 J = 2, JMAX-1
ZB(I, J) = ZB(I, J) + DZ * RATE(I, J) / RATMAX
WRITE(6, 1008) I, J, ZB(I, J)
30 CONTINUE

DO 40 I = 2, IMAX-1
ZLOC = XMAX - X(I) + (ZB(I, J) - ZB(I, I)) * SCALE
CALL PLCT(C0, ZLOC, 3)
DO 40 J = JBEGIN, JEND
Y1 = Y(J) - YMIN
ZLOC = XMAX - X(I) + (ZB(I, J) - ZB(I, I)) * SCALE
CALL PLCT(Y1, ZLOC, 2)
40 CONTINUE
CALL PLCT(Y1, ZLOC, 999)

JUMP = 1
IF (JUMP .EQ. 1) RETURN
REWIND 3
READ(3) IMAX, JMAX, KMAX, KBED, JWAG, URIVER, VTIDE, HREF, RATIO
READ(3) (DUMMY, I = 1, IMAX)
READ(3) (DUMMY, J = 1, JMAX)
READ(3) (DUMMY, K = 1, KMAX)
JJMAX = JMAX + 2
READ(3) (DUMMY, K = KBED, KMAX)
READ(3) (DUMMY, K = KBED, KMAX)
READ(3) (DUMMY, K = KBED, KMAX)
READ (3) (DUMMY, K=KBED*KMAX)
READ (3) (((DUMMY, I=1,IMAX), J=1,JMAX), K=1,KMAX)
READ (3) (((DUMMY, I=1,IMAX), J=1,JMAX), K=1,KMAX)
READ (3) (((DUMMY, I=1,IMAX), J=1,JMAX), K=1,KMAX)
READ (3) (((DUMMY, I=1,IMAX), J=1,JMAX), K=1,KMAX)
READ (3) (((DUMMY, I=1,IMAX), J=1,JMAX), K=1,KMAX)
READ (3) (((DUMMY, I=1,IMAX), J=1,JMAX), K=1,KMAX)
READ (3) (((DUMMY, I=1,IMAX), JJ=1,JJMAX), K=1,KMAX)
READ (3) (((DUMMY, I=1,IMAX), J=1,JMAX), K=1,KMAX)
WRITE (3) ((ZB(I,J), I=1,IMAX), J=1,JMAX)

CONTINUE
10 FORMAT(/,5X,7HRATMAX=E13.6,5X,6HSCALE=E13.6,5X,2HI=12,5X,2HJ=12)
1001 FORMAT(/,5X,2HX=F10.5,10X,2HY=F10.5,10X,3HRX=F10.5)
1005 FORMAT(/,5X,*TIME REQD. TO INCRE. BOTTOM BY A MAX. OF DZ IS*
1,2X,E13.6,2X,*SECONDS*)
1006 FORMAT(/,10X,*RATMAX IS ZERO*)
1007 FORMAT(/,9X,*HI,9X,*HJ,9X,*HZB,*/)
1008 FORMAT(2(8X,I2),8X,F5.3)
RETURN
END
APPENDIX D

RESULTS OF THE FLUID DYNAMICS ANALYSIS PRESENTED AS ISOCONCENTRATION CONTOURS AND VELOCITY VECTORS
Figure D-1: Side View of Isoconcentration Contours and Velocity Vectors; Baseline Run
Figure D-2: Velocity Vectors in a Top View of the Surface; Baseline Run
Figure D-3: Velocity Vectors in a Top View of the 6 Meter Depth; Baseline Run
Figure D-4: Isoconcentration Contours in a Top View of the Surface; Baseline Run
Figure D-5: End View of Isoconcentration Contours and Velocity Vectors; x = 18 Meters; Baseline Run
Figure D-6: End View of Isoconcentration Contours and Velocity Vectors; x = 200 Meters; Baseline Run
Figure D-7: Side View of Isoconcentration Contours and Velocity Vectors; Reduced Buoyancy Run
Figure D-8: Velocity Vectors in a Top View of the Surface; Reduced Buoyancy Run
Figure D-9: Velocity Vectors in a Top View of the 6 Meter Depth; Reduced Buoyancy Run
Figure D-10: Isoconcentration Contours in a Top View of the Surface; Reduced Buoyancy Run
Figure D-11: End View of Isoconcentration Contours and Velocity Vectors; x = 18 Meters; Reduced Buoyancy
Figure D-12: End View of Isoconcentration Contours and Velocity Vectors; x = 200 meters; Reduced Buoyancy
Figure D-13: Side View of Isoconcentration Contours and Velocity Vectors; Increased Surface Height
Run 3

Figure D-14: Velocity Vectors in a Top View of the Surface; Increased Surface Height
Figure D-15: Velocity Vectors in a Top View of the 6 Meter Depth; Increased Surface Height
Figure D-16: Isoconcentration Contours in a Top View of the Surface; Increased Surface Height
Figure D-17: End View of Isoconcentration Contours and Velocity Vectors; x = 18 Meters; Increased Surface Height
Figure D-18: End View of Isoconcentration Contours and Velocity Vectors; 
x = 200 Meters; Increased Surface Height
Figure D-19: Side View of Isoconcentration Contours and Velocity Vectors; Reduced Mixing
Figure D-20: Velocity Vectors in a Top View of the Surface; Reduced Mixing
Figure D-21: Velocity Vectors in a Top View of the 6 Meter Depth; Reduced Mixing

Run 4
Figure D-22: Isoconcentration Contours in a Top View of the Surface; Reduced Mixing
Figure D-23: End View of Isoconcentration Contours and Velocity Vectors; 
\( x = 18 \) Meters; Reduced Mixing
Figure D-24: End View of Isoconcentration Contours and Velocity Vectors; 
\( x = 100 \) Meters; Reduced Mixing
Figure D-25: Side View of Isoconcentration Contours and Velocity Vectors; Low River Stage
Figure D-26: Velocity Vectors in a Top View of the Surface; Low River Stage
Figure D-27: Isoconcentration Contours in a Top View of the Surface; Low River Stage
Figure D-28: End View of Isoconcentration Contours and Velocity Vectors; $x = 18$ Meters; Low River Stage
Figure D-29: End View of Isoconcentration Contours and Velocity Vectors; x = 200 Meters; Low River Stage
Figure D-30: Side View of Isoconcentration Contours and Velocity Vectors; Flood Stage
Figure D-31: Velocity Vectors in a Top View of the Surface; Flood Stage
Figure D-32: Velocity Vectors in a Top View of the 6 Meter Depth; Flood Stage
Run 6

Figure D-33: Isoconcentration Contours in a Top View of the Surface; Flood Stage
Figure D-34: End View of Isoconcentration Contours and Velocity Vectors; x = 18 Meters; Flood Stage
Figure D-35: End View of Isoconcentration Contours and Velocity Vectors; x = 200 Meters; Flood Stage
Figure D-36: Side View of Isoconcentration Contours and Velocity Vectors; Low River Stage; Maximum Coastal Current
Figure D-37: Velocity Vectors in a Top View of the Surface; Low River Stage; Maximum Coastal Current
Figure D-38: Velocity Vectors in a Top View of the 6 Meter Depth; Low River Stage and Maximum Coastal Current
Run 7

Figure D-39: Isoconcentration Contours in a Top View of the Surface; Low River Stage and Maximum Coastal Current
Figure D-40: End View of Isoconcentration Contours and Velocity Vectors; x - 18 Meters; Low River Stage and Maximum Coastal Current
Figure D-41: End View of Isoconcentration Contours and Velocity Vectors; x = 200 Meters; Low River Stage and Maximum Coastal Current
Figure D-42: Side View of Isoconcentration Contours and Velocity Vectors; Flood Stage and Maximum Coastal Current
Figure D-43: Velocity Vectors in a Top View of the Surface; Flood Stage and Maximum Coastal Current
Figure D-44: Velocity Vectors in a Top View of the 6 Meter Depth; Flood Stage and Maximum Coastal Current
Figure D-45: Isoconcentration Contours in a Top View of the Surface; Flood Stage and Maximum Coastal Current
Figure D-46: End View of Isoconcentration Contours and Velocity Vectors; 18 Meters; Flood Stage and Maximum Coastal Current
Figure D-47: End View of Isoconcentration Contours and Velocity Vectors; x = 200 Meters; Flood Stage and Maximum Coastal Current
VITA

William Rheuben Waldrop was born in Demopolis, Alabama in July, 1939. He attended public school there until graduation in 1957. He entered Auburn University the following fall and received a degree in Aerospace Engineering in 1961. After serving four years as an officer in the U.S. Air Force in aircraft development he joined the Northrop Space Laboratories in Huntsville, Alabama where he worked in the Saturn/Apollo program and attended graduate school at night. Immediately after receiving his M.S. in Aerospace Engineering in 1968, he accepted a NATO Fellowship at the von Karman Institute of Fluid Dynamics in Brussels, Belgium. After completing a one year program there, he returned to the space program with Lockheed in Huntsville.

He is married to the former Marjorie N. Kirk of Guntersville, Alabama and they have two daughters, Amy and Joan.
EXAMINATION AND THESIS REPORT

Candidate: William R. Waldrop

Major Field: Chemical Engineering

Title of Thesis: Three-Dimensional Flow and Sediment Transport at River Mouths

Approved:

[Signatures]

Major Professor and Chairman

Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:

December 8, 1972