1972

The Simulation of Large Industrial Centrifugal Compressors.

Frank Tate Davis
Louisiana State University and Agricultural & Mechanical College

Follow this and additional works at: https://digitalcommons.lsu.edu/gradschool_disstheses

Recommended Citation
Davis, Frank Tate, "The Simulation of Large Industrial Centrifugal Compressors." (1972). LSU Historical Dissertations and Theses. 2334.
https://digitalcommons.lsu.edu/gradschool_disstheses/2334

This Dissertation is brought to you for free and open access by the Graduate School at LSU Digital Commons. It has been accepted for inclusion in LSU Historical Dissertations and Theses by an authorized administrator of LSU Digital Commons. For more information, please contact gradetd@lsu.edu.
INFORMATION TO USERS

This dissertation was produced from a microfilm copy of the original document. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the original submitted.

The following explanation of techniques is provided to help you understand markings or patterns which may appear on this reproduction.

1. The sign or "target" for pages apparently lacking from the document photographed is "Missing Page(s)". If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting thru an image and duplicating adjacent pages to insure you complete continuity.

2. When an image on the film is obliterated with a large round black mark, it is an indication that the photographer suspected that the copy may have moved during exposure and thus cause a blurred image. You will find a good image of the page in the adjacent frame.

3. When a map, drawing or chart, etc., was part of the material being photographed the photographer followed a definite method in "sectioning" the material. It is customary to begin photoing at the upper left hand corner of a large sheet and to continue photoing from left to right in equal sections with a small overlap. If necessary, sectioning is continued again — beginning below the first row and continuing on until complete.

4. The majority of users indicate that the textual content is of greatest value, however, a somewhat higher quality reproduction could be made from "photographs" if essential to the understanding of the dissertation. Silver prints of "photographs" may be ordered at additional charge by writing the Order Department, giving the catalog number, title, author and specific pages you wish reproduced.

University Microfilms

300 North Zeeb Road
Ann Arbor, Michigan 48106
A Xerox Education Company
DAVIS, Frank Tate, 1942-.
THE SIMULATION OF LARGE INDUSTRIAL CENTRIFUGAL COMPRESSORS.
The Louisiana State University and Agricultural and Mechanical College, Ph.D., 1972
Engineering, chemical

University Microfilms, A XEROX Company, Ann Arbor, Michigan

© 1973
FRANK TATE DAVIS

ALL RIGHTS RESERVED
THE SIMULATION OF

LARGE INDUSTRIAL CENTRIFUGAL COMPRESSORS

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Chemical Engineering

by
Frank Tate Davis
B.S., Auburn University, 1968
M.S., Auburn University, 1969
December 1972
PLEASE NOTE:

Some pages may have
indistinct print.
Filmed as received.

University Microfilms, A Xerox Education Company
This work is dedicated to
the memory of my late father,

Sam Tate Davis, Jr.
ACKNOWLEDGEMENT

This research was conducted under the guidance of Dr. A. B. Corripio, Assistant Professor of the Department of Chemical Engineering. The author is highly appreciative of the assistance given by Dr. Corripio during this study.

The author wishes to express his appreciation for the support of this work by Project THEMIS, Contract Number F-44620-68-C-0021, administered for the U.S. Department of Defense by the U.S. Air Force Office of Scientific Research. Without this support the author's research would have been financially impossible.

The author also wishes to thank the General Electric Company, in particular Mr. W. I. Rowen, Supervisor, Control Systems Analysis, for the time and effort spent in answering questions pertaining to this research and for providing copies of several pertinent research papers. Further thanks are extended to Mr. W. C. Cutter, Sales Engineer for General Electric, for making available the papers contained within the General Electric Gas Turbine Library and for providing design data of a steam turbine centrifugal compressor installation.

Thanks are extended also to the Worthington Corporation, Mr. L. M. Anderson, Control Engineer, for providing design parameters of several steam turbine installations.

Clark Brothers, a Division of Dresser Industries, Inc. provided, through Mr. J. R. Lewis, Local Sales Engineer, the design parameters
of several centrifugal compressor installations for the author's study. Mr. Lewis spent much time answering questions dealing with the compressors, via letters and telephone conversations, and for this the author is grateful.

Cooper-Bessemer Company's local Sales Engineer, Mr. Frank P. Sims, made available design characteristics of several centrifugal compressors and graciously answered numerous questions on the subject material.

Particular thanks are extended to Mr. R. E. Ruckstuhl of Enjay Chemical Company for making available the design parameters of a centrifugal compressor and steam turbine installation.

Thanks are also due to the following people who provided technical information and encouragement in the author's research: Woodward Governor Company, Mr. George E. Parker; Crawford and Russell Incorporated, Mr. F. B. Horowitz; Shell Development Company, Dr. S. A. Shain; and the Foxboro Company, Mr. T. L. Magliozzi.

Too, the author wishes to express appreciation to the computer operators of the Computer Research Center of Louisiana State University and especially to the workers who operated the Calcomp Plotter for their patience in producing plots repeatedly in order to obtain exacting results. Deep appreciation is extended to Mr. James A. Paulsen, Associate, Chemical Engineering Department, for his continued efforts in assisting the author in writing and debugging the simulation programs.

The author wishes to thank the Dr. Charles E. Coates Memorial Fund, donated by George H. Coates, for financial assistance in the preparation of this manuscript.

Finally, a special thanks is expressed to the author's wife, Mona, for her unending encouragement and for her assistance in the typing of this manuscript.
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>ACKNOWLEDGMENT</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF TABLES</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>viii</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>xiv</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHAPTER I</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>Literature Cited</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHAPTER II</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>THE CONSTANT SPEED IDEAL GAS MODEL (CSIGM)</td>
<td>8</td>
</tr>
<tr>
<td>Introduction</td>
<td>8</td>
</tr>
<tr>
<td>The Physical System</td>
<td>9</td>
</tr>
<tr>
<td>Mathematical Model</td>
<td>11</td>
</tr>
<tr>
<td>Suction System</td>
<td>11</td>
</tr>
<tr>
<td>Discharge System</td>
<td>18</td>
</tr>
<tr>
<td>Compressor</td>
<td>20</td>
</tr>
<tr>
<td>Integration Method</td>
<td>32</td>
</tr>
<tr>
<td>Open-Loop Responses</td>
<td>33</td>
</tr>
<tr>
<td>Closing the Loop</td>
<td>42</td>
</tr>
<tr>
<td>Dynamics of the Control Valve and Transmission Line</td>
<td>43</td>
</tr>
<tr>
<td>The Controller</td>
<td>46</td>
</tr>
<tr>
<td>Closed Loop Responses</td>
<td>52</td>
</tr>
<tr>
<td>Summary</td>
<td>63</td>
</tr>
<tr>
<td>Nomenclature</td>
<td>65</td>
</tr>
<tr>
<td>Literature Cited</td>
<td>68</td>
</tr>
<tr>
<td>III</td>
<td>THE CONSTANT SPEED REAL GAS MODEL (CSRGM)</td>
</tr>
<tr>
<td>-----</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>Introduction</td>
</tr>
<tr>
<td></td>
<td>Mathematical Model</td>
</tr>
<tr>
<td></td>
<td>Suction System</td>
</tr>
<tr>
<td></td>
<td>Discharge System</td>
</tr>
<tr>
<td></td>
<td>Compressor</td>
</tr>
<tr>
<td></td>
<td>Open-Loop Responses</td>
</tr>
<tr>
<td></td>
<td>Closed-Loop Results</td>
</tr>
<tr>
<td></td>
<td>Summary</td>
</tr>
<tr>
<td></td>
<td>Nomenclature</td>
</tr>
<tr>
<td></td>
<td>Literature Cited</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IV</th>
<th>THE VARIABLE SPEED MODELS</th>
<th>119</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Introduction</td>
<td>119</td>
</tr>
<tr>
<td></td>
<td>The Plant</td>
<td>122</td>
</tr>
<tr>
<td></td>
<td>Correlating the Variable Speed Performance Map</td>
<td>124</td>
</tr>
<tr>
<td></td>
<td>Converting the Constant Speed Models to Variable Speed Models</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>The Steam Turbine Model</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>The Open Loop Responses</td>
<td>158</td>
</tr>
<tr>
<td></td>
<td>Summary</td>
<td>180</td>
</tr>
<tr>
<td></td>
<td>Nomenclature</td>
<td>182</td>
</tr>
<tr>
<td></td>
<td>Literature Cited</td>
<td>184</td>
</tr>
</tbody>
</table>

| V   | CONCLUSIONS AND RECOMMENDATIONS           | 186  |

<table>
<thead>
<tr>
<th>APPENDIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>II-1</td>
<td>CSIGM Dynamic Equations</td>
<td>28</td>
</tr>
<tr>
<td>II-2</td>
<td>Open-Loop Results for Inlet Pressure Changes (CSIGM)</td>
<td>39</td>
</tr>
<tr>
<td>II-3</td>
<td>Open-Loop Results for Inlet Temperature Changes (CSIGM)</td>
<td>39</td>
</tr>
<tr>
<td>II-4</td>
<td>Open-Loop Results for Molecular Weight Changes (CSIGM)</td>
<td>39</td>
</tr>
<tr>
<td>II-5</td>
<td>Open-Loop Results for Load Demand Changes (CSIGM)</td>
<td>43</td>
</tr>
<tr>
<td>II-6</td>
<td>Process Reaction Curve Results (CSIGM)</td>
<td>48</td>
</tr>
<tr>
<td>II-7</td>
<td>Optimization Results (CSIGM)</td>
<td>62</td>
</tr>
<tr>
<td>III-1</td>
<td>CSRGM Dynamic Equations</td>
<td>82</td>
</tr>
<tr>
<td>III-2</td>
<td>Open-Loop Comparison of CSRGM with CSIGM</td>
<td>94</td>
</tr>
<tr>
<td>III-3</td>
<td>Process Reaction Curve Results</td>
<td>99</td>
</tr>
<tr>
<td>III-4</td>
<td>Closed Loop CSRGM Optimization Results</td>
<td>112</td>
</tr>
<tr>
<td>IV-1</td>
<td>Compressor Characteristics and Correlation Results</td>
<td>132</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>II-1</td>
<td>The Constant Speed Plant</td>
<td>10</td>
</tr>
<tr>
<td>II-2</td>
<td>The Lumped Parameter Model</td>
<td>12</td>
</tr>
<tr>
<td>II-3</td>
<td>Characteristics of Butterfly Valves</td>
<td>14</td>
</tr>
<tr>
<td>II-4</td>
<td>The Compressor Performance Curve</td>
<td>22</td>
</tr>
<tr>
<td>II-5</td>
<td>Information Flow Diagram for Open Loop CSIGM</td>
<td>27</td>
</tr>
<tr>
<td>II-6</td>
<td>Open Loop Time Response of Discharge Pressure to Step Changes in Inlet Pressure</td>
<td>34</td>
</tr>
<tr>
<td>II-7</td>
<td>Open Loop Time Response of Discharge Pressure to Step Changes in Inlet Temperature for Both the Simplified and the Ideal Suction System Energy Balances</td>
<td>35</td>
</tr>
<tr>
<td>II-8</td>
<td>Open Loop Time Response of Discharge Pressure to Step Changes in Molecular Weight</td>
<td>36</td>
</tr>
<tr>
<td>II-9</td>
<td>Open Loop Time Response of Discharge Pressure to Step Changes in the Demand Load Rate</td>
<td>37</td>
</tr>
<tr>
<td>II-10</td>
<td>The Closed Loop System</td>
<td>44</td>
</tr>
<tr>
<td>II-11</td>
<td>Determination of the Process Gain K Based on the CSIGM</td>
<td>49</td>
</tr>
<tr>
<td>II-12</td>
<td>Determination of the Process Time Constant Based on the CSIGM</td>
<td>50</td>
</tr>
<tr>
<td>II-13</td>
<td>Determination of the Process Dead-Time $\theta_d$ Based on the CSIGM</td>
<td>50</td>
</tr>
<tr>
<td>II-14</td>
<td>Closed Loop Response to 5% Decrease in the Demand Load Rate</td>
<td>54</td>
</tr>
<tr>
<td>II-15</td>
<td>Closed Loop Response to 5% Increase in the Demand Load Rate $F_o$</td>
<td>55</td>
</tr>
<tr>
<td>II-16</td>
<td>Closed Loop Response to 5 psi Step Change in Set-Point for the CSIGM</td>
<td>56</td>
</tr>
</tbody>
</table>
Closed Loop Response to -5 psi Step Change in Set-Point for the CSIGM

Closed Loop Response to 5 psi Step Change in Inlet Pressure for the CSIGM

Closed Loop Response to -10°F Step Change in Inlet Temperature for the CSIGM

Closed Loop Response to 10°F Step Change in Inlet Temperature for the CSIGM

Closed Loop Response to 3 lbm/mole Step Change in Molecular Weight for the CSIGM

Information Flow Diagram for Open Loop CSRGM

Open Loop Response of the CSRGM to Step Changes in Inlet Pressure

Open Loop Response of the CSRGM to Step Changes in Inlet Temperature for (1) the Simplified, (2) the Ideal and (3) the Real Suction Energy Balances

Open Loop Response of the CSRGM to Step Changes in Molecular Weight of the Gas

Open Loop Response of the CSRGM to Step Changes in the Load Demand Rate

Determination of the Process Gain Based on the CSRGM

Determination of the Process Time Constant Based on the CSRGM

Determination of the Process Dead Time Based on the CSRGM

Closed Loop Response of the CSRGM to 5% Decrease in Load Demand Rate

Closed Loop Response of the CSRGM to 5% Increase in Load Demand Rate

Closed Loop Response of the CSRGM to 5 psi Step Change in Set-Point

Closed Loop Response of the CSRGM to -5 psi Step Change in Set-Point
<table>
<thead>
<tr>
<th>Page</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>108</td>
<td>Closed Loop Response of the CSRGM to 5 psi Step Change in Inlet Pressure</td>
</tr>
<tr>
<td>109</td>
<td>Closed Loop Response of the CSRGM to -2 psi Step Change in Inlet Pressure</td>
</tr>
<tr>
<td>110</td>
<td>Closed Loop Response of the CSRGM to -10°F Step Change in Inlet Temperature</td>
</tr>
<tr>
<td>111</td>
<td>Closed Loop Response of the CSRGM to 10°F Step Change in Inlet Temperature</td>
</tr>
<tr>
<td>123</td>
<td>The Variable Speed Plant</td>
</tr>
<tr>
<td>126</td>
<td>The Variable Speed Compressor Performance Map Used in the VSIGM and VSRGM</td>
</tr>
<tr>
<td>130</td>
<td>Computer Flow Diagram of the Correlation Procedure</td>
</tr>
<tr>
<td>131</td>
<td>Reduced Performance Curve of Compressor Used in the VSIGM and VSRGM</td>
</tr>
<tr>
<td>133</td>
<td>Reduced Performance Curve of the 1900 Horsepower Centrifugal Compressor</td>
</tr>
<tr>
<td>134</td>
<td>Variable Speed Performance Map of a 1900 Horsepower Centrifugal Compressor</td>
</tr>
<tr>
<td>135</td>
<td>Reduced Performance Curve of the 2200 Horsepower Centrifugal Compressor</td>
</tr>
<tr>
<td>136</td>
<td>Variable Speed Performance Map of a 2200 Horsepower Centrifugal Compressor</td>
</tr>
<tr>
<td>137</td>
<td>Reduced Performance Curve of the 3100 Horsepower Centrifugal Compressor with the Speed Range of 3400-5100 RPM</td>
</tr>
<tr>
<td>138</td>
<td>Variable Speed Performance Map of a 3100 Horsepower Centrifugal Compressor with the Speed Range of 3400-5100 RPM</td>
</tr>
<tr>
<td>139</td>
<td>Reduced Performance Curve of the 3100 Horsepower Centrifugal Compressor with the Speed Range of 8000-9400 RPM</td>
</tr>
<tr>
<td>140</td>
<td>Variable Speed Performance Map of a 3100 Horsepower Centrifugal Compressor with the Speed Range of 8000-9400 RPM</td>
</tr>
<tr>
<td>Section</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>IV-9A</td>
<td>Reduced Performance Curve of the 4300 Horsepower Centrifugal Compressor</td>
</tr>
<tr>
<td>IV-9B</td>
<td>Variable Speed Performance Map of a 4300 Horsepower Centrifugal Compressor</td>
</tr>
<tr>
<td>IV-10A</td>
<td>Reduced Performance Curve of the 5000 Horsepower Centrifugal Compressor</td>
</tr>
<tr>
<td>IV-10B</td>
<td>Variable Speed Performance Map of a 5000 Horsepower Centrifugal Compressor</td>
</tr>
<tr>
<td>IV-11A</td>
<td>Reduced Performance Curve of the 5400 Horsepower Centrifugal Compressor</td>
</tr>
<tr>
<td>IV-11B</td>
<td>Variable Speed Performance Map of a 5400 Horsepower Centrifugal Compressor</td>
</tr>
<tr>
<td>IV-12A</td>
<td>Reduced Performance Curve of the 5800 Horsepower Centrifugal Compressor</td>
</tr>
<tr>
<td>IV-12B</td>
<td>Variable Speed Performance Map of a 5800 Horsepower Centrifugal Compressor</td>
</tr>
<tr>
<td>IV-13</td>
<td>The Steam Turbine</td>
</tr>
<tr>
<td>IV-14</td>
<td>Performance Characteristics of the Steam Turbine</td>
</tr>
<tr>
<td>IV-15</td>
<td>Governor Valve Flow Characteristics</td>
</tr>
<tr>
<td>IV-16</td>
<td>The Steam Turbine Model</td>
</tr>
<tr>
<td>IV-17</td>
<td>VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Increases in the Inlet Pressure</td>
</tr>
<tr>
<td>IV-18</td>
<td>VSRGM and VSIGM Open-Loop Speed Responses for Step Increases in the Inlet Pressure</td>
</tr>
<tr>
<td>IV-19</td>
<td>VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Decreases in the Inlet Pressure</td>
</tr>
<tr>
<td>IV-20</td>
<td>VSRGM and VSIGM Open-Loop Speed Responses for Step Decreases in the Inlet Pressure</td>
</tr>
<tr>
<td>IV-21</td>
<td>VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Increases in the Inlet Temperature</td>
</tr>
<tr>
<td>IV-22</td>
<td>VSRGM and VSIGM Open-Loop Speed Responses for Step Increases in the Inlet Temperature</td>
</tr>
<tr>
<td>IV-23</td>
<td>VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Decreases in the Inlet Temperature</td>
</tr>
<tr>
<td>IV-24</td>
<td>VSRGM and VSIGM Open-Loop Speed Responses for Step Decreases in the Inlet Temperature</td>
</tr>
<tr>
<td>IV-25</td>
<td>VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Increases in Molecular Weight</td>
</tr>
<tr>
<td>IV-26</td>
<td>VSRGM and VSIGM Open-Loop Speed Responses for Step Increases in Molecular Weight</td>
</tr>
<tr>
<td>IV-27</td>
<td>VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Decreases in Molecular Weight</td>
</tr>
<tr>
<td>IV-28</td>
<td>VSRGM and VSIGM Open-Loop Speed Responses for Step Decreases in Molecular Weight</td>
</tr>
<tr>
<td>IV-29</td>
<td>VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Increases in the Load Demand Rate</td>
</tr>
<tr>
<td>IV-30</td>
<td>VSRGM and VSIGM Open-Loop Speed Responses for Step Increases in the Load Demand Rate</td>
</tr>
<tr>
<td>IV-31</td>
<td>VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Decreases in the Load Demand Rate</td>
</tr>
<tr>
<td>IV-32</td>
<td>VSRGM and VSIGM Open-Loop Speed Responses for Step Decreases in the Load Demand Rate</td>
</tr>
<tr>
<td>IV-33</td>
<td>VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Increases in the Governor Output</td>
</tr>
<tr>
<td>IV-34</td>
<td>VSRGM and VSIGM Open-Loop Speed Responses for Step Increases in the Governor Output</td>
</tr>
<tr>
<td>IV-35</td>
<td>VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Decreases in the Governor Output</td>
</tr>
<tr>
<td>IV-36</td>
<td>VSRGM and VSIGM Open-Loop Speed Responses for Step Decreases in the Governor Output</td>
</tr>
<tr>
<td>B-1</td>
<td>Computer Flow Diagram for Newton's Method For Solving Implicit Functions</td>
</tr>
<tr>
<td>Section</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>B-2</td>
<td>Trial and Error Procedure for the Inlet Density</td>
</tr>
<tr>
<td>C-1</td>
<td>CSIGM Process Reaction Curve for 1.4 psi Step Change in Controller Output</td>
</tr>
<tr>
<td>C-2</td>
<td>CSIGM Process Reaction Curve for 1.0 psi Step Change in Controller Output</td>
</tr>
<tr>
<td>C-3</td>
<td>CSIGM Process Reaction Curve for 0.5 psi Step Change in Controller Output</td>
</tr>
<tr>
<td>C-4</td>
<td>CSIGM Process Reaction Curve for 0.25 psi Step Change in Controller Output</td>
</tr>
<tr>
<td>C-5</td>
<td>CSIGM Process Reaction Curve for -0.25 psi Step Change in Controller Output</td>
</tr>
<tr>
<td>C-6</td>
<td>CSIGM Process Reaction Curve for -0.5 psi Step Change in Controller Output</td>
</tr>
<tr>
<td>C-7</td>
<td>CSIGM Process Reaction Curve for -1.0 psi Step Change in Controller Output</td>
</tr>
<tr>
<td>C-8</td>
<td>CSIGM Process Reaction Curve for -1.4 psi Step Change in Controller Output</td>
</tr>
<tr>
<td>C-9</td>
<td>CSRGM Process Reaction Curve for 1.4 psi Step Change in Controller Output</td>
</tr>
<tr>
<td>C-10</td>
<td>CSRGM Process Reaction Curve for 1.0 psi Step Change in Controller Output</td>
</tr>
<tr>
<td>C-11</td>
<td>CSRGM Process Reaction Curve for 0.5 psi Step Change in Controller Output</td>
</tr>
<tr>
<td>C-12</td>
<td>CSRGM Process Reaction Curve for 0.25 psi Step Change in Controller Output</td>
</tr>
<tr>
<td>C-13</td>
<td>CSRGM Process Reaction Curve for -0.25 psi Step Change in Controller Output</td>
</tr>
<tr>
<td>C-14</td>
<td>CSRGM Process Reaction Curve for -0.5 psi Step Change in Controller Output</td>
</tr>
<tr>
<td>C-15</td>
<td>CSRGM Process Reaction Curve for -1.0 psi Step Change in Controller Output</td>
</tr>
<tr>
<td>C-16</td>
<td>CSRGM Process Reaction Curve for -1.4 psi Step Change in Controller Output</td>
</tr>
</tbody>
</table>
ABSTRACT

This research effort presents the detailed development of dynamic models of the constant and variable speed centrifugal compressors. In conjunction with the variable speed model, a model of the steam turbine is developed. Each model is of the lumped parameter type and the compressor models are capable of handling any real gas or mixture of real gases for which an equation of state is known. Each model was developed so that no data other than that ordinarily supplied to the plant personnel by the compressor and turbine manufacturers is required. Use was made of the performance characteristics of a compressor and turbine which are in operation today. The resulting models were implemented on an IBM 360/65 digital computer utilizing CSMP and on an XDS 15 digital computer utilizing SL1.

The initial compressor model is based on the ideal gas equation of state and is called the constant speed ideal gas model (CSIGM). Open loop responses of the discharge pressure are given for step changes in the inlet temperature and pressure, the molecular weight of the gas and the mass rate demanded by the downstream unit. These responses are given in terms of continuous curves. The discharge pressure control loop was closed with a pneumatic controller. The dynamics of the control valve were included. The controller was tuned based on a first order lag with dead time model and based on the CSIGM. Closed loop results are given for each controller
indicating the discharge pressure responses to the above disturbances as well as changes in the discharge pressure set point.

The CSIGM was then extended so as to account for the nonidealities of the gas through the incorporation of a modified Benedict-Webb-Rubin equation of state into the model. Open and closed loop simulation results paralleling those of the CSIGM are given. The responses of the real gas model are compared to those of the CSIGM and the additional computer time and storage requirements along with the increased difficulty of programming the real gas model are pointed out.

Finally, modifications are presented which convert the constant speed models to variable speed models. The modifications center around a unique correlation of the compressor performance map in which the several head curves are made to collapse into one curve which can be adequately represented by a simple parabolic equation. The correlation is tested for nine compressors ranging in size from 1900 to 8600 brake horsepower and in speed from 3000 to 10000 rpm. Open loop responses of the discharge pressure of the compressor and the speed of the steam turbine are given for the ideal and the real gas variable speed models. The responses are for step changes in the above mentioned compressor variables in addition to step changes in the steam turbine governor output. A comparison is made of the responses produced by each model.
CHAPTER I
INTRODUCTION

The number of applications of digital computers in process control has increased in the past few years. Concurrently, the theory of advanced control concepts such as feedforward, adaptive, predictive, multivariable and optimal control have been developed and/or refined. Even though the digital computer is ideally suited for implementation of control strategies based on these concepts, which is the major advantage the digital controller has over the analog controller, the digital computer has been used in most cases merely as a substitute for the conventional analog controller. The major reason is that control strategies based on the advanced control concepts cannot be readily developed without a mathematical model of the process.

At the inception of chemical engineering it was postulated that most processes are made up of a number of operations that are similar enough to be studied by themselves, independent of the type of process of which they are a part. These were called "unit operations" and the graduates trained in their design and study were called "chemical engineers." Since each of the unit operations in a process is controlled independently of the others it appears that it would be prudent on the part of chemical engineers to follow the path of the founding fathers and develop mathematical models of each unit operation. With the mathematical models the capabilities and
flexibilities of the digital computer could then be realized through the application of the advanced concepts.

Scarcely a plant in the chemical process industries lacks at least an air compressor for supply of plant air. In many plants reciprocating, centrifugal and axial compressors serve as process equipment to help bring about reactions and phase separations, or to provide refrigeration. Many plants use them in materials handling, in liquefying or storing gases, in agitation, in lifting fluids from wells or moving them in pipes. Most plant shops use them to power air tools and, in processes with explosion hazards, they often drive air motors on agitators and other mechanical equipment.

Utilization of centrifugal compressors has become more prevalent in industry today because plant capacities have increased and because installation costs for centrifugals are lower and operating costs are less. Such units are being used in many applications. Typical of some of these are: air for blast furnaces, Bessemer converters and wind tunnels. Within the process industries they have found use in chemical processes such as nitric and sulfuric acid, synthesis of ammonia and methanol, and in refrigeration cycles handling ammonia, Freon-11 and hydrocarbon gases. Refineries have found many uses for centrifugal compressors in vapor recovery, catalytic reforming, catalytic cracking and ethylene and butadiene plants. In some processes, particularly the high-pressure industries such as ammonia and polyethylene, the compressor accounts for a sizeable part of the plant's capital cost.

Because the centrifugal compressor is a dynamic machine, its performance characteristics are such that unless adequate and proper
control is provided, through instrumentation, the compressor loop
may become unstable, causing costly surging and upsets in the com-
pressor and process. In order to more readily develop the ade-
quate and proper control strategies to prevent these upsets, the
control engineer requires a dynamic model of the centrifugal
compressor, one that might be easily implemented on either an
analog, digital, or hybrid computer. For without the computer
model, it would be almost impossible to ascertain the validity
of a proposed control scheme because in a dynamic analysis of
the centrifugal compressor too many variables come into play.

An exhaustive literature search has revealed only three
papers which present dynamic models of centrifugal compressors
and each of these appeared in 1971. The paper by Nisenfeld and
co-workers [1] does not present enough detail for a complete analy-
sis of their model but it appears that it is limited in scope.
The paper by Jeffrey [2] presents a detailed development of a dyna-
mic model of a variable speed centrifugal air compressor in the
form of an analog diagram. The appendix giving the development
of the dynamic equations was omitted from the publication. The
major disadvantage of his model is that it depends upon extensive
in-plant testing. The paper by Bergeron and Corripio [3] also
presents a detailed development in the form of an analog diagram of
the dynamic model of a multistage centrifugal compressor used in
a refrigeration system. The major drawback of their model is that
it is valid only for a pure component feed gas as it depends upon
thermodynamic data in the form of a Mollier diagram. Much detail
was omitted because of the restricted nature of the plant data.
Their analog model does present a novel way in which to simulate the surge of the compressor.

These several citations of need have provided the impetus for this research effort which involves the development of dynamic models of the constant speed centrifugal compressor and the variable speed centrifugal compressor and its driver. The compressor models are of the lumped parameter type and are an extension of the model proposed by Bergeron and Corripio in that each is capable of handling any real gas or mixture of real gases for which an equation of state is known. The models were developed in such a manner that the effects of any of the many load variables can be easily investigated on either a digital, analog or hybrid computer. In each case extensive use was made of the design data which would be supplied to the plant personnel by the compressor and turbine manufacturers. Open as well as closed loop simulation results obtained on an IBM-360/65 digital computer through use of the System/360 Continuous System Modeling Program (CSMP) and on a Xerox Data System Sigma-5 digital computer through use of a superset of the Continuous System Simulation Language specified by Simulation Councils, Inc. (SL1) will be presented.

The following chapter presents a detailed development of the dynamic model of a constant speed centrifugal compressor utilizing the ideal gas approximation. This model is designated the constant speed ideal gas model (CSIGM). Open loop simulation results for step changes in inlet pressure and temperature, in molecular weight of the feed gas and in the load demand rate are presented in the form of continuous curves. Chapter II is concluded with closed loop
simulation results for the above mentioned step changes. The loop was closed with a conventional analog controller and a comparison is made between a proportional-integral (PI) and a proportional-integral-derivative (PID) controller optimally tuned based on a linearized model and a PID controller tuned optimally through utilization of the CSIGM.

Chapter III is an extension of Chapter II in that the model is developed utilizing the equation of state of a real gas. This model is designated the constant speed real gas model (CSRGM). Open and closed loop simulation results paralleling those given in Chapter II are presented for the CSRGM. These results are compared to those of the CSIGM.

Chapter IV is a further extension of Chapters II and III in that modifications are introduced which convert the two constant speed models into variable speed models, the CSIGM into the variable speed ideal gas model (VSIGM) and the CSRGM into the variable speed real gas model (VSRGM). The modifications entail a unique correlation of the performance map which is characteristic of variable speed centrifugal compressors. Chapter IV also presents a detailed development of a variable speed driver which in this study is a steam turbine. Extensive use is made of design data supplied by the turbine manufacturer.

Open loop simulation results for the above mentioned disturbances along with step changes in the driver speed are presented for each model in the form of continuous curves. Closed loop simulation results are not presented as it is felt that a major research effort in itself is required to adequately develop them.
In summary, the purpose of this dissertation is to provide mathematical models of the constant and variable speed centrifugal compressors suitable for implementation on either an analog, digital, or hybrid computer. The models were developed so that no data is required other than that ordinarily possessed by plant personnel.

All efforts made to obtain the parameters of a specific operating plant which utilizes a centrifugal compressor failed because the companies considered the parameters proprietary. For the same reason there are no publications providing this information. Even so, the plant upon which the models are based was designed so as to approximate as nearly as possible the real life conditions. It was designed utilizing the performance curves of a centrifugal compressor which is operating today. These curves were obtained from one of the major manufacturers of centrifugal compressors.

This being the case, no comparisons are made between the simulated results and those produced by an operating compressor. It is hoped that the author will be able to provide the comparisons at a later date as he will shortly join the process control staff of a company which utilizes centrifugal compressors extensively. In any event, it is felt that the models will be useful to the industrial community, since with them the process control engineer has the potential to develop and test any control schemes required to provide adequate protection for the compressor while maintaining proper control of the plant.


CHAPTER II

THE CONSTANT SPEED IDEAL GAS MODEL (CSIGM)

Introduction

The development of a useful model is guided by two conflicting considerations. These considerations are accuracy and practicality. It is desired that the model reflect as faithfully as possible the real situation, so that reliable predictions can be made from its use, with a minimum expenditure of time and effort. There are many instances in which the control engineer must sacrifice accuracy so that results can be obtained in a specified time.

In some industrial processes involving gas compression with a centrifugal compressor, conditions are such that the ideal gas approximation is sufficiently accurate to provide reliable results. Even if the gas approaches the liquefaction region during the compression process, the ideal gas approximation may still be warranted, particularly if the control engineer can make the proper inferences from only a rough approximation. It is for these cases that this chapter is intended.

This chapter will develop the physical system upon which all three models are based. It will then present a detailed development of the constant speed centrifugal compressor model utilizing the ideal gas approximation. Open loop simulation results of the time response of discharge pressure to step changes in inlet
temperature and pressure, changes in molecular weight of the gas and changes in the load demand rate will be presented in the form of continuous curves. The chapter will be concluded with closed loop response curves of discharge pressure to similar load changes. A comparison is made between a PI and PID controller optimally tuned through utilization of a first order lag with dead time model and a PID controller optimally tuned through utilization of the nonlinear CSIGM.

The Physical System

In order to develop the mathematical model of a centrifugal compressor, the performance characteristics of the compressor and of the connected systems, upstream and downstream, must be established. It was for this reason that the relatively simple system shown in Figure II-1 was chosen for study for if the inlet and outlet systems are overly complicated the results of the simulation may hinge upon their simulation rather than the simulation of the compressor. However, this type of arrangement is one which can be found in large plant air systems, fluid catalytic cracking units and possibly in refrigeration systems [1-8].

The heart of the system is a one stage constant speed centrifugal compressor with bypass for anti-surge control and suction throttling for discharge pressure control. The compressor performance curve upon which the model is based was obtained from one of the major manufacturers of centrifugal compressors. It represents the constant speed performance characteristics of a compressor which is in operation today.
Figure II-1. The Constant Speed Plant.
The throttle valve is a butterfly valve which was sized according to the procedure given by Moore [9]. The surge control valve was sized to deliver surge flow plus twenty percent when eighty percent open. It is an equal percentage valve with a rangeability of 100:1. All piping was designed so that the pressure drop would fall in the range of 0.8 to 1.5 psi/100 feet of pipe. It was assumed that the downstream temperature of the cooler would always equal the compressor inlet design temperature. The complete steady state design is detailed in Appendix A.

Mathematical Model

The mathematical model of the compressor is based on the Helmholtz theory of resonators which was first applied to the centrifugal compressor by Emmons and co-workers [10] in their successful attempt to model surge. In order to use this theory, the suction piping between the throttle and surge control valves was lumped into a suction volume $V_s$ and the discharge piping between the compressor outlet and the first downstream restriction including the volume up to the surge control valve was lumped into a discharge volume $V_d$. This is indicated by the dashed lines in Figure II-2. The following derivations were obtained based on the lumped parameter description indicated in Figure II-2.

Suction System

The suction system is made up of the throttle control valve and the suction volume. Unsteady state energy and mass balances and the standard valve equation, which is derived from the orifice
Figure II-2. The Lumped Parameter Model.
equation [11], are required to model it. A steady state energy balance could have been used since the accumulation of energy in the suction systems of most industrial compressors is negligible. However, this yields a nonlinear algebraic loop implicit in the suction temperature which requires an excess of computer time to solve.

An unsteady state mass balance about the suction volume yields

\[ \frac{d}{dt} (\rho_s V_s) = F_i + F_{e} - \rho_s Q_c. \]  

II-1

Since the suction volume \( V_s \) is a constant, equation II-1 may be written

\[ \rho_s = \frac{(F_i + F_e - \rho_s Q_c)}{V_s}. \]  

II-2

Application of the valve equation to the throttle valve yields the following expression for the inlet mass rate, \( F_i \), in terms of the gas molecular weight, the inlet temperature and pressure and the temperature and pressure within the suction volume:

\[ F_i = 1.05 \overline{C_{vi}} C_{vi} \sqrt{\rho_{vi} \Delta P_{vi}}. \]  

II-3

The pressure drop across the valve, \( \Delta P_{vi} \), is the difference between the inlet pressure and the suction pressure. The density is taken as the average of the up- and downstream densities. \( \overline{C_{vi}} \) is the design valve coefficient and is calculated in Appendix A. \( C_{vi} \) is the function which relates the fraction of design \( C_v \) to the vane angle of the valve in degrees. This function, which is shown in Figure II-3, was extracted from the work of Chinn [12] in the form of data points and was implemented into the computer model in the same form through an arbitrary function generator which
Figure II-3. Characteristics of Butterfly Valves.
utilizes linear interpolation. Thus the inlet mass rate is calculated according to

$$F_i = 1.05 x 9541 x C_{v1} \sqrt{0.5(\rho_s + \frac{P_1 M}{(RT_1)})}(P_1-P_s).$$  

Neglecting any potential or kinetic energy changes, an unsteady state energy balance taken about the suction volume may be written

$$\frac{d}{dt}(\rho_s V_s E/M_s) = F_i h_1/M_i + F_e h_e/M_e - \rho_s Q_{ch_s}/M_s$$  

where $E$ is the molar internal energy, $h_1$, $h_e$ and $h_s$ are respectively the molar enthalpies of the inlet gas, the bypass loop gas and the gas exiting the suction volume. Utilizing the four term molar heat capacity at constant pressure relationship given by Thinh, et al [13], the internal energy and the enthalpies may be evaluated according to

$$E = \int_0^{T_s} C_{v1}dT = \int_0^{T_s} (C_p-r)dT = (a^*-r)T_s + (b^*/2)T_s^2$$  

$$+ (c^*/3)T_s^3 + (d^*/4)T_s^4$$  

$$h_1 = \int_0^{T_1} C_{p1}dT = a^*T_1 + (b^*/2)T_1^2 + (c^*/3)T_1^3$$  

$$+ (d^*/4)T_1^4$$  

$$h_e = \int_0^{T_e} C_{pe}dT = a^*T_e + (b^*/2)T_e^2 + (c^*/3)T_e^3$$  

$$+ (d^*/4)T_e^4$$  

and

$$h_s = \int_0^{T_s} C_{ps}dT = a^*T_s + (b^*/2)T_s^2 + (c^*/3)T_s^3$$  

$$+ (d^*/4)T_s^4.$$
For all cases to be considered in this study, the molecular weight of the gas will be maintained constant at its initially specified value. Consequently the M's on both sides of equation II-5 may be canceled since they are all equal and independent of time. Having done this the accumulation term may be written

\[
\frac{d}{dt}(\rho_s V_s E) = \rho_s V_s \dot{E} + V_s (h_s - rT_s) \dot{\rho}_s
\]

II-10

where use has been made of the fact that for an ideal gas, the molar internal energy is the difference between the molar enthalpy and the product of the gas constant and the absolute temperature.

Utilizing the chain rule for differentiation, the first term on the right hand side of equation II-10 may be expanded to give

\[
\rho_s V_s \dot{E} = \rho_s V_s [a^* - r + b^*T_s + c^*T_s^2 + d^*T_s^3] \dot{T}_s
\]

II-11

Substitution of this equation along with equation II-2 into equation II-10 with subsequent substitution of the results into equation II-5 yields after collecting terms

\[
\dot{T}_s = \frac{F_i(h_i - h_s + rT_s) + F_e(h_e - h_s + rT_e) - \rho_s Q_c rT_s}{\rho_s V_s [a^*-r + b^*T_s + c^*T_s^2 + d^*T_s^3]}
\]

II-12a

where the enthalpies are given by equations II-7 through II-9.

An analysis of equation II-12a indicates that for the cases considered in this study some simplifications may be in order. Utilizing equations II-7 and II-9, the difference in enthalpy of the inlet gas and the gas within the suction volume may be written as

\[
h_i - h_s = (T_i - T_s)[a^* + (b^*/2)(T_i+T_s) + (c^*/3)(T_i^2 + T_s^2 + T_iT_s)
+ (d^*/4)(T_i + T_s)(T_i^2 + T_s^2)].
\]

Because heat transfer through the walls of the suction piping has
been neglected and because $F_e$, throughout this study, will always
be maintained at approximately zero, the sum of $T_i$ and $T_s$ and $T_i^2$ and
$T_s^2$ may be approximated as just twice the suction temperature and twice
the suction temperature squared respectively. This implies that the
above equation may be written as

$$h_i - h_s = (T_i - T_s)[a^* + b^* T_s + c^* T_s^2 + d^* T_s^3]$$

where the terms in brackets are approximately equal to the bracketed
terms in the denominator of equation II-12a. Similarly, equations
II-8 and II-9 may be used to obtain

$$h_e - h_s = (T_e - T_s)[a^* + b^* T_s + c^* T_s^2 + d^* T_s^3].$$

The remaining terms in the numerator of equation II-12a may be
canceled because during the transient periods the sum of the inlet
and bypass flow rates is essentially equal to the flow of gas enter­
ing the compressor (they are exactly equal at steady state). Util­
izing these simplifications, the energy balance for the suction
system may be written as

$$\dot{T}_s = \frac{[F_i (T_i - T_s) + F_e (T_e - T_s)]/\rho_s v}{s}$$

II-12b

which, for future reference, will be termed the simplified suction
energy balance while equation II-12a is termed the ideal suction
energy balance. Equation II-12b can also be obtained from equation
II-15 if the specific heats are assumed equal and constant.

In an effort to prove that the results of these two equations
are essentially equal, the open loop results in terms of discharge
pressure versus time for step changes in the inlet temperature of
$\pm$ 30 and $\pm$ 50 degrees Fahrenheit were obtained utilizing each of
them. These results are shown in Figure II-7. The simplifications
are so good that the two sets of four curves appear to fall exactly on top of each other. This would not be the case if the surge control valve were opened. Because of this, equation II-12a has been included in the summation of dynamic equations given in Table II-1 even though all of the results for the CSIGM were obtained using the simplified suction energy balance.

**Discharge System**

The discharge system consists of the discharge volume $V_d$, the surge control valve and the cooler. To model it mathematically requires an unsteady state mass balance, the standard valve equation and an expression to evaluate the pressure drop across the cooler. An unsteady state energy balance is not required for two reasons. First, in most industrial compressor discharge systems the heat losses are negligible. Second, in modeling the compressor an expression for calculating the compressor discharge temperature is developed which, based on the first reason, is taken as the temperature within the discharge volume.

Application of the unsteady state mass balance about the discharge volume gives

$$\dot{\rho}_d = \frac{(\rho s Q_c - P_e - P_o) / V_d.}{\text{II-13}}$$

The outlet mass rate $F_o$ represents the load demand placed on the compressor system by the downstream unit. It was given the steady state design value in all cases except those in which it was perturbed.

In a manner similar to that utilized to obtain the inlet mass rate, the valve equation can be used to obtain an expression for the bypass mass rate, $F_e$, in terms of the downstream cooler
temperature and pressure, the suction temperature and pressure and the molecular weight of the gas. However, for this valve both critical or "choked" and subcritical flow were accounted for in the computer model.

In the case of choked flow for which \( P_s \leq 0.53 P_e \) [14] the valve equation yields

\[
F_e = C_f C_{ve} C_{ve} P_e \sqrt{M/T_e}/5.1
\]

\( C_f \) is a critical flow factor specified by the valve manufacturer as 0.9 and \( C_{ve} \) is the design \( C_v \) given in Appendix A. One of the manufacturers of centrifugal compressors suggested utilizing the following expression for the pressure drop across the cooler:

\[
\Delta P_c = \psi_{F_e}^2
\]

from which \( P_e \) is calculated by the following expression:

\[
P_e = P_d - \frac{\psi_{F_e}^2}{F_e}
\]

\( C_{ve} \) is the fraction of flow through the valve which may be calculated according to the following expression which holds for an equal percentage valve with rangeability of 1:1

\[
\text{fraction flow} = \frac{1}{a} \exp \left( \text{stem position} \times \ln a \right)
\]

where stem position is the ratio of the actual stem position to the stem position when the valve is wide open. Thus the bypass mass rate is calculated for critical flow according to

\[
F_e = 0.9 \times 9.74 \exp \left( 4.6 \ Z_e \right) \left( P_d - \psi_{F_e}^2 \right) \sqrt{M/T_e}/5.1
\]
since the rangeability was taken as 100:1.

In the case of subcritical flow an expression similar to equation II-3 is applicable. For this case the following is utilized:

\[ F_e = 1.05 \times 9.74 \exp (4.6 Z_e) \]

\[ \sqrt{0.5 (\rho_s + (P_d - \psi F_e)^2 M/(RT_e) (P_d - \psi F_e - P_s)} \]

**Compressor**

The equation necessary to model the compressor results from a steady state energy balance. Since it may not be clear why a steady state energy balance is used rather than an unsteady state energy balance, a complete derivation will be given indicating the assumptions implicit in the steady state balance.

An unsteady state energy balance about the compressor for one pound of gas yields

\[ \frac{d}{dt} \left( \int_{s_c} u dx \right) = H - 778 \Delta h/M \]

where the indicated integration is carried out over the length of the compressor channel. If it is assumed that the mass flow rate is constant throughout the compressor channel then equation II-20 may be written

\[ \frac{K}{s_c} \frac{dF_c}{dt} = H - 778 \Delta h/M \]

where \( K \) is designated the "channel constant" and is evaluated
according to
\[ K = \int_{c}^{L} \frac{dx}{\rho_c a_c} \]  

The left hand side of equation II-21 represents the energy required to accelerate the gas which is intuitively negligible. Indeed, the value calculated for the 'channel constant' is so small for most compressors that the whole term on the left may be assumed zero for all time. This is equivalent to saying that the work performed on the gas, the enthalpy change which the gas undergoes, is all converted to head since the difference between the kinetic energy at the entrance and that at the exit is in most cases negligible. Therefore the compressor is represented by

\[ H = 778 \Delta h/M \]  

which implies that the entire process is comprised of unsteady state balances coupled through the above steady state balance.

As implied above, the left hand side of equation II-23 is the head developed by the compressor. This represents the energy imparted to the gas by the impellers of the compressor. It is generally accepted that \( H \) is only a function of the diameter of the impeller, its angular velocity and the volumetric flow rate through the compressor \( Q_c \) [15-17]. For this case the diameter of the impeller and its angular velocity are both constant and \( H \) is a function of only \( Q_c \). The \( H-Q_c \) curve used in the simulation is shown in Figure II-4 along with the points extracted from the design curve supplied by the manufacturer.
Figure II-4. The Compressor Performance Curve.
The continuous curve shown in Figure II-4 was obtained from a linear least squares fit of the indicated data points to an equation of the form

\[ H - H_m = A(Q_c - Q_m)^2 \]

II-24

which is a parabola centered at \((Q_m, H_m)\). Even though the fit does give a conservative estimate of the surge point, the point at which the slope of the \(H-Q_c\) curve is zero, it was considered satisfactory because the first four data points were extrapolated from the manufacturer's curve. The particular constants obtained from the fit yielded a standard error of 1.3 percent or 350 ft-lbf/lbm which was termed good because the resolution of the manufacturer's curve had this order of magnitude.

The term on the right side of equation II-23 is the change in enthalpy which one pound of gas undergoes as it is compressed. Thus, based on this steady state analysis of the compressor, \(Q_c\) is an implicit function of \(\Delta h\) through the expression for the head. Given an equation of state for the gas and a heat capacity relationship, \(\Delta h\), implying \(Q_c\), can be obtained through the following sequence of calculations.

An entropy balance made about the compressor for the isentropic compression of an ideal gas yields

\[ 0 = S_d - S_g = \int_{T_S}^{T_{di}} \frac{C_p(T)dT}{T} - r \int_{P_S}^{P_d} d\ln P. \]

II-25

Application of the equation of state for an ideal gas and the four term ideal heat capacity relation yields the following expression which contains only one unknown, the isentropic discharge temperature \(T_{di}\):
\[ 0 = (a^* - r)\ln T_{di} + T_{di} (b^* + T_{di} (c^*/2 + T_{di} d^*/3)) \]
\[ - (r\ln(\rho_d/\rho_s) + (a^* - r)\ln T_s \]
\[ + T_s (b^* + T_s (c^*/2 + T_s d^*/3))). \]  

Even though this expression is very nonlinear in \( T_{di} \), its solution poses no problems since most of the digital simulation languages are provided with an implicit function solving routine. The routine used in this study is based on Newton's method for finding roots [18] and the procedure used is detailed in Appendix B.

Having obtained \( T_{di} \), the isentropic enthalpy change \( \Delta h_i \) is then calculated using the following expression:

\[ \Delta h_i = \int_{T_s}^{T_{di}} C_p(t) \, dt. \]  

II-27

Application of the ideal heat capacity relation yields

\[ \Delta h_i = T_{di} (a^* + T_{di} (b^*/2 + T_{di} (c^*/3 + T_{di} d^*/4))) \]
\[ - T_s (a^* + T_s (b^*/2 \]
\[ + T_s (c^*/3 + T_s d^*/4))). \]  

II-28

Division of \( \Delta h_i \) by the efficiency of the machine then yields the actual enthalpy change \( \Delta h \) from which \( Q_c \) can be obtained.

It should be noted that the formulation of this model forces a reversal of the roles of \( Q_c \) and \( H \) in that \( H \) is now the independent variable and \( Q_c \) is the dependent variable. As a result there are two values of \( Q_c \) for each value of \( H \), one in the surge region and one in the stable region. This can be readily seen...
if equation II-24 is solved for \( Q_c \) which gives

\[
Q_c = Q_m \pm \sqrt{(H_m - H)/(\pm A)}.
\]

II-29

In order to use this expression the negative or surge root was discarded. Logic was included in the computer program to stop the simulation and write out a message indicating that the surge region had been entered when the value of \( 778\Delta h/M \) had exceeded \( H_m \). \( 778\Delta h/M \) is used since substitution of equation II-23 into equation II-29 yields

\[
Q_c = Q_m + \sqrt{(H_m - 778\Delta h/M)/(\pm A)}.
\]

II-30

The only disadvantage of this method is that once the simulated compressor has entered surge there is no way digitally to force it out of the surge region without stopping the simulation and making parameter adjustments and then starting over again. However, Bergeron and Corripio [19] have shown that this is not the case if the model is simulated on an analog computer. In particular the implicit balance involving \( Q_c \) and \( H \) can be solved utilizing a variable diode function generator and a high gain amplifier. Then when the simulated compressor enters the surge region the high gain amplifier becomes unstable due to positive feedback and the phenomenon of surge is naturally simulated. Thus the compressor can be forced out of the surge region through proper control action, such as opening the surge control valve, without stopping the simulation. In a round about way they solve equation II-21 for \( Q_c \).

As pointed out earlier in this chapter an unsteady state energy balance is not required for the discharge system to obtain
the discharge temperature because $T_d$ is calculated from an expression used in modeling the compressor. The necessary expression which contains only one unknown, $T_d$, is obtained from the thermodynamic equation to calculate $\Delta h$ which is

$$\Delta h = \int_{T_s}^{T_d} C_p(T) dT.$$  

II-31

Substitution of the expression for the heat capacity yields, after integration, the following expression which can be solved for $T_d$ with another implicit function solving routine:

$$0 = T_d (a^* + T_d (b^*/2 + T_d (c^*/3 + T_d d^*/4)))$$

$$- (T_s (a^* + T_s (b^*/2 + T_s (c^*/3 + T_s d^*/4))))$$

$$+ \Delta h).$$  

II-32

The method used to solve this expression is also Newton's and the procedure is also detailed in Appendix B.

To facilitate a more complete analysis of the model, an information flow diagram for the entire system which has been developed to this point is shown in Figure II-5. The accompanying dynamic equations are given in Table II-1.
Figure II-5. Information Flow Diagram for Open Loop CSIGM.
## TABLE II-1

**CSIGM DYNAMIC EQUATIONS**

<table>
<thead>
<tr>
<th>System</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Suction System</strong></td>
<td></td>
</tr>
<tr>
<td>Mass balance</td>
<td>( \dot{\rho}_s = \frac{(F_i + F_e - \rho_s Q_c)}{V_s} ) (1)</td>
</tr>
<tr>
<td>Energy balance</td>
<td>( T_s = \frac{F_i (h_i - h_s + r T_s) + F_e (h_e - h_s + r T_s) - \rho_s Q_c r T_s}{\rho_s V_s [a^* - r + T_s (a^* + T_s (b^* + T_s (c^* + T_s d^*)))]} ) (2)</td>
</tr>
<tr>
<td>Inlet enthalpy</td>
<td>( h_i = T_i (a^* + T_i (b^<em>/2 + T_i (c^</em>/3 + T_i d^*/4))) ) (3)</td>
</tr>
<tr>
<td>Bypass loop enthalpy</td>
<td>( h_e = T_e (a^* + T_e (b^<em>/2 + T_e (c^</em>/3 + T_e d^*/4))) ) (4)</td>
</tr>
<tr>
<td>Suction enthalpy</td>
<td>( h_s = T_s (a^* + T_s (b^<em>/2 + T_s (c^</em>/3 + T_s d^*/4))) ) (5)</td>
</tr>
<tr>
<td>Inlet mass rate</td>
<td>( F_i = 1.05 C_v l \sqrt[3]{\rho_v l \Delta P_v l} ) (6)</td>
</tr>
<tr>
<td>Throttle valve density</td>
<td>( \rho_{v l} = 0.5 (\rho_s + P_{l1} M / (R T_{l1})) ) (7)</td>
</tr>
<tr>
<td>Throttle valve pressure drop</td>
<td>( \Delta P_{v l} = P_{l1} - P_s ) (8)</td>
</tr>
<tr>
<td>Throttle valve ( C_v )</td>
<td>( C_{v l} = \overline{C}<em>{v l} f(Z</em>{l1}) ) (*See below) (9)</td>
</tr>
<tr>
<td>Equation of state</td>
<td>( P_s = \rho_s R T_s / M ) (10)</td>
</tr>
</tbody>
</table>

\(*f(Z_{l1})\) is the function which relates the fraction of design \( C_v \) to the throttle valve vane position. It is shown in Figure II-3.
**Discharge System**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{\rho}_d = (\rho_s Q_c - F_e - F_o)/V_d )</td>
<td>Mass balance</td>
</tr>
<tr>
<td>( F_o = \text{steady state value of } F_i )</td>
<td>Outlet mass rate</td>
</tr>
<tr>
<td>( P_e = P_d - \psi P_e^2 )</td>
<td>Bypass pressure</td>
</tr>
<tr>
<td>Critical flow if ( 0.53 P_e \geq P_s ), otherwise subcritical flow</td>
<td>Bypass mass rate</td>
</tr>
<tr>
<td>( F_e = (0.9/5.1) C_{ve} P_e \sqrt{\rho_{ve}/T_e} )</td>
<td>Critical flow</td>
</tr>
<tr>
<td>( F_e = 1.05 C_{ve} \sqrt{\rho_{ve} \Delta P_{ve}} )</td>
<td>Subcritical flow</td>
</tr>
<tr>
<td>( \rho_{ve} = 0.5 (\rho_s + P_e M/(RT_e)) )</td>
<td>Surge valve density</td>
</tr>
<tr>
<td>( \Delta P_{ve} = P_e - P_s )</td>
<td>Surge valve pressure drop</td>
</tr>
<tr>
<td>( C_{ve} = 0.01 \overline{C}_{ve} \exp (4.6 Z_e) )</td>
<td>Surge valve ( C_v )</td>
</tr>
<tr>
<td>( P_d = \rho_d R T_d / M )</td>
<td>Equation of state</td>
</tr>
</tbody>
</table>

**TABLE II-1 (Cont'd)**

CSIGM DYNAMIC EQUATIONS
TABLE II-1 (Cont'd)
CSIGN DYNAMIC EQUATIONS

Compressor

Isentropic discharge temperature

\[ \Delta S = 0 = (a^* - r) \ln T_{d1} \]
\[ + T_{d1} (b^* + T_{d1}(c^*/2 + T_{d1}d^*/3)) \]
\[ - \{r \ln(p_d/p_g) + r \ln T_s \]
\[ + T_s (b^* + T_s (c^*/2 + T_s d^*/3)) \} \]

Isentropic enthalpy change

\[ \Delta h_i = T_{d1} (a^* + T_{d1}(b^*/2 + \]
\[ T_{d1}(c^*/3 + T_{d1}d^*/4))) \]
\[ - [T_s (a^* + T_s (b^*/2 + \]
\[ T_s(c^*/3 + T_s d^*/4))] \}

Actual enthalpy change

\[ \Delta h = \Delta h_i/\eta \]

Discharge temperature

\[ 0 = T_d(a^* + T_d(b^*/2 + T_d(c^*/3 + \]
\[ T_dd^*/4))) - [T_s (a^* + T_s (b^*/2 + \]
\[ T_s(c^*/3 + T_s d^*/4)) + \Delta h] \]
TABLE II-1 (Cont'd)

CSIGM DYNAMIC EQUATIONS

Energy balance

\[ H = 778 \Delta h/M \]  \hspace{1cm} (24)

Volumetric flow rate

\[ Q_c = \begin{cases} 
Q_m + \frac{\sqrt{(H_m - 778\Delta h/M)}}{(-A)} & \text{surge; } 778\Delta h/M > H_m \\
Q_{\text{max}} & \text{min; } 778\Delta h/M \leq H_{\text{min}} 
\end{cases} \]  \hspace{1cm} (25)

\( H_{\text{min}} \) and \( Q_{\text{max}} \) are respectively the head and flow rate at the "stonewall."
**Integration Method**

In order to carry out the simulation of the CSIGM on the digital computer, an integration method had to be determined. Several choices were available since SL1 is provided with Euler, trapezoidal, a four-point predictor, Adams-Moulton, fourth order Runge-Kutta-Gill and Runge-Kutta with error control [20] while CSMP is provided with rectangular, trapezoidal, Simpson's, Adams-second order, fourth order Runge-Kutta and a Milne fifth-order predictor-corrector [21].

Initially the fourth order Runge-Kutta method with a fixed integration step size of 0.01 seconds was chosen. This step size was the upper end of a range of step sizes which produced comparable open-loop simulation results. However, when the closed-loop simulation, initially at steady state, was executed with no disturbances, all of the variables slowly drifted away from their steady state values.

Because of this slow drift the lower order integration techniques, Euler, rectangular and trapezoidal, were examined. It was found that when using the optimum integration step size, each of these techniques yielded comparable open-loop as well as closed-loop simulation results and required much less computer time. Too, when the closed-loop simulation, initially at steady state, was executed with no disturbances, all of the variables remained at their steady state values. The trapezoidal method with an integration step size of 0.02 seconds was chosen over the other two lower order methods because it is the only integration technique, other than the Runge-Kutta, which CSMP and SL1 have in common.
A common integration technique was desired because initial debugging and testing was carried out on the XDS-Σ5 which is operated open shop. Subsequent runs were then carried out on the IBM-360/65 because facilities were available to produce the plots mechanically. These facilities consisted of a Calcomp plotter and the necessary tape units.

Open-Loop Responses

The open-loop responses of the process, in terms of the discharge pressure, to step changes in the inlet pressure and temperature, the molecular weight of the gas and the demand load rate are shown in Figures II-6 through II-9 respectively. Figures II-6 and II-7 show the response of the process for the following sequence of events. The system is initially operating at steady state with both controllers in the manual mode, an up-stream disturbance occurs such that either the temperature or the pressure of the gas at the inlet to the throttle valve is suddenly changed by the amount shown. The down-stream system demands the initial steady state mass rate during the time required for the process to reach the new steady state.

Figure II-8 indicates the response of the process for the same sequence of events except that the load disturbance is achieved by instantaneously replacing the gas contained within the entire system with a different gas. This different gas has a molecular weight either smaller or larger than that of the original gas by the amount shown. Even though this disturbance cannot even be approximated in a real system, it has been included so that an
Figure II-6. Open Loop Time Response of Discharge Pressure to Step Changes in Inlet Pressure.
Figure II-7. Open Loop Time Response of Discharge Pressure to Step Changes in Inlet Temperature for Both the Simplified and the Ideal Suction System Energy Balances.
Figure II-8. Open Loop Time Response of Discharge Pressure to Step Changes in Molecular Weight.
Figure II-9. Open Loop Time Response of Discharge Pressure to Step Changes in the Demand Load Rate.
evaluation may be made for a gas differing in composition from the one upon which the steady state design is based.

It should be pointed out that the more realistic disturbance of making step changes in molecular weight in the feed gas can only be achieved by changing n-1 compositions at the inlet to the throttle valve, where n is the number of components in the gas. To simulate this would require a total mole balance along with n-1 component balances for both the suction and discharge systems. This means there would be 2+2(n-1) or 2n additional nonlinear-coupled-first-order-ordinary differential equations in addition to the more coupled and more nonlinear energy balance for the suction system. Since the unrealistic disturbance does provide a qualitative indication of the system response, but with much less complication, the additional 2n differential equations have not been included in the model.

The curves shown in Figure II-9 are the response of the system to the indicated step changes in the demand load rate. They represent the response of the uncontrolled compressor to changes in the mass flow rate demanded by the unit downstream of the compressor system.

In order to obtain the responses shown in Figure II-6, the inlet pressure, which appears only in the throttle valve equation, was changed at time zero for each of the five simulation runs by the amounts shown. The remaining variables were set to their steady state values at time zero and then allowed to come to their new steady state values. Table II-2 gives the new steady state values of the discharge pressure, the head and the volumetric flow rate through the compressor. It also gives the time required for the system to reach 99.5 percent
### TABLE II-2
OPEN-LOOP RESULTS FOR INLET PRESSURE CHANGES (CSIGM)

<table>
<thead>
<tr>
<th>$\Delta P_i$ (psi)</th>
<th>$P_d^{ss}$ (psia)</th>
<th>$t_c$ (sec)</th>
<th>$H^{ss}$ (ft-lbf/lbm)</th>
<th>$Q_c^{ss}$ (acfs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>293.75</td>
<td>5.5</td>
<td>21662.</td>
<td>246.86</td>
</tr>
<tr>
<td>10</td>
<td>272.44</td>
<td>7.2</td>
<td>21342.</td>
<td>261.61</td>
</tr>
<tr>
<td>5</td>
<td>247.97</td>
<td>9.0</td>
<td>20736.</td>
<td>278.3</td>
</tr>
<tr>
<td>-4</td>
<td>194.91</td>
<td>12.9</td>
<td>18506.</td>
<td>314.74</td>
</tr>
<tr>
<td>-8</td>
<td>167.46</td>
<td>14.5</td>
<td>16783.</td>
<td>334.43</td>
</tr>
</tbody>
</table>

### TABLE II-3
OPEN-LOOP RESULTS FOR INLET TEMPERATURE CHANGES (CSIGM)

<table>
<thead>
<tr>
<th>$\Delta T_i$ (°F)</th>
<th>$P_d^{ss}$ (psia)</th>
<th>$t_c$ (sec)</th>
<th>$H^{ss}$ (ft-lbf/lbm)</th>
<th>$Q_c^{ss}$ (acfs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>181.31</td>
<td>14.1</td>
<td>17542.</td>
<td>326.26</td>
</tr>
<tr>
<td>30</td>
<td>196.22</td>
<td>13.8</td>
<td>18513.</td>
<td>314.64</td>
</tr>
<tr>
<td>-30</td>
<td>245.38</td>
<td>11.6</td>
<td>20649.</td>
<td>280.22</td>
</tr>
<tr>
<td>-50</td>
<td>263.12</td>
<td>10.7</td>
<td>21111.</td>
<td>268.85</td>
</tr>
</tbody>
</table>

### TABLE II-4
OPEN-LOOP RESULTS FOR MOLECULAR WEIGHT CHANGES (CSIGM)

<table>
<thead>
<tr>
<th>$\Delta M$ (lb/mole)</th>
<th>$P_d^{ss}$ (psia)</th>
<th>$t_c$ (sec)</th>
<th>$H^{ss}$ (ft-lbf/lbm)</th>
<th>$Q_c^{ss}$ (acfs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>251.43</td>
<td>10.4</td>
<td>20749.</td>
<td>278.00</td>
</tr>
<tr>
<td>2</td>
<td>241.00</td>
<td>10.7</td>
<td>20460.</td>
<td>284.17</td>
</tr>
<tr>
<td>1</td>
<td>230.50</td>
<td>11.0</td>
<td>20120.</td>
<td>290.63</td>
</tr>
<tr>
<td>-1</td>
<td>209.45</td>
<td>11.5</td>
<td>19261.</td>
<td>304.46</td>
</tr>
<tr>
<td>-2</td>
<td>199.39</td>
<td>11.7</td>
<td>17723</td>
<td>311.92</td>
</tr>
<tr>
<td>-3</td>
<td>189.17</td>
<td>11.9</td>
<td>15321</td>
<td>319.43</td>
</tr>
</tbody>
</table>
of the change in discharge pressure. This time is proportional to the time constant and is thus characteristic of the process. Note that as ΔP_i varies from 15 to -8 psi the characteristic time varies from 5.5 to 14.5 seconds indicating that the system time constant is strongly dependent upon the inlet pressure. Note also that Q_c increases for the same changes in ΔP_i. This explains the increase in t_c since τ ∝ V/Q_c (α means proportional) implying that Δτ ∝ -ΔQ_c and since t_c ∝ t then Δt_c ∝ ΔQ_c where Δt_c and ΔQ_c are the indicated t_c and Q_c minus the t_c and Q_c at ΔP_i = 0.

In a similar manner, the responses shown in Figure II-7 were obtained. However, in this case, the inlet temperature, which appears in both the energy balance and the throttle valve equations, was changed. Note that for this case the initial change in discharge pressure is in the opposite direction from the final change in discharge pressure. An explanation of this phenomenon results from an examination of the discharge pressure equation in which opposing changes in the discharge temperature and density alternately predominate. For an increase in the inlet temperature, the valve equation dictates that the inlet mass rate and hence the mass throughput of the compressor will decrease. Since the load demand on the outlet of the discharge system is maintained at a fixed rate, the discharge density tends to decrease to a lower final value. However, because of the assumption of no heat losses and because the bypass rate is essentially zero, the suction temperature immediately approaches the increased inlet temperature. This sudden rise in the suction temperature manifests itself as a corresponding rise in the discharge temperature which causes the discharge pressure to rise above its initial value. This
rise continues until the decreasing discharge density predominates at which time the discharge pressure begins falling as is predicted by the steady state thermodynamic head equation.

Table II-3 gives the new steady state values which were achieved by the discharge pressure, the head and the volumetric flow rate through the compressor along with the time required to reach 99.5 percent of the change in the discharge pressure. Unlike the above case, $t_c$ decreased as the disturbance was changed from its maximum positive to its maximum negative value. The same explanation as given above in terms of the change in $Q_c$ holds here also.

The responses shown in Figure II-8 were achieved by initially changing the gas from pure propane to a mixture of propane and $n$-butane for the positive $\Delta M$'s and from pure propane to a mixture of propane and ethane for the negative $\Delta M$'s. These changes were accomplished by specifying an initial mole fraction of either $n$-butane with the mole fraction of ethane set to zero or specifying an initial mole fraction of ethane with the $n$-butane mole fraction set to zero. The following equations, which are in the initial section of the computer program, were utilized to calculate the mole fraction of propane, the molecular weight of the gas and the constants of the heat capacity relation:

$$Y_p = 1 - Y_E - Y_B$$  
$$M = Y_{M_P} + Y_{M_E} + Y_{M_B}$$  
$$a^*_m = a^*_p Y_p + a^*_E Y_E + a^*_B Y_B$$  
$$b^*_m = b^*_p Y_p + b^*_E Y_E + b^*_B Y_B$$  
$$c^*_m = c^*_p Y_p + c^*_E Y_E + c^*_B Y_B$$

II-33

II-34

II-35

II-36

II-37
and

\[ d^*_M = d^*_p + d^*_e + d^*_b. \]  

The values obtained for each of the above parameters were then substituted into each of the dynamic equations containing them. The dynamic simulation was then initiated and executed until the new steady state was achieved. Table II-4 shows the steady state results as in the other two series of disturbances. As in the first case, the characteristic time increases as the disturbance is varied from maximum positive to maximum negative. The same analysis in terms of \( Q_c \) and \( \tau \) explains the trend indicated for \( t_c \).

The responses shown in Figure II-9 were obtained by changing the demand load rate from the design load rate by \( \pm 15, \pm 10 \) or \( \pm 5 \) percent of the design load rate as is shown. All other variables were set equal to their respective design values, the simulation was started and each variable was allowed to achieve its new steady state value. The new steady state values of the discharge pressure, the head and the volumetric flow rate along with the time required to reach 99.5 percent of the change in \( P_d \) are shown in Table II-5. Note that the trend indicated for \( t_c \), which can be explained by the \( \tau - Q_c \) analysis given earlier, is almost the exact inverse of the \( t_c \) trend for \( \Delta P_i \). There does not appear to be any clear-cut explanation for this because of the complicated interrelationships among the variables of the model.

**Closing the Loop**

To date the state of the art of the development and design of chemical processes does not entail the use of dynamic models though
TABLE II-5

OPEN-LOOP RESULTS FOR LOAD DEMAND CHANGES
(CSIMG)

<table>
<thead>
<tr>
<th>( % \Delta F )</th>
<th>( p_{ss}^d ) (psia)</th>
<th>( t_{ss} ) (Secs)</th>
<th>( H_{ss} ) (ft-lbf/lbm)</th>
<th>( Q_{ss}^c ) (acfs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>173.26</td>
<td>15.5</td>
<td>15491.0</td>
<td>347.03</td>
</tr>
<tr>
<td>10</td>
<td>190.58</td>
<td>14.6</td>
<td>17185.0</td>
<td>330.19</td>
</tr>
<tr>
<td>5</td>
<td>206.3</td>
<td>12.5</td>
<td>18591.0</td>
<td>313.64</td>
</tr>
<tr>
<td>-5</td>
<td>231.32</td>
<td>8.5</td>
<td>20597.0</td>
<td>281.34</td>
</tr>
<tr>
<td>-10</td>
<td>240.08</td>
<td>7.0</td>
<td>21224.0</td>
<td>265.5</td>
</tr>
<tr>
<td>-15</td>
<td>246.06</td>
<td>5.0</td>
<td>21614.0</td>
<td>249.84</td>
</tr>
</tbody>
</table>

It appears that the dynamic design is just over the horizon. The dynamic model does, however, offer unlimited possibilities in the development of control schemes for the statically designed chemical process. Most importantly, they enable the control engineer to tune control loops in some definable optimal sense and enable him to utilize disturbances which would not be permitted in the actual plant because of safety considerations, product quality considerations and the like.

The remainder of this chapter is devoted to demonstrating that the dynamic model of the centrifugal compressor can be utilized to obtain tuning constants which yield more superior responses than do tuning constants obtained from a classical tuning procedure. The discharge pressure control loop shown in Figure II-1 will be closed with a classical pneumatic controller. The loop is depicted in Figure II-10 using block diagram notation.

**Dynamics of the Control Valve and Transmission Line**

An analysis of the open-loop responses given earlier indicates that the process has a time constant in the neighborhood of a few
Figure II-10. The Closed-Loop System.
seconds. Since this is a relatively small time constant, the
dynamics of the control valve and transmission line cannot be neg­
lected as is the case for most chemical processes. Harriot has
suggested that the valve and transmission line dynamics can be ade­
quately represented by a simple first order lag with a time constant
of three seconds [22]. This is indicated in Figure II-10 by the
transfer function given in the block labeled valve.

A conversion from the Laplace domain to the time domain is
required to more easily implement these dynamics into the computer
program. This is readily accomplished and is given below.

Assume the steady state relation between the controller output
pressure and the vane position of the throttle valve is linear.
Further assume that the up- and downstream processes dictate that
the throttle valve must close while the surge valve must open upon
the loss of instrument air so as to provide maximum protection for
the compressor. Then the expression which relates the controller
output in psig to the throttle valve vane position in degrees can
be written

\[ Z = 60 - 5(U - 15). \]  

Note that the wide open position of the throttle valve is 60° which
is the value suggested by Chinn because of force considerations.
Note also that the slope of this expression is 5°/psig which is the
gain given in the valve transfer function shown in Figure II-10.

The steady state design given in Appendix A indicates a steady
state inlet mass rate of 170.5 lbm/sec. With the aid of equation
II-4, the steady state value of \( C_{v1} \) is found to be 0.65. Utilizing
the butterfly valve characteristic curve given in Figure II-3 the corresponding steady state vane position is found to be 52.6°. Substitution of this value into equation II-39 yields the steady state value for the controller output as 13.5 psig. These steady state values for the throttle valve vane position and the controller output will be subsequently used in the conversion.

The valve transfer function may be written in the time domain as

\[ \dot{z}_1 = \frac{5}{3} u_1 - \frac{1}{3} z_1 \]  

where \( z_1 \) and \( u_1 \) are respectively, the deviations of the throttle valve vane position and controller output from their steady state values and are given as

\[ z_1 = z_1^* - 52.6 \] (II-41)

and

\[ u_1 = u_1^* - 13.5 \] (II-42)

Substitution of these terms into equation II-40 yields

\[ \dot{z}_1 = \frac{5}{3} u_1 - \frac{1}{3} z_1 \] (II-43)

with

\[ z_1(o) = 52.6° \] (II-44)

which are the desired expressions.

The Controller

As indicated in Figure II-10, the controller used to close the discharge pressure control loop contains three parameters, the controller gain \( K_c \), the derivative time \( \tau_d \) and the reset rate \( K_r \). In order to obtain realistic values for these tuning parameters, the minimum error integral relationships, integral of error squared (ISE), integral of absolute error (IAE) and integral of absolute error multiplied by time (ITAE) published by Lopez and co-workers [23]
were utilized. Their formulae give the controller tuning parameters in terms of the process gain $K$, the process time constant $\tau$ and the process dead-time (transportation lag or time delay) $\Theta_d$ obtained from an open-loop process reaction curve. The latter is a plot of the discharge pressure (controlled variable) versus time for a step change in the controller output, $\Delta U_1$, with the controller in the manual mode. (Reference 23 has an excellent discussion of the process reaction curve.)

Since no actual process reaction curves were available, the model was used to generate them. This is as near to actual conditions as is possible. The nonlinearities of the process, including the control valve characteristics, caused the parameters obtained from the process reaction curve to be a function of the size and direction of the step change in the controller output. To circumvent this, eight process reaction curves covering the range $-1.4 \leq \Delta U_1 \leq 1.4$ were utilized. They are included in Appendix C along with the necessary geometric construction to obtain the parameter values which are given in Table II-6. These three sets of eight parameter values were plotted against $\Delta U_1$ and the curves obtained were extrapolated to zero step size. These plots are shown in Figures II-11, II-12 and II-13 and the parameters obtained at $\Delta U_1 = 0$ are: $K = 11.2 \text{ psi/°}$, $\tau = 4.84$ seconds and $\Theta_d = 0.87$ seconds. Substitution of these values into Lopez's formulae yielded the following best controller parameters which are based on IAE:
<table>
<thead>
<tr>
<th>$U_1$ (psig)</th>
<th>$\Delta U_1$ ($U_1 - U_1^{88}$)</th>
<th>$\Delta P_d$ ($P_d - P_d^{88}$)</th>
<th>$K$ ($\Delta P_d / \Delta U_1$)</th>
<th>$\tau$ (sec)</th>
<th>$\Theta_d$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.9</td>
<td>1.4</td>
<td>11.72</td>
<td>8.37</td>
<td>4.4</td>
<td>0.71</td>
</tr>
<tr>
<td>14.5</td>
<td>1.0</td>
<td>8.77</td>
<td>8.77</td>
<td>4.4</td>
<td>0.78</td>
</tr>
<tr>
<td>14.0</td>
<td>0.5</td>
<td>4.81</td>
<td>9.62</td>
<td>4.6</td>
<td>0.79</td>
</tr>
<tr>
<td>13.75</td>
<td>0.25</td>
<td>2.55</td>
<td>10.2</td>
<td>4.66</td>
<td>0.84</td>
</tr>
<tr>
<td>13.25</td>
<td>-0.25</td>
<td>-3.2</td>
<td>12.8</td>
<td>5.08</td>
<td>0.92</td>
</tr>
<tr>
<td>13.0</td>
<td>-0.5</td>
<td>-7.23</td>
<td>14.46</td>
<td>5.42</td>
<td>0.96</td>
</tr>
<tr>
<td>12.5</td>
<td>-1.0</td>
<td>-17.77</td>
<td>17.77</td>
<td>5.9</td>
<td>1.05</td>
</tr>
<tr>
<td>12.1</td>
<td>-1.4</td>
<td>-29.75</td>
<td>21.25</td>
<td>6.48</td>
<td>1.1</td>
</tr>
</tbody>
</table>
Figure II-11. Determination of the Process Gain $K$ Based on the CSIGM.
Figure II-12. Determination of the Process Time Constant \( \tau \) Based on the CSIGM.

Figure II-13. Determination of the Process Dead-Time \( \Theta_d \) Based on the CSIGM.
As pointed out for the control valve, implementation of equations into the computer program is more readily accomplished in the time domain. For this reason the following controller equation, which was obtained from the controller transfer function shown in Figure II-10, was utilized:

\[ U_i = K_c \left[ e + \tau_d \frac{d}{dt} e + K_R \int_0^t e \, dt \right] + 13.5 \]  

So as to conserve computer time, the computer program was written so that the time required for the process to reach steady state after having been subjected to a disturbance, this time being labeled \( t_f \), could be determined during the dynamic simulation rather than having it specified initially. This was accomplished as follows. At each time step (time step is here defined as ten times the integration step size), beginning with the tenth one, the absolute value of the error and each of its immediately past nine absolute values were compared to 0.001. If each of the ten differences were less than or equal to zero then the time at this point in the simulation was set equal to \( t_f \) and the simulation halted. This provided immense time savings, particularly in the optimization runs which are discussed next in the analysis of the closed-loop results because the required computer time was approximately equal to the simulated process time.
Closed Loop Responses

Utilizing these two sets of classically determined tuning constants, the responses in Figures II-14 through II-21 were determined. Figures II-14 and II-15 indicate the closed loop compressor responses for ± 5% changes in the demand load rate while Figures II-16 and II-17 indicate the responses for ± 5 psi changes in the discharge pressure set point. Responses to disturbance inputs in inlet pressure of 5 psi, in inlet temperature of ± 10°F and in molecular weight of 3 lbm/mole are shown in Figures II-18 through II-21.

To demonstrate that the dynamic model can be utilized to obtain tuning constants which yield more optimal responses than do the conventionally determined tuning constants, an optimization technique utilizing a pattern search [24] was implemented into the terminal section of the simulation program. Values for the 3-mode controller tuning parameters were obtained which minimized the integral of the absolute error (IAE), for each of the disturbances and an additional set was obtained which minimized the integral of the error squared (ISE) for the demand load disturbances. The results of the optimization runs are given in Table II-7, with some pertinent comments. The simulation results utilizing the optimal parameters are also shown in Figures II-14 through II-21 for easy comparison. These results are labeled the optimal controller.

Two sets of optimal parameters were obtained for the load demand rate disturbances to further demonstrate the ease and flexibility of tuning with a dynamic model. This could have just as easily been accomplished for each of the other disturbances but the responses in
Figures II-14 and II-15 indicate an insignificant amount of difference between the two bases.

It is vividly shown in these eight figures that the tuning constants optimally determined based on the model itself do yield far superior responses than do the conventionally determined tuning constants. This logically leads to the conclusion that the control engineer with the well defined dynamic model can be assured that he can obtain tuning constants which are at worst as good as the conventionally determined constants and at best far superior to them. Moreover, advanced control loops such as feedforward loops or multivariable interacting loops can be tuned with the aid of the dynamic model whereas such tasks border on being impossible utilizing conventional techniques. Thus, it is apparent that this dynamic model has the potential to be an invaluable tool for the practicing control engineer.
Figure II-14. Closed-Loop Response to 5% Decrease in the Demand Load Rate.
Figure II-15. Closed-Loop Response to 5% Increase in the Demand Load Rate $F_o$. 

Optimal (ISE and IAE) 3-Mode Controller

2-Mode Controller
Figure II-16. Closed-loop Response to 5 psi Step Change in Set-Point for the CSIGM.
Figure II-17. Closed-loop Response to -5 psi Step Change in Set-Point for the CS1GM.
Figure II-18. Closed-loop Response to 5 psi Step Change in Inlet Pressure for the CS1GM.
Figure II-19. Closed-loop Response to -10°F Step Change in Inlet Temperature for the CSIGM.
Figure II-20. Closed-loop response to 10°F Step Change in Inlet Temperature for the CSIGM.
Figure II-21. Closed-loop Response to 3 lbm/mole Step Change in Molecular Weight for the CSIGM.
## TABLE II-7
**OPTIMIZATION RESULTS**
(CSIGM)

<table>
<thead>
<tr>
<th>Disturbance</th>
<th>$K_c$</th>
<th>$K_R$</th>
<th>$\tau_d$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta F_o = 0.05 F_o^{design}$</td>
<td>20.6</td>
<td>0.002</td>
<td>0.25</td>
<td>based on ISE</td>
</tr>
<tr>
<td></td>
<td>4.62</td>
<td>0.01</td>
<td>0.44</td>
<td>based on IAE</td>
</tr>
<tr>
<td>$\Delta F_o = -0.05 F_o^{design}$</td>
<td>1.9</td>
<td>0.63</td>
<td>1.8</td>
<td>based on ISE</td>
</tr>
<tr>
<td></td>
<td>3.66</td>
<td>0.3</td>
<td>0.5</td>
<td>based on IAE</td>
</tr>
<tr>
<td>$\Delta P_{sp}^d = 5$ psi</td>
<td>2.03</td>
<td>0.025</td>
<td>0.54</td>
<td>steady state reached after only 5.6 seconds</td>
</tr>
<tr>
<td>$\Delta P_{sp}^d = -5$ psi</td>
<td>3.68</td>
<td>0.025</td>
<td>0.64</td>
<td>steady state reached after only 3 seconds</td>
</tr>
<tr>
<td>$\Delta T_i = -10^\circ F$</td>
<td>1.61</td>
<td>0.40</td>
<td>1.6</td>
<td>steady state reached after 16 seconds</td>
</tr>
<tr>
<td>$\Delta T_i = 10^\circ F$</td>
<td>0.525</td>
<td>0.325</td>
<td>1.25</td>
<td>steady state reached after 17 seconds</td>
</tr>
<tr>
<td>$\Delta P_i = 5$ psi</td>
<td>2.00</td>
<td>0.10</td>
<td>0.50</td>
<td>$\epsilon \leq 0.4$ after 3 seconds $\epsilon \leq 0.1$ after 16 seconds</td>
</tr>
<tr>
<td>$\Delta M = 3$ lbm/mole</td>
<td>3.35</td>
<td>0.175</td>
<td>0.625</td>
<td>steady state reached after only 3 seconds</td>
</tr>
</tbody>
</table>

3-mode controller settings: $K_c = 0.43, K_R = 0.148, \tau_d = 0.351$

2-mode controller settings: $K_c = 0.296, K_R = 0.2$
A lumped parameter dynamic model of the constant speed centrifugal compressor suitable for implementation on either an analog, digital or hybrid computer has been developed in terms of the compression of an ideal gas or mixture of ideal gases. It was developed so that no other data other than that provided to the plant personnel by the compressor manufacturer in the form of the characteristic performance curves is required. The performance curve utilized represents the performance of a compressor which is in operation today. It was obtained from one of the major compressor manufacturers.

The model has been successfully tested with both open and closed loop results having been obtained through the utilization of the digital simulation languages CSMP and SL1. The assumption of constant heat capacity in the suction system of the compressor has been suggested and proven to be a valid simplification. The use of an unsteady state suction energy balance for determining the suction temperature rather than the steady state suction energy balance was utilized because the steady state balance yields a very nonlinear algebraic loop implicit in the suction temperature which requires excess computer time to solve.

The open loop discharge pressure response curves for disturbances in inlet pressure, inlet temperature and load demand rate indicate that the centrifugal compressor is very nonlinear. The initial change in discharge pressure resulting from disturbances in the inlet temperature are in the opposite direction from the final change. This results because the discharge density and temperature
changes are in opposite directions with the rate of change of discharge
temperature being the larger during a very short time interval im-
mediately following the disturbance. The nonlinearities associated
with inlet pressure disturbances were found to be similar to those
associated with demand load rate disturbances with those resulting
from increases in the demand load rate being similar to those result-
ing from decreases in inlet pressure and vice versa.

The discharge pressure control loop was closed with a classical
pneumatic controller so as to demonstrate that the model can be an
invaluable tool for the control engineer. It was demonstrated very
vividly that superior controller tuning constants can be obtained
through the utilization of the model than can be obtained from con-
ventional tuning methods. It is felt that the control engineer
with this model should be able to develop any number of advanced
control strategies, the only limits being his own imagination and
intellect.

Though there are no published results available to ascertain the
reliability of the CSIGM, the model will be extended to account for
the nonidealities of the gas through the incorporation of a more
realistic equation of state. If there are discrepancies between the
CSIGM and the real compressor, it is hoped that they result only
from the ideal gas assumption. If so, the realistic equation of
state should correct them.
# NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>constant in parabolic head equation</td>
</tr>
<tr>
<td>a</td>
<td>area, ( ft^2 )</td>
</tr>
<tr>
<td>acfm</td>
<td>actual cubic feet per minute</td>
</tr>
<tr>
<td>acfs</td>
<td>actual cubic feet per second</td>
</tr>
<tr>
<td>a*,b*,c*,d*</td>
<td>constants in the heat capacity equation</td>
</tr>
<tr>
<td>( C_f )</td>
<td>critical flow factor</td>
</tr>
<tr>
<td>( C_p )</td>
<td>molar heat capacity, BTU/lb-mole °R</td>
</tr>
<tr>
<td>( C_v )</td>
<td>fraction of the design ( C_v )</td>
</tr>
<tr>
<td>( C_v' )</td>
<td>design ( C_v ), lbm/min</td>
</tr>
<tr>
<td>( dx )</td>
<td>differential length, ft.</td>
</tr>
<tr>
<td>( E )</td>
<td>molar internal energy, BTU/lbm</td>
</tr>
<tr>
<td>( F )</td>
<td>mass flow rate, lbm/min or lbm/sec</td>
</tr>
<tr>
<td>FC</td>
<td>flow controller</td>
</tr>
<tr>
<td>FX</td>
<td>flow transmitter</td>
</tr>
<tr>
<td>( g_c )</td>
<td>32.2 ( lbm-ft/lbf-sec^2 )</td>
</tr>
<tr>
<td>( H )</td>
<td>head, ft-lbf/lbm</td>
</tr>
<tr>
<td>( H_{min} )</td>
<td>head at the &quot;stonewall&quot;, 14370 ft-lbf/lbm</td>
</tr>
<tr>
<td>h</td>
<td>molar enthalpy, BTU/lb-mole</td>
</tr>
<tr>
<td>( K )</td>
<td>process gain or &quot;channel constant&quot;, ( ft^2/lbm )</td>
</tr>
<tr>
<td>( K_c )</td>
<td>controller proportional gain, psia/psig</td>
</tr>
<tr>
<td>( K_R )</td>
<td>controller reset rate, sec(^{-1})</td>
</tr>
<tr>
<td>L</td>
<td>length of channel, ft</td>
</tr>
<tr>
<td>M</td>
<td>molecular weight of mixture, lbm/lb-mole</td>
</tr>
</tbody>
</table>
molecular weight of n-butane, 58.121 lbm/lb-mole
molecular weight of ethane, 30.046 lbm/lb-mole
molecular weight of propane, 44.062 lbm/lb-mole
number of components in gas
pressure, psia
pressure controller
pressure transmitter
volumetric flow rate, acfm
gas constant, 10.731 psi-ft³/lb-mole °R
gas constant, 1.987 BTU/lb-mole °R
molar entropy, BTU/lb-mole °R
the Laplace operator
absolute temperature, °R
time, sec or min
output of controller, 3 psig ≤ U ≤ 15 psig
gas velocity, ft/sec or deviation of controller
output from steady state
volume, ft³
mole fraction
fraction of surge control valve stem position
(position/wide open position)
angular position of vane in throttle valve, °
development of throttle valve vane position from steady state
a difference operator
error = \( p_{sp}^d - P_d \)
process dead time, sec
\( \eta \) - efficiency  
\( \rho \) - specific density  
\( \tau \) - process time constant, sec  
\( \tau_d \) - controller derivative time, sec  
\( \psi \) - proportionality constant

**Subscripts and Superscripts**

- **B** - n-butane  
- **c** - refers to compressor or cooler  
- **d** - discharge  
- **di** - isentropic discharge  
- **E** - ethane  
- **e** - refers to bypass loop  
- **i** - inlet, isentropic  
- **m** - mixture  
- **o** - outlet  
- **P** - propane  
- **s** - suction  
- **ss** - steady-state  
- **v** - valve
LITERATURE CITED


3. Compressed Air and Gas Handbook, Third Edition (Revised 1966), Copyright 1961 by the Compressed Air and Gas Institute, 122 East 42nd Street, N. Y., N. Y.


5. Hancock, R., "Drivers, Controls and Accessories," Chemical Engineering, (June 1956), pp. 227-238.


CHAPTER III
THE CONSTANT SPEED REAL GAS MODEL (CSRGGM)

Introduction

In this chapter the concepts introduced in the previous chapter will be extended to account for the nonidealities of the gas. This will be accomplished by incorporating into the CSIGM the modified Benedict-Webb-Rubin (MBWR) equation of state developed by Starling [1]

\[
P = CRT + (A_o - C_o/T^2 + D_o/T^3 - E_o/T^4)C^2 \\
+ (bRT - a - \delta/T)C^3 + \alpha(a + \delta/T)C^6 \\
+ (cC^3/T^2)(1 + \gamma C^2) \exp(-\gamma C^2).
\]

III-1

Starling has reported the constants for the straight chain alkanes, C_1 through C_6[2].

This particular equation of state was chosen because, according to Starling, it is exceptionally accurate for both the liquid and vapor regions. This is a desirable trait because in many applications the model of an entire process containing a compressor will involve the liquid state. Also, all of the equations which will subsequently be derived can be easily modified so that gases other than methane, ethane, propane, n-butane, n-pentane, and n-hexane can be handled. This is done by reducing Starling's equation to the BWR equation of state by setting D_0, E_0 and \delta to zero.
Another and the more important reason the MBWR (or BWR) equation of state has been chosen is that it is one of the most complicated equations of state. A complicated equation of state was desired so that both ends of the spectrum of complexity could be examined so as to provide yardsticks for ascertaining degrees of difficulty in programming, computer storage requirements, computer time requirements, etc. relative to desired accuracy. The previous chapter examined the simplest case and this chapter will examine the most complicated and the most accurate case. Any models developed in terms of other equations of state, such as the well known two constant Redlich-Kwong equation of state, should fall somewhere, in terms of complexity, between the CSIGM and CSRGM.

Based upon the same physical system which was developed in Chapter II, a detailed development of the changes necessary to incorporate the MBWR into the constant speed centrifugal compressor model will be presented. The resulting model is designated the CSRGM and it will be summarized in terms of a table of dynamic equations and an information flow diagram.

Open loop simulation results of the time response of the discharge pressure to step changes in inlet temperature and pressure and in molecular weight of the gas will be presented in the form of continuous curves. A comparison will be made between the open loop CSRGM and CSIGM results in terms of a tabulation of the steady state head, volumetric flow rate and discharge pressure and the time required to reach these new steady states. The chapter will be concluded with a closed loop response analysis as was done in Chapter II.
Mathematical Model

The principles presented in Chapter II upon which the development of the lumped parameter CSIGM was based will be utilized to develop the CSRGM as it is also a lumped parameter model. Many of the expressions derived as a part of the CSIGM will be unchanged by the inclusion of the MBWR equation of state and hence will be presented only in brief. However, those expressions which are affected will be presented in detail.

Suction System

As in the CSIGM, the mathematical expressions required to model the suction system are unsteady state energy and mass balances and the standard valve equation. The mass balance is that given in equation II-2

\[ \dot{\rho}_s = \frac{(F_1 + F_e - \rho_s Q_C)}{V_s} \]

II-2

and the valve equation is that given in equation II-3 with slight modification. The maximum valve flow factor, \( \bar{C}_{v1} \) has been adjusted from 9541. lbm/min to 10174. lbm/min so that when the discharge pressure control loop is closed the valve will operate at the same steady state position as in the CSIGM. The other modification involves the calculation of the inlet density when the inlet temperature and pressure are known. Since the MBWR equation of state is implicit in density, a trial and error iterative procedure is required. The procedure utilized is given in Appendix B. The valve equation utilized is

\[ F_1 = 1.05 \times 10174. \bar{C}_{v1} \sqrt{0.5(\rho_1 + \rho_s)(P_1 - P_s)} \]

III-2
The unsteady state energy balance is changed by the MBWR equation of state and its derivation is as follows. The basic expression is equation II-5 which is written here with the molecular weights canceled since the molecular weight will, in the CSRGM as in the CSIGM, be maintained constant at its initially specified value

$$\frac{d}{dt}(\rho_s V_s E_s) = F_i h_i + F_e h_e - \rho_s Q_c h_s.$$  \hspace{1cm} \text{III-3}

The accumulation term may be expanded to give

$$\frac{d}{dt}(\rho_s V_s E_s) = \rho_s V_s \dot{E} + V_s \dot{E}_s.$$  \hspace{1cm} \text{III-4}

The molar internal energy for a real gas is both a function of the temperature and molar density and hence the chain rule of differentiation is required to expand the first term on the right hand side of equation III-4 to obtain the expression for calculating $T_s$. Doing this yields

$$\rho_s V_s \dot{E} = \rho_s V_s E_T \dot{T}_s + \rho_s V_s E_C \dot{C}_s / M.$$  \hspace{1cm} \text{III-5}

where

$$E_T = \left(\frac{\partial E}{\partial T_s}\right)_{C_s} \quad \text{and} \quad E_C = \left(\frac{\partial E}{\partial C_s}\right)_{T_s}.$$  \hspace{1cm} \text{III-5}

Substitution of equation III-5 into III-4 with subsequent substitution of the results into equation III-3 yields after rearrangement

$$\dot{T}_s = \frac{[F_i h_i + F_e h_e - \rho_s Q_c h_s - V_s \dot{E}_s (E + \rho_s E_C / M)] / (\rho_s V_s E_s)}{\dot{E}_s}.$$  \hspace{1cm} \text{III-6}
In view of the fact that the internal energy and the enthalpies cannot be given absolutely, choose the bypass temperature $T_e$ as the thermal datum plane. Then $h_e = 0$ and $E_s$, $h_i$ and $h_s$ can be determined from the expressions given in Appendix D.

Writing equation D-25 with $T_a = T_e$, $T_b = T_i$ and $C_a = C_b = C_i$ yields the following expression for the inlet enthalpy relative to the bypass temperature:

$$h_i = \{-B_0RT_e - 4C_0/T_e^2 + 5D_0/T_e^3 - 6E_0/T_e^4\}C_i$$

$$-\left(bRT_e - 2\delta/T_e\right)C_i^2 - 1.4\alpha\delta C_i^4/T_e$$

$$-\left(c/\gamma T_e\right)[3 - (3 + 0.5\gamma C_i - \gamma^2 C_i^2\exp(-\gamma C_i^2))]$$

$$+(B_0RT_i - 4C_0/T_i^2 + 5D_0/T_i^3 - 6E_0/T_i^4)C_i$$

$$+(bRT_i - 2\delta/T_i)C_i^2 + 1.4\alpha\delta C_i^4/T_i$$

$$+(c/\gamma T_i^2)[3 - (3 + 0.5\gamma C_i - \gamma^2 C_i^2\exp(-\gamma C_i^2))]$$

$$\exp\left(-\gamma C_i^2\right)]\frac{144}{777.649} + a^*(T_i - T_e)$$

$$+ (b^*/2)(T_i^2 - T_e^2) + (c^*/3)(T_i^3 - T_e^3)$$

$$+ (d^*/4)(T_i^4 - T_e^4).$$

III-7

If, in this expression, $T_i$ and $C_i$ are replaced respectively with $T_s$ and $C_s$, the following expression for evaluating the enthalpy of the gas within the suction volume relative to the temperature of the bypass gas is obtained:

$$h_s = \{-B_0RT_e - 4C_0/T_e^2 + 5D_0/T_e^3 - 6E_0/T_e^4\}C_s$$
\[-(bRT_e - 2\delta/T_e)C_s - 1.4\alpha\delta C_s/T_e \]
\[-(c/\gamma T_e^2)[3 - (3 + 0.5\gamma C_s^2 - \gamma C_s^3) \exp (-\gamma C_s^2)] \]
\[+ (B_0RT_s - 4C_0/T_s + 5D_0/T_s^3 - 6E_0/T_s^5)C_s \]
\[+ (bRT_s - 2\delta/T_s)C_s + 1.4\alpha\delta C_s/T_s \]
\[+ (c/\gamma T_s^2)[3 - (3 + 0.5\gamma C_s^2 - \gamma C_s^3) \exp (-\gamma C_s^2)] \]
\[\frac{144}{777.649} + a^*(T_s - T_e) + (b^*/2)(T_s^2 - T_e^2) \]
\[+ (c^*/3)(T_s^3 - T_e^3) + (d^*/4)(T_s^4 - T_e^4). \]

The expression for evaluating the internal energy of the gas within the suction volume is obtained by writing equation D-38 with \( T_A = T_e, T_B = T_s \) and \( C_A = C_B = C_s \). Doing this yields

\[ E_s = \{- (3C_0/T_s^2 - 4D_0/T_s^3 + 5E_0/T_s^5)C_s \]
\[+ \delta C_s/T_s + 0.4\alpha\delta C_s/T_s + (3c/\gamma T_s^2)[1 \]
\[- (1 + 0.5\gamma C_s^2) \exp (-\gamma C_s^2)] + (3C_0/T_e^2 \]
\[- 4D_0/T_e^3 + 5E_0/T_e^4)C_s + \delta C_s/T_e - 0.4\alpha\delta C_s/T_e \]
\[-(3c/\gamma T_s^2)[1 - (1 + 0.5\gamma C_s^2)\exp(-\gamma C_s^2)] \frac{144}{777.649} \]
\[+ (a^*-\tau)(T_s - T_e) + (b^*/2)(T_s^2 - T_e^2) \]
\[+ (c^*/3)(T_s^3 - T_e^3) + (d^*/4)(T_s^4 - T_e^4). \]

Evaluation of the two partial derivatives results in

\[ E_T = \{(6C_0/T_s^3 - 12D_0/T_s^4 + 20E_0/T_s^5)C_s \]
since it has been assumed that the cooler will always maintain $T_e$ constant, and

$$E_C = \left[ -3C_o^2 / T_s^2 + 4D_o / T_s^3 - 5E_o / T_s^4 - 2 \delta C_s / T_s \right.$$  
$$+ 2a \delta C_s / T_s + (3c C_s / T_s^2)(1 + \gamma C_s^2) \exp (-\gamma C_s^2)$$  
$$+ 3C_o / T_e^2 - 4D_o / T_e^3 + 5E_o / T_e^4 + 2 \delta C_s / T_e$$  
$$- 2a \delta C_s / T_e - (3c C_s / T_e^2)(1 + \gamma C_s^2) \exp (-\gamma C_s^2) \right] \frac{144}{777.649} \quad \text{III-11}$$

Since $h_e \approx 0$, equation III-6 may be written as

$$\dot{T}_s = \frac{F_i h_i - \dot{\rho}_s C_s h_s - V \dot{\rho}_s (E + \rho E_s / M)}{\dot{\rho}_s V E_T} \quad \text{III-12}$$

where $\dot{\rho}_s$ is obtained from equation II-2 and $h_i$, $h_s$, $E_C$ and $E_T$ are given by the above expressions.

As was done for the CSIGM, this energy balance, denoted the real suction energy balance, was investigated in an effort to determine if it could be simplified. Open loop results in terms of the discharge pressure versus time for step changes in the inlet temperature were obtained utilizing the simplified and ideal suction energy balance equations developed in Chapter II and the above
equation. The results are shown in Figure III-3 and it is apparent from them that the suction energy balance could have been derived based upon the assumption that the gas is an ideal one with constant heat capacity. This assumption is valid only because of the small temperature difference at the suction since utilization of the ideal gas law rather than equation III-1 as the suction equation of state does introduce significant errors.

**Discharge System**

The equations required to model the discharge system are exactly the same as those given in Chapter II. As a result they will not be given here but will subsequently be given in the summation of the CSRGM dynamic equations.

**Compressor**

The concepts presented in Chapter II relative to modeling the compressor proper are not changed when the nonidealities of the gas are accounted for by the introduction of the equation of state of a real gas. The basic equation is still a steady state energy balance

\[ H = 778. \frac{\Delta h}{M}. \]

The \( H-Q_c \) curve is the same as that shown in Figure II-4 and its equation is the same as that given in equation II-24

\[ H - H_m = A(Q_c - Q_m)^2. \]

The steps required to calculate the volumetric flow rate through the compressor remain unchanged and are

1. Isentropic entropy balance to obtain the isentropic discharge temperature \( t_{di} \)
2. Calculation of the isentropic enthalpy change utilizing the suction temperature and density and the discharge density and $T_{dl}$

3. Calculation of the actual enthalpy change based on the efficiency of the machine and the isentropic enthalpy change

and

4. Calculation of the volumetric flow rate utilizing equation II-30

$$Q_c = Q_m + \sqrt{H_m - 777.649 \Delta h/M}/(-A).$$  \hspace{1cm} \text{II-30}

However, the expressions required to perform these steps as well as the expression required to calculate the actual discharge temperature $T_d$ are changed considerably from those presented in Chapter II and will be presented here based on the thermodynamic derivations given in Appendix D.

For the isentropic or reversible compression of the real gas represented by the MBWR equation of state, from the suction conditions of $T_s$ and $C_s$ to the discharge conditions of $T_{di}$ and $C_d$, equation D-10 is applicable with $\Delta S = 0$, $C_A = C_s$, $T_A = T_s$, $C_B = C_d$ and $T_B = T_{di}$. After making these substitutions, equation D-10 may be expanded to

$$0 = \left\{ C_s \left( B_R + 2C_s/\gamma T_s - 3D_s/\gamma T_s^4 + 4E_s/\gamma T_s^5 \right) \right. + 0.5 C_s (b_R + 6/T_s) - 0.2a\delta C_s^2/T_s^5 $$

$$ - (2c/\gamma T_s)[1 - (1 + 0.5 \gamma C_s^2)] \exp (-\gamma C_s^2)] - C_d (B_R$$
The only unknown in this expression is the isentropic discharge temperature $T_{di}$ which may be obtained through the use of the implicit function solving routine. The routine used is Newton's, the same as that used in the CSIGM, and the required equations are given in Appendix B.

The enthalpy change $\Delta h_1$ associated with this isentropic compression is obtained from equation D-25 after making the same substitutions as those given above. Expansion of equation D-25 after making these substitutions yields:

$$\Delta h_1 = \left(C_d \left( B R T_{di} - 2A - 4C o/T_{di} + 5D o/T_{di} - 6E o/T_{di} \right) \right)^4$$

$$+ \left(b R T_{di} - 1.5a - 2\delta/T_{di} \right) C_d^2 + 0.2\alpha(6a+7\delta/T_{di}) C_d^5$$

$$+ \left(c/T_{di}\right)[3-(3 + 0.5\gamma C_d^2 - \gamma C_d^4) \exp(-\gamma C_d^2)]$$

$$+ \left(c/T_{di}\right)[3-(3 + 0.5\gamma C_s^2 - \gamma C_s^4) \exp(-\gamma C_s^2)] \frac{144.}{777.649}$$

$$+ a(T_{di} - T_s) + \frac{b}{2}(T_{di}^2 - T_s^2) + \frac{c}{3}(T_{di}^3 - T_s^3)$$

$$+ \frac{d}{4}(T_{di}^4 - T_s^4). \quad \text{III-14}$$
Division of $\Delta h$ by the efficiency of the machine then yields the actual enthalpy change $\Delta h$ which when substituted into equation II-30 yields the volumetric flow rate $Q_c$.

The actual discharge temperature $T_d$ can now be calculated since the actual enthalpy change is known. An enthalpy balance about the compressor from the suction conditions $T_s$ and $C_s$ to the discharge conditions of $T_d$ and $C_d$ is given by equation D-25 with $T_A = T_s$, $C_A = C_s$, $T_B = T_d$ and $C_B = C_d$. Since the only unknown is $T_d$, an implicit function solving routine may be utilized to solve

$$0 = \{C_d(B_oRT_d - 2A_o - 4C_o/T_d^2 + 5D_o/T_d^3 - 6E_o/T_d^4)$$

$$+ (bRT_d - 1.5a - 2\delta/T_d)C_d^2 + 0.2a(6a + 7\delta/T_d)C_d^5$$

$$+ (c/\gamma T_d)(3 - (3 + 0.5\gamma C_d^2 - \gamma C_d^4) \exp (-\gamma C_d^2))$$

$$- [C_s(B_oRT_s - 2A_o - 4C_o/T_s^2 + 5D_o/T_s^3 - 6E_o/T_s^4)]$$

$$+(bRT_s - 1.5a - 2\delta/T_s)C_s^2 + 0.2a(6a + 7\delta/T_s)C_s^5$$

$$+ (c/\gamma T_s)(3 - 2\gamma C_s^2 - \gamma C_s^4) \exp (-\gamma C_s^2)]$$

$$+ a(T_d - T_s) + (b/2)(T_d^2 - T_s^2) + (c/3)(T_d^3 - T_s^3)$$

$$+(d/4)(T_d^4 - T_s^4) - \Delta h.$$  \(\text{III-15}\)

The routine utilized is also based on that of Newton and it is detailed in Appendix B.

So that the CSRGMM can be analysed more completely and more conveniently, an information flow diagram of the open loop model is shown in Figure III-1. The accompanying dynamic equations are given in Table III-1.
Figure III-1. Information Flow Diagram for Open Loop CSRGM.
TABLE III-1
CSRGDM DYNAMIC EQUATIONS

\textbf{Suction System}

\textbf{Mass balance}
\[ \rho_s = \frac{(F_i + F_e - \rho_s Q_c)}{V_s} \]  
(1)

\textbf{Molar density}
\[ C_s = \frac{\rho_s}{M} \]  
(2)

\textbf{Energy balance}
\[ T_s = \frac{F_i h_i - \rho_s Q h_s - V_s \rho_s (E + \rho_s E_c / M)}{\rho_s V E_T} \]  
(3)

\textbf{Inlet density}
\[ 0 = C_i R T_i + \left( B_0 R T_i^3 - A_0 - C_0 / T_i^2 \right) \]  
\[ + D_0 / T_i^3 - E_0 / T_i^4 \right) C_i^2 + (b R T_i \]  
\[ - a - \delta / T_i \} C_i^3 + \alpha(a + \delta / T_i) C_i^6 \]  
\[ + \frac{c C_i^3}{T_i} \{ 1 + \gamma C_i^2 \exp(-\gamma C_i^2) - P_i \} \]  
(4)

\textbf{Inlet enthalpy}
\[ h_i = \left\{ C_i \left[ h_1(T_i) - h_1(T_e) \right] + C_i^2 \left[ h_2(T_i) - h_2(T_e) \right] \right\} \]  
\[ + C_i^5 \left[ h_3(T_i) - h_3(T_e) \right] + h_4(T_i, C_i) \]  
\[ - h_4(T_e, C_i) \frac{144}{777.649} + h_{id}(T_i) - h_{id}(T_e) \]  
(5)

\textbf{Suction enthalpy}
\[ h_s = \left\{ C_s \left[ h_1(T_s) - h_1(T_e) \right] + C_s^2 \left[ h_2(T_s) - h_2(T_e) \right] \right\} \]  
\[ + C_s^5 \left[ h_3(T_s) - h_3(T_e) \right] + h_4(T_s, C_s) \]  
\[ - h_4(T_e, C_s) \frac{144}{777.649} + h_{id}(T_s) - h_{id}(T_e) \]  
(6)
TABLE III-1 (Cont'd)

CSRGM DYNAMIC EQUATIONS

Suction energy

\[ E = \left( C_s \left[ E_1(T_e) - E_1(T_s) \right] + C_s^2 \delta \left[ 1/T_e - 1/T_s \right] \right) \left( 1 - 0.4aC_1^3 \right) + E_2(T_s, C_s) - E_2(T_e, C_s) \frac{144}{777.649} + h_{id}(T_s) - h_{id}(T_e) - r[T_s - T_e] \]  

\[ (7) \]

Energy change with temperature

\[ E_T = \left( \frac{\partial E}{\partial T_s} \right)_{C_s} = \left( C_s [6C_0/T_s^3 - 12D_0/T_s^4 \right. \]

\[ + 20E_s^2/T_s^5 + 8C_s^2 [1 - 0.4aC_3^3]/T_s^2 \]

\[ - 2E_2(T_s, C_s)/T_s \frac{144}{777.649} + (a - r) \]

\[ + b_T + c_T^2 + d_T^3 \]  

\[ (8) \]

Energy change with composition

\[ E_C = \left( \frac{\partial E}{\partial C_s} \right)_{T_s} = \left( E_1(T_e) - E_1(T_s) \right) + 26C_s \left[ 1/T_e \right. \]

\[ - 1/T_s \right] \left[ 1 - aC_s^3 \right] + 3cC_s \left( 1 + \gammaC_s^2 \right) \exp( \]

\[ - \gammaC_s^2 \left[ 1/T_s^2 - 1/T_e^2 \right] \frac{144}{777.649} \]  

\[ (9) \]

Inlet mass rate

\[ F_i = 1.05 C_{vi} \sqrt{\rho_{vi} \Delta P_{vi}} \]  

\[ (10) \]
TABLE III-1 (Cont'd)

CSRGM DYNAMIC EQUATIONS

<table>
<thead>
<tr>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Throttle valve density</td>
<td>( \rho_{vi} = 0.5 \left( \rho_s + C_i M \right) ) (11)</td>
</tr>
<tr>
<td>Throttle valve pressure drop</td>
<td>( \Delta P_{vi} = P_i - P_s ) (12)</td>
</tr>
<tr>
<td>Throttle valve flow factor</td>
<td>( C_{vi} = \overline{C}_{vi} f(Z_i) ) (*see below) (13)</td>
</tr>
</tbody>
</table>
| Equation of state                  | \[ P_s = \frac{C R_T}{s s} + \left( B_0 R_T - A_0 - \frac{C_s}{T_s} \right)^2 \]
|                                    | \[ + D_0 / T_s - E_0 / T_s \] \( C_s^2 \) + (bRT)_s \]
|                                    | \[- a - \delta / T_s^3 C_s^3 + \alpha (a + \delta / T_s) C_s^6 \]
|                                    | \[ + \frac{e C_s^3}{T_s^2} \left( 1 + \gamma C_s \right) \exp(-\gamma C_s) \] (14) |

*\( f(Z_i) \) is the function which relates the fraction of the design \( C_v \) (expected maximum valve capacity) to the throttle valve vane position. It is shown in Figure II-3.
TABLE III-1 (Cont'd)

CSRGM DYNAMIC EQUATIONS

<table>
<thead>
<tr>
<th>Discharge System</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass balance</td>
<td>( \dot{\rho}<em>d = (\rho</em>{sc} - \rho_e - \rho_o)/V_d ) (15)</td>
</tr>
<tr>
<td>Molar density</td>
<td>( C_d = \rho_d / M ) (16)</td>
</tr>
<tr>
<td>Outlet mass</td>
<td>( F_o = \text{design value of } F_i ) (17)</td>
</tr>
<tr>
<td>rate</td>
<td>( P_e = P_d - \gamma F_e^2 ) (18)</td>
</tr>
</tbody>
</table>
| Bypass pressure  | \begin{align*} 
\text{critical flow if } 0.53 P_e &> P_s' \\
\text{otherwise subcritical flow} 
\end{align*} |
| Bypass mass rate | \begin{align*} 
\text{critical flow} &\quad P_e = (0.9/5.1) C_{ve} P_e \sqrt{M/T_e} \\
\text{subcritical flow} &\quad P_e = 1.05 C_{ve} \sqrt{\rho_{ve} \Delta P_{ve}} 
\end{align*} (19) (20) |
| Surge valve      | \( \rho_{ve} = 0.5 (\rho_s + \rho_e) \) (21) |
| density          | \( \Delta P_{ve} = P_e - P_s \) (22) |
| Surge valve      | \( C_{ve} = 0.01 C_{ve} \exp (4.6 Z_e) \) (23) |
| pressure drop    | \( P_d = C_d R T_d + (B_0 R T_d - A_o - C_o/T_d^2 \\ + D_o/T_d^3 - E_o/T_d^4) C_d^2 + (b R T_d \\ - a - \delta/T_d) C_d^3 + a(a +\delta/T_d) C_d^6 \\ + \frac{c C_d^2}{T_d^2} (1 + \gamma C_d^2) \exp(-\gamma C_d^2) \) (24) |
TABLE III-1 (Cont'd)

CSRGM DYNAMIC EQUATIONS

Compressor

Isentropic discharge
temperature
\[ \Delta S = 0 = S_{id}(T_{di}) + \left\{ \begin{align*}
&+ C_d^2 S_2(T_{di}) - C_d^5 S_3(T_{di}) - S_4(T_{di}, C_d) \\
&+ [C_S S_1(T_S) + C_S^2 S_2(T_S) - C_S^5 S_3(T_S) \\
&- S_4(T_s, C_S)] \frac{144}{777.649} - S_{id}(T_S) \\
&- \rho \ln \left( \frac{\rho_d}{\rho_S} \right)
\end{align*} \right. \] (25)

Isentropic enthalpy change
\[ \Delta h_i = h_{id}(T_{di}) + \left\{ \begin{align*}
&+ C_d^5 h_3(T_{di}) + h_4(T_{di}, C_d) \right\}- [C_S h_1(T_S) \\
&+ C_S^2 h_2(T_S) + C_S^5 h_3(T_S) + h_4(T_S, C_S)] \frac{144}{777.649} \\
&- h_{id}(T_S)
\end{align*} \right. \] (26)

Actual enthalpy change
\[ \Delta h = \Delta h_i / \eta \] (27)

Discharge
temperature
\[ 0 = h_{id}(T_d) + \left\{ \begin{align*}
&+ C_d^5 h_3(T_d) + h_4(T_d, C_d) \right\} - [C_S h_1(T_S) \\
&+ C_S^2 h_2(T_S) + C_S^5 h_3(T_S) +
\end{align*} \right. \]
TABLE III-1 (Cont'd)

CSRGM DYNAMIC EQUATIONS

\[
\begin{align*}
h_u(T_s, C_s) &= \frac{144}{777.649} \cdot h_{id}(T_s) - \Delta h \\
H &= 778 \Delta h/M \\
Q_c &= \begin{cases} 
\text{surge: } & 778\Delta h/M > H_m \\
Q_m + \sqrt{(H_m - 778\Delta h/M)/(\Lambda)} & \text{for } H_m \leq 778\Delta h/M \leq H_{\text{max}} \\
Q_{\text{max}}; & 778\Delta h/M \leq H_{\text{min}} 
\end{cases}
\end{align*}
\]

Energy balance

$H$ and $Q_{\text{max}}$ are respectively the head and flow rate at the "stonewall."
### TABLE III-1 (Cont'd)

**CSRGM DYNAMIC EQUATIONS**

**Supplementary Thermodynamic Functions**

<table>
<thead>
<tr>
<th>Function</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1(T)$</td>
<td>$B_o R + 2C_o/T^3 - 3D_o/T^4 + 4E_o/T^5$</td>
</tr>
<tr>
<td>$S_2(T)$</td>
<td>$0.5(bR + \delta/T^2)$</td>
</tr>
<tr>
<td>$S_3(T)$</td>
<td>$0.2a\delta/T^2$</td>
</tr>
<tr>
<td>$S_4(T,C)$</td>
<td>$\frac{2c}{T^3} [1 - (1 + 0.5\gamma C^2) \exp(-\gamma C^2)]$</td>
</tr>
<tr>
<td>$S_{id}(T)$</td>
<td>$(a^<em>-r) \ln T + b^<em>T + (c^</em>/2)T^2 + (d^</em>/3)T^3$</td>
</tr>
<tr>
<td>$h_1(T)$</td>
<td>$B_o RT - 2A_o - 4C_o/T^2 + 5D_o/T^3 - 6E_o/T^4$</td>
</tr>
<tr>
<td>$h_2(T)$</td>
<td>$bRT - 1.5a - 2\delta/T$</td>
</tr>
<tr>
<td>$h_3(T)$</td>
<td>$0.2a(6a + 7\delta/T)$</td>
</tr>
<tr>
<td>$h_4(T,C)$</td>
<td>$\frac{c}{T^2} [3 - (3 + 0.5\gamma C^2-\gamma^2C^4)\exp(-\gamma C^2)]$</td>
</tr>
<tr>
<td>$h_{id}(T)$</td>
<td>$a^<em>T + (b^</em>/2)T^2 + (c^<em>/3)T^3 + (d^</em>/4)T^4$</td>
</tr>
<tr>
<td>$E_1(T)$</td>
<td>$3C_o/T^2 - 4D_o/T^3 + 5E_o/T^4$</td>
</tr>
<tr>
<td>$E_2(T,C)$</td>
<td>$\frac{3c}{T^2} [1 - (1 + 0.5\gamma C^2) \exp(-\gamma C^2)]$</td>
</tr>
</tbody>
</table>
Open-Loop Responses

Following the procedures given in Chapter II, under this same heading, the open-loop responses of the CSRGM to step changes in the inlet pressure and temperature, molecular weight and the load demand rate were obtained and are shown in Figures III-2 through III-5. The same integration method, the trapezoidal integration technique, was used in the simulation. However, due to the increased complexity of the CSRGM, the integration step size had to be halved resulting in a step size for the CSRGM of 0.01 seconds. This, along with the increased number of calculations, resulted in an increase in computer time such that three seconds of computer time was required for each second of the model or problem time. Also, the CSRGM required almost twice as much core as did the CSIGM in either language.

The responses shown in Figure III-2 were obtained by changing, at time zero, the inlet pressure by the amount shown. Sequentially, with time remaining at zero, the inlet density was calculated utilizing the trial and error iterative procedure described in Appendix B. The remaining variables were set to their respective steady state design values and the simulation was started. Table III-2 gives the new steady state values of the discharge pressure, the volumetric flow rate and the head developed by the compressor as well as the time required to reach 99.5 percent of the change in $P_d$. Also included in Table III-2 are the steady state values for these same variables as achieved by the CSIGM when subjected to the same disturbances.

Due to the nonidealities of the gas, including both pressure-volume and energy considerations, the discharge pressure predicted
Figure III-2. Open Loop Response of the CSRGM to Step Changes in Inlet Pressure.
Figure III-3. Open Loop Response of the CSRGM to Step Changes in Inlet Temperature for (1) the Simplified, (2) the Ideal and (3) the Real Suction Energy Balances.
Figure III-4. Open Loop Response of the CSRG1M to Step Changes in Molecular Weight of the Gas.
Figure III-5. Open Loop Response of the CSRGM to Step Changes in the Load Demand Rate.
<table>
<thead>
<tr>
<th>Disturbance</th>
<th>$P^S_d$ (psia)</th>
<th>$t_c$ (second)</th>
<th>$h^S$ (ft-lbf/lbm)</th>
<th>$Q_c^S$ (acfs)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CSRGM</td>
<td>CSIGM</td>
<td>CSRGM</td>
<td>CSIGM</td>
</tr>
<tr>
<td>$\Delta P_i = 15$ psi</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10&quot;</td>
<td>310.77</td>
<td>293.75</td>
<td>7.8</td>
<td>5.5</td>
</tr>
<tr>
<td>5&quot;</td>
<td>283.81</td>
<td>272.44</td>
<td>9.8</td>
<td>7.2</td>
</tr>
<tr>
<td>-4&quot;</td>
<td>253.58</td>
<td>247.97</td>
<td>12.1</td>
<td>9.0</td>
</tr>
<tr>
<td>-7&quot;</td>
<td>190.78</td>
<td>194.91</td>
<td>15.7</td>
<td>12.9</td>
</tr>
<tr>
<td>-8&quot;</td>
<td>167.67</td>
<td>174.53</td>
<td>16.3</td>
<td>13.8</td>
</tr>
</tbody>
</table>

| $\Delta T_i = 50$ °F |     |                 |                   |                 |                 |                 |                 |                 |
| 48.5"        | surge | 181.31          | surge             | 14.0            | surge           | 17542.          | surge           | 326.26          |
| 30"          | 167.19          | 184.97          | 14.6              | 14.0            | 16661.          | 17979.          | 335.69          | 323.34          |
| -30"         | 185.33          | 196.22          | 15.0              | 13.8            | 17972.          | 18513.          | 332.3           | 314.64          |
| -50"         | surge           | 167.46          | surge             | 14.1            | 20980.          | 20649.          | 272.4           | 280.22          |

| $\Delta M = 3$ lbm/mole |     |                 |                   |                 |                 |                 |                 |                 |
| 2"            | 267.69          | 251.43          | 17.7              | 10.4            | 20973.          | 20749.          | 272.58          | 278.00          |
| 1"            | 250.8           | 241.00          | 16.7              | 10.7            | 20635.          | 20460.          | 280.53          | 284.17          |
| -1"           | 234.96          | 230.50          | 15.3              | 11.0            | 20220.          | 20120.          | 288.8           | 290.63          |
| -1.95"        | 205.83          | 209.45          | 14.1              | 11.5            | 19133.          | 19261.          | 306.3           | 304.46          |
| -2"           | surge           | 198.86          | surge             | 11.7            | 18475.          | 18751.          | 315.13          | 311.54          |

| $\Delta F_p$  | $p_{design}$ |     |                 |                 |                 |                 |                 |                 |
| $= -0.15$     | -0.1          | 248.25          | 246.06           | 6.7             | 5.0             | 21622.          | 21614.          | 249.35          | 249.84          |
| -0.05         | 241.73         | 240.08          | 9.1              | 7.0             | 21236.          | 21224.          | 265.14          | 265.5           |
| 0.05          | 232.22         | 231.32          | 11.8             | 8.5             | 20606.          | 20597.          | 281.15          | 281.34          |
| 0.1           | 205.36         | 206.3           | 15.6             | 12.5            | 18571.          | 18591.          | 313.89          | 313.64          |
| 0.15          | 188.77         | 190.58          | 17.1             | 14.6            | 17137.          | 17185.          | 330.7           | 330.19          |
| 0.15          | 170.66         | 173.26          | 17.9             | 15.5            | 15400.          | 15491.          | 347.87          | 347.03          |
by the CSRGM is greater than that of the CSIGM for increases in the inlet pressure and less for decreases in $P_1$, the difference being the greatest when the gas is the least ideal. The CSRGM even predicts surge for a decrease in the inlet pressure of more than 7 psi while the CSIGM does not. A further indication of the differences in the predictions of the two models is that the characteristic times of the CSRGM are larger. There appears to be no concise explanation for this as an analysis is essentially impossible because of the increased complexity created by the complicated MBWR equation of state. It is important to note, however, that both models predict the same trends for $t_c$.

The responses shown in Figure III-3 were obtained in the same manner except that the variable changed was the inlet temperature which appears in the equation for calculating the inlet density, and in the suction energy balance. As previously noted, there are three sets of four curves in this figure, one set obtained utilizing the simplified suction energy balance, one set obtained using the exact suction energy balance based on an ideal gas (ideal suction energy balance) and a set obtained using the exact suction energy balance based on the MBWR equation of state (real suction energy balance). For each step change the real suction energy balance yielded a discharge pressure response noticeably different from that of the other two energy balances which yielded almost identical results. The differences between the responses obtained utilizing the simplified and the real suction energy balances were
deemed insignificant and the former was utilized in the remainder of the CSRGM simulations.

As in the above case, the characteristic time and the steady state value of \( P_d, Q_c \) and \( R \) have been included in Table III-2 for both the CSRGM and CSIGM. A point of major interest is that the CSRGM predicts an increasing \( t_c \) while the CSIGM predicts just the opposite as \( \Delta T_i \) is varied from a maximum positive to a maximum negative value though the \( t_c \)'s tend to the same values as surge is approached. There does not appear to be any simple explanation for this other than the system time constant predicted by the CSRGM is more strongly dependent upon factors which have been introduced by the real gas equation of state than it is on \( Q_c \). Note that both \( Q_c \) trends are the same.

The responses shown in Figure III-4 were obtained as in Chapter II by instantaneously replacing the pure propane contained within the system with a mixture of propane and ethane for the negative \( \Delta M \)'s and a mixture of propane and n-butane for the positive \( \Delta M \)'s. This replacement of the pure propane was accomplished by specifying either a n-butane mole fraction with the ethane mole fraction set to zero or an ethane mole fraction with the the n-butane mole fraction set to zero. In addition to equations II-33 through II-38 which were used to calculate the new mole fraction of propane, the new molecular weight and the new heat capacity constants, the following equations as specified by Benedict, et al [3] were used to calculate the new constants of the equation of state:
\[ B_{om} = \sum Y_i B_{oi} \]  
III-16

\[ \beta_m = \left[ \sum Y_i \sqrt{\beta_i} \right]^2 \]  
\[ = A_0, B_0, C_0, D_0, E_0 \text{ and } \gamma \]  
III-17

\[ \zeta = \left[ \sum Y_i (\zeta_i) \right]^{1/3} \]  
\[ \zeta = a, b, c, \alpha \text{ and } \delta \]  
III-18

where the summations are for all the components.

These results have been included so that any differences between the predictions of the two models for gases other than propane could be examined. As in the \( \Delta T_1 \) case, there is a reversal in the \( t_c \) trends with an apparent convergence to the same \( t_c \) as surge is approached. The explanation given for the \( \Delta T_1 \) case must hold here also. A point worthy of mention is that the CSRGM predicts surge for any ethane concentration greater than 13.9 mole per cent while the ethane concentration for which the CSIGM predicts surge is 40 mole per cent and greater.

The responses shown in Figures III-5 were obtained by perturbing the load demand rate as was done in Chapter II. Note that the dynamics exhibited by these responses are essentially the same as those predicted by the CSIGM. Table III-2 indicates that the new steady state values of discharge pressure as predicted by each model are the same. Thus, it is apparent that if a compressor study is to be undertaken which entails ascertaining the effects of load demand rate changes on proposed control schemes, the simpler CSIGM would be the preferred model.

**Closed-Loop Results**

As a continuation of the analysis of the CSRGM, the discharge pressure control loop was closed with the same pneumatic control
system as was utilized in the closed loop analysis of the CSIGM. Following the same procedure as was presented in Chapter II, eight process reaction curves for the range \(-1.4 \leq \Delta U_i \leq 1.4\) were determined utilizing the CSRG. These curves along with the necessary geometrical construction for obtaining the system gain \(K\), time constant \(\tau\) and dead time \(\Theta_d\) are shown in Appendix C. The results of these process reaction curves are tabulated in Table III-3 and the extrapolated curves for determining \(K\), \(\tau\) and \(\Theta_d\) for zero step change in the controller output \(U_i\) are shown in Figures III-6, III-7 and III-8 respectively. The extrapolated values are \(K = 13.6\) psi/°, \(\tau = 5.44\) seconds and \(\Theta_d = 1.1\) seconds, which differ from the CSIGM values of \(K = 11.2\) psi/°, \(\tau = 4.84\) seconds and \(\Theta_d = 0.87\) seconds. A comparison of these three curves with those of the CSIGM reveals a significant point. Even though the system parameters of the CSRG are the larger, the two sets of curves have almost the exact same shape indicating that over the range considered no additional nonlinearities of an extreme nature have been introduced.

The above values of \(K\), \(\tau\) and \(\Theta_d\) were substituted into the IAE formula of Lopez and the following controller parameters were obtained:

<table>
<thead>
<tr>
<th>PID or PI</th>
<th>3-mode controller</th>
<th>2-mode controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_c)</td>
<td>0.32</td>
<td>0.22</td>
</tr>
<tr>
<td>(K_R)</td>
<td>0.131</td>
<td></td>
</tr>
<tr>
<td>(\tau_d)</td>
<td>0.44</td>
<td></td>
</tr>
</tbody>
</table>

Each of these parameters is smaller than the respective parameters of the CSIGM except for \(\tau_d\) because of the increased \(K\), \(\tau\) and \(\Theta_d\).
### TABLE III-3. PROCESS REACTION CURVE RESULTS

<table>
<thead>
<tr>
<th>$U_i$ (psig)</th>
<th>$\Delta U_i$ ($U_i - U_{i,ss}$)</th>
<th>$\Delta P_d$ ($P_d - P_{d,ss}$)</th>
<th>$K$ ($\Delta P_d / \Delta U_i$)</th>
<th>$\tau$ (sec.)</th>
<th>$\theta_d$ (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.9</td>
<td>1.4</td>
<td>14.37</td>
<td>10.26</td>
<td>4.81</td>
<td>1.02</td>
</tr>
<tr>
<td>14.5</td>
<td>1.0</td>
<td>10.73</td>
<td>10.73</td>
<td>4.93</td>
<td>1.</td>
</tr>
<tr>
<td>14.0</td>
<td>0.5</td>
<td>5.88</td>
<td>11.76</td>
<td>5.1</td>
<td>1.05</td>
</tr>
<tr>
<td>13.75</td>
<td>0.25</td>
<td>3.12</td>
<td>12.48</td>
<td>5.17</td>
<td>1.08</td>
</tr>
<tr>
<td>13.25</td>
<td>-0.25</td>
<td>-3.89</td>
<td>15.56</td>
<td>5.72</td>
<td>1.14</td>
</tr>
<tr>
<td>13.0</td>
<td>-0.5</td>
<td>-8.76</td>
<td>17.52</td>
<td>5.96</td>
<td>1.22</td>
</tr>
<tr>
<td>12.5</td>
<td>-1.0</td>
<td>-21.42</td>
<td>21.42</td>
<td>6.42</td>
<td>1.36</td>
</tr>
<tr>
<td>12.1</td>
<td>-1.4</td>
<td>-35.63</td>
<td>25.45</td>
<td>7.05</td>
<td>1.38</td>
</tr>
</tbody>
</table>
Figure III-6. Determination of the Process Gain Based on the CSRGM.
Figure III-7. Determination of the Process Time Constant Based on the CSRGM.

Figure III-8. Determination of the Process Dead Time Based on the CSRGM.
Utilizing these two sets of controller parameters, the closed loop responses labeled the 2-mode and 3-mode controller shown in Figures III-9 through III-16 were determined. Figures III-9 and III-10 are for changes in the load demand rate of ± 5% of the design value while Figures III-11 and III-12 are for set point changes in the discharge pressure of ± 5 psi. Figures III-13 and III-14 are for disturbance inputs of 5 and -2 psi step changes in the inlet pressure and Figures III-15 and III-16 are for disturbance inputs of ± 10°F step changes in the inlet temperature. Included also in these figures are the closed loop results labeled optimal 3-mode controller which were obtained utilizing a 3-mode controller optimally tuned based on the nonlinear CSRGM as was done in Chapter II. The optimal parameters for each disturbance along with some pertinent comments are tabulated in Table III-4.

As is indicated, the controller with the optimal settings delivered superior control for each of the disturbances with the optimal responses to the step changes in set point bordering on being perfect. This was expected. However, what is not evident is the cost involved in determining the optimal settings. Because the CSRGM tended to deliver more oscillatory responses, considerably more computer time, approximately four times as much, was required to perform the optimal searches for the optimal tuning parameters. Of course this could have been obviated by easing the steady state time restraint which is, once the plant was perturbed the cost function was computed until the plant reached the new steady state. This restraint could have been eased by specifying a fixed final time. However, this would have
effected the optimal parameters somewhat and would also have clouded
the comparison of the two models.
Figure 111-9. Closed-loop Response of the CSRGM to 5% Decrease in Load Demand Rate.
Figure III-10. Closed-loop Response of the CSRG to 5% Increase in Load Demand Rate.
Figure III-11. Closed-loop Response of the CSRGM to 5 psi Step Change in Set-Point.
Figure III-12. Closed-loop Response of the CSRGM to -5 psi Step Change in Set-Point.
Figure III-13. Closed-loop Response of the CSRG to 5 psi Step Change in Inlet Pressure.
Figure III-14. Closed-loop Response of the CSRGM to -2 psi Step Change in Inlet Pressure.
Figure III-15. Closed-loop Response of the CSRGM to -10°F Step Change in Inlet Temperature.
Figure III-16. Closed-loop Response of the CSRGM to 10°F Step Change in Inlet Temperature.
<table>
<thead>
<tr>
<th>Disturbance</th>
<th>$K_c$</th>
<th>$K_R$</th>
<th>$\tau_d$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta F_o = 5%$</td>
<td>0.5</td>
<td>0.1</td>
<td>1.2</td>
<td>based on IAE</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>0.15</td>
<td>0.95</td>
<td>based on ISE</td>
</tr>
<tr>
<td>$\Delta F_o = -5%$</td>
<td>1.55</td>
<td>0.4</td>
<td>1.</td>
<td>based on IAE</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>0.5</td>
<td>1.5</td>
<td>based on ISE</td>
</tr>
<tr>
<td>$\Delta P_{sp}^d = 5 \text{ psi}$</td>
<td>3.63</td>
<td>0.01</td>
<td>0.56</td>
<td>steady state achieved after only 4 seconds.</td>
</tr>
<tr>
<td>$\Delta P_{sp}^d = -5 \text{ psi}$</td>
<td>1.77</td>
<td>0.039</td>
<td>1.12</td>
<td>steady state achieved after only 5 seconds.</td>
</tr>
<tr>
<td>$\Delta P_i = 5 \text{ psi}$</td>
<td>1.2</td>
<td>0.3</td>
<td>0.95</td>
<td>steady state reached after 14 seconds.</td>
</tr>
<tr>
<td>$\Delta P_i = -2 \text{ psi}$</td>
<td>0.25</td>
<td>0.188</td>
<td>1.06</td>
<td>steady state reached after 20 seconds.</td>
</tr>
<tr>
<td>$\Delta T_i = -10^\circ F$</td>
<td>2.13</td>
<td>0.11</td>
<td>0.71</td>
<td>$\varepsilon &lt; 0.3$ after 17 seconds $\varepsilon &lt; 0.05$ after 24 seconds steady state after 23 seconds</td>
</tr>
<tr>
<td>$\Delta T_i = 10^\circ F$</td>
<td>0.263</td>
<td>0.288</td>
<td>1.56</td>
<td>$\varepsilon &lt; 0.3$ after 17 seconds $\varepsilon &lt; 0.05$ after 20 seconds steady state after 34 seconds</td>
</tr>
</tbody>
</table>

3-mode controller settings: $K_c = 0.32$, $K_R = 0.131$, $\tau_d = 0.439$

2-mode controller settings: $K_c = 0.22$, $K_R = 0.175$
Summary

The thermodynamic model of the constant speed centrifugal compressor has been extended to account for the nonidealities of the gas by introducing into the model the modified Benedict-Webb-Rubin equation of state. As a result of the increased complexity, it would be virtually impossible to implement the resulting CSRGM on an analog computer and still maintain the same flexibility as is available from an analog model of the CSIGM. Though the CSRGM could be more readily implemented on a hybrid computer, the implementation would be much more difficult than the CSIGM. Of the three types of computers, the implementation of the CSRGM is more readily accomplished on a digital computer, though it too is much more difficult than the CSIGM.

The model has been successfully tested with both open- and closed-loop results having been determined through the use of CSMP and SL1. The only simulation modification required was that the integration step size utilized in the simulation of the CSIGM had to be halved for the CSRGM. This, along with the increased number of calculations, approximately tripled the computer time requirements for the CSRGM over that of the CSIGM. Also the amount of core required doubled.

The open loop results indicate that for the dynamic design of compressor systems and possibly in some control situations the errors introduced by utilizing the ideal gas could be significant though they are not extreme. The process reaction curves indicate that the nonlinearities of the two models are essentially the same and hence the CSIGM should be adequate in most control studies of the discharge
pressure control loop for a single stage machine. However, it is felt that the differences are significant enough to warrant using the CSRGM for multistage machines since the errors would tend to propagate and grow.

The closed loop results of the CSRGM are essentially the same as those of the CSIGM but with slightly more oscillations. The system parameters are larger and hence the controller parameters of the sub-optimal controllers are smaller except for the derivative time $\tau_d$. Due to the increased oscillatory nature, the optimal controller parameters of the two models are quite different.

Since most applications of centrifugal compressors require variable speed machines, the next chapter will develop the necessary modifications required to convert the CSIGM and CSRGM to variable speed models. It will also present the development of a prime mover model.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>constant in parabolic head equation</td>
</tr>
<tr>
<td>$A_o$</td>
<td>equation of state constant, psia$^6$ft$^2$/mole</td>
</tr>
<tr>
<td>$a$</td>
<td>equation of state constant, psia$^9$ft$^3$/mole</td>
</tr>
<tr>
<td>acfm</td>
<td>actual cubic feet per minute</td>
</tr>
<tr>
<td>acfs</td>
<td>actual cubic feet per second</td>
</tr>
<tr>
<td>$a^<em>,b^</em>,c^<em>,d^</em>$</td>
<td>constants in the heat capacity equation</td>
</tr>
<tr>
<td>$B_o$</td>
<td>equation of state constant, ft$^3$/mole</td>
</tr>
<tr>
<td>$b$</td>
<td>equation of state constant, ft$^6$/mole</td>
</tr>
<tr>
<td>$C$</td>
<td>molar density, lb-mole/ft$^3$</td>
</tr>
<tr>
<td>$C_o$</td>
<td>equation of state constant, psia$^6$ft$^{-2}$°R/mole</td>
</tr>
<tr>
<td>$C_p$</td>
<td>molar heat capacity, BTU/lb-mole °R</td>
</tr>
<tr>
<td>$C_v$</td>
<td>fraction of the design valve coefficient, heat capacity</td>
</tr>
<tr>
<td>$\bar{C}_v$</td>
<td>design valve coefficient, lbm/min</td>
</tr>
<tr>
<td>$c$</td>
<td>equation of state constant, psia$^{-2}$°R$^{2}$ft$^{-9}$/mole$^3$</td>
</tr>
<tr>
<td>$D_o$</td>
<td>equation of state constant, psia$^6$°R$^3$/mole$^2$</td>
</tr>
<tr>
<td>$E$</td>
<td>internal energy, BTU/lb-mole</td>
</tr>
<tr>
<td>$E_o$</td>
<td>equation of state constant, psia$^6$°R$^{-4}$/mole$^2$</td>
</tr>
<tr>
<td>$F$</td>
<td>mass flow rate, lbm/min or lbm/sec</td>
</tr>
<tr>
<td>$H$</td>
<td>head, ft-lbf/lbm</td>
</tr>
<tr>
<td>$H_{min}$</td>
<td>head at the &quot;stonewall&quot;, 14370 ft-lbf/lbm</td>
</tr>
<tr>
<td>$h$</td>
<td>molar enthalpy, BTU/lb-mole</td>
</tr>
<tr>
<td>$K$</td>
<td>process gain</td>
</tr>
</tbody>
</table>
\( K_c \) — controller proportional gain, psia/psig
\( K_r \) — controller reset rate, sec\(^{-1}\)
\( M \) — molecular weight of mixture, lbm/lb-mole
\( P \) — pressure, psia
\( Q \) — volumetric flow rate, acfm
\( R \) — gas constant, 10.731 psi-ft\(^3\)/lb-mole\(^\circ\)R
\( r \) — gas constant, 1.987 BTU/lb-mole\(^\circ\)R
\( S \) — molar entropy, BTU/lb-mole\(^\circ\)R
\( T \) — absolute temperature, \(^\circ\)R
\( t \) — time, sec or min
\( U \) — output of controller, 3 psig < _ U 15 psig
\( V \) — volume, ft\(^3\)
\( Y \) — mole fraction
\( \alpha \) — equation of state constant, ft\(^4\)/mole\(^3\)
\( \gamma \) — equation of state constant, ft\(^6\)/mole\(^2\)
\( \delta \) — equation of state constant, psia-ft\(^3\)-\(^\circ\)R/mole\(^3\)
\( \Delta \) — a difference operator
\( \varepsilon \) — error
\( \Theta_d \) — process dead time, sec
\( \eta \) — efficiency
\( \rho \) — specific density
\( \tau \) — process time constant, sec
\( \tau_d \) — controller derivative time, sec

**Subscripts and Superscripts**

\( B \) — n-butane
\( c \) — refers to compressor or cooler
\( d \) — discharge
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>di</td>
<td>isentropic discharge</td>
</tr>
<tr>
<td>E</td>
<td>ethane</td>
</tr>
<tr>
<td>e</td>
<td>refers to bypass loop</td>
</tr>
<tr>
<td>i</td>
<td>inlet, isentropic</td>
</tr>
<tr>
<td>m</td>
<td>mixture</td>
</tr>
<tr>
<td>o</td>
<td>outlet</td>
</tr>
<tr>
<td>p</td>
<td>propane</td>
</tr>
<tr>
<td>s</td>
<td>suction</td>
</tr>
<tr>
<td>ss</td>
<td>steady-state</td>
</tr>
<tr>
<td>v</td>
<td>valve</td>
</tr>
</tbody>
</table>
LITERATURE CITED


CHAPTER IV
THE VARIABLE SPEED MODELS

Introduction

All centrifugal compressors are in fact variable speed machines. They each have the capability to be operated over a range of speeds which encompasses the stated specific speed upon which their design is based. This range of speeds generally varies from fifty to seventy percent of the design speed to one hundred and five to one hundred and ten percent of the design speed depending upon the particular application. The criterion which determines whether they are called constant or variable speed machines is the driver. If the driver can and is operated over a range of speeds, then the associated compressor is classified as a variable speed machine while if the driver is constrained to operate at a fixed speed the associated compressor is classified as a constant speed machine.

Centrifugal compressors which are operated at a constant speed are in most cases driven by some type of electric motor. Because the motor can not in general be designed to operate at the high speeds typical of centrifugal compressors, a speed increasing gear is required to couple the motor to the compressor. This can be a highly inefficient operation and as a consequence the majority of the larger centrifugal compressors found in the process industries are driven by prime movers which are characterized by speed
ranges which match the speed range of the centrifugal compressor. Such prime movers are the steam and gas turbines.

There are, of course, other reasons for operating the compressor as a variable speed machine and each of them stems primarily from economic considerations concerning the prime mover. If a tail gas or other byproduct gas suitable for burning is available, the gas turbine is the likely choice. If high pressure steam is a byproduct of the process or is otherwise readily available then a condensing or extraction steam turbine would probably be chosen. Finally, the variable speed centrifugal compressor offers a wider range of controllable discharge pressure and hence greater flexibility through the controlled variation of the speed of the prime mover. Discharge pressure control through the manipulation of the speed of the driver also results in savings in operating costs since there are no head losses as there are across the throttle valve utilized in either suction or discharge throttle control. References [1] through [8], given at the end of this chapter, present detailed and thorough analyses of the considerations involved in choosing the prime mover and the increased capabilities obtained when a variable speed compressor is utilized.

Because of their wide use, a study involving the simulation of the centrifugal compressor would be incomplete without an analysis of the variable speed machine and this chapter is devoted to this end. The models, the CSIGM and the CSRGM, developed in Chapters II and III respectively, require only minor modifications to be converted from constant speed to variable speed models and as such only these modifications will be presented. They are presented in part through a unique correlation of the variable speed performance map. The
correlation, which involves the reduction of the family of variable speed head curves into a single curve, will be presented for the compressor simulated in this study, as well as for several other compressors in the form of plots. The compressors range in size from 1900 to 5800 horsepower and in speed from 3000 to 10000 revolutions per minute.

Unlike the constant speed centrifugal compressor, the variable speed centrifugal compressor is integrally related to its prime mover. For this reason, a mathematical model of a steam turbine will be developed. The steam turbine was chosen over the gas turbine for three reasons:

1. There seems to be a trend in industry today to utilize the steam turbine because of the increasing shortage of natural gas,

2. The data to model the steam turbine was readily available from the manufacturer, and

3. The steam turbine is much simpler to model.

A typical steam turbine-centrifugal compressor process arrangement will also be given so as to indicate that the physical system developed previously is essentially unchanged for the variable speed models.

As a continuation of the analysis given in Chapter III, a comparison will be made of the responses predicted by the model based on the MBWR equation of state, the variable speed real gas model (VSRGM) and the model based on the ideal gas law, the variable speed ideal gas model (VSIGM). This comparison will be given in terms of plots of the open loop discharge pressure and speed responses predicted by the two models when subjected to step changes in the inlet temperature.
and pressure, the gas molecular weight, the load demand rate, and
the speed of the prime mover. A closed loop analysis will not be
given as it is felt that a major research effort in itself is required
to adequately develop such an analysis.

The Plant

The plant or physical system upon which the development of the
VSIGM and VSRGM is based is shown in Figure IV-1. It consists of a
steam turbine directly coupled to a one stage, multiwheel variable
speed centrifugal compressor. With respect to the compressor and
its suction and discharge systems, this plant is exactly the same,
physically, as the plant utilized in the development of the constant
speed models. The piping and control valves are the same size
because the compressor performance curve utilized in the constant
speed models is the design curve of this variable speed compressor.
For this reason the steady state design given in Appendix A is
valid for both the constant and variable speed systems.

In an effort to present the typical arrangement for controlling
discharge pressure, the compressor has been provided with bypass for
anti-surge control and a cascaded discharge pressure to speed control
system. Even though these control loops will not be fully developed,
they have been appended so as to clarify the existence and location
of the control valves which are a part of the VSIGM and VSRGM. The
control valve in the compressor suction is, for this plant, a part
of the upstream process and as such would be controlled by some
upstream function. It is, however, the same butterfly valve utilized
in the constant speed models.
Figure IV-1. The Variable Speed Plant.
Correlating the Variable Speed Performance Map

With one exception, the variable speed compressor models, including the suction and discharge systems are exactly like their analogous constant speed models and this one exception consists of modifying equation II-30

\[ Q_c = Q_m + \sqrt{(H_m - 778\Delta h/M)/(A)} \]  

II-30

so as to account for changes in speed. In the development of the constant speed models it was tacitly assumed that the speed of the driver would always remain constant at the design speed and equation II-30 was entirely satisfactory. This is not the case for the variable speed compressor models as it is required that they be able to account for speed changes because the performance of the variable speed compressor is strongly dependent upon speed. However, this is the only change required to convert the constant speed models to variable speed compressor models and as such only the equation analogous to equation II-30 will be developed.

The development of this equation has as its basis the steady state energy balance in which the head \( H \) developed by the compressor is equated to the enthalpy change which the gas undergoes as it is compressed, equation II-23

\[ H = 778\Delta h/M. \]  

II-23

It was pointed out in Chapter II that it is generally accepted that \( H \) is only a function of the diameter \( D \) of the impeller, its angular velocity \( N \) and the volumetric flow rate \( Q_c \) through the compressor. For this case, as in the constant speed case, \( D \) is a constant.
However, \( N \) is a variable and hence \( H \) is a function of both \( Q_c \) and \( N \). This function is developed for each compressor by the compressor manufacturer based upon experimental measurements on similar machines and is expressed in the form of a compressor performance map. This map consists of plots of \( H \) versus \( Q_c \) at various speeds for the specified design suction conditions. Figure IV-2 gives the performance map for the compressor utilized in this study in terms of percent of the design point. The points represent data extracted from the manufacturer's map and the continuous curves were determined from an equation which is analogous to equation II-24. This equation will be developed subsequently.

In order to best simulate the variable speed centrifugal compressor with either a digital, an analog or a hybrid computer the relations between the three variables \( H, Q_c, \) and \( N \) must be cast into a more usable form than that given by the several curves within the performance map. Function generators, with interpolation between the curves, could be used but this requires an excessive amount of hardware for the analog simulation and an excessive number of calculations and possibly an excessive core requirement for the digital simulation. In either case inaccuracies are introduced making extrapolations in speed outside the speed range specified by the performance map unreliable enough to be prohibitive.

One such form, which has been successfully utilized in the simulation of variable speed centrifugal pumps and which, to this date, has been utilized in the simulation of the centrifugal compressor [9-12], is the set of relations known as the Fan Law. These
Figure IV-2. The Variable Speed Compressor Performance Map Used in the VSIGM and VSRGM.
relations state that power requirement is proportional to the cube of speed, the head is proportional to the square of speed and the volumetric flow rate is directly proportional to speed. The latter two relations imply that if $H/N^2$ is plotted versus $Q/N$, utilizing points taken from the performance map, the several curves should collapse into a single curve. This single curve could then be utilized in the simulation via a function generator or an equation obtained from a statistical analysis. In general, the resulting curve is a narrow band of points for the centrifugal pump but a widely scattered band of points for the centrifugal compressor.

Both Shepherd [13] and Csanady [14] point out in their excellent developments of the dimensional analysis as applied to the "pumping" of either a compressible or an incompressible fluid that the reason the widely scattered band of points is usually obtained for the compressor results from the failure of the Fan Law relations to account for compressibility effects. If the machine is a variable speed centrifugal pump these effects are second order in nature because the impeller rotates at such a slow speed and the fluid is incompressible. This is not the case for the centrifugal compressor as its impellers rotate at extremely high speeds, and the fluid is compressible. Their analyses indicate that these effects must be accounted for through the introduction of an additional term, the Mach number. However, this parameter can only be obtained from experimental studies by the manufacturer or the plant personnel and as such, efforts utilizing this concept surpass the purpose of this research.

As a result a different approach is required, one that utilizes only the data contained within the performance map.
This new approach involves an extension of the Fan Law relations to the affect that the compressibility effects are lumped into the speed exponents of the reduced head and flow parameters. That is, rather than constrain these exponents to values of 2 and 1 respectively, they are allowed to take on the values which collapse the set of head curves into one curve. One method of doing this is by "brute" force which consists of repetitively assuming values of a and b and plotting $H/N^a$ versus $Q_c/N^b$ until the correct combination is found which collapses the curves.

As can be readily visualized this could be a formidable task and initial efforts beared this out. Consequently another method, which simultaneously yields equation II-24's analog, was formulated so as to take advantage of the speed of the digital computer. This method consists of a repetitive search procedure utilizing a tandem nonlinear and linear least squares fit of the $(H,Q_c,N)$ data taken from the performance map supplied by the compressor manufacturer to the following equation:

$$y_c = y_m + A(x_c - x_m)^2$$ \hspace{1cm} IV-1

where

$$y_c = H/N^a$$ \hspace{1cm} IV-2

$$x_c = Q_c/N^b$$ \hspace{1cm} IV-3

and $y_m, x_m$ and $A$ are constants. The linear least squares (LLS) segment of the procedure is utilized to find the three constants in equation IV-1 and, based on these constants, to calculate the sum of squares (SSQ) of the errors between the given values of $y_c$ and the predicted values of $y_c$. The nonlinear least squares (NLS) segment of the procedure is utilized to find the values of the exponents.
a and b which minimizes SSQ. The technique utilized was the pattern search developed by Moore, et al. [15] which is denoted PATERN.

A flow diagram of the procedure is given in Figure IV-3. As is indicated an initial estimate of the values of a and b had to be supplied. So as to insure that the final value of SSQ was not a local minimum, sixteen different starting values were utilized. The starting value of a was sequentially set to either 0, 1, 2, or 3 and for each, the starting value of b was set to either 0, 1, 2 or 3. For each of the performance maps correlated all sixteen starting points produced the same final set of exponents and essentially the same final value of the SSQ.

The result of the fit of the data indicated by the points in Figure IV-2 is shown in Figure IV-4. The continuous curve, as well as the continuous curves shown in Figure IV-2, were determined from equation IV-1. Even though equation IV-1 does not exactly fit the data it was deemed satisfactory because of its simplicity. It is significant to note that values of a and b of 1.983 and 1.32 respectively, does produce collapsing of the curves into a very narrow band of points. Scattering occurs only at the higher flow rates at which the compressibility effects are the most significant.

In an effort to ascertain if the correlation may be considered to be of a general nature, the (H, Qc, N) data from eight additional performance maps obtained from various compressor manufacturers were correlated. The data represent compressors ranging in size from 1900 to 8600 brake horsepower and in speed from 3000 to 10,000 revolutions per minute. Table IV-1 lists the distinguishing characteristics of each compressor and Figures IV-5 through IV-12 give plots
Read the \((H,Q_c,N)\) data

Assume a value for \(a\) and \(b\)

Compute vectors of reduced head and flow parameters
\[ y_c = H/N^a; \quad x_c = Q_c/N^b \]

Perform LLS analysis on equation IV-1 obtaining
\[ y_m, x_m \text{ and } A \]

Compute
\[ SSQ = \Sigma c^2 \]

NLS Search for a new value of \(a\) and \(b\) utilizing PATERN

\( SSQ \) Minimized?

Yes

Exit with \(a, b, y_m, x_m, A\) and \(c\)

No

Figure IV-3. Computer Flow Diagram of the Correlation Procedure.
Figure IV-4. Reduced Performance Curve of Compressor Used in the VSIGM and VSRGM.

\[ a = 1.983 \]
\[ b = 1.32 \]
TABLE IV-1. COMPRESSOR CHARACTERISTICS AND CORRELATION RESULTS

<table>
<thead>
<tr>
<th>Brake Horsepower</th>
<th>Speed Range RPM</th>
<th>a</th>
<th>b</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$\sigma_3$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1900</td>
<td>8000-10000</td>
<td>2.109</td>
<td>1.113</td>
<td>1.1</td>
<td>1.3</td>
<td>1.7</td>
<td>95</td>
</tr>
<tr>
<td>2000</td>
<td>4000-5000</td>
<td>1.986</td>
<td>0.958</td>
<td>1.4</td>
<td>1.4</td>
<td>1.5</td>
<td>116</td>
</tr>
<tr>
<td>3100</td>
<td>3000-5000</td>
<td>2.018</td>
<td>1.54</td>
<td>0.5</td>
<td>0.6</td>
<td>2.4</td>
<td>208</td>
</tr>
<tr>
<td>3100</td>
<td>8000-10000</td>
<td>2.007</td>
<td>1.229</td>
<td>1.0</td>
<td>1.0</td>
<td>2.0</td>
<td>87</td>
</tr>
<tr>
<td>4300</td>
<td>8000-10000</td>
<td>1.879</td>
<td>1.128</td>
<td>1.0</td>
<td>1.3</td>
<td>1.2</td>
<td>98</td>
</tr>
<tr>
<td>5000</td>
<td>6000-8000</td>
<td>2.102</td>
<td>1.811</td>
<td>0.5</td>
<td>1.0</td>
<td>4.0</td>
<td>200</td>
</tr>
<tr>
<td>5400</td>
<td>4000-5000</td>
<td>2.019</td>
<td>1.27</td>
<td>2.2</td>
<td>2.2</td>
<td>2.8</td>
<td>124</td>
</tr>
<tr>
<td>5800</td>
<td>3000-5000</td>
<td>2.084</td>
<td>1.599</td>
<td>0.6</td>
<td>1.2</td>
<td>5.0</td>
<td>165</td>
</tr>
<tr>
<td>86005</td>
<td>4000-5000</td>
<td>1.983</td>
<td>1.32</td>
<td>2.4</td>
<td>2.4</td>
<td>3.0</td>
<td>88</td>
</tr>
</tbody>
</table>

1 The standard error is calculated via: $\sigma = \sqrt{\frac{\varepsilon^2}{(M-5)}}$; $\varepsilon = 100(H-H^P)/H$; $H^P$ = predicted head

$\sigma_1$ is the standard error based on the given values of a and b.

2 $\sigma_2$ is the standard error based on $a = 2$ and the given b.

3 $\sigma_3$ is the standard error based on $a = 2$ and $b = 1$.

4 $m$ is the total number of points used in the correlation.

5 The bottom line of the table is for the compressor utilized in the study.
Figure IV-5A. Reduced Performance Curve of the 1900 Horsepower Centrifugal Compressor.
Figure IV-5B. Variable Speed Performance Map of a 1900 Horsepower Centrifugal Compressor.
Figure IV-6A. Reduced Performance Curve of the 2200 Horsepower Centrifugal Compressor.
Figure IV-68. Variable Speed Performance Map of a 2200 Horsepower Centrifugal Compressor.
Figure IV-7A. Reduced Performance Curve of the 3100 Horsepower Centrifugal Compressor with the Speed Range of 3400-5100 RPM.
Figure IV-7B. Variable Speed Performance Map of a 3100 Horsepower Centrifugal Compressor with the Speed Range of 3400-5100 RPM.
Figure IV-8A. Reduced Performance Curve of the 3100 Horsepower Centrifugal Compressor with the Speed Range of 8000-9400 RPM.
Figure IV-8B. Variable Speed Performance Map of a 3100 Horsepower Centrifugal Compressor with the Speed Range of 8000-9400 RPM.
Figure IV-9A. Reduced Performance Curve of the 4300 Horsepower Centrifugal Compressor.
Figure IV-9B. Variable Speed Performance Map of a 4300 Horsepower Centrifugal Compressor.
Figure IV-10A. Reduced Performance Curve of the 5000 Horsepower Centrifugal Compressor.
Figure IV-10B. Variable Speed Performance Map of a 5000 Horsepower Centrifugal Compressor.
Figure IV-11 A. Reduced Performance Curve of the 5400 Horsepower Centrifugal Compressor.

\[
a = 2.019 \\
b = 1.27
\]
Figure IV-11 B. Variable Speed Performance Map of a 5400 Horsepower Centrifugal Compressor.
Figure IV-12 A. Reduced Performance Curve of the 5800 Horsepower Centrifugal Compressor.
Figure IV-12 B. Variable Speed Performance Map of a 5800 Horsepower Centrifugal Compressor.
of the resulting fits in terms of the reduced parameters and in terms of \( H, Q_c \) and \( N \). The constants and the design point data will not be given as this information has been restricted by the compressor manufacturers.

Also included in Table IV-1 are three different values for the standard error of the fit. \( \sigma_1 \) is based on the values of \( a \) and \( b \) given in the table, \( \sigma_2 \) is based on the value of two for \( a \) and the given value for \( b \) while \( \sigma_3 \) is based on the values of two for \( a \) and one for \( b \). Each of the fits yielded essentially the same values of \( x_m, y_m \) and \( A \) and hence the three values of the standard error of the fit provide a qualitative comparison of the effectiveness possessed by each set of exponents to bring about collapsing of the head curves.

A comparison of \( \sigma_1 \) and \( \sigma_2 \) indicates that the exponent \( a \) could have been given the value two for each of the compressors except the 1900, 5000 and 5800 horsepower ones. A further comparison of \( \sigma_1 \) and \( \sigma_3 \) indicates that only the 2000 and the 4300 horsepower machines possess curves which can be correlated with an \( a \) of two and \( ab \) of one. The major difference between these two machines and the others is that they were designed with a pressure ratio of two while the others were designed with pressure ratios ranging from three to seven. This explains the fit since at the lower pressure ratios the compressibility effects are second order. With the exception of these two machines, an \( a \) of two and \( ab \) of one produces an unacceptable scattering of the resulting reduced performance curve as is evidenced by the larger values of \( \sigma_3 \).
Converting the Constant Speed Models to Variable Speed Models

The conversion of the open loop constant speed models to open loop variable speed compressor models is accomplished by replacing equation 25 of Table II-1 for the CSIGM and equation 30 of Table III-1 for the CSRGM with the following set of equations:

\[ y_c = \frac{778Ah}{MN^a} \quad \text{IV-4} \]

\[ x_c = \begin{cases} 
\text{surge; } y_c > y_m \\
\frac{x_m + \sqrt{(y_m - y_c)/(-A)}}{y_m - y_c} \\
x_{c_{\text{max}}}; \quad y_c \leq y_{c_{\text{min}}}
\end{cases} \quad \text{IV-5} \]

\[ Q_c = x_c N^b \quad \text{IV-6} \]

where \( x_{c_{\text{max}}}, \) and \( y_{c_{\text{min}}} \) are respectively the reduced head and volumetric flow rate at the "stonewall". This set of equations was obtained from equations IV-1 through IV-3 and equation II-23. After making these replacements in the indicated tables, the resulting sets of equations along with the following steam turbine equations represent respectively, the VSIGM and the VSRGM.

The Steam Turbine Model

The mathematical model of the steam turbine is based upon the following description of its operation as is depicted in the simplified representation given in Figure IV-13. High pressure steam enters the turbine at mass rate \( W \) passing through the governor valve into the steam chest. It exits the steam chest through the nozzle at mass rate \( W' \) and impinges upon the buckets and passes out as exhaust. As it impinges upon the buckets it imparts a torque to the rotor assembly causing the shaft to rotate at speed \( N \).
Figure IV-13. The Steam Turbine.
The equation required to represent this operation results from a torque or momentum balance taken about the turbine rotor. This balance equates the rate of accumulation of momentum to the difference between the torque \(T\) produced by the high pressure steam as it passes through the turbine and the load torque \(T_L\) which is demanded by the compressor

\[
\frac{d}{dt} \left( \frac{2\pi NI}{g_c} \right) = T - T_L
\]

where \(I\) is the moment of inertia of the rotor assembly and which is supplied by the manufacturer. The load torque is equal to the power demand of the compressor divided by the speed of the shaft and the efficiency of the turbine which is also supplied by the manufacturer

\[
T_L = \frac{F_H}{2\pi N n_c}
\]

The torque produced by the high pressure steam as it passes through the turbine is determined from a map of the turbine's performance characteristics which is developed by the turbine manufacturer based on experimental measurements of similar machines. A typical performance map gives the steam mass rate through the throttle valve as a function of the developed horsepower with speed as a parameter. The performance map of the turbine upon which the model is based is shown in Figure IV-14 in which percent of design steam rate is plotted versus percent of design horsepower. This map was received from the General Electric Company [16] and it represents the performance of a steam turbine which is in operation today.
Figure IV-14. Performance Characteristics of the Steam Turbine.
Since the three curves are so close together, it was assumed that the middle curve would adequately represent the performance of the turbine. Also, since the curve is essentially linear, it was assumed that a straight line equation would suffice and the slope and intercept were determined from the manufacturer's map resulting in

\[ W = aL + b \]  

IV-9

where \( L \) is load in horsepower. Multiplication of the slope by the factor \( 33000/2\pi N \) converts it from horsepower to torque resulting in

\[ T = \frac{33000}{2\pi Na} (W-b) \]  

IV-10

which is the desired equation which relates the torque developed by the steam as it passes through the turbine to the rate of flow of the steam into the turbine. Thus, the equation required to describe the dynamics of the turbine's rotor is

\[ N = \frac{g_c}{2\pi} (T - T_L) \]  

IV-11

where \( T_L \) and \( T \) are given respectively by equation IV-8 and IV-10.

To maintain the speed of the turbine at a desired value, a governor assembly, which is a complex hydromechanical mechanism is provided. It senses \( N \) through a gear assembly and produces a power piston movement of \( Z \) inches if there is an error between the desired speed and \( N \). This movement causes, through a mechanical linkage, the governor valve plug to move \( Y_t \) inches which changes the inlet rate of steam and thus the speed of the shaft.

Even though the governor will not be included in the model the equations representing the dynamics of the valve and its positioner
as well as the valve flow characteristics have been included so that the open loop response of the compressor and turbine could be determined for changes in the governor output. The flow characteristics of the valve are based on data received from General Electric [16] and are shown in Figure IV-15. This function was implemented into the model via a function generator.

The equation required to describe the dynamics of the valve and its positioner is also based on information received from General Electric [16], and is obtained from the following considerations. First, assume that as the power piston moves from 0 to 1.25 inches, the normalized throttle valve position varies from 0 to 1. Further assume that the valve dynamics can be adequately described by a first order lag with a time constant of 2.6 seconds. Then the required equation is

\[
\dot{Y}_t = \frac{1}{2.6} (0.8 Z - Y_t). \quad \text{IV-11}
\]

The final equation required to model the steam turbine stems from a mass balance about the steam chest. A Control Systems Analyst of General Electric [17] has indicated that a first order lag with a time constant of 0.1 seconds adequately describes the process as determined from experimental measurements. Thus for a steam rate into the stem chest of \( \dot{W} \) and a steam rate out of \( \dot{W}' \), the equation becomes

\[
\dot{\dot{W}} = 10 (\dot{W} - \dot{W}'). \quad \text{IV-12}
\]

An information flow diagram summarizing these developments is given in Figure IV-15. It also indicates the coupling between the variable speed compressor and the steam turbine.
Figure IV-15. Governor Valve Flow Characteristics.
Figure IV-16. The Steam Turbine Model.
The Open Loop Responses

The open loop responses of the VSRGM and the VSIGM for step changes in the inlet pressure of +2, +4 and +8 psi are shown in Figures IV-17 through IV-20. Unlike the open loop responses of the constant speed models, the discharge pressure responses indicate the existence of strong nonlinearities as is apparent in Figures IV-17 and IV-19. However, the responses of the steam turbine, shown in Figures IV-18 and IV-20, do not indicate the nonlinearities. The four figures do indicate that utilization of the VSIGM rather than the VSRGM in a control study involving inlet pressure disturbances would not introduce significant errors for a single stage machine.

In Figures IV-21 through IV-24 are given the responses of the two models for step changes of +20, +10 and +5 °F in the inlet temperature. As in the above case, the discharge pressure responses shown in Figures IV-21 and IV-23 indicate extreme nonlinearities while the speed responses shown in Figures IV-22 and IV-24 do not. The variable speed models produce larger initial excursions in the discharge pressure than the constant speed models. Figure IV-21 indicates that the VSIGM yields significant errors for inlet temperature increases while Figure IV-23 indicates the opposite.

The open loop responses of the two models for step changes in the gas molecular weight of +0.5, +1, and +1.5 lbm/mole are shown in Figures IV-25 through IV-28. For this disturbance the differences between the responses of the two models are smallest for the increases in molecular weight. As in the above two cases, the discharge pressure responses indicate the existence of severe nonlinearities while the speed responses do not.
Figure IV-17. VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Increases in the Inlet Pressure.
Figure IV-18. VSRGM and VSIGM Open-Loop Speed Responses for Step Increases in the Inlet Pressure.
Figure IV-19. VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Decreases in the Inlet Pressure.
Figure IV-20. VSRGM and VSICM Open-Loop Speed Responses for Step Decreases in the Inlet Pressure.
Figure IV-21. VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Increases in the Inlet Temperature.
Figure IV-22. VSRGM and VSIGM Open-Loop Speed Responses for Step Increases in the Inlet Temperature.
Figure IV-23. VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Decreases in the Inlet Temperature.
Figure IV-24. VSRGM and VSIGM Open-Loop Speed Responses for Step Decreases in the Inlet Temperature.
Figure IV-25. VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Increases in Molecular Weight.
Figure IV-26. VSRGM and VSIGM Open-Loop Speed Responses for Step Increases in Molecular Weight.
Figure IV-27. VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Decreases in Molecular Weight.
Figure IV-28. VSRGM and VSICM Open-Loop Speed Responses for Step Decreases in Molecular Weight.
Figures IV-29 through IV-32 give the responses of the VSRGM and the VSIGM for step changes in the load demand rate of \( \pm 5 \), \( \pm 10 \) and \( \pm 15 \) percent of the design mass flow rate. For both the positive and the negative step changes, there is little difference between the predictions of the two models. The responses for the negative disturbances, shown in Figures IV-31 and IV-32 do not indicate any severe nonlinearities in terms of the discharge pressure or the speed while Figures IV-29 and IV-30 indicate extreme nonlinearities for the positive disturbances. These four figures vividly illustrate that the characteristics of the machines, the compressor and the turbine, are completely different for operation above the design point and operation below the design point.

The open loop responses of the two models for step changes in the governor output of \( \pm 0.05 \), \( \pm 0.1 \) and \( \pm 0.15 \) inches are shown in Figures IV-33 through IV-36. In this case the responses for the increased governor output, shown in Figures IV-33 and IV-34 indicate a linear process while the responses for the decreased governor output, shown in Figures IV-35 and IV-36 indicate a very nonlinear process. The more linear responses indicate the greatest amount of difference between the predictions of the two models.

The major point to be gained from an analysis of these open loop response curves is that the variable speed tandem is much more nonlinear than the constant speed machine. In all cases, the greatest difference between the predictions of the two models occurs for changes which drive the compressor beyond its normal operating point. The responses for inlet pressure decreases, inlet temperature increases, molecular weight decreases, load demand rate increases,
Figure IV-29. VSRG and VSIGM Open-Loop Discharge Pressure Responses for Step Increases in the Load Demand Rate.
Figure IV-30. VSRGM and VSIGM Open-Loop Speed Responses for Step Increases in the Load Demand Rate.
Figure IV-31. VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Decreases in the Load Demand Rate.
Figure IV-32. VSRGM and VSIGM Open-Loop Speed Responses for Step Decreases in the Load Demand Rate.
Figure IV-33. VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Increases in the Governor Output.
Figure IV-34. VSRGM and VSIGM Open-Loop Speed Responses for Step Increases in the Governor Output.
Figure IV-35. VSRGM and VSIGM Open-Loop Discharge Pressure Responses for Step Decreases in the Governor Output.
Figure IV-36. VSRGM and VSIGM Open-Loop Speed Responses for Step Decreases in the Governor Output.
and governor output increases indicate this trend. It is significant to note that increases in the load demand rate produce corresponding increases in the steam turbine speed. This contradicts the predictions which would ordinarily be obtained from an intuitive steady state analysis. This point illustrates that because of the many variables which come into play, the compressor-turbine tandem is very difficult to analyze without a dynamic model.

**Summary**

A unique correlation of the variable speed centrifugal compressor performance map which reduces the family of head curves to a single curve has been suggested in terms of reduced head and flow numbers containing variable exponents. The correlation stems from a dimensional analysis of the compressor performance but differs from the traditional approach which introduces the Mach number to account for compressibility effects. The proposed correlation accounts for the compressibility effects with no data required other than the performance map. It does so by allowing the exponents to take on the values which collapse the several head curves into a single curve. A block diagram of the search technique utilized to find the exponents has been included. The correlation has been successfully tested for nine different compressors ranging in size from 1900 to 5800 horsepower and in speed from 3000 to 10000 rpm.

Utilizing this correlation, the constant speed compressor models developed in Chapters II and III have been converted to variable speed models. Since the variable speed compressor is integrally related to its driver, a lumped parameter dynamic model of a steam
turbine has also been developed. It was developed in terms of performance data of an operating steam turbine. The performance data was received from the General Electric Company. The resulting variable speed model of the tandem arrangement has been summarized in an information flow diagram.

Open loop responses of the variable speed models in terms of the discharge pressure and the speed have been given in the form of continuous curves for step disturbances in the compressor inlet temperature and pressure, the gas molecular weight, the load demand rate and the steam turbine governor output. These disturbances indicate that the tandem arrangement possesses many nonlinearities. They also indicate significant errors between the predictions of the model based on the ideal gas law and the model based on the MBWR equation of state.

It is felt that with these models, the steam turbine manufacturer can develop more efficient governor mechanisms that those utilized in industry today. Also, the control engineer can utilize them to develop advanced control systems which are capable of delivering adequate control at all levels of operation of the tandem. Possibly the astute control engineer can even develop a suitable control scheme for starting up the system automatically.
NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>constant in parabolic head equation</td>
</tr>
<tr>
<td>a</td>
<td>exponent in reduced head number</td>
</tr>
<tr>
<td>B</td>
<td>constant in steam turbine performance equation</td>
</tr>
<tr>
<td>b</td>
<td>exponent in reduced flow number</td>
</tr>
<tr>
<td>F</td>
<td>mass flow rate, lbm/sec</td>
</tr>
<tr>
<td>FC</td>
<td>flow controller</td>
</tr>
<tr>
<td>FX</td>
<td>flow transmitter</td>
</tr>
<tr>
<td>g_c</td>
<td>32.2 lbm-ft/lbf-sec²</td>
</tr>
<tr>
<td>H</td>
<td>head, ft-lbf/lbm</td>
</tr>
<tr>
<td>h</td>
<td>molar enthalpy, BTU/lb-mole</td>
</tr>
<tr>
<td>I</td>
<td>inertia, lbm-ft²</td>
</tr>
<tr>
<td>K_t</td>
<td>constant in steam turbine performance equation</td>
</tr>
<tr>
<td>L</td>
<td>steam turbine load, hp</td>
</tr>
<tr>
<td>LLS</td>
<td>linear least squares</td>
</tr>
<tr>
<td>M</td>
<td>molecular weight, lbm/lb-mole</td>
</tr>
<tr>
<td>N</td>
<td>speed, rps</td>
</tr>
<tr>
<td>NLS</td>
<td>nonlinear least squares</td>
</tr>
<tr>
<td>P</td>
<td>pressure, psia</td>
</tr>
<tr>
<td>PC</td>
<td>pressure controller</td>
</tr>
<tr>
<td>PX</td>
<td>pressure transmitter</td>
</tr>
<tr>
<td>Q</td>
<td>volumetric flow rate, acfs</td>
</tr>
<tr>
<td>T</td>
<td>temperature, °F; torque, ft-lbf</td>
</tr>
<tr>
<td>W</td>
<td>steam rate, lbm/hr</td>
</tr>
</tbody>
</table>
\( x \) - reduced flow number

\( x_m, y_m \) - constants in parabolic fit of reduced performance

\( Y \) - normalized governor valve stem position

\( y \) - reduced head number

\( Z \) - governor output, in

\( \varepsilon \) - error

\( \Delta \) - a difference operator

\( \eta \) - efficiency

\( \sigma \) - standard error

**Subscripts and Superscripts**

\( c \) - refers to the compressor

\( d \) - discharge

\( g \) - governor

\( i \) - inlet

\( L \) - load

\( m \) - mixture

\( o \) - outlet

\( t \) - refers to the turbine
LITERATURE CITED

1. Hancock, R., "Drivers, Control and Accessories," Chemical Engineering, (June 1956), pp. 227-238.


CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The purpose of this study has been to develop dynamic models of large industrial centrifugal compressors, ones that require only the data developed by the manufacturer and which accompanies the machines. In this chapter, the conclusions drawn from the results presented in the previous chapters are summarized and recommendations for future research efforts are given.

The development of a dynamic model of the constant speed compressor utilizing the ideal gas law along with both open and closed loop simulation results were given in Chapter II. The open loop results indicated that the constant speed machine is very nonlinear. The characteristic time of the process was found to be strongly dependent upon the magnitude and direction of change of each of the load disturbances: changes in the inlet pressure, inlet temperature, gas molecular weight, and load demand rate. The system time constant, gain and dead time obtained from process reaction curves were found to be extremely sensitive to changes in the governor output. It was vividly demonstrated that tuning constants optimally determined through utilization of the nonlinear model yield far superior responses for the load disturbances than do those obtained from a conventional tuning technique.

In Chapter III, the modifications required to account for the nonidealities of the gas were given in terms of a modified Benedict-Webb-Rubin equation of state. Open and closed loop simulation results
utilizing the modified model, the CSRG, were obtained and compared with those of Chapter II. A comparison of the open loop results indicated that the assumption of an ideal gas may introduce significant errors. The same dependence of the characteristic time upon the magnitude and direction of change of the disturbance was found for changes in inlet pressure and load demand rate while an opposing dependence was found for changes in inlet temperature and molecular weight. Each of the parameters taken from the process reaction curves obtained from the CSRG were larger than those of Chapter II but their trends were the same.

The modifications required to convert the two constant speed models to variable speed models along with the development of a steam turbine model were given in Chapter IV. Open loop responses obtained from the resulting two models indicated that the variable speed centrifugal compressor is even more nonlinear than the constant speed one. These responses also indicated that the assumption of an ideal gas introduces errors which are more significant for the variable speed model than they are for the constant speed model.

Future research efforts regarding the simulation of the centrifugal compressor should be directed toward ascertaining the predictions of the models given here which can only result from a comparison with an actual machine. It is quite possible that these models do not adequately represent the actual machine but that a modification such as adding a discharge energy balance, utilizing an efficiency which is dependent on volumetric flow rate rather than a constant value or distributing the suction and discharge systems by adding resistance terms may bring them in line with real life conditions.
If it is found that the models adequately represent the actual compressor system then future research efforts should be directed toward modeling the multistage machine. A model capable of handling composition changes at the inlet of the system should also be investigated. Efforts should be made to determine if it is feasible to lump the suction and discharge volumes into a single volume. Should this be the case the model would be greatly simplified as the volumetric flow rate could then be determined directly rather than indirectly. Also, a convenient method for describing the surge of the compressor via a digital model is urgently needed.

In the area of process control, research is needed to adequately characterize the compression process in terms of the design parameters. With such a characterization the advanced control concepts such as adaptive tuning could be investigated. For the simple plant such as utilized in this study, research should be directed toward developing a multivariable control system, one capable of preventing excursions into the surge region even for decreased throughputs and at the same time providing adequate discharge or suction pressure control. Finally, research efforts should be directed toward developing nonlinear control systems, ones capable of providing adequate control at all levels of operation. With this the compressor could be started up automatically.
APPENDIX A

STEADY STATE DESIGN OF THE PLANT
Design Basis: The steady state design is based on propane being in the ideal gas state. The operating conditions have been chosen very near to those for which the compressor was originally designed. However, enough deviation has been included so that the possibly restricted details of the process for which the compressor was designed would not be revealed.

A. STEADY STATE OPERATING CONDITIONS

\[ T_g = 70^\circ F = 530^\circ R \]
\[ P_g = 75 \text{ psia} \]
\[ \rho_g = 0.581414 \text{ lbm/ft}^3 \]
\[ T_d = 109^\circ F = 569^\circ R \]
\[ P_d = 220 \text{ psia} \]
\[ \rho_d = 1.58858 \text{ lbm/ft}^3 \]
\[ Q_c = 17597 \text{ ft}^3/\text{min} @ \text{suction conditions} \]
\[ H = 19970 \text{ ft-lbf/lbm} \]
\[ W = 10231 \text{ lbm/min} \]

B. DESIGN OF PIPING

An estimate of the piping size is obtained through application of the mechanical energy balance for the steady flow of an incompressible fluid through a horizontal length of pipe. Even though the fluid is compressible the incompressible equation is used because the pressure drop will be limited to a maximum of 2 psi/100' of pipe and isothermal flow is assumed. The equations utilized are
\[ \Delta P = \frac{(4.84 f w^2)}{(\rho d^5)} \]
\[ Re = \frac{(380w)}{(d\nu)} \]

where

\( d \) = inside pipe diameter, inch

\( f \) = friction factor

\( \Delta P \) = pressure drop, psi/100'

\( Re \) = Reynolds number

\( \rho \) = density, lbm/ft\(^3\)

\( w \) = flow rate, lbm/min.

The values for \( f \), \( \mu \), and the surface roughness factor \( \epsilon \) were obtained from the fourth edition of *Perry's Chemical Engineers Handbook*, \( \mu \) from page 3-147, \( \epsilon \) from page 5-19 and \( f \) from page 5-20.

1. Suction Piping

@ the suction conditions \( \mu = 0.008 \) cp

Assuming a pipe diameter of 20"

\[ Re = \frac{(380 x 1023)}{(20 \times 0.008)} = 2.5 \times 10^7 \]

For commercial steel pipe \( \epsilon/d = 0.0001 \)

At this value of \( Re \) and \( \epsilon/d \), \( f = 0.003 \)

\[ \Delta P = \frac{(4.84 \times 0.003 \times 10231^2)}{(0.581414 \times 20.5)} \]

= 0.93 psi/100'

In a similar manner the following values of pipe size were obtained for the indicated pressure drop.

<table>
<thead>
<tr>
<th>Line Diameter</th>
<th>18&quot;</th>
<th>20&quot;</th>
<th>24&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta P/100' )</td>
<td>0.4</td>
<td>0.93</td>
<td>1.91</td>
</tr>
</tbody>
</table>
A twenty inch pipe was chosen because it produces a pressure drop near 1 psi/100' which is the value usually specified for this type of application as it is near the optimum between cost of piping and allowable pressure drop.

2. Discharge Piping

@ the discharge conditions, \( \mu = 0.009 \) cp.

Assuming a nominal pipe size of 16"

\[ Re = \frac{(380 \times 10^{231})/(16. \times 0.009)}{2.7 \times 10^7} \]

for which \( f = 0.003 \)

\[ \Delta P = \frac{(4.84 \times 0.003 \times 10^{231^2})/(1.58858 \times 16^5)}{1.5 \text{ psi/100'}} \]

Following this same procedure the values of pipe size shown below were obtained for the indicated pressure drop.

<table>
<thead>
<tr>
<th>Line Diameter</th>
<th>20&quot;</th>
<th>18&quot;</th>
<th>16&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta P/100' )</td>
<td>0.45</td>
<td>0.85</td>
<td>1.5</td>
</tr>
</tbody>
</table>

For the same reasons as stated above an eighteen inch pipe was chosen.

3. Bypass Piping

This line was sized to handle incipient surge flow (12330 acfm @ suction conditions) plus twenty per cent.

The following results were obtained

<table>
<thead>
<tr>
<th>Line Diameter</th>
<th>20&quot;</th>
<th>18&quot;</th>
<th>16&quot;</th>
<th>14&quot;</th>
<th>12&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta P/100' )</td>
<td>0.26</td>
<td>0.42</td>
<td>0.76</td>
<td>1.5</td>
<td>2.7</td>
</tr>
</tbody>
</table>

A fourteen inch pipe was chosen.
C. CALCULATION OF SUCTION AND DISCHARGE VOLUMES

In an effort to be as realistic as possible the following lengths of pipe were chosen:

1. Suction Volume
   - Length of 20" pipe = 50' . . . . . . volume = 96.5 ft$^3$
   - Length of 14" pipe = 50' . . . . . . volume = 47.0 ft$^3$
   - Total volume = $V_s = 143.5$ ft$^3$

2. Discharge Volume
   - Length of 14" pipe = 50' . . . . . . volume = 47.0 ft$^3$
   - Length of 18" pipe = 100' . . . . . . volume = 193.0 ft$^3$
   - Total volume = $V_d = 240.0$ ft$^3$

D. DESIGN OF CONTROL VALVES

Both the suction throttle valve (a butterfly valve) and the surge control valve (an equal percentage single seated valve) were sized according to the formulae given in the Instrument Engineers Handbook. These formulae were used to calculate the valve flow coefficient at the maximum expected flow rate ($\text{max. Process } C_v$). The valve $C_v$ was then calculated by dividing the $\text{max. Process } C_v$ by 0.8. The reasons for using this factor are that (1) it is not desirable to have the valve fully open at maximum flow since it is not then in a controlling position, (2) the valves supplied by a single manufacturer often vary as much as 10-20% in $C_v$, and (3) allowance must be made for pressure drop, flow rate, etc., values which differ from design.

1. Suction Throttle Valve
Utilize the procedure given by Moore\textsuperscript{1} for compressor throttle valves, take the valve pressure drop as 5% of the suction pressure.

\textbf{Max. Process} \( C_v = \frac{F_i}{(1.05 \sqrt{\rho \Delta P_v})} \) \\
\( \Delta P_v = 0.05 P_s = 3.75 \text{ psia} \) which was rounded up to 4 psia. \\
Utilize the average of the up- and downstream densities.

\( \rho = 0.5 \left( \rho_i + \rho_g \right) \) \\
\( \rho_i = \frac{P_i M}{RT_i} = \frac{(79 \times 44.09)}{(10.731 \times 530)} \) \\
\( = 0.611089 \text{ lbm/ft}^3 \) \\
\( \rho = (0.611089 + 0.581414)0.5 = 0.59625 \text{ lbm/ft}^3 \)

Design the valve to handle the flow at the "stonewall" (the maximum possible flow rate) at the above \( \Delta P_v \) and \( \rho \).

"stonewall flow" = 21300 ft\(^3\)/min \times 0.581414 \text{ lbm/ft}^3 \\
\textbf{Max. Process} \( C_v = 7633 \text{ lbm/min.} \) \\
\textbf{Valve} \( C_v = 7633/0.8 = 9541 \text{ lbm/min.} \) \\
This requires a 20" valve.

\textbf{2. Surge Control Valve} \\
Size this valve to handle 120\% of surge flow utilizing the design equation for critical flow.

\textbf{Max. Process} \( C_v = \frac{5.1 F_e}{C_f \sqrt{\frac{P_e}{M}}} \) \\
Assume the cooler maintains the outlet temperature at

530°R and that the pressure drop across the cooler is 3 psi at the maximum flow rate:

\[ F_e = 1.2 \times 12330 \text{ acfm} \times 0.581414 \text{ lbm/ft}^3 = 8602.6 \text{ lbm/min.} \]

\[ P_e = 217 \text{ psia} \]

\[ T_e = 530°R \]

\[ C_f = 0.9 \]

\underline{Max. Process} \[ C_v = 779 \text{ lbm/min} \]

\underline{Valve} \[ C_v = 779 / 0.8 = 974 \text{ lbm/min} \]

This requires a 10" valve.
APPENDIX B

IMPLICIT LOOP SOLVING Routines
Newton's Method

Given the function \( f(x) = \psi \), where \( \psi \) is a constant. The root \( a \) for which \( F(a) = f(a) - \psi = 0 \) is found according to the iterative procedure

\[
x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)} \quad ; \quad n = 0, 1, 2, \ldots
\]

where

\[
F'(x_n) = \frac{dF}{dx} \bigg|_{x=x_n}
\]

and

\[
x_0 = \text{the initial guess for } a.
\]

This iterative procedure was programmed according to the flow diagram shown in Figure B-1 and used to calculate \( T_{d1} \) and \( T_d \) in all three models utilizing the equations which follow.

In solving for \( T_{d1} \) and \( T_d \), the first initial guess was set equal to the respective steady state value of each. Each succeeding initial guess was equated to the corresponding value obtained at the immediately past integration step. In this manner, convergence to the correct root was obtained, on the average, in two iterations. To conserve computer time, the routines were programmed according to the equations shown below. The number of multiplications required is reduced drastically when utilizing these factored forms of the equations.

Solution for \( T_{d1} \) in CSIGM

For this case

\[
f(T_{d1}) = (a^* - r) \ln T_{d1} + T_{d1}(b^* + T_{d1}(c^*/2 + T_{d1} d^*/3))
\]

\[\text{B-4}\]
Figure B-1. Computer Flow Diagram for Newton's Method for Solving Implicit Functions.
\[ \psi = \ln(\rho_d/\rho_s) + (a^*-r)\ln T_S + T_S (b^*+T_S(c^*/2 + T_s d^*/3)) \quad \text{B-5} \]

\[ F'(T_{d1}) = (a^*-r)/T_{d1} + b^* + T_{d1}(c^* + T_{d1}d^*) \quad \text{B-6} \]

and

\[ N = 5. \quad \text{B-7} \]

Solution for \( T_d \) in CSIGM

For this case

\[ f(T_d) = T_d(a^* + T_d(b^*/2 + T_d(c^*/3 + T_d d^*/4))) \quad \text{B-8} \]

\[ \psi = \Delta h + T_s(a^* + T_s(b^*/2 + T_s(c^*/3 + T_s d^*/4))) \quad \text{B-9} \]

\[ F'(T_d) = a^* + T_d(b^* + T_d(c^* + T_d d^*)) \quad \text{B-10} \]

and

\[ N = 6. \quad \text{B-11} \]

Solution for \( T_{d1} \) in CSRGM and VSRGM

For this case

\[ f(T_{d1}) = (a^*-r)\ln T_{d1} + T_{d1}(b^*+T_{d1}(c^*/2 + T_{d1}d^*/3)) + [C_d(-B_o R + 1/T_{d1}(1/T_{d1}(1/T_{d1}(-2C_o + 1/T_{d1}(3D_o - 4E_o/T_{d1}))))) + C_d(0.2a6C_d/T_{d1}^2)]144/777.649 \quad \text{B-12} \]

\[ \psi = (a^*-r)\ln T_s + T_s(b^* + T_s(c^*/2 + T_s d^*/3)) + r\ln(C_d/C_s) + [C_s(B_o R + 1/T_s(1/T_s(2C_o + 1/T_s(-3D_o + 4E_o/T_s)))) + C_s(0.5(bR + \delta/T_s^2) + C_s(C_s( \]
\[ -0.2 \alpha \delta C_s / T_s^2 )) - 2c / (\gamma T_s^3 (1 - (1 + \\
0.5 \gamma C_s^2) \exp (- \gamma C_s^2)) \right) 144. / 777.649 \]

\[ F'(T_d) = (a^* - r) / T_d + b^* + T_d (c^* + T_d^d*) + ]C_d ( \\
(1/T_d (1/T_d (1/T_d (1/T_d (1/T_d (-4C_o + 1/T_d (-12D_o \\
+ 20E_o / T_d)))))) + C_d (\delta / T_d^3 + C_d (C_d ( \\
1 + 0.5 \gamma C_d^2 \exp (- \gamma C_d^2)) \right) 144. / 777.649 \]

and

\[ \psi = \left( C_s [B_o R T_s - 2A_o + 1/T_s (1/T_s (-4C_o + 1/T_s (5D_o \\
- 6E_o / T_s)))) + C_s [b R T_s - 1.5a - 26/T_s + C_s [C_s [ \\
0.2a (6a + 7\delta T_s) C_s ]] + c / (\gamma T_s^2 (3 - \\
(3 + 0.5 \gamma C_s^2 - \gamma C_s^2) \exp (- \gamma C_s^2)) \right) 144. / 777.649 \\
+ T_s (a^* + T_s (b^*/2 + T_s (c^*/3 + T_s d^*/4))) + h \]

\[ \mathbb{P}'(T_d) = \left( C_d [B R + 1/T_d (1/T_d (8C_o + 1/T_d ( \\
-15D_o + 24E_o / T_d )))) + C_d [b R + 26/T_d^2 + \\
C_d [C_d [-1.44 \alpha C_d / T_d^2 ]]] - (2c / \gamma T_d^3 (3 \\
(3 + 0.5 \gamma C_d^2 - \gamma C_d^2) \exp (- \gamma C_d^2)) \right) 144. / 777.649 \]

\[ N = 5. \]
and

\[ N = 6. \]
Iterative Procedure for Calculating Density

In the constant and variable speed real gas models formulated on the basis of the MBWR equations of state

\[ P = p(T, C) = CRT + (B_o RT - A_o - C_o / T^2 + D_o / T^3 - E_o / T^4) C^2 + (BRT - a - \delta / T) C^3 + \alpha(a + \delta / T) C^6 + (cC^2 / T)(1 + \gamma C^2) \exp(-\gamma C) \]

a trial and error iterative procedure is required to calculate the inlet density since the known variables are normally the inlet temperature and pressure. The method used is that presented by Johnson and Culver\(^1\) for the vapor phase. An initial density estimate of zero is used, with equal increments (the smaller of 0.1 \( P_i / RT_i \) and 0.01 lb-moles/ft\(^3\)) added to the density in the iterative procedure until the calculated pressure exceeds \( P_i \). The density is then reduced by the final increment, the increment is reduced by a factor of ten and then the new increment is added to the density iteratively until the calculated pressure again exceeds \( P_i \). This procedure is continued until the difference between the calculated pressure and \( P_i \) is less than or equal to 0.001 psi. It was found that convergence was obtained, on the average, in twenty-five iterations. A computer flow diagram illustrating this trial-and-error procedure is shown in Figure B-2.

Figure B-2. Trial and Error Procedure for the Inlet Density.
APPENDIX C

PROCESS REACTION CURVES
Figure C-1. CSIGM Process Reaction Curve for 1.4 psi Step Change in Controller Output.

\[ K = \frac{\Delta P_d}{\Delta U_1} = 8.37 \]
\[ \Theta_d = 0.71 \]
\[ \tau + \Theta_d = 5.11 \]
\[ \tau = 4.4 \]
Figure C-2. CSIGM Process Reaction Curve for 1. psi Step Change in Controller Output.
Figure C-3. CSIGM Process Reaction Curve for 0.5 psi Step Change in Controller Output.
Figure C-4. CSIGM Process Reaction Curve for 0.25 psi Step Change in Controller Output.
Figure C-5. CSIGM Process Reaction Curve for -0.25 psi Step Change in Controller Output.
Figure C-6. CSIGM Process Reaction Curve for -0.5 psi Step Change in Controller Output.
Figure C-7. CSIGM Process Reaction Curve for -1.0 psi Step Change in Controller Output.
Figure C-8. CSIGM Process Reaction Curve for -1.4 psi Step Change in Controller Output.
Figure C-9. CSRGM Process Reaction Curve for 1.4 psi Step Change in Controller Output.
Figure C-10. CSRM Process Reaction Curve for 1.0 psi Step Change in Controller Output.
Figure C-11. CSRGM Process Reaction Curve for 0.5 psi Step Change in Controller Output.
Figure C-12. CSRG Process Reaction Curve for 0.25 psi Step Change in Controller Output.

\[ K = \frac{\Delta P_d}{\Delta U_1} \]

\[ \theta_d = 1.08 \]

\[ \tau_0 = 6.25 \]

\[ \tau = 5.17 \]
Figure C-13. CSRGM Process Reaction Curve for -0.25 psi Step Change in Controller Output.
Figure C-14. CSRGM Process Reaction Curve for -0.5 psi Step Change in Controller Output.
Figure C-15. CSRGM Process Reaction Curve for -1.0 psi Step Change in Controller Output.
Figure C-16. CSRGM Process Reaction Curve for -1.4 psi Step Change in Controller Output.
APPENDIX D

THERMODYNAMIC DERIVATIONS
This appendix has been included to provide the derivations of the thermodynamic equations needed to calculate the changes in the molar entropy, molar enthalpy and the molar internal energy when the equation of state relates pressure as a function of temperature and molar density. The derivations are based on one mole and, up to the point at which the particular equation of state is utilized, are completely general. They are applicable for the closed system.

Consider the following change of state which is accompanied by the indicated general thermodynamic property change:

\[
\text{State A } \quad \Delta \Theta \quad \text{State B} \\
(C_A, T_A) \quad \rightarrow \quad (C_B, T_B)
\]

Propose the following reversible path:

\[
(C_A, T_A) \quad \Delta \Theta_I \quad (0, T_A) \quad \Delta \Theta_{II} \quad (0, T_B) \quad \Delta \Theta_{III} \quad (C_B, T_B)
\]

for which

\[
\Delta \Theta = \Delta \Theta_I + \Delta \Theta_{II} + \Delta \Theta_{III}.
\]  

**Entropy Change Derivation**

Utilizing equation D-1, the molar entropy change accompanying the change in state is given by

\[
\Delta S = \Delta S_I + \Delta S_{II} + \Delta S_{III}.
\]
The expression needed to evaluate each of these changes is obtained from the following considerations.

If the molar entropy is taken as a function of temperature and molar volume, its total differential is

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV.$$  \hspace{1cm} \text{D-2}

For a constant volume process, the first law of thermodynamics may be written

$$\left(\frac{\partial E}{\partial T}\right)_V = T \left(\frac{\partial S}{\partial T}\right)_V$$  \hspace{1cm} \text{D-3}

which implies

$$\left(\frac{\partial S}{\partial T}\right)_V = \frac{C_T}{T}.$$  \hspace{1cm} \text{D-4}

since by definition the left hand side of equation D-3 is the molar heat capacity at constant volume. Substitution of equation D-4 and the appropriate Maxwell's relation into equation D-2 yields after converting from molar volume to density

$$dS = \frac{C_T}{T} dT - \left(\frac{\partial P}{\partial T}\right)_C \frac{dC}{C^2}.$$  \hspace{1cm} \text{D-5}

which is the desired equation in differential form. Application of this equation to the proposed reversible path yields

$$\Delta S = \int_{C_A}^{C_B} \left(\frac{\partial P}{\partial T}\right)_C \frac{dC}{C^2} + \int_{T_A}^{T_B} \left(\frac{C_T - r}{T} \right) dT.$$  \hspace{1cm} \text{D-6}

where the substitution $C_p = C_v + r$ has been made in the second integral since this relation holds for an ideal gas. It should be noted that the first integral is $\Delta S_1$, the second $\Delta S_{II}$ and the third is $\Delta S_{III}$.

The equation of state utilized in this study is the modified BWR equation of state developed by Starling:

$$P = CRT + (B_0 RT - A_0 - C_o/T^2 + D_o/T^3 \null) - E_o/T^4)C^2 + (bRT - a - \delta/T)C^3$$

$$+ \alpha(a+\delta/T)C^6$$

$$+ \frac{cC^3}{T^3} (1 + \gammaC^2)\exp(-\gammaC^2).$$

Utilizing this equation, the change in pressure with temperature at constant density is

$$\frac{3P}{3T}_C = CR + (B_0 R + 2C_o/T^3 - 3D_o/T^4 + 4E_o/T^5)C^2$$

$$+ (bR + \delta/T^2)C^3 - \alpha\delta C^6/T^2$$

$$- \frac{2cC^3}{T^3} (1 + \gammaC^2)\exp(-\gammaC^2).$$

The relation utilized in this study to calculate the molar heat

---

capacity is that proposed by Thinh, et al.\(^2\) which has the form

\[
C_p = a^* + b^* T + c^* T^2 + d^* T^3.
\]

Substitution of equations D-8 and D-9 into equation D-6 yields after integration

\[
\Delta S = \left( C_A S_1(T_A) + C_A^2 S_2(T_A) - C_A^5 S_3(T_A) 
- S_4(T_A)^2 C_A - C_B S_1(T_B) - C_B^2 S_2(T_B) 
+ C_B^5 S_3(T_B) + S_4(T_B)^2 C_B \right)
+ S_id(T_B)^2 C_B \right)
\]

where \(\frac{144}{777.649}\) is a factor to convert from the mechanical energy units (psia-ft\(^3\)/lb-mole) to the thermal energy units (BTU/lb-mole) and the indicated functions have been defined for convenience and are

\[
S_1(T) = B_o R + 2C_o / T^3 - 3D_o / T^4 + 4E_o / T^5
\]

\[
S_2(T) = 0.5 (bR + \delta / T^2)
\]

\[
S_3(T) = 0.2 \alpha \delta / T^2
\]

\[
S_4(T,C) = \frac{2c}{\gamma T^3} \left[ 1 - (1 + 0.5\gamma^2) \exp(-\gamma T^2) \right]
\]

and

\[
S_id(T) = (a^* - r) \ln T + b^* T + (c^* / 2) T^2 + (d^* / 3) T^3
- r \ln C.
\]

Enthalpy Change Derivation

According to equation D-1, the molar enthalpy change accompanying the change of state is given by

$$\Delta h = \Delta h_I + \Delta h_{II} + \Delta h_{III}.$$  \hspace{1cm} D-16

The expression needed to evaluate each of these changes is obtained from the following derivation.

Taking the molar enthalpy as a function of temperature and pressure implies its total differential may be written

$$dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left(\frac{\partial h}{\partial P}\right)_T dP.$$  \hspace{1cm} D-17

The temperature coefficient is by definition the molar heat capacity at constant pressure and the pressure coefficient may be written

$$\left(\frac{\partial h}{\partial P}\right)_T = V + T \left(\frac{\partial S}{\partial P}\right)_T$$  \hspace{1cm} D-18

which is obtained from the first law of thermodynamics for a flow system. Substitution of the appropriate Maxwell's relation into equation D-18 with the subsequent substitution of the results into equation D-17 yields

$$dh = C_p dT + [V - T \left(\frac{\partial V}{\partial T}\right)_p] dP.$$  \hspace{1cm} D-19

where the temperature coefficient has been replaced with $C_p$. Rewriting this equation in terms of the molar density obtain

$$dh = C_p dT + \left[\frac{1}{C} + \frac{T}{C^2} \left(\frac{\partial C}{\partial T}\right)_p\right] dP.$$  \hspace{1cm} D-20

In order to place this equation in the proper form, make the following change of variables:
\[ \frac{dP}{C} = d(P/C) + \frac{P}{C^2} \; dC \quad \text{D-21} \]

and

\[ \left[ \left( \frac{\partial C}{\partial T} \right)_P \right]_T \; dP = \left[ \left( \frac{\partial P}{\partial T} \right)_C \right]_T \; dC \quad \text{D-22} \]

Equation D-22 is obtained according to the following:

\[ P = P(T,C) \]

implying the total differential of \( P \) is

\[ dP = \left( \frac{\partial P}{\partial T} \right)_C \; dT + \left( \frac{\partial P}{\partial C} \right)_T \; dC \]

which implies at constant pressure

\[ \left( \frac{\partial C}{\partial T} \right)_P = \left( \frac{\partial P}{\partial T} \right)_C \left( \frac{\partial C}{\partial P} \right)_T \]

from which equation D-22 is obtained.

Substitution of equations D-21 and D-22 into equation D-20 yields

\[ \Delta h = \left[ \left( \frac{\partial P}{\partial C} \right)_T \right]_A \; dT + \left[ \left( \frac{\partial P}{\partial T} \right)_C \right]_T \; \frac{dC}{C^2} \quad \text{D-23} \]

which is the desired equation in differential form for calculating the molar enthalpy change. Application of this equation to the reversible path yields

\[ \Delta h = \int_{C_A}^{C_B} \left[ \left( \frac{\partial P}{\partial C} \right)_T \right]_A \; dT + \int_{C_A}^{C_B} \left[ \left( \frac{\partial P}{\partial T} \right)_C \right]_T \; \frac{dC}{C^2} \]

\[ + \int_{T_A}^{T_B} \frac{dP}{C} \; dT + \int_{T_A}^{T_B} \left[ \left( \frac{\partial P}{\partial C} \right)_T \right]_B \; dT \]
Substitution of equations D-7, D-8 and D-9 into this equation yields after integration

$$
\Delta h = \left[ C_B h_1(T_B) + C_B h_2(T_B) + C_B h_3(T_B) + h_4(T_B, C_B) \right] \\
- \left[ C_A h_1(T_A) + C_A h_2(T_A) + C_A h_3(T_A) + h_4(T_A, C_A) \right] \frac{144}{777.649} \\
+ h_{1d}(T_B) - h_{1d}(T_A)
$$

where the indicated functions have been defined for convenience and are given by

$$
h_1(T) = B_o RT - 2A_o - 4C_o/T^2 + 5D_o/T^3 - 6E_o/T^4 \quad D-26
$$

$$
h_2(T) = bRT - 1.5a - 2b/T \quad D-27
$$

$$
h_3(T) = 0.2a(6a + 7b/T) \quad D-28
$$

$$
h_4(T, C) = \frac{C}{\gamma T^2} \left[ 3 \left( 3 + 0.5\gamma C \right)^2 - \gamma C^4 \right] \exp\left( -\gamma C^2 \right) \quad D-29
$$

and

$$
h_{1d}(T) = a^* T + \left( \frac{b^*}{2} \right) T^2 + \left( \frac{c^*}{3} \right) T^3 + \left( \frac{d^*}{4} \right) T^4 \quad D-30
$$

**Internal Energy Change Derivation**

The equation needed to calculate the change in the molar internal energy associated with the change in state is obtained from equation D-23 in the following manner. By definition, the molar enthalpy is given by

$$
h = E + P/C
$$

from which

$$
dE = dh = d(P/C).
$$
Substitution of this equation into equation D-23 yields the desired equation in differential form which is

$$JE = C_p \frac{dT}{p} + \frac{P}{C} \frac{dC}{T} - \frac{T}{C} \left( \frac{3P}{3T} \frac{dC}{C} \right)_T.$$  \hspace{1cm} D-32

Application of this relation to the proposed reversible path gives

$$\Delta E = \left[ \int_{C_A}^{T_B} \left( P-T \frac{3P}{3T} \frac{dC}{C} \right)_A \frac{dC}{T} \right]_{0}^{T_B}$$

$$+ \left[ \int_{T_A}^{T_B} \left( C_p - r \right) dT + \int_{0}^{C_B} \left( P-T \frac{3P}{3T} \frac{dC}{C} \right) \right]_{T_A}^{T_B}$$  \hspace{1cm} D-33

Substitution of equations D-7, D-8 and D-9 into D-33 yields

$$\Delta E = \left( C_A E_1(T_A) + C^2 E_2(T_A) - C_A^5 E_3(T_A) - E_4(T_A, C_A) \right)$$

$$- C_B E_1(T_B) - C_B^2 E_2(T_B) + C_B^5 E_3(T_B) + C_B E_4(T_B, C_B) \right) \frac{144}{777.649}$$

$$+ h_{id}(T_B) - h_{id}(T_A) - r(T_B - T_A)$$  \hspace{1cm} D-34

where the indicated functions are

$$E_1(T) = A_o + 3C_o/T^2 - 4D_o/T^3 + 5E_o/T^4$$  \hspace{1cm} D-35

$$E_2(T) = 0.5(a + 2\delta/T)$$  \hspace{1cm} D-36

$$E_3(T) = 0.2\alpha(a+2\delta/T)$$  \hspace{1cm} D-37

and

$$E_4(T, C) = \frac{3C}{T^2} \left[ 1 - (1 + 0.5C^2) \exp(-C^2) \right].$$  \hspace{1cm} D-38
NOMENCLATURE

\( a^*, b^*, c^*, d^* \) - constants in the heat capacity relation, units consistent with \( C_p \) in BTU/lb-mole °R

\( A_o, B_o, C_o, D_o, E_o, a, b, c, a, \delta, \gamma \) - constants in the equation of state, units consistent with pressure in psia, temperature in °R and density in lb-mole/ft\(^3\).

\( C \) - molar density, lb-mole/ft\(^3\)

\( C_p \) - constant pressure heat capacity, BTU/lb-mole °R

\( C_v \) - constant volume heat capacity, BTU/lb-mole °R

\( E \) - molar internal energy, BTU/lb-mole

\( h \) - molar enthalpy, BTU/lb-mole

\( P \) - pressure, psia

\( R \) - gas constant, 10.731 psia-ft\(^3\)/lb-mole °R

\( r \) - gas constant, 1.987 BTU/lb-mole °R

\( S \) - molar entropy, BTU/lb-mole °R

\( T \) - absolute temperature, °R

\( V \) - molar volume, ft\(^3\)/lb-mole

\( \Theta \) - general thermodynamic property
APPENDIX E

CSMP COMPUTER PROGRAM OF THE VSRCM

AND THE VSIGM
INITIAL
NOSORT

* WRT THE EQUATION OF STATE
  * C(1) = 80
  * C(2) = A0
  * C(3) = C0
  * C(4) = D0
  * C(5) = E0
  * C(6) = B
  * C(7) = A
  * C(8) = DELTA
  * C(9) = ALPHA
  * C(10) = C
  * C(11) = GAMMA

/ DIMENSION CPP(4), CPE(4), C PB(4), CP(5), C(11), CPR(11), CE(11), CB(11)
/ DATA CPP, CPE, CPB, CP(5), C(11), CPR(11), CE(11), CB(11) /-
  1.008568, 4.06388E-02, -1.1694E-05, 1.316498E-09, 1.292E5, 2.363078E-02, -5.114167E-06, 3.569067E-10, -5.854331E-01
  5.199243E-02, -1.496386E-05, 1.670641E-09, 1.98717/
  DATA CPR, 964762, 186347, 79617E, 04, 453708, 06, 256053, 08, 5, 46248, 40066, 4.150020, E02, 2.01402, 274461, E05, 4.56182/
  DATA CB, 1.56588, 32544, 7, 137436, E05, 333159, E06, 230902, 07, 9.14066, 71181, 8, 364238, E02, 4.00985, 700044, E05, 7.54122/
  DATA CE, .826059, 13439, 3, 295195, E04, 257477, E06, 146819, E08, 3, 11206, 22404, 5, 702189, 1, 906681, 681826, E04, 2.99656/

FIXED I, J, I1
F0 = FR*FN
YP = 1, -YE - YB
D0191 = 1, 4
19 CP(1) = YP*CPP(1) + YE*CPE(1) + YB*CPB(1)
M = YP*MP + YE*ME + YB*M
B = YP*MP + YE*ME + YB*MB
IF(IJ = 0) GO TO 71
70 RHOI = PI*M/(R*T1)
GO TO 85
71 POW = 1, 3
C(1) = YP*CPR(1) + YB*CB(1) + YE*CE(1)
C(11) = (YP*SQR(CPR(1)) + YB*SQR(CB(11)) + YE*SQR(CE(11)))*2
DO 4 I=2,5
  C(I)=(YP*SQRT(CPR(I)) + YB*SQRT(CB(I)) + YE*SQRT(CE(I)))**2
DO 5 I=6,10
  C(I)=(YP*CPR(I)*POW + YB*CB(I)*POW + YE*CE(I)*POW)**3
* TRIAL AND ERROR FOR THE INLET DENSITY.
DR=(0.1*PI)/(R*TI)
IF(DR-0.01)12,12,13
12 DELTA=DR
GO TO 14
13 DELTA=0.01
14 RHOMI=0.
15 P11=R*C(1)*TI-C(2)+1./TI*(-C(3)+1./TI*(C(4)-C(5)/TI))
PI2=R*C(6)*TI-C(7)-C(8)/TI
PI3=C(9)*(C(7)+C(8)/TI)
   PI4=C(10)/(TI*TI)*(1.+C(11)*RHOMI*RHOMI)* EXP(-C(11)*RHOMI*RHOMI)
PIC=RHOMI*(R*PI1+RHOMI*(PI1+RHOMI*(PI2+PI4+RHOMI*(RHOMI*(RHOMI*PI3))))))
IF(ABS(PIC-PI).LE.1.E-03)GO TO 16
IF(PIC-PI)17,16,18
17 RHOMI=RHOMI+DELTA
GO TO 15
18 RHOMI=RHOMI-DELTA
DELTA=DELTA/10.
GO TO 17
16 RHOMI=RHOMI*M
CI=RHOMI
85 CONTINUE
CVI=AFGEN(BUTFLY,ZIIC)
WPIC=78./60.*YTIC*WMAX
SORT
* CHARACTERISTICS OF THE BUTTERFLY VALVE.
FUNCTION BUTFLY=(0.0, 0.11, 1.5, 0.05, 0.07, 0.11, 0.03, 0.05, 0.09, 0.13, 0.17, 0.21, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 1.0, 1.05, 1.1, 1.15, 1.2)
* CHARACTERISTICS OF THE GOVERNOR VALVE
FUNCTION GUVALV=(0.6,0.78),(0.65,0.825),(0.7,0.862),(0.75,0.9),
(0.8,0.946),(0.85,0.958),(0.9,0.978),(0.95,0.992),(1.0,1.0)
PARAMETER FR=0.65
PARAMETER YE=0,YB=0.
PARAMETER PI=79,TI=530.
PARAMETER ZG=0.75622
CONSTANT J=0,J=0,KH=777.649
CONSTANT INERTA=3230,NTIC=81,YTIC=0.6
CONSTANT WMAX=74000,VTMAX=0.6
CONSTANT GC=32.2,KT=0.147,BT=1028.
CONSTANT XM=0.185,YM=1.07E-03,A=-3.24E-02,AL=2.8E=1.32
CONSTANT FN=188.7,MB=58.121,TMAX=25,CVMAX=10126.
CONSTANT EFF=.746,VD=240,VS=143.5,YCIC=7.1E-04
CONSTANT XM=0.29,TE=530,ZIIC=52.62,ZE=0.
CONSTANT MP=44.062,ME=30.046,R=10.73
CONSTANT TDERR=1.E-06,TDIERR=1.E-05,PDSP=220.
CONSTANT TSIC=530,TDIC=613,TDII=610,RHOSIC=.643642,RHODIC=1.81172
TIMER DELT=0.01,FINTIM=25,PRDEL=0.1,OUTDEL=0.1
METHDO TRAPZ
PRINT PD,TD,RHOD,PS,QC,FI,H,NT
PREPARE PD,NT
TITLE SIMULATION OF THE VSRGM AND THE VSIGM.

DYNAMIC
* TURBINE EQUATIONS
NG=0.2*NT
NTDOT=GC/(2.14159*INERTA)*(TQ-TQL)
NT=INTGRL(NTIC,NTDOT)
TQ=KT*WP-BT
TQL=FC*H/(2.14159*EFF+NT)
WP=INTGRL(WPIC,WPDOT)
WP=INTGRL(WPIC,WPDOT)
W=WFR*WMAX
YTDOT=0.8/2.6*ZG-1./2.6*YT
YT=INTGRL(YTIC,YTDOT)
PROCEDURE WFR=STEFLO(YT,VTMAX)
IF(YT.GT.YTMAX)GO TO 60
61 WFR=78./60. * YT
GO TO 62
60 WFR=AFGEN(GUVALV, YT)
62 CONTINUE
ENDPRO

`HIOTS=TS*(CP(1)+TS*(CP(2)/2.+TS*(CP(3)/3.+TS*CP(4)/4.)))`

PROCEDURE TQN. TDIN=DISS(TEMP. DELT. TDIIC. TDIIIC. TGD. TGDI)
   IF(TIME. GE. DELT) GO TO 22
21 TQN=TDIC
   TDIN=TDIIIC
   GO TO 20
22 TQN=TGD
   TDIN=TGDI
20 CONTINUE
ENDPRO

PROCEDURE DELHTS. DELS1=SUCENG(IJ. TS. CS. R)
   IF(IJ. EQ. 0) GO TO 86
87 DELHTS=0.
DELS1=0.
GO TO 88
86 CONTINUE
H1S=R*C(1)*TS-2.*C(2)+1./TS*(1./TS*(-4.*C(3)+1./TS*(5.*C(4)-
6.*C(5)/TS))))
H2S=R*C(6)*TS-1.5*C(7)-2.*C(8)/TS
H3S=0.2*C(9)*(-6.*C(7)+7.*C(8)/TS)
H4S=C(10)/(C(11)*TS)*3.-(3.+5*C(11)*CS*CS-C(11)*C(11)*CS*CS*4)...*
EXP(-C(11)*CS*CS))
   DELHTS=CS*(H1S+CS*(H2S+CS*(CS*(CS*H3S))))+H4S
S1S=R*C(1)+1./TS*(1./TS*(1./TS*(2.*C(3)+1./TS*(-3.*C(4)+4.*C(5)/TS))))
S2S=0.5*(R*C(6)+C(8))/TS*TS))
S3S=(-0.2*C(9)*C(8))/TS*TS)
S4S=-(2.*C(10))/(C(11)*TS*3.)*(1.-((1.+5*C(11)*CS*CS)*...
EXP(-C(11)*CS*CS))

235
DELSI = (CS*(S1S+CS*(S2S+CS*(CS*(CS+S3S))))+S4S)*144./777.649
88 CONTINUE
ENDPRO

CS=RHOS/M
RHOD=LIMIT(RHCS*.1.E05,RHOD1)
CD=RHOD/M
SIDTS=(CP(1)-CP(5))* ALOG(TS)+TS*(CP(2)+TS*(CP(3)/2.+TS*CP(4)/3.))
PHI=SIDTS+CP(5)*ALOG(CD/CS)-DELSI

* TRIAL AND ERROR FOR THE ISENTROPIC DISCHARGE TEMPERATURE.
PROCEDURE TD1=DITEMP(TD1N,TDIERR,PHI,CD,R,IJ)
IF(IJ.EQ.0)GO TO 72
73 CONTINUE
* FOR THE IDEAL GAS
TD1=IMPL(TD1N,TDIERR,FOFTD1)
SIDTD1=(CP(1)-CP(5))*ALOG(TD1)+TD1*(CP(2)+TD1*(CP(3)/2.+TD1*CP(4)/3.))
SDTD1P=(CP(1)-CP(5))/TD1+CP(2)+TD1*(CP(3)+TD1*CP(4))
DELS1=SIDTD1-PHI
FOFTD1=TD1-DELS1/SDTD1P
TDI=TD1
GO TO 74
72 CONTINUE
* FOR THE REAL GAS
TD2=IMPL(TDIN,TDIERR,FOFTD2)
SIDTDI=(CP(1)-CP(5))*ALOG(TD2)+TD2*(CP(2)+TD2*(CP(3)/2.+TD2*CP(4)/3.))
SDTDIP=(CP(1)-CP(5))/TD2+CP(2)+TD2*(CP(3)+TD2*CP(4))
SIDTI=-(R*C(1)+1./TD2*(1./TD2*(1./TD2*(1./TD2*(-3.*C(4)+4.*C(5)/TD2)))))
S2TDI=-0.5*(R*C(6)+C(8)/(TD2*TD2))
S3TDI=0.2*C(9)*C(8)/(TD2*TD2)
S4TDI=(2.*C(10))/(C(11)*TD2**3)*(1.-1.5*C(11)*CD*CD)*...
EXP(-C(11)*CD*CD))
DELS3=(CD*(SIDTD1+CD*(S2TDI+CD*(CD*(CD*S3TDI)))+S4TDI))144./777.649
DELS=SIDTDI+DELS3-PHI
SIDTDIP=-1./TD2*(1./TD2*(1./TD2*(-6.*C(3)+1./TD2*(12.*C(4)))))
-20.*C(5)/TD2)))))
S2TDIP=C(8)/TD2**3
S3TDIP=-(0.4*C(9)*C(8))/TD2**3
S4TDIP=-(6.*C(10))/(C(11)*TD2**4)*(1.-(1.+5*C(11)*CD*CD)*...)
EXP(-C(11)*CD*CD))
DELP=(CD*(S1TDIP+CD*(S2TDIP+CD*(CD*(CD*S3TDIP))))+S4TDIP)*...
144./777.649+SDTDIP
FOFTD2=TD2-DELS/DELP
TDI=TD2
74 CONTINUE
ENDPRO

HIDTDI=TDI*(CP(1)+TDI*(CP(2)/2.+TDI*(CP(3)/3.+TDI*CP(4)/4.)))
PROCEDURE DLHTDI=TROPIC(IJ,CD,TDI,R)
IF(IJ.EQ.0)GO TO 89
90 DLHTDI=0,
GO TO 91
89 CONTINUE
H1TDI=R*C(1)*TDI-2.*C(2)+1./TDI*(1./TDI*(-4.*C(3)+1./TDI*(5.*C(4)... -6.*C(5)/TDI))
H2TDI=R*C(6)*TDI-1.5*C(7)-2.*C(8)/TDI
H3TDI=0.2*C(9)*6.*C(7)+7.*C(8)/TDI
H4TDI=C(10)/(C(11)*TDI*TDI)*(3.-(3.+5*C(11)*CD*CD-C(11)*C(11)*CD**4... )**EXP(-C(11)*CD*CD))
DLHTDI=CD*(H1TDI+CD*(H2TDI+CD*(CD*(CD*H3TDI))))+H4TDI
91 CONTINUE
ENDPRO

DELHI=((DLHTDI-DELHTS)*144.)/777.649+(HIDTDI-HIDTS)
DELH=DELHI/EFF
H=KH*DELH/M
PROCEDURE QC,FINTIN=CMPRSRT(XM,YM,A,AL,BE,H,YCMIN,XCMAK,NT,TIME)
YSH=H/(60.*NT)**AL
IF(YC-YM)38,39,40
* SURGE
40 FINTIM=TIME
   QC=0.
   WRITE(6,600)
600 FORMAT(1X,'THE COMPRESSOR HAS ENTERED SURGE SO HALT THIS CASE',//)
   GO TO 46
*   ON THE VERGE OF SURGE
39 QC=(60.*NT)**BE*XM/60.
   GO TO 46
38 IF(YC.GT.YCMIN)GO TO 41
*   UP AGAINST THE STONE WALL
42 QC=(60.*NT)**BE*XCMAX/60.
   WRITE(6,620)
620 FORMAT(* COMPRESSOR AT STONE WALL*)
   GO TO 46
41 XC=XM+SQRT((YM-YC)/(-A))
   QC=(60.*NT)**BE*X)/60.
   CC CONTINUE
46 CC CONTINUE
ENDPRO

   PSI=DELHTS*144./777.649+HIDTS+DELH

*   TRIAL AND ERROR FOR THE ACTUAL DISCHARGE TEMPERATURE,
   PROCEDURE TD=DTMP(TDN,TDERR,PSI,CD,R,IJ)
   IF(IJ.EQ.0)GO TO 76
   77 CONTINUE
*   FOR THE IDEAL GAS
   TDIG=IMPL(TDN,TDERR,TDIG)
   HIDTGDG=TDIG*(CP(1)+TDIG*(CP(2)/2.+TDIG*(CP(3)/3.+TDIG*CP(4)/4.))))
   HIDTGP=CP(1)+TDIG*(CP(2)+TDIG*(CP(3)+TDIG*CP(4))))
   DHTDGDG=HIDTGDG-PSI
   FTDIG=TDIG-DHTDGDG/HIDTGP
   TD=TDIG
   GO TO 78
   76 CONTINUE
*   FOR THE REAL GAS
   TDR=IMPL(TDN,TDERR,FOFTDR)
HI:TDTR= TDR*(CP(1)+TDR*(CP(2)/2.+TDR*(CP(3)/3.+TDR*CP(4)/4.)))
HI:TDRP= CP(1)+TDR*(CP(2)+TDR*(CP(3)+TDR*CP(4)))
HI:TDF=R*C(1)+TDR-2.*C(2)+1./TDTR*(1./TDTR*(-4.*C(3)+1./TDTR*(5.*C(4)... -6.*C(5)/TDTR)))
H2TDR=R*C(6)*TDR-1.5*C(7)-2.*C(8)/TDTR
H3TDR=0.2*C(9)*C(6)+TDR*TDTR
H4TDR=C(10)/(C(11)*TDR+TDTR)*...
(3.-(3.+5*C(11)*CD+CD-C(11)*C(11)*CD**4)*EXP(-C(11)*CD*CD))
DLHTDR=(CD*(HI:TDTR+CD*(H2TDR+CD*(CD*(CD*HI:TDTR))))+H4TDR)*144./777.649
DHTDR=DLHTDR+HI:TDTR-PSI
H1TDAP=R*C(1)+1./TDTR(1./TDTR*(8.*C(3)+1./TDTR*(... -15.*C(4)+24.*C(5)/TDTR)))
H2TDAP=R*C(6)+2.*C(8)/(TDTR*TDTR)
H3TDRP=-1.4*C(9)*C(8)/TDTR*TDTR
H4TDRP=-2.*C(10)/(C(11)*TDR**3)*...
(3.-(3.+5*C(11)*CD+CD-C(11)*C(11)*CD**4)*EXP(-C(11)*CD*CD))
DHTDRP=(CD*(HI:TDRP+CD*(H2TDAP+CD*(CD*(CD*HI:TDAP))))+H4TDRP)... *144./777.649
DTDRP=DHTDRP+HI:TDAP
FOFTCR=TDR-DHTDR/DTDRP
TD=TDR
70 CONTINUE
ENDPRO

PROCEDURE PS=SUCP(IJ,TS,CS,R)
IF(IJ.EQ.0)GO TO 79
80 PS=CS*R*TS
GO TO 81
79 CONTINUE
PS1=R*C(1)*TS-C(2)+1./TS*(1./TS*(-C(3)+1./TS*(C(4)-C(5)/TS)))
PS2=R*C(6)*TS-C(7)-C(8)/TS
PS3=C(9)*(C(7)+C(8)/TS)
PS4=C(10)/(TS*TS)*(1.+C(11)*CS*CS)*EXP(-C(11)*CS*CS)
PS=CS*(R*TS+CS*(PS1+CS*(PS2+PS4+CS*(CS*(PS3*CS)))))
81 CONTINUE
ENDPRO
PROCEDURE PD=DISCP(IJ,TD,CD,R)
  IF(IJ.EQ.0)GO TO 82
  83 PD=CD*R*T0
  GO TO 84
  82 CONTINUE
  PD1=R*C(1)*TD-C(2)+1./TD*(1./TD*(-C(3)+1./TD*(C(4)-C(5)/TD)))
  PD2=R*C(6)*TD-C(7)-C(8)/TD
  PD3=C(9)*(C(7)+C(8)/TD)
  PD4=C(10)/(TD*TD)*(1.+C(11)*CD*CD)*EXP(-C(11)*CD*CD)
  PD=CD*(R*TD+CD*(PD1+CD*(PD2+PD4+CD*(CD*(PD3*CD)))))
  84 CONTINUE
ENDPROC

TGD1=DELAY(1.*DELT,TDI)
TGD=DELAY(1.*DELT,TD)

PROCEDURE FE=BYPASS(ZE,PD,PS,M,TD)
  PIE=PD-0.1*PD**0.7
  X2=PIE-PS
  IF(X2.LE.0.) GO TO 9
  11 X3=0.53*PIE-PS
  IF(X38.7,7
  * CRITICAL FLOW
  7 FE=1.3747/0.8*PIE*EXP(4.6*ZE)*SQRT(M/TE)
  GO TO 10
  * SUBCRITICAL FLOW
  8 RHOE=((PS+PIE)*M)/(2.*10.731*TE)
  FE=8.1795/0.8*EXP(4.6*ZE)*SQRT(RHOE*X2)
  GO TO 10
  9 FE=0.
  10 CONTINUE
  FE=FE/60.
ENDPROC

PROCEDURE FI=FLOWIN(CVI,PI,PS,RH0I,RH0S,CVIMAX)
DELP=PI-PS
IF(DELP)2.1
  1 RHC=(RHO1+RHO5)/2.
  2 FI=0.
  3 CONTINUE
ENDPRC

FS=FI+FE
RSDOT=(FS-RHOS*QC)/VS
RHOS=INTGRL(RHOSIC*RSDOT)
RDDOT=(RHOS*QC-FE-FO)/VD
RHOD=INTGRL(RHODIC,RDDOT)
TSDOT=(FI*(TI-TS)+FE*(TE-TS))/(RHOS*VS)
TS=INTGRL(TSIC,TSDOT)
FC=RHOS*QC

TERMINAL
J=J+1
  IF(J.EQ.3)FR=FR+0.05
IF(J.LT.6)GO TO 102
IF(IJ)101,101,100
  102 FR=FR+0.05
GO TO 108
101 ZG=0.69
107 J=0
  FR=0.85
IJ=1
DO 109 I=1,11
  109 C(I)=0.
YTIC=0.55
YTMAX=0.6
FN=170.3
CVIMAX=9629.7
EFF=0.846
TDIC=591.
TDIIC=600.
RHOSIC=0.581414
RHODIC=1.52945
108 FINTIM=25.
CALL RERUN
100 CONTINUE
END
STOP
ENDJOB
APPENDIX F

COMPUTER PROGRAM FOR CORRELATING

THE VARIABLE SPEED PERFORMANCE CURVES
DIMENSION P(2),S(2)
COMMON NPTS(10),SPEED(10),H(50,10),Q(50,10),
NCUR,TITLE(15),DATE(2)
DATA S/1..1./
IER=0
READ(5,103) DATE
103 FORMAT(2A4)
READ(5,100) TITLE, NCUR
100 FORMAT(15A4,I2)
DO1=1, NCUR
READ(5,101) NPTS(I),SPEED(I)
101 FORMAT(I2,8X,F10.0)
NPT=NPTS(I)
READ(5,102) (H(J,I),Q(J,I),J=1,NPT)
102 FORMAT(8F10.0)
DO6J=2,NPT
IF(H(J,I) .LE. H(J-1,I) .AND. Q(J,I) .GT. Q(J-1,I)) GO TO 6
7 WRITE(6,605) I,H(J,I),Q(J,I)
605 FORMAT(* INPUT ERROR IN CURVE NUMBER * ,I3/)
6051 * HEAD=* ,1PE12.5,* ACFM=*/E12.5)
IER=IER+1
6 CONTINUE
IF(IER .GT. 0) STOP
DO1J=1,NPT
H(J,I)=H(J,I)*1.E03
1 Q(J,I)=Q(J,I)*1.E03
DO2K=1,4
CALL SWITCH(4,JTEST)
IF(JTEST=2) 8,9,9
9 KL=K-1
JL=3-KL
P(1)=KL
P(2)=JL
PRINT 600,TITLE,DATE,P
600 FORMAT(1HL,15A4,5X,*,DATE *,2A4/
6001 * STARTING POINT ...... A= *,F3.1,2X,*8= *,F3.1//)
CALL PATTERN(2,P,S,4,1,C)
CALL PROC(P,-1.)
2 CONTINUE
8 PAUSE 'SET SENSE SWITCH 1 HIGH AND INPUT P(1) AND P(2), 2F6.0'
5 CALL SWITCH(1,ITEST)
   IF(ITEST-2) 3,4,4
3 READ(2,200) P
200 FORMAT(2F6.0)
   IF(P(1).LE.0.) GO TO 4
   PRINT 600,TITLE,DATE,P
   CALL PROC(P,-1.)
   GO TO 5
4 STOP
END
SUBROUTINE PROC(P*C)
DIMENSION P(2), B(8), Y(500), X(500)
COMMON NPTS, SPEED, H(50, 10), Q(50, 10),
1 NCUR, TITLE, DATE(2)
F(Z) = YC + A*(Z-XC)*(Z-XC)
PRNT=C
D01 I = 1, 8
1 B(I) = 0, 0
L = 0
D02 I = 1, NCUR
NPT = NPTS(I)
D02 J = 1, NPT
L = L + 1
Y(L) = H(J, I) / SPEED(I)**P(1)
X(L) = Q(J, I) / SPEED(I)**P(2)
B(1) = B(1) + Y(L)
B(2) = B(2) + X(L)
2 B(3) = B(3) + X(L) + X(L)
D03 I = 1, 3
3 B(I) = B(I) / L
D04 K = 1, L
D1 = Y(K) - B(1)
D2 = X(K) - B(2)
D4 = X(K) + x(K) - B(3)
B(4) = B(4) + D2*D1
B(5) = B(5) + D4*D1
B(6) = B(6) + D2*D2
B(7) = B(7) + D2*D4
4 B(8) = B(8) + D4*D4
B3 = B(6) + B(8) - B(7) * B(7)
B1 = (B(8) + B(4) - B(7) * B(5)) / B3
B2 = (B(6) + B(5) - B(7) * B(4)) / B3
B0 = B(1) - B1*B(2) - B2*B(3)
YC = B0 - (B1*B1) / (4 * B2)
XC = -B1 / (2 * B2)
A = B2
M=0
C=0.0
DO61=1,NCUR
NPT=NPTS(I)
DO62=1,NPT
M=M+1
HP=F(X(M))*SPEED(I)**P(I)
DIFF=H(J,I)-HP
6 C=C+DIFF*DIFF
IF(PRINT*GE.0.) RETURN
S=SORT(C/(M-5))
M=0
PRINT 600,YC,XC,A,C,S,P
600 FORMAT(///" THE EQUATION IS \( Y=\gamma \cdot C \cdot (X-XC)^2 \),
6001 \" WHERE \"/20X,' \( \gamma=\gamma,1PE12.5/20X,\)
6002 \") XC=\" \,E12.5/20X,' C=\" \,E12.5//12X,
6003 \" STATISTICS \"/20X,' SUM ERROR SQUARED=\" \,E12.5//
6004 \") STANDARD ERROR OF FIT=\" \,E12.5//12X,
6005 \" EXPONENTS \"/20X,' A=\" \,E12.5,10X,' B=\" \,E12.5)
6071=1,NCUR
DOJ=1,NPT
M=M+1
HP=F(X(M))*SPEED(I)**P(I)
DIFF=H(J,I)-HP
PCD=(DIFF*100.)/H(J,I)
7 WRITE(6,602) Q(J,I),H(J,I),HP,DIFF,PCD
602 FORMAT(4(2X,'1PE12.5'),2X,'0PF9.4)
WRITE(2,200),P
200 FORMAT(\" STD. ERR. = \" \,1PE12.5,2X,'P(1)=\" \,0PF8.5,2X,'P(2)=\" \,247
ENTRY BOUNDS(P, IG)
  ID=0
  IF(P(1) .LT. 0 .OR. P(2) .LT. 0) ID=1
  RETURN
END
SUBROUTINE PATTERN(NP,P,STEP,NPASS,IO,COST) PTRN0000
C*****************************************************************************PTRN0001
C GENERAL MULTIVARIABLE SEARCH PROGRAM TO MINIMIZE A COST FUNCTION PTRN0004
C USING PATTERN SEARCH MODIFIED TO INCLUDE CONSTRAINTS PTRN0005
C PTRN0006
C*****************************************************************************PTRN0007
C--DEFINITIONS PTRN0008
C NF----- NUMBER OF PARAMETERS TO BE SEARCHED (INTEGER) PTRN0009
C P----- PARAMETERS TO BE SEARCHED (VECTOR OF LENGTH NP) PTRN0010
C STEP-- INITIAL STEP SIZE OF EACH PARAMETER (VECTOR OF LENGTH NP) PTRN0011
C NPASS-- NUMBER OF PASSES THROUGH PATTERN WITH THE STEP SIZE OF EACH PTRN0012
C PARAMETER REDUCED BY A FACTOR OF 10 PTRN0013
C IO----- OUTPUT OPTION AVAILABLE IN SUBROUTINE PATTERN PTRN0014
C IO=0.... NO OUTPUT PTRN0015
C IO=1.... FINAL ANSWER ONLY, PRINTED PTRN0016
C IO=2.... RESULTS OF EACH ITERATION PRINTED PTRN0017
C IO=3.... RESULTS OF EACH STEP PRINTED PTRN0018
C COST-- CURRENT VALUE OF THE CRITERION FUNCTION BEING MINIMIZED PTRN0019
C BCUNDS(P,IOUT)--- SUBROUTINE WRITTEN BY USER TO CHECK FOR BOUNDARY PTRN0020
C VIOLATIONS FOR A PARTICULAR VALUE OF THE VECTOR P. PROGRAM PTRN0021
C SHOULD BE WRITTEN SO WHEN A VIOLATION OCCURS IOUT IS SET EQUAL PTRN0022
C TO ONE AND WHEN NO VIOLATION OCCURS SET EQUAL TO ZEROD PTRN0023
C PTRN0024
C PROC(P,COST)--- SUBROUTINE WRITTEN BY USER TO CALCULATE A VALUE PTRN0025
C OF COST FOR A PARTICULAR SET OF PARAMETERS, P. PTRN0026
CPTRN0027
C*****************************************************************************PTRN0028
C DIMENSION P(NP),STEP(NP),B1( 10),B2( 10),T( 10),S( 10) PTRN0029
C PTRN0030
C**********************************************************************PTRN0031
C NOTE*** VECTORS B1,B2,T,S NEED ONLY BE DIMENSIONED BY A NUMBER PTRN0032
C EQUAL TO THE NUMBER OF PARAMETERS, NP. PTRN0033
C PTRN0034
C------ STARTING POINT PTRN0035
C NRD=NPASS PTRN0036
C L=1
C
ICK=2
ITTER=0
DO5 I=1,NP
81(I)=P(I)
82(I)=P(I)
T(I)=P(I)
5 S(I)=STEP(I)*10.
C----- INITIAL BOUNDARY CHECK AND COST EVALUATION
CALL BOUNDS(P,IOUT)
IF(IOUT.LE.0)GOTO10
   IF(IOUT.LE.0)GOTO6
   WRITE(6,1005)
   WRITE(6,1000)(J,P(J),J=1,NP)
6 RETURN
10 CI=0.
   CALL PROC(P,C1)
      IF(IOUT.LE.1)GOTO11
      WRITE(6,1001)ITTER,C1
      WRITE(6,1000)(J,P(J),J=1,NP)
C----- BEGINNING OF PATTERN SEARCH STRATEGY
11 DO99 INRD=1,NRD
    DO12 I=1,NP
12 S(I)=S(I)/10.
      IF(IOUT.LE.1)GOTO20
      WRITE(6,1003)
      WRITE(6,1000)(J,S(J),J=1,NP)
20 IFAIL=0.0
C----- PRETURBATION ABOUT T
    DO30 I=1,NP
      I=0
21 P(I)=T(I)+S(I)
      IC=IC+1
      CALL BOUNDS(P,IOUT)
      IF(IOUT.GT.0)GOTO23
      CALL PROC(P,C2)
      L=L+1
IF(IO.LT.3)GOT022
WRITE(6,1002)L,C2
WRITE(6,1000)(J,P(J),J=1,NP)

22 IF(C1-C2)23,23,25
23 IF(IFC .GE. 2)GOT024
S(I)=-S(I)
GOT021
24 IF(FAIL=IFAIL+1
P(I)=T(I)
GOT030
25 T(I)=P(I)
C1=C2
30 CONTINUE
IF(IFAIL.LT.NP)GOT035
IF(ICK.EQ.2)GOT090
IF(ICK.EQ.1)GOT035
CALL PROC(T,C2)
   L=L+1
   IF(IO.LT.3)GOT031
WRITE(6,1002)L,C2
WRITE(6,1000)(J,T(J),J=1,NP)

31 IF(C1-C2)32,34,34
32 ICK=1
DO33 I=1,NP
B1(I)=B2(I)
P(I)=B2(I)
33 T(I)=B2(I)
GOT020
34 C1=C2
35 IBl=0
DO39 I=1,NP
B2(I)=T(I)
IF(abs(B1(I)-B2(I)).LT.0.1*abs(S(I))) IBl=IBl+1
39 CONTINUE
IF(IB1.EQ.NP)GOT090
ICK=0
ITER = ITER + 1
IF(IO.LT.2)GOTO40
WRITE(6,1001)ITER,C1
WRITE(6,1000) (J,T(J),J=1,NP)

C------ACCELERATION STEP
40 SJ=1.0
DO45 II=1,11
DO42 I=1,NP
42 P(I)=T(I)
SJ=SJ-.1
CALL BOUNDS(T,I.OUT)
IF(IOUT.LT.1)GOTO46
IF(II.EQ.11)ICK=1
45 CONTINUE
46 DO47 I=1,NP
47 B1(I)=B2(I)
GOTO20
90 DO91 I=1,NP
91 T(I)=B2(I)
99 CONTINUE
D0100 I=1,NP
100 P(I)=T(I)
COST=C1
IF(IO.LE.0)RETURN
WRITE(6,1004)L,C1
WRITE(6,1000) (J,P(J),J=1,NP)
RETURN
1000 FORMAT(10X,5(I7,E13.6))
1001 FORMAT(1X14HITERATION NO. ,I5/SX,5HCOST= ,E15.6,20X, 1 10HPARAMETERS)
1002 FORMAT(10X3HNO. ,I4, 8X5HCOST= ,E15.6)
1003 FORMAT(1X28HSTEP SIZE FOR EACH PARAMETER )
1004 FORMAT( 13HANSWERS AFTER ,I3,2X,23HFUNCTIONAL EVALUATIONS // 1 5X5HCOST= ,E15.6,20X,18HOPTIMAL PARAMETERS )
1005 FORMAT(1H135HINITIAL PARAMETERS OUT OF BOUNDS )
Frank T. Davis, son of Mrs. Eunice (Kea) Davis and the late Sam T. Davis, Jr. was born in Norfolk, Virginia, on February 22, 1942. He received his elementary and high school education at Crossville, Alabama, graduating in May, 1960.

He then attended Auburn University, Auburn, Alabama from January until June, 1961. In October, 1961 he enlisted in the Army of the United States. He was discharged from the Army in September, 1964 as an E-6, Staff-Sergeant. At that time he re-enrolled in Auburn University where he received the degree of Bachelor of Science, with Honor, in Chemical Engineering, in March, 1968. He received his Master of Science in Chemical Engineering from Auburn in June 1969. The author worked as a graduate assistant in Auburn University's Computer Center from March, 1968 to March, 1969.

He has worked for Enjay Chemical Company as a Process Control Engineer during the summers of 1969 and 1970 and is currently employed by Enjay.

The author is married to the former Mona Johnson of Boaz, Alabama. He is the proud father of two sons: Tate, age three and Toby, age eight months.

He is at present a candidate for the degree of Doctor of Philosophy in Chemical Engineering.
EXAMINATION AND THESIS REPORT

Candidate: Frank Tate Davis

Major Field: Chemical Engineering

Title of Thesis: The Simulation of Large Industrial Centrifugal Compressors

Approved:

[Signatures]

Major Professor and Chairman

Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:

November 27, 1972