Lumped Parameter Analysis of a Stringer Reinforced Plate Excited by Band-Limited Noise.

Dennis Joseph Bilyeu
Louisiana State University and Agricultural & Mechanical College

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LUMPED PARAMETER ANALYSIS OF A STRINGER
REINFORCED PLATE EXCITED BY BAND LIMITED
NOISE.

The Louisiana State University and Agricultural
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LUMPED PARAMETER ANALYSIS OF A 
STRINGER REINFORCED PLATE EXCITED BY BAND LIMITED NOISE

A Dissertation

Submitted to the Graduate Faculty of the 
Louisiana State University and 
Agricultural and Mechanical College 
in partial fulfillment of the 
requirements for the degree of 
Doctor of Philosophy 

in 
The Department of Mechanical, Aerospace, and 
Industrial Engineering

by

Dennis J. Bilyeu 
B. S., Louisiana State University, 1967 
M. S., Louisiana State University, 1968 August, 1972
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<td>Power spectral density of ( F(t) )</td>
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<td>Inches/pound force</td>
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<td>( F_{ko} )</td>
<td>Pounds force</td>
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<td>( \bar{x}^2 )</td>
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<td>( A )</td>
<td>(Inches)^2</td>
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<td>( J_{rst} )</td>
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The analytical and experimental investigation considers
the maximum root mean square response of a continuous elastic plate
subjected to a stationary or pseudo weakly stationary random excita-
tion. An analytical solution utilizing the lumped parameter
approach is given. The total mass of the plate and supporting
frame is divided into a fourteen mass system. The influence
coefficients associated with these fourteen masses are determined
both analytically and experimentally. A modal damping ratio matrix
is determined from the experimental response curves. These damping
ratios are found to be a function of frequency and not a function
of location on the plate. This method of solution allows one to
input a variable power spectral density excitation. The particular
excitation utilized in this investigation is bandwidth limited
on the lower end of the applied frequency spectrum at 25 hertz
and at 500 hertz on the upper end. The root mean square power
level of the random excitation over this frequency range is 149
decibels. The analytical results obtained from this method are
compared to the solution determined by other authors and to the
results of the experimental tests.

A square plate with reinforcing stringers which partition
the plate into a system of nine square panels was constructed and
instrumented. The excitation applied to the plate was random and
its power spectrum was nearly constant over the range of frequencies
spanned by the system's fourteen natural frequencies. The response
of the plate at nineteen points of interest was recorded from strain
gage outputs and transformed into power spectral density plots and root mean square values of displacement.

A quantitative comparison between the experimental and analytical results indicates a very good correlation on the root mean square displacements and a good correlation on the location in the frequency spectrum of the peaks in the power spectral density plots. Good correlation is observed regarding the particular power values of each peak at the lower frequencies, with a decrease in correlation as the frequencies approach the upper limit of the bandwidth.
CHAPTER I

INTRODUCTION

Many engineers have been focusing their attention in the past few years on the problems encountered when a random force is applied to structural materials. The advent of these problems was brought about by the development of jet engines for aircraft and rocket engines for spacecraft. It was found that the panels in the fuselage and wing structures of aircraft in the vicinity of the jet engines fail due to the acoustical random excitation they receive from the jet engine noise. A similar situation exists in the neighborhood of the nozzles on rockets engines. The basic reason this random excitation is so damaging is that it excites materials at all frequencies over the frequency range of (bandwidth) its power spectrum. If a natural frequency of the structure happens to exist in the bandwidth and the structure itself dissipates little or no energy (light damping), the resulting amplitude of vibration would become very large and failure should occur in a reasonably short period of time.

This type of response occurs in lightly damped systems because the system behaves as a narrow-band filter and absorbs energy primarily at its own natural frequency; this absorption of energy is in phase with the vibration of the system, causing the amplitude of vibration to increase with each successive cycle of vibration. The amplitude of a system with zero damping will tend to increase without bound; the amplitude of systems
with damping will tend to increase until it reaches the limiting amplitude defined by the parameters of the system, the limiting amplitude being larger with the lesser amount of damping.

Definition of the Problem

The stringer reinforced plate shown in Figure 1 is a configuration commonly found in aircraft, spaceships, and many other structures. The plate and each of the inner panel areas are square with the outer edges of the plate assumed to be fixed. The stringer reinforcements are an integral part of the plate, the panel areas being created by milling the plate into its present configuration from one sheet of metal (aluminum). The fixed edge condition was imposed by bolting an angle iron frame to the outer four inches of the plate and connecting the top and bottom frames with plates bolted into the frames. The latter plates were used to support the system during testing.

The problem is, given this plate and this excitation (Figure 2), predict the response of the plate. The problem is solved in two parts: one, a mathematical model was developed using a lumped parameter analysis; two, an experimental test was made on the particular plate shown in Figure 1.

It was decided that the form of the response should be the response power spectral density and the maximum root mean square displacements of the plate. The plate was excited by a large exponential horn measuring twelve feet square at its mouth and producing a wave front which was approximately plane with normal incidence to the plate and perfectly correlated in the horizontal and vertical
FIGURE 1: Geometric Representation of Test Plate
FIGURE 2. EXCITATION POWER SPECTRAL DENSITY OF THE RANDOM PRESSURE FIELD
directions. The random excitation was band limited from 25 Hertz to 500 Hertz. This bandwidth contains all the frequencies of interest for this particular problem and allows the attaining of a much higher root mean square power level than that attained by a wider bandwidth. A root mean square power level of approximately 149 decibels referenced to .0002 bars was attained by the horn. Figure 2 shows the power spectral density of the acoustical force exciting the plate.
CHAPTER II
PREVIOUS WORK

The purpose of this chapter is to give credit to the authors whose work is utilized as a guide in performing this investigation. The two methods of analysis commonly utilized in this type of investigation are the normal mode lumped parameter analysis and the transfer matrix technique, previously known as the Holzer-Myklestad method. Although extensive literature related to this problem was surveyed, the scope of this chapter is limited to the particular investigations which are similar to this investigation. These particular investigations are also selected because of their practicality.

The normal mode approach in the study of the response of continuous structures under random loading began with the work by Van Lear and Uhlenbeck (1)* in 1931. Recent authors using this approach includes Miles in 1954 (14), Lyon in 1956 (2), Eringen in 1957 (3), Thompson and Barton in 1957 (4), Powell in 1958 (5), Samuels in 1958 (6), Dyer in 1958, 1959 (7), (8), Bogdanoff and Goldberg in 1960 (9), Lin in 1963-1965 (10), (11), (26), Barnoski in 1967 (12), and Seireg in 1969 (13).

The work of Miles (14), Powell (5), and Lin (10), (11), (26) is the basis for the lumped parameter analysis performed by Barnoski (12) and Seireg (13). The particular method developed by Seireg is most like the analysis used in this investigation.

* Note: Numbers in parenthesis refer to references in bibliography.
The initial work in this area done by Miles (14) assumes the response of a panel is dominated by one (fundamental) mode. Consequently, he assumes the system can be represented by a single degree of freedom oscillator. His assumption leads to the very simple expression for the output power spectrum of panel response as follows:

$$\Phi_{xx}(w) = |H(w)|^2 \Phi_{FF}(w)$$  \hspace{1cm} (II-1)

where $\Phi_{FF}$ is the input power spectrum and $H$ is the transfer function. Substituting this expression into the standard mean square value equation,

$$\mathbb{E}[x^2] = \int_0^\infty \Phi_{xx}(w) \, dw$$  \hspace{1cm} (II-2)

one obtains the mean square value of the response. The work done by Miles considers only an excitation pressure field which is uniformly distributed over the panel in order that the assumption of fundamental-mode predominancy holds. If the excitation pressure field is not uniformly distributed over the panel, a more general investigation is warranted. The work by Miles, later extended by Powell (5), was eventually utilized to evaluate the response of aircraft panels to jet-engine noise. In many respects this work is similar to the analysis used in this investigation.

Powell (5) extended Miles' work to consider several modes of vibration and obtained a general expression for the output power
spectrum as follows:

$$\xi_{xx}(\omega) = \sum_{r} \sum_{s} |H_r(\omega)| \cdot |H_s(\omega)| \cdot \xi_{FF_0}(\omega) A^2 \frac{J_{rs}^2}{J_{rs_T}} \quad (II-3)$$

where, \( \xi_{FF_0} \) = excitation power spectrum at a reference point

\( A = \) overall area of the structure

\( J_{rs_T} = \) joint acceptance of the pressure field

$$J_{rs_T} = \frac{1}{A^2} \int_A R_{FF}(\omega;r,r';\tau) \alpha_r(r) \alpha_s(r') \, drdr' \quad (II-4)$$

where, \( \alpha_r \) and \( \alpha_s \) = normal mode amplitudes

\( r, r' \) = coordinates of a point on the structure

\( drdr' \) = differential area

\( R_{FF} \) = autocorrelation function of the excitation pressure

\( \tau = \) difference in response lags for two modes, \( r \) and \( s \) when excited at frequency \( \omega \).

Lin (10), (11), (26) later simplified Powell's results for the case of light damping and well separated resonant frequencies to the following equation:

$$\xi_{xx}(\omega) = \sum_{r} |H_r(\omega)|^2 \xi_{FF_0}(\omega) A^2 \frac{J_{rr}^2}{J_{rr_T}(\omega)} \quad (II-5)$$

Lin (11) eventually derived what is considered the most general form of the normal mode approach to determining the response of a linear continuous structure subjected to a random pressure. Lin also proved that the general results, Equation (II-3), arrived at by Powell can be deduced from the more general Equation (II-6) by assuming the excitation pressure field to be weakly stationary.

$$\xi_{xx}(r_1, r_2;\omega) = \int_A \Phi_{FF}(\rho_1, \rho_2;\omega) H(r_1, \rho_1;\omega) H^*(r_2, \rho_2;\omega) d\rho_1 d\rho_2 \quad (II-6)$$

where, \( r_1, r_2 \) = coordinates of the response point
\( p_1, p_2 = \text{coordinates of the excitation pressure} \)

\[ H^* = \text{complex conjugate of } H \]

Barnoski (12) applied the results determined by Lin (11) with the help of work by Crandall (28) and Roberts (29) to predict the mean square displacements and velocity response of rectangular plates subjected to a random excitation. Barnoski developed two dimensionless coefficients named (I and II), whose values range from zero to one and are to be multiplied respectively by the results of Lin for the root mean square displacement and velocity. These coefficients are determined by the particular damping ratio \((Z)\) of the system and the ratio of the cut-off frequency (upper bandwidth limit) \((\omega_c)\) of the input spectrum to the natural frequency of the system \((\omega_n)\). For damping ratio \((Z)\) less than 0.01, both the dimensionless coefficients I and II are nearly zero, for \(\omega_c/\omega_n\) somewhat less than one; they rapidly approach unity as \(\omega_c/\omega_n\) becomes slightly greater than one and converge to unity as \(\omega_c/\omega_n\) approaches infinity. Barnoski's results converge to the results given by Lin (11) for an excitation spectrum with a cut-off frequency which encompasses the natural frequencies of the system.

Seireg and Howard (30) developed an approximate normal mode method of analysis which permits any linear non-conservative system to be solved by superposition of uncoupled coordinates. The normal mode method does not generally apply to damped systems. Only a particular class of damped systems, originally defined by Rayleigh (31) and later generalized by Caughey (32), can be uncoupled by the same transformation which uncouples conservative systems. Foss (33) and O'Kelly (34) later described the complex transformations that are required to uncouple certain damped systems. These results,
although technically uncoupled, are so complicated that the primary objective of using normal modes is defeated.

Seireg and Howard (30) developed an approximate method which allows any lumped parameter linear system subjected to an arbitrary forcing function to be approximately represented by equations uncoupled by the same transformation which uncouple the conservative systems. The method developed by Seireg and Howard utilizes experimental response curves determined by exciting the systems with pure tones. These response curves are utilized to determine the damping ratios \((Z_{ik})\) as described by Bruel and Kjaer (17). The amplitude ratios at each natural frequency are used to approximate the eigenvectors of the system. Knowing the relation between the eigenvectors \((V_{ik})\) and the modal participation factors \((a_k)\) to be by definition,

\[
\sum_k V_{ik} a_k = 1 \quad (II-7)
\]

the system of simultaneous equations may be solved for the modal participation factors \(a_k\). The single-resonance assumption that the nonresonant components of the damping are to have negligible effect at the natural frequencies, produce what are called the fictitious damping ratios \((\xi_{ik})\). They are defined by,

\[
\xi_{ik} = \frac{V_{ik} a_k}{Z_{ik}} \quad (II-8)
\]

These fictitious damping ratios are used to evaluate the fictitious displacements in each mode of vibration. The summation of the independent modal displacements produces the total displacement at each point on the plate. The fictitious damping ratios reduce to modal damping ratios when the damping of the system is small. This method produces resulting displacements which vary from the
expected values by less than ten per cent when the damping ratio is less than 0.10.

The method of analysis utilized in this investigation is similar to the method used by Seireg and Howard (30). The essential difference is that the damping ratios in this investigation are sufficiently small (Figure 21) to justify utilizing modal damping ratios rather than fictitious damping ratios. In this investigation, the eigenvectors, eigenvalues, and modal participation factors are determined analytically rather than by the use of the experimental data.

The method of transfer matrices described by Lin (11), (26), McDaniel and Donaldson (27) is also a method of analysis for plates subjected to a random excitation. Dokanish (35) later expanded the transfer matrix method by combining it with the finite element technique. The general technique for applying this method of analysis is to assume the plate is composed of several rows of panels; each row of panels is assumed to be separated by inflexible stringers. The panels in each row are separated by flexible stringers which are perpendicular to the inflexible stringers. The response of this system is assumed to be harmonic in the direction normal to the inflexible stringers and to be random in the direction parallel to the inflexible stringer. Each panel may then be subdivided into strips which have their edges parallel to the stringers. The stiffness and mass matrices for each strip are then calculated. The equilibrium equations are determined to obtain the relation
between the right and left edges of each strip. Requirements of
displacement continuity and force equilibrium at the edges common
to two adjacent strips gives the transfer matrix relation.
Successive matrix multiplication finally relates the variables
of the left and right boundary of each panel and eventually of
the entire plate. Boundary conditions require the determinant
of a portion of the overall transfer matrix to vanish at the natural
frequencies of the system. By substituting values of frequency
until the determinant vanishes, the natural frequencies are determined.
The method also produces the mode shapes of the system.

Another method of analyzing the random response of plates
is the utilization of an analog computer to model and solve the
problem. Murphy and Swift (37) and Barnoski (12) are some of the
authors who have utilized this method of analysis. Through the use
of analog computers and control system techniques, a deterministic
function is generated which has the correct statistical properties
of the random excitation. This function is used to solve in a
deterministic fashion for the response, which is described
statistically in terms of displacement or stress.

For this method of analysis the kinetic and strain energies
are calculated and the equations of motion are derived by using
Lagrange's equation. A Gaussian noise generator supplies the "white
noise" which is passed through additional shaping filters to produce
volatates which have the statistical representation of the forcing
function. The differential equations are then solved by the analog
computer.

The selected investigations discussed in this chapter indicate
that a great deal of work has been done on this problem. The
primary development in this area has been in the realm of analysis. Although more work needs to be done analytically, the primary concern now is to develop techniques so that the theories may be applied to practical structures. Relative to the analytical work that has been accomplished, very little experimental work has been performed to verify the results of the analysis. A spectacular example of the additional work needed in this area is the unexpected fatigue cracks which developed on the Air Force's huge C5-A transport aircraft. These cracks are believed to be caused by the same type of random excitation which is the motivation of this investigation. Although this problem originated with modern flight vehicle structures excited by jet or rocket engine noise, it has been found to apply to many other systems.
CHAPTER III

ANALYTICAL EQUATIONS OF MOTION

This chapter establishes a derivation for the mean square response of the panel structure by using an approximate normal-mode method for a damped, lumped parameter system. A scaled drawing of the actual plate is shown in Figure 1. This structure is represented by the lumped mass system shown in Figure 3. The displacements of the masses, \( x(I), I = 1, 2, 3, \ldots, 14 \), are in a direction perpendicular to the plane of the plate and referenced to inertial coordinates.

Energy expressions for the system are as follows:

\[
\text{Kinetic energy} = T = \frac{1}{2} \sum_{i=1}^{N} m_i x_i^2
\]

\[
\text{Potential energy} = U = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} K_{ij} x_i x_j
\]

\[
\text{Dissipated energy} = D = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} C_{ij} \dot{x}_i \dot{x}_j
\]

\[
\text{Work done} = W = \sum_{i=1}^{N} f_i x_i
\]

Lagrange's equation in normalized coordinates for a multi-degree of freedom system can be written as,

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{x}_k} \right) - \frac{\partial T}{\partial x_k} + \frac{\partial U}{\partial x_k} + \frac{\partial D}{\partial \dot{x}_k} = \frac{\partial W}{\partial x_k} \quad (III-1)
\]

where the coordinate transformation is defined as,
where,

\[ \begin{align*}
  m_1 &= .000791 & m_6 &= .000791 & m_{11} &= .000791 \\
  m_2 &= .003022 & m_7 &= .000791 & m_{12} &= .000791 \\
  m_3 &= .003022 & m_8 &= .000791 & m_{13} &= .000791 \\
  m_4 &= .003022 & m_9 &= .000791 & m_{14} &= .595238 \\
  m_5 &= .003022 & m_{10} &= .000791
\end{align*} \]

FIGURE 3: Lumped Mass Model of Test Plate
\[
\begin{align*}
\mathbf{x}_i &= \sum_{k=1}^{N} V_{ik} \mathbf{x}_k \\
\dot{\mathbf{x}}_i &= \sum_{k=1}^{N} V_{ik} \dot{\mathbf{x}}_k
\end{align*}
\]

(III-2)

This transformation is necessary in order to uncouple the equations of motion for the system. It is shown in Appendix A that the \( V_{ik} \)'s necessary to perform this transformation are the eigenvectors which describe the normal modes of the free, undamped vibration system. The eigenvalues associated with each of the eigenvectors are the undamped natural frequencies of the system and can be obtained along with the eigenvectors by solving the differential equation of motion for the free undamped vibration system.

Substitution of the transformations into the work and energy terms, and utilization of the orthogonality relationship given below,

\[
\begin{align*}
\sum_{i=1}^{N} m_i V_{ik} V_{im} &= \begin{cases} 
0, & \text{for } k \neq m \\
\sum_{i=1}^{N} m_i V_{ik}^2, & \text{for } k = m
\end{cases} \\
\sum_{i=1}^{N} \sum_{j=1}^{N} K_{ij} V_{ik} V_{jm} &= \begin{cases} 
0, & \text{for } k \neq m \\
\sum_{i=1}^{N} \sum_{j=1}^{N} K_{ij} V_{ik} V_{jk}, & \text{for } k = m
\end{cases}
\end{align*}
\]
with the frequency equation for the undamped free vibration system,

\[
\sum_{i=1}^{N} \sum_{j=1}^{N} K_{ij} V_{ik} V_{jk} = \sum_{i=1}^{N} m_i V_{ik}^2 \omega_k^2
\]

and making the substitutions,

\[
\sum_{i=1}^{N} \sum_{j=1}^{N} K_{ij} V_{ik} V_{jk} = M_k \omega_k^2
\]  

(III-3)

\[
M_k = \sum_{i=1}^{N} m_i V_{ik}^2
\]

(III-3a)

\[
K_k = \sum_{i=1}^{N} \sum_{j=1}^{N} K_{ij} V_{ik} V_{jk}
\]

\[
C_k = \sum_{i=1}^{N} \sum_{j=1}^{N} C_{ij} V_{ik} V_{jk}
\]

one obtains:

\[
T = \frac{1}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} m_i V_{ik} V_k \dot{X}_k^2 = \frac{1}{2} \sum_{k=1}^{N} M_k \dot{X}_k^2
\]

\[
U = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} K_{ij} V_{ik} V_{jk} \dot{X}_k \dot{X}_k = \frac{1}{2} \sum_{k=1}^{N} M_k \omega_k^2 \dot{X}_k^2
\]

\[
D = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} C_{ij} V_{ik} V_{jk} \ddot{X}_k \ddot{X}_k = \frac{1}{2} \sum_{k=1}^{N} \sum_{i=1}^{N} \sum_{j=1}^{N} C_{ij} V_{ik} V_{jk} \ddot{X}_k \ddot{X}_k
\]
Substituting these relations into Lagrange's equation, the following set of equations are obtained:

\[
\ddot{X}_k + \omega_k^2 X_k + \frac{C_k}{M_k} \dot{X}_k = \frac{F_k}{M_k}
\]  

(III-4)

where,

\[
F_k = \sum_{i=1}^{N} f_{ik} v_{ik}
\]  

(III-4a)

Equation (III-4) is converted to the form which represents uniform viscous damping by using Equation (III-3).

\[
\ddot{X}_k + \omega_k^2 X_k + \frac{C_k}{K_k} \omega_k^2 \dot{X}_k = \frac{F_k}{M_k}
\]  

(III-5)

If \( \frac{C_k}{K_k} \) is constant for each value of \( K_k \) in Equation (III-5), the system has uniform viscous damping. This is a good approximation for systems in which the damping is an inherent property of the spring material (13). Since the physical structure utilized in the experimental work was constructed of aluminum and the inherent damping of the aluminum was the only damping considered, the above assumption of \( \frac{C_k}{K_k} \) equal to a constant is valid for this system. It is shown below that this constant \( \frac{C_k}{K_k} \) equals \( 2Z_k/\omega_k \), where \( Z_k \) is the modal damping ratio.

By definition:
\[ C_{c_k} = 2\sqrt{\frac{K_k}{M_k}}, \text{ critical damping coefficient} \]

\[ Z_k = \frac{C_k}{C_{c_k}} \]

\[ \omega_k = \sqrt{\frac{K_k}{M_k}} \]

Substitution into the above expression, one obtains:

\[ \frac{C_k}{K_k} = \frac{Z_k C_{c_k}}{\omega_k^2 M_k} \]

\[ \frac{C_k}{K_k} = \frac{2Z_k \sqrt{K_k M_k}}{\omega_k^2 M_k} \]

Equation (III-5) with the above substitutions now becomes,

\[ m_k \ddot{x}_k + \omega_k^2 x_k + 2\zeta_k \omega_k \dot{x}_k = \frac{F_k}{M_k} \]

The solution to Equation (III-6) may be determined by the well-known method of convolution (13) to be,

\[ x_k = \frac{1}{M_k \omega_d} \int_0^\infty F_k(\tau) \exp \left[ -\zeta_k \omega_d (t-\tau) \right] \sin \omega_d (t-\tau) \, d\tau \]

where,
\[ \omega_d = \omega_k \sqrt{1 - z_k^2} \]

\[ \zeta_k = z_k \sqrt{1 - z_k^2} \]

\[ t = \text{time} \]

\[ \tau = t_2 - t_1 \]

Since a function which will adequately represent the forcing function \( F_k(\tau) \) cannot be written explicitly, a probabilistic representation must be utilized. This representation essentially transforms the deterministic problem from an explicit forcing function and a resulting definite value for the response, to the probabilistic problem in which the input forcing function is given as a power spectral density and the output is the mean square value of the response.

It is assumed that the excitation is at least weakly stationary, which defines that its expected value (mean value) be a constant and that its autocorrelation function depend only on \( \tau = t_2 - t_1 \). The autocorrelation, \( R_{xx} \), is defined by the expression,

\[ R_{xx}(t_1, t_2) = E[x(t_1) x(t_2)] \]

where the \( E \) denotes expected value,

\[ E[x] = \int_{-\infty}^{\infty} x f_x(x) \, dx \]

where, \( f_x(x) \) is the probability density function. In equation form the above definition of a weakly stationary force \( F \) is:

\[ E[F(t)] = \text{constant} \]

\[ R_{FF}(t_1, t_2) = R_{FF}(t_2 - t_1) = R_{FF}(\tau) \]
From the statistical analysis of the actual random excitation that was used as an input to the structure, it was found that the excitation was indeed weakly stationary (Chapter V). The response to a weakly stationary excitation is nonstationary for small values of time \( t \). The response becomes weakly stationary after the system has been exposed to the weakly stationary excitation for a sufficiently long period of time. Lin (11) states, for example, that a sufficiently long period of time is four natural periods if the damping ratio \( Z = 0.1 \), or about twenty natural periods if \( Z = 0.02 \). These times are required for the effects of the configuration of the system at \( t = 0 \) and the resultant transient response to die out. The resulting weakly stationary response is analogous to the steady-state response in the deterministic vibration theory.

The power spectral density, \( \Phi_{xx} \), is defined as the Fourier transform of the autocorrelation function, \( R_{xx} \), of the weakly stationary random variable, \( x(t) \), as,

\[
\Phi_{xx}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{xx}(\tau) \exp(-i\omega\tau) d\tau
\]

The inversion formula can also be written as,

\[
R_{xx}(\tau) = \int_{-\infty}^{\infty} \Phi_{xx}(\omega) \exp(-i\omega\tau) d\omega
\]

where, \( i \), as used in the exponential term here and in the appropriate equations to follow, is equal to \( \sqrt{-1} \).

The significance of this expression is apparent where \( \tau \) is allowed to approach zero for large values of time \( t \).

\[
R_{xx}(0) = \int_{-\infty}^{\infty} \Phi_{xx}(\omega) \, d\omega
\]

\[
R_{xx}(0) = \mathbb{E}[x(t_1), x(t_1)] = \mathbb{E}[x^2(t_1)] = x^2(t)
\]
This last equation equates the mean-square response to the integral of the response spectral density. From the work done by Lin in reference (11), the relationship between the spectral densities of the excitation and the response is obtained.

\[ \phi_{xx}(\omega) = \phi_{FF}(\omega) |H(\omega)|^2 \]  

(III-9)

Substituting equation (III-9) into equation (III-8), one obtains,

\[ \bar{x}^2(t) = \int_{-\infty}^{\infty} \phi_{FF}(\omega) |H(\omega)|^2 d\omega \]  

(III-10)

The analysis will be accomplished in two parts: one, assuming that the power spectral density is a constant, and two, assuming that it is not a constant. The results of these two parts will be compared in Chapter V.

If the spectral density of the excitation changes very slowly in the vicinity of the natural frequency of the system, it can be assumed that this spectral density is a constant for all values of frequency and that the value of the constant is the value of the spectral density evaluated at the natural frequency. This implies that \( \phi_{FF}(\omega) \) is a constant and is equal to \( \phi_{FF}(\omega_k) \).

\[ \bar{x}^2(t) = \phi_{FF}(\omega_k) \int_{-\infty}^{\infty} |H(\omega)|^2 d\omega \]

The transfer function \( H(\omega) \) is determined by exciting the system with a sinusoidal forcing function \( f \) and arranging the resulting response \( x \) in this form:
\[ x_i(t) = H(\omega) f_i(t) \]

Utilizing Equation (III-6),
\[ \ddot{x}_k + 2z_k \omega_k \dot{x}_k + \omega_k^2 x_k = \frac{F_k}{M_k} \]

where,
\[ F_k = F_{ko} \exp(i\omega t) \]

and, \( x_k \) is assumed to be,
\[ x_k = x_{ko} \exp(i\omega t) \]
\[ \dot{x}_k = x_{ko} \exp(i\omega t) \exp(i\omega) \]
\[ \ddot{x}_k = x_{ko} \exp(i\omega t) (i\omega)^2 = -\omega^2 x_{ko} \exp(i\omega t) \]

Equation (III-6) becomes
\[ -x_{ko} \omega^2 \exp(i\omega t) + 2z_k \omega_k x_{ko} (i\omega) \exp(i\omega t) + \omega_k^2 x_{ko} \exp(i\omega t) = \]
\[ \frac{F_{ko}}{M_k} \exp(i\omega t) \]
\[ x_{ko} \left( \frac{\omega_k^2 - \omega^2 + 2i z_k \omega_k \omega}{x_{ko} \omega_k - \omega^2 + 2i z_k \omega_k \omega} \right) = \frac{F_{ko}}{M_k} \]
\[ x_k = \frac{\frac{F_{ko}}{M_k} \exp(i\omega t)}{\frac{\omega_k^2 - \omega^2 + 2i z_k \omega_k \omega}{x_{ko} \omega_k - \omega^2 + 2i z_k \omega_k \omega}} = \frac{F_k}{M_k \left[ \omega_k^2 - \omega^2 + 2i z_k \omega_k \omega \right]}\]
Substituting $X_k$ into Equation (III-2) and substitution of Equation (III-3a) and (III-4a) into the resulting equation, one obtains:

\[
x_1 = \frac{N \sum_{k=1}^{N} V_{ik} \sum_{j=1}^{N} f_j V_{jk}}{\sum_{j=1}^{N} m_j V_{jk}^2 \left[ \omega_k^2 - \omega^2 + 2i Z_k \omega \right]}
\]

\[
x_1 = \frac{N \sum_{k=1}^{N} \frac{1}{\sum_{j=1}^{N} m_j V_{jk}^2} V_{ik} \sum_{j=1}^{N} P A_j V_{jk}}{\left[ \omega_k^2 - \omega^2 + 2i Z_k \omega \right]} 
\]

Define:

\[
a_k = \frac{1}{N \sum_{j=1}^{N} m_j V_{jk}^2} 
\]

\[
\sum_{j=1}^{N} f_j V_{jk} = \sum_{j=1}^{N} P A_j V_{jk} = P \sum_{j=1}^{N} A_j V_{jk} = P \bar{A}_k
\]

where, $P = \text{uniform pressure applied to all masses}$

$A_j = \text{area of each mass}$

$\bar{A}_k = \sum_{j=1}^{N} A_j V_{jk}$

\[
x_1 = \frac{\sum_{k=1}^{N} a_k V_{ik} \bar{A}_k}{P \left[ \omega_k^2 - \omega^2 + 2i Z_k \omega \right]}
\]

It can now be seen that $H(\omega)$ is determined to be:
Substituting \( |H(\omega)|^2 \) into equation (III-10), one obtains the general equation for the steady-state mean-square response at the \( i^{th} \) location for a lumped parameter system.

\[
\sigma_i^2(t) = \sum_{k=1}^{N} \int_{-\infty}^{\infty} \phi_{FF}(\omega) \frac{a_k^2 v_{ik}^2 \bar{A}_k^2}{\omega_k^4 \left(1 - \frac{\omega^2}{\omega_k^2}\right) + \left(2z_k \frac{\omega}{\omega_k}\right)^2} \, d\omega \quad (III-12)
\]

If the power spectral density can be assumed to be a constant with respect to frequency, equation (III-12) becomes,

\[
\sigma_i^2(t) = \sum_{k=1}^{N} \phi_{FF}(\omega) \int_{-\infty}^{\infty} \frac{a_k^2 v_{ik}^2 \bar{A}_k^2}{\omega_k^4 \left(1 - \frac{\omega^2}{\omega_k^2}\right) + \left(2z_k \frac{\omega}{\omega_k}\right)^2} \, d\omega \quad (III-13)
\]

The integral in Equation (III-13) may be evaluated by the method of residues as given by references (11) and (15). See Appendix B.
Considering only the physically realizable part of the frequency spectrum, the above equation becomes,

\[ \int_0^\infty \frac{2\ a_k^2 v_{ik}^2 \ A_k^2}{\omega_k[1 - \left(\frac{\omega}{\omega_k}\right)^2 + \left(2Z_k \left(\frac{\omega}{\omega_k}\right)\right)^2]} \ d\omega = \frac{\pi a_k^2 v_{ik}^2 A_k^2}{4Z_k \omega_k^3} \]

The integrated form of Equation (III-13) becomes,

\[ \overline{\mathbf{X}_1^2(t)} = \sum_{k=1}^{N} \Phi_{PP}(\omega) \left(\frac{\pi a_k^2 v_{ik}^2 A_k^2}{4Z_k \omega_k^3}\right) \]

Utilizing Equation (III-14), one can obtain the steady state mean-square response for a system excited by a nearly constant random force. The random force in this case must be nearly constant in the neighborhood of the natural frequencies. If this latter condition is not met by the system being analyzed, Equation (III-12) must be utilized. In Chapter IV, the physical plate will be modeled in a lumped parameter format. The appropriate lumped parameter will be calculated and substituted into Equations (III-12) and (III-14) respectively. These two outputs will be compared with the intention of determining the error involved in assuming a constant power spectrum in the neighborhood of the natural frequencies for a particular input power spectrum.
CHAPTER IV
A LUMPED PARAMETER MODEL OF THE STRUCTURE
AND COMPUTER EVALUATION OF EQUATIONS OF MOTION

In this chapter, the particular plate shown in Figure 1 will be modeled as a lumped parameter system. The physical quantities such as mass, damping, and stiffness will be lumped at fourteen different locations on the plate (Figure 3). The methods used to apportion values for the lumped parameters will subsequently be discussed. Equations (III-12) and (III-14) will be evaluated using the derived values for the lumped parameters. The final results will be given in terms of the power spectral density of the output response and the root mean square maximum displacements at the fourteen locations and will be calculated by the two analytical methods discussed in Chapter III.

The particular configuration of the experimental plate was a square for fabrication convenience; however, the analysis is general for any plate configuration. Since the plate was a square, it would have been possible to model only one-eighth of the plate and maintain geometrical symmetry and obtain the deflections of the remainder of the plate through geometrical considerations; however, the small variations in the dimensions of the physical system (plate plus frame) from the dimensions which would make the system perfectly symmetrical caused some concern as to how the response may be affected, and accordingly, it was decided to model the entire plate and structure.
The number and location of the lumped masses was arbitrary but once selected became fixed and the distribution of the total mass to each of the selected lumped mass points was determined by the rule of pleasing proportions. This rule of pleasing proportions is based on knowledge obtained from numerous lumped parameter investigations of beam vibrations. One investigation (21) indicated that a beam which is proportioned such that one-half the total mass is lumped at the center and one-quarter at each of the ends, will produce analytical results which compare very favorably with the predicted values for the natural frequencies of the beam considering the beam as a continuous structure. The central area of the plate is partitioned into thirteen masses and a fourteenth mass is used to represent the outer edge of the plate and the fixed frame. (Figure 3). Each of the nine panels (Figure 1) is proportioned such that half the mass of each panel is lumped at the panel's center and one-eighth of the mass lumped at each of the four outer edges of each panel. The masses lumped at the center of each panel are the ones shown in Figure 3 with the associated numbers: 1, 6, 7, 8, 9, 10, 11, 12, 13. The lumped masses numbered 2, 3, 4, and 5 in Figure 3 are comprised of the mass of the adjoining stringers and the edge mass of the adjoining panels. The edge mass of the panels which are in direct contact with the outer frame are lumped with the mass of the frame as the mass numbered 14 in Figure 3.

The influence coefficient matrix, which is the inverse of the stiffness matrix, for the fourteen mass system is determined by two methods. Analytically, the influence coefficients are
determined by using Weaver's structural analysis programs named FR1 and FR3 (16). Experimentally, the influence coefficients are determined by measuring deflections of the lumped mass points with a dial indicator when loads are applied at the various mass points. Loads ranging from five to thirty pounds when required in increments of five pounds were applied at each mass point, and deflections at all mass points were measured for each of the applied loads (Table IIa). The value of the influence coefficients are then determined by dividing the measured value of the deflections in inches by the applied load in pounds force. An average value over all the applied loads is then calculated for each lumped mass point. For example, shown below in Table IIa are typical values of load and deflection at the center of the plate. These values are then used to fill the 14 x 14 matrix of influence coefficients. See Table II.

In order to utilize FR1 and FR3 in determining the analytical coefficients, the plate is represented by a dense network of beams. These beams are represented in Figure 3 by the symbol for a spring (\(\text{\symbol{key: spring}}\)). The results indicate, as was expected, that the more dense the network of beams used to represent the plate, the better the correlation between the analytical and the experimentally measured values of the influence coefficients. Table III shows the analytically determined influence coefficients using the network of beams.

TABLE IIa

<table>
<thead>
<tr>
<th>Loads (Pounds)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deflections (Inches)</td>
<td>.007</td>
<td>.018</td>
<td>.028</td>
<td>.038</td>
</tr>
</tbody>
</table>
## TABLE I

**Root Mean Square Amplitudes of Vibration (Inches)**

<table>
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<tr>
<th>Mass Points</th>
<th>Equation (III-14)</th>
<th>Equation (III-12)</th>
<th>Experimental</th>
<th>Error Between Exp. and (III-12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.0340</td>
<td>.0175</td>
<td>.01418</td>
<td>23.2%</td>
</tr>
<tr>
<td>2</td>
<td>.0227</td>
<td>.0117</td>
<td>.00824</td>
<td>41.0%</td>
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<tr>
<td>3</td>
<td>.0255</td>
<td>.0131</td>
<td>.01097</td>
<td>19.1%</td>
</tr>
<tr>
<td>4</td>
<td>.0250</td>
<td>.0129</td>
<td>.01059</td>
<td>21.7%</td>
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<tr>
<td>5</td>
<td>.0248</td>
<td>.0128</td>
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<td>39.1%</td>
</tr>
<tr>
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<td>.0067</td>
<td>_*</td>
<td></td>
</tr>
<tr>
<td>7</td>
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<td></td>
</tr>
<tr>
<td>8</td>
<td>.0134</td>
<td>.0069</td>
<td>_</td>
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<td>9</td>
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<td>.0067</td>
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<td>11</td>
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<td>_</td>
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</tr>
<tr>
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<td>.0029</td>
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<tr>
<td>13</td>
<td>.0059</td>
<td>.0030</td>
<td>_</td>
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<tr>
<td>14</td>
<td>.0000</td>
<td>.0000</td>
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<td></td>
</tr>
</tbody>
</table>

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*Experimental Data not available*
# Table II

## Experimental Influence Coefficients

Value Shown $\times 10^{-3} = A_{ij}$

(Inches/Pound)

<table>
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<tr>
<th>$A_{ij}$</th>
<th>j=1</th>
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<th>j=3</th>
<th>j=4</th>
<th>j=5</th>
<th>j=6</th>
<th>j=7</th>
<th>j=8</th>
<th>j=9</th>
<th>j=10</th>
<th>j=11</th>
<th>j=12</th>
<th>j=13</th>
<th>j=14</th>
</tr>
</thead>
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<tr>
<td>i=1</td>
<td>1.270</td>
<td>0.660</td>
<td>0.680</td>
<td>0.730</td>
<td>0.690</td>
<td>0.320</td>
<td>0.320</td>
<td>0.320</td>
<td>0.320</td>
<td>0.152</td>
<td>0.140</td>
<td>0.140</td>
<td>0.145</td>
<td>0.030</td>
</tr>
<tr>
<td>i=2</td>
<td>0.660</td>
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<td>0.460</td>
<td>0.380</td>
<td>0.300</td>
<td>0.400</td>
<td>0.360</td>
<td>0.150</td>
<td>0.130</td>
<td>0.279</td>
<td>0.110</td>
<td>0.085</td>
<td>0.069</td>
<td>0.030</td>
</tr>
<tr>
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<td>0.460</td>
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<td>0.300</td>
<td>0.380</td>
<td>0.420</td>
<td>0.190</td>
<td>0.430</td>
<td>0.135</td>
<td>0.105</td>
<td>0.310</td>
<td>0.069</td>
<td>0.100</td>
<td>0.030</td>
</tr>
<tr>
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<td>0.380</td>
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<td>1.000</td>
<td>0.430</td>
<td>0.160</td>
<td>0.415</td>
<td>0.150</td>
<td>0.450</td>
<td>0.105</td>
<td>0.070</td>
<td>0.295</td>
<td>0.100</td>
<td>0.030</td>
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<td>0.982</td>
<td>0.160</td>
<td>0.160</td>
<td>0.430</td>
<td>0.430</td>
<td>0.070</td>
<td>0.100</td>
<td>0.100</td>
<td>0.295</td>
<td>0.030</td>
</tr>
<tr>
<td>i=6</td>
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<td>0.420</td>
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<td>0.160</td>
<td>0.680</td>
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<td>0.123</td>
<td>0.062</td>
<td>0.145</td>
<td>0.145</td>
<td>0.045</td>
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<tr>
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<td>0.320</td>
<td>0.360</td>
<td>0.190</td>
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<td>0.160</td>
<td>0.123</td>
<td>0.690</td>
<td>0.062</td>
<td>0.117</td>
<td>0.120</td>
<td>0.050</td>
<td>0.125</td>
<td>0.045</td>
<td>0.030</td>
</tr>
<tr>
<td>i=8</td>
<td>0.320</td>
<td>0.150</td>
<td>0.430</td>
<td>0.150</td>
<td>0.430</td>
<td>0.123</td>
<td>0.062</td>
<td>0.670</td>
<td>0.123</td>
<td>0.045</td>
<td>0.135</td>
<td>0.045</td>
<td>0.135</td>
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</tr>
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<td>0.320</td>
<td>0.130</td>
<td>0.135</td>
<td>0.450</td>
<td>0.430</td>
<td>0.062</td>
<td>0.117</td>
<td>0.123</td>
<td>0.605</td>
<td>0.045</td>
<td>0.045</td>
<td>0.145</td>
<td>0.135</td>
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<td>0.125</td>
<td>0.045</td>
<td>0.145</td>
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<td>0.035</td>
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<td>0.050</td>
<td>0.030</td>
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<tr>
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<td>0.135</td>
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<td>0.050</td>
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<tr>
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<td>0.030</td>
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</table>
TABLE III
Theoretical Influence Coefficients

Value Shown x10^-3 = A_ij
(Inches/Pound)

<table>
<thead>
<tr>
<th>A_ij</th>
<th>j=1</th>
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<th>j=3</th>
<th>j=4</th>
<th>j=5</th>
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<th>j=9</th>
<th>j=10</th>
<th>j=11</th>
<th>j=12</th>
<th>j=13</th>
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<td>i=1</td>
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<td>0.747</td>
<td>0.747</td>
<td>0.747</td>
<td>0.379</td>
<td>0.379</td>
<td>0.379</td>
<td>0.379</td>
<td>0.379</td>
<td>0.113</td>
<td>0.113</td>
<td>0.113</td>
<td>0.113</td>
<td>0.030</td>
</tr>
<tr>
<td>i=2</td>
<td>0.747</td>
<td>0.978</td>
<td>0.430</td>
<td>0.430</td>
<td>0.296</td>
<td>0.396</td>
<td>0.170</td>
<td>0.170</td>
<td>0.227</td>
<td>0.075</td>
<td>0.075</td>
<td>0.055</td>
<td>0.030</td>
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</tr>
<tr>
<td>i=3</td>
<td>0.747</td>
<td>0.430</td>
<td>0.978</td>
<td>0.430</td>
<td>0.296</td>
<td>0.396</td>
<td>0.170</td>
<td>0.396</td>
<td>0.170</td>
<td>0.075</td>
<td>0.227</td>
<td>0.055</td>
<td>0.075</td>
<td>0.030</td>
</tr>
<tr>
<td>i=4</td>
<td>0.747</td>
<td>0.430</td>
<td>0.296</td>
<td>0.978</td>
<td>0.430</td>
<td>0.296</td>
<td>0.170</td>
<td>0.396</td>
<td>0.170</td>
<td>0.396</td>
<td>0.055</td>
<td>0.075</td>
<td>0.075</td>
<td>0.030</td>
</tr>
<tr>
<td>i=5</td>
<td>0.747</td>
<td>0.296</td>
<td>0.430</td>
<td>0.430</td>
<td>0.978</td>
<td>0.170</td>
<td>0.170</td>
<td>0.396</td>
<td>0.170</td>
<td>0.396</td>
<td>0.055</td>
<td>0.075</td>
<td>0.227</td>
<td>0.030</td>
</tr>
<tr>
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<td>0.396</td>
<td>0.170</td>
<td>0.170</td>
<td>0.644</td>
<td>0.130</td>
<td>0.130</td>
<td>0.089</td>
<td>0.088</td>
<td>0.088</td>
<td>0.043</td>
<td>0.043</td>
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</tr>
<tr>
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<td>0.379</td>
<td>0.396</td>
<td>0.170</td>
<td>0.396</td>
<td>0.170</td>
<td>0.130</td>
<td>0.644</td>
<td>0.087</td>
<td>0.130</td>
<td>0.088</td>
<td>0.043</td>
<td>0.088</td>
<td>0.043</td>
<td>0.030</td>
</tr>
<tr>
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<td>0.170</td>
<td>0.396</td>
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<td>0.087</td>
<td>0.644</td>
<td>0.149</td>
<td>0.043</td>
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</tr>
<tr>
<td>i=9</td>
<td>0.379</td>
<td>0.170</td>
<td>0.170</td>
<td>0.396</td>
<td>0.396</td>
<td>0.087</td>
<td>0.130</td>
<td>0.149</td>
<td>0.644</td>
<td>0.043</td>
<td>0.043</td>
<td>0.088</td>
<td>0.088</td>
<td>0.030</td>
</tr>
<tr>
<td>i=10</td>
<td>0.113</td>
<td>0.227</td>
<td>0.075</td>
<td>0.075</td>
<td>0.055</td>
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<td>0.088</td>
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<td>0.043</td>
<td>0.442</td>
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<td>0.034</td>
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<td>0.030</td>
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<tr>
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<td>0.113</td>
<td>0.075</td>
<td>0.227</td>
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<td>0.075</td>
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<td>0.088</td>
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<tr>
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<td>0.075</td>
<td>0.055</td>
<td>0.227</td>
<td>0.075</td>
<td>0.043</td>
<td>0.088</td>
<td>0.043</td>
<td>0.088</td>
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<td>0.032</td>
<td>0.442</td>
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<tr>
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<td>0.075</td>
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<td>0.043</td>
<td>0.088</td>
<td>0.088</td>
<td>0.032</td>
<td>0.034</td>
<td>0.442</td>
<td>0.034</td>
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<tr>
<td>i=14</td>
<td>0.030</td>
<td>0.030</td>
<td>0.030</td>
<td>0.030</td>
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<td>0.030</td>
<td>0.030</td>
<td>0.030</td>
<td>0.030</td>
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<td>0.030</td>
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</tr>
</tbody>
</table>
shown in Figure 3. This network of beams represents the most dense network of beams as applied to the solution of this problem, and Table III represents the best analytical approximation to the experimental data in Table II. The effect on the natural frequencies of the system is the most important factor to be considered when analyzing the difference between the two sets of influence coefficients in Table II and Table III. A comparison of the natural frequencies of the system when calculated by using the experimental and the analytical values of the influence coefficients is shown in Table IV. The final results presented in this investigation are the results determined by utilizing the experimentally measured values of the influence coefficients.

As mentioned in Chapter III, the damping associated with this system is assumed to be an inherent property of the spring material, aluminum. In this investigation, the total damping of the system was measured indirectly from forced vibration traces and assumed to conform to the restrictions imposed upon the system in Chapter III, particularly modal damping. This assumption proved to be valid for the particular system being modeled because of the relatively low values which were measured for the damping ratios (Figure 21). Authors Lin (11) and Seireg (13) give 0.04 as a sufficiently low value for the damping ratio in order that this assumption be valid. Since all measured values of the damping ratios in the frequency range of interest are below the 0.04 value, (Figure 21), the assumption is substantiated.

The damping ratios were determined from the typical
### TABLE IV

**NATURAL FREQUENCIES (Hz) DETERMINED BY USING EXPERIMENTAL AND ANALYTICAL INFLUENCE COEFFICIENTS**

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<thead>
<tr>
<th>Experimental</th>
<th>Analytical</th>
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<td>313.791260</td>
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<tr>
<td>438.847412</td>
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</table>
experimental response curves shown in Figures 17, 18, and 19. Actually nineteen curves were utilized in the investigation. The damping ratios were calculated at each natural frequency for each location on the plate. The curves showing the results of these calculations, Figure 21, indicate that the damping ratio is a function of frequency and not a strong function of location on the plate as noted by the relatively close grouping of the data points at each frequency. The particular values for the damping ratios ($Z$) at each frequency are determined by the functional relationship,

$$Z = \frac{\Delta f}{2f}$$

where $f$ is the damped natural frequency or the value of the frequency associated with each major peak in the power spectral density plots. $\Delta f$ is defined as the half power bandwidth or as the frequency range spanned by the response curve at the point on the curve which has half the power as does the peak value at the damped natural frequency (12), (17).

The particular shape of the excitation power spectral density with regard to the location in the frequency spectrum of the natural frequencies of the system will determine which equation, (II-12) or (III-14), should be used to calculate the root mean square response of the system (11). If the excitation power spectral density is constant over the range of frequencies spanned by the natural frequencies of the system, Equation (III-14) can be used; if the excitation power spectral density is anything other than a constant over the frequency range of interest, Equation (III-12)
FIGURE 4. THEORETICAL RESPONSE POWER SPECTRAL DENSITY AT LUMPED MASS NUMBER 1
FIGURE 5.
THEORETICAL RESPONSE POWER SPECTRAL DENSITY AT LUMPED MASS NUMBER 2
FIGURE 6.
THEORETICAL RESPONSE POWER SPECTRAL DENSITY AT LUMPED MASS NUMBER 3
FIGURE 7.
THEORETICAL RESPONSE POWER SPECTRAL DENSITY AT LUMPED MASS NUMBER 4

POWER SPECTRAL DENSITY (LB x 2/Hz)

FREQUENCY (Hz) × 10^1
FIGURE 9.
THEORETICAL RESPONSE POWER SPECTRAL DENSITY AT LUMPED MASS NUMBER 6
FIGURE 10.
THEORETICAL RESPONSE POWER SPECTRAL DENSITY AT LUMPED MASS NUMBER 7
FIGURE 11.
THEORETICAL RESPONSE POWER SPECTRAL DENSITY AT LUMPED MASS NUMBER 8
FIGURE 12.
THEORETICAL RESPONSE POWER SPECTRAL DENSITY AT LUMPED MASS NUMBER 9
FIGURE 13.
THEORETICAL RESPONSE POWER SPECTRAL
DENSITY AT LUMPED MASS NUMBER 10
FIGURE 14.
THEORETICAL RESPONSE POWER SPECTRAL DENSITY AT LUMPED MASS NUMBER 11
FIGURE 15.
THEORETICAL RESPONSE POWER SPECTRAL DENSITY AT LUMPED MASS NUMBER 12

POWER SPECTRAL DENSITY (LB**2/Hz).

FREQUENCY (HZ)
FIGURE 16.
THEORETICAL RESPONSE POWER SPECTRAL DENSITY AT LUMPED MASS NUMBER 13

POWER SPECTRAL DENSITY (LB^2/Hz) vs FREQUENCY (Hz)
FIGURE 17. EXPERIMENTAL RESPONSE POWER SPECTRAL DENSITY CORRESPONDING TO MASS 1
FIGURE 18.
EXPERIMENTAL RESPONSE POWER SPECTRAL DENSITY CORRESPONDING TO MASS 2
FIGURE 19.
EXPERIMENTAL RESPONSE POWER SPECTRAL DENSITY CORRESPONDING TO MASS 3
FIGURE 20.
EXPERIMENTAL RESPONSE POWER SPECTRAL DENSITY CORRESPONDING TO MASS 8
FIGURE 21: EXPERIMENTAL VALUES OF DAMPING RATIOS

- □ - MASS 1
- △ - MASS 3
- ○ - MASS 6

DAMPING RATIO (Z)

FREQUENCY (HZ)
should be used to obtain best results. For comparison purposes, the excitation shown in Figure 2 is used as the input for both Equations (III-12) and (III-14). The resulting root mean square displacements are shown in Table I, along with the corresponding values measured experimentally. In the course of evaluating Equation (III-12) the necessary data for plotting the analytical power spectral density was calculated as described in the computer solution of the problem and eventually plotted as shown in Figures 4 through 16.

A Computer Solution of the Problem

In this section, the computer program which was developed to perform the calculations necessary for evaluating Equations (III-12) and (III-14) will be discussed. This program is listed in Appendix B. The necessary inputs to this program are: (1) the mass matrix, (2) the influence coefficient matrix, (3) the modal damping ratio matrix, (4) the power spectral density of the excitation and (5) the operational parameters (Figure 22). With these inputs, the computer will calculate the stiffness matrix, the natural frequencies of the structure, the mode shapes of vibration, the modal participation factors, the power spectral density of the response, and the root mean square displacements of the system. The program is also capable of generating the necessary data for plotting the power spectral density of both the excitation and the response.

Before the progression of calculations performed by the computer program and the methods employed to perform these calculations are discussed in detail, a very basic breakdown of the program is
FIGURE 22: FLOW DIAGRAM

READ
- Titles, Cases, Dimension, Degrees of freedom, Mass Influence Coefficients, Damping Ratios, Excitation

CALL DECOMP

A = Real Symmetric Matrix

YES
- CALL INVERT
  - Combine Mass and Stiffness Matrix
    - CALL JACOB
      - Transform Eigenvalues And Eigenvectors into Real System
        - Print Eigenvalues And Eigenvectors
          - Calculate Modal Participation Factors by Equation (III-11)

NO
- STOP

CALL GRAND

I

II
Evaluate Equation III-14

Print X

Evaluate Equation (III-12)

Print WO

CALL GRAND

Print XI

CALL WEBAL

Plot WO, WI

L = K

STOP
given. The basic format on which the program is structured can be seen from these major steps: (1) the input of the appropriate data, (2) the calculation of necessary parameters, (3) substitution of the necessary parameters into Equations (III-12) and (III-14), and (4) printing the desired results. These four divisions are indicated in Figure 21 by the corresponding Roman numerals. Steps in the flow diagram which are located between the horizontal dash lines are the ones associated with each of the four major steps numbered between these lines.

The first major division of the program is devoted to reading the appropriate data necessary for calculating the parameters of the problem. The flow diagram (Figure 22) lists the particular items which are read into the computer program. The particular order and format for inputing the data is shown in the program listing in Appendix C. The first set of six 'A' format read statements are the common titles which will be printed on the power spectral density plots. Particular titles will be read in the same D0 Loops performing the particular calculations. The next read statement identifies the number of data sets ($K$) and the actual dimension size of the influence coefficient matrix ($NA$). The number of degrees of freedom ($N$) is read in next. The mass ($AMASS$), area of excitation for each mass ($FA$), the damping ratio ($Z$), the constant value of the excitation power spectral density ($W$), the variable values of the excitation power spectral density ($WI$), and the influence coefficients ($A$) are inputed in this same order. The mass, being read in, in units of pounds force, is converted to the proper inch-pound-seconds system of units by dividing it by 386.4. The values of $WI$ are also converted back to their
actual values by multiplying by 10. If the excitation power spectral density is not a constant, one inputs a zero for $W$ in the data deck. If the excitation power spectral density is a constant, one inputs a minus one for $W_I$ in the data deck.

The second major division in the program begins by calling subroutine DECOMP. This subroutine decomposes the influence coefficient matrix into an upper tri-diagonal matrix and tests the matrix to determine if it is a real symmetric matrix (18). If it is not a real symmetric matrix, the next step cannot be performed and the program is sent to stop. If it is a real symmetric matrix, the program then calls subroutine INVERT, which inverts the influence coefficient matrix and produces the stiffness matrix. The stiffness matrix ($A$) is then transformed by the inverse of the mass matrix in such a fashion that the resulting matrix ($A'$) is still symmetrical. This matrix is now in the proper form for substitution into subroutine JACOB. This subroutine calculates the eigenvalues and eigenvectors of the system by using the Jacobi Method for real symmetric matrices (20). The resulting eigenvalues are converted into units of hertz and the eigenvectors are transformed back into the real system by multiplying them by the inverse square root of the mass matrix. The total area over which the excitation is applied and the generalized area are calculated. The mass matrix and the eigenvectors are then used to calculate the modal participation factors as described in Chapter III by Equation (III-11). At this point, the displacements of the plate are referenced to the fixed frame (fourteenth mass) by subtracting the eigenvector component
associated with the fourteenth mass in all the modes of vibration, \([V(14,J)]\), from the other eigenvector components.

It can be seen by Figure 2 that the excitation power spectral density is not a constant in the particular case being investigated, but evaluation of Equation (III-14) was performed for comparison purposes by approximating the curve in Figure 2 with an average value. Subroutine GRAND is called to integrate the curve in Figure 2 from six to five hundred hertz. The average value of the curve is then obtained by dividing the integrated value by four hundred and ninety five, which is the total frequency range.

The third major division of the program, the evaluation of Equations (III-12) and (III-14), is initiated by testing the value of \(W\). If \(W\) is equal to zero, \(W\) is set equal to EXIC which is the average value calculated by subroutine GRAND. At this point, the other quantities necessary for substitution into Equation (III-14) have already been stored in the computer, and determination of the root mean square displacements \((x)\) is merely a matter of performing the required mathematical operations. The factor \(2\pi\) is used to convert the frequency units from hertz to radians per second. The constant 165.5 determined by calibration procedures discussed in Chapter V, effectively converts the excitation from volts to pounds force. The output \((x)\) is now printed as the response displacement of each lumped mass in inches.

A test is performed on \(WI\) to determine if the evaluation of Equation (III-12) is necessary in this case. If this evaluation is not necessary, \((WI = -1)\) control is shifted to the next test,
(L = K), which determines whether or not any more cases are to be calculated. If no more cases are to be evaluated the program stops. If more cases are to be evaluated, the control is sent to the beginning of the program, and the cycle is started again by reading new data for the next case. If WI is not equal to a minus one, the sequence of calculations continues. The power spectral density of the response is calculated using Equation (III-9), or equivalently the integrand of Equation (III-12). The calculation is performed in increments of one hertz over the frequency range of six to five hundred hertz. The response power spectral density (WO) is multiplied by the same conversion factors (2π and 165.5) as applied in the displacement calculations of Equation (III-14). The resulting power spectral density of the response is now in terms of pounds force squared per hertz.

The fourth major division of the program is begun by printing the response power spectral density (WO). Subroutine GRAND, which utilizes a Simpson's rule integration technique, is called to integrate WO over its range of frequencies to produce the mean square values of the displacements. The square roots of these values produce the root mean square values of the displacements (XI) in inches; XI is then printed. The particular titles for each data set are read in as 'A' format data. The subroutine WEBAL is called twice to plot the necessary power spectral density curves. The first time WEBAL is called, the power spectral density of the response at each lumped mass point (WO) is plotted (Figures 4 through 16). The second time WEBAL is called, it plots
the power spectral density of the excitation (W1). See Figure 2. The number of cases is tested (L = K). If more cases are to be calculated the program returns to the appropriate read statement and the cycle of calculations is again performed. If all the cases have been completed the program stops.
CHAPTER V

EXPERIMENTAL TEST

The object of the experimental test is to determine the root mean square values of the displacements and the response power spectral density of the plate shown in Figure 1 subjected to the random acoustical pressure shown in Figure 2. The experimental tests were performed at the Mississippi Test Facility and financially supported by NASA through the Division of Engineering Research at Louisiana State University.

The particular shape of this plate was chosen because its shape is a simple geometric form and generally found in areas where the acoustical noise may be at a sufficiently high level as to cause damage to the surrounding structures. As noted in Chapter II, other investigations in this area were made on the assumption that the stringers were sufficiently stiff in at least one direction as to give the adjoining edge of the panels a fixed boundary condition in that direction. See Lin (26) and McDaniel and Donaldson (27). This particular plate has no such restrictions and differs from other plates subjected to a random excitation in that it resists analysis by the method of transfer matrices (26), (27). The only fixed condition imposed on this plate are the fixed outer edges.

The plate is constructed of sheet aluminum alloy 6061-T6 with a modulus of elasticity of $10^7 (lb_f/in^2)$ and a shear modulus of $4 \times 10^6 (lb_f/in^2)$. These properties are taken from standard handbook values for this aluminum. The maximum thickness
of the plate is .25 inch at the boundaries and stringers. The panel thickness is .125 inch. The plate was constructed from a solid sheet of aluminum with the panel areas formed by milling away the unwanted metal. The radius of curvature at the corners where the panels and the stringers intersect is no larger than 3/16 of an inch, and the surface finish is specified as a standard 63 smooth. All tolerances were held to ± 1/64 of an inch. The plate is shown in the foreground of Figure 23 fixed on all four edges by a 4 x 1/4 angle iron frame which is bolted to the edge of the plate by 36 half inch bolts. The top and bottom frames are also joined by two 1/2 inch thick plates which were attached on opposite edges of the plate and used to support the entire system of frame and plate during the experimental test.

The plate was placed in its clamped configuration before the strain gages were attached. Biaxial and rosette type strain gages were mounted on the surface of the plate primarily around the centrally located panel. Two strain gages were also mounted on the frame. Figure 26 shows the location and alignment of the strain gages. The strain gages were calibrated in the laboratory with the use of the equipment shown in Figure 23. The dynamic part of the calibration procedure was performed in the Mobile Instrumentation Unit; this trailer is shown in Figure 24. The equipment inside the Mobile Instrumentation Unit is shown in Figure 25. The laboratory calibration was performed by attaching weights to the particular points designated as lumped mass points on the plate and measuring the strains and deflections at all points of interest. See Table VI for results of
FIGURE 23: LABORATORY CALIBRATION EQUIPMENT AND TEST PLATE
FIGURE 2: EXCITATION EQUIPMENT
FIGURE 26: Location and Direction of Strain Gages on the Frame and on the Center Panel of the Plate
the strain measurements and Table II for results of deflection measurements. It is noted that strain gages are not located at all fourteen points designated as lumped mass points in the analysis. This caused some concern until calibration was completed on the five points shown in Table I which coincided both experimentally and analytically. The five calibrated points produced the same conversion constants and gave credence to the assumption that these conversion constants were uniform for all points on the plate as shown in the calibration calculations which follow.

Two conversion constants are necessary; one to convert the volts representing the excitation power to pounds force, and a second factor to convert the root mean square volts to displacement in inches.

The conversion of volts to pounds force is accomplished through the set of linear equations,

\[ \Delta R = AV, \ \varepsilon = BR, \ \text{and} \ \L = Ce, \text{ where,} \]
\[ \Delta R = \text{Change in resistance (ohms)} \]
\[ \Delta V = \text{Change in voltage} \]
\[ \varepsilon = \text{Strain in inches/inch} \]

\[ \text{(V-1)} \]

\[ A, B, \text{ and } C \ \text{are constants to be determined through calibration procedures. The desired relationship is determined by combining the above equations to produce } \L = ABCV, \text{ where } \L \text{ is the load in pounds force and } \Delta V \text{ is the change in voltage in volts.} \]

The calibration constant \( A \) is determined by placing 50,000 and 220,000 ohm calibration resistors \( (R_p) \) in parallel with the 120 ohm \( (R_s) \) strain gages. The \( \Delta R \) is calculated from the equation \( (36) \),
## TABLE V

Change in Voltage (V) for Each Calibration Resistor at all Strain Gage Locations

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<th>Final D. C. Voltage</th>
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<th>Strain Gage Number</th>
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<th>Final D. C. Voltage</th>
<th>120.8 Ohms in Parallel with</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>+.130</td>
<td>+0.363</td>
<td>220,000 Ohms</td>
</tr>
<tr>
<td>13</td>
<td>+.025</td>
<td>+0.262</td>
<td>220,000 Ohms</td>
</tr>
<tr>
<td></td>
<td>+.024</td>
<td>+0.261</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td>+.027</td>
<td>+1.071</td>
<td>50,000 Ohms</td>
</tr>
<tr>
<td></td>
<td>+.024</td>
<td>+1.070</td>
<td>&quot;</td>
</tr>
<tr>
<td>14</td>
<td>+.114</td>
<td>+1.184</td>
<td>50,000 Ohms</td>
</tr>
<tr>
<td></td>
<td>+.120</td>
<td>+1.183</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td>+.120</td>
<td>+0.364</td>
<td>200,000 Ohms</td>
</tr>
<tr>
<td></td>
<td>+.122</td>
<td>+0.363</td>
<td>&quot;</td>
</tr>
<tr>
<td>15</td>
<td>+.068</td>
<td>+0.312</td>
<td>200,000 Ohms</td>
</tr>
<tr>
<td></td>
<td>+.070</td>
<td>+0.312</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td>+.074</td>
<td>+0.319</td>
<td>&quot;</td>
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<tr>
<td></td>
<td>+.076</td>
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<td>+.078</td>
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<td>+.079</td>
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<td>&quot;</td>
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<tr>
<td>16</td>
<td>+.110</td>
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<td></td>
<td>+.117</td>
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<td>&quot;</td>
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<tr>
<td></td>
<td>+.128</td>
<td>+0.365</td>
<td>220,000 Ohms</td>
</tr>
<tr>
<td></td>
<td>+.110</td>
<td>+0.355</td>
<td>&quot;</td>
</tr>
<tr>
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<td>-.001</td>
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<tr>
<td></td>
<td>-.028</td>
<td>+0.236</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td>-.024</td>
<td>+1.053</td>
<td>50,000 Ohms</td>
</tr>
<tr>
<td></td>
<td>-.008</td>
<td>+1.053</td>
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<td>50,000 Ohms</td>
</tr>
<tr>
<td></td>
<td>+.010</td>
<td>+1.072</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td>+.016</td>
<td>+0.240</td>
<td>220,000 Ohms</td>
</tr>
<tr>
<td></td>
<td>+.009</td>
<td>+0.243</td>
<td>&quot;</td>
</tr>
<tr>
<td>19</td>
<td>+.048</td>
<td>-0.316</td>
<td>220,000 Ohms</td>
</tr>
<tr>
<td></td>
<td>+.045</td>
<td>-0.316</td>
<td>&quot;</td>
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<tr>
<td></td>
<td>+.045</td>
<td>+1.117</td>
<td>50,000 Ohms</td>
</tr>
<tr>
<td></td>
<td>-.043</td>
<td>+1.120</td>
<td>&quot;</td>
</tr>
</tbody>
</table>
\[ R = R_G - \frac{R_G R_p}{R_G + R_p} \]  

(V-2)

In which the second term on the right hand side is the effective value of resistance created by the two resistors \( R_G \) and \( R_p \) acting in parallel. The two values of \( \Delta R \) calculated by using the two calibration resistances are \( \Delta R_5 = .29 \) ohms and \( \Delta R_{20} = .07 \) ohms. See Appendix D for sample calculations. \( \Delta R_5 \) is determined by using the 50,000 ohm calibration resistor in Equation (V-2), and \( \Delta R_{20} \) is determined by using the 220,000 ohm calibration resistor. Values of voltage were recorded on a digital volt meter by switching the calibration resistors in and out of the circuit until the change in voltage (\( \Delta V \)) was stabilized. See Table V. An average value for \( \Delta V \) was then determined. This procedure was repeated for both of the calibration resistors at each strain gage location. The calibration constant \( A \) is then calculated by the equation, \( A = \Delta R/\Delta V \). The constant \( A \) is expected to be uniform for all the strain gages since the initial value of resistance is the same for all gages. The experimental data substantiates this statement. \( A \) is equal to .274; this represents an average value calculated for all strain gages shown in Figure 27. See Appendix D for sample calculation.

The calibration constant \( B \) is determined for the equation which defines the Gage Factor (\( F \)) in terms of resistance (\( R \)), change in resistance (\( \Delta R \)), and strain (\( \varepsilon \)).

\[ F = \frac{\Delta R}{\varepsilon} \quad \text{or} \quad \varepsilon = \frac{1}{RF} \Delta R \]

Comparing the above equation with \( \varepsilon = B \Delta R \), \( B \) is seen to be defined as \( 1/RF \); \( R \) and \( F \) are given by the strain gage manufacturers (Micro-
Measurement, Inc.) to be $R = 120.8$ ohms and $F = 1.98$. $B$ is then calculated to be equal to $0.00418$.

The calibration constant $C$ is determined from the data recorded in Table VI. This data was recorded by loading the plate at the desired points of interest and recording the change in strain ($\varepsilon$) due to the applied loads ($L$). The constant $C$ is then calculated by the equation, $C = L/\varepsilon$. As noted previously, strain gages did not exist at all lumped mass points, and an average value of the five points was used for all points. A sample calculation is shown in Appendix D. The value for $C$ is determined to be equal to $0.144 \times 10^6$. The conversion constant relating load ($L$) and voltage ($AV$) may now be determined by multiplying the three calibration constants $A \times B \times C$. This calculation is performed in Appendix D, the resulting conversion constant is equal to $165.5$. The equation defining the conversion from volts to pounds force can now be written as $L = 165.5 AV$.

The second calibration constant which relates volts to inches is obtained by adding another linear equation to the set of three equations utilized in the above calibration. This equation, $\delta = DL$, relates deflections ($\delta$) and the load ($L$) through the conversion factor ($D$), which is seen to be the influence coefficients of the system. Solving this latter equation for $L = \delta/D$ and substituting in the above conversion between volts and pounds force, one obtains $= 165.5 DAV$. This conversion equation is a function of location on the plate and will be evaluated for each point at which both experimental and analytical data is available. See Table VII.

The equipment utilized in performing the experimental test
TABLE VI

Values of Strain in Micro-Inches per Inch for Various Loads at Selected Strain Gage Locations

<table>
<thead>
<tr>
<th>Strain Gage Number</th>
<th>Loads at Corresponding Strain Gage Number (Pounds Force)</th>
<th>0</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10,000</td>
<td>9,993</td>
<td>9,966</td>
<td>9,935</td>
<td>9,905</td>
<td>9,876</td>
<td>9,848</td>
<td>9,820</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10,000</td>
<td>9,993</td>
<td>9,964</td>
<td>9,927</td>
<td>9,890</td>
<td>9,859</td>
<td>9,822</td>
<td>9,790</td>
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<td>3</td>
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<td>9,967</td>
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<td>9,875</td>
<td>9,840</td>
<td>9,820</td>
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<tr>
<td>5</td>
<td>5,000</td>
<td>4,990</td>
<td>4,954</td>
<td>4,911</td>
<td>4,869</td>
<td>4,832</td>
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<td>4,961</td>
<td>4,924</td>
<td>4,889</td>
<td>4,848</td>
<td>4,818</td>
<td>4,785</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>10,000</td>
<td>9,994</td>
<td>9,966</td>
<td>9,929</td>
<td>9,890</td>
<td>9,859</td>
<td>9,824</td>
<td>9,792</td>
<td></td>
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<tr>
<td>8</td>
<td>5,000</td>
<td>4,994</td>
<td>4,965</td>
<td>4,926</td>
<td>4,889</td>
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<td>4,790</td>
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<tr>
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<td>4,988</td>
<td>4,944</td>
<td>4,888</td>
<td>4,834</td>
<td>4,780</td>
<td>4,726</td>
<td>4,674</td>
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</tr>
</tbody>
</table>
### TABLE VII

Evaluation of $\delta = 165.5D \Delta V$

<table>
<thead>
<tr>
<th>Mass</th>
<th>$D$ (from table II)</th>
<th>$165.5D$</th>
<th>$\Delta V$ (Peak Value of rms Response (Volts))</th>
<th>$\delta$ (Deflection in Inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00127</td>
<td>0.2100</td>
<td>0.0675</td>
<td>0.01418</td>
</tr>
<tr>
<td>2</td>
<td>0.00083</td>
<td>0.1372</td>
<td>0.060</td>
<td>0.00824</td>
</tr>
<tr>
<td>3</td>
<td>0.00102</td>
<td>0.1688</td>
<td>0.065</td>
<td>0.01097</td>
</tr>
<tr>
<td>4</td>
<td>0.00100</td>
<td>0.1655</td>
<td>0.064</td>
<td>0.01059</td>
</tr>
<tr>
<td>5</td>
<td>0.00098</td>
<td>0.1620</td>
<td>0.0565</td>
<td>0.00915</td>
</tr>
</tbody>
</table>
is shown in Figures 23, 24, and 25. Figure 23 shows the plate and frame in the foreground and strain sensing and recording equipment in the background. Figure 25 shows the equipment inside the Mobile Instrumentation Unit which was used to sense and record nineteen channels of dynamic strain and will be further described. Figure 24 shows the control center on the right, the Mobile Instrumentation Unit at right center, the exponential horn with the plate and frame mounted near its mouth at left center, and the pressurized air storage vessels on the left.

The strain sensing equipment seen in the background of Figure 23 consists of two Baldwin-Lima-Hamilton Strain Indicators and balancing units, a Honeywell galvanometer oscillograph, and a Tektronix Memory Oscilloscope. The equipment seen in Figure 25 consists of a bank of signal conditioners in the upper right hand corner, and a bank of amplifiers in the lower right center of the figure. A multiplexing unit is located in the lower center foreground. A nine track Lockheed tape recorder is seen on the table in the upper left hand corner along with the direct current power source located in the left center background. The control center in Figure 24 contains the necessary equipment for operating the exponential horn. Included in this equipment is a variable frequency signal generator, a bandwidth limiting noise signal generator, and an amplifier. Switches for controlling the flow of the compressed air from the pressurized air storage vessels to the vibrator baffles of the horn are also located in the control center. The exponential horn shown in Figure 24 has a profile which is described by an exponential function. It was rotated to the position shown in Figure 24 and the plate was adjusted in the crow's nest in the position shown such that the plane of the plate and the plane formed by the mouth of the
horn were parallel. This insured that an acoustical wave impinging on the plate from the horn would impact the plate with normal incidence. The radius of curvature of the wave leaving the mouth of the horn is so much greater than the dimensions of the plate that this wave can be assumed to be plane. A near field test on the acoustical pressure field impinging on the plate verified this assumption. Figure 32 (a, b, c) shows the correlation between the amplitude and the phase angle along three radial lines in the plane of the plate. The four individual curves represent the four radial distances at which the pressure was recorded. The radial distances from the center of the plate are 0, 10, 20, and 30 inches. The three radial lines along which these four pressure measurements were recorded are oriented with the horizontal y axis (a), the vertical z axis (c), and the 45 degree diagonal connecting the corners of the plate (b). These curves substantiate the assumption that the acoustical field is uniform over the entire area occupied by the plate. The curves shown in Figure 32 represent an 80 hertz pure tone. The same series of tests was performed for a 500 hertz pure tone to determine if frequency has an effect on the pressure field distribution. The relationship between amplitude and phase angles at the various locations of the 500 hertz pressure field were the same as the ones for the 80 hertz pressure field. This substantiated the assumption that the pressure field is uniform over the area occupied by the plate regardless of the frequency of the pressure field. This statement is interpreted to state that a random pressure field emitted from the horn will produce a uniform pressure field in the plane occupied by the plate.
FIGURE 27: Block Diagram of Experimental Test

Plate and Horn System → Signal Conditioners and Wheatstone Bridges → Amplifiers → Multiplex System → Tape Recorder → Data Analysis → PSD Plots and rms Displacements

19 Channels of Strain Data
The block diagram shown in Figure 27 represents the path followed by the experimental data from its generation at the plate in the form of strain to the final results represented by the power spectral density plots and the maximum root mean square deflections. The plate and horn system is composed of the plate instrumented with nineteen strain gages, the frame, the structure supporting the plate and frame, and the exponential horn. The exponential horn generates the random acoustic field which excites the plate, causing the deflections of the plate to change the resistance of the strain gages. Each strain gage is wired into a full Wheatstone bridge circuit located in the signal conditioners. The change in resistance is converted to an equivalent change in voltage as determined by the calibration constant. The resulting change in voltage is directed into amplifiers which increase the voltage from millivolts to volts (a factor of one thousand). The resulting magnified voltage is sent into the multiplexing unit along with a one volt calibration signal. The multiplex system stores the change in voltage on a high frequency carrier. The five different carrier frequencies used to record the different channels of voltage are 200, 300, 400, 500, and 600 kilohertz. The data was then recorded on the odd numbered (1, 3, 5, 7, 9) tracks of a nine track tape. Five multiplex signals were recorded on each track, except track number nine which contained three channels of excitation data. The calibration signal was recorded on each channel before each test for a period of one minute. The calibration signal was a 1 volt, 80 hertz pure tone generated from a 155 decibel sound source. See Figure 28.
FIGURE 28: Power Level and Duration of Excitation Applied to the Plate
Figure 28 shows the power level and the time duration of the four excitations produced by the exponential horn. The figure shows the last few seconds of the calibration signal starting at time equal zero. The signal drops down to the background level of 80 hertz pure tone excitation. After 45 seconds of 80 hertz excitation, the signal drops down to the noise floor for 45 seconds. The next 45 seconds of excitation consist of 139 decibels of random excitation having the same frequency spectrum as shown in Figure 2. After another 45 seconds of noise floor, the output power of the horn is raised approximately 10 more decibels. The same sequence of excitation described above is repeated at the higher power levels. The sequence of excitations is as follows: 80 hertz excitation for 45 seconds at 151 decibels, 45 seconds of noise floor, 149 decibels of random excitation for 45 seconds, 45 seconds of noise floor, and back to the calibration level. The only excitation used in the data analysis is the 149 decibels of random excitation for 45 seconds. The other tests were performed only to check the signal quality generated by the strain gages and the general behavior of the system.

The analysis of the data consists of printing the raw data and determining the root mean square value, the amplitude spectrum of the data (Figure 29), the probability density of the data (Figure 30 and 31), and the power spectral density of the data (Figures 2, 7, 18, 19, and 20).

Figure 29 is a representative slice of the data recorded by strain gage number 2. The amplitude spectrum indicates the location of the natural frequencies in the frequency spectrum and the
FIGURE 29
Amplitude Spectrum, Data, and rms Level

Amplitude

Frequency (Hertz)

Data

Time (Seconds)

rms Level (volts)
The amount of amplitude associated with each frequency. The plot of the raw data shows the particular section of data being analyzed. The root mean square plot shows the root mean square in volts of the data as a function of time. These root mean square plots are scanned to locate the maximum root mean square values of the data. See V in Table VII. These maximum values will be converted through the use of the calibration constants to the maximum root mean square deflections of the plate in inches. See δ in Table VII.

Figures 30 and 31 represent the probability density plots of the response at strain gage number 2 and of the excitation respectively. Figure 31 represents a nearly perfect Gaussian distribution which, as Lin (11) indicates, determines the excitation to be at least stationary. The probability density of the response (Figure 30) represents a near perfect Gaussian distribution, and by Lin (11) determines the response to be strongly stationary or analogously the steady state condition of the deterministic theory. Lin also states that given an excitation whose probability density has a Gaussian distribution and a structural system which is linear, the output response of the system will have a probability density which has a Gaussian distribution. Figures 30 and 31 verify this statement and also the assumption made in Chapter III regarding the excitation being at least weakly stationary.

The power spectral density plot of the excitation shown in Figure 2, along with Figure 31, contains all the information required to completely define the random excitation. The power spectral density plots of the response data shown in Figures 17, 18, 19, and 20 contain the necessary information to completely describe the response.
FIGURE 30: Probability Density Plot of the Response Data
FIGURE 31: Probability Density Plot of the Excitation Data
FIGURE 32: ACOUSTICAL PRESSURE VARIATION IN THE PLANE OF THE PLATE

a: Horizontal y axis

b: 45° Diagonal

c: Vertical z axis
These plots can be used to determine the natural frequencies of the system and the power stored at all of these frequencies. Integration of the response power spectral density curves produces the mean square response of the plate. Using the square root of this quantity and converting to the proper units, the root mean square displacements of the plate in inches are obtained. See δ in Table VII.
CHAPTER VI

COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

The results obtained through the analytical and experimental efforts described in Chapters III and IV are compared and discussed in this chapter. Plausible explanations are given for the differences between the analytical and experimental results. The particular results are discussed in terms of the power spectral density plots and the root mean square deflections. The accuracy of approximating this plate with a network of beams is also discussed.

Figures 4 and 17 represent the analytical and experimental power spectral densities of the response at the center of the plate. The general trend followed by both these curves is in good agreement. The magnitude of the peaks tend to decrease with an increase in frequency in both the analytical and experimental cases. The frequency (37 hertz) of the first natural frequency predicted analytically agrees very well with the experimentally measured value of 38 hertz. The magnitude of these two peaks also agree quite well.

In this discussion, the peaks in the power spectral density plots imply a natural frequency exists at that frequency. The peak having the greatest amount of power is the first fundamental frequency of the system. The importance of the agreement between experimental and analytical values of magnitude and frequency at the first natural frequency of the system is realized when one applies this analysis to the solution of vibration problems. The magnitude and the frequency of the first natural frequency of a structure is usually the most
important information needed to solve vibration problems associated with the structure. Good agreement of the analytical and experimental frequency of the last peak (430 hertz) on the power spectral density plots is also observed. The magnitude of this analytical peak is low as compared to the magnitude of the experimental peak. This trend of low values of magnitude for the analytical peaks is consistent throughout the entire frequency spectrum with the exception of the peaks at or near the first natural frequency of 37 (hertz). This trend becomes more pronounced as the frequencies increase.

The experimental and analytical frequencies of the system indicated by the peaks in the power spectrum tend to agree very well through the entire frequency range. The only exception is a peak at 60 hertz in the analytical power spectrum whose magnitude is two orders higher than the peak indicated by the first natural frequency noted above. The trend of the small disagreement between values of frequency for the peaks, particularly in the midband frequencies from 100 to 400 hertz, is for the analytical frequencies to be higher than the experimental frequencies. Located at approximately 10 hertz in the experimental power spectral density plot, a peak is observed which does not show up on the analytical plot. This discrepancy can be explained by realizing also the reason for the shift of natural frequencies to higher values on the analytical plots. One reason the analytical natural frequencies are slightly high is because the entire structure supporting the plate and horn system is not included in the lumped parameter model of the system. Only the crow's nest, shown in Figure 22 as the platform protruding out from under the mouth of the exponential horn is included in the model of the system. The remainder of the
supporting structure including the horn itself is not accounted for in the analysis. The effect on the natural frequencies of the system of including the entire structure which supports the horn and plate would be to shift the lowest analytical natural frequencies to a lower value and essentially leave the higher natural frequency unchanged except for a small overall shift to lower values of frequency. It is proposed that the value of frequency to which the lowest frequency would be shifted would coincide with the peak at 10 hertz shown on the experimental curve. The shift of the other analytical natural frequencies to lower values would tend to bring these values into better agreement with the experimental values. The peak at 60 hertz which would be shifted to a lower value would then exist as the only peak on the experimental curve which does not match with a corresponding peak on the experimental curve. A possible remedy for correcting this problem is to distribute the mass in such a manner that more mass is located at the center of the plate. This modification would have the effect of lowering the magnitude and frequency of the response near 60 hertz. This idea will be pursued after a comparison is made of the next two power spectral density plots.

Figures 5, 18, and 19 are representative of the analytical and the experimental response power spectral densities of a lumped mass at the intersection of two stringers on the plate. Both the analytical and experimental curves possess the same trend which is the decrease in response power as the frequencies increase. The only exception to this trend is a peak on the experimental curve at
approximately 430 hertz which is not predicted on the analytical curve. This indicates that too much mass is allocated to the lumped mass points in the peripheral area of the plate near the frame. Allocating less mass to each lumped mass point near the frame and adding more lumped mass points in this area will cause the magnitude of the response peaks to increase at the higher frequencies. This new distribution of the mass toward the center panel area will also cause the frequency of the lower modes to decrease. If this distribution of the mass is accomplished in an optimum fashion, the two peaks located at 37 and 60 hertz would be shifted to match the experimental peaks at 10 and 37 hertz.

The general tendency of the analytically determined magnitude of the peaks, particularly at the higher modes of vibration, to be higher than the corresponding experimental magnitudes would be rectified. The analytical peak at 60 hertz can be diminished in magnitude by placing several more lumped mass points in the area of the central panel. Inclusion of these mass points at the center of the stringers which surround the center panel and at points on the center panel near the stringers would have the effect of distributing the response over several frequencies in the neighborhood of 60 hertz. These additional natural frequencies could be adjusted by proper allocation of the mass to the new lumped mass points. The net effect would be to reduce the single peak at 60 hertz to a group of smaller peaks, as seen in Figures 18 and 19 in the frequencies neighboring 60 hertz.

The central portion of the frequency bandwidth from 100 to 400 hertz in Figures 5, 18, and 19 match very well on both the
analytical and experimental curves in that no major peaks are observed on either of the curves in this frequency range. This type of response is expected since the lumped mass point is located at the intersection of two stringers, which is the stiffest point in the central area of the plate. The stringers have a tendency to become node lines for all modes of vibration higher than the second mode and are exactly node lines for the third, sixth, ninth and twelfth modes of vibration. This tendency has the effect of allowing this lumped mass point to respond primarily to only the first and second modes of vibration. This tendency is observed on the curves in Figures 5, 6, 7, 8, 18, and 19.

Figures 11 and 20 represent the analytical and experimental response power spectral densities at two points which are not geometrically equivalent, but are located sufficiently close to one another to observe similarities in their power spectral densities. Figure 11 represents the response power spectral density of lumped mass number 8 (Figure 3) and Figure 20 represents the response power spectral density of strain gage 16 (Figure 26). The root mean square displacements of the two points in question are not expected to compare favorably, but the power spectral densities do show some similarities in natural frequencies between the two points. Basically, the same similarities which were observed for the two previous sets of response power spectral density plots are observed for this pair of curves. The decreasing power of the response peaks as the frequency increases is observed for both these curves. The first and last natural frequencies at 37 hertz and 430 hertz respectively, match very well on both curves. The magnitudes of these peaks do not match as is expected.
The correlation between the experimental and analytical peaks in the midband frequencies of 100 to 400 hertz is good, but has the same shift of analytical frequencies to higher values. The analytical peak of 60 hertz is again the only major difference in natural frequencies between the two curves. Although the two response points being discussed are not equivalent points, the same basic similarities noted in their power spectral densities as observed at the other points already compared indicate that the entire plate possesses these similarities between the experimental and analytical results. Figures 9, 10, 11, 12, and 20 support this observation. Figures 13, 14, 15, and 16, which represent the power spectral densities of the lumped masses at the corner panels, possess most of the same characteristics noted in the comparison between Figures 11 and 20, and, in general, also support the above observation. Correlated with this observation, is the observation that the modifications suggested to improve the analytical results would apply over the entire plate.

The modifications needed in the analytical model, as noted above, is to redistribute the mass such that more mass is included near the center of the plate. This additional mass near the center of the plate is coupled with a corresponding decrease in the mass at the lumped points near the fixed boundary of the plate. An increase in the number of lumped mass points, particularly at the midpoint of the stringers, is also needed. The entire supporting structure of the plate and horn should be included in the lumped parameter model of the system.

The effect of these analytical modifications would be to shift the analytical natural frequencies associated with each peak
in the response power spectral density plots to lower values of frequency. The first and second natural frequencies would be shifted the greatest amount, and the highest natural frequency would be shifted the least amount. The overall level of power associated with the peaks at the higher frequencies would be increased with respect to the power level at the lower frequencies. All these effects on the analytical response power spectral density would tend to make the resulting curves more similar to the experimental curves.

A comparison of the maximum root mean square deflections determined both analytically and experimentally, is displayed in Table I. The error indicated in Table I is the per cent deviation of the analytical results obtained by integrating Equation (111-12) from the experimental results. The error between the experimental results and the results of utilizing Equation (111-14) is in every case greater than the error shown for Equation (111-12). The results obtained by using Equation (111-14) is approximately 100 per cent greater than the results of Equation (111-12) and does not predict the experimental results as well as Equation (111-12). These comparisons indicate that Equation (111-14) is not a very good approximation to Equation (111-12) for the particular excitation power spectral density shown in Figure 2. The maximum deviation between the experimental and analytical [Equation (111-12)] amplitudes is only 41.0 per cent. Considering the complexity of the system involved and the fact that by definition a random vibration defies explicit definition, the 41.0 per cent error represents an excellent analytical prediction. Unfortunately, not enough experimental data
points were recorded to compare with all of the analytical points.

Table IV lists the fourteen natural frequencies calculated by using the experimentally measured and the analytically calculated influence coefficients. The analytical influence coefficients are determined by approximating the plate with a dense network of beams in which all intersections of beams are made rigid. The experimental influence coefficients are measured by recording the deflections of the plate due to calibrated point loads. The two sets of influence coefficients are listed in Tables II and III, but comparing the influence coefficients at each point in the matrix is not an efficient method of comparison in this case. A better comparison is to observe the effect these two sets of influence coefficients have on the results of this problem. The entire effect of the influence coefficients on this problem is transmitted through the eigenvalues (natural frequencies) and eigenvectors of the system. The analytical eigenvectors are altered slightly from the experimental eigenvectors, and the effect on the eigenvalues is represented in Table IV by the natural frequencies. The maximum root mean square values calculated by using the analytical influence coefficients are altered by a maximum of 6 per cent from the values calculated by using Equation (III-12), and the experimental coefficients. In most cases, these analytical influence coefficients' root mean square amplitudes are better approximations to the experimental amplitudes than the ones given by Equation (III-12) in Table I. Generally, the influence coefficients determined by the analytical method described in Chapter IV are a good approximation to the actual values and particularly for the purposes of this problem produce very good results.
CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

This chapter will enumerate the conclusions which are drawn from this investigation and point out certain areas that appear to warrant further work. The conclusions as a result of this investigation are as follows:

1. A lumped parameter analysis of a non-homogeneous plate subjected to a random excitation produced maximum root mean square displacements which compare very well to the results obtained by experimental tests. Results from the experimental work when compared to the analytical show a range of error from 19.1 to 41.0 per cent.

2. The lumped parameter analysis yields its best predictions of the experimental power spectral densities at the lower frequencies of vibration. The error of predicting the power level associated with each peak increases as the frequency increases and ranged from an error of less than one order of magnitude at the lower frequencies to four orders of magnitude at 500 hertz.

3. For the particular excitation utilized in this investigation, integration of Equation (III-12) with the actual excitation produces values of maximum root mean square displacements which compare much more favorably with the experimental results than the results produced by utilizing the "white noise" approximation of Equation (III-14) and the average value of the excitation.

4. The accuracy of predicting the peaks in the response
power spectral densities of the plate in question is strongly dependent on the proper distribution of mass to the various lumped mass points. The distribution of mass and the insufficient number of lumped mass points utilized in this investigation are the major reasons a peak in the response power spectral density plots at 60 hertz contained an erroneously large amount of power.

5. An increase in the number of lumped mass points and the proper distribution of mass to these points will also alleviate the problem of predicting low power levels at the higher frequencies.

6. The influence coefficients of the plate can be predicted sufficiently well for this investigation by representing the plate as a dense network of beams. The error in predicting the natural frequencies is less than 3.5 per cent for the first thirteen natural frequencies.

7. The damping ratio can be regarded as a constant with respect to location on the plate, and as a variable with respect to frequency of vibration. This is verified by experimentally measuring the damping ratio (Figure 21) as both a function of location and frequency.

8. This method of analysis for non-homogeneous plates has the basic qualities of simplicity and accuracy, and may be applied to a wide variety of oddly shaped structures with an ease not found among other methods of analysis.

The recommendations for future investigations in this area are:

1. A study of the effect of various distributions of mass to the lumped mass points on the response of plates should be done.
Homogeneous plates subjected to pure tones could be the first phase and the complexity of this study increased until it includes non-homogeneous plates subjected to random excitations. The object of the study would be to determine the optimum proportions for allocating mass to the lumped points in order to predict the response accurately. Hopefully, the study will lead to a set of rules or guidelines which may be used to allocate mass to the lumped points in any lumped parameter analysis of plates.

2. After the proper allocation of mass to the different areas of the plate have been determined, obtain a relationship between the number of lumped masses used in modeling the plate and the resulting error of the predicted response. This information would be extremely valuable to persons trying to make the most efficient lumped parameter analysis of a plate.

3. Study the versatility, reliability, and limitations of using a network of beams to approximate the influence coefficients of a variety of oddly shaped plates.

4. Study the tendency of the damping ratios for a plate to exhibit a sinusoidal variation in magnitude as a function of frequency. Particularly, study the behavior of the damping ratios at very low and very high frequencies.
APPENDIX A

DETERMINATION OF THE TRANSFORMATION WHICH

UNCOPLES THE EQUATIONS OF MOTION
The purpose of this appendix is to determine the conditions under which a damped dynamic system possess classical normal modes. It is shown that a necessary and sufficient condition for a damped dynamic system to possess classical normal modes is that the damping matrix be diagonalized by the same transformation which uncouples the undamped system. This transformation is accomplished by the normalized eigenvectors of the system.

In general, the coupled equations of motion for an N degree of freedom linear dynamic system with lumped parameters may be written in matrix notation as:

\[
[m] \ddot{g} + [c] \dot{g} + [k] g = f(t) \tag{A-1}
\]

where \([m] \), \([c] \), and \([k] \) are positive definite and symmetric. The homogeneous equation is simply,

\[
[m] \ddot{g} + [c] \dot{g} + [k] g = 0 \tag{A-2}
\]

The undamped homogeneous equation is determined when \([c] \) and \( \{f(t)\} \) are equal to zero.

\[
r[m] \ddot{g} + [k] g = 0 \tag{A-3}
\]

Let \([a] \) be the transformation which makes \([m] \) a diagonal matrix when multiplied in this form \([g] ^T [m] [g] \). This transformation necessarily exists because of the symmetry of \([m] \).

Define,

\[
\{x\} = [a] \{x\} \tag{A-4}
\]

and substitute into Equation (A-2).

\[
[m] [a] \ddot{x} + [c] [a] \dot{x} + [k] [a] x = 0 \tag{A-5}
\]

$$[a]^T [m] [a] \{\ddot{x}\} + [a]^T [c] [a] \{\dot{x}\} + [a]^T [k] [a] \{x\} = 0$$

(A-6)

Define,

$$[a]^T [m] [a] = [\tilde{m}] \text{ a diagonal matrix}$$

$$[a]^T [c] [a] = [\tilde{c}]$$

$$[a]^T [k] [a] = [\tilde{k}]$$

(A-7)

Since $[m]$, $[c]$, and $[k]$ are positive definite and symmetric matrices, $[\tilde{m}]$, $[\tilde{c}]$, and $[\tilde{k}]$ will also be positive definite and symmetric.

Substituting the terms defined in Equation (A-7) into Equation (A-6) one obtains:

$$[\tilde{m}] \{\ddot{x}\} + [\tilde{c}] \{\dot{x}\} + [\tilde{k}] \{x\} = 0$$

(A-8)

The $[\tilde{m}]$ matrix in Equation (A-8) is reduced to the identity matrix by defining

$$\{p\} = [R] \{x\}$$

(A-9)

where

$$[R] = \sqrt{[\tilde{m}]}$$

(A-10)

and substituting into Equation (A-8)

$$[R] \{\ddot{p}\} + [\tilde{c}] [R]^{-1} \{\dot{p}\} + [\tilde{k}] [R]^{-1} \{p\} = 0$$

(A-11)

Premultiply Equation (A-11) by $[R]^{-1}$.

$$[R]^{-1} [R] \{\ddot{p}\} + [R]^{-1} [\tilde{c}] [R]^{-1} \{\dot{p}\} + [R]^{-1} [\tilde{k}] [R]^{-1} \{p\} = 0$$

(A-12)
Define,

\[
[I] = [R]^{-1} [R] \text{ Identity matrix}
\]

\[
[A] = [R]^{-1} [\mathbf{c}] [R]^{-1}
\]

\[
[B] = [R]^{-1} [\mathbf{k}] [R]^{-1}
\]  \hspace{1cm} (A-13)

Substitute Equations (A-13) into Equation (A-12).

\[
[I] \{\ddot{p}\} + [A] \{\dot{p}\} + [B] \{p\} = 0 \hspace{1cm} (A-14)
\]

\[A\] and \[B\] are positive definite and symmetric matrices and according to Hildebrand (38) may be diagonalized simultaneously by a single transformation if and only if the two matrices \[A\] and \[B\] commute. See Bellman (39). Let \[[V]\] be the transformation such that,

\[
[V]^T [A] [V] = [a] \text{ is a diagonal matrix} \hspace{1cm} (A-15)
\]

and

\[
[V]^T [B] [V] = [b] \text{ is a diagonal matrix} \hspace{1cm} (A-16)
\]

Define,

\[
\{p\} = [V] \{n\} \hspace{1cm} (A-17)
\]

and substitute Equation (A-17) into Equation (A-14).

\[
[V] \{\ddot{n}\} + [A] [V] \{\dot{n}\} + [B] [V] \{n\} = 0 \hspace{1cm} (A-18)
\]

Premultiply by \([V]^T\).

\[
[V]^T [V] \{\ddot{n}\} + [V]^T [A] [V] \{\dot{n}\} + [V]^T [B] [V] \{n\} = 0 \hspace{1cm} (A-19)
\]

Substitute Equations (A-15) and (A-16) into (A-19).

\[
[V]^T [V] \{\ddot{n}\} + [a] [V] \{\dot{n}\} + \{b\} \{n\} = 0 \hspace{1cm} (A-20)
\]

Equation (A-20) represents the damped uncoupled system if and only is \([V]^T [V]\) is a diagonal matrix. If \([V]\) is normalized, this requirement becomes,
\[ [V]^T [V] = I \]  

(A-21)

Equation (A-21) restricts the transformation described by Equation (A-17) to be orthogonal.

It is noted that if the above transformations described by Equations (A-4), (A-9) and (A-17) were applied to the undamped system described by Equation (A-3), the required transformation would be the same as the above orthogonal transformation. It therefore follows that, if a damped system possess classical normal modes, these modes are identical with the normal modes for the undamped system. The transformation matrix which uncouples the undamped system is composed of columns which are the eigenvectors of the system. The eigenvectors of the undamped system are, therefore, the proper transformation for uncoupling the equations of motion for the damped system described above.
APPENDIX B

INTEGRATION OF THE TRANSFER FUNCTION
SQUARED BY THE THEORY OF RESIDUES
The transfer function \(H) squared is defined as,

\[
|H(\omega)|^2 = \frac{1}{M^2 \left[ (\omega_0^2 - \omega^2)^2 + (2 \xi \omega_0 \omega)^2 \right]}
\]  \hspace{1cm} (B-1)

where,

\(\xi\) = damping ratio
\(\omega_0\) = natural frequency
\(\omega\) = frequency
\(M\) = mass

The integral (I) of this function from \(-\infty\) to \(+\infty\) will be determined by the theory of residues. See James (15).

\[
I = \int_{-\infty}^{\infty} |H(\omega)|^2 \, d\omega
\]  \hspace{1cm} (B-2)

\[
I = \int_{-\infty}^{\infty} \frac{d\omega}{M^2 \left[ (\omega_0^2 - \omega^2)^2 + (2 \xi \omega_0 \omega)^2 \right]}
\]  \hspace{1cm} (B-3)

The integrand of Equation (B-3) which is a function of the real variable \(\omega\) is treated as a function of the complex variable \(z\). The complex function \(f(z)\) is defined to be,

\[
f(z) = \frac{1}{M^2 \left[ (\omega_0^2 - z^2)^2 + (2 \xi \omega_0 z)^2 \right]}
\]  \hspace{1cm} (B-4)

The complex function described by Equation (B-4) has two simple poles in the upper half complex plane and are determined by solving for the zeroes of the expression enclosed by brackets in Equation (B-4). These poles are found to be,

\[
z_1 = \sqrt{1 - \xi^2} \omega_0 + i \xi \omega_0
\]  \hspace{1cm} (B-5)

and,

\[
z_2 = \sqrt{1 - \xi^2} \omega_0 - i \xi \omega_0
\]
The integral \( I \) of Equation (B-3) is given by the Theory of Residues to be,

\[
I = 2 \pi i \{ \text{The sum of the residues of } f(z) \text{ in the upper complex plane} \}
\]

(B-6)

The residues \( R_1 \) and \( R_2 \) in the upper complex plane are determined by,

\[
R_1 = (z - z_1) |f(z)|_{z = z_1}
\]

and,

\[
R_2 = (z - z_2) |f(z)|_{z = z_2}
\]

(B-7)

(B-8)

Equation (B-6) can be represented as,

\[
I = 2 \pi i (R_1 + R_2)
\]

(B-9)

\[
I = \frac{2 \pi i}{M^2} \left[ \frac{z - \sqrt{1 - \xi^2 \omega_o - i \xi \omega_o}}{\omega_o - (\sqrt{1 - \xi^2 \omega_o + i \xi \omega_o})^2} \right] + \frac{z + \sqrt{1 - \xi^2 \omega_o - i \xi \omega_o}}{\omega_o - (\sqrt{1 - \xi^2 \omega_o + i \xi \omega_o})^2}
\]

(B-10)

Equation (B-10) is reduced to,

\[
I = \frac{2\pi i}{M^2 \omega_o} \left( \frac{\omega_o}{4i \xi} \right) = \frac{\pi}{2M \omega_o \xi}
\]

(B-11)

Equation (B-11) represents the integral of the transfer function squared from \(-\infty\) to \(+\infty\).
For the purpose of practical application the range of integration
is reduced to 0 to $\infty$. Since the integrand of Equation (B-12) is
an even function the integral may be represented by,

$$
\int_{-\infty}^{\infty} H^2(\omega) = 2 \int_{0}^{\infty} H^2(\omega) = \frac{\pi}{2 M^2 \omega_o^3 \xi}
$$

(B-13)

thus,

$$
\int_{0}^{\infty} H^2(\omega) = \frac{\pi}{4 M^2 \omega_o^3 \xi}
$$
****PROGRAM RANDOM****

This program takes an influence coefficient matrix, decomposes it into an upper tri-diagonal matrix, inverts it and produces the stiffness coefficient matrix. The mass matrix is combined with the stiff matrix and used to compute the eigenvalues and eigenvectors of the system. The eigenvectors are converted into a normalized form in the real space and used to calculate the modal participation factors. The program then reads the modal damping ratio matrix and the excitation power spectral density. The RMS displacement (in.) for each lumped mass is determined by exciting each mass and superimposing the displacements due to all excitations. The power spectral density of the output response at each lumped mass is plotted for the frequency bandwith of 6 to 50 Hz.

Prepared by Dennis J. Dilyeu

K = NO. OF CASES
NA = DIMENSION SIZE
N = DEGREES OF FREEDOM
A = INFLUENCE COEFFICIENT MATRIX
D = DIAGONAL OF INFLUENCE COEFFICIENT MATRIX
AMASS = MASS MATRIX
E = NATURAL FREQUENCIES (Hz)
F = NATURAL FREQUENCIES (rad/sec)
V = EIGENVECTORS
P = MODAL PARTICIPATION FACTORS
HZ = FREQUENCY SPECTRUM (6-50HZ)
Z = MODAL DAMPENING RATIO
W = AVERAGE VALUE OF EXCITATION PSD (VOLTS**2/Hz)
X = RMS DISPLACEMENT ASSOCIATED WITH W (IN)
WO = PSD OF RESPONSE (POUNDS FORCE**2/Hz)
XI = RMS DISPLACEMENT ASSOCIATED WITH WI (IN)
FA = AREA OVER WHICH FORCE IS APPLIED FOR EACH MASS

109
DIMENSION A(14,14), O(14,14), E(14), AMASS(14), S MASS(14), XMASS(14)

DIMENSION P(14), F(14), X(14), Z(14), V(14,14), W(502), HZ(502)

DIMENSION IAT1(18), IAF1(18), W(502), XD(502), IAX1(18), IAX2(18)

DIMENSION IAX3(18), IAT2(18), IAT3(18), IAE2(18), IAE3(18)

DIMENSION D(14), FA(14), FOR(14)

COMMON ID9(15), BUF(300), ID8(15)

LOGICAL*1 C

LOGICAL SKIP

DATA C/* * */

SKIP=*FALSE*

CALL PLOTS(UUF,300)

CALL PLOT(0.,1.5,-3)

READ(5,2) ID9

READ(5,2) ID8

2 FORMAT(15A4)

READ (5,4) IAT2

READ (5,4) IAE1

READ (5,4) IAE2

READ (5,4) IAF3

4 FORMAT(18A4)

READ(5,3) K,NA

3 FORMAT(213)

READ(5,5) N

5 FORMAT(13)

READ(5,30) (AMASS(I), I=1,N)

3: FORMAT (8F15.6)

READ(5,31) (FA(I), I = 1,N)

31 FORMAT(14F5.1)

WRITE(6,32) (FA(I), I = 1,N)

32 FORMAT(F14.7)

READ(5,130) (Z(J), J=1,N)

13 FORMAT(13F7.4)

READ(5,131) W

131 FORMAT (F15.6)

READ(5,256) (WI(M), M = 1,495)

256 FORMAT(15F5.1)
WRITE (6,13)
13 FORMAT (1H1,2X,*MASS MATRIX*,///)
   DO 35 I=1,N
      AMASS(I) = AMASS(I)/386.4
      WRITE (6,14) I, (AMASS(I))
      SMASS(I) = (SORT(AMASS(I)))/SMASS(I)
      XMASS(I) = 1./SMASS(I)
   CONTINUE
14 FORMAT (10X,12.1X,F13,6.1)
   WRITE (6,15)
15 FORMAT (1H1,15X,*DAMPING RATIO*,///)
   WRITE (6,14) (J, Z(J), J=1,N)
   DO 255 M = 1,495
      IF(W(M)*LT.GE) GO TO 26C
      W(M) = W(M)*1E-5
   GO TO 1
260 SKIP=.TRUE.,
   DO 100 K = 1,N
      READ (5,10)((A(I,J), J=1,N), I=1,N)
   10 FORMAT(F10.3)
   DO 310 I = 1,14
   310 D(I) = A(I,I)
   WRITE (6,11)
11 FORMAT (1H1,2X,*INFLUENCE COEFFICIENT MATRIX*,///)
   WRITE (6,12) ((A(I,J), J=1,N), I=1,N)
12 FORMAT(1CE12.3,///)
   CALL DECMPS(N,NA*A,EXIT)
   IF(EXIT.EQ.0) GO TO 100
   CALL INVERT(N,NA,A)
   GO TO 300
100 WRITE(6,200)
200 FORMAT(/,1X,*A ZERO OR NEGATIVE APPEARED ON THE DIAGONAL*,///)
   GO TO 500
300 WRITE(6,400)
400 FORMAT(1H1,2.1X,*STIFFNESS MATRIX*,///)
   WRITE(6,12) ((A(I,J), J=1,N), I=1,N)
PRE-MULTIPLY AND POST-MULTIPLY THE STIFFNESS MATRIX BY
THE INVERSE OF THE SQUARE ROOT OF THE MASS MATRIX

DO 40 I=1,N
DO 40 J=1,N
A(I,J)=A(I,J)*XMASS(I)*XMASS(J)
40 CONTINUE
CALL JACOB(E,Q,NA,A,NA,N,N+1)

TRANSFORM EIGENVALUES INTO FREQUENCIES AND EIGENVECTORS INTO
ORIGINAL COORDINATE SYSTEM

*RITE (6,55)
55 FORMAT(1H1,25X,*NATURAL FREQUENCIES (HZ)  */*/)
DO 60 I=1,N
F(I)=SQRT(E(I))
C(I)=F(I)/(2.9*3.1415927)
60 *RITE (6,65) I,E(I)
65 FORMAT(10X,12.1X,F13.7,/) 
DO 80 J=1,N
DO 80 I=1,N
V(I,J)=Q(I,J)*XMASS(I)
80 CONTINUE
AREA =0.
DO 81 I = 1,N
AREA = AREA+FA(I)
WRITE (6,86) AREA
86 FORMAT(10X,*TOTAL AREA OF STRUCTURE =9.2X,F9.3,2X,*INCHES**2)
DO 93 J = 1,N
SUM = SUM+FA(J)*V(J,I)
93 DUM = DUM+AMASS(J)*V(J,I)**2
FOR(I) = SUM
82 P(I) = 1.0/DUM
WRITE (6,110) (I,FOR(I),I = 1,NN)
110 FORMAT(1X,I2,12X,F14.7,/)  
WRITE (6,115)
115 FORMAT(1H1,1X,*MODAL PARTICIPATION FACTORS*,///)
DO 120 J=1,NN
120 WRITE (6,125) J,P(J)
125 FORMAT(1JX,12,1CX,F14.7,/)  
C  
C      REFERENCE PLATE DISPLACEMENT TO THE FRAME  
C  
DO 85 J = 1,N
DO 85 I = 1,N
V(I,J) = V(I,J) - V(14,J)
85 CONTINUE  
C  
90 FORMAT (1X,12,5X,(2,1)X,F13.7,/)  
WRITE (6,91)
91 FORMAT(1H1,1X,*NORMALIZED EIGENVECTORS IN U, C, S,*,///)
C  
C      CALCULATION OF RMS AMPLITUDE OF VIBRATION  
C  
CALL GRAND(WI,495,1,EXIC)
EXIC = FXIC/495,
WRITE(6,128)
128 FORMAT(1H1,1X,*RMS AMPLITUDE OF VIBRATION(IN*),///)
WRITE(6,270) EXIC
270 FORMAT(////,5X,*WAV(V**2/HZ) = '1.2X,E12.5,///)
IF (W*GT*C) GO TO 135
W = EXIC
135 DO 150 I=1,NN
SUM = SUM
DO 140 J=1,NN
XNUM = \(3.1415927 \times (P(J) \times 2) \times (V(I,J) \times 2) \times (F(J) \times 2)\)
DENUM = \(4 \times (J) \times (F(J) \times 3)\)
FRAC = XNUM/DENUM
140 SUM = SUM+FRAC
RMSS = \(W \times \text{SUM} / (2 \times 3 \times 141592)\)
X(I) = \(\text{SORT}(\text{RMSS})\)
X(I) = \(X(I) \times 165.5 / \text{AREA}\)
150 WRITE(6,155) I,X(I)
155 FORMAT(1X,12,1X,E14.7,/)
5H1L(6,215) XI
215 FORMAT(///,50X,'XI(IN) =',2X,E12.5)
READ (5,4) IAT1
READ (5,4) IAT3
195 CONTINUE
    CALL WEBAL(HZ,W1,497.5,,...5,....4,5,7,13),...IAE1,5,,IAE2,5,,
    IIAE3,55)
    D(2) = (D(2)+D(3)+D(4)+D(5))/4.
    DO 1000 M1 = 1,3
    READ(5,250) (XD(M),M = 1,495)
    WRITE(6,250) (XD(M),M = 1,495)
    DO 261 I = 1,495
    XD(I) = XD(I)*1C***(-6)*((165.5*D(M1))**2
    HZ(I) = FLOAT(I)+5.
261 CONTINUE
    READ (5,4) IAX1
    READ (5,4) IAX2
    READ (5,4) IAX3
    CALL WEBAL(HZ,XD,497.5,,...5,....4,5,7,13),...IAX1,5,,IAX2,5,,
    IIAX3,55)
1000 CONTINUE
    CALL PLOT(0,*,999)
500 STOP
END
SUBROUTINE DECMPS (N, NA, A, EXIT)

N = SQUARE MATRIX SIZE
NA = ACTUAL MATRIX SIZE
A = SQUARE MATRIX
EXIT = ERROR MESSAGE NUMBER

DIMENSION A(NA,NA)
REAL*8 SUM, DBLE
EXIT = 0.0
DO 6 I = 1,N
IM = I - 1
DO 6 J = I,N
SUM = A(I,J)
IF (I < J) GO TO 2
DO 1 K = 1,IM
1 SUM = SUM - DBLE(A(K,I)) * DBLE(A(K,J))
IF (J < I) GO TO 3
GO TO 4
3 A(I,J) = SUM * TEMP
GO TO 6
4 IF (SUM) 7, 5
5 DUM = SUM
TEMP = 1.0 / SQRT(DUM)
A(I,J) = TEMP
6 CONTINUE
RETURN
7 EXIT = 1.0
WRITE(6,12)
12 FORMAT(IX, 'SCREWED UP', //)
WRITE(6,10) I,J,SUM
10 FORMAT(15X, I2, 5X, 12, 5X, E14.7)
RETURN
END
SUBROUTINE INVERT (N, NA, U)

DIMENSION U(NA,NA)
REAL*8 SUM, DBLE

U = UPPER TRIANGULAR MATRIX (FROM DECMPS)
N = SQUARE MATRIX SIZE

DO 2 I = 1,N
  IP1 = I + 1
  IF (IP1 .GT. N) GO TO 22
  DO 2 J = IP1, N
    JM1 = J - 1
    SUM = 0.0
    DO 1 K = I,JM1
      SUM = SUM - DBLE(U(K,I)) * DBLE(U(K,J))
    1    U(J,I) = SUM * U(J,J)
  2    DO 4 J = I,N
    
    SUM = 0.0
    DO 3 K = J,N
      SUM = SUM + DBLE(U(K,I)) * DBLE(U(K,J))
    3    U(J,I) = SUM
    4    U(I,J) = U(J,I)
  
RETURN
END
SUBROUTINE GRAND(A,NA,DELTA,AREA)

DIMENSION A(NA)
AREA = 0.
H = DELTA/J.
N = NA-2
DO 1 I = 1,N,2
  AREA = AREA + H*(A(I) + A(I+1) + A(I+2))
1 RETURN
RETURN
END
SUBROUTINE WEBAL(X,Y,NP2,XO,YO,XB,YB,XL,YL,K,KHR,TIT1,TIT2,TIT3)

DIMENSION X(NP2),Y(NP2),TIT1(11),TIT2(12),TIT3(13)
COMMON ID9(15),BUF(300)

CALL PLOT(-5,-5,5,3)
CALL PLOT(-5,8,5,2)
CALL PLOT(5,5,8,5,2)
CALL PLOT(5,5,-5,2)
CALL PLOT(-5,-5,2)
N = NP2-2
CALL SCALE(X,XL,N,K)
CALL SCALOG(Y,YL,N,K)
CALL AXIS(XO,YO,109,-35,XL,C,X(N+1),X(N+2))
CALL LGAXIS(XB,YB,1H,1,YL,93,Y(N+1),Y(N+2))
CALL LGLINE(X,Y,N,K,KH)
CALL SYMBOL(0,14,TIT1,11)
CALL SYMBOL(0,14,TIT2,12)
CALL SYMBOL(0,14,TIT3,13)
CALL PLOT(-5,5.5,3)
CALL PLOT(5,5,7,5,2)
CALL PLOT(12,XO,YO,-3)
RETURN
END
SUBROUTINE JACOBI( D,QT,NQ,A,NA,N,IORD )

DIMENSION A(NA,N),QT(NQ,N),D(N)
DATA EPS/1.E-8/
DO 310 K=1,N
D(K)=A(K,K)
DO 311 M=1,N
311 QT(K,M)=D*K
310 QT(K,K)=1.

IT=0
SUM=0.0
DO 505 I=1,N
DO 505 J=1,N
505 SUM=SUM+ABS(A(I,J))
505 IF(IT-1)505,510,511
510 TH=SUM/(N*N)
GO TO 515
511 TH=TH/N
GO TO 515
512 TH=0.0
515 IT=IT+1
E=0.0
NM1=N-1
DO 10 J=1,NM1
JM=J+1
DO 10 I=JM,N
IF(ABS(A(I,J))>EPS*ABS(D(I))+ABS(D(J))) GO TO 10
L=I
M=J
IF(ABS(A(L,M))>EPS*(ABS(D(L))+ABS(D(M)))) GO TO 52
BETA=(D(L)-D(M))*S/A(L,M)
T=1.0/(BETA+SIGN(SORT(BETA**2+1.),BETA))
C2=1.0/(1.+T*T)
C=SORT(C2)
S=T*C

120
IF (ITOL.EQ.2) GO TO 512
IF (EOLT.3.EQ.0) GO TO 512
CONTINUE
L (L, M) = 0
40 C (K, L) = DUMW2 + 5 (DUM1 - DUM2 + DUM1*TAU)
DUMW2 = 0 (K, L)
DUM1 = 0 (K, M)
DO 40 K = 1, N
D (L) = D (L) + 1
303 D (M (D (M)) = -1
DO 330 K = 1, P
202 IF (LPFJ N) GO TO 303
202 A (K, L) = DUM2 + 5 (DUM1 - DUM2 + DUM1*TAU)
40 (K, M) = DUM2 + 5 (DUM1 - DUM2 + DUM1*TAU)
DUM2 = A (K, L)
DUM1 = A (K, M)
DO 202 X = M, P
DO 100 X = M, M
101 IF (MM.LT.0) GO TO 101
101 L = L + 1
L = L - 1
M = M + 1
M = M - 1
E = D + 85 (A (L, M))
L = L (M) + 1
TAU = 5 / (1 + C)
DO 306 I=1,N
DO 306 J=1,N

306 A(J,I)=A(I,J)
   IF( IORD .EQ. 00 ) RETURN
       FORD = FLOAT(IORD)
   DO 410 I=1,NM1
       IP1=I+1
       IMIN=I
   DO 465 J=IP1,N

405   IF((D(IMIN)-D(J))*FORD .GT. 0.0) IMIN = J
       IF(IMIN.EQ.I) GO TO 410
       TEMP=D(IMIN)
       D(IMIN)=D(I)
       D(I)=TEMP
   DO 415 J=1,N
       TEMP=QT(J,IMIN)
       QT(J,IMIN)=QT(J,I)
   415   QT(J,I)= TEMP

410   CONTINUE
   RETURN
END
APPENDIX D

SAMPLE CALIBRATION CALCULATION
Sample Calibration Calculations

Calculation of Constant A:

\[ \Delta R_5 = R_G - \frac{R_G + R_p}{2} \]

\[ \Delta R_5 = 120.8 - \frac{120.8 (50,000)}{120.8 + 50,000} \]

\[ \Delta R_5 = 120.8 - 120.51 \]

\[ \Delta R_5 = .29 \]

\[ \frac{\Delta R}{V} = \frac{.29 - .07}{1.051 - .254} = \frac{.22}{.797} = .276 \]

\[ \frac{\Delta R}{V} = \frac{.27 - .07}{1.056 - .246} = \frac{.22}{.810} = .2715 \]

\[ \frac{\Delta R}{V} = \frac{.29 - .07}{1.041 - .2335} = \frac{.22}{.807} = .273 \]

\[ A = \frac{\Delta R}{V} \text{ average} = (.276 + .2715 + .273) / 3 = .274 \]

Calculation of Constant B:

\[ B = \frac{1}{RF} \frac{1}{120.8 (1.98)} = .00418 \]

Calculation of Constant C:

From mass number one, L = 20 pounds force,
t = 10,000 - 9,876 = 124 \times 10^{-6} \text{ (in/in)}

C = \frac{L}{t} = \frac{20}{124 \times 10^{-6}} = .1538 \times 10^6

From mass number two, L = 20 pounds force,

t = 10,000 - 9,859 = 141 \times 10^{-6} \text{ (in/in)}

C = \frac{L}{t} = \frac{20}{141 \times 10^{-6}} = .142 \times 10^6

From mass number three, L = 20 pounds force,

\begin{align*}
t &= 5,000 - 4,856 = 144 \times 10^{-6} \text{ (in/in)} \\
C &= \frac{L}{t} = \frac{20}{144 \times 10^{-6}} = .14 \times 10^6
\end{align*}

C average = (.14 + .142 + .1538) \times 10^6 / 3 = .144 \times 10^6

A \times B \times C = .274 \times .00418 \times .144 \times 10^6 = 165.5


VITA

Dennis Joseph Bilyeu was born in Golden Meadow, Louisiana, on January 12, 1945. On May 30, 1962, he graduated from Golden Meadow High School. In September of that year he enrolled at Louisiana State University. He received his B. S. in Mechanical Engineering in January of 1967. In February of 1967, he enrolled in the Graduate School of Louisiana State University and received the Master of Science Degree in Mechanical Engineering in August of 1968. He is now a candidate for the Doctor of Philosophy Degree in Mechanical Engineering at the same University.

During the summers he worked for the following companies: Lafourche Telephone Company in Larose, Louisiana; Texaco, Inc., Leeville, Louisiana; Western Electric Company, at Cape Kennedy, Florida; and Monsanto Company in Hartford, Connecticut. He also spent two summers at the Mississippi Test Facility performing the experimental test required for his dissertation. While attending Louisiana State University, he became an active member of the following organizations: The American Society for Mechanical Engineers, Pi Tau Sigma, The American Institute of Aeronautics and Astronautics, Pi Mu Epsilon, and Theta Xi.
Candidate: Dennis J. Bilyeu

Major Field: Mechanical Engineering

Title of Thesis: Lumped Parameter Analysis of a Stringer Reinforced Plate Excited by Band Limited Noise

Approved:

[Signature]

Major Professor and Chairman

[Signature]

Dean of the Graduate School

EXAMINING COMMITTEE:

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Date of Examination: June 16, 1972