The Construction and Refinement of a Test for Analytical Cognition of Mathematical Content.

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THE CONSTRUCTION AND REFINEMENT
OF A TEST FOR ANALYTICAL COGNITION
OF MATHEMATICAL CONTENT

A Dissertation
Presented to
the Faculty of the Department of Education
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In Partial Fulfillment
of the Requirements for the Degree of
Doctor of Philosophy

by
James S. Cangelosi
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ABSTRACT

The purpose of this study was to develop a test for analytical cognition (as defined in Bloom's Taxonomy of Educational Objectives) of mathematical content that has high usability for secondary schools. The test was primarily designed for pupils who have completed at least a two-semester course in secondary-school algebra.

The motivation for undertaking such a study grew out of a concern for the disparity between stated educational objectives and those objectives which are actually measured. While statements on objectives of school mathematics curriculums usually involve the development of higher-cognitive processes (such as analysis), standardized tests which are commonly used to assess these curriculums measure no more sophisticated cognitive processes than the application level of Bloom's Taxonomy.

Ninety-three multiple-choice items were developed with each item designed to measure one of nine cognitive behaviors which are indicative of analysis of mathematical content. A panel of eight judges, all recognized experts in the field of mathematics education, judged that ninety-one of the items were valid indicators of analytical behavior.

Using sixty of the ninety-one items that were judged
valid, two trial tests were administered and item analysis was performed. The more discriminating items in each of the nine behavioral categories were chosen to form a third test. This third test consisted of forty items and was administered to one hundred six pupils who were enrolled in a secondary school algebra II course. From this administration further item analysis and a reliability study were conducted.

This third test was considered satisfactory for the purposes of the study because of the following:

(1) Each item was judged by a panel of experts to be a valid indicator of analytical cognition for those pupils for whom the test was designed; (2) thirty-four of the forty items were shown to have indices of discrimination above .40; (The other six had positive indices of discrimination.) (3) the reliability coefficient of the test, computed using the Kuder-Richardson formula twenty, was .90; (4) the test was administered by the regular classroom teachers (over a two-day period) without disrupting the normal school-day schedule; and (5) the test consisted only of multiple-choice items that were easily and reliably scored by hand.
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CHAPTER I
INTRODUCTION

Preliminary Statement

Traditionally the development of the higher mental processes has been a goal of secondary education. The demise of the long repudiated formal discipline psychology has made it difficult to place this goal on a tenable basis. Nevertheless, the literature on educational objectives is filled with such phrases as "... ability to scan or analyze a problem in a disciplined fashion ..." (Begle, 1962:648), "... habits of critical thinking ..." (Panush, 1962:439-443), "... the development of problem-solving techniques ..." (Tiegs, 1959:212), "... ability to analyze, in a particular work of art, ..." (Bloom, 1956:148), and "... ability to infer the author's purpose ..." (Bloom, 1956:148). And in addition, modern psychologists indicate that the educational community should not abandon these lofty goals; by use of proper methods, content can be taught so that intellectual skills are developed (Hartman, 1966:656-661) and non-specific transfer of learning can occur (Bruner, 1960:5-20).

While it has become increasingly necessary for the entire educational community to be evaluated, "proof
of success" has been primarily based on standardized tests that measure little deeper than recall knowledge (Robinson, 1970:67-68). Standardized achievement tests now in general use purport to measure, at best, comprehension and application. (Buros, 1965) Therefore, the most sophisticated outcomes or higher behavioral goals of education are not being measured in assessment programs. Any measurement of such goals must be done through the medium of teacher made tests. And indications are that very little, relatively speaking, is being accomplished even in teacher made tests. (Smith, 1969:4)

The purpose of this study was to develop a parameter for educational achievement in the area of one higher behavioral goal, ability to analyze mathematical content.

The items of the test were classified according to three levels of analytical cognition: analysis of elements, analysis of relationships, and analysis of organizational principles (Bloom, 1956:144-148). Numerical content and algebraic content were used, but no language or concepts that are not typical to an introductory secondary school algebra course were included. Such a scheme facilitated the attempt to construct a parameter which is comprehensive with regard to analytical levels, but which does not place a premium on content knowledge.

Statement of the Problem

The problem was to construct and refine a test for
analytical cognition of mathematical content.

Delimitation of the Problem

The test was constructed for use at the secondary school level and was primarily designed for pupils who have completed at least a two semester introductory course in secondary school algebra.

Definition of Terms

Analytical Thinking Ability. Analytical thinking ability is the intellectual skill which enables one to breakdown content into its components, detect the relationships of the components, and identify the organizational principles of the whole. (Bloom, 1956:144-148)

Analysis. Analysis is the cognitive behavior at the 4.00 level (on a 6.00 scale) in Bloom's Taxonomy. Analytical cognition has three levels (Bloom, 1956:144-148):

1. Analysis of Elements. Analysis of elements is the discerning of content components. For the purposes of this study the following behaviors were considered indicative of analysis of elements:

a) Given the definition of an unfamiliar set B such that B is a subset of the real numbers, distinguishes the elements of B from real numbers which are not elements of B.
b) Identifies the necessary (unstated) assumptions in the presentation of a mathematical argument.

c) Given a sample of arithmetic computation performed in an unknown base b, identifies b from a set of five positive integers.

2. Analysis of Relationships. Analysis of relationships is the detection of the logical or causal connection among content components. For the purposes of this study the following behaviors were considered indicative of analysis of relationships:

a) Given a function f (in abstract terms) from a set D into the reals, identifies the order relations among the elements of the image of D under f.

b) Given a premise, distinguishes between conclusions which are inferences of the given premise from conclusions which are not inferences of the given premise.

c) Given a real variable x with certain conditions imposed on x, identifies the relations among the factors and/or terms of x which are necessary in order for the given conditions to be true.

Analysis of organizational principles is the identification of the organization, systematic arrangement, and structure which holds together the components to form a whole.

For the purposes of this study the following behaviors were considered indicative of analysis of organizational principles:

a) Given a finite number of terms in an infinite real number sequence, identifies the nth terms where n is any positive integer.

b) Given the following:
   i. * is an operation on a set X.
   ii. A and B are proper subsets of X.
   iii. a*b is given for each a in A and for each b in B.
   iv. c is an element of X but not of A, and d is an element of X but not of B.

Identifies from a five element subset of X, the element e such that c*d = e.

c) Given two integer arrays each associated respectively with two given calendar dates, identifies from a group of five dates the date associated with a third integer array.

(See Appendix A, page 116)
**Numerical Content.** Numerical content is mathematical information presented primarily in the form of real number constants. For the purposes of this study, numerical content was limited to a level which requires knowledge of nothing more sophisticated than addition and multiplication on subsets of real numbers.

**Algebraic Content.** Algebraic content is mathematical information presented primarily in the form of variable symbols. For the purposes of this study, algebraic content was limited to a level no greater than one that is ordinarily associated with a secondary course in introductory algebra.

**Panel of Experts.** Panel of experts refers to the group of persons that judged the content validity of the items of $M_1$ (See "Definition of Symbols" on page 7). The following authorities in secondary school mathematics comprised this panel:

1. Dr. Sam Adams, Professor of Education at Louisiana State University.

2. Dr. Hubert S. Butts, Professor of Mathematics at Louisiana State University.

3. Dr. E. Glenadine Gibb, Professor of Education and Professor of Mathematics at the University of Texas.

4. Dr. Jack W. Garon, Supervisor of Secondary School Mathematics at Louisiana State University Laboratory School.
5. Mr. Donald W. Hammons, Supervisor of Secondary School Mathematics at Louisiana State University Laboratory School.
6. Dr. Houston T. Karnes, Professor of Mathematics at Louisiana State University.
7. Dr. Eugene D. Nichols, Head of the Department and Professor of Mathematics Education at Florida State University.
8. Dr. F. Lynwood Wren, Professor of Mathematics at San Fernando Valley State College.

**Definition of Symbols**

\( M_1: \) \( M_1 \) is the set of ninety-three multiple-choice test items developed for the purpose of measuring analytical cognition of mathematical content. Each item was classed according to the behavioral objective that it was designed to measure. \( M_1 \) formed a pool of items from which trial forms of the test of analytical cognition were drawn. (See Appendix A)

\( M_2: \) \( M_2 \) consists of \( M_1 \) less those items which were not reported favorably by the majority of the panel of experts.

\( T_1: \) \( T_1 \) is a trial form test of analytical cognition chosen from the items of \( M_2 \). (See appendix B)

\( T_2: \) \( T_2 \) is a trial form test of analytical cognition which was chosen from items of \( M_2 \). None of the items in \( T_2 \) are contained in \( T_1 \). (See Appendix C)
**Ss₁**: Ss₁ refers to the group of secondary school pupils which took T₁.

**Ss₂**: Ss₂ refers to the group of secondary school pupils, entirely distinct from Ss₁, which took T₂.

**T₃**: T₃ is the final refined form of the test for analytical cognition of mathematical content. (See Appendix E)

**Ss₃**: Ss₃ refers to the group of secondary school pupils, entirely distinct from Ss₁ and Ss₂, which took T₃.

**Procedures**

M₁ was constructed with each item designed to measure one of the nine cognitive behaviors which are indicative of analysis of numerical and algebraic content. Members of the panel of experts were provided with a questionnaire (see Appendix A) in which each item of M₁ was listed with the specific behavioral objective that it was designed to measure. For each item, each panel member was asked to give a positive or negative response to the following question: Will this item measure its corresponding specific behavioral objective? Items which were not reported valid by the majority of the panel were eliminated.

T₁ and T₂ were constructed from M₂ and administered to Ss₁ and Ss₂ respectively.

Item analysis was conducted using the scores of Ss₁ and Ss₂ on T₁ and T₂ respectively. The scores of Ss₁ and
Ss₂ were ranked. The number of times each possible response to each item was chosen by the upper 27% of Ss₁ was recorded. And the number of times each possible response to each item was chosen by the lower 27% was recorded. The difficulty level and index of discrimination for each item were computed. The same data was recorded from the scores of Ss₂ on T₂.

The more discriminating items in each of the nine behavioral categories were chosen from T₁ and T₂ to form T₃. These items were judged satisfactory because their mean index of discrimination exceeded .40. (Ebel, 1965: 346-358)

T₃ was administered to Ss₃ and a reliability coefficient of .897 was obtained using the Kuder-Richardson formula number twenty (Ebel, 1965: 310-330). This reliability coefficient obtained on T₃ was considered satisfactory and T₃ became the end product of this research.

Importance of the Study

If educators are to continue to defend curricular content and methods on the basis of developing analytical cognitive ability, then there is a need to measure this objective. An instrument which measures pupil cognitive behavior at the analysis level can function as an assessment device for answering such questions as the following: Is analytical cognition of mathematical content an outcome of
a specific mathematical course for a specific group of pupils? Is a given pupil more adept in analysis of algebraic content than he is of numerical content?

Such an instrument can serve as a model to both classroom teachers and producers of standardized tests for creating devices to measure the higher behavioral goals of education.
CHAPTER II
SURVEY OF LITERATURE

The publication of *Taxonomy of Educational Objectives* by Benjamin Bloom's committee in 1956 provided the educational community with a workable system of classifying cognitive achievement. Bloom's *Taxonomy* was selected for this study, from numerous published schemes, as the principal reference for defining levels of cognitive behavior which reflect academic achievement. In the *Taxonomy's* cognitive domain six broad levels of cognition are described: (1) knowledge (which involves the recall of facts), (2) comprehension (which involves the lowest level of understanding), (3) application (which involves the use of abstractions in concrete situations), (4) analysis, (5) synthesis (which involves the creation of unique wholes), and (6) evaluation (which involves the making of value judgments). (Bloom, 1956:62-197)

This study was involved, of course, with the *Taxonomy's* fourth broad level, analysis. Analysis is closely related to the other five levels and is not completely distinguishable from comprehension and synthesis. Bloom defined analysis as follows:

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The breakdown of a communication into its constituent elements or parts such that the relative hierarchy of ideas expressed are made explicit. Such analyses are intended to clarify the communication, to indicate how the communication is organized, and the way in which it manages to convey its effect, as well as its basis and arrangement. (Bloom, 1956:205)

The Taxonomy identifies three ordered levels of analysis: analysis of elements, analysis of relationships, and analysis of organizational principles. Analysis of elements is defined as the "identification of the elements included in a communication." Analysis of relationships is defined as the identification of "the connections and interactions between elements and parts of a communication." And analysis of organizational principles is defined as the identification of the "organization, systematic arrangement, and structure which hold the communication together." (Bloom, 1956:205-206)

Jerome Bruner's observation is consistent with the definition of analytical thinking as defined in this study:

Analytical thinking characteristically proceeds a step at a time. Steps are explicit and usually can be adequately reported by the thinker to another individual. Such thinking proceeds with relatively full awareness of the information and operations involved. It may involve careful and deductive reasoning, often using mathematics or logic and an explicit plan of attack. Or it may involve a step-by-step process of induction and experiment, utilizing principles of research design and statistical analysis." (Bruner, 1960:57)

The educational literature is replete with ambitious objectives involving cognitive achievement analogous to each of Bloom's levels. On the other hand, Buros' Mental
Measurement Yearbooks lists one hundred ninety-nine mathematical achievement examinations and not one of these is designed to measure secondary school achievement above the application level. (Buros, 1965) Throughout educational literature one point is clearly brought out: There exists a great imbalance between the desired goals of education and the measured goals of education. The present public and professional outcry for educational accountability makes the diminishing of this imbalance especially critical.

The need for more extensive pupil performance measurement is evident. If teachers, for example, are to be held responsible for what is learned by their pupils, then pupil performance must be measured at least yearly so that gains associated with each teacher can be identified. Also, if the overall effectiveness of educators and schools is to be assessed, measurement will have to be extended to many more dimensions of pupil performance than are covered by instruments in common use. This implies more comprehensive, more frequent testing than is standard practice in most school systems. In the long run, it will probably require substantial efforts to develop and validate more powerful measurement instruments. (Barro, 1970: 198)

Educators generally agree that any adequate theory of instruction must incorporate concepts such as transfer, analysis, synthesis, and evaluation. Approaches to measurement have not, however, adjusted to the "new" conception of the disciplines. A workable theory of instruction would seem to depend upon the development of measurement instruments which operationally define these concepts in a manner which will make it possible to empirically investigate the relationships of methods of instruction and learner variables to attainment of different types of educational objectives. (Smith, 1969:24)

Today's tests can ... measure vocabulary word-recognition skills with sufficient accuracy, They cannot, however, adequately measure listening comprehension or the ability to analyze the opposing
sides of an argument.

Contemporary test technology is not refined enough to meet all the demands. In performance contracting the first demand is for assessment of performance. Tests do their job well when the performance is highly specific — when, for example, the student is to add two numbers, recognize a misspelled word, or identify the parts of a hydraulic lift. When a teacher wants to measure performances that require more demanding mental processes, such as conceptualizing a writing principle or synthesizing a political argument, performance tests give us less dependable scores.

... A question for most tests specialists ... is not "Can complex educational outcomes be measured?" but "Can complex educational outcomes be measured with the time and personnel and facilities available?" (Stake, 1971:584)

... most achievement tests consist primarily of items testing specific elements of knowledge, facts, ideas, explanations, meanings, processes, procedures, relations, consequences, and so on. (Ebel, 1971:130)

In mathematics education, where the curriculum has long been defended by objectives that have never been measured, the need for developing sophisticated achievement tests is critical.

... many if not most, mathematics teachers today will suggest that one of the main reasons for teaching mathematics is to teach children to think logically. (Willoughby, 1967:13)

Research related to the development of measuring instruments for higher cognitive achievements seems to be precluded by a lack of conventions regarding levels of cognition and regarding test development standards. Without common criteria and language, three hundred researchers advance in three hundred different directions; sequence and continuity among different research projects is lost.
In a project sponsored by the U. S. Office of Education, Smith developed thirty-one scalable test items in the content area of physical science. Each set contained at least six items, each designed to measure at one of the broad cognitive levels defined in Bloom's Taxonomy. The stated purpose of the study was to construct a highly reproducible set of items which could be utilized in a study of the relationship between process and content. Administration of the test to secondary school pupils corroborated the belief that there exists a hierarchy of cognitive processes with the knowledge level items being the easiest and with items becoming more difficult as the level measured is raised. The items were found to be highly reproducible and moderately scalable. Content and process (or cognitive level) seemed to be inextricably interrelated. The success of a given individual's performance tended to depend more on the type of content than on the cognitive level. (Smith, 1969)

Recent research in mathematics education is often concerned with psychomathematics. However, the validity of these studies often depends on standardized measuring instruments which, as has been previously brought out, are designed for measurement below the analysis level.

E. G. Begle expressed the need for evaluating the outcomes of mathematics programs and consequently the need for adequate testing of mathematical skills and outcomes. (Begle, 1962:648) Begle was instrumental in developing the
National Longitudinal Study of Mathematics Abilities which had two major purposes: "... (1) a rigorous investigation of the curricular approach used by the school systems, and (2) an assessment of how successful the "new" mathematics have been in reaching their desired goals." (Romberg, 1969: 4) All levels in Bloom's Taxonomy were considered in the twenty-six test batteries which were administered over a five year period from 1962 to 1967. (Romberg, 1969:121-128) Test development was a collaborative effort of the School Mathematics Study Group Panel on Tests and numerous educators, mathematicians, psychologists, psychomathematicians, data processing specialists, and statisticians. (Romberg, 1969:211-216)

The problems involved in developing test items which measure at certain cognitive levels were realized by the developers of the National Longitudinal Study of Mathematics Abilities:

There are two fundamental problems which arise when one attempts to adapt the classification system to actual behavior and writing test items. The first is with regard to previous knowledge. That is, what might be analysis (4.00) for one student may only be comprehension (2.00) for another who had a wider background of experiences. Thus, if one attempts to write a question to test an analytical skill it may only be measuring comprehension. Ideally, one would have to analyze every test item with regard to each student's background in order to determine what behavior is being measured.

The second difficulty in classification arises from the fact that the more complex behaviors include the simpler behaviors. That is, if a student misses a question intended to measure analysis behavior, his difficulty may not be at that level but with comprehension (perhaps with translating an unfamiliar term).
These are inherent difficulties with the taxonomy. They do not invalidate its use; they simply point out its limitations. (Romberg, 1969:283)

The Advanced Placement Examination in Mathematics, under the direction of the College Entrance Examination Board, includes a number of analytical items such as the following:

Which one of the following defines a function $f$ for which $f(-x) = -f(x)$?
(A) $f(x) = x^2$. (B) $f(x) = \sin(x)$. (C) $f(x) = \cos(x)$.
(D) $f(x) = \log(x)$. (E) $f(x) = e^x$.
(Finkbeiner, 1971:502)

However, these tests are designed for students who have experienced calculus. Many of the would be analytical items, such as the one quoted above, are only simple recall or application items for pupils thoroughly schooled in calculus.

A number of psychological tests under the general heading of "field independent - field dependent" tests have been developed.

An analytical, in contrast to a global, way of perceiving entails a tendency to experience items as discrete from their backgrounds, and reflects ability to overcome the influence of an imbedded context. People differ in the extent to which their perception is analytical. This dimension of individual differences has been called field-dependence-independence. A tendency toward an analytical or global way of perceiving characterizes a person's perception in a wide variety of situations, making for marked individual self-consistency. (Witkin, 1962:57-58)

The most commonly used of these field independence - field dependence tests include the Rod-and-Frame Test, the
Tilting-Room-Tilting-Chair Test, and the Embedded-Figure Test (EFT). (Spitler, 1970:15) In each test the subject is required to differentiate either himself or an object from a surrounding field. (Spitler, 1970:15)

While these tests could be considered measures of analytical cognition, they are limited to analysis of figural or spatial content. Meaux found EFT performance to be predictable from knowledge of spatial and comparison abilities, but reasoning to be of little importance in EFT performance. (Meaux, 1960:1625)

Content and cognitive process are inextricably involved; however, analytical cognition of a wide range of content seems to be a very realistic concern of educators. Rosman studied the analytic styles of first and second grade boys and found consistent individual differences in the tendency of young children to analyze stimulus content. The sources of these individual differences were left open to question. (Rosman, 1966:2126) However, Wahler indicated that discrimination training can significantly enhance ones speed in locating stimulus figures when imbedded in a field. (Wahler, 1963:408) Witken related these individual differences to both sex and maturation level. (Witken, 1962:374)

In an in-depth analysis of the major positions of researchers of cognitive styles, Spitler found that school mathematics curriculums place a high premium on field independent (analytical) cognitive style. (Spitler, 1970:
Spitler concluded that field independent – field dependent cognitive styles are psychological constructs that have profound implications for mathematics education. (Spitler, 1970:171-173)

Statistically significant negative correlations were found between perceptual field-dependency as measured by the Embedded Figure Test and mathematical abilities; i.e., the poorer mathematics students were more field-dependent and the better were more analytic and independent in their perception. (Rosenfield, 1968:880)

The literature on test construction and test refinement is filled with varying opinion. For the purposes of this study, Robert Ebel's *Measuring Educational Achievement* and H. E. Garrett's *Statistics in Psychology and Education* will be used as principal references in determination of test reliability and content validity.
CHAPTER III
DEVELOPMENT OF THE TEST ITEMS

Formulation of the Objectives

Critical to the development of $M_1$ was the formulation of a set of measurable behavioral objectives which would be indicative of analytical cognition at each of the three levels – analysis of elements, analysis of relationships, and analysis of organizational principles. Objectives which were measurable through the medium of multiple-choice items were favored because of the usability factor for classroom situations.

Analytical thinking by definition is a slow step-by-step process. (Bruner, 1960:57-58) A speed test was not desirable as a measure of analysis, but at the same time the test was to be usable for secondary school classrooms. An untimed test of two hundred items would ordinarily give a better sampling of the universe of analytical behaviors than would a shorter test, but the longer test would have low usability for classroom situations. There was also some question as to the desirability of an excessively long test due to the fatigue factor, which would be expected to play a role in any measure of analytical behavior.

The length of the test had to be severely limited
and therefore the objectives were also limited. The content of the objectives was restricted to numerical and algebraic content as previously defined (see page 6). The objectives were then designed to include samples at each of the three levels of analysis, but limited by cognitive processes ordinarily associated with two college mathematics courses—real number analysis and number theory. This restriction was chosen with the hope of constructing an instrument which would measure cognitive processes which would be mathematically useful in the future of secondary school pupils.

Formulating objectives at the third level of analysis (analysis of organizational principles) presented a far more difficult task than for the first two levels. The lack of a unique set of organizational principles that generate a mathematical function precluded the satisfactory formulation of third-level objectives which were measurable through multiple-choice items. It was decided, however, to state third-level objectives for multiple-choice measurement and then obviate the duality of correct responses to each item by meticulous item construction.

The nine behaviors that have been used to define analysis were formulated in light of the limitations and goals previously stated. In order to avoid ambiguity and to facilitate item formulation, an attempt was made to state these nine objectives in mathematically precise terms.
The resulting objectives, to be mathematically precise, tended to be so complex that careful analysis was required by the reader for proper understanding. However, because of the mathematical expertise of those who were required to understand the objectives (namely the panel of experts), this difficulty was not considered critical.

**Development of the Items**

In light of the nine behavioral objectives, the items were developed without reference to outside resources such as text books, standardized tests, and personal consultations. An often stated complaint about standardized tests concerns a lack of originality of the test items (Buros, 1965:862-921) and as $T_3$ was to be an innovation in measuring instruments, there was a decided advantage in initially developing the items in isolation from outside resources.

The wording of the items presented the problem of brevity and simplicity versus mathematical precision. Persistence in the use of mathematically precise terminology tended to engender verbose and lengthy items which could result in undue confusion on the part of those for whom the test was intended. However, pupils who performed sophisticated analysis on the items could be penalized if the items were loosely worded. A compromise was attempted where the language of the items was carefully designed, but some mathematical exactness was sacrificed.
For example, in item VIII-8 of \( M_1 \) (See Appendix A) reference is made to an operation from the set of ordered pairs of integers cross the ordered pairs of integers into the integers, i.e., \(((a,b), (c,d)) \rightarrow e\) where \(a, b, c, d,\) and \(e\) are integers. An operation on a set \(A\) is a function from \(A \times A\) into \(A\) and therefore to be mathematically precise the operation needs to be such that \(((a,b), (c,d)) \rightarrow (e,f)\) where \(a, b, c, d, e,\) and \(f\) are integers. This could have been accomplished had the right side of each equation been an expression of the form \((e,0)\) instead of \(e\). However, it was decided that the clarity of the item would be enhanced for pupils who were unfamiliar with abstract operations (as \(S_{s_1}\) and \(S_{s_2}\) were unfamiliar) if \(\{\text{integers}\} \times \{0\}\) was replaced by a set which is isomorphic to it — namely \(\{\text{integers}\}\).

On the other hand, expressions such as "there exist," "such as," and "if and only if," so familiar to the trained mathematician but not necessarily to pupils who would take the test, were included on the premise that the meanings of such expressions were the same in the context of \(M_1\) as they are in ordinary, everyday English. A note was made to carefully instruct \(S_{s_1}\) and \(S_{s_2}\) to read each word and interpret meanings literally.

The use of shorthand symbols and mathematical terms was restricted to those which are common and typically basic to introductory algebra courses.
Third level analytical items were designed to obviate the problem of dual correct responses as previously mentioned. For example, in item VII-2 of M₁ (See Appendix A), there are an infinite number of sequences whose first four terms are $1, \frac{1}{4}, \frac{1}{9}$, and $\frac{1}{16}$. However, when given that "all the terms are developed by the same pattern," only an extremely divergent thinker would fit these first four terms into any other sequence before he recognized $\left\{ \frac{1}{n^2} \right\}_{n=1}^{\infty}$. The difficulty is further mitigated by carefully choosing the possible alternatives (a, b, c, d, and e) so there would be little chance that one of the distractors could be fit into a sequence determined by a common pattern. Each of the third level items was trial tested on four graduate mathematics students and in no case was a distractor found to be a tenable response.

Before determining what an adequate length of M₁ would be, it was necessary to estimate the number of items that members of S₁ and S₂ could be expected to answer in a typical fifty minute class period. Therefore, after ninety-three items had been constructed, six secondary school seniors were given an untimed trial test of thirty items from M₁. After fifty minutes, the number of items completed by each pupil was recorded. The pupils were then allowed to complete the remainder of the test.
Table 1

Results of the Trial Test of Thirty Items Used in Estimating the Number of Items that Can Be Completed in Fifty Minutes

<table>
<thead>
<tr>
<th>Pupil</th>
<th>Number of Items Completed After Fifty Minutes</th>
<th>Time Used To Complete All Thirty Items</th>
<th>Test Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30</td>
<td>50 minutes</td>
<td>21</td>
</tr>
<tr>
<td>B</td>
<td>22</td>
<td>63 minutes</td>
<td>20</td>
</tr>
<tr>
<td>C</td>
<td>22</td>
<td>70 minutes</td>
<td>24</td>
</tr>
<tr>
<td>D</td>
<td>21</td>
<td>83 minutes</td>
<td>25</td>
</tr>
<tr>
<td>E</td>
<td>14</td>
<td>98 minutes</td>
<td>23</td>
</tr>
<tr>
<td>F</td>
<td>14</td>
<td>100 minutes</td>
<td>19</td>
</tr>
</tbody>
</table>

The group definitely responded to the items at a more rapid rate after the first fifty minutes than they did during the initial fifty minutes. (See Table 1) The author observed that most of the pupils took about fifteen minutes to orient their thinking to the particular cognitive activity required by the items.

It was decided that $T_1$ and $T_2$ should each consist of thirty items to be administered in two fifty minute sessions.

The six pupils who took the trial test offered the following unsought comments: "Enjoyable!" "May I take another test like this?" "I've never taken a test like this before, it makes you think!" "It was different."
CHAPTER IV
JUDGMENT OF CONTENT VALIDITY

Members of the panel of experts were chosen on the basis of their reputation in mathematics education. Written letters and personal meetings were used to explain the project to the panel and to request their services. Each member agreed to judge the items of \( M_1 \) and were subsequently provided with questionnaires (See Appendix A).

The results of the panel's judgment are reported in Table 2. One judge prefaced his responses with the remark, "I should ... inform you concerning my very strong negative attitude toward the use of multiple-choice questions as a valid testing technique. To my way of thinking the situations are extremely rare in which such a technique is anything more than a measure of a combination of the power of suggestion, guesswork, and luck." This particular judge returned the only questionnaire out of the eight that was not overwhelmingly favorable.

Only two items from \( M_1 \), II-8 and VIII-2, were excluded from \( M_2 \) as there was little unfavorable agreement on any of the items.

For example, one judge marked item IV-2 favorable, but commented that it was "too tough a problem for high school students." Another judge marked IV-2 unfavorable
with the remark that it was "too easy" and indicated that it required no more than simple computation. Item I-7 was considered a "good problem" by one judge and "ambiguous" by another.

One judge did not answer sections VII, VIII, and IX and gave the following explanations: Regarding sections VII and IX, "Problems of this type do not in general have a unique solution." Regarding section VIII, "In this section you must be assuming that the binary operation * has some additional properties (e.g., commutative, associative, distributes over +, etc.), for otherwise the answers are not unique."

Another judge marked section VIII unfavorably with the comment, "You are consistently measuring the same objective, but it is not the one you stated." And yet another judge did not mark section V and wrote, "Doesn't seem that these questions carry out the objective. He isn't asked to make a distinction on the conclusion. Either the objective or the test items need to be changed."

As Table 2 indicates, other panel members judged these sections favorably and consequently all nine objectives were retained for measurement by T₁ and T₂.

In general the panel's comments corroborated the author's belief that the most difficult items were included in sections VII, VIII, and IX. However, the item analysis performed on the results of administering T₁ and T₂ proved this position to be untenable.
Table 2
The Panel of Experts' Judgment of $M_1$ for Content Validity

<table>
<thead>
<tr>
<th>$M_1$ Number</th>
<th>Number of &quot;Yes&quot; Responses</th>
<th>Number of &quot;No&quot; Responses</th>
<th>Number Not Responding</th>
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</thead>
<tbody>
<tr>
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<tr>
<td>I-2</td>
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</tr>
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</tr>
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<td>I-12</td>
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CHAPTER V
ENHANCING RELIABILITY

From the ninety-one items of $M_2$, sixty were chosen to develop $T_1$ and $T_2$ (See Appendixes B and C). Each test was designed to be administered in two fifty minute sessions. $Ss_1$ consisted of 127 pupils in two trigonometry and three algebra II sections at Broadmoor High School in Baton Rouge, Louisiana. $Ss_2$ consisted of 123 pupils in one trigonometry and four algebra II sections also at Broadmoor. The author administered both $T_1$ and $T_2$ over a four-day period. $Ss_1$ took the first fourteen items of $T_1$ in their regular mathematics class periods and the last sixteen items in their regular mathematics classes on the very next day. Similarly, $Ss_2$ took the first fifteen items of $T_1$ one day and the last fifteen on the very next day.

Care was taken to administer the test in a similar manner to all classes. However, the fact that some classes were tested in the morning, others tested in the afternoon, and all classes were tested over a two-day period increased the factors which could have lowered the indices of discrimination of the test items. Despite these consequences, testing within the everyday school class schedule was chosen over arranging for a two-hour block for all pupils in order to enhance classroom usability.
Prior to the beginning of each administration, the pupils were informed that the purpose of their taking the test was to "test the test." It was pointed out that in no way would their grades or school records be affected by their scores on this test. The pupils were requested to put forth their best efforts in order that they could contribute to the betterment of education.

Because of a prior concern that members of $S_{s_1}$ and $S_{s_2}$ would be unfamiliar with the term "sequence," as used in Section D of $T_1$ and Section D of $T_2$, each class was told the following: "A sequence is a set of ordered elements; that is, there is a first element, a second element, etc. On this test the first term is denoted by $S_1$, the second by $S_2$, and in general the nth by $S_n$.''

For similar reasons concerning Section H of $T_1$ and Section H of $T_2$, each class was also instructed as follows: "In the section involving the symbol *, the problem is to figure out what * means in each case."

The "Note" on the cover of $T_1$ and $T_2$ (See Appendixes B and C) was read aloud and the pupils were instructed to begin. On the second day, no additional instructions were given.

It became very evident after several classes had taken $T_1$ and $T_2$, that the author had over-estimated the amount of time that would be needed for $S_{s_1}$ and $S_{s_2}$ to complete the thirty items. The vast majority of the pupils
needed no more than thirty-five minutes for each one-day session.

The results of these administrations are given by Tables 3, 4, 5, 6, 7, and 8.

As Tables 7 and 8 indicate, thirty-one items had indices of discrimination above .40 and therefore would be considered "very good items" by Ebel (Ebel, 1965:364). Ten items had indices in the .30 to .39 range, a category classed as "reasonably good" by Ebel (Ebel, 1965:364). Eight items were in the .20 to .29 range, which Ebel would type as "marginal items" (Ebel, 1965:364). Ten items had positive indices below .20, and consequently were considered unsatisfactory. Only one item did not discriminate at all, and no item discriminated negatively.

As Tables 3 and 4 indicate, the distractors generally functioned in a satisfactory manner except for those items which had unsatisfactory indices of discrimination.
Table 3

The Difficulty Levels and Responses to Alternatives of the Items in $T_1$

<table>
<thead>
<tr>
<th>$T_1$ Number</th>
<th>$M_1$ Number</th>
<th>Number Responding to Each Alternative</th>
<th>Difficulty Level No Response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>1</td>
<td>I-9</td>
<td>99*</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>I-11</td>
<td>32*</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>I-10</td>
<td>23</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>I-4</td>
<td>3</td>
<td>107*</td>
</tr>
<tr>
<td>5</td>
<td>III-3</td>
<td>17</td>
<td>74*</td>
</tr>
<tr>
<td>6</td>
<td>III-4</td>
<td>11</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>V-10</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>V-11</td>
<td>11</td>
<td>90*</td>
</tr>
<tr>
<td>9</td>
<td>V-9</td>
<td>68*</td>
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</tr>
<tr>
<td>10</td>
<td>V-1</td>
<td>28</td>
<td>36</td>
</tr>
<tr>
<td>11</td>
<td>VII-1</td>
<td>13</td>
<td>107*</td>
</tr>
<tr>
<td>12</td>
<td>VII-9</td>
<td>92*</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>VII-11</td>
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<td>113*</td>
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<tr>
<td>15</td>
<td>II-3</td>
<td>22*</td>
<td>19</td>
</tr>
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</table>

* The Number of Correct Responses
Table 3 (Continued)

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<tr>
<th>$T_1$ Number</th>
<th>$M_1$ Number</th>
<th>Number Responding to Each Alternative</th>
<th>Difficulty Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>16</td>
<td>II-9</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>17</td>
<td>II-5</td>
<td>94*</td>
<td>15</td>
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<td>18</td>
<td>VI-9</td>
<td>10</td>
<td>4</td>
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<td>VI-8</td>
<td>16</td>
<td>7</td>
</tr>
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<td>20</td>
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</tr>
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<td>IV-5</td>
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<td>11</td>
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</tr>
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<td>VIII-9</td>
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<td>36</td>
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<td>29</td>
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* The Number of Correct Responses
Table 4
The Difficulty Levels and Responses to Alternatives of the Items in $T_2$

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<th>$M_1$ Number</th>
<th>Number Responding to Each Alternative</th>
<th>Difficulty Level</th>
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<td>I-5</td>
<td>92*</td>
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<tr>
<td>4</td>
<td>I-7</td>
<td>4</td>
<td>27</td>
</tr>
<tr>
<td>5</td>
<td>V-6</td>
<td>91*</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>V-7</td>
<td>92*</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>V-12</td>
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<td>6</td>
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<td>VI-10</td>
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<td>1</td>
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<td>10</td>
<td>VI-2</td>
<td>117*</td>
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<td>11</td>
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<td>11</td>
</tr>
<tr>
<td>12</td>
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<td>0</td>
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</table>

* The Number of Correct Responses
Table 4 (Continued)

<table>
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<th>$T_2$ Number</th>
<th>$M_1$ Number</th>
<th>Number Responding to Each Alternative</th>
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<th>Difficulty Level</th>
</tr>
</thead>
<tbody>
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<td>b</td>
<td>c</td>
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</tr>
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<td>17</td>
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<td>86*</td>
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<td>43*</td>
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<td>IV-1</td>
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<td>9</td>
<td>66*</td>
</tr>
<tr>
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<tr>
<td>25</td>
<td>VIII-11</td>
<td>4</td>
<td>92*</td>
<td>12</td>
</tr>
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<td>3</td>
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<td>1</td>
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<td>86*</td>
</tr>
</tbody>
</table>

* The Number of Correct Responses
Table 5

Frequency Distribution for the Scores of $Ss_1$ on $T_1$

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
<th>Score</th>
<th>Frequency</th>
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</thead>
<tbody>
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<td>29</td>
<td>2</td>
<td>18</td>
<td>7</td>
</tr>
<tr>
<td>28</td>
<td>2</td>
<td>17</td>
<td>15</td>
</tr>
<tr>
<td>27</td>
<td>1</td>
<td>16</td>
<td>13</td>
</tr>
<tr>
<td>26</td>
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<td>15</td>
<td>11</td>
</tr>
<tr>
<td>25</td>
<td>2</td>
<td>14</td>
<td>15</td>
</tr>
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<td>5</td>
<td>13</td>
<td>5</td>
</tr>
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<td>12</td>
<td>2</td>
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<td>11</td>
<td>4</td>
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<td>10</td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>9</td>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

The arithmetic mean of the scores of $Ss_1$ is 17.850.
Table 6

Frequency Distribution for the Scores of Ss₂ on T₂

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>1</td>
<td>17</td>
<td>3</td>
</tr>
<tr>
<td>29</td>
<td>2</td>
<td>16</td>
<td>9</td>
</tr>
<tr>
<td>28</td>
<td>4</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>27</td>
<td>8</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>26</td>
<td>7</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td>25</td>
<td>7</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>24</td>
<td>5</td>
<td>11</td>
<td>2</td>
</tr>
<tr>
<td>23</td>
<td>5</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>22</td>
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<td>9</td>
<td>2</td>
</tr>
<tr>
<td>21</td>
<td>6</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>11</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>12</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>8</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

The arithmetic mean of the scores of Ss₂ is 19.805.
Table 7

Indices of Discrimination for the Items in $T_1$

<table>
<thead>
<tr>
<th>$T_1$ Number</th>
<th>$M_1$ Number</th>
<th>$C_{ui}$</th>
<th>$C_{li}$</th>
<th>ID$_i$***</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I-9</td>
<td>34</td>
<td>26</td>
<td>.24</td>
</tr>
<tr>
<td>2</td>
<td>I-11</td>
<td>14</td>
<td>2</td>
<td>.35</td>
</tr>
<tr>
<td>3</td>
<td>I-10</td>
<td>17</td>
<td>4</td>
<td>.38</td>
</tr>
<tr>
<td>4</td>
<td>I-4</td>
<td>34</td>
<td>28</td>
<td>.18</td>
</tr>
<tr>
<td>5</td>
<td>III-3</td>
<td>32</td>
<td>6</td>
<td>.76</td>
</tr>
<tr>
<td>6</td>
<td>III-4</td>
<td>29</td>
<td>7</td>
<td>.65</td>
</tr>
<tr>
<td>7</td>
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<td>.26</td>
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<td>VII-11</td>
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<td>.08</td>
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<td>31</td>
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<td>15</td>
<td>II-3</td>
<td>14</td>
<td>1</td>
<td>.38</td>
</tr>
</tbody>
</table>

$C_{ui} = $ The number of correct responses to item $i$ obtained in the upper 27% (the greatest 34 scores in rank order) of $S_{S_1}$.

$C_{li} = $ The number of correct responses to item $i$ obtained in the lower 27% (the least 34 scores in rank order) of $S_{S_1}$.

ID$_i$*** = ($C_{ui} - C_{li}$)(1/34) = Index of discrimination for item $i$. 
<table>
<thead>
<tr>
<th>T1 Number</th>
<th>Mi Number</th>
<th>$C_{ui}$</th>
<th>$C_{li}$</th>
<th>ID$_i$***</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>II-9</td>
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<td>13</td>
<td>.53</td>
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<tr>
<td>17</td>
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<td>32</td>
<td>18</td>
<td>.41</td>
</tr>
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<td>19</td>
<td>VI-8</td>
<td>27</td>
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<td>.53</td>
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<td>.62</td>
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<td>.18</td>
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<tr>
<td>25</td>
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<td>.26</td>
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<td>.00</td>
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<td>IX-2</td>
<td>31</td>
<td>21</td>
<td>.29</td>
</tr>
<tr>
<td>30</td>
<td>IX-4</td>
<td>31</td>
<td>24</td>
<td>.21</td>
</tr>
</tbody>
</table>

$*C_{ui} = $ The number of correct responses to item $i$ obtained in the upper 27% (the greatest 34 scores in rank order) of Ss$_1$.

$**C_{li} = $ The number of correct responses to item $i$ obtained in the lower 27% (the least 34 scores in rank order) of Ss$_1$.

$***ID_i = (C_{ui} - C_{li})(\frac{1}{34}) = $ Index of discrimination for item $i$. 
Table 8
Indices of Discrimination for the Items in T2

<table>
<thead>
<tr>
<th>T2 Number</th>
<th>M1 Number</th>
<th>C_{ui}*</th>
<th>C_{li}**</th>
<th>ID_{i}***</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>13</td>
<td>.42</td>
</tr>
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<td>2</td>
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<td>.21</td>
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<tr>
<td>6</td>
<td>V-7</td>
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<td>18</td>
<td>.36</td>
</tr>
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*_{C_{ui}} = The number of correct responses to item i obtained in the upper 27% (the greatest 33 scores in rank order) of S_{2}.

**_{C_{li}} = The number of correct responses to item i obtained in the lower 27% (the least 33 scores in rank order) of S_{2}.

***_{ID_{i}} = (C_{ui} - C_{li})(\frac{1}{33}) = Index of discrimination for item i.
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* Cui = The number of correct responses to item i obtained in the upper 27% (the greatest 33 scores in rank order) of Ss2.

** Cl = The number of correct responses to item i obtained in the lower 27% (the least 33 scores in rank order) of Ss2.

*** IDi = (Cui - Cl)(1/33) = Index of discrimination for item i.
CHAPTER VI
SYNTHESIZING THE FINAL FORM AND EXAMINING RELIABILITY

The administration of $T_1$ to $Ss_1$ and $T_2$ to $Ss_2$ indicated that a greater number of items should be included in $T_3$ than were included in either $T_1$ or $T_2$. Also, analyses of the scores of $Ss_1$ and $Ss_2$ indicated that algebra II students had no difficulty reading and meaningfully responding to the items. Three facts corroborated this contention: (1) The vast majority of the members of $Ss_1$ and $Ss_2$ completed each of the one-day sessions in no more than thirty-five minutes; (2) a great number of items had difficulty levels in excess of 50 (See Tables 3 and 4); and (3) the arithmetic mean of the scores of $Ss_1$ was 17.85 and of $Ss_2$ the mean was 19.81.

The author judged that $T_3$ should contain forty items and that only algebra II classes needed to comprise $Ss_3$. Forty items were considered to be a number which all students could finish in two fifty-minute sessions. It is reasonable to assume that more items, possibly fifty or sixty, could have been included and thus the reliability of $T_3$ would have been enhanced. However, the author believed that to have introduced speed as a factor in determining the scores of $Ss_3$ was to lose sight of the purpose of developing $T_3$. Analysis of mathematical content,
as defined in this study, is a slow step-by-step process and no premium should be placed on speed.

In general, $T_3$ was synthesized using those items of $T_1$ and $T_2$ which had the highest indices of discrimination. However, other considerations were taken into account in choosing the items of $T_3$. First of all, it was desirable to include some items from each of the nine behavioral categories. Also, several items with especially high difficulty levels needed to be included for motivational purposes. Items which most of the members of $S_{s_3}$ would respond to correctly could serve to encourage the pupils and condition their thinking to an analytical style. Naturally, such items would be the initial items in their respective sections of $T_3$.

As in $T_1$ and $T_2$, items from the same behavioral category were grouped together under a section heading for the purpose of enhancing the clarity of the directions given in each item. (See Appendixes B, C, and E)

More items were included from categories which had a greater number of items with high indices of discrimination than from categories with less highly discriminating items. This differential ranged from only two items each from categories VII of $M_1$ (Section C of $T_3$) and IX of $M_1$ (Section I of $T_3$) to six items each in categories II of $M_1$ (Section F of $T_3$), VI of $M_1$ (Section G of $T_3$), and IV of $M_1$ (Section H of $T_3$).
The sequence of the items within each section was chosen in an attempt to obtain a favorable compromise between having the items in order of descending levels of difficulty and having similar items in adjacent positions. The sequence of the nine sections was chosen with the purpose of obtaining a balance between the time needed for Ss\textsubscript{3} to complete the first-day session and the time needed for Ss\textsubscript{3} to complete the second-day session.

Classroom usability was stressed throughout this study. In keeping with this spirit, it was decided to allow the regular mathematics teachers of Ss\textsubscript{3} to administer T\textsubscript{3}. Ss\textsubscript{3} consisted of 106 pupils in three algebra II sections at Catholic High School in Baton Rouge, Louisiana. Two sections were taught by one teacher, the third section by another teacher. Several days prior to the administration, each of the two teachers was provided with an instruction sheet (See Appendix D). These two teachers administered the test over a two-day period in a manner similar to that previously described for the administration of T\textsubscript{1} and T\textsubscript{2}. As with T\textsubscript{1} and T\textsubscript{2}, T\textsubscript{3} was hand scored. The results are reflected in Tables 9, 10, and 11.

Using the Kuder-Richardson formula number twenty, a reliability coefficient of .897 was obtained for T\textsubscript{3}. The computations are given on page 55.
### Table 9

Frequency Distribution for the Scores of Ss on T

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<th>Frequency</th>
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The arithmetic mean of the scores of Ss is 21.132.
Table 10

The Difficulty Levels of the Items in $T_3$

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<th>Number of Incorrect or No Responses</th>
<th>Difficulty Level</th>
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<th>Number of Incorrect or No Responses</th>
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Table 11

Indices of Discrimination for the Items in T₃

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</table>

*Cui* = The number of correct responses to item i obtained in the upper 27% (the greatest 29 scores in rank order) of Sₛ₃.

**Cl₁** = The number of correct responses to item i obtained in the lower 27% (the least 29 scores in rank order) of Sₛ₃.

***ID₁ = (Cui - Cl₁)(1/29) = Index of discrimination for item i.
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</tr>
</tbody>
</table>

*Cui* = The number of correct responses to item i obtained in the upper 27% (the greatest 29 scores in rank order) of Ss3.

**Cli** = The number of correct responses to item i obtained in the lower 27% (the least 29 scores in rank order) of Ss3.

***IDi*** = \((C_{ui} - C_{li})(\frac{1}{29})\) = Index of discrimination for item i.
Table 11 (Continued)

<table>
<thead>
<tr>
<th>T&lt;sub&gt;3&lt;/sub&gt; Number</th>
<th>M&lt;sub&gt;1&lt;/sub&gt; Number</th>
<th>C&lt;sub&gt;ui&lt;/sub&gt;*</th>
<th>C&lt;sub&gt;li&lt;/sub&gt;**</th>
<th>ID&lt;sub&gt;i&lt;/sub&gt;***</th>
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<tr>
<td>31</td>
<td>VI-5</td>
<td>13</td>
<td>9</td>
<td>.14</td>
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<td>32</td>
<td>VI-6</td>
<td>11</td>
<td>5</td>
<td>.21</td>
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<td>33</td>
<td>IV-12</td>
<td>24</td>
<td>12</td>
<td>.41</td>
</tr>
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<td>34</td>
<td>IV-11</td>
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<td>.59</td>
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<td>.76</td>
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<td>9</td>
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<td>.41</td>
</tr>
<tr>
<td>40</td>
<td>IX-1</td>
<td>26</td>
<td>11</td>
<td>.52</td>
</tr>
</tbody>
</table>

*<sub>C</sub><sub>ui</sub> = The number of correct responses to item i obtained in the upper 27% (the greatest 29 scores in rank order) of S<sub>3</sub>.

**<sub>C</sub><sub>li</sub> = The number of correct responses to item i obtained in the lower 27% (the least 29 scores in rank order) of S<sub>3</sub>.

***<sub>ID</sub><sub>i</sub> = (C<sub>ui</sub> - C<sub>li</sub>)\(\frac{1}{29}\) = Index of discrimination for item i.
Computation of the Reliability Coefficient

Let the scores of $S_{s_3}$ be given by $x_i$ for $i = 1, 2, ..., N-1, N$ where $N$ is the number of elements in $S_{s_3}$. And let the arithmetic mean of the scores of $S_{s_3}$ be $\mathcal{M}_{t3}$. Therefore, the following relations hold (Garrett, 1967:27):

1. $N = 106$.
2. $\mathcal{M}_{t3} = \frac{1}{N} \sum_{i=1}^{N} x_i = \frac{2240}{106} \approx 21.132$.

If the variance of the scores of $S_{s_3}$ is $\sigma^2_{t3}$, then the following holds (Garrett, 1967:50):

\[\sigma^2_{t3} = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mathcal{M}_{t3})^2 \approx \frac{7524.151}{106} \approx 70.983.\]

Let the reliability coefficient obtained on $T_3$ by applying the Kuder-Richardson formula number twenty to the scores of $S_{s_3}$ be $r_{t3}$. Let $k$ be the number of items in $T_3$. Let $p_i$ be the proportion of $S_{s_3}$ answering item $i$ correctly and $q_i$ be the proportion of $S_{s_3}$ not answering item $i$ correctly for $i = 1, 2, ..., k-1, k$. Therefore, the following relations hold (Garrett, 1967:341):

\[r_{t3} = \frac{k}{k-1} \left( 1 - \frac{1}{\sigma^2_{t3}} \sum_{i=1}^{k} p_i q_i \right) \approx \frac{40}{39} \left( 1 - \frac{8.9092}{70.983} \right) \approx .897.\]
CHAPTER VII
CONCLUSIONS AND IMPLICATIONS

Throughout this research, the author was encouraged by the enthusiastic response to the test items from many of the secondary school students who were exposed to one of the test forms. The most common comments from these students were very similar to those given by the first group of six students that are quoted on page 25.

The purpose of this study was to develop a test for analytical cognition of mathematical content that had high usability for secondary schools. The following results from this study indicate that the objective was accomplished:
(1) Each item of \( T_3 \) was judged by the panel of experts to be a valid indicator of analytical cognition for those secondary school pupils for whom the test was designed;
(2) thirty-four of the forty items of \( T_3 \) were shown to have indices of discrimination above .40; (The other six items had positive indices of discrimination. [See Table 9])
(3) the reliability coefficient, as computed using the Kuder-Richardson formula number twenty on the scores of \( S_s_3 \), was .897; (4) \( T_3 \) was administered by the regular classroom teachers without disrupting the normal school day schedule; and (5) \( T_3 \) consisted only of multiple-choice items that
were easily and reliably scored by hand. (T3 could be readily adapted for machine scoring.)

This study was limited to measuring one level of cognition in a limited content area and for a limited group of students. As pointed out in the quote from Romberg on pages 16 and 17, higher cognitive tests can only be developed in light of the background of those who are to take the test. There is a need to develop tests for analytical cognition of mathematical content for other groups besides those which have completed at least one course in high school algebra. Also, other content areas in addition to numerical and algebraic, as defined in this study, should be included in further studies.

If the educational community is to continue to defend curricular content on the basis of developing cognitive processes, then it is critical that tests measuring processes such as those measured by T3 should be developed in other content areas such as English, social studies, and the physical sciences. If one could study the correlation between scores on T3 and scores on a test of analytical cognition of social science, then light could be thrown on vital questions involving such concepts as non-specific transfer of learning.

The results of this study should serve to encourage further development of tests for measuring more sophisticated cognitive processes. Only when the differential
between educational objectives and measured objectives is diminished can ideas such as accountability become a reality.
SELECTED BIBLIOGRAPHY


APPENDIX A

QUESTIONNAIRE USED BY THE PANEL OF EXPERTS
TO JUDGE THE CONTENT VALIDITY OF M₁
MATHEMATICAL TEST ITEMS

Constructed by James S. Cangelosi

Submitted to ________________

For Judgment of Content Validity
NOTE: The test items which are reported favorably by you and the other judges will form a pool of items from which tests can be constructed and administered to groups of secondary school pupils. The results of these administrations will be evaluated for determination of test validity and reliability.

The pupils who will take these tests will have completed at least two years of secondary school mathematics, which would include a course in algebra. It is assumed that their background will be such that comprehension of the mathematical concepts involved in the items will not be a factor in their responding to the items. Pupils will be asked to respond to about fifteen of these items in a fifty-minute period.

INSTRUCTIONS:

There are nine sections to this questionnaire. At the beginning of each section, there is a description of the cognitive behavior that the items of the section are designed to measure. For each item, you are asked to answer the following question in the accompanying rectangle:

"If a pupil answers this item correctly, would he have demonstrated the cognitive behavior that the item was designed to measure?"

NOTE: An answer key is provided at the end of the questionnaire.
SECTION I
(Analysis of Elements)

The test items in this group are each designed to measure the following cognitive behavior:

Given the definition of an unfamiliar set B such that B is a subset of the real numbers, distinguishes the elements of B from real numbers which are not elements of B.
I-1. Let \( B \) be a set defined by the following:

A real number \( n \) is a member of set \( B \) if and only if \( n^2 < n \).

Which one of the following sets is a subset of \( B \)?

a. \( \{0, 1, 12\} \).

b. \( \{-\frac{1}{2}, -\frac{2}{9}, -\frac{2}{3}\} \).

c. \( \{\frac{1}{4}, \frac{2}{3}, \frac{17}{3}\} \).

d. \( \{\frac{1}{8}, \frac{2}{5}, \frac{9}{10}\} \).

e. None of these.

I-2. Let \( B \) be a set defined by the following:

A real number \( n \) is a member of set \( B \) if and only if \( n^2 \geq n \).

Which one of the following sets is a subset of \( B \)?

a. \( \{0, \frac{1}{2}, 7\} \).

b. \( \{-3, 0, \frac{1}{8}\} \).

c. \( \{-\frac{1}{4}, 1, 93\} \).

d. \( \{-3, \frac{1}{5}, \frac{7}{3}\} \).

e. None of these.
I-3. Let \( B \) be a set defined by the following:

A real number \( n \) is a member of set \( B \) if and only if \( n \) is an integer such that \( 0 < n < -\frac{9}{2} \).

Which one of the following sets is a subset of \( B \)?

a. \( \{1, 4, 4.32\} \).

b. \( \{0, 2, 4\} \).

c. \( \{\frac{1}{2}, 1, 2\} \).

d. \( \{2, 4, 7\} \).

e. None of these.

I-4. Let \( B \) be a set defined by the following:

A real number \( n \) is a member of set \( B \) if and only if \((-3)n > 0\).

Which one of the following sets is a subset of \( B \)?

a. \( \{-100, 1, 9\} \).

b. \( \{-45, -9, -\frac{3}{7}\} \).

c. \( \{-\frac{7}{8}, 0, 11\} \).

d. \( \{-4, -1, 0\} \).

e. None of these.
I-5. Let $B$ be a set defined by the following:
A real number $n$ is a member of set $B$ if and only if there exists an integer $x$ such that $x = \sqrt{n}$.

Which one of the following sets is a subset of $B$?

a. $\{1, 16, 25\}$.

b. $\left\{\frac{1}{4}, 3, 4\right\}$.

c. $\{-25, 1, 4\}$.

d. $\{0, 3, 24\}$.

---

I-6. Let $B$ be a set defined by the following:
A real number $n$ is a member of set $B$ if and only if $n^2 = n$.

Which one of the following sets is a subset of $B$?

a. $\{3, \frac{1}{4}, -1\}$.

b. $\{0, 1, -1\}$.

c. $\{\pi, \sqrt{2}, 0\}$.

d. $\{1, 10, 100\}$.

e. None of these.
I-7. Let $B$ be a set defined by the following:

An integer $n$ is a member of set $B$ if and only if there exists an integer $x$ such that $n = 3x$ and there does not exist an integer $y$ such that $n = 2y$.

Which one of the following sets is a subset of $B$?

a. $\{-3, 3, 6, 27\}$.

b. $\{0, 1, 3, 9\}$.

c. $\{-9, 0, 9, 81\}$.

d. $\{-81, -3, 9, 27\}$.

e. None of these.

I-8. Let $B$ be a set defined by the following:

A real number $n$ is a member of set $B$ if and only if there exists an integer $x$ such that $n = 8x$.

Which one of the following sets is a subset of $B$?

a. $\{-8, 16, 71\}$.

b. $\{10, 0, 18\}$.

c. $\{8, 28, 38\}$.

d. $\{-16, 64, 72\}$.

e. None of these.
I-9. Let B be a set defined by the following:

A real number n is a member of a set B if and only if there exists an integer x such that \( n = 6x \).

Which one of the following sets is a subset of B?

a. \( \{ 0, 6, 36 \} \).
b. \( \{-6, 16, 46 \} \).
c. \( \{ 6, 12, 16 \} \).
d. \( \{-3, 0, 24 \} \).
e. None of these.

I-10. Let B be a set defined by the following:

A real number n is a member of set B if and only if there exist integers x and y such that \( n = x + y\sqrt{2} \).

Which one of the following sets is a subset of B?

a. \( \{ 3 + 2\sqrt{2}, -1 + \sqrt{2}, \frac{4}{3}, 2\sqrt{2} \} \).
b. \( \{ 3 + 2\sqrt{15}, 3 + \sqrt{2}, 17 \} \).
c. \( \{ 5 + \sqrt{2}, 2, -1 + 2\sqrt{2} \} \).
d. \( \{ 0, 3 + 2\sqrt{2}, -2 \} \).
e. None of these.
I-11. Let B be a set defined by the following:
A real number \( n \) is a member of set B if and only if there exist integers \( x \) and \( y \) such that \( n = x + y\sqrt{5} \).

Which one of the following sets is a subset of B?

a. \( \{-14, 2 + 3\sqrt{5}, \sqrt{5}\} \).

b. \( \{3 + 2\sqrt{17}, 4, 1 + 4\sqrt{5}\} \).

c. \( \{3 + \frac{1}{\sqrt{5}}, 2 - \sqrt{5}, \sqrt{5}\} \).

d. \( \{4\sqrt{5}, 3 + 6\sqrt{5}, \pi\} \).

e. None of these.

I-12. Let B be a set defined by the following:
A real number \( n \) is a member of set B if and only if there exist integers \( x \) and \( y \) such that \( n = x + y\sqrt{3} \).

Which one of the following sets is a subset of B?

a. \( \{3 + \sqrt{5}, 1 - 7\sqrt{3}, 13 + \sqrt{3}\} \).

b. \( \{12 + 2\sqrt{3}, 4 + \frac{2}{3}\sqrt{3}, 1 + \sqrt{3}\} \).

c. \( \{12 + 4\sqrt{3}, 2 + 3\sqrt{3}, 9 - 2\sqrt{3}\} \).

d. \( \{\frac{1}{2} + 8\sqrt{3}, 34 + 6\sqrt{3}, 1 + \sqrt{3}\} \).

e. None of these.
SECTION II
(Analysis of Elements)

The test items in this group are each designed to measure the following cognitive behavior:

Identifies the necessary (unstated) assumptions in the presentation of a mathematical argument.
II-1. After establishing that $x$ and $y$ are real numbers such that $x>0$, the author of a mathematical proof concludes that $xy>x$.

Which one of the following conditions is necessary for this conclusion to be valid?

a. $y>0$.
b. $y<0$.
c. $y>1$.
d. $y>x$.
e. None of these.

II-2. After establishing that $x$, $y$, $z$, and $w$ are real numbers such that $x>y$, the author of a mathematical proof concludes that $x(z-w)<y(z-w)$.

Which one of the following conditions is necessary for the conclusion to be valid?

a. $z = w$.
b. $z > w$.
c. $z < w$.
d. $z \neq w$.
e. None of these.
II-3. After establishing that \( x \) and \( y \) are real numbers such that 
\[ y(x-1) = x^2 - 1, \]
the author of a mathematical proof concludes that \( y = x + 1. \)

Which one of the following conditions is necessary for this conclusion to be valid?

a. \( x \neq 1. \)
b. \( x \neq 0. \)
c. \( x \neq -1. \)
d. \( y \neq 0. \)
e. None of these.

---

II-4. After establishing that \( x, y, z, \) and \( w \) are real numbers such that 
\( x > y, \) the author of a mathematical proof concludes that 
\[ x(z-w) = y(z-w). \]

Which one of the following conditions is necessary for this conclusion to be valid?

a. \( z = w. \)
b. \( z > w. \)
c. \( z < w. \)
d. \( z \neq w. \)
e. None of these.
II-5. After establishing that \( x, y, \) and \( z \) are real numbers such that \( x \leq y \), the author of a mathematical proof concludes that \( \frac{x}{z} \leq \frac{y}{z} \).

Which one of the following conditions is necessary for this conclusion to be valid?

a. \( z > 0 \).
b. \( z \geq 0 \).
c. \( z < 0 \).
d. \( z \leq 0 \).
e. None of these.

II-6. After establishing that \( x, y, z, \) and \( w \) are real numbers such that \( x > y \), the author of a mathematical proof concludes that \( x + z > y + w \).

Which one of the following conditions is necessary for this conclusion to be valid?

a. \( z < w \).
b. \( z \leq w \).
c. \( z > w \).
d. \( z \geq w \).
e. \( z = w \).
II-7. After establishing that \( x, y, \) and \( z \) are real numbers such that \( xz = yz \), the author of a mathematical proof concludes that \( x = y \).

Which one of the following conditions is necessary for this conclusion to be valid?

a. \( x \neq z \).
b. \( z \neq 0 \).
c. \( x > 0 \) and \( y > 0 \).
d. \( x < 0 \) and \( y < 0 \).
e. None of these.

II-8. After establishing that \( x, y, z, \) and \( w \) are real numbers such that \( x = y \), the author of a mathematical proof concludes that \( x(z - w) = y(z - w) \).

Which one of the following conditions is necessary for this conclusion to be valid?

a. \( z = w \).
b. \( z - w = 1 \).
c. \( z = w \) or \( z - w = 1 \).
d. \( z \neq w \).
e. None of these.
II-9. After establishing that $x$ is a real number, the author of a mathematical proof concludes that $x^2 < x$.

Which one of the following conditions is necessary for this conclusion to be valid?

a. $0 \leq x \leq 1$.

b. $0 \leq x < 1$.

c. $0 < x \leq 1$.

d. $0 < x < 1$.

e. None of these.

II-10. After establishing that $x$ is a real number such that $|x| < 2$, the author of a mathematical proof concludes that $x < 2$.

Which one of the following conditions is necessary for this conclusion to be valid?

a. $x \geq 0$.

b. $x \leq 0$.

c. $x = 0$.

d. $x \neq -x$.

e. None of these.
II-11. After establishing that $x$ is a real number such that $|x| < 2$, the author of a mathematical proof concludes that $x > -2$.

Which one of the following conditions is necessary for this conclusion to be valid?

a. $x \geq 0$.
b. $x \leq 0$.
c. $x = 0$.
d. $x \neq -x$.
e. None of these.

II-12. After establishing that $x$ is a real number, the author of a mathematical proof concludes that $|-x| = x$.

Which one of the following conditions is necessary for this conclusion to be valid?

a. $x \geq 0$.
b. $x \leq 0$.
c. $x \neq 0$.
d. $x = 0$.
e. None of these.
SECTION III

(Analysis of Elements)

The test items in this group are each designed to measure the following cognitive behavior:

Given a sample of arithmetic computation performed in an unknown base \( b \), identifies \( b \) from a set of five positive integers.
III-1. The following addition uses numbers in a base other than 10:
\[5 + 3 = 12.\]
What is the base?

a. 4.
b. 5.
c. 6.
d. 7.
e. 8.

III-2. The following subtraction uses numbers in a base other than 10:
\[13 - 4 = 4.\]
What is the base?

a. 4.
b. 5.
c. 6.
d. 7.
e. 8.
III-3. The following addition uses numbers in a base other than 10:
\[ 4 + 3 = 12. \]

What is the base?

a. 4.
b. 5.
c. 6.
d. 7.
e. 8.

III-4. The following subtraction uses numbers in a base other than 10:
\[ 11 - 5 = 2. \]

What is the base?

a. 4.
b. 5.
c. 6.
d. 7.
e. 8.
SECTION IV
(Analysis of Relationships)

The test items in this group are each designed to measure the following cognitive behavior:

Given a function $f$ (in abstract terms) from a set $D$ into the reals, identifies the order relations among the elements of the image of $D$ under $f$. 
IV-1. Consider the variable $x$ such that $x = \frac{z}{y}$ where $z$ and $y$ are real numbers and $y \neq 0$.

In which one of the following sets of values for $z$ and $y$ will $x$ attain the least value?

a. $z = 2$ and $0 < y \leq 4$.

b. $-10 \leq z \leq 1$ and $y = 1$.

c. $z < 0$ and $y > 0$.

d. $z = 0$ and $0 < y \leq 3$.

e. $-20 \leq z \leq -1$ and $1 < y < 2$.

IV-2. Consider the variable $x$ such that $x = y^2$ where $y$ is a real number.

In which one of the following sets of values for $y$ will $x$ attain the greatest value?

a. $0 \leq y \leq 10$.

b. $-\frac{1}{2} \leq y \leq \frac{1}{2}$.

c. $-5 \leq y \leq 12$.

d. $-20 \leq y \leq 3$.

e. $-5 \leq y \leq 10$. 

Comments:
IV-3. Consider the variable $x = y^2$ where $y$ is a real number.

In which one of the following sets of values for $y$ will $x$ attain the greatest value?

a. $y$ is an element of $\{-4, -2, 3.5\}$.

b. $y$ is an element of $\{-11, -\sqrt{8}, 4\}$.

c. $y$ is an element of $\{-1, 0, \frac{17}{3}\}$.

d. $y$ is an element of $\{-6, 7, 8\}$.

e. $y$ is an element of $\{1, 2.16, 4\}$.

IV-4. Consider the variable $x$ such that $x = \frac{z}{y}$ where $z$ and $y$ are real numbers and $y \neq 0$.

In which one of the following sets of values for $z$ and $y$ will $x$ attain the greatest value?

a. $z = 2$ and $0 < y \leq 4$.

b. $1 < z \leq 100$ and $y = 1$.

c. $z < 0$ and $y > 0$.

d. $z = 0$ and $0 < y \leq 3$.

e. $-1 < z < 1$ and $\frac{1}{10} \leq y < 10$. 
IV-5. Consider the variable $x$ such that $x = \frac{z}{z - 2}$ where $0 \leq z < 2$.

What is the minimum value for $x$?

a. $-2$.

b. $-\frac{1}{2}$.

c. 0.

d. $\frac{1}{2}$.

e. There is no limit as to how small $x$ can be.

IV-6. Consider the variable $x$ such that $x = \frac{z}{z - 2}$ where $0 \leq z < 2$.

What is the maximum value for $x$?

a. $-\frac{1}{2}$.

b. 0.

c. $\frac{1}{2}$.

d. 2.

e. There is no limit as to how great $x$ can be.
IV-7. Consider the variable $x$ such that $x = \frac{z}{y}$ where $z$ is an element of the set \{-2, -1, 0, 1, 3\} and $y$ is an element of the set \{-2, -1, 1, 3\}.

What is the maximum value for $x$?

a. $-1$.

b. $1$.

c. $2$.

d. $3$.

e. There is no limit as to how great $x$ can be.

IV-8. Consider the variable $x$ such that $x = \frac{z}{y}$ where $z$ is an element of the set \{-2, -1, 0, 1, 3\} and $y$ is an element of the set \{-2, -1, 1, 3\}.

What is the minimum value for $x$?

a. $-3$.

b. $-1$.

c. $-\frac{2}{3}$.

d. $1$.

e. There is no limit as to how small $x$ can be.
IV-9. Consider the variable \( x \) such that \( x = \frac{z}{y} \) where \( z \) is an element of the set \( \{-2, -1, 0, 1, 3\} \) and \( y \) is an element of the set \( \{-2, -1, 1, 3\} \).

What is the maximum negative value for \( x \)?

a. 0.

b. \( \frac{-1}{3} \).

c. \( \frac{-1}{2} \).

d. 1.

e. None of these.

IV-10. Consider the variable \( x \) such that \( x = \frac{z}{y} \) where \( z \) is an element of the set \( \{-2, -1, 0, 1, 3\} \) and \( y \) is an element of the set \( \{-2, -1, 1, 3\} \).

What is the minimum positive value for \( x \)?

a. 0.

b. \( \frac{1}{3} \).

c. \( \frac{1}{2} \).

d. 1.

e. None of these.
IV-11. Consider the variable \( x = (3 - z)^2 \) where \( z \) is an element of the set of real numbers.

When \( x \) is at its minimum value, what is \( z \)?

a. -3.

b. 0.

c. 3.

d. 9.

e. None of these

IV-12. Consider the variable \( x = (y^2 + 3) \) where \( y \) is an element of the set of real numbers.

What is the minimum value for \( x \)?

a. -9.

b. -3.

c. 0.

d. 3.

e. None of these.
SECTION V
(Analysis of Relationships)

The test items in this group are each designed to measure the following cognitive behavior:

Given a premise, distinguishes between conclusions which are inferences of the given premise from conclusions which are not inferences of the given premise.
V-1. Let a sequence of one-hundred statements be represented by
\[ \{s_1, s_2, s_3, \ldots, s_{100} \} . \]
Suppose that the following are two true facts:
1. If \( s_n \) is true, then \( s_{n+3} \) is true for any positive integer \( n \).
2. \( s_7 \) is true.

Which one of the following is true?

a. \( s_6 \).
b. \( s_{11} \).
c. \( s_{15} \).
d. \( s_{19} \).
e. \( s_{27} \).

V-2. Let a sequence of one-hundred statements be represented by
\[ \{s_1, s_2, s_3, \ldots, s_{100} \} . \]
Suppose that the following are two true facts:
1. If \( s_n \) is true, then \( s_{n+5} \) is true for any positive integer \( n \).
2. \( s_4 \) is true.

Which one of the following is true?

a. \( s_5 \).
b. \( s_{12} \).
c. \( s_{14} \).
d. \( s_{21} \).
e. \( s_{23} \).
V-3. Let A, B, and C be statements such that the following is true:
1. If A is true, then B is false.
2. If B is true, then C is true.
Suppose that B is true, then which one of the following is a correct conclusion?

a. A is false and C is true.
b. A is true and C is true.
c. A is true and C is false.
d. No conclusion can be made about A, but C is true.
e. None of these.

____ Yes. ___ No.
Comments:

V-4. Let A, B, C, and D be statements such that the following is true:
1. If A is true, then B is true.
2. If B is true, then C is false.
3. If C is true, then D is true.
Suppose that C is true, then what can be concluded about A and D?

a. A is false and D is true.
b. A is false, but no conclusion can be made about D.
c. A is true and D is true.
d. A is true, but no conclusion can be made about D.
e. No conclusion can be made about either A or D.

____ Yes. ___ No.
Comments:
V-5. Let A, B, C, and D be statements such that the following is true:
1. If A is true, then B is true.
2. If B is true, then C is true.
3. If B is true, then D is true.
4. If D is true, then A is true.

Suppose that A is false, which other statements must also be false?

a. C and B only.
b. C and D only.
c. B and D only.
d. B only.
e. None of these.

V-6. "Set N contains at least 10 elements."

Which one of the following can be concluded from the above statement?

a. Set N contains at least 7 elements.
b. Set N contains at least 12 elements.
c. Set N is a finite set.
d. Set N is an infinite set.
e. None of these.
V-7. "Set N contains more than 10 elements."

Which one of the following can be concluded from the above statement?

a. Set N contains at least 11 elements.

b. Set N does not contain 12 elements.

c. Set N contains more than 11 elements.

d. Set N is an infinite set.

e. None of these.

Comments:

V-8. Accept the following statements to be true:

1. A is either red or green.
2. B is either blue or brown.
3. C is either red, green, blue, or brown.
4. C is never the same color as B.
5. If C is green, then A is red.
6. If C is red, then A is red.
7. If A is green, then B is blue.

Suppose that A is green. What color is C?

a. Red.

b. Green.

c. Blue.

d. Brown.

e. C cannot be determined.
V-9. Accept the following statements to be true:

1. A is either red or green.
2. B is either blue or brown.
3. C is either red, green, blue, or brown.
4. C is never the same color as B.
5. If A is red, then C is either red or blue.
6. If B is brown, then A is green.

Suppose that A is red. What color is C?

a. Red.
b. Green.
c. Blue.
d. Brown.
e. C cannot be determined.

V-10. "Set N contains more than one element, but not more than 10 elements."

Which one of the following can be concluded from the above statement?

a. Set N contains at least 4 elements.
b. Set N contains at least 10 elements.
c. Set N is a finite set.
d. Set N is an infinite set.
e. None of these.
V-11. "N and B are sets such that all the elements of N are also contained in B. x and y are elements such that x is not contained in B and y is contained in N."

Which one of the following can be concluded from the above statement?

a. x is contained in N and y is contained in B.

b. x is not contained in N and y is contained in B.

c. x is not contained in N and y is not contained in B.

d. x is contained in N and y is not contained in B.

e. None of these.

V-12. "Set N contains two and only two elements. x, y, and z are all elements of N. x ≠ y."

Which one of the following can be concluded from the above statement?

a. z = y.

b. z = x.

c. Both "a" and "b" are true.

d. Either "a" is true or "b" is true, but both are not true.

e. None of these.
SECTION VI
(Analysis of Relationships)

The test items in this group are each designed to measure the following cognitive behavior:
Given a real variable $x$ with certain conditions imposed on $x$, identifies the relations among the factors and/or terms of $x$ which are necessary in order for the given conditions to be true.
VI-1. Let \( r, s, t, \) and \( u \) be real numbers such that \( \frac{r-s}{t-u} > 0 \) and \( u > t \).

Which one of the following must be true?

a. \( r+u > t+s \).

b. \( r-s < t-u \).

c. \( s < r \).

d. \( s > r \).

e. None of these.

---

VI-2. Let \( r, s, \) and \( t \) be real numbers such that \( r+(s-t) = r \).

Which one of the following must be true?

a. \( s = t \).

b. \( s \neq t \).

c. \( s-t < r \).

d. \( s-t > r \).

e. None of these.
VI-3. Let \( r, s, \) and \( t \) be real numbers such that \( r + (s - t) > 0 \) and \( r < 0 \).

Which one of the following must be true?

a. \( s > t \).

b. \( s < t \).

c. \( s = t \).

d. \( s < 0 \).

e. None of these.

VI-4. Let \( r, s, \) and \( t \) be real numbers such that \( r - (s - t) > 0 \) and \( r < 0 \).

Which one of the following must be true?

a. \( s > t \).

b. \( s < t \).

c. \( r < s \).

d. \( r < t \).

e. None of these.
VI-5. Let \( r, s, \) and \( t \) be real numbers such that \( r + (s - t) > 0 \) and \( s < t \).

Which one of the following must be true?

a. \( r > t \).

b. \( r < t \).

c. \( r > 0 \).

d. \( r < 0 \).

e. None of these.

VI-6. Let \( r, s, \) and \( t \) be real numbers such that \( rt + st > 0 \) and \( t < 0 \).

Which one of the following must be true?

a. \( r < -s \).

b. \( r + s > t \).

c. \( -r < s \).

d. \( -r > -s \).

e. None of these.
VI-7. Let \( r, s, \) and \( t \) be real numbers such that \( rt + st < 0 \) and \( t > 0 \).

Which one of the following must be true?

a. \( r > -s \).

b. \( t > r + s \).

c. \( r - s \neq 0 \).

d. \( -r < -s \).

e. None of the these.

VI-8. Let \( r, s, \) and \( t \) be real numbers such that \( \frac{rs}{t} > 0 \), \( r < 0 \),
and \( s > 0 \).

Which one of the following must be true?

a. \( t < rs \).

b. \( t > rs \).

c. \( t < 0 \).

d. \( t > 0 \).

e. None of these.
VI-9. Let $r$ and $s$ be real numbers such that $rs = 0$ and $r = 0$.

Which one of the following must be true?

a. $s \neq r$.

b. $s > 0$.

c. $s < 0$.

d. $s = 0$.

e. None of these.

VI-10. Let $r$ and $s$ be integers such that $r + r + 1 = s$.

Which one of the following must be true?

a. $r$ is even.

b. $r$ is odd.

c. $s$ is even.

d. $s$ is odd.

e. None of these.
VI-11. Let \( r, s, \) and \( t \) be integers such that \( \frac{rs}{t} = 2 \) and \( r \) is odd.

Which one of the following must be true?

a. \( s \) is even.

b. \( s \) is odd.

c. \( s < 2t \).

d. \( s > 2t \).

e. None of these.

VI-12. Let \( r, s, \) and \( t \) be integers such that \( r+s = 2t \) and both \( r \) and \( s \) are odd.

Which one of the following must be true?

a. \( t \) is odd.

b. \( t \) is even.

c. \( t > r+s \).

d. \( t < r+s \).

e. None of these.
SECTION VII

(Analysis of Organizational Principles)

The test items in this group are each designed to measure the following cognitive behavior:
  Given a finite number of terms in an infinite real number sequence, identifies the nth term where n is any positive integer.
VII-1. \((s_1, s_2, s_3, \ldots)\) is an infinite sequence of real numbers such that \(s_1 = 0, s_2 = 1, s_3 = -2, s_4 = 3,\) and \(s_5 = -4.\) All of the terms are determined by the same pattern.

Which one of the following is \(s_{37}\)?

a. \(-38\).
b. \(-36\).
c. \(-37\).
d. \(35\).
e. \(38\).

VII-2. \((s_1, s_2, s_3, \ldots)\) is an infinite sequence of real numbers such that \(s_1 = 1, s_2 = \frac{1}{4}, s_3 = \frac{1}{9},\) and \(s_4 = \frac{1}{16}.\) All of the terms are determined by the same pattern.

Which one of the following is \(s_{9}\)?

a. \(\frac{1}{18}\).
b. \(\frac{1}{19}\).
c. \(\frac{1}{72}\).
d. \(\frac{1}{81}\).
e. \(\frac{1}{110}\).
VII-3. \((s_1, s_2, s_3, \ldots)\) is an infinite sequence of real numbers such that \(s_1 = \frac{1}{4}\), \(s_2 = \frac{1}{6}\), and \(s_3 = \frac{1}{8}\). All of the terms are determined by a common pattern.

Which one of the following is \(s_{12}\)?

a. \(\frac{1}{10}\).

b. \(\frac{1}{15}\).

c. \(\frac{1}{16}\).

d. \(\frac{1}{20}\).

e. \(\frac{1}{26}\).

VII-4. \((s_1, s_2, s_3, \ldots)\) is an infinite sequence of real numbers such that \(s_1 = \frac{3}{4}\), \(s_2 = \frac{6}{9}\), \(s_3 = \frac{15}{16}\), and \(s_4 = \frac{24}{25}\). All the terms are determined by the same common pattern.

Which one of the following is \(s_8\)?

a. \(\frac{7}{9}\).

b. \(\frac{80}{81}\).

c. \(\frac{64}{65}\).

d. \(\frac{15}{16}\).

e. \(\frac{103}{104}\).
VII-5. \((s_1, s_2, s_3, ... )\) is an infinite sequence of real numbers such that \(s_1 = 0, s_2 = \frac{1}{3}, s_3 = \frac{1}{2}, s_4 = \frac{1}{5}, s_5 = \frac{2}{3}\), and \(s_6 = \frac{5}{7}\). All the terms are determined by the same common pattern.

Which one of the following is \(s_{12}\) ?

a. \(\frac{7}{12}\).

b. \(\frac{11}{13}\).

c. \(\frac{6}{7}\).

d. \(\frac{9}{10}\).

e. \(\frac{13}{15}\).

VII-6. \((s_1, s_2, s_3, ... )\) is an infinite sequence of real numbers such that \(s_1 = \frac{1}{4}, s_2 = \frac{1}{2}, s_3 = 1, s_4 = 2, ..., s_6 = 8, ..., \) and \(s_{10} = 128\). All the terms are determined by the same common pattern.

Which one of the following is \(s_8\) ?

a. 16.

b. 24.

c. 32.

d. 64.

e. 81.
VII-7. \((s_1, s_2, s_3, \ldots)\) is an infinite sequence of real numbers such that \(s_1 = 1, s_2 = 8, s_3 = 27, \text{ and } s_4 = 64.\) All of the terms are determined by the same common pattern.

Which one of the following is \(s_7\)?

a. 87.
b. 97.
c. 125.
d. 200.
e. 343.

VII-8. \((s_1, s_2, s_3, \ldots)\) is an infinite sequence of real numbers such that \(s_1 = \frac{1}{2}, s_2 = \frac{1}{5}, s_3 = \frac{1}{10}, \text{ and } s_4 = \frac{1}{17}.\) All of the terms are determined by the same common pattern.

Which one of the following is \(s_7\)?

a. \(\frac{1}{7}\).
b. \(\frac{1}{15}\).
c. \(\frac{1}{28}\).
d. \(\frac{1}{50}\).
e. \(\frac{1}{64}\).
Section VII
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VII-9. \((s_1, s_2, s_3, \ldots)\) is an infinite sequence of real numbers such that \(s_1 = 3, s_2 = \frac{3}{2}, s_3 = 1,\) and \(s_4 = \frac{3}{4}\). All of the terms are determined by the same common pattern.

Which one of the following is \(s_{12}\)?

a. \(\frac{1}{4}\).

b. \(\frac{1}{2}\).

c. \(\frac{7}{12}\).

d. \(\frac{13}{12}\).

e. 4.

VII-10. \((s_1, s_2, s_3, \ldots)\) is an infinite sequence of real numbers such that \(s_1 = 1, s_2 = \frac{3}{2}, s_3 = 2,\) and \(s_4 = \frac{5}{2}\). All of the terms are determined by the same common pattern.

Which one of the following is \(s_{35}\)?

a. \(\frac{15}{2}\).

b. 8.

c. \(\frac{19}{2}\).

d. \(\frac{35}{2}\).

e. 18.
VII-11. \((s_1, s_2, s_3, \ldots)\) is an infinite sequence of real numbers such that \(s_1 = \frac{1}{2},\ s_2 = 4,\ s_3 = \frac{3}{2},\ s_4 = 8,\ s_5 = \frac{5}{2},\) and \(s_6 = 12.\) All of the terms are determined by the same common pattern.

Which one of the following is \(s_{20}\)?

a. \(\frac{15}{2}\).

b. 10.

c. \(\frac{25}{2}\).

d. 30.

e. 40.

VII-12. \((s_1, s_2, s_3, \ldots)\) is an infinite sequence of real numbers such that \(s_1 = 3,\ s_2 = 4,\ s_3 = 6,\ s_4 = 9,\ s_6 = 18,\) and \(s_7 = 24.\) All of the terms except for \(s_1\) are determined by a common pattern.

Which one of the following is \(s_5\)?

a. 11.

b. 12.

c. 13.

d. 14.

e. 15.
The test items in this group are each designed to measure the following cognitive behavior:

Given the following:

i. * is an operation on a set X.

ii. A and B are proper subsets of X.

iii. a*b is given for each a in A and for each b in B.

iv. c is an element of X but not of A, and d is an element of X but not of B.

Identifies from a five element subset of X, the element e such that c*d = e.
VIII-1. * is an operation on the set of integers such that 4 * 1 = 15 and 6 * 5 = 31.

Which one of the following equals 9 * 10?

a. 28.
b. 37.
c. 51.
d. 66.
e. 71.

VIII-2. * is an operation on the set of real numbers such that \( \frac{3}{2} \times \frac{1}{4} = \frac{7}{2} \) and 3 * 7 = 20.

Which one of the following equals 8 * 3?

a. \( \frac{9}{2} \).
b. 11.
c. 22.
d. 24.
e. 48.
VIII-3. * is an operation on the set of integers such that  
$10 * 3 = 16$ and $4 * 5 = 14$.

Which one of the following equals $5 * 4$?

a. 12.
b. 13.
c. 14.
d. 15.
e. 16.

---

VIII-4. * is an operation on the set of all ordered pairs of integers  
such that $(2,3)*(1,-2) = (3,-6)$ and $(0,3)*(5,4) = (5,12)$.

Which one of the following equals $(3,7)*(2,-1)$?

a. $(5,6)$.
b. $(6,-6)$.
c. $(2,14)$.
d. $(6,-7)$
e. $(5,-7)$
VIII-5. * is an operation on the set of integers such that 
\[ 5 * 8 = -6 \] and \[ 17 * 5 = 24. \]

Which one of the following equals \( 4 * 6 \)?

a. \(-4\).

b. \(-2\).

c. \(0\).

d. \(2\).

e. \(4\).

VIII-6. * is an operation on the set of all ordered pairs of integers such that \((1,2) * (5,2) = (-4,0)\) and \((12,3) * (6,1) = (4,7)\).

Which one of the following equals \((6,3) * (4,5)\)?

a. \((2,-2)\).

b. \((2,2)\).

c. \((-3,1)\).

d. \((0,8)\).

e. \((9,9)\).
VIII-7. $*$ is an operation on the set of all ordered pairs of integers such that $(1,2)*(2,10)=(-2,20)$ and $(7,3)*(2,-4)=(14,-12)$.

Which one of the following equals $(3,2)*(4,-1)$?

a. $(7,-1)$.  

b. $(5,3)$.  

c. $(12,-2)$.  

d. $(-1,-1)$.  

e. $(7,1)$.  

VIII-8. $*$ is an operation of the set of all ordered pairs of integers such that $(1,2)*(5,1)=3$, $(2,0)*(8,1)=9$ and $(6,4)*(4,4)=2$.

Which one of the following equals $(8,3)*(2,3)$?

a. $3$.  

b. $4$.  

c. $5$.  

d. $6$.  

e. $7$.  

Comments:
VIII-9. \( \ast \) is an operation on the set of real numbers such that 
\[ 16 \ast (-6) = 1 \quad \text{and} \quad 2.3 \ast 18 = 2.03. \]

Which one of the following equals \( 31.8 \ast 9.2 \) ?

a. 0.4.
b. 3.9.
c. 4.
d. 4.1.
e. 5.

VIII-10. \( \ast \) is an operation on the set of integers such that 
\[ 3 \ast 4 = 1 \quad \text{and} \quad (-8) \ast 1 = 9. \]

Which one of the following equals \( 6 \ast 2 \) ?

a. -4.
b. 0.
c. 4.
d. 8.
e. 12.
VIII-11. \( * \) is an operation on the set of integers such that
\[ 3 * 4 = 25 \quad \text{and} \quad (-8) * 1 = 65. \]
Which one of the following equals \( 6 * 2 \)?

a. 24.
b. 40.
c. 48.
d. 54.
e. 66.

VIII-12. \( * \) is an operation on the set of real numbers such that
\[ \frac{1}{16} * \frac{3}{16} = \frac{1}{2} \quad \text{and} \quad \sqrt{2} * 3 = \sqrt{\frac{2}{3} + 3}. \]
Which one of the following equals \( \frac{27}{2} * \frac{5}{2} \)?

a. 4.
b. 8.
c. \( 4\sqrt{2} \).
d. \( \sqrt{5} + \sqrt{27} \).
e. 16.
The test items in this group refer to codes devised to associate arrays of integers with calendar dates. The codes involve arithmetic operations in a fixed order on the integers in the array. Each item is designed to measure the following cognitive behavior:

Given two integer arrays and their respective dates, identifies from a group of five dates the date associated with a third integer array.
IX-1. This problem involves the determination of a coding scheme which associates a two integer array with a calendar date.

If [04 07] is November 4 and if [23 -21] is February 23, then what is [03 05]?

a. March 5.
b. May 3.
c. August 3.
d. October 3.
e. November 3.

IX-2. This problem involves the determination of a coding scheme which associates a six integer array with a calendar date.

If [45 6335] is September 17 and if [13 5410] is April 10, then what is [11 3120]?

a. February 6.
b. January 3.
d. January 30.
e. February 7.
IX-3. This problem involves the determination of a coding scheme which associates a two integer array with a calendar date.

If [20 14] is October 7 and if [06 54] is March 27, then what is [02 42]?

b. February 24.
c. March 6.
d. August 26.
e. November 12.

IX-4. This problem involves the determination of a coding scheme which associates a four integer array with a calendar date.

If [12 71] is January 17 and if [03 13] is October 31, then what is [05 42]?

c. October 21.
d. August 24.
e. May 21.
IX-5. This problem involves the determination of a coding scheme which associates an eight integer array with a calendar date.

If [01 66 73 81] is December 9 and if [82 74 74 30] is January 13, then what is [83 10 12 82]?

a. October 15.
b. November 30.
c. August 16.
d. December 10
e. March 9.
## Answer Key For Test Items

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APPENDIX B

COVER SHEET OF T₁
AND A LIST OF THE ITEMS OF T₁
ACCORDING TO M₁ NUMBERS
Please print your name: ________________________

NOTE: This test is designed to measure your ability to analyze certain mathematical situations. Although you are probably familiar with the mathematics involved, the correct response to many of the items will become obvious only after careful analysis. For each item carefully read the given information, the question, and each of the five possible responses. Choose the one correct response to each item by circling the appropriate letter (a, b, c, d, or e).

You may do your scratch work anywhere on these pages.
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APPENDIX C

COVER SHEET OF T₂
AND A LIST OF THE ITEMS OF T₂
ACCORDING TO M₁ NUMBER
TEST FOR ANALYTICAL COGNITION
OF MATHEMATICAL CONTENT

Form T-2

Please print your name: ________________________________

NOTE: This test is designed to measure your ability to analyze certain mathematical situations. Although you are probably familiar with the mathematics involved, the correct response to many of the items will become obvious only after careful analysis. For each item carefully read the given information, the question, and each of the five possible responses. Choose the one correct response to each item by circling the appropriate letter (a, b, c, d, or e).

You may do your scratch work anywhere on these pages.
## Items of $T_2$ According to $M_1$ Numbers

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APPENDIX D

INSTRUCTION SHEET FOR THE TEACHERS OF Ss_3
To: Mr Arthur and Mr. Douglas  
From: Jimmy Cangelosi  
Regarding: The administration of the test for analytical cognition to the three algebra II classes at Catholic High School on Nov, 3 and 4.

Sir:

Excuse me if I sound dogmatic in these instructions. But I am certain that you can appreciate the necessity of keeping procedure variations from class to class to an absolute minimum.

Again, thank you for allowing me to use your time. I only hope that I can repay you and your students by developing an instrument which will contribute to the cause of education.

FOR THE DAY PRIOR TO THE TEST, NOV. 2

Please explain to your students the purpose in their taking this test. Do what you can to motivate them to put forth their very best efforts on the test. It might be mentioned that experience has shown that some students who typically do poorly on mathematics tests will perform exceptionally well on this particular type of test.

They should be told that their ability to think in an orderly and logical fashion will be tested and not their knowledge of mathematical content.

There are two terms used in the test which should be explained on this day: sequence and operation *. Tell them the following:

"A sequence is a set of ordered elements, i.e., there is a first element, a second element, etc. On this test the first term is denoted by S_1, the second by S_2, and in general the nth by S_n."

"In the section involving the symbol *, the problem is to figure out what * means in each case."

FOR THE FIRST DAY OF THE TEST, NOV. 3

1. Provide each student with a copy of the test.

2. Make sure that everyone prints his name on the cover.

3. Read aloud, as the students follow, the instructions
on the front cover. Answer any reasonable questions before the students are allowed to begin work. After they begin there are to be no questions.

4. Try to allow the students as much of the 55 minute period as possible to work on the test.

5. Collect the tests as each finishes.

FOR THE SECOND DAY OF THE TEST, NOV. 4

1. (Same as #1 above)

2. Make sure that everyone prints his name in the blank at the top of page 8.

3. Answer no questions after the test begins.

4. (See #4 and 5 above,)
APPENDIX E

\( T_3 \)

THE TEST FOR ANALYTICAL COGNITION

OF MATHEMATICAL CONTENT
NOTE: This test is designed to measure your ability to analyze certain mathematical situations. You are probably familiar with the mathematics involved, because no terms, symbols, or concepts are used that were not basic to your eighth and ninth grade mathematics courses. However, the correct response to many of the items will become obvious only after careful analysis.

For each item, carefully read the given information, the question, and each of the five possible responses. Make sure that you distinguish between such terms as "integers" and "real numbers." Phrases such as "there exist," "such that," "if and only if," and "must be true" should be taken literally.

There is exactly one correct response to each item. Please indicate your choice for each item by circling the appropriate letter (a, b, c, d, or e).

If after carefully considering an item, you are uncertain of the correct response, you should mark your best guess.

Scratch work may be done anywhere on these pages.

You have until the end of this class period to complete these 20 items. Tomorrow, 20 more items will be given to you.

Thank you.
1. Let B be a set defined by the following:
   A real number \( n \) is a member of set \( B \) if and only if there exists an integer \( x \) such that \( n = 6x \).

Which of the following sets is a subset of \( B \)?
   a. \( \{0, 6, 36\} \).
   b. \( \{-6, 16, 46\} \).
   c. \( \{6, 12, 16\} \).
   d. \( \{-3, 0, 24\} \).
   e. None of these.

2. Let B be a set defined by the following:
   A real number \( n \) is a member of set \( B \) if and only if there exists an integer \( x \) such that \( x = \sqrt{n} \).

Which of the following sets is a subset of \( B \)?
   a. \( \{1, 16, 25\} \).
   b. \( \{\frac{1}{4}, 3, 4\} \).
   c. \( \{-25, 1, 4\} \).
   d. \( \{0, 3, 24\} \).
   e. None of these.

3. Let B be a set defined by the following:
   A real number \( n \) is a member of set \( B \) if and only if \( n \) is an integer such that \( 0 < n < \frac{9}{2} \).

Which of the following sets is a subset of \( B \)?
   a. \( \{1, 4, 4.32\} \).
   b. \( \{0, 2, 4\} \).
   c. \( \{\frac{1}{2}, 1, 2\} \).
   d. \( \{2, 4, 7\} \).
   e. None of these.
4. Let $B$ be a set defined by the following:

An integer $n$ is a member of set $B$ if and only if there exists an integer $x$ such that $n = 3x$ and there does not exist an integer $y$ such that $n = 2y$.

Which one of the following sets is a subset of $B$?

a. \{-3, 1, 6, 27\}

b. \{0, 1, 3, 9\}

c. \{-9, 0, 9, 81\}

d. \{-81, -3, 9, 27\}

e. None of these.

5. Let $B$ be a set defined by the following:

A real number $n$ is a member of set $B$ if and only if $n^2 < n$.

Which one of the following sets is a subset of $B$?

a. \{0, 1, 12\}

b. \{-\frac{1}{2}, -\frac{2}{9}, -\frac{2}{3}\}

c. \{\frac{1}{4}, \frac{2}{3}, \frac{17}{3}\}

d. \{\frac{1}{8}, \frac{2}{5}, \frac{9}{10}\}

e. None of these.

SECTION B

6. "Set $N$ contains more than 10 elements."

Which one of the following can be concluded from the above statement?

a. Set $N$ contains at least 11 elements.

b. Set $N$ does not contain 12 elements.

c. Set $N$ contains more than 11 elements.

d. Set $N$ is an infinite set.

e. None of these.
7. "N and B are sets such that all the elements of N are also contained in B. x and y are elements such that x is not contained in B and y is contained in N."

Which one of the following can be concluded from the above statement?

a. x is contained in N and y is contained in B.
b. x is not contained in N and y is contained in B.
c. x is not contained in N and y is not contained in B.
d. x is contained in N and y is not contained in B.
e. None of these.

8. Accept the following statements to be true:
1. A is either red or green.
2. B is either blue or brown.
3. C is either red, green, blue, or brown.
4. C is never the same color as B.
5. If A is red, then C is either red or blue.
6. If B is brown, then A is green.

Suppose that A is red. What color is C?

a. Red.
b. Green.
c. Blue.
d. Brown.
e. C cannot be determined.

9. Let a sequence of one-hundred statements be represented by $(s_1, s_2, s_3, \ldots, s_{100})$.

Suppose that the following are two true facts:
1. If $s_n$ is true, then $s_{n+5}$ is true for any positive integer n.
2. $s_6$ is true.

Which one of the following is true?

a. $s_5$.
b. $s_{12}$.
c. $s_{19}$.
d. $s_{21}$.
e. $s_{26}$.
Let a sequence of one-hundred statements be represented by 
\((s_1, s_2, s_3, \ldots, s_{100})\).

Suppose that the following are two true facts:

1. If \(s_n\) is true, then \(s_{n+3}\) is true for any positive integer \(n\).
2. \(s_7\) is true.

Which one of the following is true?

a. \(s_6\).

b. \(s_{11}\).

c. \(s_{15}\).

d. \(s_{19}\).

e. \(s_{27}\).

SECTION C

11. \((s_1, s_2, s_3, \ldots)\) is an infinite sequence of real numbers such that \(s_1 = 3\), \(s_2 = \frac{1}{2}\), \(s_3 = 1\), and \(s_4 = \frac{1}{4}\). All of the terms are determined by the same common pattern.

Which one of the following is \(s_{12}\)?

a. \(\frac{1}{4}\).

b. \(\frac{1}{2}\).

c. \(\frac{7}{12}\).

d. \(\frac{13}{12}\).

e. 4.

12. \((s_1, s_2, s_3, \ldots)\) is an infinite sequence of real numbers such that \(s_1 = 3\), \(s_2 = 4\), \(s_3 = 6\), \(s_4 = 9\), \(s_5 = 18\), and \(s_7 = 24\). All of the terms except for \(s_1\) are determined by a common pattern.

Which one of the following is \(s_5\)?

a. 11.

b. 12.

c. 13.

d. 14.

e. 15.
13. The following addition uses numbers in a base other than 10:
   \[5 + 3 = 12\]

   What is the base?
   a. 4.
   b. 5.
   c. 6.
   d. 7.
   e. 8.

14. The following addition uses numbers in a base other than 10:
   \[4 + 3 = 12\]

   What is the base?
   a. 4.
   b. 5.
   c. 6.
   d. 7.
   e. 8.

15. The following subtraction uses numbers in a base other than 10:
   \[11 - 5 = 2\]

   What is the base?
   a. 4.
   b. 5.
   c. 6.
   d. 7.
   e. 8.
SECTION E

16. * is an operation on the set of all ordered pairs of integers such that 
\((2,3)* (1,-2) = (3,-6)\) and \((0,3)*(5,4) = (5,12)\).

Which one of the following equals \((3,7)*(2,-1)\) ?

a. \((5,6)\).
b. \((6,-6)\).
c. \((2,14)\).
d. \((6,-7)\).
e. \((5,-7)\)

17. * is an operation on the set of integers such that 
\(10*3 = 16\) and \(4*5 = 14\).

Which one of the following equals \(5*4\) ?

a. 12.
b. 13.
c. 14.
d. 15.
e. 16.

d. 15.
e. 16.

18. * is an operation on the set of integers such that 
\(3*4 = 1\) and \((-8)*1 = 9\).

Which one of the following equals \(6*2\) ?

a. -4.
b. 0.
c. 4.
d. 8.
e. 12.
19. * is an operation on the set of integers such that 
3 * 4 = 25 and (−8) * 1 = 65.

Which one of the following equals 6 * 2?

a. 24.
b. 40.
c. 48.
d. 54.
e. 66.

20. * is an operation on the set of real numbers such that 
16 * (−6) = 1 and 2.3 * 18 = 2.03.

Which one of the following equals 31.8 * 9.2?

a. 0.4.
b. 3.9.
c. 4.
d. 4.1.
e. 5.
21. After establishing that $x$, $y$, and $z$ are real numbers such that $x \leq y$, the author of a mathematical proof concludes that $\frac{x}{z} \leq \frac{y}{z}$.

Which one of the following conditions is necessary for this conclusion to be valid?

- a. $z > 0$
- b. $z \geq 0$
- c. $z < 0$
- d. $z \leq 0$
- e. None of these.

22. After establishing that $x$ is a real number, the author of a mathematical proof concludes that $x^2 < x$.

Which one of the following conditions is necessary for this conclusion to be valid?

- a. $0 \leq x \leq 1$
- b. $0 < x < 1$
- c. $0 < x \leq 1$
- d. $0 < x < 1$
- e. None of these.

23. After establishing that $x$, $y$, $z$, and $w$ are real numbers such that $x > y$, the author of a mathematical proof concludes that $x(z-w) < y(z-w)$.

Which one of the following conditions is necessary for the conclusion to be valid?

- a. $z = w$
- b. $z > w$
- c. $z < w$
- d. $z \neq w$
- e. None of these.
24. After establishing that $x, y, z,$ and $w$ are real numbers such that $x > y,$ the author of a mathematical proof concludes that $x(z - w) = y(z - w)$.

Which one of the following conditions is necessary for this conclusion to be valid?

a. $z = w.$
b. $z > w.$
c. $z < w.$
d. $z \neq w.$
e. None of these.

25. After establishing that $x$ and $y$ are real numbers such that $x > 0,$ the author of a mathematical proof concludes that $xy > x$.

Which one of the following conditions is necessary for this conclusion to be valid?

a. $y > 0.$
b. $y < 0.$
c. $y > 1.$
d. $y > x.$
e. None of these.

26. After establishing that $x$ and $y$ are real numbers such that $y(x - 1) = x^2 - 1,$ the author of a mathematical proof concludes that $y = x + 1$.

Which one of the following conditions is necessary for this conclusion to be valid?

a. $x \neq 1.$
b. $x \neq 0.$
c. $x \neq -1.$
d. $y \neq 0.$
e. None of these.
**SECTION G**

27. Let \( r \) and \( s \) be integers such that \( r + r + 1 = s \).

Which one of the following must be true?

a. \( r \) is even.

b. \( r \) is odd.

c. \( s \) is even.

d. \( s \) is odd.

e. None of these.

28. Let \( r \) and \( s \) be real numbers such that \( rs = 0 \) and \( r = 0 \).

Which one of the following must be true?

a. \( s \neq r \).

b. \( s > 0 \).

c. \( s < 0 \).

d. \( s = 0 \).

e. None of these.

29. Let \( r, s, t, \) and \( u \) be real numbers such that \( \frac{r-s}{t-u} > 0 \) and \( u > t \).

Which one of the following must be true?

a. \( r + u > t + s \).

b. \( r - s < t - u \).

c. \( s < r \).

d. \( s > r \).

e. None of these.
30. Let \( r, s, \) and \( t \) be real numbers such that \( \frac{rs}{t} > 0, \ r < 0, \) and \( s > 0. \)
Which one of the following must be true?

a. \( t < rs. \)

b. \( t > rs. \)

c. \( t < 0. \)

d. \( t > 0. \)

e. None of these.

31. Let \( r, s, \) and \( t \) be real numbers such that \( r + (s-t) > 0 \) and \( s < t. \)
Which one of the following must be true?

a. \( r > t. \)

b. \( r < t. \)

c. \( r > 0. \)

d. \( r < 0. \)

e. None of these.

32. Let \( r, s, \) and \( t \) be real numbers such that \( rt + st > 0 \) and \( t < 0. \)
Which one of the following must be true?

a. \( r < -s. \)

b. \( r + s > t. \)

c. \( -r < s. \)

d. \( -r > -s. \)

e. None of these.
33. Consider the variable $x = (y^2 + 3)$ where $y$ is an element of the set of real numbers.

What is the minimum value for $x$?

a. -9.

b. -3.

c. 0.

d. 3.

e. None of these.

34. Consider the variable $x$ such that $x = (3 - s)^2$ where $s$ is an element of the set of real numbers.

When $x$ is at its minimum value, what is $s$?

a. -3.

b. 0.

c. 3.

d. 9.

e. None of these.

35. Consider the variable $x = y^2$ where $y$ is a real number.

In which one of the following sets of values for $y$ will $x$ attain the greatest value?

a. $y$ is an element of $\{-4, -2, 3.5\}$.

b. $y$ is an element of $\{-11, -\sqrt{8}, 4\}$.

c. $y$ is an element of $\{-1, 0, \frac{-17}{3}\}$.

d. $y$ is an element of $\{6, 7, 8\}$.

e. $y$ is an element of $\{1, 2.16, 4\}$.
36. Consider the variable $x$ such that $x = y^2$ where $y$ is a real number.

In which one of the following sets of values for $y$ will $x$ attain the greatest value?

a. $0 \leq y \leq 10$.
b. $-\frac{1}{2} \leq y \leq \frac{1}{2}$.
c. $-5 \leq y \leq 12$.
d. $-20 \leq y \leq 3$.
e. $-5 \leq y \leq 10$.

37. Consider the variable $x$ such that $x = \frac{z}{y}$ where $x$ and $y$ are real numbers and $y \neq 0$.

In which one of the following sets of values for $x$ and $y$ will $x$ attain the least value?

a. $z = 2$ and $0 < y \leq 4$.
b. $-10 \leq z \leq 1$ and $y = 1$.
c. $z < 0$ and $y > 0$.
d. $z = 0$ and $0 < y \leq 3$.
e. $-20 \leq z \leq -1$ and $1 < y < 2$.

38. Consider the variable $x$ such that $x = \frac{z}{z-2}$ where $0 \leq z < 2$.

What is the maximum value for $x$?

a. $\frac{-1}{2}$.
b. 0.
c. $\frac{1}{2}$.
d. 2.
e. There is no limit as to how great $x$ can be.
SECTION I

39. This problem involves the determination of a coding scheme which associates a six integer array with a calendar date.

If [45 6335] is September 17 and if [13 5410] is April 10, then what is [11 3120]?

a. February 6.

b. January 3.


d. January 30

e. February 7.

40. This problem involves the determination of a coding scheme which associates a two integer array with a calendar date.

If [04 07] is November 4 and if [23 -21] is February 23, then what is [03 05]?

a. March 5.

b. May 3.

c. August 3.

d. October 3.

e. November 3.
VITA

James S. Cangelosi was born on March 9, 1943, in Baton Rouge, Louisiana where he received his formal education. After graduating from Catholic High School in 1961, he received three degrees from Louisiana State University: B. S. in secondary education in 1965, M. Ed. in education in 1967, and M. A. in mathematics in 1971.

During the period 1965-1970, he served as mathematics teacher, athletic director, and coach at Catholic High School. In 1971, he became an instructor of mathematics at Louisiana State University.

He married Pamela Jane Woodin in 1966 and they have two children, Amy Elise and Christopher James.
EXAMINATION AND THESIS REPORT

Candidate:    James S. Cangelosi

Major Field:  Education

Title of Thesis:  The Construction And Refinement Of A Test For Analytical Cognition Of Mathematical Content

Approved:

[Signatures]

Major Professor and Chairman

Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:

March 15, 1972