Accounting for Travel Time Reliability, Trip Purpose and Departure Time Choice in an Agent-Based Dynamic Feedback-Control Toll Pricing Approach

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ACCOUNTING FOR TRAVEL TIME RELIABILITY, TRIP PURPOSE AND DEPARTURE TIME CHOICE IN AN AGENT-BASED DYNAMIC FEEDBACK-CONTROL TOLL PRICING APPROACH

A Thesis

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering

in

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by

Wan Li

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ABSTRACT

The primary goal of this study is to modify the original strategy developed by Cheng and Ishak (2013) into an agent-based dynamic feedback-control toll pricing strategy that accounts for the trip purpose, travel time reliability, departure time choice and level of income such that the toll revenue is maximized while maintaining a minimum desired level of service on the managed lanes. An external module was developed to execute the modified strategy. An agent-based modeling was applied to simulate drivers’ learning process based on their previous commuting experience. The study also analyzed how driver’s heterogeneity in value of time, value of reliability for each trip purpose will influence route decisions and thus affect the optimal toll rates. A numerical example was given to explain the modified strategy. The simulation results confirmed that under high traffic demand, drivers with urgent trip purpose have the highest probability of choosing managed lanes, and that the travel time on the managed lanes is more reliable than that on the general purpose lanes. A comparative evaluation is given between the modified strategy, the strategy currently deployed on Interstate 95 express lanes, and the original strategy. Compared to the current strategy, the increase in toll rate is steadier and the toll revenue is significantly higher for the modified strategy, while keeping the speed higher than 45 mph. On the other hand, compared to the original strategy, the modified one offers a more realistic approach that accounts for travel time reliability and delay in route choice and departure time choice, as well as generates higher toll revenue under heavy traffic demand.

Keywords: Congestion Pricing, Agent-based modeling, dynamic toll calculation, feedback control, trip purpose, travel time reliability, departure time choice, learning process.
CHAPTER 1. INTRODUCTION

In the past decade, traffic congestion has become a significantly increasing problem in the United States. Various solutions have been adopted to relieve traffic congestion but are not always feasible. Congestion pricing, which works by shifting rush-hour freeway travel demand to other transportation modes, alternative routes/lanes, or to off-peak periods, has become a cost-effective and an efficient strategy to mitigate the congestion problem on freeways. Among the few states that have already adopted congestion pricing is the state of Florida. Currently, the toll rate on I-95 in Florida is determined from traffic conditions on the managed lanes while ignoring the general purpose lanes. This study considers the travel time difference and travel time reliability on both managed lanes and general purpose lanes into a toll rate determination approach. In addition, most models that use behavioral economics to evaluate pricing strategies assume homogeneous conditions that all drivers will respond to the pricing strategies in the same manner. This study evaluate the drivers’ heterogeneity of value of time (VOT), value of reliability (VOR), trip purposes, departure time choice, and independent learning in an agent-based toll pricing model. It will provide a more accurate performance measures of impact from pricing strategies. The primary goal of this study is to modify the original strategy developed by Cheng and Ishak (2013) into an agent-based dynamic feedback-control toll pricing strategy that accounts for trip purpose, travel time reliability and departure time shift such that the toll revenue is maximized while maintaining an average speed of at least 45 mph, as required by the Federal Highway Administration (FHWA) to ensure minimum level of service.

In this study, an agent-based model is proposed to support dynamic activity-travel scheduling. Agents can adapt their behaviors and make decisions in a complex and dynamic environment by applying learning techniques and accounting for their previous experience. The
simulation network of the southbound of a segment on I-95 is selected to test the dynamic toll strategies in the morning pre-peak hour and peak hour from 5:00 AM to 7:30 AM. A comparative evaluation is given between the proposed strategy, the current strategy deployed on Interstate 95 express lanes and the original study developed by Cheng and Ishak (2013). Moreover, the impact of many factors for drivers’ route choice and departure time choice behaviors has been examined, including travel time saving, travel time reliability, trip purpose, and delay on managed lanes and general purpose lanes.

1.1 Objectives

To accomplish the goal, specific objectives are:

1. Identify the measurement of drivers’ travel time reliability, value of travel time (VOT) and value of reliability (VOR) for each trip purpose: 1) work trip, 2) urgent travel during emergency situations, and 3) leisure trip.

2. Identify the measurement of delay on managed lanes as well as general purpose lanes and investigate the impact of delay on departure time choice decision and dynamic traffic demand.

3. Analyze how drivers’ value of time (VOT), value of reliability (VOR), and trip purposes influence route choice decisions and thus affect the optimal toll rates.

4. Develop an agent-based dynamic feedback-control toll pricing strategy considering the drivers’ learning techniques.

5. Test the proposed dynamic toll strategies in microscopic simulation and compare their performance with the current toll strategy on I-95 and the previous study developed by Cheng and Ishak (2013).
1.2 Study Area

The scope of the study is limited to the state of Florida. The study area boundary includes a seven-mile segment of southbound I-95 between NW 151 Street and SR 112 Street with three general purpose lanes, as well as two managed lanes in this seven-mile segment. The general purpose lanes have multiple intermediate entrances and exits, while the managed lanes have only one entrance and one exit. Once vehicles enter the managed lanes, they have to proceed to the end and are not able to switch back to the general purpose lanes. High occupancy vehicles (HOV) can use the managed lanes for free while other vehicles have to pay toll. Based on Florida Department of Transportation (FDOT, 2011), HOV represents only 1.5% of all vehicles on I-95, so these vehicles were not included in the simulation model.

Figure 1 Map of Study Area
CHAPTER 2. LITERATURE REVIEW

This chapter provides literature review from four important perspectives. The first section reviews the research on drivers’ heterogeneity of value of time (VOT) and value of reliability (VOR) in the problem of congestion pricing; the second section provides the common and classical measurements of travel time reliability; the third section focus on how to model drivers departure time choice for individual drivers, and the fourth section highlights studies on the application of agent-based modeling in transportation research. A brief summary of literature is given after the four sections.

2.1 Drivers’ Heterogeneity of Value of Time (VOT) and Value of Reliability (VOR)

VOT and VOR are the two of the most important values obtained from travel demand studies. VOT represents the monetary values drivers place on travel time saving. Classic microeconomic theory of time allocation defines the VOT as the opportunity cost of an additional unit of leisure, i.e., the hourly wage rate. The use of the average wage rate to estimate the opportunity cost of time is an approximation to the value of time.

In the past, most studies considered VOT as a constant for all users within one user group. Jiang and Mahmassani (2013) took VOT as a continuous variable and assumed it is distributed probabilistically across the user population and can be achieved from survey data or activity based demand micro-simulation process.

He et al. (2012) proposed a new approach to estimated drivers’ VOT and VOR based on dynamic toll information collected from I-394, Minnesota. Differing from other studies, the travel time variability in this paper was defined as the difference between 90th percentile and the instantaneous travel time, in order to account for drivers’ perception of congestion at the moment when they entered managed lanes. This paper concluded that drivers traveling on Fridays have
higher VOT and VOR than other weekdays. In addition, the authors indicate that MnPASS users are willing to pay the toll more for travel time reliability than for travel time savings. The author also indicated that the heterogeneous values from the 25th to 75th percentile intervals of VOT and VOR suggest drivers’ willingness to pay the toll varies a lot.

Some papers establish a numerical relationship between VOT and average hourly rates. Lake and Ferreira (2002) studied what communities get for each dollar spent on Public Transport projects. According to them, the value of travel time (VOT) is 40% to 50% of average wage rates for non-business trips while for business related trips, it can be up to 80%-100% of the wage rate. These values are useful when considering VOT under different trip purposes in the utility function.

VOR connects the monetary values drivers place on reducing the travel time variability. Senna (1994) present a comprehensive framework for valuing travel time variability that allows for any journey purpose and arrival time constraint. The proposed model was based on the expected utility approach and the mean-standard deviation approach. The author concluded that values of time derived from the models are highly influenced by the value of travel time variability and it strongly depended on the probability distribution function drivers are faced with.

Concas and Kolpakov (2009) compiled and synthesize current and past research on the VOT and VOR. They suggest that with personal travel, VOT is equal to 50% of wage rate. With commercial (on-the-clock) travel, the VOT is equal to 100% of wage rate. As for VOR, in ordinary circumstances with no major constraints, VOR is equal to 80% to 100% of VOT. When there is non-flexible arrival/departure constraint, VOR is equal to 3 times of VOT.

Brownstone and Small (2005) conducted stated preference and revealed preference surveys and they found that VOT on the morning commute is $20-$40 per hour, or 50% -90% of average wage rate in the sample.
2.2 Measurement of Travel Time Reliability

In road networks, travel time is conventionally categorized into two parts: free flow travel time and additional travel time. Free flow travel time refers to the amount of time that takes a driver to arrive at the destination without encountering any traffic. Additional travel time refers to the increasing of travel time due to variations in the traffic conditions. The concept of travel time reliability is defined interchangeably with travel time variability in the transportation research literature. High variability means high unreliable of travel time. Under the condition, travel time reliability can be associated with the travel time distribution.

Most reliability measures are calculated from the day-to-day distribution of travel times on a particular route, that is, the variability in travel time vehicles experience on a particular time-of-day (TOD) and day-of-week (DOW) period over a longer time period. A comprehensive overview of travel time reliability measures can be found in the research of Lomax et al. (2003). They introduce four common measurements of travel time reliability:

1) Statistical range methods

Statistical range methods (Bates, et al., 2001) generally consider travel time as expected travel time plus or minus a factor times the standard deviation. This “plus or minus” type expression indicates the possible spread of travel time around some expected value, while implicitly assuming travel times to be symmetrically (e.g. normally) distributed. Statistical range estimates use the standard deviation and the coefficient of variation to describe the range of transportation conditions experienced by drivers.

2) Buffer time methods

The second approach considers the so-called buffer time, which could be explained as “the extra percentage travel time due to travel time variability on a trip that a driver should take into account in order to arrive on time” (Lomax et al., 2003). When travel time is not symmetric
distributed, both the width and the skew of the travel time distribution can provide a more robust estimate of reliability (van Lint & van Zuylen, 2005). A variation of buffer time index takes into account distributional skew $\lambda_{skew}$. It is suggested that the skew of the distribution can be measured as the ratio of the difference between the 90th and 50th percentile to the difference between the 50th and 10th percentile. In addition, buffer time is sometimes expressed as the distance between 90th or 95th percentile travel time and the average travel time.

3) The so-called “tardy-trip” measures

Tardy trip measures represent the travel time unreliability using the amount of trips that result in late arrivals (Lomax et al., 2003). The misery index takes the difference between the average travel time of the 20% worst trips and the overall travel time average. This method thus focuses on the extra delay incurred during the worst trips.

4) Probabilistic measures

Probability-based methods express travel time reliability based on probabilistic measures (Yang et al., 2000). These methods have in common that some probabilistic measure is used as measure for travel time unreliability.

Concas and Kolpakov (2009) concluded that there are two general approaches to measuring travel time reliability, the mean-variance and the scheduling approach. The mean-variance approach assumes that drivers endure inconvenience and costs when variability directly affects their travel time. This approach assumes that variability costs are symmetric with respect to the mean travel time.

The scheduling approach explicitly distinguishes between the costs associated with late and early arrivals. This approach indirectly captures the value of travel time variability through
modeling drivers’ departure time choices. Noland and Polak (2002) had indicated that scheduling models capture drivers’ reaction to travel time variability more accurately.

Van Lint and Van Zuylen (2008) studied the effective measures of travel time reliability from a driver’s viewpoint. It starts on a high level by describing a number of travel time reliability measures. From a large SP/ RP route choice experiment, it is concluded that $\lambda_{skew}$ (a measure for the asymmetry of a travel time distribution) is an important reliability measure. The larger $\lambda_{skew}$, the more the travel time distribution is skewed to the left, which implies a small amount of trips incur travel times which are much higher than the vast majority of trips. More detail analyses showed that travel information and travel goal influenced route choice behavior.

Lint et al. (2008) investigated the day-to-day distribution of travel times on the basis of empirical data from a densely used freeway in The Netherlands. They find that the travel time distribution is often wide and (left) skewed, particularly in periods where mostly congestion occurs, sets in or dissolves. Given an often skewed travel time distribution, usage of classical measures based on mean and variance of travel times may not be advisable – these measures may lead to a biased estimate of reliability.

Higatani et al. (2009) used the data from Hanshin expressway to examine the fundamental characteristics of travel time reliability. Buffer time, buffer time index, SD (standard deviation) and CV (coefficient of variation) are calculated. The result shows that the buffer time and buffer time index profiles have a similar tendency of those the SD and CV, respectively. From the paper, we can understand the latent connection between these different factors (buffer time, buffer time index, SD and CV).
2.3 Departure Time Choice

Drivers would change their departure time in reaction to level of congestion in the network system. They would either leave home earlier or later in order to avoid peak hours, leaving peak spreading in travel demand model (Zhang et al., 2011). Dynamic toll pricing strategies encourage drivers make a better tradeoff between travel time, travel cost and preferred arrival time, making the utilization of existing network maximized and more efficient. Under the conditions, it is significant to account for departure time choice in the agent based dynamic toll pricing strategies.

Zhang and Xiong (2012) developed a positive model of departure time choice for individual drivers, focusing on how individual perceives travel condition, accumulate travel experience and make final decision on departure time choice. The study incorporates reveal preference (RP) questions, memory-recall questions and stated preference (SP) design in the survey method to collect relevant data of driver behaviors in the Washington DC metropolitan area. A numerical example has been developed to illustrate how model works. The example shows the peak spreading effect that 14% of the drivers who depart between 7 am and 8 am switch to either before 7 am or after 8 am. In addition, most departure time changes occurs in the first two weeks. There are only 2% of drivers searching alternatives and 0.24% of the commuters adopt new departure time after fifty days. The authors also mention that this positive departure time model can be integrated into traffic microscopic traffic simulators or agent-based modeling for real-world application.

Many departure time choice models have been tested in the literature. Bhat et al. (2003) adopted MNL model structure to analyze departure time choice of drivers for home-based trips. The 1996 activity survey data in the Dallas-Fort Worth area have been used. The characteristics of individual, household socio-demographics, employment-related attributes have been evaluated in the model. They are found to have significant impact on departure time choice behavior.
Saleh and Farell (2005) estimated how congestion pricing on peak hours influence the departure time choice of drivers by taking the trip scheduling flexibility into account. A questionnaire survey conducted in Edinburgh has been used to investigate the potential influence. The survey data show that most of drivers depart for work leave home from 7:30 to 8:29 in the morning. Because of the congestion pricing, 70% of respondents change their departure time either earlier or later than usual. The base model estimate several variables influencing drivers’ choice of departure time between three alternatives: earlier, same as or later than usual time. The study shows that the flexibility of departure time choice has been influenced by the variables that affect the individual flexibility.

Paleti et al. (2014) used Revealed Preference (RP) data and Stated Preference (SP) data to develop a joint model and demonstrated the drivers’ departure time choice under the related policies, such as congestion pricing and parking cost. The RP data used in this study come from the Household Travel Survey (HTS) in the Jerusalem metropolitan area. The SP survey are conducted under 5 scenarios after the HTS. The data show that 50% of respondents are not willing to switch their departure time when there are tolls. SP data can better evaluate the mode choice behavior while RP data can incorporate the TOD choice. The combination of these two types of data provide rich information for analysis. A hybrid choice duration model has been developed in the study. The parameters for all LOS variables in the utility function are statistically significant. The results show that drivers would avoid both early and late arrival, but the late arrival would cause more aversion.

De Palma et al. (2004) investigated the dynamics of departure time for the afternoon commute. They found that drivers who drive alone change their departure time more than others.
In addition, younger workers and people employed in certain occupations, such as executive, scientific and service professions, would like to change their departure time more than others.

Bliemer and van Amelsfort (2006) investigated the influence of different components of schedule delay in the utility functions of time choice models and studied how drivers account travel time uncertainty into account in departure time decision making process. The utility function is shown below:

\[ U_{\text{in}}^{\text{car}} = \beta_{\theta}^{\text{car}} \theta_{\text{in}}^{\text{car}} + \beta_{\tau}^{\text{car}} \tau_{\text{in}}^{\text{car}} + \beta_{\text{SDE, arr}}^{\text{car}} \text{ASDE}_{\text{in}}^{\text{arr, car}} + \beta_{\text{SDL, arr}}^{\text{car}} \text{ASDL}_{\text{in}}^{\text{arr, car}} + \beta_{\omega}^{\text{car}} \omega_{\text{in}}^{\text{car}} + \epsilon_{\text{in}}^{\text{car}} \]

Where,

\( \theta_{\text{in}}^{\text{car}} \): Road pricing fee

\( \tau_{\text{in}}^{\text{car}} \): travel time

\( \text{ASDE}_{\text{in}}^{\text{arr, car}} \) and \( \text{ASDL}_{\text{in}}^{\text{arr, car}} \): the schedule delays for arriving early and late

\( \omega_{\text{in}}^{\text{car}} \): Travel time uncertainty

\( \epsilon_{\text{in}}^{\text{car}} \): Unobserved components

They consider preferred arrival time (PAT) as time range instead of a time point. The conclusion stated that departing on time is really important for work trip. Although respondents do think on-time arrival is important, if they depart on-time while late for the workplace due to unforeseen delays, they cannot control it nor responsible for it. The conclusion indicated that departure late may cause high disutility.

Lu and Mahmassani (2011) studied multi-criterion simultaneous route and departure time user equilibrium (MSRDUE) to model the response from heterogeneous users to dynamic pricing. They used column generation-based approach to generate possible alternatives, then used dynamic network loading model, alternative generation scheme, and multi-class alternative flow updating method to finish assignment process. Drivers are grouped by different VOT and are assumed to
choose a combination of departure time and path to minimum trip cost. The general cost function as shown below:

\[
G_{odp}^{\tau}(r; \theta, \alpha, \beta, \gamma) = TC_{odp}^{\tau} + \alpha TT_{odp}^{\tau}(r) + \beta ESD_{odp}^{\tau}(\theta) + \lambda LSD_{odp}^{\tau}(\theta)
\]

Where,

- \(C_{odp}^{\tau}\): sum of travel cost,
- \(\alpha TT_{odp}^{\tau}\): travel time weighted by VOT,
- \(\beta ESD_{odp}^{\tau}\): early schedule delay weighted by VOESD
- \(\lambda LSD_{odp}^{\tau}\): late schedule delay weighted by VOLSD.

2.4 Agent-Based Modeling

An agent-based model (ABM) is a class of computational models for simulating the actions and interactions of autonomous agents with a view to assessing their effects on the system as a whole. In traffic research, an agent-based model is proposed to support dynamic activity-travel scheduling. Given the agent model, agents can adapt their behaviors and make decisions in a complex and dynamic environment by applying learning techniques. Agents can communicate and interact with other agents so that agents are not restricted but equally treated in the system. Ma et al (2012) illustrated that agents can process different sorts of information and can schedule and predict which activities are conducted when, where, for how long, with whom, and the transport mode involved. Under the condition, activities and travels are generated by means of negotiation.

Learning process can be simulated in agent-based modeling. Learning models are introduced to improve the accuracy of drivers’ decision making. In general, the historical travel
time estimated by an agent, reflects his/her accumulated experience acquired through repeated trials on previous days.

Horowitz (1984) came up with a method by assigning the weights, which indicate the importance of the information, to different travel experiences in the past. The historical information is mainly the variability of travel time. In general, the estimation of travel time $T_{i,j,k}$ could be expressed as:

$$T_{i,j,k}^* = \sum_{l=t_0}^{j} w_{i,l} T_{i,l,k}$$

Where,

$l$ = index for days from which drivers obtain their travel experience, $l = [t_0, j]$.

$w_{i,l}$ : a set of nonnegative weights attached to the travel time of each day,

$j-t_0$ is the learning window, with $\sum_{l=t_0}^{j} w_{i,l} = 1$. The relative magnitudes of the weights indicate the relative importance of different travel experiences in the past.

Agent based modeling is a useful tool to learn the heterogeneity of drivers’ behavior. Chong et al. (2011) compared two microscopic driver behavior simulation methodologies, Gazis-Herman-Rothery (GHR) car-following model and a proposed agent-based neural network model. They conclude that the agent-based model has been proved to be a successful way of modeling driver behavior in car following. As neural agent is able to learn from different drivers’ reactions under different traffic environments, it is recommended to study the heterogeneity of actions when a driver faces various stimuli that current car-following models cannot handle.

Zhu et al. (2007) developed an Agent-based Route Choice (ARC) model to track choices of each individual decision-maker in a road network over time. They map individual choices into macroscopic flow pattern. The advantage of ARC relies on its ability to track evolutionary
decisions made by heterogeneous drivers, particularly on networks with differentiated levels of service provided by tolls. In the paper, the parameters describe how drivers perceive the available information and make route choice decisions. Upon estimation and calibration of those parameters, a fully behaviorally-based traffic assignment model can be established.

Jia et al. (2013) came up with an agent-based traffic assignment framework which will enable transportation engineers and planners to evaluate pricing strategies and support their decision-making process by incorporating driver’s heterogeneity, route choice and departure time shift model, and a Kalman Filter learning model. In drivers’ VOT heterogeneity estimation, the authors combine income and the trip purpose distributions and randomly assign each agent a vehicle type, income and trip purpose. A learning model is required in this agent-based framework. Independent learning for each individual agent is highly recommended. The gain factor attached to the agents’ historical travelling experience should vary by each individual agent and updated through day-to-day simulation.

Agent based modeling has already been applied in studying the congestion pricing. Zheng et al. (2012) combined a macroscopic modeling of traffic congestion in urban networks with an agent-based simulator to study congestion pricing schemes. In this paper, they first demonstrate by case studies in Zurich urban road network, that the output of a multi-agent based simulator is consistent with the physics of traffic flow dynamics, as expressed by a macroscopic fundamental diagram (MFD). They then apply a dynamic cordon-based congestion pricing scheme, in which tolls are controlled by an MFD, and investigate the effectiveness of the proposed pricing scheme. In this work, they introduce a new method to develop cordon-based congestion pricing scheme.
2.5 Summary

The literature review shows that it is important to include the travel time reliability into driver’s utility function in order to get a more practical simulation of driver’s route choice behavior. Besides, the agent-based modeling is capable of modeling individual drivers’ utility function based on his/her own value of time, value of reliability as well as interactions with other drivers in route decisions. Moreover, drivers can switch their departure time based on their previous traveling experience, resulting in a redistribution of vehicle flow on the network and dynamic congestion updating over time. As a result, this study develops an agent-based dynamic feedback-control toll pricing strategy that accounts for the trip purpose, travel time reliability, departure time shift and level of income of drivers such that the toll revenue is maximized while maintaining a minimum desired level of service on the managed lanes.
CHAPTER 3. METHODOLOGY

This chapter describes how to determine toll price for each time interval that considers driver’s characteristics and the network conditions. This chapter consists of four main components: external toll pricing module, modeling driver departure time choice, modeling drivers’ route choice and the feedback control mechanism. The first section introduces an external toll pricing module that was developed to obtain necessary simulation results from VISSIM so as to overcome limitation of VISSIM’s built-in Toll Pricing Calculation Model. The second section describes drivers’ learning process and how drivers switch the departure time based on their traveling experience. The third part explains the logit model that was used to model drivers route choice behavior considering drivers’ income level, toll price, travel time saving, travel time reliability and trip purpose. The fourth part explains the feedback mechanism that was used to estimate the optimal toll rate while maintain high level of service and maximum total toll revenue on managed lanes.

3.1 VISSIM Simulation

This study was conducted using a microscopic simulation platform. VISSIM, a simulation tool, was used to simulate different scenarios for the freeway network of I-95 from NW151 Street to SR112 Street and to evaluate the modified strategy. A total of 70 detectors were placed on each of the general purpose lanes and the managed lanes every half a mile which were used to collect real-time traffic data during the simulation. While different vehicle types can be simulated in VISSIM, this study was limited to passenger cars only since they comprise the majority of the traffic stream. Two input parameters were set before running the simulation on VISSIM. The free flow speed was assumed to be 70 mph. From VISSIM, the jam density was measured to be 240 passenger cars per mile per lane. From the database “Statewide Transportation Engineering
Warehouse for Archived Regional Data”, real traffic volume data on I-95 at NW151 can be retrieved.

### 3.2 External Toll Pricing Module

An external toll pricing module was developed to obtain the necessary simulation results, such as toll rate for each time interval, speeds on managed lanes and general purpose lanes, travel time saving, travel time reliability, number of vehicles entering or exiting and toll revenue. Visual Basic for Applications (VBA) programming language integrates with VISSIM through the VISSIM_COM interface. The toll price is set to be updated every 3 minutes in the external module based on traffic conditions detected from previous time intervals. The flow chart of dynamic pricing calculation is shown in Figure 2.

![Flow Chart of Dynamic Pricing Calculation](image)

**Figure 2 Flow Chart of Dynamic Pricing Calculation**

Before each simulation, the vehicle input in VISSIM has to be set up, as shown in Figure 3. For the 1st day, traffic demand data for Wednesday and Thursday in June and July 2011 were
extracted from database “Statewide Transportation Engineering Warehouse for Archived Regional Data” (STEWARD) in 15-minute intervals. The dynamic traffic demand for the following simulation days is updated based on the number of drivers choosing to shift their departure time. In the first time interval, VBA module starts and calls for VISSIM running and exporting the parameters to the external module. The new toll price can be calculated by VBA toll module in the next time interval. Then, VISSIM simulation receives the result of flow ratio of vehicles on the managed lanes and general purpose lanes from VBA route decision model and use them as input parameters for the next simulation step. The simulation results for one day from 5:00 AM to 7:30 AM are generated at the end of each simulation run.

![Vehicle Input in VISSIM](image)

**Figure 3 Vehicle input in VISSIM**

After one day simulation, relevant travel information and knowledge for each drivers update which influence drivers’ departure time choice and route choice for the next day travel. The travel time reliability, delay, number of vehicle changing departure time are estimated. The travel time reliability in each time interval is based on the travel times experienced in the previous simulation days for the same time interval. For the first simulation day, the travel time reliability
is assumed to be zero. The traffic data generated from VISSIM is exported to a spreadsheet in real-time to calculate the travel time reliability for use in the route choice process.

In this study, the simulation experiment which applies agent-based modeling has 15 simulation runs, representing 15 weekdays. The learning framework is shown in Figure 4. For each day, drivers learn traffic conditions, accumulate relevant knowledge of toll price for each time interval, travel time saving and difference of travel time reliability on general purpose lanes and managed lanes, adjust their departure time choice based on search begin rules, search rules and decision rules, and make their subjective decision of route choice. The drivers’ behavior will influence traffic conditions which make drivers experience different traveling and update the knowledge day by day.

Figure 4 Framework of Learning Progress
3.3 Modeling Drivers’ Departure Time Choice Behavior

The individual-level departure time choice model can be applied in real life application and integrated with an agent-based modeling. The heterogeneous drivers accumulate relevant knowledge on time-dependent travel conditions corresponding to different departure times, form subjective evaluation, search for alternative departure times and adjust their behaviors. Drivers can switch their departure time based on information and experience, resulting in a redistribution of vehicle flow on the network and toll rate updated over time.

(1) Search begin rules

When the utility for driver \( i \) at time interval \( t \) for the current day \( d \) is less than the average utility of all previous days, a portion of these drivers begin to search new departure time. This portion can be constant through days or decline every day. Haselkorn et al. (1991) indicate that 15.9% of drivers are willing to change their departure time, route and mode. Abdel-Aty et al. (1993) found that around 60% of drivers never change their departure time even though they listen to the traffic report. In this study, the portion of drivers who would like to search for new departure time is assumed to be 50% of drivers with disutility. The total number of drivers \( i' \) who choose to search new departure time with trip purpose \( k \) at time interval \( t \) on day \( d \) can be estimated as Eq. (2). \( N'(t) \) is the number of drivers with disutility at time interval \( t \). \( q(j, k) \) is the percentage of drivers in income category \( j \) and trip purpose category \( k \).

\[
U(i, t, d) < \frac{1}{d} [U(i, t, 1), U(i, t, 2), ..., U(i, t, d)] \quad (1)
\]

\[
N(i', t, d) = 50\% \times N'(t) \times q(j, k) \quad (2)
\]

(2) Search rules

\[
N(i', t, d) = 50\% \times N'(t) \times q(j, k)
\]
People tend to avoid delay no matter what trip purpose they have. They search for alternative departure time based on their schedule delay early (SDE), schedule delay late (SDL) and all previous traveling experience. If drivers’ prefer arrival time (PAT) is greater than their actual arrival time (AT), the drivers experience early arrival. Otherwise, drivers suffer from late arrival.

\[
SDE = \max(0, \text{PAT} - \text{AT})
\]

\[
\text{SDL} = \max(0, \text{AT} - \text{PAT})
\]

In the literature, some studies conducted revealed preference survey and stated preference survey to collect drivers’ real travel information, such as travel time, travel cost, PAT, AT, departure time, etc. (Zhang & Xiong, 2012); (Paleti, et al., 2014). In this study, because of the lack of actual survey data on the studied section of I95, search rules and departure time alternatives in VISSIM simulation are assumed to be only related to the delay, which can be estimated as the ratio of the difference between travel time of each time interval \(t\) and average travel time of the whole simulation period to the average travel time of the whole simulation period from 5:00 AM to 7:30 AM, which cover the peaks hour and off peak hours. It is assumed that all drivers, who arrive the starting point (NW151) at the same time interval \(t\) on the same day \(d\), will maintain the same travel time \(T(d,t)\) through study area on I95. Under the condition, delay can be negative or positive. Negative delay means drivers would keep their departure time or postpone it. Positive delay indicate drivers would choose an earlier departure time.

\[
\text{Delay}(d,t) = \frac{T(d,t) - \bar{T}(d,T)}{\bar{T}(d,T)} = \frac{T(d,t) - \frac{1}{5}(T(d,1) + T(d,2) + \cdots + T(d,T))}{\frac{1}{5}(T(d,1) + T(d,2) + \cdots + T(d,T))}
\]

In this study, the departure time change \(\Delta DT(i)\) is estimated below. \(\bar{T}(D, t)\) is the average travel time at time interval \(t\) of the previous days. This variable is different from \(\bar{T}(d, T)\), which is
the average travel time of the whole simulation period on day $d$. $\theta(i)$ is user specified parameter, it can be constant or varied among driver groups. If drivers get delayed by 5 minutes, they may not want to just departure ahead by 5 minutes, but ahead by 10 minutes or more. The parameters $\theta(i) \geq 1$ is to capture this nature. Besides, people may not want to change their departure time when the actual delayed time is so small, for example less than 2 minutes. As a result, the departure time change $\Delta DT(i)$ will be zero if $Delay(d, t) \times \bar{T}(D, t) < 2\text{min}$. 

$$
\text{If } Delay(d, t) > 0, \Delta DT(i) = \begin{cases} 
0, & \text{if } Delay(d, t) \times \bar{T}(D, t) < 2 \text{min} \\
-\theta(i) \times Delay(d, t) \times \bar{T}(D, t) & \text{otherwise}
\end{cases}
$$

$$
\text{If } Delay(d, t) < 0, \Delta DT(i) = \begin{cases} 
0, & \text{if } Delay(d, t) \times \bar{T}(D, t) < 2 \text{min} \\
\frac{1}{\theta(i)} \times Delay(d, t) \times \bar{T}(D, t) & \text{otherwise}
\end{cases}
$$

$$
\theta(i) = \frac{VOT(k)}{\text{Mean hourly wage}}
$$

In this study, the parameter $\theta(i)$ is estimated in Eq. (6). $VOT(k)$ is value of time for drivers with trip purpose $k$. The value has been specified in Table 1. When $Delay(d, t) > 0$, drivers are willing to keep their departure time or choose an earlier departure time. Drivers with urgent trip purpose would make larger departure time change than drivers with work and leisure trip purpose. When $Delay(d, t) < 0$, drivers would keep their departure time or postpone it. Under the condition, drivers with urgent trip purpose would not like to change the departure time as much as drivers with work or leisure trip purpose because late arrival will bring more serious penalty to them.

(3) Decision rules

If the drivers experience schedule delay early and decide to postpone departure time in the next travel, their decision rules are the same as search rules. They will choose the departure time
alternatives based on their delay. But if a driver decide he may departure early, then the estimation on whether he will actually change departure time will be made based on the decision rules below.

If the product of departure time change for driver \(i'\) at time interval \(t\) on day \(d\) and value of time for driver \(i'\) is greater than the toll price at time interval \(t\) on day \(d\), the driver will adopt the new departure time. Otherwise the driver will maintain their previous departure time. Although these drivers’ utility is less than the average utility.

\[
\Delta DT(i', t, d) \times VOT(i') > c(d, t) \tag{9}
\]

### 3.4 Travelers’ Route Choice Behavior

According to Cheng and Ishak (2013), the logit model could be applied to model drivers’ decision making process. The probability of choosing the managed lanes by a particular driver \(i\) with trip purpose \(k\) at time interval \(t\), \(P(i, k, t)\), can be defined as:

\[
P(i, k, t) = \frac{e^{U_m(i, k, t)}}{e^{U_m(i, k, t)} + e^{U_g(i, k, t)}} = \frac{1}{1 + e^{-\Delta U(i, k, t)}} \tag{10}
\]

Where \(U_m(i, k, t)\) is the utility of choosing the managed lanes for driver \(i\) with trip purpose \(k\) at time interval \(t\), \(U_g(i, k, t)\) is the utility of choosing the general purpose lanes for driver \(i\) with trip purpose \(k\) at time interval \(t\), and \(\Delta U(i, k, t) = U_m(i, k, t) - U_g(i, k, t)\) is the difference between the utility of choosing the managed lanes and the general purpose lanes for a particular driver \(i\) with trip purpose \(k\) at time interval \(t\).

Considering travel time reliability into the drivers’ utility function, the Logit model could be expressed as:

\[
P(i, j, t) = \frac{1}{1 + e^{\alpha C(t) - \beta(k)\Delta T(t) - \delta(i, k) \Delta TR(k)}} \tag{11}
\]
Where $\alpha(i)$ is the rate of change of utility for a particular driver $i$ per unit change of toll rate, $c(t)$ is the toll rate on the managed lanes at time interval $t$, $\beta(i, k)$ is the rate of change of utility for a particular driver $i$ with trip purpose $k$ per unit change of travel time saving, $\Delta T(t)$ is the travel time saving between the managed lanes and the general purpose lanes. $\partial(i, k)$ is the rate of change of utility for a particular driver $i$ with trip purpose $k$ per unit change of travel time reliability, and $\Delta TR(k)$ is the difference of travel time reliability on the managed lanes and general purpose lanes for drivers’ trip purpose $k$. The perceived travel time reliability varies among drivers with distinct trip purpose, therefore, measurements of travel time reliability are different based on the trip purposes. This will be explained in details later. It should be noted that the values of $\alpha(i)$ and $\beta(i, k)$ are user specific. In this research, the value of $\alpha(i)$ is assumed to be 1 while $\beta(i, k)$ and $\partial(i, k)$ are determined based on drivers’ mean hourly income as well as trip purpose.

### 3.4.1 Income level groups

In this study, income levels were grouped into three main categories: high-income ($80,000 or above), medium-income ($40,000-$79,999), and low-income (below $40,000). The population percentage and mean hourly income are calculated based on U.S. Census Bureau, Current Population Survey (2010). Accordingly, the percentage for each group are 10%, 24% and 66% and the mean hourly income for each group are $49.80, $28.80 and $9.60 respectively (Cheng & Ishak, 2013).

### 3.4.2 Trip purpose

The trip purpose considered in this study was limited to: (1) work trip, (2) urgent travel during emergency situations, and (3) leisure trip. In this study, 100% of mean hourly wage is assumed as the VOT for trip purpose one and 50% of mean hourly wage for trip purpose three.
according to Lake and Ferreira (2002). Since trip purpose two is a special situation wherein people would usually place a very high value on VOT, a 150% of mean hourly wage to approximate the VOT is used. Besides, Concas and Kolpakov (2009) established a numerical relationship between VOR and VOT. As suggested in their research, VOR can be estimated as follows: for trip purpose one, VOR=100%×VOT; for trip purpose two, VOR=300%×VOT, and trip purpose three, VOR=80%×VOT. This is shown in Table 1.

Table 1 VOT and VOR under different trip purposes

<table>
<thead>
<tr>
<th>Trip Purpose</th>
<th>Value of Time (VOT)</th>
<th>Value of Reliability (VOR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Work Trip</td>
<td>100% of mean hourly wage</td>
<td>VOR=100%×VOT</td>
</tr>
<tr>
<td>2. Urgent Trip</td>
<td>150% of mean hourly wage</td>
<td>VOR=300%×VOT</td>
</tr>
<tr>
<td>3. Leisure Trip</td>
<td>50% of mean hourly wage</td>
<td>VOR=80%×VOT</td>
</tr>
</tbody>
</table>

In this study, the measurement of drivers’ travel time reliability is calculated for each trip purpose as follows:

(1) Work travel: Travel time reliability is estimated based on $\lambda$, which is defined by:

$$\lambda = a\lambda_1 + b\lambda_2$$

(12)

Where $\lambda_1 = P90 - P50$, $\lambda_2 = P50 - P10$, and $Px$ is the $x^{th}$ percentile of the travel time distribution. $\lambda_1$ is the difference of travel time between the 90th percentile and the 50th percentile. Large values of $\lambda_1$ mean that the travel time distribution is skewed to the right, which implies more trips incur a longer travel time, and hence, travel time is not reliable under such condition. The constant $a$ takes on a negative value, which indicates the degree of risk aversion. Similarly, $\lambda_2$ is the difference of travel time between the 50th percentile and the 10th percentiles. Large values of $\lambda_2$ mean that the travel time distribution is skewed to the left, which implies more trips incur a shorter travel time. The constant $b$ takes on a positive value, which indicates the degree of risk
preference. Longer travel time enhances drivers’ vigilance, making them reschedule their trip in a more conservative way. Under normal conditions, people care more about the longer travel time than the shorter travel time. Thus, the absolute value of \(a\) is larger than that of \(b\). In this case, \(a\) is assigned a value of 2 and \(b\) a value of -1. And hence, the higher the value of \(\lambda\), the lower the travel time reliability.

(2) Urgent travel: This type focuses on the length of delay of the longest trips. The travel time reliability depends on the difference of maximum travel time and mean travel time.

(3) Leisure travel: The travel time reliability will be estimated by standard deviation of the travel time.

3.5. Feedback Control Mechanism

Feedback control mechanism can be applied to estimate the optimal toll rate while maintaining high level of service that keeps speed higher than 45 mph and maximizing the total toll revenue on managed lanes. This paper incorporated travel time reliability, trip purpose into the agent-based feedback control mechanism to study the toll revenue and level of service lane on I95. The difference in travel time and travel time reliability on the managed lanes and general purpose lanes is considered in this mechanism in order to address the interaction between managed lanes and general purpose lanes. The traffic condition was classified into two cases and a specific mechanism was developed for each case to maximize the toll revenue. For case one, if \(S_m(d, t)\) is slightly higher than 45 mph, the managed lanes can accommodate more traffic until speed drops to 45 mph. The toll rate could be either decreased or increased to maximize the toll revenue. If \(S_m(d, t) \gg 45\text{mph}\), the toll rate should be decreased to attract more drivers to use the managed lanes. For case two, \(S_m(d, t) \leq 45\text{mph}\), the toll rate should be increased for a relatively large
amount to discourage drivers from using the managed lanes that keeps the minimum level of service.

\[ c(d, t + 1) = c(d, t) + \Delta c^*(d, t) \]

\[ = c(d, t) + \begin{cases} 
\gamma_1^*(d, t) \Delta T(d, t) + \delta_1^*(d, t) \Delta TR(d, k) & S_m(d, t) > 45 \text{ mph} \\
\gamma_2^*(d, t) \Delta T(d, t) + \delta_2^*(d, t) \Delta TR(d, k) & S_m(d, t) \leq 45 \text{ mph} 
\end{cases} \]

(13)

Where \( c(d, t + 1) \) is the calculated optimal toll rate for the managed lanes at day \( d \) during time interval \( t + 1 \). This new rate is obtained by adjusting the current toll rate \( c(d, t) \), by \( \Delta c^*(d, t) \), where \( \Delta c^*(d, t) \) is the optimal change that maximizes the total toll revenue \( R(d, t + 1) \) of day \( d \) at time \( t + 1 \) while keeping the average speed on the managed lanes greater than 45 mph. \( \gamma_1^*(d, t) \) and \( \gamma_2^*(d, t) \) are parameters that account for the change in toll rate per unit of travel time saving, \( \Delta T(d, t) \), of day \( d \) at time interval \( t \). \( \delta_1^*(d, t) \) and \( \delta_2^*(d, t) \) are parameters that account for the change in toll rate per unit difference in travel time reliability, \( \Delta TR(d, t) \), of day at time interval \( t \). In order to obtain the optimal change in toll rate at time \( t \), \( \Delta c^*(d, t) \) is estimated as follows:

\[ \Delta c^*(d, t) = \text{Arg max}\{\hat{r}(d, t + 1), \Delta c(d, t)\} \]  

(14)

\[ \hat{r}(d, t + 1) = \hat{c}(d, t + 1) \times \hat{N}(d, t + 1) \times \sum_{j=1}^{3} \sum_{k=1}^{3} [\hat{P}(d, j, k, t + 1) \times q(j, k)] \]  

(15)

Where \( \hat{r}(d, t + 1) \) is the estimated total toll revenue of day \( d \) at time interval \( t + 1 \), \( \hat{c}(d, t + 1) \) is the estimated feasible toll rate of day \( d \) at time interval \( t + 1 \), and \( \hat{N}(d, t + 1) \) is the expected number of vehicles to choose between the general purpose lanes and the managed lanes of day \( d \) during time interval \( t + 1 \). For simplification, \( N(t) \) is used as an approximation for \( \hat{N}(t + 1) \) when estimating the total toll revenue. This is because the time interval used is relatively short (e.g. 3 minutes), and therefore, the changes in traffic demand from one time interval to the
next may not be significant. \( q(j,k) \) is the percentage of drivers in income category \( j \) and trip purpose category \( k \) and \( \hat{P}(d,j,k,t+1) \) is the estimated probability of choosing the managed lanes by drivers of income category \( j \) with trip purpose \( k \) of day \( d \) at time interval \( t + 1 \). This is estimated from the expected utility of the general purpose lanes and the managed lanes as follows:

\[
\hat{P}(d,j,k,t+1) = \frac{1}{1 + e^{\alpha(j) + \phi(\Delta T(d,t+1)) - \theta_{jk}(\Delta T(d,t+1)) - \partial_{jk}(\Delta T_R(d,t+1))}}
\]

(16)

\[
\Delta T(d,t+1) = \frac{1}{d} (\Delta T(d,t+1), \Delta T(d-1,t+1), ..., \Delta T(1,t+1))
\]

(17)

Where \( \Delta T(d,t+1) \) can be estimated as the average of all travel time savings at time interval \( t + 1 \) over the previous days that the drivers experienced. \( \Delta T_R(t+1) \) is the estimated difference of travel time reliability between managed lane and general purpose lane for time interval \( t + 1 \), which is calculated from traffic data of the previous days that the drivers experienced. The detailed measurements of travel time reliability have been previously explained.

There are three assumptions for this approximation. First, it is assumed that the number of drivers choosing between the managed lanes and the general purpose lanes does not change significantly within the relatively short three-minute time interval. Second, drivers can estimate traffic conditions based on their past travel experience which makes them learn from their past commuting experience. Third, an individual driver responds instantly to the new toll rate when they see the posted toll rate. For instance, an individual driver can know the average travel time saving as well as the difference of travel time reliability between the managed lanes and the general purpose lanes based on their previous travels. The cumulative effect of drivers’ response to the new toll rate may however take as long as 15 minutes to be realized on the managed lanes. \( \Delta T(d,t+1) \) is approximated with \( \Delta T(t) \). \( \hat{P}(d,j,k,t+1) \) can be rewritten as:

\[
\hat{P}(d,j,k,t+1) = \frac{1}{1 + e^{\alpha(j) + \phi(\Delta T(d,t+1)) - (\Delta T(d,t+1)) - \partial_{jk}(\Delta T_R(d,t+1))}}
\]

(18)
And \( \hat{r}(t+1) \) can be rewritten as:

\[
\hat{r}(d, t + 1) = \hat{c}(d, t + 1) \cdot N(d, t) \cdot \\
\sum_{j=1}^{3} \sum_{k=1}^{3} \frac{1}{1 + e^{a(j) \cdot \hat{c}(d, t + 1) - \beta(j) \cdot \Delta T(d, t + 1) - \delta(k) \cdot \Delta T_R(d, t + 1)}} \cdot q(j, k)
\]  

(19)

In addition to maximizing the toll revenue, another objective of this dynamic toll pricing strategy is to maintain a minimum level of service on the managed lanes by ensuring that the average speed on the managed lanes is at least 45 mph. Consequently, the estimated feasible toll rate \( \hat{c}(d, t + 1) \) should result in attracting the right proportion of vehicles on the managed lanes such that the estimated average speed \( \hat{S}_m(d, t + 1) \) for time interval \( t + 1 \) is maintained at or above 45 mph. The estimated average speed \( \hat{S}_m(d, t + 1) \) can be calculated from:

\[
\hat{S}_m(d, t + 1) = S_f \left(1 - \frac{\hat{k}_m(d, t + 1)}{k^*_m}\right) > 45
\]

(20)

Where \( S_f \) and \( k^*_m \) are characteristics of the freeway section and defined as: \( S_f \) is the free-flow speed on the managed lanes, \( \hat{k}_m(d, t + 1) \) is the estimated density per mile per lane for time interval \( t + 1 \), and \( k^*_m \) is the jam density for the managed lanes. \( \hat{k}_m(d, t + 1) \) can be obtained from:

\[
\hat{k}_m(d, t + 1) = \frac{\Delta n_{in}(d, t + 1) + N_m(d, t) - \Delta N_{out}(d, t)}{L \cdot n}
\]

(21)

Where \( L \) denotes the length of the managed lanes and \( n \) represents the number of managed lanes, which is equal to two for this study segment. \( \Delta n_{in}(d, t + 1) \) denotes the expected number of vehicles that enters the managed lanes at time interval \( t + 1 \), \( N_m(d, t) \) represents the number of vehicles that remained on the managed lanes at time interval \( t \), \( \Delta N_{out}(d, t) \) is the number of vehicles that exits the managed lanes at time interval \( t \). \( \Delta N_{out}(d, t) \) is also used as an approximation for the number of vehicles expected to exit the managed lanes at time interval \( t + 1 \). \( \Delta n_{in}(d, t + 1) \) can be calculated as follows:
\[
\Delta \hat{n}_{in}(d, t + 1) = N(d, t) * \sum_{j=1}^{3} \sum_{k=1}^{3} \left[ \hat{P}(d, j, k, t + 1) * q(j, k) \right]
\]

Where \( N(d, t) \) is the number of vehicles making a decision to choose between the managed lanes and the general purpose lanes at time interval \( t \). This is also used to approximate the expected number of vehicles to make a choice between the managed lanes and the general purpose lanes at time interval \( t + 1 \). Therefore,

\[
\Delta \hat{n}_{in}(d, t + 1) =
\]

\[
N(d, t) * \sum_{j=1}^{3} \sum_{k=1}^{3} \left[ \frac{1}{1 + e^{a(j) \cdot \hat{c}(d, t + 1) - \beta(j) \cdot \Delta \hat{T}(d, t + 1) - \vartheta(k) \cdot \Delta \hat{T}_{R}(d, t + 1)}} * q(j, k) \right]
\]
To illustrate how the modified toll pricing methodology works, two numerical examples are given in this section, assuming three income categories and three trip purpose categories of drivers. The first example illustrates how the departure time choice modeling works. The second example illustrates how the drivers’ route choice behavior influences the traffic condition and influences the toll pricing strategies.

### 4.1 Numerical Example on Departure time Choice Model

To illustrate how the departure time choice modeling works, a numerical example is given in this section. Assuming that drivers accumulate traveling experience in the first 10. They generate an evaluation on traffic condition and can search their preferred departure time on the 11th day based on their continuous accumulation of knowledge.

The average utility for drivers from each of 9 groups departing on 7:00 AM of their first 10 days for managed lanes ($\bar{U}_m$) and general purposed lanes ($\bar{U}_g$) are summarized in the Table 2. In the 11th day, the utility of drivers who choose departure time on 7:00 AM on managed lanes and general purposed lanes is summarized in $U_m$ and $U_g$. The traffic demand on 7:00 AM $N(t) = 7300$ veh/hr. The number of vehicles with dissatisfied utility $N'(t) = 4000$ veh/hr. The probability of choosing the managed at 7:00 AM $P(t)$ is 0.307. The toll price on 7:00 AM $c(t) = \$2$.

The length of the study area $L = 6.5$ miles. It is assumed that all drivers arriving at the starting point of study area (NW151) on 7:00 AM on the 11th day maintain the same travel time $T_{(d,t)} = 5.80$ min on managed lanes and $9.69$ min on general purposed lanes through 6.5 miles. The average travel time from 5:00 AM to 7:30 AM on the 11th day is 5.67 min on managed lanes and 7.08 min on general purposed lanes.
### Table 2 Combined Distributions of Drivers’ Income and Trip Purpose

<table>
<thead>
<tr>
<th>Driver Group</th>
<th>Work Trips</th>
<th>Income Group</th>
<th>Percentage</th>
<th>VOT ($/min)</th>
<th>$U_m$</th>
<th>$U_g$</th>
<th>Δ$DT$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Trip Purpose</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Work Trips</td>
<td>0.065</td>
<td>0.156</td>
<td>0.429</td>
<td>0.01</td>
<td>0.024</td>
<td>0.066</td>
<td>0.048</td>
</tr>
<tr>
<td>Urgent Trips</td>
<td>0.01</td>
<td>0.024</td>
<td>0.066</td>
<td>0.048</td>
<td>0.024</td>
<td>0.066</td>
<td>0.13</td>
</tr>
<tr>
<td>Leisure Trips</td>
<td>0.01</td>
<td>0.024</td>
<td>0.066</td>
<td>0.048</td>
<td>0.024</td>
<td>0.066</td>
<td>0.13</td>
</tr>
<tr>
<td>Income Group</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Percentage</td>
<td>0.065</td>
<td>0.156</td>
<td>0.429</td>
<td>0.01</td>
<td>0.024</td>
<td>0.066</td>
<td>0.048</td>
</tr>
<tr>
<td>VOT ($/min)</td>
<td>0.01</td>
<td>0.024</td>
<td>0.066</td>
<td>0.048</td>
<td>0.024</td>
<td>0.066</td>
<td>0.13</td>
</tr>
<tr>
<td>$U_m$</td>
<td>-6.37</td>
<td>-4.36</td>
<td>-2.52</td>
<td>-8.79</td>
<td>-5.74</td>
<td>-2.98</td>
<td>-1.83</td>
</tr>
<tr>
<td>$U_g$</td>
<td>-9.02</td>
<td>-5.89</td>
<td>-3.03</td>
<td>-12.77</td>
<td>-8.03</td>
<td>-3.74</td>
<td>-1.95</td>
</tr>
<tr>
<td>$U_m$</td>
<td>-6.89</td>
<td>-4.86</td>
<td>-3.01</td>
<td>-9.33</td>
<td>-6.26</td>
<td>-3.47</td>
<td>-2.31</td>
</tr>
<tr>
<td>$U_g$</td>
<td>-10.12</td>
<td>-6.73</td>
<td>-3.63</td>
<td>-14.19</td>
<td>-9.06</td>
<td>-4.41</td>
<td>-2.47</td>
</tr>
</tbody>
</table>

1. For each of the 9 driver groups, if $(U_m < \overline{U}_m')$ or $(U_g < \overline{U}_g')$, drivers $i'$ under these conditions would like to search a new departure time. The total number of drivers $i'$ can be estimated by Eq. (2):

$$N(i', t, d) = 50\% \times N(t) \times q(j, k) = 0.5 \times 4000 \times 0.065 \times 0.307 + 0.5 \times 4000 \times 0.065 \times (1 - 0.307) + \cdots + 0.5 \times 4000 \times 0.13 \times 0.307 + 0.5 \times 4000 \times 0.13 \times (1 - 0.307) = 2000\text{ veh/hr}$$

(24)

2. According to the Eq. (5), the delay on 7:00 AM on 11th day on the managed lanes and general purposed lanes can be estimated as:

$$delay(d, t, m) = \frac{5.80 - 5.67}{5.67} \times 100\% = 2.24\%$$

(25)

$$delay(d, t, g) = \frac{9.69 - 7.08}{7.08} \times 100\% = 36.73\%$$

(26)

Based on Table 2, drivers with 2.24% delay on managed lanes will keep their departure time unchanged for the trip next day. Drivers with 36.73% delay on general purposed lanes will choose a departure time chance based on Eq. (6) and Eq. (8):

1) Work trip: $\theta(1)=1$

$$\Delta DT(1) = -1 \times 0.3673 \times 7.08 = -2.6\text{ min}$$

(27)
2) Urgent trip: $\theta(2)=1.5$

$$\Delta DT(2) = -1.5 \times 0.3673 \times 7.08 = -3.9 \text{ min} \quad (28)$$

3) Leisure trip: $\theta(3)=0.5$

$$\Delta DT(3) = -0.5 \times 0.3673 \times 7.08 = -1.3 \text{ min} \quad (29)$$

3. Whether driver $i'$ will finally adopt the new departure time is based on the decision rule estimated in Eq. (9):

$$\Delta DT(i', t, d) \times VOT(i') = \{2.16, 1.25, 0.42, 4.88, 2.81, 0.94, 0.05, 0.026, 0.13\} \quad (30)$$

Comparing the results from Eq. (30) to the toll price $c(d, t)=$$2 on 7:00 AM on the 11th day, it is found that drivers $i'$ from driver group 1, 4, 5 will finally change to new departure time in the 12th day. The total number of these drivers $i'$ can be estimated as:

$$N(i', t, d) = 50\% \times N'(t) \times q(j, k) = 0.5 \times 4000 \times 0.065 \times (1 - 0.307) + 0.5 \times 4000 \times 0.01 \times (1 - 0.307) + 0.5 \times 4000 \times 0.024 \times (1 - 0.307) = 138 \text{ veh/hr} \quad (31)$$

### 4.2 Numerical Example on Toll Pricing Strategies

To illustrate how the modified toll pricing methodology works, a numerical example is given in this section, assuming three income categories of drivers. Assuming $\alpha$ is equal to 1 for all driver groups, the equivalent values of time (VOT) and reliability (VOR) for certain percentages of drivers’ mean hourly income is summarized in Table 3. The income distribution of drivers is assumed as $q_{in} = \{0.1, 0.24, 0.66\}$ for high, middle and low income. The distribution of drivers’ trip purposes is assumed as $q_{tp} = \{0.65, 0.10, 0.25\}$. The combined distribution of drivers’ trip purposes and incomes is summarized in Table 3.
Table 3 Combined Distributions of Drivers’ Income and Trip Purpose

<table>
<thead>
<tr>
<th>Driver Group</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trip Purpose</td>
<td>Work Trips</td>
<td>Urgent Trips</td>
<td>Leisure Trips</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income Group</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Percentage</td>
<td>0.065</td>
<td>0.156</td>
<td>0.429</td>
<td>0.01</td>
<td>0.024</td>
<td>0.066</td>
<td>0.048</td>
<td>0.069</td>
<td>0.13</td>
</tr>
<tr>
<td>VOT ($/min)</td>
<td>0.83</td>
<td>0.48</td>
<td>0.16</td>
<td>1.25</td>
<td>0.72</td>
<td>0.24</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>VOR ($/min)</td>
<td>0.83</td>
<td>0.48</td>
<td>0.16</td>
<td>3.74</td>
<td>2.16</td>
<td>0.72</td>
<td>0.03</td>
<td>0.02</td>
<td>0.01</td>
</tr>
</tbody>
</table>

For time interval $t$, the following conditions are assumed: the current toll rate $c(t) = $2, the number of vehicles choosing between managed lanes and general purpose lanes $N(t) = 1200$, and the travel time saving $\Delta T(t) = 5$ min. For work based trip purpose, the travel time reliability is assumed to be 6 min for general purpose lanes and 4 min for managed lanes; for trip urgent travel trip purpose, the travel time reliability is assumed to be 5 min for the managed lanes and 10 min for the general purpose lanes; and for leisure travel trip purpose, travel time reliability is assumed as 2 min for the managed lanes and 5 min for the general purposed lanes. It is also assumed that the number of vehicles occupying the managed lanes $N_m(t) = 500$, and the number of vehicles leaving the managed lanes $\Delta N_{out}(t) = 50$. Given the length of the managed lanes $L=6.5$ miles and the jam density $k_m(t) = 200$ pcpmpl, the algorithm to calculate the toll rate for the next time interval $t + 1$ is executed for case one and case two as follows:

1. Let $P_{max}=0.99$ and $P_{min}=0.01$, which denote the maximum and minimum probability of choosing the managed lanes by any particular driver.
2. From Eq. (16), \( \hat{c}(d, t + 1) \), the toll rate corresponding to \( \hat{P}(d, j, k, t + 1) \), can be determined from:

\[
\hat{c}(d, t + 1) = \frac{\ln \left( \frac{1 - \hat{P}(d, j, k, t + 1)}{\hat{P}(d, j, k, t + 1)} \right) + \beta(j) \cdot \Delta \hat{t}(d, t)}{\alpha(j)} \forall j = 1, 2, 3 \quad \forall k = 1, 2
\]  

(32)

Substituting \( P_{\text{max}} \) and \( P_{\text{min}} \) into Eq. (32), the upper and lower bounds of the toll rate can be determined from Eq. (33). Subsequently, the optimum toll rate to maximize the total revenue can be searched in the range between \( \hat{c}_{\text{min}}(d, t + 1) \) and \( \hat{c}_{\text{max}}(d, t + 1) \).

\[
\hat{c}_{\text{min}}(d, t + 1) = \frac{\ln \left( \frac{1 - \hat{P}_{\text{max}}(d, j, k, t + 1)}{\hat{P}_{\text{max}}(d, j, k, t + 1)} \right) + \beta(j) \cdot \Delta \hat{t}(d, t)}{\alpha(j)}
\]

\[
= \frac{-4.6 + \beta(j) \cdot \Delta \hat{t}(d, t)}{\alpha(j)} \quad \forall j = 1, 2, 3 \quad \forall k = 1, 2
\]  

(33)

\[
\hat{c}_{\text{max}}(d, t + 1) = \frac{\ln \left( \frac{1 - \hat{P}_{\text{min}}(d, j, k, t + 1)}{\hat{P}_{\text{min}}(d, j, k, t + 1)} \right) + \beta(j) \cdot \Delta \hat{t}(d, t)}{\alpha(j)}
\]

\[
= \frac{4.6 + \beta(j) \cdot \Delta \hat{t}(d, t)}{\alpha(j)} \quad \forall j = 1, 2, 3 \quad \forall k = 1, 2
\]  

(34)

3. From Eq. (13), the parameter \( \gamma(d, t) \) that accounts for the change in toll rate per unit of travel time saving can be estimated from

\[
\begin{align*}
\gamma_1(d, t) &= \frac{c(d, t + 1) - c(d, t)}{\Delta T(d, t)} & S_m(d, t) > 45 \text{mph} \\
\gamma_2(d, t) &= \frac{c(d, t + 1) - c(d, t)}{\Delta T(d, t)} & S_m(d, t) \leq 45 \text{mph}
\end{align*}
\]  

(35)

And therefore,
\[
\begin{align*}
Y_{1,\text{min}}(d, t) &= \frac{1}{\Delta T(d, t)} \left[ -4.6 + \beta(j) \cdot \Delta T(d, t) \right] - c(d, t) \\
Y_{1,\text{max}}(d, t) &= \frac{1}{\Delta T(d, t)} \left[ 4.6 + \beta(j) \cdot \Delta T(d, t) \right] - c(d, t)
\end{align*}
\]

\( S_{m}(d, t) > 45 \text{mph}, \forall d = 1, 2, 3 \) \hspace{1cm} (36)

\[
\begin{align*}
Y_{2,\text{min}}(d, t) &= \frac{1}{\Delta T(d, t)} \left[ -4.6 + \beta(j) \cdot \Delta T(d, t) \right] - c(d, t) \\
Y_{2,\text{max}}(d, t) &= \frac{1}{\Delta T(d, t)} \left[ 4.6 + \beta(j) \cdot \Delta T(d, t) \right] - c(d, t)
\end{align*}
\]

\( S_{m}(d, t) \leq 45 \text{mph}, \forall d = 1, 2, 3 \) \hspace{1cm} (37)

Substituting for \( \alpha = \{1,1,1,1,1,1,1,1\} \),
\[
\beta = \{0.83,0.48,0.16,1.245,0.72,0.24,0.04,0.02,0.01\},
\]
\[
\vartheta = \{0.83,0.48,0.16,3.74,2.16,0.72,0.03,0.02,0.01\}
\]
to Eq. (36) and Eq.(37),

\[
\begin{align*}
Y_{1,\text{min}}(t) &= \{-0.16, -0.65, -1.10, 3.67, 1.56, -0.36, -1.26, -1.29, -1.30\} \\
Y_{1,\text{max}}(t) &= \{1.68,1.19,0.74,5.51,3.40,1.48,0.58,0.55,0.54\}
\end{align*}
\]

\( S_{m}(d, t) > 45 \text{mph} \)

\[
\begin{align*}
Y_{2,\text{min}}(t) &= \{-0.16, -0.65, -1.10, 3.67, 1.56, -0.36, -1.26, -1.29, -1.30\} \\
Y_{2,\text{max}}(t) &= \{1.68,1.19,0.74,5.51,3.40,1.48,0.58,0.55,0.54\}
\end{align*}
\]

\( S_{m}(d, t) \leq 45 \text{mph} \)

Merging \( Y_{1,\text{min}}(t) \) and \( Y_{1,\text{max}}(t) \) into one set yields
\[
Y_{1,\text{min}}(t) \cup Y_{1,\text{max}}(t)
\]
\[
= \{-1.30, -1.29, -1.26, -1.10, -0.65, -0.36, -0.16, 0.54, 0.55\} \\
= \{0.58,0.74,1.19,1.48,1.56,1.68,3.40,3.67,5.51\}
\]

Merging \( Y_{2,\text{min}}(t) \) and \( Y_{2,\text{max}}(t) \) into one set yields
\[
Y_{2,\text{min}}(t) \cup Y_{2,\text{max}}(t)
\]
\[
= \{-1.30, -1.29, -1.26, -1.10, -0.65, -0.36, -0.16, 0.54, 0.55\} \\
= \{0.58,0.74,1.19,1.48,1.56,1.68,3.40,3.67,5.51\}
\]

4. Find the upper and lower boundaries of the two union sets. For union set of \( Y_1 \), the upper boundary is set to be \( \max\{Y_{1,\text{min}}(t) \cup Y_{1,\text{max}}(t)\} = 5.51 \) and the lower boundary is set to \( \min\{Y_{1,\text{min}}(t) \cup Y_{1,\text{max}}(t)\} = -1.30 \). However the lower boundary of union set of \( Y_2 \) should be zero since for condition B the average speed on the managed lanes is less than or
equal to 45 mph. The toll rate should be increased to discourage more vehicles from selecting the managed lanes and to restore the level of service. The upper boundary of union set of \( \gamma_2 \) is determined by \( \max \{ \gamma_{2,min}(t) \cup \gamma_{2,max}(t) \} = 5.51. \)

5. In this example, the upper boundary (1.85) and lower boundary (-1.30 or zero) define the search region for the optimum value \( \gamma_1^*(t) \) or \( \gamma_2^*(t) \) that maximizes the toll revenue or maximizes the throughput on the managed lanes during the next time interval. The search region is thus divided into small intervals with an increment (e.g., 0.01) such that:

\[
\begin{cases}
\gamma_1(t) = \{-1.30, -1.29, ..., 5.50, 5.51\} & S_m(t) > 45mph \\
\gamma_2(t) = \{0, 0.01, 0.02, ..., 5.50, 5.51\} & S_m(t) \leq 45mph
\end{cases}
\]  

(38)

6. From Eq. (13), the set of estimated toll rates \( \hat{C}'(t+1) \) for time interval \( t+1 \) can be calculated from Eq. (39). The effect of parameter \( \delta(d,t) \) and the difference of travel time reliability \( \Delta TR(d,k) \) are not taken into consideration because the drivers’ response to the new toll rate may take some time to be observed on the managed lanes.

\[
\begin{cases}
\hat{C}'(t+1) = c(t) + \gamma_1(t) \cdot \Delta T(t) & S_m(t) > 45mph \\
\hat{C}'(t+1) = c(t) + \gamma_2(t) \cdot \Delta T(t) & S_m(t) \leq 45mph
\end{cases}
\]  

(39)

Substituting for \( \gamma_1(t) = \{-1.30, -1.29, ..., 5.50, 5.51\} \), \( \gamma_2(t) = \{0, 0.01, 0.02, ..., 5.50, 5.51\} \) in Eq.(38), \( C(t) = $2, \Delta T(t) = 2 min \) for trip purpose 1, \( \Delta T(t) = 5 min \) for trip purpose 2, \( \Delta T(t) = 3 min \) for trip purpose 3 into Eq.(39), the set of estimated toll rates is obtained:
\[
\begin{align*}
\{\hat{\mathcal{C}}'(t + 1) &= \{ -4.50, -4.45, \ldots, 29.50, 29.55 \} & S_m(t) > 45\text{mph} \\
\{\hat{\mathcal{C}}'(t + 1) &= \{ 2.00, 2.05, \ldots, 29.50, 29.55 \} & S_m(t) \leq 45\text{mph}
\end{align*}
\] (40)

Discarding the negative values, the series \(\hat{\mathcal{C}}'(t + 1)\) is reduced to \(\hat{\mathcal{C}}_t'(t + 1)\), where \(\hat{\mathcal{C}}'(n, t + 1)|\hat{\mathcal{C}}'(n, t + 1) > 0, n = 1, 2, 3, \ldots, m\), where \(m\) is the length of \(\hat{\mathcal{C}}'(t + 1)\). Thus,

\[
\begin{align*}
\{\hat{\mathcal{C}}'(t + 1) &= \{ 0.05, 0.1, \ldots, 29.50, 29.55 \} & S_m(t) > 45\text{mph} \\
\{\hat{\mathcal{C}}'(t + 1) &= \{ 2.00, 2.05, \ldots, 29.50, 29.55 \} & S_m(t) \leq 45\text{mph}
\end{align*}
\] (41)

7. From Eq. (23), the set representing the number of vehicles that will likely choose the managed lanes at time interval \(t + 1\), \(\Delta\hat{N}_{in}(t + 1)\), can be estimated from:

\[
\Delta\hat{N}_{in}(d, t + 1) = N(d, t) * \sum_{j=1}^{3} \sum_{k=1}^{2} \frac{1}{1 + e^{a(j) - \hat{\mathcal{C}}_t'(t + 1) - \beta(j) - \vartheta(k) - \Delta T_t(t + 1) - \vartheta(\Delta T_t(t + 1))}} \cdot q(j, k)
\] (42)

Substituting for \(\hat{\mathcal{C}}_t'(t + 1)\) into Eq. (42) using, \(T(t) = 5\text{ min}\), \(\Delta T = \{ 2, 2, 2, 2, 5, 5, 5, 3, 3, 3 \} \text{ (min)}\),

\(q = \{ 0.065, 0.156, 0.429, 0.01, 0.024, 0.066, 0.015, 0.036, 0.099 \}\), \(N(t) = 1200\),

\(\alpha = \{ 1, 1, 1, 1, 1, 1, 1, 1 \}\), \(\beta = \{ 0.83, 0.48, 0.16, 1.245, 0.72, 0.24, 0.04, 0.02, 0.01 \}\), and \(\vartheta = \{ 0.83, 0.48, 0.16, 3.74, 2.16, 0.72, 0.03, 0.02, 0.01 \}\), yields

\[
\begin{align*}
\{\Delta\hat{N}_{in}(d, t + 1) &= \{ 383, 378, \ldots, 2, 1, 0 \} & S_m(t) > 45\text{mph} \\
\{\Delta\hat{N}_{in}(d, t + 1) &= \{ 151, 149, \ldots, 2, 1, 0 \} & S_m(t) \leq 45\text{mph}
\end{align*}
\] (43)

8. From Eq. (21), the set representing the estimated average speeds on the managed lanes at time interval \(t + 1\), \(\hat{S}_m(t + 1)\), can be calculated from

\[
\hat{k}_m(d, t + 1) = \frac{\Delta R_m(d, t + 1) + N_m(d, t) - \Delta N_{out}(d, t)}{L \cdot 2}
\] (44)

Substituting for \(\Delta\hat{N}_{in}(t + 1)\) from Eq. (25), \(N_m(t) = 500\), \(\Delta N_{out}(t) = 50\),

38
L = 6.5 miles into Eq. (44) yields

\[
\begin{align*}
\hat{k}_m(d, t + 1) &= \{64.10, 63.12, \ldots, 34.63, 34.62\} & S_m(t) > 45\text{mph} \\
\hat{k}_m(d, t + 1) &= \{46.23, 46.07, \ldots, 34.63, 34.62\} & S_m(t) \leq 45\text{mph}
\end{align*}
\]

(45)

9. From Eq. (20), the set representing the estimated average speeds on the managed lanes at time interval \( t + 1 \), \( \hat{S}_m(t + 1) \), can be calculated from

\[
\hat{S}_m(d, t + 1) = S_f(1 - \frac{k_m(d,t+1)}{k^*_m}) > 45
\]

(46)

Substituting for \( S_f = 70 \) mph (assumed free-flow speed), \( \hat{R}_m(t + 1) \), \( k^*_m = 200 \) vpmpl into Eq. (46) yields

\[
\begin{align*}
\{\hat{S}_m(d, t + 1) &= \{47.56, 47.70, \ldots, 57.87, 57.88\} \\
\hat{S}_m(d, t + 1) &= \{53.82, 53.87, \ldots, 57.87, 57.88\}
\end{align*}
\]

(47)

10. Find the set of feasible toll rates for time interval \( t + 1 \), \( \hat{C}(t + 1) \) in Eq. (41), such that the estimated average speed on the managed lanes is greater than 45 mph.

\[
\hat{C}(t + 1) = Arg \{\hat{S}_m(n, t + 1) > 45|\hat{S}_m(n, t + 1) \in \hat{S}_m(t + 1), \hat{C}^+(t + 1)\}
\]

(48)

This results in

\[
\begin{align*}
\hat{C}(t + 1) &= \{0.05, 0.10, \ldots, 12.85, 12.90\} & S_m(t) > 45\text{mph} \\
\hat{C}(t + 1) &= \{2.00, 2.05, \ldots, 12.85, 12.90\} & S_m(t) \leq 45\text{mph}
\end{align*}
\]

(49)

11. From Eq. (19), the set representing the estimated toll revenues for time interval \( t + 1 \), \( \hat{R}(t + 1) \) can be calculated from
\[
\hat{R}(d, t + 1) = \hat{C}(d, t + 1) * N(d, t) * \\
\sum_{j=1}^{3} \sum_{k=1}^{3} \left[ \frac{1}{1 + e^{\alpha(j)\hat{e}(d, t+1) - \beta(j)\Delta T(d, t+1) - \partial(k)\Delta T_R(d, t+1)}} * q(j, k) \right]^{1/3}
\]

Substituting for the set of toll rates \(\hat{C}(d, t + 1)\) from Eq. (49),

\[ q = \{0.065, 0.156, 0.429, 0.01, 0.024, 0.0660, 0.015, 0.036, 0.099\}, \quad N(t) = 1200, \]

\(\alpha = \{1, 1, 1, 1, 1, 1\}\) and \(\beta = \{0.83, 0.48, 0.16, 1.245, 0.72, 0.24, 0.04, 0.02, 0.01\}\)

and \(\partial = \{0.83, 0.48, 0.16, 3.74, 2.16, 0.72, 0.03, 0.02, 0.01\}\) and \(\Delta T(t) = 5\)min,

\(\Delta T_R(t) = \{2, 2, 2, 2, 5, 5, 3, 3, 3\}\) into Eq. (50) yields

\[
\begin{cases} 
\hat{R}(d, t + 1) = \{0.00, 0.01, ..., 333.56, 333.60\} & S_m(t) > 45mph \\
\hat{R}(d, t + 1) = \{0.00, 0.01, ..., 333.56, 333.60\} & S_m(t) \leq 45mph 
\end{cases}
\]

(51)

Calculate the maximum toll revenue \(\hat{r}(t + 1)\) such that \(\hat{r}(t + 1) = \max \{\hat{R}(t + 1)\}\). Finally, the optimal toll rate and toll revenue of this example is:

\[
\begin{cases} 
\hat{R}(d, t + 1) = $330.60, \hat{c}(t + 1) = $2.8 & S_m(t) > 45mph \\
\hat{R}(d, t + 1) = $330.60, \hat{c}(t + 1) = $2.8 & S_m(t) \leq 45mph 
\end{cases}
\]

(52)
CHAPTER 5. SIMULATION RESULTS AND ANALYSIS

This chapter tests whether the proposed strategy works well under simulations. The chapter has been divided into four sections. The first section explains the impact of drivers’ departure time choice behavior on network system. The second section addresses the impact of trip purpose and travel time reliability over drivers’ route choice behavior. The third section discusses the drivers’ learning process in agent-based modeling. The last section compares the performances of the proposed strategy and current toll pricing strategy on I-95.

5.1 Drivers’ Departure Time Choice Behavior

The departure time choice model can be combined with agent-based model to study the dynamic traffic demand and peak spreading analysis. Drivers with different trip purposes learn traffic conditions, accumulate relevant experience, choose different departure time, and in turn influence traffic volume and optimal toll rate. For the 1st day, traffic demand data for Wednesday and Thursday in June and July 2011 were extracted from database in 15-minute intervals as shown in Figure 5. It shows that from 5:00 AM to 7:30 AM, the traffic demand increases from 1500 vph to 7500 vph. After 5 days’ simulations, the dynamic traffic demand tends to be stable. Drivers stop searching for the new departure time when they satisfy the current travel experience. In addition, some drivers who originally depart at 6:50 AM to 7:30 AM switch their departure time between 6:00 AM to 6:50 AM. The effect of traffic demand spreading can be detected from Figure 5.

Figure 6 shows the number of drivers searching for new departure time under search begin rules. The principle is that drivers begin to search new departure time if their utility for the current day is less than the average utility of all previous travel days. The simulation results indicate that
from the 2nd day to the 11th day, the number of drivers in vehicles searching for new departure time increases from 90 to 400 vph. Their average utilities change and tend to stable as the number of simulations increases. In the previous several simulation days, the value of average utility for drivers are not representative, leading to an increasing number of drivers who want to adjust their behaviors. After that, it decreases to 260 vph on the 14th day and remains that value until the 16th simulation day.

![Dynamic Traffic Demand](image1)

**Figure 5 Dynamic Traffic Demand**

![Number of Travelers Searching for New Departure Time](image2)

**Figure 6 Number of Drivers searching for new Departure Time**
The traffic conditions can be evaluated by delay on both managed lanes and general purposed lanes. At low traffic demand, the value of delay is negative which indicates that drivers’ have a less travel time and would like to keep their departure time or postpone it. As the traffic demand increases, the delay becomes positive that drivers may choose an earlier departure time in order to avoid congestion. Compared to the 1st day, the delay under high demand decreases a lot for the 15th day after 6:50 AM while produce higher level of delay between 6:20 AM to 6:50 AM. The same trend can be identified in Figure 8 for general purposed lanes.

Figure 7 Delay on Managed Lanes by Departure Time Intervals

Figure 8 Delay on General Purposed Lanes by Departure Time Intervals
The departure time change for drivers with different trip purposes under search rules can be shown in Figure 9 below. Before 6:30 AM, drivers would like to postpone their departure time for 0.5 to 1.8 min. However, based on the search rule, drivers will keep their departure time if delayed travel time is so small or the departure time change is less than 2 min. After 6:30 AM, drivers with urgent trips produce highest departure time change because of their highest value of time. The optimal departure time help them achieve higher value of travel time saving. For leisure trips, the value of departure time change is less than 2 min, which indicates that they will maintain their original departure time based on the search rules in this study.

![Figure 9 Search for Alternative Departure Time](image)

The real number of drivers changing departure time is shown in Figure 10. At low demand before 6:15 AM, the behavioral adjustment is insignificant for all simulation days. The number of drivers changing departure time increases as the demand increases. Besides, a decreasing trend can be observed in Figure 10 from 2\textsuperscript{nd} day to 16\textsuperscript{th} day. As more and more drivers find their optimal departure time, the number of them adjusting departure time will decrease then maintain stable eventually.
After comparing the product of departure time change and value of time to the toll price, the final value of departure time change is determined by decision rule. The results are shown in Figure 11. At high demand, drivers with urgent trips produce highest value of departure time change, then followed by work trips. Drivers with leisure trips hardly change departure time because their value of time is low so that their schedules are flexible. Travel time saving and lower delay are not important factors for them.
5.2 Drivers’ Route Decision Behavior

Different trip purposes influence drivers’ route choice decision as the value of time and the value of reliability change by changing the trip purpose. Figure 12 shows how different trip purposes can affect drivers’ probability of choosing the managed lanes under low and high traffic demand. The probability to choose the managed lanes increases as the traffic demand increases from 5:00 AM to 7:30 AM for all trip purpose. At low traffic demand, the probability of choosing the managed lanes are similar for all three trip purposes since there are no significant difference in travel time savings or travel time reliability between the managed lanes and the general purpose lanes. While under high traffic demand, a significant difference in the probability of choosing the managed lanes is observed with the highest probability for the urgent travel. This is because drivers with urgent traveling purpose typically place more value on travel time and travel time reliability than drivers with other trip purposes.

Figure 13, Figure 14 and Figure 15 compare the probability of choosing the managed lanes for two cases of drivers’ utility function under three trip purposes. For case one that considers both travel time saving (TTS) and travel time reliability (TTR) in the driver’s utility function, the simulation results show that drivers have a higher probability to choose the managed lanes than the second case that only considers travel time saving. This implies that travel time reliability is an important factor which will influence drivers’ route choice behavior. It is higher reliability of travel time and higher value of travel time saving on the managed lanes that contribute to the higher probability of choosing managed lanes for all trip purpose.

Compared to work trip and urgent trip, the difference in the probability of choosing the managed lanes between the two cases is less obvious in leisure trip, as was shown in Figure 15. That is because drivers with work or urgent travelling trip purposes are more concerned about the
reliability of travel time as delay may cause serious consequence for them. For leisure trip, their value of time is lower. It is not worthwhile for them to pay higher toll on peak-hour to use managed lanes.

Figure 12 Probability of Choosing Managed Lanes

Figure 13 Probability of Choosing Managed Lanes for Work Trips
Figure 14 Probability of Choosing Managed Lanes for Urgent Trips

Figure 15 Probability of Choosing Managed Lanes for Leisure Trips

5.3 Learning Process in Agent-based Modeling

The travel time reliability based on previous days measured by each driver affects their route choice behavior and traffic condition on the next day, which, on the contrary, influences the travel time reliability.

Figure 16 and Figure 17 shows the travel time reliability for each simulation day at 6:30AM. Figure 16 shows that after the 10th day, the fluctuation of travel time reliability gradually
reduced for work trips. Similarly, for general purpose lanes, the same trend was found, as shown in Figure 17. Drivers accumulate their experiences of travel time reliability day by day and, eventually, the travel time reliability for individuals and the traffic system can be optimized which makes the travel time more reliable.

Figure 16 Travel Time Reliability on Managed Lanes

Figure 17 Travel Time Reliability on General Purpose Lanes
5.4 Comparative Evaluation

The performance of the modified dynamic toll pricing strategy considering the trip purpose, travel time reliability and departure time change is compared to the currently used strategy on I-95 and the original strategy developed by Cheng and Ishak (2013). The current toll determination algorithm on I-95 is based on the research from Elefteriadou et al. (2012). Figure 18 shows that the toll rate for the modified strategy increases slowly and steadily compared to the current strategy. At low demand, the toll rate from modified strategy is higher than that from current strategy and almost the same with original strategy. It aims at determining an optimal toll rate to maximize total revenue while maintain the speed on all lanes greater than 45 mph. Higher toll rate will increase the revenue while not bring any penalty on level of service under low demand. At high demand, the toll rate produced by proposed strategy is lower than the current study and higher the original study.

Figure 18 Toll Rate

Figure 19 shows that the total revenue from the modified strategy with focus on total revenue maximization is significantly higher than that from the current strategy and the original
study at high demand. The revenue differences between these two methodologies are insignificant at low demand.

![Total Revenue Chart]

**Figure 19 Total Revenue**

At the same time, the modified and current strategies all maintain a minimum level of service on the managed lanes by guaranteeing the average speed higher than 67 mph, as shown in Figure 20. Compared to the speed from the current strategy and the original strategy, the revenue maximization on managed lanes are achieved by the modified strategy without compromising the level of service on the managed lanes while produce a higher speed between 6:00AM to 7:00 AM instead.

Figure 21 shows that after 6:20 AM, the speed on the general purpose lanes declines from 70 mph and eventually remains around 40 mph for all strategies.

Travel time saving for the both strategies followed the same trend in Figure 22. The modified strategy has the advantages of higher travel time saving than the current strategy at high traffic demand while the travel time saving is insignificant under low demand for all strategies.
Figure 20 Average Speed on Managed Lanes

Figure 21 Average Speed on General Purposed Lanes

Figure 22 Travel Time Saving
CHAPTER 6. CONCLUSIONS

The primary goal of this study is to develop an agent-based dynamic feedback-control toll pricing strategy such that the toll revenue is maximized while maintaining a minimum desired level of service on the managed lanes that keeps the speed on the managed lanes above 45 mph. The impact of variations of the trip purposes, the drivers’ level of income, the value of travel time, the value of travel time reliability, and departure time change was examined using the proposed strategy. The heterogeneous drivers accumulate relevant knowledge on time-dependent travel conditions corresponding to different departure times, form subjective evaluation, search for alternative departure times and adjust their behaviors. Furthermore, the proposed strategy accounts for the impacts of drivers’ learning of traffic conditions from previous commuting experiences on their departure time choice decisions and route choice decisions. An agent-based modeling was applied to simulate drivers’ learning process based on their previous commuting experience. The heterogeneous drivers accumulate relevant knowledge on time-dependent travel conditions corresponding to different departure times, form subjective evaluation, search for alternative departure times and adjust their behaviors. Drivers can switch their departure time based on information and experience, resulting in a redistribution of vehicle flow on the network and toll rate updated over time. The study also analyze how driver’s heterogeneity in value of time, value of reliability for each trip purpose will influence route decisions and thus affect the optimal toll rates.

The study is conducted on a seven-mile section of I-95 from NW 151 Street to SR 112 Street with three general purposes and two managed lanes. The traffic demand profile included pre-peak and peak hours in the morning from 5:00 am to 7:30 am. VISSIM was used to simulate
the operation of the modified strategy on the study area. To compensate for VISSIM’s limitations, an external module integrated with VISSIM was developed to execute the process of drivers’ departure time decision, route decision and toll rate determination as well as export traffic data to an Excel spreadsheet tool in real-time during the VISSIM simulation. The Excel spreadsheet calculates the delay that is used in drivers’ departure time choice decision and dynamic traffic demand estimation. The travel time reliability was also estimated in the Excel spreadsheet that based on all previous traffic data which is used in drivers’ route choice process. The modified strategy used a Logit model to simulate drivers’ route choice behavior bases on three income levels, three trip purposes, different values of time and values of travel time reliability. An agent-based feedback control mechanism was used to calculate the optimal toll rate that would maximize the revenue every three minutes while maintaining the speed on the managed lanes larger than 45 mph. A numerical example was provided to illustrate how the modified strategy works.

Simulation results show that after considering drivers’ departure time choice behaviors, the peak spreading phenomenon was observed that some drivers who originally departure at 6:50 AM to 7:30 AM switch their departure time between 6:00 AM to 6:50 AM. Under high traffic demand, a big difference in the probability of choosing the managed lanes is noticed with the highest probability for the urgent travel. When considering both travel time saving (TTS) and travel time reliability (TTR) in the driver’s utility function, simulation results show that drivers have a higher probability to choose the managed lanes than the case that only considers travel time saving, which implies that travel time on managed lanes is more reliable than that for general purpose lanes. The comparative evaluation is given between the modified strategy, the current strategy deployed on Interstate 95 express lanes and the original strategy. Compared to the strategy currently used, the increase of the toll rate is steadier and the revenue is significantly higher for the modified strategy.
while keeping the speed at 45 mph or more on the managed lanes. The modified strategy also has the advantages of higher travel time saving. Compared to the original strategy developed by Cheng and Ishak (2013), the modified strategy offers a more realistic approach that accounts for travel time reliability and delay in route choice and departure time choice process, as well as generates higher toll revenue under heavy traffic demand.
REFERENCES


VITA

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