Air Armament Planning and Design Through Systems Analysis.

John H. Arnold
Louisiana State University and Agricultural & Mechanical College

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SYSTEMS ANALYSIS.

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AIR ARMAMENT PLANNING AND DESIGN THROUGH SYSTEMS ANALYSIS

A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Doctor of Philosophy in The Department of Mechanical, Aerospace and Industrial Engineering

by

John H. Arnold

A.B., Mercer University, 1963
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May 1972
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ABSTRACT

Closed form approximations for the probability of damaging surface targets with aerially delivered weapons are developed and analyzed for six different employment situations: single weapons against point targets, single weapons against area targets, multiple weapons against area targets, single weapons against point targets with location uncertainty, and multiple weapons against point targets with location uncertainty.

In each case, conditional damage and probability of coverage functions are developed, the product of which defines the probability of damage or probability of fractional damage depending upon whether the target is a point or an area respectively.

In addition, optimum damage probability, or rather the maximized damage probability constrained by specific design characteristics and delivery errors, is developed and compared with the capabilities of current systems. Optimum pattern radii or, pattern radii which maximize the damage probability are also developed. Methodology which leads to preliminary design characteristics is developed through determination of the number of submunitions or weapon weight required to achieve any given level of damage for given employment constraints.

Weapon preference methodology is developed which establishes a parametric evaluation procedure for weapon system employment preference and preliminary design characteristics.

Analysis of nine representative weapons systems against three representative targets, where applicable, is presented to demonstrate the usefulness and adequacy of the methodology.
The methodology developed in Chapters II and III relates specifically to continuous patterns, that is, to weapons impact patterns bounded by singly connected curves and containing a random distribution of submunitions over the patterns. The methodology of the appendix extends the principles to weapons systems whose impact patterns are annular in nature, either circular or elliptic (within established limits) which are bound by multiply connected outer and inner curves. For this application, the submunitions are constrained to lie within the annular ring or the area between the outer and inner curves.

The methodology is accurate and requires very little manpower and computer resources to employ. It is based on the Mean Area of Effectiveness (MAE) Concept and can be used to readily and accurately assess the potential of new designs and proposals if accurate estimates of the MAE can be made from the lethal performance of existing munitions and submunitions.
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<td>Range Error Probable</td>
</tr>
<tr>
<td>$R_L$</td>
<td>Lethal Radius of a Unitary Weapon</td>
</tr>
<tr>
<td>$R_{LB}$</td>
<td>Lethal Radius of an Individual Bomblet</td>
</tr>
<tr>
<td>R.M.S.</td>
<td>Root-Mean-Square</td>
</tr>
<tr>
<td>$R_P$</td>
<td>Radius of a Cluster Weapon Pattern</td>
</tr>
<tr>
<td>$R_T$</td>
<td>Radius of an Area Target</td>
</tr>
<tr>
<td>$R_V$</td>
<td>Vulnerable Radius of a Target</td>
</tr>
<tr>
<td>$R_W$</td>
<td>Radius of a Multiple Weapons Pattern</td>
</tr>
<tr>
<td>$r_1$</td>
<td>Bomblet Reliability in a Cluster Weapon</td>
</tr>
<tr>
<td>$r_2$</td>
<td>Cluster Weapon Reliability</td>
</tr>
<tr>
<td>$r_3$</td>
<td>Unitary Weapon Reliability</td>
</tr>
<tr>
<td>$W_B$</td>
<td>Weight of an Individual Bomblet</td>
</tr>
<tr>
<td>$W_{Bi}$</td>
<td>Weight of the $i$th Bomblet</td>
</tr>
<tr>
<td>$W_C$</td>
<td>Weight of a Cluster Weapon</td>
</tr>
<tr>
<td>$W_{CT}$</td>
<td>Total Weight of $N$ Multiply Delivered Cluster Weapons</td>
</tr>
<tr>
<td>Roman</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>$W_{MC}$</td>
<td>Weight of a Massive Cluster</td>
</tr>
<tr>
<td>$W_U$</td>
<td>Weight of a Unitary Weapon</td>
</tr>
<tr>
<td>$W_{UT}$</td>
<td>Total Weight of $m$ Multiply Delivered Unitary Weapons</td>
</tr>
</tbody>
</table>
A Constant Related to Unitary Weapons

A Constant of Proportionality Between Cluster Weapon Weight and Unitary Weapon Weight

Mean Value

Mean Value in Deflection

Mean Value in Range

$\pi$ 3.14159

Correlation Coefficient

Standard Deviation in Delivery Error

Standard Deviation in Aiming Error

Standard Deviation in Ballistic Error

Standard Deviation in Total Error

Standard Deviation in Deflection Error

Standard Deviation in Range Error

Standard Deviation in Target Location Error
A. Introductory Remarks

The purpose of this investigation is to develop closed form approximations for the conditional damage and the probability of coverage functions, products of which yield weapon system probability of damage functions. Several weapons systems and employment situations are investigated. Closed form solutions in terms of the various weapon, target and employment parameters, can be used both as a rapid means of accurately assessing the effectiveness of weapons systems and as a preliminary design tool for determining weapon systems design characteristics.

At present, the assessment of weapon system damage probability is accomplished by numerical integration techniques which are time consuming and require large expenditures of manpower and computer resources\(^1,2\). Duncan\(^3\) suggested the use of the Poisson distribution as a means of approximating the hit probability of at least one missile from a random circular salvo of missiles. Although narrow in scope, it was this initial work which prompted the modifications and expanded applications herein. Utilization of the Poisson distribution for approximating the conditional damage function appears in reference \(^4\). However, the probability of coverage function appears in functional form such that the probability of damage must still be computed numerically or found parametrically. Gould, Arnold and Von Waldburg\(^5\)

\(^1,2\) Superscripts are used to denote references
extended this technique, with the inclusion of the Rayleigh distribution as an approximation of coverage function, to point targets. However, efforts to date have failed to produce sufficiently accurate and general closed form approximations of both the conditional damage and probability of coverage functions to serve as useful assessment and design tools.

The problem of planning and designing air armament is extremely complex, involving a chain of decision junctions many branches of which can lead to erroneous conclusions. Preliminary design is an essential element of air armament and development planning, the decision making process of weapon system selection. An important but often neglected consideration in the decision making process is whether one system is adequate or whether more than one system is required to maintain a realistic conventional munitions inventory capable of negating existing and anticipated threats. A basic question is whether the members of a family of weapon systems complement one another sufficiently to justify the additional expenditure that multiple systems imply.  

When several measures of effectiveness that are basically different may all be important for any given set of circumstances, serious consideration should be given to more than one system. In addition, if the expected employment environments are quite different, due either to future uncertainty or enemy counteraction, more than one system could yield the flexibility of action that is essential to perform at any reasonable level of competence. A single rigid weapon system invites enemy change which may negate the value of an otherwise effective system.
Very often "analysis" situations lead to the adoption of weapon systems which will never be called upon to operate in an average environment with respect to an average measure of effectiveness. These highly specialized weapon systems are expensive luxuries and can find justification only after a basic core of effective and highly flexible general purpose systems have become a reality. The development and maintenance of large numbers of highly specialized single purpose systems is prohibitively costly for the amount of anticipated return.

The development engineer is often too close to his programs to render rational and bias free assessments pertaining to current needs and future potential. As a consequence, it is essential that a periodic review of extant exploratory, advanced and engineering development programs be made by research and development management to ascertain the viability and currency of the existing programs with existing and anticipated levels of threat and tactical operational environments.

Until a few years ago, research and development of conventional munitions within the Air Force relied almost totally on the experience and judgment of development engineers. Although the basic analytical tools for weapons effectiveness analysis had been in existence for many years, they were used primarily as a means of assessing the effectiveness of the munition after it had been developed, tested and placed into inventory. Much of this was due to the fact that the necessary a priori inputs to a system analysis were scanty, unreliable and in many cases nonexistent. Very few development programs were actually justified based on predicted performance or anticipated payoff other than the "feelings" of development engineers.
More recent emphasis on pre-development analysis, promoted mainly by a tightening research and development budget and an ever expanding weapons systems inventory, has led to the establishment of more comprehensive physical testing and of a broad weapon effectiveness data bank. As a result, sufficient target vulnerability, weapons characteristics and weapons effects data have evolved which permit sound pre-development elimination of poor designs or retention of the most promising programs.  

Current analytic efforts are extremely complex and weapon system analysis requires an expenditure of manpower comparable to that of the actual research and development of the weapon system. Current efforts involve extreme amounts of available computer resources. It is the nature of existing brute force techniques that discrimination between poor design and promising design can be made only after exhaustive computer studies.¹,² In the final analysis the choice is still dependent upon the integrity of the analyst to ascertain the validity of the input data, the viability of the employment constraints utilized in the analysis, and the interpretation of the study results.

A major deficiency with current analytical efforts is the fact that poor systems are subjected to the same detailed scrutiny as the promising systems since discrimination can be made only in the final analysis. Analyses are conducted cautiously on all systems, promoted basically by a desire in the end to be able to discriminate between the promising systems and not between the two extremes. As a result, highly sophisticated system analysis techniques have been developed in an attempt to converge to true solutions with relatively small error.¹,²
Such techniques when applied to all proposed designs and concepts, result in a needless waste of resources. Techniques which are simple and require a minimum of manpower and computer resources are needed to reduce the ever increasing number of concepts and designs to the few which offer the greatest potential. The remaining programs can then be subjected to the detailed analysis necessary for establishing a viable research and development program.

It is the purpose of this effort to develop methodology which approximates with sufficient accuracy the potential of proposed weapons systems concepts and designs to the extent that the above can be realized.

B. Pattern for Employment of Tactical Air Forces

The order of precedence in which combat air functions are accomplished cannot be prescribed by arbitrary methods and procedures. The fundamental principle governing the priority of combat air functions is the requirement to neutralize the enemy threat having the most profound and continued influence on the total mission of the combat area command. This principle is compatible with the inherent characteristics of tactical air forces, since it provides for their employment at a decisive time and place.

Tactical air forces are employed in the following tasks which produce area effects: the attainment of air superiority by destruction or neutralization of enemy air forces which threaten the area; the progressive neutralization of the enemy strength to sustain combat by isolating air and surface combat forces from their means of supply.
and battle sustenance; the disruption of enemy actions in the immediate area of engagement between the opposing surface forces. The priority of these tasks is dependent upon the effects desired in terms of the area command mission and war strategy.

Timely, offensive action against well-chosen targets is fundamental to the full exploitation of the combat potential of tactical air power. Timing of the action to destroy or neutralize a target may often be as important as the selection of the correct target.

The research and analysis of this effort will be restricted to tactical surface targets, specifically to the preliminary design and development planning methodology necessary to ascertain that a weapons system inventory is developed and maintained to meet all possible contingencies within this mission area.

C. Target Vulnerability

The determination of target vulnerability involves an analysis of a number of complex factors. Premature commitment of forces without proper consideration of target vulnerability may result in needless expenditure of effort and resources with little appreciable effect upon the enemy's ability to conduct combat operations.

A fundamental factor in considering target vulnerability is the essentiality of the target to the enemy's combat effort. The breadth of this factor lies in an examination of the entire spectrum of targets within an area of operations. Pursuant to this examination, the targets chosen should involve the most significant areas of enemy strength, without which his combat operations may need to be reduced drastically or suspended entirely.
In order to determine the susceptibility of targets to destruction or neutralization, detailed knowledge of their physical features, such as mobility, mass, construction, location, and density is required. The vulnerability of a target or target system must then be measured against existing weapons to produce varying degrees of effect depending upon the specific tactical operational environment. If voids or marginal capabilities exist, new weapons systems must be planned, designed and placed at the disposal of the tactical air forces. A detailed understanding of weapon capabilities, limitations, and effects is required in order to make the most efficient and economical selection. Targets may be vulnerable to attack, but impervious to the weapons available at the time required. Capabilities must be sufficient to insure that the effects produced are commensurate with the effort and resources expended.

D. Concepts From Weapons Systems Analysis

1. Damage and Casualty Criteria

Because of the complexity of tactical air operations against surface targets, the changing nature of order of priority from one battle area to another, logistics, and a nonlinear depletion of available local munitions stockpiles; it has been necessary to assess the effect of weapons systems at various levels below the ultimate of catastrophic target damage. Thus, a number of personnel incapacitation criteria have evolved which relate weapon effectiveness at various levels below inducing death. They are separated into three main categories, with a time dependency within each category. The most stringent category is incapacitation to the extent that personnel
can no longer defend themselves from attack. The second category
denies personnel the ability to assault, and the least stringent
criterion denies the ability to function in a supply effort.

Effects on material targets are also assessed at levels of
damage which are less than catastrophic. If a target possesses
mobility, a group of time dependent criteria relating to immobility
has evolved and, if a target possesses firepower, criteria
have evolved which relate the potential of denying the target its
firepower.

The interpretation of "kill" probabilities requires an understand-
ing of some of the basic principles underlying weapon systems
analysis. The term "kill" does not necessarily mean a kill in the
literal sense. It is defined in terms of the desired degree of damage
insofar as material targets are concerned and in terms of level of
incapacitation insofar as personnel targets are concerned. These
criteria have physical and not statistical interpretations.\textsuperscript{9}

Each weapon has a certain probability of defeating any target
(including the null probability) for any assigned damage or casualty
criterion. Some of the more commonly used criterial are listed in
Table 1-1.
TABLE 1-1

SELECTED DAMAGE AND CASUALTY CRITERIA

<table>
<thead>
<tr>
<th>Target</th>
<th>Criterion</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truck</td>
<td>A Kill</td>
<td>Vehicle Stoppage Within 2 Minutes</td>
</tr>
<tr>
<td></td>
<td>B Kill</td>
<td>Vehicle Stoppage Within 20 Minutes</td>
</tr>
<tr>
<td>Tank</td>
<td>F Kill</td>
<td>Complete or Partial Loss of Firepower</td>
</tr>
<tr>
<td></td>
<td>K Kill</td>
<td>Catastrophic Damage</td>
</tr>
<tr>
<td></td>
<td>M Kill</td>
<td>Immediate Immobilization</td>
</tr>
<tr>
<td></td>
<td>A Kill</td>
<td>Immobilization Within 2 Minutes</td>
</tr>
<tr>
<td>Aircraft</td>
<td>B Kill</td>
<td>Control Loss Within 5 Minutes</td>
</tr>
<tr>
<td></td>
<td>C Kill</td>
<td>Mission Abort</td>
</tr>
<tr>
<td></td>
<td>K Kill</td>
<td>Control Loss Within 5 Seconds Resulting in Eventual Catastrophic Damage</td>
</tr>
<tr>
<td></td>
<td>KK Kill</td>
<td>Immediate Catastrophic Damage</td>
</tr>
<tr>
<td>Bridge</td>
<td>S1</td>
<td>Drop A Single Span</td>
</tr>
<tr>
<td>Rail</td>
<td>Cut</td>
<td>Cut A Single Rail</td>
</tr>
<tr>
<td>Personnel</td>
<td>30 sec Def.</td>
<td>Ability to function in a defensive posture denied within 30 seconds</td>
</tr>
<tr>
<td></td>
<td>5 min Ass</td>
<td>Ability to function in an assault role denied within 5 minutes</td>
</tr>
</tbody>
</table>
2. **Mean Area of Effectiveness (MAE)**\(^7,9\)

The MAE concept relates the effectiveness of a weapon against a particular target, about the target centroid, in terms of the weapon's characteristics; target characteristics, both physical and vulnerable; and a specified damage or incapacitation criterion. It has been developed and is defined such that if the target is located within the mean area of effectiveness for a specified weapon and damage or incapacitation criterion, then the damage or incapacitation criterion is at least satisfied.

This definition applies directly to point targets but may be extended to include all targets whose centroids are located within the MAE. Certain targets having edge effects (unitary targets having large distributed areas such as buildings) and of a class where partial damage assessment has military significance must be approached in an entirely different manner. The reader is referred to reference 9 for further explanation.

The problem of assessing weapon system effectiveness is essentially reduced to determining the probability that a target will lie within the mean area of effectiveness subject to the constraints imposed by the weapon delivery system. The MAE concept may be modified, without loss of generality, for multiple weapons delivery, as will be shown in later developments.

3. **Delivery Accuracy**

Delivery accuracy with regard to current capabilities is a misnomer, since in the most basic sense, both the standard deviation (\(\sigma\)) and circular probable error are measures of the pilot's inability
to place his weapon or weapon pattern center on a desired point on the ground. Combat and testing experience has shown that errors in range and deflection can usually be described by a normal (or a Gaussian) distribution whose precise characteristics are well known. The range error probable (REP) and deflection error probable (DEP) measure the tendency of the impact points to differ from the target center. When the REP and DEP are identical, the resulting distribution is called circular or radial. Circular distributions can further be described in terms of the circular probable error (CEP). When REP and DEP are not equal, the CEP concept may still be employed as related in paragraph (E.3) of this chapter.

Delivery accuracy is normally divided into two categories, one being referred to as aiming error (σ_A) and the other as ballistic error (σ_B). The aiming error is attributed basically to the pilot and his ability or inability to place the mean point of impact (MPI) of the weapon system on the target center. The magnitude of this error depends significantly on pilot experience and initiative, the physical and defensive environment, and, the handling qualities of the aircraft. The ballistic errors are attributed to the weapon and are basically a measure of the divergence of impacts within a weapons pattern with respect to the MPI. The magnitude of the errors depends significantly upon the quality control in the weapons production and the ejection system.

Conversions for REP, DEP, CEP and σ are given below. For ease of tabulation, both REP and DEP are referred to as the probable error (PE).
TABLE 1-2

DELIVERY ACCURACY CONVERSIONS

<table>
<thead>
<tr>
<th>To Convert From</th>
<th>To</th>
<th>Multiply</th>
<th>By</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEP</td>
<td>σ</td>
<td>CEP</td>
<td>0.8493</td>
</tr>
<tr>
<td>CEP</td>
<td>PE</td>
<td>CEP</td>
<td>0.5729</td>
</tr>
<tr>
<td>σ</td>
<td>PE</td>
<td>σ</td>
<td>0.6745</td>
</tr>
<tr>
<td>ς</td>
<td>CEP</td>
<td>σ</td>
<td>1.1774</td>
</tr>
<tr>
<td>PE</td>
<td>σ</td>
<td>PE</td>
<td>1.4826</td>
</tr>
<tr>
<td>PE</td>
<td>CEP</td>
<td>PE</td>
<td>1.7456</td>
</tr>
</tbody>
</table>

4. Damage Probability

In the most general sense, the probability of damage (\(P_D\)) is given by:

\[
P_D = \iiint_{-\infty}^{\infty} p(x,y,z)f(x,y,z) \, dx \, dy \, dz
\]

where,

\(p(x,y,z)\) is the kill probability of a warhead detonating at (\(x,y,z\)) and,

\(f(x,y,z)\) is the probability density function for the warhead detonating at (\(x,y,z\)).

A closed form solution to this function does not exist and it must be numerically integrated at discrete points in a manner so as to converge to the optimal parameter values as efficiently as possible.
An alternative expression for the damage function is:

\[ P_D = P_{D/C} \cdot P_C \]

where \( P_{D/C} \) is the conditional damage given coverage and \( P_C \) is the probability of coverage.\(^7\)

Accurate closed form solutions for the damage probability are possible with this approach through utilization of the MAE concept and provided the conditional damage and probability of coverage functions exist. It is this approach which will be followed throughout this investigation.

For area targets, the form of the probability of fractional damage is the same and is given by:

\[ F_D = P_{D/C} \cdot F_C \]

where \( P_{D/C} \) is conditional damage given coverage and \( F_C \) is the fraction of the target covered.

E. Concepts from Statistics

1. Poisson Distribution

The discrete probability distribution

\[ p(k) = \frac{\lambda^k e^{-\lambda}}{k!} ; \quad k = 1, 2, 3, \ldots; \quad p(k) = 0 \text{ otherwise} \]

is called the Poisson distribution after Poisson who developed it in
the early part of the 19th century. The distribution has the following properties:

- **Mean** \( \mu = \lambda \)
- **Variance** \( \sigma^2 = \lambda \)
- **Standard Deviation** \( \sigma = \sqrt{\lambda} \)

2. **Bivariate Normal Distribution**

The two-dimensional bivariate normal population is the probability space induced by a pair of random variables \((x, y)\) having a joint density function given by:

\[
f(x, y) = \frac{1}{2\pi\sigma_x \sigma_y \sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2(1-\rho^2)} \left[ \frac{(x-\mu_x)^2}{\sigma_x^2} - 2\rho \frac{(x-\mu_x)(y-\mu_y)}{\sigma_x \sigma_y} + \frac{(y-\mu_y)^2}{\sigma_y^2} \right]\right\}
\]

Carrying out the required integration the marginal (Gaussian) density function \(f(x)\) of \(x\) is:

\[
f(x) = \frac{1}{\sqrt{2\pi} \sigma_x} \exp \left[ -\frac{1}{2} \left( \frac{x-\mu_x}{\sigma_x} \right)^2 \right]
\]

and \(g(y)\) of \(y\) is:

\[
g(y) = \frac{1}{\sqrt{2\pi} \sigma_y} \exp \left[ -\frac{1}{2} \left( \frac{y-\mu_y}{\sigma_y} \right)^2 \right]
\]
Thus $x$ and $y$ are normally distributed random variables with means $\mu_x$ and $\mu_y$ and, standard deviations $\sigma_x$ and $\sigma_y$.

The expectation

$$E \left( \frac{x-\mu_x}{\sigma_x} \frac{y-\mu_y}{\sigma_y} \right)$$

is the constant $\rho$, the correlation coefficient of the random variables $x$ and $y$ ($0 \leq \rho \leq 1$). If the correlation coefficient has an absolute value of unity, the joint density function is meaningless, and the variables $x$ and $y$ are said to have a singular normal distribution, the entire probability mass being concentrated on a line. There is complete linear dependence between $x$ and $y$ for this case.

If $\rho = 0$, $x$ and $y$ are uncorrelated, hence independent, and the joint density function is a product of the marginal density functions.\(\text{ }^{12}\)

$$f(x,y) = f(x)g(y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp \left\{ -\frac{1}{2} \left[ \frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{(y-\mu_y)^2}{\sigma_y^2} \right] \right\}$$

If in the bivariate normal density function above $\sigma_x = \sigma_y = \sigma$, the distribution is said to be circular normal. For mean values $\mu_x = \mu_y = 0$ the function is normalized:\(\text{ }^{12}\)

$$f(x,y) = \frac{1}{2\pi\sigma^2} \exp \left[ -\frac{1}{2} \left( \frac{x^2 + y^2}{\sigma^2} \right) \right]$$
In terms of the vector deviations from the mean value, when only the magnitude of the radial error is significant, the density function is: \[ f(r) = \frac{r}{\sigma^2} \exp\left[-\frac{1}{2} \left(\frac{r}{\sigma}\right)^2\right] \]

This function is often referred to as the radial or Rayleigh density function. These distributions have the following properties:

a) Marginal Density Functions (Figure 1-2)
   - Standard Deviations \( \sigma_x, \sigma_y \)
   - Probable Error in \( x \) \( .67449 \sigma_x \)
   - Probable Error in \( y \) \( .67449 \sigma_y \)

b) Joint Density Function \( (\sigma_x = \sigma_y) \) (Figure 1-2)
   - Standard Deviation \( \gamma = \sqrt{2} \sigma \)
   - Circular Probable Error \( \text{CEP} = \sqrt{2} \sigma \)
   - Circular Error Average \( \text{CEA} = \sqrt{\frac{\pi}{2}} \sigma \)

3. Equivalent Circular Probable Error

The circular probable error, as a parameter, is a unique function of the circular normal distribution. Although it is not associated with the non-circular (elliptic) bivariate normal distribution, there is a circle centered at the aiming point of the non-circular distribution which contains half of the sample points. The radius of this circle is often referred to as equivalent circular probable error (ECEP).
Radial distributions based on an elliptical normal distribution \((\sigma_x \neq \sigma_y)\) must be integrated numerically for accurate results. However, when the smaller distribution is at least one-third of the larger \((0.33 \leq \sigma_x/\sigma_y \leq 3.0)\), good approximations can be obtained by a straight line fit to the exact function:

\[
\text{ECEP} = 0.615 \sigma_y + 0.562 \sigma_x \quad (\sigma_y \leq \sigma_x)
\]

\[
\text{ECEP} = 0.615 \sigma_x + 0.562 \sigma_y \quad (\sigma_x \leq \sigma_y)
\]

Other common approximations used include: the geometric mean \(\sqrt{\sigma_x \sigma_y}\), which is accurate for very low values of cumulative probability and is reasonably accurate up to a cumulative probability of 0.4; the arithmetic mean \(\left(\left(\sigma_x + \sigma_y\right)/2\right)\) which is excellent at 0.6 and is often used for intermediate values including the 50% point (CEP); and, the root-mean-square \(\left(\left(\sigma_x^2 + \sigma_y^2\right)/2\right)^{1/2}\), a good approximation above a cumulative probability of 0.75. The best overall approximation is the curve fit since the interval of accuracy extends over and beyond the other three. These approximations in terms of the dimensionless parameters \((\text{ECEP}/\sigma_x; \sigma_y/\sigma_x)\) and \((\text{ECEP}/\sigma_y; \sigma_x/\sigma_y)\) are shown in Figure 1-1.
Figure 1-10

EQUIVALENT CIRCULAR ERROR PROBABLE

\begin{align*}
\text{Root Mean Square} & : 1.1774 \left( \frac{\sigma_x^2 + \sigma_y^2}{2} \right) \\
\text{Geometric Mean} & : 1.1774 \left( \sigma_x \sigma_y \right)^{\frac{1}{2}} \\
\text{Arithmetic Mean} & : 1.1774 \left( \frac{\sigma_x + \sigma_y}{2} \right) \\
\text{Exact} & : \\
\text{Straight Line Approximation} & : \\
& \begin{cases} 
0.615 \sigma_y + 0.562 \sigma_x & (\sigma_y \leq \sigma_x) \\
0.615 \sigma_x + 0.562 \sigma_y & (\sigma_x \leq \sigma_y)
\end{cases}
\end{align*}
Figure 1-2

CIRCULAR NORMAL DISTRIBUTION
(Drawn to Scale)
CHAPTER II

OPTIMUM DAMAGE PROBABILITY AGAINST POINT TARGETS

A. Singly Delivered Weapons Against a Point Target


This section contains the development of the damage equations for an area weapon delivered against a point target. An area weapon is defined as a weapon system which contains a number \(n\) of submunitions which are released in a salvo (simultaneously). In most cases, a cluster (packaged submunitions) is released as a unit and at some point along its trajectory dispenses the submunitions in a salvo.

A point target is defined as a single target containing one or more vulnerable components, any of which or any combination of which, satisfies the damage criterion if rendered inoperative.

a. The Probability of Coverage Function

The probability of coverage function \(P_c\) can be approximated by the radial distribution. It has been shown in reference 9, and is briefly discussed in the introduction, that the circular normal or radial distribution function is an excellent approximation of the elliptic normal distribution for \(0.33 \leq \sigma_x/\sigma_y \leq 3.0\). In these cases it is generally more appropriate to convert from standard deviation to circular probable error (CEP).

For the case of a point target \(R_T \ll R_p\) centered at \((0,0)\), the probability that the target centroid will lie within the weapon's pattern radius \(R_p\) is the density function integrated over \(R_p\). It can be assumed that the pilot can identify and is delivering weapons to the target.
center with standard deviation error \( \sigma \). Thus the expected position of the munition's pattern center is the target centroid.

\[
P(R_p) = \int_0^R \left[ \frac{r}{\sigma} \exp\left(-\frac{r^2}{2\sigma^2}\right) \right] \, dr
\]

(2-1)

\[
P_C = P(R_p) = \left[ 1 - \exp\left(-\frac{R_p^2}{2\sigma^2}\right) \right]
\]

(2-2)

This is a special case of the probability of covering an area target with an area munition with the area target radius degenerating to zero. The development of the more general problem will be covered in Chapter 3. The point target problem is separated from the general problem primarily due to the higher frequency with which the former appears in relation to the latter. Approximately 80% of all aerially attacked surface targets can be classed as point targets.

Equation (2-2) is not necessarily restricted to point target applications. As a matter of fact, it is an excellent approximation for most area targets of interest in tactical conventional and counter-insurgency warfare. For target radii to standard deviation ratios on the interval \( 0 \leq R_T/\sigma \leq 0.5 \), Equation (2-2) accurately describes the circular coverage function over the entire range of

\[
P_p(0 \leq R_p/\sigma \leq \infty)
\]

Restrictions on the latter interval increase the range of \( R_T/\sigma \). For example, for \( R_p/\sigma \geq 3.0 \), the function is accurate over the interval \( 0 \leq R_T/\sigma \leq 1.5 \), for \( R_p/\sigma \geq 4.0 \), it is extended over the interval \( 0 \leq R_T/\sigma \leq 3.0 \), and for \( R_p/\sigma \geq 5.0 \), it applies to \( 0 \leq R_T/\sigma \leq 4.0 \).
Even for the best contemporary combat delivery accuracies, the ratios $R_p/\sigma$ and $R_T/\sigma$ will not normally exceed 5.0. However, the advent of improved delivery accuracy necessitates an extension of the principles over larger intervals. As the standard deviation of error is improved, the range on $R_T$ must be reduced proportionately to maintain the specified intervals of $R_T/\sigma$. Developments in the next chapter will relate the extension over the interval $(0 \leq R_T/\sigma \leq \infty)$ for all $R_p (0 \leq R_p/\sigma \leq \infty)$.

b. The Conditional Damage Function

For weapons systems having a random uniform distribution of bomblets within the pattern area ($A_p$) bounded by a singly connected curve, the Poisson approximation adequately represents the conditional damage probability ($P_{D/C}$). The conditional damage probability is defined as the average damage over the weapon pattern (Figure 2-1).

Let

$$p(k \text{ successes}) = \exp (-\mu) \frac{\mu^k}{k!} \quad (2-3)$$

where

$$\mu = \frac{r_1 n \text{ MAE}_B}{A_p} = \frac{r_1 n R_{LB}^2}{R_p^2} \quad (R_{LB} \ll R_p)$$

The mean value $\mu$ is the ratio of the total area within which the damage criterion is satisfied to the total pattern area. The total area of effectiveness is the product of the number of bomblets with the system and the individual bomblet MAE. Therefore,
\[
\mu = \sum_{i=1}^{n} \frac{r_{iMAE_{B_{i}}}}{\pi R_{p}^2} = \frac{r_{1nR_{LB}^2}}{R_{p}^2}
\]

\[
P_{D/C} = 1 - \exp \left( -\frac{r_{1nR_{LB}^2}}{R_{p}^2} \right)
\]

Figure 2-1

CONDITIONAL DAMAGE FOR A SINGLE AREA WEAPON
\[ P_{D/C} = 1 - p(0 \text{ Successes}) = 1 - \exp (-\mu) \mu^0 / 0! \]

or

\[ P_{D/C} = \left[ 1 - \exp (-r_{1n} R_{LB}^2 / R_P^2) \right] \]  \hspace{1cm} (2-4)

c. The Probability of Damage Function

The product of Equations (2-1) and (2-3) leads to:

\[ P_D = \left[ 1 - \exp (-r_{1n} R_{LB}^2 / R_P^2) \right] \left[ 1 - \exp (-R_P^2 / 2\sigma^2) \right] \]  \hspace{1cm} (2-5)


Unitary weapons are divided into two separate groups for classification but can be treated identically in the development of the damage function. One class of unitary weapons is characterized by high-explosive/blast/fragmentation warheads for which the MAE concept is defined. Another class is comprised of kinetic energy and shaped-charge penetration warheads, for which the vulnerable area \( (A_v) \) concept is defined. For unitary weapons characterized by the MAE concept, the conditional damage probability is unity. The MAE is defined in such a manner that, given the target centroid is located within the MAE, the damage criterion is satisfied, thus reducing the problem to coverage expectancy. The vulnerable area concept is defined such that, given an impact within the vulnerable area, the damage criterion is satisfied thereby reducing this problem to one of hit expectancy. Thus,
the two concepts are equivalent. Since in the case of unitary warheads, \( MAE = A_p \), the damage probability based on the Rayleigh distribution becomes:

\[
PD = \left[1 - \exp\left(-\frac{MAE}{2\pi \sigma^2}\right)\right] \tag{2-6}
\]

and for the vulnerable area concept:

\[
PD = \left[1 - \exp\left(-\frac{A_V}{2\pi \sigma^2}\right)\right] \tag{2-7}
\]

and since \( MAE = \pi R_L^2 \) and \( A_V = \pi R_V^2 \),

\[
PD = \left[1 - \exp\left(-\frac{R_L^2}{2\sigma^2}\right)\right] \tag{2-8}
\]

and

\[
PD = \left[1 - \exp\left(-\frac{R_V^2}{2\sigma^2}\right)\right] \tag{2-9}
\]

3. Maximization of the Damage Functions

The damage function in Equation (2-5) may be maximized to:

\[
PD = \left[1 - \exp\left(-\frac{R_p^2}{2\sigma^2}\right)\right] \tag{2-10}
\]

simply by increasing the number of bomblets within the weapon without bound, or both. None of these alternatives is economically feasible and even if such was the case, the limiting case (2-10) becomes
analogous to the already limiting case for unitary weapons as reflected in Equations (2-8) and (2-9). Within practical employment and economic constraints, the damage function may be maximized for a fixed munitions design concept by controlling pattern size as a function of the standard deviation of error ($\sigma$), the number of submunitions ($n$) and the MAE$_B$ of the individual submunitions. This approach is practical since pattern size is a unique function of the aircraft weapon release parameters and weapon (fuze) function altitude. Thus, for a given standard deviation or error, a fixed number of submunitions each having a mean area of effectiveness characterized by ($R_{LB}$), the damage function may be maximized by determining the optimum pattern radius for the above constraints. The optimum pattern radius may be achieved by specifying aircraft release parameters and a fuze function altitude (altitude at which the cluster releases the submunitions) which may be preset or electronically set from the cockpit.

As previously related, the damage function may be specified by:

$$P_D = P_{D/C} \cdot P_C$$

Operating on the damage function

$$\frac{\partial P_D}{\partial R_p} = P_{D/C} \frac{\partial P_C}{\partial R_p} + P_C \frac{\partial P_{D/C}}{\partial R_p}$$  (2-11)
Equating \( \frac{dP_D}{dR_P} \) to zero and solving for

\[
\frac{P_C}{P_{D/C}} = - \frac{\frac{\partial P_C}{\partial R_P}}{\frac{\partial P_{D/C}}{\partial R_P}} \tag{2-12}
\]

Then in terms of Equation (2-5),

\[
\frac{\partial P_C}{\partial R_P} = \frac{R_P}{\sigma^2} \exp \left[-\frac{R_P^2}{2\sigma^2} \right] \tag{2-13}
\]

\[
\frac{\partial P_{D/C}}{\partial R_P} = - \frac{2r_1^2 R_{LB}^2}{R_P^3} \exp \left[-\frac{r_1^2 R_{LB}^2}{R_P^2} \right] \tag{2-14}
\]

Substituting these relations and the expressions for \( P_C \) and \( P_{D/C} \) into (2-12) yields:

\[
\frac{1 - \exp \left(-\frac{R_P^2}{2\sigma^2} \right)}{1 - \exp \left(-\frac{r_1^2 R_{LB}^2}{R_P^2} \right)} = \frac{R_P^4 \exp \left[-\frac{R_P^2}{2\sigma^2} \right]}{2r_1^2 R_{LB}^2 \sigma^2 \exp \left[-\frac{r_1^2 R_{LB}^2}{R_P^2} \right]} \tag{2-15}
\]

Equation (2-15) is satisfied when:

\[
R_P^4 = 2r_1^2 R_{LB}^2 / \sigma^2 \tag{2-16}
\]
The pattern radius which maximizes the damage function is:

$$R_p = \frac{1}{4} (2r_1^n R_{LB}^2 \sigma^2) \quad (2-17)$$

Substituting this expression into Equation (2-5) and simplifying, yields the damage function for the optimum pattern radius:

$$P_D = \left[1 - \exp \left(-R_{LB} \sqrt{\frac{r_1 n}{\sqrt{2} \sigma}} \right) \right]^2 \quad (2-18)$$

In many instances, it is desirable to determine the weapon system characteristics which will yield a desired level of damage ($P_D$) for any given submunition type and standard deviation of error $\sigma$. The number of bomblets required to obtain a level of damage $P_D$ with standard deviation of aiming error $\sigma$ can be determined from (2-18).

$$n = \frac{(2/r_1)^2 (\sigma/R_{LB}) \phi n \left(1 - \sqrt{P_D} \right)^2}{(2-19)}$$

The number of submunitions $n$ (of a given type) completely specifies the weight, volume, and physical characteristics of the weapon system required to yield the desired probability of damage for any given submunition design constrained by a standard deviation of error $\sigma$.

It is significant to note that the condition

$$R_p^4 = 2r_1^n R_{LB}^2 \sigma^2 \quad (2-18)$$
is equivalent to the following expressions:

\[
\frac{R_p^2}{2\sigma^2} = \frac{r_1 n R_{LB}^2}{R_p^2}
\]

This implies that the damage function is maximized when the conditional damage over the weapons pattern is equal to the probability of coverage \((P_{D/C} = P_C)\).

4. Optimum Cluster Weapons Versus Optimum Unitary Weapons
   
   a. Optimum Cluster in Terms of Circular Probable Error

   The circular probable error can be expressed in terms of the standard deviation of aiming error:

   \[
   CEP = \sqrt{2 \ln 2} \sigma
   \]

   or

   \[
   \sigma = CEP/\sqrt{2 \ln 2}
   \]

   substituting this expression into Equation (2-18)

   \[
   P_D = \left[1 - \exp \left(-R_{LB} \sqrt{r_1 n \ln 2/CEP}\right)\right]^2
   \]

   \[(2-20)\]
or, in terms of CEP

\[ \text{CEP}^2 = \frac{r_n R_{LB}^2 \varrho n^2}{\left[ \varrho n (1 - \sqrt{\frac{P_D}{D}}) \right]^2} \]  

(2-21)

\[ n = \sum_{i=1}^{n} \frac{W_{Bi}/W_B}{W_B} \]

\[ K_1 = \frac{\text{Total Cluster Weight}}{\text{Total Bomblet Weight}} = \frac{W_c}{n} = \frac{W_c}{\sum_{i=1}^{n} W_{Bi}} \]

\[ K_2 = W_B \]

b. Circular Probable Error in Terms of weapon Weight

It is desirable to compare the effectiveness of cluster weapons and unitary weapons on an equal weight basis or cost/weight basis since weapons weight is a primary penalty on aircraft performance in terms of acceleration, range and endurance. Cost may enter the problem since usually minimization of the penalty must be traded off against an increase in costs.

Define a constant \( K_1 \) such that:

\[ K_1 = \frac{\text{Total Cluster Weight}}{\text{Total Bomblet Weight}} = \frac{W_c}{n} = \frac{W_c}{\sum_{i=1}^{n} W_{Bi}} \]

\( K_1 \) is the reciprocal of the packaging efficiency. Define a constant \( K_2 \) such that:

\[ K_2 = W_B \]

Then, the number of bomblets in the cluster can be expressed as:

\[ n = \sum_{i=1}^{n} \frac{W_{Bi}/W_B}{W_B} \]
and the cluster weight as:

\[ W_C = \left( W_C' \sum_{i=1}^{n} W_{B_i} \right)(W_{B_i})^n = K_1K_2n \]

or in terms of \( n \),

\[ n = \frac{W_C}{K_1K_2} \]

Substituting this expression into Equation (2-21)

\[ \text{CEP}^2 = \frac{r_1 W_C R_{LB}^2 \ln n^2}{K_1K_2 \left[ \ln (1 - \sqrt{P_D}) \right]^2} \quad (0 < P_D < 1) \quad (2-22) \]

c. Optimum Unitary in Terms of Circular Probable Error

The expression for \( \sigma \) can be substituted into Equation (2-6) yielding:

\[ P_D = \left[ 1 - \exp \left( -\frac{\text{MAE} \ln n^2/\pi \text{CEP}^2}{\ln \left( 1 - \sqrt{P_D} \right)} \right) \right] \quad (2-23) \]

In terms of \( \text{CEP}^2 \),

\[ \text{CEP}^2 = -\frac{\text{MAE} \ln n^2}{\pi \ln \left( 1 - \frac{P_D}{P_D} \right)} \quad ; \quad 0 < P_D < 1 \quad (2-24) \]
d. Circular Probable Error in Terms of Unitary Weapon Weight

Examination of the behavior of MAE (Figure 2-2) as a function of weapon weight for unitary weapons reveals a linear logarithmic-logarithmic relationship for constant (or nearly constant) charge mass to metal mass ratios (C/M). Of particular interest is the family of general purpose unitary weapons (C/M \approx 1.0) (Figure 2-2).

The mean area of effectiveness may be expressed as:

\[ B_n \text{MAE} = \ln K + \alpha \ln W_u \]

where \( K \) is the MAE of the smallest weight being considered (equivalent to the MAE axis intercept), \( \alpha \) is the slope of the line, and \( W_u \) is the weight of the unitary weapon being evaluated.

Alternatively,

\[ \text{MAE} = K W_u^\alpha \]

substituting this expression into Equation (2-24)

\[ CEP^2 = -\frac{K W_u^\alpha B_n^2}{\pi \ln (1-P_D)} ; \quad |0 < P_D < 1| \quad (2-25) \]


e. Cluster Weight versus Unitary Weight

By equating Equations (2-25) and (2-24) the following expression is obtained:
Figure 2-2

TYPICAL UNITARY MAE

1. Light Material
2. Light Material
3. Motor Vehicle (Target 1)
4. Prone Personnel (Target 2)
5. Motor Vehicle
6. Light Armor

MAE - $F_c^2 \times 10^{-3}$

Weapon Weight - LB $\times 10^{-3}$
This approach permits the development of a weight comparison when both weapon systems are delivered with the same aiming error, or equivalently, from identical delivery aircraft.

Solving for $W_C$ in terms of $W_u^\alpha$

\[
W_C = - \left( \frac{K_1 K_2}{\pi r_1 R_{LB}} \right)^2 \left\{ \frac{\ln (1 - \sqrt{P_D})^2}{\ln (1 - P_D)} \right\} W_u^\alpha ; \quad (0 < P_D < 1)
\] (2-26)

For any specified level of damage desired ($P_D$) and for equal weapon system weights ($W_C = W_u$) where the weapon systems designs are specified:

\[
\beta = - \left( \frac{K_1 K_2}{\pi r_1 R_{LB}} \right)^2 \left\{ \frac{\ln (1 - \sqrt{P_D})^2}{\ln (1 - P_D)} \right\} ; \quad (0 < P_D < 1)
\]

and Equation (2-26) becomes

\[
W_C = \beta W_u^\alpha
\]
or

\[ W = \beta W^\alpha = \beta \left( \frac{1}{1-\alpha} \right) \]

and finally,

\[ W = -\left( \frac{K_1 K_2 K}{r_1 \pi R_{LB}} \right) \left( \frac{\ln (1 - \sqrt{P_D})}{\ln (1 - P_D)} \right)^2 \left( \frac{1}{1-\alpha} \right) \]

with either Equation (2-22) and (2-25) and parametric values of \( P_D \) (0 < \( P_D < 1.0 \)), weight and CEP; a curve is obtained which divides the CEP-weight plane into distinct areas where either cluster weapons or unitary weapons are preferred. In addition, Equations (2-22) and (2-25) will yield a family of constant \( P_D \) curves (parallel straight lines in the logarithmic CEP^2 vs weight plane) which may be utilized both as a weapon effectiveness tool or as a parametric design tool.

B. Multiply Delivered Weapons Against a Point Target

1. The Probability of Coverage, Conditional Damage and Probability of Damage Functions for Multiply Delivered Area Weapons

This section contains the development of the damage equations for multiply delivered area weapons against a point target. The development is an excellent approximation for multiple cluster patterns which are circular (Figure 1-2) and does not diverge severely from the numerically integrated solution for rectangular weapons patterns in
the interval \(0.33 \leq \frac{P_w}{P_L} \leq 3.0\) where \(P_w\) is the pattern width and \(P_L\) is the pattern length. In this case, the pattern area \(P_w P_L\) is approximated by \(\pi R_p^2\).

a. The Probability of Coverage Function

The coverage function is identically the same form as defined in paragraph (II-A-1-a). However, in this case it is desirable to determine the probability that the target lies within the multiple area weapons pattern of radius \(R_w\). The damage function will be developed later in terms of \(R_w\).

In a procedure identical to the approach in paragraph (II-A-1-a) the coverage function can be developed. It becomes:

\[
P_C' = \left[1 - \exp\left(-\frac{R_w^2}{2}\sigma^2\right)\right]
\]  

b. The Conditional Damage Function

The product of the conditional damage within a single cluster pattern and the pattern area is defined as the mean area of effectiveness of the cluster (Figure 2-3).

\[
\text{MAE}_C = \pi \frac{R_p^2}{P_D/C}
\]

The mean value can then be determined

\[
\mu = N(\text{MAE}_C) / A_w = N \frac{R_p^2}{P_D/C} R_w^2
\]
\[
\mu = \sum_{i=1}^{N} \pi R_{pL}^2 p_{D/C} / \pi R_{W}^2 = NR_{p}^2 p_{D/C} / R_{W}^2 \quad \text{(Area)}
\]

\[
\mu = \sum_{i=1}^{m} \pi R_{pL}^2 / \pi R_{W}^2 = \frac{m}{R_{W}} \quad \text{(Unitary)}
\]

\[
P_{D/C} = 1 - \exp\left(-NR_{p}^2 p_{D/C} / R_{W}^2\right) \quad \text{(Area)}
\]

\[
P_{D/C} = 1 - \exp\left(-mR_{p}^2 / R_{W}^2\right) \quad \text{(Unitary)}
\]

Figure 2-3

CONDITIONAL DAMAGE FOR MULTIPLY DELIVERED WEAPONS
It should be noted that the mean value in this sense is not the same as would be derived for the case of the product \( N \) bomblets distributed uniformly over \( A_W \) determined by

\[
\mu = r_1 N \frac{R_{LB}^2}{R_W^2}
\]

When area weapons are delivered multiply, \( n \) bomblets per pattern are constrained to lie in \( N \) pattern areas, the \( N \) patterns distributed randomly over \( A_W \). The two terms converge only for \( N \) coincident patterns of radius \( R_p = R_W \).

Therefore, the conditional damage for multiply delivered area weapons becomes:

\[
P_{D/C}' = \left[ 1 - \exp \left( -NR_p^2 \frac{P_{D/C}}{R_W^2} \right) \right] \quad \text{(2-29)}
\]

where \( R_p \) is determined by Equation (2-17) and \( P_{D/C} \) by Equation (2-4).

c. The Probability of Damage Function

The probability of damage is determined by the product of Equations (2-28) and (2-29)

\[
P'_D = \left[ 1 - \exp \left( -NR_p^2 \frac{P_{D/C}}{R_W^2} \right) \right] \left[ 1 - \exp \left( -R_W^2 / 2 \sigma^2 \right) \right] \quad \text{(2-30)}
\]
2. The Probability of Coverage, Conditional Damage and Probability of Damage Functions for Multiply Delivered Unitary Weapons

The substitution of \( R_w \) for \( R_p \) and \( R_L \) for \( R_{LB} \) in Equations (2-3), (2-4) and (2-5) yield respectively, the probability of coverage, conditional damage and probability of damage functions for \( m \) unitary weapons distributed uniformly and at random over an area \( A_w \) delivered against a point target.

\[
P_c' = \left[ 1 - \exp \left( - \frac{R_w^2}{2 \sigma^2} \right) \right]
\]

(2-31)

\[
P_{D/C}' = \left[ 1 - \exp \left( - \frac{m R_L^2}{R_w^2} \right) \right]
\]

(2-32)

\[
P_D' = \left[ 1 - \exp \left( - \frac{m R_L^2}{R_w^2} \right) \right] \left[ 1 - \exp \left( - \frac{R_w^2}{2 \sigma^2} \right) \right]
\]

(2-33)

where

\[
\mu = m \pi \frac{R_L^2}{R_w^2} = m \frac{R_L^2}{R_w^2}
\]

is the mean value over the pattern \( R_w \).

3. Maximization of the Damage Functions

a. Multiply Delivered Area Munitions

\[
\frac{\partial P_D'}{\partial R_w} \cdot P_{D/C}' + \frac{\partial P_C'}{\partial R_w} \cdot P_C' + \frac{\partial P_D'}{\partial R_p} \cdot P_D/C' = 0
\]
operation on Equations (2-28) and (2-29) results in:

\[
\frac{P_D}{P_{D/C}} = \frac{\partial P_C/\partial R_w}{\partial P_{D/C}/\partial R_w}
\]

\[
\left[1 - \exp\left(-\frac{R_w^2}{2\sigma^2}\right)\right] = \frac{R_w^4\left[\exp\left(-\frac{R_w^2}{2\sigma^2}\right)\right]}{2NR_p^2 P_{D/C} \sigma^2 \left[\exp\left(-\frac{R_w^2}{2\sigma^2}\right)\right]}
\]

which is satisfied when

\[
R_w = (2NR_p^2 P_{D/C} \sigma^2)^{1/4}
\]

(2-34)

the radius which maximizes the damage function.

Substitution of (2-34) into (2-30) yields the damage function for the optimum pattern radius

\[
P_D' = \left[1 - \exp\left(-R_p \sqrt{\frac{N P_{D/C}}{\sqrt{2} \sigma}}\right)\right]^2
\]

(2-35)

and

\[
N = \left(\frac{2}{P_{D/C}}\right) \left[\frac{(\sigma/R_p) \ln (1 - \sqrt{P_D'})}{2}\right]^2
\]

(2-36)

is the number of clusters, with individual pattern radii \(R_p\) and overall pattern radius \(R_w\), required to achieve a specified level of damage \(P_D'\) when delivered with aiming error \(\sigma\).
b. Multiply Delivered Unitary Weapons

Similar procedures applied to Equations (2-31), (2-32) and (2-33) yield:

The pattern which maximizes the damage function,

$$R_W = (2m R_L^2 \sigma^2)^{1/4}$$  \hspace{1cm} (2-37)

The maximized damage function,

$$P_D' = \left[ 1 - \exp \left( -\frac{R_L}{m} \sqrt{\frac{m}{\sigma^2}} \right) \right]^2$$  \hspace{1cm} (2-38)

and,

$$m = 2 \left( \frac{\sigma}{R_L} \right) \ln \left( 1 - \sqrt{\frac{P_D'}{P_D}} \right)^2; \hspace{0.5cm} (0 < P_D' < 1)$$  \hspace{1cm} (2-39)

is the number of bombs necessary to achieve a level of damage $P_D'$ when distributed over $R_W$ and delivered with aiming error $\sigma$.

4. Optimum Cluster Weapons vs Optimum Unitary Weapons for Multiple Weapons Against a Point Target

Converting $\sigma$ to CEP in Equations (2-35) and (2-38) and solving for CEP$^2$:

$$\text{CEP}^2 = \frac{N P_D/C R_P^2 \sigma n^2}{\ln \left( 1 - \sqrt{\frac{P_D'}{P_D}} \right)^2}; \hspace{0.5cm} (0 < P_D' < 1)$$  \hspace{1cm} (2-40)
In Equation (2-40) there are $N$ clusters each having weight $W_C = K_1 K_2 \eta n$ as defined in paragraph (II-A-4-b).

\[ NW_C = W_{CT} \]

or,

\[ N = W_{CT}/W_C \]

Therefore,

\[ CEP^2 = \frac{m R_L^2 \eta n^2}{\left[ \eta n \left( 1 - \sqrt{P_D} \right) \right]^2} ; \quad (0 < P_D < 1) \quad (2-41) \]

In Equation (2-41) there are $m$ unitary weapons each having weight $W_u$. The total weight of the $m$ unitary weapons is:

\[ W_{uT} = m W_u \]

or

\[ m = W_{uT}/W_u \]
Therefore,

\[
\begin{align*}
\text{CEP}^2 &= \frac{W_u T R_L^2 \ln 2}{W_u \ln \left(1 - \sqrt{P_D}\right)^2} ; \quad (0 < P_D < 1) \\
\end{align*}
\]  

(2-43)

To compare the effectiveness of these two systems for equal aircraft loadouts in terms of total weight of munitions expended and equal delivery accuracy, equate (2-42) to (2-43) and solve for \( W_{CT} \) in terms of \( W_u T \).

\[
W_{CT} = \left( \frac{R_L}{R_p} \right)^2 \left( \frac{W_C}{W_u} \right) \left( \frac{1}{P_{D/C}} \right) W_u T \quad ; \quad (0 < P_{D/C} \leq 1) 
\]

(2-44)

This relationship is independent of the damage probability as previously stated. It is assumed that both systems are delivered with the same accuracy. The following can be ascertained from Equation (2-44):

Clusters are preferred if:

\[
\left( \frac{R_L}{R_p} \right)^2 \left( \frac{W_C}{W_u} \right) \left( \frac{1}{P_{D/C}} \right) < 1 \quad ; \quad (0 < P_{D/C} \leq 1) 
\]

(2-45a)

Unitary weapons are preferred if:

\[
\left( \frac{R_L}{R_p} \right)^2 \left( \frac{W_C}{W_u} \right) \left( \frac{1}{P_{D/C}} \right) > 1 \quad ; \quad (0 < P_{D/C} \leq 1) 
\]

(2-45b)

where \( P_{D/C} \) is determined by Equation (2-4).
C. Single Massive Clusters versus Multiply Delivered Small Clusters

Many questions have arisen concerning the desirability of developing single massive clusters as opposed to large numbers of smaller clusters. These systems will be compared in the following paragraphs on an equal weight basis.

Let \( W_{MG} \) = Weight of the Massive Cluster

\[ W_{CT} \] = Weight of N Small Clusters

The clusters are assumed to contain identical submunitions, the number in the massive cluster will be denoted as \( M \) and each small cluster will contain \( n \) bomblets.

The damage function for the massive cluster is:

\[
P_D = \left[ 1 - \exp \left( - \frac{MR_{LB}}{R_W^2} \right) \right] \left[ 1 - \exp \left( - \frac{R_p^2}{2\sigma^2} \right) \right] \quad (2-46)
\]

and that of \( N \) smaller clusters is given by:

\[
P_D = \left[ 1 - \exp \left( - \frac{NR_p^2}{2} \frac{P_{D/C}}{R_W^2} \right) \right] \left[ 1 - \exp \left( - \frac{R_p^2}{2\sigma^2} \right) \right] \quad (2-47)
\]

where \( P_{D/C} \) is taken from (2-4), \( R_p \) is the pattern radius of a single small cluster, \( R_{LB} \) is the lethal radius of a single bomblet, and \( R_W \) is the radius of the overall munition pattern on the ground.
Equation (2-46) is maximized when $R_w = \left(2MR_{LB}^2 \sigma^2 \right)^{1/4}$

$$P_D = \left[1 - \exp \left(- \frac{R_{LB}}{\sqrt{r_1 M/\sqrt{2} \sigma}} \right) \right]^2$$  \hspace{1cm} (2-48)

and (2-47) has a maximum given by (2-35).

$$P_D = \left[1 - \exp \left(- \frac{R_p}{\sqrt{N P_{D/C} \sqrt{2} \sigma}} \right) \right]^2$$  \hspace{1cm} (2-49)

In terms of $\text{CEP}^2$ (2-48) becomes

$$\text{CEP}^2 = \frac{r_1 W_{MC} R_{LB}^2 \ln 2}{K_1 K_2 \left[ \ln (1 - \sqrt{P_D}) \right]^2}$$

and (2-49) becomes

$$\text{CEP}^2 = \frac{W_{CT} P_{D/C} R_p^2 \ln 2}{W_C \left[ \ln (1 - \sqrt{P_D}) \right]^2}$$

For equal delivery accuracy and $W_C = K_1 K_2 n$

$$\frac{W_{CT} P_{D/C} R_p^2 \ln 2}{K_1 K_2 n \left[ \ln (1 - \sqrt{P_D}) \right]^2} = \frac{r_1 W_{MC} R_{LB}^2 \ln 2}{K_1 K_2 \left[ \ln (1 - \sqrt{P_D}) \right]^2}$$
or,

\[ W_{GT} = \left( \frac{r_1 n R_{LB}^2}{R_p^2 P_{D/C}} \right) W_m C \quad (0 < P_{D/C} \leq 1) \quad (2-50) \]

The smaller clusters are preferred if:

\[ \frac{r_1 n R_{LB}^2}{R_p^2 P_{D/C}} < 1 \quad (0 < P_{D/C} \leq 1) \]

and conversely.
CHAPTER III
OPTIMUM DAMAGE PROBABILITY AGAINST AREA TARGETS

A. Singly Delivered Weapons Against an Area Target

1. The Probability of Fractional Coverage, Conditional Damage
   and Fractional Damage Functions for Area Weapons

   a. The Conditional Damage Function

   The conditional damage within the area of overlap between
   the target area and the pattern is identically the average probability
   of damage over the weapon pattern given in Equation (2-4) (Figure 3-1).
   It is repeated here for convenience.

   \[ P_{D/C} = \left[ 1 - \exp \left( - r_{1n} \frac{R_T}{R_p} \right) \right] \]  \hspace{1cm} (3-1)

   b. The Fractional Coverage Functions

   No single closed form approximation for the fractional
   coverage function could be found which yielded the desired accuracy
   over the entire \( R_T/\sigma \) range \( 0 \leq R_T/\sigma \leq \infty \). It was found that two
   expressions over appropriate intervals yielded acceptable accuracy
   over the applicable range of values.

   In the previous chapter, it was shown that the coverage function
   given in Equation (2-2) was accurate for all tactical surface targets
   \( R_T/\sigma \geq 0.5 \) and \( R_T/\sigma < \infty \). For the range \( 0.5 < R_T/\sigma < R_p/\sigma \) the coverage function is more

   For the range \( 0.5 < R_T/\sigma < R_p/\sigma \) the coverage function is more
   accurately approximated by:

   \[ F_C = C_1 \left[ 1 - \exp \left( - C_2 \frac{R_T}{\sigma} \right) \right] \]  \hspace{1cm} (3-2)
Figure 3-1
SINGLE AREA OR SINGLE UNITARY WEAPON COVER FUNCTION

\[ F_C = \frac{A_c}{\pi R_T^2} = \text{Percentage of Target Area Covered} \]
Over the interval \((R_p/\sigma \leq R_T/\sigma \leq \infty)\), the coverage function is approximated by:

\[
F_C = C_3 \frac{R_p^2}{R_T^2} \left[ 1 - \exp \left( - C_4 \frac{R_T^2}{\sigma^2} \right) \right]
\]  \hspace{1cm} (3-3)

A sequential unconstrained minimization technique employing a non-gradient parameter search was used to determine the values of the coefficients \(C_i\) which minimized the error between the computed values of the coverage function and the numerically integrated values from reference 13. The initial error function utilized was:

\[
ER = \sum_{i=1}^{n} |F_{CC_i} - F_{CT_i}|
\]

where \(F_{CC_i}\) are the calculated values of the cover function and \(F_{CT_i}\) are the numerically integrated values. The relative minimum program searched for values \(C_i\) which minimized this error function. The resulting coefficients while producing extremely accurate results over ninety-five percent of the range, produced unacceptable deviations for \(R_T/\sigma \approx R_p/\sigma\). This error function was discarded in favor of minimizing the magnitude of the largest error, such that the maximum negative error matches the maximum positive error. For this fit the coefficients have the following values.

\[
\begin{align*}
C_1 &= 1.0 & C_2 &= 0.41 \\
C_3 &= 1.0 & C_4 &= 0.436
\end{align*}
\]
c. The Fractional Damage Functions

The fractional damage functions over the two intervals are given by the products of Equation (3-1) and (3-2) or (3-3) respectively.

\[
F_D = \left[1 - \exp\left(- r_1 n R_{LB}^2 / R_p^2\right)\right] \left[1 - \exp\left(- .41 R_p^2 / \sigma^2\right)\right] \tag{3-4a}
\]

\[
F_D = \left[1 - \exp\left(- r_1 n R_{LB}^2 / R_p^2\right)\right] \frac{R_p^2}{R_T} \left[1 - \exp\left(- .436 R_T^2 / \sigma^2\right)\right] \tag{3-4b}
\]

2. The Probability of Fractional Coverage, Conditional Damage and Fractional Damage Functions for Unitary Weapons

The conditional damage for unitary warheads is unity, a consequence of the manner in which the MAE is defined for any given damage or incapacitation criterion. The fractional damage is identically the fraction of the target covered.

\[
F_D = \left[1 - \exp\left(- .41 R_L^2 / \sigma^2\right)\right] \tag{3-5a}
\]

\[
F_D = \frac{R_L^2}{R_T^2} \left[1 - \exp\left(- .436 R_T^2 / \sigma^2\right)\right] \tag{3-5b}
\]
3. Maximization of the Damage Functions for Area Targets

In a manner similar to the procedure in paragraph (II-A-4), the fractional damage functions may be maximized by determining the constrained optimum pattern radius. The fractional damage function

\[ F_D = \frac{P_D}{C} \cdot F_C \]

is maximized when

\[ \frac{F_C}{P_D/C} = -\frac{\partial F_C/\partial R_p}{\partial P_D/C/\partial R_p} \]

a. Singly Delivered Area Weapons

On the interval \((0.5 < R_p/\sigma < R_p/\sigma)\), this expression results in:

\[
\frac{\left[1 - \exp(-0.41R_p^2/\sigma^2)\right]}{\left[1 - \exp(-r_{1\text{LB}}R_p^2/\sigma^2)\right]} = \frac{0.41 R_p^4 \exp(-0.41R_p^2/\sigma^2)}{r_{1\text{LB}}R_p^2 \exp(-r_{1\text{LB}}R_p^2/\sigma^2)}
\]

which is satisfied when

\[ R_p = \left(\frac{r_{1\text{LB}}^2 \sigma^2}{0.41}\right)^{1/4} \]  

\[(3-6)\]
the pattern radius which maximizes the fractional damage. However, on the interval \( \frac{R_p}{\sigma} \leq \frac{R_T}{\sigma} \leq \infty \) the result is:

\[
\frac{\frac{R_p^2}{R_T^2} \left[ 1 - \exp\left(-\frac{.436R_T^2}{2}\right) \right]}{\left[ 1 - \exp\left(-\frac{nR_{LB}^2}{R_p^2}\right) \right]} = \frac{R_p^4 \left[ \exp\left(-\frac{.436R_T^2}{2}\right) \right]}{R_p^4 \left[ \exp\left(-\frac{nR_{LB}^2}{R_p^2}\right) \right]}
\]

or

\[
R_p^2 = \frac{r_1 nR_{LB}^2 \left[ \exp\left(-\frac{nR_{LB}^2}{R_p^2}\right) \right]}{\left[ 1 - \exp\left(-\frac{nR_{LB}^2}{R_p^2}\right) \right]}
\]

substituting \( x = \frac{r_1 nR_{LB}^2}{R_p^2} \) yields after simplification:

\[
x + 1 = \exp(x)
\]

This expression is satisfied only at \( x = 0 \) and \( x = \infty \). This implies that at \( x = 0 \), \( R_p = \infty \) corresponding to a minimum conditional damage \( P_{D/C} = 0.0 \) and a maximum coverage probability \( F_C = 1.0 \). Also implicit is at \( x = \infty \), \( R_p = 0 \) corresponding to a maximum conditional damage \( P_{D/C} = 1.0 \) and a minimum coverage probability \( F_C = 0.0 \). In both cases, \( r_1 \), \( n \) and \( R_{LB} \) are positive real numbers, and the damage probability is identically zero \( P_D = 0.0 \).

The result is expected since by the imposed constraints \( R_p \leq R_T \), at \( R_p = \infty \), \( R_T = \infty \). The two solutions form a coincident pair of global minima for the damage function, and being the only two solutions, existence of a maximum is precluded. Because of the constraints imposed
as a consequence of the interval being considered ($R_p/\sigma \leq R_T/\sigma \leq \infty$) the optimum damage function is not obtainable identically. That is, the condition ($P_D/C = F_C$) is non-existent. Therefore, the best choice of pattern radius for any given munition/target/damage criterion combination must be determined numerically by iterating the pattern radius and evaluating the damage function. An illustration of this problem is contained in Chapter 5.

This interval is academic insofar as conventional munitions design is concerned but is of considerable value for weapons effectiveness assessments purposes. Although conventional munitions cannot be designed specifically for vast area targets (due to their low yield), it is often the case that the damage potential of multiple sortie or multiple mission strikes must be predicted. If Equation (3-6) is substituted into (3-4a) the following expression is obtained:

$$ F_D = \left[1 - \exp\left(-\frac{.41r_1n R_{LB}/\sigma}{r_1n R_{LB}/\sigma}\right)\right]^2 $$ (3-7)

This is the maximum damage for the optimum pattern radius. Solving for the number of bomblets necessary to achieve a given fraction of damage for any specified target, damage criterion and aiming error, yields:

$$ n = \left(\frac{1}{.41r_1}\right)\left[\left(\sigma/R_{LB}\right) \ln \left(1 - \sqrt{F_D}\right)\right]^2; \quad (0 < F_D < 1) $$ (3-8)
4. Optimum Cluster Weapons versus Optimum Unitary Weapons

Equations (3-7) and (3-5a) are converted to terms of CEP and solved for CEP$^2$.

Equation (3-7) becomes:

$$\text{CEP}^2 = \frac{0.82 \cdot \frac{r_1 n R_{LB}^2 \ln 2}{\ln (1 - \sqrt{F_D})^2}}{(0 < F_D < 1)} \quad (3-9)$$

or, in terms of weapon weight,

$$\text{CEP}^2 = \frac{0.82 \cdot \frac{r_1 W R_{LB}^2 \ln 2}{K_1 K_2 \ln (1 - \sqrt{F_D})^2}}{(0 < F_D < 1)} \quad (3-10)$$

Equation (3-5a) becomes:

$$\text{CEP}^2 = \frac{0.82 \cdot \frac{\ln 2}{\ln (1 - F_D)}}{(0 < F_D < 1)} \quad (3-11)$$

and in terms of weapon weight

$$\text{CEP}^2 = \frac{0.82 \cdot \frac{K W^\alpha \ln 2}{\ln (1 - F_D)}}{(0 < F_D < 1)} \quad (3-12)$$

Equating (3-10) and (3-12) and solving for

$$W_C = \frac{K_1 K_2}{r_1 \pi R_{LB}^2} \left[\frac{\ln (1 - \sqrt{F_D})^2}{\ln (1 - F_D)}\right] \frac{W_u^\alpha}{(0 < F_D < 1)} \quad (3-13)$$

which is identically (2-26) for $F_D = P_D$.

For equal system weights, this expression becomes:

$$W = \frac{K_1 K_2}{r_1 \pi R_{LB}^2} \left[\frac{\ln (1 - \sqrt{F_D})^2}{\ln (1 - F_D)}\right] \frac{1}{1 - \alpha} \quad (0 < F_D < 1) \quad (3-14)$$
By comparing Equations (2-27) and (3-14), it can be concluded that target area does not influence the choice of weapons systems for singly delivered weapons as long as the target consists of a distributed simple multiple of a point target. The term simple multiple refers to an area containing a number of identical or equally vulnerable point targets. An area containing a mixture of bare trucks and ordnance laden trucks as an example is excluded.

B. Multiply Delivered Weapons Against Area Targets

1. The Probability of Fractional Coverage, Conditional Damage and Probability of Fractional Damage Functions for Area Weapons

a. The Conditional Damage Function

The conditional damage function for multiple area weapons was developed previously as reflected in Equation (2-29) and is repeated here for convenience.

\[ P_{D/C} = \left[ 1 - \exp\left( -N R^2_{w} \frac{D/C}{R_W^2} \right) \right] \]  

(3-15)

b. The Fractional Coverage Function

Two different cases must be considered since the ratio \( R_W/\sigma \) may differ significantly from the ratio \( R_T/\sigma \). For \( R_T/\sigma \leq 0.5 \) and the interval \( 0 \leq R_W/\sigma \leq \infty \) the coverage function is given by:

\[ F_C = \left[ 1 - \exp\left( -R_W^2/2\sigma^2 \right) \right] \]  

(3-16a)
and for the range \((0.5 < R_T/\sigma < R_w^2/2\sigma^2)\) it is approximated by:

\[
F'_C = \left[1 - \exp\left(-0.41 \frac{R_w^2}{\sigma^2}\right)\right]
\]

(3-16b)

For theoretical completeness, the interval \(R_w/\sigma \leq R_T/\sigma \leq \infty\) must also be treated.

\[
F'_C = \frac{R_w^2}{R_T^2} \left[1 - \exp\left(-0.436 \frac{R_T^2}{\sigma^2}\right)\right]
\]

(3-17)

Again, the interval upon which (3-17) is based is of no practical value in conventional weapons systems design, and is included only for theoretical completeness.

c. The Fractional Damage Functions

The probability of fractional damage is found by taking the products of Equations (3-15) and (3-16a), (3-16b) or (3-17) respectively:

\[
F'_D = \left[1 - \exp\left(-N R_p^2 \frac{P_D}{C/R_w^2}\right)\right] \left[1 - \exp\left(-\frac{R_w^2}{2\sigma^2}\right)\right]
\]

(3-18a)

\[
F'_D = \left[1 - \exp\left(-N R_p^2 \frac{P_D}{C/R_w^2}\right)\right] \left[1 - \exp\left(-0.41 R_w^2/2\sigma^2\right)\right]
\]

(3-18b)

\[
F'_D = \left[1 - \exp\left(-N R_p^2 \frac{P_D}{C/R_w^2}\right)\right] \frac{R_w^2}{R_T^2} \left[1 - \exp\left(0.436 \frac{R_T^2}{\sigma^2}\right)\right]
\]

(3-18c)
2. The Probability of Fractional Coverage, Conditional Damage and Probability of Fractional Damage Functions for Unitary Weapons

a. The Conditional Damage Function

The conditional damage function for multiply delivered unitary weapons was developed earlier, in Equation (2-32), and is repeated here for convenience:

\[ P_{D/C} = \left(1 - \exp\left(-\frac{mR_L^2}{R_W^2}\right)\right) \]  (3-19)

b. The Fractional Coverage Functions

The fractional coverage functions for multiply delivered weapons with pattern radius \( R_W \) are identical to those for multiply delivered area weapons developed above. Equations (3-16a), (3-16b) and (3-17) are utilized over the appropriate intervals.

c. The Fractional Damage Functions

Taking the appropriate products, the fractional damage functions become:

\[ F_D = \left[1 - \exp\left(-\frac{mR_L^2}{R_W^2}\right)\right] \left[1 - \exp\left(-\frac{R_W^2}{2\sigma^2}\right)\right] \]  (3-20a)

\[ F_D = \left[1 - \exp\left(-\frac{mR_L^2}{R_W^2}\right)\right] \left[1 - \exp\left(-.41 \frac{R_W^2}{2\sigma^2}\right)\right] \]  (3-20b)

\[ F_D = \left[1 - \exp\left(-\frac{mR_L^2}{R_W^2}\right)\right] \left[1 - \exp\left(-.436 \frac{R_W^2}{\sigma^2}\right)\right] \]  (3-20c)
3. Maximization of the Damage Function

It can be shown in the manner described in the previous chapter that the damage functions are approximated by:

\[
\frac{F'_C}{P_{D/C}} = - \frac{\partial F'_C/\partial R_w}{\partial P_{D/C}/\partial R_w}
\]

a. Multiply Delivered Area Munitions

Operation on Equations (3-15) and (3-16a) leads to Equations (3-15) and (3-16a) leads to:

\[
\frac{1 - \exp(-0.41 R_w^2/\sigma^2)}{1 - \exp(-NR_p^2 P_{D/C}/R_w^2)} = \frac{0.41 R_w^4 \exp(-0.41 R_w^2/\sigma^2)}{NR_p^2 P_{D/C} \sigma^2 \exp(-NR_p^2 P_{D/C}/R_w^2)}
\]

This relation is satisfied when:

\[
R_w = (NR_p^2 P_{D/C} \sigma^2 / 0.41)^{1/4}
\]

(3-21)

the pattern radius which maximizes the fractional damage.

Substituting Equation (3-21) into (3-18b) the following expression results:

\[
F'_D = \left[1 - \exp(-\sqrt{0.41 NP_{D/C} R_p/\sigma})\right]^2
\]

(3-22)

This represents the maximum damage commensurate with the optimum pattern
Figure 3-2
MULTIPLE AREA OR MULTIPLE UNITARY WEAPON COVER FUNCTION

\[ F_C = \frac{A_C}{\pi R_T^2} \]
radius. The number of clusters required to achieve a specified fraction of damage is given by:

\[ N = (1 / 0.41 PD / C) \ln (1 - \sqrt{F_D'})^2 ; (0 < F_D' < 1) \] (3-23)

b. Multiply Delivered Unitary Munitions

Operation on Equations (3-19) and (3-16a) leads to

Equations (3-19) and (3-16b) leads to:

\[
\frac{1 - \exp(-0.41 R_w^2 / \sigma^2)}{1 - \exp(-mR_L^2 / R_w^2)} = \frac{0.41 R_w^2 \exp(-0.41 R_w^2 / \sigma^2)}{2mR_L^2 \sigma^2 \exp(-mR_L^2 / R_w^2)}
\]

This relation is satisfied when:

\[ R_w = (mR_L^2 \sigma^2 / 0.41)^{1/4} \] (3-24)

This is the pattern radius which maximizes the fractional damage. The maximum fractional damage is obtained by substituting this expression into Equation (3-20b) yielding:

\[ F_D' = \left[ 1 - \exp(-0.41m R_L / \sigma) \right]^2 \] (3-25)

The number of unitary weapons of lethal radius \( R_L \) required to achieve a specified fraction of damage \( F_D' \) against an area target of radius \( R_T \) when delivered with aiming error \( \sigma \) is found by solving the above expression for:
4. Optimum Cluster Weapons versus Optimum Unitary Weapons

Equations (3-22) and (3-25) are converted to terms of CEP and solved for CEP^2. Equation (3-22) becomes:

\[
\text{CEP}^2 = \frac{0.82 \, N \, \frac{P_{D/C}}{R} \, R_{P}^2 \, \ln^2 \, \left( \ln \left( 1 - \sqrt{F_D} \right) \right)^2}{\ln \left( 1 - \sqrt{F_D} \right)} ; \quad (0 < F_D < 1) \tag{3-27}
\]

And in terms of weapon weight:

\[
\text{CEP}^2 = \frac{0.82 \, W_{CP} \, P_{D/C} \, R_{P}^2 \, \ln^2 \, \left( \ln \left( 1 - \sqrt{F_D} \right) \right)}{W_{C} \left( \ln \left( 1 - \sqrt{F_D} \right) \right)^2} ; \quad (0 < F_D < 1) \tag{3-28}
\]

Equation (3-25) becomes:

\[
\text{CEP}^2 = \frac{0.82 \, m \, R_{L}^2 \, \ln^2 \, \left( \ln \left( 1 - \sqrt{F_D} \right) \right)}{\left( \ln \left( 1 - \sqrt{F_D} \right) \right)^2} ; \quad (0 < F_D < 1) \tag{3-29}
\]

And in terms of weapon weight:

\[
\text{CEP}^2 = \frac{0.82 \, W_{UL} \, R_{L}^2 \, \ln^2 \, \left( \ln \left( 1 - \sqrt{F_D} \right) \right)}{W_{U} \left( \ln \left( 1 - \sqrt{F_D} \right) \right)^2} ; \quad (0 < F_D < 1) \tag{3-30}
\]
Equating (3-28) and (3-30) and solving for $W_{CT}$:

$$W_{CT} = \left( \frac{R_L}{R_p} \right)^2 \left( \frac{W_C}{W_u} \right) \left( \frac{1}{P_{D/C}} \right) \ W_{UT} \ ; \ (0 < P_{D/C} \leq 1) \tag{3-31}$$

This expression permits comparisons of the two weapons systems on an equal weight basis. The expression is independent of the fractional damage, inferring that it holds for any specified level of damage, providing the respective systems being analyzed are capable of achieving the specified level of damage. As will be related in Chapter V, the weight requirements for high fractions of damage against area targets may be prohibitively large.

It is seen from Equation (3-31) that clusters are preferred if:

$$\left( \frac{R_L}{R_p} \right)^2 \left( \frac{W_C}{W_u} \right) \left( \frac{1}{P_{D/C}} \right) < 1 \ ; \ (0 < P_{D/C} \leq 1)$$

and, unitary weapons are preferred if:

$$\left( \frac{R_L}{R_p} \right)^2 \left( \frac{W_C}{W_u} \right) \left( \frac{1}{P_{D/C}} \right) > 1 \ ; \ (0 < P_{D/C} < 1)$$

It is significant to note that Equation (3-31) is identically (2-44) and it can be concluded that target area does not influence the choice of weapons systems for multiply delivered weapons.
This conclusion, with the conclusion reached in the preceding section on singly delivered weapons against area targets leads to the general conclusion that:

The choice of weapon system for any specified target is independent of the size of the target as long as the target consists of a distributed simple multiple of a point target.

This conclusion has a far ranging impact on current weapon system effectiveness analyses. Approximately 50-70 percent of all analysis efforts are devoted to analyzing the effects of target area (area targets consisting of uniformly distributed point targets) on weapon system preference. These studies have been shown to be redundant. Elimination of these studies will eliminate a major workload in weapon system effectiveness analysis.
CHAPTER IV
OPTIMUM DAMAGE PROBABILITY AGAINST TARGETS
WITH LOCATION UNCERTAINTY

Independent mathematical derivations involving a point target located at random within a specified area, where the actual location uncertainty has some probability distribution about the area centroid, lead to mathematical relationships equivalent to those in Chapter III.

The dynamic situation occurs when aircraft on search and destroy missions, acquire a mobile target at some fixed ground coordinate, but are not in position to deliver weapons against the target immediately. During the relatively short time required for the aircraft to convert to an offensive strike attitude, the target has the latitude to maneuver or seek cover of the local terrain and vegetation. Although the pilot may not reacquire the target specifically, due to possible masking which has occurred during the lapsed time interval, there is a realistic probability of damage associated with the delivery of weapons against the original acquisition coordinates providing the lapsed time increment is small compared to the target's evasive capability.

An analogous situation is a mobile target moving through a tree-line yielding fleeting positions to the attack aircraft. The motion and exact location are distorted, but a relative area of location is identifiable.

It became apparent early in the development of the mathematics of this chapter that this dynamic situation was equivalent to the
static situation of the previous chapter. The conclusion reached is that:

The probability of damaging a point target within an area of radius $R_w$, where the location of the target is defined by standard deviation $\sigma_1$, the aircraft delivery error is defined by standard deviation $\sigma_A$, and the total error $\sigma_T$ is a convolution of the two former distributions, is identically the probability of fractional damage against an area target of radius $R_w$ attacked with aiming error $\sigma_T$. Bryant's\textsuperscript{12} work adequately treats this problem and is recommended to readers wishing to explore this subject further.

In the case of similar distributions in target location and aiming error, the variance of the total error distribution is the sum of the variances of the individual distributions. The mathematics of this section are in this respect redundant and are excluded. The results of analyses involving area targets are interpreted with regard to this analogy.
CHAPTER V
SYSTEMS ANALYSIS AND DESIGN APPLICATIONS

A. Representative Systems and Targets

Representative weapons systems consisting of both area and unitary weapons were selected to demonstrate the applications of the mathematical developments of Chapters II and III. The systems selected include an area weapon designed specifically as an anti-personnel weapon, two area weapons designed for anti-material applications, two general purpose area weapons and four general purpose high-explosive/blast/fragmentation unitary weapons.

These weapons, where applicable, were analyzed in seven employment situations against three targets selected for their size and varying degrees of difficulty. The employment situation consisted of singly delivered weapons, four, six and twelve weapons delivered in pairs, and four, six and twelve weapons delivered in salvo. Both the weapons systems and the targets considered are briefly described below. Data generated for these systems utilizing the closed form approximations were accurate to the second decimal when compared to the numerically integrated results.

1. Systems Considered in the Analysis

a. Weapon system number one (1) is an anti-personnel/materiel cluster (area weapon) containing 670, one pound bomblets. The total system weight is 830 pounds.

b. Weapon system number two (2) is an anti-materiel cluster containing 217, 2.2 pound bomblets. The total system weight is 672 pounds.
c. Weapon system number three (3) is an antipersonnel/material cluster containing 665, one pound bomblets. The total system weight is 867 pounds.

d. Weapon system number four (4) is an antipersonnel dispenser/cluster containing 2030, one-quarter pound bomblets. The total system weight is 888 pounds.

e. Weapon system number five (5) is an antimaterial dispenser containing 40, 13.2 pound bomblets. The total system weight is 708 pounds.

f. Weapon system number six (6) is a 250 pound class general purpose unitary weapon.

g. Weapon system number seven (7) is a 500 pound class general purpose unitary weapon.

h. Weapon system number eight (8) is a 1000 pound class general purpose unitary weapon.

i. Weapon system number nine (9) is a 2000 pound class general purpose unitary weapon.

2. Targets Considered in the Analysis

a. Target one (1) is a small area (3000 ft²) of prone personnel. It is classed as a point target.

b. Target two (2) is a 2 1/2 ton truck, bare, but with gas tanks partially filled.

c. Target three (3) is an area (563,000 ft²) consisting of uniformly distributed point targets of the class described in (b) above.
Table 5-1 is a summary of the weapon/target combinations selected for a representative analysis. The summary contains the physical properties of the weapons systems and their associated lethality for each applicable target. Table 5-2 is a summary of Chapters II and III. Applicable constants are related for all 9 weapon system considered.

Weapon system group 1 consists of weapons 6-9 as a family of general purpose unitary weapons. This terminology will be referred to quite frequently throughout this chapter.

Table 5-3 relates a typical comparison of the numerically integrated results obtained from The Analysis Division, Air Force Armament Laboratory to solutions generated at Louisiana State University using the close form approximations. Systems shown have annular patterns and the damage reflected is for single weapon delivery.
### TABLE 5-1

**PHYSICAL AND LETHAL CHARACTERISTICS OF ILLUSTRATIVE SYSTEMS**

<table>
<thead>
<tr>
<th>Weapon System</th>
<th>Number of Submunitions</th>
<th>Submunitions Weight (LB)</th>
<th>Total System Weight (LB)</th>
<th>Empty Weight (LB)</th>
<th>Target 1 ( \text{MAE (ft}^2 \text{)} )</th>
<th>Target 2 ( \text{MAE (ft}^2 \text{)} )</th>
<th>Target 3 ( \text{MAE (ft}^2 \text{)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>670</td>
<td>0.93</td>
<td>830</td>
<td>195</td>
<td>257</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>217</td>
<td>2.20</td>
<td>672</td>
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<td>409</td>
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<td>157</td>
<td>N/A</td>
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<td>888</td>
<td>176</td>
<td>288</td>
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TABLE 5-2
WEAPON SYSTEM CONSTANTS

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<th>System</th>
<th>K</th>
<th>α</th>
<th>$K_1$</th>
<th>$K_2$</th>
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<td>Tgt 1</td>
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<td>.95</td>
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<td>266.2</td>
<td>114.1</td>
<td>114.1</td>
<td>.6056</td>
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</table>
### TABLE 5-3

**TYPICAL COMPARISONS OF CLOSED FORM SOLUTIONS TO NUMERICALLY INTEGRATED SOLUTIONS**

<table>
<thead>
<tr>
<th>Weapon Number</th>
<th>Target Number</th>
<th>Delivery Error ((\sigma) - mils)</th>
<th>Damage Probability ((P_D))</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>40</td>
<td>20</td>
</tr>
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<td>.01</td>
<td>.015</td>
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<tr>
<td>2</td>
<td>2</td>
<td>.03</td>
<td>.034</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>.05</td>
<td>.050</td>
</tr>
</tbody>
</table>

N denotes numerically integrated results

C denotes closed form results
B. Point Target Analyses

1. Area Weapons Analysis

Composite plots of damage probability versus delivery error have been prepared for currently existing annular patterns, theoretical continuous non-optimum patterns and theoretical optimum patterns. Continuous non-optimum patterns are defined as patterns having the same radius as the outer radius of existing annular patterns but with the submunitions distributed randomly over the entire pattern as opposed to being distributed over an annular ring. Since optimum pattern radii are explicit functions of delivery error optimum continuous patterns infer that the radii must be controlled if the damage probability is to be maximized for any given delivery error.

Contrary to prevailing opinion that eliminating existing annular patterns through redesign of current cluster munitions will increase the damage probability of these systems, it is obvious from Figures 5-1, 5-2, 5-3, 5-9, 5-10 and 5-11 that any modification will result in an inconsequential increase in damage probability over the range of current combat delivery errors. However, the potential of current systems with improved delivery accuracy actually decreases for point target applications since improved accuracy places the relatively large central void of the annular pattern over the target. Considerable improvement in damage potential is possible through elimination of the annular patterns if the redesign is accompanied by substantial decrease in delivery error.

The advent of substantial improvements in delivery accuracy necessitates a new design discipline in cluster submunitions. Figure 5-4 relates the optimum pattern radius versus delivery error for
these representative systems. It is clear from this figure that simple in design, inexpensive, ballistically dispersed submunitions are contenders as replacements for current self-dispersing auto-
rotating magnus bomblets. The current systems cannot achieve patterns this small and maintain an acceptable reliability. As a matter of fact, the annular patterns contained in these analyses have the smallest radii achievable for current systems.

The pattern size of ballistically dispersed submunitions can be controlled through control of cluster function altitude or slant range. It must be reiterated that the ballistically dispersed sub­munitions are contenders only over the improved delivery accuracy range ($\sigma \leq 150$ feet) since pattern radii outside this range will be difficult to achieve by this method of dispersion.

2. Unitary Weapons Analysis

As related in Chapter 2, unitary weapons patterns (lethal areas) are not subject to maximization procedures from an employment standpoint, their lethal radii are functions only of their weight and physical design, primarily the charge mass to metal mass ratio and properties of the case metal. Damage probability can be assessed and is related in Figures 5-8 and 5-15.

3. Cluster Weapons versus Unitary Weapons

There is a continuing controversy over the design and employ­ment of cluster and unitary weapons. The controversy arises over de­sign priority, range of applicability, and flexibility of employment. Unitary weapons possess a high blast capability and with such offer a much larger range of employment (greater range of targets).
Figures 5-5, 5-6, 5-7, 5-13 and 5-14 related weapon preference in terms of CEP$^2$ and weapon weight. It is obvious from these figures that cluster munitions compensate for large delivery errors to some extent and are preferred over unitary weapons on a weight basis over a large range of delivery error. This range extends to relatively small delivery errors commensurate with what is probably the best achievable accuracy for unguided aircraft delivered munitions over the next two decades. However, it is again obvious that unitary weapons offer the greatest potential for guided weapons applications.

In addition to the above planning and selection criteria, the methodology and specifically this type of graphic presentation offers a rapid and accessible technique for the following type of employment questions:

a. Given any delivery error and weapon weight:
   What type of weapon system is preferred? What is the expected target damage?

b. Given any delivery error and desired level of damage:
   What type of weapon system is preferred?
   What weapon weight is required?

c. Given a fixed weapon weight allowable and a desired level of damage:
   What accuracy is required?

Hence, can the weapons be delivered from aircraft in a freefall mode or must they be guided?
4. Weapon Design Characteristics and Employment Parametric

The design characteristics of cluster munitions, specifically weight, volume and external configuration can be explicitly defined in terms of the number of submunitions to be contained. The methodology of Chapter 2 permits the determination of the number of bomblets required for any specified level of damage and delivery error. Figure 5-16 relates this information specifically for the submunitions contained in weapon 3 when employed against target 2.

Figure 5-17 relates the number of clusters required to achieve any desired level of damage as a function of delivery error. The information displayed pertains to the employment of weapon 3 against target 2. Figure 5-18 relates the same type of information which regards to the employment of weapon 7 against target 2. These two figures relate the inability of these systems to achieve a high degree of damage with current delivery errors without the expenditure of extremely large numbers of munitions. The number of aircraft required is implicit in the number of munitions required and can be determined simply by ratioing the number of munitions required to any specified aircraft loadout.
Figure 5-1

DAMAGE PROBABILITY

Target 1
Weapon 1
Single Weapons

1. Annular Pattern
2. Continuous Non-optimum Pattern
3. Continuous Optimum Pattern
Figure 5-2

DAMAGE PROBABILITY

Target 1
Weapon 2
Single Weapons

1. Annular Pattern
2. Continuous Non-optimum Pattern
3. Continuous Optimum Pattern

Delivery Error ($\sigma$) - Ft

$P_D$
Figure 5-3

DAMAGE PROBABILITY

Target 1
Weapon 4
Single Weapons

1. Annular Pattern
2. Continuous Non-Optimum Pattern
3. Continuous Optimum Pattern

$P_D$

Delivery Error ($\sigma$) - Ft
Figure 5-4

OPTIMUM PATTERNS

Target 1
Single Weapons

1. Weapon 4
2. Weapon 1
3. Weapon 2

Optimum Pattern Radius - Ft

Delivery Error (σ) - Ft

1
2
3
Figure 5-5
WEAPON PREFERENCE
Target 1
Weapon 1
vs
Group 1
Figure 5-6

WEAPON PREFERENCE

Target 1
Weapon 2
vs
Group 1

$C_{EP}^2 - P_e^2$

$P_D$

0.1

0.2

0.3

0.4

0.5

0.6

0.7

0.8

0.9

Weapon 2
Preferred

Weapon Weight - LB
Figure 5-7
WEAPON PREFERENCE

Target 1
Weapon 4
vs
Group 1

Weapon Weight - LB

\( CEP^2 - \eta^2 \)

\( PD \) 0.1 0.2

0.3

0.4

0.5

0.6

0.7

0.8

0.9

1

10

Preferred

Group 1

Preferred

Weapon 4
Figure 5-8

DAMAGE PROBABILITY

Target 1
Single Weapons

1. Weapon 9
2. Weapon 8
3. Weapon 7
4. Weapon 6

$P_D$

Delivery Error ($\sigma$) - Ft
Figure 5-9

DAMAGE PROBABILITY

Target 2
Weapon 1
Single Weapons

1. Annular Pattern
2. Continuous Non-Optimum Pattern
3. Continuous Optimum Pattern

Delivery Error (σ) - Ft
Figure 5-10

DAMAGE PROBABILITY

Target 2
Weapon 2
Single Weapons

1. Annular Pattern
2. Non-Optimum Continuous Pattern
3. Optimum Continuous Pattern
Figure 5-11

DAMAGE PROBABILITY

Target 2
Weapon 3
Single Weapons

1. Annular Pattern
2. Non-Optimum Continuous Pattern
3. Optimum Continuous Pattern

$P_D$

Delivery Error ($\sigma$) - Ft
Figure 5-12

OPTIMUM PATTERNS

Target 2
Single Weapons

1. Weapon 3
2. Weapon 2
3. Weapon 1
Figure 5-13

WEAPON PREFERENCE

Target 2
Weapon 1 vs
Group 1

Weapon Weight - LB
Figure 5-14

WEAPON PREFERENCE

Target 2
Weapon 3
vs
Group 1

Weapon Preference - LB
Figure 5-15

DAMAGE PROBABILITY

Target 2
Single Weapons

1. Weapon 9
2. Weapon 8
3. Weapon 7
4. Weapon 6
Figure 5-16

SUBMUNITIONS PARAMETRIC

Target 2
Weapon 3 Submunitions

Number of Submunitions Required \times 10^{-3}

Delivery Error (\sigma) - Ft
Figure 5-17

WEAPON PARAMETRIC

Target 2
Weapon 3

Number of Weapons Required

Delivery Error (σ) - Ft
Figure 5-18
WEAPON PARAMETRIC

Target 2
Weapon 7
5. Multiple Weapons Employment

Current intervalometers are designed specifically for 'stick bombing', that is, for releasing unitary weapons at specified intervals over a short segment of the flight path. Several options are available depending upon the aircraft being employed. In general, all aircraft have at least three interval options available, those being 0.06, 0.10 or 0.15 seconds.

Optimum weapons sequencing is as important as optimum release interval. However, in many cases optimum weapons sequence in terms of obtaining the most advantageous weapons pattern conflicts with the sequence necessary to maintain the weight balance and aerodynamic symmetry of the delivery aircraft. Good sequencing then, is an orderly release of external stores with generates the best possible weapons pattern constrained by the maximum acceptable weight unbalance and aerodynamic asymmetry.

Other options which are available besides interval and sequence, are the numbers of weapons released at each time interval. Some of the more common options include: single weapons, weapons in pairs, weapons in salvo, ripple single weapons and ripple pairs of weapons. Single weapon selection permits the pilot to release one weapon each time the release switch is depressed, pair selection infers the same for dual weapons release and the salvo mode clears the entire weapons complement with a single depression of the release switch. Ripple single weapons is a mode which exhausts the weapons in a string, one weapon at each time interval, the string length depending upon the intervalometer setting selected. Ripple pairs permits pairs of weapons to be released in a string.
a. Multiple Unitary Weapons

The interval of the weapons, denoted in Figure 5-19 as 'current pattern', is 0.10 seconds. As can be observed, the damage probability obtained through optimum patterns does not differ greatly from the selected standard interval, especially in the low delivery error range. These differences would have been significantly less had the 'current pattern' been made consistent with the magnitude of the delivery error. That is, had the damage at large delivery errors been assessed with 0.15 second intervals and low delivery errors with 0.06 second intervals, the differences would have been minor. This is verified in Figure 5-21 which demonstrates that large patterns are desired when large delivery errors are expected.

Optimum pattern size is highly dependent upon the number of weapons in the pattern as is shown in Figures 5-21, 5-23 and 5-25 for four, six and twelve unitary weapons released in pairs respectively. The composite plot in Figure 5-26 is indicative of the variation in desired pattern size with number of weapons contained in the pattern. The variation in expected damage as a function of the number of weapons in the pattern is amply demonstrated in Figures 5-20a, 5-22 and 5-24.

Multiple weapons released in a salvo generate patterns nearer the optimum than multiple pair releases. Figure 5-20b is illustrative of the damage achievable through this mode of release. Comparison of these results with the pairwise release in Figure 5-20a and the optimum pattern damage related in Figure 5-19 demonstrates
the degree of improvement of the salvo mode over the multiple pair mode.
Figure 5-19

DAMAGE COMPARISONS

Target 2
4 Weapons in Pairs

1. Weapon 9
2. Weapon 6

- - - - - Current Pattern
- - - - - Optimum Pattern

$P_D$

Delivery Error ($\sigma$) - Ft

97
Figure 5-20a

DAMAGE PROBABILITY

Target 2
4 Weapons in Pairs

1. Weapon 9
2. Weapon 8
3. Weapon 7
4. Weapon 6

Delivery Error (σ) - Ft
Figure 5-20b

DAMAGE PROBABILITY

Target 2
4 Weapons in Salvo

1. Weapon 9
2. Weapon 8
3. Weapon 7
4. Weapon 6
Figure 5-21

DAMAGE PROBABILITY

Target 2
4 Weapons in Pairs

Optimum Pattern Radius (Unitary) - Ft

Delivery Error (σ) - Ft

1. Weapon 9
2. Weapon 8
3. Weapon 7
4. Weapon 6
**Figure 5-22**

**DAMAGE PROBABILITY**

**Target 2**
6 Weapons in Pairs

1. Weapon 9
2. Weapon 8
3. Weapon 7
4. Weapon 6

**Graph Details:**
- **$P_D$** axis
- **Delivery Error ($\sigma$) - Ft** axis
- Curves 1 to 4 represent different weapons.
Figure 5-23

OPTIMUM PATTERNS
Target 2
6 Weapons in Pairs

Optimum Pattern Radius (Unitary) - Ft

Delivery Error (σ) - Ft

1. Weapon 9
2. Weapon 8
3. Weapon 7
4. Weapon 6
Figure 5-24

DAMAGE PROBABILITY

Target 2
12 Weapons in Pairs

1. Weapon 9
2. Weapon 8
3. Weapon 7
4. Weapon 6

Delivery Error (σ) - Ft
Figure 5-25

OPTIMUM PATTERNS

Target 2
12 Weapons in Pairs

Optimum Pattern Radius - Ft

Delivery Error (σ) - Ft

1. Weapon 9
2. Weapon 8
3. Weapon 7
4. Weapon 6
Figure 2-26

OPTIMUM PATTERNS

Target 2
Weapon 7

1. 12 Weapons in Pairs
2. 6 Weapons in Pairs
3. 4 Weapons in Pairs
b. Multiple Cluster Weapons

The effect of the central voids of existing annular patterns is lessened when cluster weapons are employed in multiples. This can be observed in Figures 5-27 through 5-32. For 12 weapons delivered in pairs, as reflected in Figures 5-31 and 5-32, the effect is insignificant. The overlapping annular rings essentially cover the central voids.

The potential increase in damage probability for weapons designed and employed with continuous optimum patterns is not lessened. On the contrary, it is amplified, as shown in Figures 5-27 through 5-32. The optimum pattern sizes commensurate with the maximized damage probability are contained in Figures 5-33 and 5-34. The optimum pattern radii for clusters is somewhat larger than those for unitary weapons, the basic difference being in the differences in cluster pattern radii and the smaller lethal radii of unitary weapons, not in the intervalometer and sequencing requirements. The intervalometer and sequencing requirements are not substantially different. The composite plot of cluster and unitary radii in Figure 5-35 is indicative of the small variations, especially over the range of low delivery error.

Weapon preference is exemplified in Figure 5-36. The trends reflected in this figure are consistent with the preference trends in single weapons employment, wherein the lower weight unitary weapons presented a greater challenge to cluster weapons.

The flattening trend in the damage curve of the continuous non-optimum patterns results from the fact that the coverage function becomes nearly unity at modestly large values of delivery
error and since the pattern radii in these patterns are fixed at
the outer radii of current annular patterns, the conditional damage
function is a constant. Only in the continuous optimum patterns
is the damage function permitted to vary and that due to the varia­
tion in the optimum pattern with delivery error.

Similarly, since the annular pattern damage is related to the
difference in the probability that the target lies in an area de­
""
Figure 5-27

DAMAGE PROBABILITY

Target 2
Weapon 1
4 Weapons in Pairs

1. Annular Patterns
2. Continuous Non-Optimum Patterns
3. Continuous Optimum Patterns

Delivery Error (σ) - Ft
Figure 5-28

DAMAGE PROBABILITY

Target 2
Weapon 3
4 Weapons in Pairs

1. Annular Patterns
2. Continuous Non-Optimum Patterns
3. Continuous Optimum Patterns
Figure 5-29

DAMAGE PROBABILITY

Target 2
Weapon 1
6 Weapons in Pairs

1. Annular Patterns
2. Continuous Non-Optimum Patterns
3. Continuous Optimum Patterns

Delivery Error (σ) - Ft
Figure 5-30

DAMAGE PROBABILITY

Target 2
Weapon 3
6 Weapons in Pairs

1. Annular Patterns
2. Non-Optimum Continuous Patterns
3. Optimum Continuous Patterns

$p_d$

Delivery Error ($\sigma$) - Ft
Figure 5-31

DAMAGE PROBABILITY

Target 2
Weapon 1
12 Weapons in Pairs

1. Annular Patterns
2. Non-Optimum Continuous Patterns
3. Optimum Continuous Patterns

Delivery Error (σ) - Ft
Figure 5-32

DAMAGE PROBABILITY

Target 2
Weapon 3
12 Weapons in Pairs

1. Annular Patterns
2. Non-Optimum Continuous Patterns
3. Optimum Continuous Patterns

Delivery Error (\( \sigma \)) - Ft
Figure 5-33

OPTIMUM PATTERNS

Target 2
Weapon 1

1. 12 Weapons in Pairs
2. 6 Weapons in Pairs
3. 4 Weapons in Pairs
**Figure 5-34**

**OPTIMUM PATTERNS**

- **Target 2**
- **Weapon 3**

1. 12 Weapons in Pairs
2. 6 Weapons in Pairs
3. 4 Weapons in Pairs
Figure 5-36
WEAPON PREFERENCE FACTOR

Target 2
Area Weapons vs Unitary Weapons

1. Weapon 1
2. Weapon 2
3. Weapon 3
C. Area Target Analyses

The methodology for analyzing area targets whose areas are less than the munitions patterns differs from the point target analysis only by a set of constant parameters and such being the case, a lengthy comparative analysis of this application would be redundant and is omitted.

Target 3 is a large area target whose total area (250 meters X 250 meters) is larger than the largest available pattern sizes of extant conventional munitions. Thus the applicable interval is \( (R_p/\sigma \leq R_T/\sigma \leq \infty) \).

Figures 5-37, 5-38 and 5-39 are illustrative of the degree of damage possible with large area munitions, designed specifically to produce widespread damage on area targets. Alternatively, it is often stated that large area munitions are designed to compensate for large delivery areas. Whatever the case may be, comparison of these results with the fractional damage generated by employment of equal numbers of unitary weapons, demonstrates the overwhelming preference of area munitions for this class of area target application. It must be emphasized that this target is susceptible to fragmentation and as such is the primary reason for the area weapon preference. The area weapon (cluster and dispenser munitions) would produce little or no damage on large industrial complexes and the preference trends would be reversed.

The fractional damage in Figures 5-37 and 5-41 results from employing the respective munitions with existing weapons patterns, that is, from weapons patterns generated by current intervalometers and release sequence. The effect of pattern radius on fractional
damage is illustrated in Figures 5-42, 5-43 and 5-44. The first two relate the damage produced by unitary weapons of varying yield for 4 and 12 weapons released in pairs respectively. It is significant to note that the lower yield weapons reach near maximum damage potential at weapon pattern radii less than the target radius \( R_p < R_T \) although the true maximum has not been reached. The higher yield weapons demonstrate a higher potential for larger pattern radii, implying patterns larger than the target area would have been beneficial if achievable. Similar trends can be observed from Figure 5-44 pertaining to cluster weapons. Here however, it is noted that their current pattern radii produce near maximum potential.
Figure 5-37

FRACTIONAL DAMAGE

Target 3
Weapon 1

1. 12 Weapons in Pairs
2. 6 Weapons in Pairs
3. 4 Weapons in Pairs
4. Single Weapon

Delivery Error (σ) - Ft
Figure 5-38

FRACTIONAL DAMAGE

Target 3
Weapon 2

1. Single Weapon
2. 4 Weapons in Pairs
3. 6 Weapons in Pairs
4. 12 Weapons in Pairs

Delivery Error (σ) - Ft
Figure 5-39

FRACTIONAL DAMAGE

Target 3
Weapon 3

1. Single Weapon
2. 4 Weapons in Pairs
3. 6 Weapons in Pairs
4. 12 Weapons in Pairs

Delivery Error (\(\sigma\)) - Ft
Figure 5-40

FRACTIONAL DAMAGE

Target 3
Weapon 7

1. 12 Weapons in Pairs
2. 6 Weapons in Pairs
3. 4 Weapons in Pairs
4. Single Weapon

Delivery Error (σ) - Ft
Figure 5-41
FRACTIONAL DAMAGE
Target 3
Weapon 9

1. 12 Weapons in Pairs
2. 6 Weapons in Pairs
3. 4 Weapons in Pairs
4. Single Weapon

$F_D$

Delivery Error ($\sigma$) - Ft
Figure 5-42

FRACTIONAL DAMAGE

Target 3
4 Weapons in Pairs

1. Weapon 6
2. Weapon 7
3. Weapon 8
4. Weapon 9

$R_T = 424 \text{ Ft}$
$\sigma = 72 \text{ Ft}$

Current Patterns

Pattern Radius - Ft

$F_D$
Figure 5-43

FRACTIONAL DAMAGE

Target 3
12 Weapons in Pairs

1. Weapon 6
2. Weapon 7
3. Weapon 8
4. Weapon 9

\( F_D \)

Pattern Radius - Ft

\( R_T = 424 \text{ Ft} \)
\( \sigma = 72 \text{ Ft} \)

Current Patterns
Figure 5-44

FRACTIONAL DAMAGE

Target 3

1. Weapon 1 (4 in Pairs)
2. Weapon 2 (4 in Pairs)
3. Weapon 3 (4 in Pairs)
4. Weapon 3 (6 in Pairs)
5. Weapon 3 (12 in Pairs)

$R_T = 424 \text{ Ft}$
$\sigma = 72 \text{ Ft}$

○ Existing Pattern

Pattern Radius - Ft
CHAPTER VI
CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

1. General Comments

It must be emphasized that the mathematical developments contained in this report were developed primarily as a means for rapid but acceptably accurate weapons systems effectiveness assessments, definition of preliminary weapons design characteristics and as a weapons development planning and programming tool. As such, perturbations induced by ejection angles and variations in ejection velocities and other aircraft interface perturbations are not considered. The methodology therefore permits the assessment of the effect of these perturbations through the comparison of ideal release conditions to actual release conditions made possible by comparing the results of this method with the numerically integrated results. Such comparisons can lead to preliminary design constraints or desired design characteristics for interface hardware.

The most basic problem confronting the efficient employment of conventional munitions today is the prevailing tendency to design and fly airframes with little regard to the external payload, other than such considerations as total payload weight, load factor design and hard point allocation and location. As a consequence, the munitions are an after-the-fact consideration, their carriage and release suffering severely from the secondary role relegated to them during airframe concept formulation and design.
It is possible, with proper design, to achieve higher damage probabilities with substantially decreased payloads. This not only decreases costs, but relieved payload requirements can be traded off for increased range, endurance, acceleration, velocity, climb, or other airframe performance characteristics. In addition, if proper design discipline is followed, the total weight of the airframe can be reduced, reducing aircraft costs or increasing the number of aircraft available for a fixed cost.

In terms of combat effectiveness, it is axiomatic that if the number of munitions required to destroy a target is reduced, sortie rate is reduced and stockpile requirements are eased. Many other tangible and intangible benefits are derived from the "domino" effect caused by a reduction in the number of weapons expended per target.

The effects of improper munitions carriage and release barely manifest themselves when current JMEN combat accuracies are considered. The capabilities commensurate with these inaccuracies are so obscure (the errors are so large) that carriage and release and ballistic dispersion errors are negligible.

The advent of improved delivery accuracy with regard to current estimates of achievability, places both the carriage and release and ballistic errors on an equal basis with delivery errors. As delivery accuracy is improved, greater emphasis must be placed on the quality control of mass production of munitions and on proper release and sequencing. Otherwise, reduction of delivery errors will be academic.
2. Specific Conclusions

a. It has been shown that in the improved delivery accuracy range, continuation of current design trends will result in sub-optimal systems. This is especially true for the delivery of cluster and dispenser munitions and for multiply delivered unitary weapons.

b. Current intervalometers and sequence become obsolete in the improved delivery accuracy range. Current multiple ejection racks (MERS) and triple ejector racks (TERS) with their adverse ejection characteristics will degrade the effectiveness of future weapons systems.

c. There appears to be almost total arbitrariness in the selection of functioning altitudes for cluster munitions. In addition to sub-optimal spacing in multiple cluster patterns through improper sequencing, the individual cluster patterns which result from arbitrary function altitudes yield substantially inferior damage probabilities to that achievable with proper regard to optimum capabilities. A high degree of inferiority is prevalent when delivery error is reduced.

d. Target area need not be a parameter in weapon systems preference studies when the area consists of distributed point targets of equal or nearly equal susceptibility to the weapon system being evaluated.

e. The approximations developed during this investigation have been shown to be of immense value in reducing the weapon systems analysis to a fraction of that currently required. This is achieved by subjecting the weapon system candidates to swift and
relatively accurate preliminary analysis and eliminating the ob-
viously undesirable weapons systems early and with little effort,
thereby permitting increased attention to evaluation of the more
promising systems or reassignment of large portions of existing
analytical manpower and computer resources.
B. Recommendations

1. Large amounts of resources are devoted to airframe and weapon system development without proper emphasis on the interface between airframe and ordnance. Detailed tradeoff studies should be made on the cost and effect of improved ejection and optimum sequencing of external and internal stores.

2. Consideration should be given to the development of variable function altitude fuzing and to the advisability of placing the choice of function altitude with the pilot or with a weapons fuzing and release computer. This would permit the pilot to select an optimum pattern radius commensurate with the type and size of target encountered. For preplanned missions, where these target characteristics are known, the proper settings could be made by the ground crew prior to mission initiation. This however, still leaves the pilot with the task of achieving a preplanned release point in multi-dimensional velocity-attitude-space for the optimum to be achieved. For search and destroy missions, armed escort and enhancement of the preplanned missions, consideration should be given to the former.

3. Consideration should be given to the design and development of inexpensive ballistically dispersed submunitions which generate a continuous distribution of munitions over the pattern and pattern size control commensurate with the above. Ballistically dispersed submunitions will permit greater flexibility in the optimum lethal design characteristics. Current spherical shapes yield the poorest lethal effects per unit weight of any possible geometric shape,
with at least half of the total fragment mass and available energy being expended fruitlessly.
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APPENDIX A

ANNULAR PATTERN METHODOLOGY

The coverage function for annular patterns may be determined by considering the coverage probabilities of the associated outer ($R_o$) and inner ($R_i$) radii. The probability that the target lies within the area generated by $R_o$ is:

$$P_{Co} = \left[1 - \exp\left(-\frac{R_o^2}{2\sigma^2}\right)\right]$$  \hspace{1cm} (A-1)

and the probability that it lies within the area generated by the $R_i$ is:

$$P_{Ci} = \left[1 - \exp\left(-\frac{R_i^2}{2\sigma^2}\right)\right]$$  \hspace{1cm} (A-2)

Finally, the probability that the target lies in the annular ring is the difference in (A-1) and (A-2):

$$P_c = \left[\exp\left(-\frac{R_i^2}{2\sigma^2}\right) - \exp\left(-\frac{R_o^2}{2\sigma^2}\right)\right]$$  \hspace{1cm} (A-3)

The conditional damage function must also be modified since the lethal effects are confined to the annular ring. The mean value becomes:

$$\mu = r_1 nMAE_B / (A_o - A_i)$$
or

\[
\mu = r_1 n R_{LB}^2 / (R_o^2 - R_1^2)
\]

and the conditional damage function is given by:

\[
P_{D/C} = \left[1 - \exp\left(-r_1 n R_{LB}^2 / (R_o^2 - R_1^2)\right)\right]
\]

(A-4)

and finally, the damage probability is given by:

\[
P_D = \left[\exp\left(-R_1^2 / 2\sigma^2\right) - \exp\left(-R_0^2 / 2\sigma^2\right)\right] X \\
\left[1 - \exp\left(r_1 n R_{LB}^2 / (R_o^2 - R_1^2)\right)\right]
\]

(A-5)

For multiple weapons employment, the area of the resulting central void must be determined and the coverage function modified in a manner similar to the above. The conditional damage function is found simply by replacing \( R_p^2 \) in Equation (2-29) with \((R_o^2 - R_1^2)\) and making the appropriate substitutions for \( R_w^2 \).

\[
P_{D/C} = \left\{1 - \exp\left[-N(R_o^2 - R_1^2) P_{D/C} R_w^2\right]\right\}
\]

where \( R_w^* \) is a function of the difference between the overall pattern area and the resulting central void if it exists.

Appropriate combinations of the above equations will yield the damage equations for area targets as demonstrated in Chapter 3.
VITA

John H. Arnold was born near Danville, Georgia on June 17, 1938. His elementary and secondary education was obtained in the public schools of Warner Robins and Macon, Georgia. He graduated from Lanier Senior High School with honors in June, 1956. Shortly after graduation from high school he enlisted in the United States Army Security Agency, serving three years in Europe, North and East Africa, and the Middle and Near East. After separating from the Army in 1959, he was employed as a cryptographer by the Air Force at Robins AFB, Georgia. In August, 1960, he enrolled at Mercer University and received his Bachelor's degree in Mathematics and Physics in June, 1963. In July of the same year, he transferred from Robins AFB to Eglin AFB, Florida and continued his employment with the Air Force in Applied Physics, specializing in rigid body mechanics and flight dynamics. He continued his education under Air Force sponsorship in September, 1965 when he enrolled in the Graduate School of the University of Notre Dame du Lac where he received his Master of Science degree in Aerospace Engineering in June, 1966. He returned to Eglin AFB and was employed as Senior Aerospace Engineer in the Analysis Division of the Air Force Armament Laboratory through 1968. In 1968, he helped form the Development Planning Activity in the Armament Laboratory and has served from that time on the Laboratory Director's Staff. In September, 1970, he entered Louisiana State University under the Air Force Graduate Training Program and is now a candidate for the degree of Doctor of Philosophy in Mechanical Engineering.
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