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A Thermodynamic Model for Wear in Sliding Contact

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A THERMODYNAMIC MODEL FOR WEAR IN SLIDING CONTACT

A Thesis

Submitted to the Graduate Faculty of
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Master of Science in Mechanical Engineering

in

The Department of Mechanical Engineering

By
Susheel Brahmeshwarkar
B.E., Osmania University, Hyderabad, India 2003
December 2006
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### Nomenclature

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A, A_r$</td>
<td>Apparent area of contact and real area of contact respectively</td>
</tr>
<tr>
<td>$a$</td>
<td>Dimensionless length of the stationary specimen</td>
</tr>
<tr>
<td>$a_c$</td>
<td>Constant used in the strain distribution</td>
</tr>
<tr>
<td>$b, b_1, b_2$</td>
<td>Dimensionless radii of the specimen $b_i = r_i/r_2, b_2 = r_5/r_2$</td>
</tr>
<tr>
<td>$c_s$</td>
<td>Factor for rate of surface strain $\frac{\delta c_s}{dt} = c_s \nu$</td>
</tr>
<tr>
<td>$H$</td>
<td>Hardness pressure of the material</td>
</tr>
<tr>
<td>$h$</td>
<td>Heat convection coefficient for the surrounding</td>
</tr>
<tr>
<td>$k_u, k_h, k_b, k_{bs}, k_s$</td>
<td>Thermal conductivities of wearing material, harder material, Bronze, Brass and Steel respectively</td>
</tr>
<tr>
<td>$K$</td>
<td>Archard’s wear coefficient</td>
</tr>
<tr>
<td>$K_{ard}$</td>
<td>Archard’s wear coefficient obtained by using Archard’s formulation</td>
</tr>
<tr>
<td>$K_{con}$</td>
<td>Archard’s wear coefficient obtained by using experimentally measured conducted heat and using entropy formulation</td>
</tr>
<tr>
<td>$K_{wor}$</td>
<td>Archard’s wear coefficient obtained by using the theoretical model and using entropy formulation</td>
</tr>
<tr>
<td>$K_{brass}, K_{brass}$</td>
<td>Archard’s wear coefficient for Brass and Bronze respectively</td>
</tr>
<tr>
<td>$K_{brass-exp}, K_{brass-exp}$</td>
<td>Archard’s wear coefficient for Brass and Bronze calculated using experimental measured conducted heat and entropy formulation</td>
</tr>
<tr>
<td>$L, l$</td>
<td>Length of the stationary specimen and dimensionless variable for it</td>
</tr>
<tr>
<td>$q$</td>
<td>Generated heat flux in the wearing process</td>
</tr>
<tr>
<td>$\dot{q}$</td>
<td>Generated heat per unit volume in the wearing process</td>
</tr>
</tbody>
</table>
$q_1$ Part of net heat generated conducted into the wearing material

$q_2$ Part of net heat generated conducted into the stationary material

$r, z$ Radial co-ordinate system

$\bar{r}, \bar{z}$ Dimensionless radial co-ordinate system

$r_1, r_2, r_3$ Radial dimensions of the stationary specimen

$S, S_{gen}$ Thermodynamic entropy of the system and entropy generated

Respectively

$S_y$ Yield strength of the material

$s$ Speed of rotation of the wearing material in rpm

$T, T_{amb}$ Temperature of the material and ambient temperature respectively

$\bar{\theta}, \bar{\theta}_w, \bar{\theta}_h$ Dimensionless temperature of region 1 for, wearing material and harder material respectively

$\bar{\Theta}, \bar{\Theta}_w, \bar{\Theta}_h$ Dimensionless temperature of region 2 for, wearing material and harder material respectively

$\tau_s, \tau_{max}$ Surface shear stress and maximum shear strength of the wearing material

$v$ Sliding velocity

$w, w_{exp}$ Wear volume and experimentally measured wear volume

$x$ Sliding length

$\delta$ Thickness of severely deformed region

$\delta \chi, \delta \chi_s$ Displacement of the material and surface displacement

$\mu$ coefficient of friction
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$, $\eta_1$, $\eta_2$</td>
<td>Heat partition factor, heat partition factor with respect to material 1 and heat partition factor with respect to material 2</td>
</tr>
<tr>
<td>$\lambda$, $\lambda_n$</td>
<td>Eigen value and nth eigenvalue</td>
</tr>
</tbody>
</table>
Abstract

Consideration of wear, an irreversible phenomenon, is a very important criterion in design. The knowledge of wear and its behavior enables one to make major considerations to conceptualize and design efficient machinery components with enhanced performance and reliability. The present work deals with the introduction of a novel approach of correlating wear with the thermodynamic properties of the system. The approach involves relating wear to thermodynamic entropy flow in the system using the laws of thermodynamics. This relation is verified experimentally and theoretically by considering a sliding contact in a disk-on-disk configuration for two sets of contacting materials namely Bronze SAE 40 on Steel 4140 and Cartridge Brass on Steel 4140. Verification of the methodology is achieved by calculating Archard’s wear coefficient using the relationships derived in this thesis and comparing it to values in literature. A theoretical model that simulates thermal response in sliding contact has been developed to theoretically verify the proposed relation. The model is based on the idea that sliding contact of two bodies would result in plastic deformation in the near surface region, that we refer here as the ‘severely deformed region’ (SDR). This plastic deformation results in heat generated in the SDR and subsequently rising the temperature of contacting bodies. The experimental analysis and the theoretical model verify the proposed relation with good agreement. The coefficient of friction has also been calculated and compared with the experimentally measured value.
1. Introduction

Wear is progressive loss of material from the surface of a solid body due to the mechanical contact or relative motion with a solid, liquid or a gaseous counter body. The importance of wear, the need for prediction of its behavior in machinery components is underlined by its effect on the functionality of the machinery. Generally speaking, material wear in engineering machinery is not necessarily catastrophic but it decreases the operating efficiency. It may result in dimensional changes of components or surface damage and this causes secondary problems such as vibration or misalignment.

Wear is a serious cause of energy and material degradation thus contributing to reducing efficiency and power. Significant interest to reduce the degradation of wear necessitated undertaking careful studies to understand its mechanism. Also important is the economic implications of material degradation which has substantially motivated the industry to pursue systematic research in this field. Intel [Competitive edge 1991] reported a projection of the increased savings in maintenance by making greater investment at the concept and design stages to reduce the manufacturing equipment life cycle cost. The potential of the research can be realized in reducing the losses by optimizing the design of the components, material selection, the transfer of load and motion, interacting environment, lubrication, surface properties, temperature, etc.

The interacting bodies, the interface material, the operating environment along with the parameters of the mechanical contact characterize a tribosystem. Wear is not an intrinsic property but is a characteristic of the tribosystem. The mode of contact viz. sliding, rolling, oscillating, impacting or grooving dictates the nature of wear undergoing in the tribosystem. Sliding contact wear is the most common type of wear in contact and rotating machinery such
as mechanical seals, clutches joints, gears, gaskets, and washers, etc. The present work studies the phenomenon of sliding wear between flat metallic surfaces in periodic sliding motion. According to Zum Gahr (1943), “Sliding wear can be characterized as a relative motion between two smooth surfaces in contact under load, where surface damage during translational sliding does not occur by deep surface grooving due to penetration of asperities or foreign particle”. Here it is vital to understand the difference between, wear occurring when two bodies are in sliding contact and sliding wear. In a realistic situation when two flat metallic surfaces are in periodic sliding motion, wear occurs due to shearing of stressed layers at the surface [Dautzenberg (1980)], grooving due to penetration of asperities or foreign particles, chemical reaction at the surface, corrosion, etc. In the present study, we deal with the ‘sliding wear’ part of the wear occurring between the surfaces in dry periodic sliding contact.

The mechanisms of sliding wear have been viewed from various perspectives by many researchers. The most common mechanisms that are associated to sliding wear when the materials are in sliding periodic motion are adhesion, surface fatigue and/or abrasion. Adhesion or bonding occurs at the asperity contacts at the interface, and these contacts are sheared by sliding which may result in detachment of a fragment from one surface and attachment to the other surface. Surface fatigue is degradation of material by the formation and propagation of cracks in the body due to the periodic stresses in the near surface. Abrasion is surface damage when asperities of a rough, hard surface or hard foreign particles slide on a softer surface. Note that the abrasion considered here is the surface damage only because of the sliding of the asperities of the hard surface on the soft surface as we omit the wear caused because of foreign particles and deep grooving of the asperities.
The present work deals with the authentication of a novel approach of correlating wear with the thermodynamic properties of the system. In this approach, wear characterized by material damage has been related to the thermodynamic entropy flow in the system using the laws of thermodynamics. Archard’s wear coefficient has been calculated from the new relation and compared with published values to verify the authenticity of the proposed relation. The verification has been done experimentally by conducting sliding contact disk-on-disk experiments at LSU, Center for Rotating Machinery (CeRoM), with two sets of contact materials Bronze SAE 40 on Steel 4140 and Cartridge Brass on Steel 4140. A theoretical model that simulates thermo-mechanical feature of sliding wear during the sliding contact has been developed to further verify the proposed relation. The theoretical model can be used to predict wear with only the loading conditions and material properties as the input. The experimental analysis and the theoretical model verify the proposed relation with good agreement.
2. Literature Review

Friction and wear are as elemental as the human race itself. The contact motion of two materials and its effects prevalent in mechanical, biological, environmental and microstructural fields has interested many researchers from early times. The first important contribution made to the understanding of friction and wear phenomenon was done by Leonardo da Vinci as reported by Dawson, (1979). He found that the friction force depended on the normal load on the sliding body but was independent of the apparent contact area. In 1699, Amontons independently postulated the ideas of friction force depending on normal load but not on apparent area of contact. The eighteenth and the nineteenth centuries followed works by Charles Augustin Coulomb, Leonhard Euler, Osborne Reynolds and many others who contributed to the historical development of the knowledge of friction, lubrication and wear.

In 1953 Archard proposed that wear in materials could be described by a simple empirical law given as \( w = K \frac{L x}{H} \), where \( w \) is the wear volume, \( L \) is the normal load, \( x \) is the sliding distance, \( H \) is the hardness and \( K \) is the constant known as the wear coefficient. The attractive simplicity of this formulation, which later came to be known as Archard’s formulation of wear, made it a popular design aid for estimating wear. But Archard’s formulation does not give any insight into the mechanism of wear, influence of test conditions like temperature, lubrication, surface roughness, mode of wear and type of relative motion between materials. Also the wear coefficient is extremely sensitive to load, velocity of relative motion, interface condition and environment. In spite of these drawbacks Archard’s law can be used conservatively if the test conditions are controlled in a definitive range. Efforts to understand the wear mechanisms have continued by many researchers since then. Bowden
and Tabor (1954) first introduced a new perspective to the theory of friction by emphasizing the role of adhesion and inter atomic forces at the surface and in the interacting materials. The main idea of this theory is to explain wear and friction behavior based on the adhesion of surface asperities to form junctions, growth of the junction area and shear at or near the junctions. The coefficient of friction is given by the considering combined effects of adhesion and surface roughness. Thus, the coefficient of friction is presented as the combination of an adhesive term and a ploughing term in their theory. It is evident that the plastic deformation plays an important role. This theory typically concentrates on deformation of asperities and on the junctions which they form, without considering the effects of the structure of the underlining material. The microstructure of the material and crystal lattice imperfections were not included.

Bukley (1977) explained that if adhesion occurs when two metals touch each other in a clean environment, then the plastic deformation is also observed. Moore and Douthwaite (1976) have concentrated on the large plastic strains observed at considerable distances from the wear surface, and they suggested that plastic deformation could account for most of the work observed. Duatzenberg and Zaat (1973), Tusya (1976), Rigney and Gleaser (1978), Rigney and Hirth (1979), and Heilmann and Rigney (1981) have all emphasized on the plastic deformation of the near surface of the contacting material as the main characteristic in sliding wear. Also, metallographic studies from performed by many researchers revealed that plastic deformation is common near the surface in sliding materials. Further Duatzenberg and Zaat (1973), Moore and Douthwaite (1976), Kennedy (1989) and Rice et al (1989) have quantitatively measured the plastic deformation at the near surface in the contacting material. They considered the softer of the interacting materials to analyze the plastic deformation.
Duatzenberg and Zaat (1973) developed a method to measure the plastic strain by considering its geometrical correlation with the deformed grain. They used microscopic observations to measure the grain boundary elongations for a wearing copper surface. Moore and Douthwaite (1976) measured the plastic strains by microscopic observations of sectioned surface of the wearing copper-silver composite solder material. The depth dependence of the plastic deformation under the wearing surface is very well understood in measurements studies done by Duatzenberg and Zaat (1973) and Moore and Douthwaite (1976). The deformation decreases exponentially along the depth below the wearing surface. Dautzenberg (1980) conducted experiments to evaluate the depth dependence of displacement (measure of deformation) in sliding by embedding a marker in the wearing material and studying the shape of the marker profile after the sliding occurs. This profile revealed that the displacement followed an exponentially decreasing trend along the depth below the wearing surface. Heilmann and Rigney (1981) proposed a relation for the deformation which they indicated is the plastic shear strain, as a function of the displacement along the wearing surface by curve fitting the individual displacements $\delta x(z)$. They obtained the relation $\delta x(z) = \delta x_s e^{-ac z}$ where $\delta x_s$ is the surface displacement and $ac$ is a constant that depends on the specific material pair and the parameters of the tribosystem. Kennedy (1989) measured the displacement in the wearing material using microscopic observations of the sectioned region along a plane perpendicular to sliding. He also theoretically verified the experimental displacements using a finite element viscoplastic model at the vicinity of the moving contact. Kennedy (1989) found that if in the finite element model the surface layers of the wearing surface were moving at 10% of the sliding velocity the displacement would agree very well with the experimental
study. This finite element model gave an excellent insight into the mechanics of the plastic
deformation in the domain of the wearing material.

Having established that sliding between two bodies is always accompanied by plastic
deformation, an effect, which is predominant in the softer of the two bodies-, many
researchers went on to propose models for estimating wear. Lacey and Torrance (1990)
applied the slip-line field model of asperity contact combined with laws of low-cycle fatigue
to calculate the Archard’s wear coefficient. The model predicts the development of surface
cracks as a result of cyclic stresses that cause fatigue. The wear coefficients were used to
predict wear and they agreed well with simple experimental designed to simulate the earliest
stages of wear (running-in process). But their model could not accurately predict the steady-
state wear. Hockenhull et al. (1993) also used the low cycle fatigue model causing surface
damage. They used plastic strain increments determined from the wave model to predict wear
that took into account the surface roughness and lubrications conditions. Kimura and Shima
(1991) used a longitudinal contact point model to evaluate the stress intensity factor at the tips
of wear cracks. Lacey and Torrance (1990), Hockenhull et al. (1993) and Kimura and Shima
(1991) whose models are based on the application of laws of mechanics to the surface and
near surface region. The surface of even accurately finished materials is non-uniform in terms
of the asperity distribution and is subject to work hardening effects at various stages of
wearing process. This very fact makes the above presented models imprecise and less
practical. The mechanical response of a wearing system which is the deformation, fracture
and fatigue failure and subsequent detachment of material (debris) is highly localized and also
time dependent. Hence, these phenomena can be modeled only with a small level of
feasibility. There was need for a global extrinsic property that could provide information about the damage during the wearing process.

Klamecki (1979) in his paper ‘Wear-an entropy production model’ idealized a wearing system to be a single body to which heat and work are applied and from which mass transfer is allowed. In a system undergoing an irreversible process like wear, entropy production is non-negative as per second law of thermodynamics. He used the entropy production to develop a constraint for the wear process. This introduced a new school of thought of characterizing wear based on the thermodynamic response of the wearing system. Bryant et al. (1999) hypothesized that a potential correlation between entropy flow and degradation of machinery components. Doelling et al. (2000) proposed a relation between the material degradation in their case ‘wear’ and the entropy flow using Archard’s wear law. The relation was obtained in a 4 step qualitative derivation as follows-

\[ w = K \frac{N x}{H} \]  \hspace{1cm} \text{[Archard’s Wear formulation]} \tag{2.1}

Differentiating eqn. (2.1) with respect to time,

\[ \frac{d w}{d t} = K \left( \frac{N}{H} \frac{d x}{d t} \right) \tag{2.2} \]

The power dissipated by friction can be given as \( P_\mu = N \mu \frac{d x}{d t} \); where \( \mu \) is the coefficient of friction. Also the rate of entropy production due to the frictional dissipation can be noted as \( \frac{P_\mu}{T} = \frac{d S}{d t} \); where \( T \) is the temperature of the control volume for which the entropy is calculated. Eqn. (2.2) was written as

\[ \frac{d w}{d t} = K \frac{T}{\mu H} \left( \frac{d S}{d t} \right) \tag{2.3} \]
Integrating eqn. (2.3) over a time interval where the temperature and coefficient of friction and hardness are constant results in-

\[
\begin{align*}
&w = \frac{KT}{\mu H}S \\
&\text{(2.4)}
\end{align*}
\]

The above relation presents the dependence of wear on the strength properties of the wearing material, the frictional compatibility of the contacting materials, the temperature of the domain and the entropy produced in the domain. For a period of the wearing process where the properties are constant and the temperature is constant, wear is linearly proportional to the entropy. Doelling et al. (2000) conducted experiments of a model machinery component pair and measured the entropy produced in the process by considering the heat conducted into the wearing material. The heat conducted into the wearing material was calculated by measuring the temperature gradient in the wearing material using thermocouples attached along the length of the wearing material, i.e. in a directional perpendicular to sliding. They assumed that the heat conducted into the harder material is negligible. For the experimental data of wear and conducted heat, by using eqn. (2.4), they calculated the wear coefficient. The wear coefficient was in good agreement with the published values of Archard’s wear coefficient [Rabinowicz (1980)] which verifies their hypothesis and the relation proposed (eqn. (2.4)). This proposed relation by Doelling et al. (2000) is used as a base line for the present work.
3. Experimental Verification of Wear and Entropy Formulation

3.1 Motivation

Doelling et al. (2000) verified their proposed relation between wear and entropy flow using a rider-on-disk configuration for C110 copper as wearing specimen sliding on AISI 1020 steel. It is assumed that copper specimen is the only body that wears. The entropy produced in the wearing material (copper) in their case is measured by considering the conducted heat in copper. As stated in the previous chapter Doelling et al. (2000) measure the temperature at two axial locations along the specimen length adjacent to contact interface in the wearing material. This temperature gradient in the direction perpendicular to sliding multiplied by the thermal conductivity is the heat flux conducted in the wearing material. They neglect the heat conducted in the harder material (steel). This gives a lower limit for the entropy produced as it is calculated by considering only the heat conducted into the wearing material. Though this can be used as the good estimate to the total entropy produced, there is a need to accurately account for the total entropy produced in the wearing process. The purpose of the experimental study in the present chapter is to account for the total entropy produced during the wearing process and verify the relation proposed by Doelling et al. (2000).

3.2 Description of Apparatus

The contact of the specimen in the experiment is in disk-on-disk configuration. This is achieved using the LRI-1a Tribometer at the Center for Rotating Machinery (CeRoM) at LSU. The LRI-1a Tribometer is a larger scale Tribometer which can apply loads in the range up to 200 pounds. Test specimens are usually of a thrust washer type, but other configurations are possible. In this research the thrust washer type design for the wearing material and cylindrical shell type design for the harder material were used. Figure 3.2 shows a picture of
the specimen used. The wearing material is fit using a screw in the spindle insert. This is the rotating part of the contact pair. The spindle is rotated using a synchronous motor. The speeds of rotating range from 0 to 5000 rpm. The machine is capable of running a constant speed and multiple speeds in a single test. A picture of the LRI-1a Tribometer used is shown below.

![LRI-1a Tribometer at CeRoM](image1)

**Figure 3.1 LRI-1a Tribometer at CeRoM**

![Function pair](image2)

**Figure 3.2 Function pair**
The spindle does not move in the axial direction. The harder specimen is affixed in a socket of lower holder and is held there without rotation by an anti-rotating pin. The holder can move in the axial direction that is perpendicular to the sliding direction. The load or the pressure between the contact pair is applied from the holder. In other words the holder presses against the rotating part (fitted to the spindle) with a force equal to the desired load. The automated loading arrangement ensures the application of the desired load and maintaining it during the test. The contact is maintained throughout the test due to the constant axial push from the holder. Fig. 3.3 shows the zoomed in view of the contact pair.

![Figure 3.3 Contact pair on the LRI-1a Tribometer](image)

### 3.2.1 Wear Measurement

The holder can move in the axial direction that is perpendicular to the sliding direction. This axial displacement of the holder is the measure of the wear between the contact specimens. A Linear Variable Differential Transformer (LVDT), which is a displacement sensor, senses the displacement of this holder during the test. The LVDT converts displacement in the axial direction into voltage. The computer connected to this machine is the user interface. Data from the LVDT can be viewed during the test. Note here that the wear displayed by the
machine is the total wearing and is the sum of the wears of each of the contacting materials. As the displacement is the measure of the wear, the machine cannot isolate thermal expansion from wear. Therefore, during the running-in time one might see wear decreasing with time. As the system reaches steady state the wear increases linearly but the actual value of wear is more than the value obtained by the machine by an amount equal to the steady state expansion of the contact bodies.

\[ w_{machine} = w_{actual} - d_{exp} \]  

(3.1)

where \( d_{exp} \) is the steady state expansion of the contacting bodies. In the present study, the wear rate or increase in wear is considered instead of actual wear. This way \( d_{exp} \) which is a constant value at steady state has no effect on the analysis. The LRI-1a measures friction, wear to 25 millionths of an inch (giving an accurate assessment of wear as a function of time).

### 3.2.2 Temperature and Friction Measurement

The tribometer is equipped with a thermocouple that can read the temperature and is interfaced with the computer to display the value during the test. Temperature of the interface is a significant parameter during friction experiments. According to the guidelines prescribed by the manufacturer the thermocouple placed in a hole bored on the curved surface of the cylindrical stationary specimen of diameter 1.6mm as close as 2.4mm from the interface, gives reasonable values that can be taken as the representative of the interface temperature. In the present experiments temperature is measured along the axial directions from the contact surface in the stationary specimen as shown in the Figure 3.3. For this reason a thermocouple reader is used. Besides the location that is closest to the interface (2.4mm), 4 more holes are made in the stationary specimen at an equal distance of 0.34 inch starting from the first hole. The value of this temperature gradient in the stationary specimen time its thermal conductivity
gives the heat flux being conducted into the stationary specimen. The machine has a torque sensor that is used to calculate the coefficient of friction.

### 3.3 Experimental Set-up

The present study verifies the formulation proposed by Doelling et al. (2000) for 2 metal pairs - SAE 40 Bronze on Steel 4140 and Cartridge Brass on Steel 4140. The verification is achieved by comparing the wear coefficient values obtained by using the formulation proposed by Doelling et al. (2000) (eqn. (2.4)) to the published values of the Archard’s wear coefficient. Copper on steel was also tried but had the following problems. The wear debris during the wear of copper when exposed to atmosphere forms copper oxide and this soft powder-like substance sticks to the interface creating waviness over the surface. Observation of the surface under the microscopic revealed a black uneven layer. This is the cause of vibration and noise during the test. Moreover the contact is non-uniform. In case of Bronze and Brass this problem was not noticed and the debris was fine metal powder which got deposited near the contact edge of the specimen. Table 3.1 shows the contacting materials and their properties.

**Table 3.1 Contacting material properties**

<table>
<thead>
<tr>
<th>Wearing material</th>
<th>Thermal conductivity W/m.K</th>
<th>Specific heat const. pressure J/kg-K</th>
<th>Density kg/m³</th>
<th>Hardness Mpa</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAE 60 Bronze</td>
<td>71.9</td>
<td>435</td>
<td>8.82 x10³</td>
<td>443.75</td>
</tr>
<tr>
<td>Cartridge Brass</td>
<td>120</td>
<td>375</td>
<td>8.53 x10³</td>
<td>390.5</td>
</tr>
<tr>
<td>Steel 4140</td>
<td>42.7</td>
<td>500</td>
<td>7.85 x10³</td>
<td>2840</td>
</tr>
</tbody>
</table>

For each metal pair four experiments are carried out that involves a combination of 2 loads and speeds except for copper as shown in the Table 3.2
Table 3.2 Loading conditions for different metal pairs

<table>
<thead>
<tr>
<th>Wearing material</th>
<th>Harder material</th>
<th>Load (N)</th>
<th>Speed (rpm)</th>
<th>Lubrication</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAE 60 Bronze</td>
<td>4140 Steel</td>
<td>17.79</td>
<td>100</td>
<td>no</td>
</tr>
<tr>
<td>SAE 60 Bronze</td>
<td>4140 Steel</td>
<td>17.79</td>
<td>200</td>
<td>no</td>
</tr>
<tr>
<td>SAE 60 Bronze</td>
<td>4140 Steel</td>
<td>13.34</td>
<td>100</td>
<td>no</td>
</tr>
<tr>
<td>SAE 60 Bronze</td>
<td>4140 Steel</td>
<td>13.34</td>
<td>100</td>
<td>no</td>
</tr>
<tr>
<td>Wearing material</td>
<td>Harder material</td>
<td>Load (N)</td>
<td>Speed (rpm)</td>
<td>Lubrication</td>
</tr>
<tr>
<td>Cartridge Brass</td>
<td>4140 Steel</td>
<td>13.34</td>
<td>100</td>
<td>no</td>
</tr>
<tr>
<td>Cartridge Brass</td>
<td>4140 Steel</td>
<td>13.34</td>
<td>200</td>
<td>no</td>
</tr>
<tr>
<td>Cartridge Brass</td>
<td>4140 Steel</td>
<td>8.89</td>
<td>100</td>
<td>no</td>
</tr>
<tr>
<td>Cartridge Brass</td>
<td>4140 Steel</td>
<td>8.89</td>
<td>100</td>
<td>no</td>
</tr>
</tbody>
</table>

Figure 3.4 shows the arrangement of a typical wear experiment carried out. The wearing material is fit in the spindle insert using an axial screw. The flat surface of the thrust washer type wearing specimen is the contact surface that is held in contact against the stationary specimen. The stationary specimen is insulated using a thermo-coal sheet wound around the specimen. Temperature is recorded by the Standard Pioneer reader that records the values of the 5 thermocouples connected to the stationary specimen. Typical experiments are run for duration of 2 hours. The attainment of steady state is decided by the temperature readings from the thermo couple. For the loads and speeds used, steady state is attained not later than 90 minutes of the test. The temperature and wear readings plotted for typical tests are shown in Figures 3.5 & 3.6.
- **Quantities measured:**

1. Wear – LVDT
2. Coefficient of friction – Force arm
3. Temperature – Thermocouple

![Image of temperature measurements on stationary specimen](image)

**Figure 3.4 Temperature measurements on the stationary specimen (harder specimen)**

Temperature values are recorded at 5 locations in the stationary specimen. For the test shown in Figure 3.6, after the 228\textsuperscript{th} data point the temperature reaches a steady state. Also the difference between temperatures at each location is maintained a constant at steady state.

Wear values from the LVDT sensor of the tribometer. Notice that the wear up to about 10 intervals of time is negative. This was because of the thermal expansion of the contacting bodies. After the steady state was reached, the thermal expansion is constant and the wear obtained increases continuously. The wear data used for analysis was taken after the system reaches a steady state. Also the first wear reading after reaching the steady state has been taken as the datum thus considering the increase in wear in each time interval rather than the absolute wear.
Figure 3.5 Temperature data from thermo-couples on the stationary specimen

Figure 3.6 Wear data in inches from LVDT for Bronze on Steel (N=17.79 N, s= 200 rpm)
3.4 Entropy Calculation

The entropy generated during the sliding process is due to the friction (an irreversible process). The measure of entropy used here is the heat that is conducted in the contacting materials. The division of the net heat between the contacting materials depends upon the relative thermal properties of the two sliding materials is given by the partition factor $\eta_1$.

![Figure 3.7 Heat partitioning during sliding contact](image)

Blok (1937), Jaeger (1942) formulated the expression for the partitioning factor between two materials designated as 1 and 2 to be

$$\eta_1 = \frac{\left\{C_{p1}k_{1}\rho_{1}\right\}^{\frac{1}{2}}}{\left\{C_{p2}k_{2}\rho_{2}\right\}^{\frac{1}{2}} + \left\{C_{p1}k_{1}\rho_{1}\right\}^{\frac{1}{2}}} \tag{3.2}$$

where $C_{p1}$, $k_1$ and $\rho_1$ are the specific heat capacity at constant pressure, thermal conductivity and density of the material 1 and so on. Therefore, heat conducted into the material 1 is given by $q_1 = \eta_1 q$ and the heat conducted in the material 2 is given as $q_2 = (1 - \eta_1)q$. Therefore, if the heat conducted in one of the materials is known and the partition factor is known the net...
heat generated can be calculated. In the experimental set-up the temperature gradient in the stationary specimen is measured using the thermocouples arrangement. Therefore, heat conducted in the stationary specimen is given by

\[ q_2 = A k_s \frac{T_i - T_{ii}}{d} \]  

(3.3)

where \( A \) is the contact area at the surface; \( T_i, T_{ii} \) are the temperatures measured by the thermocouples at two locations separated by distance \( d \) along the direction perpendicular to sliding. The net heat conducted in the contacting materials has been given as

\[ q = q_1 + q_2 = \frac{q_2}{1 - \eta} = (A \frac{k_s T_i - T_{ii}}{d (1 - \eta)}) \]  

(3.4)

If the heat conducted to the surrounding is neglected then all the dissipative heat generated during the sliding process is conducted into the sliding pair. Neglecting the effects of entropy loss due to mass transfer, the rate entropy produced in this system is therefore given as

\[ \frac{dS}{dt} = \frac{q_1 + q_2}{T}; \quad (1 - \eta) (q_1 + q_2) = A k_s \frac{T_i - T_{ii}}{d}; \quad q_1 + q_2 = A \frac{k_s T_i - T_{ii}}{d (1 - \eta)}; \]

Therefore, \[ \frac{dS}{dt} = \frac{A}{T} \frac{k_s T_i - T_{ii}}{d (1 - \eta)} \]  

(3.5)

where \( T \) is the average temperature of the interface region.

The above formulation of entropy is a modification of the one used by Doelling et al. (2000). Here the summation of heat conducted in both the materials of the sliding pair is considered as a measure of entropy generated, whereas Doelling et al. (2000) assumed that the heat conducted in wearing material is negligible. Notice here that as the entropy generated has been calculated based on the thermal response away from the interface without characterizing
the control volume. Thus, the heat conducted into the contacting materials been taken as a representative of the entropy generated in the system. However, a detail study of control volume and evaluation of the entropy generation is dealt with in the theoretical study presented in the next chapter. According to the formulation presented by Doelling et al. (2000) (eqn. 2.4) the wear coefficient is given as

\[ K = \frac{dS}{dt} \left( \frac{\mu T}{H} \right) \]  

(3.6)

In the above eqn. (3.6) the wear rate is calculated from the LVDT wear data, the entropy rate is calculated as shown in eqn. (3.5). The coefficient of friction is given by the tribometer as a function of time. The temperature \( T \) is the near surface temperature of the wearing specimen from the thermocouple reader.

The wear coefficients calculated are compared to published values of Archard’s wear coefficient. See Appendix A for the code that is used to evaluate the wear coefficients from the experiments. It is shown that for the contact pairs testing in this study the wear coefficients agree very well with the published values [Rabinowicz (1980)], [Rothbart (1996)]. The published value of wear coefficient for Brass on Steel and Bronze on Steel are

\[ 10^{-2} < K_{\text{brake}} < 10^{-3} \quad \text{[Rabinowicz (1980)]} \]  

(3.7)

\[ K_{\text{brass}} = 6 \times 10^{-4} \quad \text{[Rothbart (1996)]} \]  

(3.8)

Tables 3.3 and 3.4 shows the calculated wear coefficient values compared to the published value.
Table 3.3 SAE 40 Bronze on Steel 4140 test results

<table>
<thead>
<tr>
<th>Sliding pair</th>
<th>Load (N)</th>
<th>Speed (rpm)</th>
<th>Steady state T (K)</th>
<th>RMS Friction</th>
<th>Expt. Wear (m)</th>
<th>Wear coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bronze-Steel(4140)</td>
<td>17.79</td>
<td>100</td>
<td>302.01</td>
<td>0.484</td>
<td>0.00012</td>
<td>5.417x10^{-4}</td>
</tr>
<tr>
<td>Bronze-Steel(4140)</td>
<td>17.79</td>
<td>200</td>
<td>311.0</td>
<td>0.3353</td>
<td>0.00031</td>
<td>2.642 x10^{-4}</td>
</tr>
<tr>
<td>Bronze-Steel(4140)</td>
<td>13.34</td>
<td>100</td>
<td>302.5</td>
<td>0.4362</td>
<td>0.00013</td>
<td>5.271 x10^{-4}</td>
</tr>
<tr>
<td>Bronze-Steel(4140)</td>
<td>13.34</td>
<td>200</td>
<td>308.0</td>
<td>0.5068</td>
<td>0.00027</td>
<td>9.055 x10^{-4}</td>
</tr>
</tbody>
</table>

\[ K_{\text{bronz-expt}} = 5.5965 \times 10^{-4}; \quad 10^{-3} < K_{\text{bronz}} < 10^{-4} \]  

(3.9)

The above value is well in the range of published value given in eqn. (3.7)

![Figure 3.8 Wear coefficients comparison for the 4 tests (Bronze on Steel)](image-url)
Table 3.4 Cartridge Brass on Steel 4140 test results

<table>
<thead>
<tr>
<th>Sliding pair</th>
<th>Load (N)</th>
<th>Speed (rpm)</th>
<th>Steady state T K</th>
<th>RMS Friction</th>
<th>Expt. Wear (m)</th>
<th>Wear coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brass-Steel(4140)</td>
<td>13.34</td>
<td>200</td>
<td>305.06</td>
<td>0.458</td>
<td>0.00049</td>
<td>2.287x10^{-4}</td>
</tr>
<tr>
<td>Brass-Steel(4140)</td>
<td>13.34</td>
<td>100</td>
<td>303.05</td>
<td>0.549</td>
<td>0.00025</td>
<td>4.075x10^{-4}</td>
</tr>
<tr>
<td>Brass-Steel(4140)</td>
<td>8.89</td>
<td>100</td>
<td>302.39</td>
<td>0.564</td>
<td>0.00012</td>
<td>1.867x10^{-4}</td>
</tr>
<tr>
<td>Brass-Steel(4140)</td>
<td>8.89</td>
<td>200</td>
<td>303.56</td>
<td>0.494</td>
<td>0.00035</td>
<td>3.163x10^{-4}</td>
</tr>
</tbody>
</table>

\[ K_{\text{brass-exp}} = 2.8477 \times 10^{-4}; \quad K_{\text{brass-exp}} \approx K_{\text{brass}} \quad (3.10) \]

The above value agrees reasonably with the published value given in eqn. (3.8)

Figure 3.9 Wear coefficients comparison for the 4 tests (Brass on Steel)
Doelling et al. (2000) reported a relation between the normalized wear measured and normalized entropy calculated. They found that within first approximations, normalized wear is equal to normalized entropy. For experiments undertaken in this work the relationship has been verified to be true for both the contact pairs and at all the loads and speeds. Eqn. (3.5) gives the entropy generation rate and the wear data in the experiment which is the output of the tribometer is recorded at an interval of 20 seconds. The fractional increase in the wear as a function of time can be correlated to the fractional entropy generated in that interval. In other words, the normalized wear is given as the increase in the wear in each interval to the maximum wear during the experiment.

\[
Norm_w = \frac{w_i - w_0}{w_n - w_0}, \text{ where } w_i \text{ is the wear reading at the } i^{th} \text{ interval, } w_0 \text{ is the initial wear (which is taken as zero) and } w_n \text{ is the wear reading at the end of the experiment. The normalized entropy is the fractional increase in the entropy in an interval. Thus, } \frac{S_{\text{gen}_i}}{S_{\text{gen}_n}}, \text{ where } S_{\text{gen}_i} \text{ is the entropy generated in the } i^{th} \text{ interval and } S_{\text{gen}_n} \text{ is the total entropy accumulated at the end of the experiment in the system. Figures 3.10 and 3.11 show a plot of normalized wear against normalized entropy for Bronze on Steel and Brass on Steel for various sets of load and speed conditions. It can be observed from the Figures 3.10 and 3.11 that at steady state normalized wear is equal to normalized wear. This means that degradation (here wear) is accompanied by an equal amount of entropy which is also the measure of the magnitude of the degradation. This equality explains the basic concept of modeling degradation in terms of entropy generated.}
Figure 3.10 Normalized wear Vs. Normalized entropy showing linear dependence

Figure 3.11 Normalized wear Vs. Normalized entropy showing linear dependence
The wear coefficient is the property of the material pair in contact. It is found to vary with load and nature of contact (dry, lubricated etc). For a given harder material the wear coefficient of the wearing material increases with decrease in the hardness of the wearing material. Hardness pressure of Cartridge Brass is 390.5 MPa and that of SAE 40 Bronze is 443.75 MPa. Therefore, the wear coefficient of SAE 40 Bronze should be lower than that of Cartridge Brass when tested against a common non-wearing material (here, 4140 Steel). Figure 3.12 shows a comparison of wear coefficients for Bronze and Brass for same loading conditions, wearing against the same harder material (Steel 4140). Note that the wear coefficients of both the copper alloys used is greater than wear coefficient of copper ($10^{-4}$). This is true as the hardness of both copper alloys used here is greater than that of copper.

![Figure 3.12 Comparison of Wear Coefficient of Brass and Bronze](image-url)
3.5 Analytical Solution of Temperature in the Stationary Specimen

The temperatures recorded by the thermocouples in the experiments are the measure of net the entropy generated in the sliding system as discussed in the section 3.4. This section deals with an analytical solution for the temperatures in the stationary specimen and comparison with the experiment. The total heat frictional generated is given by product of the frictional force and velocity. The source of heat in the stationary specimen is the flux conducted at the contact interface which is part of total frictional heat. This heat conducted into the stationary specimen per unit area is given as

\[ q_2 = (1 - \eta_1)q = (1 - \eta_1) \frac{\mu N v}{A_r} \quad \text{or} \quad q_2 = \eta_2 q = \eta_2 \frac{\mu N v}{A_r} \tag{3.11} \]

where \( \eta_1 \) is the partition factor with respect to the wearing specimen and \( \eta_2 \) is the partition factor with respect to the stationary specimen. The parameter \( A_r \) is the real area of contact that is taken as 15% of the apparent area of contact [Liu et al. (2001)]. The basis for this assumption was the \( A_r \) values reported in Liu et al. (2001) for copper on steel. For the experiments presented in this work the load range was between 8.89 N to 17.79 N. As the surface preparation of the specimen used in the present work is same as that used in Liu et al. (2001), the real area of contact \( A_r \) can be extrapolated to the load range used in the experiments assuming that \( A_r \). Here the inherent implication in this assumption is that for same surface preparation and load range the real area of contact for copper would be same as that for its alloys. This resulted in an approximate range of real area between 10% and 30%. A comparison of temperatures obtained by using \( A_r = 10\%, \ 15\%, \ 20\%, \ \text{and} \ 25\% \) with the experimentally obtained temperatures (Figure 3.20 and 3.21) showed that the \( A_r = 15\% \) having the closest agreement. The apparent contact area of the stationary specimen,
(Figure 3.13) is an annular ring of inner radius \( r_1 \) and outer radius \( r_2 \). The area with inner radius \( r_2 \) and outer radius \( r_4 \) is neglected in the heat transfer analysis as its width \((r_4 - r_2)\) is of the order of 1.6 mm which is very small when compared to the width of the specimen \((30.2 \text{ mm})\).

![Figure 3.13 Contact surface of the stationary specimen](image)

Hence, the domain considered for the heat transfer analysis is cylindrical shell with outer radius \( r_2 \) and inner radius \( r_3 \) as shown in Figure 3.14.

![Figure 3.14 Schematic of the domain of the stationary specimen](image)

The heat flux generated in the wearing specimen is conducted to the harder specimen (stationary specimen) through the contact area. The area with outer radius \( r_1 \) and inner radius \( r_3 \) is considered in the analysis.
$r_3$ is exposed to natural convection. Also, the internal surface of the stationary specimen is exposed to natural convection. It is observed in the experiments that last thermocouple (the one farthest from the interface) records temperature about 5 to 6°C more than the ambient temperature. By linear extrapolation of these temperatures along the height of the stationary specimen it can be assumed to a reasonable accuracy that at 50.2 mm (2 inch) from the interface the stationary specimen attains ambient temperature. The specimen was insulated at the outer surface using a carbon foam cover as shown in figure 3.4. Hence, at radius $r_2$ insulation boundary condition is considered.

![Figure 3.15 Boundary conditions for the domain considered (side view)](image)

**Figure 3.15 Boundary conditions for the domain considered (side view)**

**Governing equation**

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0
\]

**Boundary conditions**

(i) \( \frac{\partial T}{\partial r} = 0 \) \hspace{1cm} @ \( r = r_2 \)

(ii) \( k_s \frac{\partial T}{\partial r} = h(T - T_{amb}) \) \hspace{1cm} @ \( r = r_3 \)
(iii) \(-k_s \frac{\partial T}{\partial z} = q_2\) \hspace{1cm} @ z = 0 \hspace{0.5cm} \text{for} \hspace{0.5cm} r_1 < r < r_2 \\

(iv) \(k_s \frac{\partial T}{\partial z} = h(T - T_{amb})\) \hspace{1cm} @ z = 0 \hspace{0.5cm} \text{for} \hspace{0.5cm} r_2 < r < r_1 \\

(v) \(T = T_{amb}\) \hspace{1cm} @ z = L

The boundary condition for the domain at \(z = 0\) are different over the region from \((r_2 < r < r_1)\) and \((r_1 < r < r_2)\). Hence, the domain is divided into two regions. Matching conditions of temperature and flux are used at the separation surface (at radius \(r_1\)) to obtain a continuous solution for the domain.

Figure 3.16 Region 1 and respective boundary conditions

Figure 3.17 Region 2 and respective boundary conditions
The governing equations for each regions and their respective solution is given as follows.

- **Region 1**

  Governing equation

  \[
  \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0
  \]

  Boundary conditions

  (i) \( \frac{\partial T}{\partial r} = 0 \) \hspace{1cm} @ \( r = r_2 \)

  (ii) **Matching conditions** \hspace{1cm} @ \( r = r_1 \)

  (iii) \( -k_s \frac{\partial T}{\partial z} = q_2 \) \hspace{1cm} @ \( z = 0 \)

  (iv) \( T = T_{amb} \) \hspace{1cm} @ \( z = L \)

  Using dimensionless parameters:

  \[ \bar{r} = \frac{r}{r_2}; \quad \bar{z} = \frac{z}{r_2}; \quad \bar{\theta}_h = \frac{T - T_{amb}}{T_{amb}}; \quad b = \frac{r_1}{r_2}; \quad a = \frac{L}{r_2} \]

  The governing equation and boundary conditions can be re-written as

  \[
  \frac{\partial^2 \bar{\theta}_h}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{\theta}_h}{\partial \bar{r}} + \frac{\partial^2 \bar{\theta}_h}{\partial \bar{z}^2} = 0
  \]

  (i) \( \frac{\partial \bar{\theta}_h}{\partial \bar{r}} = 0 \) \hspace{1cm} @ \( \bar{r} = 1 \)

  (ii) **Matching Conditions** \hspace{1cm} @ \( \bar{r} = b \)

  (iii) \( -k_s \frac{T_{amb}}{r_2} \frac{\partial \bar{\theta}_h}{\partial \bar{z}} = q_2 \) \hspace{1cm} @ \( \bar{z} = 0 \)

  (iv) \( \bar{\theta}_h = 0 \) \hspace{1cm} @ \( \bar{z} = a \)
At the matching surface the boundary conditions is of temperature and heat flux and hence at 
\( r = r_1 \) the boundary conditions is non-homogeneous. Thus, there are two non-homogeneous conditions in region 1. Using the superposition principle, the problem can be split into two sub-problems, each considering one non-homogeneity.

\[
\bar{\theta}_h = \bar{\theta}_{h-1} + \bar{\theta}_{h-2} \tag{3.13}
\]

First of these sub-problems are dealt as follows

\[
\frac{\partial^2 \bar{\theta}_{h-1}}{\partial \bar{F}^2} + \frac{1}{\bar{F}} \frac{\partial \bar{\theta}_{h-1}}{\partial \bar{F}} + \frac{\partial^2 \bar{\theta}_{h-1}}{\partial \bar{Z}^2} = 0 \tag{3.14}
\]

(i) \( \frac{\partial \bar{\theta}_{h-1}}{\partial \bar{F}} = 0 \) \hspace{1cm} @ \( \bar{F} = 1 \)

(ii) \( \bar{\theta}_{h-1} = 0 \) \hspace{1cm} @ \( \bar{F} = b \)

(iii) \( -k_s \frac{T_{amb}}{r_2} \frac{\partial \bar{\theta}_{h-1}}{\partial \bar{Z}} = q_2 \) \hspace{1cm} @ \( \bar{Z} = 0 \)

(iv) \( \bar{\theta}_{h-1} = 0 \) \hspace{1cm} @ \( \bar{Z} = a \)

The solution is given as

\[
\bar{\theta}_{h-1} = \sum_{n=1}^{\infty} C_n \left[ J_0(\lambda_n \bar{F}) - \frac{J_1(\lambda_n)}{Y_1(\lambda_n)} Y_0(\lambda_n \bar{F}) \right] \left( \sinh(\lambda_n \bar{z}) - \tanh(\lambda_n \bar{a}) \cosh(\lambda_n \bar{z}) \right) \tag{3.15}
\]

See Appendix C for details of the solution. The heat flux boundary condition which is the non-homogenous condition and was dealt using the orthogonal property of Bessel functions, (see Appendix C for orthogonal property), thus

\[
C_n = \frac{q_2 \sqrt{a}}{\lambda_n k_s T_{amb}} \int_{b}^{1} \bar{F} \left( J_0(\lambda_n \bar{F}) + \frac{A_2}{A_1} Y_0(\lambda_n \bar{F}) \right) d\bar{F}
\]

\[
= \frac{1}{A_1} \int_{b}^{1} \bar{F} \left( J_0(\lambda_n \bar{F}) + \frac{A_2}{A_1} Y_0(\lambda_n \bar{F}) \right)^2 d\bar{F}
\]
And Eigenvalues $\lambda_n$ are roots of eqn. (3.16). The Eigenvalues were found out using the graphical intersection method.

$$\frac{J_0(\lambda)}{Y_0(\lambda)} = \frac{J_1(\lambda b)}{Y_1(\lambda b)}$$

(3.16)

The second sub-problem is as follows-

$$\frac{\partial^2 \bar{\theta}_{h-2}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{\theta}_{h-2}}{\partial \bar{r}} + \frac{\partial^2 \bar{\theta}_{h-2}}{\partial \bar{z}^2} = 0$$

(3.17)

(i) \( \frac{\partial \bar{\theta}_{h-2}}{\partial \bar{r}} = 0 \) \( \bar{r} = 1 \)

(ii) **Matching Condition** \( \bar{r} = b \)

(iii) \( \frac{\partial \bar{\theta}_{h-2}}{\partial \bar{z}} = 0 \) \( \bar{z} = 0 \)

(iv) \( \bar{\theta}_{h-2} = 0 \) \( \bar{z} = a \)

Thus, the solution is-

$$\bar{\theta}_{h-2} = \sum_{n=1}^\infty E_n \left[ I_0(\lambda_n \bar{r}) - \frac{I_1(\lambda_n)}{K_1(\lambda_n)} \right] K_0(\lambda_n \bar{z}) \cos(\lambda'_n \bar{z})$$

(3.18)

The Eigenvalues $\lambda'_n$ are obtained as roots of the eqn. (3.19).

$$\cos(\lambda' a) = 0 \Rightarrow \lambda_n = \frac{(2n-1)\pi}{2a}$$

(3.19)

The constant $E_n$ was determined by applying the matching conditions as shown in eqn. (3.23) & (3.24). Therefore, the solution for the region 1 is as follows-

$$\bar{\theta}_h = \sum_{n=1}^\infty C_n \left[ J_0(\lambda_n \bar{r}) - \frac{J_1(\lambda_n)}{Y_1(\lambda_n)} \right] Y_0(\lambda_n \bar{r}) \left[ \sinh(\lambda_n \bar{z}) - \tanh(\lambda_n a) \cos(\lambda_n \bar{z}) \right]$$

$$+ \sum_{n=1}^\infty E_n \left[ I_0(\lambda'_n \bar{r}) + \frac{I_1(\lambda'_n)}{K_1(\lambda'_n)} \right] K_0(\lambda'_n \bar{z}) \cos(\lambda'_n \bar{z})$$

(3.20)
• Region 2

Governing equation

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0
\]  \hspace{1cm} (3.21)

Boundary conditions

(i) Matching conditions \hspace{1cm} @ r = r_1

(ii) \( k_s \frac{\partial T}{\partial r} = h(T - T_{amb}) \) \hspace{1cm} @ r = r_3

(iii) \( k_s \frac{\partial T}{\partial z} = h(T - T_{amb}) \) \hspace{1cm} @ z = 0

(iv) \( T = T_{amb} \) \hspace{1cm} @ z = L

Using dimensionless parameters:

\[
\bar{r} = \frac{r}{r_2}; \hspace{0.5cm} \bar{z} = \frac{z}{r_2}; \hspace{0.5cm} \bar{T}_h = \frac{T - T_{amb}}{T_{amb}}; \hspace{0.5cm} \bar{b}' = \frac{r_3}{r_2}; \hspace{0.5cm} a = \frac{L}{r_2}
\]

The governing equation and boundary conditions can be re-written as-

\[
\frac{\partial^2 \bar{T}_h}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{T}_h}{\partial \bar{r}} + \frac{\partial^2 \bar{T}_h}{\partial \bar{z}^2} = 0
\]

(i) Matching Conditions \hspace{1cm} @ \bar{r} = b

(ii) \( k_s \frac{1}{r_2} \frac{\partial \bar{T}_h}{\partial \bar{r}} = h\bar{T}_h \) \hspace{1cm} @ \bar{r} = b'

(iii) \( k_s \frac{1}{r_2} \frac{\partial \bar{T}_h}{\partial \bar{z}} = h\bar{T}_h \) \hspace{1cm} @ \bar{z} = 0

(iv) \( \bar{T}_h = 0 \) \hspace{1cm} @ \bar{z} = a

The solution is as follows
The constant $M_n$ was determined by using matching conditions shown in eqn. (3.23) & (3.24). And the Eigenvalues $\lambda_n^0$ are roots of the following equation-

$$\lambda^0 = -\frac{h_r}{k_b} \tan(\lambda^0 a)$$

- **Matching condition I**

Temperature at the common surface $\bar{r} = b$ is equal for both regions. Hence equating the temperatures at $\bar{r} = b$

(i) $\bar{\theta}_{h-1} + \bar{\theta}_{h-2} = \bar{\theta}_h$ \quad \text{at} \quad \bar{r} = b$

$$\sum_{n=1}^{\infty} E_n \left[ I_0(\lambda_n^0 b') + \frac{I_1(\lambda_n^0)}{K_1(\lambda_n^0 b')} K_0(\lambda_n^0 b') \right] \cos(\lambda_n^0 \bar{z}) =$$

$$\sum_{n=1}^{\infty} M_n \left[ I_0(\lambda_n^0 b') + \frac{I_1(\lambda_n^0)}{K_1(\lambda_n^0 b')} K_0(\lambda_n^0 b') \right] (\sin(\lambda_n^0 \bar{z} - \tan(\lambda_n^0 a) \cos(\lambda_n^0 \bar{z})))$$

(3.23)

- **Matching condition II**

The heat flux at the common surface $\bar{r} = b$ is equal for both regions. Hence equating the heat flux at $\bar{r} = b$

(ii) $\frac{\partial(\bar{\theta}_{h-1} + \bar{\theta}_{h-2})}{\partial \bar{z}} = \frac{\partial \bar{\theta}_h}{\partial \bar{z}}$ \quad \text{at} \quad \bar{r} = b$
\[ \sum_{n=1}^{\infty} C_n \lambda_n \left[ J_1(\lambda_n b') - \frac{J_1(\lambda_n)}{Y_1(\lambda_n)} Y_1(\lambda_n b') \right] \right] \right) \left( \sinh(\lambda_n z) - \tanh(\lambda_n a) \cosh(\lambda_n z) \right) + \]

\[ \sum_{n=1}^{\infty} E_n \left[ I_1(\lambda'_n b) - \frac{I_1(\lambda'_n)}{K_1(\lambda'_n)} K_1(\lambda'_n b) \right] \cos(\lambda'_n z) = \]

\[ \sum_{n=1}^{\infty} M_n \left[ I_1(\lambda^n b) - \frac{I_1(\lambda^n)}{K_1(\lambda^n b)} K_1(\lambda^n b) \right] \left( \sin(\lambda^n z - \tan(\lambda^n a) \cos(\lambda^n z) \right) \]

(3.24)

Solving equations (3.21) and (3.22) the constants \( E_n \) and \( M_n \) were calculated as follows-

\[ M_n = -C_n \lambda_n \left[ J_1(\lambda_n b) - \frac{J_1(\lambda_n)}{Y_1(\lambda_n)} Y_1(\lambda_n b) \right] \int_0^a \frac{\left( \sinh(\lambda_n z) - \tanh(\lambda_n a) \cos(\lambda_n z) \right) d\bar{z}}{\int_0^a \left( \sin(\lambda_n z) - \tan(\lambda_n a) \cos(\lambda_n z) \right) d\bar{z}} \frac{1}{X} \]

(3.25)

where

\[ X = \lambda^n b \left[ I_1(\lambda^n b) - \frac{I_1(\lambda^n)}{K_1(\lambda^n b)} K_1(\lambda^n b) \right] - \frac{I_1(\lambda^n b') - \frac{hr_2}{k_s \lambda^n} I_0(\lambda^n b')}{K_1(\lambda^n b') + \frac{hr_2}{k_s \lambda^n} K_0(\lambda^n b')} \]

\[ W = \lambda'_n \left[ I_1(\lambda'_n b) - \frac{I_1(\lambda'_n)}{K_1(\lambda'_n b)} K_1(\lambda'_n b) \right] - \frac{I_1(\lambda'_n b') - \frac{hr_2}{k_s \lambda'_n} I_0(\lambda'_n b')}{K_1(\lambda'_n b') + \frac{hr_2}{k_s \lambda'_n} K_0(\lambda'_n b')} \]

\[ W = \frac{[I_0(\lambda^n b) + \frac{I_1(\lambda'n)}{K_1(\lambda'_n b)} K_0(\lambda'_n b)]}{[\frac{I_0(\lambda'_n b) + \frac{I_1(\lambda'n)}{K_1(\lambda'_n b)} K_0(\lambda'_n b)]} \]

And \( E_n = M_n W \int_0^a \left( \sin(\lambda_n z) - \tan(\lambda_n a) \cos(\lambda_n z) \right) d\bar{z} \)

\[ \int_0^a \cos(\lambda_n z) d\bar{z} \]

(3.26)
The constants $E_n$ and $M_n$ were calculated as shown in the eqns. (3.25) and (3.26) and thus the solution for temperature in the stationary specimen was obtained.

- **Input data**

(i) **Dimensions**

$r_1 = 0.0127 \text{ m}$
$r_2 = 0.0143 \text{ m}$
$r_3 = 0.00794 \text{ m}$
$L = 0.05 \text{ m}$
$A = \pi(r_2^2 - r_1^2)$

(ii) $q_2 = \eta_2 \frac{\mu N v}{A_r} = \eta_2 \frac{\mu_{rms} L \frac{2\pi s}{60}}{0.1 \pi (r_2^2 - r_1^2)}$

where $\mu_{rms}$ is the RMS value of the coefficient of friction and $s$ is the speed in rpm.

(iii) $T_{\text{amb}} = 25^\circ \text{C}$

(iv) **Material properties as shown in Table 3.1**

Figure 3.18 is a contour plot of temperature in the stationary specimen when wearing material is SAE 40 Bronze for a 17.79 N load and speed of 200 rpm. The surface of region 1 at $z = 0$ is subjected to the frictional heat flux and the surface of region 2 at $z = 0$ is exposed to convection. It is observed that the isotherms become straight as the length increases indicating that away from the interface the temperature does not vary along the radial direction. The partition factor for steel when in contact with bronze is 0.4380 calculated as in eqn. (3.2).
Figure 3.18 Contour plot of Temperature °C in Stationary Specimen (Bronze-Steel)

Figure 3.19 is a contour plot of temperature in the stationary specimen when wearing material is Cartridge Brass for a 13.34 N load and speed of 100 rpm. The partition factor for steel when in contact with Brass is 0.3979 calculated as shown in eqn. (3.2). The partition factor for Brass is higher than that for Bronze. Hence, for an equal amount of heat generated in both tests, the temperature in the stationary specimen (Steel) in the Bronze test must be higher than in the Brass test. Figure 3.18 and 3.19 clearly show this difference. Also the heat generated is proportional to the load and as mentioned before the load in the Bronze test here is 17.79 N where as in Brass test it is 13.34 N. This further explains the higher temperatures in the Bronze test compared to the Brass test.
Figure 3.19 Contour plot of Temperature °C in Stationary Specimen (Bronze-Steel)

Figure 3.20 and 3.21 show a comparison of the experimentally obtained temperature with theoretical temperatures for Bronze and Brass tests respectively with $A_r = 10\%, 15\%, 20\%, \text{ and } 25\%$. The comparison showed good agreement for $A_r = 15\%$. A verification of the experimentally obtained temperatures using the above presented analytical solution shows good agreement. Note here that the real area of contact used is 15% of the apparent area of contact. Figures 3.22 and 3.23 show a comparison of temperatures obtained theoretically using the above proposed analytical solution to the temperatures read by the thermocouples in the experiment (Figure 3.3). The fact that experimental values of temperatures are in good agreement with theoretical authenticates the entropy values and wear coefficients obtained in section 3.4.
Figure 3.20 Temperature for different values of real area of contact (Bronze on Steel)

Figure 3.21 Temperature for different values of real area of contact (Brass on Steel)
Figure 3.22 Theoretical \( (A_r = 15\%) \) Vs. Experimental for SAE 40 Bronze on Steel 4140

Figure 3.23 Theoretical \( (A_r = 15\%) \) Vs. Experimental for Cartridge Brass on Steel 4140
In Bronze on Steel experiments 5 thermocouples were used but in case of Brass on Steel experiments only 3 thermocouples were used. This was because the vibrations in the Brass on Steel experiments were higher and this would displace some of the thermocouples attached to the specimen. For this reason lesser number of thermocouples was used.
4. Theoretical Model for Entropy Production

The purpose of this chapter is to theoretically calculate the entropy generated in the sliding contact system by considering the frictional energy dissipated. The energy dissipated is a result of the plastic deformation in the near surface of the wearing material. The theoretical entropy thus calculated has been used to verify the formulation (eqn. 2.4), the results of which are presented in chapter 5.

When two solid bodies slide against each other there is plastic deformation produced in and around the real areas of solid/solid contact. The deformation is substantial when the contact is dry or poorly lubricated. This deformation plays an important role in the tribological behavior of the sliding contact.

The mode of deformation and its influence on the surface properties has been studied by many researchers. Bowdon and Tabor (1954) in their adhesion theory have calculated the coefficient of friction by considering the adhesion of the surface asperities and the plastic deformation. According to the theory applied to friction and wear during sliding, the eventual formation of wear particles can be explained in six steps: “(i). loaded contact of single asperities on a pair of rubbing surfaces, (ii) the formation, (iii) growth and (iv) failure of adhesive junctions, followed by (v) the transfer and transfer back of material to the mating surface and finally (vi) detaching of transferred material, or parts of it, from the solid surface, leading to loose wear particles”. It is well established that plastic deformation plays an important role in these processes. In their theory Bowdon and Tabor (1954) considered combined effects of adhesion and surface roughness to come up with the coefficient of friction as follows

\[
\mu = \mu_{ad} + \mu_{def} \quad (4.1)
\]
where $\mu_{ad}$ is called the adhesion term and $\mu_{def}$ is called the ploughing term of the coefficient of friction. The ploughing term $\mu_{def}$ can be a dominant on very rough surfaces. Moore and Douthwaite (1976) have concentrated on the large plastic strains observed at considerable distances from the wear surface, and they suggested that plastic deformation could account for most of the work observed. Bukley (1977) in his paper explained that if adhesion occurs when two metals touch each other in a clean environment, then the plastic deformation is also observed.

Rigney and Gleaser (1978), Rigney and Hirth (1979) and Heilmann and Rigney (1981) have developed an energy-based model of friction. The basis of this model is the assumption that all the friction work is transferred into plastic deformation. Heilmann and Rigney (1981) calculated the coefficient of friction by equating the external work done by the material by the friction force to the internal resistance offered by the material.

$$\mu = \frac{A \int_0^\infty \tau(z) \Delta \gamma(z) \, dz}{L \times}$$

(4.2)

where $A$ is the total area of asperity contacts; $\tau(z)$, $\Delta \gamma(z)$ are the shear stress distribution and incremental shear strain distribution along the depth below the contact surface respectively given as

$$\tau(z) = \tau_{\text{max}} \left[ 1 - \left(1 - \frac{\tau_s^2}{\tau_{\text{max}}^2} \right)^{\exp(-a_c \, z)} \right]^{1/2}$$

(4.3)

$$\Delta \gamma(z) = a_c \, \delta \times \exp(-a_c \, z)$$

(4.4)

where $a_c$ is a constant dependent on material properties and the tribosystem, $\tau_{\text{max}}$ is the maximum shear strength of the material and $\tau_s$ is the average surface stress. A detail
discussion of the above stress-strain distribution is presented in section 4.2. The derivation of the stress-strain distribution is presented in Appendix B.

The present study makes the assumption that the all the friction work is transferred into plastic deformation and this irreversible deformation is the mechanism by which energy is dissipated. The mechanical energy is transformed into heat in the deformed region, increasing the temperature of the interface and thereby the contacting bodies. Section 4.1 deals with the size of this deformed region under the wearing surface. Section 4.2 deals with the calculation of the amount of energy dissipated as a result of plastic deformation in this deformed region.

4.1 Severely Deformed Region (SDR)

Studies on the nature of deformation and the extent of it in the wearing bodies are presented in this section. In a general sliding wear situation when a hard body slides over a soft body, plastic deformation is produced due to the applied load in an around the real areas of solid/solid contact both in the hard and the soft body. However the plastic deformation in the hard surface is minimal, if a clean environment is assumed at the interface (free of wear debris, foreign particles or abrasive particles), and can be neglected in comparison with the deformation in the soft body. Duiatzenberg and Zaat (1973) gave a quantitative determination of deformation by sliding wear in their paper. They used optical and electron microscopic observations to determine the effective deformation of in worn materials. Though they state that severe deformation occurs close to the surface, they do not emphasize in their study that plastic deformation is specifically limited to this near-surface region. Tusya (1976) was probably the first researcher to recognize that the majority of the plastic deformation which is irreversible was concentrated in a well-defined region near the surface. She called this region ‘micronized layer’. She also proposed a model of friction which is based on the work done
during plastic deformation. This model will be described later in the text. Later Rigney and Glaeser (1977) described a wear model on steady state wear. They emphasized plastic deformation near the surface, particularly in the ‘highly deformed region’ which has a fine microstructure and a high degree of preferred orientation. In metals and in some ceramic materials, this near surface microstructure consists of dislocation cells developed during an initial break-in period. They also stated that under steady state conditions the average cell structure at a given distance from the surface remains constant, and the average thickness t of the cell structure region is a constant that depends on material properties and on the details of the sliding wear test. Tsuya (1976) has also suggested that this region of severe or high plastic deformation is well-defined. This is an important result because it establishes the consistency of the highly deformed region and its thickness. This near-surface region of high or severe plastic deformation will be referred as ‘severely deformed region’ or (SDR) in this text.

It is interesting to note that these conclusions about the severely deformed region were derived after detailed observation of the microstructure and grain boundary distortion in the softer body at close vicinity to the contact surface. Duatzenberg and Zaat (1973) derived the effective deformation in sliding process from the deflection of the grain boundaries and from the change in grain thickness. Heilmann and Rigney (1981) observed the substructure under the sliding surface using Transmission electron microscopy (TEM). A TEM micrograph of the worn surface (OFHC Copper sample) from Heilmann and Rigney (1981) gave an insight into the severely deformed region.

Figure 4.1 shows the longitudinal section of a wear sample of OFHC copper worn against Steel [Heilmann and Rigney (1981)]. The substructure which results from plastic deformation varies with depth below the sliding interface. At the sliding interface, a transfer layer (dark
band) of fine particles containing Cu and Fe is visible. Below the transfer layer, well-defined elongated cells appear in the deformed copper. The elongation of grains compared to their size in the base metal is viewed as the result of plastic deformation. By comparing the grain sizes in the near-surface region and the base metal from the images a rough estimate of the thickness of the severely deformed can be obtained.

![TEM Micrograph of the worn surface of an OFHC copper sample](image)

**Figure 4.1 TEM Micrograph of the worn surface of an OFHC copper sample. Test conditions: Block of copper sliding on 440C steel ring, 66.7 N normal load, sliding speed 1cm/s, total sliding distance 12m. Courtesy of [P .Heilmann and D.A. Rigney (1981)]**

Kennedy (1989) measured the near-surface deformations due to sliding using microscopic observation of the contact region and compared these values with values predicted by his analytical model. In the analytical model, finite element visco-plasticity techniques were developed to model high rate plastic strains in the vicinity of a moving contact. The experimental values were in good agreement with the analytically predicted values. The experiments were carried out for copper wearing against tool steel. Kennedy (1989) suggested that the thickness of the severely deformed region, for the loading conditions in his
experiment was lesser than 300 µm. The thickness however is dependent on the load and speed in the experiment.

**Figure 4.2 Comparison between measured and predicted plastic displacement in copper specimen Kennedy (1989)**

Rice et al. (1989) performed a series of microscopic observation of sectioned worn surface in a Titanium alloy on Steel experiments. They reported a thickness of severely deformed layer between 10 – 15 µm. Their unique study of the thickness of severely deformed layer in the running-in process is of importance. This showed that in their experiments the thickness first increases gradually during the running-in process and then stabilizes after approximately 1000 cycles of load application and reaches a quasi-static equilibrium value. This result along with implications of the findings of Tusya (1976) establish the existence of a steady state thickness of the severely deformed region.

Microscopic observations of the worn materials to determine the thickness of the severely deformed region for the test specimen is beyond the scope of this work. Hence, the thickness is assumed to be a reasonable range of 50 – 300 µm. A parametric study comparing the
results obtained by considering the thickness to be between 50 – 300 \( \mu m \) is done. The results for which are presented in chapter 5.

### 4.3 Heat Dissipated During Plastic Deformation

The coefficient of friction between two materials gives an estimate of the heat dissipated during the sliding process. The simplest form of formulating this heat is by considering frictional work done.

\[
Q = \mu N x
\]

(4.5)

where \( \mu \) is the coefficient of friction and \( x \) is the sliding distance. The rate of heat dissipation can be calculated by considering the sliding speed instead of sliding distance. The irreversible dissipative heat as a result of plastic deformation in this severely deformed region has been studied by Suh and Sridharan (1975), Tsuya (1977), Heilmann and Rigney (1981) and others. Tsuya (1977) calculated this deformation energy for a contact width of \( w \) and a depth \( t \) for a sliding distance of \( S \)

\[
W = \int \int \int_0^w \int_0^S \int_0^t \rho E_w \, dx \, dy \, dz
\]

(4.6)

where \( E_w \) is the deformation energy per unit mass and \( \rho \) is the density. Tsuya used data on micro hardness profiles to estimate \( E_w \). Rigney and Hirth (1979) pointed out that there are problems with the details of her calculation as \( E_w \) is not a derived function of the material properties.

It had been proven using plasticity theory Dautzenberg (1977) that the displacement of the material during wear is caused by simple shear. Heilmann and Rigney (1981) formulated the plastic work during the deformation as the total work done by the shearing stress over the volume of the deformed region. The present study uses the ideas presented by Heilmann and
Rigney (1981) (eqn. (4.7)) to model the energy dissipation in the softer material. According to them the plastic work is equal to the virtual work done by the shear stresses.

\[
W = A \int_{0}^{\infty} \tau(z) \Delta \gamma(z) \, dz
\]  

(4.7)

where \( A \) is the total area of asperity contacts; \( \tau(z) \), \( \Delta \gamma(z) \) are the shear stress distribution and incremental shear strain distribution along the depth below the contact surface respectively given by the following expressions-

\[
\tau(z) = \tau_{\text{max}} \left[ 1 - \left(1 - \frac{\tau_{s}^2}{\tau_{\text{max}}^2} \right) \exp(-a_{c} z) \right]^{1/2}
\]

\[
\Delta \gamma(z) = a_{c} \delta \chi_{s} \exp(-a_{c} z)
\]

where \( a_{c} \) is a constant dependent on material properties and the tribosystem, \( \tau_{\text{max}} \) is the maximum shear strength of the material and \( \tau_{s} \) is the average surface stress and \( \delta \chi_{s} \) is the surface displacement. For the derivation of the stress and the incremental strain equation please refer Appendix B.

Thus the integral in eqn. (4.7) is as follows

\[
W = A \tau_{\text{max}} a_{c} \delta \chi_{s} \int_{0}^{\infty} \left[ 1 - \left(1 - \frac{\tau_{s}^2}{\tau_{\text{max}}^2} \right) \exp(-a_{c} z) \right]^{1/2} \exp(-a_{c} z) \, dz
\]  

(4.8)

This gives the total dissipative heat generated. The integral in eqn. (4.8) is from \( 0 \) to \( \infty \).

This means the entire domain of the wearing material is accounted for to calculate the work done in plastic deformation. According to literature presented in section 4.2, it has been already discussed that the irreversible plastic deformation is limited to only the SDR. Hence, the work done is plastic deformation has been calculated for the SDR. The eqn. (4.8) has been re-written as-
\[ W = A \tau_{\max} a_c \delta \chi \int_0^\delta \left[ 1 - \left( 1 - \frac{\tau_s^2}{\tau_{\max}^2} \right)^{1/2} \exp(-a_c z) \right] \exp(-a_c z) \, dz \]  

(4.9)

The distribution of this heat in the severely deformed region can be understood by considering the heat dissipated per unit volume in this region. From eqns. (2.5) and (2.6) it can be easily stated that the plastic work done per unit volume is given by

\[ dW = \tau(z) \Delta y(z) = \tau_{\max} \left[ 1 - \left( 1 - \frac{\tau_s^2}{\tau_{\max}^2} \right)^{1/2} \right]^{1/2} a_c \delta \chi_s \exp(-a_c z) \]  

(4.10)

The plastic work done in the deformation region is released as heat, raising the temperature of the contacting bodies. As stated earlier the heat generated rate per unit volume is equal to the rate of plastic work done per unit volume.

\[ \dot{q}(z) = \frac{dW}{dt} \]  

(4.11)

Therefore, using eqn. (4.8), the work done by the shear stresses is given as-

\[ dW = \tau_{\max} \left[ 1 - \left( 1 - \frac{\tau_s^2}{\tau_{\max}^2} \right)^{1/2} \right]^{1/2} a_c \delta \chi_s \exp(-a_c z) \]  

(4.12)

The rate of work done

\[ \frac{dW}{dt} = \tau_{\max} \left[ 1 - \left( 1 - \frac{\tau_s^2}{\tau_{\max}^2} \right)^{1/2} \right]^{1/2} a_c \frac{\delta \chi_s}{dt} \exp(-a_c z) \]  

(4.13)

where \( \frac{\delta \chi_s}{dt} \) is the rate of plastic strain at the surface. Kennedy (1989) suggested from his finite element analysis of the plastic strains in the wearing material that the rate of the surface strain can be taken as 10% of the sliding speed. Therefore, equation (4.13) can be written as

\[ \dot{q}(z) = \tau_{\max} \left[ 1 - \left( 1 - \frac{\tau_s^2}{\tau_{\max}^2} \right)^{1/2} \right]^{1/2} a_c (c_s \nu) \exp(-a_c z) \]  

(4.14)

where \( c_s = 0.1 \); [Kennedy (1989)]
The values of $\tau_z$, $\delta x_z$ are surface shear stress and surface displacement. The surface shear stress which is another unknown is taken as between $0.2 \tau_{\text{max}}$ and $0.8 \tau_{\text{max}}$. This is based on work hardening studies presented by Zum Gahr (1943). A parametric study of the resulting wear coefficient for surface shear stress ranging from $0.2 \tau_{\text{max}}$ and $0.8 \tau_{\text{max}}$ has also been carried out. To obtain the value of constant $a_c$ the displacements (plastic strain) as a function of the depth of the severely deformed region has been used to curve fit the values of $\delta x(z)$ in eqn. (4.4) to obtain the constant $a_c$. To calculate these displacements Dautzenberg (1980) & Moore and Douthwaite (1976) used a marker embedded in OFHC copper (wearing material) along the depth. The deviation in the shape of the marker profile after wearing occurs was then used as a measure to calculate the individual displacements under the wearing surface. Therefore, one method to calculate individual displacements is to use the marker profile; the other method is to use a finite element solution of the nodal displacements along the thickness of the severely deformed region. Both the marker profile technique and a finite element solution of the wearing domain are beyond the scope of this work. Hence the value of $a_c$ is calculated here by curve fitting the nodal displacements of the copper on steel experiment presented in Kennedy’s (1989) paper. This is a reasonable assumption as the shape of the curve of $\delta x(z)$ for copper alloys would be very close to that of copper. Figure 4.3 shows a comparison of the values of displacements from the work of Kennedy (1989) to the curve fit function given in eqn. (4.4). The value of $a_c = 9000$ agrees well with the published data and has been used in the present work.
Having known the value of $a_c$, $\gamma(z)$ and $\tau(z)$ are known. It was assumed that the plastic deformation in the harder material is negligible and that the rise in temperature in the harder material is due to the heat flux conducted from the interface. The energy dissipated in the highly deformed region is divided, according to the thermal conductivities, between the sliding materials. In the subsequent sections the temperature distribution in the softer material as a result of this energy dissipated due to plastic deformation is estimated theoretically.

4.7 Calculation of Entropy Flow

This section deals with the calculation of rate of entropy produced during the steady state wearing process. As stated in the beginning of this chapter the irreversibility in the tribo-system is assumed to occur in the severely deformed region. Hence, this severely deformed region is taken as the control volume. Work is done by external forces on this control volume.
and all of this work is assumed to be released as heat due to the irreversibility. As the boundary surfaces of the control volume are conducting, this released heat is transferred to the surroundings. From a thermodynamic point of view, this control volume can be categorized as an open system that witnesses an irreversible non-equilibrium process. At steady state the gradient of temperature remains a constant. Figure 4.4 shows a schematic of the control volume illustrating its boundaries and location.

![Figure 4.4 Control volume during the wearing process](image)

The wear debris which is part of the wearing material is not included in the control volume. Therefore, there is mass transfer from the system at a constant rate from the system. However, the part of the entropy lost by mass transfer is small when compared to the net entropy generated. Thus, this quantity is neglected as will be discussed later in this section. Work is done on the control volume by the source causing the relative motion. The interfacial material which is a lubricant is just shown for generalization purposes and not used in the analysis here as the contact is dry.
In the case of the disk-on-disk configuration considered here it is the work done by the spindle that provides a constant torque \( T \) as it rotates at a constant angular velocity \( \omega \). For a rotation \( \theta \) the work done by the spindle is given as

\[
W = T \theta \tag{4.15}
\]

This work done is equal to the virtual work done by the shear stresses in the severely deformed region Heilmann and Rigney (1981). The shear stress and strain at a depth \( z \) from the surface is given by Heilmann and Rigney (1981) as follows-

\[
\tau(z) = \tau_{\text{max}} \left[ 1 - \left\{ 1 - \frac{\tau_s^2}{\tau_{\text{max}}^2} \right\} \exp(-a_c z) \right]^{1/2}
\]

\[
\Delta \gamma(z) = a_c \frac{\delta \tau}{\delta \gamma} \exp(-a_c z)
\]

Thus, the work done by the shear stresses over the volume of the severely deformed region is given as-

\[
W = A \tau_{\text{max}} a_c \frac{\delta \tau}{\delta \gamma} \int_0^\delta \left[ 1 - \left\{ 1 - \frac{\tau_s^2}{\tau_{\text{max}}^2} \right\} \exp(-a_c z) \right]^{1/2} \exp(-a_c z) \; dz
\]

\[
T \theta = A \tau_{\text{max}} a_c \frac{\delta \tau}{\delta \gamma} \int_0^\delta \left[ 1 - \left\{ 1 - \frac{\tau_s^2}{\tau_{\text{max}}^2} \right\} \exp(-a_c z) \right]^{1/2} \exp(-a_c z) \; dz \tag{4.16}
\]

Eqn. (4.16) explains the assumption in this study. The external work done is assumed to be equal to the work of plastic deformation. In other words the work done by the spindle has no other effect besides the plastic deformation of the material in the severely deformed region.

Further, the work of plastic deformation is the irreversible work that causes a release of equivalent amount of heat. At steady state this heat is conducted to the surroundings at a constant rate. The rate of this work done is given as

\[
\frac{dW}{dt} = \dot{q}(\tau) = \tau_{\text{max}} \left[ 1 - \left\{ 1 - \frac{\tau_s^2}{\tau_{\text{max}}^2} \right\} \exp(-a_c z) \right]^{1/2} a_c \frac{\delta \tau}{\delta \gamma} \exp(-a_c z)
\]
The mass and energy interactions of the control volume as illustration in the Figure (4.5)

For the disk-on-disk setup considered in this study, part of the released heat in the severely deformed region of the wearing material is conducted into the harder material, part of it is convected to the surrounding atmosphere and the rest is conducted into the remaining of the wearing material.

Therefore, the total heat released due to plastic deformation is

\[ Q = Q_1 + Q_2 + Q_{\text{conv}} \]

Applying First law of thermodynamics to this system

\[ Q = W + dU \quad (4.17) \]

Applying the second law to evaluate the entropy generation in a time \( \Delta t \) we have

\[ S_{\text{gen}} = \int_{\Delta t} \left( \frac{dS}{dt} - \frac{dQ}{T} + s_m \frac{dm}{dt} \right) dt > 0 \quad (4.18) \]
where $s_m$ is the specific entropy of the material leaving the system (here its wear debris) and 
\[
\frac{dm}{dt}
\] is the rate of mass transfer.

Klamecki (1980) proposed an entropy production model for wear. In his model he defines entropy of the system as a function of thermodynamic variables of internal energy $U$, deformation gradient $F$, area $A$ and mass $M$:

\[
S = S(U, F, A, M)
\]  

(4.19)

According to Klamecki (1980), the irreversible process like that of wear is a non-equilibrium process. It can be analyzed by considering a companion equilibrium process Keller (1976). The equilibrium process is characterized by a series of equilibrium states in which the values of the thermodynamic variables are those of the actual process. If a time interval of $\Delta t$ is considered starting at a particular instant during steady state in the actual process, then the entropy generation relation is

\[
S_{gen} = \int_0^t \left( \frac{dS_e}{dt} - \frac{dQ}{T} + s_m \frac{dm}{dt} \right) dt
\]

(4.20)

where $S_e$ is the entropy of the accompanying equilibrium process and according to Klamecki (1980) the entropy $S_e$ of the accompanying equilibrium process from equation (4.19) is

\[
dS_e = \frac{1}{T}(dU + b \gamma dF - \gamma_e dA - \mu_e dM)
\]

(4.21)

In which the partial derivatives are defined as

\[
\left(\frac{\partial S_e}{\partial U}\right) = \frac{1}{T_e} ; \quad \left(\frac{\partial S_e}{\partial F}\right) = \frac{b_e}{T_e} ; \quad \left(\frac{\partial S_e}{\partial A}\right) = -\frac{\gamma_e}{T_e} ; \quad \left(\frac{\partial S_e}{\partial M}\right) = -\frac{\mu_e}{T_e}
\]

where $b$ is a stress tensor, $\gamma$ and $\mu$ are the surface energy and chemical potential respectively. Using the conservation of energy
\[ dU = dQ - dW \]

Writing \[ dW = b dF - \gamma dA + \mu dM \]

And including the effects of mass transfer we have

\[ dU = dQ + b dF - b_m dF_m - \gamma dA - \gamma_m dA_m + \mu dM - \mu_m dm - u_m dm \]

(4.22)

The subscripts \(m\) indicates mass leaving the system. According to Klamecki (1980) using eqns. (4.20), (4.21) and (4.22) the entropy generation relation can be written as

\[
S_{gen} = \int \left[ \left( \frac{1}{T_e} - \frac{1}{T} \right) \frac{dQ}{dt} + \frac{1}{T_e} \left( b - b_e \right) \frac{dF}{dt} - b_m \frac{dF_m}{dt} - \left( \gamma + \gamma_e \right) \frac{dA}{dt} - \gamma_m \mu_e \frac{dA_m}{dt} + \left( \mu - \mu_e \right) \frac{dM}{dt} 
- \mu_m \frac{dm}{dt} \right] \, dt
\]

Neglecting the effects of surface energy, chemical potential and mass transfer we have

\[
S_{gen} = \int \left[ \left( \frac{1}{T_e} - \frac{1}{T} \right) \frac{dQ}{dt} + \frac{1}{T_e} \left( b - b_e \right) \frac{dF}{dt} \right] \, dt
\]

\( (b - b_e) \frac{dF}{dt} \) is the rate of release of strain energy \( \frac{dE}{dt} \). The heat transfer term can be neglected with a reasonable approximation of the difference of temperatures of the equilibrium and actual process being small. The entropy generation term can now be written as

\[
S_{gen} = \int \left[ \frac{1}{T_e} \frac{dE}{dt} \right] \, dt
\]

Integrating we have

\[
S_{gen} = \frac{E}{T_e}
\]

The total strain energy released is the volume integral of the plastic stress and strain in the deformed region. Therefore, from equation (4.8) we have
It is interesting to note that the expression of entropy generation does not depend on whether the system is a steady state or not. This explains the universal application of entropy generation and its potential for modeling complicated processes such as wear. This implies that entropy generation relation eqn. (4.24) can also be applied to the transient stage in the wearing process. Also during a steady state wearing process, the rate of entropy is independent of time. That means at steady state wear, for a given constant load and speed the shear stress and strain distribution, the thickness of the severely deformed region and local temperature remain constant yielding a constant value for rate of entropy as given by eqn. (4.24). The net entropy generated in a given time interval is the simple the product of the time and entropy rate.

### 4.4. Orientation and Geometry of Contact Surface

Contact specimen in the presented study are cylindrical shells of Bronze and Steel 4140 respectively. Wear due to contact occurs on the flat ring surface of the metal which is oriented as disk on disk configuration. The softer metal, which is the material considered for analysis, is the bronze specimen with its wearing side ground and polished. Figure 4.6 shows the contact specimens and Figure 4.7 shows their orientation. LRI-tribometer ensures a constant contact and load between the two metals, during the test. Thermocouples on the surface of the steel specimen used are to record the temperature along the length of this specimen. Tests can
be conducted at variable speeds over a period of time or constant speed throughout the test time period. The weights of both the specimen are measured before and after the test using an accurate electronic balance.

Figure 4.6 Steel 4140 and Bronze specimen  
Figure 4.7 Contact orientation

4.5. Theoretical Model

The purpose of this model is to solve for the temperature distribution in the wearing specimen and eventually calculate the entropy produced in the control volume considered. The domain to solve for the temperature distribution is the complete wearing specimen but the control volume considered for entropy calculation is the part of the specimen where the irreversibility is assumed to be produced. The following paragraph explains about the two domains and the reason for characterizing them accordingly.

According to Rigney and Hirth (1979), Rigney and Gleaser (1978), Tsuya (1976) and others, the plastic deformation in the wearing specimen is greatly concentrated in the near surface ‘micronised’ layer or the ‘highly deformed zone’. The irreversible heat dissipated as a result of plastic deformation in the highly deformed zone is the source of frictional heat generated in the wearing contact Rigney and Hirth (1979). The heat generation per unit volume in this region is a function of the strain function in the highly deformed region Tsuya (1976). In a
thermal problem, this zone can be considered as the volume in which there is an internal heat generation. This heat generation per unit volume is basically equal to the rate of plastic work done per unit volume given in eqn. (4.14).

The domain considered to solve the temperature distribution can be divided into two regions. The region close to the interface or the severely deformed region (Region 1), is the one in which there is internal heat generation as a result of plastic deformation of the metal. The remainder region, Region 2, is the one in which heat is conducted from Region 1, the source being the internal heat generated in that region. It is primary to understand that this model suggests that the only source of heat in the whole domain including the harder material is the internal generated in the severely deformed region as a result of the plastic deformation.

It is also assumed that the wearing specimen is semi-infinite in length for generalization purposes. Therefore, the temperature at the far end of the specimen is assumed to be ambient. Convection is assumed on the internal and external curved surfaces of the cylindrical specimen. Of the heat generated in the highly deformed region part of it is conducted in to the harder material. Therefore, at the interface a constant heat flux that is partitioned from the total heat generated is assumed to leave the wearing specimen.

4.6. Solution

The solution of the temperature presented in this section is for the wearing specimen. At steady state the part of the total heat generated in the wearing material is conducted into the wearing material. The fraction that is conducted into the harder material $q_2$ is determined by the partition factor for the pair of contacting materials (section 3.4).

$$q_2 = \eta_2 q$$

(4.25)

The internal heat generated in the SDR as shown in eqn. (4.14) is
\[ \dot{q}(z) = \tau_{\text{max}} \left[ 1 - \left( 1 - \frac{r^2}{\tau_{\text{max}}^2} \right)^{\exp(-a_e z)} \right]^{1/2} a_e(c, \nu) \exp(-a_e z) \]

Figure 4.8 shows the geometry of the domain considered.

The light border represents the cross-sectional view of the wearing specimen of which the shaded portion represents the highly deformed region. The dark border represents the harder material.

Governing equation for the heat conduction in wearing material

\[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}(z)}{k_h} = 0 \quad r_1 < r < r_2; \quad 0 < z < \delta; \]

\[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \quad r_1 < r < r_2; \quad \delta < z < \infty; \]

The following are the boundary conditions

\[ (ii) \quad -k_h \frac{\partial T}{\partial r} = h(T - T_{\text{amb}}) \quad @ r = r_2 \]
To solve the temperature distribution analytically, the two regions are analyzed separately as the governing equation for them differ. Appropriate matching conditions of temperature and flux are imposed at the plane separating the two regions to satisfy the continuity at the interface. Figure 4.9 shows the two regions and the boundary conditions. The boundary condition at the plane B-B is considered unknown. The temperature solution for each region is then obtained from the three other boundary conditions in terms of unknown constants to be determined. Matching conditions for equal temperature and heat flux are then used to obtain the unknown constants.

Figure 4.9 Boundary conditions and geometry of the specimen

The details of the formulation of the problem for both region 1 and 2 are shown below.
• Region -2

The governing equation is given as follows

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \quad r_1 < r < r_2; \quad \delta < z < \infty; \quad (4.26)
\]

The following boundary conditions for region 2 are-

(i) \[-k_h \frac{\partial T}{\partial r} = h(T - T_{amb}) \quad \text{at } r = r_2\]

(ii) \[k_h \frac{\partial T}{\partial r} = h(T - T_{amb}) \quad \text{at } r = r_i\]

(iii) \[\text{Matching Conditions} \quad \text{at } z = \delta\]

(iv) \[T = T_{amb} \quad \text{at } z = \infty\]

Using dimensionless parameters:

\[
\bar{r} = \frac{r}{r_2}; \quad \bar{z} = \frac{z}{r_2}; \quad \Theta = \frac{T - T_{amb}}{T_{amb}}; \quad b = \frac{r_i}{r_2}
\]

The governing equation and boundary conditions can be re-written as

\[
\frac{\partial^2 \Theta}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \Theta}{\partial \bar{r}} + \frac{\partial^2 \Theta}{\partial \bar{z}^2} = 0 \quad (4.27)
\]

(i) \[-k_h \frac{\partial \Theta}{\partial \bar{r}} = h \Theta \quad \text{at } \bar{r} = 1\]

(ii) \[k_h \frac{\partial \Theta}{\partial \bar{r}} = h \Theta \quad \text{at } \bar{r} = b\]

(iii) \[\text{Matching Conditions} \quad \text{at } \bar{z} = a\]

(iv) \[\Theta = 0 \quad \text{at } \bar{z} = \infty\]

The solution for the above region after applying the boundary conditions is then-
\[ \overline{\Theta}_n(\xi, \zeta) = \sum_{n=1}^{\infty} C_n e^{-\lambda_n z} \left( J_0(\lambda_n \xi) - \frac{h r_2}{k_n \lambda_n} J_0(\lambda n \lambda n \xi) + J_1(\lambda n \lambda n \xi) \right) \]

\[ = \sum_{n=1}^{\infty} C_n e^{-\lambda_n z} \frac{h r_2}{k_n \lambda_n} \left( J_0(\lambda n \lambda n \xi) + J_1(\lambda n \lambda n \xi) \right) \]

\[ Y_i(\lambda n \lambda n \xi) + \frac{h r_2}{k_n \lambda_n} Y_0(\lambda n \lambda n \xi) \] \hspace{1cm} (4.28)

where \( C_n \) is a constant that was determined by applying the matching conditions (4.34) and (4.36). The Eigen values are roots of the following equation-

\[ \frac{h r_2}{k_n \lambda_n} J_0(\lambda n \lambda n \xi) - J_1(\lambda n \lambda n \xi) = - \frac{h r_2}{k_n \lambda_n} J_0(\lambda n \lambda n \xi) + J_1(\lambda n \lambda n \xi) \]

\[ \frac{Y_i(\lambda n \lambda n \xi) - \frac{h r_2}{k_n \lambda_n} Y_0(\lambda n \lambda n \xi)}{Y_i(\lambda n \lambda n \xi) + \frac{h r_2}{k_n \lambda_n} Y_0(\lambda n \lambda n \xi)} \]

- **Region 1**

The governing equation can be written as

\[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{\hat{q}(z)}{k_h} = 0 \]

\[ r_1 < r < r_2; \quad 0 < z < 1; \]

The following are the boundary conditions for region 1.

(i) \( -k_h \frac{\partial T}{\partial r} = h(T - T_{amb}) \) \hspace{1cm} @ \( r = r_2 \)

(ii) \( k_h \frac{\partial T}{\partial r} = h(T - T_{amb}) \) \hspace{1cm} @ \( r = r_1 \)

(iii) \( -k_h \frac{\partial T}{\partial z} = q_z \) \hspace{1cm} @ \( z = 0 \)

(iii) **Matching Conditions** \hspace{1cm} @ \( \zeta = a \)

The matching condition at \( z = a \) is a non-homogeneous condition as at the common interface the temperature and flux are finite and functions of radial co-ordinate. The governing equation has one non-homogenous term and two of the boundary conditions are non-homogenous. Using the superposition principle, the problem can be split into three sub-problems, each considering one non-homogeneity. The problem can be split as a 1-D solution which is a
function of $z$ (translational) co-ordinate and considers the heat generation term and the other two as a 2-D solution which is function of radial and translational co-ordinate that considers the one non-homogeneity each in the translational direction [KaKac & Yener (1993)].

$$T(r, z) = T_1(z) + T_2(r, z) + T_3(r, z) \quad \text{(4.29)}$$

Using dimensionless parameters:

$$\bar{r} = \frac{r}{r_2}; \; \bar{z} = \frac{z}{r_2}; \; \bar{\theta}_w = \frac{T - T_{amb}}{T_{amb}}; \; b = \frac{\eta}{r_2}; \; a = \frac{\delta}{r_2}$$

$$\bar{\theta}_w = \bar{\theta}_{w-1} + \bar{\theta}_{w-2} + \bar{\theta}_{w-3}$$

Each of these problems has been dealt as shown. The first part of the problem is

(i) \quad \frac{T_{amb}}{r_2^2} \frac{\partial^2 \bar{\theta}_{w-1}}{\partial \bar{z}^2} + \frac{\bar{q}(z)}{k_h} = 0

(ii) \quad \frac{\partial \bar{\theta}_{w-1}}{\partial \bar{r}} = 0 \quad @ \bar{z} = 0

(iii) \quad \bar{\theta}_{w-1} = 0 \quad @ \bar{z} = a

The solution has been found by integrating the governing equation twice as follows

$$\frac{\partial \bar{\theta}_{w-1}}{\partial \bar{r}} = \int \dot{q}(\bar{z}) \, d\bar{z} + C_1$$

$$\bar{\theta}_{w-1} = \int \int \dot{q}(\bar{z}) \, d\bar{z} + C_1 \bar{z} + C_2 \quad \text{(4.30)}$$

where $C_1$ and $C_2$ are constants determined by the boundary conditions. The heat generation function $\dot{q}(\bar{z})$ is the product of the shear stress at a particular depth in the wearing material and strain at that location in the severely deformed region (eqn 4.14).

$$\dot{q}(\bar{z}) = \tau_{max} \left[1 - \left\{1 - \frac{\tau_2}{\tau_{max}} \right\} e^{-a_r \cdot r_2 \bar{z}} \right]^{1/2} \; a_r \cdot c_r \cdot v \cdot e^{-a_r \cdot r_2 \bar{z}} \quad \text{Since} \quad z = \bar{z} \cdot r_2$$
\[ \frac{\partial \bar{\theta}_{w-1}}{\partial \bar{r}} = -\frac{r_s^2 \tau_{\text{max}} a_c C_s v}{k_b T_{\text{amb}}} \left[ -e^{-a_c \tau_{r_2}} + \frac{(1 - \frac{\tau_{r_2}^2}{\tau_{\text{max}}^2})e^{-a_c \tau_{r_2}}}{2 a_c r_2 \ln(1 - \frac{\tau_{r_2}^2}{\tau_{\text{max}}^2})} \right] + C_1 \]

\[ \bar{\theta}_{w-1} = -\frac{r_s^2 \tau_{\text{max}} a_c C_s v}{k_b T_{\text{amb}}} \left[ e^{-a_c \tau_{r_2}} - e^{-a_c \tau_{r_2} \ln(1 - \frac{\tau_{r_2}^2}{\tau_{\text{max}}^2})} + \frac{Ei\{1, -e^{-a_c \tau_{r_2} \ln(1 - \frac{\tau_{r_2}^2}{\tau_{\text{max}}^2})}\}}{2 a_c r_2 \ln(1 - \frac{\tau_{r_2}^2}{\tau_{\text{max}}^2})} \right] + C_1 \bar{z} + C_2 \]

where \( Ei(1, x) \) is the Exponential Integral function defined as

\[ Ei(1, x) = \int e^{-\frac{m x}{m}} \, dm \] where \( m \) is arbitrary constant. Also

\[ C_1 = \frac{r_s^2 \tau_{\text{max}} a_c C_s v}{a_c k_b T_{\text{amb}}} \left[ \frac{(1 - \frac{\tau_{r_2}^2}{\tau_{\text{max}}^2})}{2 \ln(1 - \frac{\tau_{r_2}^2}{\tau_{\text{max}}^2})} - 1 \right] \]

\[ C_2 = \frac{\tau_{\text{max}} a_c C_s v}{2 a_c k_b T_{\text{amb}}} \left[ e^{-a_c \tau_{r_2}} - e^{-a_c \tau_{r_2} \ln(1 - \frac{\tau_{r_2}^2}{\tau_{\text{max}}^2})} + \frac{Ei\{1, -e^{-a_c \tau_{r_2} \ln(1 - \frac{\tau_{r_2}^2}{\tau_{\text{max}}^2})}\}}{2 \ln(1 - \frac{\tau_{r_2}^2}{\tau_{\text{max}}^2})} \right] - C_1 a \]

The detail evaluation of the integral in (4.30) is shown in the Appendix D.

The second part of the problem is

\[ \frac{\partial^2 \bar{\theta}_{w-2}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{\theta}_{w-2}}{\partial \bar{r}} + \frac{\partial^2 \bar{\theta}_{w-2}}{\partial \bar{z}^2} = 0 \]

(i) \[ -\frac{k_h}{r_2} \frac{\partial \bar{\theta}_{w-2}}{\partial \bar{r}} = h \bar{\theta}_{w-2} \] \( @ \bar{r} = 1 \)

(ii) \[ \frac{k_h}{r_2} \frac{\partial \bar{\theta}_{w-2}}{\partial \bar{r}} = h \bar{\theta}_{w-2} \] \( @ \bar{r} = b \)

(iii) \[ \frac{k_h T_{\text{amb}}}{r_2} \frac{\partial \bar{\theta}_{w-2}}{\partial \bar{z}} = q_2 \] \( @ \bar{z} = 0 \)
(iv) \( \bar{\theta}_{w-2} = 0 \) \hspace{1cm} (@ \bar{z} = 0)

The solution is given as follows-

\[
\bar{\theta}_{w-2}(\bar{r}, \bar{z}) = \sum_{n=1}^{\infty} E_n (J_0(\lambda_n \bar{r} b) + J_1(\lambda_n \bar{r} b)) - \frac{hr_2}{k_h \lambda} \frac{Y_0(\lambda_n \bar{r})}{Y_1(\lambda b) + \frac{hr_2}{k_h \lambda} Y_0(\lambda b))}.
\] 

(4.32)

\[. \text{cosh}(\lambda_n \bar{z}) - \text{coth}(\lambda_n a) \sinh(\lambda_n \bar{z}) \]

where \( E_n \) is determined by using the orthogonal property of Bessel functions resulting in

\[
E_n = - \frac{q_3 r_2}{k \lambda \text{coth}(\lambda_n a) \lambda_n} \int_{\bar{r}}^{1} \bar{r} (J_0(\lambda_n \bar{r}) + C^1 Y_0(\lambda_n \bar{r})) \frac{1}{\bar{z}^2} \int_{\bar{r}}^{1} \bar{r} (J_0(\lambda_n \bar{r}) + C^1 Y_0(\lambda_n \bar{r}))
\]

The third part of the problem is

\[
\frac{\partial^2 \bar{\theta}_{w-3}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{\theta}_{w-3}}{\partial \bar{r}} + \frac{\partial^2 \bar{\theta}_{w-3}}{\partial \bar{z}^2} = 0
\]

(i) \[- \frac{k h}{r_2} \frac{\partial \bar{\theta}_{w-3}}{\partial \bar{r}} = h \bar{\theta}_{w-3} \] \hspace{1cm} (@ \bar{r} = 1)

(ii) \[ \frac{k h}{r_2} \frac{\partial \bar{\theta}_{w-3}}{\partial \bar{r}} = h \bar{\theta}_{w-3} \] \hspace{1cm} (@ \bar{r} = b)

(iii) \[ \frac{\partial \bar{\theta}_{w-3}}{\partial \bar{z}} = 0 \] \hspace{1cm} (@ \bar{z} = 0)

(iv) Matching Conditions \hspace{1cm} (@ \bar{z} = a)

The solution is

\[
\bar{\theta}_{w-3}(\bar{r}, \bar{z}) = \sum_{n=1}^{\infty} G_n (J_0(\lambda_n \bar{r} b) + J_1(\lambda_n \bar{r} b)) - \frac{hr_2}{k_h \lambda} \frac{Y_0(\lambda_n \bar{r})}{Y_1(\lambda b) + \frac{hr_2}{k_h \lambda} Y_0(\lambda b))} \text{cosh}(\lambda_n \bar{z})
\]

(4.33)
The constant \( G_n \) was determined by using matching conditions shown in eqn. (4.34) & (4.36).

- **Matching condition I**

Temperature at the common surface \( \bar{z} = a \) is equal for both regions. Hence equating the temperatures at \( \bar{z} = a \)

\[
(i) \quad \bar{\theta}_{w-1} + \bar{\theta}_{w-2} + \bar{\theta}_{w-3} = \bar{\Theta}_{w} \quad \text{at} \quad \bar{z} = a
\]

\[
-r^2 \tau_{\max} a^2 c_s v \left[ e^{-a_r r^2} + \frac{Ei\{1 - e^{-a_r r^2} \ln(1 - \frac{r^2_{\max}}{r^2_2})\}}{a_r^2 r^2_2} + C_1 a + C_2 \right]
\]

\[
+ \sum_{n=1}^{\infty} G_n R(\bar{r}) \cosh(\lambda_n a) = \sum_{n=1}^{\infty} C_n e^{-\lambda_n a} R(\bar{r})
\]

where the following is used for notation convenience:

\[
R(\bar{r}) = (J_0(\lambda_n \bar{r}) - \frac{h r^2}{k_n \lambda}) \frac{Y_0(\lambda_n \bar{r})}{Y_1(\lambda b) + \frac{h r^2}{k_n \lambda}}
\]

From the definition of the sub-problem for \( \bar{\theta}_{w-1} \),

\[
\bar{\theta}_{w-1} = 0 \quad \text{at} \quad \bar{z} = a
\]

Therefore rearranging terms and rewriting in (2.14) we have

\[
0 = \sum_{n=1}^{\infty} [C_n e^{-\lambda_n a} - G_n \cosh(\lambda_n a)] R(\bar{r})
\]

\[
[C_n e^{-\lambda_n a} - G_n \cosh(\lambda_n a)] = 0
\]

\[
C_n e^{-\lambda_n a} = G_n \cosh(\lambda_n a)
\]

- **Matching condition II**

The heat flux at the common surface \( \bar{z} = a \) is equal for both regions. Hence, equating the heat flux at \( \bar{z} = a \).
\[
(ii) \quad \frac{\partial (\theta_{w-1} + \theta_{w-2} + \theta_{w-3})}{\partial \bar{z}} = \frac{\partial \Theta_w}{\partial \bar{z}} \quad (\text{at } \bar{z} = a)
\]

\[
- \frac{r_s^2 \tau_{\text{max}}}{k_b T_{\text{amb}}} \frac{ac c_s v}{a c r_s} \left[ -e^{-ac ar_s} + \frac{(1 - \frac{\tau_s^2}{\tau^2_{\text{max}}})e^{-ac ar_s}}{2ac r_s \ln(1 - \frac{\tau_s^2}{\tau^2_{\text{max}}})} \right] + C_1 +
\]

\[
- \sum_{n=1}^{\infty} E_n R(\bar{r}) \frac{\lambda_n (\cosh(\lambda_n a) - \coth(\lambda_n a) \sinh(\lambda_n a)) + \sum_{n=1}^{\infty} G_n R(\bar{r}) \lambda_n \sinh(\lambda_n a)}{\lambda_n \sinh(\lambda_n a)} (4.36)
\]

\[
= - \sum_{n=1}^{\infty} C_n \lambda_n e^{-\lambda_n a} R(\bar{r})
\]

Rearranging terms and rewriting in (2.16) we have

\[
- \frac{r_s^2 \tau_{\text{max}}}{k_b T_{\text{amb}}} \frac{ac c_s v}{a c r_s} \left[ -\exp(-ac ar_s) + \frac{(1 - \frac{\tau_s^2}{\tau^2_{\text{max}}})\exp(-ac ar_s)}{2a c r_s \ln(1 - \frac{\tau_s^2}{\tau^2_{\text{max}}})} \right] + C_1 =
\]

\[
- \sum_{n=1}^{\infty} \left[ C_n \lambda_n e^{-\lambda_n a} + \lambda_n E_n (\cosh(\lambda_n a) - \coth(\lambda_n a) \sinh(\lambda_n a)) + G_n \lambda_n \sinh(\lambda_n a) \right] R(\bar{r})
\]

Using orthogonal property of Bessel function we have

\[
[C_n \lambda_n e^{-\lambda_n a} + \lambda_n E_n (\cosh(\lambda_n a) - \coth(\lambda_n a) \sinh(\lambda_n a)) + G_n \lambda_n \sinh(\lambda_n a)]
\]

\[
\left( - \frac{r_s^2 \tau_{\text{max}}}{k_b T_{\text{amb}}} \frac{ac c_s v}{a c r_s} + \frac{(1 - \frac{\tau_s^2}{\tau^2_{\text{max}}})e^{-ac ar_s}}{2ac r_s \ln(1 - \frac{\tau_s^2}{\tau^2_{\text{max}}})} \right) + C_1 \int_{b}^{h} R(\bar{r}) \, d\bar{r}
\]

\[
= - \frac{\int_{b}^{h} R(\bar{r}) \, d\bar{r}}{\int_{b}^{h} R(\bar{r})^2 \, d\bar{r}}
\]

In eqns. (4.35) and (4.37) all the terms are known except the constants $C_n$ and $G_n$. The integrals in these equations were calculated and by simplifying the two equations for the two unknowns, $C_n$ and $G_n$ are calculated.

\[
G_n = \frac{Y - \lambda_n E_n (\cosh(\lambda_n a) - \coth(\lambda_n a) \sinh(\lambda_n a))}{\lambda_n (\cosh(\lambda_n a) + \sinh(\lambda_n a))}
\]

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\[ C_n = \frac{G_n \cosh(\lambda_n a)}{e^{\lambda_n a}} \]

where \( Y \) is the value of the right hand side of eqn. (4.37).

The temperature solution applied for the experimental parameters described in section 4.4 is shown as follows. The values of surface shear stress \( t_s \) and thickness of the severely deformed region \( \delta \) are not known for a given load and speed. Hence, a parametric study has been done (see chapter 5) to compare results obtained by various combination of values of \( t_s \) and \( \delta \).

- **Input data**

  (i) **Dimensions**
  \[ r_1 = 0.0127 \text{ m}; \quad r_2 = 0.0143 \text{ m}; \quad r_3 = 0.00794 \text{ m} \]
  \[ L = 0.05 \text{ m} \]
  \[ A = \pi (r_2^2 - r_1^2) \]

  (ii) **Ambient Temperature** \( T_{\text{amb}} = 25 \text{ C} \)

  (iii) **Material properties** as shown in Table 3.1

  (iv) **Shear Strength** \( \tau_{\text{max}} = \frac{S_y}{2} \) (Maximum shear stress theory)

  (v) \( c_s = 0.1 \) (Section 4.3)

  (vi) **Convection coefficient** \( h = 28.6 \text{ W/m}^2\text{K} \) (At room temperature and pressure)

  (vii) **Surface shear Stress** \( \tau_s \in (0.2 \tau_{\text{max}}, 0.8 \tau_{\text{max}}) \) (Section 4.3)

  (vii) **Thickness of SDR** \( \delta \in (50 \mu m, 300 \mu m) \) (Section 4.2)

Figure 4.10 shows a typical temperature contour in the region 1 for a Bronze on steel test. The surface shear stress used here is \( \tau_s = 0.5 \tau_{\text{max}} \) and the thickness of the severely deformed
region $\delta = 100 \mu m$. It can be observed that the temperature variation in the radial direction is less when compared to the gradient in the $z$ direction.

![Figure 4.10 Contour plot of Temperature $^0C$ for Region 1 (Bronze on Steel)](image)

Figure 4.10 shows the temperature contour plot for region 2. It can be noted that the temperature variation in the translational direction $z$ is more pronounced than in the radial direction. The temperature at the splitting plane B-B is observed to be equal in both regions as per the matching condition. Figure 4.12 shows the temperature plot for entire domain of the wearing specimen that includes both the region 1 and 2. The region 1 which is the SDR is of the order of microns in practice. The gradient of temperature in the $z$ direction is higher for region 2 than region 1.
Figure 4.11 Contour plot of Temperature °C for Region 2 (Bronze on Steel)

Figure 4.12 Temperature °C for the whole domain (Bronze on Steel)
5. Results and Discussion

5.1 Wear Coefficient Comparison

Verification of the proposed formulation (eqn. (2.4)) has been done by comparison of Archard’s wear coefficient obtained in the experimental and theoretical models presented in chapter 3 and 4 respectively. The wear coefficient using the Archard’s formulation (eqn. (2.1)) is given as

\[ K = \frac{w_{\text{exp}}}{N} \frac{H}{x} \quad \text{or} \quad K = \frac{dw_{\text{exp}}}{dt} \frac{H}{N} v \]  \hspace{1cm} (5.1)

Thus the wear coefficient can be determined for given loading conditions (Load \( N \), velocity \( v \)) by measuring the wear rate experimentally \( w_{\text{exp}} \), where \( H \) is the hardness of the wearing material. This has been done to give a datum for comparison of wear coefficient obtained using the entropy formulation (eqn. (2.4)). From eqn. 3.6 the wear coefficient using the formulation is given as

\[ K = \frac{dw_{\text{exp}}}{dS} \frac{\mu T}{H} \]  \hspace{1cm} (5.2)

In chapter 3, the temperature is measured using the thermocouple and entropy is calculated by considering the gradient of the temperatures measured by the thermocouple. In the theoretical model presented in chapter 4, the temperature has been calculated theoretically by solving an appropriate heat conduction problem in the domain of the wearing specimen and by considering the internal heat generated in SDR to be equal to the virtual work done the shear stresses. To distinguish between the wear coefficients obtained by the above described approaches the following notation had been used.

I. \( K_{\text{ard}} \) - Applying Archard’s formulation (eqn. (5.1)).
• **Input data required**

(i) *Dimensions*

\[ r_1 = 0.0127 \, m, \quad r_2 = 0.0143 \, m, \quad r_3 = 0.00794 \, m \]

\[ A = \pi (r_2^2 - r_1^2) \]

\[ l = 0.05 \, m \]

(ii) *Load* \( N \)

(iii) *Speed* \( s \)

(iv) *Material properties* as shown in Table 3.1

• **Measured variables**

(i) *Coefficient of friction*

(ii) *wear volume* \( w_{\text{exp}} \)

• **Expression used for calculating wear coefficient (Archard’s formulation)**

\[ K = w_{\text{exp}} \frac{H}{N \, x} \quad \text{or} \quad K = \frac{dw_{\text{exp}}}{dt} \frac{H}{N \, v} \]

II. *\( K_{\text{con}} \)* - Applying the proposed formulation (eqn. (5.2)). Where the temperatures are measured using the thermocouples and entropy calculated considering the conducted heat into the contacting materials (chapter 3). The subscript represents ‘conducted heat’ which is used in this approach for calculating entropy.

• **Input data required**

(i) *Dimensions*

\[ r_1 = 0.0127 \, m, \quad r_2 = 0.0143 \, m, \quad r_3 = 0.00794 \, m \]

\[ A = \pi (r_2^2 - r_1^2) \]

\[ l = 0.05 \, m \]

(ii) *Load* \( N \), *Speed* \( s \)
(iii) Material properties as shown in Table 3.1

(iv) Ambient Temperature \( T_{amb} = 25°C \)

- Measured variables

(i) Temperature in stationary specimen by thermocouples

(ii) Coefficient of friction

(iii) Wear volume \( w_{exp} \)

- Calculated variables

\[
\frac{dS}{dt} = \frac{1 - \eta}{T} \left( \frac{A k_s}{S} \frac{T_i - T_{ii}}{d} \right)
\]

(i) Rate of Entropy generation

- Expression used for calculating wear coefficient (entropy formulation)

\[
K_{con} = \frac{dw_{exp}}{dS} \left( \frac{\mu}{H} \right)
\]

III.  \( K_{war} \) - Applying the proposed formulation (eqn. (5.2)), where the temperature is calculated in theoretical model by considering the virtual work done by the shear stresses. The entropy is calculated by considering the ratio of the virtual work done and local temperature in the SDR (chapter 4). The subscript represents ‘Plastic work’ which is considered here to calculate the entropy.

- Input data required

(i) Dimensions

\[ r_1 = 0.0127 \, m, \quad r_2 = 0.0143 \, m, \quad r_3 = 0.00794 \, m \]
\[ A = \pi (r_2^2 - r_1^2) \]
\[ l = 0.05 \text{ m} \]

(ii) Load \( N \), speed \( s \)

(iii) Material properties as shown in Table 3.1

(iv) Ambient Temperature \( T_{\text{amb}} = 25^\circ \text{C} \)

(v) Shear Strength \( \tau_{\text{max}} = \frac{\sigma_u}{2} \)

(vi) \( c_s = 0.1 \)

(vii) Convection coefficient \( h = 28.6 \text{ W/m}^2.\text{K} \)

(viii) Surface shear Stress \( \tau_s \in (0.2 \tau_{\text{max}}, 0.8 \tau_{\text{max}}) \)

(ix) Thickness of SDR \( \delta \in (50 \mu\text{m}, 300 \mu\text{m}) \)

- Measured variables

(i) Coefficient of friction

- Calculated variables

(i) Temperature in wearing specimen and stationary specimen

(ii) Rate of Entropy generation

\[
\frac{dS_{\text{gen}}}{dt} = \frac{NA \tau_{\text{max}} \sigma_a \frac{\delta^3}{a} \left[ (1 - \frac{\tau_s^2}{\tau_{\text{max}}^2}) \exp(-a_z) \right]^{1/2} \exp(-a_z) \ dz}{T}
\]

- Expression used for calculating wear coefficient (entropy formulation)

\[
K_{\text{wor}} = \frac{dW_{\text{exp}}}{dS} \left( \frac{\mu T}{H} \right)
\]
Note that for calculating $K_{wor}$ the input data for surface shear stress is not a specific value but a range $\tau_s \in (0.2\ \tau_{\text{max}}, \ 0.8\ \tau_{\text{max}})$. The same is the case with the thickness of SDR $\delta \in (50\ \mu m, \ 300\ \mu m)$. Hence, the wear coefficient obtained is dependent on the values chosen for the set $\{\tau_s, \delta\}$. However, the choice of value for $\{\tau_s, \delta\}$ can be justified by comparing the temperatures calculated in the stationary specimen obtained in the theoretical model to the temperatures recorded by thermo-couples in the experiment. Figure 5.1 shows a comparison of the wear coefficients for a set of 4 values of surface shear stress and a thickness of SDR as 100 $\mu m$.

![Figure 5.1](image)

**Figure 5.1 Wear coefficient $K_{wor}$ for Bronze on Steel; $\delta = 100\ \mu m$; (N=17.79 N, s=200 rpm)**

The wear coefficient is of the order $2 \times 10^{-5}$ to $1.2 \times 10^{-4}$. This is close but lower than the values obtained in eqn. 3.9. From Figure 5.1 the surface stress relation $\tau_s = 0.2\ \tau_{\text{max}}$ is closest.
to the value in eqn. 3.9. But the appropriate stress relation can be justified by comparing the temperatures in the stationary specimen using that relation to the experimental values. Figure 5.2 shows a comparison of temperatures in the stationary specimen obtained by using $\tau_s \in (0.2\tau_{\text{max}}, 0.8\tau_{\text{max}})$ in the theoretical model to the experimental temperature values. The comparison shows that temperatures obtained using $\tau_s = 0.4\tau_{\text{max}}$ are in good agreement. Thus, the choice of $\{\tau_s = 0.4\tau_{\text{max}} \& \delta = 100 \mu m\}$ can be justified. However, there can be more than one such set of values for $\{\tau_s, \delta\}$ that may yield good agreement to the comparison of temperatures. Hence, determining an unique set of values for $\{\tau_s, \delta\}$ is beyond the scope of this work. A similar comparison for Brass on Steel is also shown in Figure 5.3 and 5.4.

![Figure 5.2 Temperature in Stationary specimen: Bronze on Steel (17.79 N, 200 rpm)](image-url)

Figure 5.2 Temperature in Stationary specimen: Bronze on Steel (17.79 N, 200 rpm)
Figure 5.3 Wear coefficient $K_{wor}$ for Brass on Steel $\delta = 100 \mu m$ ($N=13.34 N$, $s=100 \text{ rpm}$)

Figure 5.4 Temperature in Stationary specimen: Brass on Steel ($13.34 N$, $100 \text{ rpm}$)
From Figure 5.4 the values of \( \{ \tau_s \text{ & } \delta \} \) that are justified are \( \{ \tau_s = 0.2 \tau_{\text{max}} \text{ & } \delta = 100 \ \mu m \} \).

Thus, the justified value of wear coefficient is \( 1.2 \times 10^{-1} \) which is close to one in eqn. (3.10).

Tables 5.2 and 5.3 show values of wear coefficient each calculated for a combination of surface shear stress \( \tau_s \in (0.2 \tau_{\text{max}} , \ 0.8 \tau_{\text{max}} \) ) and thickness of SDR \( \delta \in (50 \ \mu m , \ 300 \ \mu m \) ) for Bronze on Steel and Brass on Steel respectively.

**Table 5.1 Wear coefficient \( K_{wbr} \) for Bronze on Steel for Load 17.79 N, s=200 rpm**

<table>
<thead>
<tr>
<th>( \tau_s \rightarrow )</th>
<th>( \tau_s = 0.2\tau_{\text{max}} )</th>
<th>( \tau_s = 0.4\tau_{\text{max}} )</th>
<th>( \tau_s = 0.6\tau_{\text{max}} )</th>
<th>( \tau_s = 0.8\tau_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta \downarrow )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta = 50 \ \mu m )</td>
<td>1.4641x10^{-4}</td>
<td>7.279 x10^{-5}</td>
<td>4.800 x10^{-5}</td>
<td>3.529 x10^{-5}</td>
</tr>
<tr>
<td>( \delta = 100 \ \mu m )</td>
<td>9.696 x10^{-5}</td>
<td>4.799 x10^{-5}</td>
<td>3.144 x10^{-5}</td>
<td>2.283 x10^{-5}</td>
</tr>
<tr>
<td>( \delta = 150 \ \mu m )</td>
<td>8.278 x10^{-5}</td>
<td>4.081 x10^{-5}</td>
<td>2.663 x10^{-5}</td>
<td>1.919 x10^{-5}</td>
</tr>
<tr>
<td>( \delta = 200 \ \mu m )</td>
<td>7.711 x10^{-5}</td>
<td>3.788 x10^{-5}</td>
<td>2.465 x10^{-5}</td>
<td>1.769 x10^{-5}</td>
</tr>
<tr>
<td>( \delta = 250 \ \mu m )</td>
<td>7.458 x10^{-5}</td>
<td>3.650 x10^{-5}</td>
<td>2.372 x10^{-5}</td>
<td>1.698 x10^{-5}</td>
</tr>
<tr>
<td>( \delta = 300 \ \mu m )</td>
<td>7.344 x10^{-5}</td>
<td>3.581 x10^{-5}</td>
<td>2.324 x10^{-5}</td>
<td>1.662 x10^{-5}</td>
</tr>
</tbody>
</table>
Table 5.2 Wear coefficient $K_{w or}$ for Brass on Steel for Load 13.34 N, s=100 rpm

<table>
<thead>
<tr>
<th>$\tau_s \rightarrow$</th>
<th>$\tau_s = 0.2 \tau_{max}$</th>
<th>$\tau_s = 0.4 \tau_{max}$</th>
<th>$\tau_s = 0.6 \tau_{max}$</th>
<th>$\tau_s = 0.8 \tau_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta = 50 \mu m$</td>
<td>1.925 x10^{-4}</td>
<td>9.575 x10^{-5}</td>
<td>6.315 x10^{-5}</td>
<td>4.643 x10^{-5}</td>
</tr>
<tr>
<td>$\delta = 100 \mu m$</td>
<td>1.274 x10^{-4}</td>
<td>6.312 x10^{-5}</td>
<td>4.135 x10^{-5}</td>
<td>3.003 x10^{-5}</td>
</tr>
<tr>
<td>$\delta = 150 \mu m$</td>
<td>1.086 x10^{-4}</td>
<td>5.366 x10^{-5}</td>
<td>3.502 x10^{-5}</td>
<td>2.525 x10^{-5}</td>
</tr>
<tr>
<td>$\delta = 200 \mu m$</td>
<td>1.010 x10^{-4}</td>
<td>4.978 x10^{-5}</td>
<td>3.242 x10^{-5}</td>
<td>2.327 x10^{-5}</td>
</tr>
<tr>
<td>$\delta = 250 \mu m$</td>
<td>9.753 x10^{-5}</td>
<td>4.795 x10^{-5}</td>
<td>3.118 x10^{-5}</td>
<td>2.233 x10^{-5}</td>
</tr>
<tr>
<td>$\delta = 300 \mu m$</td>
<td>9.581 x10^{-5}</td>
<td>4.700 x10^{-5}</td>
<td>3.054 x10^{-5}</td>
<td>2.185 x10^{-5}</td>
</tr>
</tbody>
</table>

Figure 5.5 shows a comparison of the wear coefficient obtained using the theoretical model to the wear coefficient obtained by using Archard’s formulation. The surface shear stress $\{\tau_s, \delta\}$ and thickness of the SDR used are $\tau_s = 0.4 \tau_{max}$ and $\delta = 100 \mu m$ as this combination for Bronze on Steel (Load 17.79 N and Speed 200 rpm) has been justified as shown in Figure 5.2. Similarly Figure 5.6 shows the comparison for Brass on Steel $\tau_s = 0.2 \tau_{max}$ and $\delta = 100 \mu m$ for a 13.34 N and 100 rpm test.

Note here that comparison of the wear coefficient of the three approaches is done by using a constant value of the wear (from experiment). The comparison shows that for the same wear rate the wear coefficient determined using the theoretical model is lower than the wear coefficient calculated using Archard’s formulation. In other words, the entropy formulation presented here (eqn. (2.4)), if used to calculate the wear rate with a given wear coefficient would predict higher wear rate than the wear rate obtained using the same wear coefficient in
Archard’s formulation. Hence, if Archard’s formulation is treated as the reference, then the entropy formulation presented here would predict more wear than that would actually occur. In design applications, the entropy formulation discussed here would be a conservative approach. The factor of safety obtained using entropy formulation would be higher than using Archard’s formulation. This is a safer approach but not the most optimized approach for design. Further improvements to the model, as suggested in the section 5.3, can be made to make this approach (entropy formulation presented here) more realistic and optimum.

![Graph comparing Archard's formulation and entropy formulation](image)

**Figure 5.5 Comparison of $K_{ard}$, $K_{con}$ and $K_{wor}$ for Bronze on Steel (N= 17.79 N, s=200 rpm)**
5.3 Discussion of Uncertainty

In section 5.1, the wear coefficient is calculated using three approaches. Approach I is via direct application of Archard’s wear formulation (eqn. (2.1)). Approach II and III use the entropy formulation (eqn. (2.4)) to calculate the wear coefficient. Both approaches II and III are based on the assumption that the total work done during plastic deformation is equal to the heat generated in the severely deformed region. But there is a distinction in calculation of entropy in approaches II and III. Approach II considers the heat conducted into the contacting materials to calculate the entropy whereas approach III considers the work done during plastic deformation to calculate the entropy.

\[
dS_{\text{gen}} = \frac{A k_s T_i - T_{ii}}{d} \frac{1 - \eta}{T} \quad \text{[Approach II]}
\]
In the realistic case the work done during plastic deformation is not totally realized as heat generation. Some part of this work is sound, vibration, etc. Hence, approach II gives a lower limit to the entropy generated.

\[
\frac{dS_{gen}}{dt} = \frac{NA \tau_{max} ac \frac{\delta}{dt} \int_{0}^{\delta} \left[1 - \left(1 - \frac{\tau_s^2}{\tau_{max}^2}\right) \exp(-ac z)\right]^{1/2} \exp(-ac z) \, dz}{T} \quad \text{[Approach III]}
\]

However, in approach III the assumption is used to simulate the temperature in the severely deformed region. Thus, the temperature simulated in approach III is higher than in the realistic case and hence the entropy calculated in the approach III should be lower than the realistic case. Hence, both approaches II and III give a lower limit of entropy generated. This means that the wear estimated by approaches II and III should be lower than the actual wear. In other words the wear coefficient \(K_{con}\) and \(K_{wor}\) should be higher than \(K_{arg}\). Figures 5.5 and 5.6 show the contrary. This disparity \(K_{con}\) can be accounted to the following reasons.

The temperatures recorded by the thermocouples may not accurately represent the actual temperatures as the tips of the thermocouples are not glued into the specimen. The heat partition factor used here is calculated using eqn. (3.2) which is independent of the relative velocity of the contacting materials. The resulting entropy calculated is very sensitive to the value of partition factor. Hence, a more inclusive relation that considers the effect of relative velocity is needed. And finally, it is difficult to accurately determine the heat losses due to convection. The disparity in \(K_{wor}\) can be explained based on the following arguments. Even though the stress-strain relation used in approach III gives a good estimate of the mechanics of the sliding contact may not be an accurate relation. Also the range of values considered for the surface shear stress and thickness of the severely deformed region \(\{\tau_s, \delta\}\) are based on an assumption that the copper and its alloys have an exactly similar mechanical response.
Figure 5.7 Comparison of $K_{ard}$ and $K_{wor}$ for Brass on Steel

Figure 5.7 shows a comparison of wear coefficients obtained by using various surface shear stress values to the wear coefficient obtained by Archard’s wear formulation. For the surface shear stress value of $\tau_s = 0.05 \tau_{\text{max}}$ the agreement between the wear coefficient obtained using Archard’s formulation and the theoretical model is best. But Figure 5.2 suggest that the surface shear stress value of $\tau_s = 0.4 \tau_{\text{max}}$ is the one that satisfies the experimental verification.

If the experimental values of temperature obtained used the thermocouples are used as a reference then $\tau_s = 0.4 \tau_{\text{max}}$ is the surface shear stress relation which gives a lower wear coefficient value than the Archard’s formulation. In case of Brass on Steel, $\tau_s = 0.2 \tau_{\text{max}}$ agrees well with the experimental verification but $\tau_s = 0.05 \tau_{\text{max}}$ agrees well with the Archard’s formulation.
There are some questions about the accuracy the correct surface shear stress relation. The same is the case with the thickness of the SDR. If a wider range of values of $\{\tau_s, \delta\}$ are chosen then there might exist a combination which would agree with both the Archard’s wear formulation and the experimental temperatures. This uncertainty exists in the theoretical model presented here. But this uncertainty has been addressed to some extent by doing the parametric study for the values of $\{\tau_s, \delta\}$ and experimental temperature verification. Finite element solution of the nodal displacements and forces or marker technique for calculating displacements in the SDR can be used to accurately determine the values of $\{\tau_s, \delta\}$ that would remove this uncertainty. This aspect forms the basis for the future of this work.

Figure 5.8 Comparison of $K_{\text{arch}}$ and $K_{\text{war}}$ for Brass on Steel
5.4 Theoretical Calculation of Coefficient of Friction

As stated in chapter 4, the basis of the theoretical model is the assumption that the virtual work done by shear stresses in the SDR is assumed equal to energy lost by friction. This section deals with calculation of the coefficient of friction based on this assumption and comparing it with measured values of coefficient of friction in the experiment. Heilmann and Rigney (1981) calculated the coefficient of friction by equating the external work done by the material by the friction force to the internal resistance offered by the material.

\[
\mu = \frac{A \int_0^\infty \tau(z) \Delta \gamma(z) \, dz}{N x} \quad (5.3)
\]

In the present study the internal resistance or the virtual work done by the shear stresses has been assumed to be limited only in the severely deformed region. Hence, the eqn. (5.3) can be written as

\[
\mu_t = \frac{A \int_0^\delta \tau(z) \Delta \gamma(z) \, dz}{N x} \quad (5.4)
\]

where \( \mu_t \) stands for the coefficient of friction using the theoretical model. The sliding length \( x \) in eqn. (5.4) has been replaced by velocity at steady state and the numerator has been differentiated with respect to time to consider the rate of work done. Hence, the eqn. (5.4) has been modified to

\[
\mu_t = \frac{A \int_0^\delta \dot{q}(z) \, dz}{N v} \quad (\text{Since the rate of work done is equal to heat generated}) \quad (5.5)
\]

Thus, \( \mu_t \) has been calculated as follows.
\[
\mu_i = \frac{A \int_0^\delta \tau_{\text{max}} \left[ 1 - \left(1 - \frac{\tau_s^2}{\tau_{\text{max}}^2}\right)^{\frac{1}{2}}\right] \exp(-a_c z) \exp(-a_\kappa z) \, dz}{N \nu}
\]

Figure 5.7 shows a comparison of the coefficient of friction calculated from eqn. (5.6) in which \( \tau_s = 0.4 \, \tau_{\text{max}} \) and \( \delta = 100 \, \mu m \) to the experimental values for Bronze on Steel test (Load 17.79 N (4lb), speed 200 rpm).

**Figure 5.7 Coefficient of friction for Bronze on Steel (Load 17.79 N (4lb), speed 200 rpm)**

Figure 5.8 shows the coefficient of friction comparison for Brass on Steel tests (Load 3 lb, 100 rpm). In both Figure 5.7 and 5.8 the coefficient of friction calculated using the theoretical model is higher than the experimental values. This means that the energy dissipation calculated in higher in the theoretical model. This also justifies the low wear coefficients obtained in Figure 5.5 and 5.6. Note that the theoretical value of coefficient of friction is calculated by considering the values of \( \{\tau_s, \delta\} \) such that the temperatures simulated in the
stationary specimen by the theoretical model, agrees well with the experimentally measured temperatures.

**Figure 5.10 Coefficient of friction for Brass on Steel (Load 13.34N (3 lb), speed 100 rpm)**

Figure 5.11 shows the comparison of the coefficient of friction obtained by using different values of surface shear stress $\tau_s$, with the experimentally recorded values for bronze on steel experiments. The figure shows that the value which is closed to the experimental coefficient of friction is for $\tau_s = 0.4 \tau_{\text{max}}$. This is indeed the value that is justified by comparing the temperatures in the stationary specimen (Figure 5.2). Hence, the justification of the surface shear stress value by comparing the coefficient of friction and temperatures in the stationary specimen gives the same result. Thus, the wear coefficient shown in Figures 5.5 and 5.6 are justified. Though there are some limitations because of the inherent assumptions as discussed in section 5.3, the entropy formulation presented here provides a reasonable estimate of the
wear in sliding contact. The theoretical model can be improved to get more accurate results if the uncertainties are addressed as discussed in section 5.3.

**Figure 5.11** Coefficient of friction comparison for different values of surface shear stress for Bronze on Steel (N=17.79 N, s=200 rpm)
6. Conclusions

Wear is an irreversible phenomenon of great importance in design of contact machinery. A novel approach of correlating wear with the thermodynamic properties of the system has been presented. The approach involves relating the wear to thermodynamic entropy flow in the system using the laws of thermodynamics. This relation has been verified experimentally and theoretically by considering a sliding contact in disk-on-disk configuration for two sets of contacting materials namely Bronze SAE 40 on Steel 4140 and Cartridge Brass on Steel 4140. The verification, which basically involves comparison of the Archard’s wear coefficient calculated using the theoretical model with the published values, revealed a comparable agreement. In the experimental study the entropy is calculated by measuring the conducted heat into the contacting bodies. The wear coefficient calculated using the formulation in the experimental study has been found to be lower than the published value. In the theoretical model the entropy calculation is done by estimating the work of deformation during the sliding process. It is assumed in the model that the virtual work done by the shear stress is equal to the energy dissipated in plastic deformation and that this energy is realized as heat conducted into the contacting materials. The wear coefficient obtained in the theoretical model has been found to be lower than the published values and also one obtained in the experimental study. That is, experimental comparison gave closer agreement than the theoretical model. Both the experimental and theoretical model discussed here have been found to be conservative in design application as they predict higher wear for a given value of wear coefficient. Coefficient of friction has also been calculated theoretically and compared with experimental values showing good agreement.
References


Appendix A: Scheme for Experimental Verification

Code used to evaluate the wear coefficients using the experimental data.

Measured data-

Wear \( w \), Temperature \( T \) and Coefficient of friction \( \mu \)

% Testing Conditions

\[
L=\frac{3 \times 17.861}{4}; \quad \text{Load in N : } 3 \text{lbf}=1.8144 \text{Kgf}=17.861 \text{ N}
\]

\[
s=2 \times 10.467; \quad \text{Speed in rad/s : } 200 \text{rpm}=2 \pi \times \frac{200}{60}
\]

\[
r_1=0.0286; \quad \text{% Outer radius: } r_1=1.125''
\]

\[
r_2=0.0254; \quad \text{%inner radius: } r_2=1''
\]

\[
A=\pi \times (r_1^2-r_2^2); \quad \text{%Contact Area}
\]

%Material Properties from Matweb.com

\[
wst=w(233:341,1); \quad \% \text{ Steady state wear}
\]

\[
Tst=(89.7-32) \times \frac{5}{9}+273; \quad \% \text{ Steady state Temperature in Kelvin}
\]

\[
fst=f(233:341,1); \quad \% \text{ Steady state Coefficient of friction}
\]

\[
kbs=120; \quad \% \text{ Thermal conductivity for Brass cartridge from Matweb.com}
\]

\[
Cpbs=375; \quad \% \text{ Specific Heat Capacity from Matweb.com } Cpbs=375 \text{ J/kg-C}
\]

\[
robs=8.53 \times 10^3(3); \quad \% \text{Density of brass from Matweb.com: } robs =8.53 \times 10^3 \text{ kg/m3}
\]

\[
ks=42.7; \quad \% \text{ Thermal conductivity for steel}
\]

\[
Cps=500; \quad \% \text{ Specific Heat Capacity from Matweb.com } Cps_{avg}=500 \text{ J/kg-C}
\]

\[
ros=7.85 \times 10^3(3); \quad \% \text{Density of 4140 Steel from Matweb.com}
\]

\[
A=\pi \times (r_1^2-r_2^2);
\]

\[
Eta=1/(1+(Cpbs*kbs*robs/Cps/ks/ros)\times(0.5)); \quad \% \text{partitioning factor ...Eta*Q into wearing material}
\]

%%%%Knew value finding

\[
n=\text{length(wst)};
\]

%friiction coefficient in a certian time interval is the average of the
%friiction coefficients at the begining and the end of the interval.
for i=1:n-1
    fu(i)=(fst(i+1)+fst(i))/2;
end

t_int=20;
Q=L*s*fu*(r1+r2)/2*t_int;  %Heat generated by frictional heat generation
wstm=wst*0.0254;

% Making the first data of wear as zero we get n-1 wear values
for i=1:n
    wm(i)=wstm(i)-wstm(1);
end

wv=A*wm';  %Wear volume

% Normalized wear and entropy calculation
for (i=1:n-1)
    S(i,1)=Q(i)/Tst;
    S1(i,1)=sum(S(1:i,1));
    Norm_w(i,1)=(wv(i+1,1)-wv(i,1))/wv(n,1);
end;
Norm_S=S1/max(S1);

for (i=1:n-1)
    mm(i)=abs((wv(i+1)-wv(i))/S(i));
end;

HardnessBs=10.65*10^8; % hardness pressure is 3.55*ultimate tensile strength (got from matworld)
fst_rms=sqrt(sum(fst(1:length(fst),1).^2)/length(fst));
Tst_rms=sqrt(sum(Tst(1:length(Tst),1).^2)/length(Tst));

knew=mm.*fu*HardnessBs/Tst;
knew_rms=sqrt(sum(knew(1,1:length(knew)).^2)/length(knew));
x=length(wv);
knew_avg=(wv(x)-wv(1))/S1(n-1)*fst_rms*HardnessBs/Tst_rms
% USING THE CONDUCTED HEAT CONCEPT

T1_st = 89.7; % Temperature data from thermocouples
T2_st = 87.9;
T3_st = 87.2;
Td1_st = T1_st - T2_st; % Temperature gradient
Td2_st = T2_st - T3_st;
Td_st = (Td1_st + Td2_st) / 2;
T1T2_st = (T1_st + T2_st) / 2;
Qc = (1/(1-Eta)) * Td_st / 0.0086 * ks * A * 20; % distance between thermocouple : 0.0086 = 0.34 * 0.0254

for (i=1:n-1)
    Sc(i,1) = Qc / T1T2_st;
    S1c(i,1) = sum(Sc(1:i,1));
    Norm_wc(i,1) = abs((wstm(i+1,1) - wstm(1,1))) / abs((wstm(n,1) - wstm(1,1)));
end;
Norm_Sc = S1c / max(S1c);

for (i=1:n-1)
    mmc(i) = abs((wv(i+1) - wv(i)) / Sc(i));
end;
knewc = mmc .* fu * HardnessBs / Tst;
knewc_rms = sqrt(sum(knewc(1,1:length(knewc)).^2) / length(knewc));
x = length(wv);
knewc_avg = (wv(x) - wv(1)) / S1c(n-1) * f_st_rms * HardnessBs / Tst_rms

% USING THE CONDUCTED HEAT CONCEPT with Curve fit wear data

x = [1:1:length(wstm)']
wcf=(3*10^(-5)*x+0.0057)*0.0254;  \% Curve fit equation from excel

for i=1:n
    wmcf(i)=wcf(i)-wcf(1);
end

wvcf=A*wmcf';

for (i=1:n-1)
    Sc(i,1)=Qc/T1T2_st;
    S1c(i,1)=sum(Sc(1:i,1));
    Norm_wc(i,1)=abs((wcf(i+1,1)-wcf(1,1)))/abs((wcf(n,1)-wcf(1,1)));
end;

Norm_Sc=S1c/max(S1c);

for (i=1:n-1)
    mmc(i)=abs((wvcf(i+1)-wvcf(i))/Sc(i));
end;

knewc_cf=mmc.*fu*HardnessBs/Tst;
knewc_cf_rms=sqrt(sum(knewc(1,1:length(knewc)).^2)/length(knewc))
x=length(wv);
knewc_avg_cf=(wvcf(x)-wvcf(1))/S1c(n-1)*fst_rms*HardnessBs/Tst_rms

plot(knewc_cf)

**Cartridge Brass**

Thermal conductivity kbs=120 W/m.K

Specific heat at constant pressure (from Mat web) Cpbs=375 J/kg-C

Density rob=8.53 x10^3 kg/m3

**SAE 60 Bronze**

Thermal conductivity kb=71.9 W/m.K

Specific heat at constant pressure (from Mat web) Cpb=435 J/kg-C

Density rob=8.82 x10^3 kg/m3
Steel 4140

Thermal conductivity $ks = 42.7 \text{ W/m.K}$

Specific heat at constant pressure (from Mat web) $C_p = 500 \text{ J/kg-C}$

Density $\rho = 7.85 \times 10^3 \text{ kg/m}^3$
Appendix B: Derivation of the Shear Stress-Strain Relation

[Voce, 1947-48] introduced an empirical relation for compression stress stain curves, incorporates a saturation stress $\tau_{\text{max}}$:

$$\tau = \tau_i + (\tau_{\text{max}} - \tau_i)\{1 - \exp\left(-\frac{\gamma - \gamma_i}{\gamma_c}\right)\}$$  \hspace{1cm} (B-I)

Here $\tau_i$ and $\gamma_i$ are the shear stress and strain respectively at the beginning of the test. Since $\tau_i$ and $\gamma_i$ are usually small compared with $\tau_{\text{max}}$, they can be set equal to zero. Also for small $\gamma$ values, the exponential function can be expanded easily and eq. (I) can reduce to the power law: $\tau \approx \gamma^{1/2}$. Therefore eqn. (I) can be re-written as

$$\tau = \tau_{\text{max}}\{1 - \exp\left(-c\gamma\right)\}^{1/2}$$  \hspace{1cm} (B-II)

Experimental data are available in the form of marker profiles developed during sliding [Dautzenberg, 1980] and [Moore and Douthwaite, 1976]. A marker is embedded in the material in such a way that, at the start of the testing, the projection of the marker, viewed longitudinal section is perpendicular to the sliding surface and parallel to the $z$-axis. After sliding occurs, the marker is bent over in the direction of sliding, and its shape may be described roughly by an exponential curve. This observed profile is the result of many small displacements $\delta x$, at the surface and smaller values of $\delta x_i(z)$ below the surface. If the displacement profile can be described by a simple exponential function, e.g. $\approx \exp(-az)$, then the individual displacements $\delta x_i(z)$ can be written as

$$\delta x_i(z) = \delta x_s \exp(-az)$$  \hspace{1cm} (B-III)

The constant $a$ can be determined by fitting an exponential curve to the appropriate measured marker profile.
The incremental strain in the direction perpendicular to sliding is given by the equation,

$$\Delta \gamma(z) = \frac{\partial x(z)}{\partial z} = a \frac{\partial x(z)}{\partial z} \exp(-az) \quad (B-IV)$$

Since displacements decrease with depth, the associated shear strain increments $\Delta \gamma(z)$ would also decrease. Since $\Delta \gamma(z)$ satisfies eqn.(IV) then it is reasonable to assume that $\gamma(z)$ decreases exponentially in the same way:

$$\gamma(z) = \gamma_s \exp(-az) \quad (B-V)$$

Using eqn. (II), the average surface strain $\gamma_s$ may be expressed in terms of the average surface stress $\tau_s$:

$$\gamma_s = -\frac{1}{c} \ln\left\{1 - \left(\frac{\tau_s}{\tau_{\text{max}}}\right)^2 \right\} \exp(-az) \quad (B-VI)$$

Combining eqns. (II), (V) and (VI) the shear stress distribution is given as

$$\tau(z) = \tau_{\text{max}} \left[1 - \left(\frac{\tau_s}{\tau_{\text{max}}}\right)^2 \exp(-az) \right]^{1/2} \quad (B-VII)$$
Appendix: C Solution in Terms of Bessel Function and Their Orthogonality

The differential equation
\[
\frac{d^2 y}{dx^2} + \frac{1}{x} \frac{dy}{dx} + \left(1 - \frac{n^2}{x^2}\right)y = 0
\]  
(C-I)

is called Bessel’s differential equation of order \( n \). Two linearly independent solutions of this equation for all values of \( n \) are \( J_n(x) \), the Bessel function of the first kind of order \( n \) and \( Y_n(x) \), the Bessel function of the second kind of order \( n \). Thus the solution of equation (C-I) is given as
\[
y(x) = c_1 J_n(x) + c_2 Y_n(x) \]  
(C-II)

The Bessel function \( J_n(x) \) in series form is defined as
\[
J_n(x) = \left(\frac{x}{2}\right)^n \sum_{k=0}^{\infty} \frac{(-1)^{\kappa}}{k! \Gamma(n + k + 1)} (\frac{x}{2})^{2\kappa}
\]  
(C-III)

where \( \Gamma(n + k + 1) \) is the gamma function.

The differential equation
\[
\frac{d^2 y}{dx^2} + \frac{1}{x} \frac{dy}{dx} - \left(1 + \frac{n^2}{x^2}\right)y = 0
\]  
(C-IV)

is called Bessel’s modified differential equation of order \( n \). Two linearly independent solutions of this equation for all values of \( n \) are \( I_n(x) \), the modified Bessel function of the first kind of order \( n \) and \( K_n(x) \), the modified Bessel function of the second kind of order \( n \). Thus, the solution for (C-IV) is given as
\[
y(x) = c_1 I_n(x) + c_2 K_n(x) \]  
(C-V)
$I_n(x)$ and $K_n(x)$ are real and positive when $n > -1$ and $x > 0$. The Bessel function $I_n(x)$ in series form is given by

$$I_n(x) = \left(\frac{1}{2}x\right)^n \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}x\right)^{2n}}{k! \Gamma(n + k + 1)}$$  \hspace{1cm} (C-VI)

Hence, the solution of the equation

$$\frac{d^2 R}{d\bar{r}^2} + \frac{1}{\bar{r}} \frac{dR}{d\bar{r}} + \lambda R = 0 \text{ where } n = 0 \text{ is give as}$$

$$R(\bar{r}) = c_1 J_0(\lambda \bar{r}) + c_2 Y_0(\lambda \bar{r})$$  \hspace{1cm} (C-VII)

And the solution of the equation

$$\frac{d^2 R}{d\bar{r}^2} + \frac{1}{\bar{r}} \frac{dR}{d\bar{r}} - \lambda R = 0 \text{ is given as}$$

$$R(\bar{r}) = c_1 I_0(\lambda \bar{r}) + c_2 K_0(\lambda \bar{r})$$  \hspace{1cm} (C-VIII)

c_1 \text{ and } c_2 \text{ are determined using the boundary conditions.}

**Orthogonal property**

The orthogonal property of the function $R(\bar{r}) = c_1 J_0(\lambda_m \bar{r}) + c_2 Y_0(\lambda_m \bar{r})$, for $a < \bar{r} < b$ is given as

$$\int_a^b [c_1 J_0(\lambda_m \bar{r}) + c_2 Y_0(\lambda_m \bar{r})] [c_1 J_0(\lambda_p \bar{r}) + c_2 Y_0(\lambda_p \bar{r})] d\bar{r} = 0$$

$$= N(\lambda_m) \text{ for } m \neq p$$

$$= N(\lambda_m) \text{ for } m = p$$

where

$$N(\lambda_m) = \int_a^b [c_1 J_0(\lambda_m \bar{r}) + c_2 Y_0(\lambda_m \bar{r})]^2 d\bar{r}$$  \hspace{1cm} (C-IX)
Appendix D: Integration of Heat Generation Function

To evaluate this integral in eqn. (4.30).
\[
\bar{\theta}_{w-1} = \int \int \dot{q}(\bar{\xi}) \, d\bar{\xi} + C_1 \bar{\xi} + C_2
\]

We know from eqn. (4.14)
\[
\dot{q}(\bar{\xi}) = \tau_{\text{max}} \left[ 1 - \left( 1 - \frac{\tau_{s}^2}{\tau_{\text{max}}^2} \right) \exp(-ac \, r_{2} \, \bar{\xi}) \right]^{1/2} \, ac \, c_s \, v \, \exp(-ac \, r_{2} \, \bar{\xi})
\]

\[
\int \dot{q}(\bar{\xi}) \, d\bar{\xi} = \int \tau_{\text{max}} \left[ 1 - \left( 1 - \frac{\tau_{s}^2}{\tau_{\text{max}}^2} \right) \exp(-ac \, r_{2} \, \bar{\xi}) \right]^{1/2} \, ac \, c_s \, v \, \exp(-ac \, r_{2} \, \bar{\xi}) \, d\bar{\xi}
\]

(D-I)

Making an substitution
\[
y = \exp(-ac \, r_{2} \, \bar{\xi})
\]

(D-II)

We have
\[
dy = -ac \, r_{2} \, \exp(-ac \, r_{2} \, \bar{\xi}) \, d\bar{\xi}
\]

Eqn (D-I) can thus be written as
\[
\int \dot{q}(\bar{\xi}) \, d\bar{\xi} = -\frac{\tau_{\text{max}} \, ac \, c_s \, v}{ac \, r_{2}} \int \left[ 1 - \left( 1 - \frac{\tau_{s}^2}{\tau_{\text{max}}^2} \right) \right]^{1/2} \, dy
\]

The power series in the integral can be expanded and written as
\[
\left[ 1 - \left( 1 - \frac{\tau_{s}^2}{\tau_{\text{max}}^2} \right) \right]^{1/2} = 1 - \frac{\left( 1 - \frac{\tau_{s}^2}{\tau_{\text{max}}^2} \right)^{y}}{2} + \frac{\left( 1 - \frac{\tau_{s}^2}{\tau_{\text{max}}^2} \right)^{2y}}{3!} - \frac{\left( 1 - \frac{\tau_{s}^2}{\tau_{\text{max}}^2} \right)^{3y}}{3!} + \ldots
\]

Considering the first two terms as the other terms are negligible we have
\[
\left[ 1 - \left( 1 - \frac{\tau_{s}^2}{\tau_{\text{max}}^2} \right) \right]^{1/2} = 1 - \frac{\left( 1 - \frac{\tau_{s}^2}{\tau_{\text{max}}^2} \right)^{y}}{2}
\]

(D-III)

\[
\int \dot{q}(\bar{\xi}) \, d\bar{\xi} = -\frac{\tau_{\text{max}} \, ac \, c_s \, v}{ac \, r_{2}} \int \left[ 1 - \frac{\left( 1 - \frac{\tau_{s}^2}{\tau_{\text{max}}^2} \right)^{y}}{2} \right] \, dy
\]
\[ \int \dot{q}(\bar{z}) \, d\bar{z} = -\tau_{\text{max}} \frac{ac \, c \, v_y}{ac \, r_2} \left[ y - \frac{(1 - \frac{\tau_s^2}{\tau_{\text{max}}^2})}{2 \ln(1 - \frac{\tau_s^2}{\tau_{\text{max}}^2})} \right] + C_1 \]

Re-substituting for the primary variable as in (D-II)

\[ \int \dot{q}(\bar{z}) \, d\bar{z} = -\tau_{\text{max}} \frac{ac \, c \, v_y}{ac \, r_2} \left[ \exp(-ac \, r_2 \, \bar{z}) - \frac{1 - \frac{\tau_s^2}{\tau_{\text{max}}^2}}{2 \ln(1 - \frac{\tau_s^2}{\tau_{\text{max}}^2})} \right] \] (D-IV)

Integrating (D-IV) again we have

\[ \int \int \dot{q}(\bar{z}) \, d\bar{z} = \int \left( -\tau_{\text{max}} \frac{ac \, c \, v_y}{ac \, r_2} \left[ \exp(-ac \, r_2 \, \bar{z}) - \frac{1 - \frac{\tau_s^2}{\tau_{\text{max}}^2}}{2 \ln(1 - \frac{\tau_s^2}{\tau_{\text{max}}^2})} \right] \right) d\bar{z} \]

\[ \int \int \dot{q}(\bar{z}) \, d\bar{z} = -\tau_{\text{max}} \frac{ac \, c \, v_y}{ac \, r_2} \left[ -\frac{\exp(-ac \, r_2 \, \bar{z})}{ac \, r_2} + \frac{Ei\{1 - e^{-ac \, r_2 \, \bar{z}} \, \ln(1 - \frac{\tau_s^2}{\tau_{\text{max}}^2})\}}{2 \, ac \, r_2 \, \ln(1 - \frac{\tau_s^2}{\tau_{\text{max}}^2})} \right] \]

where \( Ei(1, x) \) is the Exponential Integral function defined as

\[ Ei(1, x) = \int e^{\frac{m \, x}{m}} \, dm \text{ where } m \text{ is arbitrary constant.} \]

Hence, the solution is

\[ \bar{\theta}_{n-1} = -\frac{r_2^2 \, \tau_{\text{max}} \, ac \, c \, v_y}{k_b \, T_{\text{amb}}} \left[ \frac{e^{-ac \, r_2 \, \bar{z}}}{ac^2 \, r_2^2} + \frac{Ei\{1 - e^{-ac \, r_2 \, \bar{z}} \, \ln(1 - \frac{\tau_s^2}{\tau_{\text{max}}^2})\}}{2 \, ac^2 \, r_2^2 \, \ln(1 - \frac{\tau_s^2}{\tau_{\text{max}}^2})} \right] + C_1 \bar{z} + C_2 \] (D-VII)
Vita

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