2013

Using formative assessment to enhance student performance on geometric proof writing

Benjamin Hargrave
Louisiana State University and Agricultural and Mechanical College

USING FORMATIVE ASSESSMENT TO ENHANCE STUDENT PERFORMANCE ON GEOMETRIC PROOF WRITING

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural College
in partial fulfillment of the
requirements for the degree of
Master of Natural Sciences

in

The Interdepartmental Program in Natural Sciences

by
Benjamin Hargrave
B.S., Louisiana State University 2006
August 2013

ACKNOWLEDGEMENTS

I would like to thank my lovely wife, Andrea McCarthy, without whom I would not be half the man I am. I would like to extend my sincere appreciation to the people of the National Science Foundation whose support of the Louisiana Math and Science Teachers Institute continues to develop competent professionals who are leaders in their field. I would like to acknowledge Dr. James Madden, my thesis advisor, for his guidance and support in the program. Finally I'd like to thank my mother, an educator for over forty years, and my father for all they have done for me.

TABLE OF CONTENTS

List of Figures

ABSTRACT

The purpose of this research is to uncover best practices to create competent proof writers. Studies have shown the best setting to do this is in the high school geometry classroom. Throughout a yearlong study of geometry, students were exposed to theorems and their demonstrations. Despite constant exposure, students were still unable to produce their own proof of propositions. The questions then became how can an educator provide critical feedback that encourages student reasoning and develops logical argumentation skills? With the goal in mind, twenty-five students enrolled in a geometry course at Baton Rouge High School in Baton Rouge, Louisiana were given the task of replicating certain proofs in their own words. More than the students' performance, this research focuses on the teacher's role as a feedback source of students' proof-writing ability. Submitted are the efforts of one educator to establish norms in the construction of geometric proof writing, provide a method of feedback in the form of a student checklist and student teacher interviews, and adapt these efforts into an evaluation tool. The research will show students' original writing, teacher feedback, final product, and the evaluation results in the hopes of establishing best practices to increase student performance on proof writing tasks.

CHAPTER 1: INTRODUCTION

Typically in high school geometry proofs of fundamental geometric theorems are modeled by the textbook throughout the year and proof writing tasks are assigned for students to practice their craft. Often student don't know where to begin and struggle to present a valid argument even when the conclusion is known. As evident in student reflections, many feel that they need to study and memorize each and every theorem that comes up in class. In contrast to this memorization/regurgitation formula, current school reform aims to create critical thinkers in a classroom that can understand the task asked of them and respond appropriately. In geometry this means using previous ideas and accepted truths to build an axiomatic library of truth and then build upon those truths to explore new ideas – in essence, deductive reasoning skills. To assess these skills, teachers charge their students with the task of proof writing. Often, when charged with such a task, it is unclear to students what is expected of them. It falls to the teacher to inculcate the norms of proof-writing while still allowing learners to reason for themselves.

Can educators provide targeted feedback to develop authentic proof writing from students while also employing an efficient and consistent way to assess student work? If so, can this process identify trends among larger populations of learners that can indicate adjustments that need to be made in the teaching of proof? The purpose of this research is to introduce a formative and summative assessment method that can pinpoint individual student weaknesses and also identify the challenges common to a larger population. This will be accomplished by creating criteria by which the students' ability to express ideas in proof writing can be supported by means of explicit instruction and formative feedback. The feedback tools will be a proof writing checklist that defines exactly what goes into a geometric proof and a consultation format in which students receive feedback on how best to adapt their writing to the checklist. The aim

of the checklist is to establish norms that all valid proofs must possess. This includes a relevant diagram, proper use of necessary definitions, postulates, and theorems, as well as a clear logical thought process. As evidence of the effectiveness of these tools, this thesis presents students prewriting of certain geometric theorems, explains the interview process and checklist editing, displays final drafts of students' interpretation of these fundamental theorem, and shapes the checklist into a rubric to provide data in the hopes of identifying larger trends in the strengths and weakness common to a class.

This study is largely based on the implementation of the new Common Core State Standards (CCSS). Established in 2009 through joint effort from the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO), the CCSS is a national initiative that seeks to "provide a consistent, clear understanding of what students are expected to learn" (NGACBP 2010). The CCSS outlines eight Standards of Mathematical Practices. Mathematical Practice Standard 3 states the students will be able to "Construct viable arguments and critique the reasoning of others." It says:

> Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures... They justify their conclusions, communicate them to others, and respond to the arguments of others.

A central job of geometry teachers is to build such communication skills. The goal of this thesis is to provide a feedback tool and process by which to support students' critical thinking and deductive reasoning skills to construct geometric proofs and a rubric to fairly assess those skills. In doing so, they will be demonstrating a very important element of the Common Core Standards.

Chapter Two will present literature related to the problem of student proof writing, the historical context of the need for such a task, the current educational standards on the subject, and recent classroom studies. Chapter Three will set the stage for the research in this thesis and who is involved. Chapter Four will detail the creating of the tools used and how they were implemented in the class. Chapter Five will present evidence of those tools' effectiveness and the data to support such claims. Finally, Chapter Six provides a conclusion to this thesis by reiterating the goals and reflecting on the process.

CHAPTER 2: LITERATURE REVIEW

The Principles and Standards for School Mathematics (2000) of the National Council of Teachers of Mathematics (NCTM) defines the term *mathematical proof* to mean "arguments consisting of logically rigorous deductions of conclusions from hypotheses" (56). It goes on to state, "proof should be a significant part of high school students' mathematical experience, as well as an accepted method of communication" (349). To be prepared for proof writing, students must be confident in their own reasoning ability. They must be secure enough to question mathematical notions presented to them (Fussell 2005: 32). To facilitate this critical skepticism, educators at all levels are responsible for teaching students the reasoning and logic skills necessary to justify and explain their solutions. Ideally, when a student enters a high school geometry classroom, they should have the skills and background necessary for the production of a well-organized geometric proof. Proof writing is considered to be a way to "unify the mathematical experience" (Vavilis 2003: 2; Fussell 2005: 1).

Section 2.1 of this chapter uses popular literature to explore and define the role of the teacher in the geometry classroom as the instructor, coach, and judge of a student's proof writing. The next section, 2.2, focuses on the historical changes in high school curriculum through the nineteenth and twentieth century that brought the art of demonstration to the forefront of geometry instruction. Section 2.3 focuses on current school reform and states explicitly what the new Common Core State Standards expects high school students to be able to prove by the end of their geometry studies. Section 2.4 explores previous studies in the field of student proof writing.

2.1: The Teacher's Role

While there has been a clear curricular focus on proof in geometry studies, the initiative does not explain how to incorporate the practice into the day-to-day classroom setting. It is up to the teacher to "organize the atmosphere of the classroom so that students are encouraged to produce logical arguments and proof" (Herbst 2002a; Fussell 2005: 2). This is a difficult task. In most mathematics classrooms, a topic or process is presented to a learner, and they are more than "willing to trust whatever brilliant mathematician thought up the theorem" (Harel & Sowder 1998; Fussell 2005: 41). However, if proof-producing students is the goal of high school geometry, the students must be made to understand that purpose of proof writing is not in the application of the theorem, but in the presentation of the thought process.

Most mathematics textbooks assume that when students study proof, they are experiencing a well-formed deductive argument system, and that their analytical abilities are sophisticated enough to engage in rigorous mathematical thinking (Herbst 2002a; Fussell 2005). However, some students lack the perspective to see the larger body of knowledge being created with each proof building off of the last. Students tend to get frustrated with the technical terms and the precision of language that must be used when constructing a valid argument, and teachers are often limited in the time they can devote to proof studies. This combined with a lack of curricular clarity forces teachers to "rely heavily on their own mathematical knowledge base and their own beliefs of proof's role in the mathematical world" (Martin & McCrone 2001; Knuth 2002; Fussell 2005: 39).

In establishing the axiomatic language of geometry, it may seem fruitless to spend time proving old ideas that have become intuitively obvious, but the students must be made to understand that these postulates and definitions form the building blocks of the unintuitive

theorems explored in the course. "Using proof as a tool to explain new and difficult concepts increases not only the usefulness of proof but also students' opinion of proof" (Fussell 2005: 13; McCrone et al. 2002). The accepted truth of the postulate and the proper use of the definition lend their truth to each argument, and eventually, to the conclusion of a theorem. This is the foundation of deductive reasoning.

So how does a teacher ease a class of uncomfortable students into the rigor of proof writing? The teacher's task is two-fold. They must design and present activities modeling the framework of what is mathematically accepted as a formal proof while at the same time equipping students with the resources necessary to be successful in their own construction of an argument (Herbst 2002b: 178). The tasks created cannot be trivial nor can they be too difficult. To scaffold their learning, teachers will often give diagrams and auxiliary lines or even *plans for proof* to shift students' reasoning to comply with their own theoretical framework of how to demonstrate a proposition. In addition, they must supply the class with formative feedback that is not too critical. Lastly, the teacher must use what has transpired in the classroom setting and the relative difficulty of the proof to "reduce possible ambiguities in interpreting student performances" (199) – they must evaluate.

Teacher feedback is unsettling for students and must be handled with care. It is the first time in the students' mathematics career that they are asked to justify their reasoning.

> Asking a student why they included a particular statement or reason is often regarded as an attack on their answer. Students believe that a teacher would only ask for an explanation of their answer if it were wrong, students assume that the correct answers are never questioned; only accepted (Fussell 2005: 34).

While teachers are responsible for instructing students on how to reach a conclusion from a set of premises using logical deduction, it is also their responsibility to teach students how to use the conclusion (Herbst, 2002a). In addition to the time restraints in a given school year,

teachers are faced with the dilemma of preparing students for standardized tests. These assessments often address the application of a geometric theorem rather than its place in a larger body of knowledge.

2.2: History of Proof Writing

University of Michigan's Professor of Mathematics Education Patricio Herbst characterizes the evolution of Geometric proof instruction in three waves. He identifies "a baseline period characterized by students replicating the proofs given by a text, a transitional period of students crafting proofs for propositions, and a final stage of students learning how to do proofs" (Herbst 2002a: 287).

American high schools began to offer geometry courses in the 1840s as universities began to make it a requirement for admission. Herbst calls this time of instructions the Era of Text, as "the study of geometry entailed mastering the Euclidean body of knowledge as developed by a text" (288). The textbooks of the time followed the popular mathematics notion of developing as much as possible of the geometrical body of knowledge with the fewest possible postulates. The word demonstration was used, but neither to denote an object of study nor to identify a skill. In this time period "the study of geometry would be done through reading and reproducing a text; such work would train the reasoning faculties of students" (289).

A few decades closer to the turn of the century, more high schools offered courses in geometry, and the textbooks multiplied. The new texts evolved in part due to a new consideration of students' learning role. More than merely readers, students were viewed as untrained intellects who had to acquire the facts of geometry as well as the logical reasoning that connected those facts (290). In addition to replicating the proofs of a proposition in a text, students were given the opportunities to craft proofs for "original propositions." Professor

Herbst calls this transitional period *the Era of Originals*. The texts of these times presented the traditional propositions, but left interesting corollaries up to the learner to prove. “The presence of originals presumed that students would learn to reason by reasoning” (Herbst 2002a: 290).

As these originals became more popular, so did the need for explicit instructions on how to prove. This need called for the “texts [to develop] the bare bones of a norm for proving” (291). This time period introduced the use of *hypothetical constructions*. These given diagrams were often included with the proposition that needed to be proven and the auxiliary lines necessary for demonstration “afford[ed] students some of the elements that they would need in furnishing a proof” (292). The method of their actual construction, as in the classical text, was less important than their role in the logical reasoning necessary to extend a line of thought to its desired conclusion.

In addition to the use of these hypothetical constructions, the demonstrations included in the text “were developed in more succinct sentences” often taking no more than a single page to “facilitate its understanding” (293). Unlike the proofs in the era of text that kept explanations in paragraphs and provided only some of the reasons for statements, these new proofs established the practice of numbering lines of the demonstration and giving a short justification for each statement. In this way, a student could see the progression of logic in every step, readily refer to previous theorems to see how they apply to new ones, and gain insight into the logical construction of truth from previous propositions. The proofs of this era “would thus adhere to a more general norm of exposition that ended to make explicit what logical reasoning was” (293).

The students had been charged with the task of creating original proofs, given the resources with which to do that, and shown a normative framework by which to model so that “the spelling out of the proof may (if needed) be judged as understood” (293). The elements of

student proving were in place, and its practice was employed in high school geometry courses across the country. This was the educational setting at the end of the nineteenth century and the framework for the findings of the educational reform panel known as the Committee of Ten.

The Committee of Ten was a group of educational leaders called to serve by the National Educational Association with the mandate to study the problems related to college requirements for admission. It was part of their mission to examine what "should be the high school course of studies and best methods of teaching each subject" (Herbst 2002a: 286). This committee found the "educational value of mathematics [was the] training to the mind's power of conceiving, judging, and reasoning" and that "in formal geometry we have the best possible arena for training in deductive reasoning" (Hill 1895: 353-354; Herbst 2002a: 295).

One of the changes recommended by the committee was to extend the study of geometry into elementary school. The new course of concrete geometry would "familiarize the pupil with the facts of plane and solid geometry…to be subsequently employed in abstract reasoning," and in this new course, "accommodate the tension between training the mental faculties and [transmitting] the culturally valued geometrical knowledge" (Newcomb et al. 1893: 106; Herbst 2002a: 295). In this recommendation, the Committee of Ten cemented the purpose of the high school geometry course as not just the learning of the Euclidean body of geometric truths, but instead the "…art of demonstration (i.e., proving) thus became a main objective of the study of geometry" (Herbst 2005: 296).

From the *Era of Originals* it was "understood that the way to acquire the art of demonstration would be by demonstrating, by actually dealing with the geometric truths as a pretext to acquire the art" (296). The challenge now would be to produce students who could not only construct proof, but also understand the process of proving. To meet this challenge, the

geometry textbooks of this time period separated propositions into fundamentals and exercises. "The distinctions between the fundamental propositions and exercises played an important role in making possible the teaching of the art of proving" (Herbst 2002a: 301). These fundamentals would be presented by the text and reproduced by the students. In this way, students would be instructed on the body of geometric knowledge as well as provided tools and templates with which to construct their own proofs in the exercises. Following the demonstration of a fundamental proposition would be a series of exercises. The students would supply the proofs of the exercises in the same manner the propositions were presented. They were to use the fundamental propositions as justification in their demonstration, thereby developing new knowledge from old. Professor Herbst calls this time at the turn of the twentieth century the *Era of Exercises*.

In this *Era of Exercises,* the exercises themselves were "extremely important" as the "opportunity to prove … had to be actually taken and its purpose had to be accomplished for the course to fulfill its purpose as the place to acquire the art of proving" (299). The problem with the "originals" of the previous era was that they were too few, and these were too difficult. The goal of the originals was to allow the students to reason with a theorem to the point of creating new knowledge based on that proposition. The exercises were to be "many, easy, and carefully graded" (Young et al. 1899: 136; Herbst 2002a: 299). These frequent exercises afforded opportunities to construct proof and to be successful, and rather than necessarily create new knowledge, it was a way to practice what had been already learned. This was a major shift from "proving as a means to know new things to proving to practice using known things in proofs" (Herbst 2002a: 300). This *Era of Exercise* framed the instruction of Geometry throughout the twentieth and into the twenty-first centuries.

2.3: The Common Core State Standards

The Common Core State Standards (CCSS) is an initiative led by the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO). Since its release in 2009, it has gathered the support of such professional organizations as American Council on Education, the College Board, the Coalition for a College and Career Ready America, The National Council of Teachers of Mathematics (NCTM), The National Mathematics Education Organization, and the boards of education in numerous states across the country. It is currently being implemented in 45 states, the American territories of Samoa, the Virgin Islands, and Guam, as well as the District of Columbia. The goal of the CCSS is to ensure that all students during their primary and secondary education experience will have developed the reasoning skills that will prepare them for a collegiate or workforce career of the twenty-first century (NGACBP 2010).

The mission statement of the CCSS is to "provide a consistent, clear understanding of what students are expected to learn." To provide this for Mathematics educators, it defines and differentiates Standards for Mathematical Practice and Standards of Mathematical Content. Whereas the Standards of Mathematical Practice (MP) apply across all primary and secondary grade levels, the Standards of Mathematical Content are grade and subject level specific. There are eight standards of Mathematical Practice: make sense of a problem and persevere in solving them, reason abstractly and quantitatively, construct viable arguments and critique the reasoning of others, model with mathematics, use appropriate tools strategically, attend to precision, look for and make use of structure, and to look for and express regularity in repeated reasoning (NGACBP 2010).

The CCSS breaks down Mathematical Content from Kindergarten to Grade 8, and high school content standards are broken into subjects: number and quantity, algebra, functions, modeling, geometry, and statistics and probability. The focus of this research is based on the high school geometry content standards, which are broken into six strands, and students are expected to be able to construct proofs in four of the six strands: congruence, similarity, right triangles, and trigonometry, circles, expressing geometric properties with equations, geometric measurement and dimensions, and modeling with geometry.

In the geometry strand labeled Congruence*,* students are asked to prove theorems about lines and angles, triangles, and parallelograms. In terms of lines and angles, students need to show that vertical angles are congruent. This require an understanding that when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and that points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. The theorems about triangles include: the measures of the interior angles of a triangle sum to 180 degrees, base angles of isosceles triangles are congruent, the segment joining the midpoints of two sides of a triangle are parallel to the third side and half the length, and that the medians of a triangle meet at a point. The proofs of parallelogram properties are that opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and that rectangles are parallelograms with congruent diagonals (NGACBP 2010).

The proofs addressed in the strand labeled Similarity, Right Triangles, and Trigonometry include more theorems about triangles. These are that a line parallel to one side of a triangle divides the other two proportionally, and that the Pythagorean Theorem can be proven using

similar right triangles, and finally, building off the previous strand, using congruence and similarity for triangles to solve problems and to prove relationships in geometric figures.

The only proof students are asked to demonstrate in the Circles strand is that all circles are similar. The proofs demonstrated in Expressing Geometric Properties with Equations lie in the realm of analytical Geometry. The use of coordinates, distance formula, etc is employed in this strand to reinforce previous topics such as properties of parallelograms and congruence (NGACBP 2010).

The final two stands deal with understanding Geometric properties such as area and volume and the relationship between two-dimensional and three-dimensional figures. At no point in these strands are students asked to create proof so they were not used in my research design.

2.4: Previous Studies

There are a number of significant studies that focus on geometric proof writing, but few address the problem of having a consistent and reliable assessment technique that can aid in both the teacher's ability to evaluate students proof writing skills, as well as the student's ability to understand what is expected of them in the construction of a proof. One particularly relevant study comes from mathematics professors at Utah State University, who developed a rubric for formative assessment of proof writing in the college classroom. The purpose of the study was to create an efficient and consistent way to assess proofs that could easily be adopted by teaching assistants, and would provide summative assessments and feedback on student written work. The goal was to be able to process large amounts of student work efficiently, promote good writing skills and measure student improvements (Brown and Michel 2010: 1).

The rubric created for the research used three axes: validity, readability, and fluency then assessed correctness, ability to interpret, and ability to clearly communicate technical concepts respectively. At the beginning of the semester, students were shown examples of “good” and “not-so-good” responses, and these were discussed so that the students were aware of what was expected from them. The authors scaled each category of the rubric, and assignments were graded either by the professor or teaching assistant, awarding 0-3 points depending on the quality of the work. All of the data was recorded and an overall shift in performance was noted. Students began to perform better after the first three assignments. They also turned in higher quality assignments and appeared to take more pride in their work. In addition, the rubric created a way to easily communicate the technical errors found in student work. On the professor’s end, he was able to grade assignments twice as fast, and was able to share the burden with graduate assistants who were trained in the use of rubric (Brown & Michel 2010: 8)

Another study by Yvonne Chimwaza from the Department of Natural Sciences at Louisiana State University designed a similar checklist that presented questions derived from the Common Core State Standards of Mathematical Practice that students must ask themselves as they respond to a prompt (Chimwaza 2012). Amanda McAllister, also in the Department of Natural Sciences from Louisiana State University, assigned informal proof writing on a regular basis. She showed how a student’s informal writing of geometric ideas improved their ability to create formal proofs in a high school geometry course (McAllister 2013).

CHAPTER 3: NATURE OF THE STUDY

This chapter presents the facts and nature of the research. The first section gives information about the students around which the research was designed. The second section gives the teacher's perspectives of the problem that needs addressing and the rationale behind the research. Lastly, the third section explains what was done to aid students in their proof writing in the form of a proof writer's checklist and how that checklist was adapted to make a rubric to evaluate student work.

3.1: Population and Setting

The data collected for this research came from students at Baton Rouge Magnet High School in Baton Rouge, Louisiana. This high school is an academic magnet that attracts students with strong grade point averages (GPA) from the elementary and middle school grades. For students to stay at the high school, they must maintain at least a 2.5 GPA at the end of every school year. The data in this study came from the twenty-five ninth and tenth grade students enrolled in the author's 2012-2013 Geometry class. Of these twenty-five students, ten were male and fifteen female. Sixteen were African-American, two were minority other, and the remaining seven were white. All students had completed at least one year in the study of algebra and they all went on to pass the End of Course (EOC) Test in geometry with only one scoring *Fair* while all others scored *Good* or *Excellent*. Due to the nature of the school environment, all students were highly motivated and college bound.

3.2: Rationale

Though much has been said about the necessity for proof writing and many initiatives implemented, very little has been said as to the best way to evaluate student work and provide formative feedback. When constructing a proof, there is no right or wrong answer, therefore any

rubric used for assessment must be flexible yet consistent. Due to the challenge of assessing student proof writing performance, it is difficult to produce quantitative data that can identify broader trends in high school students' ability to demonstrate understanding of proof. The purpose of this research is to introduce a formative and summative assessment method that not only can pinpoint individual student weaknesses but also can identify the challenges common to a larger population. If we expect students to prove we must show them how to construct a proof, give them the opportunities to prove, evaluate their work, and provide critical feedback in order for them to improve their writing.

As is typical in most textbooks, the McDougal Littell *Geometry* textbook – used in geometry classes across East Baton Rouge Parish – covers the fundamental theorems and provides proof writing exercises at the end of each section. The class went over these fundamental theorems' demonstrations in the fashion presented by the text and the exercises were assigned as homework. Students and teacher would review these proof writing assignments as any other homework assignment. The solutions, as written by the text's authors, were presented, and students' questions were answered. This is a common practice of any mathematics classroom: a process of teacher modeling, practicing, and reviewing. However, despite daily review of proof writing, students were failing to produce improve based on test results. When asked to present a proof on a test or quiz, the students were often at a loss for how to reach a desired conclusion from given conditions or how the statements were justified. It appeared that giving them the assignment to prove on their own and then presenting a proof was not enough scaffolding for them to be able to create correct proofs on their own.

3.3: Design of the Study

If the goal of high school geometry is to teach students how to prove, they must be told what goes into a geometric proof. To facilitate this recipe, students were given a checklist explaining what elements must go into a geometric proof (Appendix A). Though each demonstration of a proposition is slightly different, all of them require a diagram, the proper use of givens, definition, postulates and theorems, and clarity of thought that relates all of these aspects to one another.

The first requirement is a diagram. For students to be able to show their reasoning with figures in a plane, they must provide a clearly labeled diagram. In the event that a hypothetical construction is necessary, such as a parallel line or a perpendicular bisector, the student should show this construction in the diagram to demonstrate its relation to the other parts of the figure.

In proof writing, a student must use all pertinent givens, definitions, postulates, and theorems and avoid non-pertinent ones. Students must introduce the given information based on what the proposition is stating. Postulates and definitions can be used at any time, and their truth-value is unquestioned. In the event that a theorem is necessary, the student is expected to be able to provide a demonstration; the checklist required that to be included that in the writing. This creates a complete proof with no doubt of its validity as well as the assurance that the student is in full control of the material.

Finally, the checklist asks if there is a clear progression of thought. The argument must have structure. Each statement must lead to the next with a mindfulness of how it is reaching the conclusion. For each theorem used, the conditions of the theorem must be clearly outlined. The entire argument must be understandable with the goal that an incoming geometry student could follow it and understand it validity.

From this checklist, the teacher can create rubrics for specific proofs. With the three categories being: (1) diagrams, (2) givens, definitions, postulates, and theorems, and (3) clarity of thought, teachers can allocate points into the different categories to create formative and summative assessment tools. In this way the teachers can identify student's strengths and weaknesses and develop them as proof writers.

CHAPTER 4: PROCESS

This chapter presents the method for data collection used in this research. Section 4.1 discusses the students' assignment of creating a portfolio of propositions based on the text and the expectations set by the Common Core State Standards. Section 4.2 describes the teacher's method of evaluation and leads to the following chapter, which details the results of the process.

4.1: The Proof Portfolio

At then end of the class's study in Euclidean geometry, students were given a project to see if they could reproduce proofs from the text and explain them in their own words. Students were given choices of which theorems they wished to demonstrate. The categories were drawn from the Common Core State Standards (CCSS), which explicitly state what students need to be able to prove by the end of their high school geometry studies. Students had to pick one proof from each category, submit a rough draft in two-column form, review with the teacher using the checklist, receive feedback, and recreate the proof in paragraph form to be submitted in a portfolio.

The categories of proofs were: a) theorems about lines and angles, b) theorems about triangles, c) theorems about parallelograms, d) theorems about circles, and e) a category labeled "miscellaneous theorems" that are difficult in nature. Each theorem was stated in the classical proposition form, and from that form students were to interpret the necessary givens and what needed to be demonstrated to prove the hypothesis.

In the category labeled "theorems about lines and angles," students could elect to prove that vertical angles were congruent, when a transversal crosses parallel lines alternate interior angles are congruent, or that points on a perpendicular bisector are those exactly equidistant from the segment's endpoints. In the category of "theorems about triangles," students could

demonstrate that the measures of the interior angles of a triangle sum to 180 degrees, that the base angles of an isosceles triangle are congruent, or that the segment joining the midpoints of two sides of a triangle are parallel to the third side and half its length. The category labeled "theorems about parallelograms" included opposite sides are congruent, opposite angles are congruent, or that the diagonals of a parallelogram bisect each other. The category "theorems about circles" included if two chords intersect inside of a circle the product of the lengths of a segment of one chord is equal to the product of the lengths of the segment of the other chord, if two secants intersect outside of a circle the product of the length of one secant segments and the length of its external segment equals the product of the lengths of the other secant segment and the length of its external segment, or that if a secant and a tangent segment intersect outside of a circle the product of the length of the secant segment and the length of its external segment equals the square of the length of the tangent segment. Finally, students had to select one of the "miscellaneous theorems," that a line parallel to one side of a triangles divides the other two proportionally, the Pythagorean Theorem proved using similar triangle, or the converse of the Pythagorean Theorem.

Once the student had chosen their five theorems, they used any resources they felt pertinent to construct a two-column proof and had one week to submit them to the teacher. Afterwards, each student met individually with the teacher to receive one-on-one feedback on how closely their proof adhered to the checklist and what aspects needed to be improved upon or explained further. After those interviews students had one week to adapt their demonstrations and create final drafts in paragraph form to be submitted for final grading.

4.2: The Rubrics

The teacher designed a rubric based on the checklist described in the previous section, and used this to assess the work in each written proof. The rubric relied on the deductive framework of the textbook and the teacher's own view on how best to demonstrate a proposition. A certain number of points were allotted to the students' diagram, their use of the givens, definitions, postulate and theorems, as well as the clarity of thought. The diagram points were awarded based on whether it was provided, and whether it visually demonstrated the necessary hypothetical constructions. A point was awarded for each given, definition, postulate, or theorem stated in the proof. Finally, points were awarded for the clarity of thought process. One point was awarded each time the student was able to clearly state the conditions that necessitate the postulate or theorem. Additional points were awarded for a clear progression leading to the conclusion based on the results of the postulate or theorem. The following chapter presents the results of the students' work on five of the propositions.

CHAPTER 5: FINDINGS

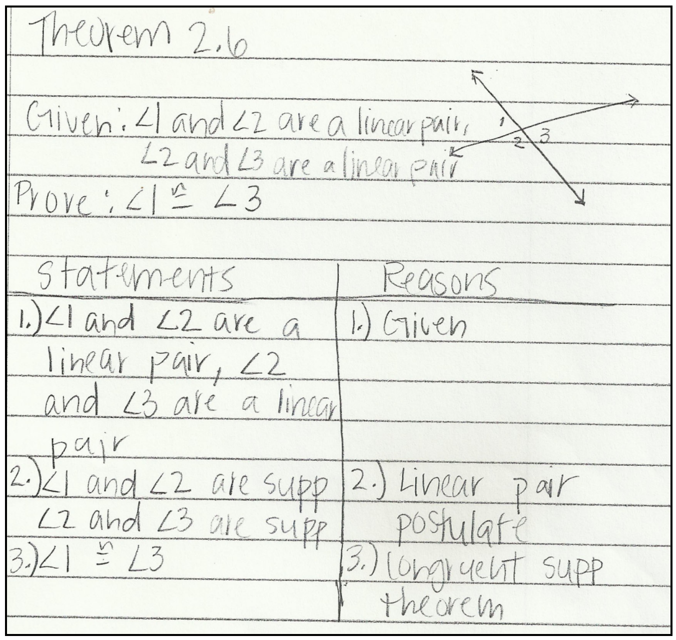
This chapter presents five of the most common student submissions to the proof portfolio project, which are inspected in detail. For each section, the theoretical framework of the proof is presented, student work is shown, common errors are explored, and the data supporting the rubric is explained.

5.1 The Vertical Angles Theorem

Of twenty-five students, twenty-one of them elected to demonstrate the Vertical Angles Theorem, which states that vertical angles are congruent. The McDougal Littell *Geometry* textbook presents this proof in chapter 2, early in the student's study of formal proof writing, in a section titled "Proving Statements About Angles." It defines vertical angles as two angles whose sides form opposite rays, i.e. lines. Its demonstration hinges on the use of the Linear Pair Postulate – that if two angles form a straight angle then they are supplements – and the Congruent Supplements Theorem – if two angles are supplementary to the same angle they are congruent. It lists the givens as $\angle 5$ and $\angle 6$ form a linear pair, $\angle 6$ and $\angle 7$ form a linear pair and then sets out to prove $\angle 5 \cong \angle 7$.

When presenting this proof in their rough draft, many students copied the text's demonstration; however, when asked what the congruent supplements theorem means, they were at a loss for an explanation. In order for the students to create a full proof that relies only on postulates, givens, and definition they would have to include the proof of the congruent supplements theorem in their own demonstration. This is done by using the concept of the measure of an angle. When presenting proof about sums or equivalences, the understanding of that theorem can best be facilitated by translating the geometric terms into numbers or variables. The geometric term $\angle 1$ refers to a figure in a plane whereas the numerical term $m\angle 1$ refers to

the number that is the measure of the angle marked *1*; therefore, if two angles are said to be supplementary then the sum of the measure of their angles adds to be 180 degrees, i.e. $m\angle 1 + m\angle 2 = 180$. Figure 5-1 is a typical reproduction of the text's proof and Figure 5-2 is the student's final draft with the inclusion of the requested equations.

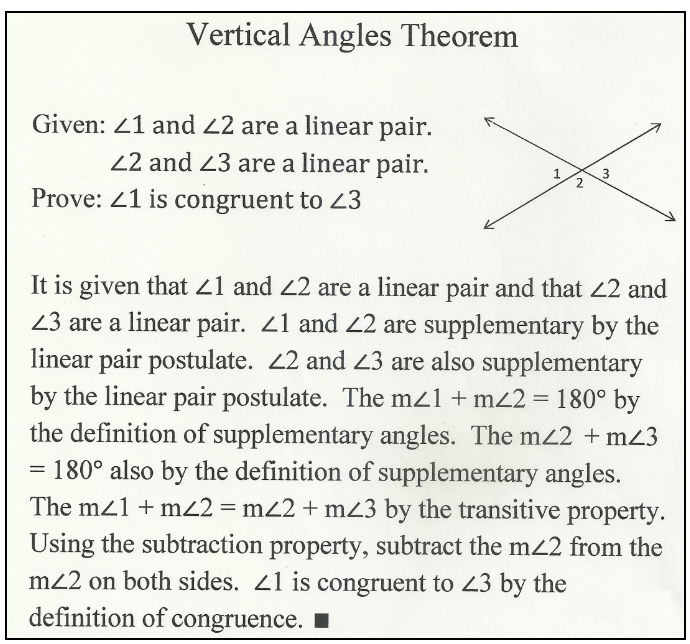

Theorem 2.6

Given: ∠1 and ∠2 are a linear pair, ∠2 and ∠3 are a linear pair

Prove: ∠1 ≅ ∠3

Statements	Reasons
1.) ∠1 and ∠2 are a linear pair, ∠2 and ∠3 are a linear pair	1.) Given
2.) ∠1 and ∠2 are supp ∠2 and ∠3 are supp	2.) Linear pair postulate
3.) ∠1 ≅ ∠3	3.) Congruent supp theorem

Figure 5-1. Student's rough draft of the Vertical Angles Theorem.


Vertical Angles Theorem

Given: ∠1 and ∠2 are a linear pair.
∠2 and ∠3 are a linear pair.
Prove: ∠1 is congruent to ∠3

It is given that ∠1 and ∠2 are a linear pair and that ∠2 and ∠3 are a linear pair. ∠1 and ∠2 are supplementary by the linear pair postulate. ∠2 and ∠3 are also supplementary by the linear pair postulate. The m∠1 + m∠2 = 180° by the definition of supplementary angles. The m∠2 + m∠3 = 180° also by the definition of supplementary angles. The m∠1 + m∠2 = m∠2 + m∠3 by the transitive property. Using the subtraction property, subtract the m∠2 from the m∠2 on both sides. ∠1 is congruent to ∠3 by the definition of congruence. ■

Figure 5-2. Student's final draft of the Vertical Angles Theorem.

The diagram provided by the students shows that two angles whose sides are opposite rays are adjacent to the same angle and both form linear pairs with that angle. The proof then uses the postulate to conclude they are supplements, and the definition of supplements to generate an equation like the one above. With the subtraction value the same – the common angle measure – it concludes that the two angles have the same measure. The proof is only completed when the student has demonstrated their congruence by showing that they have the same measure, which is how the text defines congruence. Presented in figure 5-3 is a side-by-side comparison of a rough draft and corrections to vocabulary and the conclusion made to complete the proof. Figure 5-4 shows the same student's final draft.

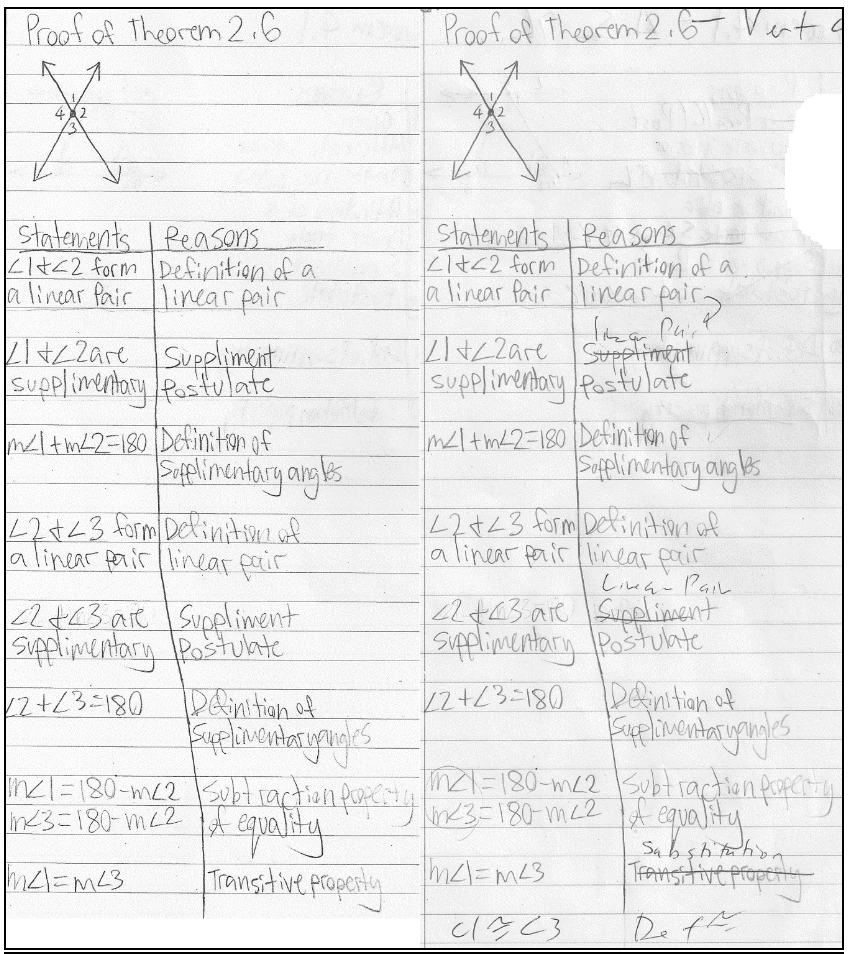
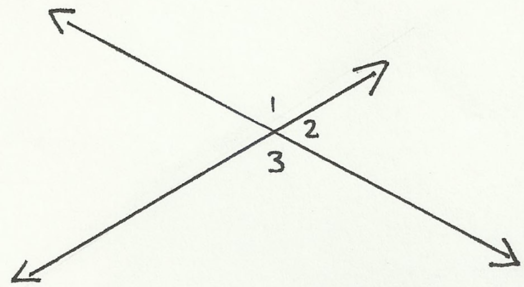


Figure 5-3. Student's rough draft of the Vertical Angles Theorem alongside teacher's formative feedback.

Vertical angles theorem



Angles one and two form a linear pair by definition of linear pair. Angles one and two are supplementary by the linear pair postulate. The measures of angles one and two add to 180 because of the definition of supplementary angles. Angles 2 and 3 form a linear pair because of the definition of a linear pair. Angles two and three are supplementary by the linear pair postulate. The measures of angles two and three add to 180 because of the definition of supplementary angles. The measure of angle one is equal to 180 minus the measure of angle two by the subtraction property of equality. By the same property, the measure of angle three is equal to 180 minus the measure of angle two. The measure of angle one is equal to the measure of angle three by the substitution property. Angle one is congruent to angle three by the definition of congruence.

Figure 5-4. Student's final draft of the Vertical Angles Theorem.

The rubric assigns eight total points to this demonstration. It scores the diagram with one point, which includes the presentation of two lines intersecting with at least three named angles. There are four points allocated to the use of givens, definitions, postulates, and theorems. The only given required is that when the two lines meet linear pairs are formed. Stating this earns the student one point. The Linear Pair Postulate must be used to state that these linear pairs are supplementary and from the definition of "supplementary," an equation must be created equaling 180 degrees. For the correct use of the postulate, the correct application of the definition, and the

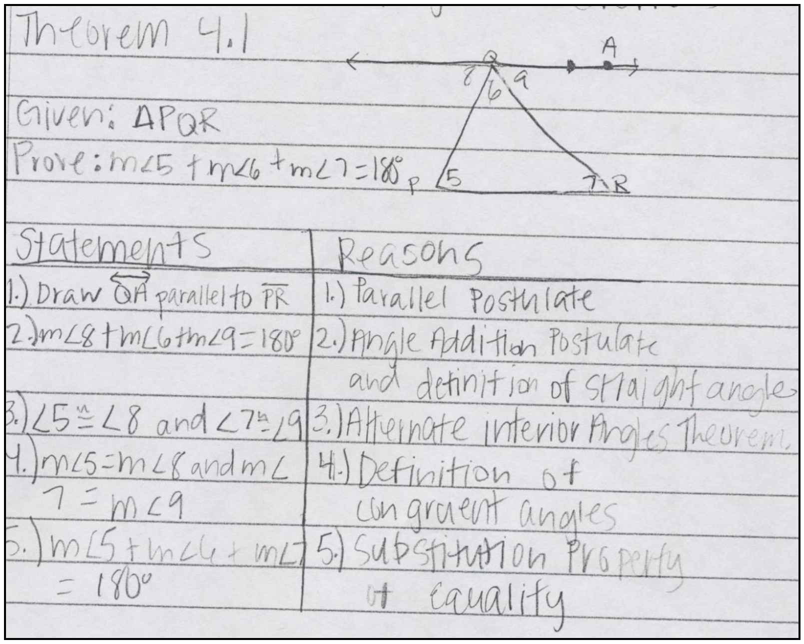
translation to an algebraic equation the student is awarded four points. Finally, the clarity of thought comes when the writer uses algebra to set the equations equal to one another, substitute the common angle measure, and demonstrate the equality of the measure of the desired angle. Once the student has used the definition of congruence to establish the congruence of the angles based on their equal measure, the writer can earn three points for this clarity of thought. Table 1 shows the student's scores on this rubric for their rough draft and Table 2 is their scores on the final draft (Appendix).

The students' rough drafts only earned an average of 57% of the total possible eight points, and the final draft, after student-teacher consultations, shows an increase to 82%. As the diagram used in this proof was very basic, 100% of students met the requirements of this aspect in both the rough and final draft. The use of givens, definitions, postulates, and theorems increased from 52% to 82% and the clarity of thought from 49% to 76%. This increase is mostly due to students' misguided use of the congruent supplements theorem, which is due to a poor explanation of the proof on the textbook. While this is not a difficult proof to understand and most students employed it effectively, it is not a fundamental theorem; therefore, the reliance on the theorem does not sufficiently validate of the proof. After a on-on-one intervention most students understood that its proof needed to be contained in the whole proof, allowing them to improve their grade in the final draft.

5.2: The Triangle Sum Theorem

The Triangle sum theorem was included in the portfolios of seventeen students. It is the first proof that requires an auxiliary line in its demonstration. The textbook presents a proof in the first section of Chapter Four: Congruent Triangles. As it does in many demonstrations, the book includes in this demonstration a "plan for proof." It indicates that a line can be drawn

through a vertex of the triangle that is parallel to the opposite side. The book justifies this construction by reference to "The Parallel Postulate," which reads, "If there is a line and a point on the line, then there is exactly one line through the point parallel to the given line." (Note that this is not the classical Parallel Postulate, which includes only the uniqueness statement.) Consider the vertex angle of the triangle together with each of the two adjacent angles formed by the sides of the triangle incident to the vertex and the rays along the auxiliary line. Together, these three angles form a straight angle, or as one student phrased it, a "linear triple." Since the three angles form a straight angle their measure must add to 180°. This proof uses a numeric equation in its demonstration. The crux of the proof lies in the alternate interior angles theorem, a theorem unproven in some student portfolios. By creating this auxiliary line parallel to the opposite side of the triangle, the other two sides act as transversals and the angles on either side of the transversal in between the parallel lines. In other words, the other two interior angles, and the angles adjacent to the original vertex angle, must be congruent. Establishing their congruence means that the measures are equal, thereby permitting substitution of the values into the original equation and showing that if the straight angle measures 180° so must the three interior angles of the triangle. Figures 5-5 and 5-6 are one student's reproduction of the proof in rough draft and final draft form respectively.



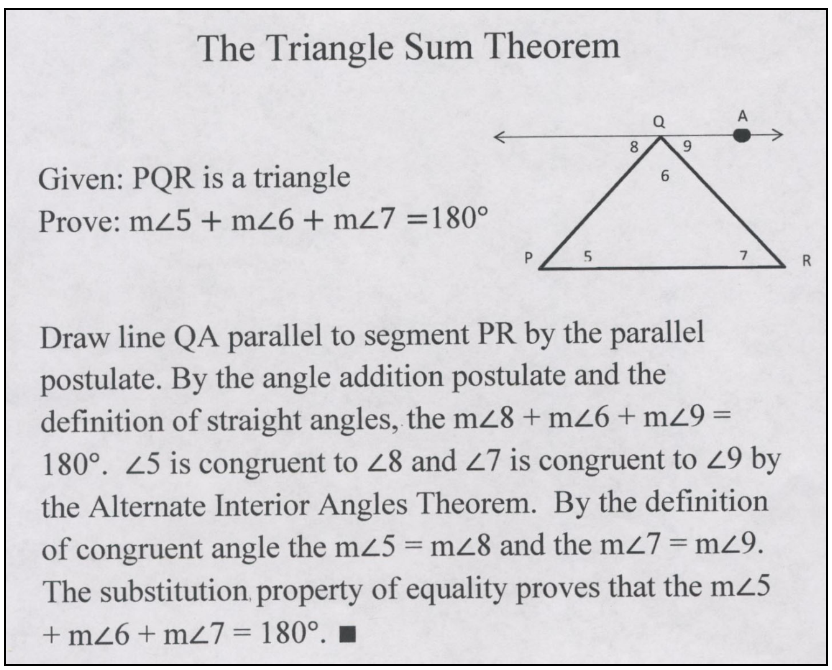
Theorem 4.1

Given: ΔPQR

Prove: $m\angle 5 + m\angle 6 + m\angle 7 = 180°$

Statements	Reasons
1.) Draw $\overleftrightarrow{QA}$ parallel to $\overline{PR}$	1.) Parallel Postulate
2.) $m\angle 8 + m\angle 6 + m\angle 9 = 180°$	2.) Angle Addition Postulate and definition of straight angle
3.) $\angle 5 \cong \angle 8$ and $\angle 7 \cong \angle 9$	3.) Alternate interior Angles Theorem.
4.) $m\angle 5 = m\angle 8$ and $m\angle 7 = m\angle 9$	4.) Definition of congruent angles
5.) $m\angle 5 + m\angle 6 + m\angle 7 = 180°$	5.) Substitution Property of equality

Figure 5-5. Student's rough draft of the Triangle Sum Theorem.



The Triangle Sum Theorem

Given: PQR is a triangle

Prove: $m\angle 5 + m\angle 6 + m\angle 7 = 180°$

Draw line QA parallel to segment PR by the parallel postulate. By the angle addition postulate and the definition of straight angles, the $m\angle 8 + m\angle 6 + m\angle 9 = 180°$. $\angle 5$ is congruent to $\angle 8$ and $\angle 7$ is congruent to $\angle 9$ by the Alternate Interior Angles Theorem. By the definition of congruent angle the $m\angle 5 = m\angle 8$ and the $m\angle 7 = m\angle 9$. The substitution property of equality proves that the $m\angle 5 + m\angle 6 + m\angle 7 = 180°$. ■

Figure 5-6. Student's final draft of the Triangle Sum Theorem.

The important concepts the student must show in their demonstrations are the parallel postulate and the translation from geometric notation into algebraic notation. Many students did not understand the concept of the hypothetical construction and most attributed it as a given.

Students had to be made aware of the importance of the parallel postulate in this early example so that they can use it in later proofs. In addition, many students did not see the difference in the statements that the angles were congruent and that they had the same measure. Though the statements are equivalent, algebraic properties such as substitution, addition and subtraction apply only to numerical values, thus the need for their representation in the proof. As the teacher's edition of the textbook points out, "It is necessary to make this switch from geometric notation to algebraic notation because what we are attempting to prove requires an algebraic statement" (Larson et al. 2005: 196). Figure 5-7 is a rough draft submitted by a student along with notes to correct the proof. Figure 5-8 is the final draft from the same student with the correct given, and the equations showing the sum of the measures of the angles.

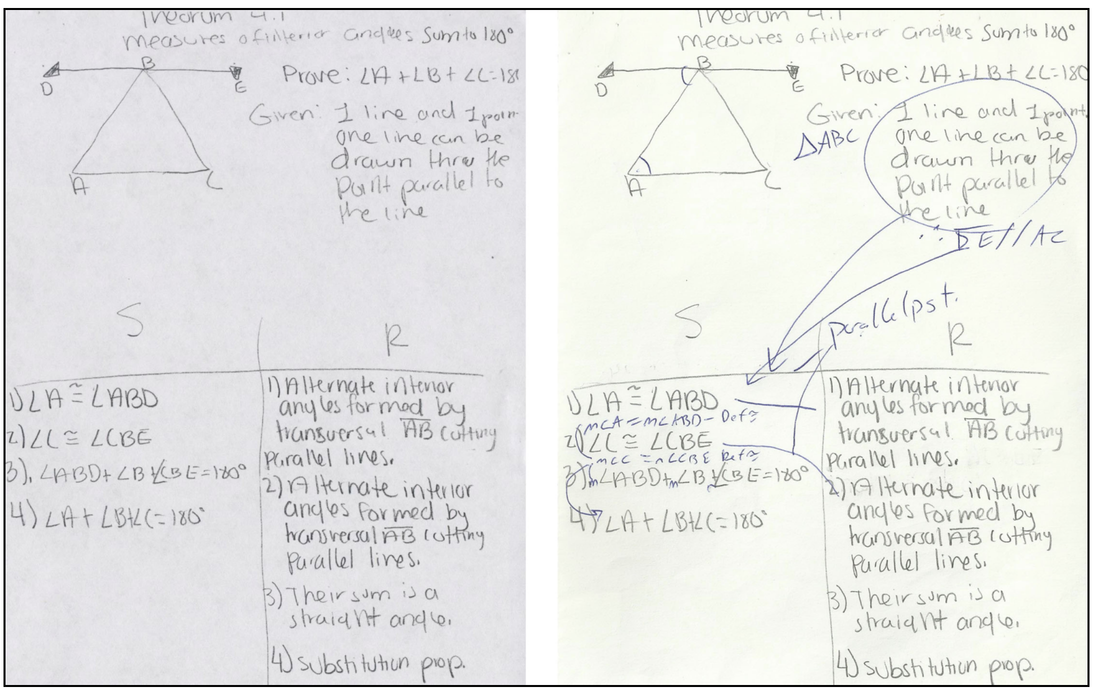
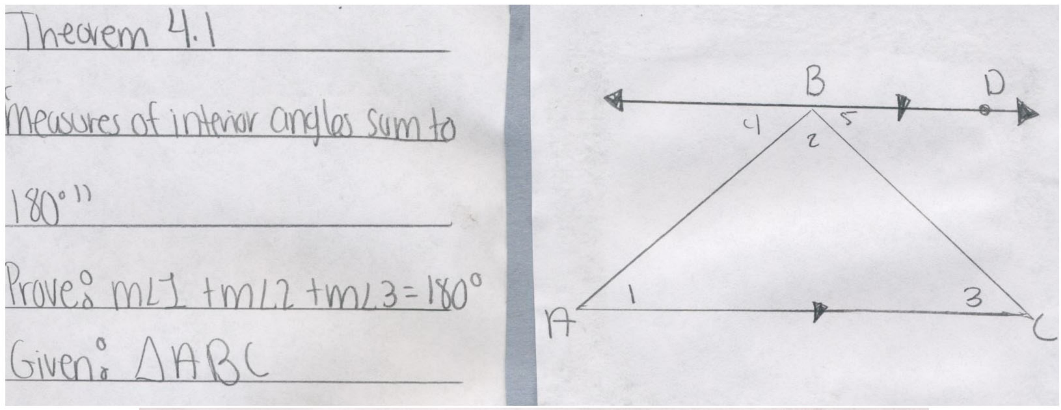


Figure 5-7. Another student's rough draft of the Triangle Sum Theorem.



By the parallel postulate, you draw line BD parallel to line AC. Then by the Angle Addition Postulate & definition of a straight angle, the measure of Angle 4 + the measure of Angle 2 + the measure of Angle 5 = 180 degrees. By Alternate Interior Angles, Angle 1 is congruent to Angle 4 & Angle 3 is congruent to Angle 5. The measure of Angle 1 is equal to the measure of Angle 4 & the measure of Angle 3 is equal to the measure of Angle 5, by the definition of congruent angles. Lastly, the measure of Angle 1 + the measure of Angle 2 + the measure of Angle 3 = 180 degrees by the substitution property of equality.

Figure 5-8. Another student's final draft of the Triangle Sum Theorem.

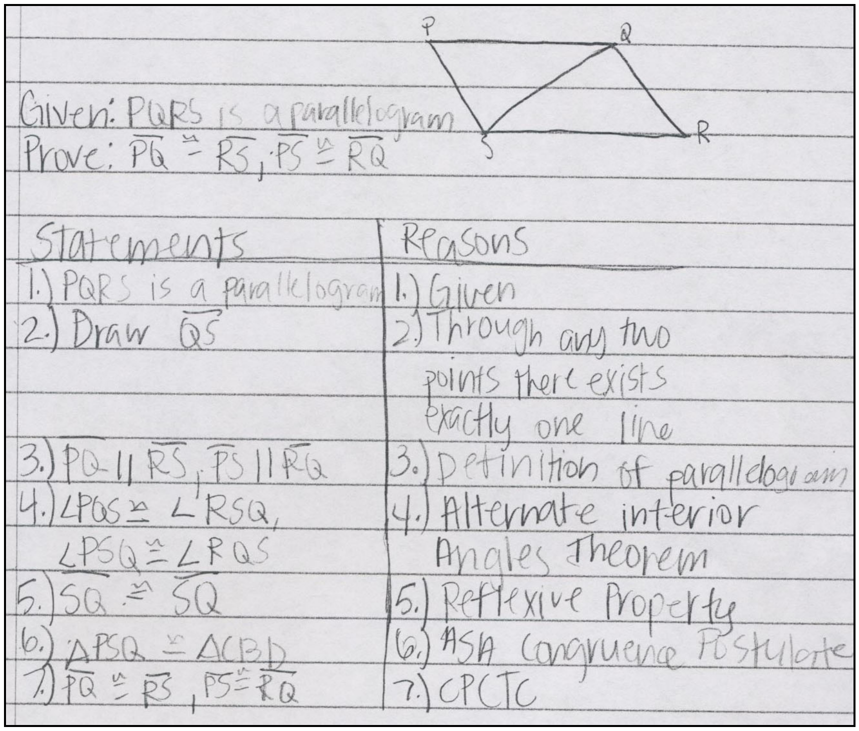
The Rubric allotted nine points for this demonstration. Since an auxiliary line is necessary, two points are given in the diagram category – one for the labeled triangle and another for the parallel line and numbered angles. Four points are awarded in the category of givens, definitions, postulates, and theorems. The student must be able to articulate that the only given is any arbitrary triangle, and from there, employ the parallel postulate. The parallel postulate is of fundamental importance in this proof, for without it, and the resulting alternate interior angles theorem showing congruence of angles, this proof will not hold. Once the student has pointed out the existence of this line and the congruence of the angles, they must correctly use the definition of a straight angle to translate this equation into one equaling 180 degrees. The clarity of thought earns three points, and those come from providing a clear translation from congruent angles to equal measures and then the substitution of those measures into the equation totaling 180°.

Tables 3 and 4 show the student's rough final draft scores of the rubric respectively. Students had only slight errors with this proof and averaged 73% of the total points in the rough draft. This average increased to 86% in the final draft after one-on-one consultation. Because some students neglected to include a diagram in their rough draft we can see an increase from 94% to 100% in the category of diagrams. Unfortunately, perhaps due to the difficulty of the diagram, students had a hard time interpreting the given of this proof. The only given is an arbitrary triangle with labeled vertices. Only 35% of students mentioned this in their rough draft, and that increased to only 41% in the final. This could be a careless omission, or it could reveal students misunderstanding that this property applies to all triangles. Students used the parallel postulate more in the final draft. Perhaps because of the way it is presented in the textbook. Some students said this auxiliary line is given or made no mention of it at all, with only 59% of students pointing out that this line exists because of the parallel postulate. After consultation that stressed the importance of this postulate, 94% of the students included it in their proof. For ease of explanation, and because it is a fundamental theorem in geometry, the alternate interior angle theorem was admissible in the demonstration and 88% of students employed it correctly in the rough and final drafts. The greatest gains in student performance were in the clarity of thought category. Only 59% of students used the definition of congruence in the rough draft, whereas 82% used it in the final to create equivalence of measure. The definition of straight angle was used by 71% of students in the rough, and 88% in the final, thereby creating an equation equaling 180°. The demonstration concluded with students correctly substituting these measures into the equation, which 71% did in the rough and 82% in the final.

5.3: Opposite Sides of a Parallelogram are Congruent

The proof of this nameless theorem was included in eighteen portfolios. It is displayed in chapter six of the text, the chapter concerning quadrilaterals and parallelogram. It is the first fundamental proof that uses triangle congruence to reach its conclusion; fundamental in the sense that it is not a trivial exercise that practices using triangle congruence to reach an inconsequential conclusion, but one whose conclusion will play a major role in further demonstrations and in the formation of new knowledge.

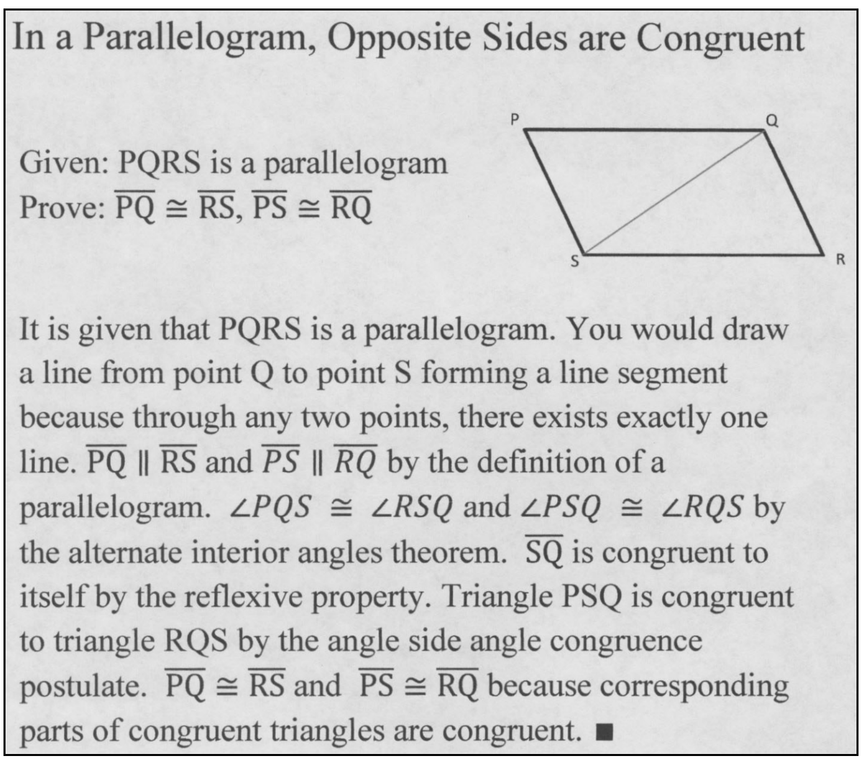
The texts lists the given as only some arbitrary parallelogram labeled *ABCD*, and seeks to prove that the opposite sides (sides *AB* and *CD* as well as *AD* and *BC*) are congruent. This demonstration points out that the sides to be proven congruent must also be parallel by definition of a parallelogram. From there, Euclid's first postulate, the text's fifth, is employed to draw line segment BD through the interior of the parallelogram. "Through any two points exists exactly one line" is the justification given in the two-column proof of the theorem, and many students quoted this in their own demonstrations. With the sides parallel and the newly constructed diagonal acting as a transversal, the proof points out the congruence of the pairs of alternate interior angles on either side of BD. Those angles, along with the included side *BD* congruent to itself by the reflexive property, outline the congruence of the two triangles forming the parallelogram (triangles *ADB* and *CBD*) by the Angle-Side-Angle Triangle Congruence Postulate. From there, the proof concludes that since the triangles are congruent, the corresponding parts of the congruent triangle must also be congruent. Since sides *AB* and *BC* correspond, they are congruent, and since *AD* and *BC* correspond, they are as well. These sides are also the opposite sides of the original parallelogram *ABCD*, which is what was to be proven in the first place. Figures 5-9 and 5-10 show a student's rough draft and final draft.



Given: PQRS is a parallelogram
Prove: $\overline{PQ} \cong \overline{RS}, \overline{PS} \cong \overline{RQ}$

Statements	Reasons
1.) PQRS is a parallelogram	1.) Given
2.) Draw $\overline{QS}$	2.) Through any two points there exists exactly one line
3.) $\overline{PQ} \parallel \overline{RS}, \overline{PS} \parallel \overline{RQ}$	3.) Definition of parallelogram
4.) $\angle PQS \cong \angle RSQ$, $\angle PSQ \cong \angle RQS$	4.) Alternate interior Angles Theorem
5.) $\overline{SQ} \cong \overline{SQ}$	5.) Reflexive Property
6.) $\triangle PSQ \cong \triangle CBD$	6.) ASA Congruence Postulate
7.) $\overline{PQ} \cong \overline{RS}, \overline{PS} \cong \overline{RQ}$	7.) CPCTC

Figure 5-9. Student's rough draft of Opposite Sides are Congruent in a Parallelogram Proof.



In a Parallelogram, Opposite Sides are Congruent

Given: PQRS is a parallelogram
Prove: $\overline{PQ} \cong \overline{RS}$, $\overline{PS} \cong \overline{RQ}$

It is given that PQRS is a parallelogram. You would draw a line from point Q to point S forming a line segment because through any two points, there exists exactly one line. $\overline{PQ} \parallel \overline{RS}$ and $\overline{PS} \parallel \overline{RQ}$ by the definition of a parallelogram. $\angle PQS \cong \angle RSQ$ and $\angle PSQ \cong \angle RQS$ by the alternate interior angles theorem. $\overline{SQ}$ is congruent to itself by the reflexive property. Triangle PSQ is congruent to triangle RQS by the angle side angle congruence postulate. $\overline{PQ} \cong \overline{RS}$ and $\overline{PS} \cong \overline{RQ}$ because corresponding parts of congruent triangles are congruent. ■

Figure 5-10. Student's final draft of Opposite Sides are Congruent in a Parallelogram Proof.

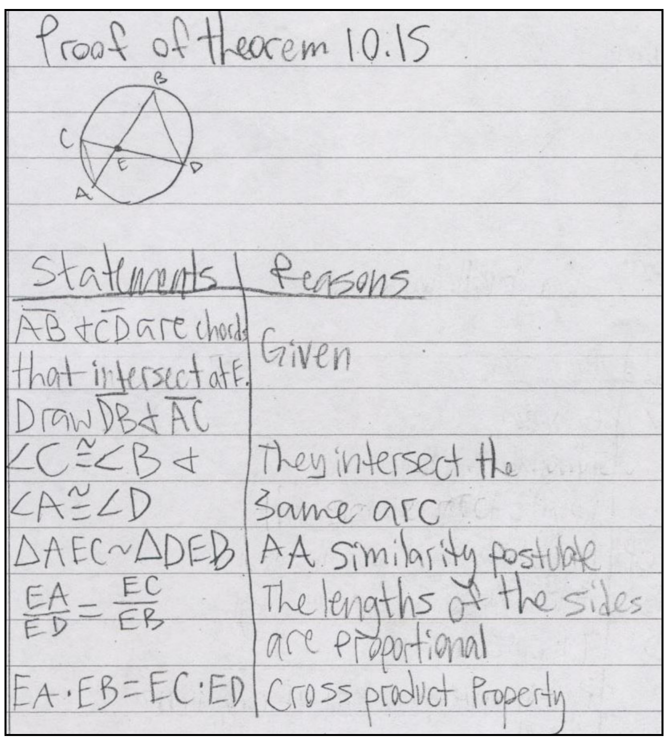
The adaptation of the rubric from the checklist designates eight total points for this demonstration. Because of the necessary diagonal, two points are allotted to the diagram category; one for the labeled parallelogram and another for the constructed diagonal. The givens, definitions, postulates, and theorems accounted for four of the total points on the rubric. The given is the parallelogram, and by its definition, the sides are parallel. It is necessary to point out the parallel sides, as that is a condition for the alternate interior angles theorem used in the demonstration. The student's clarity of thought accounts for the remaining two points and it come from pointing out, along with the alternate interior angles, that the constructed diagonal is a side shared by two triangles. Pointing this out, the final postulate the student must use is the angle side angle postulate. The student explaining that since the triangles are congruent, the pieces that form them are congruent is essential to earn points for clarity of thought.

Students did very well with this demonstration with an average of 89% score on the rough and 99% on the final. Students scored an average of 78% for the correct diagram, and that score increased to 97% on the final. The scores reflect that some students neglected to include a diagram at all in their presentation, and the subsequent 97% resulted from a single student who forgot to include the diagonal. The given in this case, as opposed to the previous proof of triangles, was well documented. In the rough draft, all but one student identified the given as any arbitrary parallelogram, and in the final, 100% supplied that information in their demonstration. The necessary alternate interior angle theorem was accounted for in 89% of the rough drafts followed by 100% in the final draft, and the triangles congruence postulate was up from 94% to 100% from rough to final. Because of the consultations the category of givens, definitions, postulates, and theorems saw in increase from 92% in the rough to 99% in the final. Additionally, the student's ability to clarify the triangle congruence and its implication in the

proof increased the clarity of thought score from 94% to 100%. Tables 5 and 6 show the scores of the rough and final draft respectively.

5.4: Product of Chords Equality

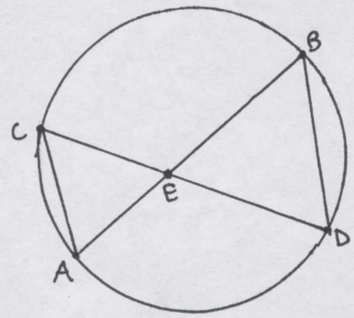
This states that if two chords intersect inside of a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of another chord. Twenty students included this demonstration in their portfolios. The text demonstrates this proof in Chapter 10: Circles. It presents the proof in a paragraph form and that may be some of the cause of the lack of justification in many of the students' rough drafts. The proof relies on the fact that similar triangles have proportional sides and then uses the cross product property to show the products are equal. The difficulty demonstration rests in showing the similarity of the triangles. Figure 5-11 is a rough draft of a student's proof and 5-12 is the final draft.


Proof of theorem 10.15

Statements	Reasons
$\overline{AB}$ + $\overline{CD}$ are chords that intersect at E.	Given
Draw $\overline{DB}$ + $\overline{AC}$	
$\angle C \cong \angle B$ + $\angle A \cong \angle D$	They intersect the same arc
$\triangle AEC \sim \triangle DEB$	AA similarity postulate
$\frac{EA}{ED} = \frac{EC}{EB}$	The lengths of the sides are proportional
$EA \cdot EB = EC \cdot ED$	Cross product Property

Figure 5-11. Student's rough draft of Product of Cords Equality Proof.

Product of Chord Segments Theorem



It is given that lines AB and CD are chords that intersect at point E. Lines DB and AC are drawn because through any two points there is a line. Angles C and B are congruent because they intersect the same arc. Angles A and D are also congruent for the same reason. Triangles AEC and DEB are similar by the angle angle similarity postulate. EA over ED is equal to EC over EB because the lengths of the sides are proportional by definition of similarity. EA multiplied by EB is equal to EC multiplied by ED by the cross product property.

Figure 5-12. Student's final draft of the Product of Cords Equality Proof.

To prove the triangles are similar, the student must first identify the triangles. They must join the endpoints of the chords using the hypothetical construction of joining two points to form a line. From there, students must show at least two angles are congruent and in that task they have options. They can point out the vertical angles created by the chords' intersection are congruent, or they can use a theorem stating that inscribed angles, angles whose vertex is on the circumference of a circle, intersecting the same arc of a circle are congruent. This theorem itself requires a lengthy proof. For ease of explanation, and because it is intuitively apparent, it is acceptable to point out that if the sides of two angles intercept the same arc they must be the same size, and therefore, congruent. Figure 5-13 shows student's rough draft along-side formative feedback. This student went on to use the idea of the measure of an inscribed angle equaling one-half the measure of the arc it intercepts, and since two angles intercept the same

arc, they must be the same size, and therefore, congruent. This is an acceptable informal proof and does lead to the correct conclusion that the triangles are similar. Figure 5-14 shows the same student's final draft.

The proof of this circle theorem requires similar triangles. Because it deals with the intersection of chords that form two sides of a triangle the third needs to be constructed. So two points are given for a diagram. It must contain a circle in which two chords intersect and the endpoints must be joined to form two triangles. The givens, definitions, postulates and theorems come from the given chord intersection, the proof of similar triangles by the angle-angle similarity postulate, and the definition of similar triangles to create the side proportions. The inclusion of these three statements earns the proof writer three points. The remaining three points come from the clarity of thought this is the explanation of which angles make the triangles similar and why they are congruent, and the use of the proportions to establish the equality of the products. A student can earn eight total points for this demonstration. Tables 7 and 8 show the point distribution of the rough draft and final draft respectively.

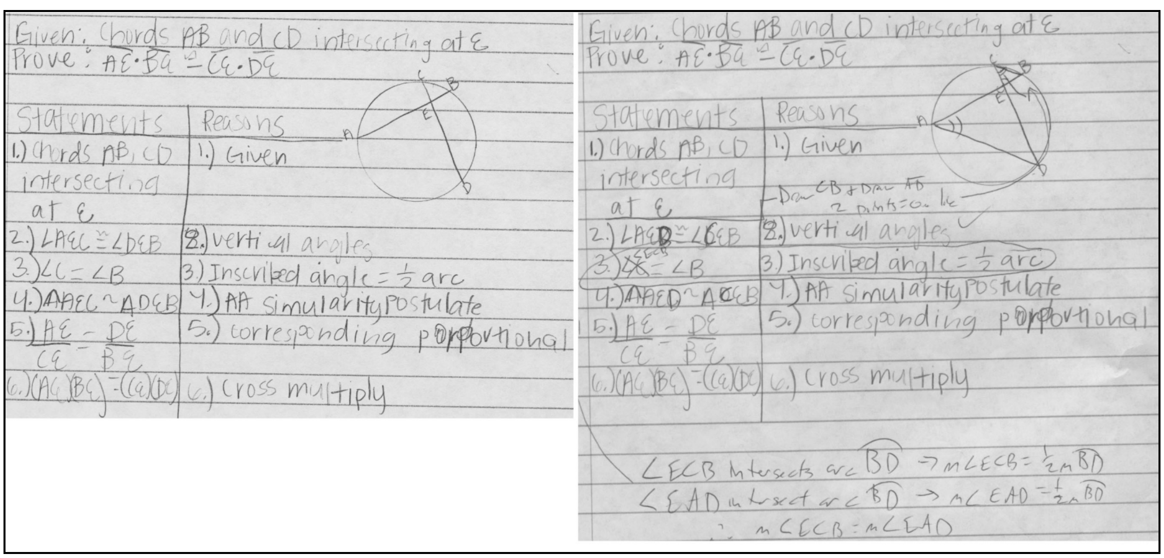
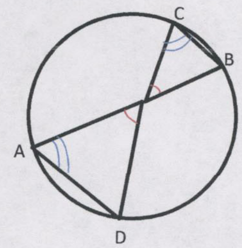


Figure 5-13. Student's rough draft of the Product of Cords Equality Proof alongside formative teacher's assessment.

The Product of Chords Equality

Given: Chords AB and CD intersecting at E

Prove: $(\overline{AE})(\overline{BE}) \cong (\overline{CE})(\overline{DE})$



It is given that chords AB and CD are intersecting at E. Angle AED is congruent to angle CEB because they are vertical angles. Angle ECB intersects arc BD, so the measure of angle ECB = $\frac{1}{2}$ the measure of arc BD. Angle EAD intersects arc BD so the measure of angle EAD = $\frac{1}{2}$ the measure of arc BD. The measure of angle ECB is congruent to the measure of angle EAD. Triangle AED is similar to triangle CEB by the angle angle similarity postulate. Because $\frac{AE}{CE} = \frac{DE}{BE}$ they correspond proportionally. Using the cross product property $(\overline{AE})(\overline{BE}) \cong (\overline{CE})(\overline{DE})$. ■

Figure 5-14. Another student's final draft of Product of Cords Equality Proof.

Students earned an average of 71% on the rough draft, and 84% after consultation. Though some students omitted diagrams entirely on their rough draft, 90% did show a circle with intersecting chords; however, only 50% of students included the constructions of the third side of the triangle. It took quite a lot of explanation during consultation to get students to see the need for this joining of the endpoints. They had to see that if they wanted to reference an angle, they needed to have two segments to make that angle, not a segment and an arc. After consultation, 100% of students included a circle and its intersecting chords, and 80% showed the constructed sides. Again, perhaps because of the diagram, only 45% of students included the given information that two circles intersect inside of a segment in their rough draft, and only 50%

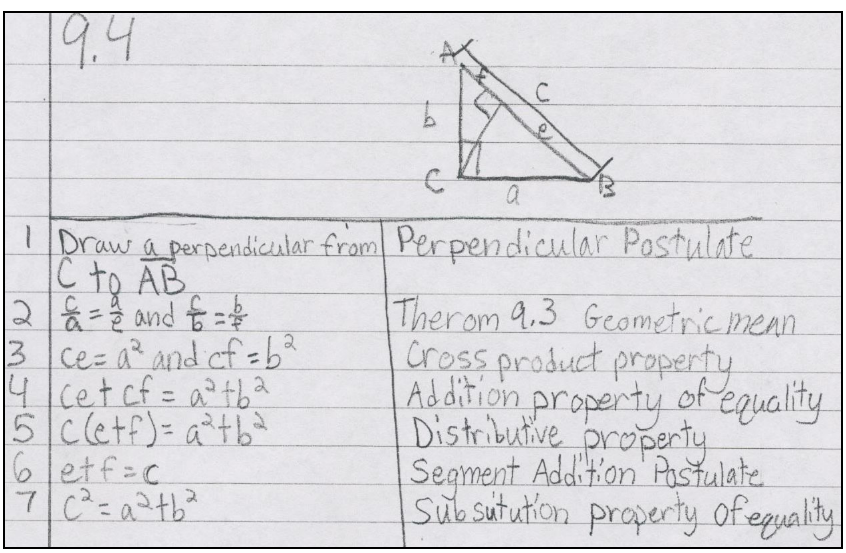
included this in their final draft. Most students realized that this demonstration relied heavily on similar triangles, and 95% of them included this in their rough and final draft. However, students struggled with clearly explaining the reasoning for this similarity. The postulate that states the similarity of triangles relies on the congruence of two angles. In the rough draft, 80% of students stated the congruence of one set of angles, but only 60% of students included the second set. In the final draft, those numbers increased to 95% for one pair and 75% for the other. Once this similarity is established, 80% of students used the definition of similar triangles to establish the proportions of the sides, and that number increased to 95% in the final draft. The conclusion of this proof relies on the cross product of these proportions and though only 60% of students were able to clarify this cross product from the proportion in the rough draft, and 95% of students were able to make this point clear in the final.

5.5: The Pythagorean Theorem Proved Using Similar Triangles

By the time they arrive at a high school Geometry class, students should be intimately familiar with the Pythagorean Theorem and its application. Its proof may seem trivial and unnecessary, but if the geometry class is a place to exercise deductive reasoning there should be no higher ideal than demonstrating the proof of the Pythagorean Theorem; thereby answering the question of "why" instead of just "how" that all intellectually curious students should be harboring. Though there are numerous proofs and explanations of the Pythagorean Theorem, the proof given by the text and the one sixteen students submitted in their own portfolios was the proof using similar triangles. This is a difficult proof and requires algebraic reasoning and geometric interpretation as well as keen focus of what is to be proven.

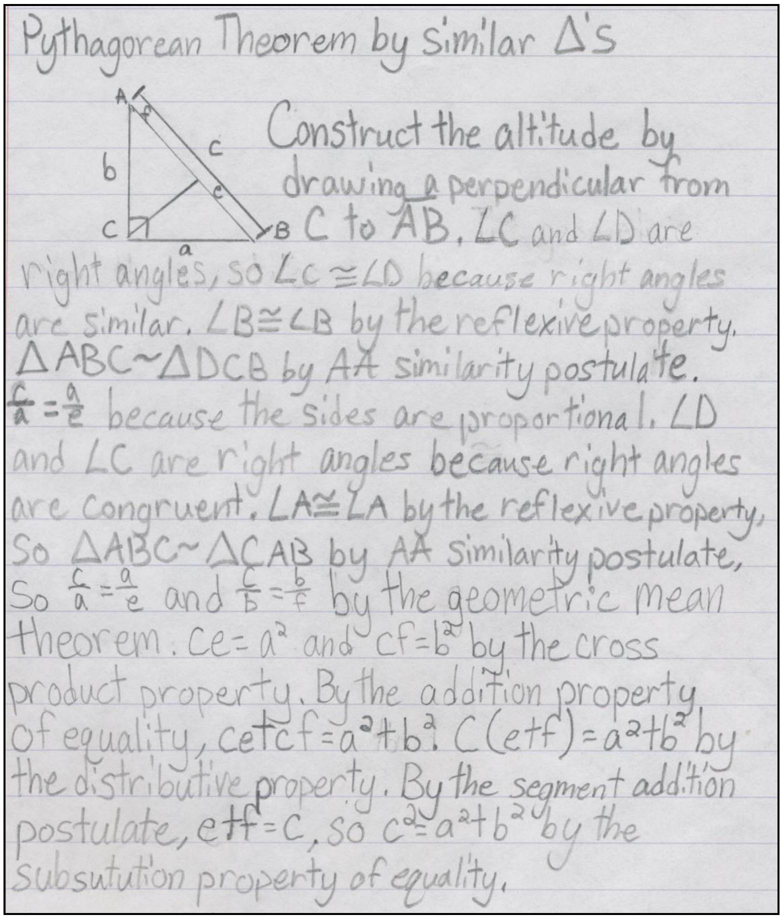
The proof given by the text is in Chapter Nine: Right Triangles and Trigonometry. Its givens are a triangle labeled ABC and that $\angle ACB$ is a right angle. It sets out to prove $a^2+b^2=c^2$.

It follows a section that focuses on the geometric mean theorem and the similarity of right triangles. The text defines the geometric mean of two numbers *a* and *b* to be a number *x* such that the ratio of *x* to *a* is equal to the ratio of *b* to *x*. In other words, $x^2 = ab$ and $x = \sqrt{(ab)}$. This defines the geometric mean as a numeric value. The difficulty in this proof is using geometric concepts to derive this numerical value. The text also presents a proof that if an altitude is drawn to the hypotenuse of a right triangle, the two right triangles formed are similar to the whole right triangle and also similar to each other. The proof of this proposition is left as an exercise to prove at the end of the section. In order for a student to create a fully formed proof of the Pythagorean Theorem, they must demonstrate the similarity of these right triangles, use their proportional sides to establish the geometric mean proportions, and equate the correct proportion to derive the conclusion algebraically. Figure 5-15 is a student's submission of the proof presented in the textbook. In it, the student relies on the aforementioned geometric mean theorem to create the desired proportions. With those proportions, the algebraic manipulations lead to the desired conclusion. Figure 5-16 is the same student's final draft incorporating the proof of similarity to extract the desired geometric mean proportions.


9.4

1	Draw a perpendicular from C to $\overline{AB}$	Perpendicular Postulate
2	$\frac{c}{a} = \frac{a}{e}$ and $\frac{c}{b} = \frac{b}{f}$	Therom 9.3 Geometric mean
3	$ce = a^2$ and $cf = b^2$	Cross product property
4	$ce + cf = a^2 + b^2$	Addition property of equality
5	$c(e+f) = a^2 + b^2$	Distributive property
6	$e + f = c$	Segment Addition Postulate
7	$c^2 = a^2 + b^2$	Subsutution property of equality

Figure 5-15. Student's rough draft of the Pythagorean Theorem Proof.



Pythagorean Theorem by similar Δ's

Construct the altitude by drawing a perpendicular from C to $\overline{AB}$. $\angle C$ and $\angle D$ are right angles, so $\angle C \cong \angle D$ because right angles are similar. $\angle B \cong \angle B$ by the reflexive property. $\triangle ABC \sim \triangle DCB$ by AA similarity postulate. $\frac{c}{a} = \frac{a}{e}$ because the sides are proportional. $\angle D$ and $\angle C$ are right angles because right angles are congruent. $\angle A \cong \angle A$ by the reflexive property, So $\triangle ABC \sim \triangle CAB$ by AA similarity postulate, So $\frac{c}{a} = \frac{a}{e}$ and $\frac{c}{b} = \frac{b}{f}$ by the geometric mean theorem. $ce = a^2$ and $cf = b^2$ by the cross product property. By the addition property of equality, $ce+cf = a^2+b^2$. $c(e+f) = a^2+b^2$ by the distributive property. By the segment addition postulate, $e+f = c$, so $c^2 = a^2+b^2$ by the subsutution property of equality.

Figure 5-16. Student's final draft of the Pythagorean Theorem Proof.

Many students were unable to grasp the subtitles inherent in this proof. After formative feedback, all students recognized they were relying on an unproven theorem, the Geometric Mean Theorem, to construct their demonstration. The goal then was to provide enough feedback so that the student would be able to first prove that the two smaller right triangles are similar to the larger by showing angles' congruence, and then to relate the proper sides into proportions. To do this, the students were instructed to point out that since the altitude was perpendicular to the hypotenuse of the original right triangle, all three triangles share a right angle, so there is a congruence of angles that can be used. After that, all that needs to be done is to recognize by constructing the smaller triangles within the larger, each shares one angle with the larger. After pointing this out, the similarity is thus proven by the angle-angle similarity postulate. Even after

formative feedback, many students were unable to demonstrate a viable argument for their similarity. Many stated the similarity or the proportions were "given," and some stated that they were similar but did not point out which angles were congruent to justify their similarity. Figure 5-17 is a side-by-side comparison of one student's rough draft and feedback, and Figure 5-18 is the final draft. Despite the demonstration of how to show similarity, the student simply stated that they were similar by the postulate without justification. Figure 5-19 is the final draft of a students effort to create the proportions as a "given" and another as "the definition of geometric mean."

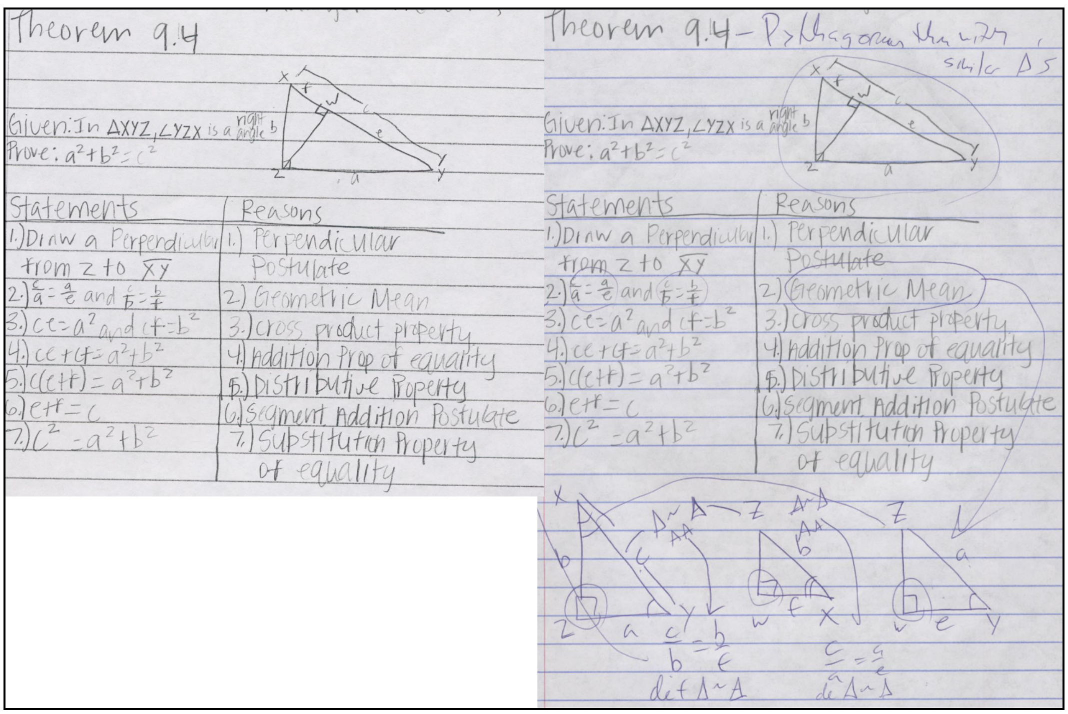
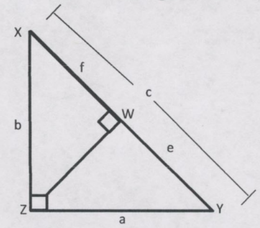


Figure 5-17. Another student's rough draft of the Pythagorean Theorem Proof alongside teacher's formative assessment.

Pythagorean Theorem with Similar Triangles



Given: In triangle XYZ, angle YZX is a right angle.
Prove: $a^2 + b^2 = c^2$

By the perpendicular postulate, draw a perpendicular from Z to $\overline{XY}$. Triangle XYZ is similar to triangle ZXW and similar to triangle ZYW by the angle angle similarity postulate. $\frac{c}{b} = \frac{b}{f}$ and $\frac{c}{a} = \frac{a}{e}$ by the definition of triangles are similar to triangles and by the geometric mean. Using the cross product property, $ce = a^2$ and $cf = b^2$. $ce + cf = a^2 + b^2$ by the addition property of equality. Using the distributive property, $c(e+f) = a^2 + b^2$. $e + f = c$, by the segment addition postulate and $c^2 = a^2 + b^2$ by the substitution property of equality. ■

Figure 5-18. Another student's final draft of the Pythagorean Theorem Proof.

- In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs

To begin this proof draw a perpendicular line from C to Line AB, the spot will be called D, D is an altitude of Triangle ACB. To separate the two smaller triangles, but still prove them proportional Line CD must equal Line CD through the Reflexive Property. CD is now the geometric mean in both triangles and therefore Line f / Line CD = Line CD / Line e. Line a and Line b are also geometric means due to the Definition of a Geometric Mean. Therefore it is given that Line c / Line a = Line a / Line e. By crossing the products ce = a*a and cf = b*b. The Addition Property furthers this equation by making ce + cf = a*a + b*b. Distributive property then shows us that c(e + f) = a*a + b*b. e + f = c by the Segment Addition Postulate. Finally c*c = a*a +b*b by the Substitution Property of Equality, Proving the Pythagorean Theorem.

Draw a perpendicular from C to D	Definition of an Altitude
Line CD = Line CD	Reflexive Property of Equality
f/CD = CD/e	CD is the geometric mean of BD and AD
Line a and Line b are also geometric means	Definition of Geometric Mean
c/a = a/e	Given
ce = a*a and cf = b*b	Cross Product Property
ce + cf and a*a + b*b	Addition Property of Equality
c(e + f) = a*a + b*b	Distributive Property
e + f = c	Segment Addition Postulate
c*c= a*a + b*b	Substitution property of Equality

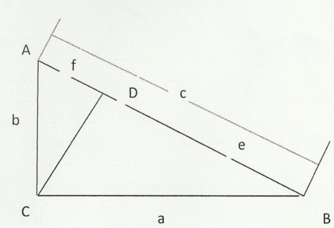


Figure 5-19. Final draft of student's effort to create the proportions as a "given" and another as "the definition of geometric mean."

The proof of the Pythagorean Theorem was by far the most intricate of any submitted to the portfolio. The adaptation of the checklist to a rubric produced sixteen points the students were expected to meet. The diagram is simply a right triangle with the constructed altitude from the right angle, so it made up only two points of the total. Six points are allotted to the givens, definitions, postulates, and theorems. There is only one point for the given information, a triangle with a right angle. The remaining five points come from the two similarity statements, the two resulting proportions, and the use of the angles addition postulate to relate the hypotenuse to the sum of its pieces. The clarity of thought makes up the bulk of the rubric's points, totaling eight points. To earn these points, the student must point out two pairs of angles in two separate triangles to establish the similarity, the results of the cross products, the addition of the two results, and the substitution of the sum of the pieces for the hypotenuse to complete the theorem.

Tables 9 and 10 show the point breakdown of the rough draft and final draft. The difficulty of this proof resulted in two students not turning in rough drafts at all. Their data is not included in the rough draft data, but their submission to a final draft is. Students earned an average of 59% of the total points in the rough draft, a score that increase to 67% in the final. The greatest gains were in the use of givens, definitions, postulates, and theorems from 58% to 71%. However, the clarity did not see such improvements, with scores only increasing from 52% to 56%. This can be attributed to the student's ability to recognized that the triangles needed to be similar for the proportions to hold, but their inability to demonstrate that similarity through the use of angle congruence as seen in student's submission in Figure 5-18.

5.6: Student Reflections

Students were asked, but not required, to submit personal reflections in their portfolios. This optional free writing assignment was intended to allow students to express their opinion on the point of proof writing, and describe how their views changed over the course of the school year. Most students indicated that at the beginning of their studies, they felt proofs were "pointless." As one student reflected, "I thought that somebody somewhere else had already proven these theorems to be true, so why did I have to?" This sentiment seems to be common among all the students. Memorizing all of the postulates and theorems was a major concern for some students, but two students reflected that this was not a requirement at all. One student pointed out that rather than memorizing the formal names of a postulate or theorem, one could just know the conditions and that would be acceptable. Another noted that there was no need to memorize when a geometry textbook contained them all to be referenced whenever needed. Given a geometry book, they could prove any theorem asked of them. It is important to note that at no point in the reflections did the students realize they were constructing proofs not to learn the truth of the theorems, which is well established, but to learn how to demonstrate a conclusion given a set of premises.

5.7: Summary of Findings

The data compiled on the five geometric proofs using the established rubric show that students' performances increased after consultations with the teacher where the rubric and checklist were discussed. Not surprisingly, the majority of students struggled with two main aspects: providing the appropriate givens, postulates, definitions and theorems, as well as the clarity of thought in their argument. After consultation, students showed significant improvement in providing the appropriate givens, postulates, definitions, and theorems, but did

not show as great an improvement in presenting a clear and concise thought process connecting the arguments. This demonstrates that while teaching an abstract concept such as proof writing, it is easy to instruct what concepts to use, but it does not guarantee a student's fluency in deductive reasoning. The rubric was successful in providing a consistent and clear method of evaluating student work as well as making them aware of what is expected of them in their own proof writing.

Chapter 6: Conclusion

One major goal of the Common Core State Standards (CCSS) is to create competent critical thinkers who can articulate and explain their ideas using mathematics. A good setting for instruction addressing these goals is in the high school geometry class. Students should have a firm and correct geometric intuition from their elementary studies in concrete geometry. In high school, they are given the task of articulating this knowledge in definitions and axioms. Based on this, they must create deductive proofs as a way of organizing and systematizing their knowledge. When students write their own proofs, they are learning to express their mathematical ideas with utmost clarity and precision and trace the connections to other knowledge. However, students do not always grasp the goals of proof writing. Often they do not know what is being asked of them when they are given a proof writing assignment. The textbook utilized across East Baton Rouge Parish presents proofs throughout, but it never offers any meaningful explanation of the purpose for writing proofs and (consequently, perhaps) does not offer any consistent norms for proof writing. In order to teach students to reproduce proofs, to interpret proofs in their own words, and create their own proofs, the teacher must establish a strategy of intervention and assessment. It falls to the teacher to provide perspective on not only why students must construct proofs, but also how to construct proof. The teacher must then offer feedback to sharpen the students' reasoning skills while still allowing them opportunities to reason for themselves.

This thesis sets out to develop classroom procedures that could be used to achieve this goal. In the present chapter, the evidence is gathered concerning the effectiveness of the procedure and the implications of the research is discussed. Conclusions will be stated in the form of recommendations regarding practices, for which evidence will be provided. The

recommendations are as strong as the evidence and no stronger. Finally, this chapter discusses some of the limitation of the research, and suggests possible future directions to pursue with similar studies.

6.1: Evidence

The evidence presented in this thesis concerns one teacher's attempt to support and develop students' proof writing. After a yearlong study in geometry, students were still struggling with proof writing and an intervention was needed. To aid student's writing, a checklist was developed to detail explicitly a set of standards for proof. This gave the students a template for their writing. Once completed, the writing assignments were evaluated and formative feedback was provided. The teacher met with each student individually for five to ten minutes and took care to assure that the students understood that the feedback was not an attack on their work but a meeting to point out which aspects of the proof needed further development. Through the pre- and post-writing exercises, the students worked toward meeting a set of explicitly stated expectations. They submitted five demonstrations of fundamental geometric propositions. Their growth was measured with an adapted rubric, and the findings are detailed showing growth in all areas.

The evidence gathered in this thesis suggests that students will benefit if the norms for proof writing are written down in a simple, brief format, and if proofs are graded by direct reference to these norms. This study suggests that there ought to be routines for supplying feedback that is supportive and not negative. Brief one-on-one meetings worked very well for this research, and were one of the most productive aspects in the entire course. Finally, students should be given an opportunity to use the feedback to improve their writing and resubmit. The teacher should take time prior to the beginning of the course to write down the norms for proof

writing and a rubric to go with it. Supplying formative feedback should become part of the routine, provided at least once per chapter or unit of study. Following the feedback, giving students opportunities for editing their work will solidify the concepts introduced in the proofs.

6.2: Implications

The checklist created is straightforward, simple and can be utilized by any geometry teacher. However, simply having this tool is not enough. There must be a system of formative feedback built into the classroom that can improve student's writing without being critical to the point of mistrust. This method of intervention and assessment is not limited to this research but has applications for all classrooms that teach proof. It can be used as a building block to establish a solid proof-based geometry curriculum that adheres to the Common Core State Standards.

6.3: Limitations and Further Research

The data collected in this research was a response to students' struggles in proof writing after a yearlong study in geometry. The timeframe was limited to the last few weeks of school, after end of course testing and before summer break. Ideally, rather than an end of the year intervention strategy, this practice could be employed throughout the school year. Students' work could be catalogued and growth charted as the year progressed. Additionally, checklists can be given to students for exercises in peer editing. Once students have become comfortable with the method of deductive reasoning, the checklist could be adapted to different types of proof. When proof by contradiction or analytical geometric proofs are introduced, students will gain a deeper level of understanding of the proof writing process.

Proofs are challenging. Some students and teachers question the relevance of proving something already known; however, it is not the truth that is the lesson, it is the process. By

developing an intervention and assessment method, students will better understand this purpose, and their proof writing skills will improve.

REFERENCES

Brown, David E. and Shayla Michel
2010 "Assessing Proofs With Rubrics: the RVF Method." Utah State University, Logan, UT. <http://sigmaa.maa.org/rume/crume2010/Archive/Brown_D.pdf>

Cirillo, Michele and Patricio G. Herbst
2012 "Moving Toward More Authentic Proof Practiced in Geometry" in *The Mathematics Educator*, 2011/2012 Vol. 21, No. 2: 11-33.

Chimwaza, Yvonne Mariki
2012 *Writing in Geometry With the Common Core State Standards Developing Mathematical Thinkers*. Thesis submitted to the Interdepartmental Program in Natural Sciences. On file at the Louisiana State University.

Fussell, Karen Nicole Hilton
2005 The Educational Purposes of Geometric Proof in the High School Curriculum. A thesis submitted to the Graduate Faculty of North Carolina State University.

Herbst, Patricio
2002 Establishing a Custom of Proving in American School Geometry: Evolution of the Two-Column Proof in the Early Twentieth Century in Educational Studies in Mathematics, Vol. 49: 283-312.

Herbst, Patricio
2002 "Engaging Students in Proving: A Double Bind on the Teacher" in the *Journal for Research in Mathematics Education*, Vol. 33, No. 3: 176-203.

Hill, F
1895 "The educational value of mathematics", in the *Educational Review* 9, 349–358.

Knuth, E.J.
2002 "Secondary School Mathematics Teacher's Conceptions of Proof." in *Journal for Research in Mathematics Education*. Vol. 22: 379-405.

Larson, Ron, Laurie Boswell & Lee Stiff
2005 *McDougal Littell Geometry* Houghton Mufflin Company. Evanston, Illinois.

Martin, T.S. & S.S. McCrone
2001 "Investigating the Teaching and Learning of Proof: First Year Results." (ERIC Document Reproduction Service ED466962).

McCrone, S. S., T. S. Martin, J. Dindyal, & M. L. Wallace
2002 "An Investigation of Classroom Factors that Influence Proof Construction Ability" (ERIC Document Reproduction Service No. ED471776)

National Council of Teachers of Mathematics (NCTM)
2000 *Principals and Standards for School Mathematics*.

National Governors Association Center for Best Practices (NGACBP)
2010 The Common Core State Standards <http://www.corestandards.org/). National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO). Washington, D.C.

Newcomb, S.
1893 "Reports of the conferences: Mathematics", in *National Education Association, Report of the Committee on Secondary School Studies*, Arno Press, New York, pp. 104–116. (Original work published 1893)

Vavilis, Bob
2003 Implications of John Dewey's and Lev Vygotsky's Theoretical Frameworks for the Teaching and Learning of 10th Grade Geometric Proofs. Chicago State University.

Young, J.W.A.
1899 "Report of the Committee of the Chicago section of the American Mathematical Society", in *National Education Association, Report of Committee on College Entrance Requirements*. July 1899, NEA, Chicago, pp. 135–149.

Appendix A: Proof Writing Checklist

Relevant Diagram:

- Clearly labeled
- Necessary auxiliary lines displayed

Definitions, Givens, and Postulates

- Givens pertinent to the proposition
- Postulates used correctly
- Definitions used correctly
- Theorems used are proven to be true

Clarity of Thought Progression

- Logical progression of thought
- Mindfulness of conclusions in each statement
- Justification of each statement

APPENDIX B: DATA TABLES

Table 1. Vertical Angles Theorem Rough Draft

Number	Diagram	Givens	LinePairPost	Def of Supp	Eq = 180	Eq set equal	Subtraction	Conclusion	Total Score
1	1	1	1	1	0	0	0	1	5
2	1	0	0	0	0	0	0	1	2
3	1	1	0	1	0	0	0	1	4
5	1	1	1	0	0	0	0	1	4
6	1	1	1	1	1	1	1	1	8
7	1	1	1	0	0	0	0	0	3
8	1	1	1	0	0	0	0	1	4
9	1	1	1	0	0	0	0	1	4
10	1	1	0	0	1	1	1	1	6
11	1	1	0	0	1	1	1	1	6
14	1	1	0	1	1	0	0	1	5
15	1	0	0	1	1	1	1	0	5
16	1	0	0	0	1	0	0	1	3
17	1	1	1	0	0	0	0	1	4
18	1	0	0	1	1	0	0	0	3
19	1	1	1	1	1	1	1	1	8
20	1	1	1	0	0	0	0	1	4
21	1	0	0	0	1	1	1	1	5
22	1	1	1	0	0	0	0	1	4
24	1	0	0	1	1	1	1	0	5
25	1	1	1	0	0	0	0	1	4
	100%	71%	52%	38%	48%	33%	33%	81%	57%

Diagram	100%
G, D, P, T	52%
Clarity	49%

Table 2.Vertical Angles Theorem Final Draft

Number	Diagram	Givens	LinePairPost	Def of Supp	Eq = 180	Eq set equal	Subtraction	Conclusion	Total Score
1	1	1	1	0	0	1	1	1	6
2	1	1	1	1	1	1	1	0	7
3	1	1	0	0	0	0	0	1	3
5	1	1	1	0	1	1	1	1	7
6	1	1	1	1	1	1	1	0	7
7	1	1	1	0	0	0	0	1	4
8	1	1	0	1	1	1	1	0	6
9	1	1	1	1	1	1	1	1	8
10	1	1	1	0	0	0	1	1	5
11	1	1	1	1	1	1	1	0	7
14	1	1	1	1	1	1	1	0	7
15	1	1	1	1	1	1	1	1	8
16	1	1	1	1	1	1	1	0	7
17	1	1	0	1	1	1	1	1	7
18	1	1	1	1	1	0	1	1	7
19	1	1	1	1	1	1	1	1	8
20	1	1	0	1	1	0	1	1	6
21	1	1	1	1	1	1	1	1	8
22	1	1	1	0	0	0	0	1	4
24	1	1	1	1	1	1	1	1	8
25	1	1	1	1	1	1	1	1	8
	100%	100%	81%	71%	76%	71%	86%	71%	82%

Diagram	100%
G, D, P, T	82%
Clarity	76%

Table 3.Triangle Sum Theorem Rough Draft

Number	diagram	paralell line	Given	Parallel Post	Def Straight	Alt int thm	Def Congr	Eq = 180	Subst	Total
1	1	1	0	1	1	1	1	1	1	8
2	1	0	0	0	0	0	0	0	0	1
4	1	1	0	1	1	1	1	1	1	8
5	1	1	1	1	1	1	0	1	0	7
6	1	1	1	1	1	1	1	1	1	9
7	1	1	0	1	1	1	1	1	1	8
9	1	1	1	1	1	1	1	1	1	9
14	1	1	0	0	1	1	1	1	1	7
15	1	1	0	0	0	1	0	1	0	4
16	1	1	0	0	0	1	0	0	0	3
17	1	1	1	1	1	1	1	1	1	9
19	1	1	1	0	0	1	0	1	1	6
20	0	0	0	1	1	1	1	1	1	6
21	1	1	0	0	0	1	0	1	1	5
22	1	1	1	1	1	1	1	1	1	9
24	1	1	0	0	1	0	0	1	1	5
25	1	1	0	1	1	1	1	1	0	7
	94%	88%	35%	59%	71%	88%	59%	88%	71%	73%
	Diagram	91%								
	G, D, P, T	63%								
	Clarity	73%								

Table 4. Triangle Sum Theorem Final Draft

Number	diagram	paralell line	Given	Parallel Post	Def Straight	Alt int thm	Def Congr	Eq = 180	Subst	Total
1	1	1	0	1	1	1	1	1	1	8
2	1	1	0	1	1	1	0	0	1	6
4	1	1	0	1	1	1	1	1	1	8
5	1	1	1	1	1	1	1	1	1	9
6	1	1	1	1	1	1	1	1	1	9
7	1	1	0	1	1	1	1	1	1	8
9	1	1	1	1	1	1	1	1	1	9
14	1	1	0	1	0	1	1	1	1	7
15	1	1	0	1	1	1	0	1	1	7
16	1	1	0	0	1	1	0	1	0	5
17	1	1	1	1	1	1	1	1	1	9
19	1	1	1	1	1	1	1	1	1	9
20	1	1	0	1	1	1	1	1	0	7
21	1	1	1	1	1	1	1	1	1	9
22	1	1	1	1	1	0	1	1	1	8
24	1	1	0	1	1	0	1	1	1	7
25	1	1	0	1	0	1	1	1	0	6
	100%	100%	41%	94%	88%	88%	82%	94%	82%	86%

Diagram	100%
G, D, P, T	78%
Clarity	86%

Table 5. Opposite Sides of Parallelogram are Congruent Rough Draft

Number	Diagram	Diagonal	Given	paralellogram	Alt int angles	Reflevive	Tri cong	CPCTC	Total
1	0	0	1	1	1	1	1	1	6
2	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	8
4	1	1	1	1	1	1	1	1	8
5	0	0	1	1	1	1	1	1	6
6	1	1	1	1	1	1	1	1	8
7	1	1	1	1	1	1	1	1	8
8	1	1	1	0	1	1	1	1	7
9	1	1	1	1	1	1	1	1	8
12	1	1	1	1	1	1	1	1	8
14	1	1	1	1	1	1	1	1	8
15	1	1	1	1	1	1	1	1	8
17	1	1	1	1	1	1	1	1	8
18	1	1	1	1	1	1	1	1	8
19	1	1	1	1	0	1	1	1	7
20	0	0	1	1	1	1	1	1	6
22	1	1	1	1	1	1	1	1	8
25	1	1	1	1	1	1	1	1	8
	78%	78%	94%	89%	89%	94%	94%	94%	89%

Diagram	78%
G, D, P, T	92%
Clarity	94%

Table 6. Opposite Sides of Parallelogram are Congruent Final Draft

Number	Diagram	Diagonal	Given	paralellogram	Alt int angles	Reflevive	Tri cong	CPCTC	Total
1	1	1	1	1	1	1	1	1	8
2	1	1	1	1	1	1	1	1	8
3	1	1	1	1	1	1	1	1	8
4	1	1	1	1	1	1	1	1	8
5	1	1	1	1	1	1	1	1	8
6	1	1	1	1	1	1	1	1	8
7	1	1	1	1	1	1	1	1	8
8	1	1	1	1	1	1	1	1	8
9	1	1	1	1	1	1	1	1	8
12	1	1	1	1	1	1	1	1	8
14	1	0	1	1	1	1	1	1	7
15	1	1	1	1	1	1	1	1	8
17	1	1	1	1	1	1	1	1	8
18	1	1	1	1	1	1	1	1	8
19	1	1	1	0	1	1	1	1	7
20	1	1	1	1	1	1	1	1	8
22	1	1	1	1	1	1	1	1	8
25	1	1	1	1	1	1	1	1	8
	100%	94%	100%	94%	100%	100%	100%	100%	99%

	Diagram	97%
	G, D, P, T	99%
	Clarity	100%

Table 7. Product of Chords Equality Rough Draft

Number	Diagram	Aux lines x2	Given	1 angle	2 angle	Similar	poroportion	cross prod	Total
1	1	0	0	0	0	1	1	1	4
2	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	1	1	1	4
5	1	0	1	1	1	1	1	0	6
7	1	1	0	1	0	1	1	1	6
8	1	1	1	1	0	1	1	1	7
9	1	0	1	1	1	1	1	1	7
10	1	1	1	1	0	1	0	0	5
12	1	0	1	0	0	1	1	1	5
14	1	1	0	1	1	1	1	0	6
15	1	1	1	1	1	1	1	1	8
16	1	1	1	1	1	1	1	0	7
17	1	1	1	1	1	1	1	1	8
18	1	1	0	1	0	1	1	1	6
19	1	1	1	1	1	1	1	1	8
20	0	0	0	1	1	1	1	0	4
21	1	0	0	1	1	1	1	1	6
22	1	1	1	1	1	1	0	0	6
24	1	0	0	1	1	1	0	0	4
25	1	1	0	1	1	1	1	1	7
	90%	55%	50%	80%	60%	95%	80%	60%	71%
	Diagram	73%							
	G, D, P, T	75%							
	Clarity	67%							

Table 8. Product of Chords Equality Final Draft

Number	Diagram	Aux lines x2	Given	1 angle	2 angle	Similar	poroportion	cross prod	Total
1	1	1	0	1	1	1	1	1	7
2	1	0	1	1	1	1	1	1	7
3	1	0	0	1	1	1	1	1	6
5	1	1	1	1	1	1	1	1	8
7	1	1	0	1	0	1	1	1	6
8	1	1	1	1	0	1	1	1	7
9	1	1	1	1	1	1	1	1	8
10	1	0	1	1	0	1	1	1	6
12	1	1	1	1	0	1	1	1	7
14	1	0	0	1	1	1	1	1	6
15	1	1	1	1	1	1	1	1	8
16	1	1	0	0	0	1	1	1	5
17	1	1	1	1	1	0	1	1	7
18	1	1	0	1	1	1	1	1	7
19	1	1	0	1	1	1	1	1	7
20	1	1	0	1	1	1	0	1	6
21	1	1	1	1	1	1	0	0	6
22	1	1	0	1	1	1	1	1	7
24	1	1	0	1	1	1	0	1	6
25	1	1	0	1	1	1	1	1	7
	100%	80%	45%	95%	75%	95%	85%	95%	84%

	Diagram	90%
	G, D, P, T	75%
	Clarity	88%

Table 9. Pythagorean Theorem Rough Draft

Number	diagram	altitude	Given	angle	angle	sim tri	proportion	cross prod	angle	angle	sim tri	proportion	cross prod	Add	seg add	subst	Total
1	1	1	0	0	0	0	1	1	0	0	0	1	1	1	1	1	9
5	1	1	1	0	0	0	1	1	0	0	0	1	1	1	1	1	10
6	1	1	1	0	0	0	1	1	0	0	0	1	1	1	1	1	10
7	1	1	0	0	0	0	1	1	0	0	0	1	1	1	1	1	9
8	1	1	1	0	0	0	1	1	0	0	0	1	1	1	1	1	10
9	1	1	1	0	0	0	1	1	0	0	0	1	1	1	1	1	10
13	1	1	0	0	0	0	1	1	0	0	0	1	1	1	1	1	9
15	1	1	0	1	0	1	1	1	1	0	0	1	1	1	1	1	12
17	1	1	1	0	0	0	1	1	0	0	0	1	1	1	1	1	10
18	1	1	0	0	0	0	1	1	0	0	0	1	1	1	1	1	9
19	1	1	0	0	0	0	1	1	0	0	0	1	1	1	1	1	9
20	0	0	0	0	0	0	1	1	0	0	0	1	1	1	1	1	7
21	1	1	1	0	0	0	1	1	0	0	0	1	1	1	1	1	10
22	1	1	0	0	0	0	1	1	0	0	0	1	1	1	1	1	9
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	93%	93%	43%	7%	0%	7%	100%	100%	7%	0%	0%	100%	100%	100%	100%	100%	59%
	Diagram	**93%**															
	G, D, P, T	**58%**															
	Clarity	**52%**															

Table 10. Pythagorean Theorem Final Draft

Number	diagram	altitude	Given	angle	angle	sim tri	proportion	cross prod	angle	angle	sim tri	proportion	cross prod	Add	seg add	subst	Total
1	1	1	0	0	0	0	1	1	0	0	0	1	1	1	1	1	9
5	1	1	1	0	0	0	1	1	0	0	0	1	1	1	1	1	10
6	1	1	1	0	0	1	1	1	0	0	1	1	1	1	1	1	12
7	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	15
8	1	1	1	0	0	0	1	1	0	0	0	1	1	1	1	1	10
9	1	1	1	0	0	1	1	1	0	0	1	1	1	1	1	1	12
13	1	1	1	0	0	1	1	1	0	0	1	1	1	1	1	1	12
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	16
17	1	1	1	0	0	1	1	1	0	0	1	1	1	1	1	1	12
18	1	1	0	0	0	0	1	1	0	0	0	1	1	1	1	1	9
19	1	1	0	0	0	0	1	1	0	0	0	1	1	1	1	1	9
20	1	1	0	0	0	0	1	1	0	0	0	1	1	1	1	1	9
21	1	1	1	0	0	0	1	1	0	0	0	1	1	1	1	0	9
22	1	1	0	0	0	0	1	1	0	0	0	1	1	1	1	1	9
23	1	1	0	0	0	0	1	1	0	0	0	1	1	1	1	1	9
24	1	1	0	0	0	0	1	1	0	0	0	1	1	1	1	1	9
	100%	100%	50%	13%	13%	38%	100%	100%	13%	13%	38%	100%	100%	100%	100%	94%	67%
	Diagram	**100%**															
	G, D, P, T	**71%**															
	Clarity	**56%**															

APPENDIX C: IRB APPROVAL

Application for Exemption from Institutional Oversight

LSU
Institutional Review Board
Dr. Robert Mathews, Chair
131 David Boyd Hall
Baton Rouge, LA 70803
P: 225.578.8692
F: 225.578.5983
irb@lsu.edu
lsu.edu/irb

Unless qualified as meeting the specific criteria for exemption from Institutional Review Board (IRB) oversight, ALL LSU research/ projects using living humans as subjects, or samples, or data obtained from humans, directly or indirectly, with or without their consent, must be approved or exempted in advance by the LSU IRB. This Form helps the PI determine if a project may be exempted, and is used to request an exemption.

– Applicant, Please fill out the application in its entirety and include the completed application as well as parts A-F, listed below, when submitting to the IRB. Once the application is completed, please the completed application to the IRB Office or to a member of the Human Subjects Screening Committee. Members of this committee can be found at http://research.lsu.edu/CompliancePoliciesProcedures/InstitutionalReviewBoard%28IRB%29/item24737.html

– A Complete Application Includes All of the Following:
- **(A)** A copy of this completed form and a copy of parts B thru F.
- **(B)** A brief project description (adequate to evaluate risks to subjects and to explain your responses to Parts 1&2)
- **(C)** Copies of all instruments to be used.
  - *If this proposal is part of a grant proposal, include a copy of the proposal and all recruitment material.
- **(D)** The consent form that you will use in the study (see part 3 for more information.)
- **(E)** Certificate of Completion of Human Subjects Protection Training for all personnel involved in the project, including students who are involved with testing or handling data, unless already on file with the IRB. Training link: (http://phrp.nihtraining.com/users/login.php)
- **(F)** IRB Security of Data Agreement: (http://research.lsu.edu/files/item26774.pdf)

1) Principal Investigator: Benjamin Hargrave **Rank:** Teacher

Dept: Natural Sciences **Ph:** (337) 257-1769 **E-mail:** hargrave.benjamin@gmail.com

IRB# E8317 LSU Proposal # ____
- Complete Application
- Human Subjects Training
- IRB Security of Data Agreement

2) Co Investigator(s): please include department, rank, phone and e-mail for each
*If student, please identify and name supervising professor in this space

Dr. James Madden

3) Project Title: Classical Proof Writing in a Modern Geometry Classroom

Study Exempted By:
Dr. Robert C. Mathews, Chairman
Institutional Review Board
Louisiana State University
203 B-1 David Boyd Hall
225-578-8692 | www.lsu.edu/irb
Exemption Expires: 6/6/2016

4) Proposal? (yes or no) No **If Yes, LSU Proposal Number** ____

Also, if YES, either
- **This application completely matches the scope of work in the grant**

OR
- **More IRB Applications will be filed later**

5) Subject pool (e.g. Psychology students) children < 18

*Circle any **"vulnerable populations" to be used:** (children <18; the mentally impaired, pregnant women, the ages, other). Projects with incarcerated persons cannot be exempted.

6) PI Signature B. H[illegible] **Date** 5/29/13 (no per signatures)

**** I certify my responses are accurate and complete.** If the project scope or design is later changes, I will resubmit for review. I will obtain written approval from the Authorized Representative of all non-LSU institutions in which the study is conducted. I also understand that it is my responsibility to maintain copies of all consent forms at LSU for three years after completion of the study. If I leave LSU before that time the consent forms should be preserved in the Departmental Office.

Screening Committee Action: Exempted ✓ Not Exempted ____ Category/Paragraph 1

Signed Consent Waived?: Yes / No

Reviewer Mathews **Signature** Robert C Mathews **Date** 6/7/13

Parental Permission Form

Project Title: Classical Proof Writing in Modern Geometry Classroom

Performance Site: Baton Rouge Magnet High School

Investigators: The following investigator is available for questions,

M-F (7:05am – 2:45 pm)

Benjamin Hargrave

High School Mathematics Teacher

Bhargrave1@ebrschools.org

Purpose of the study: The purpose of this research project is to investigate and highlight the difficulties and successes students have when writing Geometric Proof. This study also seeks to develop effective strategies and methods for educators to apply when teaching Geometric Proof Writing.

Inclusion Criteria: Students enrolled in High School Geometry classes.

Description of the study: Over a period of the school year, the investigator will assign, collect, and study students' proof writing. The assignments will be assigned as individual work, small group, project, or a mixing of different techniques. This information will be used to adjust lessons to improve student understanding.

Benefits: Research shows that if students develop the ability to explain mathematical properties through proof writing they will develop a deeper understanding of the mathematics, be able to organize their thoughts more clearly, and learn the skills of logic to demonstrate truth.

Risks: There are no known risks

Right to Refuse: Participation is voluntary. A child will become part of the study only if both child and parent agree to the child's participation. At any time, either the subject may withdraw from the study or the subject's parent may withdraw the subject from the study without penalty or loss of any benefit to which they might otherwise be entitled. Students participating will not be asked to do anything outside of normal class procedures; their work will simply not be included in the data recorded.

Privacy: Investigator may review the school records of participants in this study. Results of the study may be published, but no names or identifying information will be included for publication. Subject identity will remain confidential unless law requires disclosure.

Financial Information: There is no cost for participation in the study, nor is there

any compensation to the subjects for participation.

Signatures:

The study has been discussed with me and all my questions have been answered. I may direct additional questions regarding study specifics to the investigator. If I have questions about subjects' rights or other concerns, I can contact Robert C. Mathews, Chairman, Institutional Review Board, (225) 578-8692, irb@lsu.edu, www.lsu.edu/irb. I will allow my child to participate in the study described above and acknowledge the investigator's obligation to provide me with a signed copy of this consent form.

Parent's Signature:__________________________

Date:________________

The parent/guardian has indicated to me that he/she is unable to read. I certify that I have read this consent from to the parent/guardian and explained that by completing the signature line above he/she has given permission for the child to participate in the study.

Signature of Reader:__________________________

Date:________________

Study Exempted By:
Dr. Robert C. Mathews, Chairman
Institutional Review Board
Louisiana State University
203 B-1 David Boyd Hall
225-578-8692 | www.lsu.edu/irb
Exemption Expires: 6/6/2016

Investigator: B. Hargrave
Project: Classical Proof Writing in a Modern Geometry Classroom

Child Assent Form

I, ________________________________, agree to be in a study that is focused on classical proof writing in a modern Geometry classroom. As a participant in the study, I agree to have the project investigator (Mr. Hargrave) refer to the following areas in a written report at the conclusion of the study:

a) My performance on formal assessments (pre- and post-writing)
b) My thoughts reflected in writing assignments
c) My performance and comments on mini-projects, teacher created tasks, and surveys
d) My comments shared in whole class and small-group/peer discussions

I understand my workload and participation in class will not increase or decrease as a participant in the study and my math class will operate in a manner I am accustomed. I can decline participation in the study at any time without being penalized.

Child's Signature: ____________________________ Age: ______

Date: __________________

Witness* ________________________________ Date: __________________

* (Witness must be present for the assent process, not just the signature by the minor.)

Study Exempted By:
Dr. Robert C. Mathews, Chairman
Institutional Review Board
Louisiana State University
203 B-1 David Boyd Hall
225-578-8692 | www.lsu.edu/irb
Exemption Expires: 6/6/2016

VITA

Benjamin Hargrave was born in Lafayette, LA to Deborah and Hubert Hargrave. He graduated with a BA in Education and a Minor in Mathematics from Louisiana State University in 2006 and has been a math educator in East Baton Rouge Parish ever since. He has previously taught at McKinley High School and Robert E. Lee High School and is currently a teacher at Baton Rouge Magnet High School.