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Evaluating the Influence of Moisture Variation on Resilient Modulus for Unsaturated Pavement Subgrades

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EVALUATING THE INFLUENCE OF MOISTURE VARIATION ON RESILIENT MODULUS FOR UNSATURATED PAVEMENT SUBGRADES

A Thesis

Submitted to the Graduate Faculty of the Louisiana State University Agricultural & Mechanical College in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering

in

The Department of Civil & Environmental Engineering

by

Ayan Mehrotra
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ABSTRACT

The cost of repairing pavements that fail before the end of their service life is enormous in the United States and it is continuously rising. Premature pavement failure is often associated with loss of support in the subgrade layer, especially in regions with fine-grained subgrade soils. Resilient modulus ($M_r$) is analogous to the stiffness of a subgrade and a key design parameter for pavements. Due to seasonal variation, $M_r$ varies periodically due to changes in moisture content. However, $M_r$ depends not only on moisture content but also the stress state of the subgrade, which for unsaturated soils is dependent on the matric suction. This study attempts to reinforce the relationship between $M_r$ and matric suction by conducting Repeated Load Triaxial testing on four (4) different soil types and evaluating the Soil Water Characteristic Curves (SWCC) for each soil type. The SWCC curves are evaluated through the entire range of saturation by combining the axis-translation and chilled mirror hygrometer techniques. It is evident that $M_r$ depends on matric suction, which also varies with moisture content, thereby, a $M_r$-matric suction relationship provides a sound theoretical framework to account for moisture variation in unsaturated subgrade soils. A modified constitutive $M_r$-matric suction relationship is proposed to accurately capture the stress state of the soil, and account for the variation in contribution of matric suction to $M_r$. For practical purposes, other methods to incorporate the effect of moisture variation on $M_r$ values are also evaluated. The variation in $M_r$ was captured in terms of changes in degree of saturation. Statistical regression models, with the ability to incorporate moisture variation, are proposed to predict regression constants, $k_1$, $k_2$, and $k_3$, for $M_r$ constitutive models.
CHAPTER 1 INTRODUCTION

1.1 Background

Pavement design has evolved over the years in terms of how the support provided by different pavement layers is quantified. Initially, the modulus of subgrade reaction (k) was utilized to quantify the support provided by the subgrade. However, K was obtained under static loading conditions, which does not adequately represent the cyclic loading experienced by a pavement due to vehicular traffic load. The resilient Modulus (Mr), which is the measure of the stiffness of a material, represents a fundamental material property, which is especially important for pavement design since it serves as a key input parameter for the Mechanistic Empirical Pavement Design Guide (MEPDG). The Mr value is analogous to the elastic stiffness of a material under cyclic loading. The Mr value for unbound base and subgrade materials depends on several parameters such as density, moisture content and stress state. Pavement design involves determining the materials and thicknesses of the various layers involved (e.g. asphalt, base, subbase, etc.), in which the Mr values play an important role in determining the thicknesses of different layers. Design Engineers will generally use a design Mr value that represents either the as-compacted Mr value or the in-situ Mr value if the subgrade is not prepared during construction. However, during the service life of a pavement, the Mr value is not constant and fluctuates with seasonal variations. The moisture content may increase during the rainy season and decrease during a dry season, which will affect the Mr value of the subgrade layer.

The moisture condition of a subgrade can be defined utilizing several different approaches: gravimetric water content, degree of saturation, and matric suction. The
MEPDG introduced an adjustment factor, $F_u$, as part of its’ Enhanced Integrated Climatic Model (EICM) to adjust the $M_r$ value of subgrade from optimum conditions to a desired condition based on changes in degree of saturation. Pavement design Engineers can utilize this method to obtain a design value of $M_r$ that accounts for expected changes in moisture content due to seasonal variation. Designing the pavement utilizing a singular $M_r$ value without taking into account the effect of seasonal moisture fluctuations during the service life of the pavement can lead to un-conservative design and significant loss of subgrade support for the pavement. Therefore, it is essential to incorporate the variations in $M_r$ value for subgrade soils due to seasonal changes in the design of pavements. However, it is also known that $M_r$ is also dependent on the stress state of the soil, which cannot be captured by the degree of saturation. For unsaturated soils, the stress state of the soil implies the effects of matric suction. Since matric suction varies with moisture content, it is possible to incorporate the effects of seasonal variation on $M_r$ by incorporating matric suction in a $M_r$ prediction model. This provides a sound theoretical approach since the effective stress in unsaturated soils is also dependent on the matric suction, and it has long been shown that the effective stress controls the stress and deformation characteristics of soils.

1.2 Problem Statement

The support provided by subgrade layers is critical to maintaining a serviceable pavement throughout its’ service life. The support provided by the subgrade layer for the pavement is generally quantified in terms of resilient modulus ($M_r$). The $M_r$ of a subgrade is dependent upon several different factors that need to be considered during pavement design to provide an accurate assessment of the support provide by the
subgrade, and consequently other pavement layers can be designed accordingly. In places where fine-grained subgrades, especially those with high plasticity index (PI), are prevalent, such as in southern Louisiana presents a unique challenge because to design Engineers. This is due to the Mr value of fine-grained subgrade is highly dependent on the moisture content. The moisture content usually varies throughout the service life of a pavement due to seasonal variation. An increase in the moisture content, past the optimum conditions, can be detrimental to fine-grained subgrade soils. If the pavement design fails to account for changes in the Mr value due to moisture fluctuations, it could lead to a decrease in the service life of a pavement.

Highway subgrade soils, because of their shallow depths, are generally under unsaturated conditions; i.e., the Groundwater Table depth is below the depth of the subgrade in consideration for pavement design. In traditional soil mechanics, the soils are mostly assumed to be under saturated conditions. This means, that the soil mass under consideration consists of two phases, solids (soil particles) and water. This assumption is generally acceptable because it leads to a conservative geotechnical design and makes possible to develop simple analytical solutions that lay the foundation for geotechnical engineering. However, since subgrade soils for pavements exist largely in an unsaturated state, it is important to employ the fundamentals of unsaturated soil mechanics to explore soil strength, and deformation characteristics in subgrades.

In unsaturated soil mechanics, an important parameter which plays an integral role in the strength and deformation characteristic of unsaturated soils is the matric suction. The matric suction is defined as the difference between the air pressure and
the pore water pressure inside the pores of an unsaturated soil medium. This relationship is presented in Equation 1.1.

\[ \Psi = U_a - U_w \]  

(1.1)

Where:

- \( U_a \): Pressure of gas (usually air) inside soil pore
- \( U_w \): Pore water pressure
- \( \Psi \): Matric suction

The matric suction has a unique relationship with water content for each soil type, which is usually determined by the Soil Water Characteristic Curve (SWCC). Considering, the seasonal affect on moisture variation of highway subgrades and the subsequent affects on resilient modulus and matric suction, it is useful to explore a relationship between the matric suction and resilient modulus for subgrade soils. This is the main objective of this research project. Previous research has shown that matric suction generally decreases with increasing moisture content in a soil. This could also explain why the resilient modulus decreases with increasing moisture content in an unsaturated soil. By evaluating the relationship between the resilient modulus and matric suction for unsaturated highway subgrade soils, it will be possible to predict the changes that would occur in the \( M_r \) values of a subgrade during the service life of a pavement due to seasonal variations. Currently, the MEPDG utilized the EICM to consider the changes in the water content of the subgrade during the design life of a pavement. The EICM incorporates an adjustment factor, \( F_U \), to predict \( M_r \) for unfrozen unbound materials based on variations in moisture content while utilizing the \( M_r \) value at optimum conditions (\( M_{rOPT} \)). The adjustment factor, \( F_U \), however, does not incorporate matric suction. Utilizing matric suction, which has a direct impact on the stress state of
unsaturated soils, better predictions of changes in Mr due to changes in moisture content may be possible.

1.3 Objective of the Study

The underlying theme of this study is to evaluate the impact of changes in moisture conditions of the subgrade soil have on the Mr value. This study focuses on exploring this concept by utilizing the principles from unsaturated soil mechanics since subgrades generally exist above the groundwater table.

The following points identify the key objectives of this study:

- Establish the relationship between Mr and stress state defined by cyclic stress and confining pressure for four different soil types, representative of common existing subgrade soils in southern Louisiana.
- Evaluate the impact of as-compacted moisture content on the Mr values for the soils tested in this study.
- Determine the Soil Water Characteristic Curves for the four soil types to establish the relationship between water content and matric suction.
- Evaluate the matric suction values for Mr test specimens through correlation with the degree of saturation.
- Explore the Mr-moisture content relationship with respect to matric suction, and evaluate the impact of matric suction on Mr values.
- Compare the measured matric suction – Mr results with available Mr constitutive models that incorporate matric suction in evaluating Mr values.
- Propose/modify a constitutive relationship to best capture the Mr-matric suction relationship for the four different soil types tested.
Correlate, utilizing statistical regression models, regression constants (e.g. $k_1$, $k_2$, and $k_3$) for the $M_r$ constitutive relationships with typical soil physical properties, thereby allowing Engineers’ a method to obtain regression constants without performing Repeated Load Triaxial tests to obtain $M_r$ values at different moisture contents.

1.4 Scope of Study

The scope of this study encompassed two different stages, a laboratory testing stage and a data/regression analysis stage. The laboratory testing stage involved soil characterization, Repeated Load Triaxial $M_r$ testing, and measuring SWCC curves. The data analysis stage focused on evaluating the link between the moisture variation and $M_r$ values in terms of gravimetric water content, degree of saturation, and matric suction. The data analysis was utilized to develop a modified $M_r$ constitutive relationship that incorporates matric suction to predict $M_r$ for unsaturated subgrade soils. Also, statistical analysis was performed to link the regression constants ($k_1$, $k_2$, and $k_3$) from constitutive models to soil physical properties.

The laboratory testing stage involved performing Repeated Load Triaxial tests to evaluate $M_r$ values, at different states, for the four soil types tested in this study. The specimens for each soil type were tested at various as-compacted moisture contents to obtain $M_r$ values. SWCC curves were determined for each soil type by measuring matric suction utilizing the axis-translation and chilled mirror hygrometer techniques. Tube Suction Tests were also performed to assess the moisture susceptibility of the four soils tested.
Data analysis involved comparing the Mr values for the different soil types at a specified stress state for various moisture contents. The effect of moisture variation on Mr values was evaluated in terms of matric suction. Non-linear regression analysis was performed on measured Mr data to obtain regression constants for Mr constitutive models (e.g., k₁, k₂ and k₃). Statistical Analysis Software (SAS) was utilized to conduct model selection and multiple regression analysis to evaluate statistical models that can predict regression constants (from Mr) constitutive relationships based on typical soil physical properties. The measured data was also utilized to develop a modified Mr-matric suction constitutive model that is capable of capturing the effect of moisture variation on Mr in terms of matric suction, while creating an explicit link between the SWCC and Mr values.

1.5 Outline

This thesis is presented in a six chapter format, beginning with an introduction chapter followed by chapters presenting the literature review, methodology, results, analysis of results, and conclusions. The introduction chapter provides an overview of the study, while stating the problem the study attempts to solve and introduces the steps taken to solve stated problem.

Following the introduction the methodology taken to conduct the study, specifically the laboratory testing is presented. Subsequently, results from the laboratory testing program are presented. The presented laboratory data is then analyzed and various relationship are discussed and evaluated. Based on the analysis of the data obtained from this study, certain conclusion are formulated and presented.
CHAPTER 2 LITERATURE REVIEW

2.1 Unsaturated Subgrades and Seasonal Variation

A pavement structure generally consists of several different layers, which act collectively to support the applied vehicular loads while maintaining certain serviceability criteria. The subgrade is the native material underneath a constructed pavement. It may be overlain by different layers such as a base, subbase, and/or asphalt. Due to the relatively shallow influence depth for the loads carried by roadways, the subgrade of interest in pavement engineering is usually present in an unsaturated state condition, i.e. located above the depth of groundwater table. As it can be seen in Figure 2.1, the near surface unsaturated soil are considered to be in the Active Zone, where they undergo periodic moisture fluctuations caused by seasonal variations.

Figure 2.1: Seasonal phenomena in the near surface deposit of an unsaturated soil.
Previous studies have been conducted to monitor the moisture changes beneath pavements. As expected, it was demonstrated that the changes in moisture conditions usually correspond to the seasonal changes. Figure 2.2 presents results from a study in Iowa (White, 2008) where Time Domain Reflectometer (TDR) probes were installed beneath the pavement, in the subgrade, to measure the volumetric water content. As seen in the figure, fluctuation of the water content can be observed over time.

![Figure 2.2: Volumetric Moisture Content measurements obtained from TDR probes (1.1 and 1.3 ft below pavement surface) (White, 2008).](image)

Since the subgrade soils are usually under unsaturated condition, the fluctuation of water content is expected to impact other soil properties, mainly soil suction. Therefore it can be expected that the soil suction also varies seasonally in unsaturated subgrades. Nguyen et al. (2010) conducted a study to evaluate the seasonal pattern of suction in subgrade soils beneath pavement. Figure 2.3 shows results from a sensor installed directly beneath the centerline of the pavement to measure suction. It can be seen from the figure not only that the suction varies through the course of the year but
also varies with the same pattern over a period of 5 years, which is evidence of a specific seasonal pattern.

Figure 2.3: Matric suction at sensor T1-5 for years 2001 – 2005 (Nguyen et al. 2010)

2.2 Unsaturated Soil Mechanics

2.2.1 Stress in the Unsaturated State

To better understand the effect of moisture variation on unsaturated subgrades, it is important first to understand the fundamentals of unsaturated soil mechanics and the differences between saturated state and unsaturated state soil behavior. The main difference is the existence of three phases within unsaturated soils: air, water, and soil particles. Saturated soil only consists of two (2) phases: water and soil particles. The state of stress in saturated soil can be defined by effective stress, total stress and pore water pressure. In unsaturated soils the contribution of pore pressure to total stress is not always 100%, depending on the degree of saturation, making the effective stress analysis in unsaturated soil complicated. Bishop (1959) presented an effective stress equation for unsaturated soils, which takes into account the pore-air pressure and the degree of saturation dependent contribution pore-water pressure; this relationship is presented in Equation 2.1
\[ \sigma_v' = (\sigma_z - U_a) + \chi (U_a - U_w) \]  \hspace{1cm} (2.1)

Where:

\( \sigma_v' \) – Effective Stress
\( \sigma_z \) – Total Stress
\( U_a \) – Pore-air pressure
\( U_w \) – Pore-water pressure
\( \chi \) – Bishop’s effective stress parameter; 0 for dry soils and 1 for saturated soils

Generally, the pore-air pressure is assumed to be at atmospheric conditions (gauge pressure = 0) in the field. Another unique property of unsaturated soils is that the pore-water pressure in unsaturated soils has a negative value. It can be seen that the effective stress for unsaturated soils increase as the pore-water pressure becomes negative.

2.2.2 Capillarity

To gain a better understanding of the physical phenomenon of how the negative pore-water pressure (PWP) increases the effective stress in unsaturated soils, it is important to understand the capillary forces. Using capillary tubes to describe the phenomenon in unsaturated soils is a useful tool since the surface tension due to air-water-soil interface result in negative pore water pressure, which leads to the redistribution of water in a capillary tube or unsaturated soil. Capillary rise in soil describes the upward movement of water from the water table due to the presence of a pressure gradient between air-water interface.
Figure 2.4 presents a mechanical equilibrium diagram for capillary rise in a small diameter tube $u_a - u_w$ acting over the area of the meniscus, and the vertical projection of $T_s$ acting over the circumference of the meniscus leads to the relationship given in Equation 2.2.

$$(U_a - U_w) \frac{\pi}{4} d^2 = T_s \pi d \cos \alpha$$

(2.2)

Where:

- $U_a = \text{Pore Air pressure}$
- $U_w = \text{Pore Water pressure}$
- $d = \text{diameter of tube}$
- $T_s = \text{Surface tension}$
- $\alpha = \text{contact angle}$

From the above equation, it is evident that the term $u_a - u_w$, matric suction, is a function of the tube diameter, or the size of the pore(s) in a soil, and the smaller the diameter, the larger the matric suction value will be. This concept is important when we discuss the suction values achieved by certain soil types.

Figure 2.4: Capillary rise in a small tube (Lu and Likos, 2004)
The effect of negative PWP and surface tension give rise to suction stress in unsaturated soils. Suction stress refers to the net interparticle force generated within a matrix of unsaturated soil particles due to the combined effects of negative pore water pressure and surface tension, which occur at the pore water-air-soil grain interface. The suction stress tends to pull soil particles towards each other.

2.2.3 Suction

Suction is a fundamental property of unsaturated soils. It is usually divided into two parts, osmotic suction and matric suction. The 'Total Suction', which can be considered as the potential energy of the water in the soil, describes the difference between the thermodynamic potential of the soil pore water compared to free water. The thermodynamic potential of the soil pore water is mainly reduced by capillarity effects, short-range adsorption, and effect of dissolved salts. Matric suction can be attributed to capillarity and short-range adsorption effects; while osmotic suction can be attributed to the effect of dissolved salts. The osmotic suction is only present in marine and leached soils. The effects of short-range adsorption are only prominent at low water contents when the adsorbed water is mainly in the form of thin films around the soil particles.

The potential energy of the soil pore water is the algebraic sum of the different potentials and can be represented by the following equation (Lu and Likos, 2004):

\[ \Delta \mu_t = \Delta \mu_c + \Delta \mu_o + \Delta \mu_e + \Delta \mu_f \]  

(2.3)

Where the 1st term represents the change due to curvature of the air-water interface (capillarity), the 2nd term is the change due to osmotic effects (dissolved solutes), the 3rd term is due to the electrical field, and the 4th term is due to van der Waals forces.
Excluding the effects of dissolved solutes, all the other terms added up represent the
matric suction. While all the terms represent a negative value, the value of matric
suction is positive because it represents the change in potential from the free water
state. Matric potential is often the largest contributor to the potential of pore water in
unsaturated soils (Nam et al., 2009). Matric suction is generally defined by the value \((u_a - u_w)\), which is a term in Equation 2.1. It represents the magnitude of negative pore
water pressure in unsaturated soils.

Since the total suction is related to the thermodynamic potential of the soil pore
water, using this principal, a relationship was developed (Fredlund and Rahardjo, 1993)
to represent the total suction as a function of the partial vapor pressure of the pore
water.

\[
\psi_T = -\frac{RT}{V_0 * \omega_v} \ln\left(\frac{U_v}{U_{v0}}\right)
\]

Equation (2.4)

Where:

- \(R = \) Universal gas constant \((\text{J/mol K})\),
- \(T = \) absolute temperature \((\text{K})\),
- \(V_0 = \) Specific
  volume of water \((\text{m}^3/\text{kg})\),
- \(\omega_v = \) molecular mass of water vapor \((\text{g/mol})\),
- \(U_v = \) partial
  pressure of pore-water vapor \((\text{kPa})\), and
- \(U_{v0} = \) saturation pressure of water \((\text{kPa})\).

It should be noted that the term \((U_v / U_{v0})\) represents a measure of the relative humidity
\((\text{RH})\). Therefore it is possible to obtain a measure of the total suction by measuring the
relative humidity of pore water vapor.

2.2.4 Relationship between suction and water content

The reduction of thermodynamic potential of the pore water is related to the
amount of pore water present in the soil, therefore a relationship does exist between soil
suction and water content of the soil. This relationship is described by the Soil Water
Characteristic Curve (SWCC). The SWCC consists of different ranges of suction, where the main water holding mechanism is different, and differs by soil type.

Figure 2.5 illustrates a generalized SWCC, which is separated into three (3) different ‘regimes’. It can be seen that when the soil is near saturation, the main mechanism causing the decrease in thermodynamic potential of the pore water is the capillary force. From there, the SWCC transitions into the ‘Adsorbed Film’ regime, in which the water is retained in thin films due to electrical field polarization, van der Waals attraction, and exchangeable cation hydration. In the 'Tightly Adsorbed' regime, the water is retained by molecular bonding, specifically hydrogen bonding with oxygen or hydroxyls on the surface of the soil particles. In the latter two regimes, the water content is low and suction values are high, and the water mainly exists in thin films around the soil particles. At this stage, the high suction values are mainly due to short range adsorption effects, which are controlled by the properties of the surface of the soil solids. In the capillary stage, the water content is high and suction is mainly controlled by the curved interface of the air-water-soil interaction. This mechanism is mainly governed by the pore-size distribution. The transition between the high suction and low suction range is different for each soil type. In soils such as sands and silts, where the soil particles have little to no surface charge properties, the SWCC is dominated by the capillary regime. In clays, where the particles have significant surface charges and interactions, the SWCC is dominated by the surface adsorbed forces, while the capillary represents a small portion of the SWCC. It should also be noted that the ‘Air-entry suction’ is the suction value at which air starts to enter the largest pores in the soil and the soil begins to de-saturate.
Figure 2.5: Conceptual model for general behavior of SWCC (McQueen and Miller, 1974)

Figure 2.6 illustrates the generalized SWCCs for three (3) different soil types. The differences amongst the SWCCs for the different soil types (sand, silt, and clay) are evident in this figure. For sand, the SWCC is dominated by the capillary suction region, while the tightly adsorbed and adsorbed film regimes are limited for sandy soil. This is due to the fact that sandy soils lack surface charge properties and have a relatively small specific surface area. Silty soils also lack surface charge properties, however, they have large specific surface areas and this allows them to adsorb a significant amount of water. It can be seen that the capillary regime is limited for clay soils, and their SWCCs are dominated by the tightly adsorbed and adsorbed film regime. This is expected since clay soils have significant surface charge properties, cation hydration, and large specific surface areas. As far as air-entry values go, sandy soils have the lowest air-entry value (AEV), while clay soils have the largest. Since AEV signifies air
entering the largest pores in the soil, they are largely a function of the pore size
distribution so it is not surprising that sandy soils, which have the largest pores, have
the lowest air-entry values.

Figure 2.6: Representative SWCC's for sand, silt, and clay (Lu and Likos, 2004)

2.2.5 Hysteric behavior of Soil Water Characteristic Curves

The SWCCs that have been depicted so far, illustrate only drying curves, in
which a soil is initially saturated and begins to de-saturate as suction values increase
past the AEV. However, under field conditions, soils undergo wetting and drying cycles.
Under the wetting cycle, soils imbibe water (increase in water content) as suction values
decrease. However, the SWCC for the drying and wetting cycles are unique. At the
same suction value the soil can have two different water contents depending on
whether it is undergoing drying or wetting. This is important to consider for highway
subgrades, since they undergo cyclical drying and wetting due to seasonal variation.
Figure 2.7 represents a conceptual visualization of hysteresis behavior in a SWCC (Lu and Likos, 2004). It is evident that there is no unique equilibrium between moisture contents, and soils undergoing drying tend to retain a larger amount of water, at the same suction value, than soils undergoing wetting. From Figure 2.7, one can notice that the hysteresis effect is most pronounced in the region of rapid de-saturation which coincides with the capillary regime, while at the higher suction range, the effect of hysteresis is less pronounced. Also, a soil undergoing wetting may never reach full saturation due to the presence of entrapped air bubbles (Pham, 2005). There are several different reasons why the hysteresis effect exists for SWCCs. The main reasons include: changes in the geometry of the pore-size distribution or ink-bottle effect (Lu and Likos, 2004), variation in contact angle between air-water-soil interface for advancing versus receding meniscus, and presence of entrapped air during wetting. Hysteresis is mainly present in the capillary regime, which explains the importance of the ink-bottle effect.

Figure 2.7: Conceptual visualization for hysteresis in a SWCC (Lu and Likos, 2004)
The ink-bottle effect and the variation in contact angle due to drying/wetting cycles are usually identified as the primary reasons for hysteric behavior. The ink-bottle effect can best be explained by Figure 2.8. As drying progresses, the matric suction is described by the meniscus formed in Part a. The magnitude of matric suction is a function of the radius of curvature of the meniscus, which is determined by the size of the pore throat. Wetting is shown in Part b of Figure 2.8, and similar to drying, the magnitude of matric suction is determined by the meniscus formed at the air-water-soil interface/size of the pore throat. Recalling Equation 2.2, the matric suction is inversely proportional to pore size diameter, i.e. matric suction increase with decreasing pore size diameter. Since the meniscus is more severe, i.e. radius of curvature is smaller, during the drying process, a larger suction value is needed to continue to advance the meniscus and drain the pore. While during the wetting process, Part b, the meniscus is less severe and has a larger radius of curvature, therefore a lower suction value is needed to advance the meniscus past the pore throat and allow the pore to fill with water. During wetting, the lower pore throat tends to keep the rest of the pore from
filling, which results in a lower water content at the same suction as compared to the drying cycle.

Another cause of hysteresis is the difference in contact angle. The contact angle is defined as the angle formed by air-water-soil interface if a water droplet was placed on the soil surface. As evidenced in Figure 2.9, the contact angle for an advancing meniscus (wetting) is larger than the contact for a receding meniscus (drying). Considering the relationship provided in Equation 2.2, a smaller contact angle leads to a larger matric suction value.

![Diagram of drying and wetting contact angles](image)

Figure 2.9: Water droplet on inclined surface illustrating difference between wetting and drying contact angles (Lu and Likos, 2004).

### 2.2.6 Laboratory method for measuring suction

Suction is one of the most difficult soil properties to measure. While devices exist to measure suction in the field and laboratory, devices that measure suction in the field have a very limited range, and laboratory measurement methods are limited to research labs and rarely utilized in engineering practice. However, even with these challenges, it is important to measure suction to understand the behavior of unsaturated soils. There are several different methods to measure suction. However, most of these methods only cover a certain suction range and a combination of multiple methods may be needed to
generate a complete SWCC across the entire suction range for a certain soil type.

Certain methods also measure matric suction and osmotic suction separately, while others only measure the total suction. Figure 2.10 gives a brief snapshot of the different suction measurements available and their applicability.

![Image of suction measurement techniques](image-url)

**Figure 2.10**: Approximate ranges for different suction measurement techniques (Lu and Likos, 2004)

This objective of this study was to measure matric suction in unsaturated soils, therefore we will be focusing on the Axis-Translation Technique and Chilled-Mirror Hygrometer. The Axis-Translation technique is one of the most popular laboratory methods for measuring suction, especially in the range of 0-1500 kPa, and it only measures matric suction. It involves the separation of the air and water phase by utilizing a High-Air Entry (HAE) material. In this method a positive air pressure is applied to the specimen, while the water pressure is maintained at atmospheric conditions.
Since the matric suction is defined by the value \((u_a - u_w)\), the applied air pressure is equal to the applied matric suction. A visual description of this method is shown in Figure 2.11. It can be seen that the HAE material, usually a ceramic disk, allows for the separation for air-water phase. Therefore, as the air-pressure is increased the water can drain out from the specimen till equilibrium is reached. The saturated HAE provides a connection between the water in the soil and that in the reservoir where measurements are made. At equilibrium, the pressure difference across the air-water interface is the same for the air-water interface in the specimen and air-water interface in the HAE material. When the pressure is increased, the water drains out from the soil pores till equilibrium is reached. If the testing device has a water reservoir to collect the water that drains, the water content of the soil specimen can be tracked over a range of suction values. By conducting this process over a series of suction values, and measuring the amount of water released at each suction value, a SWCC can be constructed.

Figure 2.11: Equilibrium position(s) for air-water interface in axis-translation technique (Lu and Likos, 2004)
While the axis-translation technique is quite useful, and provides a direct measurement of the suction-water content relationship, it is limited to a suction range of 0-1500 kPa. This limitation is mainly due to the lack of availability of a HAE material with an air-entry value greater than 1500 kPa. While this range is useful for most non-plastic soils, i.e. sands and silts, where most of the SWCC is located in the capillary regime, it is not sufficient to create a SWCC for medium to high plasticity soils. Therefore, it is important to use either another technique for plastic soils or use a combination of two measurement methods. The filter paper method is a common and inexpensive technique for measuring matric suction over a large range, the range covered by the filter paper method is displayed in Figure 2.10; titled “Contact Filter Paper Method.”

In the filter paper technique, a dry filter paper is placed in contact with a soil specimen and water is transferred from the soil specimen to the filter paper via capillary flow due to a difference in matric suction (Nam, 2009). After a period of equilibration, the water content of the filter paper is measured and then utilizing a calibration curve, a matric suction value can be obtained utilizing the water content. There are some common filter papers that are widely utilized, Whatman No. 42, and the Schleicher and Schuell No. 589 papers, and their calibration curves are presented in ASTM D 5298 (2003). While this technique is appealing due to its’ applicability over a wide range of suction value, and its’ relatively easy and inexpensive setup, it has some drawbacks. One of the drawbacks is the fact that one specimen is needed to generate just one point on the SWCC, specimens have to be prepared at various moisture contents and then allowed to equilibrate with the filter paper to be able to generate an entire SWCC. This makes the process of constructing an entire SWCC quite time consuming. Also, as the
soil specimens become dry (high suction values), it becomes difficult to achieve and maintain good contact between the filter paper and soil specimen, which is essential to obtain an accurate measurement of matric suction. These problems combined with an equilibration period of anywhere between 7-14 days, might warrant a different approach for generating a SWCC.

Another method that can be useful for measuring suction, especially in the high suction range for highly plastic soils, is the chilled-mirror hygrometer technique. This technique is accurate in measuring the total suction in the range of 1 MPa and 450 MPa. This method draws on the relationship provided in equation 2.4, where the total suction can be measured based on the relative humidity of the water vapor from a soil specimen. A popular chilled mirror hygrometer device is the WP4C Dewpoint Potentiometer manufactured by Decagon Devices Inc. The WP4C allows for the measurement of suction by bringing the liquid phase water of the soil specimen in equilibrium with the vapor phase water in a closed chamber inside the device, once equilibrium is achieved, the device can then measure the vapor pressure of the headspace (Decagon Device, 2013). The device has dew point sensor, a fan, a temperature sensor, and infrared thermometer. There is also a small mirror in the chamber where the specimen is placed for measurement. Once the sample is placed inside the chamber it is cooled till condensation forms on the mirror. An infrared laser that is directed at the mirror scatters light once condensation forms on the mirror, at this point the devices measures the temperature (to obtain saturation vapor pressure) and the vapor pressure of the air inside the chamber (by measuring dewpoint temperature). From these measurements, the relative humidity of the sample can be obtained. At
equilibrium the relative humidity of the sample is the same as the relative humidity of the air in the chamber, and the total suction value for the soil can be obtained by measuring the vapor pressure of the air in the chamber.

While the chilled-mirror hygrometer technique is useful due to its ability to measure high suction values and take relatively fast measurements, it is very sensitive to changes in temperature; which could present a drawback if temperature is not adequately controlled and measured. The WP4C has an internal temperature control function that helps combat the problem of close control of temperature. Accurate measurements are observed when the sample is near chamber temperature before being inserted in the device for measurement. Another drawback associate with this technique is the decrease in accuracy when the sample is near saturation, i.e. low suction values. There is a rapid increase in suction as relative humidity decreases at low suction. The WP4C has an accuracy of +/- 0.05 MPa for a range of 0-5 MPa, which means that at a suction of 0.1 MPa, the error could be as large as 50%. Also, an increase in the scatter of data measured below 1000 kPa was observed in previous studies when measuring suction using the chilled-mirror hygrometer technique (Lu and Likos, 2004).

With this in mind, it may be appropriate to use two measurement techniques, one for suctions below 1500 kPa (e.g., axis-translation technique) in combination with the chilled-mirror hygrometer technique to produce complete SWCC’s. Figure 2.12 presents the results reported from a study conducted by Nam (2009). It can be seen that the suction measurements from different techniques can be combined to produce a singular SWCC. Specifically, once can see that the chilled-mirror hygrometer technique
measurements in the high suction range can be combined with measurements from a different technique in the lower suction range (i.e., axis-translation) to create SWCCs for soils that achieve a large magnitude of suction values.

![SWCC graphs](image)

Figure 2.12: SWCC’s for soils using different methods. Open symbols are for matric suction and solid symbols are for total suction (Nam, 2009)

2.2.7 Models for Soil Water Characteristic Curves (SWCC)

While experimental methods are important for measuring suction, it is also important to utilize models to fit analytical functions through experimental results to obtain a continuous SWCC. A good model allows for the construction of a well-defined SWCC, which can be utilized to obtain several important values; air-entry value, saturated water content, and the residual water content. These terms and their location in reference to a SWCC are provided in Figure 2.13.
Figure 2.13: Definition of terms for a typical SWCC (Fredlund and Xing, 1994)

Most of the popular models used to define a SWCC exist in either a 3 parameter or 4 parameter form. Two of the most popular, and widely utilized models, were proposed by Fredlund and Xing (1994), as discussed in Equations 2.5 and 2.6; and van Genuchten (1980), as discussed in Equation 2.7. These models allow for the development of an analytical relationship between the soil suction and the volumetric water content. It is important to note that both of these models were originally derived based on relationships from the pore-size distribution of a soil specimen, hence their sigmoidal shape is similar to the shape of a particle size distribution curve.

\[
\theta_w = \frac{\theta_s}{\left( \ln\left( e + \left( \frac{\psi}{\alpha} \right)^b \right) \right)^c} \quad (2.5)
\]

\[
\theta_w = \left[ 1 - \frac{\ln \left( 1 + \frac{\psi}{\psi_r} \right)}{\ln \left( 1 + \frac{1.000.000}{\psi_r} \right)} \right] \frac{\theta_s}{\left( \ln\left( e + \left( \frac{\psi}{\alpha} \right)^b \right) \right)^c} \quad (2.6)
\]
\[ \theta_w = \theta_r + \frac{\theta_s - \theta_r}{1 + (\frac{\Psi}{a})^b} \]  

(2.7)

Where:

\( \theta_w \) – Volumetric Water content
\( \theta_s \) – Saturated water content
\( \theta_r \) – Residual water content
\( \Psi \) – Matric suction
\( \Psi_r \) – Residual suction

a, b, and c – fitting parameters (Alternatively, b = n; c = m)

Owing to their similar origins, the fitting parameters in the above equation affect the shape of the SWCC in similar ways for the different models. In the three models presented above, the parameter ‘a’ is related to the suction value at the inflection point of the curve (i.e., air-entry value), parameter ‘b’ effects the slope of the curve in the desaturation zone, and the parameter ‘c’ effects the symmetry of the slope of the curve about the inflection point (Leong and Rahardjo, 1997). It is important to note that

Equation(s) 2.6 and 2.7 are four parameter models, having four unknowns; while Equation 2.5 is a three parameter model having just three unknowns. The saturated water content is generally assumed to be a known value, while the residual water content (\( \theta_r \)) is not a known value and is defined as the water content at which a large increase in suction causes only a small decrease in water content. Similarly, the residual suction represents the suction value at which a large increase in suction is needed for a small decrease in water content.

The difference between Equation 2.5 and Equation 2.6 (both proposed by Fredlund and Xing (1994)), lies in the usage of the correction factor, C (\( \Psi \)), which is
present in Equation 2.6. In Equation 2.5, $C(\psi) = 1$. The reason for including the
correction factor in Equation 2.6 is to force $\theta_w$ to 0 at a suction of 1,000,000 kPa. The
relationship for the correction factor is given in Equation 2.8. This suction value is
supported by thermodynamic considerations (Equation 2.4), which indicates that as
Relative Humidity approaches 0 as suction approaches 1,000,000 kPa. However,
forcing $\theta_w$ to 0 at a suction of 1,000,000 kPa has no theoretical basis (Leong and
Rahardjo, 1997). It should be noted that in Equation 2.5, when $C(\Psi) = 1$, $\theta_w$ is not equal
to 0 at a suction of 1,000,000 kPa.

$$C (\psi) = \left[ 1 - \frac{\ln(1+\frac{\psi}{\psi_r})}{\ln(1+\frac{1,000,000}{\psi_r})} \right]$$ (2.8)

Leong and Rahardjo (1997) conducted study examining and comparing some
of the popular models available to represent SWCCs, including those listed above. It
was found that Equation 2.6 gave the best fit to the experimental data. However,
sensitivity analysis tended to favor Equation 2.5 and it was also deemed favorable due
to requiring less computational effort, since it only has 3 parameters. Nam (2009)
compared the 3 parameter version of Equation 2.7 (van Genuchten) with Equation 2.5,
and found that both gave identical fits to experimental data, withstanding some
deviation from each other in the high suction range. The identical results are to be
expected since both models are derived from pore-size distribution functions.

**2.3 Repeated Loading and Deformation Characteristics of Highway Subgrades**

2.3.1 Pavement Design

An important aspect of pavement design is quantifying the support provided by
different pavement layers; i.e surface asphalt layer, base, subbase, and subgrade. The
support provided by the subgrade layer is key in determining the thicknesses of the other layers. The subgrade is generally defined as having an infinite thickness. Initially the support provided by the subgrade layer was quantified in terms of the modulus of subgrade reaction \((k)\) which is defined as the pressure sustained by the soil under a rigid plate at a specified settlement.

Over time, pavement design has evolved in terms of quantifying support provided by pavement layers, including the subgrade, under cyclic loading conditions instead of static loading conditions. Initially, plate load tests under static loading conditions were utilized to determine the modulus of subgrade reaction and quantify the support provided by the subgrade. However, pavements experience loading due to vehicular traffic load. Loading due to vehicular traffic can best be captured in terms of cyclic stresses. Therefore, with the introduction of design methods such as AASHTO 1993 and MEPDG (NCHRP, 2003) there was a move to quantify support provided by the subgrade layer in terms of resilient modulus \((M_r)\). \(M_r\) is calculated under cyclic loading conditions, and better describes the loading conditions due to vehicular traffic as opposed to the modulus of subgrade reaction.

2.3.2 Resilient Modulus

The \(M_r\) is a key property in the pavement design, and an input design parameter especially when the MEPDG pavement design procedures are utilized. \(M_r\) is defined as the ratio of maximum cyclical stress to elastic strain under repeated cyclical loading (AASHTO, 1993).

\[
M_r = \frac{\sigma_{cyc}}{\varepsilon_r}
\]

Equation (2.9)
The Mr is mainly used to quantify the support the pavement receives from the subgrade layer. It was initially introduced as an input parameter in the 1986 AASHTO Guide for Design of Pavement Structures. Its' popularity was preceded by the usage modulus of subgrade reaction. However, since pavement experience cyclical loading (due to moving traffic loads), Mr was thought to better describe the support provided by the subgrade for a pavement. Mr is similar to modulus of elasticity as it is determined based only on elastic deformation. This also leads to Mr being analogous to the stiffness of a subgrade. As can be seen in Figure 2.14, under repeated loading Mr is determined based on the recoverable strain (i.e., elastic strain). However, it should also be noted that as the number of loading cycles increases, there is an accumulation of plastic strain (i.e., non-recoverable deformation). While Mr is based just on the elastic strain, pavements under repeated loading experience both elastic and plastic strain. Plastic deformation manifests itself in a pavement as rutting (permanent deformation) and is an undesirable property as it could lead to a loss in serviceability and/or failure. Generally, the larger the Mr value the better the subgrade soil would be considered. A large Mr value would indicate that the subgrade can handle certain cyclical loading with little deformation (i.e., subgrade is stiff).

Figure 2.14: Graphical Representation of Resilient Modulus (Kim and Kim, 2007)
It has been shown that a decrease in $M_r$ during the service life of a pavement results in increased deflection of the pavement, which can shorten its’ service life. As stated earlier, the $M_r$ is a key input parameter in the MEPDG, which has a significant effect on the design of base course and asphalt layers thicknesses (Darter et al., 1992). Therefore, it is important to evaluate an appropriate $M_r$ value for a pavement subgrade. It is also important to account for the fact that the $M_r$ value of a subgrade is not a constant value. In reality, $M_r$ is continually changing due to the effects of seasonal variation. The AASHTO Guide for the Design of Pavement Structures (1993) recommends using a $M_r$ value dubbed as the ‘effective roadbed soil resilient modulus’ so that the effect of seasonal variation can be properly considered.

2.3.3 Evaluating Resilient Modulus for Subgrade Soils

The MEPDG $M_r$ input values are rated at different levels, ranging from Level 1 to Level 3. With a Level 1 input being the most reliable, obtained from comprehensive laboratory or field tests, and Level 3 being the least reliable, usually estimated by the designer based on previous experience and with little to no testing. Level 2 inputs are estimated through correlation with various material properties obtained from field or laboratory testing. Since $M_r$ is a widely used input parameter for pavement design, and one which is difficult to evaluate in the laboratory due to the need for special equipment that is not widely available outside of research laboratories, several correlations have been developed between $M_r$ and field tests. Several different correlations have been proposed to evaluate $M_r$ values based on results from field testing such as (but not limited to): Dynamic Cone Penetration (DCP) test, Geogauge, and Light Falling Weight
Deflectometer (LFWD). Another widely utilized correlation for $M_r$ is obtained from a laboratory test method, such as California Bearing Ratio (CBR).

While methods of obtaining $M_r$ values for subgrades are acceptable and very useful in engineering practice, in research the widely accepted method for obtaining $M_r$ is direct laboratory testing due to its accuracy and ability to control multiple factors that directly affect $M_r$. Laboratory evaluation of $M_r$ involves conducting a Repeated Loading Test (RLT). The test is generally conducted in a triaxial environment with a cylindrical (disturbed or undisturbed) soil specimen. One of the main advantages of laboratory RLT is the ability to apply multiple stress states to a soil specimen by utilizing a combination of confining and deviatoric stresses. Also, RLT tests directly mimic the repeated loading experienced by pavement subgrades due to vehicular traffic. However, it should be noted that even laboratory RLT cannot duplicate certain loading conditions experienced by subgrade under traffic loading, such as rotation of principal stresses. Currently, the AASHTO T-307 standard is utilized to specify the laboratory testing method for obtaining $M_r$ values via RLT.

The AASHTO T-307 serves as a protocol for direct measurement of $M_r$, resulting in Level I input for MEPDG. The protocol recommends different procedures depending on the type of material (e.g., base, subgrade). For subgrade soils, the samples are tested at three different confining pressures with five different deviatoric stresses at each confining pressure, this results in $M_r$ being evaluated at 15 different stress states. The stress states required, when performing a $M_r$ test on subgrade soils, by AASHTO T-307 will be presented in Chapter 3.
2.4 Factors Effecting Resilient Modulus

2.4.1 Stress State

It has been shown that the resilient modulus ($M_r$) of a subgrade is affected by the stress state experienced by the subgrade. Generally, the stress state of a subgrade is defined by the confining pressure and deviatoric stress experienced by the subgrade. However, it should be noted that for unsaturated subgrades, matric suction is an important factor in defining the stress state of a subgrade (Yang, 2008). Increasing confining pressure serves to increase the $M_r$ of soils, as it increases the bulk stress experienced by the soil, therefore providing a stiffening effect to the specimen. Increasing deviatoric stress, tends to decrease the $M_r$ of soil specimens because it increases the shear stresses experienced by the soil specimen. The effect of confining pressure is more pronounced on granular soils, while the effect of deviatoric stress is more pronounced on cohesive soils. Figure 2.15 illustrates the effect confining pressure and deviatoric stress have on subgrade soil at different moisture contents.

![Figure 2.15: Effect of stress state on resilient modulus of a A-4 soil (Liang et al., 2008)](image-url)
2.4.2 Moisture Conditions

Subgrades for pavements can generally be prepared two different ways, if possible they are compacted at close to optimum moisture content (OMC) and maximum dry density (MDD) but sometimes this is not feasible and pavements are constructed on subgrades under existing in-situ conditions. Either way, post-construction the moisture content of the subgrade comes to equilibrium with its’ surrounding conditions (Yang, 2005; Uzan, 1998) and then varies thereafter due to seasonal variation. Considering that moisture content of a subgrade is not constant, even if the subgrade is prepared at a specified moisture content and density, it is important to realize the impact of moisture content on the Mr value.

Fine-grained subgrades generally experience a decrease in Mr with an increase in moisture content (Drumm, 1997). Subsequently, a decrease in Mr leads to increased deflection of the pavement, which results in a shortening of the service life of the pavement. To study the impact of moisture changes on Mr, laboratory studies have been performed where either the specimens are compacted at varying moisture contents or compacted at one moisture content and then subjected to post-compaction moisture changes. It should be noted, that in the field, subgrades are subjected to post-compaction moisture changes and varying moisture contents at compaction may not accurately simulate the field conditions. This is because the changes in compaction moisture content effects the soil structure (Drumm, 1997). Figures 2.16 and 2.17 serve to display the impact of moisture content on the Mr value.
While, it is well understood that the increase in moisture content results in a decrease in $M_r$, it should also be realized that the effect of changes in moisture content on $M_r$ is different for different soil types. Drumm (1997) found that while A-7 soils tended to have larger $M_r$ values at optimum conditions, compared to A-4 and A-6 soils, they also exhibited a larger decrease in $M_r$ once the moisture content increased to values greater than optimum.

Figure 2.16: Impact of post-compaction moisture increase on $M_r$ (Drumm, 1997)

Figure 2.17: Variation of resilient modulus with moisture content for a sandy soil (Khoury, 2004)
Another route of evaluating the impact of moisture condition on $M_r$ is by considering the relationship between $M_r$ and degree of saturation ($S$) of a soil specimen. While, at first glance, the relationship between $M_r$ and degree of saturation may seem to be similar to that between moisture content and $M_r$, it is important to observe that evaluating the degree of saturation also involves the effect of density. The degree of saturation is dependent on both the moisture content of the soil and its density, and therefore provides a better description of the soil state (Drumm, 1997). Figure 2.18, presents the relationship between degree of saturation and $M_r$ for different soil types. The figure demonstrates that, for different soil types, the $M_r$-moisture content relationships are different.

Figure 2.18: Effect of post-compaction saturation on resilient modulus (Drumm, 1997)
2.4.3 Matric Suction

For unsaturated subgrades, it is imperative to evaluate the impact of suction on $M_r$ since suction is a fundamental property of unsaturated soils that affects the stress state of unsaturated soils. In a field study conducted by Sauer and Monosmith (1968), it was observed that there is a relationship between suction and deflection of a pavement, such that deflection decreased with increasing suction values. As described in Equation 2.1, the matric suction impacts the effective stress for unsaturated soils; and since effective stress controls the strength and deformation characteristics for soils, it is expected that suction will also impact $M_r$ for unsaturated soils. An increase in suction will increase the stiffness of the soil and hence increase $M_r$ of unsaturated soils. As mentioned earlier, soil suction is composed of two components, matric suction and osmotic suction. However, Khoury et al. (2003) demonstrated that the changes in $M_r$ for unsaturated soils are mainly attributed to changes in matric suction.

Knowing that, matric suction is defined as the difference between pore-air pressure and pore-water pressure ($u_a - u_w$). The magnitude of suction present in a soil is related to the moisture content, which is varying in a pavement subgrade over time. It is therefore important to evaluate the impact of suction in order to account for the effect of seasonal variation on $M_r$ for unsaturated subgrades. Figure 2.19 provides a useful illustration of the dependency of $M_r$ on moisture content and suction, and the interdependence of the two relationship ($M_r$-moisture content, $M_r$-suction).
Previous research studies have taken several different routes to evaluate the impact of matric suction on $M_r$ utilizing laboratory testing. Khoury and Zaman (2004) prepared laboratory specimens at specified moisture contents and subsequently subjected them to either post-compaction drying or wetting processes. Following achievement of the target moisture contents, the specimens were subjected to laboratory $M_r$ testing. After $M_r$ testing, a sample from the specimens was taken to obtain the matric suction using the filter paper method. Their findings showed that not only the $M_r$-suction relationship was dependent on suction but also on the initial compaction moisture content and the extent of drying.

Yang et al. (2005) employed a similar approach. They stated that since subgrade soils were prepared at OMC, and allowed to come to equilibrium with the
surrounding soils, an increase in moisture content for the subgrade soils was to be expected post-construction. Therefore, to replicate field conditions, specimens (A-7 laboratory compacted soils) were prepared at OMC and then subjected to wetting. In this study, suction was measured using the Filter Paper Method, following $M_r$ testing. The soil samples were wetted to two (2) different stages, EMC (equilibrium moisture content) and TMC (moisture content between OMC and EMC). It was seen that there was a drastic decrease in $M_r$ when moisture content increased from OMC to TMC. However, the decrease in $M_r$ from TMC to EMC was minimal. It was also observed that at OMC, $M_r$ values were relatively insensitive to changes in deviator stress but at TMC and EMC, $M_r$ tended to decrease with increasing deviator stress. This would indicate that subgrades on the wet side of optimum are less stiff, indicating a decrease in suction. Figure 2.20 presents the change in $M_r$ due to changes in moisture content and changes in deviatoric stress. Figure 2.21 depicts the relationship between suction and moisture content, which shows a decrease in suction with the increase in moisture content.

![Graphs showing variation of $M_r$](image)

**Figure 2.20:** Variation of $M_r$ due to moisture content and deviator stress at different relative compaction levels; a.) 100% RC b.) 95% RC (Yang et al., 2005)
Yang et al. (2008) realized that the previous research focused on either evaluating suction values after conducting $M_r$ testing or preparing $M_r$ specimens at a certain moisture content and then obtaining the suction utilizing a SWCC created for the same soil type. It was hypothesized that a specimen may not maintain a constant suction value as it is subjected to repeated loading during a resilient modulus test. A technique was needed to be able to control or measure suction during the course of a $M_r$ test. Yang et al. (2008) developed a suction controlled $M_r$ testing system, where the specimen could be subjected to dynamic loading, while the suction was controlled via the axis-translation technique. Figure 2.22 display the device utilized to conduct such testing. To control matric suction in this device, positive air pressure was introduced from the top of the specimen and a HAE ceramic disc was installed at bottom of the pedestal to allow water to pass but not air.

Figure 2.21: Variation of Matric Suction w/ moisture content, A-7-5 Soil (Yang et al., 2005)
Yang et al. (2008) found that the matric suction tends to decrease with increasing the number of load application during Mr testing. For high initials suctions (e.g., 450 kPa), it was seen that matric suction kept increasing gradually with increasing load application, however for low suctions (e.g. 50 kPa) it was seen that matric suction leveled after 10,000 load cycles. It was noted that the decrease in suction was accompanied by the development of excess pore-water pressure (PWP) during loading. To evaluate the effect of matric suction on stiffness, the authors compared the relationship between deviator stress and resilient strain at different matric suctions. It can be seen, from Figure 2.23, that the resilient strain increases significantly with
increasing deviator stress at low suctions, but the increase in resilient strain with increasing deviator stress is minimal at large magnitudes of matric suction. It was concluded that an increase in matric suction has a stiffening effect on the soil specimen.

![Graph showing variation of resilient strain with increasing deviator stress at different matric suction values](image)

Figure 2.23: Variation of resilient strain with increasing deviator stress at different matric suction values (Yang, 2008)

2.4.4 Effect of Hysteresis on Mr-Matric Suction Relationship

It is well known that a SWCC displays a hysteric relationship with soils having different suction values at similar water contents, depending on whether the soils are undergoing drying or wetting. Since Mr for unsaturated soil depends on suction, it is expected that the Mr-suction relationship is also hysteric (Khoury et al., 2011). They conducted a study, utilizing a suction controlled triaxial Mr testing system, to study the effect of subjecting a specimen to a drying or wetting path on the Mr-suction relationship. In their study, the specimens were placed in the triaxial cell and subsequently subjected to increasing values of suction (drying path). Once the equilibrium was reached at a target suction value, the specimen was subjected to a Mr test (AASHTO T-307-99 protocol). Subsequently, the suction values were decreased
(wetting path) and the specimen was once again subjected to $M_r$ testing at the same target suction values. The results of such a testing program are shown in Figure 2.24.

![Figure 2.24: $M_r$-Suction relationship for drying and wetting paths (Khoury et al., 2011)](image)

As seen in Figure 2.24, the specimen was also subjected to secondary drying and wetting paths following the completion of primary drying and wetting. Two trends can be noticed: (a) $M_r$ increases with increasing matric suction, and (b) $M_r$ at a given suction value is higher for primary wetting (PW) and secondary wetting (SW) compared to secondary drying (SD) and primary drying (PD). This is an indication that the $M_r$-suction relationship is also hysteric with respect to drying/wetting. It is worth noting that the hysteresis observed on a SWCC shows that a soil retains more water at the same suction value when undergoing drying compared to when undergoing wetting. This may indicate that the soil is less stiff, at a particular suction value, when undergoing drying due to the presence of more water.
2.4.5 Effect of Pore-water Pressure Buildup during Cyclical Loading

Subgrade soils that contain fines can be expected to experience undrained conditions (allowing for development of excess PWP) under traffic loading (cyclical loading) (Cary and Zapata, 2011). Per AASHTO T-307-99, Mr testing is conducted under drained conditions. Generally, no excess pore-water pressure (PWP) development is expected in soils subjected to triaxial testing under drained conditions. However, considering the amount of repeated loading applications and the short duration of time between each load application, it can be expected that fine-grained soils develop excess PWP even when a repeated load triaxial test is performed under drained conditions, i.e., the drainage valves remain open during the test.

Cary and Zapata (2011) investigated the effects of PWP development, in a soil containing fines, when a specimen was subjected to repeated dynamic loading with the loading applied under a haversine load pulse. The test results showed that PWP reached a peak value at the peak of the load pulse, followed by dissipation during the rest period between loading applications. However, all of the PWP developed under the loading sequence did not dissipate during the rest period and there was a small accumulation of excess PWP with increasing number of loading applications. The accumulation of excess PWP becomes significant and results in decreasing the effective stress in the soil, which decreases the stiffness of the soil and hence its’ Mr value. This process is illustrated in Figure 2.25.
Figure 2.25: PWP characteristic under dynamic loading (Cary and Zapata, 2011)

Figure 2.25 illustrates the accumulation of PWP at the rest period following the loading cycle. It should be noted that the peak pressure (reached at the apex of the loading cycle) also accumulates over time with increasing number of load application. Figure 2.26 give a global perspective (PWP w/ respect to time) of accumulation of excess PWP during the loading period (peak) and at the end of the rest period (cycle end). It should be noted that the excess PWP is not only generated in saturated specimens but also generated in unsaturated specimens subjected to cyclical loading. In saturated specimens, the development of excess PWP serves to create positive a PWP, which decreases the effective stresses. In unsaturated specimens, the PWP is initially negative. As unsaturated specimen are subjected to cyclical loading, they develop excess (positive) PWP. The development of excess PWP causes a reduction in
the magnitude of the negative PWP, for unsaturated specimens, which consequently leads to a reduction in the magnitude of the matric suction (Cary, 2011). A reduction in the matric suction causes a decrease in the effective stress of the unsaturated specimen.

![Diagram](image.png)

Figure 2.26: Development of peak and cycle-end excess PWP (Cary and Zapata, 2011)

### 2.5 Models for Evaluating Resilient Modulus

#### 2.5.1 Universal Model

The model proposed by Witczak and Uzan (1998) is widely referred to as the universal model, and it has been adopted by MEPDG to represent $M_r$ behavior with respect to stress state. The generalized model adopted by MEPDG is presented in Equation 2.9.

$$M_r = k_1 P_a \left( \frac{\theta}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{k_3}$$

(2.9)
Where:

\[ P_a = \text{atmospheric pressure} \]

\[ \theta = \text{bulk stress} = \sigma_1 + \sigma_2 + \sigma_3 \]

\[ \tau_{\text{oct}} = \text{octahedral shear stress} = \frac{\sqrt{2}}{3} (\sigma_1 - \sigma_3) \text{ when } \sigma_2 = \sigma_3 \]

\( k_1, k_2, k_3 = \text{regression constants} \)

While Equation 2.9 is useful and widely utilized, it only takes into account the effect of stress state on \( M_r \) but does not consider the effect of moisture variation on \( M_r \) caused by seasonal variation. Researches have tried to combat this problem by relating the regression constants to soil physical properties, trying to incorporate the effects of seasonal variation into \( M_r \) predictions (Nazzal and Mohammad, 2010; Yau and Von Quintus 2002). Nazzal and Mohammad (2010) introduced physical meanings for the regression constants by evaluating them across different moisture conditions to evaluate how the changes in moisture conditions can effect the regression constants. It concluded that \( k_1 \) is related to the stiffness of the material, which increases with increasing effective stress. \( k_2 \) describes the stiffening effect an increase in bulk stress has on the soil, \( k_2 \) decreases with increasing moisture content; \( k_3 \) describes the softening of the material with increasing shear stress, such that \( k_3 \) decreases (becomes more negative) as moisture content increases.

2.5.2 Models Incorporating Moisture Variation

The importance of developing a model that has the ability to predict changes in \( M_r \) due to changes in moisture content has been recognized, and several models have been developed to accomplish this task. The MEPDG introduced the Enhanced Integrated Climatic Model (EICM) to predict changes in properties of pavement
structures due to environmental effects, specifically the seasonal variation. For flexible asphalt concrete pavements, EICM requires the user to input $M_r$ at a specified moisture condition, and subsequently EICM evaluates the expected changes in moisture content. To evaluate the impact of the seasonal changes on the user input value, EICM creates a set of adjustment factors that account for moisture changes, freezing, thawing, and effects of post thawing. MEPDG then combines the adjustment factors obtained from EICM with the effects of loading due to traffic, and applies the total effect to the material properties. Once this accomplished, MEPDG makes use of transfer functions to predict pavement performance taking into account the effect of EICM adjustment factor and the external loading on material properties. The relationship for the EICM adjustment factor is presented in Equation 2.10.

$$\log \frac{M_r}{M_{ropt}} = a + \frac{b-a}{1+EXP\left(\frac{b-a}{k_m}\cdot(S-S_{opt})\right)}$$

(2.10)

Where:

$M_r / M_{ropt} =$ resilient modulus ratio

$a =$ minimum of log ($M_r/M_{ropt}$)

$b =$ maximum of log ($M_r/M_{ropt}$)

$k_m =$ regression parameter

$(S - S_{opt}) =$ variation in degree of saturation (expressed as a decimal)

The right side of Equation 2.10 represents the adjustment factor, $F_u$, which when solved by applying the anti-logarithm to obtain the adjusted $M_r$ by multiplying the adjustment factor by the $M_r$ value at optimum moisture condition. The MEPDG recommends values of -0.5934, 0.4, and 6.1324 for $a$, $b$, and $k_m$, respectively, for fine-grained soils. A graphical presentation of Equation 2.10 is given in Figure 2.27 for fine-
grained soils. Note that, MEPDG provides a different set of values for $a, b$, and $k_m$ for coarse-grained soils.

![Figure 2.27: Effect of moisture changes on $M_r$ utilizing EICM adjustment factor, $F_u$ (NCHRP, 2004)](image)

Cary and Zapata (2010) conducted a study to evaluate the validity of Equation 2.10 for a wide range of moisture conditions. It was found that the EICM models tends to under-predict $M_r$ in dry/arid conditions, especially for high PI soils; however, insufficient data was available to evaluate the validity of the model for wetter conditions. Figure 2.28 illustrates how the EICM model fits the data collected by Cary and Zapata (2010). It can be seen that there is significant data scatter when the degree of saturation is well below the optimum condition.
Cary and Zapata (2010) stated that the effect of soil type is critical when considering increases in the $M_r$ value due to a decrease in moisture content, especially for soils on the dry side of optimum. This assumption is valid since soils with high plasticity index (PI) values tend to reach much higher suction values at lower degrees of saturation as compared to lower PI soils. They proposed a model to incorporate the effect of soil type on $M_r$ changes. The model is presented in Equation 2.11. The model incorporates the effects of soil type by including the term $wPI$, which is the product of PI and % passing the No. 200 sieve expressed as a decimal.

$$
\log F_u = mx \left[ (\alpha + \beta \ast e^{-wPI})^{-1} + \frac{\left( (\delta + \gamma \ast wPI)^{5} - (\alpha + \beta \ast e^{-wPI})^{-1} \right)}{\left( \ln (\frac{-\delta + \gamma \ast wPI^{5}}{\alpha + \beta \ast e^{-wPI}}) + \rho + \omega \ast e^{-wPI} \frac{S - S_{opt}}{100} \right)} \right]^{1+e} \right]
$$

(2.11)
Where:
\[ a = \alpha + \beta \times e^{-w_{PI}} \]
\[ b = \delta + \gamma \times w_{PI}^{0.5} \]
\[ k_m = \rho + \omega \times e^{(-w_{PI})} \times 0.5 \]
\[ m = \text{correction factor} = 1.002 \]
\[ \alpha = -0.600, \beta = -1.87194, \delta = 0.800, \gamma = 0.080, \rho = 11.96518, \text{and} \ \omega = -10.19111 \]

Equation 2.11 was utilized to create the model presented in Figure 2.29, similar to the EICM model presented in Figure 2.27. The authors (Cary and Zapta, 2010) suggested that this model allows for more accurate predictions in the dry range by taking into account the additional stiffness gain by higher PI soils in the lower saturation range.

Figure 2.29: Variation of Fu as a function of (S - S_{opt}) and w_{PI} (Cary and Zapata, 2010)

2.5.3 Models Incorporating Matric Suction

Yang et al. (2005) recognizes the need to develop a model that incorporates soil suction in predicting Mr, since suction has a direct impact on the stiffness of unsaturated
soils. The model proposed by Yang et al. (2005) is a variation of the deviator stress model initially introduced by AASHTO T 292-91. The original deviator stress model for $M_r$ is presented in Equation 2.12.

$$M_r = k_1 (\sigma_d)^{k_2} \tag{2.12}$$

Where:

$\sigma_d$ = deviatoric stress

$k_1, k_2$ = regression constants

Utilizing the unsaturated soils effective stress concept (Equation 2.1), Yang et al. (2005) proposed a new relationship based on Equation 2.12 that accounts for soil suction as:

$$M_r = k_5 (\sigma_d + \chi \psi_m)^{k_6} \tag{2.13}$$

Where:

$\chi$ = Parameter representing contribution of suction to effective stress (0 for completely dry soil and 1 for saturated soils)

$\psi_m$ = matric suction

$k_5, k_6$ = regression constants

It is believed that Equation 2.13 accurately captures the effect of suction, especially at low moisture contents when its' effect is very significant, and the effect of deviator stress, which is significant at higher moisture content. Since changes in moisture content affect suction, the effect of seasonal variation on $M_r$ is implicitly included in Equation 2.13. It can be seen in Figure 2.30 that Equation 2.13 provides a good fit between the measured and predicted $M_r$ data.
Figure 2.30: Predicted versus Measured $M_r$ values based on model presented in Equation 2.13 (Yang et al., 2005)

Gupta et al. (2007) developed a model to predict $M_r$ for unsaturated soils, based on three (3) stress variables the bulk stress, matric suction ($U_a - U_w$), and deviator stress. The model proposed by Gupta (2007) was based on the principles of a model proposed by Vanapalli et al. (1996), which describes the shear strength of unsaturated soils across the entire SWCC range (Equation 2.14).

$$
\tau_{us} = c' + (\sigma_n - U_a)tan\phi' + (U_a - U_w)(\theta^k tan\phi')
$$

Where:

- $C'$ = effective cohesion of a saturated soil
- $\Phi'$ = effective friction angle of saturated soil
- $(\sigma_n - U_a)$ = net normal stress
- $(U_a - U_w)$ = matric suction
- $\Theta = $ normalized volumetric water content $= \frac{\theta}{\theta_s}$
- $k$ = fitting parameter
In Equation 2.14, the 1\textsuperscript{st} part of the model represents the shear strength when the soil is saturated, the 2\textsuperscript{nd} part represents the contribution to shear strength due to matric suction. \( \Theta \) was incorporated into the model to reflect the amount of water in the soil, and it varies from unity (when the soil is saturated) to a very small value at residual conditions. The authors model includes normalizing water content to properly evaluate the contribution of suction, since the area of contact between the soil particles, which is wetted, decreases with an increase in suction and vice-versa when suction is decreased. The increase or decrease of the wetted area of contact between soil particles is related to the rate at which shear strength changes under unsaturated conditions. Considering this, it can be said that there is a significant relationship between the strength of unsaturated soil and the SWCC, which describes the relationship between water content and suction (Vanapalli et al, 1996).

Gupta et al. (2007) stated that explicitly including one of the parameters that describes the SWCC (e.g. Fredlund and Xing, 1994) into Equation 2.14 will create a power relationship between the soil suction and the shear strength similar to the one presented in Equation 2.15.

\[
\tau_{us} = (\sigma_n - U_a) \tan \phi' + c'(\theta \tan \phi')^\beta
\] (2.15)

The advantage of Equation 2.15 over Equation 2.14 is that there is no need to evaluate normalized water content and soil suction. Utilizing the relationship presented in Equation 2.15 and using the Universal Mr model (NCHRP 2003), the following relationship (Equation 2.16) was presented to incorporate suction in evaluation Mr.

\[
Mr = \left(k1P_a \left(\frac{\theta}{P_a}\right)^{k2} \left(\frac{\tau_{sat}}{P_a} + 1\right)^{k3}\right) + \alpha(U_a - U_w)^\beta
\] (2.16)

Where:
α = Intercept of $M_r$ at given $\theta$ $\tau_{oct}$ Vs. suction relationship

$\beta$ = Slope of $M_r$ at given $\theta$ $\tau_{oct}$ Vs. suction relationship

Liang et al. (2008) attempted to improve the model presented by Yang et al. (2005) because they believed that Yang et al. (2005) model requires calibration of regression constants at each moisture content, for the same soil type, to be effective. Liang et al. (2008) also intended to propose a model which can incorporate effects of seasonal variation in predicting $M_r$. This model is based upon the Universal Model utilized by MEPDG (NCHRP 2004), which is presented here again for clarity.

$$M_r = k_1 P_a \left( \frac{\theta}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{k_3}$$

By incorporating the effective stress equation for unsaturated soils (Bishop, 1959), Liang et al. (2008) was able to propose a new model to include suction in evaluating $M_r$ as follows:

$$M_r = k_1 P_a \left( \frac{\theta + \chi \psi_m}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{k_3}$$

(2.17)

Where:

$P_a$ = atmospheric pressure

$\chi$ = Bishop’s Effective stress parameter

$\psi_m$ = Matric Suction

$\tau_{oct}$ = octahedral shear stress

$\theta$ = bulk stress

$k_1$, $k_2$, $k_3$ = regression constants

To evaluate $\chi$, Liang et al. (2008) recommended a model introduced by Khalili and Khabbaz (1998), which is presented in Equation 2.18. In Liang et al. (2008), $\chi$ was only
evaluated at suction values greater than the air-entry value since prior to that the soil would be saturated and $\chi = 1$.

$$\chi = \left( \frac{(U_a - U_w)b}{U_a - U_w} \right)^{0.55}$$  

Equation (2.18)

Where:

$(U_a - U_w)b = \text{air-entry pressure}$

$U_a - U_w = \text{matric suction}$

To validate the model, the Liang et al. (2008) conducted repeated load triaxial tests to obtain $M_r$ values and filter paper method to obtain suction values. Data from previous literature was utilized. They conducted regression analysis at OMC for $M_r$ tests to obtain the regression constants, the obtained regression constants, along with the model in Equation 2.17, were applied to specimens at different moisture contents to predict $M_r$ values. Liang et al. (2008) also compared the total stress approach, neglecting suction, versus the effective stress approach, including suction, to predict $M_r$ values. It was seen that $M_r$ predictions were significantly better when suction was included. A comparison between the total stress approach and effective stress approach for A-6 soil is displayed in Figure 2.31. It can be seen that including matric suction helps in improving the prediction of $M_r$ values of the soil.

Cary and Zapata (2011) also presented a model that included the effect of suction in evaluating $M_r$ for unsaturated soils. However, unlike the other models, this model included the effects of pore-water pressure buildup during cylical loading. Excess soil PWP is usually generated under moving vehicle loads, while dissipation occurs in the lag time between applied loads. When the lag time is long (i.e. slow moving traffic) there may be no accumulation of PWP between load cycles. However, When the lag
time is long (i.e. slow moving traffic) there may be no accumulation of PWP between load cycles.

However, if the lag time is short (i.e., fast moving traffic) there may be significant accumulation of excess PWP as the number of applied loads increases (Cary and Zapata, 2011). The dissipation of PWP is dependent upon the hydraulic conductivity of the soil and the lag time between load repetitions. When the soil has a high hydraulic conductivity or there is large lag time between load repetitions, this condition can be modeled in the laboratory through performing a drained $M_r$ test. However, if the soil has a low hydraulic conductivity or there is a short lag time between load repetitions, an undrained $M_r$ test would need to be performed to accurately depict field conditions. Cary and Zapata (2011) proposed the following model:

$$M_r = k_1 P_a \left( \frac{\theta_{net} - 3 + \Delta u_{w - sat}}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{k_3} \left( \frac{(\psi_m - \Delta \psi_m)}{P_a} + 1 \right)^{k_4} \quad (2.19)$$

Figure 2.31: Predicted versus Measured Values for A-6 soil (Liang et al., 2008)
Where:

\[ \Delta U_{w-sat} = \text{build up of PWP under saturated conditions; } \Psi_m = 0 \]

\[ \Psi_m = \text{initial matric suction} \]

\[ \Delta \Psi_m = \text{relative change in matric suction w/ respect to } \Psi_m \text{ due to buildup of PWP under unsaturated conditions; in this case } \Delta U_{w-sat} = 0 \]

It should be noted that the model presented in Equation 2.19 was developed utilizing Mr testing conducted using an unsaturated soil triaxial cell, which allowed for the usage of the axis-translation technique to apply matric suction during Mr testing and also for measurement/control of porewater pressure. Hence, the usage of \( \theta_{net} \) instead of \( \theta \) to represent the bulk stress( \( \theta_{net} = \theta - U_a \) as the soil approaches saturation \( U_a \) tends towards 0 and \( \theta_{net} \) becomes \( \theta \). To validate the model in Equation 2.19, Cary and Zapata (2011) performed several different comparisons.

Witicizak et al. (2000) proposed a model that incorporates the environmental adjustment factor along with Mr at an applied external stress to predict changes in Mr as a function of changes in degree of saturation. The model is described as follows:

\[ Mr = 10^{\left( a + \frac{b-a}{1+\exp\left(\frac{\ln\left(b-a\right)+k_m\left(S-S_{opt}\right) - \frac{\tau_{opt}}{P_a} + 1\right)}{k_1 P_a \left( \frac{\theta}{P_a} \right) + k_2 \left( \frac{\tau_{opt}}{P_a} + 1 \right)} \right)^{k_3}} \]  

(2.20)

Cara and Zapata (2011) utilized Mr test results to obtain regression the constants, \( k_1 \) to \( k_4 \), in Equation 2.19. The predicted Mr results from Equation 2.19 were compared to those obtained using Equation 2.20. The comparison, which is presented in Figure 2.32, shows that Equation 2.19 tends to give a better prediction of measured Mr values.
a.) Using Equation 2.20

b.) Using Equation 2.19

Figure 2.32: Goodness of fit for measured versus predicted $M_r$ values for soil with PI = 5 (Cary and Zapata, 2011)
Cary and Zapata (2011) compared the model presented in Equation 2.10 with Liang et al. (2008) suction dependent $M_r$ model. The results obtained by fitting the data to Liang et al. (2008) as shown in Figure 2.33. When compared to Figure 2.32 (part b) it can be seen that Cary and Zapata (2011) model provides a better prediction of $M_r$ for this soil type.

![Graph showing goodness of fit for plastic soil with PI = 5 using Liang et al. (2008) model (Cary and Zapata, 2011)](image)

Figure 2.33: Goodness of fit for plastic soil with PI = 5 using Liang et al. (2008) model (Cary and Zapata, 2011)

Nokkaew et al. (2014) conducted a study to evaluate the effects of matric suction on $M_r$ of Recycled Asphalt Pavement (RAP) and Recycled Asphalt Material (RAM) in a postcompaction state. While a relationship between matric suction and $M_r$ has been well established for traditional base course material, the authors wanted to investigate the relationship further for RAP and RAM since they are hydrophobic materials. To evaluate the relationship, specimens were prepared at OMC and 95% of
maximum dry density, subsequently saturated, and then dried to a target suction value before Mr testing. To analyze the results obtained from Mr testing, the authors utilized the model proposed by Liang et al. (2008) to predict Mr values, but a slight modification was made by using the definition of \( \chi \) presented in Equation 2.21, whereas Liang et al. (2008) utilized the definition presented in Equation 2.18.

\[
\chi = \theta^k = \left( \frac{\theta - \theta_r}{\theta_s - \theta_r} \right)^k
\]  

(2.21)

Where:

- \( \theta \) = volumetric water content
- \( \theta_r \) = residual water content
- \( \theta_s \) = saturated water content
- \( k \) = fitting parameter to fit measured values to predicted values of \( \chi \)

This resulted in Equation 2.22 being utilized for the prediction of Mr.

\[
M_r = k_1 Pa \left( \frac{\theta + \theta_r \psi}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{k_3}
\]

(2.22)

It can be seen that Equation 2.22 (Nokkaew et al., 2014) provided a similar fit with the measured data when compared to Liang et al. (2008) as shown in Figure 2.34. However, Nokkaew et al. (2014) contended that the Liang et al. (2008) model cannot predict Mr near saturation and at residual condition because of the definition of \( \chi \) utilized by Liang et al. (2008) assumes a linear relationship between \( \chi \) and soil suction in a logarithmic scale when the suction value is greater than the air-entry pressure.
2.5.4 Mr-Suction Model Incorporating Hysteresis

As discussed earlier, the SWCC curves display hysteric behavior. Since the Mr is dependent on suction; the Mr-suction relationship is also expected to experience hysteric behavior, i.e. dependent upon the path followed by the soil (wetting or drying). Khoury et al. (2011) proposed a model that captures this hysteric behavior when predicing Mr. Recalling Figure 2.24, it was seen that the hysteric Mr-suction relationship means that Mr values at similar moisture contents differ based on the path followed to achieve that moisture content. The model proposed by Khoury et al. (2011) is given in Equation 2.23.

\[
M_r = \left[ \left( k_1 P_a X \left( \frac{\theta_p}{\theta_a} X \left( \frac{I}{P_a} + 1 \right)^{k_3} \right) \right) + (\Psi - \Psi_0) X \left( \frac{\theta_d}{\theta_a} \right)^k \right] X \left( F_{dw} \right) \]  

(2.23)

Where:
\[
\left( k_1 p_a X \frac{\theta_b k_2}{p_a} X \left( \frac{x}{p_a} + 1 \right)^{k_3} \right) = \text{Universal Model}
\]

\[\Psi = \text{Suction}\]

\[\Psi_o = \text{low suction corresponding to Mr test (e.g. Wet of Optimum)}\],

\[\theta_d = \text{Volumetric water content along drying curve},\]

\[\theta_s = \text{Volumetric water content corresponding to 0 suction, i.e., saturated water content}\]

\[k = 1/n\]

\[n = \text{model parameter ‘b’ from Fredlund and Xing’s fitting model (Equation 2.5)}\]

\[F_{dw} = \frac{\theta_d}{\theta_w}\]

\[\theta_w = \text{volumetric water content corresponding to wetting curve at same suction as } \theta_d\]

\[k_1, k_2, k_3 = \text{model regression constants}\]

The first part of Equation 2.23 is equivalent to the Universal Model. The 2nd part of Equation 2.23, \((\Psi - \Psi_o) X \left( \frac{\theta_d}{\theta_s} \right)^k\) tries to capture the impact of suction Mr. The term, \(\left( \frac{\theta_d}{\theta_s} \right)\) accounts for the change in water content along the drying curve; while the exponent, n, relates this change in water content to the SWCC via the fitting parameter b from Fredlund and Xing (1994) equation, which represents the rate of change (slope) of suction due to a change in water content. Utilizing the 1st and 2nd terms of Equation 2.23, Mr along a drying curve can be predicted provided that \(\Psi_o\) is known. The values obtained can then be multiplied by the factor, \(F_{dw}\), to obtain Mr values along the wetting curve. The parameter \(F_{dw}\) allows for the prediction of Mr along the wetting curve on the basis of drying tests.

To validate the model presented in Equation 2.23, Khoury et al. (2011) subjected specimens to Mr testing at selected points on the SWCC corresponding to
drying/wetting cycles. The apparatus utilized for Mr testing allowed for control of suction via the axis-translation technique. The comparison between measured and predicted values is presented in Figure 2.35.

![Comparison of measured and predicted values](image)

Figure 2.35: Comparison of measured and predicted values at net confining pressure of 41 kPa and deviator stress of 28 kPa. (Khoury et al., 2011)

Khoury et al. (2011) prepared additional set of specimens for testing at a predetermined suction value. They also followed a second method of Mr testing; In the second method multiple Mr tests were conducted on the same specimen as different suction(s) were progressively applied to the specimen to achieve either drying or wetting before Mr testing was conducted. Their model was fitted to measured values from this second method of Mr testing. It was seen that the model tended to underpredict Mr values for this method. They believed this was due to specimens in the second method undergoing a hardening effect due to multiple Mr tests being conducted.
on the same specimen. The fit for the measure versus predicted values for this method is displayed in Figure 2.36.

![Figure 2.36: Measured versus Predicted values (using Equation 2.23) for specimens subjected to multiple $M_r$ tests along wetting and drying paths (Khoury et al., 2011)](image-url)
CHAPTER 3 METHODOLOGY

In this chapter, the laboratory testing program will be discussed in detail; mainly the methodology followed to conduct the different laboratory tests will be explained. Criteria was established for soil properties/classification to guide the selection of soils to be utilized in this study. The objective was to select soil types displaying a range of PI values representative of subgrade soils found in southern Louisiana. Generally, unless otherwise stated, laboratory testing was performed in accordance with the standards presented by American Society of Testing and Materials (ASTM) or American Association of State Highway and Transportation Officials (AASHTO). The testing program was carefully crafted before laboratory testing commenced.

3.1 Selection/Classification of Soils

Four (4) different soils were utilized in the laboratory testing program, selected physical properties, Atterberg Limits and percent of fines, for the soils utilized are proved in Table 3.1. From now on, for the duration of this document, the different soil types will be referred to in accordance with the column titled ‘Soil Name’, column 2, in Table 3.1.

<table>
<thead>
<tr>
<th>Soil No.</th>
<th>Soil Name</th>
<th>Liquid Limit</th>
<th>Plastic Limit</th>
<th>Plasticity Index</th>
<th>% Passing No. 200 Sieve</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P-7</td>
<td>31</td>
<td>24</td>
<td>7</td>
<td>68.9</td>
</tr>
<tr>
<td>2</td>
<td>P-17</td>
<td>38</td>
<td>21</td>
<td>17</td>
<td>43.8</td>
</tr>
<tr>
<td>3</td>
<td>P-26</td>
<td>44</td>
<td>18</td>
<td>26</td>
<td>95.4</td>
</tr>
<tr>
<td>4</td>
<td>P-53</td>
<td>88</td>
<td>35</td>
<td>53</td>
<td>95.7</td>
</tr>
</tbody>
</table>

*Soil passing No. 4 Sieve was utilized for -200 Wash
Table 3.2 provides the AASHTO and Unified Soil Classification System (USCS) classifications for the four soils listed in Table 3.1, along with data from the moisture-density relationships and specific gravity test results. As can be seen in Table 3.2, the study covers a broad range of soil types to accurately evaluate the effect of soil types on the different relationships examined in this study. The results from the hydrometer test are reported in Table 3.3.

Table 3.2: Soil classification, moisture-density relationship, and specific gravity of soils

<table>
<thead>
<tr>
<th>Soil</th>
<th>MDD* (pcf)</th>
<th>OMC (%)*</th>
<th>Specific Gravity</th>
<th>AASHTO</th>
<th>UCS</th>
<th>Visual Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-7</td>
<td>108.3</td>
<td>17</td>
<td>2.67</td>
<td>A-4</td>
<td>ML</td>
<td>Dark brown, clayey silt</td>
</tr>
<tr>
<td>P-17</td>
<td>110.1</td>
<td>16</td>
<td>2.65</td>
<td>A-6</td>
<td>SC</td>
<td>Light brown, sandy clay w/ traces of gravel</td>
</tr>
<tr>
<td>P-26</td>
<td>100.6</td>
<td>22</td>
<td>2.71</td>
<td>A-7-6</td>
<td>CL</td>
<td>Brown, lean clay</td>
</tr>
<tr>
<td>P-53</td>
<td>78.2</td>
<td>35</td>
<td>2.66</td>
<td>A-7-6</td>
<td>CH</td>
<td>Dark clay</td>
</tr>
</tbody>
</table>

*Maximum Dry Density (MDD) and Optimum Moisture Content (OMC) based on Standard Proctor test (ASTM D698)

Table 3.3: Hydrometer Analysis

<table>
<thead>
<tr>
<th>Soil</th>
<th>% Sand</th>
<th>% Silt</th>
<th>% Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-7</td>
<td>35</td>
<td>52</td>
<td>13</td>
</tr>
<tr>
<td>P-17</td>
<td>19</td>
<td>63</td>
<td>18</td>
</tr>
<tr>
<td>P-26</td>
<td>4</td>
<td>61</td>
<td>35</td>
</tr>
<tr>
<td>P-53</td>
<td>2</td>
<td>13</td>
<td>84</td>
</tr>
</tbody>
</table>

*Soil passing No. 10 sieve was utilized for Hydrometer Analysis

### 3.2 Repeated Load (RLT)/Resilient Modulus (M_r) Tests

The moisture-density relationships evaluated for each soil type provided pertinent information which was utilized to prepare laboratory compacted specimens. M_r samples were laboratory compacted at a specified density and moisture obtained, obtained from the moisture-density relationship. The study attempts to explore the relationship between matric suction and M_r, and subsequently, the effects of seasonal variation on
Mr. Therefore, Mr specimens were compacted and tested at different moisture contents corresponding to different degrees of saturation. The moisture contents were varied by changing the compaction moisture content during specimen preparation. Table 3.3 provides the different moisture contents utilized for each soil type when conducting Mr tests.

Table 3.4: Factorial for Resilient Modulus (Mr) testing

<table>
<thead>
<tr>
<th>Soil</th>
<th>Classification (AASHTO)</th>
<th>Moisture Contents selected for Mr testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-7</td>
<td>A-4</td>
<td>OMC -3%, OMC, OMC +3%</td>
</tr>
<tr>
<td>P-17</td>
<td>A-6</td>
<td>OMC -3%, OMC, OMC +3%</td>
</tr>
<tr>
<td>P-26</td>
<td>A-7-6</td>
<td>OMC -6%, OMC-3%, OMC, OMC+3%, OMC +6%</td>
</tr>
<tr>
<td>P-53</td>
<td>A-7-6</td>
<td>OMC -6%, OMC -3%, OMC, OMC +3%, OMC +6%</td>
</tr>
</tbody>
</table>

*For P-26, the initial testing program called for Mr testing at OMC +6, however, the sample was too weak and unable to sustain the loading experienced during the Mr test.

Samples were prepared, and tested, in accordance with the procedures presented in AASHTO T-307-10 for fine-grained subgrade soils. The prepared specimens had the following approximate dimensions; 2.8” (in.) diameter and 5.6” (in.) height. Figure 3.1 displays a laboratory prepared specimen that was subjected to Mr testing.

Figure 3.1: Laboratory compacted specimen for Mr testing (Soil P-53 at OMC)
Prior to specimen preparation, bulk soil samples were dried in a 60° Celsius oven. Following drying, the soil samples were processed through a No. 4 Sieve. Only the material passing No.4 Sieve was utilized for preparation of the Mr specimens. Once the sample had been processed through the No. 4 sieve, an appropriate amount of demineralized/de-aired water was added to achieve the target moisture content in accordance with Table 3.4. Following the addition of water, the samples were thoroughly mixed and subsequently covered and left overnight to achieve homogenous moisture conditions.

The specimens were compacted in a laboratory mold utilizing Standard Proctor procedure. The target density for compacted specimens was obtained from the moisture-density curves and corresponded to the target moisture content. The specimens were compacted in five (5) layers of equal weight. Equation 3.1 was utilized to determine the number of blows required per layer by a 5.5 lb hammer, falling 12 inches, to achieve a compaction energy similar to Standard compaction energy (12,400 ft-lbf/ft^3). Following compaction, the specimens were prepared for placement inside the device utilized to conduct the RLT Mr test. Figure 3.2 displays the items needed to prepare the Mr specimen for placement inside the triaxial cell.

\[
\text{Compaction Energy} = \frac{\text{(# of blows/layer)(# of layers)(Wt.of hammer)(drop height)}}{\text{volume of mold}}
\]

Equation (3.1)

Following preparation the specimen was weighed and appropriate dimensions were measured. The height and diameter were measured using a caliper to ensure the specimen maintained a 2:1 height to diameter ratio. As can be seen in Figure 3.2, the specimen is then prepared for placement inside the triaxial cell. A porous stone and
filter paper were placed on the bottom base plate. The specimen is then placed on top of the filter paper and porous stone, and another filter paper and porous stone are placed on top of the specimen. Once the specimen, porous stones, and filter papers are in place on the base plate, a latex membrane is placed around the specimen to protect it inside the triaxial cell.

Figure 3.2: Preparation of sample prior to placement in MTS device for Mr testing (Dhakal, 2012)

The device utilized for Mr testing was the Material Testing Systems, MTS 810, with a closed loop servo hydraulic system. The device is pictured in Figure 3.3. The device measures the applied load utilizing a load cell, which is installed inside the triaxial cell. This setup helps minimize the errors related to the measured loads. The capacity of the load cell was 5,000 lbf. Two (2) linear variable differential transducers (LVDTs) were placed between the top platen and the base plate to measure the axial displacements. Utilizing the internal LVDTs is thought to decrease the amount of error in the measured axial deformation when compared to external LVDTs. Air was used to apply confining pressure to the specimens. As can be seen in Figure 3.3, the device
contains drainage valves that are attached to the top platen and the base plate; hence the usage of filter paper and porous stone. In this study, the drainage valves were kept open during Mr testing and therefore, the tests were conducted under drained conditions.

Figure 3.3: MTS 810 RLT Device

As mentioned earlier, the AASHTO T-307-10 protocol was followed for Mr testing. The procedure specifies loading conditions applied to subgrade specimens during a Mr test. The loading conditions are a function of three (3) different confining pressures with five (5) different cyclic deviatoric stresses applied at each confining pressure. Therefore, the subgrade soil specimen is subjected to 15 different stress states during the course of a Mr test. The loading procedure is provided in Table 3.5. In this study, 1000 cycles were applied during the conditioning stage to remove imperfections on the top and bottom surface that might occur during compaction. The conditioning phase also helped eliminate most of the initial plastic deformation. As seen
in Table 3.5, a constant load equal to 10% of the maximum axial load was maintained on the specimen at all time. The cyclical load was applied in the form of a haversine shaped load pulse, which is illustrated in Figure 3.4. A haversine shaped load pulse is thought to best represent the loading conditions experienced by a pavement layer under vehicular loading. During vehicular loading, a point in the pavement experiences minimal deviatoric stresses when the wheel load is a considerable distance away from that point. The point experiences the maximum deviatoric stress when the wheel load is directly on top. Per AASHTO T-307-10, the loading period/per pulse was 0.1 second, while the rest/dwelling period was 0.9 second.

Table 3.5: Testing Sequence for Subgrade Soil (AASHTO T-307-10)

<table>
<thead>
<tr>
<th>Sequence No.</th>
<th>Confining Pressure (psi)</th>
<th>Max Axial Stress (psi)</th>
<th>Cyclic Stress (psi)</th>
<th>Constant Stress (psi)</th>
<th>No. of Load Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditioning</td>
<td>6</td>
<td>4</td>
<td>3.6</td>
<td>0.4</td>
<td>500-1000</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>2</td>
<td>1.8</td>
<td>0.2</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>4</td>
<td>3.6</td>
<td>0.4</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>6</td>
<td>5.4</td>
<td>0.6</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>8</td>
<td>7.2</td>
<td>0.8</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>10</td>
<td>9</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>2</td>
<td>1.8</td>
<td>0.2</td>
<td>100</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>4</td>
<td>3.6</td>
<td>0.4</td>
<td>100</td>
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<td>8</td>
<td>4</td>
<td>6</td>
<td>5.4</td>
<td>0.6</td>
<td>100</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>8</td>
<td>7.2</td>
<td>0.8</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>10</td>
<td>9</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>2</td>
<td>1.8</td>
<td>0.2</td>
<td>100</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>4</td>
<td>3.6</td>
<td>0.4</td>
<td>100</td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>6</td>
<td>5.4</td>
<td>0.6</td>
<td>100</td>
</tr>
<tr>
<td>14</td>
<td>2</td>
<td>8</td>
<td>7.2</td>
<td>0.8</td>
<td>100</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>10</td>
<td>9</td>
<td>1</td>
<td>100</td>
</tr>
</tbody>
</table>
Figure 3.4: Haversine shaped load pulse utilized during Mr testing (NCHRP I-28A, 2004).

The MTS 810 data acquisition system records the data from the last five (5) load cycles at each stress state. The data obtained from the last five (5) cycles, at each stress state, is averaged to provide a Mr value. The Mr value is calculated utilizing the relationship presented in Equation 3.2. Each test provides fifteen (15) Mr values, at different stress states, for each specimen.

\[
Mr = \frac{\sigma_{cyc}}{\varepsilon_r} \quad (3.2)
\]

Where:

\( \sigma_{cyc} = \text{Applied cyclical stress} \)

\( \varepsilon_r = \text{Resilient strain (based on recoverable/elastic deformation)} \)

Following completion of testing, the specimens were carefully removed from the triaxial cell and removed from the latex membrane. Moisture content of the specimens was measured after Mr testing was completed. A test was considered admissible if the moisture content was within 0.5% of the target moisture content and the dry density was within 2% of the target dry density. It should be noted here that three (3) replicate specimens were required at each moisture content for each soil type in order to perform
statistical analysis, and to take into account the variations that might occur when testing laboratory prepared specimens.

3.3 Soil Water Characteristic Curve

The SWCC was established utilizing two (2) different techniques, axis-translation and chilled-mirror hygrometer. Separate devices were utilized for each technique. The objective was to measure the SWCC for each specimen that spanned the entire range of moisture conditions, from saturation to residual.

3.3.1 Axis-Translation Technique

The axis-translation technique for measuring matric suction relies on independent control of applied pore-air pressure and pore-water pressure (PWP). A positive pore-air pressure (PAP) is applied while PWP is maintained at atmospheric conditions, since (Ua-Uw) is the applied matric suction. The technique allows us to increase the applied suction by increasing the applied pore-air pressure. The SWC-150 device, manufactured by GCTS Testing Systems was one of the devices used in this study to measure the matric suction. The version of the device utilized in this study, at Louisiana Transportation and Research Center (LTRC) is pictured in Figure 3.5.

Figure 3.5: SWC-150 Fredlund SWCC Device by GCTS Testing Systems
The SWC-150 also allows for measuring the changes in water content of the specimen. As can be seen in Figure 3.5, the device features volume tubes, which allow for accurate measurement of inflow/outflow of water from the specimen, so moisture content can be tracked during the duration of the test. A flushing device is also provided with the specimen, which allows for flushing of diffused air that builds up over time. Therefore, eliminating any errors in measuring water content due to diffused air affecting the readings of the water level in the volume tubes. The device also features two (2) pressure gauges/regulator knobs, one which controls the lower suction range (0 - 200 kPa) with 2 kPa divisions and the other controls the high suction (200-2000 kPa) with 20 kPa divisions. Loading under K₀ conditions can also be applied to the specimen since the device features a weight plate with a loading shaft that is in contact with the specimen during the test. To apply a load, additional dead weights can be added to the load plate. However, in this study, only a small contact load was applied to the specimen to ensure good contact between the specimen and the ceramic stone.

The ceramic stone is inserted into the base plate to separate the air-phase and water phase during the course of test. Generally, the following ceramic stones are utilized: 5-bar, 10-bar, and 15-bar. The 15-bar ceramic stone has an Air-Entry Value (AEV) of 1,500 kPa, which is the largest AEV value ceramic stone commercially available. Therefore, the largest suction that can be applied using the SWC – 150 device is 1500 kPa. A 15-bar ceramic stone was utilized for all tests conducted as a part of this study. The air-supply initially available for this study was not able to supply the pressure needed to apply and maintain a suction of up to 1500 kPa. To alleviate this issue, an air-pressure booster was utilized to achieve the air pressure desired. The
incoming air supply was routed through the air pressure booster to the SWC-150 device. The maximum air pressure available without the pressure booster was approximately 800 kPa (116 psi). Therefore, it was necessary to utilize an air pressure booster to achieve applied pressures up to 1500 kPa. The pressure booster utilized was the RL 00S manufactured by Midwest Pressure Systems, which features the ability to boost pressure at a 2:1 ratio with a supply range of 15 (103kPa) -150 psi (1,030 kPa). The air pressure booster is pictured in Figure 3.6.

![Air Pressure Booster](image)

Figure 3.6: RL00S Bootstrap Compressor 2:1 Ratio Air Pressure Booster (Midwest Pressure Systems)

The Fredlund device also allows for the measurement of both adsorption (wetting) and desorption (drying) curves. Since the specimen is in contact with the ceramic stone, which is also in contact with the water volume tubes through a reservoir below the ceramic stone, the specimen can easily imbibe water or release water due to suction changes. Generally, an increase in suction results in release of water from the specimen, while a decrease in suction results in the specimen imbibing water.

3.3.2 Specimen Preparation for SWC-150 Fredlund Device

While the SWC-150 device allows for testing undisturbed and disturbed samples, in this study remolded (disturbed) soil specimens were utilized. Initial soil preparation was similar to that for Mi specimens. Bulk samples were dried in a 60° C oven and subsequently processed through a No. 4 sieve, only material passing the No. 4 sieve
was utilized for testing. Demineralized/de-aired water was then added to the soil to achieve the target moisture content, and the soil was thoroughly mixed and left to equilibrate overnight. All Fredlund device samples were prepared at OMC and MDD. The compacted samples were 2.8" in diameter and 5.6" in height and were compacted using standard Proctor effort, which is similar to Mr samples described earlier (Figure 3.1).

Following compaction, the samples were hand trimmed into a stainless steel consolidation ring (sample ring). The samples needed to be trimmed into the sample ring in order to be able to be placed in the Fredlund device for testing. The dimensions of the sample ring were as follows: Diameter = 2.5" and Height = 1.0". Subsequently, the samples were saturated. To saturate the sample, the sample was placed on top of a filter paper and porous stone and another filter paper and saturated porous stone were placed on top of the sample. The sample, along with the filter papers and porous stone, was placed in a small container, which was filled with demineralized/de-aired water. Care was exercised to prevent the sample from being inundated, instead the water level was kept approximately 1/8" below the top of the specimen to allow for the release of entrapped air. A small dead weight was placed on top of the sample during saturation to discourage swelling. Saturation of specimens took various durations of time depending on the soil type, from overnight for low PI soils to several days for high PI soils. Following saturation, the specimen was trimmed again to ensure it was flush with the sample ring. The specimen was then placed on top of a previously saturated ceramic stone and placed inside the Fredlund device. Once the sample was placed inside the Fredlund device, the apparatus was assembled by attaching all of the appropriate
hoses, bolts, O-rings, and screws to ensure that the device was air-tight and no leakage would occur during testing (See Figure 3.7 for SWCC specimen).

Figure 3.7: Soil sample trimmed into sample ring prior to saturation for SWCC testing

3.3.3 Testing Procedure for Fredlund Device

In this study, all specimens prepared for testing via Fredlund device were saturated prior to testing. Therefore, the desorption (drying) curve was measured initially. If an adsorption (wetting) curve was intended to be measured, this was done after the drying curve had already been measured. After the saturated specimen was placed inside the device, and device assembly was complete, an initial suction was introduced by increasing the applied air pressure to a value above 0 kPa.

Applied suction values were determined prior to initiating the test. The initial suction applied was determined to allow for accurately capturing the Air Entry Value (AEV). The family of curves shown in Figure 3.8, which are based on the PI of the soil and the percent passing the No. 200 sieve (expressed as a decimal), were utilized to determine the applied suction values during the test. The objective was to accurately capture the AEV along with the desaturation zone.
Figure 3.8: Family of SWCC’s (Zapata, 1999)

Once a suction was applied to the specimen, changes in water content of the specimen were tracked by observing the volume change tubes. A specimen had to be considered at equilibrium under an applied suction before the next increment (of suction) could be applied. The equilibrium was considered achieved if no change in the water volume tubes occurred over a period of 24 hours. Once equilibrium was achieved, the next increment was applied and held till equilibrium was achieved. Equilibrium times varied depending on the soil type of the specimen, with low PI soils having the shortest equilibrium time and high PI soils having the longest equilibrium period. The volume tubes were ‘flushed’ using the flushing device provided with the Fredlund device to ensure no diffused air remained when water level measurements were taken. The test was considered complete once the last suction increment was applied and equilibrium achieved. Subsequently, the device was de-pressurized, disassembled, and the specimen was removed and oven dried to obtain a water content measurement.
To evaluate the hysteric behavior displayed by the SWCC, select samples were subjected to a wetting path following completion of the test along the drying path. To evaluate the wetting path, once equilibrium had been reached following the last suction increment, the suction values were decremented and the specimen was allowed to imbibe water. Similar to the method for measuring the drying curve, the specimen was allowed to come to equilibrium following each suction decrement. Equilibrium was considered to be achieved once there was no change in water levels for a 24 hour period. Suction values were decremented till the last decrement, which usually corresponded to the first suction applied during the drying process, was applied and equilibrium achieved. Subsequently, the specimen was removed and oven dried. It should be noted here that the equilibrium times for a specimen undergoing wetting cycle were significantly longer than the equilibrium time for the same specimen undergoing drying cycle. To adequately measure hysteresis, specimens needed to achieve sufficient desaturation during the drying path. Considering the limited suction range of the Fredlund device, higher PI soils did not show significant desaturation (up to 1500 kPa) to be able to capture the effect of hysteresis. Therefore, only soil P-7 was selected for application of a drying and subsequent wetting path to capture the effect of hysteresis on the SWCC.

3.3.4 Measuring Suction utilizing Chilled-Mirror Hygrometer Method

As mentioned earlier, to evaluate the SWCC in the suction range above 1,500 kPa, the WP4C Dewpoint Potentiometer manufactured by Decagon Devices, Inc. was utilized. The device is pictured in Figure 3.9. The WP4C uses the chilled mirror dew point technique to measure the water potential (suction) of a sample. The device has a
range of 0 – 300 MPa with an accuracy is +/- 0.5 MPa in the 0 – 5 MPa range and +/- 1% from the 5- 300 MPa range (Decagon Devices, 2007). The device measures the total suction, which is the sum of matric and osmotic suction.

Figure 3.9: WP4C Dewpoint Potentiometer manufactured by Decagon Devices, Inc.

The device is accompanied by sample cups, which are 15 mL in volume, in which the specimen being tested is placed before being inserted into the sample drawer for measurement. The sample chamber in the device is temperature controlled. The temperature can be controlled by the user within the range of 15 to 40°C with an accuracy of +/- 0.2°C. Sample readings can be taken in three (3) different modes offered by the device; ‘Precise Mode’, ‘Continuous Mode’, or ‘Fast Mode’. In ‘Precise Mode, the device takes several subsequent reading on a sample until the successive readings occur within a pre-determined tolerance, which ensures greater accuracy. Obtaining a measuring in ‘Precise Mode’ generally takes 10 – 15 minutes. ‘Continuous Mode’ is useful for long term monitoring of samples, as reading are taken continuously

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till the sample is removed from the device. ‘Fast Mode’ offers quick measurements as a sample is only measured once, however this causes less accurate measurements.

3.3.5 Specimen Preparation for WP4C

Preparing remolded specimens to be tested utilizing the WP4C device was challenging due to the small size of the cups (15 mL). Also, the sample cups could not be filled to the top because once the sample cup is placed inside the chamber for measurement, the top of the cup comes in contact with sensors inside the device. Therefore, it was imperative to ensure there was no sample residue on top of the sample cup to avoid contamination of sensors.

Considering the small size of the sample, approximately half the volume of the sample cup, it was not possible to evaluate the effects of density on the WP4C samples. Therefore, it was determined that a larger bulk specimen would have to be prepared and then trimmed into the sample cup. A 2.8” (diameter) by 5.6” (height) specimen was prepared at OMC and MDD, it was possible to evaluate the density utilizing the larger specimen. Also, it was thought that the soil structure would be similar to that for the Fredlund device specimens since they were also prepared at OMC and MDD. Similar to specimen preparation for Fredlund device, the soil was initially dried and processed through a No. 4 sieve and material passing the No.4 sieve was utilized. Demineralized/de-aired water was used to achieve target moisture contents, this process helps in eliminating the effects of osmotic suction which are caused by the presence of dissolved solutes in the pore water. Specimens prepared for the WP4C device were not saturated after compaction.
After compaction, the specimen dimensions and weight were recorded. Then, an approximately 1 in. (in height) portion was taken from the middle half of the bulk specimen, to obtain the sample for the WP4C device. The sample was obtained by carefully ‘pushing’ the sample cup into 1 in. specimen till the sample cup was approximately ½ inserted into the larger specimen. Subsequently, the soil around the sample cup was trimmed and the cup was carefully cleaned to ensure no sample residue remained on the outer edges of the sample cup. This technique allowed the sample cup to be kept approximately ½ full which, is a recommended practice by the manufacturer.

Figure 3.10: Stainless steel sample cups, utilized in this study, for WP4C

3.3.6 Testing Procedure for WP4C Device

The procedure followed in this study to obtain suction measurements utilizing the WP4C device is in general accordance with ASTM D6846-07 along with the procedure utilized by Nam et al. (2009). Before taking any sample measurements, it was important to verify the calibration of the device. A 0.5 molal KCl solution of known water potential was utilized for calibration verification. The calibration of the device was verified each
day the device was utilized. If the calibration needed adjustment, the device allowed for calibration adjustment by the user.

After trimming of the sample into the sample cup, the cup and sample were weighed and the sample cup was sealed by placing the plastic cap on the cup and using paraffin tape to ensure a good seal. Subsequently, the sample was allowed to equilibrate for 24 hours. It was important to achieve water vapor equilibrium in the headspace above the sample in the sample cup. Following equilibration, the sample cup was placed inside the device and suction measurements were obtained under the ‘Precise Mode’. Following the initial reading, the sample was allowed to air-dry till a pre-determined weight change was attained corresponding to a target moisture content. Once the target moisture content was attained, the sample cup was sealed again using the plastic cap and allowed to equilibrate for one (1) hour. Following the equilibration, the sample cup was once again placed in the device to obtain suction measurements. This process was repeated to obtain suction measurements at several pre-determined moisture contents. The test was stopped once the minimal weight change was observed over a 24 hours period of air drying. Following the last measurement, the sample was oven dried and the data was used to back-calculate the previous moisture contents. It should be noted here that the temperature of the sample had to be maintained close to that of the same chamber, the measurement times will be long. However, the sample has to be cooler than the chamber, otherwise condensation may occur once the sample was placed inside the chamber. The manufacturer recommends the sample temperature to be between 0 and 0.5 degrees cooler than the chamber temperature.
3.4 Tube Suction Test

The Tube Suction (TS) test was developed by the Finnish National Road Administration and presented by Scullion and Saarenketo (1997). It allows for evaluating the moisture susceptibility of a soil/aggregate by measuring its' surface dielectric values. This provides a measure of ‘free moisture’ in the soil. In pavement engineering, the material with less free moisture are expected to perform better than those with more free moisture (Zhang and Tao, 2008). The measurement obtained from the TS test is supposed to give an indication of the durability of the soil. Scullion and Saarenketo (1997) proposed that the results from this test could be used to classify ‘good-performing’ and ‘poor-performing’ pavement materials.

The method utilized for soil preparation, and subsequent TS testing in this study are an adaptation of the method(s) presented by Zhang and Tao (2008). Samples, obtained from material passing No. 4 sieve, were prepared at OMC and MDD in 4 in. (diameter) by 8 in. (height) mold. While the mold was 8 in. in height, the sample were constituted such that the height of the samples was approximately 7 in. Following compaction, the soils were dried in a 60° C oven till no further weight change was observed. The molds utilized had small holes punctured at the bottom to allow the sample to imbibe water once the test commenced.

Once the samples had achieved constant weight in the oven, they were removed and placed on top of porous stones in a water bath. The water level in the bath rose to approximately 1 in. higher than the bottom of the sample cylinders (placed on porous stones). Figures 3.11a and 3.11 b depict the molds utilized for TST test and setup for TS tests, respectively. Subsequently, dielectric value (DV) readings were taken
periodically till the readings and sample weight became constant. DV readings were taken utilizing a Percometer v.3 (Adek LLC, Estonia). To obtain measurements, five (5) readings were taken at the surface of each specimen (per time interval), in which the highest and lowest readings were eliminated. The remaining three (3) readings were averaged to obtain one (1) DV value for the tested specimen at the specified time interval.

Figure 3.11: a.) Molds utilized for TS test; b.) Setup for TS testing
CHAPTER 4 RESULTS

The results from the laboratory testing program will be presented in this chapter, which will be followed by a brief discussion of the results. Due to space limitations not all of the Resilient Modulus ($M_r$) test results may will be presented in this chapter. However, Appendix I will contain the rest of the $M_r$ laboratory test results.

4.1 Moisture Density Relationship

The moisture-density relationship was obtained for each soil type utilizing Standard Proctor effort. These curves were utilized to obtain the optimum moisture content and maximum dry density and the target moisture content and target densities for laboratory prepared samples for each soil type. From Figure 4.1, one can realize that the optimum moisture content (OMC) increases with increasing the PI of the soils. A minimum of four (4) points, preferably, two (2) on the dry side and two (2) on the wet side of optimum were required to complete the curve. The test was performed in general accordance with the ASTM D698.

Figure 4.1: Moisture-Density relationships for the four soil types utilized in study
4.2 Evaluation of Resilient Modulus

The resilient modulus (M_r) values for the different soil types were evaluated by performing Repeated Load Triaxial (RLT) tests. As mentioned earlier, table 4.1 provides the target moisture contents for each soil type. Three (3) identical specimens were tested at each moisture content to obtain a representative M_r value. M_r specimens were evaluated at different moisture contents to observe the impact of moisture content on M_r values for unsaturated soils; specimens tested were under unsaturated condition (i.e. degree of saturation, S < 1). Table 4.2 provides the estimated degree of saturation (S) corresponding to target the moisture contents for each soil type. The values were obtained by averaging values for three (3) identical specimens. Figures 4.2 through 4.5 provide example results obtained from M_r testing of the four soil types. The rest of the M_r results are provided in Appendix I. Soil P – 26, was originally planned to be tested at OMC +6%; however, the specimens were found to be too weak to withstand M_r testing without failure.

<table>
<thead>
<tr>
<th>Soil</th>
<th>OMC -6%</th>
<th>OMC -3%</th>
<th>OMC</th>
<th>OMC +3%</th>
<th>OMC +6%</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-7</td>
<td>N/A</td>
<td>14%</td>
<td>17%</td>
<td>20%</td>
<td>N/A</td>
</tr>
<tr>
<td>P-17</td>
<td>N/A</td>
<td>13%</td>
<td>16%</td>
<td>19%</td>
<td>N/A</td>
</tr>
<tr>
<td>P-26</td>
<td>16%</td>
<td>19%</td>
<td>22%</td>
<td>25%</td>
<td>N/A</td>
</tr>
<tr>
<td>P-53</td>
<td>29%</td>
<td>32%</td>
<td>35%</td>
<td>38%</td>
<td>41%</td>
</tr>
</tbody>
</table>

*The moisture contents above are target values, specimens were considered acceptable if moisture content was within +/- 0.5% of the target moisture content.
Table 4.2: Degree of Saturation (S) for Target Moisture Contents

<table>
<thead>
<tr>
<th>Soil</th>
<th>S (%) – OMC -6%</th>
<th>S (%) – OMC -3%</th>
<th>S (%) – OMC</th>
<th>S (%) – OMC +3%</th>
<th>S (%) – OMC +6%</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-7</td>
<td>N/A</td>
<td>63.6</td>
<td>82.1</td>
<td>87.9</td>
<td>N/A</td>
</tr>
<tr>
<td>P-17</td>
<td>N/A</td>
<td>67.5</td>
<td>84.1</td>
<td>91.5</td>
<td>N/A</td>
</tr>
<tr>
<td>P-26</td>
<td>58.6</td>
<td>76.6</td>
<td>88.3</td>
<td>92.3</td>
<td>N/A</td>
</tr>
<tr>
<td>P-53</td>
<td>68.9</td>
<td>79.5</td>
<td>85.4</td>
<td>91.7</td>
<td>94.2</td>
</tr>
</tbody>
</table>

Table 4.3 provides a snapshot of $M_r$ values obtained for each specimen and the statistical analysis that was performed on the three (3) identical specimens tested for the four soil types at different moisture contents. This table provides summary of $M_r$ values evaluated at a deviatoric stress of approximately 4.0 psi and a confining pressure of 6.0 psi. This stress state corresponds approximately to a bulk stress of 22.5 psi, and an octahedral shear stress of 1.9 psi, which is the recommended stress state for highway subgrade by Strategic Highway Research Program Protocol P-46 (Drumm et al., 1997). In Figure 4.2 through 4.5 the displayed $M_r$ values will correspond to the average value obtained from the three triplicate specimens.

Figures 4.2 – 4.5 provide results from individual $M_r$ tests. In these figures, $M_r$ values are plotted as a function of deviatoric stress. The figures intend to show the impact deviatoric stress and confining pressure have on the $M_r$ value. Fifteen $M_r$ values are reported from each individual test. Due to space limitations, only a total of two (2) individual tests from each soil type are displayed here, one corresponds to dry of optimum, the other corresponds to wet of optimum. Generally, $M_r$ values decrease with increasing the deviator stress and with decreasing the confining pressure.
Table 4.3: Summary Mr Values

<table>
<thead>
<tr>
<th>Soil No.</th>
<th>Soil Type</th>
<th>Pl</th>
<th>Moisture Content</th>
<th>Resilient Modulus (ksi)</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Coeff. Of Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>I</td>
<td>II</td>
<td>III</td>
<td></td>
</tr>
<tr>
<td>P-53</td>
<td>A-7</td>
<td>53</td>
<td>OMC - 6%</td>
<td>11.1</td>
<td>12.6</td>
<td>11.3</td>
<td>11.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OMC - 3%</td>
<td>9.4</td>
<td>9.6</td>
<td>9.6</td>
<td>9.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OMC</td>
<td>9.3</td>
<td>9.0</td>
<td>9.5</td>
<td>9.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OMC +3%</td>
<td>6.0</td>
<td>7.3</td>
<td>6.1</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OMC +6%</td>
<td>6.7</td>
<td>6.0</td>
<td>5.0</td>
<td>5.9</td>
</tr>
<tr>
<td>P-17</td>
<td>A-6</td>
<td>17</td>
<td>OMC - 3%</td>
<td>11.5</td>
<td>12.6</td>
<td>10.3</td>
<td>11.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OMC</td>
<td>6.6</td>
<td>5.1</td>
<td>5.8</td>
<td>5.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OMC +3%</td>
<td>2.4</td>
<td>2.1</td>
<td>2.0</td>
<td>2.1</td>
</tr>
<tr>
<td>P-7</td>
<td>A-4</td>
<td>7</td>
<td>OMC - 3%</td>
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<td>8.7</td>
<td>8.6</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>OMC</td>
<td>4.9</td>
<td>5.0</td>
<td>5.0</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>OMC +3%</td>
<td>3.3</td>
<td>3.1</td>
<td>3.0</td>
<td>3.1</td>
</tr>
<tr>
<td>P-26</td>
<td>A-7</td>
<td>26</td>
<td>OMC - 6%</td>
<td>12.5</td>
<td>11.3</td>
<td>12.0</td>
<td>12.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OMC - 3%</td>
<td>9.3</td>
<td>8.4</td>
<td>9.0</td>
<td>8.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OMC</td>
<td>6.9</td>
<td>7.3</td>
<td>7.0</td>
<td>7.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OMC +3%</td>
<td>2.4</td>
<td>2.9</td>
<td>3.0</td>
<td>2.8</td>
</tr>
</tbody>
</table>

OMC - 6% **Sample Failed**

Figure 4.2: Resilient Modulus test Results; a.) P-7 at OMC -3%, b.) P-7 at OMC +3%
Figure 4.2 continued

(Figure 4.2 continued)

Figure 4.3: Resilient Modulus test results; a.) P-17 at OMC -3%, b.) P-17 @ OMC +3%
Figure 4.3 continued

(a) 6 psi confining
(b) 4 psi confining
(c) 2 psi confining

Figure 4.4: Resilient Modulus test results; a.) P-26 at OMC -6%, b.) P-26 at OMC +3%
Figure 4.5: Resilient Modulus test results; a.) P-53 at OMC -6% b.) P-53 at OMC +6%
While the detailed analysis of the results presented in this chapter will follow in Chapter 5, it is prudent to discuss some of the trends noted in the Figures 4.2 through 4.5. From Table 4.3, it is important to note that while higher PI soils, P-26 and P-53 had a higher $M_r$ value at optimum, they displayed a more dramatic decrease in $M_r$ at wet of optimum. This is in agreement with the results reported by Drumm et al. (1997). The results of $M_r$ tests on the dry side of optimum showed that the effect of confining pressure is more pronounced on lower PI soils. Soils, P-7 and P-17, both have >30% retained on the No. 200 sieve, indicating that they contain a significant sand fraction, which may help explain the significance of confining pressure on these samples when on the dry side of optimum. The effect of confining pressure is least pronounced for higher PI soils. Moreover, the effect of deviator stress for the soils tested in this study is
more pronounced with increasing moisture content, as evidenced by the slope of the $M_r$ versus deviator stress line. This is expected since the specimens compacted at wet of optimum display a weaker soil fabric, and consequently, are less stiff than specimens compacted at dry of optimum. For soils P-7 and P-17 compacted at wet of optimum, there was a tendency of increasing $M_r$ value with increasing deviator stress after a certain point. During testing, it was observed that these specimens tended to bulge radially with increasing deviator stress which helped stiffen the specimens axially, hence results of increasing $M_r$ values with increasing deviator stress.

4.2.1 Effect of Moisture Conditions on Resilient Modulus

As expected, the moisture condition has a significant impact on $M_r$ values. Figure 4.6 displays the effects of moisture content on $M_r$ for the different soil types tested in this study. It can be seen that $M_r$ value decreases with increasing the moisture content. $M_r$ is analogous to the stiffness of a soil, and it is well known that stiffness usually decreases with increasing the moisture content. The effect of moisture conditions on $M_r$ are explored in Figures 4.6 through 4.8.

![Figure 4.6: $M_r$ vs. moisture content](image)

Figure 4.6: $M_r$ vs. moisture content
In Figures 4.7 and 4.8, the $M_r$ values are normalized with respect to $M_r$ values obtained at optimum moisture contents (OMC). The normalized $M_r$ values are plotted.
against the moisture content variation from OMC and variation in degree of saturation (S – \(S_{\text{opt}}\)). Increase in \(\frac{M_r}{M_{\text{r opt}}}\) indicates an increase in \(M_r\) with respect to \(M_r\) at OMC and vice versa when \(\frac{M_r}{M_{\text{r opt}}}\) decreases. Both figures, Figures 4.7 and 4.8, show that \(\frac{M_r}{M_{\text{r opt}}}\) decreases as the specimens conditions get more to the wet side of optimum moisture condition. There is a strong linear relationship, as shown in Figure 4.7, between \(\frac{M_r}{M_{\text{r opt}}}\) and variation in water content for the four soil types. However, Figure 4.8 shows a strong non-linear relationship between \(\frac{M_r}{M_{\text{r opt}}}\) and changes in degree of saturation (S). It should be noted here that the relationship in Figure 4.7 seems to be dependent on the soil type; while the relationship in Figure 4.8 seems to be independent of the soil type.

### 4.3 Soil Water Characteristic Curves

An important part of this study was measuring the Soil Water Characteristic Curves (SWCC), in order to assess the relationship between variation in moisture and matric suction for the four soil types. Figures 4.9 through 4.12 present the SWCC curves obtained for the four soil types. The SWCC curves are plotted as a function of the degree of saturation (S) versus matric suction. These figures display the desorption (drying) curves and the suction values obtained from the Fredlund device and WP4C device. The figures generally show that the matric suction increases as the S value decreases. Measured values are presented along with the predicted curves obtained by performing non-linear least squares optimization. The predicted curves were obtained with Equations 4.1 and 4.2, the Fredlund and Xing (1994) relationship, which is presented here again in Equations 4.1 and 4.2 for the reader’s convenience. Equation 4.1 presents the four parameter version of the Fredlund and Xing (1994) model, while
Equation 4.2 presents the three parameter version of the model. Table 4.4 provides the values of the fitting constants obtained for both versions of the model.

\[
\theta_w = \left[1 - \frac{\ln\left(1 + \frac{\psi_r}{\psi}\right)}{\ln\left(1 + \frac{1,000,000}{\psi_r}\right)}\right] \frac{\theta_s}{\ln\left(e + \left(\frac{\psi}{a}\right)^b\right)^c}
\]  

(4.1)

\[
\theta_w = \left[\frac{\theta_s}{\ln\left(e + \left(\frac{\psi}{a}\right)^b\right)^c}\right] \quad (4.2)
\]

Table 4.4: Fitting Parameters for Fredlund and Xing (1994) Model

<table>
<thead>
<tr>
<th></th>
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<th></th>
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</thead>
<tbody>
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<td></td>
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<td>4.2</td>
<td>4.1</td>
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<tr>
<td></td>
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<td>P-26</td>
<td>P-53</td>
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<td></td>
</tr>
<tr>
<td>(\theta_s)</td>
<td>0.34</td>
<td>0.34</td>
<td>0.32</td>
<td>0.32</td>
<td>0.41</td>
<td>0.41</td>
<td>0.56</td>
</tr>
<tr>
<td>a</td>
<td>268.45</td>
<td>677.76</td>
<td>1431.33</td>
<td>844.05</td>
<td>364.75</td>
<td>912.38</td>
<td>175.45</td>
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<tr>
<td>b</td>
<td>0.73</td>
<td>0.71</td>
<td>23.81</td>
<td>1.05</td>
<td>0.78</td>
<td>0.77</td>
<td>0.74</td>
</tr>
<tr>
<td>c</td>
<td>0.90</td>
<td>1.78</td>
<td>0.13</td>
<td>1.18</td>
<td>0.46</td>
<td>1.30</td>
<td>0.45</td>
</tr>
<tr>
<td>(\psi_r)</td>
<td>655.30</td>
<td>-</td>
<td>92.90</td>
<td>-</td>
<td>425.53</td>
<td>-</td>
<td>3025.43</td>
</tr>
</tbody>
</table>

Figure 4.9: SWCC for soil P-7
Figure 4.10: SWCC for soil P-16

Figure 4.11: SWCC for soil P-26
Figure 4.12: SWCC for soil P -53

In Figures 4.9-4.12, the suction measurements from the Fredlund device and WP4C were combined to create a singular SWCC presenting the relation between degree of saturation and matric suction. However, while the Fredlund device measures the matric suction directly via the axis-translation technique, the WP4C measures the total suction, which is the combination of both matric and osmotic suctions. Since osmotic suction arises from the effects of dissolved solutes, WP4C specimens were prepared utilizing demineralized/distilled water to minimize the impact of osmotic suction. Soils mainly bind water through matric forces, with the absence of dissolved salts in a soil, it can be assumed that the matric suction compromises the majority component of the total suction in soils (Decagon Devices, 2013).

Based on Figures 4.9 through 4.12, it is clear that the matric suction-degree of saturation relationship is unique for each soil type due to the dependence of the
relationship on the pore size distribution and the physiochemical interactions, which are highly dependent on the soil type. Higher PI soils tend to have larger ranges of suction where they undergo desaturation as compared to granular soils that undergo desaturation over a narrow range of suction values. This can be attributed to the fact that capillary forces, which dominate the lower suction range, are the main water holding mechanism in granular/low PI soils; while surface adsorptive forces play a large role in holding water in high PI soils. In addition, the Air-entry Value (AEV) of soils depends on the pore size distribution, such that soils with smaller pores have higher AEV than soils with larger pores. This is evidenced in Figures 4.9 – 4.12, with P-7 a low PI soil with a significant sand content, has the lowest AEV value amongst the soils evaluated. However, the P-53 soil, which has the largest clay fraction amongst the soils tested, and therefore the smallest pores, has the largest AEV value. The SWCC curves for the four soils are plotted together in Figure 4.13 for comparison and to give the reader a clear view of the above discussed fundamentals.

Figure 4.13: Comparison of SWCC’s for all soil tested.
4.3.1 Hysteric Behavior of SWCC

As mentioned in Chapter 2, the Mr – matric suction relationship captured by the SWCC is dependent on the moisture change path (i.e., drying or wetting) followed by the specimen. At a given matric suction, a specimen can have different water contents, depending on whether the soil undergoes drying or wetting path. This phenomenon was explored in this study by utilizing the Fredlund device to conduct a specific test in which the soil specimen was subjected to a drying path followed by a wetting path. Soil P-7 was selected for this test due to its' favorable characteristics. It has the lowest PI of the soils tested in this study, and therefore, it was possible to achieve substantial desorption by utilizing the Fredlund device with a matric suction limit of 1500 kPa. Also, P-7 has shorter equilibrium time than the other three soil types, which was a key factor since equilibrium times on the wetting path tend to be much longer than those on the drying path.

The test was performed by initially saturating the specimen, which was then subjected to a drying path utilizing the Fredlund device by incrementally increasing the applied matric suction. Once the equilibrium was achieved at the final increment, the applied matric suction was decreased. As matric suction was decreased, the specimen began to imbibe water. Equilibrium was considered achieved when there was no change in water levels. The applied matric suction during the wetting path was decremented following the same interval in which it was incremented on the drying path. Figure 4.14 presents the hysteresis behavior of SWCC for P-7 soil. It should be noted that during imbibition (wetting) soil does not reach saturation due to the presence of entrapped air. The water content at '0' suction is approximately equal to 90% of the
water content at saturation (Rogowski, 1971). The predicted drying path in the figure is obtained utilizing Equation 4.2.

![Graph showing the relationship between degree of saturation and matric suction.](image)

**Figure 4.14: Measured Drying and Wetting Path for P-7 Soil Utilizing Fredlund Device**

### 4.3.2 Resilient Modulus – Matric Suction

An important aspect of this study was to evaluate the relationship between the resilient modulus (\( M_r \)) and matric suction. Since the matric suction changes as the moisture conditions of the soil change. It is expected that the change in matric suction will also impact the \( M_r \) value. In order to evaluate the \( M_r \) – matric suction relationship, matric suction values for each \( M_r \) test were obtained by correlating the degree of saturation of the \( M_r \) specimen during testing to a corresponding degree of saturation on the SWCC. Table 4.5 provides the matric suction values obtained for the \( M_r \) tests for each soil type. It was assumed that compacting the soil in an unsaturated state would induce a suction corresponding to a similar degree of saturation on the SWCC. Figure
4.15 and 4.16 present the $M_r$ – suction relationships for the soil types tested in this study.

Figure 4.15: Resilient Modulus versus Matric Suction for all soils tested

Figure 4.16: $M_r / M_{opt}$ Vs. Matric Suction for the Four Soil Types Tested
Figure 4.15 displays the dependence of $M_r$ on matric suction for each soil type individually. It can be seen that $M_r$ increases with an increase in matric suction, which can be attributed to the stiffening effect a soil specimen experiences as matric suction increases. As seen in Figure 4.15, there is a power relationship trend between $M_r$ and matric suction, such that $M_r$ increases with increasing matric suction. Figure 4.16 illustrates the relationship between the normalized $M_r$ values ($M_r / M_{r\text{opt}}$) and matric suction. While each soil individually displays a power relationship between $M_r$ and matric suction, when the results from all soils are combined, an acceptable logarithmic linear relationship can be seen. This indicates that the matric suction could serve as a good predictor variable for observing the increase/decrease in $M_r$ due to variation in moisture content for unsaturated soils, regardless of the soil type. Table 4.5 provides a summary of the magnitude of induced matric suctions for the different $M_r$ specimens, obtained for each soil type.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Moisture Content</th>
<th>Degree of Saturation (%)</th>
<th>Matric Suction (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-7</td>
<td>OMC -3%</td>
<td>63.6</td>
<td>435</td>
</tr>
<tr>
<td></td>
<td>OMC</td>
<td>82.1</td>
<td>104</td>
</tr>
<tr>
<td></td>
<td>OMC +3%</td>
<td>87.9</td>
<td>54</td>
</tr>
<tr>
<td>P-17</td>
<td>OMC -3%</td>
<td>67.5</td>
<td>1169</td>
</tr>
<tr>
<td></td>
<td>OMC</td>
<td>83.6</td>
<td>405</td>
</tr>
<tr>
<td></td>
<td>OMC +3%</td>
<td>91.5</td>
<td>199</td>
</tr>
<tr>
<td>P-26</td>
<td>OMC -6%</td>
<td>58.6</td>
<td>1954</td>
</tr>
<tr>
<td></td>
<td>OMC -3%</td>
<td>76.6</td>
<td>563</td>
</tr>
<tr>
<td></td>
<td>OMC</td>
<td>88.3</td>
<td>192</td>
</tr>
<tr>
<td></td>
<td>OMC +3%</td>
<td>92.3</td>
<td>95</td>
</tr>
<tr>
<td>P-53</td>
<td>OMC -6%</td>
<td>68.9</td>
<td>1497</td>
</tr>
<tr>
<td></td>
<td>OMC -3%</td>
<td>79.5</td>
<td>499</td>
</tr>
<tr>
<td></td>
<td>OMC</td>
<td>85.4</td>
<td>245</td>
</tr>
<tr>
<td></td>
<td>OMC +3%</td>
<td>91.7</td>
<td>83</td>
</tr>
<tr>
<td></td>
<td>OMC +6%</td>
<td>94.2</td>
<td>43</td>
</tr>
</tbody>
</table>
4.4 Tube Suction Test

Tube suction tests (TST) were performed on the soils evaluated in this study to assess their moisture susceptibility. Evaluating the moisture susceptibility of a sample via tube suction tests involves allowing the test specimen to undergo capillary soaking while simultaneously measuring its' surface dielectric (DV) value utilizing a probe. Poorly performing specimens tends to reach saturation quickly while attaining a high maximum DV value. Figure 4.17 presents results of the tube suction tests that were performed as part of this study. The results are given in terms of maximum DV achieved by the different soil specimens. The DV value of a specimen under capillary soaking increases gradually till reaching a maximum DV value and then stabilize. Scullion and Saarenekto (1997) proposed a maximum DV criteria for identifying the quality of base materials, as described in Figure 4.17. However, it should be noted that this criteria was proposed to evaluate the suitability of base materials, while the soils utilized in this study represent subgrade materials. The time needed to reach the maximum DV varied for the different soil types, with P-7 soil reaching the maximum DV quicker than the other soils in this study. The maximum DV for P-53 soil is not presented in Figure 4.17 due to an impractical amount of time needed for the specimen to reach the maximum DV. Presumably, the P-53 soil with a PI of 53 has an extremely low hydraulic conductivity, which may be the main determining factor that the P-53 soil is not able to reach a maximum DV after long monitoring period. During testing, no noticeable change was observed in the surface DV measurements for the P-53 soil, even after a considerably prolonged monitoring period.
Figure 4.17: Maximum DV values from TST for soils tested with Scullion and Saarenketo (1997) classification criterion
CHAPTER 5 ANALYSIS

The results obtained from the laboratory testing, presented in Chapter 4, were utilized to examine existing relationships between $M_r$ and moisture conditions and to propose new relationships, which will be discussed in this chapter. The effect of moisture conditions on $M_r$ was evaluated in terms of changes in gravimetric water content, degree of saturation, and matric suction. Existing constitutive relationships between $M_r$ and matric suction will be evaluated in this chapter. A proposed modified $M_r$-matric suction relationship will be presented. Statistical models to evaluate regression constants for $M_r$ constitutive models, based on soil physical properties, and with the ability to incorporate the effect of moisture variation on $M_r$, will be presented in this chapter as well.

5.1 Effect of Stress State on Resilient Modulus

As shown in figures presented in Chapter 4, resilient modulus ($M_r$) is affected by the stress state experienced by the soil. During laboratory testing stress state of a specimen being tested is varied by changing the confining pressure and deviatoric stresses. Generally, a decrease in $M_r$ is seen with decreasing confining pressure and increasing deviatoric stress. For simplicity, $M_r$ values are generally presented at a specific stress state. While there are several suggested stress states for subgrades, the stress state recommended by SHRP P-46 for subgrades is utilized in this study. This stress state corresponds approximately to a bulk stress of 22.5 psi and an octahedral shear stress of 2.0 psi.

\[ \sigma_b = \sigma_1 + \sigma_2 + \sigma_3 \]  

(5.1)

Where:
\[ \sigma_b = \text{bulk stress} \]

\[ \sigma_1, \sigma_2, \sigma_3 = \text{Major, intermediate, and minor stresses respectively} \]

\[ \tau_{oct} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \]  \hspace{1cm} (5.2)

Where:

\[ \tau_{oct} = \text{octahedral shear stress} \]

### 5.1.1 Universal \(M_r\) Model

Since it is evident \(M_r\) is impacted by stress state, constitutive models created to predict \(M_r\) incorporate the stress state experienced by the soil. One of the most widely recognized models was proposed by Witczak and Uzan (1998), subsequently adopted by MEPDG and dubbed the “Universal” model. For the readers’ convenience the model is presented here again in Equation 5.3.

\[ M_r = k_1 P_a \left( \frac{\theta}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{k_3} \]  \hspace{1cm} (5.3)

Where:

\[ P_a = \text{atmospheric pressure} \]

\[ \tau_{oct} = \text{octahedral shear stress} = \frac{\sqrt{2}}{3} (\sigma_1 - \sigma_3) \text{ when } \sigma_2 = \sigma_3 \]

\[ k_1, k_2, k_3 = \text{regression constants} \]

This model is widely accepted because it generally provides a good fit when \(M_r\) is assessed for a single test for most soil types. For this study, nonlinear regression analysis was performed for each individual \(M_r\) test to obtain the regression constants \((k_1, k_2, k_3)\) and also to assess the adequacy of the fit to the test data by evaluating the coefficient of determination (\(R^2\)). Regression analysis was performed by utilizing the Solver Add-in in Microsoft Excel to minimize the sum of square errors (SSE), and
subsequently $R^2$ was calculated. It should be noted that coefficient of determination, $R^2$, refers to the goodness of fit for a linear regression. However, for nonlinear regression $R^2$ does not have a clear definition. Therefore, the $R^2$ values discussed in this chapter will refer to the Psuedo $R^2$ value. Equation 5.5 provides the relationship utilized to evaluate the pseudo $R^2$ value.

$$SSE = \sum(Y - Y_i)^2$$  \hspace{1cm} \text{Equation (5.4)}

Where:

$Y = \text{Measured value}$,  $Y_i = \text{Predicted value}$

$$R^2 = 1 - \frac{SSE}{SST}$$  \hspace{1cm} \text{Equation (5.5)}

Where:

SSE = sum of square errors

SST = Total Sum of squares;  $SST = \sum(Y_i - \overline{Y})^2$ where $\overline{Y}$ is the mean.

Table 5.1 presents the results obtained from the Regression Analysis performed on the measured $M_r$ values by utilizing Equation 5.3. The values of the regression constants, along with $R^2$, are shown for each soil and moisture content tested, the values correspond to averages obtained from the three (3) replicate specimens. Some general trends due to changes in moisture conditions can be noted amongst the regression constants. Generally, it can be seen that the $k_1$ coefficient achieves its’ maximum value on the dry side and its’ value decreases with increasing moisture content. This is similar to Nazzal and Mohammad (2010), they stated that $k_1$ is proportional to the stiffness of the material which is dependent upon the effective stress of the soil. In unsaturated soils, effective stress is dependent on matric suction, and matric suction increases with decreasing water content, therefore the increase in $k_1$ can
be attributed to an increase in matric suction. The coefficient for $k_2$ is a little more variable but generally tend to decrease with increasing moisture content. Generally, the value of the $k_3$ coefficient is negative. The $k_3$ coefficient describes the softening of the material with increasing octahedral shear stress Nazzal and Mohammad (2010). It should be noted that generally, $k_3$ values become more negative with increasing moisture content. This could imply that the materials at a higher moisture content are more susceptible to weakening due to an increase in shear stress (Nazzal and Mohammad, 2010). The $k_3$ coefficients display a positive value for P-17 at OMC +3 and P-7 at OMC and OMC +3, these specimens tended to deform axially while bulging radially with increasing deviator stress. This had a stiffening effect on the specimen with increasing deviator stress, hence the positive $k_3$ coefficients.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Moisture Condition</th>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$k_3$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-53</td>
<td>OMC -6</td>
<td>805.86</td>
<td>0.30</td>
<td>-1.09</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>OMC -3</td>
<td>706.06</td>
<td>0.24</td>
<td>-1.40</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>OMC</td>
<td>636.91</td>
<td>0.38</td>
<td>-1.45</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>OMC +3</td>
<td>476.30</td>
<td>0.09</td>
<td>-0.95</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>OMC +6</td>
<td>393.71</td>
<td>0.14</td>
<td>-1.94</td>
<td>0.96</td>
</tr>
<tr>
<td>P-26</td>
<td>OMC -6</td>
<td>794.46</td>
<td>0.24</td>
<td>-0.61</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>OMC -3</td>
<td>631.89</td>
<td>0.19</td>
<td>-0.90</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>OMC</td>
<td>548.70</td>
<td>0.16</td>
<td>-1.90</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>OMC +3</td>
<td>258.56</td>
<td>0.08</td>
<td>-2.68</td>
<td>0.89</td>
</tr>
<tr>
<td>P-17</td>
<td>OMC -3</td>
<td>812.91</td>
<td>0.37</td>
<td>-1.52</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>OMC</td>
<td>400.50</td>
<td>0.36</td>
<td>-1.26</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>OMC +3</td>
<td>119.98</td>
<td>0.22</td>
<td>1.28</td>
<td>0.78</td>
</tr>
<tr>
<td>P-7</td>
<td>OMC -3</td>
<td>568.53</td>
<td>0.39</td>
<td>-1.07</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>OMC</td>
<td>303.26</td>
<td>0.24</td>
<td>0.15</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>OMC +3</td>
<td>143.15</td>
<td>0.20</td>
<td>1.89</td>
<td>0.94</td>
</tr>
</tbody>
</table>
5.2 Effect of Moisture Variation on Resilient Modulus Values

The moisture content has a significant impact on the resilient modulus ($M_r$) of subgrade soils. The $M_r$ of a subgrade is critical in quantifying the support provided to the pavement by the underlying subgrade. Considering that the moisture conditions of a subgrade are cyclically varying due to seasonal changes, it is important to observe the impact moisture changes have on $M_r$ for subgrade soils. Figures and tables were presented in Chapter 4 showed the decrease in $M_r$ with increasing moisture content. An increase in moisture content weakens the soil fabric which in turn leads to an increased susceptibility to deformation. While the behavior can be discussed in terms of matric suction for unsaturated soils, generally it is more practical to discuss it in terms of moisture variation. The Mechanistic Empirical Pavement Design Guide (MEPDG) utilized the Enhanced Integrated Climatic Model (EICM) to account for the effect of seasonal variation on $M_r$; and Equation 5.6 presents the relationship utilized by EICM.

$$\log\left(\frac{M_r}{M_{rop}}\right) = a + \frac{b-a}{1+EXP\left(\log\left(\frac{b}{a} + km^* (S-S_{opt})\right)\right)}$$  \hspace{1cm} (5.6)

Where:

$M_r / M_{rop} =$ resilient modulus ratio

$a =$ minimum of log ($M_r/M_{rop}$)

$b =$ maximum of log ($M_r/M_{rop}$)

$km =$ regression parameter

$(S - S_{opt}) =$ variation in degree of saturation (expressed as a decimal)

MEPDG has recommended values of a, b, and km for fine-grained soils. By utilizing these values, Figure 5.1 provides presents the relationship between measured $M_r$ values, obtained from the laboratory testing in this study, versus predicted $M_r$ values
obtained utilizing Equation 5.6. The $R^2$ (pseudo) value, presented in Figure 5.1, indicates Equation 5.6 provides an adequate fit to the measured data from this study.

Figure 5.2 provides a graphical representation of Equation 5.6, where the ratio $M_r / M_{opt}$ is plotted against variation of degree of saturation $S$; the measured data from this study is plotted on the figure as well. Figure 5.7 shows that the trend obtained from Equation 5.6, for fine-grained soils, is represented well by the measured data from this study.

![Figure 5.1: Measured versus Predicted $M_r$ values with Equation 5.6](image1)

![Figure 5.2: Graphical representation of Equation 5.6 for fine-grained soils with scatter of measured data](image2)
While Equation 5.6 provides a good prediction for the measured data from this study, as evidenced by Figure 5.2, it was worthwhile trying to establish a separate relationship between the normalized Mr values (Mr/M_{opt}) versus variation of degree of saturation from optimum, utilizing the data from this study; this relationship is presented in Figure 5.3. As shown in the figure, a polynomial regression function was the best to describe this relationship. It provided a $R^2$ of 0.81, which indicates a good fit. The best fit polynomial equation from Figure 5.3 (Equation 5.7) was evaluated with data from this study, and from Drumm et al. (1997). Figure 5.4 shows the trend between measured Mr values from this study, and Drumm et al. (1997), and predicted Mr from Equation 5.7. As evidenced by the $R^2$ value, Equation 5.7 gives a good prediction to the data.

$$
\frac{Mr}{M_{opt}} = -0.0009x^2 - 0.0511x + 1
$$

Equation 5.7

Where:

$x = S - S_{opt}$ ($\%$); Valid for $-30 \leq x \leq 10$

![Figure 5.3: Mr / M_{opt} Vs. S - S_{opt} (%) for measured data with a polynomial best fit line](image-url)
Figure 5.4: Predicted $M_r$ values from Equation 5.7 versus measured $M_r$ values

The impact of moisture variation on the $M_r$ value of unsaturated subgrade values can be evaluated utilizing several different methods. As discussed, one method to capture the effect of moisture variation on the $M_r$ value is utilizing degree of saturation. Another simple, and popular, approach is considering the effect of moisture variation on the $M_r$ value in terms of gravimetric water content. Figure 5.5 shows the relationship between $M_r/M_{r_{opt}}$ versus $w - w_{opt}$ (%), which represents the variation of water content with respect to OMC. Based on Figure 5.5, a discernible relationship does not exist between the two quantities; this is in contrast to the relationship observed between $M_r/M_{r_{opt}}$ and $S - S_{opt}$. However, in Figure 5.6 the values for $w - w_{opt}$ are normalized with respect to plasticity index (PI), and a good relationship can be observed. This indicates that the relationship between $M_r/M_{r_{opt}}$ and variation in water content must include the effect of soil type to provide a good fit.
Based on the figures above, it can be established that variations in moisture conditions have a definite impact on $M_r$ and a relationship can be established between the variations of $M_r/M_{ropt}$ and variation of moisture conditions from conditions at OMC. However, based on the data obtained in this study, it is better to represent changes in moisture in terms of degree of saturation ($S$) compared to representing the changes in terms of gravimetric water content. The $R^2$ obtained for the $M_r/M_{ropt}$ versus $S - S_{opt}(\%)$ relationship was 0.82. While there was no trend when $M_r/M_{ropt}$ was compared to $w - w_{opt}$.
$w_{opt}(\%)$, only when $w - w_{opt}(\%)$ is normalized with respect to PI is a relationship observed. It appears as if the $M_r/M_{ropt}$ versus $S - S_{opt}(\%)$ is not greatly affected by soil type. Degree of saturation ($S$) maybe a better predictor for changes in $M_r$, compared to gravimetric water content, because it includes the effects of dry density. Also, degree of saturation ($S$) is directly related to matric suction via the SWCC, therefore changes in $S$ also imply a change in the suction value. Considering degree of saturation ($S$) is an easily accessible soil property, it may be advantageous to predict changes in $M_r$, due to changes in moisture conditions, in terms of changes in degree of saturation ($S$) instead of changes in water content.

5.3 Resilient Modulus – Matric Suction Relationship

The moisture dependence of resilient modulus $M_r$ can be viewed through the lens of matric suction for unsaturated soils. As the moisture content of the soil varies so does the matric suction. For unsaturated soils, matric suction contributes to the effective stress, therefore, the $M_r$-matric suction relationship has a sound theoretical framework. In this section, several existing $M_r$ relationships which incorporate suction will be reviewed and their fit to the data measured in this study will be analyzed.

5.3.1 $M_r$ – Matric Suction Relationship – Stress Dependent

The literature reveals that several constitutive models have been proposed to incorporate the effect of matric suction in predicting $M_r$. Generally, the purpose of these models is twofold, they provide a better theoretical framework for unsaturated soils by incorporating suction, and they also take into account the effect of moisture variation on changes in $M_r$. The following models are generally based on the Universal $M_r$ model
(Equation 5.3), and the effective stress equation for unsaturated soils (Bishop, 1959) which is presented here again for the readers' convenience.

\[
\sigma'_v = (\sigma_z - U_a) + \chi (U_a - U_w)
\]  

(5.8)

Gupta et al. (2007) proposed a model (Equation 5.9) to incorporate suction in evaluating \(M_r\) by proposing the addition of a new term to the Universal \(M_r\) model to account for suction. By using data obtained from the laboratory testing program of this study, the Gupta et al. (2007) model was evaluated. The results of the measured versus predicted values are presented in Figure 5.7, along with the fitting parameters. The matric suction values were obtained by correlating the degree of saturation (S) of the \(M_r\) specimen to the SWCC for the soil type being tested. Subsequently, regression analysis was performed for each soil type, across all moisture contents tested, to obtain one set of fitting parameters for the soil type in question. Figure 5.8 presents the results of the measured and predicted values for soil P-26. From Figure 5.8 it can be seen that Equation 5.9 provides a good fit to the measured data for P-26. Table 5.2 presents the results of the nonlinear regression analysis performed on the three other soil types utilizing Equation 5.9. Based on Figure 5.8 and Table 5.2, Equation 5.9 provides a good fit to the measured \(M_r\) data from this study. However, Equation 5.9 also presents a significant drawback due to the need for evaluating five (5) regression constants.

\[
M_r = \left( k_1 P_a \left( \frac{\theta}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{k_3} \right) + \alpha (U_a - U_w)^\beta
\]  

(5.9)

Where:

\(\theta\) - bulk stress

\(\tau_{oct}\) – octahedral shear stress

\(U_a - U_w\) – matric suction
\( k_1, k_2, k_3, \alpha, \beta \) – fitting parameters

### Table 5.2: Regression Constants Obtained From Equation 5.9

<table>
<thead>
<tr>
<th>Soil No.</th>
<th>P-7</th>
<th>P-17</th>
<th>P-53</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 )</td>
<td>51.96</td>
<td>71.74</td>
<td>225.41</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>1.73</td>
<td>3.17</td>
<td>1.19</td>
</tr>
<tr>
<td>( k_3 )</td>
<td>-0.88</td>
<td>-7.71</td>
<td>-6.06</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>968.80</td>
<td>119.47</td>
<td>2472.74</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.46</td>
<td>0.86</td>
<td>0.24</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.92</td>
<td>0.95</td>
<td>0.94</td>
</tr>
</tbody>
</table>

![Figure 5.7: Measured Vs. Predicted Mr for soil P-26 values utilizing Equation 5.9](image)

Liang et al. (2008) also proposed a model which incorporated suction into the Universal \( M_r \) model. However, instead of utilizing an additional term to incorporate suction, Liang et al. (2008) proposed including the effect of suction as a part of the bulk stress. This approach has some validity since an increase in matric suction is thought to have a stiffening effect on the soil, which is similar to what occurs when there is an increase in bulk stress. Equation 5.10 provides the model proposed by Liang et al. (2008).
\[ M_r = k_1 P_a \left( \frac{\theta + \chi_w \psi_m}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{k_3} \]  

(5.10)

Where:

\( \theta = \) bulk stress

\( \chi_w = \) Bishop’s parameter

\( \psi_m = \) matric suction

\( \tau_{oct} = \) Octahedral shear stress

\( P_a = \) atmospheric pressure

\( k_1, k_2, k_3 = \) regression constants

To evaluate Bishop’s \( \chi \) parameter for Equation 5.10, Liang et al. (2008) used the definition of \( \chi \) developed by Khalili and Khabbaz (1998), this relationship is presented in Equation 5.11. The relationship in Equation 5.11 was developed on the premise that the relationship between \( \chi_w \) and suction is linear on a log-log scale, when the suction being evaluated is greater than the suction at air-entry.

\[ \chi_w = \left( \frac{(U_a - U_w)b}{U_a - U_w} \right)^{0.55} \]  

Equation (5.11)

Where:

\( (U_a - U_w)b = \) Air-entry value

\( U_a - U_w = \) matric suction

Measured data from this study was evaluated utilizing Equation 5.10, data was evaluated by performing regression analysis across all moisture contents each specimen was tested \( (M_r) \) at. Figure 5.8 provides the results obtained via regression analysis for soil P-26. The Air-Entry Value (AEV) was evaluated utilizing the procedure shown in Figure 5.9. As presented, two (2) asymptotic lines were drawn and their intersection was taken as the air-entry value. The procedure presented in Figure 5.9 is
similar to the one utilized by Liang et al. (2008) to evaluate AEV. Once the AEV was determined, it was possible to calculate Bishop’s parameter via Equation 5.11 for different moisture contents.

Figure 5.8: Measured Vs. Predicted Mr for soil P-26 values utilizing Equation 5.10

Figure 5.9: Illustration of method to obtain Air-entry value via SWCC
As evidenced by the $R^2$ value of 0.81 obtained by fitting the measured data to Equation 5.10 Liang et al. (2008) provides a good fit. However, Equation 5.10 performed inconsistently when measured data from the other three (3) soil types tested in this study, with $R^2$ values ranging from 0.17 to 0.85. The relationship utilized by Liang et al. (2008) to evaluate Bishop’s parameter poses some uncertainties since Khalili and Khabbaz (1998) developed this relationship based on static triaxial shear strength testing of unsaturated soils. Resilient modulus testing is obviously performed under dynamic loading. Also, Equation 5.11 is only valid for suction above the AEV. This may pose some concerns when considering soils near saturation.

Cary and Zapata (2011) also proposed a model to incorporate suction in predicting $M_r$ for unsaturated soils. This model, presented in Equation 5.12, is also a variation of the Universal $M_r$ model proposed by Witczak and Uzan (1988). The main difference between the model given in Equation 5.12 and other matric suction dependent $M_r$ models is the utilization of the change in matric suction due to pore-water pressure (PWP) buildup during repeated loading. Cary and Zapata (2011) believed that the effect of PWP buildup is significant under long term dynamic loading, and it is prudent to include its’ effect in predicting $M_r$ because PWP build up decreases effective stress which can negatively impact $M_r$.

$$M_r = k_1 P_a \left( \frac{\theta_{net} - 3 \Delta U_w - sat}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{k_3} \left( \frac{(\psi_m - \Delta \psi_m)}{P_a} + 1 \right)^{k_4} \quad (5.12)$$

Where:

- $k_1 \geq 0$, $k_2 \geq 0$, $k_3 \leq 0$, and $k_4 \geq 0$ = regression constants
- $\theta_{net} = \theta - 3 u_a$, net bulk stress where $u_a$ is pore air pressure
- $\Delta U_w - sat$ = build up of PWP under saturated conditions; $\Psi_m = 0$
- $\Psi_m$ = initial matric suction
\[ \Delta \Psi m = \text{relative change in matric suction w/ respect to } \Psi m \text{ due to buildup of PWP under unsaturated conditions; in this case } \Delta U_{w-sat} = 0 \]

Regression analysis was performed to fit the measured data from this study to Equation 5.12. However, buildup of PWP during Mr testing was not measured during this study. Therefore, Equation 5.12 was utilized with \( \Delta U_{w-sat} = 0 \) and \( \Delta \Psi_m = 0 \). Mr testing for this study was conducted in general accordance with AASHTO T-307-99, which calls for Mr testing to be performed under drained conditions. Generally, when testing a soil under drained conditions it is expected that there will be no PWP buildup. This assumption is generally safe for soils with good permeability (e.g. granular soils). However, all the soils tested in this study would be classified as fine-grained soils (per AASHTO classification) and it may not be safe to assume that there was no PWP buildup during Mr testing. Nonetheless, the number of loading cycles applied per AASHTO T-307-99 are relatively small compared what may be needed for significant PWP buildup (Cary, 2011). Figure 5.10 shows the results obtained from regression analysis to fit measured data for soil P-26 to Equation 5.12.

Judging from the \( R^2 \) value (0.85) in Figure 5.11, Equation 5.12 provides a good fit for the measured data for soil P-26. This was indeed also true for the other three (3) soil types in this study, with \( R^2 \) ranging from 0.91 – 0.97. Table 5.3 presents the \( R^2 \) values obtained for the four soil types utilizing the three different models analyzed up to this point. While Equation 5.12 certainly seems to be a valuable model, with validity to the approach of accounting for the effects of PWP buildup under dynamic loading, it does have some drawbacks. PWP buildup under repeated loading can be difficult to measure, requiring a specialized system, and it is also difficult to predict since it
depends on a variety of factors (e.g. # of loading cycles, duration of load, dwelling time between repetitions). Also, Equation 5.12 does not account for the variation in contribution of matric suction to effective stress as moisture content changes. The variation in contribution is generally assessed via Bishop’s parameter. However, judging from the results obtained from this study, it may be possible to utilize Equation 5.12 without including the effect of PWP buildup if the number of load cycles is relatively small.

Figure 5.10: Measured versus Predicted values for P-26 utilizing Equation 5.12

Table 5.3: Results of Non-Linear Regression Analysis

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P-53</td>
<td>0.94</td>
<td>0.85</td>
<td>0.92</td>
</tr>
<tr>
<td>P-26</td>
<td>0.88</td>
<td>0.81</td>
<td>0.85</td>
</tr>
<tr>
<td>P-17</td>
<td>0.95</td>
<td>0.69</td>
<td>0.97</td>
</tr>
<tr>
<td>P-7</td>
<td>0.91</td>
<td>0.64</td>
<td>0.9</td>
</tr>
</tbody>
</table>
5.3.2 $M_r$ – Matric Suction Relationship – Stress Independent

As shown previously, the $M_r$ value is dependent upon the stress state of the soil, however in pavement design, a designer can generally select an acceptable stress state that the subgrade will experience over its’ service life. The equations presented previously, relating $M_r$ to matric suction, can then be greatly simplified if a specific stress state is utilized. The stress state for $M_r$ throughout this study specified by Strategic Highway Research Program (SHRP) Protocol P-46 (Drumm, 1997) was adopted. This stress state corresponds to a deviatoric stress of 4 psi (28 kPa) and a confining pressure of 6 psi (41 kPa).

With single stress state, Equation 5.9 (Gupta et al., 2007) can then be reduced to Equation 5.13. Per Equation 5.13, it can be seen that $M_r$ varies with suction as a power function relationship. Equation 5.13 was utilized to establish a relationship between $M_r$ and matric suction for the soils tested in this study. For each soil type, $M_r$ values, from each moisture content tested, were selected to correspond to the SHRP P-46 stress state. Subsequently, linear regression analysis was performed utilizing the measured data, and the predicted data utilizing Equation 5.13 to obtain the fitting parameters $\alpha$ and $\beta$. Figure(s) 5.11-5.4 provide the $M_r$ versus matric suction relationship obtained utilizing Equation 5.13.

$$M_r = \alpha \Psi^\beta$$  \hspace{1cm} \text{Equation (5.13)}

Where:

$\alpha, \beta =$ fitting parameters

$\Psi =$ matric suction

\hspace{1cm} 126
Figure 5.11: Mr Vs. Matric Suction for P-53 utilizing Equation 5.13

Figure 5.12: Mr Vs. Matric Suction for P-26 utilizing Equation 5.13

Figure 5.13: Mr Vs. Matric Suction for P-17 utilizing Equation 5.13
Figure 5.14: $M_r$ versus Matric Suction for P-7 utilizing Equation 5.13

By examining the coefficient of determination, $R^2$, it can be seen that the model provides a good fit for the measured data. From Figures 5.11-5.14, it can be observed that there indeed exists a non-linear relationship between matric suction and $M_r$, with $M_r$ increasing as suction increases. However, the trend between $M_r$ increase with increasing suction seems to differ slightly between the higher PI (P-53 and P-26) and lower PI (P-17 and P-7) soils.

A similar approach was also applied to Liang et al. (2008) model. Equation 5.14 provides the version of Liang et al. (2008) which would occur if $M_r$ values were evaluated at different suction values but at the stress state represented by SHRP P-46. The $k_3$ term is neglected since octahedral shears stress remains constant for the different suction values. Regression analysis was performed to fit the measured $M_r$ data to Equation 5.14 and evaluate the regression constants, $k_1$ and $k_2$. Figure(s) 5.15 – 5.18 provide the relationship between matric suction and $M_r$ obtained by utilizing Equation 5.14, for each soil type tested in this study.

$$M_r = k_1 \left( \frac{22.5 + \chi_w \psi_m}{14.7} \right)^{k_2}$$  \hspace{1cm} \text{Equation (5.14)}
Where:

\( \chi_w \) = Bishop’s Parameter

\( \Psi_m \) = matric suction

\( k_1, k_2 \) = fitting parameters

Figure 5.15: \( M_r \) versus Matric Suction utilizing Equation 5.14 for P-53

Figure 5.16: \( M_r \) versus Matric Suction utilizing Equation 5.14 for P-26
Based on the results above, the version of Liang et al. (2008) presented in Equation 5.14 provides a good fit to the measured laboratory data from this study. The
results from Figure(s) 5.15 – 5.18 are similar to those presented in Figure 5.11-5.14, with \( M_r \) increasing as matric suction increases. Also, the relationship(s) seen in Figure 5.15-5.18 show a non-linear trend of increasing \( M_r \) versus increasing suction; the trend(s) are similar to those seen in Figure(s) 5.15-5.18. This reinforces that the \( M_r \)-matric suction relationship may be best represented by a power function.

5.4 Proposed Constitutive Model to Capture Effect of Matric Suction in Predicting \( M_r \)

It has been demonstrated throughout this study that matric suction has a significant impact on \( M_r \) of unsaturated subgrade soils, also, matric suction is a component of the stress state of unsaturated soils. Therefore, a sound theoretical approach for predicting \( M_r \) of unsaturated soils should incorporate matric suction. As an added benefit, it is possible to incorporate the effect of seasonal moisture changes in unsaturated subgrade soils on \( M_r \) since matric suction varies with moisture content. In this section, a constitutive model, which incorporates the effect of suction in predicting \( M_r \) for unsaturated soils will be proposed.

5.4.1 Constructing Model

There has long been an effort to place emphasis on incorporating matric suction when developing constitutive relationships for unsaturated soils. Generally, these efforts have resulted in better predictive capability of strength and deformation characteristics of unsaturated soils. Fredlund et al. (1978) proposed a linear function which incorporated matric suction in predicting the shear strength of unsaturated soils. This relationship is present in Equation 5.15.

\[
\tau_f = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b
\]  

(5.15)

Where:
\[ \tau_r = \text{shear strength of unsaturated soil} \]

\[ c' = \text{effective cohesion of saturated soil} \]

\[ \phi' = \text{effective angle of shearing resistance for saturated soil} \]

\[ \phi^b = \text{angle of shearing resistance with respect to matric suction} \]

\[ (\sigma_n - u_a) = \text{net normal stress on the plane of failure} \]

\[ (u_a - u_w) = \text{matric suction on plane of failure} \]

The relationship in Equation 5.15 derives the shear strength of an unsaturated soil by adding the contribution of matric suction to shear strength when the soil is unsaturated, \((u_a - u_w) \tan \phi^b\), to the shear strength of a saturated soil, \(c' + (\sigma_n - u_a) \tan \phi'\). Subsequent research (Gan et al., 1988; Escario and Juca, 1989), however, found that the shear strength-matric suction relationship for unsaturated soils shows a non-linear relationship. This is expected since the contribution of matric suction to effective stress of unsaturated soils varies with the amount of water in the soil. Bishop (1959) took this into account when proposing the effective stress equation for unsaturated soil by incorporating Bishop’s parameter, \(\chi\), which represented the contribution of suction to effective stress and varied from 1 when the soil is saturated to 0 when the soil is completely dry. Based on this, it can be inferred that the contribution of matric suction to effective stress increase with an increase in the amount of water in the soil.

The contribution of matric suction to effective stress, and consequently shear strength of unsaturated soils is thought to vary with the area of water, i.e. the area of water menisci which is in contact with soil/aggregate particles (Vanapalli et al., 1996). Initially, when the soil is saturated the area of water is equal to unity, however, as the
soil begins to de-saturate (as suction increases) and air begins to enter the soil pores, the area of water in contact with the soil particles begins to decrease. Figure 5.20 (Vanapalli, 1994), illustrates the changes in the area of water that occur as the soil de-saturates. Initially, the soil is saturated and the area of water in contact with soil particles is continuous. As suction increases to values about the air-entry value, the water content of the soil decreases with increasing suction and consequently, the area of water in contact with the soil particles also reduces. This continues till residual saturation condition is achieved. Under residual saturation conditions, a large increase in suction only causes a small decrease in water content. Also, under residual saturation conditions the area of water in contact with the soil particles is discontinuous and very small.

Considering that the impact of PWP on effective stress varies with the amount of water in contact with the soil particles, Vanapalli et al. (1996) thought to the include the effect of area of water when considering the relationship between suction and shear strength for unsaturated soils. Therefore, the contribution of suction to shear strength was represented in terms of the area of water. Vanapalli et al. (1996) saw similarities between the area of water and the normalized water content, and subsequently represented the area of water as the normalized water content, the relationship for normalized water content is presented in Equation (5.16).
Figure 5.19: Variation of area of water with increase suction (Vanapalli, 1994)

\[
\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} 
\]  

(5.16)

Where:

\( \Theta \) = normalized water content

\( \theta \) = volumetric water content

\( \theta_r \) = volumetric water content at residual condition

\( \theta_s \) = volumetric water content at saturated condition

Based on the principles presented above, Vanapalli et al. (1996) proposed the non-linear relationship presented in Equation 5.17 to evaluate the shear strength of unsaturated soils. While similar to the linear relationship presented in Equation 5.15, there are two distinct differences between the linear shear strength-suction relationship
proposed by Fredlund (1976) and the one presented in Equation 5.17. The contribution of suction to shear strength varies in terms of the normalized water content, while the relationship still has two parts, one for the saturated shear strength and the other for the contribution of suction to shear strength for unsaturated soils. The angle of shearing resistance ($\phi'$) is the same for the saturated state and the unsaturated state, by utilizing the normalized water content, i.e., Vanapalli et al. (1996) did not consider changes in the angle of shearing resistance due to changes in suction. The relationship in Equation 5.17, presents a clear relationship between the SWCC and shear strength by relating the contribution of suction to shear strength in terms of the water content of the soil. Also, the residual water content needed to evaluate the normalized water content can be obtained from the SWCC. The work done to establish the relationship between suction and shear strength for unsaturated soils laid the groundwork for the approach taken to establish a relationship between $M_r$ and suction.

$$\tau = [c' + (\sigma_n - u_a) \tan \phi'] + (u_a - u_w)[(\theta^k)(tan\phi')]$$

Equation (5.17)

Where:

- $k$ = fitting parameter to obtain better agreement amongst measured and predicted values

Khoury et al. (2011) proposed a relationship to capture the hysteric behavior of $M_r$ with respect to moisture variation. $M_r$ shows a hysteric behavior similar to that experienced by SWCC tests; $M_r$ values at similar moisture contents tend to differ depending on whether the soil is undergoing drying or wetting. Figure 5.21 shows results obtained through laboratory testing conducted by Khoury et al. (2012); this figure illustrated the variation of $M_r$ with respect to specimens undergoing drying (represented
by IDC – Initial Drying Curve, and MDC – Main Drying Curve) and those undergoing wetting (represented by MWC – Main Wetting Curve).

Figure 5.20: $M_r$ – hysteresis behavior (Khoury, 2012)

Based on laboratory test results, Khoury (2011) proposed a model to capture the hysteric behavior of $M_r$, the proposed model is presented in Equation 5.19. This model attempts to predict the increase in $M_r$ along the drying curve, and adjust the value on the drying curve by utilizing $F_{DW}$ to obtain a $M_r$ value along the wetting path at similar moisture condition. The model presented in Equation 5.19 creates a direct and indirect link to the SWCC to establish the hysteric relationship. It utilizes volumetric water contents for the $M_r$ specimens to obtain suction values from the SWCC. Subsequently, it creates a direct link to the SWCC by utilizing the fitting parameter, $n$, from the Fredlund and Xing (1994) model.

$$M_r = \left[ k_1 P_a \frac{\theta_b^k}{\theta_a} \left( \frac{\tau_{oc} + 1}{P_a} \right)^{k_3} + (\Psi - \Psi_o) \left( \frac{\theta_d}{\theta_o} \right)^k \right] F_{DW} \tag{5.19}$$

Where:
\[
\left( k_1 P_a \frac{\theta_2^k}{\bar{P}_a} \left( \frac{\bar{P}_o c_t}{P_a} + 1 \right)^{k/3} \right) = \text{Universal model}
\]

\( \Psi \) – suction

\( \Psi_0 \) – low suction, corresponding to a \( M_r \) test at wet of optimum

\( \theta_d \) – volumetric water content along drying curve

\( \theta_s \) – saturated volumetric water content; obtained from SWCC

\( F_{DW} = \frac{\theta_d}{\theta_w} \); where \( \theta_w \) = volumetric water content along wetting curve corresponding to same suction as \( \theta_d \)

\( k = \frac{1}{n} \); where \( n \) is a fitting parameter from Fredlund and Xing (1994) model to describe SWCC

Borrowing ideas from Liang et al. (2008) and Khoury et al. (2011), this study proposes the model presented in Equation 5.20 to evaluate \( M_r \) of unsaturated subgrade soil while taking into account the effect of matric suction. This model takes into account the role the area of water has in determining the contribution of matric suction to \( M_r \) by incorporating the normalized water content. The normalized water content accounts for the effect of residual water content when the water phase is discontinuous and the area of water in contact with soil particles is negligible. The model also creates a direct link to the SWCC by utilizing the fitting parameter \( n \). The fitting parameter \( n \), is obtained from the Fredlund & Xing (1994) model, which is presented again in Equation 5.21 for the readers’ convenience. The \( n \) parameter represents the slope of the SWCC, i.e. the rate of suction change due to change in water content. An added benefit of including the \( n \) parameter is that it implicitly takes into consideration soil type. The rate of change of suction due to changes in water content differs for different soil types, with certain soils (e.g. sands, silts) experiencing large changes in water content with small changes in suction, especially in the lower suction range. While other soil types (e.g. clays), tend to display a much ‘flatter’ SWCC having a milder slope due to the soil having a higher
water holding capacity. Figure 5.22 (Leong and Rahardjo, 1997) shows the effect of the $n$ parameter on the shape of the SWCC. It can be seen that the ‘zone of desaturation’ is significantly affected by the $n$ parameter, it should be understood that the ‘zone of desaturation’ varies by soil type, with soils with little water holding capacity (e.g. sand) having a narrow zone of desaturation when compared to clays.

$$M_r = k_1 P_a \left( \frac{(\theta + \theta^k \psi)}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} \right)^{k_3} \quad (5.20)$$

Where:

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r}, \text{ where } \theta = \text{volumetric water content, } \theta_r = \text{residual water content (from SWCC),}$$

$$\theta_s = \text{saturated water content (from SWCC)}$$

$$k = \frac{1}{n}$$

$$\theta_w = \frac{\theta_s}{\left( \ln(e + (\frac{\psi}{\alpha})^n) \right)^m} \quad (5.21)$$

Figure 5.21: Effect of $n$ parameter with $a$ and $m$ held constant (Leong and Rahardjo, 1997)
5.4.2 Validation of Proposed Model

In order to validate the model proposed in Equation 5.20, laboratory data obtained from this study, and data from external sources were utilized. To utilize the model presented in Equation 5.20, it is imperative to obtain SWCC curves which have been evaluated over the entire range of saturation. An SWCC curve spanning the entire range of saturation allows for accurate evaluation of the \( n \) parameter, and the residual water content. Identifying a specific value for the residual water content can be challenging. At the residual condition, the water phase in the soil pores is discontinuous and exists mainly as thin films surrounding soil particles. Another popular definition of the residual condition is the water content at which a large increase in suction causes only a small decrease in the water content. This definition is arbitrary which makes it difficult to identify a residual water content. Figure 5.22 provides an illustration of the different stages of water obtained from SWCC.

![SWCC Stages](image_url)

Figure 5.22: Stages of a SWCC for entire range of saturation (Vanapalli et al., 1999)
While some relationships used to describe the SWCC use the residual water content as a fitting parameter, it is important to obtain the residual water content from the SWCC by observing the suction – water content relationship and determining where the residual condition occurs. For this study, to evaluate the residual water content, the volumetric water content was plotted against matric suction and the residual water content was determined graphically. Figures 5.23 and 5.24 show the relationships obtained for the four soil types to determine the residual water content. Once the residual water content was determined, it was possible to evaluate the normalized water content needed for Equation 5.20. Also, from Figures 5.23 and 5.24, it can be seen that the residual water content value generally decreases as PI decreases which is expected since higher PI soils tend to have a higher water holding capacity.

Figure 5.23: a.) Residual Water Content – P-53; b.) Residual Water Content – P-26
To validate the model utilizing laboratory data from this study, non-linear regression analysis was performed to fit the measured data and evaluate regression constants $k_1$, $k_2$, and $k_3$. The objective was to evaluate whether the model could capture changes in $M_r$ due to changes in moisture content, and consequently changes in matric suction. Measured $M_r$ values were obtained as an average of the three replicate specimens, with 15 measured $M_r$ values per moisture content. Then regression analysis was performed across all moisture contents tested per soil type, and one set of regression constants was obtained per soil type. Figures 5.25 – 5.28 present the results of the regression analysis in term of measured versus predicted $M_r$ values by utilizing Equation 5.20 for the four soil types tested.
Figure 5.25: Measured Versus Predicted $M_r$ values utilizing Equation 5.20 for Soil P-53

$R^2 = 0.84$
$k_1 = 368.1$
$k_2 = 0.45$
$k_3 = -1.1$
$n = 0.59$

Figure 5.26: Measured Versus Predicted $M_r$ values utilizing Equation 5.20 for Soil P-26

$R^2 = 0.89$
$k_1 = 169.6$
$k_2 = 0.9$
$k_3 = -1.0$
$n = 0.76$
Based on the figures above, it can be seen that the proposed model provided a good fit to the laboratory data from this study, the model has the capability to accurately predict $M_r$ across different moisture contents with varying matric suction. For the lower PI soils, P-17 and P-7, there is more scatter for $M_r$ values obtained on the wet side optimum. This can attributed to the strain-hardening results, positive $k_3$ coefficient, obtained during $M_r$ testing for P-7 and P-17 on the wet side of optimum. A positive $k_3$ value contradicts results from other $M_r$ tests which display strain-softening behavior. To
To further validate the proposed model, the model was applied to data from other sources in literature. One of the challenges in obtaining external data was the amount of information needed to apply the model. The required data included Mr values at different moisture contents, and a SWCC curve spanning the entire range of saturation. Data obtained from Gupta et al. (2007) and Liang et al. (2008) was utilized to validate the model. Figures 5.34 and 5.35 present the results obtained from the validation process. For Liang et al. (2008) data for one soil type across two different moisture contents was obtained, with Mr values at multiple stress states. Regression analysis was performed to obtain one set of regression constants to represent the soil across both moisture

Table 5.4: Coefficient of Determination (R²) Obtained from Non-Linear Regression Analysis

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Liang (2008)</th>
<th>Proposed Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-7</td>
<td>0.64</td>
<td>0.92</td>
</tr>
<tr>
<td>P-17</td>
<td>0.69</td>
<td>0.97</td>
</tr>
<tr>
<td>P-26</td>
<td>0.81</td>
<td>0.89</td>
</tr>
<tr>
<td>P-53</td>
<td>0.85</td>
<td>0.84</td>
</tr>
</tbody>
</table>
contents. A similar procedure was applied to Gupta et al. (2007) data, in this case regression analysis was conducted across three moisture contents for one soil type. Based on the results in Figures 5.30 and 5.31, it can be observed that Equation 5.20 provides a good fit to the external data, and is able to predict $M_r$ across different moisture contents for unsaturated soils while taking into account the effect of matric suction.

![Figure 5.29: Measured Versus Predicted $M_r$ Values Obtained by Applying Equation 5.20 to Data Obtained from Gupta et al. (2007)](image1)

![Figure 5.30: Measured Versus Predicted $M_r$ Values Obtained by Applying Equation 5.20 to Data Obtained from Liang et al. (2008)](image2)
5.5 Prediction of Regression Constants Based on Soil Physical Properties

Up to this point several constitutive $M_r$ models have been applied, discussed, and shown to demonstrate the capability to capture the stress dependent and moisture dependent behavior of $M_r$ for unsaturated subgrade soils. However, application of these models involves performing RLT triaxial tests to measure $M_r$ values, which must be utilized to obtain regression constants. Unfortunately, the ability to perform RLT triaxial tests is generally limited to research laboratories due to the expensive equipment and skilled personnel required to perform the test. It is necessary to provide Engineers with a method to apply to $M_r$ constitutive models which will not require performing RLT tests. For this study, a correlation was created between the regression constants from the $M_r$ constitutative models and soil physical properties. Relationships between soil physical properties and regression constants were developed for two $M_r$ constitutive models, the ‘Universal’ (Witczak and Uzan, 1998) model and the model proposed in this study, Equation 5.20.

5.5.1 Predicting Regression Constants for ‘Universal’ $M_r$ Model

A Stepwise regression analysis was performed utilizing SAS (Statistical Analysis Software) to correlate soil physical properties with regression constants $k_1$, $k_2$, and $k_3$; the ‘Universal’ $M_r$ model is presented here again in Equation 5.22 for the readers’ convenience. Stepwise regression is a sequential model building process which identifies appropriate independent variables to evaluate a dependent variable (i.e. $k_1$, $k_2$, $k_3$). Once the most important variables that contribute to the response variable were identified, the adequacy of the model to evaluate the regression constants was evaluated and the model was subsequently validated by comparing measured $M_r$ data.
from this study with the predicted $M_r$ values obtained by utilizing the Stepwise regression model.

\[ M_r = k_1 P_a \left( \frac{\theta}{P_a} \right)^{k_2} \left( \frac{t_{oc}}{P_a} + 1 \right)^{k_3} \] (5.22)

As mentioned previously, Stepwise regression is a sequential process which is a combination of the backward and forward model selection methods. Stepwise regression begins with fitting all possible simple (i.e. one independent variable) models and the initial model selected represents the one with the largest F-statistic. Subsequently, all possible two variable models are evaluated and compared, and the model with the largest F-statistic chosen to continue the model building process. At each step, the significance of all variables in the model is checked and if the significance of a variable falls below a specified threshold then that variable is removed. The process of adding and removing variables continues till no variables outside of the model have the significance to enter the model. Key user input parameters for this method in SAS are the specified significance for a variable to enter the model (SLentry) and specified significance for a variable to be removed from the model (SLexit). For this study a significance value equal to 0.15 was chosen as the threshold value for SLentry and SLexit.

Stepwise regression is a popular technique for identifying key parameters which could serve as predictor variables, however, it is at times prone to over-fitting to the data utilized to create the model. The objective of this study was to conduct purposeful selection to create a model which included only significant variables, and to ensure the number of independent variables utilized to predict a dependent variable accounted for the sample size (i.e. avoid having too many independent variables for the sample size.
utilized to create the model). To combat this issues, several statistical methods were
used in conjunction with Stepwise regression to create purposeful model able to predict
regression constants $k_1, k_2,$ and $k_3$ based on soil physical properties.

The initial step was gathering the data required to begin the model selection
process, this required performing nonlinear regression analysis on the results of each
$M_r$ test utilizing Equation 5.22. Non-linear regression analysis was performed separately
on $M_r$ tests for each soil type, at each moisture content tested. This yielded a total of 45
data points for each regression constant, the input data is summarized in Table 5.1.
Table 5.1 provides the average value of the regression constants, obtained from three
replicate specimens, at each moisture content. However, in the statistical analysis data
from each replicate was utilized. In creating the statistical model, it was important to
realize the trends noticed in the regression constants as moisture content varied, which
were discussed earlier.

![Figure 5.31: a.) $M_r$ test for P-17 at OMC +3; b.) $M_r$ test for P-7 at OMC +3](image)
Once the input data (dependent variables) were identified and defined, the next step was to identify independent variables which could serve as predictor variables in the model. Since only one model will be created for the different soil types, it was important to include physical soil properties in the independent variables. Also, properties defining the moisture condition of the soil were also included such as gravimetric water content and degree of saturation. By including soil moisture conditions, the model will be able to predict regression constants across different moisture contents, thereby having the ability to include the effects of seasonal variation when evaluating the $M_r$ value. The following soil physical properties and their interactions were included in evaluating possible predictor variables; degree of saturation (S), gravimetric water content (w), PI, liquid limit (LL), % passing No.200 sieve, % clay, and % silt.

The following Equations define the independent variable which were initially identified, statistical analysis was utilized to choose appropriate variables from the list below to predict the regression constants. The following variables were initially selected to be able to capture the trends displayed by the regression constants with respect to moisture content, as mentioned earlier.

\[
ssa = S - S_{opt} \times \left( \frac{PL}{p200} \right) \tag{5.23}
\]

\[
ssopt = S - S_{opt} \tag{5.24}
\]

\[
ssc = \frac{S - S_{opt}}{\%clay} \tag{5.25}
\]

\[
wp i = \frac{w - w_{opt}}{pI} \tag{5.26}
\]

\[
opt = \frac{w}{w_{opt}} \tag{5.27}
\]
$\text{sss} = \left( S - S_{\text{opt}} \right) \times \% \text{Silt}$ (5.28)

$\text{wwa} = \left( \frac{w - w_{\text{opt}}}{w_{\text{opt}}} \right) \frac{\text{PI}}{p_{200}}$ (5.29)

$\text{wwc} = \left( \frac{w - w_{\text{opt}}}{w_{\text{opt}}} \right) \% \text{clay}$ (5.30)

$\text{ll} = \text{Liquid Limit}$ (5.31)

$p_{200} = \% \text{ passing No. 200 Sieve}$ (5.32)

$\text{pi} = \text{Plasticity Index}$ (5.33)

$\text{wcw} = \frac{w - w_{\text{opt}}}{\% \text{clay}}$ (5.34)

$\text{sp}_s = \left( \frac{S - S_{\text{opt}}}{S_{\text{opt}}} \right) \text{pi}$ (5.35)

Where:

$w = \text{gravimetric water content}$

$w_{\text{opt}} = \text{optimum moisture condition}$

$S = \text{degree of saturation}$

$S_{\text{opt}} = \text{degree of saturation at optimum condition}$

$\frac{\text{pi}}{p_{200}} = \text{Activity parameter}$

After gathering the data, an initial Stepwise regression was performed utilizing PROC GLMSelect in SAS. The results were output with respect to different statistical fit criterion. By being able to view different fit criterion, it was possible compare several different models obtained from Stepwise regression process simultaneously, an example of the output from SAS for this step is shown in Figure 5.32. From this figure, it can be seen that it was possible to evaluate the model at each step of the Stepwise regression process based on seven different fit criterion. For this study, the model
selected by the AICC criterion was select in the initial Stepwise regression process. In Figure 5.32, the optimum value for the different fit criterion coincides with the model selected at the end of the Stepwise regression process. However, this was not always the case and the model showing the optimum value for the AICC fit criterion did not coincide with the model selected at the end of the Stepwise regression.

Figure 5.32: SAS PROC GLMSelect Output for Stepwise Regression with Different Fit Criterion for ln k1

The AICC selection criterion is a variation of the Akaike Information Criterion (AIC) which is a measure of the relative quality of a statistical model. AIC is evaluated based on formulation presented in Equation 5.36. AIC credits the model for providing a good fit to the data but also penalized the model for additional parameters. AICC adds a correction factor to the original AIC formulation to account for a finite sample size. Simply, it penalizes a model which has too many parameters with respect to the sample size of the data utilized to create the model. This was useful since the sample size
utilized in this study was finite with approximately 45 measured data points. The formulation for AICC is provided in Equation 5.37. For both AIC and AICC, a smaller (i.e. more negative) value indicates a better fit.

\[ AIC = 2k - \ln(L) \]  
(5.36)

Where:

\[ k = \text{# of parameters in model} \]

\[ L = \text{maximized values for the likelihood function for the estimated model} \]

\[ AICC = AIC + \frac{2k(k+1)}{n-k-1} \]  
(5.37)

Where:

\[ n = \text{sample size} \]

Once a model was selected from the initial Stepwise regression process utilizing AICC as the selection criterion, the selected model was subjected another Stepwise regression with data partitioning. This method was utilized to avoid over-fitting of the model and to provide a model with improved prediction capabilities. In this step, the data was partitioned to where 70% of the data was used for training the model and 30% was used for validating the model. PROC GLMSelect was utilized again with a partition statement. In this step, only the variables selected from the initial Stepwise regression are input and SAS conducts another Stepwise regression utilizing those variables. However, during this step 70% of the data is utilized to train the model but the model is selected based on its’ performance on the validation data (30%). The selected model minimizes errors between measured and predicted values from the validation data. Figure 5.38 provides an example of the SAS output for this step. It can be seen the model selected is at step 5 and it is selected based on the minimizing error of the
validation data. Comparing Figure 5.33 to 5.32, it can be seen that the model selected in Figure 5.33 has one less predictor variable than the model selected in Figure 5.32, this is because the removed predictor variable (sps) did not decrease ASE value of the validation data any further (as seen in Figure 5.33).

![Progression of Average Squared Errors by Role for ln k1](image)

Figure 5.33: Averaged Squared Errors (ASE) From Training and Validation Data for ln k₁

Once a model was selected from the validation process, the next step was to check for multi-collinearity issues. It is important to avoid multi-collinearity, which indicates a linear dependence amongst two or more independent variables, in a statistical model because a multi-collinearity problem could result in contradictory results of the F-test and t-test. To evaluate multi-collinearity SAS was utilized to output the Variance Inflation Factor (VIF) value, generally a VIF value less than 10 indicates no severe multi-collinearity problems exist. Figure 5.34 provides an example of the SAS output with VIF values for ln k₁. It can be seen that for two variables, opt and wwa, the
VIF value is greater than 10. Therefore, one of the variables needs to be removed from the model. In this instance, wwa was removed from the model and a subsequent VIF analysis revealed that the VIF of the other variables in the model were less than 10 once wwa was removed. The procedure described above was also utilized to create statistical model to predict \(k_2\) and \(k_3\), Table 5.5 provides the selected models for each regression constant and the statistical parameters associated with those models. Equations 5.38 – 5.40 provide the selected models to predict \(k_1\), \(k_2\) and \(k_3\) for the Universal Mr model.

\[
\begin{align*}
\ln k_1 & = 12.97 - 7.02opt + .0075ll + .036wwc + .036ssopt \\
\ln k_2 & = 0.92 - 0.42opt - 0.00334p200 \\
k_3 & = 5.33 - 7.97opt + 10.00wwa + 0.014ll
\end{align*}
\]
Table 5.5: Results of Stepwise and Multiple Regression Analysis

$k_1$ (ln $k_1$) coefficient prediction model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter Estimate</th>
<th>Standard Error</th>
<th>t-value</th>
<th>Pr&gt;t</th>
<th>R^2</th>
<th>VIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept</td>
<td>12.97564</td>
<td>0.65445</td>
<td>19.83</td>
<td>&lt;.0001</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>opt</td>
<td>-7.0236</td>
<td>0.60662</td>
<td>-11.87</td>
<td>&lt;.0001</td>
<td>9.2</td>
<td></td>
</tr>
<tr>
<td>ll</td>
<td>0.00753</td>
<td>0.00132</td>
<td>5.73</td>
<td>&lt;.0001</td>
<td></td>
<td>1.2</td>
</tr>
<tr>
<td>wwc</td>
<td>0.03626</td>
<td>0.00776</td>
<td>4.67</td>
<td>&lt;.0001</td>
<td>3.1</td>
<td></td>
</tr>
<tr>
<td>ssopt</td>
<td>0.03629</td>
<td>0.00772</td>
<td>4.7</td>
<td>&lt;.0001</td>
<td>0.9</td>
<td>8.9</td>
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</tbody>
</table>

$k_2$ coefficient prediction model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter Estimate</th>
<th>Standard Error</th>
<th>t-value</th>
<th>Pr&gt;t</th>
<th>VIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept</td>
<td>0.92071</td>
<td>0.09371</td>
<td>9.83</td>
<td>&lt;.0001</td>
<td>0</td>
</tr>
<tr>
<td>opt</td>
<td>-0.42317</td>
<td>0.07826</td>
<td>-5.41</td>
<td>&lt;.0001</td>
<td></td>
</tr>
<tr>
<td>p200</td>
<td>-0.00336</td>
<td>0.00056296</td>
<td>-5.96</td>
<td>&lt;.0001</td>
<td>0.59</td>
</tr>
</tbody>
</table>

$k_3$ coefficient prediction model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter Estimate</th>
<th>Standard Error</th>
<th>t-value</th>
<th>Pr&gt;t</th>
<th>VIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept</td>
<td>5.33299</td>
<td>1.27678</td>
<td>4.18</td>
<td>0.0002</td>
<td>0</td>
</tr>
<tr>
<td>opt</td>
<td>-7.96924</td>
<td>1.37046</td>
<td>-5.82</td>
<td>&lt;.0001</td>
<td></td>
</tr>
<tr>
<td>wwa</td>
<td>10.00257</td>
<td>3.34971</td>
<td>2.99</td>
<td>0.0054</td>
<td>6.7</td>
</tr>
<tr>
<td>ll</td>
<td>0.01429</td>
<td>0.00314</td>
<td>4.55</td>
<td>&lt;.0001</td>
<td>0.68</td>
</tr>
</tbody>
</table>

5.5.2 Analysis of Proposed Statistical Model

Based on Table 5.6, the model(s) met statistical requirements such as significance levels less than .05 and lack of multi-collinearity. However, the objective is to accurately predict $k_1$, $k_2$, and $k_3$ and subsequently, $M_r$. Figures below compares measured versus predicted values of $k_1$, $k_2$, and $k_3$; it should be noted that for $k_3$ the data points with positive values were left out for this evaluation.
Figure 5.35: Measured Versus Predicted Values for $k_1$ (Universal Model)

Figure 5.36: Measured Versus Predicted Values for $k_2$ (Universal Model)
Figure 5.37: Measured Versus Predicted Values for $k_3$ (Universal Model)

Figure 5.38: Measured Versus Predicted $M_r$ Values for All Soils at All Moisture Contents Tested Obtained From Predicted Regression Constants for Universal Model
Based on the figures above, Equation 5.28-5.30 provide adequate predictions for the values of the regression constants from the ‘Universal’ $M_r$ model. Figure 5.39 reinforces that the model is unable to capture strain-hardening behavior, however the model is conservative in predicting $M_r$ values that display strain-hardening. Based on Figures 5.38 and 5.39, the statistical models presented in Equations 5.28-5.30 can accurately predict $M_r$ values and also have the ability to capture the effect of moisture variation on the $M_r$ values. Another advantage is that model incorporates soil physical properties, therefore, one model can be utilized satisfactorily on different soil types.

5.5.3 Predicting Regression Constants for Proposed $M_r$ – Matric Suction Model

A statistical model was created to predict regression constants $k_1$, $k_2$, and $k_3$ for the model proposed in this study incorporating the effects of matric suction in predicting $M_r$ values. The model is presented again in Equation 5.41 for the readers’ convenience. A statistical method similar to the one applied to predict regression constants for the ‘Universal’ $M_r$ was utilized here also. The independent (predictor) variables presented in
Equations 5.23-5.35 were also chosen to represent the regression constants for the proposed $M_r$ – matric suction model. Once again, the idea was to create a singular model to predict each regression constant separately for all soil types and across different moisture contents. The input data for the $M_r$ – matric suction model is summarized in Table 5.7, where the values of the regression constants at each moisture content are the average of three replicate specimens.

\[ M_r = k_1 x P_a \left( \frac{\theta + \theta^k \psi}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} \right)^{k_3} \]  
Equation (5.41)

<table>
<thead>
<tr>
<th>Soil No.</th>
<th>Moisture Content</th>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$k_3$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P - 53</td>
<td>OMC -6</td>
<td>73.83</td>
<td>1.56</td>
<td>-1.09</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>OMC -3</td>
<td>224.65</td>
<td>0.92</td>
<td>-1.36</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>OMC</td>
<td>286.39</td>
<td>0.92</td>
<td>-1.40</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>OMC +3</td>
<td>440.97</td>
<td>0.14</td>
<td>-0.95</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>OMC +6</td>
<td>449.93</td>
<td>0.19</td>
<td>-2.19</td>
<td>0.96</td>
</tr>
<tr>
<td>P-26</td>
<td>OMC -6</td>
<td>65.51</td>
<td>1.37</td>
<td>-0.58</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>OMC -3</td>
<td>225.12</td>
<td>0.71</td>
<td>-0.90</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>OMC</td>
<td>400.68</td>
<td>0.37</td>
<td>-1.91</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>OMC +3</td>
<td>236.44</td>
<td>0.15</td>
<td>-2.69</td>
<td>0.89</td>
</tr>
<tr>
<td>P-16</td>
<td>OMC -3</td>
<td>7.19</td>
<td>2.45</td>
<td>-1.52</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>OMC</td>
<td>62.30</td>
<td>1.44</td>
<td>-1.26</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>OMC +3</td>
<td>69.43</td>
<td>0.49</td>
<td>1.28</td>
<td>0.78</td>
</tr>
<tr>
<td>P-7</td>
<td>OMC -3</td>
<td>568.53</td>
<td>0.94</td>
<td>-1.10</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>OMC</td>
<td>303.26</td>
<td>0.87</td>
<td>0.15</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>OMC +3</td>
<td>143.15</td>
<td>0.78</td>
<td>1.91</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Similar to the approach utilized for predicting regression constants for the ‘Universal’ model, it was important to note the trends displayed by the regression constants with
respect to moisture content. Analyzing Table 5.6, it can be seen that certain trends do exist for the regression constants with respect to changes in moisture content, however, these trends are not as distinct or strong as those displayed by the regression constants for the ‘Universal’ model. For $k_1$, generally there is an increase in value as moisture content increases, except for soil P-7 which displays a decrease in $k_1$ with increasing moisture content. Regression constant $k_2$ displays the strongest trend amongst the three regression constants. There is a distinct decrease in $k_2$ with increasing moisture content, which is expected since $k_2$ includes the effects of matric suction which decreases as moisture content increases. Also, the values of $k_2$ are generally higher than those for the ‘Universal’ model. This can be attributed to $k_2$ including the effects of bulk stress and matric suction in the proposed model. For this model, $k_2$ is best able to capture the stiffening effect on the soil specimen due to an increase in matric suction.

Regression constant $k_3$ generally displays a decreasing trend with increasing moisture content, in this model $k_3$ also captures the effect of softening with increasing shear stress and this effect becomes more pronounced as moisture content increases. Positive $k_3$ values are encountered for the same soils at the same moisture contents which showed positive $k_3$ values when utilizing the ‘Universal’ model, once again the statistical model created to predict regression constants will be unable to capture strain-hardening behavior.

Once the input data was analyzed and outliers removed, the statistical procedure described for ‘Universal’ model earlier was applied to obtain Equations 5.42 – 5.44 which correlate regression constants $k_1$, $k_2$, $k_3$ (for proposed Mr – matric suction model) to soil physical properties. Once again, by incorporating soil properties and moisture
conditions, the model is able to predict regression constants for different soil types and $M_r$ at different moisture conditions. It should be noted that in the model(s) presented in Equations 5.42-5.44, degree of saturation as a predictor variable is prevalent. This is expected since the regression constants are being predicted for a model which incorporates matric suction in predicting $M_r$ and degree of saturation has a substantial impact on matric suction as evidenced by the SWCC. Table 5.8 provides the statistical measures for the models presented in Equations 5.42-5.44, all of the predictor variables are significant with P-values less than 0.05 for Student's t-test, and severe multicollinearity problems are avoided as evidenced by VIF values less than 10.

$$
\ln k_1 = 2.4489 + 0.3546ssa + 0.0354p200 - 0.12221wwc
$$

(5.42)

$$
k_2 = 1.75644 - 0.08682ssa - 0.53478ssc - 0.01931p200 + 0.01311pi
$$

(5.43)

$$
k_3 = 5.65346 + 0.1337ssa - 7.80061wopt + 0.01649pi
$$

(5.44)

Based on Table 5.7, the model(s) proposed in Equation 5.42-5.44 satisfy statistical requirements and provide a good fit to the measured values of $k_1$, $k_2$, and $k_3$ as evidenced by the $R^2$ values presented in Tables 5.6. Subsequently, it was important to evaluate the ability of the predicted regression constants to predict $M_r$ values utilizing the proposed $M_r$-matric suction model. Figures 5.40 and 5.41 evaluate the measured versus predicted values relationship for the proposed $M_r$-matric suction model utilizing predicted regression constants from Equations 5.42-5.44. In Figure 5.40, the models' inability to capture strain-hardening behavior is evident. However, even after accounting for the strain-hardening $M_r$ results the model still under-predicts a wide range of $M_r$ values, as evidenced by Figure 5.41. When comparing Figure 5.41 to Figure 5.39, it can be seen that the predicted regression constants for the 'Universal' $M_r$ model provide a
better fit to the measured $M_r$ values. This can be attributed to the lack of a well-defined trend amongst the regression constants obtained from the measured data when applying the proposed $M_r$ – matric suction model.

Table 5.7: Multiple Regression Analysis for Proposed $M_r$-Matric Suction Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter Estimate</th>
<th>Standard Error</th>
<th>t-value</th>
<th>Pr&gt;t</th>
<th>$R^2$</th>
<th>VIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>2.44899</td>
<td>0.27269</td>
<td>8.98</td>
<td>&lt;.0001</td>
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<tr>
<td>ssa</td>
<td>0.3546</td>
<td>0.04338</td>
<td>8.17</td>
<td>&lt;.0001</td>
<td>6.59</td>
<td></td>
</tr>
<tr>
<td>p200</td>
<td>0.0354</td>
<td>0.0032</td>
<td>11.06</td>
<td>&lt;.0001</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>wwc</td>
<td>-0.12221</td>
<td>0.02667</td>
<td>-4.58</td>
<td>&lt;.0001</td>
<td>0.88</td>
<td>6.59</td>
</tr>
</tbody>
</table>

**k2 coefficient prediction model**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter Estimate</th>
<th>Standard Error</th>
<th>t-value</th>
<th>Pr&gt;t</th>
<th>$R^2$</th>
<th>VIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept</td>
<td>1.75649</td>
<td>0.12917</td>
<td>13.6</td>
<td>&lt;.0001</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>ssa</td>
<td>-0.08682</td>
<td>0.01023</td>
<td>-8.49</td>
<td>&lt;.0001</td>
<td>1.67</td>
<td></td>
</tr>
<tr>
<td>ssc</td>
<td>-0.53478</td>
<td>0.07832</td>
<td>-6.83</td>
<td>&lt;.0001</td>
<td>1.75</td>
<td></td>
</tr>
<tr>
<td>p200</td>
<td>-0.01931</td>
<td>0.00198</td>
<td>-9.75</td>
<td>&lt;.0001</td>
<td>1.76</td>
<td></td>
</tr>
<tr>
<td>pi</td>
<td>0.01311</td>
<td>0.00239</td>
<td>5.49</td>
<td>&lt;.0001</td>
<td>0.92</td>
<td>1.84</td>
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</tbody>
</table>

**k3 coefficient prediction model**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter Estimate</th>
<th>Standard Error</th>
<th>t-value</th>
<th>Pr&gt;t</th>
<th>$R^2$</th>
<th>VIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept</td>
<td>5.65346</td>
<td>1.08906</td>
<td>5.19</td>
<td>&lt;.0001</td>
<td>0.00</td>
<td></td>
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<tr>
<td>ssa</td>
<td>0.1337</td>
<td>0.03367</td>
<td>3.97</td>
<td>0.0004</td>
<td>5.62</td>
<td></td>
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<tr>
<td>wopt</td>
<td>-7.80061</td>
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<td>-6.76</td>
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<tr>
<td>pi</td>
<td>0.01649</td>
<td>0.00419</td>
<td>3.93</td>
<td>0.0004</td>
<td>0.69</td>
<td>1.37</td>
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</tbody>
</table>
Figure 5.40: Measured Versus Predicted Mr Values for the Four Soil Types at Different Moisture Contents Utilizing Proposed Mr-Matric Suction model

Figure 5.41: Measured Versus Predicted Mr Values Excluding Mr Tests Displaying Strain-Hardening Behavior (P-7 at OMC, OMC+3 and P-17 at OMC+3)

5.6 MEPDG Sensitivity Analysis

The goal of this study has been to study the impact of moisture variation on the Mr value of unsaturated subgrade soils. The laboratory testing, and subsequent analysis of the data, revealed that Mr is significantly dependent on moisture conditions. Also, models, constitutive and statistical, are proposed with the intention of providing a method to incorporate moisture variation when predicting Mr values. However, from a
practical standpoint, it is important to discuss the importance of incorporating variation in the $M_r$ value when designing a pavement. Pavement can be designed utilizing guidelines and procedures proposed in the MEPDG; as noted, MEPDG utilizes the EICM to adjust an input ‘representative’ $M_r$ value to account for moisture variation. However, as an alternative, the design engineer can also input varying $M_r$ values over the course of the year based on expected moisture conditions. By employing the models discussed in this study, it would be possible to evaluate different input $M_r$ values for different moisture conditions (i.e. wet, dry, and optimum). This part of the study intends to evaluate whether the different $M_r$ input methods would have an impact on design outcomes using the MEPDG design guidelines.

AASHTOWARE® Pavement ME Design software was utilized to evaluate the impact of different input methods, for $M_r$, on design outcomes for a 2-lane roadway modeled as a rural highway with low traffic volume and a ‘typical’ section design with 2-inch thick asphalt layer, underlain by 8-inch base course layer and semi-infinite subgrade layer. Initially, results were obtained for an input ‘representative’ $M_r$ value corresponding to OMC, for each soil type. Subsequently, monthly varying $M_r$ values were input for three seasons; wet, dry, and optimum. 12 months were sections into the three seasons, four months/per season, and an $M_r$ value corresponding to each season was input. Therefore three (3) different $M_r$ values were input for a year corresponding to wet, dry, and optimum conditions. Till this point, the input $M_r$ values were those obtained from the laboratory testing program. The results, presented in Table 5.9, indicate that, generally, inputting monthly varying $M_r$ value leads to higher permanent deformation
(entire pavement) as compared to versus inputting a single representative $M_r$ value corresponding to OMC.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Representative $M_r$</th>
<th>Seasonal $M_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-7</td>
<td>1.54</td>
<td>1.75</td>
</tr>
<tr>
<td>P-17</td>
<td>1.47</td>
<td>1.98</td>
</tr>
<tr>
<td>P-26</td>
<td>1.31</td>
<td>1.65</td>
</tr>
<tr>
<td>P-53</td>
<td>1.25</td>
<td>1.29</td>
</tr>
</tbody>
</table>

*Note: Representative $M_r$ – Input was Measured $M_r$ value at OMC; Seasonal $M_r$ – Input was Measured $M_r$ values corresponding to OMC, wet of OMC, and dry of OMC.

AASHTOWARE analysis was also performed by utilizing $M_r$ values predicted from $M_r$-matric suction constitutive relationships such as Liang et al. (2008) and proposed model of this study. The objective was similar to the analysis performed on the measured $M_r$ values, evaluate the difference in MEPDG response, in terms of permanent deformation, when inputting seasonally varying $M_r$ values versus inputting representative $M_r$ value. Again, three (3) $M_r$ values were input, each corresponding to a four (4) month time period and representing wet, dry, and optimum conditions. The input $M_r$ values were obtained from the regression analysis performed on the measured data utilizing the Liang et al. (2008) and the proposed $M_r$-matric suction models. Table 5.10 presents the permanent deformation results obtained by utilizing $M_r$ values from the two different models. To compare the results in Table 5.10, obtained from using $M_r$ models that account for matric suction, versus laboratory measured $M_r$ values; the data from column 2 in Table 5.9 is presented again in Table 5.10. Utilizing $M_r$ values obtained from the models leads to lower permanent deformation results as compared to inputting measured $M_r$ values. The
permanent deformation results based on $M_r$ values obtained from the proposed model are closer to the results obtained by utilizing measured $M_r$ values, but still under predicted.

Table 5.9: AASHTOWARE Permanent Deformation Results for $M_r$-Matric Suction Models

<table>
<thead>
<tr>
<th>Soil</th>
<th>Measured $M_r$</th>
<th>Liang (2008)</th>
<th>Proposed Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-7</td>
<td>1.75</td>
<td>1.34</td>
<td>1.46</td>
</tr>
<tr>
<td>P-17</td>
<td>1.98</td>
<td>1.34</td>
<td>1.64</td>
</tr>
<tr>
<td>P-26</td>
<td>1.65</td>
<td>1.35</td>
<td>1.42</td>
</tr>
<tr>
<td>P-53</td>
<td>1.29</td>
<td>1.27</td>
<td>1.29</td>
</tr>
</tbody>
</table>

Based on Tables 5.9 and 5.10, it can be seen that MEPDG permanent deformation results vary based on the input method for the $M_r$ value. Accounting for seasonal variation in $M_r$ values can have an impact on pavement design as evidenced by the varying results in the magnitude of permanent deformation when utilizing different approaches for inputting the $M_r$ value.
CHAPTER 6 SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

6.1 Summary

This study was conducted to study the effects of seasonal variation on the resilient modulus ($M_r$) of unsaturated subgrade soils. For this purpose, a comprehensive laboratory testing program was carried out to evaluate the impact of moisture changes on the $M_r$ value of unsaturated subgrade soils of different plasticity indices. Repeated Load Triaxial (RLT) tests were conducted to assess the $M_r$ values for four different subgrade soils, representing the range of subgrade soils found in southern Louisiana, prepared at various as-compacted moisture contents. The effect of moisture variation on $M_r$ was evaluated in terms of changes in gravimetric water content, changes in degree of saturation, and variation of matric suction. Soil Water Characteristic Curves (SWCC) were established to evaluate the relationship between water content and matric suction for the four different soil types. Once this information was available it was possible to correlate the changes in moisture conditions in terms of matric suction for unsaturated soils. SWCC curves for the entire range of saturation were established for each soil types by utilizing a combination of the axis-translation and chilled-mirror hygrometer techniques to measure matric suction.

Once the laboratory data was evaluated, relationships between $M_r$ and moisture conditions were established in terms of gravimetric water content, degree of saturation, and matric suction. Several existing constitutive models that incorporate matric suction to predict $M_r$ for unsaturated soils were analyzed and their ability to capture the changes in $M_r$ due to changes in moisture content were evaluated. A modified constitutive relationship was proposed to incorporate the effect of matric suction in predicting $M_r$ for
unsaturated soils. Statistical analyses were carried out to develop models to predict the regression constants \((k_1, k_2, k_3)\) from \(M_r\) – matric suction constitutive relationships, i.e., Liang et al. (2008) and proposed model from this study, based on basic physical properties of soils.

**6.2 Conclusions**

Based on the results of the laboratory testing program and the subsequent analyses, the following conclusions can be drawn:

- The \(M_r\) value for subgrade soils is found to be is dependent on the stress state of the soil. Results of \(M_r\) testing demonstrated that \(M_r\) decreases with increasing deviatoric stress and increases with increasing confining pressure.

- The effect of confining pressure is more pronounced on high plasticity A-7 (AASHTO Classification) soils as compared to A-6 and A-4 soils.

- The moisture content has a significant impact on the \(M_r\) value for subgrade soils with the \(M_r\) values decreasing as the moisture content increases. A-7 soils were the most susceptible to decreases in \(M_r\) when moisture content increased to wet of optimum values. A-7 soils also displayed significant strain-softening behavior on \(M_r\) tests conducted on specimens prepared at the wet side of optimum moisture contents.

- Utilizing the degree of saturation to assess changes in \(M_r\) values due moisture variations was found to provide acceptable predictions with respect to the measured \(M_r\) values. A non-linear relationship was proposed to evaluate the normalized \(M_r\) \((M_r/M_{ropt})\) in terms of the deviation of degree of saturation from optimum condition \((S - S_{opt})\).
• The SWCC curves were established, for each soil type, utilizing a combination of axis-translation and chilled mirror hygrometer techniques. This allowed for the representation of the matric suction-water content relationship through the entire range of saturation.

• SWCC curves that establish the relationship between the degree of saturation and matric suction demonstrate that the matric suction increases with decreasing the degree of saturation.

• The laboratory results show that the Plasticity Index (PI) of a soil has a significant effect on the shape of its’ SWCC. For the four soils tested in this study there was a general trend of a shift to the left for the SWCC curves as the PI decreased (i.e., soils with lower PI had a narrower range of de-saturation compared to higher PI soils). This was expected since higher PI soils have a higher water holding capacity due to surface charges and short-range adsorption.

• The SWCC curves also display hysteric behavior, the matric suction-degree of saturation relationship is dependent on whether the soil is undergoing drying or wetting cycle.

• The M-matric suction relationship was evaluated for each soil type by correlating the degree of saturation between the SWCC and M specimen, thus obtaining the matric suction values for the as-compact M specimen.

• An increase in matric suction results in stiffening the unsaturated soil specimens as evidenced by the effective stress model for unsaturated soils, where effective stress increases with increasing matric suction. For unsaturated soils, incorporating matric suction in predicting M provides the best theoretical
approach since matric suction is a key parameter in describing the stress state of unsaturated soils.

- The contribution of matric suction to the effective stress and \( M_r \) values varies with respect to the area of water in contact with the soil particles. The normalized water content can be utilized to capture the varying contribution of matric suction to effective stress and the \( M_r \) value.

- Existing constitutive models that incorporate matric suction for predicting \( M_r \) values for unsaturated soils, e.g., Gupta et al. (2007), Liang et al. (2008) and Cary and Zapata (2011), were evaluated by comparing the predicted \( M_r \) values with measured \( M_r \) values from this study, and by performing non-linear regression analysis. The results showed that the existing models had the ability to capture the effect of moisture variation on \( M_r \) through incorporating matric suction. The existing \( M_r \) models generally provided a good fit to the measured \( M_r \) data from this study; however, only the Liang et al. (2008) model accounts for the variation in contribution of matric suction to \( M_r \) value due to changes in water content.

- An \( M_r \) – matric suction constitutive model was proposed in this study. The model includes the effect of water content on the contribution of matric suction to \( M_r \) and also establishes an explicit link to the SWCC by incorporating an SWCC fitting parameter into the model. Incorporating the SWCC parameter also allows for the effect of soil type to be implicitly included in the \( M_r \) – matric suction relationship. The proposed model was able to accurately capture the effect of moisture variation on \( M_r \) values of tested specimens in terms of changes in matric suction.
- Statistical models were developed to predict the regression constants, i.e., \(k_1\), \(k_2\) and \(k_3\), for \(M_r\) constitutive relationships based on basic physical properties of soils. This method allows Engineers to utilize \(M_r\) constitutive relationships without having to perform laboratory \(M_r\) tests to obtain regression constants. These models incorporate the effect of soil type and moisture conditions, allowing the models to predict the regression constants for different soil types and at different moisture contents.

- The regression constant models developed for the ‘Universal’ \(M_r\) constitutive relationship and the proposed \(M_r\) – matric suction relationship were compared with measured \(M_r\) values. The comparison shows that the predicted \(M_r\) values utilizing these model are in good agreement with the measured \(M_r\) values. However, the statistical models cannot capture the strain-hardening behavior experienced by some of the soil specimens when wet of optimum.

6.3 Recommendations

Based on this study, the following recommendations are made to incorporate the effects of seasonal variation in assessing a design value of \(M_r\) for unsaturated subgrades, and for the future work to better evaluate the impact of matric suction on \(M_r\) of unsaturated soils:

- Subgrade soils are generally not prepared during construction, they are used under in-situ conditions. For this study, the \(M_r\) specimens were laboratory prepared utilizing remoulded soils. For an accurate representation of in-situ \(M_r\) conditions, undisturbed soil samples from the subgrade layer should be obtained and tested in the laboratory.
• The matric suction for \( M_r \) specimens was obtained by correlating the degree of saturation for the prepared \( M_r \) specimen with the SWCC curve. However, the SWCC curves were obtained for a drying path while \( M_r \) specimens were prepared at as-compacted moisture contents. Considering the effect of drying/wetting path have on the SWCC relationship. The \( M_r \) specimens should also be subjected to the same path as the SWCC to achieve the target moisture conditions. This approach will provide a more accurate assessment of the magnitude of matric suction.

• It is important to evaluate the effect of hysteresis when considering the impact of moisture variation on \( M_r \) of subgrade soils.

• The proposed constitutive \( M_r \)-matric suction relationship was validated based on the results from the four soil types utilized in this study. A wide range of soil types are needed to validate the applicability of the model for a variety of soil types with different PI’s and different moisture contents.

• The statistical models proposed in this study to evaluate the regression constants based on soil physical properties are unable to capture the strain-hardening behavior. Another statistical model needs to be developed to capture the behavior of soils that display strain-hardening \( M_r \) behavior.
REFERENCES


Proctor, R. R. “Standard Effort (12 400 Ft-lbf/ft³ (600 kN·m/m³)).”


Vanapalli, S.K. “Simple Test Procedures and Their Interpretation in Evaluating the Shear Strength of an Unsaturated Soil” (1994)


Figure 1: Results of $M_r$ Tests for Soil P-53 at OMC -6%
Figure 2: Results of $M_r$ Tests for Soil P-53 at OMC -3%

Figure 3: $M_r$ Test Results for Soil P-53 at OMC
Figure 4: $M_r$ Test Results for Soil P-53 at OMC +3%
Figure 5: $M_r$ Test Results for Soil P-53 at OMC+6%

Figure 6: $M_r$ Test Results for Soil P-26 at OMC -6%
Figure 7: $M_r$ Test Results for Soil P-26 at OMC -3%
Figure 8: $M_r$ Test Results for Soil P-26 at OMC

Figure 9: $M_r$ Test Results for Soil P-26 at OMC+3%
Figure 10: Mr Test Results for P-17 at OMC -3%
Figure 11: M_r Test Results for P-17 at OMC

Figure 12: M_r Test Results for P-17 at OMC +3%

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Figure 13: Mr Test Results for Soil P-7 at OMC -3%
Figure 14: Mr Test Results for Soil P-7 at OMC

Figure 15: Mr Test Results for Soil P-7 at OMC +3%
## APPENDIX 2 NON-LINEAR REGRESSION ANALYSIS RESULTS

Table 1: Regression Constants for ‘Universal’ Mₚ Model (Witczak and Uzan, 1988) Obtained Via Non-Linear Regression Analysis

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Moisture Content</th>
<th>Regression Constants</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>k1</td>
<td>k2</td>
</tr>
<tr>
<td>OMC -6%</td>
<td>I</td>
<td>729.83</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>713.80</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>676.19</td>
<td>0.26</td>
</tr>
<tr>
<td>OMC -3%</td>
<td>I</td>
<td>731.06</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>731.22</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>655.90</td>
<td>0.35</td>
</tr>
<tr>
<td>OMC</td>
<td>I</td>
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<td>0.56</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>648.91</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>645.75</td>
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<td>0.08</td>
</tr>
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<td></td>
<td>II</td>
<td>534.69</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>436.22</td>
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<tr>
<td>P-53</td>
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<td></td>
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</tr>
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</tr>
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<td></td>
<td>III</td>
<td>730.04</td>
<td>0.26</td>
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<tr>
<td></td>
<td>OMC -3%</td>
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</tr>
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<td></td>
<td>II</td>
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<td>III</td>
<td>629.79</td>
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<td>I</td>
<td>511.19</td>
</tr>
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<td></td>
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<td>602.82</td>
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</tr>
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<td></td>
<td>III</td>
<td>532.08</td>
<td>0.12</td>
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<tr>
<td>P-26</td>
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<td>I</td>
<td>217.31</td>
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<td></td>
<td>II</td>
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<td>Soil Type</td>
<td>Moisture Content</td>
<td>Regression Constants</td>
<td>$R^2$</td>
</tr>
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<td>-----------</td>
<td>-----------------</td>
<td>----------------------</td>
<td>------</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_1$</td>
<td>$k_2$</td>
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<td></td>
<td>III</td>
<td>720.55</td>
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<tr>
<td>OMC</td>
<td>I</td>
<td>720.55</td>
<td>0.43</td>
</tr>
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<td></td>
<td>II</td>
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<td>0.54</td>
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<tr>
<td></td>
<td>III</td>
<td>663.65</td>
<td>0.96</td>
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<tr>
<td>OMC +3%</td>
<td>I</td>
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</tr>
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<td></td>
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<td>117.27</td>
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<tr>
<td></td>
<td>III</td>
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<td>0.21</td>
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<td>OMC +3%</td>
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<td></td>
<td>II</td>
<td>304.04</td>
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</tr>
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<td></td>
<td>III</td>
<td>310.28</td>
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</tr>
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<td></td>
<td>OMC +3%</td>
<td>148.45</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>140.14</td>
<td>0.20</td>
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<tr>
<td></td>
<td>III</td>
<td>140.86</td>
<td>0.14</td>
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</table>
Table 2: Regression Constants for Mr-Matric Suction Constitutive Model Proposed in this Study Obtained Via Non-Linear Regression Analysis

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Moisture Content</th>
<th>Regression Constants</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td>$k1$</td>
<td>$k2$</td>
</tr>
<tr>
<td>OMC -6%</td>
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<td>12.52</td>
<td>2.21</td>
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<tr>
<td></td>
<td>II</td>
<td>130.51</td>
<td>1.13</td>
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<tr>
<td></td>
<td>III</td>
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<td>1.34</td>
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APPENDIX 3 SAS INPUT

dm 'log;clog;output;clear';
options nodate nocenter pageno=1 ls=90 ps=56;
ODS listing;
ods graphics on;
ods html close; ods html;
title1 "Thesis Research";
title2 "Ayan Mehrotra";
title3 'k1 vs dependent variables';
data k1;
   INFILE 'F:\SAS Analysis\lk1.csv' dlm=',' dsd missover firstobs=2;
   input lnk1 k2 ssa ssopt ssc wpi opt sss wwa wwc ll p200 pi sps wcw;
datalines;
run;
*proc print data=k1; run;
*proc reg data=k1;
   model lnk1=ssa ssopt ssc wpi opt sss wwa wwc ll p200 pi wcw / selection=stepwise slentry=0.25 slstay=0.15 details; run;
*proc reg data=k1;
   model k2=ssa ssopt ssc wpi opt sss wwa wwc ll p200 pi/ selection=stepwise slentry=0.25 slstay=0.15 details; run;
*proc glmselect data=k1 plot=CriterionPanel;
   model lnk1=ssa ssopt ssc wpi wopt sss wwa wwc ll p200 pi sps wcw
      / selection=stepwise(select=SL) stats=all;
run;
%include 'allsubreg.sas';
*%allsubsreg (data=k1,
   depvar=lnk1,
   indepvar=wopt ssopt wwc wwa ll,
   sortvar=press,
   printvar=_CP_ _PRESS_ _RMSE_ _MSE_ _RSQ_ _ADJRSQ_ _AIC_ _BIC_ _SBC_);
*%allsubsreg (data=k1,
   depvar=k2,
   indepvar=p200 wopt sss ll ssopt wpi,
   sortvar=press,
   printvar=_CP_ _PRESS_ _RMSE_ _MSE_ _RSQ_ _ADJRSQ_ _AIC_ _BIC_ _SBC_);
*proc glmselect data=k1 plots=(CriterionPanel ASE) seed=1;
   partition fraction(validate=0.30);
   model lnk1 = ssopt opt wwa wwc ll sps
      / selection=stepwise(choose=validate stop=10);
run;

*PROC GLM DATA = k1 ;
MODEL lnk1 =wopt ssopt ll wwc wwa / SOLUTION CLPARM;
PROC REG DATA = k1 ;
MODEL lnk1 = opt ll wwc ssopt wwa / VIF ;
RUN ;
VITA

Ayan Mehrotra was born in India and raised in Baton Rouge, Louisiana. He completed his undergraduate degree in Spring 2011 at Louisiana State University receiving a Bachelor of Science in Civil Engineering. At Louisiana State University, he was a member of the Iota Tau chapter of Phi Beta Sigma Fraternity, Inc., a Resident Assistant, and a member of the S-Stem scholarship program. Upon completion of his undergraduate degree, he worked as a project manager in the construction services department for Professional Service Industries.

Ayan returned to Louisiana State University in Fall 2012 to pursue a Masters of Science degree in Civil Engineering with a concentration in geotechnical engineering. During his time as a graduate student, he was a graduate research assistant at Louisiana Transportation Research Center, a mentor to high school students through the Upward Bound Science Mentors summer program, and a member of the Louisiana State University Student Incubator program through the Louisiana Business and Technology Center.