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E2 POLYNOMIAL STRENGTHS : A COMPARISON WITH SHELL-MODEL RESULTS*

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Polynomial-expansion E2 strengths and sum rules involving $(J,T) = (0,0)$ and $(2,0)$ states of $(ds)^6$ are compared with shell-model results. The agreement is excellent. The strength distribution is describable as a narrow linear ridge in the energy space. The level-to-level fluctuations in the total strength are small, as expected from a theorem of Dyson and Mehta; fluctuations in the first and second moments are also small. The effect of collectivity and the partitioning of the sum rules into exothermic and endothermic parts are discussed.

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DISCLAIMER

To illustrate the polynomial theory[1)] for strength distributions and to assess the accuracy of various orders of approximation, we have chosen an example for which detailed shell-model results could be generated to provide a standard for comparison: namely, E2 transitions involving the (J,T) = (0,0) and (2,0) states of $(ds)^6$, which are of matrix dimensionality 71 and 307 respectively. The interaction is that of Kuo[2)] with ^{17}O single-particle energies; the shell-model energetics are shown in the inset of Fig. 1d. For scalar theories there are two polynomial sets defined by the total 0^+ and 2^+ state densities; a partitioning of each space into jj configurations leads to distinct polynomial sets associated with each configuration density, 20 for 0^+ and 22 for 2^+. All density moments as well as the required strength moments, were calculated from the shell-model matrices for the Hamiltonian and transition operator. (A direct evaluation of these quantities, in terms of the parameters of the Hamiltonian and model space, would of course be required in practical applications of the theory.*)

Writing W for the initial states and W' for the final we have, for the p'th energy-weighted sum-rule quantity, $M_p(W) = \sum_{W'} R(W',W)(W')^p = \int R(W',W)(W')^p \rho(W')dW'$. $M_o(W)$, M_1/M_o and M_2/M_o are respectively the strength, in this case BE(2), originating with a state at W, and the centroid and second moment of its distribution. These quantities are given in Figs. 1(a-c) for the 0^+ starting states, as calculated via shell-model, scalar polynomials of orders 2 and 4, and 2nd-order configuration polynomials. Figures 1(d-f) gives the same for the $(2^+ \rightarrow 0^+)$ strength; besides that the upper plots in Fig. 1d gives, with a scaling factor 1/5, the <u>total</u> 2^+ strength $(0^+ \rightarrow J = 0\text{-}4)$.

The agreements are remarkably good; in fact even the very simple first-order (linear) configuration theory, not shown, does very well also.

The strength function $R(W',W)$ is shown in log histogram displays in Fig. 2, the intensity levels 0-7 covering a strength variation of about 10^3. "Second-order scalar" for example, implies a double expansion with polynomial orders 0-2 in each space, and similarly for configurations. Given also are equivalent results with the state densities multiplied in, i.e. for $S(W',W) = \rho'(W')R(W',W)\rho(W)$; for fixed 0^+ or 2^+ starting energy $S(W',W)$ gives the relative strength/MeV to the final 2^+ or 0^+ states (divide by the density of the starting states to arrive at the absolute strengths per MeV). The agreement of scalar fourth-order and configuration second-order with the shell-model results is good.

Observe (Figs. 1b,1e) that the centroid moves linearly with the energy, and, since the 0^+ and 2^+ spectral spans are almost equal and only very slightly displaced with respect to each other, the peak of the strength corresponds to a very small (0-2 MeV) energy transfer (energy of the emitted γ ray in the case of exothermic reactions). But beyond that the spread in the transferred energy, as measured by the width of the strength distribution $(M_2 - M_1^2)^{1/2}$, is also small (σ^2(strength) ~ σ^2(density)/10), so small in fact that a polynomial determination of the width is impractical. The width can be determined of course from the $S(W',W)$ display (note again that the histogram is logarithmic; a reduction by one unit corresponds to reducing the strength by a factor 2.7). In general terms the strength distribution is describable as a narrow linear ridge in the (W,W') space.

Shown as an example in Fig. 1a is the "exothermic" part of the

$0^+ \rightarrow 2^+$ strength, i.e. that part of the strength, $\int_{-\infty}^{W} R(W',W)\rho(W')dW'$, for which $W > W'$. Subtracting this from $\mathcal{M}_0(W)$ gives the "endothermic" part. This kind of decomposition, and for the first and second moment as well, is of course entirely essential for many purposes including the study of cascade processes.

The level-to-level fluctuations in the total strength $\mathcal{M}_0$ originating in a given state are small, ~10-15% for the 0^+ states (Fig. 1a), about 5% for 2^+ (upper curves in Fig. 1d). This result, important in limiting the errors generated by the statistical smoothing, is easily understood. A theorem of Dyson and Mehta[3)] asserts that energy-level fluctuations involving states of the same exact symmetries are small, of the order of the local spacing. The same is then necessarily true for any operator K closely correlated with H, as of course the Q·Q operator is, this then implying that the strength fluctuations are also small. Quantitatively the appropriate correlation, in terms of the centroids and widths for K and H, is

$$\zeta = \langle (K - \mathcal{E}_K)(H - \mathcal{E}) \rangle / \sigma_K \sigma \qquad (1)$$

and then, in the linear approximation, $(K - \mathcal{E}_K)/\sigma_K \equiv \zeta(H - \mathcal{E})/\sigma$, ζ being of course the slope of $K(W)/\sigma_K$. When ζ is close to unity the expectation value varies strongly with W and the fluctuations, like those of the energy levels, must be small; for small ζ the energy variation is weak and the fluctuations may be large. For both the $0^+ \rightarrow 2^+$ strength and the total strength originating with $(2^+ \rightarrow J = 0\text{-}4)$ we find the very strong correlation $\zeta \approx 0.58$ and the small fluctuations described. On the other hand, for the $2^+ \rightarrow 0^+$ transitions alone, the transition operator is no longer Q·Q but is instead the weakly correlated operator (ζ =0.09) Q·$P(0^+)$Q (where P is a projection operator) and we have a slow M_o variation and large fluctuations (the lower curves of Fig. 1d). The $2^+ \rightarrow 0^+$ strength must then be only a small fraction of the almost-non-fluctuating total 2^+ strength; the fraction is in fact about 4%. The $(2^+ \rightarrow 2^+)$ strength, for which we do not show the M_o plots, accounts for 22% of the strength; for this case we have ζ = 0.35, the slope and fluctuations being intermediate between $(2^+ \rightarrow 0^+)$ and $(2^+ \rightarrow J = 0\text{-}4)$. The fluctuations in M_1 and M_2 are of the same order of magnitude as in M_o, but happily, though for reasons not as yet understood, are correlated with those of M_o so that the strength moments, M_1/M_o, M_2/M_o fluctuate much less and are in fact remarkably smooth (Figs. 1b,1c,1e,1f). The fluctuations in the BE2 values are well described by the Porter-Thomas distribution.

It might be expected a priori that quadrupole collective effects, of major consequence in (ds)-shell E2 transitions, might invalidate a statistical analysis. Though this obviously does not happen, the collective effects are very plainly to be seen. To begin with, the very large Q·Q expectation value (~325) in the $(0^+, T = 0)$ ground state domain (Fig. 1a) is compatible only with the "most collective" contributing SU(3) representations, $(\lambda,\mu) = (6,3), (7,1)$, $(4C = Q\cdot Q + 3L^2 + 4(\lambda + \mu + 3)(\lambda + \mu) - 4\lambda\mu = 360, 324$ respectively). For the 2^+ states $(\lambda,\mu) = (8,2)$, for which $4C \rightarrow 456$, is also contributing; we see (Fig. 1d) the Q·Q expectation value to be ~400. The smallness of the strength width is another indication; for $\zeta = 1$ (H a multiple of Q·Q) we would have σ(strength) = 0. More striking are the two low-lying "spikes" in the $(2^+ \rightarrow 0^+)$ strength (Fig. 1d) each corresponding to a strong collectivity predominantly involving single transitions, $(2_5^+ \rightarrow 0_1^+)$, $(2_{11}^+ \rightarrow 0_2^+)$, of almost zero energy as shown in the inset to Fig. 1a. No anomalies however, show up in the centroids and second moments for these states. Two other low-lying 2^+ states (#2,9) have extremely low $(2^+ \rightarrow 0^+)$ strength (Fig. 1d); for these the spikes which are seen (Figs. 1e,f) in the centroid and width may not, because of the smallness of M_o, represent real effects. As explained above no spikes can be expected in the total 2^+ strength, nor in the 0^+ strength, and none is seen (Fig. 1d, upper curve, and Fig. 1a). On the other hand the relative failure of the polynomial theory for the exothermic part of the $(0^+ \rightarrow 2^+)$ strength at low energy is ascribable to the collective concentration of the strength into 2^+ states lying just above their 0^+ partners (inset and the lower plots in Fig. 1a).

The essential simplicity of the results, the almost linear energy dependence of the moments and so forth, is due of course to the operation of the central limit theorem which filters out most of the input information. It is important however, and not surprising, that strong low-lying collective effects are not dominated by the CLT; but neither do they interfere in any serious way with its operation. We see a true co-existence of the two phenomena.

Finally we remark on the relative simplicity of the calculations. The second-order scalar approximation has made use of 19 input traces, 5 density moments for each space and 9 transition moments; scalar-4 has used 44 numbers. The configuration approximations are by no means as economical, but they should be applicable in spaces of indefinitely large dimensionality to study, for example, processes at higher energy as well as effective-charge and similar renormalizations.

References

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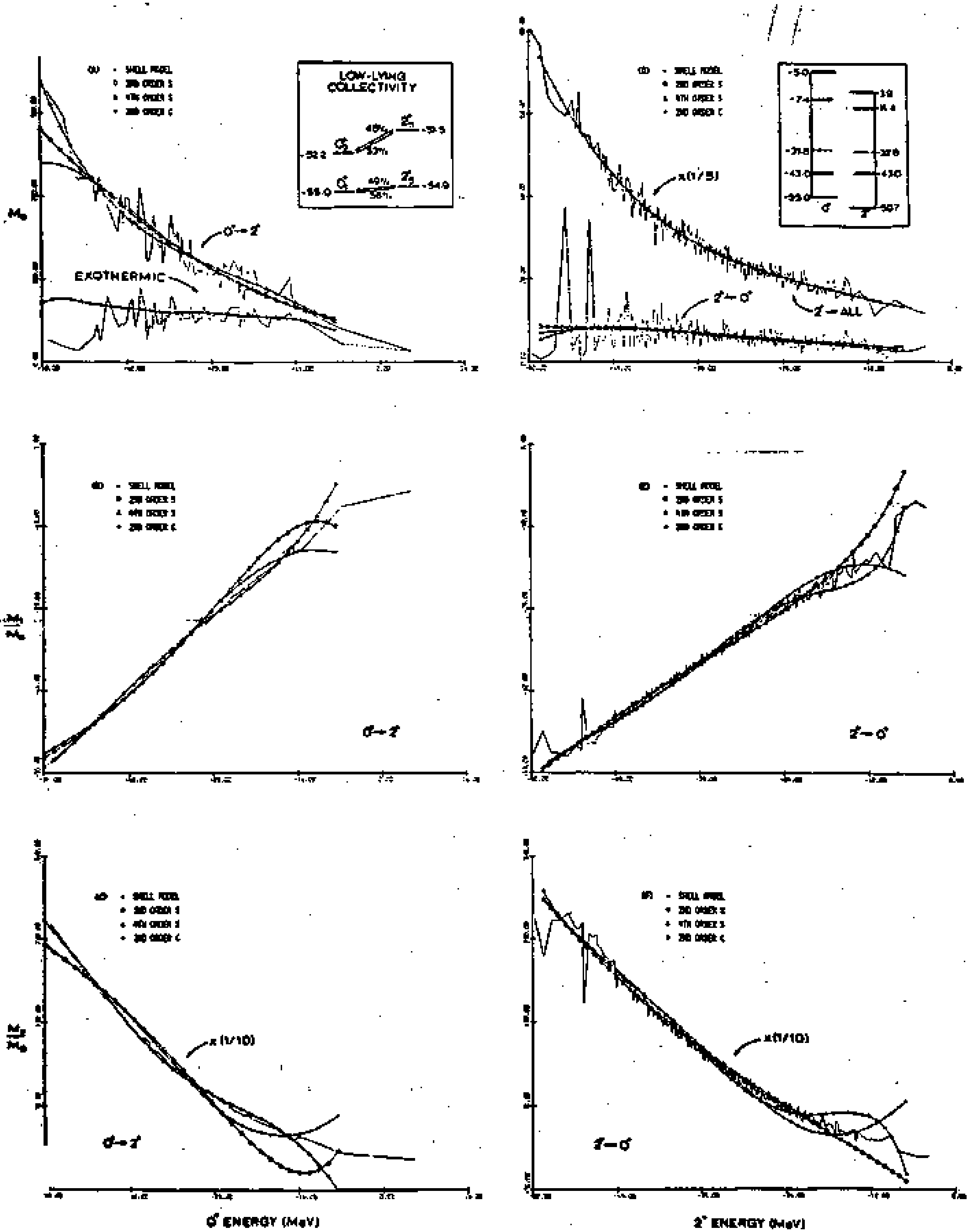
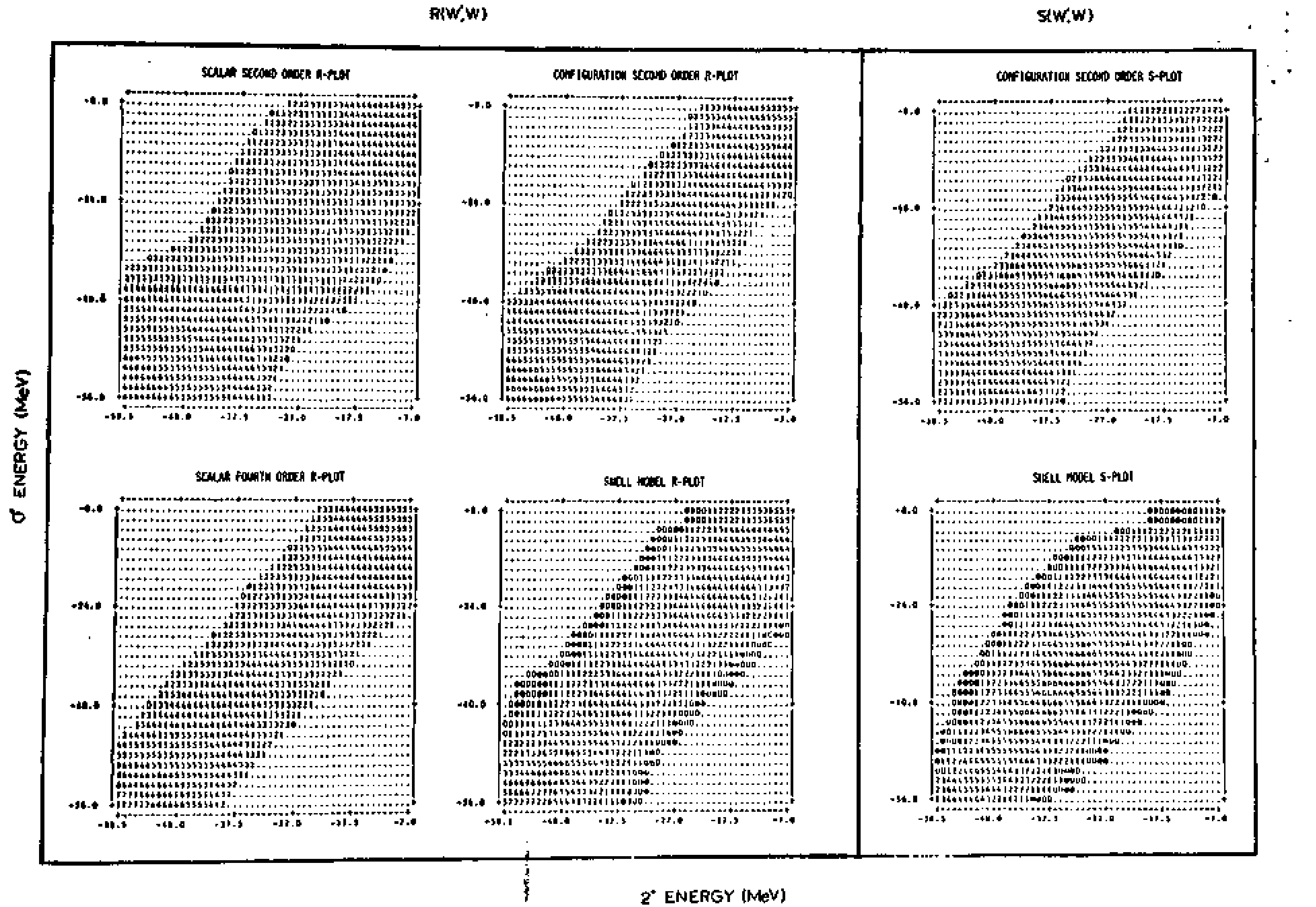
*Results for sum-rule quantities associated with single nucleon transfer (occupancies and centroids) are available and will be published in the near future.

Figure Captions

Fig. 1 Strengths, centroids and second moments for $(0^+ \rightleftarrows 2^+)$ transitions in $(ds)^6$, and scalar (S) and configuration (C) polynomial approximations to them. The strength is normalized to $Q \cdot Q = 4C - 3L^2$ where C, the SU(3) Casimir operator, has eigenvalues $(\lambda + \mu + 3)(\lambda + \mu) - \lambda\mu$. Inset (a) shows the two dominant collective $(0^+ - 2^+)$ pairs and the percentages of the $(0^+ \rightleftarrows 2^+)$ strengths which they account for; (a) gives also the exothermic part of the $(0^+ \rightarrow 2^+)$ strength (which at low energies is very much influenced by the strength concentration in the collective pairs) and a polynomial approximation to it. The total 2^+ strength, $(2^+ \rightarrow J = 0\text{–}4)$, scaled down by a factor 5, is given in (d); included in this strength is a diagonal contribution $(2^+ \rightarrow 2^+$ with zero energy transfer), proportional to the square of the quadrupole moment. The shell-model energies for the highest and lowest states, the scalar centroids (---), and the highest and lowest configuration centroids (∿∿∿) are displayed in inset (d).

Fig. 2 Log-histogram displays of the strength distributions $R(W',W)$ and $S(W',W) = \rho^{\dagger}(W')\, R(W',W)\rho(W)$. The (71 × 307)-dimensional shell-model results have been reduced to (31 × 51) by a running average. The intensity level is given by $L = \ln\,(x/x_{min})-1$; where, for $x = R$, $x_{min} = 0.0047$ and, for $x = S$, $x_{min} = 2.36$. For example, $L = 1$ represents strengths (R) between 0.035 and 0.095 whereas $L = 6$ goes from 5.2 to 14. The total spread in the strengths is $\sim 10^3$.


SHELL MODEL
2ND ORDER S
4TH ORDER S
2ND ORDER C
LOW-LYING COLLECTIVITY
EXOTHERMIC
0+ → 2+
x(1/5)
2+ → 0+
2+ → ALL
x(1/10)
0+ ENERGY (MeV)
2+ ENERGY (MeV)


R(W,W)
S(W,W)
SCALAR SECOND ORDER R-PLOT
CONFIGURATION SECOND ORDER R-PLOT
CONFIGURATION SECOND ORDER S-PLOT
SCALAR FOURTH ORDER R-PLOT
SHELL MODEL R-PLOT
SHELL MODEL S-PLOT
0' ENERGY (MeV)
2' ENERGY (MeV)