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Abstract

An extended version of the pseudo-SU(3) model which includes both spin and proton-neutron degrees of freedom is used to study the influence of the pairing interaction on K-band mixing, B(E2) values and quadrupole moments. Using the asymmetric rotor model as a backdrop, specific consequences of a many-particle shell-model based description of these collective properties are demonstrated and fundamental limits of the collective model's approach are investigated. Finally, the pseudo-SU(3) model, including representation mixing induced by pairing, is used to calculate the energies of ^{140}Ce and the results are compared to experimental data and other theories.

1 Introduction

Since the discovery of the pseudo-spin symmetry 25 years ago [1, 2] it has attracted the attention of numerous physicists and led to many successful applications of the theory (see Ref. [3] for a review). The pseudo-SU(3) model [4] takes full advantage of the pseudo-spin symmetry and has been used in the description of a wide spectrum of nuclear physics phenomena, ranging from collective excitation spectra [5] and the scissors mode [6], to identical superdeformed bands [7], and most recently to $\beta\beta$-decay [8].

In spite of these successful applications, most calculations that have used the pseudo-SU(3) model are very schematic in the sense that: A) the Hamiltonian has a rather simple structure as it usually includes only a long-range quadrupole-quadrupole term plus a residual interaction designed to accommodate certain special properties of low-energy collective spectra, and B) the configuration space was severely truncated, usually down to only one irreducible representation (irrep) of $SU(3)$. The reason for invoking these simplifications was technical, in particular, it was not possible to calculate the matrix elements between states of different $SU(3)$ irreps. Very recently a code was released that lifts this limitation [9]. It allows: A) the inclusion of short-range interactions, like pairing correlations, which other shell-model theories indicate to be essential, and B) larger configuration spaces which include many $SU(3)$ irreps.

As a consequence of these developments, in a recently published paper [10] an extended version of the pseudo-SU(3) model, which takes the spin degrees of freedom in a full

proton-neutron formalism into account, was reported. Specifically, the pairing interaction was expressed in terms of $SU(3)$ tensor operators and the effects of pairing correlations on the low-energy collective spectra and moments of inertia were demonstrated.

It is the purpose of this contribution to extend these studies by investigating the influence of pairing on K-band mixing, B(E2) values, and quadrupole moments. These studies will be done by comparing the pseudo-SU(3) model results to those of the asymmetric rotor model. The latter is a simple version of the geometric collective model which explains low-energy nuclear properties in terms of rotations and surface vibrations of a liquid-drop type nucleus and neglects all single-particle degrees of freedom. As a phenomenological model it has been very successful in the description of a variety of low-energy properties of even-even nuclei [11]. By comparing results of the pseudo-SU(3) and the asymmetric rotor models, it is possible to identify fundamental limitations of the collective model approach on the one hand, while demonstrating the ability of the pseudo-SU(3) model to describe collective nuclear properties correctly on the other. These schematic studies provide an understanding of the intrinsic properties of the pseudo-SU(3) model that is the basis upon which extended application of the theory for describing and predicting nuclear structure phenomena will rest. As a first attempt in this direction, the excitation energies of the semi-magic nucleus ^{140}Ce are calculated and compared to experimental data and the corresponding results for other theories.

This paper is organized as follows: In the next section a brief review of the pseudo-SU(3) model and the asymmetric rotor model is given, including a listing of the wavefunctions, Hamiltonians, and transition operators for each theory. In Section 3.1, the importance of K, the intrinsic z-axis angular momentum projection is discussed, and how states of different K values are coupled in both the asymmetric rotor model and in the pseudo-SU(3) model is explored. In Sections 3.2 the dependence of B(E2) values and quadrupole moments on the pairing strength is investigated; specifically, these measures as revealed by the pseudo-SU(3) model are compared to the corresponding asymmetric rotor model results. In Section 4 the pseudo-SU(3) model is used to describe the experimental energy spectrum of ^{140}Ce and the results are compared to other theories. A summary and conclusion is given in Section 5.

2   Models and Observables

An outline of the assumptions and definitions of the wave functions and Hamiltonian of the pseudo-SU(3) model (Subsection 2.1) and asymmetric rotor model (Subsection 2.2) are given in this section. Since the behavior of B(E2) rates and quadrupole moments are presented in Section 3, definitions of these observables are given in Subsection 2.3.

2.1   Pseudo-SU(3) model

In a recent publication, an extended version of the pseudo-SU(3) model was introduced. The formulation that was introduced included general expressions for basis states and for

matrix elements of generic operators between them [10]. Therefore, the present discussion is restricted to a brief summary of the essential ingredients of the model. The interested reader can find additional details in Refs. [5, 6].

The extended version of the pseudo-SU(3) model introduced in Ref. [10] and which is employed here, explicitly includes the spin degree of freedom and the Pauli Exclusion Principle in a full proton-neutron fermion formalism. Specifically, the pseudo-SU(3) model is a $0\hbar\omega$ theory; vertical couplings to higher shells are only allowed in an extension of the pseudo-SU(3) model called the pseudo-symplectic model [3]. In this contribution the protons (π) and neutrons (ν) occupy the real-space shells $N_\pi = 4$ and $N_\nu = 5$ or, equivalently, the pseudo-space shells $\tilde{N}_\pi = 3$ and $\tilde{N}_\nu = 4$ which is characteristic of rare-earth nuclei.

The wavefunctions of the pseudo-SU(3) model are classified by Casimir invariants of the group chain (see Ref. [10])

$$
\begin{aligned}
U(2\Omega_\pi) \times U(2\Omega_\nu) \;\supset&\; \left[U(\Omega_\pi) \times U(2)^{(\pi)}\right] \times \left[U(\Omega_\nu) \times U(2)^{(\nu)}\right] \\[2mm]
\supset&\; \left[SU(3)^{(\pi)} \times SU(2)^{(\pi)}\right] \times \left[SU(3)^{(\nu)} \times SU(2)^{(\nu)}\right] \\[2mm]
\supset&\; \left[SU(3)^{(\pi)} \times SU(3)^{(\nu)}\right] \times \left[SU(2)^{(\pi)} \times SU(2)^{(\nu)}\right] \\[2mm]
\supset&\; SU(3) \times SU(2)_S \supset SO(3)_L \times SU(2)_S \supset SU(2)_J
\end{aligned}
\tag{1}
$$

where $2\Omega_\sigma$ denotes the total number of single-particle levels in the N_σ shell for protons ($\sigma = \pi$) and neutrons ($\sigma = \nu$), respectively. The irreducible representations (irreps) of $U(2\Omega_\sigma)$, labeled by $[1^{m_\sigma}]$, where m_σ denotes the number of σ-type particles, are the anti-symmetrized many- particle proton or neutron wavefunctions which separate into spatial ($\rightarrow U(\Omega_\sigma)$, irrep label $[f_\sigma]$) and spin ($\rightarrow U(2)^{(\sigma)}$, irrep label $\left[\bar{f}_\sigma\right]$) degrees of freedom. All calculations presented in this contribution are restricted to the most symmetric and energetically lowest irrep of $U(\Omega_\sigma)$, which means that couplings to ($S \neq 0$) modes are excluded. This seems reasonable since those spin-flip excitations are significantly higher in energy than the low-energy excitations which are of primary interest in this contribution. All $SU(3)$ irreps (λ_σ, μ_σ) that are contained in a given $[f_\sigma]$ are determined from the reduction $U(\Omega_\sigma) \supset SU(3)^{(\sigma)}$ where possible multiple occurrences of the same (λ_σ, μ_σ) are labeled by the integer α_σ. The reduction $\left[SU(3)^{(\pi)} \times SU(3)^{(\nu)}\right] \rightarrow SU(3)$ leads to the $SU(3)$ irreps of the total wavefunction, labeled by (λ, μ), which are determined by taking all possible products $\{(\lambda_\pi, \mu_\pi) \times (\lambda_\nu, \mu_\nu)\} \rightarrow \rho(\lambda, \mu)$ into account, where the running index $\rho = 1, \ldots, \rho_{\max}$ numbers possible multiplicities. The orbital angular momentum values L which are contained in a fixed (λ, μ)-irrep are determined through the reduction $SU(3) \supset SO(3)_L$ and multiple occurrences are numbered by the index κ. The total spin S is derived from the SU(2) product $S_\pi \times S_\nu \rightarrow S$ where proton and neutron spins S^π

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and S^{ν} label the irreps of $SU(2)^{\pi}$ and $SU(2)^{\nu}$ respectively. Finally the total angular momentum J is obtained through the SU(2) coupling $L \times S \to J$ as symbolized by the group reduction $[SO(3)_L \times SU(2)_S] \to SU(2)_J$.

The eigenvalues of the Casimir invariants provided by this group reduction scheme, augmented with the necessary multiplicity labels, allows for a unique classification of the pseudo-SU(3) model wavefunctions,

$$| \{m_\pi[f_\pi]\alpha_\pi (\lambda_\pi, \mu_\pi), m_\nu[f_\nu]\alpha_\nu (\lambda_\nu, \mu_\nu)\} \rho (\lambda, \mu)\kappa L \{S_\pi, S_\nu\} S ; JM\rangle$$

$$= \sum_{\{-\}} \langle (\lambda_\pi, \mu_\pi) \kappa_\pi L_\pi M_{L_\pi} ; (\lambda_\nu, \mu_\nu) \kappa_\nu L_\nu M_{L_\nu} | (\lambda, \mu) \kappa L M_L \rangle_\rho$$

$$\times \langle S_\pi M_{S_\pi}, S_\nu M_{S_\nu} | SM_S \rangle \times \langle LM_L, SM_S | JM \rangle$$

$$\times |m_\pi[f_\pi]\alpha_\pi (\lambda_\pi, \mu_\pi) \kappa_\pi L_\pi M_{L_\pi} S_\pi M_{S_\pi}\rangle \times |m_\nu[f_\nu]\alpha_\nu (\lambda_\nu, \mu_\nu) \kappa_\nu L_\nu M_{L_\nu} S_\nu M_{S_\nu}\rangle$$

$$(2)$$

with the abbreviation $\{-\} = \{M_{S_\pi}, M_{S_\nu}, M_S, M_{L_\pi}, M_{L_\nu}, M_L, \kappa_\pi, \kappa_\nu, L_\pi, L_\nu\}$. In this result $\langle L_1 M_{L_1}, L_2 M_{L_2} | LM_L \rangle$ denotes SU(2) Clebsch-Gordan coefficients and the $SU(3)$ coupling coefficient $\langle \ldots ; \ldots | \ldots \rangle$ is defined in Ref. [12].

2.1.1 Hamiltonian and Transition Operators

The pseudo-SU(3) model Hamiltonian, $H_{PSU(3)}$, used for both the schematic calculations in Section 3 and a comparison to experimental data in Section 4, is a many-particle extension of the Nilsson-model Hamiltonian,

$$H_{PSU(3)} = H_0 - \frac{\chi}{2} Q^a \cdot Q^a - G_\pi H_P^\pi - G_\nu H_P^\nu + aK_J^2 + bJ^2 + D_\pi \sum_{i_\pi} l_{i_\pi}^2 + D_\nu \sum_{i_\nu} l_{i_\nu}^2 \quad (3)$$

where Q^a denotes the algebraic quadrupole operator and χ the quadrupole-quadrupole interaction strength parameter. (H_0 denotes the spherical harmonic oscillator which, however, is trivial since the configuration space of the pseudo-SU(3) model is restricted to a single shell in both proton and neutron spaces. Since only *excitation* energies are of interest, the H_0 contribution, which is equal for every configuration, is of no physical importance.) H_P^π (H_P^ν) stands for the pairing interaction for protons (neutrons) which is multiplied by the pairing strength parameter G_π (G_ν). The term K_J^2 is introduced to accommodate the K-band splitting observed in the low-energy spectra of heavy deformed nuclei and J^2 is the square of the total angular momentum. These five terms constitute the Hamiltonian for the schematic calculations in Section 3 and have already been discussed to some extend in previous publications so the reader is referred to Refs. [3, 13, 14] for additional details. The parameter set is identical to the one used in Ref. [10], namely, $\chi = 4.32$ keV, $a = 202$ keV, and $b = 9.26$ keV, $D_\pi = D_\nu = 0$ and is compatible with sets obtained from best-fit calculations for rare earth nuclei [15].

4

In the Nilsson Hamiltonian the squared single-particle angular momentum operator, l^2, is of great importance since it effectively flattens the harmonic oscillator potential for states of higher angular momentum and in so doing mimics a radial potential shape of the Woods-Saxon type. It is therefore necessary to add analogous terms to $H_{PSU(3)}$ if the pseudo-SU(3) model is used for the description and prediction of experimental data. This is why the Hamiltonian of Ref. [10] has been extended to include the terms $D_\pi \sum_{i_\pi} l_{i_\pi}^2$ and $D_\nu \sum_{i_\pi} l_{i_\pi}^2$ for protons and neutrons, respectively. The particular choice of parameters used for the pseudo-SU(3) model application to ^{140}Ce is discussed in Section 4.

An expression for the electric quadrupole transition operator and the magnetic dipole operator for the pseudo-SU(3) model is given in Ref. [6]. The result for the real-space electric quadrupole transition operator is

$$T_{2\mu}(E2) \equiv b_0^2 \left(e_\pi \sum_{i \epsilon \pi} r_\pi^2(i) Y_{2\mu}(\vartheta_i^\pi, \varphi_i^\pi) + e_\nu \sum_{i \epsilon \nu} r_\nu^2(i) Y_{2\mu}(\vartheta_i^\nu, \varphi_i^\nu) \right) \tag{4}$$

where $b_0 = A^{\frac{1}{3}}$fm is the harmonic oscillator size parameter, e_σ is the effective nuclear charge for protons ($\sigma = \pi$) and neutrons ($\sigma = \nu$), respectively (see Section 4), the spherical single-particle coordinates of the valence nucleons are denoted by $r_\sigma(i), \vartheta_i^\sigma$, and φ_i^σ, and $Y_{2\mu}$ stands for a spherical harmonic [16]. Since all calculations of the pseudo-SU(3) model are performed in the pseudo-space, this real-space operator must be expressed in terms of pseudo- space quantities. The explicit procedure and the results of this real-space $\rightarrow$ pseudo-space transformation are given in Ref. [6].

2.2 Asymmetric Rotor Model

For the sake of completeness, the most important definitions of the asymmetric rotor model are given next. The reader is referred to Refs. [17, 18, 19, 20, 21] for a more detailed account. The asymmetric rotor model Hamiltonian H_{ARM} is defined as

$$\begin{aligned} H_{ARM} &= \sum_{k=1}^{3} \frac{I_k^2}{\mathcal{J}_k} \\ &= \left(I^2 - I_3^2 \right) / \left(\frac{1}{4\mathcal{J}_1} + \frac{1}{4\mathcal{J}_2} \right) + \frac{I_3^2}{2\mathcal{J}_3} \\ &\quad + \left(I_+^2 + I_-^2 \right) / \left(\frac{1}{8\mathcal{J}_1} - \frac{1}{8\mathcal{J}_2} \right), \end{aligned} \tag{5}$$

where I_k^2 denotes the intrinsic k-component of the total angular momentum I and I_+ (I_-) the corresponding raising (lowering) operators. The hydrodynamic moment of inertia $\mathcal{J}_k \equiv 4B\beta^2 \sin(3\gamma - \frac{2k\pi}{3})$ depends on the mass parameter B and the quadrupole deformation variables (β, γ) (with $0^o \leq \gamma \leq 60^o$, $\beta > 0$, see Ref. [21]) all of which are treated as adjustable model parameters. The eigenfunctions of H_{ARM}, $\Psi^{ARM}(\vartheta_i)$, are determined by diagonalization within an orthonormal basis of symmetrized wave functions

$$|IMK> \equiv \sqrt{\frac{2I+1}{16\pi^2(1 + \delta_{K0})}} \left(D_{MK}^{I*}(\vartheta_i) + (-1)^I D_{M-K}^{I*}(\vartheta_i) \right) \tag{6}$$

where the Wigner functions $D_{MK}^{I*}(\vartheta_i)$ [22] depend on the three Euler angles $(\vartheta_1, \vartheta_2, \vartheta_3)$ and M (K) denotes the laboratory (intrinsic) angular momentum z-axis projection. For a fixed angular momentum I the basis dimensionality is given by the number of possible K values: $K = 0, 2, 4, \ldots, I$ if I even and $K = 2, 4, \ldots, I - 1$ if I odd.

Figure 1

Finally, recall that the asymmetric rotor model eigenenergies have a characteristic γ dependence (see Fig. 1) which is symmetric around $\gamma = 30^o$. (The other two model parameters β and B induce only an overall scaling of the eigenenergies.)

The quadrupole operator $Q_{2\mu}^{ARM}$ of the ARM is given in lowest order by

$$Q_{2\mu}^{ARM} = A\beta \left(D_{\mu 0}^{2*}(\vartheta_i) \cos\gamma + \left(D_{\mu 2}^{2*}(\vartheta_i) + D_{\mu-2}^{2*}(\vartheta_i) \right) \frac{\sin\gamma}{\sqrt{2}} . \right) \tag{7}$$

In this expression the normalization coefficient $A = 3ZR_o^2/4\pi$ depends upon the charge Z and the nuclear radius R_o.

2.3 Physical Observables

For the sake of completeness, and before continuing with a more detailed consideration of the pseudo-SU(3) wave functions, the definitions of some physically important quantities will be given. Reduced matrix elements of the SO(3) tensor operator, $Q_{\lambda\mu}$, between states with initial (final) angular momentum and projection J_i and M_i (J_f and M_f) are defined by [22]

$$\langle \gamma_f J_f || Q_\lambda || \gamma_i J_i \rangle \langle J_i M_i, \lambda\mu | J_f M_f \rangle \equiv \langle \gamma_f J_f M_f | Q_{\lambda\mu} | \gamma_i J_i M_i \rangle$$

where γ_i and γ_f represent additional quantum numbers that are required to uniquely define the initial and final state, and $\langle \gamma_f J_f || Q_\lambda || \gamma_i J_i \rangle$ stands for the reduced matrix element. (The use of J for the total angular momentum is customary in microscopic work while I is normally used in macroscopic theories. Here and in what follows, J and I will be used interchangeably for the total angular momentum.) The definition of the reduced transition probability for electric quadrupole radiation [21] is then given as

$$B(E2; \gamma_i J_i \rightarrow \gamma_f J_f) \equiv \frac{2J_f + 1}{2J_i + 1} \langle \gamma_f J_f || T_2(E2) || \gamma_i J_i \rangle^2 \tag{8}$$

and the definition of the electric quadrupole moment is

$$Q(\gamma J) \equiv \sqrt{\frac{16\pi}{5}} \sqrt{\frac{J(2J-1)}{(J+1)(2J+3)}} \langle \gamma J || T_2(E2) || \gamma J \rangle . \tag{9}$$

3 Pairing and Observables

The consequences of the pairing interaction within the pseudo-SU(3) model is explored in this section by considering its effect on K-band mixing and on B(E2) values and quadrupole moments. The K-band mixing is considered first because the information provided by this effect is easier to realize and simpler to understand than the corresponding B(E2) and quadrupole moment results.

3.1 K-band Mixing

The quantum number K_J is the projection of the total angular momentum J on the system's intrinsic z-axis. Since only $S = 0$ states are considered in this contribution, $J \equiv L$ and $K_J \equiv K_L$ and thus the symbol K can and will be used for both. The K label is important in the classification of low- energy collective bands of prolate or near-prolate even-even nuclei. Specifically, the ground-state band of these nuclei is associated with $K = 0$, the so-called one-phonon γ band with $K = 2$, likewise the two-phonon γ-band with $K = 4$ or $K = 0$, and so on.*

The quantum number K is exactly conserved for prolate nuclei only; couplings between bands with different K values are connected with deviations from axial symmetry, that is, with $\gamma \neq 0$. From the point of the view of the Geometric Collective Model [11, 25], the K-band mixing can be understood within the framework of the very simple asymmetric rotor model picture: The Hamiltonian in Eq. 5 is diagonal so long as $\gamma = 0$, since in this case $\mathcal{J}_1 = \mathcal{J}_2$ and K is trivially conserved. Increasing γ (introducing triaxiality) generates non-vanishing matrix elements between basis states with $\Delta K = \pm 2$. For example, the $K = 0$ eigenstate of the symmetric rotor gains non-vanishing contributions from the $K = 2$ basis function, and so on. This β-independent phenomenon is illustrated quantitatively in Fig. 2 which depicts the expectation value of K^2 for the even J asymmetric rotor model yrast states, $\langle ARM|K^2|ARM\rangle_y$, as a function of γ.

Figure 2

The results show that $\langle ARM|K^2|ARM\rangle_y$ increases with both higher angular momentum and larger γ values: For a fixed value of the γ deformation, $\langle ARM|K^2|ARM\rangle_y$ increases with increasing angular momentum because the higher the angular momentum, the higher the K values ($K \leq I$) of basis functions $|IMK >$ that can contribute to the eigenfunction. And for a fixed value of the angular momentum, $\langle ARM|K^2|ARM\rangle_y$ increases with increasing γ-value because the non-diagonal term in H_{ARM} becomes more and more dominant. On the far right-hand-side of Fig. 2, the expectation value of K^2 is indicated for the case of complete mixing, that is, if all the diagonalization coefficients have the same $\frac{1}{\sqrt{d}}$

---

*Recently the K=4 band received some attention through a systematic measurement of γ-vibrational anharmonicities in the rare-earth region [23, 24]

magnitude and thus $\langle ARM|K^2|ARM\rangle_y = \frac{1}{d}\sum_K K^2$, where d denotes the dimensionality. At $\gamma = 60^o$ these estimates are about 25 percent below the results of the diagonalization, indicating that the yrast states favor basis state with higher K values than that of a uniform distribution.

It is now interesting to use these simple results as a backdrop for a study of pairing induced K-band mixing in the pseudo-SU(3) model. To accomplish this it is necessary to have a shell-model expression for the $\hat{K}^2$ operator [13, 14, 26]. For completeness the argument that leads to such a result is repeated here. The basic idea follows from a frame-independent shell- model expression for the asymmetric rotor model Hamiltonian (see Eq. 5). To achieve this one can introduce two SO(3) scalars

$$
\begin{aligned}
X_3^a &\equiv \sum_{i,j} L_i Q_{ij}^a L_j = \sum_i \lambda_i I_i^2 \,, \\
X_4^a &\equiv \sum_{i,j,k} L_i Q_{ij}^a Q_{jk}^a L_k = \sum_i \lambda_i^2 I_i^2
\end{aligned}
$$

where L_i (I_i) and Q_{ij}^a (λ_i) denote, respectively, projections of the total angular momentum and the algebraic quadrupole tensor in either the laboratory or the intrinsic body-fixed principal axis system [13]. (The sums in these expression insure rotational invariance and thereby the frame-independent character of the operators.) Adding the total angular momentum $L^2 = I^2 = \sum_i I_i^2$ to X_3^a and X_4^a means a mapping onto the asymmetric rotor model Hamiltonian can be established:

$$
\frac{I_1^2}{2\mathcal{J}_1} + \frac{I_2^2}{2\mathcal{J}_2} + \frac{I_3^2}{2\mathcal{J}_3} \equiv aL^2 + bX_3^a + cX_4^a,
$$

where the coefficients a, b, and c are real numbers. The geometrical expression for the $\hat{K}^2 = I_3^2$ operator emerges trivially from this expression by setting $1/2\mathcal{J}_1 \equiv 1/2\mathcal{J}_2 \equiv 0$ and $1/2\mathcal{J}_3 \equiv 1$. Taking advantage of this observation, it is straightforward to derive a shell-model expression for $\hat{K}^2$ [13]

$$
\hat{K}^2 = \frac{\lambda_1\lambda_2 L^2 + \lambda_3 X_3^a + X_4^a}{2\lambda_3^2 + \lambda_1\lambda_2} \tag{10}
$$

where the principle axis quadrupole components can be shown to be given by [13]

$$
\lambda_1 = \frac{1}{3}(-\lambda + \mu)\,, \ \ \lambda_2 = \frac{1}{3}(-\lambda - 2\mu - 3)\,, \ \ \lambda_3 = \frac{1}{3}(2\lambda + \mu + 3)\,.
$$

The matrix elements of $\hat{K}^2$ in the extended version of the pseudo-SU(3) model which is used in this contribution can be easily derived using the generic formulas in [14].

To investigate the influence of the pairing force strength on K-band splitting in the pseudo-SU(3) model, consider a configurations with two protons and two neutrons

$(2\pi, 2\nu)$ in the real [pseudo] space shells $(N_\pi = 4, N_\nu = 5)$ $[(\tilde{N}_\pi = 3, \tilde{N}_\nu = 4)]$. This configuration exhibits a rich structure and its dimensionalities are small enough that basis space truncation measures need not be invoked, that is, all possible couplings of proton and neutron $SU(3)$ irreps can be taken into account for all angular momentum values.

Figure 3

Figure 3 shows the expectation value of $\hat{K}^2$ for even angular momentum yrast states of the $(2\pi, 2\nu)$ configuration, $\langle 2\pi 2\nu \,|\, K^2 \,|\, 2\pi 2\nu \rangle_y$, as a function of the pairing strength parameter G, where for simplicity G_π and G_ν were set equal $(G_\pi = G_\nu \equiv G)$ in the calculations. Before entering into a detailed discussion of these results, note that only the pairing terms in Eq. 3 couple states belonging to different $SU(3)$ irreps (λ, μ). If $G = 0$ the quadrupole-quadrupole interaction $Q^a \cdot Q^a \sim 4C_2 - 3L^2$ dominates and forces states of the $SU(3)$ irrep with largest second order Casimir invariant C_2 (Eq. 12) to lie energetically lowest. For the $(2\pi, 2\nu)$ configuration this leading irrep is $(\lambda, \mu) = (14, 0)$ which contains $K = 0$ states only and is the reason why the results show $\langle 2\pi 2\nu | \hat{K}^2 | 2\pi 2\nu \rangle_y = 0$ for all value of the angular momentum (Fig. 3).[†]

In a recent paper (see Figures 3-5 in [10]) the intensity distribution of the yrast eigenstates of $H_{PSU(3)}$ was shown to spread over more and more basis states with increasing pairing strength. As a consequence, the $(2\pi, 2\nu)$ yrast states have contributions from $(\lambda \neq 0, \mu \neq 0)$ $SU(3)$ irreps which contain $K \neq 0$ configurations. This admixture shows up (Fig. 3) as an increase in the $\langle 2\pi 2\nu | \hat{K}^2 | 2\pi 2\nu \rangle_y$ value with increasing pairing strength G. (The $L = 10, 12, 14\hbar$ states are unaffected since they are unique.)

Another mechanism that acts in the same direction is the coupling to higher K states within one $(\lambda \neq 0, \mu \neq 0)$ irrep through the X_3^a and X_4^a operators contained in $\hat{K}^2$ (Eq. 10). This effect increases the expectation value of $\hat{K}^2$ for states of higher angular momentum as long as there are states with higher K values available.

For fixed G the $\langle 2\pi 2\nu | \hat{K}^2 | 2\pi 2\nu \rangle_y$ value increases up to $J = 6\hbar$ and goes down for higher angular momenta due to the decreasing availability of higher K basis states. This mechanism counteracts the two aforementioned effects and is illustrated in Fig. 4 where the average K value, $\bar{K}$, for a fixed even angular momentum value is given for the $(2\pi, 2\nu)$ configuration.

Figure 4

Note that the $(2\pi, 2\nu)$ distribution has its maximum at $J = 4$ and clearly indicates the decreasing availability of higher K basis states with increasing J.

[†]The selection rule for K in a fixed (λ, μ)-irrep is given by $K = \min(\lambda, \mu), \min(\lambda, \mu)$ -2, $\min(\lambda, \mu) - 4, \ldots, 0$ or 1 [27].

Figures 5a) and 5b)

General results of this type are shown in Figs. 5 where $\bar{K}$ values for identical particle configurations in the $\tilde{N} = 3$ (Fig. 5a) and $\tilde{N} = 4$ (Fig. 5b) shells are given. Only results for particle configurations are given since the $\bar{K}$ values for holes are identical to those for particles if the hole and particle numbers are equal. The latter is the reason why within a single shell the maximum $\bar{K}$ value, $\bar{K}_{max}$, increases until the shell is half full and decreases in a particle-hole symmetric fashion when the shell is more than half full. By comparing results for the $\tilde{N} = 3$ and $\tilde{N} = 4$ shells one finds that the $\bar{K}$ distributions are shifted towards higher K values with increasing shell number and that as a general rule the maximum average $\bar{K}$ value for each configuration is found at roughly one third of the maximum possible angular momentum. This feature is a consequence of the selection rule on K values in a fixed $SU(3)$ irrep, which in turn follows from the Pauli Exclusion Principle (see Ref. [28]). One does not expect to find this effect in the phenomenological asymmetric rotor model which knows nothing about the single-particle structure. And, indeed, there are no rules other than $(K \leq I)$ which limit the K projection in the asymmetric rotor model and thus higher $\bar{K}$ values can be obtained.

At the beginning of this section it was noted that a deviation from prolate axial symmetry is a necessary condition for the generation of K-band mixing. This result is depicted in Fig. 2 where the γ dependence of the $\langle ARM|K^2|ARM\rangle_y$ value is shown. It is therefore interesting to consider changes in the γ deformation that result when the pairing strength is increased. The γ of a particular pseudo-SU(3) model wave function can be determined by taking advantage of the mapping between the nuclear deformation variables (β, γ) and the $SU(3)$ irrep labels (λ, μ) [29, 30]:

$$\beta = \sqrt{\frac{4\pi}{5}} \frac{1}{A\bar{r}^2} (C_2 + 3)^{\frac{1}{2}},$$

$$\gamma = \frac{1}{3} \cos^{-1}\left(\frac{C_3}{2(C_2 + 3)^{\frac{3}{2}}}\right),$$

(11)

where C_2 and C_3 denote the second and third order Casimir invariants of $SU(3)$ with eigenvalues

$$C_2(\lambda, \mu) = (\lambda + \mu + 3)(\lambda + \mu) - \lambda\mu,$$

$$C_3(\lambda, \mu) = (\lambda - \mu)(\lambda + 2\mu + 3)(2\lambda + \mu + 3).$$

(12)

Since C_2 and C_3 are diagonal in the pseudo-SU(3) model basis functions their expectation values are easily calculated.

Using Eqs. 11, the dependence of the γ deformation on the strength of the pairing interaction was determined for the even J yrast states of the $(2\pi, 2\nu)$ configuration. The

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results are shown in Fig. 6.

Figure 6

With no pairing all members of the yrast band which are pure $(\lambda, \mu) = (14, 0)$ states with the same $\gamma = 3.2^o$ deformation. But when the strength of the pairing interaction is turned on, the system is driven towards triaxiality. As long as the spectrum is dominated by the quadrupole- quadrupole interaction ($G \leq 0.1\text{MeV}$) and the rotational character of the eigenstates is more or less conserved, the yrast eigenstates are less triaxial the higher their angular momentum. Since in this domain the change with increasing angular momentum is a relatively small effect, it might be attributable to rotational stretching. For larger G values, however, the triaxiality is found to increases more or less uniformly for all of the yrast states. For a very large pairing term, where the corresponding γ values are indicated as "Asymptotic Values" on the far right side of Fig. 6, the states show about the same γ-deformation.

3.2 Quadrupole Moments and B(E2) Values

The effect of the pairing interaction on quadrupole moments and B(E2;$I \rightarrow I + 2$) transition rates is considered in this subsection. Recall that increasing the pairing strength corresponds to increasing the triaxiality of the calculated yrast states of a system. To further probe the effect pairing has on collective behavior it is helpful to look at the γ dependence of quadrupole moments and B(E2) values. And as above, it is useful to start with the complementary asymmetric rotor model results.

Figure 7

Figure 8

Figure 7 depicts the γ dependence of the spectroscopic quadrupole moments of even J yrast states of the asymmetric rotor model. For a range of γ values induced by increasing the pairing strength from weak to strong in the pseudo-SU(3) model , that is, for γ values between about $3^o \leq \gamma \leq 17^o$ (Fig. 6), the negative quadrupole moments are found to decrease in magnitude for all values of the angular momentum. Comparing this with the corresponding pseudo-SU(3) model results shown in Fig. 8, shows that the quadrupole moments of the even yrast states in the pseudo-SU(3) model also decrease with increasing pairing strength. The similarity of these results confirms that an increase in the pairing strength goes hand-in-hand with an increase in the γ deformation.

The situation is less clear if the yrast intra-band B(E2) values of the two models are compared, see Figures 9 and 10.

Figure 9

Figure 10

These figures depict, respectively, the dependence on γ of the intra-yrast band transitions B(E2;$I \rightarrow I + 2$) in the asymmetric rotor and pseudo-SU(3) models. While for $3^o \leq \gamma \leq 17^o$ the asymmetric rotor model results show a moderate decrease in the B(E2;$0 \rightarrow 2$) strength, for all pratical purposes the asymmetric rotor model B(E2;$I \rightarrow I + 2$) values are independent of γ. In contrast with this, Fig. 10 shows that the pseudo-SU(3) model yields a very pronounced drop in the B(E2;$I \rightarrow I+2$) transition strengths with increasing pairing strength. So while diagonal measures suggest that increasing γ in the asymmetric rotor model and the pairing strength in the pseudo-SU(3) model produce similar effects, they predict quite different results for intra-yrast band B(E2) transitions.

4 Application to $^{140}_{58}$Ce$_{82}$

The results of the previous section form a backdrop for pseudo-SU(3) model applications that describe and predict nuclear properties like excitation energies, B(E2) values, quadrupole moments, and g_R-factors. A more complete and systematic study of rare-earth nuclei will be reported elsewhere [15]; this section focuses on the semi-magic $^{140}_{58}$Ce$_{82}$ nucleus which has eight valence protons outside the $Z = 50$ core. This means the model space does not have to be truncated, nevertheless, the $^{140}_{58}$Ce$_{82}$ system is sufficiently well-studied experimentally to allow for a thorough examination of the pseudo-SU(3) model description of its properties.

The $^{140}_{58}$Ce$_{82}$ nucleus has six valence protons in the usual ($N_\pi = 4$) shell, or equivalently, six protons in the pseudo ($\tilde{N}_\pi = 3$) space, and no valence neutrons. The neutron part of the total wave function is therefore trivial and this in turn implies some simplifications in the Hamiltonian of Eq. 3. Dropping all terms with vanishing matrix elements yields

$$H_{PSU(3)} = -\frac{\chi}{2} \, Q^a \cdot Q^a - G_\pi H_P^\pi + aK_J^2 + bJ^2 + D_\pi \sum_{i_\pi} l_{i_\pi}^2 + c \, C_3 \tag{13}$$

where the third order Casimir operator, C_3, of $SU(3)$ has been added. The physical meaning of this operator, see Eqs. 11 and 12, can be easily understood through the relation of the pseudo-SU(3) model to the geometric collective model. As mentioned in the introduction, this liquid-drop type model describes the low-energy collective excitations of even-even nuclei in terms of quadrupole surface vibrations and rotations of the nucleus as a whole.

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A number of publications show that a phenomenological treatment of the geometric collective model provides a very successful means for describing the low-energy properties (energies, B(E2) values, and quadrupole moments) of even-even nuclei throughout, including phenomena like shape coexistence and shape transition [11, 21]. The basis of such an approach is an expansion of the model Hamiltonian in powers of the collective (β, γ) variables. A typical ansatz for the nuclear potential in such a scheme is

$$V(\beta, \gamma) = x_2\beta^2 + x_3\beta^3 \cos 3\gamma + x_4\beta^4 + \ldots \tag{14}$$

where the real numbers $x_2, x_3, x_4, \ldots$ are usually determined in a least-square fitting procedure aimed at an optimal description of the experimental data of the nucleus under consideration.

To understand the physical effect of C_3 in Eq. 13, it is useful to recall the mapping between the pseudo-SU(3) labels (λ, μ) and the deformation variables (β, γ) [29] that is given in Eq. 11. The mapping is based on an associated of the invariants of the $SU(3)$ and geometric collective models,

$$C_2 \sim \beta^2, \qquad C_3 \sim \beta^3 \cos 3\gamma. \tag{15}$$

These relations also provide an explanation for the physical meaning of C_3 in Eq.13: Depending on the positive or negative sign of c in Eq. 13, or equivalently on the sign of x_3 in Eq. 14, the nuclear system is driven towards more oblate or more prolate deformations. So in contrast with the quadrupole-quadrupole interaction, the C_3 term adresses the γ degree-of-freedom and helps determine the triaxiality of the system.

Six parameters (see Table 1) are required to fix the pseudo-SU(3) model Hamiltonian (Eq. 13) for $^{140}_{58}\text{Ce}_{82}$. These values were determined in a best-fit calculation based on known experimental energies and B(E2) values of the system.

Table 1

There are a number of theoretical and experimental studies which focused on the ^{140}Ce nucleus [31]. The current study focuses only on its excitation spectrum; the reader is referred to [15] for other measures as well as a systematic comparison of the properties of several different rare earth nuclei.

Figure 11

Pseudo-SU(3) model results for ^{140}Ce energies up to about 3 MeV are shown in Fig. 11. A comparision with experimental excitation energies that was taken from [32] is given to the immediate right of the pseudo-SU(3) results followed by the spectrum obtained by Wildenthal [33] and the results of a model study by Waroquier and Heyde [34].

The Wildenthal results were obtained in a conventional shell model calculation that used a modified surface δ interaction in a truncated configuration space. The single-particle energies and the parameters of the modified surface δ interaction were adjusted to describe the excitation energies of nuclei in the mass range from $A = 136$ to $A = 145$ [33]. In addition to a surface δ interaction, Waroquier and Heyde [34] used a Gaussian residual nucleon-nucleon interaction. The single-particle energies were determined by solving the inverse-gap equations.

While the agreement between the three theories is of approximately the same quality for energies below about 2.5 MeV, the pseudo-SU(3) model does not account for a low-lying 5^+ state which experimental evidence suggests lies at 2.349 MeV. Otherwise, both the pseudo-SU(3) model and the calculations by Wildenthal account for virtually all levels up to about 3 MeV while the results by Waroquier and Heyde seem to miss a few of the experimentally established states. In addition, note that the two former models predict a few levels, like two 4^+ states at about 2.8 MeV, which have not been found experimentally. The pseudo-SU(3) spectrum also predicts a 0^+ state at about 2.25 MeV that cannot be found in the experimental results.

5   Summary, Conclusion and Outlook

The effect of pairing on K-band mixing, B(E2) values, and quadrupole moments has been studied within the framework of a complete (untruncated) pseudo-SU(3) model theory. The asymmetric rotor model, which is a collective model theory that describes nuclear properties by means of rotations only, was introduced as a backdrop for helping to identify collective properties of the pseudo-SU(3) model and, in a complementary way, for discovering limitations of the collective model approach:

- From general considerations it is clear that deviations from axial symmetry are a necessary condition for K-band mixing. This was seen explicitly for the asymmetric rotor model by noting that for $\gamma \neq 0$ the non-diagonal part of the Hamiltonian is non-zero, and for the pseudo-SU(3) model since an increase in the pairing strength not only induces K-band mixing but also generates an increase in γ deformation.

- The K label of the pseudo-SU(3) model is limited by a $SU(3)$ to $SO(3)$ selection rule. This restriction is inherent to the Pauli Exclusion principle and not part of the asymmetric rotor model where, in principle, there is no maximum K value.

- The quadrupole moments of yrast states of the asymmetric rotor model decrease in magnitude with increasing γ deformation ($0 \leq \gamma \leq 30^o$) and in the pseudo-SU(3) model with increasing pairing strength. Since an increase in the pairing strength was found to be correlated with an increase in γ deformation from about $\gamma = 3^o$ to $\gamma = 17^o$, the physics behind the two phenomena appears to be very similar.

- The yrast intra-band B(E2;$I \rightarrow I + 2$) strengths decrease rather strongly with increasing pairing strength while the asymmetric rotor model values show only a

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moderate decrease with increasing γ deformation.

Finally, the pseudo-SU(3) model was used for a description and prediction of experimental energies of ^{140}Ce and the results were compared to other theories. The Hamiltonian parameters were determined in a best-fit calculation that used as input the experimental energies and the B(E2)-values of the low-lying states. The pseudo-SU(3) model was found to describe the known experimental energies satisfactorily, in a addition, a few new level were predicted. A more complete and systematic comparison of the predictions of the pseudo-SU(3) model with experimental data and other theories will be reported in a forthcoming contribution [15].

Regarding future developments it should be obvious that the presented formalism can be easily applied to $(S \neq 0)$ states and hence to a description of not only even-even but also odd-even, even-odd, and odd-odd nuclei. To put this study into perspective, it is also important to note that the results learned from pseudo-SU(3) model studies apply as well to the pseudo- symplectic model which is a natural extension that takes couplings to higher shell fully into account.

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Parameter	^{140}Ce
χ [keV]	3.97
D_π [keV]	-101.05
G_π [keV]	157.5
a [keV]	130.6
b [keV]	0.085
c [keV]	5.078

Table 1: List of parameters used in the pseudo-SU(3) calculations of ^{140}Ce (see text).

values

Figure captions

Figure 1: The γ dependence of the lowest asymmetric rotor model eigen- energies in units of $\frac{\hbar^2}{4B\beta^2}$. The angular momentum values are indicated on the far right and along the central vertical line.

Figure 2: The γ dependence of the expectation value $\langle ARM|\hat{K}^2|ARM\rangle_y$ of the $\hat{K}^2$ operator is shown for even angular momentum yrast states of the asymmetric rotor model. The angular momentum of the various curves are indicated on the far right. The values for $\langle ARM|\hat{K}^2|ARM\rangle_y$ that are obtained for complete mixing, that is, when all the diagonalization coefficients are set equal to one another, are indicated by horizontal lines at the far right labeled above with the corresponding value of the angular momentum.

Figure 3: The expectation value of the $\hat{K}^2$ operator in even angular momentum yrast states is shown as a function of the pairing strength parameter G, where for simplicity the latter were all set equal, $G_\pi = G_\nu = G$. The angular momentum value of each curve is indicated on the far right. The horizontal lines on the far right indicate asymptotic values for the matrix elements $\langle ARM|\hat{K}^2|ARM\rangle_y$ that are obtained in the limit of a very large pairing strength.

Figure 4:   This figure illustrates an average property of the pseudo-SU(3) model basis states, namely, the average K value for all basis states of a fixed even angular momentum J. The $\bar{K}$ distribution shown is for the $(m_\pi = 2, m_\nu = 2)$ configuration.

Figure 5:   This figure is similar to Figure 4. The $\bar{K}$ values were calculated for all even angular momentum states J and different particle numbers m in the $N = 3$ shell [5a), left] and the $N = 4$ shell [5b), right]. (Particle distributions corresponding to more than half- filled shells can be obtained by invoking particle-hole symmetry.)

Figure 6:   The expectation value of the γ deformation, $\langle \gamma \rangle$, is shown for even angular momentum yrast states of the $(m_\pi = 2, m_\nu = 2)$ configuration. The abscissa is the pairing strength parameter G where for simplicity the proton and neutron strengths were set equal, $G_\pi = G_\nu \equiv G$. On the far right the $\langle \gamma \rangle$ values for a very large value of the pairing strength (" Asymptotic Values") are indicated as bars, each of them labeled by its angular momentum value.

Figure 7:   Quadrupole moments of yrast states of the asymmetric rotor model for even values of the angular momentum are shown as a function of the γ deformation where the numbers on the far left and right denote the angular momentum in units of $\hbar$.

Figure 8:   Quadrupole moments of yrast states of the $(m_\pi = 2, m_\nu = 2)$ configuration for even values of the angular momentum are shown as a function of the pairing strength G. The numbers next to the curves on the right denote the value of the angular momentum in units of $\hbar$. For comparison, values for the prolate rotor are indicated on the far left. Note that these are almost identical to the pseudo-SU(3) model values. On the far right quadrupole moments for the case of very large G are indicated by dashed horizontal lines (" Asymptotic Values ").

Figure 9:   The γ dependence of the intra-band transition probabilities, B(E2;$I \to I + 2$), for the asymmetric rotor model yrast states are shown, with the initial and final values of the angular momentum indicated on the far right.

Figure 10:   The intra-band reduced transition probabilities, B(E2;$I \to I + 2$), for eigenfunctions of the pseudo-SU(3) model are shown as a function of increasing pairing strength. On the far right the horizontal lines labeled "Asymptotic Values" are for a very large value of the pairing strength. The dotted lines from the smaller G results are included to help guide the eye. Similarly, on the far left the horizontal lines labeled "Rotor Values" are the B(E2;$I \to I + 2$) strengths of the prolate rotor.

Figure 11: From left to right the figure depicts the excitation spectrum of ^{140}Ce as calculated within the pseudo-SU(3) model , the experimental values (left center), the results of a calculation by Wildenthal (right center), and, on the very right, the energies as obtained in the model used by Waroquier and Heyde (see text). Note that the thin dashed lines between the levels are supplied to guide the eye between corresponding levels and that the spacings between the ground and the first excited states are not to scale.)