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Longitudinal Development of Nuclear-Electromagnetic Cascades at Energies Around Trillion Ev.

Charles Richard Gillespie

Louisiana State University and Agricultural & Mechanical College

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CASCADeS AT ENERGIES AROUND 10^{12} eV

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
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requirements for the degree of
Doctor of Philosophy

in

The Department of Physics

by
Charles Richard Gillespie
B.S. University of Kansas, 1959
M.S. Louisiana State University, 1961
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ABSTRACT

An experiment was performed in which the longitudinal development of nuclear-electromagnetic cascades resulting from 71 hadron-nucleus interactions was measured. The hadrons had energies greater than 0.5 TeV and were incident on a glass-scintillator ionization spectrometer which was 11 interaction lengths deep. With this spectrometer the development of the nuclear-electromagnetic cascade was measured in terms of the ionization energy loss by the cascade in the spectrometer. A calibration technique was developed which allowed the ionization energy loss measurements to be expressed in energy units. The average energy of the single hadrons incident on the apparatus was 1 TeV. The integral energy spectrum of the hadrons was described by a power law with an exponent of $2.1 \pm 0.3$ in the energy interval from 0.5 TeV to 1.0 TeV. The average nuclear-electromagnetic cascade ionization energy loss for all incident hadrons was found and the results compared with corresponding three dimensional Monte Carlo nuclear-electromagnetic cascade calculations. By describing the decrease of the nuclear-electromagnetic cascade with an exponential decay, the effective average
inelasticity of the hadron-nucleus interactions was indicated to be slightly greater than the value of 0.5 assumed in the Monte Carlo calculations for proton-glass interactions.
I. INTRODUCTION

A significant feature of strong interactions at energies greater than $10^{10}$ eV is multiple particle production. At energies around $10^{12}$ eV the number of secondaries resulting from the strong interaction of a hadron with a nucleus may be several or as many as 180.\textsuperscript{1-3} The secondaries are mainly hadrons. When the initial hadron-nucleus interaction takes place in a material medium (absorber) and if the secondaries have sufficient energy, then these hadronic secondaries can strongly interact with the nuclei of the absorber. From these secondary interactions more hadrons may be produced which can strongly interact. The cascade of strong interactions in the absorber will continue until each hadron has insufficient energy to strongly interact with the nuclei of the absorber. Because of the cascade of particles from multiple particle production, there will be an increase in the number of energetic particles as the depth of observation in the absorber increases. The number of particles will begin to decrease at larger depths in the absorber because more particles are being absorbed than are being created.
The energetic proton emerging from a proton-nucleus interaction is called the surviving proton and on the average it retains about 1/2 of the energy of the incident proton. The produced parties ($\pi^\pm$, $\pi^0$, $K^\pm$, etc.) share the remaining energy according to a distribution in which the average transverse momentum is about 0.3 GeV/c. About 2/3 of these produced particles are the $\pi^\pm$ meson which constitute virtually all of those particles which undergo secondary interactions.

The $\pi^0$ meson decay rapidly ($\sim 10^{-15}$ sec) into pairs of gamma rays. These gamma rays typically have sufficient energy to initiate electromagnetic (EM) cascades. Since about one third of the particles produced in the strong interactions are $\pi^0$ there will be EM cascades initiated at various depths in the absorber. The resulting cascade of particles (electrons from EM cascades and hadrons from the strong interactions) is called a nuclear-electromagnetic (NEM) cascade.

The development of the EM cascade in an absorber is characterized by the radiation length of the absorber. The average distance between strong interactions of the same particle in an absorber is characterized by the interaction length. In absorbers with medium to high atomic numbers the interaction length is several times
longer than the radiation length. For this reason and because of the relatively small energy required to generate an energetic electron the number of electrons of the EM cascades exceeds the number of hadrons at all depths in the absorber after the first interaction. At large depths in the absorber the particles constituting the NEM cascade are essentially all electrons.

As the charged particles of the NEM cascade propagate in the absorber they lose energy through ionization and creation of nuclear evaporation particles. Because of these energy loss mechanisms and the sharing of the energy of the initial hadron by all produced particles, further production of particles will eventually terminate and the existing particles will cease to propagate in the absorber. The decrease in the number of particles in a NEM cascade begins at depths of 2-5 interaction lengths in the absorber for $10^{12}$ eV$^4$ incident protons interacting with the absorber nuclei.

In recent experiments to study directly the secondary particles arising from hadron-nucleus interactions at energies greater than $10^{11}$ eV the properties of the resulting NEM cascade have been used to determine the energy of the incident hadron. The only existing source of hadrons having these energies is
cosmic rays. Since the hadrons are randomly incident, the energy of each hadron must be estimated or measured. The two techniques of the ionization spectrometer and the burst counter are those in use to measure the energy of the incident hadron.

The ionization spectrometer was proposed by Grigerov in 1958. The secondary particles interact in an absorber whose depth is large enough to contain a great fraction of the NEM cascade. Ionization detectors are placed at increasing depths in the absorber. As the NEM cascade passes through the detectors the cascade's ionization energy lost in the detectors is measured. This measurement is usually expressed in terms of the number of cascade particles passing through the detectors.

By sampling the cascade at various depths in the absorber the energy of the incident hadron can be determined by:

1. integrating over the depth of the absorber the sampled ionization loss for an absorber sufficiently deep to contain almost all of the cascade, and

2. using hadrons of known energy to determine the relation between detector measurements and the hadron's energy.
In both methods the efficiency of techniques of sampling the ionization energy from evaporation fragments is generally so low that corrections for this energy lost must be made,\textsuperscript{12, 13}

The burst counter utilizes the fact that the NEM cascade has a maximum number of particles which is strongly dependent on the energy of the incident hadron.\textsuperscript{1, 14} The average depth at which this maximum occurs is weakly dependent on the energy of the incident hadron. In the burst counter the number of particles is observed with a single detector after the cascade has developed in a fixed thickness of absorber. The number of particles measured is then related to the energy of the incident hadron. Because of inherent fluctuations in the development of the NEM cascade this method has a significantly higher uncertainty in determining the energy of the incident hadron than does the ionization spectrometer.

The extensive use of the properties of the NEM cascade development has stimulated the generation of models to describe the NEM cascade.\textsuperscript{1, 4, 13-16} The characteristics of the NEM cascade development depends on the interaction cross-sections, inelasticity and multiplicity of the strong hadron-nucleus interaction. It has been shown that the development of the NEM
cascade resulting from the incident hadron interacting with the nucleus of the absorber is significantly dependent on the characteristics of the first interaction, namely depth in the absorber where the first interaction occurs, inelasticity and multiplicity.

Two models have been proposed which deal with the complete NEM cascade in various absorbers. One model treats the NEM cascade analytically and shows the average dependence of the number of particles in the cascade on such parameters as the inelasticity and multiplicity of the first interaction. The second model uses a Monte Carlo method to determine the number of particles at any depth in the absorber. In both cases the NEM cascade development is described in terms of particle numbers where the term particle refers to that which defined in Rossi's Approximation B, i.e. an electron which loses energy at a constant rate.

The development of NEM cascades has been measured with ionization spectrometers in events with incident hadron energies less than 905 GeV. There has been some effort in testing the models but little effort in providing a comprehensive experiment by which the models can be tested. A particular difficulty is the particle concept of Approximation B. In the experiments performed the quantities that were
measured were (1) the number of ion pairs created in an ionization chamber at different depths in an absorber\(^2, 6, 13\) and (2) the ionization energy loss of the cascade particles as they passed through a scintillator. This energy loss was then converted to numbers of particles.\(^2, 9\) In the first case the number of ion pairs is dependent on the amount of energy available to ionization, not necessarily the number of Approximation B particles. Because of the energy distribution of the particles in the NEM cascade the energy loss of these particles passing through a scintillator as in (2) cannot accurately be related to the number of particles in the cascade unless the distribution is well known. The nature of this distribution has yet to be measured even for the case of a single EM cascade.

All of the spectrometers previously reported have one common characteristic. The cosmic ray flux of hadrons having energies greater than \(10^{12}\) eV is rather low. In order to have a large sensitive area to intercept the hadrons and a practical absorber depth, the spectrometers were constructed with high-Z absorbers. The detectors consisted naturally of low-Z materials. It has been predicted that the difference in \(Z\) between the absorber and detector would strongly
perturb the cascade development leading to significant errors in determining the cascade development. This phenomena is called the transition effect. In the case of a spectrometer having many layers of detectors, the cascade perturbations due to the transition effect accumulate resulting in a cascade development significantly different from that in a pure absorber. In addition, predictions have been made which show that the fraction of primary energy lost by nuclear evaporation is greater in a high-Z absorber than in a low-Z absorber. The sampling efficiency of this energy loss is less in a high-Z spectrometer because the range of the evaporation fragments is short. In addition, the frequency of fragmentation in the high-Z absorber is much greater than that in the low-Z detectors.

The purpose of this experiment is to measure accurately the energy of the incident hadron in the energy range from 0.5 TeV to 1.5 TeV and to determine the development of the NEM cascade with an ionization spectrometer. The apparatus for this experiment is located at an altitude of 3.7 km in Climax, Colorado. This apparatus is part of an effort to investigate the properties of the secondary particles emitted in the
primary hadron-nucleus interaction. The construction of the ionization spectrometer is unique in that the absorber is a low-Z material, specifically glass. This construction reduced the transition effect to negligible proportions and the fraction of energy going into nuclear evaporation fragments is greatly diminished. In order to accurately predict the transition effect an experiment was performed to measure it. Utilizing the results obtained from the transition effect experiment a calibration technique was devised which allowed the NEM cascade development to be presented in standard units of energy loss (eV).
II. DESCRIPTION OF APPARATUS

The apparatus is contained in the Louisiana State University Cosmic Ray Physics Laboratory located near Climax, Colorado at an altitude of 3.7 km. The configuration of the apparatus is shown in Fig. 1. The main sections of the apparatus pertinent to this experiment are the spectrometer and the flash tubes. The emulsion chamber and the carbon target do not actively participate in this experiment. They become important in the study of secondary particles from primary hadron-nucleus interactions.

A. Ionization Spectrometer

The spectrometer is constructed using a low-Z (glass) absorber and regularly spaced layers of plastic scintillator. The spectrometer has a total thickness of 1134 gm/cm² of glass and 11 gm/cm² of scintillator giving a depth of 10.6 interaction lengths. Each sheet of Pilot Y scintillator has dimensions of 91.5 cm x 183 cm x 2.54 cm. Each glass absorber module has dimensions of 91.5 cm x 183 cm x 22 cm. The glass absorber at the top of the spectrometer is one half module thick.
Fig. 1
The geometrical factor of the spectrometer is 0.1 m²sr.

The light from the scintillators is detected by photomultipliers arranged as shown in Fig. 2. Two 5 inch diameter RCA 8055 photomultipliers view a pair of scintillators which are separated by a glass module. This geometry was found to give a uniform (3.7%) photomultiplier signal for muons of nearly the same energy passing anywhere through the 91.5 cm x 183 cm area of the scintillator sheets.20 This geometry has an advantage over light piping techniques in that the light from the scintillators can also be viewed by an image intensifier system.21 The 10 groups of two scintillators viewed by two photomultipliers are optically isolated from each other.

The ionization energy loss of the NEM cascade in the spectrometer is sampled by the scintillators. Each group of two scintillators and two photomultipliers constitutes an independent information channel. The term "sampling layer" is used to denote the material within which the NEM cascade loses the amount of energy through ionization indicated by the information channel. A sampling layer shown in Fig. 3 consists of the two scintillators, one module of glass between the scintillators, one half of a glass module above the
Fig. 2
Fig. 3
top scintillator, one half of a glass module below the lower scintillator and the two photomultipliers. There are 10 sampling layers in the spectrometer.

The 20 scintillators are viewed by 20 five inch diameter photomultipliers to measure the NEM cascade energy loss at different depths in the spectrometer. Each of three of these scintillators (labeled S1, S19, S20 in Fig. 3) are also viewed by 6 two inch diameter photomultipliers, 3 on each 91.5 cm side. The tubes are optically coupled tightly to the scintillators. The system of a scintillator with the 6 coupled two inch photomultipliers constitutes another independent information channel. Corresponding to scintillators S1, S19 and S20 these information channels are labeled Muon 1, Muon 19 and Muon 20 respectively. This arrangement gives a well resolved single minimum ionizing particle signal from each scintillator with good (~10%) uniformity. These scintillators are used in calibration and the signal from the two inch diameter tubes of scintillators S1 is used as the event timing reference.

Four 21 cm x 21 cm x 2.54 cm scintillators are placed in a horizontal plane at the corners of the top flash tube row. These scintillators are used to detect particles of a shower accompanying an incident hadron.
They are used to veto the triggering of the electronics if the number of particles passing through them exceed a threshold.

B. Neon Flash Tubes

To locate the lateral position and angle of incidence of the hadron on the spectrometer, its track is indicated by rows of neon flash tubes of the kind developed by Conversi. These tubes all have 1.75 cm diameters. There are four rows of tubes for the 183 cm side and three rows for the 91.5 cm side. The arrangement of these tubes relative to the top of the spectrometer, emulsion chamber and carbon target is shown in detail in Fig. 4. The rows of flash tubes have longer dimensions than the spectrometer so that the particle acceptance aperture is defined by the spectrometer and not by the flash tubes. As seen in Fig. 4 the tubes of the 183 cm side have the largest track resolving power because of the large number of rows of tubes.

The tubes are made of glass and filled with a neon-helium gas. An ionizing particle passing through the tube creates ion pairs in the gas. A high electric field applied across the tubes accelerates the ions causing an avalanche resulting in an arc discharge.
When the row of tubes is viewed "end-on" the track of a particle passing through the row is seen as a series of dots when the arc lights up the end of the tubes. The tubes are arranged in rows with aluminum sheets above and below the rows. These sheets are connected to high voltage pulser which supply the electric field upon command from the trigger logic. The light from the ends of the tubes is steroscopically photographed. An example of the track of a single particle through the flash tubes is shown in Fig. 5.

C. Electronics

The block diagram of the spectrometer electronics is shown in Fig. 6. The spectrometer has two modes of operation; event mode and calibration mode. The units used only in the calibration mode are drawn in dotted lines. All other units are common to both modes of control. The abbreviations established in the following discussion pertain to those used in Fig. 6.

The amplitudes of the pulse heights from the photomultiplier tubes measuring the cascade energy loss are measured using a specially built pulse height to time converter (PHC). These converters operate in the following manner. As shown in Fig. 7 the voltage rise of the pulse sets the converter. The low level is set if the pulse amplitude exceeds 3 mV and the upper
Fig. 6
OUTPUT OF CONVERTER

INPUT \( t_0 \) HEIGHT-TO-TIME CONVERTER

-200 MV

+200 MV

3 MV

300 MV

SET

\( t_k \)

\( t_k \)

\( t_0 \)

\( t_0 \)

432 NSEC TIME CONSTANT

Fig. 7
level is set if the pulse amplitude exceeds 300 mV. As the pulse decays below the set level, the converter is reset. The amplitude of the photomultiplier pulse is then logarithmically related to the time $t_k$ between when the converter was set and when it was reset. Measurement of this time interval $t_k$ is done with an oscilloscope. The oscilloscope sweep is started at the time when the converter levels are set. When the converter is reset a pulse is generated. This pulse then vertically deflects the scope beam. The time interval $t_k$ is then the time between the start of the scope sweep and vertical deflection. For pulse amplitudes greater than 3 mV but less than 300 mV, i.e. low level, the scope beam deflection is positive. For pulse amplitudes larger than 300 mV the scope beam deflection is negative. In this way a pulse amplitude dynamic range of four decades can be measured in one sweep of an oscilloscope beam.

In order to display the converter outputs of all 10 layers a dual beam scope and pulse shaping network (APG) are used. On one beam of the scope (upper) the amplitude information of layer 1 through 5 is displayed. On the other beam (lower) the amplitude information of layers 6 through 10 is displayed. The pulse shaping networks uniquely shape the pulse from
each converter so that they can be identified on the scope.

Since the pulse height converters measure the time it takes for a pulse to decay, it is important that a stable time reference be established. This reference is provided by detecting the zero crossing of the differentiated and delay line clipped signal from Muon 1. In any mode of operation of the spectrometer there is at least one ionizing particle passing through scintillator S1. This insures that there will be a signal from Muon 1 and detection of the zero crossing insures the time stability of the signal.

D. Logic

When a hadron is incident on the spectrometer and associated apparatus (flash tubes, carbon target and emulsion chamber), there will be at least one particle passing through scintillator S1. The signal from Muon 1 in conjunction with other signals indicates that an event has occurred. The logic circuit is satisfied if there is a signal from Muon 1, the sum of all of the signals of cascade energy loss photomultipliers is greater than some value and there has been no more than 1 particle passing through any of the shower
counters. If the logic is satisfied then an event is considered to have occurred and the scope sweep is started so that the 10 pulse heights from the sampling layer photomultipliers will be recorded. After the pulse heights have been recorded (elapsed time of about 2.5 μsec) the flash tubes are triggered. The pulse height and the flash tube information are recorded photographically.

In calibration there are two objectives, to determine the average signal from the sampling layer photomultipliers for muons having energies within a known range, and to find the relationship between the sampling layer photomultiplier pulse amplitudes and the pulse height converter pulses on the scope.

The average muon signal is found from the sampling layer pulse height distribution obtained when muons penetrate the spectrometer. This is determined from coincidence requirements on the single minimum ionizing particle signals from Muon 1, 19, 20. When these scintillators detect a penetrating muon the pulse height from a sampling layer is recorded. The average muon signal is obtained from the pulse height distribution resulting from about 500 muon events.
The single minimum ionizing particle signal distributions from Muon 1, 19 and 20 are well resolved from the photomultiplier noise. The signals from Muon 1 and 19 are sent to single channel analyzers (SCA) and those from Muon 20 are sent to a threshold discriminator. The low level of the single channel analyzers and the threshold of the discriminator are set to reduce the rate of random coincidences between the units due to photomultiplier noise. The upper levels of the single channel analyzers are set to reduce the rate of coincidences between these units due to such phenomena penetrating the spectrometer as: a NEM cascade, several muons, obliquely incident air showers, etc.

The relationship between the sampling layer pulse amplitudes and the pulse height converter pulses is obtained by simulating an event with an artificial light pulser. For each light pulse of 2-3 nsec duration the linear pulse height of a sampling layer is recorded simultaneously with the pulse height converter pulse. By varying the intensity of the light pulse the relationship is obtained for the operating range of the converter.
E. Significance of the Transition Effect

Because of predicted extreme consequences of the influence of the transition effect on energy measurements in spectrometers it was decided to perform an experiment to measure this effect. Two experiments were run and reported together. One at the Cambridge Electron Accelerator (CEA) using single electrons of 5 GeV energy and one using a current of 1 GeV electrons from the Stanford Mark III linear accelerator. In both cases the electrons were incident on a primary absorber of given thickness. Behind the primary absorber was a secondary absorber of Plexiglass. At certain depths in the Plexiglass the properties of the EM cascades were measured.

In the case of the CEA experiment the detector was a single sheet of 2 mm thick plastic scintillator viewed by two photomultipliers. The detector used in the Stanford run was a small cylinder of anthracene at one end of a probe which radially moved across the Plexiglass. The photomultiplier detecting the light from the anthracene cylinder was at the exterior end of the probe. The results from these experiments for lead and Plexiglass are shown in Fig. 8. It is noticed that the effect is much smaller than predicted. Also, introduction of a backscatterer tended to diminish the
TRANSITION EFFECT IN LEAD PLEXIGLASS

E N E R G Y  D E P O S I T E D

THEORETICAL 3.1 r.l. 1.0 GeV

5.2 r.l. 5.0 GeV

5.6 r.l. 1.0 GeV

11.7 r.l. 1.0 GeV

Fig. 8
effect. The magnitude of the transition effect is directly related to the amount of deviation from a constant value of 1.0 (arbitrary units) of the energy deposited as a function of depth in Plexiglass. This can be seen from measurement points 2 and 3 and 7 and 8 of the complete results of the Stanford measurements shown in Table I. The experiments showed that the transition effect was quite small for glass. If one takes into account the appropriate backscattering for glass then for practical purposes the transition effect can be neglected from consideration in the glass spectrometer.
<table>
<thead>
<tr>
<th>Measurement Point No.</th>
<th>Type</th>
<th>Absorber Thickness cm</th>
<th>Transition into Plexiglass cm</th>
<th>Energy Deposited (Normalized) 1 GeV Electrons with 5.08 cm Pb</th>
<th>Backscattering</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Pb</td>
<td>1.75</td>
<td>0.3</td>
<td>1.0 ± 0.03</td>
<td>no</td>
</tr>
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<td>2</td>
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<td>1.75</td>
<td>2.9</td>
<td>0.86 ± 0.03</td>
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<td>2.9</td>
<td>0.98 ± 0.04</td>
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</tr>
<tr>
<td>4</td>
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<td>12.8</td>
<td>0.55 ± 0.02</td>
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</tr>
<tr>
<td>5</td>
<td>Pb</td>
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<td>44.8</td>
<td>0.30 ± 0.01</td>
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</tr>
<tr>
<td>6</td>
<td>Pb</td>
<td>3.17</td>
<td>0.3</td>
<td>1.0 ± 0.03</td>
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</tr>
<tr>
<td>7</td>
<td>Pb</td>
<td>3.17</td>
<td>2.9</td>
<td>0.72 ± 0.03</td>
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</tr>
<tr>
<td>8</td>
<td>Pb</td>
<td>3.17</td>
<td>2.9</td>
<td>0.98 ± 0.04</td>
<td>yes</td>
</tr>
<tr>
<td>9</td>
<td>Pb</td>
<td>3.17</td>
<td>12.8</td>
<td>0.43 ± 0.02</td>
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</tr>
<tr>
<td>10</td>
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<td>44.8</td>
<td>0.22 ± 0.02</td>
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</tr>
<tr>
<td>11</td>
<td>Pb</td>
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<td>1.0 ± 0.03</td>
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</tr>
<tr>
<td>12</td>
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<td>13</td>
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<td>0.56 ± 0.03</td>
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<tr>
<td>14</td>
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<td>0.3</td>
<td>1.0 ± 0.03</td>
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</tr>
<tr>
<td>15</td>
<td>Fe</td>
<td>5.08</td>
<td>12.8</td>
<td>0.64 ± 0.03</td>
<td>no</td>
</tr>
<tr>
<td>16</td>
<td>Fe</td>
<td>5.08</td>
<td>44.8</td>
<td>0.37 ± 0.04</td>
<td>no</td>
</tr>
<tr>
<td>17</td>
<td>Glass</td>
<td>26.0</td>
<td>0.3</td>
<td>1.0 ± 0.03</td>
<td>no</td>
</tr>
<tr>
<td>18</td>
<td>Glass</td>
<td>26.0</td>
<td>12.8</td>
<td>0.76 ± 0.03</td>
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</tr>
<tr>
<td>19</td>
<td>Glass</td>
<td>26.0</td>
<td>44.8</td>
<td>0.52 ± 0.03</td>
<td>no</td>
</tr>
</tbody>
</table>
III. EXPERIMENTAL PROCEDURE

The sampling layer signal resulting from a known ionization energy loss in the layer is found using a specific trigger logic, called muon signature, applied to the signals from Muon 1, 19 and 20. The muon signature permits a sampling layer signal to be recorded only when a single muon having an energy within narrow limits has traversed the spectrometer. The energy lost in a sampling layer by this muon is essentially constant and can be calculated.

In a nuclear interaction event, the energy lost in the scintillator and sampling layers by the NEM cascade is usually considerably larger than that of a muon. The energy loss of an event is related to the energy loss of a muon through: (1) the linear dependence of scintillator light emission on ionization energy loss and (2) the known dependence of the photomultiplier signal on the intensity of incident light.

Events were selected in which a single hadron was incident on the apparatus and the resulting NEM
cascade lost an amount of ionization energy in the spectrometer greater than a given threshold. Since the incident hadron would not lose all of its energy through ionization in the spectrometer the unmeasured energy losses were estimated.

A. Calibration Using Muons

The signal output of Muon 1, 19 and 20 are used to define the muon signatures. Two signatures are in use. One is SM standing for stopping muon and the other is PTM standing for passing through Muon. The SM signature is defined by a coincidence (50 nsec resolving time) between Muon 1 and 19 with Muon 20 acting as a veto. The PTM signature is defined by a coincidence of Muon 1, 19 and 20 signals.

The PTM signature requires that within 50 nsec a minimum ionizing particle passes through S1 and S19 and that a charged particle penetrates S20. This signature is satisfied by: (1) a weakly interacting charged particle passing through the spectrometer, or (2) several charged particles obliquely traversing S1, S19 and S20 within 50 nsec. Because of the geometry of S1, S19 and S20 and the amount of material between them, essentially all of the PTM signatures are due
to cosmic ray muons passing through the entire spectrometer.

The SM signature requires that within 50 nsec minimum ionizing particles pass through S1 and S19 and that no charged particle penetrates S20. In this case virtually all of the SM signatures are due to cosmic ray muons that (1) pass through S1, (2) continue through the spectrometer, passing through S19 and (3) stop before they reach S20.

The use of single channel analyzers to define Muon 1 and 19 eliminates contamination of PTM or SM signatures by showers of particles. Contamination of the muon signatures by electrons from muon decay, knock-on electrons and delta rays is small because for these events to satisfy the signatures they must occur at particular depths in the spectrometer and within small depth intervals. This requirement reduces the possibility of their occurrence to a negligible amount. The large amount of material between S1 and S19 removes the possibility of particles other than muons satisfying the PTM and SM signatures.

There is a small fraction (~5%) of the PTM and SM signatures that are satisfied by several obliquely incident charged particles. By a suitable correlation of simultaneous signals from pairs of sampling layers,
95% of the signals from multiple, obliquely-incident particles can be rejected.

There are 52 gm/cm² of glass between S19 and S20. Since there are about 900 gm/cm² of glass between S1 and S19, the range of energies of the muons that give a SM signature is small. The range of energies of muons which give a PTM signature extends from a minimum energy (which is slightly more than the maximum energy of a SM muon) to the highest muon energies possible. For layers SL1-SL9 the sampling layer signal is calibrated in terms of the energies lost in the sampling layer by muons giving an SM signature. Since the range of energies of muons giving an SM signature is small, the corresponding energy losses in each sampling layer are well defined. Sampling layer 10 cannot be calibrated using muons giving a SM signature because these muons do not pass through scintillator S20. For this reason layer 10 is calibrated using muons that give a PTM signature. Since the range of energies of these muons is large, the corresponding energy losses in layer 10 are not as accurately determined and the energy losses used to calibrate the other layers. By using these cosmic ray muons a particular signal amplitude for the sampling layer can be related to a particular average muon energy loss in the layer.
The muons incident on the spectrometer are distributed in energy and incident angle. The energy lost in a sampling layer by a muon is the average of these distributions over the range of energies allowed by the muon signatures and the acceptance geometry of the spectrometer.

Consider a vertical muon with energy $E$ penetrating the spectrometer. If $E_1 \leq E \leq E_2$ where $E_1$ and $E_2$ are the energy limits defined by the muon signature then the muon will lose energy $\Delta E_k$ in the $k^{th}$ sampling layer. For a normalized distribution $N(E)$ of energies of incident muons the average energy lost $\langle \Delta E_k \rangle$ in the $k^{th}$ sampling layer is

$$\langle \Delta E_k \rangle = \int_{E_1}^{E_2} \Delta E_k N(E) \, dE$$

The energy $\Delta E_k$ that a single muon loses as it passes through a sampling layer was determined using range-energy relations of muons in glass and scintillator. These range-energy relations were calculated using the method of Barkas and Berger. The constants to be used in this method for glass were taken from the molecular composition data supplied by the manufacturer of the glass used in the spectrometer. The constants for scintillator were taken from Hayman and Crispin.
For muons incident on the spectrometer at angles other than vertical the limits \( E_1 \) and \( E_2 \) are larger since the muons must pass through more material to give an acceptable signature. The muons used in calibrating the spectrometer have energies \( \gtrsim 2 \text{GeV} \). At these energies the muon range becomes nearly a linear function of energy. Therefore, for a muon incident at an angle \( \theta \) from the vertical, let the energy limits become \( E_1 \sec \theta \) and \( E_2 \sec \theta \). For the small acceptance angle \( (\theta \leq 20^\circ) \) of the spectrometer let the angular distribution\(^2^7\) \( J(\theta) \) of the muon flux be related to the vertical flux \( J(0) \) by

\[
J(\theta) = J(0) \cos^2 \theta
\] (2)

The average of the energy lost in the \( k^{\text{th}} \) sampling layer over the acceptance geometry of the spectrometer by a muon giving an acceptable signature becomes

\[
\langle \Delta E_k^\mu \rangle = \int \int_{\Omega} \int_{E_1 \sec \theta}^{E_2 \sec \theta} \Delta E_k^\mu N(E) \, dE \cos^2 \theta \, d\Omega \, dA
\] (3)

where \( \Omega \) is the acceptance solid angle of the spectrometer and \( A \) is the sensitive area of the spectrometer.
The distribution $N(E)$ at the top of the spectrometer was found from measured sea level distributions\textsuperscript{28}, \textsuperscript{29} using survival probability calculations.\textsuperscript{15} The distribution $N(E)$ is given in Table II.

Equation 3 was evaluated numerically by modifying the geometrical factor calculation procedure used in Ref. 30. In addition to calculating $\langle E_k^\mu \rangle$, $\langle (\Delta E_k^\mu)^2 \rangle$ was calculated in order to find the standard deviation $\sigma_k^\mu$ where

$$\sigma_k^\mu = \sqrt{\langle (\Delta E_k^\mu)^2 \rangle - \langle E_k^\mu \rangle^2}$$ \hspace{1cm} (4)

For each sampling layer and for scintillators S1, S19 and S20 $\langle \Delta E_k^\mu \rangle$ and $\sigma_k^\mu$ were found for energies ranging from 2.05 GeV to 50 GeV. This energy range corresponds to the range of energies of vertical muons which satisfy the PTM signature. These values are listed in Table III. Similarly $\langle \Delta E_k^\mu \rangle$ and $\sigma_k^\mu$ were found for SM muons corresponding to the vertical muon energy range of 1.93 GeV to 2.04 GeV. These values are also listed in Table III. Because of the definition of SM signature no values of $\langle \Delta E_k^\mu \rangle$ are defined for S20 and sampling layer 10. Through variation of the energy limits it was found that the values of $\langle \Delta E_k^\mu \rangle$ and $\sigma_k^\mu$
TABLE II

Energy Distribution of Muons at the Top of the Spectrometer

<table>
<thead>
<tr>
<th>E GeV</th>
<th>N(&gt;E) x 10^{-3} sec^{-1}</th>
<th>cm^{-2}sr^{-1}(GeV/c)^{-1}</th>
</tr>
</thead>
<tbody>
<tr>
<td>.351</td>
<td>3.73</td>
<td></td>
</tr>
<tr>
<td>.433</td>
<td>4.93</td>
<td></td>
</tr>
<tr>
<td>.598</td>
<td>4.70</td>
<td></td>
</tr>
<tr>
<td>.815</td>
<td>3.94</td>
<td></td>
</tr>
<tr>
<td>.999</td>
<td>3.62</td>
<td></td>
</tr>
<tr>
<td>1.11</td>
<td>3.14</td>
<td></td>
</tr>
<tr>
<td>1.24</td>
<td>2.95</td>
<td></td>
</tr>
<tr>
<td>1.39</td>
<td>2.67</td>
<td></td>
</tr>
<tr>
<td>1.64</td>
<td>2.21</td>
<td></td>
</tr>
<tr>
<td>1.96</td>
<td>1.81</td>
<td></td>
</tr>
<tr>
<td>2.50</td>
<td>1.37</td>
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</tr>
<tr>
<td>4.85</td>
<td>.4.49</td>
<td></td>
</tr>
<tr>
<td>5.45</td>
<td>.347</td>
<td></td>
</tr>
<tr>
<td>6.3</td>
<td>.228</td>
<td></td>
</tr>
<tr>
<td>7.45</td>
<td>.184</td>
<td></td>
</tr>
<tr>
<td>9.15</td>
<td>.122</td>
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</tr>
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<td>10.8</td>
<td>.0817</td>
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<td>14.6</td>
<td>.0422</td>
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<td>17.8</td>
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<tr>
<td>31.3</td>
<td>.00577</td>
<td></td>
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<td>42.3</td>
<td>.00285</td>
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<tr>
<td>56.1</td>
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<tr>
<td>70.0</td>
<td>.000664</td>
<td></td>
</tr>
<tr>
<td>88.1</td>
<td>.000325</td>
<td></td>
</tr>
</tbody>
</table>
TABLE III

Calculated Average Energy Lost by Muons in Sampling Layer k

<table>
<thead>
<tr>
<th></th>
<th>For Muons satisfying PTM signature</th>
<th>For Muons satisfying SM signature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>\langle \Delta E_k^\mu \rangle (MeV)</td>
<td>\sigma(\Delta E_k^\mu) (MeV)</td>
</tr>
<tr>
<td>1</td>
<td>228</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>235</td>
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</tr>
<tr>
<td>3</td>
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<td>12</td>
</tr>
<tr>
<td>4</td>
<td>233</td>
<td>15</td>
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<tr>
<td>5</td>
<td>231</td>
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<td>6</td>
<td>229</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>228</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>225</td>
<td>18</td>
</tr>
<tr>
<td>9</td>
<td>224</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>177</td>
<td>16</td>
</tr>
<tr>
<td>S1</td>
<td>6.1</td>
<td>.3</td>
</tr>
<tr>
<td>S19</td>
<td>5.8</td>
<td>.4</td>
</tr>
<tr>
<td>S20</td>
<td>15.4</td>
<td>.6</td>
</tr>
</tbody>
</table>
for the sampling layers listed in Table III were insensitive to a 5% variation in either limit.

B. Ionization Energy Lost in the Spectrometer

The energy lost in each sampling layer by muons that satisfy the SM signature requirement has been calculated. In calibrating the spectrometer, the photomultiplier signals from SL 1-9 are recorded only when the SM signature is satisfied. The signals from SL10 are recorded only when the PTM criteria are met. From the distribution of pulse heights obtained for each sampling layer k (SLk) the average pulse height \( \langle V_k \rangle \) is found. This average pulse height corresponds to the average energy lost in SLk by muons satisfying the SM or PTM criteria. The distributions of pulse heights obtained are determined mainly by Landau fluctuations in energy losses and photoelectron statistics. An example of the pulse height distribution obtained from a sampling layer by muons giving a SM signature is shown in Fig. 9.

With NEM cascades where larger energy losses in the sampling layers occur, the pulse height of the sampling layer signal is larger than that caused by muons. The pulse heights of these signals are not
measured directly as when \( \langle V_k^u \rangle \) is determined. Instead
the pulse heights from each sampling layer are logarithmically transformed to a time interval by the pulse
height converter. For the \( k^{th} \) sampling layer the pulse
height \( V_k \) is related to a time interval \( t_k \) by

\[
V_k = V_k^0 \exp \left( \frac{t_k}{\tau_k} \right)
\]

(5)

Where \( V_k^0 \) and \( \tau_k \) are constants of the pulse height
converter.

The constants \( V_k^0 \) and \( \tau_k \) are determined by using
an artificial light pulse and observing the resultant pulse height and time interval. The intensity of the
light pulse is varied so that pulse heights and corresponding time intervals are observed over the
operating range of the spectrometer. From the pulse height and time interval data obtained, the constants
\( V_k^0 \) and \( \tau_k \) are determined.

An example of the high level (\( V \geq 300 \text{ mV} \)) pulse
height and time interval relationship is shown in
Fig. 10. An example of the low level (\( V \geq 3 \text{ mV} \)) relation-
ship is shown in Fig. 11. Here one sees that the effect
of photomultiplier noise is to increase the time interval
for a given pulse height. Because of the exponential
distribution of photomultiplier noise, the low level
relationship is much more sensitive to the noise than
is the high level. The lines drawn in Figs. 10 and 11 are best fit lines to the points and bounded by error lines within which are contained 68% of the data points. The best fit was found using the least squares method. Typically the errors in obtaining $V_k$ from Eq. 5 is 20% for the low level and 8% for the high level.

Knowing that the average signal $\langle V_k^H \rangle$ corresponds to a particular energy loss and that the pulse height $V_k$ from other energy losses is related to the observed time interval $t_k$ then the ratio $\frac{V_k}{\langle V_k^H \rangle}$ becomes

$$\frac{V_k}{\langle V_k^H \rangle} = \frac{V_k^0 \exp \left( \frac{t_k}{\tau_k} \right)}{\langle V_k^H \rangle}$$

In the next section it will be shown that this ratio determines the energy lost in sampling layer $k$.

The primary purpose of the spectrometer is to measure the energy lost in each sampling layer by a NEM cascade. Therefore, the relation between the muon energy loss, which is now well known, and the cascade energy loss must be found.

Let $\frac{dU}{dx}$ denote the ionization energy loss function for a NEM cascade at depth $x$ in the spectrometer. The energy $\Delta E_k^C$ lost by the cascade in sampling
layer \( k \) between depths \( x_1 \) and \( x_6 \) is then

\[
\Delta E_c^k = \int_{x_1}^{x_6} \frac{dU}{dx} \, dx
\]

(7)

where \( x_1 \) and \( x_6 \) are the depths corresponding to the boundaries of the sampling layer. Since the cascade loses energy at different rates in glass and scintillator, \( \Delta E_c^k \) becomes

\[
\Delta E_c^k = \int_{x_1}^{x_2} \frac{dU_g}{dx} \, dx + \int_{x_2}^{x_3} \frac{dU_s}{dx} \, dx + \int_{x_3}^{x_4} \frac{dU_g}{dx} \, dx + \int_{x_4}^{x_5} \frac{dU_s}{dx} \, dx + \int_{x_5}^{x_6} \frac{dU_g}{dx} \, dx
\]

(8)

where \( \frac{dU_g}{dx} \) denotes the energy loss rate function for the cascade in glass, \( \frac{dU_s}{dx} \) denotes the energy loss rate for the cascade in scintillator and the integration limits correspond to the depths of the boundaries of the different materials as illustrated in Fig. 12. It will be shown that if glass of the same thickness in gm/cm\(^2\) as the scintillator is substituted for the scintillator then in that depth there exists a simple
Fig. 12
relation between the cascade energy loss rate functions of the two materials. This relation will be used for finding the energy loss between $x_2$ and $x_3$ and between $x_4$ and $x_5$.

In Approximation B a fundamental assumption is that each electron of the cascade loses energy by ionization in an absorber at a constant rate which is equal to the critical energy of the absorber. Subsequent calculations and experiments have shown the inadequacy of Approximation B. For lead absorbers, the average energy loss for an electron in an EM cascade was found to be significantly different from the values predicted by Approximation B. A Monte Carlo program was used to determine the average energy loss on an electron in an EM cascade in glass and scintillator. This program was developed to give rapid calculations of EM cascades in any material and any combinations of materials. The validity of the program can be seen in the agreement of the calculations with experiment for the transition curve and the transition effect illustrated in Fig. 13. Using this program the average energy loss was found as a function of depth for various energies of electrons and gamma rays initiating the cascade. The results obtained are listed in Tables IV and V. These values can be compared with the critical energies of 45 MeV and 88 MeV for glass and scintillator.
ENERGY DEPOSITED - ARBITRARY SCALE

PLEXIGLASS THICKNESS - r.i.

Fig. 13
TABLE IV

Average Energy Loss of Electrons in 1 GeV Electromagnetic Cascades

<table>
<thead>
<tr>
<th>Depth in Absorber r.l.</th>
<th>Average Energy Loss - MeV/r.l.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Glass</td>
</tr>
<tr>
<td>2</td>
<td>47</td>
</tr>
<tr>
<td>4</td>
<td>47</td>
</tr>
<tr>
<td>6</td>
<td>47</td>
</tr>
<tr>
<td>8</td>
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<td>10</td>
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<td>22</td>
<td>49</td>
</tr>
<tr>
<td>24</td>
<td>49</td>
</tr>
</tbody>
</table>
TABLE V

Average Energy Loss of Electrons in Electromagnetic Cascades at 24 r.l. Depth in the Absorber

<table>
<thead>
<tr>
<th>Energy of Incident Particle GeV</th>
<th>Average Energy Loss - MeV/r.l. Glass</th>
<th>Scintillator</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>49</td>
<td>88</td>
</tr>
<tr>
<td>1.0</td>
<td>48</td>
<td>87</td>
</tr>
<tr>
<td>5.0</td>
<td>49</td>
<td>88</td>
</tr>
</tbody>
</table>
respectively. It can be seen from Tables IV and V that for each material the average energy loss for an electron is quite constant with respect to variations in depth in the material and in energy.

Since the interaction length-radiation length ratio is low in the glass, scintillator spectrometer, there will not be large fluctuations in the NEM cascade caused by rapid growth and decay of individual EM cascades. Therefore, the average energy loss of an electron in a NEM cascade in this spectrometer will be the same as for an EM cascade.

Consider the NEM cascade as it emerges from the glass absorber and penetrates the scintillator. There will be a particular distribution of particles passing through the scintillator. If the scintillator was replaced by an equivalent thickness in gm/cm\(^2\) of glass this distribution of particles would not change significantly. This is confirmed by the negligible transition effect for the thickness of scintillator used in the spectrometer. The ionization energy lost by the particles passing through the scintillator can be expressed as the product of the average energy loss \(\langle \frac{dE}{dx} \rangle\) of an electron in the material and the number \(n_e\) of electrons. Since the scintillator is sufficiently thin so the number of electrons does not
change as they pass through, then the cascade can be related through the expressions

\[
\frac{dU_g}{dx} = n_e \langle \frac{dE_g}{dx} \rangle \quad \frac{dU_s}{dx} = n_e \langle \frac{dE_s}{dx} \rangle \quad (9)
\]

where \( n_e \) is the number of electrons. Then

\[
\frac{dU_s}{dx} = \frac{\langle \frac{dE_s}{dx} \rangle}{\langle \frac{dE_g}{dx} \rangle} \frac{dU_g}{dx} \quad (10)
\]

This relation should be valid for the thickness of the scintillator used in the spectrometer. Now Eq. 8 becomes

\[
\Delta E_k^c = \int_{x_1}^{x_6} \frac{dU_g}{dx} \, dx + \left[ \frac{dE_g}{dx} \frac{dE_s}{dx} \right] - 1 \left[ \int_{x_2}^{x_3} \frac{dU_g}{dx} \, dx \right] + \int_{x_4}^{x_5} \frac{dU_g}{dx} \, dx 
\]

\[
(11)
\]

It was found for special cases that the term

\[
\left[ \frac{\langle \frac{dE_s}{dx} \rangle}{\langle \frac{dE_g}{dx} \rangle} \right] - 1 \left[ \int_{x_2}^{x_3} \frac{dU_g}{dx} \, dx + \int_{x_4}^{x_5} \frac{dU_g}{dx} \, dx \right] 
\]

\[
(12)
\]
contributes less than 1% to the value of \( \Delta E_K^C \). The cases considered were (1) a NEM cascade having an exponential decay of its energy loss rate from 100 MeV/gm cm\(^{-2}\) to 25 MeV/gm cm\(^{-2}\) in 4 radiation lengths and (2) a parabolic increase from 10 MeV/gm cm\(^{-2}\) to \(10^3\) MeV/gm cm\(^{-2}\) and then a parabolic decrease from \(10^3\) MeV/gm cm\(^{-2}\) to 10 MeV/gm cm\(^{-2}\), all in 6 radiation lengths. When one neglects the terms given by (12), Eq. (11) becomes

\[
\Delta E_K^C = \int_{x_1}^{x_6} \frac{dU_g}{dx} \; dx
\]

(13)

Suppose Eq. 12 is approximated by

\[
\Delta E_K^C = \frac{1}{2} \left[ \left( \frac{dU_g}{dx} \right)_{x_a} + \left( \frac{dU_g}{dx} \right)_{x_b} \right] \Delta x
\]

(14)

where \( \left( \frac{dU_g}{dx} \right)_{x_a} \) and \( \left( \frac{dU_g}{dx} \right)_{x_b} \) denote the values of \( \frac{dU_g}{dx} \) at the depths \( x_a \) and \( x_b \) of Fig. 12, and \( \Delta x \) denotes the sampling layer thickness (\( x_6 - x_1 \)).

Since the thicknesses (\( x_3 - x_2 \)) and (\( x_5 - x_4 \)) are small let

\[
x_a = x_2 = x_3
\]

(15)
and

$$x_b = x_4 = x_5$$  \hspace{1cm} (15)

i.e. the cascade is assumed to be unchanged in its passage through a scintillator layer. The Eq. (14) becomes

$$\Delta E^C_k = \frac{1}{2} \left[ \left( \frac{dU_g}{dx} \right)_{x_2} + \left( \frac{dU_g}{dx} \right)_{x_4} \right] \Delta X_k$$  \hspace{1cm} (16)

The relative error \( \zeta \) introduced when Eq. (14) is approximated by Eq. (16) is

$$\zeta = \frac{5}{96} \left( \frac{d^3U_g}{dx^3} \right)_{x_0} \int_{x_1}^{x_6} \frac{dU_g}{dx} \, dx$$  \hspace{1cm} (17)

where \( x_0 \) is the depth of the center of the sampling layer. The derivation of Eq. (17) is shown in the Appendix. Using results from a Monte Carlo simulation of a cascade resulting from a 1 TeV proton interaction, \( \zeta \) was found to have characteristic values of 0.3%. Therefore, Eq. (16) is believed to be a good approximation.
The signal from each sampling layer is strictly proportional to the ionization energy lost by the cascade in the scintillators, but not to the total energy lost in the entire sampling layer. Hence, the cascade sampling layer signal \( V_k \) for layer \( k \) is not explicitly proportional to \( \Delta E_k^c \) of Eq. (16).

The cascade sampling layer signal is

\[
V_k = C \left[ \left( \frac{dU_s}{dx} \right)_{x_2} + \left( \frac{dU_s}{dx} \right)_{x_4} \right] \Delta s \quad (18)
\]

where \( \Delta s \) is the thickness of a scintillator, \( \frac{dU_s}{dx} \) is the energy loss rate for the cascade in scintillator and \( C \) is a proportionality constant whose value depends on scintillator efficiency, photomultiplier geometry and the electronic characteristics of the photomultipliers.

By using Eq. (10) the expression for \( V_k \) becomes

\[
V_k = C \left( \frac{dE_s}{dx} \right) \left[ \left( \frac{dU_g}{dx} \right)_{x_2} + \left( \frac{dU_g}{dx} \right)_{x_4} \right] \Delta s \quad (19)
\]
By using Eqs. 16 and 19 one finds

\[ V_k = 2C \frac{\Delta s}{\Delta x_k} \frac{\langle \frac{dE_s}{dx} \rangle}{\langle \frac{dE_g}{dx} \rangle} \Delta E_k^c \quad (20) \]

The result is that the cascade sampling layer signal is proportional to the ionization energy lost in the sampling layer by the cascade.

The average energy lost in each sampling layer by calibration muons has been tabulated in Table III. The corresponding photomultiplier signal \( \langle \nu_k^\mu \rangle \) arises from the energy lost in the scintillators by the muons. When Eq. (18) is applied to the passage of muons through the sampling layer, one has

\[ \langle \nu_k^\mu \rangle = C \left[ \left( \frac{dU_s}{dx} \right)_{x_2} + \left( \frac{dU_s}{dx} \right)_{x_4} \right] \Delta s \quad (21) \]

where \( \frac{dU_s}{dx} \) is the energy loss function for muons in scintillator. Since light emission and detection characteristics are the same for the passage of muons through the sampling layer as for the passage of cascades, the constant \( C \) is the same in Eqs. (18) and (21).
Suppose that the constant $D_k$ given by

$$D_k = \frac{\Delta E_k^\mu}{\left[ \left( \frac{dU_S}{dx} \right) x_2 + \left( \frac{dU_S}{dx} \right) x_w \right]}$$

(22)

can be evaluated for each sampling layer $k$. Then the average signal obtained from calibration is related to the average energy loss of calibration muons by

$$<\nu_k> = \frac{C \Delta S \Delta E_k^\mu}{D_k}$$

(23)

The energy lost in a sampling layer by a cascade can then be related to the energy lost by the calibration muons using Eqs. (20) and (23). The result is

$$\Delta E_k^C = \frac{\Delta X_k}{2D_k} \frac{\langle dE_g \rangle}{\langle dE_g \rangle} \frac{\nu_k}{\langle \nu_k \rangle} \Delta E_k^\mu$$

(24)

Since $D_k$ may be dependent on muon energy and sampling layer, the term $\frac{\Delta X_k}{2D_k}$ has been evaluated for many muon energies and for all sampling layers. For sampling layers SL1 through SL9, where $\Delta E_k^\mu$ and $\langle \nu_k \rangle$ were
determined using muons giving a SM signature, 
\[ \frac{\Delta X_k}{2D_k} = 1.14 \pm 0.01. \] 
For SL10 where the same quantities were found using muons giving the PTM signature 
\[ \frac{\Delta X_k}{2D_k} = 1.04 \pm 0.01. \]

Using the values of 1.84 MeV/gm cm\(^{-2}\) and 1.98 MeV/gm cm\(^{-2}\) for the average ionization energy loss rates of electrons in glass and scintillator respectively, the energy lost in a sampling layer by a cascade may be found from

\[ \Delta E_k^C = 1.06 \frac{V_k}{\langle V_k \rangle} \Delta E_k^\mu k = 1-9 \tag{25} \]

and

\[ \Delta E_k^C = 0.96 \frac{V_k}{\langle V_k \rangle} \Delta E_k^\mu k = 10 \tag{26} \]

where

\[ V_k = V_k^0 \exp \left( \frac{t_k}{\tau_k} \right) k = 1-10 \tag{27} \]

The standard deviation \(\sigma_k\) of the measurement error of \(\Delta E_k^C\) is explicitly dependent on the measured time interval \(t_k^\mu\) and on which level of the pulse height converter was used to measure \(V_k\). The measurement errors of \(\langle V_k^\mu \rangle\), \(V_k^0\) and \(\tau_k\) were determined through successive calibrations. With the values
obtained the average $\langle \sigma \rangle$ of all $\sigma_k$ was found to be

$$\langle \sigma \rangle = 25\%$$  \hspace{1cm} (28)

for $3mV \leq V_k \leq 300mV$ i.e. pulse height converter low level and

$$\langle \sigma \rangle = 15\%$$  \hspace{1cm} (29)

for $300mV \leq V_k \leq 30V$ i.e. pulse height converter high level, with a $t_k$ of 1 $\mu$sec.

C. Event Selection

Data from the spectrometer and flash tubes were recorded when the following conditions were satisfied.

1. The sum of the signals from the sampling layer photomultipliers exceeded the value which corresponded to an ionization energy loss of 200 GeV in the spectrometer.

2. At least one minimum ionizing particle passed through scintillator S1.

3. At most one minimum ionizing particle passed through any of the shower counters.

The events accepted for analysis of the NEM cascade development were those in which the incident
hadron had energies greater than 0.5 TeV. Since the amount of ionization energy lost in the spectrometer is less than the energy of the incident hadron, the threshold of 200 GeV insured that the likelihood of not recording a 0.5 TeV event would be negligible.

As mentioned in Chapter II, the method of recording the pulse height of the sampling layer demands a stable time reference provided by the second condition.

Since the shower counters do not have a large surface area the possibility of the incident hadron being accompanied by shower particles exists. Therefore, the following criteria were used for accepting an event on the basis of the flash tube data.

1. An unambiguous track through all of the flash tube layers was discernable.

2. The projection of the track would be completely contained in the spectrometer.

3. The only tubes in the layers below the emulsion chamber that had fired were those associated with the track.

4. At most 4 tubes in the layer of flash tubes below the emulsion chamber had fired.
These criteria were applied equally to the 183 cm side tubes and to the 91.5 cm side tubes. An example of an acceptable track is shown in Fig. 14.

After the event was accepted on the basis of the flash tube data, the spectrometer data were read. From angular measurements made on the flash tube pictures the angle \( \theta \) between the hadrons path and the normal to the spectrometers sensitive area was determined. The energy \( \Delta E_k^C \) lost in each layer \( k \) was normalized to determine the longitudinal energy lost \( \Delta E_k^L \) by

\[
\Delta E_k^L = \Delta E_k^C \cos \theta
\]  

(30)

Typically the angle \( \theta \) had values of 5°, so this constituted a small correction. Then the longitudinal development of the NEM cascade as a function of depth in the spectrometer was found. An example of this development is illustrated in Fig. 15.

D. Energy of the Incident Hadron

The energy \( E_0 \) of the incident hadron is determined on the basis of (1) measurement of the ionization energy lost in the spectrometer (2) estimation of energy lost by the NEM cascade before it enters the spectrometer (3) estimation of the energy of the
Fig. 15
NEM leaks out the sides and bottom of the spectrometer and (4) estimation of the energy going into nuclear evaporation fragments.

For every event recorded, the total longitudinal ionization energy $E_I$ lost in the spectrometer is found by taking the sum of the ionization energy lost in each sampling layer. Since the arguments of the previous section pertaining to the measurement of the ionization energy lost apply to each sampling layer, $E_I$ is found by

$$E_I = \sum_{k=1}^{10} \Delta E_k^L$$

(31)

When a hadron is incident on the apparatus the first interaction may take place in the flash tube section, carbon target, emulsion chamber or in the spectrometer. For first interactions occurring in parts of the apparatus other than the spectrometer, the NEM cascade will lose energy $E_T$ between the point of the first interaction and the spectrometer. The energy $E_T$ is estimated by linearly extrapolating the values of $\Delta E_k^L$ as a function of depth for $k = 1, 2$ i.e. the two uppermost sampling layers. This process is illustrated in Fig. 15.
The energy leaking out the sides of the spectrometer is negligible because acceptable events are those in which the core of the NEM cascade is always several radiation lengths inside the spectrometer. Energy escapes from the bottom of the spectrometer in the form of muons and electrons. The energy carried out by the muons is very small. Electrons carry energy out the bottom as a result of the NEM cascade not having decayed completely in the spectrometer. In this case the energy $E_B$ going out the bottom is estimated by extrapolating an exponential decay which is fit, using the least squares method, to the values of $\Delta E_k^L$ for $k = 8, 9, 10$.

Theoretical estimates\textsuperscript{12, 13} of the fraction of energy going into evaporation fragments have been made for various materials. This energy has been measured only in emulsion. For an accepted event, the energy $E_D$ lost through evaporation processes is estimated from the published results considered to be the most accurate.\textsuperscript{12} In Ref. 12 one notices that at the energies between 0.5 TeV and 1.5 TeV the largest value of $E_D$ is 19% of $E_0$.

Then the energy $E_0$ of the incident hadron is determined by

$$E_0 = E_I + E_T + E_B + E_D$$  \hspace{1cm} (32)
At any depth in the spectrometer the energy lost in a sampling layer by a NEM cascade is dependent on the cascade structure before it enters the sampling layer as well as the cascade development inside the layer. Therefore, the contribution of the error in measuring the ionization energy $E_I$ lost in the sampling layers to the uncertainty $\Delta E_0$ in determining $E_0$ is not the accumulation of independent measurement errors. The contribution to $\Delta E_0$ by $E_I$ is estimated to be 15% of $E_0$. This estimation was obtained from the results of Ref. 14 and Ref. 35 assuming an individual sampling layer error of 25% of the measured value. The contribution of the errors in the extrapolations made to determine $E_T$ and $E_B$ to $E_0$ are estimated to be 3% of $E_0$ and 7% of $E_0$ respectively. The uncertainty in determining the energy going into fragmentation has been reported to be 6% of $E_0$ in the energy region of this experiment. Since $E_0$ is obtained from the sum of independently determined quantities ($E_I$, $E_T$, $E_B$, and $E_D$) the uncertainty $\Delta E_0$ becomes 18% of $E_0$. 
IV. RESULTS AND CONCLUSIONS

During the operation time of 1463 hours 71 events having incident hadron energies greater than 0.5 TeV were accumulated. About 90% of these events showed only a single charged particle track in the flash tubes. The rest of the events were accompanied by low energy shower particles which were stopped in the emulsion chamber.

A. Integral Spectrum

The integral spectrum of the accepted events as a function of the incident hadron's energy $E_0$ is shown in Fig. 16. This spectrum does not strictly follow a power function of energy. Previous reports$^{36-38}$ of the integral spectra of hadrons at nearly the same altitude described the spectra as power functions. The reason for the difference between this experiment and others lies in the requirement that this experiment accept only events where the accompanying shower particles do not enter the spectrometer.
Most of the hadrons incident on the spectrometer come from an interaction high in the air above the apparatus. From this interaction there will be shower particles accompanying the hadron. For low energy hadrons that satisfy the acceptance criteria the height of the interaction is sufficient that the shower particles have diverged enough to miss the spectrometer. For high energy hadrons there is a greater likelihood that they will be accompanied by shower particles which have not diverged sufficient to miss the spectrometer. In addition, the shower particles accompanying the high energy events have more energy and will be more likely to penetrate the emulsion chamber and enter the spectrometer. Therefore, a bias against accepting high energy events has been introduced in order to eliminate air shower particles which would perturb the measurement of the NEM cascade development if they penetrated the spectrometer.

It is possible that some of the accepted events involved from hadrons which did not come from interactions in the atmosphere. However, these hadrons cannot be distinguished in this experiment from those that do come from interactions in the atmosphere.
A previous experiment\textsuperscript{10} to detect hadrons that do not come from interactions in the atmosphere indicates that the shower detectors of this experiment are inadequate to make the distinction.

A power function of energy was fit to the low energy region of the integral spectrum in the interval 0.5 TeV \( \leq E \leq 1.0 \) TeV. The fit was made using the least squares method and is illustrated by the line in Fig. 16. The value of the exponent obtained was 2.1 \pm 0.3. The agreement of this value with others is shown in Table VI. The energy range and the criteria for accompanying particles is listed in Table VI. One notices that for the energy range around 1 TeV all experiments agree.

The steepening of the integral spectrum obtained in this experiment was explained by the rejection of events accompanied by air shower particles. This is substantiated by noticing that Kamata,\textsuperscript{10} et al. obtained a spectrum exponent of 2.4 \pm 0.2 when observing hadrons that were unaccompanied over a large area by shower particles.

B. Average NEM Cascade

The average ionization energy loss as a function of depth in the spectrometer of all of the accepted
### TABLE VI

Comparison of Incident Hadron Spectra

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Exponent of Power Law</th>
<th>Energy Range (TeV)</th>
<th>Accompanied by Shower Particles</th>
</tr>
</thead>
<tbody>
<tr>
<td>This Experiment</td>
<td>2.1 ± .3</td>
<td>0.5 - 1.5</td>
<td>Partial</td>
</tr>
<tr>
<td>Akashi (Ref. 38)</td>
<td>1.9 ± .3</td>
<td>2 - 10</td>
<td>Yes</td>
</tr>
<tr>
<td>Grigorov (Ref. 37)</td>
<td>1.86 ± .04</td>
<td>5 - 30</td>
<td>Yes</td>
</tr>
<tr>
<td>Jones (Ref. 36)</td>
<td>2.0</td>
<td>0.07 - 1</td>
<td>Partial</td>
</tr>
<tr>
<td>Kamata (Ref. 10)</td>
<td>2.4 ± .2</td>
<td>3 - 10</td>
<td>No</td>
</tr>
<tr>
<td>Raghavan (Ref. 42)</td>
<td>2.0 ± .2</td>
<td>0.01 - .5</td>
<td>No</td>
</tr>
</tbody>
</table>
events is shown in Fig. 17. The extrapolations of the cascade development were made from the average values. The energy of the average NEM cascade was found to be 1 TeV. The fluctuations in the cascade development are shown in Fig. 18. In the events accepted the fluctuations are due to the incident hadrons interacting in either the carbon target, emulsion chamber or spectrometer and due to normal cascade fluctuations. An example of a cascade showing unusual development in the spectrometer is shown in Fig. 19.

The exact point where the incident hadron initially interacted could not be determined since the spatial resolution of the flash tubes is 1.5 cm. The width of the NEM cascade resulting from a high energy hadron interacting in the carbon target or the emulsion chamber may be sufficiently small so that only one tube in a row of flash tubes may fire. In this way a developing cascade may appear to be a single track.

The average cascade is compared with the results of a three dimensional Monte Carlo calculation\textsuperscript{39} for protons interacting in a pure glass absorber. This comparison is shown in Fig. 20. In the Monte Carlo calculation the average number of particles in depth intervals of 4 r.l. is given.
Fig. 18

STD. DEV. of FLUCTUATIONS - GeV

SAMPLING LAYER
Fig. 20

**RADIATION LENGTHS**

**NUMBER of PARTICLES**

- THIS EXPERIMENT
- --- MONTE CARLO

**SAMPLING LAYER**
To obtain the number of particles from the measured cascade, the average energy loss was divided by the average energy loss in glass of an electron. Since the average NEM cascade was obtained from an energy spectrum of events, this spectrum was simulated in the calculations. The simulation of the experimental spectrum of incident hadrons by the calculations was done as follows:

1. The Monte Carlo calculations were done for three energies, 0.5 TeV, 1.0 TeV and 1.5 TeV. The number of individual Monte Carlo NEM cascades calculated to obtain the average cascades for these energies were 200, 100 and 100 respectively. For each energy the number $n$ of particles at a depth $d$ was found. Corresponding to the three energies are the number $n_1$, $n_2$, and $n_3$ of particles at depth $d$.

2. The fractions $f_1$, $f_2$ and $f_3$ of the experimental events in the respective energy ranges $0.5 \text{ TeV} \leq E_0 \leq 0.725 \text{ TeV}$, $0.725 \text{ TeV} < E_0 < 1.25 \text{ TeV}$ and $E_0 \geq 1.25 \text{ TeV}$ were determined to be $f_1 = \frac{36}{71}$, $f_2 = \frac{24}{71}$ and $f_3 = \frac{11}{71}$.

3. The number $N$ of particles at the depth $d$ of the composite Monte Carlo NEM cascade was found from

$$N = f_1 n_1 + f_2 n_2 + f_3 n_3$$

(33)
In addition the calculations simulated the effect of protons interacting in the carbon target or emulsion chamber by requiring that 20% of the Monte Carlo protons interact at the beginning of the Monte Carlo spectrometer.

The agreement between the experimental and calculated results is not exact. The major reasons for the imperfect agreement are:

1. low experimental statistics.
2. inability of the Monte Carlo calculations to simulate exactly the effect of hadrons interacting in the carbon target and emulsion chamber.
3. a fraction of the incident hadrons being pions.

If the hadron had interacted in the carbon target or the emulsion chamber then the NEM cascade would have begun development prior to entering the spectrometer. By the cascade being partially developed, energy is lost more rapidly in the shallow depths of the spectrometer than when the hadron interacts in the glass. Part of the incident hadrons being pions also increases the energy loss at shallow depths. This is due to the possibility of the inelasticity of pion-nucleus interactions being larger than that of proton-nucleus interactions.
Comparison of the average cascade with the results obtained from other experiments whose absorbers were of greatly different materials is improper. A strong dependence of the NEM cascade development on the absorber material has been predicted.\textsuperscript{1, 14}

At large depths (\geq 7 interaction lengths) in the absorber the decrease of the NEM cascade can be approximated by an exponential decay. From Ref. 1 one notices that the decay constant is related strongly to the inelasticity of the proton-nucleus interaction. This relation is of the form that the decay constant increases and the inelasticity increases. The decay constant of the average NEM cascade measured in this experiment is $4.6 \pm 0.4 \times 10^{-3}$ (gm/cm\textsuperscript{2})\textsuperscript{-1}. In the Monte Carlo calculations which used an average inelasticity of 0.5, the decay constant was $4.2 \pm 0.2 \times 10^{-3}$ (gm/cm\textsuperscript{2})\textsuperscript{-1}. Comparison of these decay constants indicates a slight increase in effective inelasticity of the experimental results. The inelasticity of hadrons interacting in the emulsion chamber and the inelasticity of pion-nucleus interactions are greater than that of proton-nucleus interactions in glass. Inclusion of these types of events increases the effective inelasticity of the experimental results.
APPENDIX

Error in Approximation of Integral

Given a function \( f(x) \) of a variable having at least a continuous second derivative, \( f''(x) \). Let \( I \) be the definite integral of their function between the limits \( x_0 - \frac{1}{2} \) and \( x_0 + \frac{1}{2} \). Then

\[
I = \int_{x_0 - \frac{1}{2}}^{x_0 + \frac{1}{2}} f(x) \, dx
\]

Suppose \( I \) is to be approximated by \( I_G \) where

\[
I_G = \frac{1}{2} \left[ f(x_0 - \frac{1}{4}) + f(x_0 + \frac{1}{4}) \right] \Delta x
\]

Since \( \Delta x = 1 \) then:

\[
I_G = \frac{1}{2} \left[ f(x_0 - \frac{1}{4}) + f(x_0 + \frac{1}{4}) \right]
\]

The error \( z \) where:

\[
z = I - I_G
\]
can be found in the following way:

\[ I = \int_{x_0 - \frac{1}{2}}^{x_0 + \frac{1}{2}} f(x) \, dx = F(x_0 + \frac{1}{2}) - F(x_0 - \frac{1}{2}) \]  

(5)

then \( z = F(x_0 + \frac{1}{2}) - F(x_0 - \frac{1}{2}) - \frac{1}{2} [f(x_0 - \frac{1}{4}) + f(x_0 + \frac{1}{4})] \)  

(6)

Since \( F(x) \) and \( f(x) \) have at least continuous second derivatives, then \( F(x) \) and \( f(x) \) can be expanded around \( x_0 \) giving:

\[ F(x_0 + \frac{1}{2}) = F(x_0) + \frac{1}{2} f(x_0) + \frac{1}{8} f'(x_0) + \frac{1}{24} f''(x_0) + \ldots \]  

(7)

\[ F(x_0 - \frac{1}{2}) = F(x_0) - \frac{1}{2} f(x_0) + \frac{1}{8} f'(x_0) - \frac{1}{24} f''(x_0) + \ldots \]  

(8)

\[ f(x_0 + \frac{1}{4}) = f(x_0) + \frac{1}{4} f'(x_0) + \frac{1}{32} f''(x_0) + \ldots \]  

(9)

\[ f(x_0 - \frac{1}{4}) = f(x_0) - \frac{1}{4} f'(x_0) + \frac{1}{32} f''(x_0) + \ldots \]  

(10)

If one neglects derivatives higher than the second, then

\[ z = \frac{5}{36} f''(x_0) \]  

(11)
and the relative error $\zeta$ becomes:

$$
\zeta = \frac{5}{96} \frac{f''(x_0)}{\int_{x_0 - \frac{1}{2}}^{x_0 + \frac{1}{2}} f(x) \, dx}
$$

(12)
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Charles Richard Gillespie was born September 18, 1936 in Topeka, Kansas. After graduating from Topeka High School, Topeka, Kansas in 1954, he entered the University of Kansas. Receiving the B.S. degree in 1959 he entered the Graduate School of Louisiana State University. After being awarded the M.S. degree, he held teaching positions at Cumberland College, Murray State University and Vanderbilt University. He again entered the Graduate School of Louisiana State University in 1964. The requirements for the Doctor of Philosophy in the Department of Physics were completed in January, 1971.
EXAMINATION AND THESIS REPORT

Candidate: Charles Richard Gillespie

Major Field: Physics

Title of Thesis: Longitudinal Development of Nuclear-Electromagnetic Cascades at Energies Around $10^{12}$ eV

Approved:

[Signature]
Major Professor and Chairman

[Signature]
Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:

January 12, 1971