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The Fermi Surface of Thallium: Radio-Frequency Size Effect.

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THE FERMI SURFACE OF THALLIUM:
RADIO-FREQUENCY SIZE EFFECT

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in

The Department of Physics and Astronomy

by

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ABSTRACT

Radio-frequency size effect measurements were made on high-purity, single crystal, thallium plates approximately 0.1 mm to 0.25 mm thick. Extremal calipers of the Fermi surface obtained from the data were used to construct an internally consistent Fermi surface model consisting of a third-band hole surface and fourth and fifth-band electron surfaces. The Fermi surface model developed from the data is appreciably different from either the free-electron model or the model arising from a previous energy band calculation. In particular, no calipers were observed that could be assigned to pockets of third-band holes at the M point in the Brillouin zone. The fourth-band electron hexagonal network is found to be considerably distorted from the free-electron model with no connection along the ΓA direction and no pockets in the ΓMK plane. Comparisons between the present model and recent de Haas-van Alphen measurements are made.

I. INTRODUCTION

In the past, the Fermi surface (FS) of thallium has been investigated through magnetoresistance measurements,¹⁻⁵ the magnetoacoustic effect,⁶⁻⁸ cyclotron resonance,^{9,10} and the de Haas-van Alphen (dHvA) effect.¹¹⁻¹⁵ The results of these experiments have been interpreted in terms of the single orthogonalized plane wave (OPW) model of the FS and on a FS model developed from a relativistic OPW (ROPW) energy band calculation by Soven.¹⁶ The agreement between these measurements and the two FS models is at best only fair. The two theoretical models of the Tl FS consist of a third-band hole surface centered at A and a smaller third-band hole surface at each M point in the Brillouin zone (BZ). The electron surfaces consist of a fourth-band hexagonal network surrounding the larger hole surface in the ALH plane and smaller fourth-band pieces along the Γ M line of the BZ. In addition, there are fifth and sixth-band posts centered at H in the BZ. The above experiments imply that the large hole surface, the hexagonal electron network and the fifth-band posts have been observed. The resulting FS dimensions have conflicted with the theory and other experiments.

The lack of agreement among the various experiments is due to the ambiguity of the existing theoretical models resulting in conflicting analysis of the data. The present investigation was performed in order to obtain extensive and detailed dimensions of the Tl FS from which an accurate model could be developed. The major result of this experiment is an internally consistent model of the FS which displays general agreement with past experiments.

Classically, the equation of motion for a charge q at position $\vec{r}$, velocity $\vec{v}$, in a constant, external magnetic field $\vec{H}$ is

$$\frac{d\vec{p}}{dt} = \frac{q}{c} \vec{v} \times \vec{H}. \quad (1)$$

If $\vec{p}$ is expressed semiclassically in terms of the charge carriers wave vector, $\vec{k}$, equation (1) becomes

$$\hbar \frac{d\vec{k}}{dt} = \frac{q}{c} \vec{v} \times \vec{H}. \quad (2)$$

The integration of equation (2) with respect to time yields

$$\hbar \vec{k} = \frac{q}{c} \vec{r} \times \vec{H}. \quad (3)$$

For the specific case of charge carriers in a metallic crystal the allowed momentum values can be determined, if $\vec{r}$ can be determined, for a given $\vec{H}$.

The determination of these FS dimensions using the radio-frequency size effect (RFSE) has been

described in detail earlier.^{17,18} In the RFSE experiment a thin, single crystal plate of the sample is placed in a magnetic field applied parallel to the surface of the sample. Discontinuities in the rf surface impedance of the sample are exhibited when the strength of the magnetic field is such that extremal electronic orbits on the FS exactly span the sample thickness. The extremal FS dimensions are then obtained from equation (3) using the relation $k_c = 0.015194 Hd$, where H is the value of the applied field in gauss at the point where the extremal orbits just span the sample, d is the sample thickness in mm, and k_c is the FS extremal dimension in $\text{\AA}^{-1}$.

In the sections which follow, an explication of the experimental technique relevant to RFSE in thallium is presented, a presentation of experimental results, and, finally, the data analysis and caliper assignments are given.

II. EXPERIMENTAL PROCEDURE

The samples used in this experiment were cut from a zone-refined thallium bar (purity of 99.9999%) which was procured from Cominco Products Incorporated. The Tl bar contained crystals of sufficient size to facilitate sample preparation without further treatment. Large crystals cut from this bar had a resistance ratio ($R_{4.2^{\circ}\text{K}} / R_{300^{\circ}\text{K}}$) of $\sim 5000:1$. After orienting the selected crystals, thin slabs were cut from the bar with an acid string saw using a 60% acetic acid and 40% hydrogen peroxide solution. A table was planed by spark erosion to insure orientation and the sample cemented to the table with conducting silver paint. After planing, the sample was removed and the table was replaned to assure that the sides of the sample would be parallel. The sample was affixed once more and the remaining side planed to the desired thickness. The slabs were then spark cut to the fixed dimensions. After spark planing and cutting, the samples were lapped in a solution of 40% acetic acid, 10% hydrogen peroxide, and 50% water to remove the surface strain caused by the thinning process and then stored in glycerine to avoid oxidation. The final sample thicknesses were

typically between 0.1 and 0.25 mm. Samples this thin were necessary so that most of the calipers could be obtained at field strengths well above the superconducting critical field of ~ 95 G at 1.2°K . Since thallium plated of this thickness are extremely susceptible to strain, preformation of the rf oscillator tank coils was necessary. The coils were made by winding forty gauge wire around dummy samples of varying thickness mica. The windings were bonded together using Duco household cement and the mica removed after the cement had hardened. The coils were then matched with the sample dimensions to insure that the thin Tl plates would not be strained due to the shrinkage of the windings during the cooling process. The samples were first washed in H_2O to remove the glycerine, then inserted in the preformed tank coil, and immediately placed in an evacuated dewar for twenty-four hours at 300°K to remove all traces of the H_2O . The sample thickness was not measured nor was it rechecked for orientation until the experiment was completed to circumvent the possibility of strain during these operations. The sample orientation by Laue back reflection X-ray pictures, was always found to be within $\pm 0.5^\circ$ of the desired direction after the measurements. Slab

dimensions were measured by using a traveling microscope to find the area of the face of the plate. Then the mass of the sample was divided by the density and the area to determine the thickness d . The value of d at 1.1°K was then calculated by compensating for the thermal contraction of Tl along its respective crystallographic axes. The experimental apparatus represented schematically in Fig. 1 has been described previously by Steenhaut and Goodrich.¹⁸ This apparatus allows both the first and second harmonic of the real part of the surface impedance to be recorded. The preformed tank coils are the inductive element in a modified version of a high level marginal oscillator which provides the bilateral antisymmetric excitation for the sample. The present measurements were performed at rf frequencies ranging from 8 to 15 MHz. The exact rf frequency used in each run was chosen to optimize the sensitivity of the marginal oscillator for the particular tank coil and sample used. The temperature of the sample was maintained at a constant value by pumping on the helium bath using a diaphragm type pressure regulator. The method of recording the data and sample thickness measuring techniques have an over-all reproducibility of 0.5% determined by comparing the caliper values of the same orbit on

Fig. 1

different samples.

III. EXPERIMENTAL RESULTS

The RFSE line shapes observed in Tl display the same general characteristics observed earlier in Cd¹⁷ and Zn¹⁸. Of the more than fifty samples prepared, many exhibited rather weak RFSE signals. The reason for the low signal intensity in many of the samples is that the thin samples were extremely susceptible to strain which decreases the mean free path of the carriers. Signal strengths decreased rapidly with increased temperature and the characteristic RFSE line shapes were not discernible on any of the samples above the superconducting transition temperature of Tl at 2.2°K. Consequently, some of the orbits associated with the smallest segments of the FS expected at low magnetic fields may not have been observed. The intensity of the RFSE signals is also dependent upon the geometry of the orbit. This fact aids in the caliper assignment and will be discussed in the next section.

All of the caliper values reported here were calculated by choosing the point in the field of maximum slope preceding the first major peak in the RFSE line shape obtained on a plot of the derivative of the real part of the surface impedance with respect

to the magnetic field, (dR/dH) . This point corresponds to the first peak in the second derivative curve (d^2R/dH^2) . This has been shown to be a reliable point on the line shape from which to compute the caliper value in past RFSE experiments devoted to FS determinations.^{17,18} In angular regions where many caliper series are present and the RFSE lines overlap, a slightly different procedure was used to determine the caliper point on each line. It was observed that the difference in field values between peaks on the second derivative curves (ΔH) was a constant for a given line. In Fig. 2 a recorder tracing of d^2R/dH^2 versus H taken with $\vec{H}$ applied 12° from $[0001]$ direction in an $\vec{n}$ parallel to $[11\bar{2}0]$ sample is shown. The line shapes associated with series m and o are shown in this plot. It is seen that ΔH is a constant for each line and the caliper point can be determined by extrapolating to the first peak in the line shape. This procedure was applied to determine the caliper values reported here even when the first peak in the second derivative was observable. In the latter cases the agreement between the extrapolated position of the first peak and the measured position agreed to within the measuring accuracy. The experimental error in the determination

Fig. 2

of the calipers by this method is $\pm 0.008 \text{ \AA}^{-1}$ for all but two caliper series which were always weak and could only be determined to $\pm 0.02 \text{ \AA}^{-1}$. This accuracy was checked by comparing the caliper values obtained from different samples for the same orbit and by comparing the determination of points of the FS obtained from different sample orientations. These independent determinations always agreed to within the quoted accuracy. Since the third and fourth bands of Tl are degenerate along the AL line in the BZ, the calipers of the third and fourth band sheets in this direction should add to give the BZ dimension from L to L. As can be seen from Table I and the dimensions of the BZ given in the caption accompanying Fig. 4, the agreement is within the quoted experimental error when the caliper point on the RFSE line is chosen in the manner described above. This point on the line has also been shown to be the caliper point for cylindrical Fermi surfaces in a recent calculation of line shapes by Juras.²⁰ Further support for the choice of this point on the line as the caliper point has recently been given by Boudreau and Goodrich²¹ from line shape studies on Cd.

Fig. 3 is a plot of the FS calipers obtained

TABLE I: The FS dimensions of Tl along symmetry directions.

Direction	Band	Extremal Calipers ($\text{\AA}^{-1}$)					
		Measured				Calculated	
		This Experiment	Rayne	Eckstein	Coon	Soven	Free Electron
A Γ	3	0.50		0.52	0.50	0.49	0.52
HK	4	0.49			0.51		
HK	5	0.44					
LM	4	0.67			0.67	0.64	
AL	3	1.43	1.46	1.43	1.47	1.44	1.55
AL	4	0.67	0.62		0.61	0.64	
AH	3	1.73	1.66	1.81	1.75	1.81	1.92
AH	4	0.67	0.64		0.72	0.62	
AH	4 [*]	1.80				1.91	

* Dimension of the inside of the fourth-band electron hexagonal network in ALH direction.

Fig. 3

for magnetic field rotations in the plane of the Tl plates having a normal, $\vec{n}$, parallel to the $[0001]$, $[10\bar{1}0]$, and $[11\bar{2}0]$ symmetry axes. Calipers that could be definitely identified as resulting from the addition of orbits are not shown. Appendix I (Tables IV, V, and VI) is a numerical list of these caliper values grouped according to their respective series.

IV. DATA ANALYSIS AND CALIPER ASSIGNMENT

In the following discussion, the term caliper will be construed to mean an extremal maximum or minimum length caliper in the direction of $\vec{n} \times \vec{H}$ with a magnitude equal to the projection of an extremal orbit in k space onto a plane perpendicular to $\vec{n}$. There are three possible types of orbits that may give rise to RFSE calipers: (1) orbits having true maximal or minimal extent in the $\vec{n} \times \vec{H}$ direction; (2) truncated orbits which are orbits in a plane perpendicular to $\vec{H}$ that increase in extent until they no longer exist in the same form; and (3) broken orbits which give rise to calipers between two turning points on an orbit on the FS. (In this case carriers do not complete a closed orbit before being scattered at the sample surface.²²) All three types of calipers have been reported previously^{16,17} and were also observed in the present investigation. Table II lists the caliper series according to the type of orbits responsible for each series.

The BZ for Tl is illustrated in Fig. 4 with the dimensions along the symmetry directions indicated. The interpretation of the data is in terms of a FS

TABLE II: Identification of the type of orbit responsible for the observed caliper series.

Type of Orbit	Series
1 - maximum or minimum	a c d e f g h j k l m o p q v w x y z α θ ξ γ
2 - truncated	s t u λ b δ
3 - broken	i n r ϕ π ω

Fig. 4

having a third-band hole surface centered at A, a fourth-band electron hexagonal network around the third-band hole surface in the ALH plane, and a fifth-band electron surface centered at H. No evidence exists to suggest that either the third band or the fourth band crosses the Fermi level along the Γ M direction or that the fourth band is connected to the adjacent BZ by posts lying in the Γ AHK plane. As will be discussed later, the results of this investigation and recent dHvA experiments suggest that these posts do not extend as high as the top of the fourth zone along HK. Extremal cross sectional areas for the various orientations were calculated numerically from the extremal calipers where possible. In the absence of central calipers which give the radii directly, the dimensions were taken from scaled drawings constructed from the data. In Figs. 5, 6, and 7 the solid lines represent the cross section of the FS in the respective planes, the dashed lines represent the projection of the extremities of the FS projected onto these planes, and the dotted lines are estimated connections between the experimentally determined sections of the FS. Also in Fig. 7 an extremal noncentral orbit is represented. The map of this orbit was obtained by using all three

Fig. 5

Fig. 6

Fig. 7

symmetry plane FS projections to construct a cross sectional view of the FS on a plane parallel to the $AL\Gamma$ plane at a distance of $0.432 \text{ \AA}^{-1}$ along AH.

A. Third-Band Holes

Caliper series a was readily assignable as being due to minimal, central orbits around the third-zone hole surface as indicated in Fig. 5. This series is very weak at angles less than 10° and not observed at all within 5° of the $[10\bar{1}0]$ axes due to the degeneracy existing between the third and fourth bands along the AL line.¹⁹ Orbits spanning both the third and fourth bands and passing through these degeneracy points were observed in all three orientations. Such orbits are represented in Figs. 5, 6, and 7 (labeled c, α , and n, respectively).

The assignment of series a completely determines the outline of the third band in the ALH plane. Series e confirms this outline and is due to a noncentral maximum closed orbit (Fig. 3). Series e has been observed and assigned to the same noncentral orbit by Coon *et al.*⁸ Series b arises from an inflection point in a plot of calipers versus distance along the direction of $\vec{H}$, k_H . The resultant plot for each of the caliper directions shows good agreement between the point of inflection of the k_c versus k_H curves and the distance along $\vec{H}$ at which the calipers fit the FS. The corresponding dHvA area for the cross section in

the ALH plane was calculated and is in excellent agreement with prior experiments (Table III).

The assignment of series o and p to central maxima calipers on the projection of the third-band surface in the ALM plane implies that neither Soven's ROPW or the single-OPW models accurately predicted the topology of the T1 FS. Series o and p indicate that the maximum distance that the central third-band hole surface extends in the $A\Gamma$ direction is $0.44 \text{ \AA}^{-1}$ directly above a point $0.40 \text{ \AA}^{-1}$ from A towards L (Fig. 7). The further assignment of series p, n, t, s, r, u, ω , π , ϕ , γ , ξ , θ , α , λ , and δ (Figs. 6 and 7) gives a completely self-consistent model for the third-band hole surface. The resultant model is depicted in Fig. 8. It has twelve projections in the $A\Gamma$ direction -- six major ones in the ALM planes and six minor ones in the AHK planes. The minor projections extend $0.35 \text{ \AA}^{-1}$ in the $A\Gamma$ direction above a point $0.50 \text{ \AA}^{-1}$ from A towards H.

In addition to requiring that the model developed from the calipers be topologically consistent, the intensity of the RFSE peaks aid in the assignment of caliper series. The intensity of the peaks is, to a large extent, dependent upon the geometry of the orbit. That is, extremal orbits that spend

TABLE III: A comparison of experimental dHvA areas.

Field Direction	Band	Experimental Area in $\text{\AA}^{-2}$			
		This Experiment	Ishizawa ^a	Anderson ^b	Priestley ^c
0001	3	1.96	1.95	1.95	1.96
10 $\bar{1}$ 0	3	0.943	0.936		0.936
11 $\bar{2}$ 0	3 (central)	0.961			
11 $\bar{2}$ 0	3 (noncentral)	0.900 [*]	0.889	~ 0.889	0.876
0001	4	2.01	2.02	2.02	2.04
0001	5	~ 0.018	~ 0.019	~ 0.019	

^aY. Ishizawa and W. R. Datars (Ref. 16).

^bJ. R. Anderson et al. (Ref. 15).

^cM. G. Priestley (Ref. 13).

^{*}Shown in Fig. 5 as outlined by — · — · —

Fig. 8

proportionately longer time in the skin depth of the sample, and whose trajectories are parallel to the surface while in the skin depth, have a greater effect on the surface impedance than do extremal orbits associated with more pointed segments of the FS. An example of this fact is that the most intense signal observed in this investigation was series o between 9° and 15° from the $[10\bar{1}0]$ direction and had a signal to noise ratio of more than 1000 to 1. As can be seen in Figs. 7 and 5, it is in this region that the extremal orbits associated with these calipers have a flat and extensive trajectory in the skin depth of the sample. Another example in the consideration of signal intensity as a guide for caliper assignment is the assignment of series p and q. The intensity of RFSE peaks decreased with increased angle very rapidly indicating the FS associated with these series became more pointed with increased angle.

Series s, t, and u result from truncated orbits due to the projection of the major peaks in the ALM plane above the main body of the third-band hole surface (Fig. 7). Estimates of the central and noncentral dHvA areas for this cross section are given in Table III.

For $\vec{n}$ parallel to $[10\bar{1}0]$, only two series, Θ and ξ , with relatively weak signals, were assigned to closed central orbits on the third-band surface. The reason for the lack of observable central orbits in this orientation is that they must pass through the point of degeneracy between the third and fourth-band surfaces at least twice during one orbit. That is, the number of electronic carriers spanning the third-zone hole surface is diminished due to some of the carriers switching to orbits on the fourth zone as a result of magnetic breakdown near the point of degeneracy. Although ξ is due to central orbits on a pointed section of the FS, it is observable since there are no other calipers in this region of magnetic field and angle. Orbits associated with caliper series Θ contain a relatively large number of carriers since for a given k_c there exists a large range of values for k_H . That is, orbits contributing to the change in surface impedance resulting in this observed caliper series do not all pass through the degeneracy point.

Series π (Fig. 6) is additional evidence for the assignment of series o and p and indicates that the third-band surface extends approximately $0.25 \text{ \AA}^{-1}$ along the $A\Gamma$ line. The assignment of the remaining

series α , γ , and δ (Fig. 6) allows a dHvA area for this cross section of the third-band surface to be calculated from scaled drawings and is in excellent agreement with previous experiments (Table III).

B. Fourth-Band Electrons

In all three orientations, the RFSE lines corresponding to extremal closed orbits on the fourth-band hexagonal network arms were very intense, indicating that these segments of the FS are relatively smooth. The outline of this network in the ALH plane is shown in Fig. 5. Caliper series f is plotted as a noncentral closed minimal orbit around the arms. It is topologically consistent with plots of series g and d, interpreted as resulting from closed maximal and minimal orbits on the electron surface (Fig. 5). Both of these calipers have their midpoint at the L point in the BZ. The dHvA area of hole orbits on the inside of this cross sectional view of the fourth-band electron surface is compared with previous experiments in Table III.

Series k and l (Fig. 7) result from maximal closed orbits on the fourth-band network. The outline shown in Fig. 7 due to these orbits is a projection of their extent onto the ALM plane. Series m is due to a closed-hole orbit on the inside of the network, while series j is a minimal closed orbit at the junction of the arms.

Series w, x, and y are noncentral, closed,

maximal and minimal calipers which have been consistently plotted (Fig. 6). Series z is tentatively assigned to be calipers from truncated orbits on the fourth-zone electron surface, but they could also be interpreted as maximal orbits on the third-band hole surface. These calipers furnish a projection of the network in the AHK plane. Caliper series y is a result of closed minimal Y-shaped orbits as viewed along HK. The plot of caliper series α results in a profile of the fourth-zone similar to Soven's ROPW prediction, but without the projections in the HK direction (posts). No orbits similar in nature to m were seen in this orientation due to the degeneracies along AL between the third and fourth-band surfaces.

The absence of the three posts occurring in both the single-OPW and the ROPW models in the HK direction, centered about H, is clearly indicated by the assignment of series x, y, w, k, j, and l. In addition to this investigation, recent dHvA experiments¹⁵ fail to show any evidence supporting the existence of the posts predicted by the ROPW and single-OPW models.

Series i is tentatively assigned as resulting from electronic carriers whose trajectories are nearly parallel to the sample surface. As can be

seen in Fig. 7, orbits around the fourth-band arms could result in carriers running parallel to the $A\Gamma$ direction over a large part of this orbit. Since the cross section of these arms varies greatly as a function of distance from the ALM plane, it is not possible to definitely assign the position of series i in Fig. 5.

C. Fifth-Band Electrons

Series v and h are assigned to pockets of fifth-band electrons centered at H. Series v shows signs of splitting indicating that more than one extremal orbit can exist on this portion of the surface. Effects of orbits like those responsible for series v were seen in the data taken on samples with $\bar{n}$ parallel to $[10\bar{1}0]$. However, calipers could not be assigned because the signals associated with series w interfered with the weaker RFSE lines of the fifth band. Both v and h agree very well with the model proposed in Ref. 15 (Figs. 5 and 7). In Fig. 5 the surface is drawn to be essentially round. This construction is not unique and the data could easily as well be fit to a triangular shape using noncentral orbits. In addition, the data shows some deviation from a constant caliper which is not noticeable on the scale of Fig. 6.

V. CONCLUSION

Table I is a comparison between the FS calipers along the principal symmetry direction obtained in the present investigation and past experiments. In addition to the experimental values, calipers resulting from the single-OPW and Soven's ROPW calculations are given. Although there is little numerical agreement between experiment and theory, Soven's model accurately predicts the shape of the FS in the ALH plane. The primary topological differences between the present model and the FS predicted by the ROPW calculation are the absence of fourth-zone posts in the AHK plane and the position and magnitude of the third-zone projections in the $A\Gamma$ direction. The ROPW model of the third-band hole surface has twelve projections in the $A\Gamma$ direction of equal extension, with 3 m symmetry in planes 15° from the ALM and AHK planes. The model resulting from the present investigation (Fig. 8) has six minor projections in the AHK planes and six major ones in the ALM planes. Calipers assigned to the fifth-band pockets at H are in excellent agreement with a recently proposed model by Ishizawa and Datars.¹³ This pocket of electrons shows the expected effect of spin-orbit coupling in

the AH direction (Fig. 5). No calipers were observed that could be interpreted as resulting from orbits on third or fourth-band pockets of the FS in the Γ MK plane. Also no effects due to the proposed sixth-band electron pockets centered at H were seen. The present investigation does not exclude the presence of these segments of the FS however. The relatively short mean free path at the higher temperatures necessary to avoid the superconducting phase in Tl results in very weak signals. The weak signals and the large slope in the surface impedance at low fields are possible explanations for the absence of calipers for these small pieces of the FS.

Calipers obtained in the present investigation are currently being used to adjust the nonlocal and spin-orbit parameters occurring in a pseudopotential calculation of the energy bands for Tl.

APPENDIX

TABLE IV: RFSE caliper values and directions obtained
on Tl samples with $\vec{n}$ parallel to [0001].

Angle	a	b	c	d	e	f	g	h	i
10 $\bar{1}$ 0			2.11		1.536	.555	.670	.17	.497
2			2.12		1.536	.555	.670		.497
4			2.13		1.538	.555	.670	.17	.497
6	1.466		2.14		1.538	.558	.670	.17	.497
8	1.489		2.16		1.538	.564	.670		.497
10	1.511		2.17			.567		.16	.497
12	1.537		2.17			.571		.17	.497
14	1.570		2.18			.578			.496
16	1.592					.582		.16	.494
18	1.615					.592		.15	
20	1.630	1.414				.603		.15	

Angle	a	b	c	d	e	f	g	h	i
22	1.658	1.382		2.04		.613			
24	1.680	1.356		1.99		.623		.16	
26	1.701	1.322		1.95		.634			
28	1.716	1.287		1.89		.649	.659		.17
1120	1.731	1.255					.669	.17	
32							.681		
34							.695		
36							.709		
38							.729		
40							.754		
43							.772		
48							.844		
53							.930		
58							1.060		

TABLE V: RFSE caliper values and directions obtained
on Tl samples with $\vec{n}$ parallel to $[10\bar{1}0]$.

Angle	w	x	y	z	α	γ	δ	θ	ϕ	π	η	ω	ξ
11 $\bar{2}0$												1.430	
3							1.120		1.122			1.415	
6							1.149		1.106			1.388	
9					1.689		1.165						
							1.190		1.092			1.372	
12					1.664		1.228						
							1.253		1.083			1.348	
15					1.648		1.288						
							1.315		1.071				
18									1.067				
21									1.067				
24									1.067		.813		
27									1.053		.774		
30											.736		
33			.979							.335	.709		

Angle	w	x	y	z	α	γ	δ	θ	ϕ	π	ρ	ω	ξ
36			.898							.301	.700		
39			.812							.269	.675		
42			.760							.250			
45			.723							.223			
48			.693							.215			
51			.666							.203			
54	.639									.191			
57	.613									.180			
60	.586							.793		.164			
63	.559							.763		.162			
66	.538	.717						.736		.156			
69	.529	.706						.707		.149			
72	.516	.691						.690		.146			
75	.507	.681								.142			

Angle	w	x	y	z	α	γ	δ	θ	ϕ	π	ρ	ω	ξ
78	.502	.675				.628				.141			.755
81	.496	.672				.615				.140			.804
84	.494	.669		.643		.597				.140			.857
87	.494	.669		.657		.590				.140			
0001	.494	.669		.665		.588				.140			

TABLE VI: RFSE caliper values and directions obtained
on Tl samples with $\vec{n}$ parallel to $[11\bar{2}0]$.

Angle	j	k	l	m	n	o	p	q	r	s	t	u	v
$10\bar{1}0$						1.536				1.276			
3				1.614		1.521				1.281	.332		
6				1.665		1.506					.336		
9				1.714		1.494					.347		
12				1.763		1.471					.360		
15				1.836		1.459					.375		
18					1.213	1.439					.387		
21					1.243	1.430					.404		
24					1.257	1.422					.413		
27						1.399							
30						1.369							
33						1.340							

Angle	j	k	l	m	n	o	p	q	r	s	t	u	v
36						1.305			1.078				
39						1.288			1.068				
42						1.267			1.051			.547	
45									1.019			.574	
48							.831		.996			.608	
51		.790					.856		.972			.634	
54		.774					.876		.946			.655	
57		.756					.899					.680	.266
60		.742					.926					.705	.296
63		.726											.315
66		.717	.663										.360
69		.709	.663										.392
72		.703	.663										.412
75	.613	.688	.663										.422

Angle	j	k	l	m	n	o	p	q	r	s	t	u	v
78	.597	.680	.668										.432
81	.585	.674	.670					.753					.435
84	.574		.671					.780					.436
87	.562							.815					
0001	.553							.854					

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VITA

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EXAMINATION AND THESIS REPORT

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