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Experimental Study of a Cascade of Low Pressure Turbine Blades with Upstream Periodic Stator Wakes

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by
Carlos Rene Gonzalez Rodriguez
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Nomenclature

Acronyms

2D2C PIV  2-Dimensional, 2-Component Particle Image Velocimetry

2D3C PIV  2-Dimensional, 3-Component (Stereoscopic) Particle Image Velocimetry

3D3C PIV  3-Dimensional, 3-Component (Volumetric) Particle Image Velocimetry

ABS  Acrylonitrile Butadiene Styrene

AC  Alternating Current

AFRL  Air Force Research Laboratory

APU  Auxiliary Power Unit

BNC  Bayonet Neill-Concelman Connector

BOD  Bi-Orthogonal Decomposition

CAD  Computer Aided Design

CCD  Charged Coupled Device

CFD  Computational Fluid Dynamics

CMOS  Complementary Metal Oxide Semiconductor

CRVP  Counter Rotating Vortex Pair

CTA  Constant Temperature Anemometry

DFT  Discrete Fourier Transform

DMD  Dynamic Mode Decomposition

DNS  Direct Numerical Simulation

FFT  Fourier Fast Transform

GB  Gigabyte
<table>
<thead>
<tr>
<th>Acronym</th>
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<tbody>
<tr>
<td>IRT</td>
<td>Infrared Thermography</td>
</tr>
<tr>
<td>JICF</td>
<td>Jet in Crossflow</td>
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<tr>
<td>LPT</td>
<td>Low Pressure Turbine</td>
</tr>
<tr>
<td>Nd:YAG</td>
<td>Neodymium-Doped, Yttrium Aluminum Garnet</td>
</tr>
<tr>
<td>NI</td>
<td>National Instruments</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PIV</td>
<td>Particle Image Velocimetry</td>
</tr>
<tr>
<td>POD</td>
<td>Proper Orthogonal Decomposition</td>
</tr>
<tr>
<td>PSP</td>
<td>Pressure Sensitive Paint</td>
</tr>
<tr>
<td>QR</td>
<td>Orthogonal Triangular Decomposition</td>
</tr>
<tr>
<td>RANS</td>
<td>Reynolds-Averaged Navier Stokes</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>ROM</td>
<td>Reduced Order Model</td>
</tr>
<tr>
<td>SI</td>
<td>Système International D’Unités</td>
</tr>
<tr>
<td>SLA</td>
<td>Stereolithography</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular Value Decomposition</td>
</tr>
<tr>
<td>TBC</td>
<td>Thermal Barrier Coating</td>
</tr>
<tr>
<td>TIT</td>
<td>Turbine Inlet Temperature</td>
</tr>
<tr>
<td>TR PIV</td>
<td>Time-Resolved Particle Image Velocimetry</td>
</tr>
<tr>
<td>TTL</td>
<td>Transistor-Transistor Logic Interface Standard</td>
</tr>
<tr>
<td>UV</td>
<td>Ultraviolet</td>
</tr>
<tr>
<td>VI</td>
<td>Virtual Instrument</td>
</tr>
<tr>
<td>YSZ</td>
<td>Yttria-Stabilized Zirconia</td>
</tr>
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</table>
Greek Symbols

\( \alpha \)  
DMD Modal Amplitudes

\( \Delta \)  
Camera Depth of Field [mm]

\( \eta \)  
Film Cooling Effectiveness

\( \dot{\phi} \)  
Eigenfunctions in POD Direct Method

\( \lambda \)  
POD Eigenvalue or Laser Wavelength [m]

\( \mu \)  
DMD Ritz Values

\( \mu \)  
Dynamic Viscosity [kg/ms]

\( \nu \)  
Fluid Kinematic Viscosity [m\(^2\)/s]

\( \Omega \)  
POD Spatial Domain

\( \omega \)  
Simpson’s Weight Function Used in POD or DMD Mode Frequency [rad/s]

\( \omega_z \)  
Z-Component of Vorticity [1/s]

\( \Phi \)  
Matrix of POD Modes

\( \phi \)  
Dimensionless Temperature Parameter

\( \Psi \)  
DMD Mode

or \( \phi \)  
POD Eigenfunction

\( \rho \)  
Density [kg/m\(^3\)]

\( \rho(\cdot,\cdot) \)  
Correlation or Autocorrelation Coefficient

\( \Sigma \)  
Singular Values of DMD Linear Mapping Matrix

\( \sigma \)  
Standard Deviation or Growth Rate in DMD Analysis [1/s]

\( \sigma_l, \sigma_{ll} \)  
Normalization Factors for PIV Cross Correlation Direct Calculation

\( \tau_{ij} \)  
Reynolds Stress Tensor Per Unit Mass [m\(^2\)/s\(^2\)]

\( \Theta \)  
PIV Cross Correlation in Frequency Domain
Roman Symbols

$A$ DMD Linear Mapping Matrix or One of Its Members

$a$ POD Coefficient

$A$ DMD Linear Mapping Matrix

$\mathring{A}$ DMD Approximate Linear Mapping Matrix

$b$ Modal Coefficient of Any Arbitrary Orthonormal Basis

$B(.)$ Probability Density Function

$BR$ Blowing Ratio

$C$ Autocovariance Approximation in POD Direct Method

$c$ DMD Linear Combination Coefficients

$c$ Matrix of Eigenvectors in POD Analysis

$C$ DMD Vector of Linear Combination Coefficients

$C(s)$ Cross Correlation Function

$C_x$ Blade Axial Chord Length [mm]

$cov(.,.)$ Covariance

$D$ Diagonal Matrix

$D$ Diameter of Film Cooling Holes [mm]

$DC$ Duty Cycle

$DR$ Density Ratio

$E$ Autocovariance Matrix of Function $u$ in POD Analysis

$F$ Dimensionless Forcing Frequency or Camera Lens F Number

$f$ Forcing Frequency [Hz]

$F_l$ In-Plane Loss of Pair Term in Discrete PIV Cross Correlation

$Fo, Fo^{-1}$ Fourier Transform and Inverse Fourier Transform
\( Fr \) Frossling Number

\( I \) Image Intensity or Momentum Flux Ratio

\( \bar{I} \) Local Average Image Intensity in PIV Cross Correlation

\( I \) DMD Unit Vector

\( I' \) Image 1 Intensity in PIV Cross Correlation

\( I'' \) Image 2 Intensity in PIV Cross Correlation

\( IA \) Interrogation Area \([\text{pixels}^2]\)

\( K \) Kurtosis

\( k \) Turbulent Kinetic Energy per Unit Mass \([\text{m}^2/\text{s}^2]\)

\( L \) Characteristic Length \([\text{m}]\)

\( L^2 \) Hilbert Space of Square Integrable Complex Valued Functions

\( M \) Lens Magnification

\( N \) Total Number of Pixels in PIV Cross Correlation or Number of Snapshots

\( N_T \) Number of Realizations of \( u \) in POD Analysis

\( N_x \) Number of Pixels in \( x \) Direction in PIV Cross Correlation

\( N_x \) Number of Spatial Points at Which Each Member of \( u \) is Defined in POD and DMD Analysis or Horizontal Size of Individual Domains in Image Patching \([\text{pixels}]\)

\( N_y \) Number of Pixels in \( y \) Direction in PIV Cross Correlation or Vertical Size of Individual Domain in Image Patching \([\text{pixels}]\)

\( Nu \) Nusselt Number

\( P \) Fluid Pressure \([\text{Pa}]\)

\( p \) Number of Modes to Use in Modal Reconstruction

\( Pr \) Prandtl Number

\( q''_f \) Heat Flux \([\text{W/m}^2]\)
\( \hat{R} \) Discretized Cross Correlation

\( r \) DMD Residual Vector

\( \vec{R} \) Particle Displacement in Image Plane [pixels]

\( \vec{r} \) Particle Displacement in Object Plane [m]

\( R(x,x') \) Autocorrelation Function in POD Analysis

\( r,s \) Interrogation Spot Displacement in x,y Directions in PIV Cross Correlation [pixels]

\( R_{ul} \) Numerator of Discrete PIV Cross Correlation Direct Calculation

\( Re \) Reynolds Number

\( S \) Skewness

\( S \) DMD Companion Matrix

\( \vec{s} \) Displacement Vector [pixels]

\( St \) Stanton Number or Strouhal Number

\( T \) Temperature [K]

\( \Delta t \) Snapshot Time Separation in DMD [s]

\( t,t' \) Two Time Instances [s]

\( t_0 \) Initial Time [s]

\( t_{JET,ON} \) Time a Jet is On During Film Cooling Cycle [s]

\( Ts \) Temperature - Specific Entropy Diagram

\( U \) Right Singular Vectors of DMD Linear Mapping Matrix

\( u \) X-Component of Velocity [m/s] or General Function for POD Analysis

\( \hat{u} \) Element of Function \( u \) in POD Direct Method

\( \| \vec{\pi}_i \| \) Averaged Velocity Magnitude [m/s]

\( \| u_i \| \) Velocity Magnitude [m/s]

\( \vec{\pi} \) Averaged X-Component of Velocity [m/s]
\( \overline{u} \) Averaged Velocity [m/s]

\( u' \) Fluctuating X-Component of Velocity [m/s]

\( u_i \) Velocity [m/s]

\( u_i' \) Fluctuating Velocity [m/s]

\( V \) Member of DMD Snapshot Matrix

\( v \) Y-Component of Velocity [m/s]

\( \mathbf{V} \) DMD Snapshot Matrix

\( \overline{v} \) Averaged Y-Component of Velocity [m/s]

\( v' \) Fluctuating Y-Component of Velocity [m/s]

\( V_1 \) First DMD Snapshot Matrix

\( V_2 \) Second DMD Snapshot Matrix

\( V_{vand} \) DMD Vandermonde Matrix

\( VR \) Velocity Ratio

\( \mathbf{W} \) Left Singular Vectors of DMD Linear Mapping Matrix

\( w \) Z-Component of Velocity [m/s]

\( \overline{w} \) Averaged Z-Component of Velocity [m/s]

\( w' \) Fluctuating Z-Component of Velocity [m/s]

\( x \) and \( x' \) Two Different Positions in Spatial Domain for POD

\( \bar{x} \) Position Vector [pixels]

\( \Delta x \) Horizontal Overlap in Image Patching [pixels]

\( x_i \) Position in the Cartesian coordinate system [m]

\( Y \) Eigenvectors of DMD Matrix \( \tilde{A} \)

\( \Delta y \) Vertical Overlap in Image Patching [pixels]

\( \Delta Z_0 \) Light Sheet Thickness [mm]
$Z_0$ Distance from Object Plane to Lens [m]

$z_0$ Distance from Lens to Image Plane [m]

**Subscripts**

0 Without Film Cooling

$\infty$ Freestream

$aw$ Adiabatic Wall

$c$ Coolant

$f$ With Film Cooling

$h$ High

$i, j$ Pixel Spatial Position Indices in PIV Cross Correlation

$i, j$ Spatial Position Indices in POD Analysis

$i$ DMD Snapshot Index

$j$ DMD Mode Index

$k$ Member of $u$ Function in POD Analysis

$l$ Low

$m$ Mean

$pp$ Peak to Peak

$rms$ Root Mean Square

$x$ X-Component

$y$ Y-Component
Abstract

The objective of this study is to experimentally study film cooling flows. A closed-loop wind tunnel with a four passage linear cascade of US Air Force Research Laboratory (AFRL) ultra-high-lift L1A low pressure turbine (LPT) blades and upstream wake generator is used in conjunction with Particle Image Velocimetry (PIV) flow visualization technique to study turbulent film cooling flows due to the interaction between vanes and blades. Further post-processing in the form of Proper Orthogonal Decomposition (POD) and Dynamic Mode Decomposition (DMD) modal analyses is performed to determine the relevant modes that characterize the coherent structures in the flow. An image patching algorithm is also implemented. The results obtained are used to characterize the periodic wake on the cascade flow.

The periodic wake has been studied in detail near the leading edge of the suction side. The velocity data led to the mean velocity profile and maximum velocity deficit in the wake. The POD identified the most energetic modes representing the vortex shedding wavelength, and its harmonics, of the wake generator plates. The DMD confirmed the wake passage frequency.

Implementation of the image patching algorithm with four domains was presented. The technique was successful in computing the average vector field. Further downstream of the leading edge, the POD modes are shown to become more chaotic and less energetic. The leading order mode pair loses close to half of their energy to lower order modes due to the cascading of turbulent kinetic energy to lower spatial scales and to viscous dissipation losses. When the wake is impinging on the leading edge, the boundary layer separates near the transition point. The boundary layer remains completely attached to the trailing edge when the wake is not impinging on the leading edge.
Chapter 1
Introduction

1.1 Background

A gas turbine is a type of continuous combustion engine designed to produce either shaft power or thrust. The three main components of every gas turbine are the compressor, the combustor, and the turbine. A gas turbine increases the pressure of the airflow in the compressor. The fuel is then sprayed into the airflow and the mixture is ignited in the combustor generating a very high temperature, high pressure flow. The turbine extracts the energy from the flow by expanding the flow to the exhaust pressure and temperature. There are two main applications for gas turbines, depending on whether the energy extracted from the flow is used to produce mostly shaft power or mostly thrust. Gas turbines used to produce thrust are used in aircraft propulsion. In this application, shaft power is only produced for driving the compressor and the aircraft generators. Most of the energy is recovered by expanding the flow through a nozzle to produce thrust. Gas turbines that produce mostly shaft power are used to drive electric generators, ships, trains, and tanks. Most of the energy is used to drive the shaft that can then be used to drive a generator, ship propeller, or tank tracks. Figure 1.1 shows a cut-out view of a thrust producing gas turbine in the form of a turbojet. Figure 1.2 shows a cut-out view of a land-based gas turbine used in electric power generation.

There are dozens of applications and types of gas turbines. Most modern commercial aircraft use either the high-bypass turbofan or a turboprop for propulsion. Military fighter jets of the 1950s and 1960s used afterburning turbojets, though more recent designs have changed to low-to-medium bypass afterburning turbofans. Helicopters are mostly equipped with lightweight gas turbines called turboshfts, used to optimize the shaft power that drives the helicopter rotor. Another application of gas turbines in aircraft is the Auxiliary Power Unit (APU). The APU is a small power generating gas turbine that powers auxiliary systems when the main engines are shut down and also provides the electrical power to start the main engines. Gas turbines that produce mostly shaft power are used to generate electricity by driving a generator. The systems can range from small portable devices to complex combined cycle power plants that produce several hundred megawatts of electric power. Gas turbines can also be used as purely a source of shaft power to drive industrial compressors and pumps. Figure 1.3 shows an afterburning turbofan during testing. Figure 1.4 shows a GE power generating gas turbine during assembly.

1.1.1 The Brayton Cycle

The ideal gas turbine cycle is the Brayton Cycle (Figure 1.5). Ambient air is drawn in at state 0. The compressor raises the pressure of the air isentropically to state 3. Fuel is added to the flow and the combustion adds heat isentropically to the mixture between states 3 and 4. The turbine then expands the combustion products isentropically to state 5. Further expansion to state 8 can be accomplished in a thrust-producing gas turbine via a nozzle. An isobaric heat rejection to state 0 in the heat exchanger completes the cycle. Figure 1.5 shows the cycle and its $T_s$ diagram. Note that in Figure 1.5a the general closed loop Brayton cycle components are pictured while a propulsion gas turbine cycle is shown in Figure 1.5b. A real gas turbine is an open loop Brayton process with the heat rejection step (state 8 to state 0) being replaced by the continuous intake of fresh air.

No real gas turbine follows the ideal Brayton cycle. In particular, real gas turbines show significant nonisentropic compressions and expansions. One of the most important performance parameters is the thermal efficiency. The thermal efficiency of a gas turbine is heavily dependent on the turbine inlet temperature (TIT), which is $T_4$ in Figure 1.5b. An increase in the TIT leads to an increase in the turbine thermal efficiency, the net work output, and the pressure ratio across the compressor (Figure 1.6). Thus the TIT is the single most important parameter that characterizes the performance of the gas turbine.
Figure 1.1: Thrust-Producing Gas Turbine (U.S. FAA Flight Standards Service, 2004)

Figure 1.2: Power Generation Gas Turbine (Florida International University, 2004)
Figure 1.3: Test of a Pratt and Whitney F-100 Afterburning Turbofan (Pratt & Whitney, 2003)

Figure 1.4: GE Land-Based Power Generating Gas Turbine During Assembly (General Electric, 2014)
Figure 1.5: Gas Turbine Cycle

(a) Components (Smirnov, 2008)  
(b) Ts Diagram (NASA Glenn Research Center, 2008)

Figure 1.6: Gas Turbine Thermal Efficiency and Work Output as a Function of the Compressor Pressure Ratio (Bidan, 2013)

(a) Thermal Efficiency vs. Compressor Pressure Ratio for Different Turbine Inlet Temperatures  
(b) Normalized Work Output vs. Compressor Pressure Ratio for Different Turbine Inlet Temperatures
1.1.2 Turbine Temperature Management Technologies

Increasing the TIT is an important focus of current gas turbine research. Because the TIT occurs directly aft of the combustor, this is one of the highest temperatures in the gas turbine. Currently, the highest TIT reported is around 1600 °C in the M501J power generation gas turbine (Mitsubishi Heavy Industries, 2011). This temperature is higher than the melting temperature of the nickel superalloys used in turbine airfoils. Thus the TIT is one of the principal factors that sets the maximum combustion temperature well below the stoichiometric combustion temperature. Indeed, an increase in the TIT of as small as 50 °C can significantly increase the thermal efficiency. But, if the TIT is above the operational temperature of the turbine, the life of the blades and vanes can be significantly decreased. Therefore, increasing the allowed TIT has been a continuous area of research and development by industry, universities, and government entities.

The first major breakthrough was the implementation of nickel-based superalloys. Directionally solidified cast alloys, followed by the more recent introduction of single-crystal cast alloys, were significant improvements to the survivability of the superalloys. The increased material survivability has led to increased firing temperatures. Both have led to the need for protecting the airfoils against hot corrosion and oxidation. This lead to the development of coatings. Diffusion coatings and overlay coatings have led to the development of the current generation of thermal barrier coating (TBC) (Alvin et al., 2007).

Another concurrent area of research has been turbine cooling methods. Thermal barrier coatings based on Yttria-Stabilized Zirconia (YSZ) represent the current state of materials development. TBCs have allowed increased TIT but cooling methods are still needed to protect the airfoils and to increase the TIT even further. The first cooling mechanism to be implemented was internal cooling. In internal cooling, relatively cool air is drawn from the compressor. The cool air bypasses the combustor and is channeled to the inside of the turbine airfoils and casing walls. Inside the airfoils, the cool air flows through channels cooling the walls of the airfoils. Straight radial channels were the first to be used. An improvement was the use of serpentine channels. In some local hot spots, the cool air can be used as a jet to impinge the interior surface of the airfoil, improving the cooling effects even further. Film cooling, the method of interest in this study, consists of an array of holes on the surface of the airfoil that reach into the interior air passages and therefore the holes expel the cool air to the outside flow (Figure 1.7). The expelled jets form a thin film of cooler air around the airfoil that protect it against the high temperature flow through the turbine. A combination of thermal barrier coatings, internal cooling, and film cooling has allowed the gradual increase in TIT to temperatures that were not conceivable fifty years ago. Figure 1.8 shows a historical timeline of the development of turbine airfoil thermal management technologies. Cooling mechanisms became readily used by the 1980s. Thermal barrier coatings were introduced in the 1990s. The next generation of predicted material improvements could increase the TIT to 1800 °C by 2020.

1.2 Motivation

Numerous studies have focused on film cooling and its potential to increase the turbine efficiency by increasing the heat transfer effectiveness of the film and reducing the amount of coolant used. A higher film effectiveness allows for a higher TIT, thereby increasing the efficiency. A reduction of the amount of coolant used will lead to higher volume of airflow through the turbine and thus more working fluid will be available increasing the work output of the turbine. Modern gas turbines extract between 20% and 30% of the compressor flow for cooling purposes (Han & Ekkad, 2001). This large amount of flow extracted imposes a penalty on the work output but it allows a higher TIT. Research has focused on increasing the margin between the benefit of film cooling and the reduction in work output from the decreased turbine flow.

In this study, film cooling under turbulent flow conditions is studied. In a real gas turbine, two types of turbulence can be identified: freestream turbulence and turbulence due to the wakes produced by upstream airfoils. The second type is studied here. Wakes generated by upstream airfoils impinge on the aft film-cooled airfoils. In some cases, the wakes can increase the film cooling efficiency, in others the wakes may disrupt the film reducing the film cooling performance (Womack et al., 2008b).
Figure 1.7: Film Cooling of a Turbine Airfoil (Swiss Federal Institute of Technology, Zurich, 2005)

Figure 1.8: Turbine Inlet Temperature Timeline (Wadley Research Group, 2013)
1.3 Literature Survey

1.3.1 Film Cooling

Film cooling has been an important focus of gas turbine engine research for the past forty years. One of the first aircraft engines to incorporate film cooling was the Pratt and Whitney J58 engine used in the SR-71 Blackbird. Before the J58, Pratt and Whitney produced engines with uncooled turbine airfoils. The TIT was limited to 900 °C, near the limiting temperature of the airfoil alloys of the time. The J58 could reach a TIT of 1100 °C with cooled first stage turbine airfoils (Langston, 2013). From the 1960s to the 1990s, research focused on the geometry of the film cooling holes. The principal achievement during this period was the experimental testing, development, and implementation of shaped film cooling holes in production gas turbines (Bunker, 2005). As such research is frequently funded by government agencies, military engines were the first to implement shaped film cooling holes. Today, both commercial and military engines benefit from the early research in shaped film cooling holes. Various cooling hole geometries and injection angles were experimentally studied from the 1960s to 1980s. A comprehensive review of early research in shaped film cooling holes is given by Goldstein (1971). Among the parameters studied were jet spacing, internal blade geometry and fluid dynamics, injection angle, hole length, and blowing ratio. A more recent complete study on the effects of these parameters on film cooled blades was performed by Bunker (2005). In the study, different geometries are studied with emphasis on angled fan diffuser exits with expansion into the blade surface. This geometry is the most commonly used in gas turbines due partially to increased film cooling coverage and manufacturing constrains. Figure 1.9 shows such a geometry (marked A). The other geometries are less common and there is a smaller amount of available data for these geometries.

Over the last two decades film cooling research has focused on increasing the film cooling effectiveness and decreasing the amount of coolant extracted from the compressor. At the turn of the century, leading edge film cooling was identified as a new area of research (Han & Ekkad, 2001). In their study of the effects of free-stream turbulence on leading edge film cooling, Ou & Rivir (2001) showed that under high freestream turbulence, film cooling effectiveness increases for blowing ratios (BR) between 1.0 and 2.0. Furthermore, the film cooling effectiveness increased with increasing Reynolds number up to a blowing ratio of 2.0. For higher blowing ratios, no dependence on the Reynolds number was encountered.

Insteadiness in the freestream has also been an important area of research. For instance, Ligrani et al. (1996), in a two part report, were one of the first to study the effects of freestream turbulence in a film-cooled blade. The study focused on the effects of sinusoidal variations of static pressure and freestream
velocity in a single row of film cooling holes. The results show that there is a significant change in the film cooling effectiveness when the freestream turbulence moves from quasi-steady to non-quasi-steady state. The change is attributed to instantaneous changes in the boundary layer when the pulsations are activated. In a side note to this report, it is important to notice the relative unimportance given to wake passing on film cooling performance. Ligrani et al. (1996) stated that the static pressure variations in a wake produced by an upstream turbine airfoil are smaller than those produced by freestream disturbances. Furthermore, it is stated that the wakes disrupt the film protection to a lesser extent than freestream disturbances. Further studies of the interactions between vanes and blades have become necessary to quantify wake effects on the film. Extensive focus has been placed lately on studying wake effects on film cooling performance. Ekkad et al. (1997) studied the combined effects of freestream turbulence and upstream wakes on the heat transfer in a turbine blade. Upstream turbulence grids and a spoked wheel wake generator were used to generate the unsteadiness. The test blade was instrumented with thermocouples to measure the temperature of the blade surface. Figure 1.10 shows the test geometry. The results show that the best film effectiveness for mean turbulence intensities up to 21.2% is at blowing ratios of 1.2 (CO$_2$) and 0.8 (air). The grid turbulence was also found to dominate over the wake turbulence, especially at higher mean turbulence intensities. Even at lower intensities, the presence of the unsteady wake yielded lower film effectiveness and higher heat transfer coefficients. Other interesting studies in wake effects have been conducted by Heidmann et al. (1997), Womack et al. (2008b), Olson et al. (2010).

A recent trend in film cooling research has been pulsed film cooling, also called forced filmed cooling. The objective of the research has been to identify optimal forcing parameters that will decrease the amount of coolant used while still maintaining proper blade protection. Bons et al. (2001) and Bons et al. (2002) studied the effects of forced jets on the flow separation on a low-pressure turbine blade. In the studies, a reduced coolant flow was found to provide as much coverage as an unforced jet at low duty cycles and low forcing frequencies. Since then, forced film cooling research has focused on studying the effects of duty cycle, forcing frequency, and blowing ratio on the forced film cooling performance. Flow field and heat transfer results were obtained by Coulthard et al. (2007a) for a flat plate model (Figure 1.11). A single row of film cooling holes was studied with anemometry techniques, thermocouples, and infrared imaging. Surprisingly, the results showed that steady cooling at moderate blowing ratios was more effective than forced cooling. Forced film cooling increased the Stanton number ratios. Increased duty cycles and dimensionless forcing frequencies only exacerbated the effect. A few exceptions were found at high blowing ratios. It was found that high dimensionless forcing frequencies tended to mitigate jet lift-off and therefore increase the film effectiveness, though not on a level comparable to cases with lower blowing ratios. Another interesting study in forced

Figure 1.10: Test Section Geometry (Ekkad et al., 1997)
film cooling was performed by Ekkad et al. (2006). In this study, infrared thermography (IRT) technique was used to study the forced film cooling performance of a leading edge model. The results obtained are very promising: the forced jets act as a jet with lower average momentum. This led to less jet lift-off and more lateral spreading. Both increased the film effectiveness and decreased the heat transfer coefficients. Interestingly, the forcing frequency was not found to have an effect on the film cooling performance. The duty cycle had a larger effect with duty cycles of 10% or less providing inadequate coverage. It is a clear sign of the lack of research in forced film cooling the fact that two similar experiments by Ekkad et al. (2006) and Coulthard et al. (2007a) yielded contradictory results as to the effect of jet forcing on the film cooling performance. Besides the difference in the experimental apparatus, it is very difficult to determine the cause of the discrepancies. The empirical nature of film cooling research leaves researchers with an insufficient knowledge of the fluid dynamics at play that may shed some light on why similar experiments can yield vastly different results.

More recently, Womack et al. (2008a), studied the combined effects of wakes and jet forcing using the same wind tunnel as Coulthard et al. (2007a) with the addition of a spoked wheel wake generator. At low blowing ratios, the effect of the wakes and forcing is catastrophic to the film cooling effectiveness. At higher blowing ratios, there is still a reduction in film cooling effectiveness but the wake tends to reattach the jet to the blade thereby slightly improving the otherwise ineffective film. The wake timing with respect to the forcing also played a significant role in the results. The wake timing had a significant effect at low wake Strouhal numbers. This effect was steadily reduced as the Strouhal number increased. Overall, no benefit was found to forcing the coolant flow under the presence of periodic wakes.

### 1.3.2 Film Cooling Parameters

The effectiveness of a film is based on the degree of blade cooling and the reduction of heat transfer between the freestream and the blade. Film cooling effectiveness is defined in Equation 1.1 in terms of the relevant temperatures in a film-cooled blade.

\[
\eta = \frac{T_{aw} - T_\infty}{T_c - T_\infty}
\] (1.1)
The adiabatic wall temperature is denoted by $T_{aw}$, the coolant temperature is denoted by $T_c$ and the freestream temperature is denoted by $T_\infty$. High film cooling effectiveness (close to unity) is desirable for film cooling applications. Two other important parameters in film cooling are the Stanton number $St$ (Equation 1.2) and the Frossling number $Fr$ (Equation 1.3), both non-dimensional heat transfer coefficients. The more practical measure of film cooling efficiency is the heat flux ratio, defined in Equation 1.5 (Ekkad et al., 2006), (Coulthard et al., 2007a).

$$St = \frac{Nu}{RePr}$$ (1.2)

$$Fr = \frac{Nu}{Re^{1/2}}$$ (1.3)

$$\phi = \frac{T_w - T_\infty}{T_c - T_\infty}$$ (1.4)

$$\frac{\dot{q}_f}{\dot{q}_0} = \frac{St_f}{St_0} \left(1 - \frac{\eta}{\phi}\right)$$ (1.5)

The non-dimensional parameters used are defined as: Nusselt number ($Nu$), Reynolds number ($Re$), and Prandtl number ($Pr$). The non-adiabatic wall temperature is denoted by $T_w$. Both the Stanton number and the Frossling number should be as low as possible (close to zero) for maximum film cooling effectiveness. The heat flux ratio is defined as the ratio of wall heat flux with film cooling ($\dot{q}_f$) to the wall heat flux without film cooling ($\dot{q}_0$). The heat flux ratio should also be as close to zero as possible for the least heat transfer between the freestream and the airfoil. The heat flux ratio is a function of the Stanton number with film cooling ($St_f$), the Stanton number without film cooling ($St_0$), the film cooling effectiveness ($\eta$), and a dimensionless temperature ($\phi$) (Bidan, 2013).

The blowing ratio (Equation 1.6) is the main parameter that defines the film cooling jets. It is the ratio of coolant mass flux to the freestream mass flux. The velocity ratio, $VR$, density ratio, $DR$, and momentum flux ratio, $I$, are defined similarly in Equations 1.7-1.9. The parameters are related in Equations 1.10-1.11. The symbols are as follows: $\rho_c$ is the coolant density, $V_c$ is the coolant velocity, $\rho_\infty$ is the freestream density, and $V_\infty$ is the freestream velocity.

$$BR = \frac{\rho_c V_c}{\rho_\infty V_\infty}$$ (1.6)

$$VR = \frac{V_c}{V_\infty}$$ (1.7)

$$DR = \frac{\rho_c}{\rho_\infty}$$ (1.8)

$$I = \frac{\rho_c V_c^2}{\rho_\infty V_\infty^2}$$ (1.9)

$$BR = DR \times VR$$ (1.10)

$$I = BR^2 / DR$$ (1.11)

The parameters defined above are meaningful only for steady cooling. For forced, or pulsed, film cooling, five parameters are needed to characterize a cooling cycle. The low coolant flow (or off) part of the cycle is defined by the low blowing ratio, $BR_l$. The high coolant flow part of the cycle is defined by the high blowing ratio, $BR_h$. The two coolant states are related by the peak to peak blowing ratio, $BR_{pp}$. The mean blowing ratio, $BR_m$, is the time averaged blowing ratio over a cycle. Duty cycle, $DC$, (Equation 1.14) is defined as the fraction of the forcing period that the jet is blowing. Two equations (Equations 1.12 and 1.13) relate the five variables ($BR_l$, $BR_h$, $BR_{pp}$, $BR_m$, and $DC$) which means three variables must be independently fixed (Bidan, 2008). $F$ (Equation 1.15) is a dimensionless forcing frequency parameter frequently called the
Strouhal number \((St)\). In the equations below, \(t_{JET,ON}\) is the time the jet is on during a cooling cycle, \(f\) is the forcing frequency, and \(L\) is a characteristic length.

\[
BR_{pp} = BR_h - BR_l \tag{1.12}
\]

\[
BR_m = DC \cdot BR_h + (1 - DC) BR_l \tag{1.13}
\]

\[
DC = \frac{t_{JET,ON}}{t_f} \tag{1.14}
\]

\[
St = F = \frac{fL}{V_\infty} \tag{1.15}
\]

1.4 Objectives

The objective of this study is experimentally study film cooling. A closed-loop wind tunnel with a four passage linear cascade of US Air Force Research Laboratory (AFRL) ultra-high-lift, aft-loaded L1A low pressure turbine (LPT) blades and upstream wake generator is used in conjunction with Particle Image Velocimetry (PIV) flow visualization technique to study turbulent film cooling flows due to the interaction between vanes and blades. Further post-processing in the form of Proper Orthogonal Decomposition (POD) and Dynamic Mode Decomposition (DMD) modal analyses is performed to determine the relevant modes that characterize the coherent structures in the flow. An image patching algorithm is also implemented to obtain average vector fields. The objectives are as follows:

1. Perform PIV experiments near both surfaces of the central test blade in order to characterize the flow field dynamics of the upstream periodic, turbulent wake in the cascade flow.

2. Develop a POD algorithm to study the turbulent coherent structures in the wake.

3. Develop a DMD algorithm to study the temporal evolution of the coherent structures in the wake.

4. Implement an image patching algorithm to obtain an overall average flow field using individual PIV domains.

The structure of the thesis is as follows:

- Chapter 2 focuses on the experimental and analysis techniques used in the experiment: PIV, POD, DMD, and image patching.

- Chapter 3 gives a detailed description of the experiment: wind tunnel (including test section) and the PIV acquisition equipment.

- Chapter 4 is an in-depth explanation and description of the results in the suction side, near the leading edge.

- Chapter 5 presents a selection of the most important results at certain domains in the suction side, downstream of the leading edge.

- The appendix contains an explanation of the results obtained on the pressure side of the blade.
Chapter 2
Experimental Measurement and Analysis Techniques

2.1 Particle Image Velocimetry (PIV)

PIV is a particle displacement method used to obtain the velocity field in a wide flow domain. The concept behind the PIV is as follows: if the flow is seeded with small particles, the position of these particles can be recorded over a short time and thus obtain a velocity vector that describes the particle motion. From the measured displacement and the given time interval, the particle velocity can be determined. This process can be repeated for thousands of particles in a given flow to obtain the velocity field in the flow. From the velocity field, other important quantities such as vorticity, mean velocities, and RMS velocities can be derived.

The basic procedure for PIV is as follows: The flow is first seeded with particles that faithfully follow the behavior of the flow. As the seed passes through the test section, a light source fires for a short instant. An optics system forms the light beam into a thin sheet. The thin sheet illuminates the seed and a camera takes a picture of the seed. A short time later, the light source fires again and another picture is taken. The two images of the seed now correspond to the particles’ positions at two different times separated by a known time delay. Thus, the particles’ velocities can be obtained using statistical correlations. From the particles’ velocities, a velocity field for the flow can be derived. The main components of a PIV system are discussed below. The information below was compiled from Raffel et al. (2007), Nikitopoulos & Garrison (2005), and Jahanmiri (2011). Figure 2.1 shows the main PIV components.

Figure 2.1: PIV Components (Jahanmiri, 2011)
2.1.1 Light Source and Optics

Lasers are the most common light source used in PIV because of their high energy density and monochromatic light. Furthermore, laser light is easily formed into light sheets and their timing can be easily controlled. Two types of lasers are used: continuous emission, and pulsed. For the continuous emission type, a pulsing mechanism needs to be added. Therefore, the mechanical complexity of continuous emission laser systems for PIV is a disadvantage. Another disadvantage is their low energy emission which means only larger particles will scatter light and thus continuous emission lasers are usually limited to low speed water flows. Pulsed lasers are the more common type because of their higher energies and the ease of timing. Pulsed laser can be pulsed to almost any time interval, usually limited by the speed of the timing electronics. A simple way to create the pulsed laser is using two laser heads integrated in a single housing. The two laser heads can be pulsed with almost any time delay and the optics inside the housing combine the two lasers into a single beam. A common PIV laser system is a single housing, two-head system with frequency-doubled neodymium-doped, yttrium aluminum garnet (Nd:YAG) crystals. Nd:YAG solid state lasers naturally emit light at 1064 nm wavelength, which is in the UV spectrum. For PIV applications, the laser beam is passed through a frequency-doubling crystal, called a second harmonic generator, that outputs a laser beam at 532 nm wavelength. The timing of the laser pulses can be accurately controlled by independently adjusting the flash-pump trigger and the Q-switch of each laser head. The laser beam is formed into a sheet by means of a plano-concave cylindrical lens followed by a spherical lens. The height of the beam is set by the cylindrical lens while the thickness is set by the spherical lens. For PIV applications, a 1 mm light sheet thickness is commonly used.

2.1.2 Camera and Lens

Charged coupled device (CCD) cameras are the most commonly used though complementary metal oxide semiconductor (CMOS) devices have been in recent use for high speed flows. A CCD camera consists of an electronic sensor that contains thousands of pixel elements. The signal that each pixel outputs is proportional to the light intensity that the pixel is exposed to. The main advantage of CCD cameras is that the pixel intensities can be easily converted to digital data and analyzed by a computer. High spatial resolutions and short inter-exposure times on the order of 10 $\mu$s can be achieved using CCD cameras.

In single frame imaging, the particle image pairs are burned in the same frame and the analysis is done on a single frame per image pair. The most common imaging method is double-frame where the image pairs are taken in two separate frames. The analysis then consists of cross correlating the two frames per image pair.

Figure 2.2 shows the imaging configuration in a PIV system. The distances $Z_0$ and $z_0$ are known and thus the magnification $M = z_0/Z_0$ is known. The camera records a particle displacement $\vec{R}$ in the image plane, then the particle displacement $\vec{r}$ in the object plane is $\vec{r} = \vec{R}/M$. Another consideration is that the camera lens should be set to a small depth of field $\Delta$, close to the thickness of the laser sheet, $\Delta Z_0$, to adequately focus on the laser sheet. This can be achieved by using a short distance to the object plane, a wide aperture, and a long focal length. The depth of field can be calculated from the lens F number, $F$, and the laser wavelength, $\lambda$, using Equation 2.1.

$$\Delta = 4\lambda \left[ \left( 1 + \frac{1}{M} \right) (F) \right]^2$$  \hspace{1cm} (2.1)

2.1.3 Seeding Particles

For seeding particles, there are two opposing requirements: the particles should be large enough to scatter the laser light but should also be small and light enough to faithfully follow the flow. For gaseous flows these requirements mean that the particle diameter should be about 0.1 – 10 $\mu$m. The size of the particles must be as uniform as possible to prevent any bias towards larger particles that scatter more light.
Commonly used seeding materials for gaseous flows are given in Table 2.1. For a more thorough reference on seeding techniques, see Melling (1997).

### 2.1.4 Timing Electronics

The image acquisition sequence begins with a trigger. The trigger can be an internal electronically-generated trigger or the trigger can be set to coincide with some external event. The trigger sets off a sequence of events beginning with any preset time delay, which may or may not be zero, to take the image. This first user-defined time delay is useful when imaging periodic flow structures because it allows the imaging of any phase position in the period. The timing electronics send a signal to fire the first laser shot and the camera takes an image. A second time delay is defined as the time between the first and second laser shots. When the second laser shot happens, a second image is taken which may be burned in the same frame as
the first image (single-frame imaging) or may be burned in a second frame (double-frame imaging). This completes the data acquisition necessary to produce a single velocity vector field.

### 2.1.5 Advanced PIV Techniques

Thus far, only basic two-dimensional, two-component (2D2C) PIV has been discussed. Stereoscopic PIV, also known as 2D3C PIV, is a more recent technique whereby the out-of-plane velocity component of the flow can be calculated using two cameras that image the same location in the flow. Due to the stereoscopic effect, the out-of-plane component can be recovered from the two image pairs (Figure 2.3).

Time resolved (TR) PIV is used to study the temporal evolution of a flow field. This is achieved by using CMOS cameras with frame rates varying from 1 kHz to 30 kHz and laser systems with repetition rates of 1 kHz to 20 kHz. Another technique is volumetric PIV, or 3D3C PIV, where a volume of flow is illuminated and three or more cameras focused on the volume from different angles record the seed passing through the volume. All three velocity components in the volume can be recovered. This technique is specially useful for unsteady three-dimensional flows like vortices and wakes. Volumetric PIV consists of a family of measurement techniques such as holographic PIV and tomographic PIV.

### 2.1.6 Previous Research Using PIV

By the early 1990s, PIV had evolved to a proven whole field quantitative visualization technique. Early research by Willert et al. (1996) used a high speed dual-sensor CCD video camera to study a free jet exhausting from a nozzle. The cross correlation parameters and methodology was also explained. In the same publication, Willert et al. (1996) studied helicopter tip vortices with a different camera system. Other geometries presented in the publication are the wake of a lifting wing and a turbulent boundary layer. The results presented were the velocity fields of the geometries studied. No further analysis or physical significance was studied. Rather, the publication serves to showcase some of the earlier applications of PIV. Willert (1997) presented a detailed study on the future application of stereoscopic PIV to wind tunnel
flows. Emphasis was placed on the stereoscopic PIV methods like image dewarping, calibration, image pre-
processing, and analysis software. The methods presented were applied to a vortex ring. Another excellent
source of early PIV applications in wind tunnel flows is Kompenhans et al. (2000).

2.1.7 Film Cooling Research Using PIV

Peterson & Plesniak (2002) used PIV to study a jet in crossflow (JICF) geometry for film cooling
applications. PIV was used to study the internal velocity fields within the injection holes. A double-pulsed
laser and laser optics were used to generate measurement planes in the spanwise direction at different depths
along the injection holes. The results showed that the jets had strong in-hole vortices that enhanced the
counter rotating vortex pairs (CRVPs) outside the holes. The CRVPs caused the jets to lift off resulting
in lower film effectiveness. A counter flow plenum which inhibits the strength of the CRVPs was found to
result in better downstream film coverage. A more recent film cooling jet study using PIV was performed
by Bernsdorf et al. (2006). A single row of seven film cooling holes was studied using stereoscopic PIV
for flow field characterization and heat flux gauges for surface heat flux measurements. Figure 2.4 shows
the test section geometry. Figure 2.5 shows the PIV setup. The two cameras for stereoscopic PIV can be
clearly seen along with a 45° mirror to focus the sheet on the surface of the jets. The study showed that
at higher $BRs$, the jet tended to lift off. The CRVP was also well documented using the PIV technique.
The vorticity field clearly shows the CRVP which is expected to affect the film cooling effectiveness. Note
that no heat transfer results are presented. An interesting study by Wright et al. (2011) also uses PIV to
study the effects of freestream turbulence on film cooling structure. Jessen et al. (2011) also conducted a
study using 2D2C and 2D3C PIV to study reverse pressure gradient effects on a row of film cooling holes.
More recently, Eberly & Thole (2013) provided 2D2C time-averaged and time-resolved measurements on a
row of five film cooling holes at different blowing ratios (0.25 to 2.00) and two different density ratios ($DR$
(1.2 and 1.6). They also performed adiabatic effectiveness measurements using infrared thermography. The
time-averaged measurements showed that the only case were the jets remained attached was at $BR = 0.6$.
Even at $BR = 1.0$, the jet lifted off the surface. The time-resolved measurements captured the formation of
the shear layer vortices. Eberly & Thole (2013) identified the roll-up of the shear layer as the source of the
shear layer vortices. Another recent study in film cooling jets was performed by Johnson et al. (2014). In
this study, PIV was used in conjunction with Pressure Sensitive Paint (PSP) technique to characterize both
the flow field and the heat transfer characteristics of a row of circular jets in crossflow. This study confirms
the general result that at low blowing ratios, the jets remain attached to the surface and that higher blowing
ratios led to jet lift-off. The study also noted that for the same blowing ratio, film cooling flows with higher
density ratios remained closer to the wall than those with lower density ratios. The PIV measurements were
especially useful in relating the heat transfer measurements to the behavior of the flow as measured by the
PIV system.
2.1.8 PIV Data Processing

Assuming a double frame acquisition method, two images are necessary to calculate a vector field. Each image is broken into a grid. Each cell is called an interrogation spot that will produce one velocity vector. The interrogation spot should be small enough so that the velocity gradients in the spots are negligible. However, each interrogation spot should contain several particles, usually on the order of 10-25 particles per interrogation spot to yield an accurate velocity vector. To prevent overlapping of the same particles on image 1 and image 2, the minimum displacement should be at least two particle diameters. These rules serve as guidelines for choosing an adequate interrogation area.

2.1.8.1 Pre-Processing

For a given interrogation spot, the two images will give the position of the particles at two instants in time. The images are pre-processed by stretching the intensity of the brightest particles to the maximum intensity available so that they show as pure white. Similarly, the background is stretched to the minimum intensity possible so that it shows as pure black. If several snapshots of the same experiment are taken, then a mean intensity for each pixel can be calculated and subtracted from each image to yield an image with as less noise as possible.
2.1.8.2 Cross Correlation

The next step is to determine the position of the particles. If a particle is captured by four pixels, the intensities of the four pixels are used to calculate the center of the particle to within a fraction of a pixel. This procedure becomes less accurate if the particle is only seen by two or three pixels. The positions of the particles are recorded and a matrix of the distances between each particle is created. For instance, if there are 15 particles in an interrogation spot, then the positions of each particle in both images is recorded. Then the distance from each particle in image 1 to each particle in image 2 is calculated and the matrix is created. From the matrix, a histogram of particle displacements is calculated. The displacement that occurs the most is the actual displacement. The process by which the actual displacement of the particles in the interrogation spot is calculated is called the cross-correlation (Equation 2.2).

\[ C(s) = \int_{I_A} I_1(\vec{x}) I_2(\vec{x} + \vec{s}) \, dx \]  \hspace{1cm} (2.2)

The image intensity at a particular position \( \vec{x} \) is \( I_i(\vec{x}) \). Subscript 1 refers to the first image and subscript 2 refers to the second image. The vector \( \vec{s} \) is the displacement of the second image relative to the first. The integral is taken over the interrogation area, \( I_A \). The highest peak in \( C(s) \) is the displacement peak. This displacement along with appropriate optical parameters and the time between images yields the velocity vector of the interrogation spot. This procedure is performed by the analysis software and is repeated for all interrogation spots in the images to yield the velocity vector field. If several image pairs are taken, the results can be ensemble-averaged to compute mean and RMS velocities. Further parameters like vorticity and shear stresses can be calculated using either the instantaneous velocity field or the averaged velocity field.

**Calculation of the Cross Correlation**

If a CCD or CMOS camera is used, the images are stored digitally as pixels with pixel values proportional to the light intensity recorded by the pixel. Thus the images are defined discretely by the pixel values. The appropriate spatial cross correlation is thus a sum rather than an integral. Let image 1 and image 2 be denoted by \( I' \) and \( I'' \), respectively. Both images are defined by \( N_x \times N_y \) pixels with position indices \( i \) and \( j \), respectively. The displacement of the second image with respect to the first image is \( r \) and \( s \), in the \( x \) and \( y \) directions, respectively. The local mean image intensity is \( \bar{I} \). Then, the discrete cross correlation is given by Equation 2.3 (Westerweel, 1997).

\[ \hat{R} \left[ r, s \right] = \frac{1}{N_x N_y} \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \left( I' \left[ i, j \right] - \bar{I} \right) \left( I'' \left[ i + r, j + s \right] - \bar{I} \right) \]  \hspace{1cm} (2.3)

The discretized cross correlation can be computed directly or through the use of a discrete Fourier transform (DFT) or the Fourier Fast Transform (FFT) algorithms.

**Direct Calculation**

Dabiri (2014) described two methods to obtain a displacement measurement. One method is to divide the Equation 2.3 by an in-plane loss-of-pair term, \( F_l \) (Equation 2.4), to get an unbiased displacement measurement.

\[ F_l \left[ r, s \right] = \left( 1 - \frac{|r|}{N_x} \right) \left( 1 - \frac{|s|}{N_y} \right) \]  \hspace{1cm} (2.4)

The second method is to represent Equation 2.3 as Equation 2.5. Equations 2.7-2.8 are the normalization factors. The term \( \bar{I} \left[ r, s \right] \) represents the average of \( I' \) that is coincident with \( I' \) (Dabiri, 2014).

\[ \hat{\bar{R}} \left[ r, s \right] = \frac{R_{ll} \left[ r, s \right]}{\sigma_l \left[ r, s \right] \sqrt{\sigma_{ll} \left[ r, s \right]}} \]  \hspace{1cm} (2.5)
\[ R_{ll}[r,s] = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \left[ I'[i,j] - \bar{I} \right] \left[ I'[i+r,j+s] - \bar{I} \right] \]  
\[ \sigma_l[r,s] = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} (I'[i,j] - \bar{I})^2 \]  
\[ \sigma_{ll}[r,s] = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} (I'[i+r,j+s] - \bar{I})^2 \]

**Fourier Transform Method**

The main advantage of the Fourier transform is that the computation is reduced to a \( \sim N^2 \log_2 (N) \) for \( N \) pixels as opposed to \( \sim N^4 \) for the direct method. Using the correlation theorem, a cross correlation in the temporal domain is equal to the complex conjugate multiplication of the Fourier transform of the functions in the frequency domain. For PIV data, the cross correlation in the frequency domain, \( \Theta \), becomes Equation 2.9. The Fourier transform is denoted by \( F_o \) and \( .^* \) denotes a complex conjugate. The cross correlation in the temporal domain, \( R \), is found by applying an inverse Fourier transform, \( F_o^{-1} \), on \( \Theta \) (Equation 2.10). It has been argued that the use of windowing functions to deal with the non-periodicity of the image data is unnecessary in digital PIV analysis because the windowing functions introduce systematic errors and decrease the signal to noise ratio of the cross correlation. Furthermore, as long as the particle displace less than \( 1/4 \) of the interrogation area, zero-padding of the image data is unnecessary and the results of a FFT-based cross correlation and a direct cross correlation will be almost identical (McKenna & McGills, 2002).

\[ \Theta = F_o(I') F_o^* (I') \]  
\[ R = F_o^{-1} (\Theta) \]

**2.1.8.3 Autocorrelation**

The auto correlation is used in single-frame imaging where the two images are burned in the same frame. The image pre-processing is the same as for double-frame imaging. Once the particles are located, the distance from each particle to all particles (including itself) in an interrogation spot is calculated and written in a matrix. The process by which the actual displacement of the particles in the interrogation spot is calculated is the autocorrelation function. The displacement that occurs the most is always the self-correlation corresponding to a displacement of \((0,0)\). This generates the highest peak in the autocorrelation, the self-correlation peak. The next two highest peaks are symmetric about the self-correlation peak. These two peaks are displacement peaks. The displacement ambiguity is eliminated by shifting the second image with respect to the first image. Due to the added complexity necessary, the double-frame imaging is preferred.

**2.1.8.4 Post Processing**

Some issues arise with PIV data processing. For instance, some of the particles that appear in image 1 might have moved out of the interrogation spot in image 2. These are called lost pairs. A solution to this problem is the overlapping of interrogation spots to minimize the number of lost pairs. Another important effect is image noise which is defined by the height of the background peaks in the cross-correlation. Applying a minimum ratio of displacement peak magnitude to noise peak RMS magnitude sets a minimum threshold on the viable peaks. In areas of high velocity gradients, the shape of the interrogation spots can be changed automatically by the analysis software to obtain more accurate results. Another validation scheme is neighborhood validation. The vector given by an IA is checked against its neighbor vectors. If the vectors differ by more than a preset percentage, the vector is discarded. An important consequence of discretizing the cross correlation is the fact that the cross correlation will only exist at integer values of position. In order to accurately determine the location of the displacement peak, curve-fitting algorithms are implemented to
obtain sub-pixel accuracy. The displacement peak is curve-fitted to its two next highest peaks in both $x$ and $y$ directions to obtain the location of the displacement peak with sub-pixel accuracy. Common curve-fitting algorithms are parabolic and Gaussian algorithms. For further information on the data processing for PIV, see Westerweel (1997) and Dantec Dynamics A/S (2013).

### 2.1.9 Errors in PIV

Due to the complexity of PIV experiments there are many sources of error. Among these sources are: nonuniform seed concentrations, nonuniform seed size and mass, in and out of plane lost pairs, sub-pixel peak fitting, and noise. The information below was compiled from Dabiri (2014) and Huang et al. (1997).

For sub-pixel fitting the main sources of error are the errors introduced by the algorithm used and errors due to larger than normal particle sizes. The Gaussian interpolation scheme, for instance, has a maximum 3% interpolation error for a pixel shift of 0.3 pixels. The effects of particle size can be quantified as an RMS uncertainty in the interrogation spot displacement. This RMS uncertainty depends on the size of the interrogation area with the error minimized by larger IAs since more particles are used to calculate the cross correlation. With a $16 \times 16$ pixel IA, the optimum particle diameter is 2.0 pixels which corresponds to an RMS uncertainty of $\sim 0.04$ pixels.

The peak-locking effect happens when the particle size is too small to accurately determine the location of the displacement peak. This error is reduced by using a Gaussian sub-pixel interpolation and is a minimum for a particle diameter of 2.0 pixels. Smaller particle diameters lead to a higher bias error in the location of displacement peak.

The errors due to seed density are quantified by the probability of a valid vector output from the PIV analysis. For an effective particle density (number of particles in an IA after eliminating the particles that have move out the IA) of 10 particles and a probability of detecting at least 5 particles, valid vectors are produced for 95% of cases.

The quantization of the pixel intensities can also introduce significant errors. The optimal level was found to be an 8-bit (256 levels) quantization of the pixel intensities. Reducing this number to 4 bits increases the error a maximum of 3.5 times the error achieved with 8 bits.

The effects of noise in the pixel intensities are significant only for particle displacements less than 0.4 pixels. For instance, if the particle displacement is greater than 0.4 pixels, noise levels of 5% and 10% yield negligible values of RMS uncertainty compared to 0% noise levels.

Velocity gradients also contribute significant error if the IAs are too large. Since the CCD pixel grid is square, the cross correlation does not take into account higher order terms that lead to rotation and deformation. Minimizing the IA mitigates the effects of velocity gradients. Another factor, albeit with a smaller impact, is the seed density. A higher density leads to a lower RMS uncertainty than a lower density.

As seen from this short review on PIV errors, the complex nature of PIV measurements introduces many sources of error. There have been studies on how certain parameters introduce errors into determining the location of the displacement peak. Even though some of these parameters can yield significant error, such errors can be minimized with a careful design of the experiment and thoughtful selection of the PIV analysis parameters. It is claimed that, ideally, the accuracy of the PIV analysis is 1/10 of pixel for the particle displacement (Jenssen, 2014).

### 2.2 Image Patching

The objective of this technique is to combine the individual PIV spatial domains into a single, large spatial domain for visualization purposes. The technique and algorithm were developed by Upadhyay (2015). Suppose the individual PIV domains have a known amount of overlap between them. The raw acquisition images are exported to a standard image format such as .tiff.

Each domain has known dimensions in pixels and the overlap between them is known in units of length. Through the calibration, the overlap is known in terms of pixels. The image patching technique allows the combination of two or more overlapping domains into a single average field spanning the overall size of the area covered by the domains. The details of the technique are the work of Upadhyay (2015) who provided
the basic algorithms and training in the technique. Due to a verbal agreement, details of the technique and algorithm cannot be reproduced in this publication.

2.3 Proper Orthogonal Decomposition

Proper Orthogonal Decomposition (POD) is a procedure for extracting a set of modes that best represent the flow field. In other fields, it is called the Karhunen-Loève decomposition or principal component analysis. In the context of fluid dynamics it was introduced by Lumley (1967). POD can be described using the definitions and derivations of Berkooz et al. (1993), Holmes et al. (2012), and Cazemier (1997).

Suppose there is a scalar field \( u(x) \), \( x \in \Omega \). \( \Omega \) may be the length of the flow or the computational domain. It belongs to the Hilbert space of square integrable complex-valued functions \( L^2(\Omega) \). The goal is to identify the functions that form an orthonormal basis that yield the most similar representation of the members of the ensemble \( u \). Mathematically, the most similar representation consists of maximizing the projection of the functions \( \psi \) on \( u \). In the \( L^2 \) Hilbert space, the projection of \( \psi \) on \( u \) is given by Equation 2.11.

\[
\max_{\psi} \left\langle \frac{|(u, \psi)|^2}{(\psi, \psi)} \rightangle = \left\langle \frac{|(u, \phi)|^2}{(\phi, \phi)} \right\rangle
\]  
(2.11)

The averaging operation (temporal, spatial, ensemble, or phase), the modulus, the inner product, and the complex conjugate transpose are denoted by \( \langle , \rangle \), \( | \| \), \( (,.) \), and \( ^* \), respectively. The projection is also subjected to the constrain that the inner product \( (\psi, \psi) = 1 \). In order to find the optimal orthogonal basis \( \psi = \phi \) that maximizes the inner product with \( u, \phi \) must be an eigenfunction of the two-point correlation tensor given in Equation 2.12.

\[
\int_{\Omega} \langle u(x) \otimes u^*(x') \rangle \phi(x') \, dx' = \int_{\Omega} R(x, x') \phi(x') \, dx' = \lambda \phi(x')
\]  
(2.12)

Note that the autocorrelation of \( u \) is defined as \( R(x, x') = \langle u(x) \otimes u^*(x') \rangle \) where \( \otimes \) denotes the tensor product. Spectral theory guarantees that the maximum of Equation 2.12 corresponds to the largest eigenvalue \( \lambda_1 \). The empirical eigenfunctions are given by \( \phi_k(x) \). It is also guaranteed that \( \lambda \geq 0 \) because the two-point correlation tensor is positive definite. After solving the eigenvalue problem, the eigenvalues are ordered in descending order. The field \( u(x) \) can then be reproduced by a modal decomposition in the eigenfunctions (Equation 2.13) where \( a_k \) are called the POD coefficients (Equation 2.14). Since the POD coefficients \( a_k \) are calculated based on the inner product of \( u_k \) and \( \phi_k \), by definition, \( a_k \) is the projection of \( u_k \) on \( \phi_k \). The autocorrelation \( R(x, x') \) guarantees that the POD coefficients are uncorrelated and thus that each eigenvalue is larger than the corresponding eigenvalue in any other arbitrary orthonormal decomposition with modal coefficients \( b_k \) (Equations 2.15-2.16).

\[
u(x) = \sum_{k=1}^{\infty} a_k \phi_k(x) \]
(2.13)

\[
a_k = \langle u_k(x), \phi_k(x) \rangle \]
(2.14)

\[
\langle a_k a^*_k \rangle = \langle \psi_k, \psi_k' \rangle \lambda_k \]
(2.15)

\[
\sum_{k=1}^{n} \langle a_k a^*_k \rangle = \sum_{k=1}^{n} \lambda_k \geq \sum_{k=1}^{n} \langle b_k b^*_k \rangle \]
(2.16)

2.3.1 Direct Method

The direct method to solve the eigenvalue problem in Equation 2.12 was first introduced by Lumley (1970). If \( u_k \) is an \( N_x \)-dimensional matrix, then the function \( R(x, x') = \langle u(x) \otimes u^*(x') \rangle \) yields an \( N_x \times N_x \)
eigenvalue problem. The matrix \( u_k \) has \( N_T \) realizations such that the averaging operation \( \langle u_k \rangle \) over \( N_T \) realizations can be estimated by an arithmetic average. The remaining integral can be approximated by Simpson’s rule. The two-point correlation tensor (Equation 2.12) then becomes Equation 2.17 where \( \omega_i \) are the weight functions for the quadrature method used. Multiplying Equation 2.17 by \( \sqrt{\omega_i} \) yields Equation 2.18 where \( C, \hat{u}_k \) and \( \hat{\phi} \) are given by Equations 2.19-2.21. Equation 2.18 represents the symmetric \( N_x \times N_x \) eigenvalue problem. The components of all eigenvectors must be multiplied by \( 1/\sqrt{\omega_i} \) to obtain the eigenfunctions which can then be normalized to obtain the POD modes.

\[
\frac{1}{N_T} \sum_{k=1}^{N_T} \left[ u_k \left( x \right) \sum_{i=1}^{N_x} u_k^* \left( x_i \right) \phi \left( x_i \right) \omega_i \right] = \lambda \phi \left( x \right) \tag{2.17}
\]

\[
C \hat{\phi} = \lambda \hat{\phi} \tag{2.18}
\]

\[
C = \frac{1}{N_T} \sum_{k=1}^{N_T} \hat{u}_k \hat{u}_k^* \tag{2.19}
\]

\[
\hat{u}_k = \begin{bmatrix}
\sqrt{\omega_1} u_k \left( x_1 \right) \\
\sqrt{\omega_{i+1}} u_k \left( x_{i+1} \right) \\
\vdots \\
\sqrt{\omega_{N_x}} u_k \left( x_{N_x} \right)
\end{bmatrix} \tag{2.20}
\]

\[
\hat{\phi} = \begin{bmatrix}
\sqrt{\omega_1} \phi \left( x_1 \right) \\
\sqrt{\omega_{i+1}} \phi \left( x_{i+1} \right) \\
\vdots \\
\sqrt{\omega_{N_x}} \phi \left( x_{N_x} \right)
\end{bmatrix} \tag{2.21}
\]

### 2.3.2 Snapshot Method

For two-dimensional flow fields, \( N_x \approx 10^4 \), which yields a problem size of \( \sim 10^8 \). This is acceptable for computation purposes. With three-dimensional flow fields the problem size is \( \sim 10^{13} \) and requires a different formulations of the eigenvalue problem (Bidan, 2013). An alternative lower order problem called the snapshot method was proposed by Sirovich (1987). The following explanation is derived from Meyer et al. (2007), Smith et al. (2005), and Meyer et al. (2008). The snapshot method starts by defining an \( N_T \)-dimensional matrix \( c_k \) (Equation 2.22). Equation 2.12 is multiplied by \( u_k^* \left( x \right) \) and integrated over \( \Omega \) to yield Equation 2.23.

\[
c_k = \int_{\Omega} u_k^* \left( x' \right) \phi \left( x' \right) dx' \tag{2.22}
\]

\[
\frac{1}{N_T} \sum_{k=1}^{N_T} c_k \int_{\Omega} u_k^* \left( x \right) u_k \left( x \right) dx = \lambda \int_{\Omega} u_k^* \left( x \right) \phi \left( x \right) dx \tag{2.23}
\]

The eigenvalue problem has a size of \( N_T \times N_T \). This problem is easily solved by realizing that \( E_{ik} = \int_{\Omega} u_k^* \left( x \right) u_k \left( x \right) dx \) is the autocovariance matrix of \( u \). Thus \( E = u^T u \) and Equation 2.23 can be rewritten in matrix notation as Equation 2.24.

\[
\begin{bmatrix}
E_{1,1} & \cdots & E_{1,N_T} \\
\vdots & \ddots & \vdots \\
E_{N_T,1} & \cdots & E_{N_T,N_T}
\end{bmatrix}
\begin{bmatrix}
c_1 \\
\vdots \\
c_{N_T}
\end{bmatrix}
= \lambda
\begin{bmatrix}
c_1 \\
\vdots \\
c_{N_T}
\end{bmatrix}
\rightarrow
Ec = \lambda c \tag{2.24}
\]
From the solution of the eigenvalue problem, the eigenfunctions are given by Equation 2.25. Since each member of \( u_k(x) = [u_1^k \cdots u_{N_x}^k]^T \) is specified at \( N_x \) points, the discretized eigenfunctions are given by Equation 2.26.

\[
\phi^n(x) = \frac{1}{\lambda^N T} \left[ e_1^n \cdots e_{N_x}^n \right] \begin{bmatrix} u_1(x) \\ \vdots \\ u_{N_x}(x) \end{bmatrix}
\]

\[
\begin{bmatrix} \phi_1^1 & \cdots & \phi_N^1 \\ \vdots & \ddots & \vdots \\ \phi_1^N & \cdots & \phi_N^N \end{bmatrix} = \frac{1}{N_T} \left[ \frac{e_1^1}{\lambda_1} \cdots \frac{e_{N_x}^1}{\lambda_1} \right] \begin{bmatrix} u_1^1 & \cdots & u_{N_x}^1 \\ \vdots & \ddots & \vdots \\ u_1^{N_T} & \cdots & u_{N_x}^{N_T} \end{bmatrix}
\]

The eigenvalues and the corresponding eigenfunctions are then arranged in descending order. The eigenfunctions form an orthogonal basis of \( u \). To obtain the orthonormal basis, the eigenfunctions are normalized by the \( L^2 \) norm (denoted by \( \| \cdot \| \) ) which is equal to the square root of the inner product. The matrix \( \Phi \) contains all normalized eigenfunctions which are hereafter called the POD modes (Equation 2.27). The POD coefficients in Equation 2.13 are then given by projecting \( u \) onto the POD modes (Equation 2.28). Since the eigenvalues are arranged in descending order, the POD modes are automatically ranked in descending order of contribution to the projection of \( u \). Knowledge of the POD modes and POD coefficients allows one to reconstruct all members (snapshots) of \( u \) (Equation 2.29). Any number of POD modes with corresponding coefficients can be used to reconstruct the members. The evolution of the members can thus be studied as more POD modes are used in the reconstruction. The same derivation can be used for scalars and vectors defined in one, two or three spatial dimensions.

\[
\Phi = \begin{bmatrix} \phi_1^1 & \cdots & \phi_N^1 \\ \|\phi_1^1\| & \cdots & \|\phi_N^1\| \\ \vdots & \ddots & \vdots \\ \phi_1^N & \cdots & \phi_N^N \\ \|\phi_1^N\| & \cdots & \|\phi_N^N\| \end{bmatrix}
\]

\[
a = \Phi^T \begin{bmatrix} u_1^1 & \cdots & u_{N_x}^1 \\ \vdots & \ddots & \vdots \\ u_1^{N_T} & \cdots & u_{N_x}^{N_T} \end{bmatrix}
\]

\[
u = \Phi a
\]

2.3.3 Advantages and Disadvantages

The study of turbulent fluid flows is an enormous challenge because of the lack of general solutions to the Navier-Stokes equations. For fully-developed turbulent flows, there exists a way to simplify the problem by the study of organized spatial features called coherent structures. These structures undergo a temporal life cycle. The proper orthogonal decomposition is a way to extract the coherent structures in turbulent flows. Its strength lies in the fact that it is a statistically-based method that nevertheless provides the analytical understanding of the answers it provides. Experimental or Computational Fluid Dynamics (CFD) data is used to obtain the coherent structures of the flow with the added advantage of a clear understanding of what the results mean due to its analytical foundations. The results can also lead a deterministic reduced order model (ROM) of the turbulent flow (Berkooz et al., 1993).

From the mathematical point of view, one of the principal advantages of POD is that it forms an empirical basis that spans the smallest possible linear subspace necessary to characterize the function studied. Since the POD is a linear operation, then the empirical basis inherits all linear properties of the function. For instance, if the flow is incompressible, then the empirical eigenfunctions will also be incompressible. Another principal advantage is the fact that the empirical basis is able to reconstruct the
members of the ensemble it was calculated from. This can be said of any modal basis such as Fourier series but the truly powerful advantage of the POD is that the POD is in fact the optimal empirical basis for reconstruction of ensemble members. This follows from Equation 2.16, which states that the eigenvalues of the POD basis are of greater magnitude than the corresponding eigenvalues of any other arbitrary orthonormal basis. Furthermore, Equation 2.15, effectively states that the POD coefficients are uncorrelated. Since it is the optimal orthonormal basis, convergence of the square of the function is achieved with a smaller number of modes than any other orthonormal basis. This makes the POD basis preferable for reduced order modeling (Berkooz et al., 1993).

In turbulent flows, the function analyzed is the velocity fluctuations, \( u'_i \). By Equation 2.28, the POD coefficients have dimensions of velocity. By Equation 2.15, it is seen that the eigenvalues are proportional to the square of the POD coefficients. Thus, for turbulent flows, the POD eigenvalues are proportional to \( u'^2_i \). This is equivalent to stating that the POD eigenvalues are proportional to the turbulent kinetic energy per unit mass of the flow. The turbulent kinetic energy associated with a given POD mode is proportional to the corresponding eigenvalue. The total turbulent kinetic energy in the flow is given in Equation 2.30. The turbulent kinetic energy associated with the \( n^{th} \) POD mode is given in Equation 2.31 (Gurka et al., 2006).

\[
k = \sum_{i=1}^{N_T} \lambda^i
\]
\[
k^n = \frac{\lambda^n}{k}
\]

One of the disadvantages of the POD is that it can overestimate the dimensionality of some nonlinear data sets. If a nonlinear data set can be represented parametrically with one variable, then the POD represents the data set in a linear space of higher dimension. Another disadvantage is the fact that POD describes the data set using global measures. Thus for large domains with varying characteristics, POD can overlook some of the local features present in only a subset of the domain (Kerschen et al., 2005). A useful model to combine local POD analyses to form a POD of the larger domain is needed to adequately represent both local and global coherent structures. Another disadvantage is that not all fluid dynamics will be represented by the POD analysis. Some dynamics are predominantly represented by lower energy POD modes which may be truncated if they are not identified earlier as important in the dynamical representation of the flow. POD favors the modes with highest energy and thus may inadequately represent the dynamic behavior of certain flows (Rowley, 2005).

A further disadvantage is that the less deterministic the flow is, the greater the number of modes is necessary to reconstruct the flow. Thus a more turbulent flow will need hundreds of modes to reconstruct most of the turbulent kinetic energy. Random events, such as those occurring in turbulent boundary layers are also less likely to be captured in the POD modes. Another significant limitation is the fact that only time-averaged information about the flow is deduced from the POD (Kostas et al., 2005).

### 2.3.4 Previous Research

Andrianne et al. (2009) performed POD post-processing to PIV data from a cylinder wake and separated flow from an airfoil. This study is of particular interest as it gives a detailed interpretation of the PIV data and the POD modes extracted from the data. Hanfeng (2012) studied the wake of a square cylinder using both PIV and POD post-processing. Gurka et al. (2006) performed a POD analysis of vorticity data to identify large scale coherent structures. If the input function to the POD analysis is a vorticity vector, then the eigenvalues are proportional to the enstrophy associated with a given POD mode. If only 2D2C PIV data is available, then the vorticity will be in the out-of-plane direction and thus the eigenvalues will only represent the enstrophy in that direction or “quasi-enstrophy.” Gurka et al. (2006) cite a few experimental and computational studies that the identification of coherent structures is more accurate with the POD modes of vorticity fields. The main reason to chose vorticity over velocity as the POD function is the fact that vorticity is a Galilean invariant variable. If there are streamwise velocity fluctuations, the coherent structures may be smeared because the POD modes are capturing the phenomenon that is not part of the coherent structures. Galilean invariant quantities are not susceptible to this effect. To compare the effects of the choice of variable, Gurka et al. (2006) performed a PIV study on a turbulent boundary layer with subsequent POD analysis on
the fluctuating velocity and the fluctuating spanwise vorticity. To compare the results of the velocity POD to the vorticity POD, the curl of the velocity POD modes was calculated and compared to the vorticity POD modes. The direct calculation of vorticity POD modes was identified as the preferred method to identify coherent structures in the boundary layer because they provided a clearer pattern with less smeared boundaries than either velocity POD modes or derived vorticity POD modes. Kostas et al. (2005) supports the claim that vorticity should be the variable of choice in the POD analysis. Furthermore, if the POD modes form the basis of a reduced order model (ROM), then the vorticity equation (Equation 2.32, \( \nu \) is the fluid kinematic viscosity) should be used for the Galerkin projection. The problem with using the vorticity equation for ROMs is that requires knowledge of both the velocity and vorticity POD modes. The ideal case is to determine both the POD velocity basis and the POD vorticity basis. If this is not possible, calculating the curl of the velocity POD modes is computationally easier than the calculating the velocity POD basis from the POD vorticity basis. If 2D2C PIV data is used, the vorticity POD basis is less computationally intensive since only one scalar field is used in the POD analysis as opposed to a 2D vector field. Even though 3D data is ideal in the identification of coherent structures, a quasi-entrainment identification is preferred over a 2D2C velocity POD basis. This was the case for their study on a backward facing step flow. As mentioned earlier, the same superiority of vorticity POD basis was found in the identification of coherent structures in turbulent boundary layers.

\[
\frac{\partial \omega_i}{\partial t} + u_j \omega_{i,j} = \omega_j u_{i,j} + \nu \omega_{i,j} \tag{2.32}
\]

### 2.4 Dynamic Mode Decomposition

Dynamic Mode Decomposition (DMD) is a modal decomposition used to extract dynamical and stability information of the coherent structures in a flow. It was introduced by Schmid (2010). DMD can be described using the definitions and derivations published by Schmid et al. (2011), Schmid (2012), Tissot et al. (2014), Tu et al. (2014b), and Jovanović et al. (2014). In the present discussion, the definitions and derivations will be developed using a single, common notation.

Suppose there is an ensemble of \( N \) snapshots represented in a matrix \( V \) (Equation 2.33). The snapshots are separated by a constant \( \Delta t \) and each snapshot is defined at \( N_g \) grid points. The goal is to find an operator \( A \) that produces a linear mapping between the snapshots (Equation 2.34). Thus the snapshot sequence can be represented as a Krylov sequence (Equation 2.35).

\[
V = \{V_1, V_2, V_3, ..., V_N\} \tag{2.33}
\]

\[
V_{i+1} = AV_i \quad i = 1, ..., N - 1 \tag{2.34}
\]

\[
V = \{V_1, AV_1, A^2V_1, A^3V_1, ..., A^{N-1}V_1\} \tag{2.35}
\]

It is assumed that the snapshot sequence becomes linearly dependent as the number of snapshots increases. Adding further snapshots will not significantly contribute to determination of the coherent structures in the flow. If \( N - 1 \) snapshots are assumed to be linearly independent, then the \( N^{th} \) snapshot can be expressed as a linear combination of the other snapshots (Equations 2.36-2.37).

\[
V_N = c_1 V_1 + c_2 V_2 + c_3 V_3 + \cdots + c_{N-1} V_{N-1} + r \tag{2.36}
\]

\[
V_N = V_1^{N-1} \mathbf{c} + r \tag{2.37}
\]

The residual vector is \( r \) and the vector of linear combination coefficients is \( \mathbf{c} \). The linear mapping for all the snapshots is thus given by Equation 2.38. In matrix notation, the linear mapping is given by Equation 2.39 where \( I_{N-1}^T \) is a \( N - 1 \) unit vector. The matrix \( \mathbf{S} \) is called the companion matrix. Since the \( i^{th} \) element in \( V_1^{N-1} \) is identical to the \( (i - 1)^{th} \) element in \( V_2^N \), all the columns in \( \mathbf{S} \), except for the last one, have 0 or 1 elements. The last column is the vector \( \mathbf{c} \). If the residual \( r \) is small, then the companion matrix closely approximates the linear mapping matrix \( A \). Thus the objective of the DMD is to calculate the companion matrix while minimizing the residual in the least-squares sense (Equation 2.40).
\[ A \{ V_1, V_2, V_3, \ldots, V_{N-1} \} = \{ V_2, V_3, V_4, \ldots, V_N \} = \{ V_2, V_3, V_4, \ldots, V_N - 1 \} + r I_{N-1}^T \] (2.38)

\[ AV_1^{N-1} = V_2^N = V_1^{N-1} S + r I_{N-1}^T \] (2.39)

\[ S = \min_S \| V_2^N - V_1^{N-1} S \| \] (2.40)

### 2.4.1 Direct Method

The direct method (Tu, 2013) calculates the linear mapping matrix \( A \) directly by taking the Moore-Penrose inverse (denoted by \(^\dagger\)) of \( V_1^{N-1} \) and multiplying by \( V_2^N \), as shown in Equation 2.41. Hereafter the bold matrix notation will be dropped and all symbols represent matrices. The snapshot matrices \( V_1^{N-1} \) and \( V_2^N \) will be denoted by \( V_1 \) and \( V_2 \), respectively.

A singular value decomposition (SVD) of \( V_1 \) is performed and the \( A \) matrix is rewritten in terms of the left singular vectors, \( W \), the singular values, \( \Sigma \), and the right singular vectors, \( U \) (Equation 2.42). The projection of \( A \) in \( U \), \( \tilde{A} \), (Equation 2.43) is diagonalized via an eigenvalue decomposition (Equation 2.44) with Ritz eigenvalues \( \mu \) and Ritz eigenvectors \( Y \). Finally the dynamic modes \( \Psi \) are given by Equation 2.45.

\[ A = V_2 V_1^\dagger \] (2.41)

\[ V_1 = U \Sigma W^* \quad \rightarrow \quad A = V_2 W \Sigma^{-1} U^* \] (2.42)

\[ \tilde{A} = U^* A U \] (2.43)

\[ eig (\tilde{A}) = [Y \mu] \] (2.44)

\[ \Psi = U Y \] (2.45)

### 2.4.2 Snapshot Method

For most experimental and computational data, the number of grid points \( N_x \) is significantly greater than the number of snapshots \( N \). The snapshot matrices \( V_1 \) and \( V_2 \) have dimensions of \( N_x \times N \). Thus \( A \) will have dimensions of \( N_x \times N_x \). Similar to the POD direct method, the DMD direct method becomes computationally prohibitive for large spatial domains. Thus a snapshot method is used. Two common algorithms are based on the economy-sized orthogonal-triangular (QR) decomposition and on the economy-sized SVD, respectively. If the snapshot matrix \( V_1 \) is full-rank, then the QR decomposition may be used. But for experimental data, this may not be the case because of measurement errors and noise. The SVD algorithm is more robust and numerically stable and thus more suitable for experimental data than the QR algorithm (Tu, 2013). Thus the SVD algorithm presented by Jovanović et al. (2012) is summarized here.

In the snapshot method, the approximate \( A \) matrix, \( \tilde{A} \), is computed directly from the economy-sized SVD of the snapshot matrix \( V_1 \) (Equation 2.46). The eigendecomposition and calculation of the dynamic modes follows the procedure for the direct method.

\[ V_1 = U \Sigma W^* \quad \rightarrow \quad \tilde{A} = U^* V_2 W \Sigma^{-1} \] (2.46)

### 2.4.3 Eigenspectrum and Snapshot Reconstruction

The temporal evolution of the dynamic modes is governed by the Ritz eigenvalues of \( \tilde{A}, \mu \) (Tissot et al., 2014). The eigenvalues (and hence the dynamic modes) are imaginary. Thus each dynamic mode
has a corresponding growth rate $\sigma$ and frequency $\omega$ as shown in Equation 2.47. The index $j$ is the eigendecomposition index, the index $i$ is the snapshot index, and $\Delta t$ is the time between snapshots.

$$
\mu_j = \exp (\sigma_j + i\omega_j) t_i, \quad \sigma_j = \frac{\log (|\mu_j|)}{\Delta t}, \quad \omega_j = \frac{\arg (\mu_j)}{\Delta t} \tag{2.47}
$$

In the snapshot reconstruction, the time dependence is included in the Ritz eigenvalues and the dynamic modes represent the spatial dependence. To complete the reconstruction, the modal amplitudes need to be determined. There are many ways to rank the modes. They can be ranked based on their 2-norm (which can also be used as their amplitude) or based on their frequency or growth rate (Tissot et al., 2014). Jovanović et al. (2012) proposed another method to determine the amplitudes. The amplitudes $\alpha$ are derived from an optimization problem such that the Frobenius norm of the difference between $V_1$ and the reconstructed snapshots $\Psi D_\alpha V_{and}$ is minimized (Equation 2.48). $D_\alpha$ is a diagonal matrix with the elements of $\alpha$ and $V_{and}$ is the Vandermonde matrix defined in Equation 2.49, where $r$ is the rank of $V_1$. It can be shown that the vector of optimal amplitudes is given by Equation 2.50, where $^\top$, $\alpha$, and $^{-1}$ are the complex conjugate, the element-by-element multiplication, and the inverse, respectively. The diag(.) operation creates a diagonal matrix with elements given by its argument. Thus the snapshot reconstruction is performed using Equation 2.51, where $p$ is the number of modes to use in the reconstruction.

$$
\min \alpha \|V_1 - \Psi D_\alpha V_{and}\|_F^2 \tag{2.48}
$$

$$
V_{and} = \begin{bmatrix}
1 & \mu_1 & \cdots & \mu_1^{N-1} \\
1 & \mu_2 & \cdots & \mu_2^{N-1} \\
\vdots & \vdots & \ddots & \vdots \\
1 & \mu_r & \cdots & \mu_r^{N-1}
\end{bmatrix} \tag{2.49}
$$

$$
\alpha = [(Y^*Y) \circ (V_{and} V_{and}^*)]^{-1} \text{diag}(V_{and} W Y^* Y) \tag{2.50}
$$

$$
V_{1,i} \approx \sum_{j=1}^p \Psi_j \mu_j^i \alpha_j, \quad i \in \{1, 2, 3, ..., N - 1\} \tag{2.51}
$$

### 2.4.4 Advantages and Disadvantages

One of the principal objectives of the DMD is to extract dynamic features that would result from a global stability analysis. In classical stability analysis, the system matrix is required to create artificial flow fields for the numerical method. Thus, only data from numerical simulations is used. In experimental data, there is no system matrix just flow field data. DMD uses the flow field data to extract the modes that govern the stability of the flow. Thus it can be applied to both experimental and numerical data (Schmid, 2010).

In classical POD only the spatial orthogonal modes (topos) are calculated. In Bi-Orthogonal Decomposition (BOD), the temporal modes (chronos) are also calculated. In DMD, the BOD modes are calculated when performing the SVD of the snapshot matrix. The approximate linear mapping matrix thus represents the temporal correlation of the topos (Schmid, 2010). An advantage of DMD over POD and BOD is that each dynamic mode has a distinct frequency and growth rate. In BOD, the chronos act across different frequencies. Another advantage of DMD over POD is that the dynamic modes are not ranked by their energy content. It is well known that some low energy modes are dynamically important and may not be captured by POD but may be captured by DMD. Each dynamic mode has an energy associated with it but it is given only at its distinct frequency (Bistrian & Navon, 2014). A disadvantage of DMD over POD is that while the POD modes are orthogonal in space, the DMD modes are orthogonal in time. Thus the dynamic modes are unique in the temporal domain but are correlated in the spatial domain and the projection error is higher in DMD than in POD. The residual of the linear mapping only depends on the last snapshot, thus noise in the last snapshot will significantly affect the DMD analysis and increase the projection error. Furthermore, the energy of the flow is not obtained from DMD, as opposed to POD (Tissot et al., 2014).
Another advantage of DMD occurs when analyzing 2D slices of 3D flow fields. When compared to a 3D DMD, the 2D DMD analysis will only change the spatial dependence of the decomposition but not the temporal dependence of the dynamic modes. Thus the stability information can be extracted from the 2D DMD. Furthermore, the spatial structures of the 2D dynamic modes closely approximate the projections of the 3D dynamic modes on the slice (Schmid, 2010).

A source of discussion among researchers using DMD is the ranking of the modes. There is no clear way to rank the modes as in POD. They can be ordered by their 2-norm, by frequency, or by growth rate. Jovanović et al. (2014) proposed a method that minimizes the Frobenius norm of the projection error. Tissot et al. (2014) introduced a ranking based on the energy content of dynamic modes. Each dynamic modes is multiplied by its Ritz eigenvalue and the 2-norm of the result is calculated. The integral over the temporal domain yields the energy of each dynamic mode. Note that this is a criterion for ranking the modes and does not correspond to the energy of the flow, as discussed above.

### 2.4.5 Previous Research

To provide a proof of concept of the analysis, Schmid (2010) provided three examples on numerical and experimental data: flow over a square cavity, flow in the wake of a flexible membrane, and a jet between two cylinders. Important aspects of the analysis such as convergence of the residuals, the eigenspectrum, and sub-domain analysis, are also discussed.

Sayadi et al. (2012) performed DMD analysis on a turbulent flat plate boundary layer. The skin friction coefficient was used as the basis for the decomposition. The results show that the spanwise average of the first DMD mode closely matches the time and spanwise averaged Direct Numerical Simulation (DNS) result. This a direct result of the DMD since as the number of snapshot increases the first DMD mode will converge to the mean field.

The wake of bluff body was analyzed with by DMD by Tu (2013). The geometry was a thick flat plate with elliptical leading edge and blunt trailing edge. The Reynolds number was 50,000 and the data was gathered from TR-2D2C PIV measurements. The wake shedding frequency is captured by the dominant peak in the eigenspectrum. The first three pairs of modes show the antisymmetry of the modes about the centerline of the body. An important result is that the antisymmetry is not perfect due to noise in the experimental data. The results were compared to a computational simulation of a similar configuration and found the antisymmetry to be more exact for the computational results.

Tissot et al. (2014) applied both DMD and an optimized DMD algorithm to the the canonical flow of a 2D cylinder wake at $Re = 1,300$. The data was obtained from TR-2D2C PIV measurements. The optimized DMD corresponds to the ranking of the dynamic modes based on their energy content rather than their 2-norm. Reconstruction of the snapshot clearly showed a better convergence of the projection error with the optimized DMD than the classical DMD. Chen et al. (2012) also applied DMD to a 2D cylinder wake $Re = 60$. The DMD identifies the modes that grow from the unstable Hopf equilibrium. The behavior and applicability of the DMD in the transient regime between the Hopf bifurcation and the limit cycle is also discussed. The DMD modes are also compared to the POD modes at the limit cycle. They were found to be very close to one another, after appropriate selection of the dynamic modes.

### 2.5 Turbulent Flow Statistics

The most obvious characteristic of turbulent flow is its irregularity — its randomness. This makes a deterministic analysis difficult and statistical analysis is preferred. Turbulent flow occurs at high Reynolds numbers and is characterized as an unsteady flow with three dimensional vorticity fluctuations. It is also characterized by high rates of mass, momentum and heat transfer. Turbulence is always a dissipative phenomenon because viscous shear stresses always convert the turbulent kinetic energy into internal energy. Turbulence is nonetheless a continuum phenomenon, even in the smallest scales (Tennekes & Lumley, 1972).

The following discussion is derived from Tennekes & Lumley (1972) and Batchelor (1967).
2.5.1 Statistical Moments of Velocity

Suppose a turbulent flow is measured continuously from time $t_0$ to time $t_0 + T$ and the velocity $u_i$ is recorded. The moment of $u_i$ can be defined as the mean values of the powers of $u_i$. Thus the first moment of velocity is identically the time average of velocity (Equation 2.52). In experimental measurements, the velocity is sampled at $N$ discrete points, with equal time intervals between them. The discretized first moment of velocity is given in Equation 2.53. The velocity vector will hereafter be denoted by either $u_i$ or $u_j^i$.

$$\overline{u}_i = \lim_{T \to \infty} \frac{1}{T} \int_{t_0}^{t_0+T} u_i(t) \, dt \quad (2.52)$$

$$\overline{u}_i = \frac{1}{N} \sum_{j=1}^{N} u_j^i \quad (2.53)$$

The central moments are defined as the moments formed with $u_i' = u_i - \overline{u}_i$. By Equation 2.52, the first central moment is equivalent to the time average of the velocity fluctuations. By definition, this is identically zero. The second central moment, $\sigma^2$, is called the variance of $u_i$. It is a measure of the mean square departure of $u_i$ from $\overline{u}_i$. It determines the width of the probability density function (PDF) of $u_i$. The third central moment determines the amount of asymmetry of the PDF of $u_i$. The fourth central moment is a measure of the flatness of the PDF of $u_i$. A smaller fourth central moment leads to a flatter PDF. The third and fourth central moments are usually normalized by powers of $\sigma$. The normalized third and fourth central moments are called the skewness, $S$, and the kurtosis, $K$, of $u_i$, respectively. Equations 2.54-2.61 define the central moments of $u_i$ for both continuous sets and discretized sets as well as skewness and kurtosis. The PDF of $u_i$ is defined as $B(u_i) = B(u_i')$.

$$\sigma^2 \equiv \overline{u}_i^2 = \int_{-\infty}^{\infty} u_i'^2 B(u_i') \, du_i' = \lim_{T \to \infty} \frac{1}{T} \int_{t_0}^{t_0+T} [u_i'(t)]^2 \, dt \quad (2.54)$$

$$\overline{u}_i^3 = \int_{-\infty}^{\infty} u_i'^3 B(u_i') \, du_i' = \lim_{T \to \infty} \frac{1}{T} \int_{t_0}^{t_0+T} [u_i'(t)]^3 \, dt \quad (2.55)$$

$$\overline{u}_i^4 = \int_{-\infty}^{\infty} u_i'^4 B(u_i') \, du_i' = \lim_{T \to \infty} \frac{1}{T} \int_{t_0}^{t_0+T} [u_i'(t)]^4 \, dt \quad (2.56)$$

$$\sigma^2 \equiv \overline{u}_i^2 = \frac{1}{N} \sum_{j=1}^{N} [(u_j')^2]^j \quad (2.57)$$

$$\overline{u}_i^3 = \frac{1}{N} \sum_{j=1}^{N} [(u_j')^3]^j \quad (2.58)$$

$$\overline{u}_i^4 = \frac{1}{N} \sum_{j=1}^{N} [(u_j')^4]^j \quad (2.59)$$

$$S \equiv \frac{\overline{u}_i^3}{\sigma^3} \quad (2.60)$$

$$K \equiv \frac{\overline{u}_i^4}{\sigma^4} \quad (2.61)$$

The square root of the variance of $u_i$ is called the standard deviation, $\sigma$, of $u_i$. Another name for $\sigma$ is the RMS velocity, $u_i^{rms}$. The standard deviation of velocity may be called the RMS velocity since it is a measure of the velocity’s variation about a reference value, in this case the mean velocity.
Joint moments between two functions can also be defined. In fluid dynamics the joint moment of two velocity components, $u$ and $v$ for instance, is common. The joint moment is also called the covariance or correlation. It is a measure of how the PDF of $u$ depends on the PDF of $v$, and vice versa. The covariance is also equal to the time average of the product of the fluctuating velocity components. For a discretized velocity field the covariance of $u$ and $v$, $\text{cov}(u,v)$, is given in Equation 2.62. The covariance can also be normalized by the product of the standard deviations of the velocity components. This quantity is called the correlation coefficient, $\rho(u,v)$ (Equation 2.63).

$$\text{cov}(u,v) = \frac{1}{N-1} \sum_{i=1}^{N} (u'_i)(v'_i)$$ (2.62)

$$\rho(u,v) = \frac{\text{cov}(u,v)}{\sigma_u \sigma_v}$$ (2.63)

Another quantity called the autocorrelation can also be defined for fluctuating velocity components. It is a measure of how the values of the fluctuating velocity component at different times are related to each other. The autocorrelation of $u'_i$ at times $t$ and $t'$ is $\overline{u'_i(t) u'_i(t')}$, The normalized autocorrelation, called the autocorrelation coefficient $\rho(u'_i,u'_i)$, is defined in Equation 2.64.

$$\rho(u'_i,u'_i) = \frac{\overline{u'_i(t) u'_i(t')}}{\overline{u'^2_i}}$$ (2.64)

So far, only time averages have been discussed. A common averaging technique in experimental fluid dynamics is the ensemble average. An ensemble average is an average over multiple snapshots of identical processes as opposed to averaged snapshots over a long period of time. The difference between time averaging and ensemble averaging is seen in Figure 2.6.

In the same way central moments, covariance, and autocorrelation are defined with respect to time, the same quantities can be defined for ensembles. A statistically stationary process is a process that is invariant under a time shift. For a statistically stationary process, the time average and the ensemble average are
the same. If this holds, then the process is called an ergodic process. Turbulence is assumed to be an ergodic process (Goldstein, 1996). Thus, any time averaged quantities may be replaced for ensemble-averaged quantities.

### 2.5.2 RANS Equations

If the flow field is statistically steady, then the average of flow quantities such as velocity and pressure, are not functions of time and the quantities may be expressed using the Reynolds decomposition. Consider a vector quantity such as velocity that in general is dependent on position within the flow, \( x_i \), and time, \( t \). Then the Reynolds decomposition is expressed as Equation 2.65. The time average and the fluctuations are denoted by \( \overline{\cdot} \) and \( \cdot' \), respectively. The Reynolds decomposition can also be applied to scalar quantities like pressure and temperature.

\[
u_i(x_i, t) = \overline{\nu_i(x_i)} + \nu_i'(x_i, t) \tag{2.65}\]

The Reynolds decomposition of velocity and a time averaging operation can be used to rewrite the Navier Stokes equations in terms of average and fluctuating components. The resulting equations are called the Reynolds-Averaged Navier Stokes (RANS) equations. An incompressible flow of a Newtonian isotropic fluid, constant dynamic viscosity and no body forces is assumed. The Navier Stokes equations are given in Equation 2.66. The RANS equations are presented in Equation 2.67. The pressure and density are denoted by \( P \) and \( \rho \), respectively.

\[
\begin{align*}
\text{Continuity Equation:} & \quad \overline{\nu_i} \overline{\nu_j} = 0 \\
\text{Momentum Equation:} & \quad \rho \left( \frac{\partial \overline{\nu_j}}{\partial t} + \overline{\nu_i} \overline{\nu_j} \right) = -P \overline{\nu_j} + \mu \overline{\nu_j} \overline{\nu_j} \\
\text{RANS Continuity Equation:} & \quad \overline{\nu_i} \overline{\nu_j} = 0 \\
\text{RANS Momentum Equation:} & \quad \rho \overline{\nu_i} \overline{\nu_j} = -P \overline{\nu_j} + \mu \overline{\nu_j} \overline{\nu_j} - \rho \left( \overline{\nu_i} \overline{\nu_j} \right) \overline{\nu_j} \\
\end{align*}
\tag{2.66}
\]

\[
\begin{align*}
\text{RANS Continuity Equation:} & \quad \overline{\nu_i} \overline{\nu_j} = 0 \\
\text{RANS Momentum Equation:} & \quad \rho \overline{\nu_i} \overline{\nu_j} = -P \overline{\nu_j} + \mu \overline{\nu_j} \overline{\nu_j} - \rho \left( \overline{\nu_i} \overline{\nu_j} \right) \overline{\nu_j} \\
\end{align*}
\tag{2.67}
\]

The RANS momentum equation contains an additional term, the last one. It is called the Reynolds stress tensor (per unit mass), \( \tau_{ij} \), given in Equation 2.68. It describes the diffusive nature of momentum in a turbulent flow. The Reynolds stress tensor is generally not know and thus leads to the closure problem in turbulence: a separate model is necessary to approximate the Reynolds stress tensor. Note that the averaging of 2D2C PIV data yields the average and fluctuating components of \( \overline{\nu} \) and \( \overline{\nu}' \): \( \overline{\nu}, \overline{\nu}', u', \text{and} \ u' \). Thus three components of the Reynolds stress tensor can be calculated: \( \overline{u'^2}, \overline{u'v'}, \text{and} \ \overline{v'^2} \).

\[
\tau_{ij} = \overline{\nu_i' \nu_j'} = \begin{pmatrix}
\overline{u'^2} & \overline{u'v'} & \overline{u'w'} \\
\overline{v'u'} & \overline{v'^2} & \overline{v'w'} \\
\overline{w'u'} & \overline{w'v'} & \overline{w'^2}
\end{pmatrix} \tag{2.68}
\]
Chapter 3
Experimental Apparatus

3.1 Wind Tunnel

The experiments are conducted in the cascade wind tunnel located in the Engineering Laboratory Annex Building at Louisiana State University, Baton Rouge, LA, US. The cascade wind tunnel is a closed-loop, low turbulence wind tunnel that was specifically built to study film cooling flows in a cascade of low pressure turbine blades. The purpose of the wind tunnel is to study the effects of wakes generated by upstream nozzle guide vanes on the forced film cooling performance of a cascade of turbine blades. The wind tunnel was designed and built to provide a versatile platform for the film cooling studies. The wind tunnel is capable of accommodating Constant Temperature Anemometry (CTA), Pitot-Static tubes, thermocouples, pressure-tapped blades, PIV, and Infrared Thermography (IRT). A 3D CAD rendering of the wind tunnel is shown in Figure 3.1. The principal components of the wind tunnel can be seen in Figure 3.2. The following discussion will focus on the test section, as that is the more relevant component to this study. The reader is referred to Junca-Laplace (2011) and Foreman (2013) for details on the design and construction of the wind tunnel.

3.1.1 Test Section

The test section (Figures 3.4-3.5) of the wind tunnel consists of a four-passage linear cascade of United States Air Force Research Laboratory (US AFRL) Ultra High Lift L1A low pressure turbine blades and an upstream wake generator. Directly upstream of the test section is the contraction cone with a contraction ratio of 6.16. The 3D profile of the contraction cone is made of 2 matched cubic polynomials to smoothly
accelerate the flow to the desired test section freestream velocity of 50 m/s. The contraction cone connects to a test section with a cross section of 304.8 mm × 520.7 mm (12 in × 20.5 in). At the inlet of the test section, the turbulence intensity was measured to be ~ 0.2% of the freestream velocity (Junca-Laplace, 2011).

The cascade is made of three full and two shaped wall L1A blades with an axial chord length, \( C_x \), of 152.4 mm (6 in) and a span of 304.8 mm (12 in). The cascade solidity, defined as the ratio of axial chord length to blade spacing, is 1. The blade inlet and exit angles are 35° and 60° from the axial, respectively. Thus the blade turning angle is 95°. With a 50 m/s freestream velocity, the Reynolds number based on the axial chord is approximately 500,000. The wake generator consists of a translating 9.53 mm (3/8 in) thick plates powered by a conveyor belt. The leading edges and trailing edges are half-cylinders. The chord and span of the plates match those of the blades. Furthermore, the pitch (spacing between each plate) is 152.4 mm (6 in). The distance between the trailing edges of the plates and the leading edge of the blades is 0.40\( C_x = 60.96 \) mm (2.4 in). The design speed of the wake generator is 1 m/s. At this speed, the wake generator creates a 1° deviation angle for a total cascade inlet angle of 36° and cascade turning angle of 96°.

Thus, the conveyor belt is mounted at an angle of 36° with respect to the freestream (Junca-Laplace, 2011).

### 3.1.2 Cascade

The three full blades were made using a process called stereolithography (SLA) with Accura 55 acrylonitrile butadiene styrene (ABS) plastic. Hereafter, the center blade will be referred to as the test blade and the lower blade will be referred to as the PIV blade. The upper blade was designed as a second PIV blade to illuminate the pressure side of the test blade.

The test blade has three rows of film cooling holes on each side and an air supply plenum inside. All film cooling holes have a diameter of \( D = 3.175 \) mm (1/8 in). Each row of film cooling holes has five holes with a spacing of \( 3 \times D \). The center hole in each row is placed in the midspan of the blade. The holes are angled 35° with respect to the local tangent. Figure 3.6 is a schematic of the test blade. Table 3.1 states the chord location of the three rows of film cooling holes with the respective length \( L \) to diameter ratios for the holes. The data was taken from Foreman (2013).

The PIV blade is made of five sections as seen in Figure 3.7. Note that the optics windows is a flat glass panel placed in a region of low curvature. On the inside is a slot used to traverse the optics inside the blade. The optics assembly is screwed into a periscopic rod that runs on the slot. The optics assembly was designed in-house by Dr. Guillaume Bidan to convert the incoming laser beam into a thin sheet that illuminates the suction side of the test blade. The laser beam comes in the direction of the positive \( y \) axis in Figure 3.8
Table 3.1: Locations and Length to Diameter Ratios of Film Cooling Holes (Foreman, 2013)

<table>
<thead>
<tr>
<th>Side</th>
<th>Row</th>
<th>% Chord Location (x/C_c) × 100</th>
<th>Length to Diameter Ratio (L/D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suction</td>
<td>1</td>
<td>12.50</td>
<td>1.816</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>29.17</td>
<td>1.824</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>50.00</td>
<td>1.880</td>
</tr>
<tr>
<td>Pressure</td>
<td>1</td>
<td>16.67</td>
<td>1.694</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>33.33</td>
<td>1.664</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>50.00</td>
<td>1.664</td>
</tr>
</tbody>
</table>
Figure 3.4: Test Section Components (Junca-Laplace, 2011)

Figure 3.5: Test Section Details (Junca-Laplace, 2011)
through a small hole in the plastic base spacer (the section of the blade secured to the acrylic window). The laser hits the optics assembly. The optics assembly consists of a right-angle prism to turn the beam 90° followed by a plano-concave cylindrical lens with a -12.5 mm focal length to generate the laser sheet. The traverse system was also designed in-house. The laser sheet created by the optics is seen in Figure 3.8. The laser beam is traveling into the page, directly normal to the optics. The optics then create the sheet. The optics assembly allows rotation about the z and y axes and translation on the y axis. This provides great flexibility as the entire suction side of the test blade can be covered at any spanwise location.

Dr. Bidan’s design for the bottom PIV blade was adapted to a new design for the top PIV blade which was started by the author and finished by Mr. Kevin Wood. The objective was to illuminate the pressure side of the test blade. The internal design of the bottom PIV blade and the window design was used in the top PIV blade. The optics assemblies remain the same as the one used in the bottom PIV blade. Figure 3.9 shows a 2D view of the redesigned central section of the top PIV blade and a 3D CAD render of the cascade with the new lasers. Note that there are two optics assemblies inside the blade. One creates a laser sheet on the pressure side of the test blade while the other creates a laser sheet in the wake of the central test blade for future PIV studies in this region.

3.2 PIV System

Figure 3.10 is a schematic of the PIV system and supporting systems necessary for running an experiment. A discussion of the principal components follows.

3.2.1 PIV Laser

The PIV system consists of a frequency-doubled, dual head, single housing New Wave Research Solo 120XT Nd:YAG laser. It consists of two laser heads in a single housing along with a second harmonic generator and the optics to combine the laser into a single beam. The laser heads fire a wavelength of
Figure 3.7: PIV Blade (Foreman, 2013)

Figure 3.8: Laser Sheet
1064 nm, which is in the UV range, but the second harmonic generator outputs light at 532 nm wavelength. The laser has a maximum energy of 120 mJ with a beam diameter of 4.5 mm. The laser can fire at a rate of 15 Hz with a pulse width of 3 – 5 ns. The laser housing is powered by a separate power supply. The power supply serves three functions: power supply, cooling system, and controls. The power supply brings AC power to the laser housing. The cooling system is a closed-loop water system. The deionized water is circulated via a pump to the Nd:YAG rods and returned to an air-water heat exchanger and passed through a deionization filter before being put back in circulation. The controls are located in a front panel of the power supply. The energy level, the repetition rate, and on/off controls for each laser head are available. The laser can also be triggered externally by connecting the external trigger signal through BNC connectors or a TTL port in the back of the power supply. The BNC connectors give the greatest timing flexibility because both the flash-pump firing and Q-switch can be independently controlled for each laser head. Theoretically, the timing can be set so that the width of the first laser shot overlaps the width of the second laser shot (e.g. the shots are separated by only a few ns).

3.2.2 Light Guiding Arm and Beam Traverse

A separate LaVision light guiding arm is placed next to the laser output to guide the laser beam to a fine-adjustment laser beam traverse. The laser guiding arm connects to the laser beam traverse. The traverse is placed directly in front of the PIV blade such that only fine adjustments at the traverse are necessary to align the laser beam to the optics inside the PIV blade. The traverse provides adjustments in the plane perpendicular to $y$ axis in Figure 3.8. The traverse is capable of adjusting for displacement on and rotations about two axes in this plane. Furthermore, the beam traverse contains a LaVision light sheet optics device to control the diameter of the beam and thus control the thickness of the laser sheet.

3.2.3 Camera and Lens

The camera used is a Dantec Dynamics NanoSense 3E CMOS camera with a resolution of $1,280 \times 1,000$ pixels and frame rate of up to 1,000 frames per second. It is a 10-bit camera but only 8 bits are used during the acquisition. The buffer size is 4 GB. The lens is a Nikon 50 mm prime lens with a maximum aperture of
Figure 3.10: Experimental Setup
A C-mount adapter is used to attach the lens to the camera. The lens allows for manual adjustment of the focusing ring and aperture to focus accurately on the laser sheet.

### 3.2.4 Seeding System

The seeding system consists of a High End Systems FQ-100 fog generator with Atmospheres HQ fog fluid. The fog generator was originally designed to be used in stage performances, thus it produces a non-toxic fast-dissipating fog. It can produce a fog with particle sizes between $0.25 - 6 \mu m$. The fog generator is controlled via a remote control. The fog is fed into the wind tunnel by means of a hose that connects to the output of the fog generator. The hose feeds the fog directly aft of the cascade to ensure maximum mixing with the flow by the time the fog flows to the test section.

### 3.2.5 Traverse

The traverse system is three axis linear traverse that is placed directly in front of the test section, as seen in Figure 3.11. The camera is mounted at the end of the $y$ axis. This allows positioning the camera to any position overlooking the test section. The beam traverse is also placed on the frame of the traverse, not on the traverse itself. The beam traverse can be moved to any position in the frame of the linear traverse.

### 3.2.6 Timing and Monitoring Electronics and Computers

The acquisition components are the following: an acquisition computer, a monitoring computer, a stack of National Instruments (NI) monitoring electronic modules, a timer box, a trigger. The interfaces are as follows. The monitoring computer is used to monitor the operation of the wind tunnel by means of a LabView virtual instrument (VI). The velocity, temperature and pressure in the test section inlet are measured via a Pitot-Static tube and a type K thermocouple. These measurement signals are processed by the NI modules and monitored in the VI. The trigger is a sensor placed in such a way that it outputs a trigger signal every time a wake generator plate is abeam the leading edge of the test blade. The trigger signal is processed by the NI modules and forwarded to the acquisition computer. The acquisition computer is the one that controls the timing of the acquisition sequence. The entire sequence is controlled in Dantec Dynamics’ DynamicStudio. Once the trigger signal arrives, the acquisition sequence starts after a user defined time delay. The timer box (Dantec Dynamics 80N77) controls the required signals and delays to the PIV laser and to the camera.
Chapter 4
Wake Characterization at the Leading Edge of the Suction Side

4.1 Test Conditions

The conditions for PIV tests are kept as constant as possible between different tests. The ambient conditions are given in Table 4.1. Dantec Dynamics’ DynamicStudio is used for the acquisition of the images and subsequent PIV analysis. The time delay between frame 1 and frame 2 is set to 3 µs. This ensures that the maximum particle displacement in a 16 × 16 pixels (the minimum interrogation area used in the analysis) interrogation spot is about a 1/4 of the interrogation area. The image format used is .tiff. The maximum repetition rate of the laser, 20 Hz, is used, though the actual triggering rate is controlled by DynamicStudio to get the phase-locked positions studied. 400 frame pairs are acquired per test.

One condition is not constant, the seed concentration. The fog generator is controlled manually which means that to produce fog, a knob needs to be turned. The knob controls the volume of fog output. In order to maintain an optimal seed concentration, a short burst of 1 – 2 s of fog at approximately 20% volume output is used when needed. This setting was determined through trial and error to maintain as close to a constant seed concentration for all snapshots. Due to the manual operation of the fog input, a uniform concentration cannot be maintained all the time, though the current setting generates as close to a uniform concentration as possible.

For any given spatial domain (e.g. the field of view and position of the camera), several tests take place. The wake is a periodic phenomenon. For a 1 m/s plate speed and 152.4 mm plate spacing, the period of the plate and thus of the wake passage is 0.1524 s. Thus every 0.1524 s, a plate is abeam the leading edge of the test blade. This point may be defined as the 100% phase position. This is also the point when the trigger sends it signal. If the delay between the trigger and the acquisition sequence is 0 s, then the test is characterizing the cycle at the 100% phase position. 400 frame pairs are taken in 400 cycles. The same test can be repeated with a nonzero time delay. Thus, by setting the time delay between the trigger and the acquisition sequence, the entire cycle can be characterized at discrete phase positions. Figure 4.1 is a schematic depicting the phases and Figure 4.2 is an example at 80% phase.

4.2 Analysis Conditions

4.2.1 Calibration

Prior to the analysis, a calibration must be performed. A calibration target with white background and black dots is aligned with the laser sheet and a picture is taken with the camera. The separation between the dots is defined in DynamicStudio and the software finds the origin and defines a position grid in millimeters for every pixel in the camera. Thus the displacement in pixels output by the analysis is transformed to a displacement in millimeters and then meters to obtain a velocity vector in SI units. For illustration, the

<table>
<thead>
<tr>
<th>Property</th>
<th>Lower Value</th>
<th>Upper Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>101.0</td>
<td>103.0</td>
<td>kPa</td>
</tr>
<tr>
<td>Temperature</td>
<td>19.0</td>
<td>21.0</td>
<td>°C</td>
</tr>
<tr>
<td>Relative Humidity</td>
<td>35</td>
<td>60</td>
<td>%</td>
</tr>
</tbody>
</table>
Figure 4.1: Schematic of Phases in the Wake Passage Cycle

Figure 4.2: Example at 80% Phased-Locked Position
calibration image for the suction side of the leading edge is shown in Figure 4.3. The parameters of the calibration target are given in Table 4.2.

4.2.2 PIV Analysis

The pre-processing consists of stretching the images such that the brightest particles have a bright white color and the background has a very dark color. If the image have a noisy background or if the background is not very dark, a further step is performed. The ensemble-averaged pixel intensity is calculated and subtracted from each image. The images are then stretched again. The PIV analysis is an adaptive cross correlation between the two frames. The method used by DynamicStudio is based on the Fourier Fast Transform. Furthermore, there are certain improvements over the basic cross correlation. A grid step size is defined as the distance between the centers of adjacent IAs. The method determines an appropriate IA for each IA within a maximum and minimum user defined sizes. The first iteration of the cross correlation uses the maximum IA defined. Subsequent iterations reduce the IA if the local seed density requires it. The IAs may also overlap 25% of the IA in both the vertical and horizontal directions to reduce the number of lost pairs. High accuracy sub pixel refinement based on a two-directional Gaussian fit is used to determine the location of the displacement peak. The IA shape and size is set to adapt to the velocity gradients in between the iterations to obtain more accurate results in areas of high velocity gradients. A central difference scheme is used to calculate the mean particle displacement in the IAs.
Table 4.3: PIV Analysis Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid Step Size</td>
<td>16</td>
<td>pixels</td>
</tr>
<tr>
<td>Minimum IA Size</td>
<td>(16 \times 16) pixels</td>
<td></td>
</tr>
<tr>
<td>Maximum IA Size</td>
<td>(64 \times 64) pixels</td>
<td></td>
</tr>
<tr>
<td>Minimum Peak Height</td>
<td>0.30</td>
<td>-</td>
</tr>
<tr>
<td>Ratio of Peak Height to Second Highest Peak</td>
<td>1.50</td>
<td>-</td>
</tr>
<tr>
<td>Peak Height Relative to RMS Noise Level</td>
<td>5.0</td>
<td>-</td>
</tr>
<tr>
<td>Neighborhood Size</td>
<td>(5 \times 5) IAs</td>
<td></td>
</tr>
<tr>
<td>Neighborhood Residual Normalization Factor</td>
<td>0.10</td>
<td>pixels</td>
</tr>
<tr>
<td>Neighborhood Residual Acceptance Limit</td>
<td>2.00</td>
<td>-</td>
</tr>
<tr>
<td>Particle Detection Limit</td>
<td>5.0</td>
<td>-</td>
</tr>
<tr>
<td>Desired # of Particles / IA</td>
<td>12</td>
<td>-</td>
</tr>
<tr>
<td>Maximum Magnitude of Individual Velocity Gradients</td>
<td>0.10</td>
<td>-</td>
</tr>
<tr>
<td>Maximum 2-Norm of Velocity Gradients</td>
<td>0.20</td>
<td>-</td>
</tr>
</tbody>
</table>

The validation scheme used is based on peak validation and outlier detection. A minimum peak height ratio between the two highest peaks is used in the peak validation. The outlier detection based on a moving average uses a neighborhood of IAs to check whether the central IA displacement differs by a certain amount from its neighbors. If the displacement of the central IA differs by a given amount from its neighbors, then the vector is invalid and substituted with an interpolated vector. The statistics of each phase positions were calculated using the ensemble average. Table 4.3 list the values used in the analysis. These values were empirically determined to be the most suitable for the experimental data collected.

### 4.2.3 POD and DMD Post Processing

The POD analysis is done in DynamicStudio using the snapshot method on the velocity data. The result is the energy content of each POD mode and a vector field for each POD mode. The POD modes can then be projected into the velocity vectors to reconstruct the snapshots based on any combination of modes or based on the necessary modes to achieve a certain energy on the projected snapshot. Furthermore, DynamicStudio can also calculate a time history of the POD coefficients. A MATLAB script was also created to calculate the phase plots and to study the convergence of the decomposition.

The DMD analysis is done with an in-house MATLAB script that uses the snapshot method. The modal amplitudes are calculated using the algorithm by Jovanović et al. (2014). The result is the eigenspectrum and the DMD modes. Furthermore the DMD script was adapted to calculate the decomposition across the phases of the wake passage (the original script calculates the decomposition with phase-locked data).

### 4.3 Instantaneous Results

The first set of tests on the wake were made on the leading edge. The entire phase of the wake at this location has been characterized. Given the camera field of view, the wake itself is only seen in phase positions 0% to 22% and 64% to 100%. These intervals were mapped in steps of 2% of the phase or 3080 µs. This means that in these intervals, 400 snapshots were taken for every 2% step in the phase. The choice to use 2% was made to adequately characterize the phase of the wake. A smaller step size would lead to significantly longer testing time to map the interval. The remaining phase positions, 24% to 56% were mapped with a step size of 8%. What follows is a series of images and velocity contours of some of the phase positions. The velocity vectors were output to Tecplot 360 for contouring.

The instantaneous results consist of the velocity field, both horizontal \(u\) and vertical \(v\) components, normalized by the freestream velocity of 48 m/s. The \(xy\) grid has been normalized by the axial chord, \(C_x = 0.1524\) m. Furthermore, Tecplot 360 calculates the \(z\) vorticity using a second-order accurate central
difference scheme to calculate the velocity gradients. The $z$ vorticity (Equation 4.1) is normalized by the axial chord and the freestream velocity. Figures 4.4-4.8 show the results for phase positions of 48%, 70%, 80%, 90%, and 100%. Each phase has the raw image from frame 1, the $u$ and $v$ components of velocity, the velocity magnitude, and the $z$ vorticity. Frame 2 is not shown since the difference between frame 1 and frame 2 is not clearly seen unless the frames are cycled on top of each other. Note that the gray areas on the contour plots represent the blade.

$$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad \text{(4.1)}$$

The 48% phase position is the baseline. The wake is not in the PIV domain. The flow is traveling from right to left, thus the $u$ velocity is negative. The $u$ velocity shows that any given $y$, the flow accelerates as it goes from right to left. This acceleration is more pronounced near the blade than at the bottom of the domain. The $v$ velocity is negative at the inlet but transitions to positive values near $x/C_x = -0.1$. The combined results from the $u$, $v$ velocities suggest that from the inlet, the flow accelerates in a $(-x, y)$ direction first and then accelerates in a $(-x, -y)$ direction. This is consistent with the flow accelerating due to the favorable pressure gradient due to the contoured shape of the suction side of the blade. The velocity magnitude contour plot clearly shows the accelerating flow while the streamlines show the direction of the acceleration. Since the wake is not in the domain, the vorticity plot does not show significant values of vorticity compared to the phases where the wake is in the domain.

In the other phases, the velocity signatures are significantly changed compared to the 48% phase signature. The wake is in the domain and causes a reduction in the velocity in its region of influence. The regions of low velocities travel upward as the phase increases. At 70% phase, the wake is near the lower edge of the domain. At 80% phase, the wake is in the middle of the domain. At 90%, the wake is close to the surface of the blade. At 100% phase, the wake is impinging on the leading edge of the blade. The exact signature of the wake cannot be discerned in the instantaneous snapshots because of the high turbulence levels in the wake. The passage of the wake is more clearly seen in the instantaneous vorticity plots. These show the cores of positive and negative vorticity that form the vortices shed from the plates. Since the data acquisition is not time-resolved neither in the wake passage time scale (due to a compromise between testing time and wake passage resolution) nor in the vortex shedding time scale (due to limitations with the available laser system), the vortex cores are not ordered in time. Rather, each individual snapshot captures the vortices at a random time in the vortex shedding cycle.

### 4.4 Phased-Averaged Statistics

#### 4.4.1 Convergence

The instantaneous velocity fields were phased-averaged to obtain converged statistics. The phase average was calculated by ensemble averaging all phase-locked snapshots for a given phase. The procedure was repeated for all phases. This averaging allows for meaningful turbulence statistics since the phase-averaged quantities include the periodic component and thus the quantities based on the central moments arise from the turbulent random fluctuations in the flow.

The first step was to ensure that the statistics were converged. For this a MATLAB script was written to calculate the average and RMS velocities using an increasing number of snapshots in each calculation. All snapshots used are phase-locked at 80% phase since in this phase the wake is in the center of the domain. A total of 400 snapshot are available. The snapshots are read into MATLAB and the average and RMS velocities are calculated using two snapshots. The mean and standard are calculated as defined in Section 2.5.1. The calculation is then performed with three snapshot, then four, and so on. The last calculation using 400 snapshots is the accurate solution used in calculating the errors. The errors are defined in Equations 4.2-4.5, where the $\bar{\cdot}$ and $\cdot_{\text{rms}}$ represent the mean and RMS velocities, respectively. The number of snapshots used in the calculation is $m$ and $\|\cdot\|_2$ represents the $L^2$ norm. The $L^2$ norm is used in order to get the magnitude of the error across all spatial gridpoints. For example, if $m = 54$, the difference between the mean at each gridpoint calculated with 54 snapshots is subtracted from the mean at each gridpoint calculated with 400 snapshots. Then the $L^2$ norm of the error is calculated. The result is normalized by the $L^2$ norm, across
Figure 4.4: Suction Side Near the Leading Edge - Instantaneous Contours - 48% Phase Position. $U_\infty = 48.0$ m/s.
Figure 4.5: Suction Side Near the Leading Edge - Instantaneous Contours - 70% Phase Position. $U_{\infty} = 48.0$ m/s.
Figure 4.6: Suction Side Near the Leading Edge - Instantaneous Contours - 80% Phase Position. $U_\infty = 48.0$ m/s.
Figure 4.7: Suction Side Near the Leading Edge - Instantaneous Contours - 90% Phase Position. $U_\infty = 48.0$ m/s.
Figure 4.8: Suction Side Near the Leading Edge - Instantaneous Contours - 100% Phase Position. $U_\infty = 48.0$ m/s.
the spatial grid, of the mean calculated with 400 snapshots. The procedure is applied to both the mean and RMS velocities. Figure 4.9 shows the convergence of the error as the number of snapshots increases. The results show that with 200 snapshots the errors are within 5% of the solution with 400 snapshots. Using more snapshots increases the accuracy but the rate of convergence is slowing down. Adding more snapshots will not significantly increase the accuracy of the results but it will increase the data acquisition time.

\[ \bar{u}_{\text{error}} = \frac{\| \bar{u}_{400} - \bar{u}_m \|_2}{\| \bar{u}_{400} \|_2} \]  

\[ \bar{v}_{\text{error}} = \frac{\| \bar{v}_{400} - \bar{v}_m \|_2}{\| \bar{v}_{400} \|_2} \]  

\[ u_{\text{rms, error}} = \frac{\| u_{\text{rms,400}} - u_{\text{rms,m}} \|_2}{\| u_{\text{rms,400}} \|_2} \]  

\[ v_{\text{rms, error}} = \frac{\| v_{\text{rms,400}} - v_{\text{rms,m}} \|_2}{\| v_{\text{rms,400}} \|_2} \]  

### 4.4.2 Contour Plots

The phased-averaged results are shown in Figures 4.10-4.16. The results are grouped by variables. Each figure contains different phases of one variable, \( u \) velocity for example. The phases chosen were 10%, 48%, 64%, 76%, 80%, 86%, 90%, 96%, and 100%. The wake is in view of the domain during phases 64% through 100%. Phases 10% and 48% represent phases without the wake in the domain. The grid is again normalized by the axial chord, \( C_x = 0.1524 \text{ m} \), and the velocities by the freestream velocity, \( U_\infty = 48.0 \text{ m/s} \); the vorticity is normalized by both. The RMS velocities are made non-dimensional by the local velocity magnitude, \( \| U \| \), to obtain the local measure of the turbulent intensities. The turbulent kinetic energy per unit mass, \( k \), in Figure 4.16 is defined in Equation 4.6 and is non-dimensionalized by the square of the local velocity magnitude. In Figure 4.16, the dimensionless \( k \) is multiplied by 100 to obtain the percent turbulent kinetic energy.

\[ k = \frac{1}{2} (u_{\text{rms}}^2 + v_{\text{rms}}^2) \]  

The \( u \) velocity plots are shown in Figure 4.10. At 10% phase, the images are acquired when the plates are slightly above the leading edge of the test blade. Thus the wake might still affect the flow in the suction side. This is not captured in the plot since there is no significant difference between the 10% and 48% phases. At 64% phase, the wake is slightly below the domain. There is slight decrease in the \( u \) velocity at the bottom of the domain. At 76% phase the signature of the wake is clear but not entirely in the domain. The velocity in the wake is significantly decreased in the bell-shaped area that makes the signature of the wake. At 80% and 86% phases the wake is passing through the middle of the domain and the wake is nearly horizontal. The wake starts to curve around the airfoil at 90% phase. At 96% and 100% phases a region of acceleration below and to the left of the wake can be identified as the flow accelerates due to the pressure gradients in the cascade. The extend to which the wake extends downstream is limited by the interaction between the wake and the leading edge region of the blade. The \( v \) velocity plots shown in Figure 4.11 show less disturbance due to the wake passage than the \( u \) velocity plots. The \( v \) velocity field changes only slightly due to the wake passage. The velocity magnitude plots (Figure 4.12) show that the wake has an average velocity below the freestream velocity. With the wake outside the domain, the cascade flow accelerates to \( 1.75U_\infty \) near the top left of the domain. With the wake at 100% phase, the flow only reaches \( 1.60U_\infty \) at the same location.

The \( z \) vorticity plots (Figure 4.13) show islands of vorticity at 10%, 48%, and 64% phases. These are believed to be a result of the PIV analysis. The RMS velocity plots (Figures 4.14-4.15) show similar, albeit lower magnitude, signatures where the islands of vorticity are located. The magnitudes are amplified due to the noise introduced in the numerical differentiation required to calculate the vorticity. The other phases show bands of high positive vorticity and negative vorticity separated by a small neutral region. This is characteristic of a turbulent wake. A turbulent wake is a boundary-free shear flow and thus the gradients in
Figure 4.9: Convergence of the velocity statistics as the number of snapshots $m$ increases.
cross-stream direction ($\partial/\partial y$) are significantly larger than those in the streamwise direction ($\partial/\partial x$) (Tennekes & Lumley, 1972). Thus the $z$ vorticity signature is dominated by the $\partial u/\partial y$ component. The normalized $\partial u/\partial y$ velocity gradient for the 86% phase is shown in Figure 4.17 as well as for 48% phase (when the wake is not in the domain) for comparison. A similar signature to the vorticity is seen with a sign change since the velocity gradient is multiplied by a negative sign in the vorticity calculation. Furthermore, the vorticity is dissipated as the distance from the plates increases. Near the blade, the vorticity seems to dissipate slower than away from the blade. This is due to the cascade flow increasing the magnitude of the $\partial u/\partial y$ velocity gradient as shown in the 48% phase in Figure 4.17.

The RMS velocity plots (Figures 4.14-4.15) clearly show the regions of high turbulence in the wake. Given the maximum $u_{rms}$ value of 0.16, the number of snapshots (400), and a 95% confidence interval on the Gaussian distribution, the expected uncertainty in the mean $u$ velocity is 1.57% of the velocity magnitude. A similar analysis of the $v_{rms}$ component yields a 3.14% uncertainty in the mean $v$ velocity. These uncertainties are lower if the freestream velocity is considered instead of the velocity magnitude. The turbulent kinetic energy seen in Figure 4.16 clearly shows the dissipation of the turbulent components in the wake as the distance from the plates increases. A maximum value of 6.2% is observed. If $k$ is normalized by the freestream velocity, a more common practice in turbulence studies, the maximum value drops to 3.6%.
Figure 4.11: Suction Side - Near the Leading Edge - Phased-Averaged Contour Plots - $v$ Velocity
Figure 4.12: Suction Side - Near the Leading Edge - Phased-Averaged Contour Plots - Velocity Magnitude, $\|U\|$
Figure 4.13: Suction Side - Near the Leading Edge - Phased-Averaged Contour Plots - Vorticity, $\omega_z$
Figure 4.14: Suction Side - Near the Leading Edge - Phased-Averaged Contour Plots - $u_{rms}$
Figure 4.15: Suction Side - Near the Leading Edge - Phased-Averaged Contour Plots - $v_{rms}$
Figure 4.16: Suction Side - Near the Leading Edge - Phased-Averaged Contour Plots - Turbulent Kinetic Energy per Unit Mass, $k$. 

(a) 10% Phase  
(b) 48% Phase  
(c) 64% Phase  
(d) 76% Phase  
(e) 80% Phase  
(f) 86% Phase  
(g) 90% Phase  
(h) 96% Phase  
(i) 100% Phase
4.4.3 Wake Profile

The mean wake profile can be extracted from the velocity field. The mean field at 80% phase is read into MATLAB and the $u$ component along the $x/C_x = 0.12$ line is extracted. The $u$ component is plotted against $y$ in Figure 4.18 to obtain the bell-shaped profile of the wake. The notation used is taken from Tennekes & Lumley (1972). The velocity outside the wake ($y/C_x = 0.06$) is denoted by $U_0 = 52.9$ m/s. The difference between $U_0$ and the minimum velocity in the wake is the maximum velocity deficit, $U_s = 7.4$ m/s. The $y$ location where the velocity deficit is $1/2U_s$ is denoted by $y_{1/2}$. The important result of the wake profile is that $U_s/\|U\| \approx 0.17$ which is on the same order as $u_{rms}/\|U\| \approx 0.16$. This is the expected result for a turbulent wake far away from the wake generator (Tennekes & Lumley, 1972).

4.5 Turbulent Component Results

The turbulent components of the velocity and vorticity are presented in Figures 4.19-4.23. The results were taken from the same snapshots used in the instantaneous results in Section 4.3. The results at 48% phase show smooth contour levels of velocity and vorticity but the instantaneous results are contaminated with the random fluctuations shown in Figure 4.19. The results for phases 70%-100% show much higher turbulent levels since the wake is in the domain. The turbulent components of velocity reach up to $u' \approx 0.3U_\infty$ and $v' \approx 0.4U_\infty$ denoting high levels to turbulence intensity in the wake. The comparison between the turbulent velocity and vorticity components reveal the locations of the vortices shed from the wake generator plates. For instance, in Figure 4.21, the $u'$ velocity plot shows a region in the center of the domain with a negative velocity on top of a region of positive velocity. Furthermore, the $v'$ velocity plot shows a region of negative velocity to the left of a region of positive velocity. The combined $u'$ and $v'$ results indicate the presence of counterclockwise vortex near the center of the domain. This is confirmed by the $\omega'_z$ plot which a small but strong core of positive vorticity at this location. A similar interpretation of the other plots reveals the location of other vortices. The vortices appear in counterclockwise-clockwise pairs. This is due to the vortices shedding from the top and bottom surfaces of the wake generator plates. It is important to note that the turbulent vorticity ($\omega'_z$) signatures are very close to the instantaneous vorticity ($\omega_z$) signatures and that both have levels well above the mean vorticity ($\bar{\omega}_z$) levels ($\omega'_z \approx 10\bar{\omega}_z$). These high levels of turbulent vorticity is a characteristic of turbulent flows.
4.6 POD Results

4.6.1 Convergence and Energies

The POD analysis was performed for the 80% phased-locked snapshots. This phase was chosen because most of the wake passes through the center of the domain without impinging on the leading edge of the test blade. The first step was to verify the convergence of the POD code. The convergence of the energy captured by the POD mode 1 as the number of snapshots used in the decomposition increases is shown in Figure 4.24. It is seen that approximately 100 snapshots are needed to obtain an accurate energy content of the leading mode. Thus a decomposition with 400 snapshots is already converged.

The resulting energies and cumulative energies for an analysis with 400 snapshots is given in Figure 4.25. The results for the first six modes are also tabulated in Table 4.4. The results clearly show that the flow is predominantly determined by the first two modes. Between them, 70% of the turbulent kinetic energy of the flow is captured and both are one order of magnitude larger than the subsequent modes (modes 3-399). Modes 3-4 have energies around 2.3% each and modes 5-6 have approximately 1.7% energy each. This grouping of the energy in mode pairs is characteristic of periodic flow and will be explained below in greater detail.
Figure 4.19: Suction Side Near the Leading Edge - Fluctuating Component Contours - 48% Phase Position. $U_\infty = 48.0$ m/s.

Table 4.4: Suction Side Near the Leading Edge - POD energies for first six modes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Energy Content (%)</th>
<th>Cumulative Energy Content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>38.18</td>
<td>38.18</td>
</tr>
<tr>
<td>2</td>
<td>32.53</td>
<td>70.71</td>
</tr>
<tr>
<td>3</td>
<td>2.38</td>
<td>73.10</td>
</tr>
<tr>
<td>4</td>
<td>2.33</td>
<td>75.43</td>
</tr>
<tr>
<td>5</td>
<td>1.83</td>
<td>77.26</td>
</tr>
<tr>
<td>6</td>
<td>1.69</td>
<td>78.95</td>
</tr>
</tbody>
</table>
Figure 4.20: Suction Side Near the Leading Edge - Fluctuating Component Contours - 70% Phase Position.

$U_\infty = 48.0 \text{ m/s}$. 

(a) $u'$ Velocity
(b) $v'$ Velocity
(c) $\sqrt{u'^2 + v'^2}$, Velocity Magnitude
(d) $\omega'_z$, Vorticity
Figure 4.21: Suction Side Near the Leading Edge - Fluctuating Component Contours - 80% Phase Position. $U_\infty = 48.0 \text{ m/s}$. 

(a) $u'$ Velocity 

(b) $v'$ Velocity 

(c) $\sqrt{u'^2 + v'^2}$, Velocity Magnitude 

(d) $\omega_z$, Vorticity
Figure 4.22: Suction Side Near the Leading Edge - Fluctuating Component Contours - 90% Phase Position. $U_\infty = 48.0$ m/s.
Figure 4.23: Suction Side Near the Leading Edge - Fluctuating Component Contours - 100% Phase Position. $U_\infty = 48.0$ m/s.
Figure 4.24: Suction Side Near the Leading Edge - Convergence of the POD energy of mode 1 as the number of snapshots ($N$) increases.

Figure 4.25: Suction Side Near the Leading Edge - POD energies. The decomposition index (mode index) is denoted by $m$. 
4.6.2 POD Modes

The signature of the POD modes is shown in Figures 4.26-4.29. The white area on top above $y/C_x \approx 0.07$ is the blade surface. Note that the POD modes are orthonormal, thus they are dimensionless vector fields. The mode signatures confirm that the energy pairings correspond to mode pairs. Modes 1-2 are the most energetic modes and correspond to the vortex shedding of the wake generator plates. The streamwise distance between cores of the same sign corresponds to the vortex shedding wavelength. The streamwise shift between mode 1 and mode 2 is approximately 1/4 of the wavelength of the 1,2 mode pair. The similar amounts of energy and the streamwise shift present in mode pairs correspond to the streamwise convection of the large scale structures in the flow. For modes 1-2, the large scale structures being convected are the periodic vortices shed from the wake generator plates. For the higher order modes, the large scale structures correspond to harmonic components of the vortex shedding wavelength. As the mode index increases, the coherent structures represented in the modes decrease in energy and length scale and become more chaotic. This is a result of the turbulent kinetic energy cascading to increasingly smaller energies and length scales. This effect is readily seen in modes 5-6. The coherent structures are of similar length scales as modes 3-4 but modes 5-6 are more chaotic. This is particularly evident in the POD mode magnitudes (Figure 4.28) and in the vorticity of the POD modes (Figure 4.29) plots. Figure 4.30 shows the magnitude of higher order POD modes. It is clear that the length scales decrease as the decomposition index increases.

4.6.3 POD Coefficients

Another way to interpret the POD results is by examining the behavior of the POD coefficients as a function of time, $a(t)$. The POD coefficients are the projection of the POD modes onto the velocity basis. Thus the coefficients have dimensions of velocity. There are $N - 1$ coefficients ($N$ is the number of snapshots) and each coefficient has a different value for each snapshot. Thus, if the snapshots are time-resolved, the POD coefficients represent the amplitude of the modes at each time in the snapshot sequence. For example, coefficient 2 is the second coefficient in the decomposition order and takes 399 different values (for $N = 400$) depending on the time step. The same is true for all other coefficients. Thus the temporal behavior of the coefficients indicate how much each mode contributes to each snapshot in the sequence. For non-time-resolved data, the evolution of the coefficients is with respect to the snapshot index. Figure 4.31 presents the snapshot history of the first four coefficients and for coefficient 20 for snapshots 1-25. It is seen that coefficients 1-2 have significantly higher values than the other. Similarly, coefficients 3-4 have higher values than coefficient 20. This is again a consequence of the turbulent kinetic energy cascading to smaller length scales. The points themselves seem to be random. This is due to the fact that the data is not time-resolved. Each snapshot captures the vortex shedding at random points in the vortex shedding cycle. Thus the coefficient values reflect the random behavior of the acquisition. If the data were time-resolved with respect to the vortex shedding cycle, the curves for each coefficient history would be sinusoids. Furthermore, the wavelength shift between each mode pair would be clearly seen in the phase shift between the sinusoids of each coefficient pair. Bidan (2013) shows the above-mentioned features of the POD coefficient time history for a time-resolved LES simulation of a cylinder in crossflow in the limit cycle regime.

4.6.4 POD Phase Plots

A second way to interpret the POD coefficients is by means of phase plots. The coefficients are first normalized by square root of twice the corresponding eigenvalue. As each coefficient has $N - 1$ values, the normalized coefficients can be plotted against one another to determine how they are correlated. Figures 4.32-4.36 show different phase plots correlating the most energetic modes in the decomposition. Figure 4.32 shows the phase plot for modes 1,2. This plot shows that the points are arranged in a circle indicating similar amplitudes and a 90° shift between the two coefficients. This shift is seen in the signatures of the coherent structures in modes 1 and 2. The correlation between modes 3 and 4 in Figure 4.33 seems to be weak due to the scattering of the data points. The same behavior is observed on the correlation between coefficients 5 and 6. The correlation between modes 1 and 3 in Figure 4.34 reveals the relative wavelength
Figure 4.26: Suction Side Near the Leading Edge - POD $u$ modes 1-6.
Figure 4.27: Suction Side Near the Leading Edge - POD $v$ modes 1-6.
Figure 4.28: Suction Side Near the Leading Edge - Magnitude of POD modes $\|\Psi\| = \sqrt{\Psi_u^2 + \Psi_v^2}$ 1-6.
Figure 4.29: Suction Side Near the Leading Edge - Vorticity of POD modes $\frac{\partial \Psi}{\partial x} - \frac{\partial \Psi}{\partial y}$ 1-6.
Figure 4.30: Suction Side Near the Leading Edge - Magnitude of higher order POD modes, $||\Psi|| = \sqrt{\Psi_u^2 + \Psi_v^2}$. 
of mode 3 with respect to the wavelength of mode 1. The black solid lines are where the points are more closely grouped together. The red dotted lines are vertical and horizontal lines. It is seen that the horizontal red line crosses the black lines six times while the vertical red line crosses the black lines twice. Thus the relative wavelength of mode 1 to mode 3 is $6/2 = 3$. Thus mode 3 has $1/3$ of the wavelength of mode 1. This quantifies the relative sizes of the coherent structures in mode 3 with respect to those in mode 1. Since mode 3 and mode 4 represent an energy pair, it is expected that the phase plot correlating modes 1 and 4 has a similar behavior as Figure 4.34. This is shown in Figure 4.35 where the relationship shown in the black and red lines is $3:1$ regarding mode 1 to mode 4. A similar analysis of Figure 4.36 reveals a $2:1$ correlation between modes 1 and 5.

### 4.7 DMD

#### 4.7.1 Acquisition Technique

The DMD is primarily used to study the temporal evolution of the underlying dynamics of the flow. Thus the DMD spectrum is limited by the Nyquist-Shannon criterion. Chen et al. (2012) clearly identified the aliasing effects of running DMD on data with dynamics that act beyond the Nyquist frequency of the acquisition. The PIV data is not time resolved with respect to neither the wake passage time scale nor the vortex shedding time scale. The plates translate at $1 \text{ m/s}$ which, given the pitch of the plates, results in a wake passage frequency of $6.56 \text{ Hz}$. The vortex shedding frequency was identified by Foreman (2013) as $\sim 950 \text{ Hz}$, using hot wire anemometry surveys in the space between the plates and the leading edge of the cascade. Each PIV double-frame acquisition happens every other plate passage. Thus the acquisition frequency is $3.28 \text{ Hz}$ which results in a Nyquist frequency of $1.64 \text{ Hz}$. Given the low Nyquist frequency of the phased-locked acquisitions, phased-locked data is not used in the DMD. The PIV laser available is capable of a maximum of $10 \text{ Hz}$ firing frequency in double-frame mode. This results in a $5 \text{ Hz}$ Nyquist frequency. Thus neither the wake passage frequency nor the vortex shedding frequency would be resolved using a constant acquisition frequency of $10 \text{ Hz}$.

A different approach was implemented. The wake passage was broken into 10 equidistant phases: $10\%$, $20\%$, $30\%$, ..., $100\%$. 450 phased-locked double-frame images were taken at each phase. The velocity matrix

![Figure 4.31: Suction Side Near the Leading Edge - POD coefficients. The snapshot index is denoted by $m$.](image-url)
Figure 4.32: Suction Side Near the Leading Edge - POD coefficient phase plots - Correlation between modes 1 and 2.

Figure 4.33: Suction Side Near the Leading Edge - POD coefficient phase plots - Correlation between modes 3 and 4.
Figure 4.34: Suction Side Near the Leading Edge - POD coefficient phase plots - Correlation between modes 1 and 3.

Figure 4.35: Suction Side Near the Leading Edge - POD coefficient phase plots - Correlation between modes 1 and 4.
was formed by taking one snapshot from the 10% phase, then one snapshot from the 20% phase and so on until the snapshot from the 100% phase. Then a new snapshot was taken from the 10% phase and the process was repeated. Thus the velocity matrix was ordered in cycles of the wake passage. In total 50 cycles were used. The time scale for the DMD is the temporal separation between adjacent phases, 0.01524 seconds which results in an “acquisition” frequency of 65.62 Hz and a Nyquist frequency of 32.81 Hz. This frequency is enough to identify the wake passage frequency. The vortex shedding frequency was determined to be too high to apply this acquisition technique. It would require phased-locked acquisitions at least 2000 different phases in the wake passage to apply the DMD. Unless otherwise specified, all the following results use acquisition data gathered and ordered as explained in this paragraph.

### 4.7.2 Use of Mean-Subtracted Data

As explained by Chen et al. (2012), using mean subtracted data has unexpected consequences on the DMD. The snapshot matrix becomes rank deficient, even if the original data is full rank. All Ritz values are then determined completely by the number of snapshots rather than by the content of the snapshots. This is seen in Equation 4.7 where the Ritz values are only dependent on $N$, the number of snapshots. In Equation 4.7, $j$ is the decomposition index and $i$ is the imaginary number. Equation 4.7 shows that the Ritz values are purely imaginary and thus the DMD modes have no growth/decay rate. Chen et al. (2012) emphasizes the consequence of performing the DMD on a linearly-independent data set with no mean: the DMD is equivalent to a temporal discrete Fourier transform (DFT). This severely limits the output of the DMD. No growth/decay information is gained and the spectrum is composed of equidistant and predetermined frequencies as is the norm in DFT analyses. In contrast, DMD on mean-added data has a spectrum that can contain any frequencies below the Nyquist frequency. On mean-subtracted DMD, the Ritz values are purely imaginary with a magnitude of 1, thus all points in the eigenspectrum will be on the unit circle. Thus the predetermined frequencies and lack of growth/decay information is a direct consequence of the mean subtraction and the results are then wholly independent of the fluid dynamics captured in the snapshots.

The use of mean-subtracted data was tried on both phased-locked data and data taken across the phases of the wake passage. The results are not presented in this publication but both sets yielded an eigenspectrum.
concentrated on the unit circle and a stability spectrum with all modes being marginally stable. Furthermore the Ritz values were very close to those theorized by Equation 4.7. Thus the use of mean-subtracted data will no longer be considered and all the following results used mean-added data.

\[ \mu_j = \exp \left( \frac{2\pi ij}{N} \right) \rightarrow \sigma_j = 0 \] 

\[ (4.7) \]

### 4.7.3 Scaling of the Modes

The reconstruction of the snapshots using the DMD modes and eigenspectrum requires knowledge of the modal amplitudes. Through analysis of the data, it was found that if the modes are not scaled by the amplitudes, then all the modes have similar magnitudes and the coherent structures in the flow appear at any modal index. Thus it is hard to discern which modes are more important to the dynamics of the flow. The modal amplitudes are needed to make this distinction. The modal amplitudes can be calculated in different ways. In this study, they were calculated using the method presented by Jovanović et al. (2014). The amplitudes are calculated using an algorithm that minimizes the square of the Frobenius norm of the error in the snapshot reconstruction. All the following results have the modes scaled by the amplitudes.

The mean flow is mode 1. The other modes represent coherent structures in the flow with varying degrees of spatial wavelength. The most coherent and largest scale structures can appear at any modal index but, due to the scaling, the other modes have lower scale structures. Thus scaling the modes identifies the modes that represent the largest scale structures. The scaled modes can be sorted in descending order of the complex modulus of the amplitudes to identify the most important modes. If the modes are sorted, then the mean mode (mode 1 of the unsorted modes) can appear at any modal index, even though typically it appears within the first 20 modes. Sorting the modes allows the ranking of the modes in terms of the size of the coherent structures in them. Since the sorting only rearranges the modes, the following results are sorted modes.

### 4.7.4 Results

#### 4.7.4.1 Convergence

The convergence of the decomposition is seen in how the DMD residual behaves as the number of snapshots decreases. The DMD residual arises due to the assumption that the last snapshot is a linear combination of the previous snapshots. The residual \( R \) is defined in Equation 4.8 as the difference between the second snapshot matrix and the reconstructed snapshots. Note that the residual arises entirely from the reconstruction of the last snapshot. In Figure 4.37, the Frobenius norm of the residual \( R \) is normalized by the Frobenius norm of the second snapshot matrix, \( V_2 \), is plotted versus the number of wake passage cycles, up to 100 cycles. It is clearly seen that the decomposition error reaches 1% after only 10 cycles and by 50 cycles the error is \( \sim 0.15\% \). The fluctuations in the error happen because of the high susceptibility of the DMD to changes in the last snapshot. Every time a new cycle is added, a new last snapshot is used and this results in fluctuations in the residual.

\[ R = V_2 - D_{\alpha} \Psi V_{\text{and}} \] 

\[ (4.8) \]

#### 4.7.4.2 Spectral Quantities

The eigenspectrum shown in Figure 4.38, where the imaginary and real parts of the Ritz values are plotted against each other. Most of the modes are on the unit circle and are thus marginally stable. Four modes are unstable (outside the unit circle) and a few modes are stable (inside the unit circle). This is confirmed by the stability spectrum in Figure 4.39 which shows two unstable modes \( (\sigma > 0) \). Since the stability spectrum is symmetric about the frequency axis only two unstable modes appear. The other two
unstable modes appear on the negative frequency side which is not shown in Figure 4.39. The symmetry of
the spectrum is a result of using real data as input to the decomposition.

The amplitude spectrum shown in Figure 4.40 shows the complex modulus of the amplitude of the modes
versus the frequency. The wake passage frequency of 6.56 Hz is shown in a green circle as having a smaller
magnitude than neighboring peaks. Another spectral quantity is the energy spectrum formed with the $L^2$

norm of the unscaled dynamic modes (Figure 4.41). This quantity is normalized by the number of snapshots,
$N$, and the Frobenius norm of the matrix of snapshots $U$. Again the peak representing the wake passage is
obscured by other peaks. In order to mitigate the effects of high energy but fast-decaying modes, Tu et al.
(2014b) proposed scaling the mode norms by the magnitude of their Ritz value to the power $N − 1$. This
reduces the peak height of those modes with large energy (large norm) but which decay quickly as quantified
in their Ritz values. Using this scale, the wake passage frequency is now a clear peak above the background
frequencies. Thus the wake passage frequency has been clearly identified with the DMD. The peak at the
origin ($f = 0$) is the mean flow while the other peaks are believed to be aliased peaks since the flow contains
important spectral content beyond the Nyquist frequency.

### 4.7.4.3 Dynamic Modes

The first mode (Figure 4.43) is the mean flow. It is a real mode (no imaginary component). The mode
is marginally stable ($\sigma = 0$) and has no oscillation ($f = 0$) as expected. Mode 2 (Figure 4.44) is also a real
mode with no oscillation. It has the highest decay rate of the decomposition. Thus it is not an important
mode since it does not contribute to the long term dynamics of the flow. Modes 3 and 4 are shown in Figures
4.45-4.47. These modes are conjugate pairs of each other. This means the real components are exactly the
same and the imaginary components are the negatives of each other. This is true for most modes in the
decomposition. The modes that correspond to the wake passage frequency peak in Figure 4.42 are modes
39 and 40. The real and imaginary component of mode 39 are shown in Figures 4.48-4.49.

Beyond the usefulness of the spectral quantities, the modes don’t provide much information. The
signatures of the modes are chaotic and thus are difficult to interpret. This is probably due to the
Figure 4.38: Suction Side Near the Leading Edge - DMD eigenspectrum. The green circle is the unit circle denoting marginally stable modes.

Figure 4.39: Suction Side Near the Leading Edge - DMD stability spectrum.
Figure 4.40: Suction Side Near the Leading Edge - DMD amplitude spectrum. The green circle denotes the wake passage frequency.

Figure 4.41: Suction Side Near the Leading Edge - DMD energy spectrum. The green circle denotes the wake passage frequency.
contamination of the spectrum with aliased frequencies arising from the faster dynamics in the flow. DMD is best implemented on time-resolved data.

4.7.5 Concluding Remarks on DMD

The DMD has proved to be capable of identifying the wake passage frequency but the spectrum contains aliased frequencies which make interpretation of the dynamic modes difficult. The principal issue is that the DMD is being applied to sub-Nyquist-rate PIV data when the DMD is restricted by the Nyquist-Shannon criterion. The Nyquist frequency of the PIV data is too low to adequately resolve the temporal evolution of the coherent structures that dominate the dynamics of the flow. To deal with this issue, Tu et al. (2014a) developed a technique based on compressive sampling and an orthogonal matching pursuit algorithm to perform spectral analysis on sub-Nyquist-rate data. This algorithm is going to be investigated in the future.

4.8 Concluding Remarks

The periodic wake has been studied with all the available experimental and post-processing techniques near the leading edge of the suction side. This location provides the spatial domain where the clearest signature of the wake can be obtained, given the equipment available. The wake is starting to enter the cascade and thus has been minimally affected by the pressure gradient in the cascade flow. Furthermore, all important phases in the wake passage can be studied. An in-depth acquisition, analysis, and post-processing campaign was conducted at this spatial domain to fully characterize the wake.

The velocity data yielded the mean and RMS velocity fields plus the vorticity field. This is turn led to the mean velocity profile and maximum velocity deficit in the wake. The POD identified the most energetic modes representing the vortex shedding wavelength and its harmonics of the wake generator plates. The DMD confirmed the wake passage frequency but the contamination of the data with high frequency content beyond the Nyquist frequency yielded little information on the dynamic modes.
Figure 4.43: Suction Side Near the Leading Edge - DMD - Mode 1. $\mu = 0i + 0.999$
Figure 4.44: Suction Side Near the Leading Edge - DMD - Mode 2. $\mu = 0i + 0.133$
Figure 4.45: Suction Side Near the Leading Edge - DMD - Mode 3 - Real Components. $\mu = 0.813i - 0.435$
Figure 4.46: Suction Side Near the Leading Edge - DMD - Mode 3 - Imaginary Components. $\mu = 0.813i - 0.435$
Figure 4.47: Suction Side Near the Leading Edge - DMD - Mode 4 - Imaginary Components. $\mu = -0.813i - 0.435$
Figure 4.48: Suction Side Near the Leading Edge - DMD - Mode 39 - Real Components. $\mu = 0.589i + 0.805$
Figure 4.49: Suction Side Near the Leading Edge - DMD - Mode 39 - Imaginary Components. \( \mu = 0.589i + 0.805 \)
Chapter 5
Wake Characterization at Downstream Domains of the Suction Side

5.1 Domain B: LE - x45mm

The next domain to study is Domain B. For Domain B, the camera was shifted 45 mm to the left (in the negative $x$ direction) from the original domain at the leading edge (denoted as LE). For this domain only the phased-averaged velocity results will be presented.

The results are shown in Figures 5.1-5.7. The results are grouped by variables. Each figure contains different phases of one variable, $u$ velocity for example. The phases chosen were 8%, 24%, 64%, 76%, 80%, 86%, 90%, 96%, and 100%. The wake is in view of the domain during phases 64% through 100%. Phases 8% and 24% represent phases without the wake in the domain. The grid is again normalized by the axial chord, $C_x = 0.1524$ m, and the velocities by the freestream velocity, $U_\infty = 48.0$ m/s; the vorticity is normalized by both. The RMS velocities are made non-dimensional by the local velocity magnitude, $\|U\|$, to obtain the local measure of the turbulent intensities. The turbulent kinetic energy per unit mass, $k$, in Figure 5.7 is defined in Equation 4.6 and is non-dimensionalized by the square of the local velocity magnitude. In Figure 5.7, the dimensionless $k$ is multiplied by 100 to obtain the percent turbulent kinetic energy. The curved surface at the top of each plot is the blade surface. The two straight lines bounding the domain are the limits of laser sheet illumination.

The primitive variables in the flow field are shown in Figures 5.1-5.2 show only slight disturbances in the passage of the wake. The wake is in the domain in phases 64%-100%. The $v$ velocity field shows more clearly the overall direction of the wake as it is convected downstream by the main flow. To visualize the position of the wake, the vorticity field or the RMS velocities are more useful. In Figure 5.4, the 8% and 24% phase show a slight, but noticeable, vorticity in the base flow. The source of this is unknown but it is believed to be related to the slight background RMS fluctuations seen on the same phases on Figures 5.5-5.6. The RMS velocities show a decrease on the higher phases (86%-100%) with respect to the other phases with the wake on the domain. This is due to the wake impinging on the leading edge of the test blade. At phase 86%, the wake is starting to “feel” the presence of the leading edge and part of the wake is impinging on the leading edge. This leads to the decreased RMS values downstream. Another contributing factor is the increased velocity near the blade. Since the RMS plots are normalized by the local velocity magnitude, the plots show a measure of the turbulent intensity. As the flow accelerates near the blade, the turbulence intensity drop down accordingly, as seen in the $k$ plot, Figure 5.7. This effect is increasingly noticeable as the wake moves closer to the leading edge.

5.2 Image Patching Results

The image patching technique was applied to four domains (Table 5.1). These domains are very close together thus they have significant overlap between them. This was chosen because this would reduce the error in the unreliable areas in the patched domain. Given the overlap of the domains, the resultant size of the patched domain is not much larger ($\sim 0.6x/C_x$ and $\sim 0.3y/C_x$) than the size of the individual domains ($\sim 0.4x/C_x$ and $\sim 0.25y/C_x$).

The areas of unreliable data are the seams, where the domains meet. Even after careful pre-processing of the vector fields, where all edges of the individual domains were generously masked from the averaging operation, some mismatch in velocities at the seams is still present. This effect is seen in the patched $u$ velocity field in Figure 5.8. The vertical seam at $x/C_x \approx 0.05$ shows a slight jump of velocity (in negative values). The jump is also seen in the velocity magnitude plot in Figure 5.9. The $v$ velocity plot does not show this jump. The vorticity plot (5.9) clearly shows the location of the seams. The calculation of the
Figure 5.1: Suction Side - LE - x45 mm (Domain B) - Phased-Averaged Contour Plots - $u$
Figure 5.2: Suction Side - LE - x45 mm (Domain B) - Phased-Averaged Contour Plots - $v$
Figure 5.3: Suction Side - LE - x45 mm (Domain B) - Phased-Averaged Contour Plots - $\|U\|$
Figure 5.4: Suction Side - LE - x45 mm (Domain B) - Phased-Averaged Contour Plots - $\omega_2$
Figure 5.5: Suction Side - LE - x45 mm (Domain B) - Phased-Averaged Contour Plots - $u_{rms}$
Figure 5.6: Suction Side - LE - x45 mm (Domain B) - Phased-Averaged Contour Plots - $v_{rms}$
Figure 5.7: Suction Side - LE - x45 mm (Domain B) - Phased-Averaged Contour Plots - $k$
Table 5.1: Image patching domains. The domain shifts are measured with respect to the leading edge domain (LE Domain).

<table>
<thead>
<tr>
<th>Domain Name</th>
<th>Domain Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.1 (LE Domain)</td>
<td>0</td>
</tr>
<tr>
<td>P.2</td>
<td>z10 mm</td>
</tr>
<tr>
<td>P.3</td>
<td>x30 mm</td>
</tr>
<tr>
<td>P.4 (Domain B)</td>
<td>x45 mm</td>
</tr>
</tbody>
</table>

partial derivatives introduces more noise in the signal and thus significant erroneous values are found at the seams. The RMS quantities are shown in Figure 5.10. The primary purpose of these plots is to show the dissipation of the turbulent fluctuations in the streamwise direction of the base cascade flow. This is most evident in the \( k \) plot where near the blade leading edge \( k \approx 5\% \) while near the left end of the domain \( k \approx 2.0\% \). Overall, the image patching technique has been validated and its limitations have been exposed.
Figure 5.10: Suction Side - Image Patching - 80% Phase - Phased-Averaged Contour Plots - $u_{rms}$, $v_{rms}$, and $k$

### 5.3 Domain C: LE - x75 mm

#### 5.3.1 Phased-Averaged Results

In this domain, the blade surface is located on the top right corner of the contour plots. The baseline phase is 8% and the wake is visible for phases 48% to 100%. The wake passage is hard to discern in the $u$ velocity plots (Figure 5.11). The position of the wake is best seen at 64% phase. The $v$ velocity contours in Figure 5.12 clearly show the velocity deficit in the wake as it approaches the blade. At 95% and 100% phase, most of the wake is outside the domain and the profile not seen in the $u,v$ plots. The velocity magnitude plots (Figure 5.13) show only a slight acceleration of the flow in the streamwise direction. The vorticity plots in Figure 5.14 show the passage of the wake. For phases 96% and 100%, the vorticity signature of the wake is not seen in the domain. The RMS quantities again show the decreased turbulence intensities as the wake approaches the blade. This is due to the mean flow accelerating near the blade. The turbulent kinetic plot shows a peak of 4% at 48% phase. The maximum at 64% is about 3% which compares well to the results on Domain B.
Figure 5.11: Suction Side - LE - x75 mm (Domain C) - Phased-Averaged Contour Plots - $u$
Figure 5.12: Suction Side - LE - x75 mm (Domain C) - Phased-Averaged Contour Plots - \( \nu \)
Figure 5.13: Suction Side - LE - x75 mm (Domain C) - Phased-Averaged Contour Plots - $\|U\|$
Figure 5.14: Suction Side - LE - x75 mm (Domain C) - Phased-Averaged Contour Plots - $\omega_z$
Figure 5.15: Suction Side - LE - x75 mm (Domain C) - Phased-Averaged Contour Plots - $u_{rms}$
Figure 5.16: Suction Side - LE - x75 mm (Domain C) - Phased-Averaged Contour Plots - $v_{rms}$
Figure 5.17: Suction Side - LE - x75 mm (Domain C) - Phased-Averaged Contour Plots - $k$
Table 5.2: Suction Side - LE - x75 mm (Domain C) - 60% Phase - POD energies for first six modes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Energy Content (%)</th>
<th>Cumulative Energy Content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28.66</td>
<td>28.66</td>
</tr>
<tr>
<td>2</td>
<td>27.86</td>
<td>56.52</td>
</tr>
<tr>
<td>3</td>
<td>5.70</td>
<td>62.22</td>
</tr>
<tr>
<td>4</td>
<td>4.53</td>
<td>66.75</td>
</tr>
<tr>
<td>5</td>
<td>3.91</td>
<td>70.66</td>
</tr>
<tr>
<td>6</td>
<td>2.27</td>
<td>72.93</td>
</tr>
</tbody>
</table>

5.3.2 POD Results

The POD results are presented for this domain because this is the first domain to show a breakdown of the mode pairing beyond the leading order pair. The POD was calculated with the phased-locked vector fields at 60% phase since at this phase, the wake passes through the middle of the domain.

5.3.2.1 Energies

The modal energies for the leading six modes are presented in Table 5.2. A graph of all the energies is shown in Figure 5.18. The first two modes are clearly a mode pair. They have very similar energies and together account to close to 50% of the total turbulent kinetic energy in the flow. A significant drop in energy occurs for the other modes. Based only on the energy content of modes 3-6, it is difficult to ascertain which modes form mode pairs.

Compared to the energy distribution of the modes calculated at the leading edge (Table 4.4), the energies of the leading mode pair have decreased ∼25% for mode 1 and ∼15% for mode 2. Furthermore, the energy distribution of the higher order modes is uneven and random. Whereas at the leading edge, modes 1-2 accounted for ∼71% of the energy, at this domain, modes 1-2 only account for ∼57%. This suggests a redistribution of the turbulent kinetic energy among the modes. Given the much smaller energy content of the leading order modes and the higher number of modes required for energy convergence, this suggests the energy redistribution happens through the energy cascade to smaller length scales (and thus higher modal index). This redistribution is uneven as seen in the energy of modes 3-6. A similar pattern in the energy distribution was found in an experiment with a stationary, thick plate with blunt trailing edge by Doddipatla (2010).

5.3.2.2 Modes

Modes 1-6 are shown in Figures 5.19-5.22. Modes 1-2 appear to be of the same wavelength as Modes 1-2 in Domain A. These are the leading order modes and thus have the wavelength corresponding to the vortex shedding frequency of wake generator plates. The modes appear to have been deformed in the streamwise direction for the $u$ modes and in the cross-stream direction in the $v$ modes. The mechanism for this may be the pressure-gradient flow present in the cascade. In the $u$ modes, modes 3 and 6 appear to have no companion modes, while modes 4-5 appear to be mode pairs. In the $v$ modes, again modes 4-5 appear as shifted pairs while mode 6 has no companion. The signature of mode 3 is similar to those of modes 4-5 but is less elongated in the cross-stream direction. This is most likely due to the similar energies of modes 3-5. The vorticity of the POD modes (Figure 5.22) confirms this. Mode 3 has areas with slightly less negative rotation than similar areas in modes 4-5. The lone mode 3 was also observed by Doddipatla (2010).

5.4 Domain D: LE - x75 mm + z90 mm

In this domain, the blade surface is located on the gray areas of the contour plots (Figures 5.23-5.29). The baseline phase is 30% and the wake is visible for phases 70% to 100%. The significance of phases 10%
and 20% will be explained below. This domain is located near the transition point. The pressure gradient in the cascade flow becomes an adverse pressure gradient at the transition point.

At 30%, the $u$ velocity plot (Figure 5.23) shows a change in sign due to the turning of the flow around the profile of the blade. The $v$ velocity plot in Figure 5.24 shows a deceleration from $\sim 1.9v/U_\infty$ near the bottom the domain (at the transition point) to $\sim 1.7v/U_\infty$ near the top of the domain. The same deceleration is seen in Figure 5.25 for the velocity magnitude. Negligible values of vorticity (Figure 5.26), RMS quantities (Figures 5.27-5.28), and turbulent kinetic energy (Figure 5.29) are seen at 30% phase. For phases 70% to 90% the wake is visible in the domain and disturbs the base flow accordingly.

At 100% phase, velocity field is significantly changed from the 90% flow field. The $u$ velocity direction change is abruptly inhibited near $y/C_x = 0$. The $v$ velocity is also significantly more reduced than at other phases. In the velocity magnitude plot, a small region develops near the top right corner of very low velocity ($\|U\| \lesssim 1.5U_\infty$). This indicates that the boundary layer has separated from the surface of the blade. This is clearly seen in the significant increase in vorticity near the surface for 100% phase. The RMS and $k$ plots confirm this. This boundary layer separation is due to the wake impinging on the leading edge. The cascade is designed for an angle of attack of 0°. Previous coefficient of pressure measurements at the blade surfaces performed by Foreman (2013) confirm that even a slight angle of attack of 1.5°-2° leads to boundary layer separation. The wake impinging on the blade causes unpredictable changes on the direction of the inlet flow into the cascade, thus changing the angle of attack and causing boundary layer separation. When the wake is not directly impinging on the leading edge (phases 30% to 90%), the boundary layer remains attached. For 100% phase, the boundary layer separates. This is clearly seen in the vorticity, RMS, and $k$ plots. At 10% phase, the effects of the wake impingement cause earlier boundary layer separation. At 20% phase the boundary layer separates slightly downstream. For 30% phase, the effects of the wake have passed and the boundary layer remains attached.
Figure 5.19: Suction Side - LE - x75 mm (Domain C) - 60% Phase - POD $u$ modes 1-6. Note the different contour scales on each mode.
Figure 5.20: Suction Side - LE - x75 mm (Domain C) - 60% Phase - POD \( v \) modes 1-6.
Figure 5.21: Suction Side - LE - x75 mm (Domain C) - 60% Phase - Magnitude of POD modes \( \|\Psi\| = \sqrt{\Psi_u^2 + \Psi_v^2} \) 1-6.
Figure 5.22: Suction Side - LE - x75 mm (Domain C) - 60% Phase - Vorticity of POD modes \( \frac{\partial \Psi_u}{\partial x} - \frac{\partial \Psi_v}{\partial y} \) 1-6.
Figure 5.23: Suction Side - LE - x75 mm + z90 mm (Domain D) - Phased-Averaged Contour Plots - $u$
For this domain, the blade surface is located on the gray areas of the contour plots (Figures 5.30-5.36). The baseline phase is 30% and the wake is visible for phases 70% to 100%. This domain is located near the trailing edge of the blade. The field of view is small at this location because the laser sheet cannot travel any further to the left.

The flow in this domain is dominated by a fast growing boundary layer. The boundary layer is thickening due to the adverse pressure gradient in the cascade flow at this location. The thickness of the boundary layer is clearly seen in the vorticity and RMS plots. In the vorticity plots (Figure 5.33), the blue areas are areas of high vorticity (below the contour scale) in the boundary layer. The deceleration of the flow in the boundary layer is shown in the $v$ velocity plots. The wake passage is shown in phases 70% to 90%.

As in Domain D, the wake triggers boundary layer separation for phases 100% to 20%. This effect is most clearly seen in the $v$ velocity plots, Figure 5.31. In the contour plots, the color of all negative values was replaced by black to identify flow reversal. If the $v$ velocity becomes negative near the surface of the blade, then there is flow reversal in the boundary layer and flow separation occurs. As seen in Figure 5.31, for the
Figure 5.25: Suction Side - LE - x75 mm + z90 mm (Domain D) - Phased-Averaged Contour Plots - $||U||$
Figure 5.26: Suction Side - LE - x75 mm + z90 mm (Domain D) - Phased-Averaged Contour Plots - $\omega_z$
Figure 5.27: Suction Side - LE - x75 mm + z90 mm (Domain D) - Phased-Averaged Contour Plots - $u_{rms}$
Figure 5.28: Suction Side - LE - x75 mm + z90 mm (Domain D) - Phased-Averaged Contour Plots - $v_{rms}$
Figure 5.29: Suction Side - LE - x75 mm + z90 mm (Domain D) - Phased-Averaged Contour Plots - \( k \)
baseline (30% phase) and phases 70% to 96% with the wake visible in the domain, there is no flow reversal. The first reversal occurs at 100% phase, with the wake directly impinging on the leading edge. The flow reversal also occurs for 10% phase and slightly for 20% phase as seen in Domain D. Thus the boundary layer separates due to wake impingement on the leading edge. Otherwise, the boundary layer remains attached throughout the entire suction side of the airfoil.

5.6 Concluding Remarks

The results on the suction side downstream of the leading edge were presented in this chapter. An initial implementation of the image patching algorithm with four domains was also presented. The technique was successful in computing the average vector field. Slight discontinuities are present near the seams of the individual domains.

Further downstream of the leading edge, the POD modes are shown to become more chaotic and less energetic. The leading order mode pair loses close to half of its energy to lower order modes due to the cascading of turbulent kinetic energy to lower spatial scales and to viscous dissipation losses. The transition point in the switch in the sign of the pressure gradient in the cascade flow is identified. The wake passage is
Figure 5.31: Suction Side - LE - x75 mm + z150 mm (Domain E) - Phased-Averaged Contour Plots - $v$
Figure 5.32: Suction Side - LE - x75 mm + z150 mm (Domain E) - Phased-Averaged Contour Plots - $\|U\|$
Figure 5.33: Suction Side - LE - 75 mm + z150 mm (Domain E) - Phased-Averaged Contour Plots - $\omega_z$
Figure 5.34: Suction Side - LE - x75 mm + z150 mm (Domain E) - Phased-Averaged Contour Plots - $u_{rms}$
Figure 5.35: Suction Side - LE - x75 mm + z150 mm (Domain E) - Phased-Averaged Contour Plots - $v_{rms}$
Figure 5.36: Suction Side - LE - x75 mm + z150 mm (Domain E) - Phased-Averaged Contour Plots - $k$
found to severely disturb the boundary layer. When the wake is impinging on the leading edge, the boundary layer separates near the transition point. This is due to the chaotic and unpredictable change in the angle of attack of the test blade. As expected for an ultra-high-lift, aft-loaded LPT airfoil at high Reynolds number, the boundary layer remains completely attached to the trailing edge when the wake is not impinging on the leading edge.
Chapter 6
Concluding Remarks and Future Work

The objective of this study was to experimentally study film cooling. A closed-loop wind tunnel with a four passage linear cascade of US Air Force Research Laboratory (AFRL) ultra-high-lift, aft-loaded L1A low pressure turbine (LPT) blades and upstream wake generator was used in conjunction with Particle Image Velocimetry (PIV) flow visualization technique to study turbulent film cooling flows due to the interaction between vanes and blades. Further post-processing in the form of Proper Orthogonal Decomposition (POD) and Dynamic Mode Decomposition (DMD) modal analyses was performed to determine the relevant modes that characterize the coherent structures in the flow. An image patching algorithm was also implemented. The results obtained are used to characterize the periodic wake on the cascade flow.

The periodic wake has been studied with all the available experimental and post-processing techniques near the leading edge of the suction side. An in-depth acquisition, analysis, and post-processing campaign is conducted at this spatial domain to fully characterize the wake. The velocity data led to the mean velocity profile and maximum velocity deficit in the wake. The POD identified the most energetic modes representing the vortex shedding wavelength, and its harmonics, of the wake generator plates. The DMD confirmed the wake passage frequency but the contamination of the data with high frequency content beyond the Nyquist frequency yielded little information on the dynamic modes.

An initial implementation of the image patching algorithm with four domains is presented. The technique was successful in computing the average vector field. Slight discontinuities are present near the seams of the individual domains. Further downstream of the leading edge, the POD modes are shown to become more chaotic and less energetic. The leading order mode pair loses close to half of their energy to lower order modes due to the cascading of turbulent kinetic energy to lower spatial scales and to viscous dissipation losses. When the wake is impinging on the leading edge, the boundary layer separates near the transition point. The boundary layer remains completely attached to the trailing edge when the wake is not impinging on the leading edge.

The next step in the acquisitions is to perform steady blowing film cooling studies. A concurrent CFD study needs to be conducted to verify the results presented here and to compare to the steady film cooling studies at different blowing ratios. Following the steady blowing parametric studies, forced film cooling is the end objective of the flow field studies. Timing software for acquisitions and experiment modifications are prerequisites for the forced film cooling studies. The time-resolved CFD study would provide the necessary data for developing reduced order models based on the POD and/or DMD. An active film cooling control scheme of the boundary layer separation due to the wake passage is the ultimate goal of the flow field studies.
References


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General Electric 2014 Gas Turbine During Assembly. Picture.


MEYER, K. E., CAVAR, D. & PEDERSEN, J. M. 2007 POD as Tool for Comparison of PIV and LES Data. 7th International Symposium on Particle Image Velocimetry 07 (1), 1–12.


Nikitopoulos, D. E. & Garrison, T. J. 2005 Particle Based Methods. Class Notes, Louisiana State University, Baton Rouge, LA, US.


PRATT & WHITNEY 2003 F-100 During Test. Picture.


Swiss Federal Institute of Technology, Zurich 2005 Film Cooling of Turbine Blades in Gas Turbines. Picture.


Appendix
Wake Characterization on the Pressure Side

The flow in the pressure side is very different from that in the suction side. The pressure-gradient is an adverse pressure gradient near the leading edge, followed by an almost zero pressure gradient and finally a favorable pressure gradient to the trailing edge. The freestream velocity for all pressure side domains is $U_\infty = 46.5$ m/s.

6.1 Domain F: Pressure Side Leading Edge

6.1.1 Phased-Averaged Results

In the first domain, the baseline flow is 60% phase. The wake is visible for phases 100% to 30%. The plots are shown in Figures A.1-A.5. The blade is the curved surface at the bottom of the domain while the left edge of the laser is also masked gray. The baseline flow shows a decelerating flow in the streamwise direction. The wake is captured in its entirety at the 15% phase. The passage of the wake captured by the vorticity fields shows a highly distorted wake signature. Figure A.4 shows 100% to 10% phases to have unexpected signature. By 15% phase, the wake starts to return to its characteristic banded signature. It can be seen that from 15% phase to 25% phase, the positive and negative vorticity bands come closer together. This behavior is because of the presence of the blade. As the wake passes through 90% phase, it starts to feel the presence of the blade and is disturbed accordingly. As explained below, this has a significant effect on the POD modes. The $k$ results (Figure A.5) also show significantly different results than on the suction side. The maximum $k$ value seen at 10% is close 60%. This is due to very low velocity ($\|U\|/U_\infty \approx 0.25$) in the flow near the blade. Since $k$ is non-dimensionalized by the local velocity magnitude, the $k$ values are an order of magnitude over those in the suction side. This is confirmed by the fact that the $k$ values decrease as the wake moves farther from the blade surface.

6.1.2 POD Results

The POD analysis was performed on the 15% phased-locked velocity snapshots. 450 snapshots were used in the decomposition.

6.1.2.1 Energies

The leading order energies are tabulated in Table A.1 and graphed in Figure A.6. The energy content of the modes seems random and no mode pairs can be identified by means of the energies alone. This is due to the interruption of the wake by the leading edge. The wake is disturbed and changed unpredictably and this is reflected in the POD modes.

6.1.2.2 Modes

The modes are shown in Figures A.7-A.10. Only the first four modes are shown. The modes are all chaotic and the coherent structures are skewed and deformed. The signature of $u$ mode 2 seems to correspond to the $\frac{\partial v}{\partial y}$ velocity gradient field while $v$ modes 3-4 are similar to the harmonics of the vortex shedding mode.
Figure A.1: Pressure Side - LE (Domain F) - Phased-Averaged Contour Plots - $u$

Table A.1: Pressure Side - LE (Domain F) - 15% Phase - POD energies for first six modes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Energy Content (%)</th>
<th>Cumulative Energy Content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.54</td>
<td>24.54</td>
</tr>
<tr>
<td>2</td>
<td>10.35</td>
<td>34.90</td>
</tr>
<tr>
<td>3</td>
<td>9.14</td>
<td>44.04</td>
</tr>
<tr>
<td>4</td>
<td>7.61</td>
<td>51.65</td>
</tr>
<tr>
<td>5</td>
<td>3.98</td>
<td>55.63</td>
</tr>
<tr>
<td>6</td>
<td>3.05</td>
<td>58.68</td>
</tr>
</tbody>
</table>
Figure A.2: Pressure Side - LE (Domain F) - Phased-Averaged Contour Plots - $v$

(a) 5% Phase  
(b) 10% Phase  
(c) 15% Phase  
(d) 20% Phase  
(e) 25% Phase  
(f) 30% Phase  
(g) 60% Phase  
(h) 95% Phase  
(i) 100% Phase
Figure A.3: Pressure Side - LE (Domain F) - Phased-Averaged Contour Plots - $\|U\|$
Figure A.4: Pressure Side - LE (Domain F) - Phased-Averaged Contour Plots - $\omega_z$
Figure A.5: Pressure Side - LE (Domain F) - Phased-Averaged Contour Plots - $k$
6.2 Domain G: Upstream of Leading Edge

Due to the abrupt change in the wake structure due the presence of the blade, a new domain that captures this change was studied. The domain is focused as far upstream from the leading edge as the laser optics will allow. The fore optics slot on the top PIV blade were used to illuminate this domain, thus the area below the blade is not reachable by the laser sheet. The plots are shown in Figures A.11-A.15. The baseline flow is 50% phase since the wake is not visible either above or below the blade in this domain. As seen in the velocity plots, as the wake approaches the leading edge from the bottom of the domain, the flow on the pressure side changes abruptly. At 100% the velocity on the pressure side decreases significantly near the surface compared to the 95% phase. At 5%, the velocity in the velocity reaches its minimum in the wake passage period. The $k$ plot shows a high value $k \approx 90\%$. This was not uncovered in the previous domain because it was too close to the blade and was assumed to be due to edge effect. The flow field recovers to its baseline flow as the wake moves away from the blade from 10% phase.

6.3 Domain H: Pressure Side LE - x50 mm + z55 mm

In this domain, the baseline flow is 80% phase. The wake is visible for phases 100% to 35%. The plots are shown in Figures A.16-A.20. The blade is the curved gray surface at the bottom left of the domain while the left edge of the laser is also masked gray. The flow is slightly accelerating in the streamwise direction. The wake passage is clearly seen in all three velocity plots, Figures A.16-A.20. The vorticity signatures shown in Figure A.19 show the characteristic positive and negative bands of vorticity. The $k$ plots in Figure A.20 show a maximum of 25% at 10% phase. This is again an effect of the low velocity in the domain. The maximum velocity in the domain is $\|U\|/U_\infty \approx 1.0$ while in the suction side, the maximum velocity at this axial chord location is closer to $\|U\|/U_\infty \approx 2.0$. 

Figure A.6: Pressure Side - LE (Domain F) - 15% Phase - POD energies.
Figure A.7: Pressure Side - LE (Domain F) - 15% Phase - POD $u$ modes 1-4.
Figure A.8: Pressure Side - LE (Domain F) - 15% Phase - POD $v$ modes 1-4.
Figure A.9: Pressure Side - LE (Domain F) - 15% Phase - Magnitude of POD modes $||\Psi|| = \sqrt{\Psi_u^2 + \Psi_v^2}$.
Figure A.10: Pressure Side - LE (Domain F) - 15% Phase - Vorticity of POD modes $\frac{\partial \Psi_x}{\partial x} - \frac{\partial \Psi_y}{\partial y}$. 1-4.
Figure A.11: Upstream of LE (Domain G)- Phased-Averaged Contour Plots - $u$
Figure A.12: Upstream of LE (Domain G)- Phased-Averaged Contour Plots - $v$
Figure A.13: Upstream of LE (Domain G) - Phased-Averaged Contour Plots - $\|U\|$
Figure A.14: Upstream of LE (Domain G)- Phased-Averaged Contour Plots - $\omega_z$
Figure A.15: Upstream of LE (Domain G)- Phased-Averaged Contour Plots - $k$
Figure A.16: Pressure Side - LE - x50 mm + z55 mm (Domain H) - Phased-Averaged Contour Plots - $u$
Figure A.17: Pressure Side - LE - x50 mm + z55 mm (Domain H) - Phased-Averaged Contour Plots - $v$
Figure A.18: Pressure Side - LE - x50 mm + z55 mm (Domain H) - Phased-Averaged Contour Plots - $||U||$
Figure A.19: Pressure Side - LE - x50 mm + z55 mm (Domain H) - Phased-Averaged Contour Plots - $\omega_z$
Figure A.20: Pressure Side - LE - x50 mm + z55 mm (Domain H) - Phased-Averaged Contour Plots - $k$

6.4 Domain I: Pressure Side LE - x50 mm + z110 mm

The baseline flow is 80% phase. The wake is visible for phases 100% to 35%. The plots are shown in Figures A.21-A.25. The blade is the curved gray surface at the bottom left of the domain. The flow is still accelerating up until the trailing edge since the pressure gradient is favorable. The maximum velocity is $\|U\|/U_\infty \approx 1.6$. The wake passage is best seen in the $v$ velocity and velocity magnitude plots, Figures A.22 and A.23, respectively. The vorticity plots show an alternating vorticity near the trailing edge of $\omega_z \approx \pm 2.0U_\infty/C_x$. The maximum $k$ value is 3% at 10% phase. Note that no boundary layer is detected in the contour plots. The boundary layer in the pressure side remains thin since most of the flow is in a zero or favorable pressure gradient.

6.5 Concluding Remarks

Near the leading edge, the wake is affected by the presence of the blade in its passage. This causes unpredictable changes in the flow on the pressure side, specially at phases near the surface of the blade (95%
Figure A.21: Pressure Side - LE - x50 mm + z110 mm (Domain I) - Phased-Averaged Contour Plots - $u$
Figure A.22: Pressure Side - LE - x50 mm + z110 mm (Domain I) - Phased-Averaged Contour Plots - $v$
Figure A.23: Pressure Side - LE - x50 mm + z110 mm (Domain I) - Phased-Averaged Contour Plots - $||U||$
Figure A.24: Pressure Side - LE - x50 mm + z110 mm (Domain I) - Phased-Averaged Contour Plots - $\omega_z$
Figure A.25: Pressure Side - LE - x50 mm + z110 mm (Domain I) - Phased-Averaged Contour Plots - $k$
phase to 10% phase). This effect and the low velocity near the leading edge combine to yield exceptionally high turbulent kinetic energy values. Furthermore, the POD basis becomes chaotic on the pressure side due to the effects explained above. The energy distribution and modes appear random and no mode pairs can be identified. The boundary layer remains thin throughout the pressure side and is not detected in the PIV contour plots.
Vita

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