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Dynamical symmetry of isobaric analog 0^+ states in medium mass nuclei

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An algebraic $\mathfrak{sp}(4)$ shell model is introduced to achieve a deeper understanding and interpretation of the properties of pairing-governed 0^+ states in medium mass atomic nuclei. The theory, which embodies the simplicity of a dynamical symmetry approach to nuclear structure, is shown to reproduce the excitation spectra and fine structure effects driven by proton-neutron interactions and isovector pairing correlations across a broad range of nuclei.

I. INTRODUCTION

A recent renaissance of interest in pairing correlations in atomic nuclei is linked to the search for a reliable microscopic theory for describing the structure of medium mass nuclei around the $N = Z$ line where protons and neutrons occupy the same major shell and their mutual interactions are expected to strongly influence the structure and decay modes of such nuclei. The revival of interest in pairing correlations is also prompted by radioactive beam experiments, which are advancing the exploration of ‘exotic’ nuclei, such as neutron-deficient or $N \approx Z$ nuclei far off the valley of stability. Likewise, such a microscopic framework is important for astrophysical applications, for example a description of the rp -process in nucleosynthesis, which runs close to the proton-rich side of the valley of stability through reaction sequences of proton captures and competing β decays [1, 2].

In this paper we show that a simple but powerful group theoretical approach, with $\text{Sp}(4)$ the underpinning symmetry, can provide a microscopic description and interpretation of the properties of pairing-governed 0^+ states in the energy spectra of the even- A nuclei with mass numbers $32 \leq A \leq 100$ where protons and neutrons are filling the same major shell. In this regard, it is important to recall that $\text{SO}(5)$ [3, 4] (with a Lie algebra that is isomorphic to $\mathfrak{sp}(4)$) has been shown to play a significant role in the structure of fp -shell $N = Z$ nuclei [5]. Indeed, a model based on this symmetry group can be used to track the results of an isospin-invariant pairing plus quadrupole shell-model theory [6].

A theory that invokes group symmetries is driven by an expectation that the wave functions of the quantum mechanical system under consideration can be characterized by their invariance properties under the corresponding symmetry transformations. But even if the symmetries are not exact, if one can find near invariant operators, the associated symmetries can be used to help reduce the dimensionality of a model space to a tractable size. Within the framework of the $\text{Sp}(4)$ symplectic group, an approximate symmetry drastically reduces the model space, which allows the model to be applied in a broad region of the chart of the nuclides [7]. This symmetry is adequate only for a certain class of phenomena, which in our inves-

tigation is related to significant isovector (isospin $T = 1$) pairing correlations in even- A nuclei. While the model is not valid for all the states in the energy spectra of the 319 nuclei we consider (nor can any model achieve this), it does yield a realistic reproduction (with only 6 parameters) of the pairing-governed isobaric analog 0^+ state spectra in medium mass nuclei where the valence protons and neutrons occupy the same major shell. The validity and reliability of the model with respect to the interactions it includes are confirmed additionally via a finite energy difference method that we employed to reproduce detailed nuclear structure, including $N = Z$ anomalies, isovector pairing gaps and staggering effects [8].

The present investigation shows the advantage of the algebraic $\mathfrak{sp}(4)$ approach over other theoretical studies. Namely, the $\text{Sp}(4)$ model, which is based on Helmer’s quasi-spin scheme [9], is both simpler and provides a better understanding of the fundamental nature of the nuclear interaction compared, for example, to the more general but also more elaborate $U(4\Omega)$ model [10] based on the conventional seniority scheme of Racah and Flowers [11, 12]. It also yields a better and more detailed microscopic description of isovector pairing correlations and proton-neutron interactions than mean-field theories and results extracted from semi-empirical mass formulae. In addition, it can be used in higher-lying shells where other approaches cannot be applied.

II. PAIRING MODELS: FROM $\text{SU}(2)$ TO $\text{Sp}(4)$

It has been recognized for a long time that ground states of even-even nuclei reflect strongly on the nature of the nuclear interaction, especially its propensity to form correlated, angular momentum $J = 0$ pairs [13, 14]. It is also well known that the low-lying energy spectrum of isotopes of a doubly magic core (such as ^{40}Ca or ^{56}Ni) can be well reproduced in terms of neutron pair [nn] addition to the spectrum of states of the core [49]. In complete analogy with this, if one adds to the ground state of the core $J = 0$, $T = 1$ pairs of nucleons (two protons pp , two neutrons nn , or a proton and a neutron pn), one would construct fully paired states that in general reflect the close interplay of like-particle and pn pairs. Provided

that the isospin is (almost) a good quantum number, which is typically the case for low-lying states in light and medium mass even- A nuclei with valence protons and neutrons simultaneously filling the same major shell, fully paired 0^+ states built in this way describe isobaric analog 0^+ states (*IAS*) in nuclei across the entire shell. In the mass range $32 < A < 100$ where the influence of shape deformation on these 0^+ *IAS* states is relatively weak, this notion of fully paired (seniority zero) states is a valid, albeit approximate picture. While in even-even nuclei within this region the ground states are such fully paired 0^+ states, this it is not always the case for odd-odd nuclei. The strong proton-neutron interaction usually drives the state with the least symmetry energy ($\sim T^2$, where T is nuclear isospin) lowest. However, it is not the purpose of this article to investigate such states. Rather, we consider the isobaric analog 0^+ state, which is typically higher in energy and is strongly influenced by isovector pairing correlations. Even though the states are represented as $J = 0$ pairs, the interaction that governs them is not exclusively pairing in the $J = 0$ channel, but must also include the important pn $J_{\text{odd}} \geq 1$ isoscalar ($T = 0$) interaction. The significant interplay between these isovector and isoscalar interactions is evident in the low-lying structure of $N = Z$ odd-odd nuclei and has been the focus of a large number of experimental [15, 16, 17, 18] and theoretical studies [6, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28].

The zero seniority 0^+ states can be constructed as ($T = 1$)-paired fermions

$$|n_{+1}, n_0, n_{-1}\rangle = \left(\hat{A}_{+1}^\dagger\right)^{n_{+1}} \left(\hat{A}_0^\dagger\right)^{n_0} \left(\hat{A}_{-1}^\dagger\right)^{n_{-1}} |0\rangle, \quad (1)$$

where $n_{+1,0,-1}$ are the numbers of $J = 0$ pairs of each kind, pp , pn , nn , respectively, and $|0\rangle$ denotes the vacuum state. The transition operator, which changes the number of particles in a pairwise fashion, $\hat{A}_{0,+1,-1}^\dagger$, creates a proton-neutron (pn) pair, a proton-proton (pp) pair or a neutron-neutron (nn) pair of total angular momentum $J^\pi = 0^+$ and isospin $T = 1$. Each operator \hat{A}_μ^\dagger , $\mu = 0, +1, -1$, together with its conjugate pair-annihilation operator, \hat{A}_μ , and a pair number operator generate an $SU^\mu(2)$ subgroup of $Sp(4)$, which in the case of $\mu = \pm$ is the standard like-particle pairing Kerman's $SU(2)$ group [29].

From a microscopic perspective, the pair-creation (pair-annihilation) operators, \hat{A}^\dagger (\hat{A}), are realized in terms of creation $c_{jm\sigma}^\dagger$ and annihilation $c_{jm\sigma}$ single-fermion operators with the standard anticommutation relations $\{c_{jm\sigma}, c_{j'm'\sigma'}^\dagger\} = \delta_{j,j'}\delta_{m,m'}\delta_{\sigma,\sigma'}$, where these operators create (annihilate) a particle of type $\sigma = \pm 1/2$ (proton/neutron) in a state of total angular momentum j (half integer) with projection m in a finite space $2\Omega = \Sigma_j(2j+1)$. There are ten independent scalar products

(zero total angular momentum) of the fermion operators:

$$\begin{aligned} \hat{A}_{\mu=\sigma+\sigma'}^\dagger &= \frac{1}{\sqrt{2\Omega(1+\delta_{\sigma\sigma'})}} \sum_{jm} (-1)^{j-m} c_{jm\sigma}^\dagger c_{j,-m,\sigma'}^\dagger, \\ \hat{A}_\mu &= (\hat{A}_\mu^\dagger)^\dagger, \\ \hat{T}_\pm &= \frac{1}{\sqrt{2\Omega}} \sum_{jm} c_{jm,\pm 1/2}^\dagger c_{jm,\mp 1/2}, \\ \hat{N}_{2\sigma} &= \sum_{jm} c_{jm\sigma}^\dagger c_{jm\sigma}, \end{aligned} \quad (2)$$

which form a fermion realization of the symplectic $\mathfrak{sp}(4)$ Lie algebra. Such an algebraic structure is exactly the one needed to describe isovector (like-particle plus pn) pairing correlations and isospin symmetry in nuclear isobaric analog 0^+ states. In (2), $\hat{N}_{\pm 1}$ are the valence proton (neutron) number operators. The generators \hat{T}_0 and \hat{T}_\pm are associated with the components of the isospin of the valence particles and close on an $\mathfrak{su}^T(2)$ subalgebra of $\mathfrak{sp}(4)$. In terms of the generators of the $Sp(4)$ group (2), the operator that counts the total number of valence particle n is expressed as $\hat{N} = \hat{N}_{+1} + \hat{N}_{-1}$ and the third isospin projection operator is $\hat{T}_0 = (\hat{N}_{+1} - \hat{N}_{-1})/2$.

While the $Sp(4)$ symplectic group embeds in itself the well-known symmetry of like-particle pairing, $Sp(4) \supset SU^+(2) \otimes SU^-(2)$, it brings into the theory the significant interaction between protons and neutrons through the reduction chains $Sp(4) \supset U^\mu(2) \supset U^\mu(1) \otimes SU^\mu(2)$ with $\mu = 0$ (proton-neutron pairing symmetry) and $\mu = T$ (isospin symmetry). These group reductions allow the $Sp(4)$ -invariant degenerate energy states to split, which is the case of physical interest. Such a dynamical symmetry that our model possesses provides for a natural classification scheme of nuclei as belonging to a single- j level or a major shell (multi- j), which are mapped to the algebraic multiplets. This classification also extends to the corresponding ground and excited states of the nuclei.

The general model Hamiltonian with $Sp(4)$ dynamical symmetry consists of one- and two-body terms and can be expressed through the $Sp(4)$ group generators,

$$\begin{aligned} H = & -G \sum_{i=-1}^1 \hat{A}_i^\dagger \hat{A}_i - F \hat{A}_0^\dagger \hat{A}_0 - \frac{E}{2\Omega} (\hat{T}^2 - \frac{3\hat{N}}{4}) \\ & - D(\hat{T}_0^2 - \frac{\hat{N}}{4}) - C \frac{\hat{N}(\hat{N}-1)}{2} - \epsilon \hat{N}, \end{aligned} \quad (3)$$

where $\hat{T}^2 = \Omega\{\hat{T}_+, \hat{T}_-\} + \hat{T}_0^2$ is the isospin operator, G, F, E, D and C are interaction strength parameters and $\epsilon > 0$ is the Fermi level energy. This Hamiltonian conserves the number of particles (n) and the third projection (T_0) of the isospin, while it includes scattering of a pp pair and a nn pair into two pn pairs and vice versa.

As we have shown in [7], the algebraic model Hamiltonian (3) arises naturally within a microscopic picture. Using relations (2), it can be rewritten in standard second quantized form, which in turn defines the physical nature of the interaction and its strength. For example, in this

way, one can identify the parameters G/Ω and $(G+F)/\Omega$ in (3) with the strength of the $J = 0$ $T = 1$ pairing interaction between two protons (neutrons) and a proton and a neutron, respectively. The C , D , and E parameters are related to the expectation value of an average J -independent interaction, which includes for example high- J like-particle interaction with a strength specified by the parameters $C + \frac{D}{2} + \frac{E}{4\Omega}$ [7].

Furthermore, the E term in (3), together with the C term, is related to the microscopic nature of the odd J isoscalar ($T = 0$) interaction between a proton and a neutron, $-\frac{E}{2\Omega}(\hat{T}^2 - \frac{3\hat{N}}{4}) - C\frac{\hat{N}(\hat{N}-1)}{2} = -\frac{E}{2\Omega}(\hat{T}^2 - \frac{\hat{N}}{2} - \frac{\hat{N}^2}{4}) - (C + \frac{E}{4\Omega})\frac{\hat{N}(\hat{N}-1)}{2}$, where the first part comprises the J -independent pn isoscalar force. It is diagonal in the isospin basis and can be compared to [30, 31]. In addition, the quadratic in \hat{N} term can be understood as an average two-body interaction between the valence particles (note that for n equivalent particles there are $\binom{n}{2} = \frac{n(n-1)}{2}$ particle couplings).

From another perspective, the E -term can be related to the symmetry energy [3, 10] as its expectation value in states with definite isospin is of the form $T(T+1)$, which enters as a symmetry term in many nuclear mass relationships [32, 33]. We refer to the E -term as a *symmetry term*, although it is common to address the symmetry energy in a slightly different way: the $T(T+1)$ term together with the isospin dependence of the isovector pairing term yield both symmetry ($\sim T^2 \sim (Z-N)^2$) and Wigner ($\sim T$) energies [46]. The first one was originally included in the Bethe-Weizsäcker semi-empirical mass formula [34, 35] and implies that the nuclear symmetry energy has the tendency toward stability for $N = Z$. The Wigner energy is associated with proton-neutron exchange interactions and is responsible for a sharp energy cusp at $N = Z$ leading to an additional binding of self-conjugate nuclei [14]. In short, the symmetry energy together with the terms that are linear in \hat{N} (3) can be directly related to a typical mass formula [34, 35], while in addition our model improves the description of isovector pairing correlations and high J identical-particle and pn interactions and uses an advanced Coulomb repulsion correction [37].

In this way, Hamiltonian (3) includes an isovector ($T = 1$) nn , pn , and pp pairing interaction ($G \geq 0$ for attraction) and a diagonal (in an isospin basis) isoscalar ($T = 0$) proton-neutron force, which is related to the so-called symmetry term (E). Hence, the model Hamiltonian (3) includes the dominant interactions that govern the 0^+ states under consideration and provides for an exact solution of the present problem.

In addition, the D -term in (3) introduces isospin symmetry breaking and the F -term accounts for a plausible, but weak, isospin mixing. While both terms are significant in the investigation of certain types of phenomena [36], the study of their role is outside the scope of this paper. These parameters yield quantitative results that are better than the ones with $F = 0$ and $D = 0$: for

example, in the case of the $1f_{7/2}$ level the variance between the model and experimental energies of the lowest isobaric analog 0^+ states increases by 85% when the D and F interactions are turned off. At the same time, the latter provide only fine adjustments compared to the main driving forces incorporated in (3). In this sense, a simpler isospin invariant $SO(5)$ model is suitable for a qualitative description of the isobaric analog 0^+ state energy spectra of nuclei.

III. DESCRIPTION OF ISOBARIC ANALOG 0^+ STATES

A. Interaction strength parameters

The interaction strength parameters are estimated in a fit of the minimum eigenvalue ($-E_0$) of the H energy operator (3) to the Coulomb corrected [37] experimental energies [38, 39] of the lowest isobaric analog 0^+ states of even- A nuclei (ground states for even-even nuclei and some [$N \approx Z$] odd-odd nuclei). We use the Coulomb correction $V_{Coul}(A, Z)$ derived in [37] so that the Coulomb corrected energies are adjusted to be $E_{0,exp}(A, Z) = E_{0,exp}^C(A, Z) + V_{Coul}(A, Z) - E_{0,exp}(A_c, Z_c)$, where $E_{0,exp}^C(A, Z)$ is the total measured (positive) energy including the Coulomb energy and $E_{0,exp}(A_c, Z_c) = E_{0,exp}^C(A_c, Z_c) + V_{Coul}(A_c, Z_c)$ is the corrected energy of a nuclear core. Analogously, the theoretically predicted energies that include the Coulomb repulsion can be obtained as $E_0^C(A, Z) = E_0(A, Z) - V_{Coul}(A, Z) + E_{0,exp}(A_c, Z_c)$.

TABLE I: Parameters and statistics for three regions (**I**, **II**, and **III**) specified by the valence model space. G , F , C , D , E , ϵ , and χ are in MeV, SOS is in MeV^2 .

Parameters	I	II	III
and statistics	($1d_{3/2}$)	($1f_{7/2}$)	($1f_{5/2}2p_{1/2}$ $2p_{3/2}1g_{9/2}$)
G/Ω	0.702	0.453	0.296
F/Ω	0.007	0.072	0.056
C	0.815	0.473	0.190
D	0.127	0.149	-0.307
$E/(2\Omega)$	-1.409	-1.120	-0.489
ϵ	9.012	9.359	9.567
SOS	1.720	16.095	300.284
χ	0.496	0.732	1.787

The fitting procedure was performed separately for three groups of nuclei with valence nucleons occupying (**I**) the $1d_{3/2}$ level with a ^{32}S core, (**II**) the $1f_{7/2}$ level with a ^{40}Ca core, and (**III**) the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ shell with a ^{56}Ni core [7]. The results reveal that the model interaction accounts quite well for the available experimental energies for a total of 149 nuclei (refer to the small

value of the χ -statistics in Table I, where χ^2 is the sum of squares, SOS , divided by the difference between the number of data cases and the number of fit parameters) [7]. The values of the parameters in H (3) are kept fixed hereafter (Table I).

In the case of the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ shell (**III**), the parameters of the effective interaction in the $Sp(4)$ model with degenerate multi- j levels are likely to be influenced by the non-degeneracy of the single-particle orbits. Nevertheless, as the dynamical symmetry properties of the two-body interaction in nuclei from this region are not lost, the model remains a good multi- j approximation [7], which is confirmed with the use of various discrete derivatives of the energy function [8].

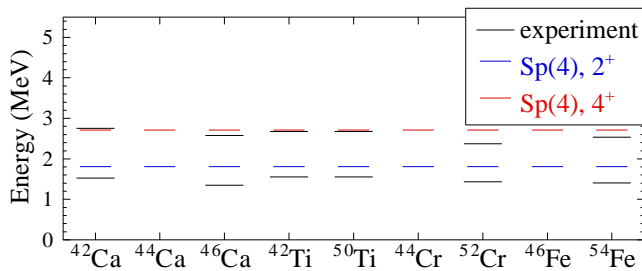


FIG. 1: (Color online) Low-lying energy spectra of near closed-shell nuclei in the $1f_{7/2}$ level as described by the like-particle pairing limit of the $Sp(4)$ model and compared to the experiment.

While the optimum fit is statistically determined solely by χ , the physical aspect of the nuclear problem requires, in addition, the estimate for the parameters to be physically valid. Indeed, the values of the like-particle pairing strength G , obtained by our $Sp(4)$ model, yield consistent results [8] with the experimental pairing gaps derived from the odd-even mass differences [40, 41]. In this way, the G values are expected to reproduce the low-lying vibrational spectra of near closed-shell nuclei in the $SU^\pm(2)$ limit of the model (like-particle pairing) (Figure 1). When the results from the three nuclear regions (**I**, **II**, and **III**) are considered, the pairing strength parameter is found to follow the well-known $1/A$ trend [14, 42, 43, 44, 45],

$$\frac{G}{\Omega} = \frac{23.9 \pm 1.1}{A}, \quad R^2 = 0.96, \quad (4)$$

where R^2 is a coefficient of correlation and represents the proportion of variation in the strength parameter accounted for by the analytical curve (Figure 2). Similarly, the values of the other strength parameters lie on a curve that decreases with nuclear mass A (Figure 2)

$$\begin{aligned} \frac{E}{2\Omega} &= \frac{-50.2 \pm 3.3}{A}, & R^2 &= 0.93, \\ C &= \left(\frac{32.30 \pm 0.02}{A} \right)^{1.887 \pm 0.004}, & R^2 &= 0.99. \end{aligned} \quad (5)$$

As expected for a symmetry energy term, the $1/A$ dependence holds for the parameter E . The dependence

of C on the mass number A suggests that the quadratic correction $C \frac{n(n-1)}{2}$ to the mean field may change slowly from one nucleus to another, which is consistent with the saturation of the nuclear force. Although the data set used in the fitting procedure was rather small, the trend toward a smooth functional dependence of the interaction strength parameters on the mass number A reveals their *global* character, namely the interactions in the model Hamiltonian (3) are related to an overall behavior common to all nuclei.

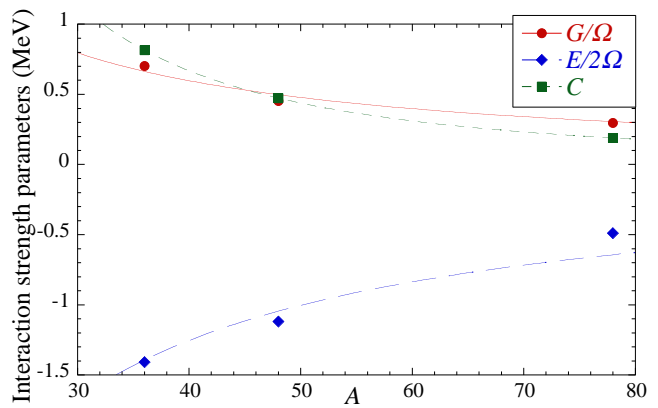


FIG. 2: (Color online) Dependence of the interaction strength parameters G/Ω , $E/2\Omega$, and C (in MeV) on the mass number A with values from the three regions, (**I**), (**II**) and (**III**).

Furthermore, the Wigner energy [46], $-W2T$, is implicitly included in the $Sp(4)$ theoretical energy, which in turn makes the estimation of its strength possible. The Wigner energy appears as the term that is the linear in T in the pn isoscalar force (proportional to the symmetry term) and in the isovector pairing through the second-order Casimir invariant of $\mathfrak{sp}(4)$. In a good-isospin regime, the symmetry energy contribution is $-\frac{E}{2\Omega}T(T+1)$ [due to the \hat{T}^2 -term in (3)], and the W interaction strength parameter can be expressed through the model parameters (Table I) as $W = \frac{E-G}{4\Omega}$. In the framework of the $Sp(4)$ model, the estimated values for W from the three regions (**I**, **II** and **III**) are found to lie on a curve

$$W = \frac{-31 \pm 2}{A}, \quad R^2 = 0.96, \quad (6)$$

with a very good correlation coefficient R^2 and a remarkably close value to most other estimates: $W = -30/A$ [47], $W = -37/A$ [33], $W = -37.4/A$ [31], $W = -42/A$ [48] and $W = -47/A$ [28].

In short, the outcome of the optimization procedures shows that the effective interaction with $Sp(4)$ dynamical symmetry provides a reasonable description of the lowest isobaric analog 0^+ states, retaining the physical meaning and validity of its microscopic nature.

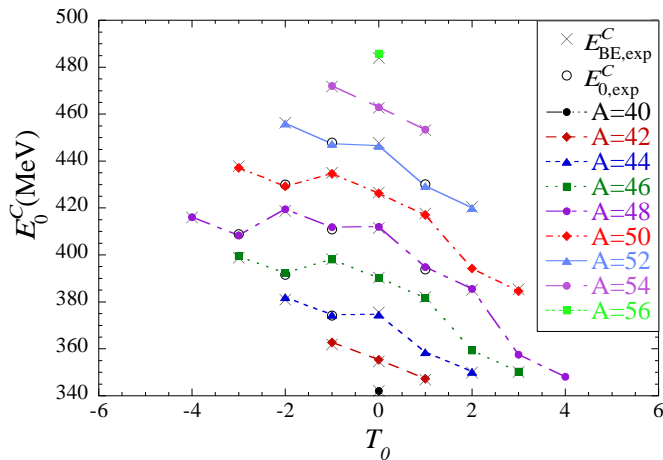


FIG. 3: (Color online) Isobaric analog 0^+ state energy, E_0^C , in MeV (including the Coulomb energy) versus the isospin projection T_0 for the isobars with $A = 40$ to $A = 56$ in the $1f_{7/2}$ level, $\Omega_{\frac{7}{2}} = 4$. The experimental binding energies $E_{BE,exp}^C$ (symbol “x”) are distinguished from the experimental energies of the isobaric analog 0^+ excited states $E_{0,exp}^C$ (symbol “o”). Each line connects theoretically predicted energies of an isobaric sequence.

B. Energy spectra of the isobaric analog 0^+ states

In all three nuclear regions, there is good agreement with experiment (small χ -statistics), as can be seen in

Figure 3 for the isobars $A = 40 - 56$ in the $1f_{7/2}$ level (II). The theory predicts the lowest isobaric analog 0^+ state energy of nuclei with a deviation ($\chi/\Delta E_{0,exp} \times 100[\%]$) of 0.7% for (I) and 0.5% for (II) and (III) in the corresponding energy range considered, $\Delta E_{0,exp}$.

The fitting procedure not only estimates the magnitude of the interaction strength and determines how well the model Hamiltonian “explains” the experimental data, it also can be used to predict nuclear energies that have not been measured. This includes energies of nuclei with odd number of protons and neutrons and as well nuclei away from the valley of stability with $N \approx Z$ or proton-rich that are of great interest in modern astrophysical studies. From the fit for the $1f_{7/2}$ case, the binding energy of the proton-rich ^{48}Ni nucleus is estimated to be 348.19 MeV, which is 0.07% greater than the sophisticated semi-empirical estimate of [47]. Likewise, for the odd-odd nuclei that do not have measured energy spectra the theory can predict the energy of their lowest isobaric analog 0^+ state: 358.62 MeV (^{44}V), 359.34 MeV (^{46}Mn), 357.49 MeV (^{48}Co), 394.20 MeV (^{50}Co) (Figure 3). The Sp(4) model predicts the relevant 0^+ state energies for an additional 165 even- A nuclei in the medium mass region (III) plotted in Figure 4. The binding energies for 25 of them are also calculated in [47]. For these even-even nuclei, we predict binding energies that on average are 0.05% less than the semi-empirical approximation [47].

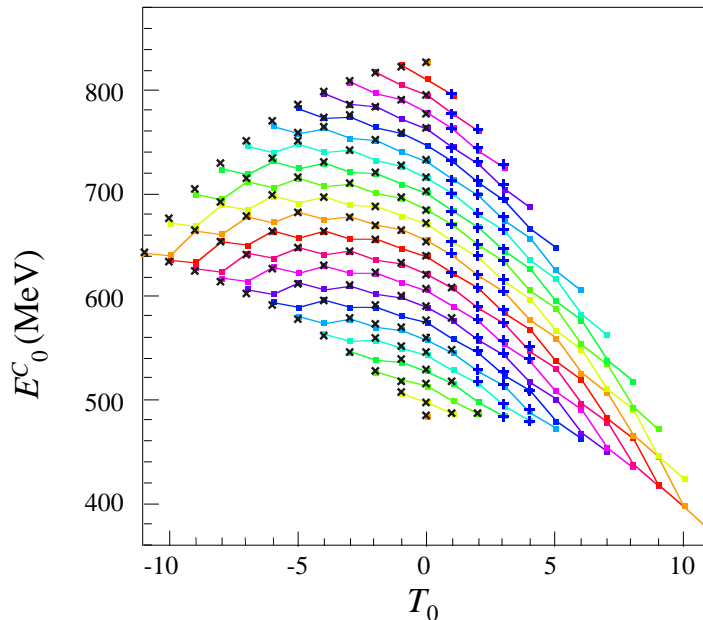


FIG. 4: (Color online) Theoretical energies E_0^C (including the Coulomb energy contribution) of the lowest isobaric analog 0^+ states for isobars (connected by lines in different colors) with mass number $A = 56, 58, \dots, 100$ in the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ major shell (^{56}Ni core), compared to experimental values (black ‘x’) and semi-empirical estimates in [47] (blue ‘+’).

Without varying the values of the interaction strength parameters (Table I), the energy of the higher-lying pairing-governed isobaric analog 0^+ states in nuclei under consideration can be theoretically calculated. These states are eigenvectors of the model Hamiltonian (3) and differ among themselves in their pairing modes due to the close interplay between like-particle and pn pairs. The theoretical energy spectra of these isobaric analog 0^+ states agree remarkably well with the available experimental values [50] (Figure 5). This agreement, which is observed not only in single cases but throughout the shells, represents a valuable result. This is because the higher-lying 0^+ states under consideration constitute an experimental set independent of the data that enters the statistics to determine the model parameters in (3). Such a result is, first, an independent test of the physical validity of the strength parameters, and, second, an indication that the interactions interpreted by the Sp(4) model Hamiltonian are the main driving force that defines the properties of these states. In this way, the Sp(4) dynamical symmetry of the zero-seniority IAS 0^+ states of even- A nuclei reveals a simple and fundamental aspect of the nuclear interaction related to isovector $J = 0$ pairing correlations and higher- J proton-neutron interactions. Moreover, the simple Sp(4) model can be used to provide a reasonable prediction of the (ground and/or excited) pairing-governed isobaric analog 0^+ states in proton-rich nuclei with energy spectra not yet experimentally fully explored.

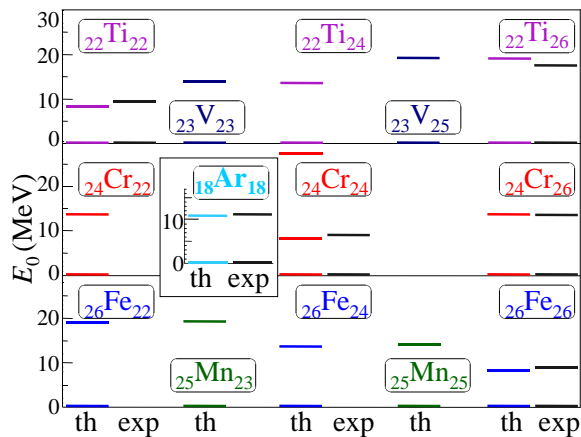


FIG. 5: (Color online) Theoretical and experimental (black lines) energy spectra of the higher-lying pairing-governed isobaric analog 0^+ states for isotopes in the $1f_{7/2}$ shell (^{40}Ca core). Insert: First excited isobaric analog 0^+ state energy in ^{36}Ar in the $1d_{3/2}$ shell (^{32}S core) in comparison to its experimental value.

Such a conclusion is based furthermore on our com-

plementary investigation [8] on the fine structure phenomena among the isobaric analog 0^+ states. A study of this kind is quite necessary because it is well-known that a good reproduction of the experimental nuclear energies does not guarantee straight away agreement of the fine structure of nuclei in comparison to the experiment. We have examined such detailed features by discrete approximations of derivatives of the energy function (3) filtering out the strong mean-field influence [8]. In short, this investigation revealed a remarkable reproduction of the two-proton and two-neutron separation energies, the irregularities found around the $N = Z$ region, the like-particle and pn isovector pairing gaps, the significant role of the symmetry energy and isovector pairing correlations in determining the fine nuclear properties, and a prominent staggering behavior observed between groups of even-even and odd-odd nuclides [8]. This study confirmed additionally the validity and reliability of the group theoretical Sp(4) model and the interactions it includes.

IV. CONCLUSIONS

In this paper we presented a simple Sp(4) model that achieved a reasonable prediction of the pairing-governed isobaric analog 0^+ state energy spectra of a total of 319 even-even and odd-odd nuclei with only six parameters. The model Hamiltonian is a two-body effective interaction, including proton-neutron and like-particle pairing plus symmetry terms (the latter is related to a proton-neutron isoscalar force). We compared the theoretical results with experimental values and examined in detail their outcome. While the model describes only the pairing-governed isobaric analog 0^+ states of even-even medium mass nuclei with protons and neutrons occupying the same shell, it reveals a fundamental feature of the nuclear interaction, which governs these states. Namely, the latter possess clearly a simple Sp(4) dynamical symmetry. Such a symplectic Sp(4) scheme allows also for an extensive systematic study of various experimental patterns of the even- A nuclei.

Acknowledgments

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[1] K. Langanke, *Nucl. Phys.* **A630**, 368c (1998).
 [2] H. Schatz *et. al*, *Phys. Rep.* **294**, 167 (1998).

[3] K. T. Hecht, *Nucl. Phys.* **63**, 177(1965); *Phys. Rev.* **139**, B794 (1965); *Nucl. Phys.* **A102**, 11 (1967).

- [4] J. N. Ginocchio, *Nucl. Phys.* **74**, 321 (1965).
- [5] J. Engel, K. Langanke and P. Vogel, *Phys. Lett.* **B389**, 211 (1996).
- [6] K. Kaneko, M. Hasegawa and J. Zhang, *Phys. Rev.* **C59**, 740 (1999).
- [7] K. D. Sviratcheva, A. I. Georgieva and J. P. Draayer, *J. Phys. G: Nucl. Part. Phys.* **29**, 1281 (2003).
- [8] K. D. Sviratcheva, A. I. Georgieva and J. P. Draayer, *Phys. Rev.* **C69** 024313 (2004).
- [9] K. Helmers, *Nucl. Phys.* **23**, 594 (1961).
- [10] I. Talmi, *Simple Models of Complex Nuclei: The Shell Model and Interacting Boson Model* (Harwood Academic Publishers GmbH, Switzerland, 1993).
- [11] G. Racah, *Phys. Rev.* **62**, 438 (1942); *Phys. Rev.* **63**, 367 (1943).
- [12] B. H. Flowers, *Proc. Roy. Soc. (London)* **A212**, 248 (1952).
- [13] A. Bohr, B. R. Mottelson and D. Pines, *Phys. Rev.* **110**, 936 (1958).
- [14] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York), Vol. I (1969); Vol. II (1975).
- [15] N. Zeldes and S. Lirin, *Phys. Lett.* **62B**, 12 (1976).
- [16] D. Rudolph *et al.*, *Phys. Rev. Lett.* **76**, 376 (1996).
- [17] S. M. Vincent *et al.*, *Phys. Lett. B* **437**, 264 (1998).
- [18] J. Garces Narro *et al.*, *Phys. Rev.* **C63**, 044307 (2001).
- [19] O. Civitarese, M. Reboiro and P. Vogel, *Phys. Rev.* **C56**, 1840 (1997).
- [20] K. Langanke, D. J. Dean, S. E. Koonin and P. B. Radha, *Nucl. Phys.* **A613**, 253 (1997).
- [21] D. J. Dean *et al.*, *Phys. Lett.* **B399**, 1 (1997).
- [22] W. Satula, D. J. Dean, J. Gary, S. Mizutori and W. Nazarewicz, *Phys. Lett.* **B407**, 103 (1997).
- [23] A. Poves and G. Martinez-Pinedo, *Phys. Lett.* **B430**, 203 (1998).
- [24] G. Martinez-Pinedo, K. Langanke and P. Vogel, *Nucl. Phys.* **A651**, 379 (1999).
- [25] W. Satula and R. Wyss, *Phys. Lett.* **B393**, 1 (1997); *Nucl. Phys.* **A676**, 120 (2000).
- [26] A. O. Macchiavelli *et al.*, *Phys. Lett.* **B480**, 1 (2000).
- [27] A. O. Macchiavelli *et al.*, *Phys. Rev.* **C61**, 041303(R) (2000).
- [28] P. Vogel, *Nucl. Phys.* **A662**, 148 (2000).
- [29] A. K. Kerman, *Ann. Phys.* **12**, 300 (1961).
- [30] M. Hasegawa and K. Kaneko, *Phys. Rev.* **C59**, 1449 (1999).
- [31] K. Kaneko and M. Hasegawa, *Phys. Rev.* **C60**, 024301 (1999).
- [32] J. Jänecke and H. Behrens, *Phys. Rev.* **C9**, 1276 (1974).
- [33] J. Dufflo and A. P. Zuker, *Phys. Rev.* **C52**, R23 (1995).
- [34] C. F. von Weizsäcker, *Z. Physik* **96**, 431 (1935).
- [35] H. A. Bethe and R. F. Bacher, *Rev. Mod. Phys.* **8**, 82 (1936).
- [36] K. D. Sviratcheva, A. I. Georgieva and J. P. Draayer, *Phys. Rev.* **C72** 054302 (2005).
- [37] J. Retamosa, E. Caurier, F. Nowacki and A. Poves, *Phys. Rev.* **C55**, 1266 (1997).
- [38] G. Audi and A. H. Wapstra, *Nucl. Phys.* **A595**, 409 (1995).
- [39] R. B. Firestone and C. M. Baglin, *Table of Isotopes*, 8th Edition (John Wiley & Sons, 1998).
- [40] S. G. Nilsson and O. Prior, *Mat. Fys. Medd. Dan. Vid. Selsk.* **32**, No.16 (1961).
- [41] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag New York Inc., 1980).
- [42] A. K. Kerman, R. D. Lawson and M. H. Macfarlane, *Phys. Rev.* **124**, 162 (1961).
- [43] L. S. Kisslinger and R. A. Sorensen, *Rev. Mod. Phys.* **35**, 853 (1963).
- [44] A. M. Lane, *Nuclear Theory* (W. A. Benjamin, Inc., 1964).
- [45] J. Dudek, A. Majhofer and J. Skalski, *J. Phys. G: Nucl. Part. Phys.* **6**, 447 (1980).
- [46] E. Wigner, *Phys. Rev.* **51**, 106 (1937).
- [47] P. Möller, J. R. Nix and K.-L. Kratz, *LA-UR-94-3898* (1994); *At. Data Nucl. Data Tables* **66**, 131 (1997).
- [48] W. D. Myers and W. J. Swiatecki, *Nucl. Phys.* **A601**, 141 (1996).
- [49] Without compromising the theory, one can consider closed shells as part of an inert core that is spherical and does not affect directly the single-particle motion of the valence nucleons in the last unfilled shell.
- [50] The energy spectra of nuclei in the (III) region with nuclear masses $56 < A < 100$ is not yet completely measured, especially the higher-lying 0^+ states. This makes a comparison of the theory to the experiment impossible.