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Computational modeling of geogrid reinforced soil foundation and geogrid reinforced base in flexible pavement

Jie Gu
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COMPUTATIONAL MODELING OF GEOGRID REINFORCED SOIL FOUNDATION AND GEOGRID REINFORCED BASE IN FLEXIBLE PAVEMENT

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

In

The Department of Civil Engineering and Environmental Engineering

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ABSTRACT

The objectives of this study are to investigate and evaluate the benefits of inclusion of geogrids in two types of geosynthetic reinforced soil/aggregate structures—reinforced soil foundations (RSF) and reinforced base aggregate in flexible pavements, thus shedding the light on the design of these reinforced structures.

Two different finite element models were developed using ABAQUS software. The first model was used to investigate the bearing capacity and settlement of RSF and to perform parametric study on the effect of different design parameters on the performance of RSF. The second model was used to analyze the performance of geogrid reinforced bases in flexible pavement in terms of surface rutting, which was also used to perform parametric study on the effect of different design parameters on the performance of reinforced pavements. Based on the results of finite element analyses, multiple regression models were developed to estimate the benefit of reinforced geomaterial structures under different combination of design parameters.

The results of finite element analysis on RSF showed that the inclusion of reinforcement, in general, results in increasing the bearing capacity and reducing the settlement of the reinforced soil. The benefit increases with increasing the tensile modulus and/or number of reinforcement layers. The results also showed that the effective reinforcement depth is about 1.5 times the footing width, and there exists an optimum depth of first reinforcement layer where the highest bearing capacity can be achieved.

The results of finite element analysis on geogrid reinforced bases in flexible pavements showed that the use of geogrid reinforcement reduces the lateral strains within the base and subgrade layers, reduces the vertical strains on top of subgrade layer, and hence significantly reduces the surface permanent deformation (or rutting) of pavements. In terms of traffic benefit
ratio (TBR), the geogrid base reinforcement helps increasing the service life of pavements, with TRB values of up to 3.4 were obtained for pavement sections over weak subgrades. The finite element analysis clearly demonstrated that the geogrid improvement increases with increasing the geogrid tensile modulus and with decreasing of both the base course layer thickness and the subgrade strength.
CHAPTER 1 INTRODUCTION

1.1 Background

The techniques used for ground improvement using geosynthetics have been developed extensively over the last few decades, in particular those applied in pavement and foundation engineering. The concept of reinforced soil as construction material is based on the existence of soil-reinforcement interaction due to tensile strength, frictional and the adhesion properties of the reinforcement and was first introduced by the French architect and engineer Henri Vidal in the 1960s (Vidal, 1978). Since then, this technique has been widely used in geotechnical engineering practice.

The reinforcing materials that have been developed over the years range from stiff to flexible geosynthetic materials and can be classified as either extensible or inextensible reinforcements (McGown et al., 1978). Recently, geosynthetics have been used extensively as reinforcements for improving the load-settlement characteristics of soft foundation soils. Their use has been proven to cost-effectively improve the bearing capacity and settlement performance of earth structure (Basudhar et al., 2007; Ghazavi and Lavasan, 2008). The most common types of geosynthetics include Geogrids, Geotextiles, Geomembranes, Geosynthetic Clay Liners, Geonets, and Geopipes (Koerner, 1997), whereby Geogrids are one of the most commonly used forms of reinforcement, which, as they offer superior interface shear resistance due to interlocking.

In the present study, two types of geogrid reinforced structures—geogrid reinforced foundations and geogrid reinforced bases in pavement—will be examined. Extant studies have shown that geogrid reinforced foundations can increase the ultimate bearing capacity or/and reduce the settlement of shallow footings, compared to the conventional methods, such as
replacing natural soils or increasing footing dimensions. In addition, Geogrid can provide tensile reinforcement through frictional interaction with base course materials, thereby reducing the applied vertical stresses on the subgrade, resulting in prevention of rutting that stems from subgrade overstress.

1.2 Statement of the Problem

1.2.1 Geogrid Reinforced Foundation

In many coastal areas of the United States earth embankments have to be built over weak subsurface soils. Since high quality embankment soils are not always available locally, marginal cohesive soils tend to predominate in the composition of the ground structure. The presence of such a marginal soil often results in low load bearing capacity as well as the construction of embankments over weak soil results in excessive settlements for overlying structures, which can cause damage in structure, reduction in the durability, and/or deterioration in the performance level. Of particular importance is excessive differential settlement of the concrete approach slab in highway engineering, as it causes the significant bridge “bump” problem (Figure 1.1). This results in uncomfortable rides, dangerous driving conditions, and requires frequent.

![Figure 1.1 Illustration of approach slab and its interaction with soil (Chen, 2007)](image)

In an attempt to solve this problem, the state of Louisiana recommended changing the design of approach slab by increasing its rigidity. By implementing this design, the gravity of the
slab and traffic loads will be transferred to the two ends of the slab, rather than being distributed over the entire slab length. Accordingly, a shallow foundation is needed at the far end of the approach slab to carry that part of the load reaction (Figure 1.2). In addition, the soil underneath the footing must be treated to improve the bearing capacity and to prevent excessive settlement of underlying weak soil through distributing the applied load over a wide area. Conventional treatment methods applied to address this issue either replace part of the weak cohesive soil by an adequately thick layer of stronger granular fill, increase the dimensions of the footing, or use a combination of both approaches. An alternative and more economical solution is the use of geosynthetics to reinforce the soil underneath the strip footing.

Figure 1.2 Reinforced soil foundation applied to approach slab (Chen, 2007)

This can be achieved by either directly reinforcing marginal embankment soil or replacing it with stronger granular fill (e.g. crushed limestone) in combination with the inclusion of geosynthetics. The resulting composite zone (reinforced soil mass) will improve the load carrying capacity of the footing (or improve the soil's bearing capacity) and provide better pressure distribution in the layer above the underlying weak soils, hence reducing the associated total and differential settlements.
Given that potential benefits of a footing with a geosynthetic-reinforced foundation soil depend on multiple factors, a rational design methodology is needed to fully exploit these benefits. Thus, identifying optimum design parameters through conducting a parametric study of the influence of each factor on the behavior of a strip footing on geosynthetics-reinforced soil is a prudent and cost effective approach.

The cost of constructing and monitoring full-scale reinforced foundations on embankments soil is rather high. Hence, a suitable alternative, such as a numerical simulation by means of appropriate methods, must be sought. In that respect, finite-element analysis has been proven most effective in conducting complex numerical studies of many geotechnical problems.

This part of study will present the finite element parametric analysis performed as a part of the present study in order to investigate the influence of various factors on the bearing capacity and the settlement of strip foundations. Based on these findings, a regression model that can readily be employed in footing reinforcement design was developed and will be subsequently described.

1.2.2 Geogrid Reinforced Bases in Flexible Pavement

Pavement structures are built to support loads induced by traffic vehicle loading and to distribute them safely to the underlying subgrade soil. A conventional flexible pavement structure consists of a surface layer of asphalt (AC) and a base course layer of granular materials built on top of a subgrade layer. One of the common types of pavement failures (or distress) is the excessive surface rutting. Rutting is the permanent surface depression along the wheel path. An example is shown in Figure 1.3.

In order to address this issue that requires frequent and costly road resurfacing (leading to disruption of traffic flow or even road closures), polymer geogrids have recently been introduced
with the aim to improve the performance of paved and unpaved roadways. The reinforcement layer is usually placed between the base course and sub grade interface, as shown in Figure 1.4. Due to the wide application of this technique, many experimental and analytical studies have been conducted to assess and potentially quantify the improvements associated with geogrid base reinforcement of roadways. The findings suggest that the use of geogrid reinforcement in flexible pavement structure has three main benefits: help in construction pavements over soft subgrades, improv or extend the pavement’s projected service life, and reduce the thickness of pavement structural cross section (basically the base course layer) for a given service life.

Figure 1.3 Illustration of rutting in pavement

Owing to its popularity, several design methods of geogrid-reinforced pavement have recently emerged, typically based on empirical or analytical approaches. Empirical design methods rely on obtaining a performance level from a laboratory model test and then extrapolated to the field conditions for practical application in the design (Berg et al., 2000). Thus, these methods are limited to the conditions that can be simulated experimentally, which may not fully describe practical usage on the road. While, the key shortcoming of design methods based on analytical solution is that they typically do not address all the variables (e.g., geogrid location and stiffness, base course layer thickness, strength/stiffness of subgrade) that
affect the performance of these pavements, which have been validated by experimental data (e.g., Perkins and Ismeik, 1997a, 1997b).

Thus, in order to overcome these limitations, a mechanistic design procedure for reinforced pavement structures should be developed, which requires a better understanding and characterization of the geogrid-reinforced mechanisms. Despite extensive work in this field, the behavior of the geogrid reinforced road system is still not fully understood. In particular, more research is needed in quantifying the structural contribution by geogrid reinforcement and incorporating it into the design methodology. As a part of this process, factors that affect the performance of geogrid reinforced pavement structures should be determined and evaluated. It is likely that finite element method will remain the most practical and cost effective approach, due to the high cost associated with constructing and monitoring geogrid-reinforced bases in pavement. That is why the present study will aim to address these issues by developing improved model for analysis and design of geogrid-reinforced bases in flexible pavement under cyclic loads.

![Figure 1.4 The flexible pavement section](image_url)
1.3 Objectives and Scope of the Study

1.3.1 Objectives of the Study

The present study aims at evaluating the performance of reinforced soil foundations and reinforced bases in flexible pavement. Its two specific objectives are:

1. Assessment of the benefits of reinforcing embankment soil of low to medium plasticity with geogrids beneath a strip footing from the perspective of improving the ultimate bearing capacity and reducing footing settlement. In addition, study of the effects of contributing parameters and variables in order to develop a statistical regression model that can readily be employed in design of reinforced soil foundations for Louisiana.

2. Assessment of the benefits of reinforcing the base course layer in a flexible pavement structure with geogrid reinforcement from the perspective of extending the life of pavements, and evaluating the influence of the different variables and parameters on the degree of improvement in the performance of these structures. The ultimate goal is to develop a statistical regression model that can readily be employed in design of geogrid-reinforced bases for Louisiana.

1.3.2 Scope of the Study

In order to meet the study objectives, separate analyses were conducted to address each one in turn.

The first objective was achieved through finite element analyses, which included development of finite element model for the reinforced soil foundations under strip footing, choice of material model, verification with small-scale laboratory tests, and statistical regression based on the finite element analysis. The finite element model was verified by laboratory model
tests, and the results used to analyze the strip footing positioned on the reinforced soil in order to identify an optimum reinforcement design with the aid of statistical analysis.

The parameters studied as a part of analysis related to the first objective include:

a. Effective length of reinforcement,
b. Effective depth of reinforcement zone,
c. Spacing between reinforcement layers,
d. Optimum top spacing for first reinforced layer,
e. Stiffness or tensile modulus of reinforcement,
f. Footing width,
g. Embedment depth of footing,
h. Friction of soil,
i. Cohesion of soil,
j. Elastic modulus of soil.

The second objective was achieved by conducting numerical modeling programs of pavement system in which base course layer was reinforced with geogrid layer. Suitable material model was implemented to simulate different material in the system. Using the developed model, a parametric study was performed to identify the key factors affecting the design of reinforced flexible paved roads. Once these factors were quantified, an improved design method for reinforced pavement structure was proposed, based on statistic regression analyses.

The parameters being studied for the second objective include:

1. The location of the reinforcement material, whereby three different locations were investigated to determine the optimum location:

   a. Bottom of the base course layer,
   b. Middle of the base course layer,
c. Upper one third of the base course layer.

2. The thickness of the base course layer: four base course layer thicknesses were investigated—150 mm, 200 mm, 250 mm, and 300 mm.

3. The stiffness or tensile modulus of reinforcement material: four geogrid types with different stiffness properties were evaluated.

4. The strength of the subgrade material: three subgrades with different strength properties were investigated; representing materials that are, weak, moderate, and stiff.

1.4 Dissertation Outline

The dissertation is comprised of seven chapters.

Chapter 2 presents an extensive literature review of experimental and numerical studies of reinforced soil foundations and reinforced bases in flexible pavements. Focus is given to the finite element study method and results reported by other researchers in this field.

Chapter 3 outlines, in detailed, the research methods employed during the evaluation the benefits of the reinforced soil structure.

Chapter 4 provides verification of the research methods by comparing the results obtained through the numerical analysis with the small-scale results.

Chapter 5 and Chapter 6 present the numerical results of reinforced soil foundation and reinforced bases in flexible pavement respectively.

Chapter 7 summarizes the key findings and concludes the study, as well as providing some suggestions for future research.
CHAPTER 2 LITERATURE REVIEW

2.1 Reinforced Soil Foundation

2.1.1 Introduction

In the past three decades, reinforced soil foundations (RSF) have been widely used in various geotechnical engineering applications, such as bridge approach slab, bridge abutment, building footings, and embankment.

Researchers have shown that the inclusion of reinforcement in soil foundations is a cost-effective solution to increase the ultimate bearing capacity and/or reduce the settlement of shallow footings compared to the conventional methods, such as replacing natural soils or increasing footings’ dimensions. The most common type of reinforcement used in soil foundation applications are geogrids, as shown in Figure 2.1.

(a) Uniaxial geogrid      (b) Biaxial geogrid         (c) Triaxial geogrid

Figure 2.1 Typical geogrids used as soil reinforcement

A typical reinforced soil foundation and the descriptions of various geometric parameters are shown in Figure 2.2. The geometric parameters in the figure are denoted as follows: (1) top layer spacing, or depth to first reinforcement layer \(u\), (2) number of reinforcement layers \(N\), (3) total depth of reinforcement \(d\), (4) vertical spacing between reinforcement \(h\), (5) length of reinforcement \(l\), (6) embedment depth of footing \(D_f\).
Figure 2.2 Geometric parameters for a reinforced soil foundation

During the past thirty years, many experimental, numerical, and analytical studies have been performed to investigate the behavior of reinforced soil foundation (RSF) for different soil types (e.g., Binquet and Lee, 1975a,b; Huang and Tatsuoka, 1990; Kurian et al., 1997; Chen 2007). Researchers introduced two concepts to evaluate the benefits of RSF (e.g., Chen 2007, Abu-Farsakh et al. 2007): one is the bearing capacity ratio (BCR), which is defined as the ratio of the bearing capacity of the RSF to that of unreinforced soil foundation. The other one is the settlement reduction factor (SRF), which is defined here as the ratio of the immediate settlement of the footing on a RSF to that on an unreinforced soil foundation at a specified surface pressure.

2.1.2 Reinforcement Mechanism of Reinforced Soil Foundation

The improve performance of reinforced soil foundation can be attributed to three fundamental reinforcement mechanisms as described below (Chen 2007).

(1) Rigid boundary (Figure 2.3a): if the top layer spacing (u) is greater than a certain value, the reinforcement would act as a rigid boundary and the failure would occur above the
reinforcement. Binquet and Lee (1975b) were the first who reported this finding. Experimental study conducted by several researchers (Akinmusuru and Akinbolade, 1981; Mandal and Sah, 1992; Khing et al., 1993; Omar et al., 1993b; Ghosh et al., 2005) confirmed this finding subsequently.

(2) Membrane effect (Figure 2.3b): Under loading, the footing and soil beneath the footing move downward. As a result, the reinforcement is deformed and tensioned. Due to its tensile stiffness, the curved reinforcement develops an upward force to support the applied load. A certain amount of settlement is needed to mobilize tensioned membrane effect, and the reinforcement should have enough length and stiffness to prevent it from failing by pull out and rupture. Binquet and Lee (1975b) were perhaps the first who applied this reinforcement mechanism to develop a design method for a strip footing on reinforced sand with the simple assumption made for the shape of reinforcement after deformation. Kumar and Saran (2003) extended this method to a rectangular footing on reinforced sand.

(3) Confinement effect (lateral restraint effect) (Figure 2.3c): Due to relative displacement between soil and reinforcement, the friction force is induced at the soil-reinforcement interface. For grid reinforcement, the interlocking can be developed by the interaction of soil and reinforcement. Consequently, lateral deformation or potential tensile strain of the soil is restrained. As a result, vertical deformation of soil is reduced. Since most soils are stress-dependent materials, improved lateral confinement can increase the compressive strength of soil and thus improve the bearing capacity. Huang and Tatsuoka (1990) substantiated this mechanism by successfully using short reinforcement having a length (L) equal to the footing width (B) to reinforce sand in their experimental study. Michalowski (2004) applied this reinforcing mechanism in the limit analysis of reinforced soil.
foundation and derived the formula for calculating the ultimate bearing capacity of strip footings on reinforced soils.

Figure 2.3 Reinforcement mechanisms (Chen 2007)

2.1.3 Experimental Studies on Reinforced Soil Foundation

2.1.3.1 Small-Scale Laboratory Tests

The early experimental study on reinforced soil foundation (RSF) was conducted by Binquet and Lee (1975) to evaluate the benefits of metal strips on the bearing capacity of reinforced sand soil. Since then, numerous researchers have conducted small-scale laboratory studies to investigate the potential benefits of reinforced soil foundations (e.g., Ramswamy and Purushothaman, 1992; Mandal and Sah, 1992; Das and Omar, 1994; Chen et al. 2007, 2009).
Binquet and Lee (1975) conducted a series of small-scale model tests to simulate three different foundation conditions: (1) a deep homogeneous sand foundation, (2) sand above a deep soft layer of clay or peat (simulated by using a 2.25 in. thick layer of Pack Lite foam rubber), and (3) sand above a soft pocket of material such as clay (simulated by using a 2 in. thick of finite soft pocket Sears foam rubber). Their model tests were conducted in a 60 in. (1,500 mm) long, 20 in. (510 mm) wide, and 13 in. (330 mm) high box. The model footing was a 3 in. (76 mm) wide strip footing. The foundation soil consisted of Ottawa No. 90 sand having a uniformity coefficient \((C_u)\) of 1.5 and a coefficient of curvature \((C_c)\) of 0.75. All the model tests were conducted at a dry density of 1,500 kg/m\(^3\). The corresponding friction angles for triaxial and plane conditions were 35º and 42º, respectively. The reinforcement was the household aluminum foil prepared at 0.5 in. (13 mm) wide strips placed along the length of the box, at a linear density of 42.5%, a tensile strength of 0.57kN/m, and a vertical spacing of 1.0 in. (25 mm). The pull out test results showed the peak and residual soil-tie friction angles were 18º and 10º, respectively.

The test results indicated that the bearing capacity could be improved by a factor ranging from 2 to 4 by reinforcing the soil foundation. They reported that a minimum critical number of reinforcement layers would be required, and that increasing the number of layers would definitely result in better improvements. Their test results also suggested that the reinforcement placed below the influence depth which was about 2B in their study had negligible effect on the increase of the bearing capacity. The depth to the first reinforcement layers was found to be another significant factor to the success of process. Based on their model tests, and in most cases, placing the first layer at \(u = 1.0\) in. (25 mm) \((u/B = 0.3)\) below the base of the footing resulted in the greatest improvement. They observed that the broken locations of reinforcements were below the edges or toward the center of the footing rather than near the classical slip surface.
Guido et al. (1986) conducted an experimental study on the comparison of the bearing capacity of geogrid and geotextile reinforced earth slabs. Their model tests were conducted in a square Plexiglas box with dimensions of 1.22 m (width) × 0.92 m (height). A 305 mm wide square footing was used in the test. The foundation soil consisted of sand having an effective particle size ($D_{10}$) of 0.086 mm, a uniformity coefficient ($C_u$) of 1.90, and a coefficient of curvature ($C_c$) of 1.23. All the model tests were conducted at a dry unit weight of 14.39 kN/m$^3$ ($D_r = 55\%$) with friction angle of 37º. The geogrid and geotextile used in the tests were tensar SS1 geogrid and Du Pont Typar 3401 geotextile. The ratio of soil-reinforcement friction to soil-soil friction determined by direct shear test was 0.985 for geotextile at a relative density of 55%. The geogrid failed by tension in pull-out test at a normal stress of 50 kPa and a relative density of 55%.

Guido et al. (1986) showed that the BCR decreased with the increase of $u/B$, but the increase was not significant for $u/B$ greater than 1.0. Decreasing the vertical spacing of the reinforcement resulted in an increase in the BCR values. Their test results also showed that the improvement in bearing capacity was negligible when the number of reinforcement layers was increased beyond 3 layers which correspond to an influence depth of 1.0B for $u/B$, $h/B$, and $l/B$ ratios of 0.5, 0.25, and 3. Negligible improvement on the BCR was observed while increasing the length ratio ($l/B$) of the reinforcement beyond 2 for geogrid reinforced sand and 3 for geotextile reinforced sand with two reinforcement layers and $u/B$ and $h/B$ ratios of 0.25 and 0.25, respectively. Better performance of geogrid reinforced sand than geotextile reinforced sand was observed in their tests. Generally, the BCRs for the geogrid reinforced sand were approximately 10% greater than those for geotextile reinforced sand. The BCR achieved in their studies for geogrid reinforced sand ranged from 1.25 to 2.8.
Ramaswamy and Purushothaman (1992) conducted an experimental study of model footings on the geogrid reinforced clayey soil foundation using a 40 mm-diameter circular footing. The foundation soil consisted of clay (CL) having 100% passing the 0.075 mm opening sieve with a specific gravity of 2.66, and liquid and plastic limits equal to 31 and 18, respectively. The maximum dry density of the soil was 1800 kg/m³ with an optimum moisture content of 18% as determined by the Standard Proctor test. Three moisture contents, 14%, 18%, and 20%, were used in the model tests. The corresponding dry densities were 1725 kg/m³, 1810 kg/m³, and 1765 kg/m³, respectively.

The results showed that the optimum top layer spacing ratio was about 0.5 and the effective length ratio ($l/D$) of the reinforcement was about 4. The BCR increased from 1.15 to 1.70 as the number of layers increased from 1 to 3. The bearing capacity of both unreinforced and reinforced clay decreased with increasing the moisture content. The BCR of reinforced clay with two layers of geogrid at optimum moisture content (=1.47) was higher than those at wet and dry sides (=1.11 and 1.26, respectively).

Mandal and Sah. (1992) conducted a series of model tests on model footings supported by geogrid reinforced clay. The tests were conducted in a 460 mm wide, 460 mm long, and 460 mm deep steel box. A hardwood with dimensions of 100 mm long, 100 mm wide, and 48 mm thick was used as model footing. The foundation soil consisted of clay (CL) having liquid and plastic limits equal to 72 and 41, respectively. The model tests for clay were conducted at a moisture content of 28%. The corresponding undrained shear strength was about 27 kN/m².

The model test results showed that a maximum BCR was obtained at $u/B=0.175$, while the minimum settlement reduction factor (SRF) at the ultimate bearing pressure of unreinforced clay was obtained at $u/B=0.25$. The settlement reduction factor (SRF) is defined here as the ratio of
the immediate settlement of the footing on a reinforced clay to that on an unreinforced clay at a specified surface pressure. The maximum BCR of 1.36 was achieved at $u/B = 0.175$ in their study. With the use of geogrid reinforcement the settlement could be reduced up to 45%.

Das and Omar (1994) studied the effects of footing width on BCR of model tests on geogrid reinforced sand. Six different sizes of model strip footings with widths of 50.8 mm, 76.2 mm, 101.6 mm, 127 mm, 152.4 mm, and 177.8 mm were used in model tests. The length of all footings is 304.8 mm. Model tests were performed in a box with dimensions of 1.96 m (length) × 0.305 m (width) × 0.914 m (height). The foundation soil consisted of sand having an effective particle size ($D_{10}$) of 0.34 mm, a uniformity coefficient ($C_u$) of 1.53, and a coefficient of curvature ($C_c$) of 1.10. The sand was poured into the box with the relative density of 55%, 65%, and 75%.

From test results, they observed that the settlement ratio ($s/B$) at ultimate load was about 6-8% for unreinforced soil foundation and 16-23% for RSF at ultimate load. The test results also showed that the magnitude of bearing capacity ratio (BCR) increased from 2.5~4.1 to 3~5.4 with the decrease of the relative density. Based on the test results, they reached the conclusion that the magnitude of BCR decreased from 4.1~5.4 to 2.5~3 with the increase of the footing width and was practically constant (BCR = 2.5, 2.9, and 3.0 for reinforced sand at the relative density of 75%, 65%, and 75%, respectively) when the width of footings is equal to or greater than 130 ~ 140 mm.

Yetimoglu et al. (1994) investigated the bearing capacity of rectangular footings on geogrid reinforced sand using both laboratory model tests and numerical analyses. The model tests were conducted in a 70 cm wide, 70 cm long, and 100 cm deep steel box. A 127 mm long, 101.5 mm wide, and 12.5 mm thick rectangular steel plate was used as model footing. The foundation soil
consisted of uniform sand having an effective particle size ($D_{10}$) of 0.15 mm, a uniformity coefficient ($C_u$) of 2.33, and a coefficient of curvature ($C_c$) of 0.76. The model tests were conducted at an average dry unit weight of 17.16 kN/m$^3$ ($D_r = 70\text{~}73\%$). The corresponding friction angle determined by direct shear tests was about 40º.

The test results showed that the settlement ratio ($s/B$) was about 0.03 ~ 0.05 at failure for all the unreinforced and reinforced sand, while the BCR ranged from 1.8 to 3.9. Therefore it seems the settlement of the footing at failure might not be influenced significantly by the geogrid reinforcement.

This observation of test results is different from that of Das and Omar’s (1994). Based on both the model test results and numerical study, the following findings were reported: (1) the optimum top layer spacing ratio ($u/B$) was found to be around 0.3 and 0.25 in reinforced sand with single layer and multi-layer reinforcement, respectively, (2) the optimum vertical spacing ratio ($h/B$) between reinforcement layers was determined as 0.2 to 0.4 depending on the number of reinforcement layers, (3) the influence depth was approximately 1.5B and the effective width ratio ($b/B$) of reinforcement was around 4.5, (4) increasing the reinforcement stiffness beyond a certain limit would result in insignificant increase in the BCR value.

Yetimoglu et al. (1994) believed the disagreement in the results reported by different researchers might be attributed to the different material properties used in their model tests.

As Yetimoglu et al. (1994) pointed out, and Jewell et al. (1984) and Milligan and Palmeira (1987) indicated, the geogrid-soil interaction was influenced significantly by the ratio of minimum grid aperture size ($d_{\text{min}}$) to mean particle size ($D_{50}$). Based on this study, they reached the conclusion that the reinforcement layout had a very significant effect on the bearing capacity of reinforced foundation, especially for the number of reinforcement layers.
Gabr, et al. (1998) used the plate load tests with instrumentation to study the stress distribution in geogrid-reinforced sand. The model tests were conducted in a 1.52 m wide, 1.52 m long, and 1.37 m deep steel box. A 0.33 m wide square footing was used in the test. The foundation soil consisted of Ohio River sand having a uniformity coefficient (\(C_u\)) of 8, and a coefficient of curvature (\(C_c\)) of 1. The model tests were conducted at a unit weight ranged from 17.3 kN/m\(^3\) to 17.9 kN/m\(^3\). The corresponding friction angle determined by triaxial tests was about 38.6°.

Their results showed a better attenuation of the stress distribution due to the inclusion of the reinforcement. The stress distribution angle (\(\alpha\)) estimated using the measured stress beneath the center of the footing indicates higher values of the angle (\(\alpha\)) for reinforced sand as compared to unreinforced sand. The stress distribution angle (\(\alpha\)) decreased with increasing the surface pressure; while the rate of reduction for unreinforced sand is higher than that for reinforced sand.

Gabr and Hart (2000) evaluated the test results in terms of elastic modulus instead of the bearing capacity as traditionally used by other researchers. Test results showed the load-settlement response with the inclusion of geogrid was stiffer than that without geogrid. The elastic modulus of reinforced sand decreased with increasing the top layer spacing (\(u\)) for SR1 geogrid. On the other hand, optimum top layer spacing ratio of \(u/B = 0.65\) was observed for the stiffer geogrid SR2. For one layer of SR1 geogrid, the ratio of modulus constant of reinforced sand (\(E_{ref}\)) to that of unreinforced sand (\(E_{unref}\)), \(E_{ref}/E_{unref}\), decreased from 1.6 to 1.05 and 1.5 to 1.15 at \(s/B = 1.5\%\) and 3\%, respectively as \(u/B\) increased from 0.5 to 1.0. At \(s/B = 1.5\%\) and 3\%, \(E_{ref}/E_{unref}\) for one layer of stiffer geogrid SR2 ranged from 1.0 to 1.3 and 0.96 to 1.3, respectively.

Shin et al. (2002) investigated the influence of embedment on BCR for geogrid reinforced sand. The model tests were conducted in a 174 mm wide, 1000 mm long, and 600 mm deep steel
box. A 172 mm long, 67 mm wide and 77 mm thick wood was used as a model strip footing. The foundation soil consisted of a poorly graded sand having an effective particle size ($D_{10}$) of 0.15 mm, a specific gravity of 2.65, and the uniformity coefficient ($C_u$) and coefficient of curvature ($C_c$) equal to 1.51 and 1.1, respectively. The model tests were conducted at an average relative density of $D_r = 74\%$. The corresponding friction angle determined by direct shear tests was about 38$^\circ$. The top layer spacing ratio ($u/B$), vertical spacing ratio between reinforcement ($h/B$) layers, and length ratio ($l/B$) of the reinforcement were constants for all tests and had a value of 0.4, 0.4, and 6, respectively.

The model test results showed the influence depth for placing reinforcement was about 2B. The BCR at ultimate bearing capacity increased with the increase of the embedment depth of the footing. For footing with embedment depth ratio ($D_f/B$) of 0, 0.3 and 0.6, the ultimate BCR increased from 1.13 to 2.0, 1.25 to 2.5, and 1.38 to 2.65 as the number of reinforcement layer increased from 1 to 6, The BCR values measured at a settlement ratio ($s/B$) less than 5$\%$ were smaller than those at ultimate bearing capacity. The BCR with embedment was greater than that without embedment. Although, the magnitude of the ratio of BCR at settlement less than 0.05B to BCR at ultimate bearing capacity decreased with the increase of the depth of the footing.

Chen et al. (2007) conducted a series of model tests on square footings supported by geosynthetic reinforced clay. The model tests were conducted inside a steel box with dimensions of 1.5 m (length) × 0.91 m (width) × 0.91 m (height). The model footings used in the tests were 25.4 mm thick steel plates with dimensions of 152 mm × 152 mm and 152 mm × 254 mm. The foundation soil consisted of low to medium plasticity marginal embankment soil having a liquid limit of 31 and a plastic index of 15. This soil contains 72$\%$ silt and 19$\%$ clay. The maximum dry density of the soil is 1,670 kg/m$^3$ with an optimum moisture content of 18.75$\%$ as
determined by Standard Proctor test. This silty clay soil was classified as CL according to the Unified Soil Classification System (USCS), and A-6 according to the AASHTO classification system. The shear strength parameters determined by large scale direct shear test at optimum moisture content of 18.75% with densities ranging from 1,525 kg/m³ to 1,763 kg/m³ revealed internal friction angles range between 25.96° and 24.13° and cohesion intercept range between 5.06 kPa and 24.58 kPa. Three types of geogrids and one type of geotextile were used as reinforcement in their tests.

Chen et al. (2007) reported that the optimum spacing of the top layer was found to be 0.33B for the square footing on geogrid reinforced clay. The bearing capacity increases with increasing number of reinforcement layers. The significance of adding a new reinforcement layer decreases with the increase in number of layers, which becomes negligible below the influence depth. The influence depth was obtained at approximately 1.5 B for geogrid reinforced clay and 1.25 B for geotextile reinforced clay in this study. The bearing capacity increases with the decrease of the vertical spacing of reinforcement layers. Geogrids with higher stiffness perform better than geogrids with lower stiffness.

Chen et al. (2009) conducted an experimental study of plate load tests on the reinforced crushed limestone. Their model tests were conducted in a 0.91 m wide, 1.5 m long, and 0.91 m deep steel box. A 152 mm wide square footing was used in the test. The foundation soil consisted of a crushed limestone with a uniformity coefficient of 20.26 and a coefficient of curvature of 1.37. The maximum dry density of the crushed limestone was 2,268 kg/m³, with an optimum moisture content of 7.5%, as determined by Standard Proctor test. This crushed limestone was classified as GW according to the Unified Soil Classification System (USCS) and A-1-a according to the AASHTO classification system. Large scale (304.8 mm × 304.8 mm ×
130.9 mm) direct shear tests on this crushed limestone at its maximum dry density under three different normal stresses (25 kPa, 50 kPa, 75 kPa) indicated an internal friction angle of 53°.

Five types of geogrids, one type of steel wire mesh (SWM), and one type of steel bar mesh (SBM) were used in the test. The model test results showed that the reinforcement appreciably improved the bearing capacity of crushed limestone and reduced the footing settlement. The bearing capacity was increased up to a factor of 2.85 at a settlement ratio of 10%, and the footing settlement was reduced up to 75% at a surface pressure of 5,500 kPa for crushed limestone reinforced with three layers of steel bar mesh. The BCR increases and the SRF decreases with an increase in the number of reinforcement layers. Geogrids with higher tensile modulus performed better than geogrids with lower tensile modulus. The structure and aperture size of geogrid have minimal influence on the performance of the geogrid reinforced crushed limestone sections tested in their study. The performance of footings on crushed limestone sections reinforced with steel wire mesh (SWM) and steel bar mesh (SBM), which have much higher tensile modulus than the geogrids used in their study, is much better than that on geogrid reinforced crushed limestone sections.

2.1.3.2 Large-Scale Field Studies

A series of large scale model tests on reinforced sand has been reported by Adams and Collin (1997). The tests were conducted in a 6.9 m (length) × 5.4 m (width) × 6 m (height) concrete box with 0.3 m, 0.46 m, 0.61 m, and 0.91 m square footings. Poorly graded fine concrete mortar sand was selected for tests. The sand had a mean particle size ($D_{50}$) of 0.25 mm, and a uniformity coefficient ($C_u$) of 1.7. The parameters investigated in the tests included the number of reinforcement layers, spacing between reinforcement layers, the top layer spacing, plan area of the reinforcement, and the density of soil.
The test results indicated that three layers of geogrid reinforcement could significantly increase the bearing capacity and that the ultimate bearing capacity ratio (BCR) could be increased to more than 2.6 for three layers of reinforcement. However the amount of settlement required for this improvement is about 20 mm (s/B = 5%) and may be unacceptable on some foundation application. The results also showed that the beneficial effects of reinforcement at low settlement ratio (s/B) could be achieved maximally when top layer spacing is less than 0.25B. They found that the improvement of the RSF was related to the density of sand. Large settlement ratio was required to mobilize the reinforcement in loose sand. The researchers also pointed out the fact that a general failure was less likely to happen if the top layer spacing was less than 0.4B. Adams and Collin (1997) recommended future research to be oriented towards determining the relation between the footing size and the thickness of the reinforced soil mass and comparing the behavior of the different reinforced soils.

Abu-Farsakh et.al (2008) conducted six large-scale field tests on geogrid-reinforced silty clay marginal embankment soil to study the behavior of reinforced soil foundation under the field condition. The stress distribution in the soil with and without reinforcement and the strain distribution along the reinforcement were also evaluated in their study. The tests were performed in an outdoor test pit having a dimension of 3.658 m (12 ft) (length) × 3.658 m (12 ft) (width) × 1.829 m (6 ft) (height). The model footing was 203 mm (8 in.) thick steel-reinforced precast concrete block with dimensions of 457 mm (1.5 ft) × 457 mm (1.5 ft). The foundation soil consisted of low to medium plasticity marginal embankment soil having a liquid limit of 31 and a plastic index of 15. This soil contains 72% silt and 19% clay. The maximum dry density of the soil is 1,670 kg/m³ with an optimum moisture content of 18.75% as determined by Standard Proctor test. This silty clay soil was classified as CL according to the Unified Soil Classification
System (USCS), and A-6 according to the AASHTO classification system. The shear strength parameters determined by large scale direct shear test at optimum moisture content of 18.75% with densities ranging from 1,525 kg/m³ to 1,763 kg/m³ revealed internal friction angles range between 25.96° and 24.13° and cohesion intercept range between 5.06 kPa and 24.58 kPa. Three types of biaxial geogrids with different tensile modulus were used in their study. Three reinforcement layers/spacing combinations (i.e., three layers placed at 305 mm spacing, four layers placed at 203 mm spacing, and five layers placed at 152 mm spacing) were investigated.

The test results showed that the inclusion of geogrid reinforcements results in increasing the soil’s bearing capacity and reducing the footing immediate settlement. The reinforcement benefits improve with the increase in number and tensile modulus of geogrids and with the decrease in layers’ spacing. The inclusion of reinforcements helps in redistributing the applied load to a wider area. The test results also showed that the developed strain along the geogrids is directly related to the footing settlement. The effective length of the geogrid reinforcement was estimated to be about 4 based on the strain measurement.

2.1.4 Finite Element Studies on Reinforced Soil Foundations

Reinforced soil consists of two constituents, namely, the soil and the reinforcement. Finite element modeling of reinforced soil foundation presented by researchers can be categorized into two groups: the first group treats reinforced soil as an equivalent homogeneous continuum media (eg, Pruchnicki and Shahrour, 1994, Sawicki, 1999 and Chen et al., 2000). In this category of method, the reinforced soil was treated as a macroscopically homogeneous composite material, the properties of which depend on respective properties of the constituents, their volume fractions and geometrical arrangement. The results obtained on the bases of this kind method can determine the failure mode and bearing capacity of RSF.
The second group models the reinforcement and soil as two separate components (e.g., Yetimoglu et al., 1994; Kurian et al., 1997; Maharaj, 2002). The reinforcement is generally treated as a linear elastic material in these studies. The soil model used by different researchers includes Modified Duncan hyperbolic model (Yetimoglu et al., 1994), Ducan-Chang model (Kurian et al., 1997), Mohr-Coulomb model (Boushehrian and Hataf, 2003), and Drucker-Prager model (Maharaj, 2002). Soil-reinforcement interface are generally modeled using two approaches: constraint approach and contact elements. The constraint approach generally assumes that separation is not allowed between the soil and reinforcement in normal direction, while in tangential direction slip can occur. In the use of the contact element, the normal stiffness is often given a very high value to prevent interpenetration of nodes. Material models of RSF used in literature are summarized in Table 2.2. This group of finite element analysis is typical and a lot of researchers performed their analysis following this approach. The finite modeling of this study will also be based on this approach. (What is the advantage and disadvantage of this approach).

Yetimoglu et al. (1994) performed finite element study on rectangular footing sitting on top of geogrid-reinforced sands for the small-scale model footing tests they conducted. In this study, the analyses were conducted under axi-symmetric conditions. The rectangular footing was treated as an equivalent circular plate of the same footing area. The geogrid reinforcement and the footing were represented by a series of discrete axi-symmetric shell elements and the sand was represented by an assembly of axi-symmetric quadrilateral and triangular elements. Vertical loads were applied sequentially in an equal increment and each load increment was divided into five steps to achieve better computational accuracy. The stress-strain behavior of the sand was simulated by the modified Duncan hyperbolic model (Duncan et al. 1980) and was represented...
by an assembly of axi-symmetric quadrilateral and triangular elements. The strains developed in
the grogrid reinforcement at failure were thought to be very small and hence a constant secant
modulus determined at an axial strain of 1% was used for the geogrid in the analyses. The
nominal thickness of the geogrid was 0.95 mm.

Their studies indicated that the optimum value of depth ratio at which the BCR was the
highest is around 0.3 for single-layer reinforced sand and is around 0.25 for the multilayer
reinforced sands. Their studies also indicated that the optimum vertical spacing of reinforcement
layers on geogrid-reinforced sands varied between 0.2B and 0.4B. The BCR increased with
increasing number of the reinforcement layers; however, the rate of increase in BCR was less
significant beyond a depth of 1.5B. And the BCR generally increased slightly with increasing
reinforcement size. The increase was more pronounced for reinforcement size ratio (L/B) up to
approximately 4.5, beyond which the BCR remained more or less constant. The BCR increases
with the increase of reinforcement stiffness and an axial stiffness beyond 1,000 kN/m would not
result in significant increases in the BCR.

Kurian et al. (1997) investigated the settlement of footing on reinforced sand by using 3D
finite element analysis. The results of the analysis were then compared with those from
laboratory model tests. 8-node brick element was used to discretize the soil, while 3D truss
element was used to discretize the reinforcement. A 3D soil-reinforcement interface friction
element developed on the basis of Goodman element was used in the analysis. Both the
reinforcement and the interface elements were geometrically 3D line elements. The stress-strain
behavior of sand was modeled by Ducan-Chang model, while the footing and the reinforcements
were assumed to be linearly elastic. The sand used in their study had an effective particle size
The friction angle determined by triaxial tests was about 38°.

Kurian et al. (1997) reported that there was a clear reduction of settlement in the reinforced sand at higher loads as compared to unreinforced sand. The numerical results also indicated that a small increase in settlement occurred in reinforced sand at the initial stage of loading process. A possible explanation of this phenomenon given by Kurian et al. (1997) was that the normal load was too small to mobilize enough friction between soil and reinforcement, i.e. a weak plane was initially presented with the inclusion of reinforcement. The relative movement between soil and reinforcement increased with the increase of load and decreased with increase of reinforcement depth. The maximum shear stress at the soil-reinforcement interface occurred at a relative distance ($x/B$) of about 0.5 from the center of the footing. The tension developed in reinforcement was maximum at the center and gradually decreased towards to the end of the reinforcement. As compared to unreinforced sand, the vertical stress contours shifted downwards in reinforced sand, i.e. spreading the stress deeper.

Yoo (2001) investigated the bearing capacity behavior of strip footing on geogrid-reinforced sand slope through finite element method. In his study, a footing pressure producing a footing settlement of 10% of the footing width ($0.1B$) at the footing center was taken as the ultimate bearing capacity. In the finite element modeling the initial stress condition was established first by applying the gravity force due to soil in steps with geogrid reinforcements in place. In Yoo’s (2001) finite element analysis of strip footing on geogrid-reinforced sand slope, the non-linear behavior of the fill was modeled using the modified version of hyperbolic stress-strain and bulk modulus model proposed by Duncan et al. while the foundation and the geogrid were treated as a linear elastic material.
The results showed the failure zone of the reinforced slope loaded with a footing was wider and deeper than that for the unreinforced slope. More geogrid reinforcement benefit was observed when placed at a depth equivalent to the footing width below the footing base than at shallower depths. Critical geogrid layout parameters for which maximum benefit was achieved were independent of the footing location with respect to the slope face, except the effective length of geogrid and the geogrid tensile modulus.

Maharaj (2002) investigated the influences of top layer spacing, vertical spacing of reinforcement layers, size of the reinforcement and the number of layers on the settlement of strip footing on reinforced clay using two dimensional nonlinear finite element analysis. The footing and soil were discretized into four node isoparametric finite elements while the reinforcement was discretized into four node one dimensional isoparametric elements. Drucker-Prager yield criteria was used to model clay. The footing and reinforcement were assumed to be linear elastic material. The clay used in the model had a Poisson’s ratio of 0.45 and an elasticity modulus of 13,000 kN/m². The cohesion intercept and friction angle of clay were 10.84 kN/m² and 0°. The stiffness of reinforcement used in the model ranged from 500 kN/m to 20,000 kN/m.

From finite element analysis of strip footing on reinforced clay, Maharaj concluded that in the case of single layer of reinforcement, the optimum top layer spacing ratio \((u/B)\) was found to be around 0.125 in reinforced clay. He also found that the effective length ratio \((l/B)\) of reinforcement was around 2.0, the influence depth depended on the stiffness of reinforcement and the increase in geosynthetics’ stiffness resulted in reducing the settlement of the footing.

Boushehrian and Hataf (2003) performed numerical analyses with Plaxis (Version 7.12) to investigate the bearing capacity of circular and ring footings on reinforced sand. The Mohr-Coulomb model was used for soil; the axi-symmetric condition and 15-node triangular elements
were used for the analysis. To model the slip between the soil and the reinforcement these elements are combined with interfaces. Reinforcements are slender objects with a normal stiffness but with no bending stiffness and can only sustain tensile forces and no compression. They are simulated by the use of special tension elements. The only material property of reinforcement is an elastic normal (axial) stiffness.

The effects of the depth of the first layer of reinforcement, vertical spacing and number of reinforcement layers on bearing capacity of the footings were investigated. Their results indicated that, when a single layer of reinforcement is used there is an optimum reinforcement embedment depth for which the bearing capacity is greatest. There also appeared to be an optimum vertical spacing of reinforcing layers for multi-layer reinforced sand. The bearing capacity was also found to increase with increasing number of reinforcement layers, if the reinforcements were placed within a range of effective depths. In addition, the analysis indicated that increasing reinforcement stiffness beyond a threshold value does not result in a further increase in the bearing capacity.

El Sawwaf’s (2007) used the computer code Plaxis to perform two-dimensional plane strain finite element analyses of RSF. In his analysis, the initial stress condition was established first by applying the gravity force due to soil in steps with the geogrid reinforcements in place. A prescribed footing load was then applied in increments (load control method) accompanied by iterative analysis up to failure. The modeled boundary conditions were assumed such that the vertical boundaries are free vertically and constrained horizontally while the bottom horizontal boundary is fully fixed. The non-linear behavior of sand was modeled with hardening soil model, which is an elastoplastic hyperbolic stress – strain model, formulated in the framework of friction hardening plasticity. The interaction between the geogrid and soil is modeled at both
sides by means of interface elements, which allow for the specification of a reduced wall friction compared to the friction of the soil. Effect of number of geogrid layers, Effect of geogrid layer length Effect of depth to top layer, vertical spacing of the geogrid were studied and he concluded that for optimum response, the recommended depth of reinforcing geogrid and geogrid spacing are 0.6 and 0.5 of the footing width. The geogrid length should be greater than or equal to five times the footing width and the recommended number of geogrid layers is 3.

Ahmet et al. (2008) used finite element analysis to investigate the performance of embankment construction over weak subgrade soil. Two-dimensional plain strain condition was adopted and only half of the physical model is considered due to an axisymmetric at the center of the footing. The displacement is restricted to zero in x-direction along the centerline of the model due to symmetry and the right side of the model for subgrade layer only. Also, the displacement is zero in x- and y- directions along the exposed bottom of the model and the displacement for the exposed ground surface of the sample are free to move in x- and y- directions. All the dead load was defined and separate solved as a linear elastic model. They adopted modified cam-clay model for clay and non-linear elastic-plastic (i.e hyperbolic) model for sand. Linear elastic model was used to represent geosynthetics materials and slip surface model was used to model the interaction characteristics between soil and reinforcement.

They reported that geogrid performed much better than geotextile. The best performance was achieved when geosynthetic reinforcement were located nearest to the footing. The strain within geosynthetics became almost negligible after a distance equal six times of the footing width. Better stress distribution and deformation pattern within embankment were obtained when the geosynthetics were introduced.
Basudhar et al. (2008) analyzed the behavior of a geotextile-reinforced sand-bed subjected to strip loading using the finite element method. The problem domain is divided into a finite number of four nodded rectangular elements for soil and two nodded linear elements for geotextile. The geotextile element is modeled as an axial element with linear approximation for the displacement in x-direction. No interface element is used but the interface is modeled as a contact problem. Mohr-Coulomb criterion is adopted for the slip between the soil and geotextile. Their results showed that for a single layer of geotextile reinforcement the optimal placement depth is 0.6B and the shear force in the geotextile increases till the reinforcement length from the central line is equal to 0.5 of footing width and then decreases.

Latha and Somwanshi (2009) performed numerical simulations on square footings resting on sand. The elastic-perfectly Mohr-Coulomb model was used to model the behavior of sand. Reinforcement was modeled. Effect of the type and tensile stiffness of geosynthetic material, the depth of reinforced zone, the spacing of reinforcement layers and the width of reinforcement were studied and they concluded that the layout and configuration of reinforcement play a vital role in bearing capacity improvement rather than the tensile strength of the geosynthetic material. They also reported that the effective depth of the zone of reinforcement below a square footing is twice the width of the footing. Within the effective reinforcement zone, the optimum spacing of reinforcing layers is about 0.4 times the width of the footing. Optimum width of reinforcement is about 4 times the width of the square footing.

Alamshahi and Hataf (2009) carried out a series of finite element analyses with PLAXIS software package (Professional version 8, Bringkgreve and Vermeer, 1998) to assess strip footings on sand slopes reinforced with geogrid. The initial stress condition was implemented first by applying the gravity force due to soil weight in steps with the geogrid reinforcements in
A prescribed footing load was then applied in increments accompanied by iterative analysis up to failure. The model boundary conditions showed that the vertical boundary is free vertically and constrained horizontally while the bottom horizontally is fully fixed. Six node triangle plane strain elements are selected for the soil and three node tensile elements are used for the footing and the geogrid. They used the non-linear Mohr–Coulomb criteria to model the sand for its simplicity, practical importance and the availability of the parameters needed. The interaction between the geogrid and soil was modeled at both sides by means of interface elements, which enabled for the specification of a decreased wall friction compared to the friction of the soil.

Results showed that the load-settlement behavior and bearing capacity of the rigid footing can be considerably improved by the inclusion of a reinforcing layer. The depth to the top geogrid layer, number of geogrid layers, vertical spacing of the geogrid were all investigated and based on their particular case, the optimum embedment depth and vertical spacing of the reinforcement layer was about 0.75 times the width of the footing. The optimum number of reinforcements was 2.

Chen and Abu-Farsakh (2011) performed a series of finite element analyses (FEA) on footings with different sizes using the commercial ABAQUS program to numerically study the scale effect of reinforced soil foundations. The soil is simulated as an isotropic elasto-perfectly plastic continuum. The yield criterion is described by the extended Drucker-Prager model with a linear form. The reinforcement is simulated as a membrane, which transmits in-plane force only and has no bending stiffness. The stress-strain behavior of reinforcement is modeled by a linear elastic model. The surface-based contact interaction is used in their study to model the soil-reinforcement interface.
Results showed that the load-settlement curves of unreinforced soil are the same if the settlement is expressed in a nondimensional settlement ratio of \( s/B \). The bearing capacity of reinforced soil decreases with increasing footing size if the total depth ratio of reinforcement \( (d/B) \), the vertical spacing ratio \( (h/B) \) of reinforcement layers, and hence the number of reinforcement layers \( (N) \) are kept constant. However, the difference in the bearing capacity is negligible if the total depth ratio \( (d/B) \) of reinforcement and the reinforced ratio \( (R_r) \) remain constant for all sizes of footing. The FEM analysis also indicates that the scale effect is mainly related to the reinforced ratio \( (R_r) \) of the reinforced zone, which is defined as:

\[
R_r = \left( \frac{E_r A_r}{E_s A_s} \right)
\]  

Where: 
- \( E_r \) is the elastic modulus of the reinforcement = \( J/t_R \);
- \( J \) is the tensile modulus of reinforcement;
- \( A_r \) is the area of reinforcement per unit width = \( Nt_r \times 1 \);
- \( t_R \) is the thickness of the reinforcement;
- \( N \) is the number of reinforcement layers;
- \( E_s \) is the modulus of elasticity of soil;
- \( A_s \) is the area of reinforced soil per unit width = \( d \times 1 \);
- \( d \) is the total depth of reinforced zone = \( u + (N-1)h \).

### 2.1.5 Summary of Literature Findings

Based on the above literature review, it is clearly demonstrated that the improved performance of reinforced soil foundations depends on a number of factors.

To maximize the reinforcement benefit, the optimum top layer spacing ratio \( (u/B) \), the optimum vertical spacing ratio \( (h/B) \), the effective length of reinforcement \( (L/B) \) and the influence depth ratio \( (d/B) \) are reported by different researchers and summarized in Table 2.2.
### Table 2.1 Summary of optimum parameters for reinforced soil foundations

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2.2 Geogrid Reinforced Bases in Flexible Pavements

2.2.1 Introduction

Pavement structures are built to support loads induced by traffic vehicle loading and to distribute them safely to the subgrade soil. A conventional flexible pavement structure consists of a surface layer of asphalt (AC) and a base course layer of granular materials built on top of a subgrade layer. Pavement design procedures are intended to find the most economical combination of AC and base layers' thicknesses and material types, taking into account the properties of the subgrade and the traffic loading to be carried during the service life of the roadway.

The major structural function of a base layer is to provide a stable platform for the construction of the asphalt layer and to reduce the compressive stresses on top of the subgrade and the tensile stresses in the asphalt layer. The base layer should be able to distribute the stresses applied to the pavement surface by traffic loading. These stresses must be reduced to levels that do not overstress the underlying subgrade soil.

The Base course layer can be the cause of pavement failures, due to inadequate capacity of support to upper layers or to being insufficiently stiff, such that they fail to transfer the load uniformly to the subgrade, leading to localized overloading of the subgrade, and resulting in excessive pavement rutting. These pavement failures usually necessitate complete pavement reconstruction, and not only remedial treatment of the pavement surface where the problem is visible. Therefore, when constructing a pavement structure on a weak subgrade soil layer, it may be required to increase the thickness of base layers, or use good quality base course material. However, the depletion of high quality aggregates is at a rapid pace as a consequence of the increasingly demands on highway systems. In addition, there are usually limitations on the
thickness of the pavement structures. These problems provide a motivation for exploring alternatives to existing methods of building and rehabilitating roads. Geogrid reinforcement in base course layer offers one such alternative.

The use of geogrids reinforcement in roadway applications started in the 1970s. Since then, the technique of geogrid reinforcement has been increasingly used and many studies have been performed to investigate the behavior of geogrid reinforcement in roadway applications (e.g., Hass et al., 1988; Al-Qadi et al., 1994; Perkins, 1999, 2001, 2002; and Berg et al., 2000). The results of experimental, analytical, and numerical studies reported in literature showed that geogrid reinforcement in pavement structures can extend the pavement’s service life, reduce base course thickness for a given service life, delay rutting development and help construction of pavements over soft subgrades (Al-Qadi et al. 1997, Cancelli and Montanelli 1999, Wasage et al. 2004, Montanelli et al. 1997, Moghaddas-Nejad and Small 1996, Kinney et al. 1998). The increase in service life of pavement structure is usually evaluated using the Traffic Benefit Ratio (TBR), which is defined as the ratio of the number of load cycles to achieve a particular rut depth in reinforced section to that of an unreinforced section of identical thickness, material properties, and loading characteristics. Meanwhile, the reduction in base thickness is usually evaluated using the Base Course Reduction (BCR) factor, which is defined as the ratio of reinforced base thickness divided by the unreinforced base thickness of same performance under a given traffic load level.

In providing reinforcement, the geosynthetic material structurally strengthens the pavement section by changing the response of the pavement to applied loading (Koerner, 2005). Geogrid reinforcement provided a more uniform load distribution through distributing the load to larger area and a deduction in the rut depth at the surface of the asphalt course (Wasage et al., 2004).

Earlier studies (Anderson and Killeavy 1989, Barksdale et al, 1989 and Cancelli et al., 1996) have demonstrated that geogrids are superior to geotextiles when used as a reinforced member. So geogrids will be used as the only reinforcement for the study of reinforced bases in flexible pavements in this dissertation.

2.2.2 Reinforcement Mechanisms of Reinforced Geogrid Base Reinforced Pavement

The reinforcement mechanisms in geogrid base reinforced pavement sections include lateral restraint, increased bearing capacity and tension membrane effect.

2.2.2.1 Lateral Confinement Mechanism

The principle mechanism responsible for reinforcement in paved roadways is the base course lateral restraint and is schematically illustrated in Figure 2.3. Verticular loads applied to the roadway surface create a lateral spreading motion of the base course. Tensile lateral strains are created in the base below the applied load as the material moves down and out away from the load. The geosynthetic restrains the base thus reducing or restraining this lateral movement. The term lateral restraint involves several components of reinforcement including: (i) restraint of lateral movement of base aggregate (Perkins, 1999a); (ii) increase in stiffness of the base course aggregate layer (Bender and Barenberg, 1978), (iii) reduction of shear stress in the subgrade soil (Love et al., 1987), (iv) improved vertical stress distribution on the subgrade (Milligan et al., 1989). This mechanism of reinforced bases pavement is illustrated in Figure 2.3.
2.2.2.2 Increase of the Bearing Capacity Mechanism

The improved bearing capacity is achieved by shifting the failure envelope of the pavement system from the relatively weak subgrade to the relatively stiff base layer as illustrated in Figure 2.4. Such that, the bearing failure model of subgrade may change from punching failure without reinforcement to general failure with ideal reinforcement. Binquet and Lee (1975) initially established this finding.

2.2.2.3 Tension Membrane Mechanism

The tension membrane effect (Giroud and Noiray, 1981) develops as a result of vertical deformation creating a concave shape in the tensioned geogrid layer; this is demonstrated in Figure 2.5. The vertical component of the tension membrane force can reduce the vertical stress acting on the subgrade. Some displacement is needed to mobilize the tension membrane effect. Generally, a higher deformation is required for the mobilization of tensile membrane resistance as the stiffness of the geogrid decreases. Significant rut depth and high stiffness of the geosynthetic must be provided to initiate the membrane effect and thus to enhance the bearing
capacity of the subgrade (Som and Sahu, 1999 and Gobel et al., 1994). In order for this type of reinforcement mode to be significant, there is a consensus that the subgrade CBR should be less than 3 (Barksdale et al., 1989).

Figure 2.5 Schematic illustration of improved bearing capacity (Perkins 2001)

Figure 2.6 Schematic illustration of tension membrane mechanism (Perkins 2001)
2.2.3 Experimental Studies on Geosynthetic Reinforced Pavement

2.2.3.1 Small-Scale Laboratory Studies

In order to better understand the reinforcement mechanisms acting in a large-scale reinforced soil structure, studies were also conducted to evaluate such mechanisms at a small-scale controlled laboratory environment. These studies have investigated the effect of geosynthetics on the deformation and strength behavior of reinforced materials using both monotonic and cyclic triaxial tests. Gray and Al-Refeai (1986) conducted triaxial compression tests on dry reinforced sand using five different types of geotextile. Test results demonstrated that reinforcement increased peak strength, axial strain at failure, and, in most cases, reduced post-peak loss of strength. At very low strain (<1%), reinforcement resulted in a loss of compressive stiffness. Failure envelope of the reinforced sand showed a clear break with respect to the confining pressure. After the point of break, failure envelope for the reinforced sand paralleled the unreinforced sand envelope.

Ashmawy and Bourdeau (1997, 1998) conducted monotonic and cyclic triaxial tests on geotextile-reinforced silt and sand samples which was 71 mm in diameter and 170 mm in length. The results of these studies had shown that the presence of geosynthetics had significantly improved the strength of tested samples. In addition the geosynthetic layer tended to reduce the accumulated plastic strains under cyclic loading. Ashmawy and Bourdeau (1997) investigated the effects of reinforcement layers spacing and reinforcement material properties on the achieved improvement. Their results showed that the amount of improvement depends on the spacing of the geotextile layers, and to a lesser extent on the geotextile and interface properties.

Perkins et al. (1999) have performed laboratory tests on reinforced and unreinforced sections that mimicked pavement layer materials and geometry and loading conditions encountered in the
field. Overall results from the test sections have demonstrated significant improvement in pavement performance, as defined by surface rutting, results from the inclusion of geosynthetic reinforcement. And substantial improvement was seen a soft clay subgrade was used. For stronger subgrade, it appears that little to no improvement occurred under these conditions. With the geogrid products used, the stiffer geogrid provided better pavement performance. And they also concluded that geogrids provide better improvement than geotextile product.

Moghaddas-Nejad and Small (2003) also conducted drained repeated triaxial compression tests on two granular materials (sand and fine gravel) reinforced by geogrid. The geogrid layer was placed at the mid-height of that sample which was 200 mm in diameter and 400 mm in length. The results of this study showed that for a particular confining stress, the effect of a geogrid on the reduction in permanent deformation increases rapidly with an increase in the deviator stress, until a peak is reached, then decreases gradually. However, the geogrid did not have a considerable effect on the resilient deformation of the tested materials.

Perkins et al. (2004) have performed cyclic triaxial tests on reinforced and unreinforced aggregate specimens. The specimens were 600 mm in height and 300 mm in diameter and were compacted inside a rigid compaction mould using a vibrating plate compactor. For the reinforced specimens, a single layer of reinforcement was placed at mid-height of the sample. Four different types of reinforcements were used in the tests (two geogrids, one geotextile and one geocomposite). Their findings supported the previous work reported by Moghaddas-Nejad and Small (2003), where it showed that the reinforcement does not have an effect on the resilient modulus properties of unbound aggregates, while showed appreciable effect on the permanent deformation properties of unbound aggregate as measured in repeated load permanent deformation tests. Perkins et al. (2004) also indicted that the relatively poor repeatability seen in
permanent deformation tests made it difficult to distinguish between tests with different reinforcement products. Their results also showed that the reinforcement did not have an appreciable effect on the permanent deformation until a mobilized friction angle of approximately 30 degrees is reached.

Nazzal et al. (2007) conducted an experimental study to evaluate the strength properties as well as permanent and resilient behavior of crushed limestone with and without geogrid reinforcement under monotonic and cyclic loading. The crushed limestone has a maximum size of 19 mm, and a $D_{10}$ of 0.18 mm, a $D_{60}$ 6 mm, and a uniformity coefficient of 30. It is classified as A-1-a and GW-GC according to the American Association of State Highway and Transportation (AASHTO) classification system, and the Unified Soil Classification System (USCS), respectively. The maximum dry unit weight and optimum moisture content, as determined by the standard Proctor test, were 17.2 kN/m$^3$ and 7.0%, respectively. Five types of biaxial geogrid with different tensile modulus were used in their study.

The results showed that the geogrid reinforcement apparently increased the strength and stiffness parameters (e.g., the secant elastic moduli, the ultimate shear strength) of crushed limestone under the monotonic loading and reduced the permanent deformation under the cyclic loading. The higher tensile modulus geogrid achieved better performance, as compared to the lower tensile modulus geogrid. The benefit of geogrid reinforcement was more pronounced at a higher strain level.

Subaida (2009) conducted an experimental study to investigate the beneficial use of woven coir geotextiles as reinforcing material in a two-layer pavement section. Monotonic and repeated loads were applied on reinforced and unreinforced laboratory pavement sections through a rigid circular plate. The effects of placement position and stiffness of geotextile on the performance of
reinforced sections were investigated using two base course thicknesses and two types of woven coir geotextiles. The test results indicate that the inclusion of coir geotextiles enhanced the bearing capacity of thin sections. Placement of geotextile at the interface of the subgrade and base course increased the load carrying capacity significantly at large deformations.

Considerable improvement in bearing capacity was observed when coir geotextile was placed within the base course at all levels of deformations. The plastic surface deformation under repeated loading was greatly reduced by the inclusion of coir geotextiles within the base course irrespective of base course thickness. The optimum placement position of coir geotextile was found to be within the base course at a depth of one-third of the plate diameter below the surface.

Abu-Farsakh and Chen (2011) conducted an experimental study to investigate the potential benefits of using geogrid base reinforcement in flexible pavement. A series of cyclic plate load tests were conducted on flexible pavement sections that were constructed inside a test box with inside dimensions of 2.0 m (length) × 2.0 m (width) × 1.7 m (height).

A cyclic load was applied through a steel rod that fit into a concave-shaped hole on a 25.4-mm-thick and 305 mm-diameter steel plate. The maximum applied load in tests was 40 kN (9,000 lb), which resulted in a loading pressure of 550 kPa (80 psi) and simulated dual tires under an equivalent 80-kN (18,000-lb) single-axle load. The subgrade consisted of a silty clay, having a liquid limit (LL) of 31 and a plasticity index (PI) of 15. The base course consisted of a Kentucky crushed limestone with an effective particle size \( (D_{10}) \) of 0.382 mm, a mean particle size \( (D_{50}) \) of 3.126 mm, a uniformity coefficient \( (C_u) \) of 11.80, and a coefficient of curvature \( (C_c) \) of 1.07. Four different geogrids were used to reinforce the base layer in the pavement test sections. The HMA mix used in the construction of the pavement test sections was a 19.0 mm design level 2 super pave mixtures.
The results indicated that the inclusion of geogrid base reinforcement can significantly reduce the rut depth and extend the service life of pavement sections. The traffic benefit ratio (TBR) was increased up to 15.3 at a rut depth of 19.1 mm. The surface rutting curves obtained showed that the performance of pavement sections was improved with the increase of tensile modulus of geogrid. The inclusion of geogrid base reinforcement resulted in redistributing the applied load to a wider area. The best performance was observed when the geogrid layer was placed at the upper one third of base layer.

Abu-Farsakh et al. (2011) conducted repeated load triaxial (RLT) tests on unreinforced and geogrid reinforced crushed limestone specimens. The crushed limestone have an effective particle size ($D_{10}$) of 0.28 mm, a mean particle size ($D_{50}$) of 5 mm, a uniformity coefficient ($C_u$) of 24, and a coefficient of curvature ($C_c$) of 1.97. Two groups of geogrids, TX with triangle aperture and BX with rectangle aperture, were used as reinforcement in their study. The results showed that the inclusion of geogrid reinforcement helped in reducing the accumulation of permanent deformation of granular base specimens under the RLT tests. The improvement was found to be a function of geogrid tensile modulus, geogrid arrangement/location, and with less effect the geogrid geometry. Geogrid arrangement proved to have the greatest influence on the reduction of permanent strain. They also reported that the geogrid reinforcement did not show any significant improvement in the resilient deformation or resilient modulus of crushed limestone specimen. The improvement was higher for geogrid-reinforced granular specimens prepared at the optimum and dry of optimum than those prepared at the wet of optimum.

### 2.2.3.1 Large-Scale Field Studies

Cancelli and Montanelli (1999). The results suggested that at a given maximum rut depth, a geogrid reinforced gravel base is equivalent to a much thicker unreinforced base. High strength
woven geotextiles provide good separation functions but limited reinforcement action. A rut depth of 5 mm is reached within 200 ESAL while with geogrids up to 80000 ESAL. Thus the relative traffic improvement ratio of geogrids is up to 400 times greater than woven geotextile. The higher tensile modulus geogrids have shown better contribution at CBRs 3% or lower. The percent reduction of rutting, between reinforced and unreinforced sections, increases with reducing the subgrade CBR, for all geosynthetics. A traffic improvement factor of 10 for a rut depth of 5 and 10 mm can be used for most of the soil conditions and appropriate geogrid type. The structural layer coefficient of the aggregate, when calculated in agreement with Cancelli et. al(1996), can be increased by a geogrid layer, having a layer coefficient ratio ranging from 1.5 to 2.0. The magnitude of elastic strains for geogrids is less than 0.2% for most of the sections monitored.

Tingle and Webster (2003) did four field test sections: one unreinforced with 20 in. thick base layer, One reinforced with woven PP geotextile and 15 in. thick base layer, One reinforced with nonwoven PP geotextile and 15 in. thick base layer, One reinforced with composite (geogrid/nonwoven geotextile) and 10 in. thick base layer. The results showed that the unreinforced section revealed no distinct rutting of the subgrade. However, evidence of subgrade intrusion into the base extended 5 in. above the interface, and aggregate from the base had punched into the subgrade to a depth of 1.5 in. below the interface. Reinforced section with woven geotextile showed no damage to the geotextile, but the subgrade did show up to 3 in. of rutting. There was no evidence of significant subgrade intrusion. Reinforced section with nonwoven geotextile showed that the subgrade had rutted in excess of 3 in. Approximately 0.25 to 0.75 in. of subgrade intrusion was identified only along the the vertical edges of both rutted wheelpaths, where the nonwoven geotextile had stretched almost to the point of rupture. No
evidence of subgrade intrusion was noted along the longitudinal overlap. Reinforced section with composite revealed that the subgrade had rutted approximately 2 in. The nonwoven geotextile had stretched slightly in one wheelpath, allowing 0.25 in. of subgrade intrusion along the vertical edge of the wheelpath. The geogrid had also torn under the center of the same wheelpath. No other damage to the geosynthetics was noted. Base course reduction factor was 0.75 for geotextile reinforced sections and 0.5 for composite reinforced sections.

Helstrom et al. (2007) evaluated the reinforcement and drainage capabilities of geosynthetics in roadways. Two series of test sections were constructed. One was constructed with a 300 mm (12 in.) thick base layer whereas the other one were constructed with a 600 mm (24 in.) thick base layer. Five test sections were constructed for each series: control, geogrid at base/subgrade interface, geogrid in the middle of base layer, drainage geocomposite at the base/subgrade interface with geogrid in the middle of base layer, and drainage geocomposite at the base/subgrade interface. Asphalt thicknesses of 150 mm (6 in.) were used for all test sections. The subgrade soil has standard penetration field blow counts as low as 7 and natural water contents approaching the liquid limit. Helstrom et al. (2007) reported that geogrid reinforcement and drainage geocomposite increased the effective structural number by between 5% and 17% for sections with a 300 mm (12 in.) thick base course and had no apparent effect on the sections with a 600-mm (24-in.) thick base course. They also reported that the drainage geocomposite had no effect on pore water pressure in the subgrade soils and 18–83% of the long term force in the geogrid was developed during placement and compaction of the base aggregate. Based on the analysis of the results, Helstrom et al. (2007) further concluded that the improved performance brought by geogrid reinforcement and drainage geocomposite in the 300-mm (12-in.) base
sections is equivalent to adding 25~75 mm (1~3 in.) of base aggregate to an unreinforced section.

Al-Qadi et al. (2008) investigated the geogrid effectiveness in a low-volume flexible pavement. Nine pavement test sections were constructed with base thickness of 203 mm (8 in.), 305 mm (12 in.), and 457 mm (18 in.) for this purpose. Asphalt thicknesses of 76 m (3 in.) were used, except in one section, where asphalt thickness of 127 mm (5 in.) was used. The pavement test sections were constructed on subgrade with a California bearing ratio (CBR) of 4 percent. Based on the accelerated testing results, Al-Qadi et al. (2008) concluded that for a thin base course layer, placing geogrid at the subgrade/base course interface gives better performance and that the geogrid should be placed at the upper one third of the base course layer for a thicker base course layer.

Henry et al. (2009) assessed the potential benefits of geogrid base course reinforcement in flexible pavements. The subgrade material used in their study was silt (ML under Unified Soil Classification System (USCS) guidance or A-4 under AASHTO soil classification system). Two asphalt and base thicknesses were used: 102 mm (4 in.) and 152 mm (6 in.) for the asphalt; and 300 (12 in.) and 600 mm (24 in.) for the base layer. Each combination of asphalt and base thickness was constructed with and without geogrid. As such, the total of eight test sections has been evaluated. The subgrade has modulus values of approximately 55 MPa (CBR of about 5 percent). Geogrids were placed at the base/subgrade interface for all stabilized sections. The results reported by (Henry et al. 2009) showed TBRs of 1.3 to 1.4. No benefit was observed for the test section with 600 mm (24 in.) thick base and 150 mm (6 in.) thick asphalt.

Hossain and Schmidt (2009) assessed the benefit of using a geotextile as a separator in low-volume roadways. Two pavement test sections were constructed on subgrade with a CBR of 5
percent. The test sections consisted of an 8 in layer of HMA and a 12 in of aggregate base layer. Laboratory repeated load triaxial test were also conducted on cylindrical specimens, which consist of 4 in thick subgrade soil and 2 in thick base aggregate. Hossain and Schmidt (2009) reported additional service life gained from geotextile reinforcement. They also reported that aggregate-soil compatibility strongly influences the magnitude and mechanism of benefit that can be provided by a geotextile placed at the base/subgrade interface.

2.2.4 Numerical Analyses of Geosynthetic Reinforced Pavement

Several numerical studies were performed to analyze pavement sections and assess the improvements due to the geosynthetic reinforcement. Most of the numerical studies were performed using the finite element method. Different constitutive models were used to determine the model that is most capable of representing the stresses and deformations in a reinforced pavement. Table 2.3 summarizes the constitutive models to model the asphalt concrete layers, base course layers, subgrade layers, reinforcement and interface that were reported in literature to investigate reinforced flexible pavement.

Conventional plasticity models with isotropic hardening rules are well suited for the prediction of permanent strain under a single cycle of load application. Repeated application of stress to the same level as that experienced during the initial load cycle results in purely elastic behavior with no accumulation of permanent strain. Plasticity based material models with kinematic hardening rules need to predict the accumulation of permanent strain in pavement layers under the application of repeated traffic loads.

The use of plasticity models with kinematic hardening rules allows for the growth of permanent surface deformation to be better predicted. Plasticity models of this type have been available since the 1970’s (Dafalias, 1975) but have only recently been applied to pavement
modeling. McVay and Taesiri (1985) described a bounding surface plasticity model that was developed and compared to results from repeated load triaxial tests.

The bounding surface concept is general and permits the inclusion of any type of formulation for a yield surface, which is taken to represent the formulation for the bounding surface.

The bounding surface plasticity has been widely used in modeling soils over past years. It can successfully simulate the behavior of sand (e.g. Bardet, 1986), clay (e.g. Voyiadjis and Kim, 2003) and crushed limestone aggregate (e.g. Perkins, 1999). Some of its application form has been summarized in Table 2.4.

The finite element analyses results of Barksdale et al. (1989) showed that the BCR value increased by increasing the stiffness of the reinforcement. Increasing the thickness of the AC or base course layers reduced the magnitude of the BCR. The optimal location of the reinforcement was found to be between the bottom of the base course layer and 1/3 up into the base layer.

Miura et al. (1990) performed a finite element analysis on reinforced and unreinforced pavement sections. They compared the results of the finite element analysis with the experimental measurements on similar sections. The results indicated that the prediction of finite element analysis was not in agreement with the behavior observed in the tests. The predicted reduction in surface displacements was 5% compared to an actual displacement reduction of 35% measured by the tests.

Dondi (1994) used ABAQUS software package to conduct a three dimensional finite element analysis to model the geosynthetic reinforced pavements. The results of this study indicated that the use of the reinforcement resulted in an improvement in the bearing capacity of the subgrade layer and a reduction in the shear stresses and strains on top of it. In addition, the
vertical displacements (rutting) was also reduced by 15 to 20 % due to the intrusion of geosynthetic reinforcement. With an empirical power expression, the life of the reinforced sections was estimated to be increased by a factor of 2 to 2.5 as compared to the unreinforced section.

Wathugala et al. (1996) used ABAQUS finite element software package to formulate the finite element model for pavements with geogrid reinforced bases. The results of the analysis were compared with an unreinforced pavement sections at the same geometry and material properties. The comparisons indicated that the inclusion of geogrid reinforcement reduced the permanent deformations (rutting) by 20% for a single load cycle. This level of improvement was related to the flexural rigidity of the geosynthetics caused by the model presentation used by the authors (Perkins, 2001).

Leng and Gabr (2005) conducted a numerical analysis using ABAQUS to investigate the performance of reinforced unpaved pavement sections. Their previous experimental work was used to validate the performance of the developed finite element model of the geosynthetic-reinforced pavements Leng and Gabr (2002). The researchers reported that the performance of the reinforced section was enhanced as the modulus ratio of the aggregate layer to the subgrade decreased. The critical pavement responses were significantly reduced for higher modulus geogrid or better soil/aggregate-geogrid interface property.

Kwon et al. (2005) developed a finite element model for the response of geogrid reinforced flexible pavements. The results of this study indicated that the benefits of including geogrids in the granular base-subgrade interface could be successfully modeled by considering residual stresses concentrations assigned just above the geogrid reinforcement. These residual stresses were found to considerably increase the resilient moduli predicted in the base and subgrade of a
pavement section modeled. In addition, the study indicated that low subgrade vertical strains were also predicted to demonstrate a lower subgrade rutting potential when residual stress concentrations were assigned in the vicinity of the geogrid.

Nazzal et al. (2010) developed a finite-element model with ABAQUS software package to investigate the effect of placing geosynthetic reinforcement within the base course layer on the response of a flexible pavement structure. Finite-element analyses were conducted on different unreinforced and geosynthetic reinforced flexible pavement sections. The results of this study demonstrated the ability of the modified critical state two-surface constitutive model to predict, with good accuracy, the response of the considered base course material at its optimum field conditions when subjected to cyclic as well as static loads. The results of the finite-element analyses showed that the geosynthetic reinforcement reduced the lateral strains within the base course and subgrade layers. Furthermore, the inclusion of the geosynthetic layer resulted in a significant reduction in the vertical and shear strains at the top of the subgrade layer. The improvement of the geosynthetic layer was found to be more pronounced in the development of the plastic strains rather than the resilient strains. The reinforcement benefits were enhanced as its elastic modulus increased.

2.2.5 Summary of Literature Findings

Based on the above literature review, it is clearly demonstrated that geogrid base reinforcement benefits depend on a number of factors. These include: location of geogrid layer within the base course layer, base course thickness, strength/stiffness of subgrade layer, and the geometric and engineering properties of the geogrids. Studies in the literature have shown that the weaker the subgrade, the higher the percent reduction of rutting, and there was little improvement obtained for subgrades with high CBR (Cancelli and Montanelli 1999, Perkins
1999). The benefit of geogrid generally decreases with an increase in the thickness of the base course and becomes insignificant when the base course is very thick (Kinney et al. 1998, Collin et al. 1996). The location of geogrid within the base layer in the pavement system is very important to its reinforcement effectiveness (Perkins 1998, Webster 1993). The optimum location of geogrid depends on many factors, such as subgrade strength and base course thickness. For a thin base course layer, placing geogrid at the subgrade/base course interface gives better performance and that the geogrid should be placed at the upper one third of the base course layer for a thicker base course layer (Hass et al. 1988, Al-Qadi et al. 2008). However, no benefits were expected when a single layer of geogrid was placed at the midpoint or higher within the base layer for a thick base course over very soft flexible subgrades (Hass et al. 1988). Current available information does not provide clear quantifiable relationship between the performance of geogrid base reinforcement and any of the geogrid properties, such as aperture geometry, stability modulus, flexural stiffness, junction strength, and tensile modulus (Berg et al. 2000). It is believed that these properties work together to determine the performance of geogrid. Any property alone may not be enough to characterize the performance of geogrid.
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<td>D-P</td>
<td>Isotropic E-P-P</td>
<td>Elastoplastic M-C</td>
</tr>
</tbody>
</table>

Axi-sym: Axi-symmetric
M-C: Mohr-Coulomb Model
D-P: Drucker-Prager Model
Table 2.4 References on application of bounding surface model

<table>
<thead>
<tr>
<th>Author</th>
<th>Bounding Surface Formulation</th>
<th>Flow Rule</th>
<th>Hardening Rule</th>
<th>Projection Center</th>
<th>load</th>
<th>Soil Type</th>
<th>N.O.</th>
</tr>
</thead>
<tbody>
<tr>
<td>McVay and Taesiri (1985)</td>
<td>[ \bar{I}^2 + \left( \frac{Q-1}{N} \right)^2 \bar{J} - \frac{2}{Q} \bar{I} I_0 + \left( \frac{2-Q}{Q} \right) I_0^2 = 0 ] [ \sqrt{\bar{J}} + N(e-1) \left[ \frac{e}{(e-1) Q} \bar{I} - \bar{I} \right] \ln \left[ 1 - \frac{(e-1) Q I}{e I_0} \right] = 0 ]</td>
<td>Non-asso</td>
<td>anisotropic</td>
<td>The origin of stress</td>
<td>Cyclic</td>
<td>Pavement base material</td>
<td>10</td>
</tr>
<tr>
<td>Dafalias, Herrmann (1986)</td>
<td>( (\bar{I} - I_0) \left( \bar{I} + \frac{R-2}{R} I_0 \right) + (R-1)^2 \frac{\bar{J}}{N} = 0 ) ( \frac{(\bar{I} - I_0)}{R} \left( \frac{\bar{J}}{N} - I_0 \left( \frac{\bar{J}}{N} - I_0 \left( 1 + 2 \frac{RA}{N} \right) \right) \right) = 0 ) ( \frac{(\bar{I} - T I_0)}{R} (\bar{I} - (T + 2\xi) I_0) + \rho \bar{J}^2 = 0 )</td>
<td>Asso</td>
<td>Isotropic</td>
<td>Along the hydrostatic line</td>
<td>Cyclic</td>
<td>Cohesive soil</td>
<td>14</td>
</tr>
<tr>
<td>Anandarajah, Dafalias (1985,1986)</td>
<td>( (\bar{I} - I_0^{a}) \left( \bar{I}^a + \frac{R-2}{R} I_0^{a} \right) + (R-1)^2 \frac{\bar{J}^a}{N(\alpha^a)} = 0 ) ( (\bar{I} - I_0^{a}) \left( \bar{I}^a + \frac{R-2}{R} I_0^{a} \right) + (R-1)^2 \frac{\bar{J}^a}{N(\alpha^a)} = 0 )</td>
<td>Asso</td>
<td>Anisotropic</td>
<td>Along the ( I_0^{a} )-axis</td>
<td>Cyclic</td>
<td>Cohesive soil</td>
<td>14</td>
</tr>
<tr>
<td>Kaliakin, V.N. and Dafalias, Y. F. (1989,1990)</td>
<td>( (\bar{I} - I_0) \left( \bar{I} + \frac{R-2}{R} I_0 \right) + (R-1)^2 \frac{\bar{J}}{N} = 0 )</td>
<td>Asso</td>
<td>Isotropic</td>
<td>Along the hydrostatic line</td>
<td>Cyclic</td>
<td>Cohesive soil</td>
<td>12</td>
</tr>
<tr>
<td>Ling, Yue and Kaliakin, V. N.(2002)</td>
<td>( (\bar{I} - I_0) \left( \bar{I} + \frac{R-2}{R} I_0 \right) + (R-1)^2 \frac{J_0^2}{\chi / 27} = 0 )</td>
<td>Asso</td>
<td>Isotropic, Rotational and distortional</td>
<td>Along the ( K_0 ) line</td>
<td>Cyclic</td>
<td>Cohesive soil</td>
<td>12</td>
</tr>
</tbody>
</table>
(Table 2.4 Continued)

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Type of Shape</th>
<th>Equation</th>
<th>Assumption</th>
<th>Source</th>
<th>Origin of Stress</th>
<th>Load Type</th>
<th>Sand Type</th>
<th>N.O.P.</th>
</tr>
</thead>
<tbody>
<tr>
<td>J.P. Nardet (1986)</td>
<td>Single ellipse</td>
<td>[ \left( \frac{I/3 - A}{\rho - 1} \right)^2 + 3 \left( \frac{J}{M(\alpha)} \right)^2 - A^2 = 0 ]</td>
<td>Asso</td>
<td>Isotropic</td>
<td>The origin of stress</td>
<td>Cyclic</td>
<td>Sand</td>
<td>9</td>
</tr>
<tr>
<td>Saleeb, A. F. and Lou, K. A. (1988)</td>
<td>Bullet-shaped</td>
<td>[ \bar{q} - M_f \bar{p} \left( 1 + \frac{1}{\alpha_f} \right) \left[ 1 - \left( \frac{\bar{p}}{\bar{p}_c} \right)^{a_f} \right] = 0 ]</td>
<td>Non-Asso</td>
<td>Isotropic</td>
<td>The origin of stress</td>
<td>Cyclic</td>
<td>Sand</td>
<td>13</td>
</tr>
<tr>
<td>Dafalias, Y. F. and Manzari, M. T. (2004)</td>
<td>Tear drop shape</td>
<td>[ \left[ (s - p\alpha) : (s - p\alpha) \right]^{1/2} - \sqrt{2/3} pm = 0 ]</td>
<td>Non-Asso</td>
<td>Anisotropic</td>
<td>The origin of stress</td>
<td>Cyclic</td>
<td>Sand</td>
<td>15</td>
</tr>
<tr>
<td>Russell, Adrian R. and Khalili, Nasser (2004)</td>
<td>Combination of segments of a hyperbola and an ellipse</td>
<td>[ \bar{q} - M_{cs} \bar{p} \left[ \ln \left( \frac{\bar{p}_c / \bar{p}}{\ln R} \right) \right]^{1/N} = 0 ]</td>
<td>Non-asso</td>
<td>Isotropic</td>
<td>Center of Homology</td>
<td>mono</td>
<td>Sand</td>
<td>10</td>
</tr>
<tr>
<td>Crouch, R. S. and Wolf, J. P. (1994)</td>
<td>Combination of segments of a hyperbola and an ellipse</td>
<td>[ \bar{J} - \frac{\rho N I_0}{R} \left( 1 - \left( \frac{R \bar{I} / I_0 - 1}{R - 1} \right)^2 \right) = 0 ]</td>
<td>Non-asso</td>
<td>Anisotropic</td>
<td>varies</td>
<td>Cyclic and mono</td>
<td>unified</td>
<td>25</td>
</tr>
</tbody>
</table>

N.O.P: Number of Parameters; Asso: Associated flow rule; Mono: Monotonic load
CHAPTER 3 RESEARCH METHODOLOGY

3.1 Numerical Modeling of Reinforced Soil Foundations

Finite element modeling of reinforced soil foundation (RSF) includes geometry modeling, load modeling and material modeling. The commercial FEM program ABAQUS (ABAQUS, 2004) was used in this study. ABAQUS is a powerful finite element software package. It has been used in many different engineering fields throughout the world. ABAQUS software performs static and/or dynamic analysis and simulation of complex engineering and non-engineering problems. It can deal with bodies with various loads, temperatures, contacts, impacts, and other environmental conditions. In this section, the ABAQUS will be used to analyze the behavior and performance of strip footing over reinforced soils relevant to solving the approach slab problem discussed earlier in Chapter 1.

3.1.1 Geometry Model

The proposed strip footing supporting the bridge approach slab in the study has a width ranging from 4 ft to 6 ft and a length of 40 ft to 60 ft correspondingly. The length to width ratio of the footing is equal to 10, and thus the strip footing problem can be treated as a plane strain condition.

Two-dimensional plane strain model was adopted to simulate the performance of strip footing over reinforced soil. The boundary dimensions for the finite element model were determined by conducting several analyses on different mesh sizes to select the dimensions of the mesh in which the footing’s bearing capacity is not affected by the boundary conditions. Sensitivity analysis was also conducted to find the appropriate mesh size to minimize mesh-dependent effects. Number of finite element meshes with different degrees of refinement were tried first in order to obtain the appropriate mesh for the analysis of strip footing on reinforced
soil that converges to a unique solution. A refined mesh was adopted to minimize the effect of mesh dependency on the finite element modeling of cases involving changes in the number, length, and the location of geogrid layers.

The finally adopted finite element model is illustrated in Figure 3.1, which has the dimensions of $7.5B \times 7.5B$ and includes 16500 elements. The soil was discretized using eight-noded isoparametric elements and geogrid was modeled with 3-node truss element. The interaction between the soil and geogrid was modeled with surface element following Coulomb friction law. In the Figure 3.1, the width of the footing, depth of the first reinforcement layer (also called top layer spacing), depth of last reinforcement layer (or influence depth), and the vertical spacing between reinforcement layers is designated as $B, u, d,$ and $h,$ respectively. The boundary conditions are also presented in the figure.

3.1.2 Load Model

The footing is regarded as rigid, so applying load on the footing is equal to applying uniform vertical downward displacements at the nodes immediately underneath the footing (Yetimoglu et al., 1994). Horizontal displacements at the interface between the footing and the soil were restrained to zero assuming perfect roughness of the interface and symmetry of the footing. The vertical displacement was applied in 1000 increments to achieve a smooth response curve. In the loading process, a footing pressure producing a footing settlement of 10% of the footing width ($s/B = 10\%,$ here $s$ is the footing settlement) at the footing center was taken as the ultimate bearing stress (Chungsik, 2001). The embedment of a footing was simulated by applying a uniform vertical pressure ($\sigma = \gamma_s \cdot D_f$ with $\gamma_s$ is the soil’s unit weight, and $D_f$ is the embedment depth of footing) at the bottom level of the footing. The initial condition (geostatic stress) of the
reinforced soil was established by applying the gravity force due to soil in the first step of the analysis.

Figure 3.1 Finite element model of the strip footing sitting on geogrid-reinforced soil

3.1.3 Material Models

The material models of geogrid-reinforced soil are composed of soil model, geogrid model and soil-geogrid interaction model, which are discussed in the following sub-sections.

3.1.3.1 Soil Model

The soil (both embankment silty clay and crushed limestone) was discretized using eight-noded isoparametric elements and was modeled as an isotropic elasto-plastic continuum
described by the extended Drucker-Prager model. The Drucker-Prager plasticity model is an isotropic elasto-plastic model that has been used in many studies in the literature to represent the behavior of granular base course aggregate and cohesive subgrade soils. The linear Drucker-Prager criterion was used in this study and is written as:

\[ F = t - p \cdot \tan \beta - d = 0 \]  

(3.1)

Where:

\[ t = \frac{1}{2} \cdot q \cdot \left[ 1 + \frac{1}{K} - \left( 1 - \frac{1}{K} \right) \left( \frac{r}{q} \right)^3 \right] \]  

(3.2)

\( \beta \) is the slope of the linear yield surface in the P-t stress plane and is commonly referred to as the friction angle of the material;

\( d \) is the cohesion of the material;

\( K \) is the ratio of the yield stress in tri-axial tension to that in tri-axial compression;

\( p \) is the mean effective stress, and it can be calculated, as

\[ p = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}) \]  

(3.3)

Where \( \sigma_{11}, \sigma_{22}, \sigma_{33} \) are the normal stress of stress vector \( \sigma \).

\( q \) is the Mises equivalent stress, and it is determined through Equation 3.4,

\[ q = \sqrt{\frac{3S \cdot S}{2}} \]  

(3.4)

Where, \( S \) is the deviatoric stress, and its component is obtained through \( s_{ij} = \sigma_{ij} - p \delta_{ij}, \delta_{ij} = 1 \) when \( i \) is equal to \( j \) and \( \delta_{ij} = 0 \) when \( i \) is not equal to \( j \). and \( r \) is the third invariant of deviatoric stress, \( r = \left( \frac{9}{2} S \cdot S \cdot S \right) \).

The yield surface of this model in the p-t plane is illustrated in Figure 3.2.
Generally, the experimental data are only available for Mohr-Coulomb model with the friction angle and cohesion. If the experimental data for Drucker-Prager yield line is not readily available, the yield line can be obtained from Mohr-Coulomb friction angle, $\phi$, and cohesion, $c$, which has been specified in ABAQUS/Standard manual. Under plane strain condition, the relationship for matching these two models can be determined as follows:

$$
\beta = \arctan \frac{\sqrt{3} \cdot \sin \varphi}{\sqrt{1 + \frac{1}{3} \cdot \sin^2 \varphi}}; \quad \text{and} \quad d = \frac{c \cdot \sqrt{3} \cdot \cos \varphi}{\sqrt{1 + \frac{1}{3} \cdot \sin^2 \varphi}}
$$

(3.5)

Where $\varphi$ is the Mohr-Coulomb frictional angle of the soil determined from laboratory direct shear tests and $c$ is the Mohr-Coulomb cohesion of the soil. The material properties of the soil used in the study are presented in Table 3.1.
Table 3.1 Material properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Friction Angle</th>
<th>Cohesion (psi/kPa)</th>
<th>Elastic Tensile Modulus (psi/kPa)</th>
<th>Poisson ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Embankment soil</td>
<td>30</td>
<td>11.6/80.0</td>
<td>37700/259932</td>
<td>0.3</td>
</tr>
<tr>
<td>Crushed limestone</td>
<td>48</td>
<td>-</td>
<td>17420/120000</td>
<td>0.35</td>
</tr>
<tr>
<td>Geogrid I</td>
<td>-</td>
<td>-</td>
<td>10380/71568</td>
<td>0.3</td>
</tr>
<tr>
<td>Geogrid II</td>
<td>-</td>
<td>-</td>
<td>21760/150030</td>
<td>0.3</td>
</tr>
<tr>
<td>Geogrid III</td>
<td>-</td>
<td>-</td>
<td>43580/300474</td>
<td>0.3</td>
</tr>
<tr>
<td>Geogrid IV</td>
<td>-</td>
<td>-</td>
<td>47440/327087</td>
<td>0.3</td>
</tr>
<tr>
<td>Geogrid V</td>
<td>-</td>
<td>-</td>
<td>54620/376592</td>
<td>0.3</td>
</tr>
<tr>
<td>Geogrid VI</td>
<td>-</td>
<td>-</td>
<td>91280/629353</td>
<td>0.3</td>
</tr>
<tr>
<td>Geogrid VII</td>
<td>-</td>
<td>-</td>
<td>102100/703955</td>
<td>0.3</td>
</tr>
<tr>
<td>Geogrid VIII</td>
<td>-</td>
<td>-</td>
<td>131800/908729</td>
<td>0.3</td>
</tr>
<tr>
<td>Steel Wire</td>
<td>-</td>
<td>-</td>
<td>171000/1178000</td>
<td>0.3</td>
</tr>
<tr>
<td>Steel Bar</td>
<td>-</td>
<td>-</td>
<td>204200/1407000</td>
<td>0.3</td>
</tr>
</tbody>
</table>

*Parameters taken from previous research study (Cai, et al., 2005)
**Parameters from large-scale shearing test
***Parameters from the manufactures

3.1.3.2 Reinforcement Model

According to Abu-Farsakh et al. (2008), typical strain distribution of geogrid in reinforced soil foundations is less than 2%, so geogrid can be assumed to be linear elastic. The geogrids was modeled with three-nodded isoparametric truss elements. The material properties of the reinforcement used in the study are also presented in Table 3.1, which were provided by the manufacturers.

3.1.3.3 Soil-Refinement Interaction Model

The soil-reinforcement interaction was simulated using two contact surface pairs above and below the reinforcement layer. The ABAQUS contact interaction feature uses the constraint approach to model the interaction between two deformable bodies or between a deformable body and a rigid body. With this feature one surface definition provides the master surface and the other surface provides the slave surface. After this contact pair is defined, a family of surface contact elements is automatically generated. At each node, these elements construct series...
measures of clearance and relative shear sliding. The simulation interaction consists of two components: one normal to the surfaces and one tangential to the surfaces.

For embankment soil-geogrid interface, Normal interaction between geogrid-soil was simulated by a “hard” contact while shear interaction between them was modeled with two contact surface pairs above and below the geogrid. Master/slave surface definitions were used for the top and bottom contact surfaces of the geogrid. Basic Coulomb friction model was used to model the shear interaction, which relates the maximum allowable frictional (shear) stress across an interface to the contact pressure between the contacting bodies. The general form of the coulomb friction model is given below:

$$\tau_{\text{crit}} = \mu \sigma$$

(3.6)

Where $\tau_{\text{crit}}$ is the critical shear stress along the interface; $\sigma$ is the normal stress along the interface; $\mu$ is the interface friction coefficient ($\mu = \tan \delta$, where $\delta$ is the interface friction angle).

Figure 3.3 Basic Coulomb friction model

In the study of reinforced embankment soil, the value of $\mu$ is varied to investigate the effect of interface on the performance of reinforced soil foundations.
The above mentioned two contacting surfaces can carry shear stresses up to a certain magnitude across their interface before they start sliding relative to one another. The shear stress versus shear displacement relationship is illustrated in Figure 3.4. The relationship has an elastic region,

\[
\tau = G_f \Delta \tag{3.7}
\]

Where, \( \tau \) is the shear stress along the interface; \( \Delta \) is elastic slip along the interface; \( G_f \) is the interface shear modulus, which depends on parameters of \( \tau_{\text{max}} \) and \( E_{\text{slip}} \), as shown as in Figure 3.4. The \( E_{\text{slip}} \) is the limitation of the relative shear displacement before the allowable interface shear stress is reached. An elastic slip of 1mm (\( E_{\text{slip}} = 1\text{mm} \)) was selected to prescribe the allowable relative displacement along the interface of embankment soil and reinforcement.

For the interface of crushed limestone and embankment soil, a full interlocking was assumed between the reinforcement and the surrounding material, i.e. crushed limestone and
reinforcement are tied together at the interface so that there is no relative motion (or slip) between them. This was modeled by using the tie-condition in ABAQUS interaction feature. Each node of the slave surface is tied to the nearest node on the master surface.

3.2 Numerical Modeling of Reinforced Base Pavements

The numerical finite element model of reinforced bases in flexible pavements needs to be capable of describing the dynamic stresses and strains responses and the accumulation of permanent strains in the system. To accomplish these objectives, finite element models that allow for the development and accumulation of permanent strains with applied cyclic load are required. The following sub-section will describe the numerical method adopted to simulate the stress-strain behavior of geogrid reinforced bases in flexible pavements.

3.2.1 Geometry Model

A two-dimensional axisymmetric finite element model was developed using ABAQUS finite element software package (ABAQUS, 2004) to analyze the flexible pavement structure with and without geogrid base reinforcement. Hua (2000) showed that rutting in a flexible pavement can be modeled using two-dimensional finite element models rather than three-dimensional models without significant loss in accuracy.

The radius of the mesh was selected based on the distance at which the vertical and horizontal strains become insignificantly small in all layers; and the depth of the mesh was chosen to be at the depth at which the maximum induced vertical stress in the subgrade became insignificantly small (<0.01% of the applied pressure). Mesh sensitivity was studied to determine the level of fine mesh needed for a stable finite element analysis that converges to a unique solution. The final mesh used in the study has a radius of 4500 mm and a depth of 4000 mm, shown in Figure 3.5. The figure also shows the different layers of pavement structure (AC layer,
base course layer and subgrade layer) and the geogrid reinforcement as well as the bounding conditions. Based on this analysis, 60, 360, 1800, 3961 elements were used for the geogrid, AC, base course layer and subgrade layer, respectively.

Eight-noded biquadratic axisymmetric quadrilateral elements were used for the subgrade, base, and asphalt concrete layers, while a three-noded quadratic axisymmetric membrane element with thickness of 1 mm was used for the geogrid reinforcement.

Figure 3.5 Finite element model for reinforced pavement
3.2.2 Load Model

The wheel load was simulated by applying the contact pressure on a circular area with a diameter of 305 mm (12 inch) on the surface of pavement section. A haversine-shaped load is adopted in the finite element analyses, which simulates the approaching and departing of wheel load and is presented in Figure 3.6. It has the following form:

\[
F = \frac{P[1 - \cos\left(\frac{2 \cdot \pi \cdot t}{T}\right)]}{2}
\]

Where \( P \) is the peak pressure (\( P = 550 \) kPa) and \( T \) is the total time for one full load cycle. The load was implemented with the use of a user subroutine (DLOAD).

![Figure 3.6 Haversine-Shaped load form used in the finite element analysis](image)

3.2.3 Materials’ Models

Typical reinforced pavement system consists of hot-mix asphalt layer, base course layer and subgrade layer as well as reinforcement layer. Different constitutive models are used to describe the behavior of the different materials.
3.2.3.1 Asphalt Concrete Model

Given that AC is a viscous material and that it exhibits permanent strain, ideally a visco-plastic material model would be perfect. However, a number of factors precluded the use of a model of this type. However, many studies suggested that a linear elastic model is suitable for modeling the AC layer. Harold (1994) indicated that the AC layer behaves elastic or visco-elastic at low to moderate temperature, the plastic response of AC mixtures can be neglected. Also, Benedetto and La Roche (1998) concluded that AC mixtures exhibit a complex elasto-visco-plastic response but at small strain magnitude the plastic component can be neglected. Saad (2005) suggested that when the time duration of this load affecting a pavement structure is small, the viscoelastic behavior of this AC layer becomes almost equivalent to an elastic structure.

Since only a 2 inch (50 mm) layer of AC was adopted in the analysis, which is very thin, it has small contribution to permanent deformation. In this study, the plasticity of AC layer was introduced by specification of an ultimate yield stress corresponding to a perfect plasticity hardening law. The parameters used for the AC layer is presented in Table 3.2.

<table>
<thead>
<tr>
<th>Material</th>
<th>υ</th>
<th>Elastic Modulus(kPa)</th>
<th>Yield Stress (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>0.35</td>
<td>3,450,000</td>
<td>770</td>
</tr>
</tbody>
</table>

3.2.3.2 Base Course Model

Granular material is the main source for base course layer. The cyclic response of granular materials is complex due to its highly nonlinear behavior. Many constitutive models have been proposed on incremental theory, shear strain and kinematic hardening theories (Prager, 1955; Ziegler, 1959), multiple yield surfaces (Mroz, 1967), and the bounding surface plasticity (Dafalias, 1975). The bounding surface plasticity has great attraction due to the ease of use and
accuracy of simulation. Perkins (1999) has used the bounding surface model to conduct finite element analysis of geogrid reinforced bases in flexible pavement system. The bounding surface plasticity model proposed by Dafalias and Herman (1986) is used in this finite element study to simulate the behavior of granular base material under cyclic load.

The bounding surface, as illustrated in Figure 3.7, is a smooth surface consisting of two ellipses and one hyperbola with continuous tangents. The inner surface in the figure is elastic zone. Stress state within the elastic zone produce purely elastic behavior. Stress states lying between the elastic zone and the bounding surface are capable of producing both elastic and inelastic behavior. As the stress state approaches the bounding surface, the rate of plastic strain increases.

Formulation, implementation, calibration and verification of the bounding surface model will be discussed later in details in Chapter Four.
3.2.3.3 Subgrade Model

The subgrade was modeled using the Modified Cam Clay model (Roscoe and Burland, 1968). The Modified Cam-clay model is expressed in terms of three variables: the mean effective pressure $p$, the deviator stress $q$, and the specific volume $v$.

The yield function of the modified Cam Clay model corresponding to a particular value $p_c$ of the pre-consolidation pressure has the form shown in Equation 3.9, and is represented by an ellipse in the $q$-$p$ plane as shown in Figure 3.8.

$$f = q^2 - M_c^2 (p_c - p)$$

(3.9)

Where:

- $M_c$ is the slope of critical state line in the $q$-$p$ plane;
- $p_c$ is the preconsolidation pressure;
- Again, $p_c$ is the mean effective stress, can be calculated using Equation 3.3; and $q$, is the deviator stress, that can be calculated with Equation 3.4.

![Figure 3.8 Modified Cam Clay yield Surface in p-q plane](image-url)
In the modified Cam-Clay model, associated plastic flow is assumed. Thus the yield surface is also the plastic potential. The size of the yield surface is controlled by the hardening rule, which depends only on the volumetric plastic strain component. Thus, when the volumetric plastic strain is compressive, the yield surface grows in size; however when there is a dilative plastic strain, the yield surface contracts.

The modified cam-clay model is ready for use in ABAQUS. Parameters needed as input include shear modulus (\(G\)), slope of critical state line (\(M\)), virgin compression slope (\(\lambda\)), swell/recompression slope (\(\kappa\)), initial void ratio (\(e_0\)). The cam-clay model parameters that used in this study are presented in Table 3.3.

Table 3.3 Modified Cam-Clay Model Parameters for Different Subgrade Soils (Nazzal, 2007)

<table>
<thead>
<tr>
<th>Subgrade</th>
<th>G (kPa)</th>
<th>M</th>
<th>(\lambda)</th>
<th>(\kappa)</th>
<th>(e_0)</th>
<th>CBR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft</td>
<td>5170</td>
<td>0.65</td>
<td>0.225</td>
<td>0.11</td>
<td>1.35</td>
<td>1.5</td>
</tr>
<tr>
<td>Medium</td>
<td>20000</td>
<td>1</td>
<td>0.11</td>
<td>0.084</td>
<td>0.95</td>
<td>7</td>
</tr>
<tr>
<td>Stiff</td>
<td>35000</td>
<td>1.56</td>
<td>0.022</td>
<td>0.005</td>
<td>0.54</td>
<td>15</td>
</tr>
</tbody>
</table>

3.2.3.4 Geogrid Model

Since dynamic strains induced in the geosynthetic are relatively small and are considered within the elastic range in the pavement system (Perkins, 1999) a linear elastic model was used to describe the behavior of geogrid material. Such model was proved to be efficient when used by other researchers (e.g., Dondi, 1994; and Ling and Liu, 2003; and Perkins, 2001). A secant modulus value for a low value of strain is believed to the most descriptive design parameters. Five geogrid types with different tensile modulus (at axial strain of 2%) were used.

However, since the geogrid has an orthotropic linear elastic behavior, it was required to determine the equivalent isotropic elastic properties that can be used in the finite element analysis. The following section describes the method used to convert the orthotropic to isotropic
linear elastic properties. Direction dependence of elastic properties was prescribed through the use of a linear, orthotropic elastic constitutive matrix. Orthotropic linear elasticity is described by three modulus ($E_{ij}$), three independent Poisson’s ratio ($\nu_{ij}$), and three shear modulus ($G_{ij}$).

The constitutive equation for an orthotropic linear-elastic material containing the elastic constants described is given by Equation 3.10.

$$
\begin{bmatrix}
    E_{xm} \\
    E_m \\
    E_n \\
    \gamma_{xm-m} \\
    \gamma_{xm-n} \\
    \gamma_{m-n}
\end{bmatrix}
\begin{bmatrix}
    1/E_{xm} & -\nu_{xm-m} / E_m & -\nu_{x-nxm} / E_n & 0 & 0 & 0 \\
    -\nu_{xm-m} / E_m & 1/E_m & -\nu_{n-m} / E_n & 0 & 0 & 0 \\
    -\nu / E_{xm} & -\nu_{m-n} / E_m & 1/E_n & 0 & 0 & 0 \\
    0 & 0 & 0 & 1/G_{xm-m} & 0 & 0 \\
    0 & 0 & 0 & 0 & 1/G_{xm-n} & 0 \\
    0 & 0 & 0 & 0 & 0 & 1/G_{m-n}
\end{bmatrix}
\begin{bmatrix}
    \sigma_{xm} \\
    \sigma_m \\
    \sigma_n \\
    \tau_{xm-m} \\
    \tau_{xm-n} \\
    \tau_{m-n}
\end{bmatrix}
$$

(3.10)

Where the subscripts “xm” and “m” denote the in-plane cross-machine and machine directions, and “n” denotes the direction normal to the plane of the geosynthetic. The model contains 9 independent elastic constants, of which 4 (Exm, Em, vxm-m, Gxm-m) are pertinent to a reinforcement sheet modeled by membrane elements in a pavement response model.

Poisson’s ratio, $\nu_{m-xm}$, is related to these other constants through Equation 3.11.

$$
\nu_{m-xm} = \nu_{xm-m} \frac{E_m}{E_{xm}}
$$

(3.11)

When using membrane elements, values for the remaining elastic constants can be set to any values that ensure stability of the elastic matrix. Stability requirements for the elastic constants are given by Equations 3.12 through 3.16 (Hibbitt, 2004).

$$
E_{xm}, E_m, E_n, G_{xm-m}, G_{xm-n} > 0
$$

(3.12)

$$
|\nu_{xm-m}| = \left( \frac{E_{xm}}{E_m} \right)^{1/2}
$$

(3.13)
The constitutive matrix for an isotropic linear-elastic constitutive matrix is given by Equation 3.17 and contains 2 independent elastic constants \((E, \nu)\). The third elastic constant in Equation 3.18 is the shear modulus \((G)\), which is expressed in terms of \(E\) and \(\nu\) by Equation 3.14.

\[
\varepsilon_{xm} = \begin{bmatrix} 1/E & -\nu/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & 1/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & -\nu/E & 1/E & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G \end{bmatrix} \begin{bmatrix} \sigma_{xm} \\ \sigma_m \\ \sigma_n \\ \tau_{xm-m} \\ \tau_{xm-n} \\ \tau_{m-n} \end{bmatrix}
\]

(3.17)

\[
G = \frac{E}{2(1+\nu)}
\]

(3.18)

An equivalency of measured orthotropic elastic constants \((E_{xm}, E_m, \nu_{xm-m}, G_{xm-m})\) to isotropic constants \((E, \nu)\) can be determined using work-energy equivalency formulation, such that two materials, one containing orthotropic properties and the second containing isotropic properties, are assumed to experience an identical general state of stress.

The work energy produced by the application of the stress state shown in can be determined in general by Equation 3.19. Substitution of Equations 3.10 and 3.17 into Equation 3.19 results in the work energy for the orthotropic and isotropic materials given by Equations 3.20 and 3.21, respectively.
\[ W = \frac{1}{2} (\sigma e_{sm} + a \sigma e_m + b \sigma e_{sm-m}) \]  
(3.19)

\[ W = \frac{1}{2} \left( \frac{1}{E_{sm}} + \frac{a^2}{E_m} - \frac{2av_{m-sm}}{E_m} + \frac{b^2}{G_{sm-m}} \right) \]  
(3.20)

\[ W = \frac{\sigma^2}{2E} \left( 1 - 2av + a^2 + 2b^2(1 + v) \right) \]  
(3.21)

Setting Equations 3.20 and 3.21 equal to each other and solving for equivalent isotropic elastic modulus \((E_{equ})\) that produce same work energy by the orthotropic and isotropic materials results in Equation 3.18.

\[ E_{equ} = \frac{1 - 2av + a^2 + 2b^2(1 + v)}{\frac{1}{E_{sm}} + \frac{a^2}{E_m} - \frac{2av_{m-sm}}{E_m} + \frac{b^2}{G_{sm-m}}} \]  
(3.22)

Assuming a value Pioson’s ratio of \(v = 0.25\) and substitution of Equation 3.11 into Equation 3.22 results in Equation 3.23.

\[ E_{equ} = \frac{1 - 0.5a + a^2 + 2.5b^2}{\frac{1}{E_{sm}} + \frac{a^2}{E_m} - \frac{2av_{m-sm}}{E_m} + \frac{b^2}{G_{sm-m}}} \]  
(3.23)

Based on finite element and field testing programs, Perkins et al. (2004) suggested that \(a = 0.35\) and \(b = 0.035\) values were appropriate. Equivalent isotropic elastic properties for the geogrids used in this chapter were computed and summarized in Table 3.4

<table>
<thead>
<tr>
<th>Geogrid Type</th>
<th>Reference Name</th>
<th>Elastic Modulus (kPa)</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geogrid Type I</td>
<td>GGI</td>
<td>585100</td>
<td>0.25</td>
</tr>
<tr>
<td>Geogrid Type II</td>
<td>GGII</td>
<td>660000</td>
<td>0.25</td>
</tr>
<tr>
<td>Geogrid Type III</td>
<td>GGIII</td>
<td>860000</td>
<td>0.25</td>
</tr>
<tr>
<td>Geogrid Type IV</td>
<td>GGV</td>
<td>886500</td>
<td>0.25</td>
</tr>
<tr>
<td>Geogrid Type V</td>
<td>GGV</td>
<td>950000</td>
<td>0.25</td>
</tr>
</tbody>
</table>
3.2.3.5 Interaction Model

Full bonding was assumed between the different pavement layers and between the reinforcement layer and soil, base course and subgrade layer, AC layer and base course layer. This assumption is acceptable for the case of a paved system where the allowed surface rutting of such a system surface is small and the slippage is not likely to occur unless excessive rutting takes place (Barksdale 1989; Espinoza 1994; Nazzal, 2007).

3.2.4 Mechanistic-Empirical Rutting Model

The improvement of the inclusion of the geogrid layer within the base course layer was evaluated using the mechanistic empirical approach. Mechanistic-empirical modeling of flexible pavements relies upon the use of a numerical model to describe the response of the pavement system to an externally applied load representative of the traffic to which the roadway will be subjected. The response parameters computed from the results of finite element analysis are used to determine the pavement structure’s permanent deformation (rutting) based on empirical models.

The permanent deformation of pavement structures was determined by first dividing each pavement layer into sub-layers. Damage models are then used to relate the vertical compressive strain, computed from the finite element analysis, at the mid-depth of each sub-layer and the number of traffic applications to layer plastic strains. The overall permanent deformation is then computed using Equation 3.24 as sum of permanent deformation for each individual sub-layer.

\[ D_p = \sum_{i=1}^{N} \varepsilon^i_p \cdot h^i \] (3.24)

Where:

\( D_p \) : Permanent deformation of pavement
$N_s$ : Number of sub-layers

$\varepsilon_{ip}^i$ : Total plastic strain in sub-layer i

$h^i$ : Thickness of sublayer i

Three main damage models were used in the study, namely, one for the asphalt concrete material (Equation 3.25), one for the base layer (Equation 3.27), and one for subgrade layer (Equation 3.28). The parameters of these models were determined through national calibration efforts using the Long-Term Pavement Performance (LTPP) database, and laboratory tests conducted on the different pavement materials used.

For Asphalt concrete layer:

$$\frac{\varepsilon_p}{\varepsilon_{vd}} = k_1 10^{-3.4488} T^{1.5606} N^{0.473844}$$  \hspace{1cm} (3.25)

Where

$\varepsilon_p$ : Accumulated plastic strain at $N$ repetitions of load;

$\varepsilon_{vd}$ : Vertical strain of at mid-depth of the asphalt layer;

$N$ : Number of load repetitions;

$T$ : Pavement temperature ($^\circ F$);

$k_1$ : Function of total asphalt layer(s) thickness and depth to computational point, is used to correct for the variable confining pressures that occur at different depths and is expressed as:

$$k_1 = (C_1 + C_2 \cdot depth) \cdot 0.328196^{depth}$$  \hspace{1cm} (3.26)

Where:

$$C_1 = -0.1039 \cdot h_{ac}^2 + 2.4868 \cdot h_{ac} - 17.342$$

$$C_1 = 0.0172 \cdot h_{ac}^2 - 1.7331 \cdot h_{ac} - 27.428$$
\( h_{ac} \): Asphalt layer thickness.

For base course layer:

\[
\frac{\varepsilon_p}{\varepsilon_{vb}} = \beta_{GB} \left( \frac{\varepsilon_0}{\varepsilon_r} \right) \cdot e^{-\left( \frac{\rho}{N} \right)^\beta}
\] (3.27)

Where:

\( \varepsilon_{vb} \): Vertical strain at mid-depth of the base course material;

\( \beta_{GB} \): is national model calibration factor for unbound base course material and is equal to 1.673;

\( \varepsilon_0, \beta \) and \( \rho \) are material parameters;

\( \varepsilon_r \): Resilient strain imposed in laboratory test to obtain material properties.

For subgrade layer:

\[
\frac{\varepsilon_p}{\varepsilon_{vS}} = \beta_{SG} \left( \frac{\varepsilon_0}{\varepsilon_r} \right) \cdot e^{-\left( \frac{\rho}{N} \right)^\beta}
\] (3.28)

Where:

\( \varepsilon_{vS} \): Vertical strain at mid-depth of the subgrade layer;

\( \beta_{SG} \) is a national model calibration factor for subgrade material and is equal to 1.35.

### 3.3 Statistical Analysis

Based on the methodology described earlier, finite element models of reinforced soil foundation and geogrid base reinforced pavements were developed. Independent design parameters of RSF and reinforced pavement were varied to study their effects on the performance of the whole reinforced structures in terms of Bearing Capacity Ratio (BCR) for Reinforced Soil Foundations and Traffic Bearing Ratio (TBR) for reinforced pavements in
statistical models. Thus the vital parameters influencing the performance of the RSF and those affecting the behavior of reinforced pavement will be identified and quantified. A regression model will be developed to estimate the BCR of RSFs and TBR of geogrid base reinforced pavements. All statistical analysis will be conducted using the Statistical Analysis Software (SAS) package.

3.3.1 Multiple Linear Regression Analysis

Multiple regression analysis is a statistical methodology to discover the relationship between a dependent variable and a set of independent variables.

3.3.1.1 Regression Model

In multiple linear regression analysis, it is hypothesized that this relationship is linear and has the following form:

\[ y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_k x_{ik} + \epsilon_i \]  

(3.29)

Where, \( i=1, 2, \ldots, n \) and \( n \) is the number of observations;

\( y_i \) is dependent variable;

\( x_{i1}, x_{i2}, \ldots, x_{ik} \) are independent variables;

\( \beta_0, \beta_1, \ldots, \beta_k \) are unknown parameters;

\( \epsilon_i \) is random error.

It should be noted that this model is called “linear” because of it’s linearity in \( \beta \)’s, not in the \( x \)’s. Applying matrix notation, multiple regression model can be presented in a compact form:

\[ y = X\beta + \epsilon \]  

(3.30)

Where, \( y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \), \( X = \begin{bmatrix} 1 & x_{11} & \cdots & x_{i1} \\ 1 & x_{21} & \cdots & x_{i2} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & \cdots & x_{nk} \end{bmatrix} \), \( \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix} \), \( \epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix} \).
3.3.1.2 Fitting the Model

Least squares of error method can be used to fit the model. The least squares estimate of \( \beta \) can be obtained by minimizing:

\[
S = \sum_{i=1}^{n} \left[ y_i - (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_k x_{ik}) \right]^2 = (y - X\hat{\beta})' (y - X\hat{\beta})
\]  

(3.31)

Minimizing \( S \) by setting the derivative of it with respect to \( \beta \) to zero:

\[
\frac{\partial S}{\partial \beta} = -2X'(y - X\hat{\beta}) = 0
\]  

(3.32)

Where, \( \hat{\beta} \) are the least squares estimate of \( \beta \).

Now, \( \hat{\beta} \) can be obtained:

\[
\hat{\beta} = (X'X)^{-1}X'y
\]  

(3.33)

3.3.1.3 Significance test for the overall model

Significance test for the overall model is a test to determine the effectiveness of the entire model, i.e. whether the linear relationship exists between the dependent variable and independent variables.

This is generally done by testing the null hypothesis: \( H_0: \beta_1 = \beta_2 = \cdots = \beta_k = 0 \) against the alternative hypothesis \( H_1: \) at least one of the \( \beta_j \) is non-zero.

The null hypothesis implies that none of the independent variables are linearly related to the dependent variable in the assumed multiple regression equation.

The alternative hypothesis suggests at least one of the independent variables is linearly related to the dependent variable.

This hypothesis can be tested by a comparison of MSR (Mean Square Regression) and MSE (Mean Square Error).
This test is an F statistic. The best way for this test is to use Analysis of Variance (ANOVA). ANOVA table are generally used for the ANOVA calculations; and it has the following general form.

### TABLE 3.5 ANOVA table for Multiple Linear Regression

<table>
<thead>
<tr>
<th>Degrees of freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>k</td>
<td>SSR</td>
<td>MSR</td>
</tr>
<tr>
<td>Error</td>
<td>n-k-1</td>
<td>SSE</td>
<td>MSE</td>
</tr>
<tr>
<td>Total</td>
<td>n-1</td>
<td>SST</td>
<td></td>
</tr>
</tbody>
</table>

The terms displayed in Table 3.5 are defined and computed as follows:

\[
SST = \sum_{i=1}^{n} (y_i - \bar{y})^2 , \quad \text{total sum of squares}
\]

\[
SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 , \quad \text{sum of squares due to error}
\]

\[
SSR = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2 , \quad \text{sum of squares due to regression}
\]

\[
MSE = \frac{SSE}{n - k - 1}, \quad \text{mean square due to error}
\]

\[
MSR = \frac{SSR}{k}, \quad \text{mean square due to regression}
\]

Where \( \hat{y}_i \) are the predicted values,

\( \bar{y} \) is the mean of dependent variables.

The three sums of squares are related by the formula:

\[
SST = SSR + SSE
\]

Rejecting null hypothesis (H\(_0\)) if \( F > F_{\alpha, k, n-k-1} \);

failing to reject null hypothesis (H\(_0\)), if \( F \leq F_{\alpha, k, n-k-1} \);

\( \alpha \) is the significance level.
3.3.1.4 Goodness of Fit of the Model

The quality of the fit can be measured by the sum of the squares of the residuals, which is defined as: \( e_i = y_i - \hat{y}_i \) \hspace{1cm} (3.35)

A good fit should have small residuals. However, this quantity is dependent on the units of \( y_i \). Thus the coefficient of determination, \( R^2 \), is generally used to measure the goodness of fit.

\[
R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}
\] \hspace{1cm} (3.36)

\( R^2 \) ranges from 0 to 1. The closer it is to 1, the better the fit. When \( R^2 \) is equal to 1 it means perfect linear relationship exists between the dependent variable and independent variables, while \( R^2 \) is equal to 0 it indicates independent variables have no impact on the dependent variable. \( R^2 \) can only increase by adding more independent variables to a model.

This is because SST is always the same for a given set of observations and SSE never increases with the inclusion of an additional independent variable. Since a large value of \( R^2 \) made by adding more dependent variables means nothing, it is often advisable to use the adjusted coefficient of multiple determinations (\( R_a^2 \)) as an alternative measure of fit.

\[
R_a^2 = 1 - \frac{SSE/(n-k-1)}{SST/(n-1)} = 1 - \frac{MSE}{MST}
\] \hspace{1cm} (3.37)

MSE is the estimate of standard error (\( \sigma^2 \)), i.e. \( s^2 = MSE \). It is easy to show when the number of observations \( n \) is large, the approximate width of 95% confidence interval for a future observation is \( 4s \). Therefore, the quality of the fit can also be assessed by \( s^2 \). The smaller the values of \( s^2 \) are, the better the fit. This measurement provides an excellent indication of the quality of the fit when the prediction is a very important function for the model. In most cases, both \( R^2 \) (and \( R_a^2 \)) and \( s^2 \) needs to be considered to assess the goodness of fit.
3.3.1.5 Significance Tests for Individual Regression Coefficients

If null hypothesis \( H_0 \) in significance test for the entire model is rejected, it only indicates at least one of the \( \beta_j \) is non-zero. The additional tests are needed to determine which these \( \beta_j \) are. Significance tests for individual regression coefficients would be useful for this determination.

This is generally done by testing the null hypothesis: \( H_0: \beta_j = 0 \) against the alternative hypothesis \( H_1: \beta_j \neq 0 \).

If null hypothesis \( (H_0) \) is not rejected, it indicates the independent variable \( x_j \) can be removed from the regression model. This test is a \( t \) statistic and can be written as,

\[
t = \frac{\hat{\beta}_j}{SE_{\hat{\beta}_j}} = \frac{\hat{\beta}_j}{\sqrt{c_{jj}MSE}}
\]

(3.38)

Where \( SE_{\hat{\beta}_j} \) is the standard error of the regression coefficient \( \hat{\beta}_j \), \( c_{jj} \) is diagonal element of \((X'X)^{-1}\) corresponding to \( \hat{\beta}_j \).

Rejecting null hypothesis \( (H_0) \), if \(|t| > t_{\alpha/2,n-k-1} \); failing to reject null hypothesis \( (H_0) \), if \(|t| \leq t_{\alpha/2,n-k-1} \). \((1-\alpha)\%\) Confidence Interval (CI) for \( \beta_j \) can be constructed as following:

\[
(1-\alpha)\%CI(\beta_j) = \hat{\beta}_j \pm T_{\alpha/2,n-k-1} \sqrt{c_{jj}MSE}
\]

(3.39)

3.3.2 Selection Technique

The goal of multiple regression analysis is often to determine which independent variables are important in predicting values of the dependent variable. The ideal multiple regression model in this context provides the best possible fit while using the fewest possible parameters. Different selection methods can be used to determine the 'best' model for the data.
Backward variable selection starts with the full model and removes one variable at a time based on a user-defined selection criterion. In SAS, the default is to remove the variable with the least significant F-test for Type II sum square error. Then the model is refit and the process is repeated. When all of the statistical tests are significant (i.e. none of the parameters are zero), the reduced model will be chosen. The default level of significance for this method is 0.10, rather than the 0.05 we usually use.

Forward selection fits all possible simple linear models, and chooses the best (largest F statistic) one. Then all possible 2-variable models that include the first variable are compared, and so on. The problem with this method is that once a variable is chosen, this variable remains in the model, even if it becomes non-significant.

Stepwise selection works in much the same way as forward selection, with the exception that the significance of each variable is rechecked at each step along the way and removed if it falls below the significance threshold. In this study, the stepwise selection method is used.

Finally, the $R^2$ selection method reports $R^2$ and MSE for all possible models. Such that, the differences between the models are compared, and the best model with the highest $R^2$ and lowest $s^2 = \text{MSE}$ is selected.
CHAPTER 4 IMPLEMENTATION AND VERIFICATION OF FINITE ELEMENT MODELS

4.1 Verification of the Finite Element Model Used for RSF

Finite element model of RSF was established according to the description in previous chapter (Chapter 3). The verification of the model is presented in this part. Geogrid was treated as linear elastic and the secant elastic modulus at a strain of 2% was used as its elastic modulus. Both the silty clay embankment soil and crushed limestone were modeled with Drucker-Prager model. Interaction between geogrid and silty clay embankment soil was modeled using the friction model available in ABAQUS and interaction between geogrid and crushed limestone was modeled with the tie option in ABAQUS.

4.1.1 Determination of Drucker-Prager Model Parameters

Due to the fact that the experimental data for Drucker-Prager yield line is not readily available, the yield line was obtained from the Mohr-Coulomb friction angle, \( \varphi \), and cohesion, \( c \).

Under plane strain condition, the relationships can be expressed as follow:

\[
\beta = \arctan \frac{\sqrt{3} \cdot \sin \varphi}{\sqrt{1 + \frac{1}{3} \cdot \sin^2 \varphi}}; \quad \text{and} \quad d = \frac{c \cdot \sqrt{3} \cdot \cos \varphi}{\sqrt{1 + \frac{1}{3} \cdot \sin^2 \varphi}}
\] (4.1)

Where \( \varphi \) is the Mohr-Coulomb frictional angle of the soil;

\( c \) is the Mohr-Coulomb cohesion of the soil.

The material properties of the soil used in the study are already presented in Table 3.1.

4.1.2 Verification of the Material Model with Small-Scale Laboratory Tests for RSF

In order to verify the suitability of the adopted material models for the soil, geogrids, and geogrid-soil interaction, finite element analyses were first checked against the results from laboratory model tests for a square footing on reinforced silty clay embankment soil reported by
Chen (2007) and reinforced crushed limestone reported by Abu-Farsakh et al. (2007). The soil properties were summarized in Table 3.1.

The model footings used in the laboratory were steel plates with dimensions of 6 inch (150 mm) x 6 inch (150 mm) x 1 inch (150 mm) (length x width x height), and the model tests were conducted in a 60 inch (1.5 m) long, 36 inch (0.9 m) wide and 36 inch (0.9 m) deep steel box. The geogrid used has an equivalent thickness of 0.04 inch and an elastic modulus of 538 psi (3709 kPa). Figures 4.1a to 4.1d show the comparison between the finite element analyses and the laboratory model footing tests for unreinforced and one-layer geogrid reinforced clay soil, unreinforced crushed limestone and 3-layer reinforced limestone, respectively.

As can be seen, the finite element analyses have a reasonable agreement with the results of model footing tests, although there are some discrepancies between them that are less remarkable near the footing’s ultimate bearing pressure. Therefore, the developed finite element model can be used with confidence to perform parametric study to evaluate the effect of different variables and parameters contributing to the performance of RSF.

4.2 Implementation and Verification of the Finite Element Model for Geogrid Base Reinforced Pavement

Cyclic response of granular base course materials is complex due to its highly nonlinear behavior. The bounding surface plasticity has great attraction due to the ease of use and its capacity of describing the behavior of granular material under cyclic loading (Manzari, 1997). In this study, the bounding surface model proposed by Dafalias and Herrman (1986) was used to model the behavior of base course material within the pavement structures.

4.2.1 Implementation of Bounding Surface Model

A user material subroutine (UMAT) for ABAQUS finite element analysis was developed and implemented into a Fortran code under the concept of Bounding Surface Plasticity (Dafalias
Slight modification was done to normalize all the directions in the model. The formulation of Dafalias and Herrman (1986) is explained below.

(a) Unreinforced embankment soil

(b) One-layer reinforced embankment soil

(c) Unreinforced crushed limestone

(d) One-layers of reinforced crushed limestone

Figure 4.1 Footing stress-settlement curves
4.2.1.1 Description of Bounding Surface Model

The model is described in terms of two surfaces represented in the stress space shown in Figure 4.2. The large surface represents the bounding surface, which in a conventional plasticity model is equivalent to a yield surface. The small surface denotes an elastic zone. In the scheme of the bounding surface theory, the stress state is denoted as $\sigma(I, J, \theta)$. And its projection stress on the bounding surface is expressed as $\sigma(I, J, \tilde{\theta})$.

The bounding surface $F(I, J, \theta, I_0) = 0$ is a smooth surface consisting of two ellipses and one hyperbola with continuous tangents. The formulation specifies the current size of the bounding surface in terms of the parameter $I_0$, the value of which reflects the amount of preloading or pre-consolidation of the material, which will be updated by the end of every integration step through a function of the accumulated plastic strain developed.

Stress state within the elastic zone produce purely elastic behavior. Stress states lying between the elastic zone and the bounding surface are capable of producing both elastic and inelastic behavior. As the stress state approaches the bounding surface, the rate of plastic strain increases. As the plastic strains develop the bounding surface expands. The stress state can only stay inside of the bounding surface. The projection center is denoted as $I_c$ and $s_c$ is a elastic factor. The distance between the projection center to the image point on the bounding surface of current stress point is denoted as $r$. The distance between current stress point and its image point on the bounding surface is denoted as $\delta$. And thus the distance between the projection center to the current stress point can be calculated as $(r - \delta)$, which are all shown in Figure 4.2. The ratio of the image stress and the current stress state (denoted as $\beta$) can be calculated as the ratio of $r$ and $(r - \delta)$ (i.e. $\beta = r/(r - \delta)$).
With the stress state approaching the bounding surface $\beta$ decreases and thus the plastic modulus decreases accordingly (refer to Equation 4.40 in the following section). And thus the plastic strain rate increases based on Equations 4.21 and 4.24 (presented in the following section).

The formulation of the constitutive matrix will be described in the following section.

Figure 4.2 Schematic Illustration of Bounding Surface Model
(after Dafalias and Herrman, 1986)

4.2.1.2 Formulation of Bounding Surface Model

All the formulation here is expressed in matrix form. “{ }” denotes an N-element column vector and “[ ]” denotes an N x N matrix. A superposed dot indicates the rate. The superscript T indicates the transposed matrix. A comma followed by a subscripted variable implies the partial derivative with respect to that variable. Bar over the stress quantities refer to points on the bounding surface. The formulation is described below in steps:

(1) Stress and strain vector and stress invariants

The effective stress vector $\{\sigma\}$, the strain vector $\{\varepsilon\}$, the vector equivalent of the Kronecker
delta tensor \{ \delta \}, and the second deviatoric stress vector \{ s s \} are defined in the matrix form as,

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{13} \\
\sigma_{23}
\end{bmatrix}, \quad
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\varepsilon_{12} \\
\varepsilon_{13} \\
\varepsilon_{23}
\end{bmatrix}, \quad
\begin{bmatrix}
1 \\
1 \\
1 \\
0 \\
0 \\
0
\end{bmatrix}, \quad
\begin{bmatrix}
\sigma_{11}s_{11} + \sigma_{12}s_{12} + \sigma_{13}s_{13} \\
\sigma_{12}s_{12} + \sigma_{22}s_{22} + \sigma_{23}s_{23} \\
\sigma_{13}s_{13} + \sigma_{23}s_{23} + \sigma_{33}s_{33} \\
\sigma_{11}s_{12} + \sigma_{12}s_{22} + \sigma_{13}s_{32} \\
\sigma_{11}s_{13} + \sigma_{12}s_{23} + \sigma_{13}s_{33} \\
\sigma_{12}s_{13} + \sigma_{22}s_{23} + \sigma_{33}s_{32}
\end{bmatrix}
\]

(4.2)

Where, \( \sigma_{11}, \sigma_{22} \) and \( \sigma_{33} \) are normal stresses;

\( \sigma_{12}, \sigma_{13}, \sigma_{23}, \sigma_{31}, \sigma_{21}, \sigma_{32} \) are shear stresses.

The deviatoric stress vector is given as

\[
\{ s \} = \{ \sigma \} - \left( \frac{1}{3} \{ \sigma \}^T \{ \delta \} \right) \{ \delta \}
\]

(4.3)

The first, second and third stress invariants, denoted as \( I, J \) and \( S \) respectively, are given by

\[
I = \{ \sigma \}^T \{ \delta \},
\]

(4.4)

\[
J = \sqrt{\frac{1}{2} \{ s \}^T \{ s \}}
\]

(4.5)

\[
S = \left( \frac{1}{3} \{ s s \}^T \{ s \} \right)^{\frac{1}{3}}
\]

(4.6)

The Lode angle is defines as

\[
\theta = \frac{1}{3} \sin^{-1} \left[ \frac{3\sqrt{3}}{2} \left( \frac{S}{J} \right)^3 \right]
\]

(4.7)

The corresponding first, second, third stress invariants and Lode angle of the image stress state are denoted as \( \tilde{I}, \tilde{J}, \tilde{S}, \) and \( \tilde{\theta} \) respectively.

(2) The total strain rate can be decomposed of an elastic part and a plastic part based on the basic kinematical assumption as follows:
\[ \{ \dot{e} \} = \{ \dot{e}^e \} + \{ \dot{e}^p \} \]  

(4.8)

Where the superscripts e and p refer to the elastic and plastic parts.

(3) The elastic strain rate comply to the hypoelastic constitutive relations, specified as,

\[ \{ \dot{\varepsilon}^e \} = [ C^e ] \{ \varepsilon^e \} \]  

(4.9)

\[ \{ \varepsilon^e \} = [ D^e ] \{ \dot{\varepsilon}^e \} \]  

(4.10)

\[ [ D^e ] = 2G[I] + \left( K - \frac{2G}{3} \right) \{ \delta \} \{ \delta \}^T \]  

(4.11)

Where \( K \) is the tangent elastic bulk modulus and \( G \) is the tangent elastic shear modulus. The quantity \([ C^e ]\) is elastic compliance matrix and \([ D^e ]\) is elastic stiffness matrix.

(4) The bounding surface \( F(I,J,I_0) = 0 \) is a smooth surface consisting of two ellipses and one hyperbola with continuous tangents as shown in Fig 4.2.

For ellipse 1

\[ F = (\overline{I} - I_0) \left( \overline{I} + \frac{R - 2}{R} I_0 \right) + (R - 1)^2 \left( \frac{\overline{J}}{N} \right)^2 = 0 \]  

(4.12)

For the hyperbola

\[ F = (\overline{I} - I_0)^2 - \left( \frac{\overline{J}}{N} - \frac{I_0}{R} \right) \left( \frac{\overline{J}}{N} - \frac{I_0}{R} \left( 1 + \frac{RA}{N} \right) \right) = 0 \]  

(4.13)

For ellipse 2

\[ F = (\overline{I} - TI_0) \left[ \overline{I} - (T + 2\xi) I_0 \right] + \rho J^2 = 0 \]  

(4.14)

The formulation specifies the current size of the bounding surface in terms of the parameter \( I_0 \), the value of which reflects the amount of preloading or pre-consolidation of the material.

(5) The variables used in the bounding surface is described here
\[ \xi = -\frac{T(Z + TF')}{Z + 2TF'} ; \rho = \frac{T^2}{Z(Z + 2TF')} \]  (4.15)

\[ y = \frac{RA}{N}, \quad F' = \frac{N}{\sqrt{1 + y^2}}, \quad Z = \frac{N}{R \left(1 + y - \sqrt{1 + y^2}\right)} \]  (4.16)

Where, \( R, A, T \) are the shape parameters of bounding surface,

\( N \) represents the slope of the classical creptical state line. The value of \( N, R \) and \( A \) varied with different Lode angle, \( \theta \).

The dependence on \( \theta \) through the parameters \( N(\theta), R(\theta) \) and \( A(\theta) \) according to

\[ Q(\theta) = g(\theta, c)Q_c \]  (4.17)

\[ c = \frac{Q_e}{Q_c}, \quad g(\theta, c) = \frac{2c}{1 + c - (1 - c)\sin 3\theta} \]  (4.18)

Where \( Q \) stands for any value of \( N, R \) and \( A \) corresponding to a certain lode angle. The quantity of \( Q_c \) is its value at the state of compression and the quantity of \( Q_e \) is its value at extension. Equation 4.17 and 4.18 defined a possible interpolation law between \( Q_c \) and \( Q_e \).

(6) Radial mapping rule is adopted here. The mapping rule associates an image point with the current stress point. The image stress point is obtained by projecting a ray from the projection center, through the current stress point, onto the bounding surface.

The distance between current stress point and its image point on the bounding surface is denoted as \( \delta \) and can be calculated as,

\[ \delta = \left[ (\bar{\sigma}_{ij} - \sigma_y)(\bar{\sigma}_{ij} - \sigma_y) \right]^{1/2} \]  (4.19)

The distance between the projection center to the image point on the bounding surface of current stress point is denoted as \( r \), and \( s_p \) is an elastic factor as described in Figure 4.2. The
stress state inside the elastic nuclear produces purely elastic strain while stress state outside 
the elastic nuclear produces both elastic and plastic strains.

The ratio of the image stress and the current stress is defined as $\beta$ and thus the radial 
mapping rule can be expressed as below,

$$\bar{T} = \beta(I - CI_0) + CI_0 \quad (4.20)$$

$$\bar{J} = \beta J \quad (4.21)$$

$$\bar{S} = \beta S \quad (4.22)$$

$$\bar{\theta} = \theta \quad (4.23)$$

Where $CI_0$ specifies the location of the projection center on the $I$ axis.

(7) The flow rule for the plastic strain rate is assumed to be expressed as

$$\{\dot{\varepsilon}^p\} = < L > \{P\} \quad (4.24)$$

Where, $\{P\}$ is the normalized direction of the plastic strain rate and $L$ is the loading index.

$<>$ is the Macauley brackets. $< L > = L$, if $L > 0$ and $< L > = 0$ if $L \leq 0$. The associated 
flow rule is adopted in this study.

(8) The hardening rule has a similar expression to that of the flow rule as given in the following 
form,

$$\dot{I}_0 = < L > V \quad (4.25)$$

Where $V$ collects and scales the appropriate plastic strain rate direction (The determination 
of $V$ value will be discussed in the following section).

Given that the bounding surface is a smooth surface which is expressed as $F(\bar{\sigma}, I_0) = 0$, the 
stress point may lie inside ($F < 0$) or on the surface ($F = 0$), however, it will never be outside
of the bounding surface \((F = 0)\). From the consistency condition the following equation can be derived as:

\[
\vec{F} = \{F_{\sigma}\}^T \{\vec{\sigma}\} + \vec{F}_{\sigma} \hat{t}_0 = 0
\] (4.26)

The normalized loading direction (normal to the bounding surface or loading surface) is given as,

\[
\overline{Q} = \frac{\{F_{\sigma}\}}{\|\{F_{\sigma}\}\|}
\] (4.27)

Using Equations 4.25 and 4.27 in Equation 4.26 leads to

\[
<L> = -\frac{\|\{F_{\sigma}\}\|}{\vec{F}_{\sigma}V} \{\overline{Q}\}^T \{\vec{\sigma}\}
\] (4.28)

Assume \(\overline{H}\) is defined as

\[
\overline{H} = -\frac{\vec{F}_{\sigma}V}{\|\{F_{\sigma}\}\|}
\] (4.29)

Then Equation 4.28 can be rewritten as:

\[
<L> = \frac{1}{\overline{H}} \{\overline{Q}\}^T \{\vec{\sigma}\}
\] (4.30)

Using Equations 4.9 and 4.24, Equation 4.8 can be rewritten here as

\[
\{\dot{\epsilon}\} = [C^e] \{\dot{\sigma}^e\} + <L> P
\] (4.31)

When \(<L>\) is a non-zero value, multiplying Equation 4.31 by \(\{\overline{Q}\}^T [D^e] \) and then substituting Equation 4.30 leads to

\[
\{\overline{Q}\}^T [D^e] \{\dot{\epsilon}\} = \{\overline{Q}\}^T [D^e] [C^e] \{\dot{\sigma}^e\} + \{\overline{Q}\} [D^e] \{P\} <L> = <L> (\overline{H} + \{\overline{Q}\} [D^e] \{P\})
\] (4.32)

Therefore,

\[
<L> = \frac{\{\overline{Q}\} [D^e] \{\dot{\epsilon}\}}{\overline{H} + \{\overline{Q}\} [D^e] \{P\}}
\] (4.33)
(9) Evolution of internal variable \( I_0 \)

As mentioned earlier, the variable, \( I_0 \), is a function of accumulated plastic strain and its evolution will be discussed here. From linear hydrostatic e-ln p consolidation and swelling/rebound relations, the following relationship can be obtained,

\[
I_{0,e_0} = \frac{<I_0 - I_I > + I_I}{\lambda - \kappa} (1 + e_m)
\]  

(4.34)

\[
e = -(1 + e_m)\{\delta\}^T \{\dot{e}\} = -(1 + e_m)\{\delta\}^T \{\dot{e}^e\} - (1 + e_m)\{\delta\}^T \{\dot{e}^p\} = \dot{e} + \dot{e}^p
\]  

(4.35)

\[
\frac{dl}{de^e} = \frac{-<I_0 - I_I > + I_I}{\kappa}, \quad \frac{dl_0}{de^e} = \frac{-<I_0 - I_I > + I_I}{\kappa}
\]  

(4.36a)

or

\[
\frac{dl_0}{de} = \frac{-<I_0 - I_I > + I_I}{\lambda}, \quad \frac{dl_0}{de^p} = \frac{-<I_0 - I_I > + I_I}{\lambda - \kappa}
\]  

(4.36b)

(10) Elastic constants \( K \) And \( G \)

\[
K = \frac{1+e_m}{3\kappa} (<I - I_I > + I_I)
\]  

(4.37a)

\[
G = \frac{3(1 - 2\mu)}{2(1 + \mu)} K
\]  

(4.37b)

Where \( \lambda \) and \( \kappa \) are the typical swelling and consolidation slopes in the \( e - \ln p \) plot, and the newly introduced \( I_I \) is a lower-limit value of \( I \) and \( I_0 \) (\( I_I = 3p_I \), \( p_I \) is the atmosphere pressure). \( \mu \) is the Poisson’s ratio.

(11) Bounding and additive plastic moduli

From equation (4.23), \( \dot{\varepsilon}_0^p = \{\delta\}^T \{\dot{e}^p\} = <L > \{\delta\}^T \{P\} \)  

(4.38)

Substituting the Equations 4.34 and 4.38 into the hardening rule (Equation 4.25), the variable \( V \) can be written as
\[ V = \frac{<I_o - I_i> + I_i}{\lambda - \kappa} (1 + e_{in}) \{\delta\}^T \{P\} \] (4.39)

(12) Shape-Hardening Function

The actual plastic modulus \( H \) is related to \( \bar{H} \) via \( \delta \), the distance between current stress point and its image point on the bounding surface, and can be expressed as

\[
H = \bar{H} + \hat{H} \frac{\delta}{<r - s_p \delta>} = \bar{H} + \hat{H} \frac{\beta}{\beta - 1} s_p^{-1} \] (4.40)

\[
\hat{H} = \frac{1 + e_{in}}{\lambda - \kappa} \, p_a \left[ z^{0.02} h(\theta) + (1 - z^{0.02}) h_0 \right] \] (4.41)

Where,

\( h(\theta) \), called as the shape-hardening factor, is a function of \( \theta \), \( h_c \), and \( h_e \), and can be calculated using Equation 4.17 and 4.18;

\( h_c \) and \( h_e \) are material parameters;

\( r \) is the distance between the projection center to the image point on the bounding surface of current stress point;

\( s_p \) is an elastic factor;

and \( \beta \) is the ratio of the image stress and the current stress.

\[
h_0 = \frac{h_c + h_e}{2} \] (4.42)

(13) Then the elasto-plastic constitutive matrix can be expressed as,

\[
[D^{ep}] = [D^e] - u(L) \left[ \frac{[D^e] \{P\} \{Q\}^T [D^e]}{H + \{Q\}^T [D^e] \{P\}} \right] \] (4.43)

(14) The stress-rate is then obtained as follows:

\[
\{\dot{\sigma}\} = [D^{ep}] \cdot \{\dot{e}\} \] (4.44)
4.2.1.2 Integration of the Bounding Surface Constitutive Model

A key step in the implementation of any elasto-plastic model involves integrating the constitutive relations to obtain the unknown increment in the stresses. These relations define a set of ordinary differential equations and methods for integrating them are usually classified as explicit or implicit.

The basic idea of explicit integration is that these integrators are written in a way that we can update all unknown values independently. In an explicit integration scheme, the yield surface, plastic potential gradients and hardening rule are all can be determined at the known stress states. All the variants can be calculated and updated at the end of every integration step. In a fully implicit method, the gradients and hardening law are evaluated at unknown stress states and the resulting system of non-linear equations must be solved iteratively.

The implicit method is powerful because the resulting stresses automatically satisfy the yield criterion to a specified tolerance; however, it is difficult to implement it for complex constitutive relations because it can lead to tedious algebra (Schofield and Wroth, 1968). While, according to Wissmann and Hauck (1983) and Sloan (1987), the accuracy and efficiency of explicit methods is significantly enhanced by combining them with automatic sub-stepping and error control. So in this study, the explicit sub-stepping integration scheme was used to implement the bounding surface model in ABAQUS finite element software with subroutines.

The integration scheme for the bounding surface model integrates the constitutive law by automatically dividing the strain increment into a number of sub-steps. An appropriate size of the sub-step is controlled by two criterions: first one is $\beta \geq 1$; second is $\varepsilon < 1\%$ ($\varepsilon$ is stress ratio).

More details are discussed as below:

(1) Basic integration scheme
For a given solution (time) step the relationship between stress and strain increments in elasto-plasticity can be obtained from Equation 4.44 by integrating over the step \( t_{N-1} \rightarrow t_N \):

\[
\int_{t_{N-1}}^{t_N} \{\sigma\} dt = \int_{t_{N-1}}^{t_N} \{D^{ep}\} \{\varepsilon\} dt
\]  

(4.45)

For increment \( N \) the strain rate is approximated by the finite difference expression:

\[
\{\varepsilon\} = \frac{\{\Delta \varepsilon\}}{\Delta t_N}
\]

(4.46)

Substituting Equation 4.46 into Equation 4.45 gives

\[
\{\Delta \sigma\} = \frac{\{\Delta \varepsilon\}}{\Delta t_N} \int_{t_{N-1}}^{t_N} \{D^{ep}\} dt
\]

(4.47)

Letting \[\overline{D} = \frac{1}{\Delta t_N} \int_{t_{N-1}}^{t_N} \{D^{ep}\} dt\]

(4.48)

Equation 4.47 can be written as

\[
\{\Delta \sigma\} = [\overline{D} \{\Delta \varepsilon\}]
\]

(4.49)

(2) Substepping

\( M \) substeps were used along with the assumption of proportional strain components, leading to the following equation:

\[
[\overline{D}] = \sum_{m=1}^{M} [\overline{D}]_m
\]

(4.50)

In which

\[
[\overline{D}]_m \approx \frac{1}{2\Delta t_m} \left\{ [D^{ep}]_{m-1} + [D^{ep}]_m \right\}
\]

(4.51)

The quantities \([\overline{D}]_m\) and \([\overline{D}]_{M-1}\) represent the values of \([D^{ep}]\) corresponding to the stress and strain states at the beginning and end of a substep, respectively.

Substeps of equal length are used, i.e.
\[ \Delta t_m = \Delta t_N / M , \quad t_m = t_{m-1} + (m-1) \Delta t_m , \quad t_m = t_{m-1} + \Delta t_m \]  \hspace{1cm} (4.52)

Approximately the strain at time \( t_m \) can be expressed as follows,

\[ \{ \varepsilon \}_m \approx \{ \varepsilon \}_{N-1} + \frac{m}{M} \{ \Delta \varepsilon_N \} \]  \hspace{1cm} (4.53)

The stress estimate at the corresponding time is initially taken to be

\[ \{ \sigma \}_m \approx \{ \sigma \}_{m-1} + \{ \Delta \sigma \}_{m-1} = \{ \sigma \}_{N-1} + \sum_{i=1}^{m-1} \{ \Delta \sigma_i \} + \{ \Delta \sigma \}_{m-1} \]  \hspace{1cm} (4.54)

At the beginning of each iteration, an attempt was made to use only one substep integration (\( M=1 \)). If the value of \( \beta \) for the calculated stress at the end of the solution step is less than 1, an attempt of \( M=2 \) will be used. And if at the end of either the first or second substep, \( \beta \) value does not meet the criterion (\( \beta \geq 1 \)), the number of substeps (\( M \)) will be doubled again, which means an attempt of \( M=4 \) will start. The process should continue until the criterion is met, which means the number of substeps (\( M \)) arrived at by the process will be 1, 2, 4, 8, 32 …

Classical radial return method (Hughes, 1983) has been adopted to bring a point back the bounding surface whenever a stress state (at the beginning of the step, or at the end of one of the substeps) is found to be outside of the bounding surface. It scales back the stress state along the line connecting the current stress state to the projection center. The scaled stress state is then used to calculate the plastic modulus. However, the scaled stress is not used to update the size of the bounding surface. The importance of using the unscaled stress for this operation stems from the fact that the size of the bounding surface is really controlled by strain considerations and the stains are not scaled.

Once the \( \beta \) criterion is met, another criterion will be used to determine the end of the iteration. Since we are integrating Equation 4.47, it is an obvious error control to compare the
resulting $\overline{D}$ or the predicted $\Delta \sigma$. In this study, the predicted stress increment at M and 2M substeps were compared. The ratio of $\varepsilon = \left| \frac{\{\delta \sigma\}_{2M} - \{\delta \sigma\}_M}{\{\delta \sigma\}_{2M}} \right|$, used as another error control, is required to be less than 1% (Herrmann, 1986) in this study. $\{\delta \sigma\}_M$ represents incremental stress vector at the end of the increment with different number of substeps.

At the end of each load increment, the stress states and strain states were updated. With the integration of elasto-plastic matrix, the incremental stress can be calculated at each substep, stress increment was accumulated and updated over the full step. Correspondingly, the $I_0$ was also updated as follows:

$$\{I_0\}_{N+1} = \{I_0\}_N + dI_0$$  \hspace{1cm} (4.55)

And thus the bounding surface changes accordingly with $I_0$ based on Equations 4.11 to 4.13.

### 4.2.1.3 Flow Chart of the Bounding Surface Model

The bounding surface was implemented into the USER MATERIAL (UMAT) subroutines for ABAQUS finite element program by a FORTRAN code. The general structure of the FORTRAN code is presented in Figure 4.3.

### 4.2.2 Calibration of the Bounding Surface Model

#### 4.2.2.1 Description of Model Parameters

There are 14 parameters in total needed to be calibrated for the use of this model on base course material in Louisiana. The full set of parameters can be grouped into the following four categories: (i) the elastic response parameters ($\kappa$ and $\mu$), (ii) the consolidation parameters ($\lambda$), (iii) critical state parameters, (iv) bounding surface parameters.

The values of some parameters can be determined directly from the results of triaxial drained tests and some parameters can take assumed values, and some of them should be
determined from the best fitting curves from triaxial tests. The detailed descriptions of the model parameters, their determination methods and typical values are presented in Table 4.1

The model contains the ability to define separate material constants for $M, R, A$ and $h$ for stress paths in compression and extension. In the absence of data to support a proper selection of these terms, values of these parameters were taken to be equal in extension and compression.

In this study, the value of $\lambda = 0.018$, $\kappa = 0.0018$ were selected based on the study conducted by Nazzle (2007) on similar crushed limestone aggregate material. A default Poisson’s ratio value of $\mu = 0.3$ was used, which was reported by Heath (2002) for the crushed limestone materials.

The critical state line (CSL) for crushed limestone material is drawn in Figure 4.4 according to consolidated undrained triaxial tests. Based on the test results, an $N_c$ value of 0.37 ($N_c = \frac{M_c}{3\sqrt{3}}$, $M_c$ is the slop of CSL in q-p plane) was determined. Due to the practical difficulty in triaxial extension test, $M_c / M_c = 0.8$ can be assumed (Kaliakin, 1985). Same default value of $N_c / N_c = 0.8$ correspondingly was also assumed based on a research study conducted by Perkins (2001) on crushed limestone material.

The $R_c, R_c, A_c, A_e$ and $T$ parameters determine the shape of the bounding surface in compression and extension, the first two for ellipse 1, the second two for the hyperbola and the $T$ for ellipse 2.

$R_c$ and $R_c$ has a default value of 2 when no experimental data is available (Roger, 1994). A nominal value of $T = 0.01$ was taken based on Perkins (2001) study since the crushed lime stone has almost no strength in extension. Values of $A_c = A_e = 0.02$ were also selected based on Perkins (2001) study. The $S_p$ parameter determines the size of the elastic nucleus. An $S_p = 1$ was
used here which was used by the other researchers (Dafalias, 1986, Rogers, 1994, and Perkins, 2001).

The value of $\phi = 1$ means the elastic nucleus shrinks to the projection center (refer to Figure 4.2). The value of $C$ determines the projection center. In this study, a value of 0.3 was used. The $H_e$ and $H_c$ are the shape hardening factor in compression and extension, respectively, which were calibrated using a trial and error procedure to fit the stress-strain curve of the crushed limestone material, as shown in Figure 4.5. Based on their typical range, values of 5-50 was tried, and a value of $H_e = H_c = 20$ was selected.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Determination Method</th>
<th>Typical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>Virgin compression slop</td>
<td>CTC test</td>
<td>0.1-0.2</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Swell/recompression slope</td>
<td></td>
<td>0.02-0.08</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Poisson’s ratio</td>
<td>Assumed value</td>
<td>0.15-0.3</td>
</tr>
<tr>
<td>$N_c$</td>
<td>Slope of CSL in compression</td>
<td>CTC test</td>
<td>0.1-0.8</td>
</tr>
<tr>
<td>$N_e$</td>
<td>Slope of CSL in extension</td>
<td></td>
<td>0.1-0.8</td>
</tr>
<tr>
<td>$R_c$</td>
<td>Bounding surface shape parameters for ellipse 1</td>
<td></td>
<td>2-3</td>
</tr>
<tr>
<td>$R_e$</td>
<td>Bounding surface shape parameters for hyperbola</td>
<td></td>
<td>0.02-0.2</td>
</tr>
<tr>
<td>$A_c$</td>
<td>Bounding surface shape parameters for ellipse 2</td>
<td>Best fitting the curve</td>
<td>0.05-0.15</td>
</tr>
<tr>
<td>$A_e$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>Projection center parameter</td>
<td></td>
<td>0.0-0.5</td>
</tr>
<tr>
<td>$S$</td>
<td>Elastic zone parameter</td>
<td></td>
<td>1-2</td>
</tr>
<tr>
<td>$H_e$</td>
<td>Shape hardening parameters</td>
<td></td>
<td>5-50</td>
</tr>
<tr>
<td>$H_c$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

CTC=Conventional Triaxial Compression
CSL=Critical State Line
Figure 4.3 Schematic Flow Chart for the UMAT
Figure 4.4 Critical state line (CSL) in p-q space for the Crushed Limestone

Figure 4.5 Illustration of effect of parameter Hc
4.2.2.2 Description of the Studied Material

The Kentucky crushed limestone aggregate, typically used in the construction of base course layers in Louisiana, will be used in the study. Sieve analyses tests for crushed limestone were performed, and the grain size distribution was shown in Figure 4.6. The medium grain size \((D_{50})\) of the material was found to be 5 mm and the effective size \((D_{10})\) was found to be 0.28 mm. The coefficient of uniformity \((C_u)\) and coefficient of curvature \((C_c)\) were found to be 24 and 1.97, respectively. The Unified Soil Classification System (USCS) classify this material as gravel well-graded and the American Association of State Highway and Transportation (AASHTO) as an “A-1-a” soil. The values of maximum dry density and optimum moisture content obtained from the Standard Proctor test analysis are 140.3 lb/ft\(^3\) (2247 kg/m\(^3\)) and 6.6%, respectively. The friction angle and cohesion obtained from monotonic triaxial compression test are 49\(^\circ\) and 26 kPa, respectively.

4.2.3 Verification of the Bounding Surface Model

The material parameters that will be used in the UMAT for crushed limestone material are summarized as follows, \(\mu =0.3, \lambda =0.018, \kappa =0.0018, \mu = 0.3, N_c = N_e = 0.37, R_e = R_e = 2, A_c = A_e = 0.02, T =0.01, C =0, S =1, H_c = H_e = 20\).

To verify the prediction of the bounding surface model implemented with the aid of user subroutine in ABAQUS both monotonic and cyclic undrained triaxial compressive tests were conducted on the crushed limestone material. In the monotonic tests, the soil sample was first consolidated at a confining pressure of 21 kPa and then load was applied to the sample at a constant strain rate until failure was reached. While for the repeated load triaxial test, the same confining pressure of 21 kPa was first applied and the sample was then subjected to 100 cycles of loading and unloading.
The following sections provide detailed information on the materials used and their properties. The laboratory procedures for the triaxial tests performed were also highlighted.

The AASHTO recommends that a split mold should be used for compaction of granular materials. Therefore, all samples were prepared using a split mold with an inner diameter of 150 mm and a height of 350 mm. The material was first oven dried at a pre-specified temperature and then mixed with water at the optimum moisture content (6.6%). The achieved water contents were within ±0.5 percent of the target value. The material was then placed within the split mold and compacted using a vibratory compaction device to achieve the maximum dry density (140.3 \( lb/ft^3 = 2247 \ kg/m^3 \)) measured in the standard Proctor test. To achieve a uniform compaction throughout the thickness according to ASTM, samples were compacted in six-50 mm layers. Each layer was compacted until the required density was obtained; this was done by measuring the distance from the top of the mold to the top of the compacted layer. Then the smooth surface on top of the layer was lightly scratched to achieve good bonding with the next layer. The achieved dry densities of the prepared samples were within ±1 percent of the target value. Samples were enclosed in two latex membranes with a thickness of 0.3 mm.

The model simulation of the monotonic triaxial compression test on unreinforced crushed limestone under strain controlled static load is shown in Figure 4.7. It can be seen that the implemented bounding surface model has a very good agreement with the experimental results. The model simulation of triaxial undrained compression test on unreinforced soil under repeated load is shown in Figure 4.8. Figure 4.8 also showed a good match between the experimental results and finite element analysis using the bounding surface model. Therefore, the selected model parameters for bounding surface model were able to describe the behavior of crushed limestone base material, and consequently will be used in the finite element model to perform
parametric study (Chapter 6) to evaluate the effect of different parameters on the performance of geogrid reinforced bases in flexible pavement.

![Particle Size Distribution Curve of Crushed Limestone](image1)

Figure 4.6 Particle Size Distribution Curve of Crushed Limestone

![Verification of Model Simulation for static triaxial test](image2)

Figure 4.7 Verification of Model Simulation for static triaxial test
Figure 4.8 Verification of Model Simulation for Repeated Load Test

(a) Displacement versus time

(b) Strain versus load cycles
CHAPTER 5 RESULTS AND ANALYSES ON REINFORCED SOIL FOUNDATION

5.1 Results of Finite Element Analyses for Reinforced Embankment Soil

Finite element analyses were conducted on unreinforced and reinforced silty clay embankment soil to evaluate the influence of various factors affecting the performance of strip footing on reinforced studied soils. The factors included in this study are: the effective depth of reinforced zone, spacing between reinforcement layers, tensile modulus of reinforcement, soil-reinforcement interaction coefficient, optimum top spacing for the single-layer and multi-layer reinforced soil, footing width, the embedment depth of footing, soil friction angle and soil elastic modulus.

For each case, the load-deformation curve, obtained from the finite element simulation, was used to determine the ultimate bearing capacity and settlement of the footing. The ultimate bearing capacity of the footing was defined as the bearing capacity that corresponds to a settlement ratio \((s/B)\) of 10\% (Yoo, 2001). The influence of these factors will be discussed in the following sections. The benefits of RSF were assessed in terms of the bearing capacity ratio (BCR) and/or settlement reduction factor (SRF). The material properties used in this part of study are presented in Table 5.1.

5.1.1 Stress and Strain Distribution

Stress and strain distributions as well as developed plastic zones within the foundation soil with and without geogrid reinforcement layers are presented first, which will shed some lights on the reinforcement mechanisms. The vertical stress distributions within unreinforced soil and soil
reinforced with 3-layer and 6-layer of type VI geogrid, are shown in Figure 5.1a and b, respectively. All of these stresses correspond to the moment when the footing sitting on unreinforced soil reaches its ultimate bearing capacity (s/B=10%).

The vertical stress shown in Figure 5.1a is along a horizontal line at a distance of 1.5B underneath the footing bottom. The inclusion of reinforcement results in a significant reduction in the magnitude of vertical stress compared to the unreinforced soil, and more reduction is achieved with more reinforcement layers.

A similar trend, as illustrated in Figure 5.1b, is observed in the distributions of vertical stress along the central axis of the footing. The inclusion of reinforcement layers spreads the load applied on the footing into a wider area of the foundation soil, and thus helps reduce the ultimate consolidation settlement of the footing that will be developed. Also, the more reinforcement layers included in the foundation soil, the more remarkable the reinforcement effect in the sense of reducing stresses in the foundation soil.

<table>
<thead>
<tr>
<th>Material</th>
<th>Friction Angle</th>
<th>Cohesion (psi/kPa)</th>
<th>Elastic Modulus (psi/kPa)</th>
<th>Poisson ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silty clay soil*</td>
<td>30.0</td>
<td>11.6/80.0</td>
<td>37700/259932</td>
<td>0.3</td>
</tr>
<tr>
<td>Geogrid I**</td>
<td>NA</td>
<td>NA</td>
<td>10380/71568</td>
<td>0.3</td>
</tr>
<tr>
<td>Geogrid II**</td>
<td>NA</td>
<td>NA</td>
<td>21760/150030</td>
<td>0.3</td>
</tr>
<tr>
<td>Geogrid III**</td>
<td>NA</td>
<td>NA</td>
<td>43580/300474</td>
<td>0.3</td>
</tr>
<tr>
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<td>47440/327087</td>
<td>0.3</td>
</tr>
<tr>
<td>Geogrid V**</td>
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<td>NA</td>
<td>54620/376592</td>
<td>0.3</td>
</tr>
<tr>
<td>Geogrid VI**</td>
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<td>NA</td>
<td>91280/629353</td>
<td>0.3</td>
</tr>
<tr>
<td>Geogrid VII**</td>
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<td>NA</td>
<td>102100/703955</td>
<td>0.3</td>
</tr>
<tr>
<td>Geogrid VIII**</td>
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<td>NA</td>
<td>131800/908729</td>
<td>0.3</td>
</tr>
</tbody>
</table>

* Parameters from previous research study (Cai, C.S. et al., 2005)
**Parameters from the manufacturers
(a) Vertical stress distribution along a horizontal line 1.5B beneath the footing

(b) Vertical stress distribution along the central axis

Figure 5.1 Vertical stress distribution at p=400 psi (2800 kPa)
(B=4ft, \(D_f=0\) and \(u/B=h/B=0.25\))
The failure of the footing with or without a reinforcement layer can be traced with the aid of the FEM analyses. Figure 5.2a shows the plastic zones developed in the unreinforced soil when the footing reaches its ultimate bearing capacity while Figure 5.2b shows the plastic zone developed in the soil reinforced with 3 layers of Type VI geogrid (u/B=h/B=0.5) under the same footing pressure (p=400 psi=2800 kPa). A salient distinction in the reinforced case is a small and isolated plastic zone compared to a large and continuous one in the unreinforced case. It appears that the inclusion of reinforcement layers helps minimizing the plastic zone from converging into a continuous body and thus delays the soil failure. As a consequence, a foundation on reinforced soil exhibits higher bearing capacity and reduced settlement.

![Figure 5.2 Plastic zones developed in unreinforced and reinforced soil foundations](image)

(a) in unreinforced soil  (b) in reinforced soil(u/B=h/B=0.5)

Figure 5.2 Plastic zones developed in unreinforced and reinforced soil foundations (B=4 ft, Df=0)

When the footing reinforced with Type VI geogrid reaches its ultimate bearing capacity, axial strain developed in geogrid layers within the half of the reinforced soil is shown in Figure 5.3a and 5.3b. Figure 5.3a shows the geogrid strain distribution in a three-layer reinforced soil, and Figure 5.3b shows the geogrid strain distribution in a five-layer reinforced soil. In both cases
the bottom geogrid layer (i.e., the 3rd or the 5th layer in Figure 5.3) is embedded 1.5B beneath the footing bottom. Figure 5.3a and 5.3b indicate that the largest strain occurs at the geogrid location underneath the axis of the footing, and dramatically drops off at geogrid locations further away from the footing center. As would be expected, the 1st geogrid layer always experiences the largest strain, and the 2nd geogrid layer experiences the second largest strain, and so on in both cases. In addition, the strains in the three-layer case are larger than their counterparts in the five-layer case. Figure 5.3a and 5.3b indicate that a tensile strain (positive in the figure) is developed within a length equal to 2B range in the geogrid of the half system of RSF. The development of a tensile strain in geogrids implies the mobilization of geogrids and thus the reinforcing benefits of geogrids in foundation can be realized. It follows that the reinforcement effect of a geogrid in a foundation soil can fully be mobilized provided that its full length is larger than 4B. This finding agrees well with other researchers’ result (Adams and Collin, 1994, Shin et al., 2002 and Maharaj, 2003). In which, the effective length of reinforcement under strip footing is 4B for clay (Maharaj, 2003) and is around 4.5 B for sand (Adams and Collin, 1994 and Shin et al., 2002).

5.1.2 Optimum Location of First Reinforcement Layer

The influence of the location of first reinforcement layer (u) on the BCR is discussed in this section, based on the FEM analyses for the footing (B=4ft) placed on single-layer, two-layer and three-layer geogrid-reinforced soil systems at varying depth ratios. The typical variations of the BCR with varied depth ratios (u/B) for single-layer, two-layer and three-layer type VI reinforced soil are shown in Figure 5.4.
Figure 5.3 Strain developed in geogrid of Type VI (B=4ft, $D_f=0$)

(a) Three-layer system ($u/B=h/B=0.5$)   
(b) Five-layer System ($u/B=h/B=0.3$)

Figure 5.4 Variation of BCR with depth ratio ($u/B$) in single-layer, two-layer and three-layer reinforcement reinforced embankment soil (B=4ft, $D_f=0$ and $h/B=0.5$)

- 3-layer type VI geogrid reinforced soil, $h/B=0.25$
- 2-layer type VI geogrid reinforced soil, $h/B=0.25$
- One-layer reinforced soil
In the single-layer reinforcement case, the BCR increases first with the increase of the depth ratio ($u/B$) and then decrease after a threshold value of $u/B$. This threshold depth ratio ($u/B$) is around 0.5, where the BCR is the highest. The variation of the BCR with depth ratios ($u/B$) is similar in the two-layer and three-layer reinforcement cases. However, the threshold depth ratio slightly decreases with the number of reinforcement layers—around 0.4 in two-layer reinforcement case and around 0.3 in three-layer reinforcement case. The threshold depth ratio is used in the following sections in which the influence of other reinforcement factors on the reinforced footing is investigated. The findings of the present study on the effect of the depth ratio are similar to those reported by other researchers (Yetimoglu et al, 1994, Maharaj, 2003), in which the optimum location of multi-layer reinforced clay under square footing is 0.25-0.3 $B$ (Yetimoglu et al., 1994) and the optimum location of single-layer reinforced clay under strip footing is about 0.5 $B$.

5.1.3 Effective Depth of Reinforced Zone

The design of reinforced soil foundations requires the determination of the effective (or influence) depth of the reinforced zone, below which reinforcement inclusion will not have appreciable benefit on footing performance. To identify the effective depth, finite element analyses were conducted on a footing ($B=4$ ft) using three types of geogrid reinforcements placed uniformly either at 12 inch interval ($u/B=h/B=0.25$) or 24 inch interval ($u/B=h/B=0.5$).

Three types of geogrid (III, VI, and VIII) were included to investigate the dependence of effective depth on geogrid’s properties. Type III geogrid has a relatively low tensile modulus and
Type VI has a medium tensile modulus, while type VIII represents a stiff geogrid. For each type of geogrid, a series of finite element analyses were conducted with the number (N) of reinforced layers increasing till the reinforced depth reaches 2.5B (N*h/B+u/B=2.5B).

The load-deformation curves for each case was determined and used to calculate the bearing capacity ratio (BCR) at s/B equal to 10%. An example of the load-deformation curves obtained for the four-foot wide strip footing on clay embankment soil reinforced with Type III and Type VI geogrids placed at varying vertical spacings are presented in Figures 5.5a and 5.5b, respectively.

Figure 5.6a presents the BCRs at s/B = 10%, for the 12 inch reinforcement spacing, as the number of reinforcement layers increases from one to ten. The variations in the BCRs at s/B = 10% versus the number of reinforcement layers for the 24 in reinforcement spacing are shown in Figure 5.6b. As expected, the BCR of the reinforced footing increased as the number of reinforcement layers increased, but at a decreasing rate. For the 12 inch spacing cases, there is no significant improvement in the BCR when the number of reinforcement layers exceeds 6, which corresponds to a depth of 1.5B= 6 ft (1.8 m). Similarly, no further significant improvement in BCR was achieved for the 24 in (0.6 m) spacing as the number of layers exceeds 3, which is also corresponds to 1.5B =6 ft (1.8 m) depth. Accordingly, the effective reinforcement depth expressed as the strip footing’s width will be equal to 1.5B for the soil in question. This finding is also similar that of Das’s observation (Das, 1994). In his work on reinforced sand under strip footing the effective reinforcing zone is around 0.175 B.
Figure 5.5 Footing stress versus footing settlement (B=4ft, \(D_f=0\), \(u/B=h/B=0.25\))

(a) Type III geogrid reinforcement

(b) Type VI geogrid reinforcement
Figure 5.6 Variation of BCR with reinforcement layers for multi-layer reinforced soil of geogrid (B=4ft and $D_f=0$)

(a) 12 inch spacing ($u/B=h/B=0.25$)

(b) 24 inch spacing ($u/B=h/B=0.5$)
As seen in Figure 5.6 the improvement trend in BCR is the same for all the three types of geogrid that been investigated, which also indicated that the effective reinforcement depth is independent of the geogrid type.

5.1.4 Effect of Reinforcement Spacing

The effect of reinforcement spacing (h) on the footing’s bearing capacity and settlement was investigated by changing the number/spacing of reinforcement layers within the effective reinforcement depth of 1.5B. A series of finite element analyses were conducted on the footing-reinforced soil model using three geogrid types (III, VI, and VIII) placed at five different spacing. The following reinforcement layers-spacing configurations were examined: three layers placed at 24 in. spacing, four layers placed at 18 in. spacing, six layers placed at 12 in. spacing, nine layer placed reinforcement at 8 in. spacing, and twelve layers placed at 6 in. spacing. For each case, the BCR at $s/B =10\%$ and the settlement reduction factor (SRF) at the ultimate load capacity of a 3-layer Type VI reinforced soil were calculated. Figures 5.7a and 5.7b depict the relationship between the reinforcement spacing and the BCR and SRF, respectively. For the three geogrids used, the figures show that at a given settlement the load carrying capacity of the footing decreases with the increase in reinforcement spacing, with larger decrease rates at small spacings. Besides, the footing settlement at the ultimate load capacity of a 3-layer Type VI reinforced soil is smaller for closer reinforcement spacings. The reduction effect of footing settlement is more remarkable when the spacing ratio ($h/B$) reduced from 0.5 to 0.2. Therefore, smaller reinforcement spacing should always be desirable provided that its cost is justified.
(a) Variation of BCR with reinforcement spacing

(b) Variation of SRF with reinforcement spacing

Figure 5.7 Effect of reinforcement spacing (B=4ft and $D_f=0$)
5.1.5 Effect of Reinforcement Tensile Modulus

Tensile modulus is one of the most important properties of geogrids, which have significant influence on the performance of footing on reinforced soils. In this study, eight types of uniaxial geogrids with varying tensile modulus were analyzed to examine the influence of their tensile modulus from the perspective of the ultimate bearing capacity and settlement of the footing. The properties of geogrids are presented in Table 5.1. The geogrid’s elastic modulus was taken as its tensile modulus (at 5% strain) per unit width divided by its thickness. A series of finite element analysis were conducted for each tensile modulus using 3, 6 and 12 layers of reinforcement placed within the effective depth of 1.5 B at a uniform spacing. The calculated BCR values at s/B=10% and SRF versus the geogrid tensile modulus are presented in Figure 5.8a and b. respectively. Regardless of the number of reinforcement layers, the footing with geogrids of higher tensile modulus has a larger bearing capacity than that with weaker geogrids. However, this modulus-related increase in the BCR is more remarkable at low normalized geogrid stiffness and gradually decreases as the geogrid tensile modulus exceeds 35,000 kN/m.

On the other hand, the SRF decreases with the increase in reinforcement tensile modulus, at a gradually reducing rate. In general, the figures indicate that a better reinforcement effect can be achieved in terms of higher ultimate bearing capacity and smaller settlement when the geogrid has higher tensile modulus. For the soil studied herein, a geogrid with a tensile modulus ranging from 5,000 to 25,000 kN/m will maximize the benefits of the reinforced soil footing. No more significant improvement is achieved when the tensile modulus of geogrid exceeds 35,000 kN/m.
(a) Variation of BCR with reinforcement stiffness

(b) Variation of SRF with reinforcement stiffness

Figure 5.8 Effect of reinforcement tensile modulus for the footing overlying multi-layer reinforcement soil (B=4ft and $D_r=0$)
5.1.6 Effect of Geogrid-Soil Interaction

The geogrid-soil interaction coefficient measures the interface friction between the geogrid and soil. Its effect on the reinforced footing is examined herein by modeling a footing placed on a six-layer reinforced soil at 1 ft reinforcement spacing. The investigated geogrids included type III, VI, and VIII. The interaction coefficients range from 0.4 to 0.8 with equal interval of 0.1. The influence of the interaction coefficient on the BCR and SRF is illustrated in Figures 5.9a and 5.9b, respectively. As the interaction coefficient increases, the BCR increases and SRF decreases for all studied geogrid reinforced soil footings, which means that better interaction between soil and geogrids always provide better performance of RSF. Figures 5.9a and 5.9b indicate that the increase rate in the BCR or the decrease rate in the SRF is relatively independent of the type of geogrids. It can also be noticed that the variations of the BCR and SRF are relatively small as the interaction coefficient varies from 0.6 to 0.8, which represents typical interaction coefficient values in most geogrid-reinforced soils used in engineering applications.

5.1.7 Effect of Footing Embedment Depth

Embedment depth of an unreinforced footing has significant effect on its performance, which has been extensively studied and is well understood. However, its influence on the reinforced footing is less understood and is discussed in this section.

A footing (B=4 ft) with different embedment depths (including 0B, 0.25B, 0.5B, 0.75B, and 1B) placed on a multi-layer reinforced soil was analyzed using the FEM model presented in a previous section (Chapter 3).
(a) Variation of BCR with geogrid-soil interaction coefficient

(b) Variation of SRF at p=400 psi with geogrid-soil interaction coefficient

Figure 5.9 Effect of geogrid soil interaction for a footing overlying 6-layer reinforced soil (B=4ft and Df=0)
The variation of the BCR and the variation of SRF with footing embedment depth are shown in Figures 5.10a and 5.10b, respectively. With the increase in the embedment depth of the footing, both the BCR and the SRF slightly decrease at an approximately linear manner. The slight reduction trend of the BCR with the increase in the embedment depth, as illustrated in Figure 5.10a, can be explained by the fact that the increase in the embedment depth increases the bearing capacity of the unreinforced footing more than that of the reinforced footing.

### 5.1.8 Effect of Footing Width

The influence of the footing’s width (or scale effect) on the performance of reinforced soil footings was investigated by several researchers (Das and Omar, 1994, Elvidge and Raymond, 2001). In this study, the effect of footing width on the BCR and SRF of reinforced soil footings was studied by changing the width of strip footing from 3ft (0.9 m) to 6ft (1.8m), and the results are shown in Figures 5.11 a and b, respectively.

It can be observed from Figure 5.11 a and b that with the increase in footing width, both the BCR and the SRF decrease at a linear manner. This result is similar to the findings of Das and Omar (1994) and Elvidge and Raymond (2001), which that the increase in footing width resulted in a decrease in the BCR. Again, this is due to larger increase in the bearing capacity of the unreinforced footing compared to the reinforced footing brought up by the increase in the footing’s width, which consequently causes a decrease trend in the BCR, as illustrated in Figure 5.11a.

And thus it can be concluded that increase in footing width cannot bring more benefit in increasing BCR, however, it can reduce more footing settlement. The benefit in reducing the settlement of RSF can be explained by the fact that wider footing can distribute overlying load in a wider range and thus the corresponding settlement reduces and thus the SRF of RSF decreases.
Figure 5.10 Effect of footing embedment depth for footing reinforced with type VI geogrid 
(B=4ft)

(a) Variation of BCR with footing embedment

(b) Variation of SRF with footing embedment

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Figure 5.11 Effect of footing width for footing reinforced with Type VI geogrid
(B=4ft and $D_f=0$)

(a) Variation of BCR with footing width

(b) Variation of SRF with footing width
5.1.9 Effect of Soil Friction Angle

Soil friction angle $\phi$ is an important factor affecting the clay soil behavior. Soil’s shear strength is related to its friction angle through Mohr-Coulomb equation (Terzaghi, 1942), shown in Equation 5.1.

$$\tau = \sigma \cdot \tan \phi + c$$

(5.1)

In this study the effect of soil friction angle on the BCR and SRF of reinforced soil footings was studied by varying it with 25°, 30° and 35° while all the other parameters of the soil remained unchanged.

The results are shown in Figure 5.12a and b respectively. With the increase in friction angle, both the BCR and SRF decreased, which means that the increase in friction angle of soil (or shear strength) cannot bring benefit in increasing its bearing capacity. This can be explained that the increase in soil friction angle results more increase in the bearing capacity of the unreinforced soil compared to the reinforced soil, which consequently causes a decrease in the BCR as shown as in Figure 5.12a.

5.1.10 Effect of Soil Cohesion

Cohesion of clay, $c$, is also an important factor affecting the behavior of clay.

In this study the effect of soil friction angle on the BCR and SRF of reinforced soil footings was studied by varying it with 9 psi (63 kPa), 11.6 psi (80 kPa) and 14 psi (98 kpa) while all the other strength parameters of the soil remained the same.

The results are shown in Figure 5.12a and 5.12b respectively. With the increase in cohesion, both the BCR and SRF decreased. Again, this can be explained that the increase in cohesion results more increase in the bearing capacity of the unreinforced soil compared to the reinforced soil, which consequently causes a decrease in the BCR as shown as in Figure 5.13a.
Figure 5.12 Effect of soil friction angle for footing reinforced with type VI geogrid (B=4ft and D_f=0)

(a) Variation of BCR with soil friction angle

(b) Variation of SRF with soil friction angle
(a) Variation of BCR with soil cohesion

(b) Variation of SRF with soil cohesion

Figure 5.13 Effect of soil cohesion for footing reinforced with type VI geogrid
  \((B=4\text{ft and } D_f =0)\)
5.2 Statistical Regression Analysis of Reinforced Embankment Soil

5.2.1 Development of BCR Regression Model

As confirmed by the finite element analyses, the behavior of a strip footing sitting on geogrid-reinforced soil depends on multiple factors including the geogrid spacing, geogrid tensile modulus, soil-geogrid interaction, top spacing of first geogrid layer, footing width, footing embedment depth, soil friction angle and cohesion.

The effect of these factors should be appropriately determined to ensure a rational design of a geogrid-reinforced footing. Therefore, based on the finite element results a multi-regression statistical analysis was conducted to develop a BCR model that can facilitate the design of a reinforced soil footing.

Fifty nine cases of finite element results were used for the regression model. All the geogrid layers were assumed to lie within the effective reinforced depth and have enough length to fully mobilize its tensile contribution in all the cases.

The Statistical Analysis Software (SAS) package was used in this study. The full model described in Equation 5.2 was first analyzed that includes the effects of all variables and their interactions.

\[
BCR = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \beta_7 X_7 + \beta_8 X_1 X_2 + \\
\beta_9 X_1 X_3 + \beta_{10} X_1 X_4 + \beta_{11} X_1 X_5 + \beta_{12} X_1 X_6 + \beta_{13} X_1 X_7 + \beta_{14} X_2 X_3 + \beta_{15} X_2 X_4 + \\
\beta_{16} X_2 X_5 + \beta_{17} X_2 X_6 + \beta_{18} X_2 X_7 + \beta_{19} X_3 X_4 + \beta_{20} X_3 X_5 + \beta_{21} X_3 X_6 + \beta_{22} X_3 X_7 + \\
\beta_{23} X_4 X_5 + \beta_{24} X_4 X_6 + \beta_{25} X_4 X_7 + \beta_{26} X_5 X_6 + \beta_{27} X_5 X_7 + \beta_{28} X_6 X_7
\]  \hspace{1cm} (5.2)

Where:

BCR: is the bearing capacity ratio of the reinforced soil at s/B=10%,

X1: is the spacing ratio between geogrid layers (h/B),
X2: is the stiffness ratio of reinforcement included in the reinforced soil (i.e. \( T_r \cdot t / E_s \)),

X3: is the interaction coefficient between reinforcement layers and soil;

X4: is the footing embedment ratio \((D_f / B)\),

X5: is the footing width ratio \((B / 4ft)\),

X6: is the normalized soil friction angle \((\varphi / 30)\),

X7: is the normalized soil cohesion \((c / 11.5 \text{ psi})\),

\( \beta_0 - \beta_{28} \): Statistical parameters,

\( T_r \) is the tensile modulus of reinforcement, \( t \) is the thickness of the reinforcement and \( E_s \) is the elastic modulus of soil.

A stepwise variable selection procedure was then performed on the general model shown in Equation 5.2 to remove insignificant variables from the general model. The statistical variable selection procedure showed that no interaction between these variables is significant and that geogrid spacing, geogrid stiffness, soil-geogrid interaction coefficient, footing embedment, and footing width, soil friction angle and cohesion are all statistically significant variables for the BCR at the 95% confidence level.

The multiple regression analysis was then conducted on the reduced model and the results yielded the following model:

\[
\text{BCR} = 3.84848 - 1.99668 \times X_1 + 0.12434 \times X_2 + 0.57453 \times X_3 - 0.01057 \times X_4 - 0.06924 \times X_5 - 1.28114 \times X_6 - 0.81625 \times X_7
\]  

(5.3)

The analyses of variance of the proposed BCR model are presented in Table 5.2. The high R-Square value and adjusted R-Square value suggested a good regression of the data. Significance tests for individual parameters are conducted by using t statistics. The results of these t statistics are summarized in Table 5.3. It can be seen that with a 95% confidence level, X1, X2, X3, X4,
X5, X6 and X7 all have significant effect on the BCR values, which means that they all have their independent effect on the BCR.

5.2.2 Verification of the BCR Regression Model

The regression BCR model in Equation 5.3 was further verified by comparing the results of regression model with the results from additional 20 finite element analysis cases. The detailed variables and comparison are presented in Table 5.4 The absolute error in predicting the BCR value was calculated for each case and presented in the table. The absolute errors range from 0.16 % to 4.79 %, which suggests that the BCR values predicted by the regression model in Equation 5.3 have acceptable accuracy. The verification was also illustrated in Figure 5.14, which shows the good match between the calculated BCR from FEM and Predicted BCR from statistical regression method as well.

Table 5.2 Summary of the Analysis of Variance of the BCR Model

<table>
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<th>Source</th>
<th>Degree of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr&gt;F</th>
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<td>264.26</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Error</td>
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<td>0.21524</td>
<td>0.00422</td>
<td></td>
<td></td>
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<td>Corrected Total</td>
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<td>R-Square</td>
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<td>Adj R-Sq</td>
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Table 5.3 Summary of the BCR Model Parameters Estimate

| Variable | Parameters Estimate | Standard Error | Pr>|t|  | Variance Inflation |
|----------|---------------------|----------------|------|---------------------|
| Intercept| 3.84848             | 17.33          | <.0001 | 0                   |
| X1       | -1.99668            | 33.35          | <.0001 | 1.061               |
| X2       | 0.12434             | 0.0034         | <.0001 | 1.105               |
| X3       | 0.57453             | 0.0137         | <.0001 | 1.058               |
| X4       | 0.01057             | 0.0268         | <.0001 | 1.029               |
| X5       | -0.06924            | 0.0304         | <.0001 | 1.064               |
| X6       | 1.28114             | 0.15913        | <.0001 | 1.000               |
| X7       | 0.81625             | 0.122          | <.0001 | 1.001               |
### Table 5.4 Verification of Regression Models

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<th>X2</th>
<th>X3</th>
<th>X4</th>
<th>X5</th>
<th>X6</th>
<th>X7</th>
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<th>BCR (REG)</th>
<th>ABS (Error) (%)</th>
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Figure 5.14 Comparison between the BCR calculated from FEM and BCR predicted from statistical prediction
5.3 Results of Finite Element Analyses for Reinforced Crushed Limestone over Silty Clay Embankment Soil

As confirmed in section 5.1 the effective reinforcing zone is 1.5 B under the footing for the studied silty clay embankment soil, thus in this section we will replace the embankment soil under a proposed strip footing to 1.5 B with crushed limestone then reinforce it. Comprehensive finite element parametric study was conducted to evaluate the influence of various factors on the performance of strip footing on reinforced crushed limestone soil. The performance of footing was assessed in terms of bearing capacity ratio (BCR) and/or settlement reduction factor (SRF) of the footing. The material properties used in this part of study are presented in Table 5.5.

Table 5.5 Material properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Friction Angle ($\varphi$)</th>
<th>Cohesion (kPa)</th>
<th>Elastic modulus (MPa)</th>
<th>Elastic Tensile Modulus (kN/m)</th>
<th>Poisson Ratio</th>
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<td>48</td>
<td>-</td>
<td>120</td>
<td>-</td>
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<td>Embankment soil **</td>
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<td>80</td>
<td>260</td>
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<td>0.3</td>
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<tr>
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<td>-</td>
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<td>Reinforcement II***</td>
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<td>-</td>
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<td>-</td>
<td>-</td>
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<td>-</td>
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<td>Reinforcement VII***</td>
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<td>-</td>
<td>141895</td>
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</tbody>
</table>

SWM: Steel wire mesh  
SBM: Steel bar mesh  
* Parameters from large-scale shearing test  
** Parameters from previous research study (Cai, C.S. et al., 2005)

5.3.1 Stress and Strain Distribution

Stress distributions under different footing pressure within the foundation soil with and without reinforcement layers are first presented, which will shed some lights on the
reinforcement mechanisms. The vertical stress distributions within unreinforced soil and soil reinforced along the horizontal line 1.5B below the footing with 3-layer and 6-layer steel wire mesh at final state of unreinforced one are shown in Fig 5.15 respectively. The inclusion of a reinforcement layer reduces significantly the magnitude of vertical stress compared to the unreinforced soil, and more reduction is achieved with more reinforcement layers. The inclusion of reinforcement layers spreads the load applied on the footing onto a wider range of the foundation soil, and thus help reduces the ultimate consolidation settlement of the footing that will be developed.

When the footing reinforced with SWM reaches its ultimate bearing capacity, axial strain developed in geogrid layers within the half of the reinforced soil is shown in Figure 5.16a and b. Figure 5.16a shows the geogrid strain distribution in a three-layer reinforced soil, and Figure 5.16b shows the geogrid strain distribution in a five-layer reinforced soil. In both cases the bottom geogrid layer (i.e., the 3rd or the 6th layer in Figure 5.16) is embedded 1.5B beneath the footing bottom. Figure 5.16a and b indicate that the largest strain occurs at the geogrid location underneath the axis of the footing, and dramatically drops off at geogrid locations further away from the footing center. As would be expected, the 1st geogrid layer always experiences the largest strain, and the 2nd geogrid layer experiences the second largest strain, and so on in both cases. In addition, the strains in the three-layer case are larger than their counterparts in the six-layer case.

5.3.2 Optimum Location of First Reinforcement Layer

Based on the FEM analyses for the strip footing placed on single-layer, two-layer and three-layer SWM reinforced studied soil at varying depth ratios, the typical variations of the BCR with depth ratios (u/B) for SWM reinforced soil was shown in Figure 5.17.
(a) Vertical stress distribution along a horizontal line 1.5B beneath the footing

(b) Vertical stress distribution along central axis

Figure 5.15 Vertical Stress Distributions at p=750 psi (5250 kPa) 
((B=4ft, D_f=0 and u/B=h/B=0.25))
Figure 5.16 Strain developed in reinforcement of SWM (B=4ft, $D_r=0$ and $u/B=h/B=0.25$)
From Figure 5.17 it can be seen for the single-layer reinforcement case, the BCR increases with the increase of the depth ratio (u/B) and then decreases after reaching a threshold value of u/B. This threshold depth ratio (u/B) for the single-layer of reinforcement, at which a peak BCR is obtained, is found to be u/B = 0.35. The variation of the BCR with depth ratios (u/B) is similar in the two-layer and three-layer reinforcement cases. The threshold depth ratio slightly decreases with the increase in the number of reinforcement layers. In these cases, the u/B ratio is about 0.33, which is the same ratio adopted in the FE parametric study.

The findings of the present study on the optimum u/B ratio are similar to those reported by other researchers (e.g., Shin et al., 2002; and Abu-Farsakh et al., 2007a, b). The results of laboratory model footing tests conducted by Abu-Farsakh et al (2007a) showed that the u/B = 0.33 for geogrid-reinforced silty clay and sand. Shin et al. (2002) showed that for strip footings on geogrid-reinforced sand and clay the ultimate BCR can be achieved when u/B is around 0.3 for reinforced sand and 0.4 for reinforced clay. From the results of rectangular footing on geogrid-reinforced sand, Yetimoglu et al. (1994) observed that the BCR varied from 1.6 to 1.0.
when the u/B changed from 0.3 to 1.2 in single-layer reinforced sand and that the BCR varied from 3.0 to 1.0 when u/B changed from 0.15 to 1.2 in the multi-layer reinforced sand. The results of laboratory strip footing tests conducted by Sakti and Das (1994) on geotextile-reinforced clay showed that the most beneficial effect of geotextile reinforcement on the bearing capacity is realized when the first layer is placed at a (u/B) of 0.35 to 0.4 below the footing. These studies suggest that the optimum u/B value depends on the type of footings, type of soils, and type of reinforcement.

5.3.3 Effect Length of Reinforcement Layers

As discussed in the previous section about strain distribution in reinforced soil, the maximum strain along reinforcement occurs directly beneath the center of the footing and decreases as the distance away from the center of footing increases. And thus the length of reinforcement also can affect the performance of the reinforced soil.

A series of finite element analysis on 3-layer SWM and SBM reinforced soil was performed with different reinforcement length. The variations of the BCR and the SRF as a function of length of reinforcement layers are presented in Figures 5.18a and 5.18b, respectively. The figures show that the BCR increases and the SRF reduces with increase in the length of reinforcement, however, the trend became stable when $L$ was larger than $4B$, which is consistent with our findings in the previous section. To fully mobilize the benefits bought by the reinforcement, all the length of reinforcement layers was always $6B$ for the reinforced cases in the rest of the study.

5.3.4 Effect of Reinforcement Spacing

The effect of reinforcement spacing on the footing’s bearing capacity and settlement was investigated by changing the number/spacing of reinforcement layers within the effective reinforcement depth of $1.5B$. 
(a) Variation of BCR with reinforcement length

(b) Variation of SRF with reinforcement length

Figure 5.18 Effect of length of reinforcement layers (B=4ft, $D_r=0$)
A series of finite element analyses were conducted on the footing-reinforced soil model at five different spacing. Within the 1.5 B depth under the strip footing (B=4ft), the following reinforcement layers-spacing configurations were examined: three layers placed at 24 in. spacing, four layers placed at 18 in. spacing, six layers placed at 12 in. spacing, nine layer placed reinforcement at 8 in. spacing, and twelve layers placed at 6 in. spacing. The corresponding pressure-settlement curves are shown in Fig 5.19. In this figure, all the reinforcements are steel bar mesh.

![Figure 5.19 Typical curves of footing pressure versus footing settlement (B=4ft, Df=0)](image)

Figure 5.19 Typical curves of footing pressure versus footing settlement (B=4ft, Df=0)

For each case, the BCR at s/B =10% and the SRF at a footing pressure of 700 psi (4823 kPa) were calculated. Figures 5.20a and b depict the relationship between the reinforcement spacing and the BCR and SRF, respectively. For the reinforcements used, the figures show that at a given settlement the load carrying capacity of the footing decreases with the increase in reinforcement spacing, with larger decrease rates at small spacings. Besides, the footing settlement at the same
load is smaller for closer reinforcement spacings. Therefore, smaller reinforcement spacing should always be desirable provided that its cost is justified.

Figure 5.20 Effect of reinforcement spacing (B=4ft, $D_f=0$)

(a) BCR versus reinforcement spacing

(b) SRF versus reinforcement spacing
5.3.5 Effect of Reinforcement Tensile Modulus

As already confirmed in previous section tensile modulus of reinforcement has important effects on BCR and SRF of RSF. Different uniaxial reinforcements with varying tensile modulus were analyzed to examine the influence of their tensile modulus from the perspective of the ultimate bearing capacity and settlement of the footing.

A series of finite element analysis were conducted for 6 different reinforcement tensile modulus using 3, 6 and 12 reinforcement layers at a uniform spacing. The calculated BCR values at s/B=10% versus the tensile modulus of reinforcement are presented in Figure 5.21a. The relationship between the footing’s SRF (at p=700psi=4823 kPa) and the tensile modulus of reinforcement is also presented in Figure 5.21b.

Regardless of the number of reinforcement layers, the footing on reinforcements with higher tensile modulus has a larger bearing capacity than that with weaker reinforcements. However, this modulus-related increase in the BCR is more remarkable at low tensile modulus of reinforcement and gradually decreases as the reinforcement’s tensile modulus exceeds 15,000 kN/m.

On the other hand, the SRF decreases with the increase in tensile modulus of reinforcement, at a gradually reducing rate, which means less settlement, can be achieved if reinforcement with higher tensile modulus are provided.

In general, the figures indicate that a better reinforcement effect can be achieved in terms of higher ultimate bearing capacity and smaller settlement when the reinforcement has higher tensile modulus. For the soil studied herein, reinforcement with a tensile modulus ranging from 5,000 kN/m to 10,000 kN/m will maximize the benefits of the reinforced soil footing. Reinforcement with tensile modulus higher than 15,000 kN/m has no significant improvement.
Fig 5.21 Effect of reinforcement tensile modulus on SWM reinforced soil (B=4ft, Df=0)

(a) BCR versus reinforcement’s tensile modulus

(b) SRF versus reinforcement’s tensile modulus
5.3.6 Effect of Footing Embedment Depth

Finite element analysis were conducted on a strip footing (B=4ft) placed at different embedment depths (including 0B, 0.25B, 0.5B, 0.75B, and 1B) on top of a multi-layer reinforced soil.

The variations of the BCR and the SRF as a function of footing embedment depth are presented in Figure 5.22a and b, respectively. The figures show that the BCR reduces and the SRF increases, at approximately linear manners, with the increase in the embedment depth of the footing.

The reduction trend of the BCR with the increase in the embedment depth (Figure 5.22a) can be attributed to a larger increase in the bearing capacity of the unreinforced soil foundation compared to that of reinforced soil foundation. The figure also shows that the variations of BCR and SRF with embedment ratio were similar for different layers of geogrid, which means that the number of geogrid layers has minimal effect on the trend of variation of BCR and SRF with embedment ratio.

5.3.7 Effect of Footing Width

The effect of the footing’s width (or scale effect) on the performance of reinforced soil foundations was in terms of BCR and SRF of reinforced soil footings was studied by changing the width of strip footing from 3 ft to 6 ft with an interval of 1 ft, and the results are presented in Figures 5.23a and b respectively.

With the increase in footing width, both the bearing capacity and the settlement reduce at a linear manner. Again, this is due to larger increase in the bearing capacity of the unreinforced footing compared to the reinforced footing brought up by the increase in the footing’s width, which consequently causes a decrease trend in the BCR, as illustrated in Figure 5.23a.
Figure 5.22 Effect of footing embedment depth (B=4ft)
Figure 5.23 Effect of footing width for footing reinforced with SWM ($D_f = 0$)

(a) BCR versus footing width

(b) SRF versus footing width
5.3.8 Effect of Friction Angle of Crushed Limestone

As clayey soil, the friction angle of crushed limestone also has very important effect on its strength properties. In this study the effect of soil friction angle on the BCR and SRF of reinforced soil footings was studied by varying it with 48°, 50° and 52° while all the other parameters of the soil remained unchanged.

The results are shown in Figure 5.24a and b respectively. With the increase in friction angle, both the BCR and SRF decreased, which means that no extra benefit will be gained if only the friction angle of the crushed limestone is increased. This can be explained that the increase in soil friction angle results in more increase in the bearing capacity of the unreinforced soil compared to the reinforced soil, which consequently causes a decrease in the BCR as shown as in Figure 5.24a.

5.3.9 Effect of Elastic Modulus of Crushed Limestone

Since crushed limestone only has a nominal cohesion the effect of cohesion is not going to be studied here. Instead, the elastic modulus of the soil was studied by varying it with 14510 psi (100 MPa), 17420 psi (120 MPa), and 20320 psi (140 MPa) while all the other strength parameters of the soil remained the same.

The results are shown in Figure 5.25a and b respectively. With the increase in elastic modulus of soil, both the BCR and SRF increased, which means by increasing the the elastic modulus of reinforced soil (i.e. by means of compacting) the bearing capacity of it can be increased.

This can be explained that the increase in elastic modulus results in more increase in the bearing capacity of the reinforced soil compared to the unreinforced soil, which consequently causes an increase in the BCR as shown as in Figure 5.25a.
Figure 5.24 Effect of soil friction angle for footing reinforced with SWM
(B=4ft and $D_f=0$)

(a) Variation of BCR with soil friction angle

(b) Variation of SRF with soil friction angle
(a) Variation of BCR with soil elastic modulus

(b) Variation of BCR with soil elastic modulus

Figure 5.25 Effect of soil elastic modulus for footing reinforced with SWM
(B=4ft and $D_f = 0$)
5.4 Statistical Regression Analysis of Reinforced Crushed Limestone

5.4.1 Development of BCR Regression Model

As confirmed by the finite element analyses, the behavior of a strip footing sitting on reinforced soil depends on multiple factors including the reinforcement spacing, reinforcement stiffness, top spacing of first reinforcement layer, footing width, and footing embedment depth. The effect of these factors should be appropriately determined to ensure a rational design of a reinforced footing. Therefore, based on the results of finite element analysis a multi-regression statistical analysis was conducted to develop a BCR model that can facilitate the design of footing on reinforced crushed limestone. In developing the BCR model, all the reinforcement layers were assumed to lie within the effective reinforced depth and have enough length to fully mobilize its tensile contribution. The Statistical Analysis Software (SAS) package was used in this study. The full model described in Equation 5.4 was first assumed that includes the effects of all variables and their interactions.

\[
BCR = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \beta_7 X_1 X_2 + \beta_8 X_1 X_3 + \beta_9 X_1 X_4 + \beta_{10} X_1 X_5 + \beta_{11} X_1 X_6 + \beta_{12} X_2 X_3 + \beta_{13} X_2 X_4 + \beta_{14} X_2 X_5 + \beta_{15} X_2 X_6 + \beta_{16} X_3 X_4 + \beta_{17} X_3 X_5 + \beta_{18} X_3 X_6 + \beta_{19} X_4 X_5 + \beta_{20} X_4 X_6 + \beta_{21} X_5 X_6
\]  

(5.4)

Where:

- BCR: is the bearing capacity ratio of the reinforced soil at \( s/B = 10\% \),
- X1: is the spacing ratio between reinforcement layers (h/B),
- X2: is the normalized stiffness of reinforcement included in the reinforced soil,
- X3: is the footing embedment ratio (\( D_f/B \)),
- X4: is the footing width ratio (B/4ft),
- X5: is the normalized friction angle of soil (\( \phi/48 \));
X6: is the normalized elastic modulus of soil (E/17420 psi or E/120 MPa).

A stepwise variable selection procedure was then performed on the general model shown in Equation 5.4 to remove insignificant variables from the general model. The statistical variable selection procedure showed that no interaction between these variables is significant and that reinforcement spacing, reinforcement stiffness, footing embedment, and footing width, soil friction angle, and soil elastic modulus are the all statistically significant variables for the BCR at the 95% confidence level. The multiple regression analysis was then conducted on the reduced model and the results yielded the model shown in Equation 5.5.

\[
BCR=3.17874-0.41281*X1+0.07947*X2-0.21817*X3-0.37297*X4-1.65633*X5+0.41759*X6
\]

The analyses of variance of the proposed BCR model are presented in Table 5.6. The high R-Square value and adjusted R-Square value suggested a good regression of the data. Significance tests for individual parameters are conducted by using t statistics. The results of these t statistics are summarized in Table 5.7. It can be seen that with a 95% confidence level, X1, X2, X3, X4, X5, and X6 all have significant effect on the BCR values, which means that they all have their independent effect on the BCR.

5.4.2 Verification of the BCR Regression Model

The regression BCR model in Equation 5.5 was further verified by comparing the results of regression model with the results from additional 20 finite element analysis cases. The detailed variables and comparison are presented in Table 5.8, which is also illustrated in Figure 5.26. The absolute error in predicting the BCR value was calculated for each case and presented in the table. The absolute errors range from 0.12 % to 2.318 %, which suggests that the BCR values predicted by the regression model in Equation 5.5 have acceptable accuracy.
Table 5.6 Summary of the Analysis of Variance of the BCR Model

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr&gt;F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>6</td>
<td>0.492</td>
<td>0.123</td>
<td>164.53</td>
<td>&lt;.0001</td>
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<tr>
<td>Error</td>
<td>56</td>
<td>0.0254</td>
<td>0.000758</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>62</td>
<td>0.517</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Root MSE</td>
<td>0.027</td>
<td>R-Square</td>
<td>0.96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependent Mean</td>
<td>1.41</td>
<td>Adj R-Sq</td>
<td>0.95</td>
<td></td>
<td></td>
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<tr>
<td>Coeff Var</td>
<td>1.94</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.7 Summary of the BCR Model Parameters Estimate

| Variable | Parameters Estimate | Standard Error | Pr>|t| | Variance Inflation |
|----------|---------------------|----------------|----|------------------|
| Intercep | 3.17874             | 0.314          | <.0001 | 0                |
| X1       | -0.41281            | 0.289          | <.0001 | 1.061            |
| X2       | 0.07947             | 0.0034         | <.0001 | 1.105            |
| X3       | -0.21817            | 0.0137         | <.0001 | 1.058            |
| X4       | -0.37297            | 0.0268         | <.0001 | 1.029            |
| X5       | -1.65633            | 0.1712         | <.0001 | 1.116            |
| X6       | 0.41759             | 0.0552         | <.0001 | 1.006            |

Figure 5.26 Comparison between the BCR calculated from FEM and BCR predicted from statistical prediction
Table 5.8 Verification of Regression Models

<table>
<thead>
<tr>
<th>No.</th>
<th>X1</th>
<th>X2</th>
<th>X3</th>
<th>X4</th>
<th>X5</th>
<th>X6</th>
<th>BCR (FEM)</th>
<th>BCR (REG)</th>
<th>ABS (Err) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1875</td>
<td>0.147</td>
<td>1.00</td>
<td>1.00</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.2816</td>
<td>1.2832</td>
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<td>2</td>
<td>0.1875</td>
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<td>0.00</td>
<td>0.75</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.6218</td>
<td>1.5946</td>
<td>1.676</td>
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<td>3</td>
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<td>1.0000</td>
<td>1.3977</td>
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<td>1.0000</td>
<td>1.3215</td>
<td>1.3149</td>
<td>0.502</td>
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<td>5</td>
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<td>1.00</td>
<td>1.0000</td>
<td>0.8333</td>
<td>1.3236</td>
<td>1.3543</td>
<td>2.318</td>
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<td>0.1475</td>
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<td>0.8333</td>
<td>1.2765</td>
<td>1.2853</td>
<td>0.685</td>
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<td>1.0000</td>
<td>1.3391</td>
<td>1.3549</td>
<td>1.177</td>
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<td>1.00</td>
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<td>1.1667</td>
<td>1.3981</td>
<td>1.4245</td>
<td>1.887</td>
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<td>0.8333</td>
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<td>1.2163</td>
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<td>11</td>
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<td>1.00</td>
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</tr>
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<td>1.00</td>
<td>1.0833</td>
<td>1.1667</td>
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</tr>
<tr>
<td>13</td>
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<td>0.00</td>
<td>1.00</td>
<td>1.0000</td>
<td>0.8333</td>
<td>1.4103</td>
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</tr>
<tr>
<td>14</td>
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<td>1.00</td>
<td>1.0000</td>
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<td>1.00</td>
<td>1.0417</td>
<td>0.8333</td>
<td>1.3457</td>
<td>1.3627</td>
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<tr>
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<td>0.8333</td>
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<td>1.1667</td>
<td>1.4145</td>
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</table>
This chapter presents the results of the numerical modeling study that was conducted to capture the impacts of the base course layers’ parameters reflected by the granular base thickness, subgrade strength, as well as the stiffness and location of the geogrid reinforcement layer on the structural performance of geogrid reinforced flexible pavement systems.

6.1 Finite Element Model

The finite element model was developed using the ABAQUS finite element software package (ABAQUS, 2004) to analyze the flexible pavement structure with geogrid reinforced bases.

As described in Chapter 3, the geogrid reinforced pavement system was modeled as two-dimensional (2D) axisymmetric finite element model. The multi-layered geogrid base reinforcement pavement system is analyzed by assigning different material models to different materials. The AC layer was modeled as elastic-perfectly-plastic material, the base course layer was modeled using the bounding surface model (Dafalias and Herrman, 1986), the subgrade layer was modeled with modified Cam-Clay model available in ABAQUS and the geogrid layer was modeled as linear elastic material. The following section describes the features of the finite element model.

6.1.1 Finite Element Mesh

The radius of the mesh was selected based on the distance at which the vertical and horizontal strains become insignificantly small in all layers. And the depth of the mesh was chosen to be at the depth at which the maximum induced vertical stress in the subgrade became insignificantly small (<0.01% of the applied pressure). The mesh used in the study has a radius of 4.5 m and a depth of 4 m, shown in Figure 6.1.
To determine the suitable element size for the 2D axisymmetric model, a series of finite
element analyses were performed with increasing element numbers. Mesh sensitivity was studied
to determine the level of fine mesh needed for a stable finite element analysis that converges to a
unique solution. Based on this analysis, 60, 360, 1800, 3961 elements were used for the geogrid,
AC, base course layer and subgrade layer, respectively.

Eight-noded biquadratic axisymmetric quadrilateral elements were used for the subgrade,
base, and asphalt concrete layers, while a three-noded quadratic axisymmetric membrane
element with thickness of 1 mm was used for the geogrid reinforcement.

Conventional kinematic boundary conditions were adopted, such that the horizontal
movement along the left and right boundaries and the vertical movement along the bottom
boundary were restrained by using roller supports. Such boundary conditions have been
successfully used by Zaghloul and White (1993), Kuo et al. (1995) and Nazzal (2007).

A three-noded axisymmetric membrane element is used in the FE mesh to model the geogrid.
The membrane elements are capable of resisting loads in tension but they have no resistance to
bending. This membrane element is really a bar element in the axisymmetric analysis plane. As
the axisymmetric r-z plane is rotated around the pavement centerline, the geogrid can be
modeled as a membrane due to the nature of the axisymmetric stress analysis.

6.1.2 Load Model

The loading model in this study included applying gravity loads in the first load step of the
analysis, then applying 100 cycles of loading representative of a 80 kN (18 kips) single axle
wheel loading, which is the standard load known as equivalent single axle load (ESAL)
recommended by AASHTO (1993).

The wheel load was approximately simulated by applying the uniform contact pressure on a
circular area with a radius of 152 mm (6 inch) at the surface. A harversine-shaped load was coded with user subroutine DLOAD. ABAQUS will recall the user subroutine automatically when it is needed. More information about the loading model used in this study is provided in Chapter 3.

![Finite element model for reinforced pavement](image)

Figure 6.1 Finite element model for reinforced pavement

### 6.1.3 Residual Stress

The application of large vertical stresses required during construction of the pavement system are reported to cause horizontal stresses to develop that become locked into the granular bases and subgrades (Sowers, et al., 1957, Uzan, 1985 and Selig, 1987) Residual stresses develop
in the base course layer as a result of the initial compaction. These residual stresses should be properly quantified and taken into account for determining the initial stress state of a flexible pavement system.

Almeida et al. (1993) recognized that a pavement in its original state (after compaction) has horizontal residual stresses that are likely to be able to increase the elastic stiffness of the base course layer. A residual stress of 21 kPa was assumed to exist throughout the depth of the unreinforced base course layer in accordance with the field measurements of Barksdale and Alba (1993).

Usually, the residual stresses around the geogrid would be higher than the other part because the inclusion of the geogrid naturally causes development of stiffer layer associated with the interlocking action that develops around geogrid reinforcement (Perkins et al., 2004; Konietzky et al., 2004 and 2005).

Though the distribution of the locked-in horizontal residual stresses in the base course around the geogrid reinforcement are still not yet fully understood (Kwon, 2007); a recent discrete element analyses of the geogrid base course layer conducted by McDowell et al. (2005) showed that the zone of lateral confinement effect of geogrid tends to extend to approximately 100 mm from geogrid side, which is already been adopted by Nazzal (2007) in his research study of geogrid base reinforcement pavement system.

The residual stress distributions for unreinforced and reinforced flexible pavement systems used in this study are shown in Figure 6.2. All the residual stresses were applied as initial stress conditions in this finite element model of geogrid base reinforcement pavement system. And since the residual stresses distribution for reinforced cases is not readily available in ABAQUS, a user defined subroutine (SIGNI) was developed to simulate the distribution of the residual stress.
6.1.4 Material Constitutive Models’ Parameters

Typical geogrid base reinforced pavement system consists of hot-mix asphalt concrete layer, base course layer, subgrade layer and geogrid reinforcement layer. Different material models need to be employed to describe the behavior of different materials and the geogrid interface in the pavement system. The following section will describe the models’ parameters used in the finite element analysis.

6.1.4.1 Asphalt Concrete (AC) Layer

In this study, an elastic-perfectly plastic model was used to describe the behavior of asphalt concrete (AC) layer. The plasticity was introduced by specification of an ultimate yield stress corresponding to a perfect plasticity hardening law. The parameters used for the AC layer is presented in Table 6.1.

Table 6.1 Material Parameters for AC layer (Masad et al., 2005)

<table>
<thead>
<tr>
<th>Material</th>
<th>$\nu$</th>
<th>Elastic Modulus (kPa)</th>
<th>Yield Stress (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>0.35</td>
<td>3,450,000</td>
<td>770</td>
</tr>
</tbody>
</table>
6.1.4.2 Base Course (BC) Layer

The bounding surface model developed by Dafalias and Herrman (1986) was used to model the crushed limestone base material. The features of this model were described in details in Chapter 4 of this dissertation. Furthermore, the calibration of the model parameters and verification of the model prediction were also presented in Chapter 4. With triaxial undrained testing data of the soil and best fitting curve method, 14 parameters needed for the model were determined. Table 6.2 presents a summary of the calibrated model parameters used in the finite element analysis conducted in this chapter.

Table 6.2 Bounding Surface Model Parameters for Crushed Limestone Base Material

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Values used in the study</th>
</tr>
</thead>
<tbody>
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<td>λ</td>
<td>Virgin compression slope</td>
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</tr>
<tr>
<td>κ</td>
<td>Swell/recompression slope</td>
<td>0.0018</td>
</tr>
<tr>
<td>μ</td>
<td>Poisson’s ratio</td>
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</tr>
<tr>
<td>$M_c$</td>
<td>Slope of CSL in compression</td>
<td>0.37</td>
</tr>
<tr>
<td>$M_e$</td>
<td>Slope of CSL in extension</td>
<td>0.37</td>
</tr>
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<td>Bounding surface shape parameters for ellipse 1</td>
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</tr>
<tr>
<td>$R_e$</td>
<td>Bounding surface shape parameters for hyperbola</td>
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<tr>
<td>$A_c$</td>
<td>Bounding surface shape parameters for ellipse 2</td>
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</tr>
<tr>
<td>$A_e$</td>
<td>Bounding surface shape parameters for hyperbola</td>
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<td>$C$</td>
<td>Projection center parameter</td>
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<tr>
<td>$S$</td>
<td>Elastic zone parameter</td>
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</tr>
<tr>
<td>$H_c$</td>
<td>Shape hardening parameters</td>
<td>20</td>
</tr>
</tbody>
</table>

6.1.4.3 Subgrade Layer

The subgrade was modeled using the Modified Cam clay model. Three sets of the Modified Cam clay model parameters were selected to describe the behavior of subgrade materials from
other study (Nazzal, 2007) to represent weak, moderate and stiff subgrades. The selected parameters are presented in Table 6.3.

Table 6.3 Modified Cam-Clay Model Parameters for Different Subgrade Soils (Nazzal, 2007)

<table>
<thead>
<tr>
<th>Subgrade</th>
<th>G (kPa)</th>
<th>M</th>
<th>λ</th>
<th>κ</th>
<th>e₀</th>
<th>CBR</th>
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</thead>
<tbody>
<tr>
<td>Soft</td>
<td>5170</td>
<td>0.65</td>
<td>0.225</td>
<td>0.11</td>
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<tr>
<td>Medium</td>
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<td>0.11</td>
<td>0.084</td>
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<td>7</td>
</tr>
<tr>
<td>Stiff</td>
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<td>1.56</td>
<td>0.022</td>
<td>0.005</td>
<td>0.54</td>
<td>15</td>
</tr>
</tbody>
</table>

6.1.4.4 Geogrid Layer

A linear elastic model was used to describe the behavior of geogrid material since the induced strain in the geogrid is very small (<1%) and is considered within the linear elastic range of the geogrid layer. Five types of geogrid with different equivalent elastic modulus were used in the finite element analysis to investigate the effect of geogrid tensile strength on pavement response and performance. A summary of these properties are shown in Table 6.4.

Table 6.4 Geogrid Material Properties

<table>
<thead>
<tr>
<th>Geogrid Type</th>
<th>Reference Name</th>
<th>Elastic Modulus (kPa)</th>
<th>ν</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geogrid Type I</td>
<td>GGI</td>
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</tr>
<tr>
<td>Geogrid Type II</td>
<td>GGII</td>
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</tr>
<tr>
<td>Geogrid Type III</td>
<td>GGIII</td>
<td>860000</td>
<td>0.25</td>
</tr>
<tr>
<td>Geogrid Type IV</td>
<td>GGVI</td>
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<td>0.25</td>
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<td>Geogrid Type V</td>
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</tbody>
</table>

6.2 Parametric Study Matrix

The finite element model developed in this chapter was used to investigate the effects of different variables on the degree of improvement achieved by reinforcing the base course layer with a single layer of geogrid reinforcement. These variables included the strength of the subgrade material, the thickness of the base course layer, as well as the stiffness and location of the geogrid reinforcement layer. To study these variables, finite element analyses were first
conducted on twelve (12) unreinforced sections with three different subgrade strength properties and four base course layer thicknesses for use as references. The three different subgrades included: a weak subgrade with a CBR value less than 1.5, a moderate subgrade with a CBR value of 7, and a stiff subgrade with a CBR of 15. While the five different base course layer thicknesses varied from 150 mm (6 in.) to 300 mm (12 in.), and included: 150 mm (6 in.), 200 mm (8 in.), 250 mm (10 in.), 300 mm (12 in.) base layer thicknesses. Table 6.5 presents a summary of the different sections investigated in this study. It should be noted that the different section will be identified using the reference names provided in Table 6.5. Finite element analyses were then conducted on the different pavement sections reinforced with geogrid layer placed at upper 1/3, middle or the bottom of the base course layer. The three different locations of geogrid in the base course layer of the pavement system are illustrated in Figure 6.3.

Table 6.5 Pavement sections studied

<table>
<thead>
<tr>
<th>Section</th>
<th>Thickness of AC layer (mm)</th>
<th>Thickness of Base course (mm)</th>
<th>Subgrade Quality</th>
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</thead>
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<tr>
<td>Section 1a</td>
<td>50</td>
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</tr>
<tr>
<td>Section 1b</td>
<td>50</td>
<td>150</td>
<td>Moderate</td>
</tr>
<tr>
<td>Section 1c</td>
<td>50</td>
<td>150</td>
<td>Stiff</td>
</tr>
<tr>
<td>Section 2a</td>
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</tr>
<tr>
<td>Section 2c</td>
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</tr>
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<tr>
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<td>Moderate</td>
</tr>
<tr>
<td>Section 4c</td>
<td>50</td>
<td>300</td>
<td>Stiff</td>
</tr>
</tbody>
</table>

6.3 Results of Finite Element Analysis

The following sections summarize the results of the finite element analysis of reinforced bases in flexible pavements.
(a) Location at upper 1/3 of the base course layer

(b) Location at middle of the base course layer

(c) Location at bottom of the base course layer

Figure 6.3 Geogrid locations in the parametric study (hbc varies as shown in Table 6.5)
6.3.1 Stresses and Strain Distribution

The lateral strains profiles at different distances from the center of the wheel load predicted from the finite element analysis within the subgrade layer for unreinforced and reinforced sections 1a and 4c are shown in Figures 6.4 and 6.5, respectively. In these sections, the geogrid layer was placed at the bottom of base course layer. It can be seen that the geogrid layer significantly constrained the lateral strains within the base course layer and subgrade layer. And geogrid with higher tensile modulus has more reduction in lateral strains developed in the reinforced sections. It is also noted that the constraining effect was mainly below the wheel loading area and it decrease with increasing distance from the center of the wheel load. The reduction of lateral strain provided by geogrid reinforcement was more appreciable in section with thin base layer build on top of weak subgrade layers compared to sections built with thick base layer over stiff subgrade soils.

Figures 6.6 and 6.7 present the lateral strain profiles computed at different distances from the center of the wheel load for unreinforced sections 1a and 4c, and same sections reinforced sections with one layer of type V geogrid placed at different locations. In general, for pavements with base course thickness of less than 300 mm, geogrid placed at upper one third of base course layer has the least constraint effect in lateral strains developed; while, geogrid placed at the bottom of base course layer has the greatest reduction. Beyond a distance of 300 mm from the wheel load center, the location of geogrid almost has no effect on the lateral strains, mainly since the geogrid layer did not have any contribution to the lateral strain beyond this point.

It should be noted here that the effect of geogrid location, or on the other words, the optimum location of geogrid might be different for pavements with base course layer thickness larger than 300 mm.
Figures 6.8 and 6.9 show the vertical strain profiles at different locations within the subgrade layer for unreinforced and reinforced sections 1a and section 4c, respectively, for geogrid layer placed at the bottom of base course layer. It is noted from these figures that the inclusion of the geogrid layer resulted in significant reduction in the vertical strain at the top of subgrade layer and this kind of effect decrease with the increasing in the distance from the top. And this kind of reduction in vertical strains increases with the increase of geogrid stiffness. Furthermore this reduction is influenced by the base course thickness and subgrade stiffness. Greater reduction in vertical strain is noticed for weaker subgrade and thinner base course layer.

Figures 6.10 and 6.11 show the profile of vertical strains computed at different depths within the subgrade layer for sections 1a and 4c, respectively, reinforced with one layer of type IV geogrid placed at the different locations studied. It is noted that sections reinforced with a geogrid layer placed at the bottom of the base course had much greater reduction in vertical strain when compared to the other locations. Furthermore, the reduction geogrid locations were more pounced in sections with thin base course layer built over weak subgrade layers.

The shear strain distributions developed at the top of the subgrade layer for sections 1a and 4c are shown in Figures 6.12 and 6.13, respectively. It can be seen that the geogrid resulted not only in decreasing the shear strains development at the top of the subgrade layer but also in providing a better distribution of these strains. And again, geogrid with higher tensile modulus provide more reduction in the shear strains development and better distribution of the strains.

The Shear strain distributions developed at the top of the subgrade layer for section 1a and 4c with geogrid type V placed at different locations in the base course layer are shown in Figures 6.14 and 6.15, respectively. It can be seen that the geogrid placed at the bottom of base course
layer has greater reduction in shear strain developed at the top of the subgrade layer as compared to other locations.

Figures 6.16 and 6.17 show the plastic strain distribution on the top of subgrade layer for unreinforced and reinforced sections 1a, 4c, respectively. It can be seen that plastic strains developed in the unreinforced section was greater than those in reinforced sections. And reinforced sections reinforced with geogrid of higher tensile modulus undergoes less plastic strains than the reinforced sections reinforced with geogrid of lower tensile modulus.

Figures 6.18 and 6.19 show the profiles of vertical plastic strain obtained at the top of the subgrade layer for sections 1a and 4c reinforced with one layer of type VI geogrid placed at different locations. It can be seen that among all the location considered, the location at the bottom is the most efficient in reducing the vertical plastic strain for pavements with base course thickness of less than or equal to 300 mm.

Figure 6.20 present the plastic vertical strain contours of one typical section (section 1a) for base course layer with thickness of 150 mm on top of weak subgrade. Figure 6.21 present the plastic vertical strain contours of one typical section (section 4c) for base course layer with thickness of 300 mm on top of stiff subgrade.

It can be seen from Figures 6.20 and 6.21, that the plastic strains developed in the unreinforced sections are larger and wider than those developed in the reinforced sections. It appears that the inclusion of one layer of geogrid reinforcement located at the bottom of base course layer helps reducing the plastic strains developed in the subgrade layer; and thus the pavement deformation induced in the subgrade layer will be reduced. Accordingly, surface rutting, as the total displacement of AC layer, base course layer and subgrade layer, will be decreased.
(m)  (a) At wheel center    (b) 152 mm from center    (c) 304 mm from center

Figure 6.4 Lateral Strain Profile of Unreinforced and Reinforced system with Geogrid Layer Placed at Bottom of Base Course Layer for Section 1a
Figure 6.5 Lateral Strain Profile of Unreinforced and Reinforced system with Geogrid Layer Placed at Bottom of Base Course Layer for Section 4c
(m)         (a) At wheel center           (b) 152 mm from center     (c) 304 mm from center

Figure 6.6 Lateral Strain Profile of Unreinforced and Reinforced system with one layer of GGV Placed at Different Location within Base Course Layer for Section 1a
Figure 6.7 Lateral Strain Profile of Unreinforced and Reinforced system with one layer of GGV Placed at Different Location within Base Course Layer for Section 4c

(m) (a) At wheel center (b) 152 mm from center (c) 304 mm from center
(a) At top of subgrade

(b) 0.152 m below top of subgrade

(c) 0.304 m below top of subgrade

Figure 6.8 Vertical Strain Profiles within Subgrade Layer for Unreinforced Section 1a and Reinforced with a Layer of Geogrid Placed at Bottom of BC layer
Figure 6.9 Vertical Strain Profiles within Subgrade Layer for Unreinforced Section 4c and Reinforced with a Layer of Geogrid Placed at Bottom of BC layer

(a) At top of subgrade

(b) 0.152 m below top of subgrade

(c) 0.304 m below top of subgrade
Figure 6.10 Vertical strain profiles within subgrade layer for unreinforced and reinforced section 1a with one Layer of GGV placed at different location in BC layer
Figure 6.11 Vertical Strain Profiles within Subgrade layer for Unreinforced and Reinforced Section 4c with One Layer of GGV Placed at Different Location in BC Layer.
Figure 6.12 Shear Strain Profile at Top of Subgrade Layer for Unreinforced and Reinforced section 1a with Geogrid Placed at the bottom of Base Course Layer

Figure 6.13 Shear Strain Profile at Top of Subgrade Layer for Unreinforced and Reinforced section 4c with Geogrid Placed at the bottom of Base Course Layer
Figure 6.14 Shear Strain Profile at Top of Subgrade Layer for Unreinforced and Reinforced Section 1a with One Layer of GGV Placed at Different Location in BC Layer

Figure 6.15 Shear Strain Profile at Top of Subgrade Layer for Unreinforced and Reinforced Section 4c with One Layer of GGV Placed at Different Location in BC Layer
Figure 6.16 Vertical Plastic Strain Profile at Top of Subgrade Layer for Unreinforced and Reinforced Section 1a with Geogrid Placed at the Bottom of Base Course Layer

Figure 6.17 Vertical Plastic Strain Profile at Top of Subgrade Layer for Unreinforced and Reinforced Section 4c with Geogrid Placed at the Bottom of Base Course Layer
Figure 6.18 Vertical Plastic Strain Profile at Top of Subgrade Layer for Unreinforced and Reinforced Section 1a with One Layer of GGV Placed at Different Location in BC Layer

Figure 6.19 Vertical Plastic Strain Profile at Top of Subgrade Layer for Unreinforced and Reinforced Section 4c with One Layer of GGV Placed at Different Location in BC Layer
Figure 6.20 Plastic strain contours for section 1a (150 mm BC layer over weak subgrade)

(a) Unreinforced section

(b) GGV placed at the bottom of BC layer
Figure 6.21 Plastic strain contours for section 4c (300 mm BC layer over Stiff subgrade)
6.3.2 Permanent Deformation

Finite element analysis of the sections (which reflects the individual and crossing effect of base course thickness and subgrade strength) described in Table 6.5 were developed. Among the analysis the geogrid location was varied from upper 1/3, middle and bottom of the base course layer to investigate its effect on the permanent deformation of the reinforced pavement system and three tensile modulus of geogrid (GGI, GGIII and GGV) were also used to study its effect.

Figures 6.22 through 6.29 depict the accumulated permanent deformation curves computed using the finite element analysis for unreinforced and geogrid reinforced sections at 100 load cycles. It can be clearly seen that the use of geogrid reinforcement results in reducing the permanent deformation for reinforced pavement sections. However, the magnitude of reduction depends on the geogrid location and tensile modulus (stiffness), the subgrade strength, and the base course thickness; such that the permanent deformation decrease with increasing the geogrid tensile modulus, the base course thickness and the subgrade strength.

From Figures 6.22 through 25 it is clear that the smallest surface deformation of the selected sections was always achieved when the geogrid reinforcement was placed at the bottom of the base course layer. Since the largest base course thickness in the study is 300 mm (12 inch), the conclusion that the optimum location of geogrid in the pavement scheme is at the bottom of the base course layer, is applied for base course thickness of less than 300 mm (12 inch). Different optimum location might occur for base course thickness of more than 300 mm (12 inch), which is not investigate in this study.

Figures 26 through 29 shows that the accumulated permanent deformation reduces with increase in the tensile modulus of geogrid when the geogrid layer is placed at the same location; and that the effect of geogrid tensile modulus increases with decreasing the base course thickness.
Figure 6.22 Rutting Curves of Different Pavement Section 1 with different geogrid location
Figure 6.23 Rutting Curves of Different Pavement Section 2 with different geogrid location
Figure 6.24 Rutting Curves of Different Pavement Section 3 with different geogrid location
Figure 6.25 Rutting Curves of Different Pavement Section 4 with different geogrid location
Figure 6.26 Rutting Curves of Different Pavement Section 1 with different geogrid type located at the bottom of base course layer
Figure 6.27 Rutting Curves of Different Pavement Section 2 with different geogrid type located at the bottom of base course layer
Figure 6.28 Rutting Curves of Different Pavement Section 3 with different geogrid type located at the bottom of base course layer.
Figure 6.29 Rutting Curves of Different Pavement Section 4 with different geogrid type located at the bottom of base course layer
6.4 Evaluation of the Reinforcing Effect Using a Mechanistic Empirical Approach

The improvement achieved by the inclusion of geogrid layer within the base course layer in flexible pavement was evaluated using the mechanistic empirical approach. In this approach, the response parameters (i.e. strains) computed from the finite element analyses (mechanistic part) are used to determine the pavement structure distresses (i.e. surface rutting) based on empirical damage models following the Guide for Mechanistic Empirical Design (2004).

The surface rutting or permanent deformation of pavement structures was determined by first dividing each pavement layer into sub-layers. Damage models are then used to relate the vertical compressive strain, computed from the finite element analysis, at the mid-depth of each sub-layer, and the number of traffic applications to layer plastic strains. The overall permanent deformation is then computed using Equation 6.1 as the sum of permanent deformation for all individual sub-layers.

\[
D_p = \sum_{i=1}^{N_s} \varepsilon_{p}^i \cdot h^i
\]  

(6.1)

Where:

\(D_p\) : Permanent deformation of pavement section

\(N_s\) : Number of sub-layers

\(\varepsilon_{p}^i\) : Total plastic strain in sub-layer \(i\)

\(h^i\) : Thickness of sub-layer \(i\)

Three main damage models were used in this study, one model for the asphalt concrete material (Equation 6.2), one model for the base course layer (Equation 6.4), and one model for the subgrade materials (Equation 6.5). The parameters of these models were determined through
national calibration efforts using the Long-Term Pavement Performance (LTPP) database, and laboratory tests conducted on the different pavement materials used.

For Asphalt concrete layer:

\[
\frac{\varepsilon_p}{\varepsilon_{vA}} = k_1 10^{-3.4488} T^{1.5606} N^{0.473844}
\]  
(6.2)

Where

- \( \varepsilon_p \): Accumulated plastic strain at \( N \) repetitions of load;
- \( \varepsilon_{vA} \): Vertical strain of the asphalt material;
- \( N \): Number of load repetitions;
- \( T \): Pavement temperature (° F);
- \( k_1 \): Function of total asphalt layer(s) thickness and depth to computational point, is used to correct for the variable confining pressures that occur at different depths and is expressed as:

\[
k_1 = (C_1 + C_2 \cdot depth) \cdot 0.328196^{depth}
\]  
(6.3)

Where:

\[
C_1 = -0.1039 \cdot h_{ac}^2 + 2.4868 \cdot h_{ac} - 17.342
\]

\[
C_1 = 0.0172 \cdot h_{ac}^2 - 1.7331 \cdot h_{ac} + 27.428
\]

- \( h_{ac} \): Asphalt layer thickness

For base course layer:

\[
\frac{\varepsilon_p}{\varepsilon_{vB}} = \beta_{GB} \left( \frac{\varepsilon_0}{\varepsilon_r} \right) \cdot e \left( \frac{\rho}{N} \right)^\beta
\]  
(6.4)

Where

- \( \varepsilon_{vB} \): Vertical strain of the base course material
\( \beta_{gb} \) : is national model calibration factor for unbound base course material and is equal to 1.673.

\( \varepsilon_0, \beta \) and \( \rho \) are material parameters

\( \varepsilon_r \) : Resilient strain imposed in laboratory test to obtain material properties.

For subgrade layer

\[
\frac{\varepsilon_p}{\varepsilon_{vS}} = \beta_{sg} \left( \frac{\varepsilon_0}{\varepsilon_r} \right) e^{-\left( \frac{\rho}{N} \right) \alpha} \tag{6.5}
\]

Where

\( \varepsilon_{vS} \) : Vertical strain of the subgrade material

\( \beta_{sg} \) is a national model calibration factor for subgrade material and is equal to 1.35.

The number of traffic passes to reach a 25 mm (1 inch) of permanent surface deformation for different unreinforced and reinforced pavement sections with geogrid ranging from Type I, III and V located at the bottom of the base course layer was calculated using the aforementioned mechanistic empirical approach and was summarized in Table 6.6.

As were discussed earlier, the geogrids were able to extend the service lives of the reinforced sections by reducing the amount of permanent deformation (rutting) in these sections. The increase in service life of pavement structure is usually evaluated by using the Traffic Benefit Ratio (TBR). The TBR is defined as the ratio of the number of load cycles to achieve a particular rut depth in the reinforced section to that of unreinforced section of identical thickness, material properties, and loading characteristics. The TBR values obtained at 25 mm rutting depth for the different sections studied in this research project were also calculated and summarized in Table 6.6.
Table 6.6 Summary of Rutting of Different Unreinforced and Reinforced Sections

<table>
<thead>
<tr>
<th>Section</th>
<th>Geogrid</th>
<th>Nf Rutting</th>
<th>TBR</th>
<th>Section</th>
<th>Geogrid</th>
<th>Nf Rutting</th>
<th>TBR</th>
</tr>
</thead>
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<td>Section 3a</td>
<td>None</td>
<td>3.77E+05</td>
<td>NA</td>
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<tr>
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<td>Section 3a</td>
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<td>7.50E+05</td>
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</tr>
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<td>Type III</td>
<td>1.93E+05</td>
<td>3.37</td>
<td>Section 3a</td>
<td>Type III</td>
<td>8.97E+05</td>
<td>2.38</td>
</tr>
<tr>
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<td>2.51</td>
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<tr>
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<td>9.84E+05</td>
<td>NA</td>
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<tr>
<td>Section 1b</td>
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<td>2.37</td>
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<td>Type I</td>
<td>1.59E+06</td>
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<tr>
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<td>NA</td>
</tr>
<tr>
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<td>2.18</td>
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<tr>
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<td>Type III</td>
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<td>Section 4c</td>
<td>Type V</td>
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</tbody>
</table>

The resulted TBR values in Table 6.6 with Types I, III and V geogrid reinforced sections were also illustrated in Figures 6.30 through 6.32. It can be seen from Table 6.6 and the figures that the increase of the geogrid tensile modulus resulted in greater reduction in the permanent deformation of reinforced pavement system and hence increasing the number of load repetitions is needed to reach the 25 mm surface rutting. This is consistent with the finite element findings proceeded earlier in this chapter which showed that higher geogrid tensile modulus resulted in larger reduction in vertical strains. It also can be seen that the improvement provided by the
geogrid reinforcement decreased with the increase of base course thickness and increase in subgrade strength.

Once the location of geogrid reinforcement is fixed at the bottom of the base course layer, the reinforcement mechanism in reducing the permanent deformation will then depend on three main factors: base course thickness, geogrid tensile modulus, subgrade strength and their interaction with each other. So further analysis need to be performed to evaluate the combined effect on the performance of geogrid base reinforced pavement sections.

Figure 6.30 TBR of pavement system with reinforced bases over weak subgrade

Figure 6.31 TBR of pavement system with reinforced bases over moderate subgrade
6.5 Development of TBR Model with Statistical Regression

To quantify the effect of different factors on improvement of reinforced pavement system, all the reinforced cases from Table 6.6 were used to develop a statistical TBR regression model.

6.5.1 TBR Model with Statistical Regression

As confirmed in the previous section of this chapter, the base course layer thickness, tensile modulus of geogrid and strength of subgrade all have effect on the TBR of geogrid base reinforced pavement system. Initially, a general model that includes all of the investigated variables and their interactions was selected. Then multiple regression analysis was conducted to remove the insignificant variables and finally a statistical regression model was obtained to predict the TBR.

The general TBR model is given as:

\[
TBR = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_1 X_2 + \beta_5 X_2 X_3 + \beta_6 X_3 X_4
\]  (6.6)

Where

\( X_1 \): is the reinforced base layer thickness in mm,
\( X_2 \): is the geogrid modulus (kPa) used in the finite element models normalized to a modulus value of 135000 (kPa),

\( X_3 \): is subgrade CBR value, representing the strength of subgrade,

\( X_1X_2 \): is the interaction between the effect of the reinforced thickness and normalized geogrid modulus,

\( X_2X_3 \): is the interaction between the effect of the reinforced thickness and subgrade CBR value;

\( X_3X_1 \): is the interaction between the effect of the subgrade strength and normalized geogrid modulus.

A stepwise variable selection procedure was conducted on the selected model to eliminate any insignificant variable. Based on the results of this procedure, only the normalized geogrid modulus, the interaction between the reinforced thickness and normalized geogrid modulus, the interaction between the subgrade strength and normalized geogrid modulus were found to be significant.

Based on the results of stepwise selection analysis, multiple regression analysis was conducted on finite element data to develop a TBR prediction model. Table 6.7 and Table 6.8 present the results of the regression analysis. The results showed that the final model has \( R^2 \) of 0.96 and a Root MSE of 0.15, which suggested that the model well fits the data used. The Pr>|t| values for all parameters are all small enough to show their significant effect on the prediction of the model, and all the Variance Inflation Factors for these variables are less than 10, which indicate they are not collinear.

Equation 6.7 presents the final TBR model obtained from the statistical regression analysis.

\[
TBR = 1.28 + 2.25X_2 - 0.91X_1X_2 - 0.023X_3X_1 
\]  

\text{(6.7)}
The regression model of TBR indicates that the predicted TBR values increases with the increase of the geogrid tensile modulus and with the decrease in the base layer thickness and the subgrade strength. Furthermore, it is noted that the beneficial effect of the grogrid tensile modulus decreases with the increase in the base course layer thickness and the increase of subgrade strength.

Table 6.7 Summary of the Analysis of Variance of the TBR Model

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<th>Source</th>
<th>DF</th>
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<th>Mean Square</th>
<th>F Value</th>
<th>Pr&gt;F</th>
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<td>0.46</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
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<td>10.51</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Root MSE</td>
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<td>R²</td>
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Table 6.8 Summary of the TBR Model Parameters Estimate

| Variable | Parameters Estimate | Standard Error | Pr>|t| | Variance Inflation |
|----------|---------------------|----------------|------|-------------------|
| Intercept| 1.29                | 0.14           | <.0001 | 0                 |
| X2       | 2.25                | 0.14           | <.0001 | 1.95              |
| X1X2     | -0.91               | 0.065          | <.0001 | 2.06              |
| X3X1     | -0.028              | 0.0036         | <.0001 | 1.11              |

6.5.2 Verification of the Statistical TBR Model

Additional 20 cases of geogrid reinforced sections were run using the finite element model and the corresponding TBR values for these cases were calculated using the mechanistic empirical method. These TBR values are used to verify the regression model shown in Equation 6.7. The TBR values of these cases calculated from the finite element analysis using mechanical-empirical method and those predicted from the statistical regression model were compared and
summarized in Table 6.9. The absolute errors range from 0.12 % to 5.03 %, which suggest that the TBR values predicted by the regression model are within acceptable accuracy. The verification results of the regression model are also presented in Figure 6.33.

Table 6.9 Verification of Regression Models

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<th>No.</th>
<th>X1</th>
<th>X2</th>
<th>X3</th>
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<th>Predicted TBR</th>
<th>Absolute Error(%)</th>
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Figure 6.33 Comparison between the TBR calculated from FEM and TBR predicted from statistical prediction
CHAPTER 7 SUMMARY, CONCLUSIONS AND SUGGESTIONS

7.1 Summary

The benefits of using geosynthetics to improve the load bearing characteristics and longevity of reinforced soil structures are wildly recognized. Geogrids, as one of the most commonly used forms of reinforcement, offers improved interface shear resistance due to interlocking, in particular when compared to commonly used alternatives.

In the present study, two types of geogrid reinforced structures—geogrid reinforced foundations and geogrid reinforced bases in flexible pavement system—were modeled by applying finite element analysis to investigate their potential benefits. For this purpose, the study, comprised of two distinctive parts, assessed the influences of different variables and parameters contributing to the improved performance of reinforced soil foundation (RSF) and reinforced bases in flexible pavement system.

Based on the findings of the above analysis, finite element models that can simulate the behavior of RSF were established. Finite element analyses of different parameter combinations were run to investigate their influence on the RSF in terms of bearing capacity ratio (BCR). The parameters studied included: effective length and depth of reinforcement zone, spacing between reinforcement layers, optimum top spacing for the first reinforced layer, tensile modulus (or stiffness) of reinforcement, footing width, and embedment depth of footing, friction angle and cohesion of the studied silty clay embankment soil, the friction angle and elastic modulus of the studied crushed limestone.

Finite element analyses that can simulate the behavior of flexible pavement with reinforced bases were also conducted in order to assess the benefits in terms of traffic bearing ratio (TBR)
provided by different variables, such as the thickness of the base course layer, the location and tensile modulus (or stiffness) of the reinforcement layer and the subgrade strength.

7.2 Conclusions

7.2.1 The Key Conclusions from Study of Reinforced Soil Foundation (RSF)

Based on the comprehensive FEM analyses of a strip footing sitting on a cohesive soil or crushed limestone reinforced with multiple-layers of commonly used geogrids in addition to steel wire mesh and steel bar mesh, the following conclusions can be drawn:

a. The ultimate bearing capacity of the reinforced soil footing increases with the increase in number of reinforcement layers up to a certain influence depth. The depth of influence, also called effective depth of reinforcement, was found to be about 1.5 times the footing width. No appreciable improvement was achieved by the inclusion of additional reinforcement layer below the depth of influence.

b. The optimum depth ratio of the first reinforcement layer (u/B) at which the bearing capacity ratio (BCR) was the highest is around 0.5~0.6 and 0.3~0.4, for a single-layer and multi-layer reinforced soil system, respectively.

c. Within the effective depth of reinforcement, the ultimate bearing capacity of the reinforced soil footing increases and the settlement decreases with the decrease in reinforcement spacing. However, the effect of reinforcement spacing becomes less significant as the spacing is reduced to below 12 inch (300 mm).

d. The ultimate bearing capacity of the reinforced soil increases and settlement decreases with the increase in the geogrid tensile modulus (or stiffness). However, the stiffness-related increase is more pronounced at geogrid tensile modulus in the 5,000 -25,000 kN/m, and gradually decreases above its upper boundary.
e. The increase in footing embedment depth and/or footing width improves the ultimate bearing capacity of the unreinforced soil more than that of the reinforced soil, resulting in a slight decrease in the BCR.

f. Regression models that predict the benefits of RSF in terms of BCR were successfully developed and can be readily used in design of RSF structures. In general, these models show that the geogrid improvement increases with the increase in the geogrid stiffness and decreasing in spacing ratio, footing embedment ratio and footing width ratio.

g. From the strain distributions of geogrids and the study of the effect of geogrid length, the length of the geogrid has to be at least four times of the footing width \((L=4B)\) to fully mobilize the benefits.

### 7.2.2 The Key Conclusions from Study of Reinforced Bases in Flexible Pavement System

Based on the results of the numerical modeling analysis of geogrid reinforced bases in flexible pavement system, the following conclusions can be drawn:

a. The geogrid reinforcement of base course layer results in reducing the lateral strains within the base course and subgrade layers.

b. The geogrid benefits in improving the developed total and plastic strains are more appreciable in sections with weak subgrades compared to those with moderate or stiff subgrades. In addition, these benefits are reduced as the thickness of the base course layer increases, and vice versa.

c. The increase in the geogrid tensile modulus (or stiffness) results in significant reduction of permanent deformation; however, the geogrid stiffness effect decreases
with the increase in the thickness of the reinforced base course layer and the strength of subgrade layer.

d. Analysis of finite element results based on the mechanistic empirical approach demonstrated that the geogrid reinforcement can extend the service life of pavements, with traffic benefit ratios (TBR), at 25 mm surface rut, of up to 3.4 were obtained for pavement sections over weak subgrade; the TBR values tends to increase with increasing the geogrid tensile modulus, with decreasing of base course layer thickness and with decreasing of subgrade strength.

e. Regression models that predict the benefits of reinforcing base course layers in terms of traffic benefit ratio (TBR) were successfully developed for use in design of reinforced flexible pavement structure. In general, these models indicate that the geogrid improvement increases with the increase in the geogrid stiffness and the decrease in base course layer thickness and subgrade strength.

7.3 Suggestions for Future Studies

Based on the findings of the present study, it is evident that further research in this field can yield practical and valuable result. Hence, future studies should focus on:

a. Using the finite element model for the reinforced soil foundation to simulate a full-scale geogrid reinforced approach slab embankment and compare the finite element results with field measurements from monitoring instrumental embankments.

b. Developing advanced material models in order to better simulate the behavior of soil and unbound granular material, as well as the interaction between clay soil/crushed limestone and geogrid.
c. Extend the finite element work on geogrid reinforced pavement to include base thickness more than 300 mm and reassess the geogrid location, possibly using double geogrid layer, etc.

d. Given that the work carried out in the dissertation was based on finite element analysis of geogrid reinforced soil foundation and geogrid reinforced pavements, there is a need to verify the findings of this study using full-scale geogrid reinforced soil/base structures, such as static loading of geogrid reinforced approach slab embankments, and accelerated load testing of geogrid base reinforced test lane pavement sections.
REFERENCES


VITA

Jie Gu was born on August 7th, 1978, in Qinhuangdao, Hebei Province, China. She was the only daughter of her parents, Tiecheng Gu and Xuefen Cai. She received her bachelor’s degree in civil engineering from Hebei University of Technology, Tianjin, China, in 2001 and her master’s degree in structural engineering from the same university in 2004. She married Chongyang Man in the summer of 2004. Then she came to the United States to pursue a doctoral degree in civil engineering at Louisiana State University in August 2004. She got her three sweet kids (Addison Bliss, Cameron Dylan, and Emma Ferris) during this period. The degree of Doctor of Philosophy in civil engineering will be conferred to her in December 2011.