10-8-2004

Physical significance of q deformation and many-body interactions in nuclei

K. D. Sviratcheva
Louisiana State University

C. Bahri
Louisiana State University

A. I. Georgieva
Louisiana State University

J. P. Draayer
Louisiana State University

Physical Significance of q-Deformation and Many-body Interactions in Nuclei

K.D. Sviratcheva,[1] C. Bahri,[1,2] A.I. Georgieva,[1,3] and J.P. Draayer[1]

[1]Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, USA
[2]Departemen Fisika, FMIPA, Universitas Indonesia, Depok 16424, Indonesia
[3]Institute of Nuclear Research and Nuclear Energy,
Bulgarian Academy of Sciences, Sofia 1784, Bulgaria

The quantum deformation concept is applied to a study of pairing correlations in nuclei with mass $40 \leq A \leq 100$. While the nondeformed limit of the theory provides a reasonable overall description of certain nuclear properties and fine structure effects, the results show that the q-deformation plays a significant role in understanding higher-order effects in the many-body interaction.

1. Introduction — First used in applications of quantum inverse scattering [1], quantum (or q-) deformation [2, 3] has been the focus of considerable attention in various fields of physics in recent years. In addition to purely mathematical examinations, recent studies of interest include applications in string/brane theory, conformal field theory, statistical/quantum mechanics, and metal clusters [4, 5, 6, 7, 8], as well as in nuclear physics [9, 10].

Mathematically, a deformation parameter (q) is used to realize a mapping of c-numbers (or operators) X into their q-equivalents: $[X]_p \doteq \frac{q^{pX}-q^{-pX}}{q^p-q^{-p}} \stackrel{q\to 1}{\to} X$ (denoted $[X]$ when $p=1$). $[X]_p$ is nonlinear in X. It is invariant under the transformation $q \to q^{-1}$, i.e. with respect to the sign of a real parameter $\varkappa$ (where $q = e^{\varkappa}$ for q real), and hence depends only on even powers of $\varkappa$. A feature of any quantum algebra is that in the $\varkappa \to 0$ ($q \to 1$) limit, one recovers the nondeformed results, just as classical mechanics (Galilean relativity) is restored from quantum mechanics (Einsteinian relativity) when $\hbar \to 0$ ($\frac{1}{c} \to 0$).

The earliest applications of the quantum algebraic concept to nuclear structure were related to an $\mathrm{SU}_q(2)$ description of rotational bands in axially deformed nuclei [11] and of like-particle pairing [12]. Even though optimum values of the q-parameter have achieved an overall improved fit to the experimental energies, the question on the physical nature of q-deformation when applied to the nuclear many-body problem remains open.

It is well-known that effective two-body interactions in nuclei are dominated by pairing and quadrupole terms. The former accounts for pair formation and gives rise to a pairing gap in nuclear spectra, and the latter is responsible for enhanced electric quadrupole transitions in collective rotational bands. Indeed, within the framework of the harmonic oscillator shell-model, both limits have a clear algebraic structure in the sense that the spectra exhibit a dynamical symmetry. In the pairing limit the "quasi-spin" symplectic Sp(4) ($\sim$ SO(5)) [13, 14, 15, 16] together with its dual Sp(2Ω), for 2Ω shell degeneracy, use the seniority quantum number [17, 18] to classify the spectra. On the other hand, in the quadrupole limit symplectic $\mathrm{Sp}(6,\mathbb{R})$ [19] governs a shape-determined dynamics.

Pairing, introduced in physics for describing superconductivity, is fundamental to condensed matter, nuclear, and astrophysical phenomena of recent interest. In nuclear physics, a two-body microscopic model with Sp(4) dynamical symmetry allows one to focus on like-particle (pp and nn) and proton-neutron pn isovector (isospin $T=1$) pairing correlations and, in addition, to include a pn isoscalar ($T=0$) interaction. Nuclear properties are generally well-described within this framework [20, 21], with labels of the Sp(4) scheme yielding a basic understanding of the overall systematics.

The new feature reported on in this article is an extension of the theory to include nonlinear local deviations from the pairing solution as realized through q-deformation of the $\mathfrak{sp}(4)$ algebra [27]. An important property of the q-deformed model is that it does not violate physical laws fundamental to a quantum mechanical nuclear system and conserves the angular momentum, the total number of particles, and the isospin projection.

The quantum extension of $\mathfrak{sp}(4)$ makes possible the analytical modeling of a set of many-body interactions. In general, the latter are rather complicated to handle, nevertheless, they introduce an overall improvement of the theory [22]. We aim to show that the $q \neq 1$ results are uniformly superior to those of the nondeformed limit and that the q-parameter varies smoothly with nuclear characteristics. The results of this study suggest that q-deformation has physical significance over-and-above the simple pairing gap concept, extending to the very nature of the nuclear interaction itself. The role of q-deformation is not model limited, it can extend to include a description of various many-body effects.

2. Nonlinear pairing model — The $\mathfrak{sp}_q(4)$ deformed algebra [23, 24, 25] is realized in terms of q-deformed fermion operators, $\alpha^\dagger_{\nu=\{jm\sigma\}}$ and α_ν, each of which creates and annihilates a nucleon with isospin σ ($\pm\frac{1}{2}$ for proton/neutron) in a single-particle state of total angular momentum j (half-integer) with third projection m. The q-operators are defined through their anticommutation relations, $\{\alpha_{jm\sigma}, \alpha^\dagger_{kn\tau}\}_{q^{\pm\delta_{\sigma\tau}}} = q^{\pm\frac{N_{2\sigma}}{2\Omega}}\delta_{jk}\delta_{mn}$ $\delta_{\sigma\tau}, \{\alpha^{(\dagger)}_\nu, \alpha^{(\dagger)}_{\nu'}\} = 0$, where the q-anticommutator is $\{A,B\}_{q^p} = AB + q^p BA$, $N_{2\sigma=\pm 1}$ is the proton (neutron) number operator and $2\Omega = \sum_j (2j+1)$ is the space dimension for given σ [25]. The nondeformed $c_\nu^{(\dagger)}$ operators ($\alpha_\nu^{(\dagger)} \overset{q\to 1}{\to} c_\nu^{(\dagger)}$) obey the usual anticommutation relations. The basis operators, $T_\pm$ and $A^{(\dagger)}_{1,0,-1}$, of the $\mathfrak{sp}_q(4)$ algebra are constructed as eight bilinear products of the fermion q-operators coupled to total angular momentum and parity $J^\pi = 0^+$, in addition to the nucleon number operators $N_{\pm 1}$, which remain undeformed [25]. The isospin projection operator $T_0 = \frac{1}{2}(N_{+1} - N_{-1})$ and the total nucleon number operator $N = N_{+1} + N_{-1}$ are also undeformed. In the $q \to 1$ limit, $T_{0,\pm}$ are associated with isospin and $A^{(\dagger)}_{1,0,-1}$ create (annihilate) a proton-proton, proton-neutron, or neutron-neutron $J = 0$ pair.

As for the microscopic nondeformed approach, the most general Hamiltonian [20] of a system with q-deformed symplectic dynamical symmetry ($\mathfrak{sp}_q(4) \supset \mathfrak{su}_q(2)$) and conserved proton and neutron particle numbers can be expressed as

$$H_q = -\varepsilon_q N - G_q \sum_{k=-1}^{1} A_k^\dagger A_k - F_q A_0^\dagger A_0 - \frac{E_q}{2\Omega}\left(\mathbf{T}^2 - \Omega\left[\frac{N}{2\Omega}\right]\right) - D_q \Omega \left[\frac{1}{\Omega}\right][T_0]^2_{\frac{1}{2\Omega}} - C_q 2\Omega\left[\frac{1}{\Omega}\right]\left[\frac{N}{2}\right]_{\frac{1}{2\Omega}}\left[\frac{N}{2} - 2\Omega\right]_{\frac{1}{2\Omega}}, \quad (1)$$

where $\mathbf{T}^2 = \Omega(\{T_+, T_-\} + \left[\frac{1}{\Omega}\right][T_0]^2_{\frac{1}{2\Omega}})$ and the definitions of $[X]_p$ and $[X]$ are used. In principle, the deformation parameters $\gamma_q = \{\varepsilon_q, G_q, F_q, E_q, D_q, C_q\}$ can differ from their nondeformed counterparts $\gamma = \{\varepsilon, G, F, E, D, C\}$, which we assume to be constant for all nuclei within a major shell. The model describes the behavior of N_{+1} valence protons and N_{-1} valence neutrons in the mean-field of a doubly-magic nuclear core. The basis states, specified by the numbers of pn and like-particle pairs, are constructed by the action of $A^\dagger_{0,\pm 1}$ on the vacuum.

The nondeformed Hamiltonian H, $H_q \overset{q\to 1}{\to} H$, is an effective two-body interaction that includes isovector pairing (parameter G) and a so-called symmetry term (E), which together with the N^2-term arise naturally from a general two-body rotational and isospin invariant microscopic interaction. Both C- and E-terms account for an isoscalar pn interaction that is diagonal in an isospin basis [28]. These interactions govern the lowest 0^+ isobaric analog states of light and medium mass even-A nuclei ($40 \le A \le 100$) with protons and neutrons occupying the same major shell, where the seniority zero limit is approximately valid [20, 21, 26]. For these states, the nondeformed model has already proven to provide a reasonable overall description for a total of 136 nuclei [20]. This includes a remarkable reproduction of the energy of the states and their detailed structure reflecting observed $N_{+1} = N_{-1}$ irregularities and staggering patterns [21]. As a consequence, any deviation within a nucleus from the reference global behavior can be attributed to local effects which although typically small can be important for determining the detailed structure of individual nuclei and hence need to be taken into account [22].

As a group theoretical approach, the quantum extension of H includes many-body interactions in a very prescribed way, retaining the simplicity of the exact solution. Moreover, the quantum model not only has the $\mathfrak{sp}_q(4) \supset \mathfrak{su}_q(2)$ dynamical symmetry, it contains the original dynamical Sp(4) symmetry.

3. Novel properties of q-deformation — From an undeformed perspective, the deformation introduces higher-order, many-body terms into a theory that starts with only one-body and two-body interactions. The way in which the higher-order effects enter into the theory is governed by the $[X]$ form. In terms of $\varkappa$, everything is tied to the deformation with $[X] = \frac{\sinh(\varkappa X)}{\sinh(\varkappa)} = X(1 + \varkappa^2 \frac{X^2-1}{6} + \varkappa^4 \frac{3X^4 - 10X^2 + 7}{360} + \ldots) \overset{\varkappa\to 0}{\to} X$. An illustrative example is the expansion in $\varkappa$ of the last term in H_q (1), $-C_q 2\Omega\left[\frac{1}{\Omega}\right]\left[\frac{N}{2}\right]_{\frac{1}{2\Omega}}\left[\frac{N}{2} - 2\Omega\right]_{\frac{1}{2\Omega}} = -2C_q \frac{N}{2}(\frac{N}{2} - 2\Omega) - \frac{C_q \varkappa^2 \{(16\Omega^2 - 24\Omega + 5)(V^{(1)} + V^{(2)}) + 6V^{(2)} + (6 - 8\Omega)V^{(3)} + V^{(4)}\}}{96\Omega^2} - \ldots$, with $V^{(1)} = \sum_{\nu_1} c^\dagger_{\nu_1} c_{\nu_1}$, $V^{(2)} = \sum_{\nu_1\nu_2} c^\dagger_{\nu_1} c^\dagger_{\nu_2} c_{\nu_2} c_{\nu_1}$, $V^{(3)} = \sum_{\nu_1\nu_2\nu_3} c^\dagger_{\nu_1} c^\dagger_{\nu_2} c^\dagger_{\nu_3} c_{\nu_3} c_{\nu_2} c_{\nu_1}$, and $V^{(4)} = \sum_{\nu_1\nu_2\nu_3\nu_4} c^\dagger_{\nu_1} c^\dagger_{\nu_2} c^\dagger_{\nu_3} c^\dagger_{\nu_4} c_{\nu_4} c_{\nu_3} c_{\nu_2} c_{\nu_1}$. The zeroth-order approximation corresponds to the nondeformed two-body force and coincides with it for a strength C_q equal to C, and the higher-order terms introduce many-body interactions. The latter may not be negligible, for example, our results show that the energy contribution of the four-body interaction in the expansion above can reach a magnitude of several MeV in nuclei in the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ shell.

Similarly, the zeroth-order term of H_q (1) coincides with the H nondeformed interaction only if the strength parameters are equal, $\gamma_q = \gamma$. This term must remain unchanged when deformation is introduced, since H has been shown to reproduce reasonably well the overall behavior common for all the nuclei in a shell. This is why we fix the values of the parameters $\gamma_q = \gamma$ and allow only $\varkappa$ to vary. The decoupling of the deformation from the γ parameters that are used to characterize the two-body interaction itself, means that the latter can be assigned best-fit global values for the model space under consideration without compromising overall quality of the theory. This in turn underscores the fact that the deformation represents something fundamentally different, a feature that cannot be "mocked up" by allowing the strengths of the nondeformed interaction to absorb its effect. In short, the q-deformation adds to the theory, which describes quite well the overall nuclear behavior, a mean-field correction along with two-, three-, and many-body interactions of a local character that can be responsible for residual single-particle and many-body effects.

4. Analysis of the role of the q-deformation — Since the q-parameter is associated with local phenomena, it is expected to vary from one nucleus to another. The possible presence of local effects built over the global properties of the 0^+ states under consideration can be recognized within an individual nucleus by the deviation of the predicted nondeformed energy $\langle H\rangle$ from the experimental value $E_{\rm exp}$, namely, the solution of the equation $\langle H_q\rangle = E_{\rm exp}$ provides a rough estimate for $\varkappa$ (see Fig. 1). However, in nuclei where $\langle H\rangle \geq E_{\rm exp}$ there is no solution (see Fig. 1) and the theoretical prediction closest to the experiment occurs at the nondeformed point, $\varkappa = 0$.

The analysis yields values for the deformation parameter $|\varkappa|$ for each nucleus that fall on a smooth curve (see Fig. 2(a)). The observed smooth behavior of $\varkappa$ reveals its functional dependence on the model quantum numbers. This result, even though qualitative, underscores the fact that the q-deformation as prescribed by the $\mathfrak{sp}_q(4)$ model is not random in character but rather fundamentally related to the very nature of the nuclear interaction.

This, in turn, allows us to assign a parametrized functional dependence of the deformation parameter on the total particle number N and the isospin projection T_0,

$$\varkappa(N,T_0) = \xi_1(\tfrac{N}{2\Omega}-1)(\tfrac{N}{2\Omega}+\xi_2-2\theta(N-2\Omega))e^{-0.5(\frac{2T_0}{\xi_3})^2} + \xi_4\theta(N-2\Omega)|T_0|\sqrt{\tfrac{N}{2\Omega}-1}, \quad \theta(x) = \begin{cases} 1, & x\geq 0 \\ 0, & x<0 \end{cases}, \qquad (2)$$

which reflects the complicated development of nonlinear effects observed in Figure 2(a). As a next step, we use the $\varkappa(N,T_0)$ deformation function (2) to fit the minimum eigenvalues of H_q (1) to the relevant experimental energies of the even-even nuclei in the $1f_{7/2}$ and $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ shells. In doing this, we minimize any renormalization of the q-deformed parameter due to a possible influence of other local effects that are not present in the model. In the fitting procedure, only the four parameters $(\xi_{1},_2,_3,_4)$ of $\varkappa(N,T_0)$ in Eq.(2) are varied. Determined statistically, they provide an estimate for the overall significance of q-deformation within a shell.

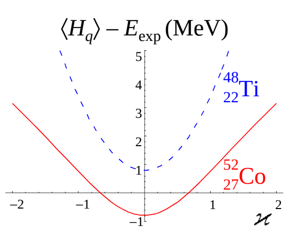

FIG. 1: Differences between theoretical and experimental energies vs. the $\varkappa$ parameter for a typical near-closed shell nucleus (solid line) and for a mid-shell nucleus (dashed line).

The $q\neq 1$ results are uniformly superior to those of the nondeformed limit. In the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ shell, for example, the q-deformed model, $SOS_q = 130.21$ MeV^2 ($\chi_q = 1.28$ MeV) [29], clearly improves the nondeformed theory, $SOS = 271.63$ MeV^2 ($\chi = 1.79$ MeV). The optimum results are achieved for:

$$\xi_1 = -2.13, \quad \xi_2 = 0.37, \quad \xi_3 = 3.07, \quad \xi_4 = 0.15. \qquad (3)$$

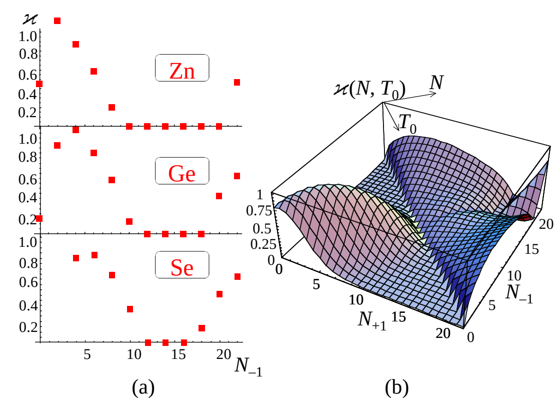

FIG. 2: $\varkappa$-Parameter estimation: (a) within each nucleus, and (b) $\varkappa(N,T_0)$ within the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ shell (global parameters: $\varepsilon = 13.851$, $G/\Omega = 0.296$, $F/\Omega = 0.056$, $E/(2\Omega) = -0.489$, $D = -0.307$, and $C = 0.190$ in MeV).

The behavior of the q deformation (as prescribed by Eq. (2)) is consistent in both of the regions considered (shells $1f_{7/2}$ and $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$), with a general trend of higher values above mid-shell, where the increase in particle number can lead to stronger nonlinear effects. As a whole, the model with the local q improves the energy prediction compared to the nondeformed global model and reproduces more closely the experiment (see Fig. 3). One reason may be that the q-deformed fermions, unlike usual quasiparticles, indeed obey the fundamental laws.

The many-body nature of the interaction is most important away from mid-shell and for many even-even nuclei tends to peak [with significant values of q] when $N_{+1} = N_{-1}$ where strong pairing correlations are expected (see Fig. 2). Values of the deformation parameter $q \approx 1$ may be found in nuclei with only one or two particle/hole pairs from a closed shell. For these nuclei the number of particles is insufficient to sample the effect of higher-order terms in a deformed interaction and the nondeformed limit gives a good description.

Around mid-shell ($N \approx 2\Omega$) the deformation adds little improvement to the $\varkappa = 0$ theory. This suggests that for these nuclei the many-body interactions as prescribed by $\varkappa(N, T_0)$ in Eq. (2) are negligible and the model is not sufficient to describe other types of local effects that may be present. The results imply that even though the q-parameter gives additional freedom for all the nuclei, it only improves the model around regions of dominant pairing correlations. In short, the pair formation favors the nonnegligible higher-order interactions between the pair constituents that are detected via the $\mathfrak{sp}_q(4)$ model.

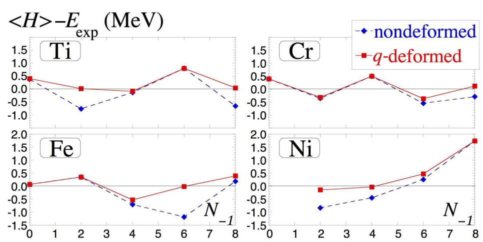

FIG. 3: The q-deformed and nondeformed energies compared to experimental values for even-even isotopes in the $1f_{7/2}$ shell (global parameters: $\varepsilon = 13.149$, $G/\Omega = 0.453$, $F/\Omega = 0.072$, $E/(2\Omega) = -1.120$, $D = 0.149$, and $C = 0.473$ in MeV).

A q-deformed nonlinear extension of the Sp(4) model, which is the underlying symmetry for describing isovector pairing correlations and pn isoscalar interactions in atomic nuclei, has been investigated. When compared to experimental data, the theory shows a smooth functional dependence of the deformation parameter q on the proton and neutron numbers. In addition, the $q \neq 1$ results are uniformly superior to those of the nondeformed limit. The outcome suggests that the deformation has physical significance related to the very nature of the nuclear interaction itself and beyond what can be achieved by simply tweaking the parameters of a two-body interaction. The specific features of the nuclear structure can be investigated through the use of a local q that detects the presence and importance of many-body interactions accompanying dominant pairing correlations in nuclei. This is in addition to the good description of the global properties of the nuclear dynamics provided by the nondeformed two-body interaction. Although the physical significance of q-deformation is presently approached within the Sp(4) theoretical framework, it is clearly model independent and can reveal various many-body phenomena. The results also underscore the need for additional studies to achieve a more comprehensive understanding of q-deformation in nuclear physics.

In summary, the concept of quantum deformation has been linked to the smooth behavior of physical phenomena in atomic nuclei.

This work was supported by the US National Science Foundation, Grant Number 0140300.

[1] P.P. Kulish and N.Yu. Reshetikhin, J. Soviet Math. **23**, 2435 (1983).
[2] M. Jimbo, Lett. Math. Phys. **10**, 63 (1985); **11**, 247 (1986).
[3] V. Drinfeld, Proc. Int. Congress on Math, Vol. 1 (UC-Press, Berkeley: 1986).
[4] A. Ballesteros, N.R. Bruno and F.J. Herranz, Phys. Lett. **B574**, 276 (2003).
[5] F.A. Bais, B.J. Schroers, and J.K. Slingerland, Phys. Rev. Lett. **89**, 181601 (2002).
[6] A. Algin, M. Arik, and A.S. Arikan, Phys. Rev. E **65**, 026140 (2002).
[7] Jian-zu Zhang, Phys. Lett. **B477**, 361 (2000).
[8] D. Bonatsos et al., Phys. Rev. A **62**, 013203 (2000).
[9] D. Bonatsos, B.A. Kotsos, P.P. Raychev, and P.A. Terziev, Phys. Rev. C **66**, 054306 (2002).
[10] A. Ballesteros, O. Civitarese, F.J. Herranz, and M. Reboiro, Phys. Rev. C **66**, 064317 (2002); A. Ballesteros, O. Civitarese, and M. Reboiro, Phys. Rev. C **68**, 044307 (2003)
[11] P.P. Raychev, R.P. Roussev, and Yu.F. Smirnov, J. Phys. G: Nucl. Part. Phys. **16**, L137 (1990).
[12] S. Shelly Sharma, Phys. Rev. C **46**, 904 (1992); D. Bonatsos, J. Phys. A: Math. Gen. **25**, L101 (1992).
[13] A.K. Kerman, Ann. Phys. (NY) **12**, 300 (1961).
[14] K. Helmers, Nucl. Phys. **23**, 594 (1961).
[15] K.T. Hecht, Nucl. Phys. **63**, 177 (1965); Phys. Rev. **139**, B794 (1965); Nucl. Phys. A **102**, 11 (1967).
[16] J.N. Ginocchio, Nucl. Phys. **74**, 321 (1965).
[17] G. Racah, Phys. Rev **63**, 367 (1943).
[18] B.H. Flowers, Proc. Roy. Soc.(London)**A212**, 248 (1952).
[19] G. Rosensteel and D.J. Rowe, Phys. Rev. Lett. **38**, 10 (1977); Ann. Phys. (NY) **126**, 343 (1980).
[20] K.D. Sviratcheva, A.I. Georgieva, and J.P. Draayer, J. Phys. G **29**, 1281 (2003).
[21] K.D. Sviratcheva, A.I. Georgieva, and J.P. Draayer, Phys. Rev. C **69**, 024313 (2004).
[22] P. Navrátil and B.R. Barrett, Phys. Rev. C **59**, 1906 (1999); P. Navrátil and W.E. Ormand, Phys. Rev. C **68**, 034305 (2003); Feng Pan, V.G. Gueorguiev, and J.P. Draayer, Phys. Rev. Lett. **92**, 112503 (2004).
[23] T. Hayashi, Commun. Math. Phys. **127**, 129 (1990).
[24] B. Abdesselam, D. Arnaudon, and A. Chakrabarti, J. Phys. A: Math. Gen. **28**, 3701 (1995).
[25] K.D. Sviratcheva et al., J. Phys. A **34**, 8365 (2001).
[26] J. Engel, K. Langanke, and P. Vogel, Phys. Lett. **B389**, 211 (1996)
[27] We use lowercase (capital) letters for algebras (groups).

[28] In addition, the F-term accounts for a possible, even if extremely small, isospin mixing.

[29] SOS is defined as the sum of the squared differences in the theoretical and experimental energies, and χ^2 is the averaged SOS per a degree of freedom in the statistics.