

1-1-2005

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Recommended Citation

Sviratcheva, K., Georgieva, A., & Draayer, J. (2005). Isospin symmetry breaking in an algebraic pairing $Sp(4)$ model. *Physical Review C - Nuclear Physics*, 72 (5) <https://doi.org/10.1103/PhysRevC.72.054302>

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Isospin Symmetry Breaking in an Algebraic Pairing $\text{Sp}(4)$ Model

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(Dated: July 12, 2018)

An exactly solvable $\mathfrak{sp}(4)$ algebraic approach extends beyond the traditional isospin conserving nuclear interaction to bring forward effects of isospin symmetry breaking and isospin mixing resulting from a two-body nuclear interaction that includes proton-neutron (pn) and like-particle isovector pairing correlations plus significant isoscalar pn interactions. The model yields an estimate for the extent to which isobaric analog 0^+ states in light and medium mass nuclei may mix with one another and reveals possible, but still extremely weak, non-analog β -decay transitions.

I. INTRODUCTION

A fundamental feature of nuclear structure is the basic symmetry between neutrons and protons, namely, the *charge independence* of the nuclear force, that is evident in the striking similarity in the energy spectra of nuclear isobars [1]. This implies that the proton-proton (pp) interaction and the neutron-neutron (nn) interaction are equal to the isospin $T = 1$ pn interaction and leads to ‘rotational’ invariance in isotopic space. However, the isospin invariance is violated by the electromagnetic interaction, mainly the Coulomb repulsion between nucleons, which has become the focus of many phenomenological and microscopic studies [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20].

The primary effect of the Coulomb force is to introduce into the theory a dependence on the third isospin projection, T_0 , resulting in energy splitting of isobaric analog states (a T -multiplet) without coupling different isospin multiplets. At the same time, the isospin-violating part of the Coulomb interaction leads to small isospin mixing in nuclear ground states increasing with Z and largest for $N = Z$. The ground state isospin impurity is theoretically estimated to be as small as a percent for nuclei in the $1f_{7/2}$ level [1], and up to 4 – 5% toward the fp shell closure [9]. Another source of mixing probability is the isospin non-conserving part of the nuclear Hamiltonian, which includes effects due to the proton-neutron mass difference and small charge dependent components in the strong nucleonic interaction [4]. Experimental results clearly reveal the existence of isospin mixing [21, 22]. The increase in isospin mixing towards medium mass nuclei has been detected in novel high-precision experiments [23, 24, 25, 26, 27], which continue to push the exploration of unstable nuclei with the advent of advanced radioactive beam facilities.

The violation of the charge independence of the nuclear interaction is well established. The purely nuclear parts of the pp force and the $T = 1$ pn force differ from each other, which appears to be associated with the electromagnetic structure of the nucleons [4]. An analysis of the 1S scattering in the pn system and the low-energy pp scattering lead to the estimate that the nuclear inter-

action between protons and neutrons ($V_{pn}^{T=1}$) in $T = 1$ states are more attractive than the force between the protons (V_{pp}) by 2%, $|V_{pn}^{T=1} - V_{pp}|/V_{pp} \sim 2\%$ [28]. In addition, the charge asymmetry between the pp and nn interactions was found to be smaller; namely, less than 1% [29]. More recent investigations confirm isospin violation in light nuclei [13, 30, 31, 32, 33] starting with modern charge dependent realistic interactions [34, 35, 36, 37] and including valuable input information from particle physics (see for example, [38, 39]). Furthermore, after the Coulomb energy is taken into account the discrepancy in the isobaric-multiplet energies is bigger for the seniority zero levels as compared to higher-seniority states indicating the presence of a short range charge dependent interaction [7]. Indeed, the $J = 0$ pairing correlations have been recently shown to have an overwhelming dominance in the isotensor energy difference within isobaric multiplets [40], which manifests itself in the charge dependent $T = 2$ nature of the pairing interaction.

The findings mentioned above point out the need for a charge dependent microscopic description of $J = 0$ pairing correlations. An algebraic $\mathfrak{sp}(4)$ approach is ideally suited for this purpose [41, 42] for it combines, on the one hand a microscopic modeling of the pairing interaction and its charge dependence, and on the other hand, a straightforward scheme for estimating the significance of the isospin mixing due to pairing correlations without the need for carrying out large-dimensional matrix diagonalizations. Strong isospin breaking in pair formation, if found, implies a significant presence of isospin admixture among the seniority-zero isobaric analog 0^+ states including 0^+ ground states. This in turn will affect the predictive power of precise studies of superallowed $0^+ \rightarrow 0^+$ Fermi β -decay transitions. This is because the latter provide reliable tests of isospin mixing (see [43] for a review), and as well furnish a precise test of the unitarity condition of the Cabibbo-Kobayashi-Maskawa matrix [44] (for a review of this subject, see [45]).

Our objective is to explore isospin mixing beyond that due to the Coulomb interaction, which is isolated with the help of an advanced Coulomb correction formula [46]. Specifically, we focus on the isospin non-conserving part of the *pure* nuclear interaction, which recently has been

found to be at least as important as the Coulomb repulsion [40]. The outcome of this study shows the significance of the pairing charge dependence and its role in mixing isospin multiplets of pairing-governed isobaric analog 0^+ states.

II. THEORETICAL FRAMEWORK: THE REASONABLE APPROXIMATION

We employ a simple but powerful group-theoretical model, which is based on the $\mathfrak{sp}(4)$ algebra (isomorphic to $\mathfrak{so}(5)$ [47, 48, 49]). The $\text{Sp}(4)$ microscopic model is precisely suitable for the qualitative study of isospin violation in isobaric analog 0^+ states because it naturally extends the isospin invariant nuclear interaction to incorporate isospin non-conserving forces, while it retains the $\text{Sp}(4)$ dynamical symmetry of the Hamiltonian (see [50] for a review on dynamical symmetries).

A comparison with experimental data demonstrates that the $\text{Sp}(4)$ model provides a reasonable description of the pairing-governed isobaric analog 0^+ states [61] in light and medium mass nuclei, where protons and neutrons occupy the same shell [41, 42, 51]. The two-body model interaction includes proton-neutron and like-particle pairing plus symmetry terms and contains a non-negligible implicit portion of the quadrupole-quadrupole interaction [52]. Moreover, the $\text{Sp}(4)$ model interaction itself, which relates to the whole energy spectrum rather than to a single $J^\pi = 0^+ T = 1$ state, was found to be quite strongly correlated (0.85) with the realistic CD-Bonn+3terms interaction [53] in the $T = 1$ channel and with an overall correlation of 0.76 with the realistic GXPF1 interaction [54] for the $1f_{7/2}$ orbit [52]. In short, the relatively simple $\text{Sp}(4)$ model seems to be a reasonable approximation that reproduces especially that part of the interaction that is responsible for shaping pairing-governed isobaric analog 0^+ states.

The $\text{Sp}(4)$ model reflects the symplectic dynamical symmetry of isobaric analog 0^+ states [42] determined by the strong nuclear interaction. The weaker Coulomb interaction breaks this symmetry and significantly complicates the nuclear pairing problem. This is why, in our investigation we adopt a sophisticated phenomenological Coulomb correction to the experimental energies such that a nuclear system can be regarded as if there is no Coulomb interaction between its constituents. The *Coulomb corrected* experimental energy, E_{exp} , for given valence protons N_{+1} and neutrons N_{-1} is adjusted to be

$$E_{\text{exp}}(N_{+1}, N_{-1}) = E_{\text{exp}}^C(N_{+1}, N_{-1}) - E_{\text{exp}}^C(0, 0) + V_{\text{Coul}}(N_{+1}, N_{-1}), \quad (1)$$

where [62] E_{exp}^C is the total measured energy including the Coulomb energy, $E_{\text{exp}}^C(0, 0)$ is the binding energy of the core, and $V_{\text{Coul}}(N_{+1}, N_{-1})$ is the Coulomb correction for a nucleus with mass A and Z protons taken relative to the core $V_{\text{Coul}}(N_{+1}, N_{-1}) = V_{\text{Coul}}(A, Z) -$

$V_{\text{Coul}}(A_{\text{core}}, Z_{\text{core}})$. The recursion formula for the $V_{\text{Coul}}(A, Z)$ Coulomb energy is derived in [46] with the use of the Pape and Antony formula [55]. The Coulomb corrected energies (1) should reflect solely the nuclear properties of the many-nucleon systems.

Assuming charge independence of the nuclear force, the general isoscalar Hamiltonian with $\text{Sp}(4)$ dynamical symmetry, which consists of one- and two-body terms and conserves the number of particles, can be expressed through the $\text{Sp}(4)$ group generators,

$$H_0 = -G \sum_{i=-1}^1 \hat{A}_i^\dagger \hat{A}_i - \frac{E}{2\Omega} (\hat{T}^2 - \frac{3\hat{N}}{4}) - C \frac{\hat{N}(\hat{N}-1)}{2} - \epsilon \hat{N}, \quad (2)$$

where $\hat{T}^2 = \Omega \{ \hat{T}_+, \hat{T}_- \} + \hat{T}_0^2$ and 2Ω is the shell dimension for a given nucleon type. The generators \hat{T}_\pm and \hat{T}_0 are the valence isospin operators, $\hat{A}_{0,+1,-1}^{(\dagger)}$ create (annihilate) respectively a proton-neutron (pn) pair, a proton-proton (pp) pair or a neutron-neutron (nn) pair of total angular momentum $J^\pi = 0^+$ and isospin $T = 1$, and $\hat{N} = \hat{N}_{+1} + \hat{N}_{-1}$ is the total number of valence particles with an eigenvalue n . The G, E and C are interaction strength parameters and $\epsilon > 0$ is the Fermi level energy (see Table I in [42] for estimates). The isospin conserving Hamiltonian (2) includes an isovector ($T = 1$) pairing interaction ($G \geq 0$ for attraction) and a diagonal isoscalar ($T = 0$) force, which is related to a symmetry term (E).

Charge dependent but charge symmetric nucleon-nucleon interaction ($V_{pp} = V_{nn} \neq V_{pn}$) brings into the nuclear Hamiltonian a small isotensor component (with zero third isospin projection so that the Hamiltonian commutes with T_0). This is achieved in the framework of the $\text{Sp}(4)$ model by introducing the two additional terms,

$$H_{\text{IM}} = -F \hat{A}_0^\dagger \hat{A}_0, \quad H_{\text{split}} = -D (\hat{T}_0^2 - \frac{\hat{N}}{4}), \quad (3)$$

to the isospin invariant model Hamiltonian (2) in a way that the Hamiltonian

$$H = H_0 + H_{\text{IM}} + H_{\text{split}} \quad (4)$$

possesses $\text{Sp}(4)$ dynamical symmetry. In other words, charge dependence is introduced into the model pairing Hamiltonian (2) by allowing the strength of two of the underlying interactions to vary. The interaction strength parameters F and D (3) determined in an optimum fit over a significant number of nuclei (total of 149) [41] are given in Table I and yield non-zero values. These parameters yield quantitative results that are superior than the ones with $F = 0$ and $D = 0$; for example, in the case of the $1f_{7/2}$ level the variance between the model and experimental energies of the lowest isobaric analog 0^+ states increases by 85% when the D and F interactions are turned off. For the present investigation the parameters in (2) along with F and D (3) are not varied as their values were fixed to be physically valid and to yield reasonable energy [41, 42] and fine structure [51] reproduction for light and

medium mass nuclei with valence protons and neutrons occupying the same shell. For these nuclei in the mass range $32 \leq A \leq 100$, the pairing-governed isobaric analog 0^+ states are well described, but still approximately, by the eigenvectors of the effective Hamiltonian (4) in a basis of fully-paired 0^+ states [42].

TABLE I: Interaction strength parameters related to the isospin problem for three regions of nuclei specified by the valence model space. F , D , and E are in MeV.

Strength parameters	Model space	$(1d_{\frac{3}{2}})$	$(1f_{\frac{7}{2}})$	$(1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2})$
F/Ω		0.007	0.072	0.056
D		0.127	0.149	-0.307
$ D/\frac{E}{2\Omega} $		0.090	0.133	0.628

While the second interaction (H_{split}) in (3) takes into account only the splitting of the isobaric analog energies, the first correction induces small isospin mixing (IM). The isospin mixing interaction (3) does not account for the entire interaction that mixes states of same angular momentum and parity but different isospin values. It only describes a possible $\Delta T = 2$ mixing between isobaric analog 0^+ states due to a pure nuclear pairing interaction. While the extent of such isospin admixing is expected to be smaller than the total mixing due to isospin non-conserving terms [6, 10, 11, 13, 43], it may influence precise model calculations depending on the importance of the charge dependence in pairing correlations.

III. ISOSPIN INVARIANCE BREAKING AND ISOSPIN MIXING

The estimate for the model parameters (Table I) can determine the extent to which the isospin symmetry is broken while T remains a good quantum number. Breaking of the isospin invariance $|D/\frac{E}{2\Omega}|$ (Table I) is in general negligible for light nuclei ($1d_{\frac{3}{2}}$ and $1f_{\frac{7}{2}}$ levels) in agreement with the experimental data. For medium mass nuclei in the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ major shell the isospin breaking is significantly greater. Furthermore, as expected from observations, for the $1d_{\frac{3}{2}}$ level the interaction strengths of all pn , pp and nn pairing are almost equal (T is a good quantum number), $F \approx 0$ (Table I), and they differ for the $1f_{\frac{7}{2}}$ and for the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ shells, with the pn isovector strength being more attractive, $F > 0$. Indeed, the F isospin mixing interaction strength is extremely small and hence a charge independent nuclear interaction (where F is neglected) comprises a quite reasonable approximation. The latter yields major simplifications to the pairing problem and consequently most of isovector

pairing studies have been done assuming good isospin.

The question regarding how strong individual isospin non-conserving nuclear interactions are [such as (3)] still remains open – there are no sharp answers at the present level of experimental results and microscopic theoretical interpretations. It is only their overall contribution that is revealed by the free nucleon-nucleon data [28] to be slightly (by 2%) more attractive in the pn $T = 1$ system than the pp one. Within the framework of the $\text{Sp}(4)$ model, the charge dependence of the pure nuclear interaction can be estimated through the comparison of the $T_0 = 0$ two-body model interaction [(4) with $\varepsilon = 0$] relative to the $T_0 = 1$ in the $T = 1$ multiplets, which, for example in the $1f_{\frac{7}{2}}$ level, is on average $\sim 2.5\%$. This estimation does not aim to confirm the charge-dependence, which is very difficult at this level of accuracy compared to the broad energy range considered in the model for nuclei with masses $32 \leq A \leq 100$. Nonetheless, it reflects the fingerprints of the experimental data in the properties of the model interaction (4).

In addition, the $\text{Sp}(4)$ model reproduces reasonably well the c -coefficient in the well-known isobaric multiplet mass equation [2, 8, 56]

$$a + bT_0 + cT_0^2, \quad (5)$$

for the binding energies of isobaric analogs (of the same mass number A , isospin T , angular momentum J , etc.), where the coefficient c (b) depends on the isotensor (isovector) component of the nuclear interaction [i.e., of rank 2 (1) with respect to isospin ‘rotations’]. The c -coefficient is indeed an energy filter, $2c = E(T_0 + 1) + E(T_0 - 1) - 2E(T_0)$, for a given mass number A and isospin T . In the framework of our model, this energy function for the lowest isobaric analog 0^+ states was found to be in a good agreement with observed fine-structure effects (where data was available) [51] and it reproduces the experimental staggering behavior with respect to A (Figure 1). Both theoretical and experimental results show that this finite energy difference when centered at an $N = Z$ odd-odd nucleus ($T_0 = 0$ and $A/2$ odd), and hence c , is negative and very close to zero for $T = 1$ multiplets in the $1f_{\frac{7}{2}}$ shell (see Figure 1 for $A/2$ odd). Such an agreement of the $\text{Sp}(4)$ model outcome with experimental evidence is a valuable result. The requirement that the coefficients of (5) are well reproduced is essential for the isospin non-conserving models [6, 10, 40], which has been achieved in [6] by increasing (approximately by 2%) of all the $T = 1$ pn matrix elements relative to the nn ones and which has led to a conclusion in [40] that the isotensor nature of the nuclear interaction is dominated by a $J = 0$ pairing term.

In short, the freedom allowed by introducing additional parameters (as F and D) reflects the symmetries observed in light nuclei (good isospin) and the comparatively larger symmetry-breaking as expected in medium-mass nuclei. Hence, the charge dependence of the nuclear force, being a very challenging problem, yields results,

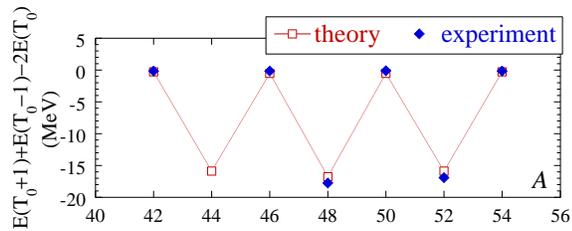


FIG. 1: Energy difference, $E(T_0 + 1) + E(T_0 - 1) - 2E(T_0)$ with $T_0 = 0$, for the lowest isobaric analog 0^+ states in nuclei in the $1f_{7/2}$ level (or twice the c -coefficient of (5) for the $A/2$ -odd $T = 1$ multiplets) according to the $Sp(4)$ model (red solid line with open squares) in comparison to the experiment (blue diamonds).

based on a simple group-theoretical approach, that are qualitatively consistent with the observations.

A. Near Isospin Symmetry of the Isobaric Analog 0^+ States

Both empirical evidence (such as scattering analysis and finite energy differences) and the comparison of the model to experimental data (Table I) does not yield equal pairing strengths ($F \gtrsim 0$) resulting in a coupling of isospin eigenstates $|n, \tilde{T}, T_0\rangle$ from different isospin multiplets with a degree of mixing expected to be very small. Therefore, the eigenvectors of the total Hamiltonian (4), $|n(\tilde{T})T_0\rangle$, have an almost good isospin \tilde{T} quantum number. Their overlap with the states of definite isospin values yields an estimate for the magnitude of the isospin admixture (see Table II for the $1d_{3/2}$ and $1f_{7/2}$ orbits):

$$\delta_{\tilde{T}, T} = \left| \langle n, T, T_0 | n(\tilde{T})T_0 \rangle \right|^2 * 100[\%]. \quad (6)$$

The overlap percentages in Table II confirm that the nuclear lowest isobaric analog 0^+ states have primarily isospin $T = |T_0|$ for even-even, and $T = |T_0| + 1$ for odd-odd nuclei, with a very small mixture of the higher possible isospin values. As it is expected, the $\delta_{\tilde{T}, T}$ isospin mixing increases as Z and N approach one another and towards the middle of the shell. For nuclei occupying a single- j shell, the mixing of the isospin states is less than 0.17%. Although the isospin mixing is negligible for light nuclei in the $j = 3/2$ orbit, it is clearly bigger for the $j = 7/2$ level. The mixing is expected to be even stronger in multi-shell configurations.

B. Non-Analog β -Decay Transitions

For a superallowed Fermi β -decay transition ($0^+ \rightarrow 0^+$) the ft comparative lifetime is nucleus-independent

TABLE II: $Sp(4)$ model estimate for the overlap [%] of isobaric analog 0^+ states of almost good isospin \tilde{T} with the states of definite isospin for ^{36}Ar in the $1d_{3/2}$ and the nuclei in the $1f_{7/2}$ level. The table is symmetric with respect to the sign of $n - 2\Omega$.

${}^A\text{X}(\tilde{T})$	(N_{+1}, N_{-1})	$T = 0$	$T = 1$	$T = 2$	$T = 3$	$T = 4$
${}^{36}\text{Ar}^{(0)}$	(2, 2)	99.9999	-	0.0001	-	-
${}^{44}\text{Ti}^{(0)}$	(2, 2)	99.90	-	0.10	-	-
${}^{46}\text{Ti}^{(1)}$	(2, 4)	-	99.98	-	0.02	-
${}^{46}\text{V}^{(1)}$	(3, 3)	-	99.98	-	0.02	-
${}^{46}\text{Cr}^{(1)}$	(4, 2)	-	99.98	-	0.02	-
${}^{48}\text{Ti}^{(2)}$	(2, 6)	-	-	99.997	-	0.003
${}^{48}\text{V}^{(2)}$	(3, 5)	-	-	99.994	-	0.006
${}^{48}\text{Cr}^{(0)}$	(4, 4)	99.83534	-	0.16465	-	10^{-5}
${}^{48}\text{Cr}^{(2)}$	(4, 4)	0.143	-	99.849	-	0.008
${}^{48}\text{Mn}^{(2)}$	(5, 3)	-	-	99.994	-	0.006
${}^{48}\text{Fe}^{(2)}$	(6, 2)	-	-	99.997	-	0.003

according to the conserved-vector-current (CVC) hypothesis and given by

$$ft = \frac{K}{G_V^2 |M_F|^2}, \quad K = 2\pi^3 \hbar \ln 2 \frac{(\hbar c)^6}{(m_e c^2)^5}, \quad (7)$$

where $K/(\hbar c)^6 = 8.120270(12) \times 10^{-7} \text{ GeV}^{-4} \text{ s}$ (m_e is the mass of the electron) and G_V is the vector coupling constant for nuclear β decay (see for example [10]). M_F is the Fermi matrix element $\langle \text{F} | \sqrt{2\Omega} T_{\pm} | \text{I} \rangle$ between a final (F) state with isospin projection T_0^{F} and an initial (I) states with T_0^{I} in a decay generated by the raising (for β^- decay) and lowering (β^+) isospin transition operator [63] $\sqrt{2\Omega} T_{\pm}$, which in the framework of our model is given as

$$|M_F|^2 = 2\Omega \left| \langle \text{F}; n(\tilde{T})T_0 \pm 1 | T_{\pm} | \text{I}; n(\tilde{T})T_0 \rangle \right|^2. \quad (8)$$

Typically, the isospin impurity caused by isospin non-conserving forces in nuclei is estimated as a correction to the Fermi matrix element $|M_F|^2$ of the superallowed \tilde{T} analog $0^+ \rightarrow 0^+$ transition, $\delta_C = 1 - |M_F|^2 / \left\{ \tilde{T}(\tilde{T} + 1) - T_0^{\text{F}} T_0^{\text{I}} \right\}$. For more than two-state mixing, the degree of isospin admixture between isobaric analog 0^+ states should be estimated using the normalized transition matrix element between non-analog (NA) states (e.g. [43]),

$$\delta_{IAS} = \frac{|M_F^{\text{NA}}|^2}{\left\{ \tilde{T}(\tilde{T} + 1) - T_0^{\text{F}} T_0^{\text{I}} \right\}}, \quad (9)$$

where \tilde{T} is the almost good isospin of the parent nucleus (see Table III for $1f_{7/2}$). In general, the δ_{IAS} correction may be very different than the order of the $\delta_{\tilde{T}, T}$ overlap quantity (6) presented in Table II because in decays

the degrees of isospin mixing between non-analog states within both the parent and daughter nuclei are significant.

TABLE III: Non-analog β -decay transitions to energetically accessible 0^+ states under consideration and the corresponding isospin mixing estimates δ_{IAS} (9), in %, according to the Sp(4) model for nuclei in the $1f_{7/2}$ level.

β -decay	δ_{IAS} , %
${}^A_Z X(\tilde{T}_X) \rightarrow {}^A_{Z-1} Y(\tilde{T}_Y)$	%
${}^{44}_{23}\text{V}(2) \rightarrow {}^{44}_{22}\text{Ti}(0)$	0.098
${}^{46}_{25}\text{Mn}(3) \rightarrow {}^{46}_{24}\text{Cr}(1)$	0.0169
${}^{46}_{24}\text{Cr}(3) \rightarrow {}^{46}_{23}\text{V}(1)$	0.0104
${}^{46}_{23}\text{V}(3) \rightarrow {}^{46}_{22}\text{Ti}(1)$	0.00447
${}^{48}_{27}\text{Co}(4) \rightarrow {}^{48}_{26}\text{Fe}(2)$	0.00327
${}^{48}_{26}\text{Fe}(4) \rightarrow {}^{48}_{25}\text{Mn}(2)$	0.00280
${}^{48}_{25}\text{Mn}(4) \rightarrow {}^{48}_{24}\text{Cr}(2)$	0.00189
${}^{48}_{24}\text{Cr}(4) \rightarrow {}^{48}_{23}\text{V}(2)$	0.00103
${}^{48}_{23}\text{V}(4) \rightarrow {}^{48}_{22}\text{Ti}(2)$	0.00038
${}^{48}_{25}\text{Mn}(4) \rightarrow {}^{48}_{24}\text{Cr}(0)$	4.5×10^{-7}
${}^{48}_{25}\text{Mn}(2) \rightarrow {}^{48}_{24}\text{Cr}(0)$	0.14328
${}^{50}_{27}\text{Co}(3) \rightarrow {}^{50}_{26}\text{Fe}(1)$	0.0169
${}^{50}_{26}\text{Fe}(3) \rightarrow {}^{50}_{25}\text{Mn}(1)$	0.0104
${}^{50}_{25}\text{Mn}(3) \rightarrow {}^{50}_{24}\text{Cr}(1)$	0.00447
${}^{52}_{27}\text{Co}(2) \rightarrow {}^{52}_{26}\text{Fe}(0)$	0.098

The analysis of the results shows that the mixing between isobaric analog 0^+ states (which is at least $\Delta T = 2$ mixing) is on average 0.006% excluding even-even $N = Z$ nuclei. This is on the order of a magnitude less than the mixing of the first excited 0^+ non-analog state due to isospin non-conserving interaction, which is typically about 0.04% for the $1f_{7/2}$ level [21, 43]. In addition, this yields nonanalog β -decays weaker than possible Gamow-Teller transitions; the strength of the latter is found to be less than 0.02% of the total β -decay strength for the nuclei in the $1f_{7/2}$ shell [21] and to substantially increase with increasing mass number A [14, 25, 57]. This makes δ_{IAS} mixing very difficult to be detected especially when the isospin-symmetry breaking correction (δ_C) to analog Fermi matrix elements in this level is on the order of a percent [9, 14].

Not surprising, the largest values for the δ_{IAS} correction are observed for $\Delta T = 2$ β^\pm -decays to energetically accessible 0^+ ground states of even-even $N = Z$ nuclei (Table III). While for these decays δ_{IAS} is extremely small, namely less than 0.14%, as expected for the contribution of the higher-lying 0^+ states [43], it is comparable

to the order of isospin-symmetry breaking corrections for the $1f_{7/2}$ orbit that are typically taken into account [43]. The reason may be that for the even-even $N = Z$ nuclei the second-lying isobaric analog 0^+ states are situated relatively low due to a significant pn interaction (Figure 2).

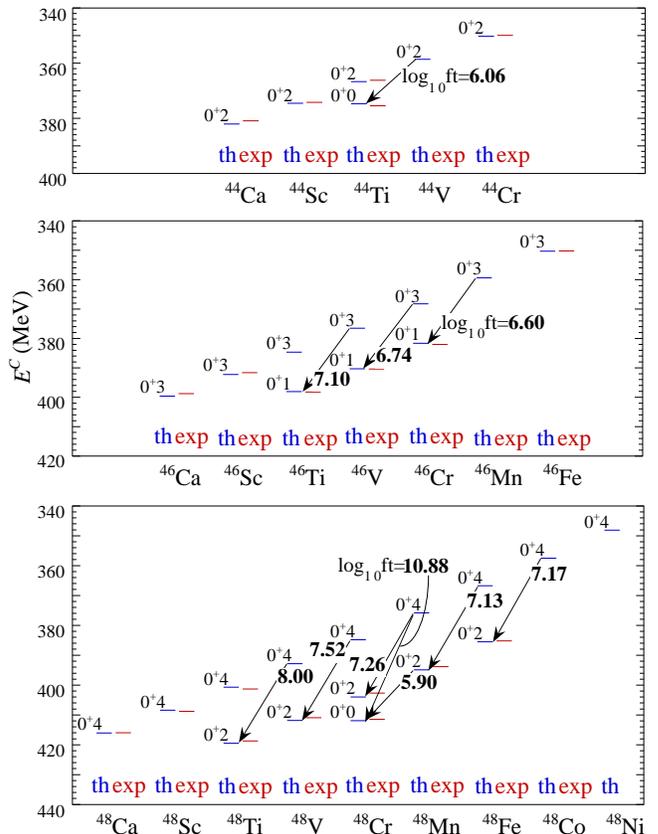


FIG. 2: Non-analog $0^+ \rightarrow 0^+$ β^+ -decay transitions to energetically accessible $J^\pi \tilde{T}$ states under consideration indicated by arrows and by the corresponding theoretically calculated $\log_{10} ft$ values (there are no available experimental ft values for comparison). The theoretical (blue, “th”) and experimental [58, 59] (red, “exp”) binding energies E^C in MeV (including the Coulomb potential energy) are shown for the isobar sequences with $A = 44$ to $A = 48$ in the $1f_{7/2}$ level.

Above all, the δ_{IAS} results in Table III clearly show the overall pattern and the order of significance of the isospin mixing under consideration. This is evident within the first-order approximation in terms of the F parameter ($F \ll 1$) of δ_{IAS} , which for $1f_{7/2}$ deviates on average by only 2% from its exact calculations in Table III. The δ_{IAS} isospin mixing correction is then proportional to F^2 and one finds out that its order of magnitude remains the same for large variations of the F parameter of more than 60%. In addition, greater F values are not very likely because the δ_{IAS} estimates (Table III) fall close below an upper limit, which does not contradict experimental and theoretical results for other types of isospin mixing.

Moreover, in this first-order approximation the ratio of any two isospin corrections for $1f_{7/2}$ is independent of the parameters of the model interaction. This implies that such a ratio does not reflect at all the uncertainties of the interaction strength parameters but rather it is characteristic of the relative strength of both decays. It identifies the decay, for which the maximum isospin mixing correction is expected in the $1f_{7/2}$ orbit, namely ${}^{48}_{25}\text{Mn}^{(2)} \rightarrow {}^{48}_{24}\text{Cr}^{(0)}$, and as well as the amount by which δ_{IAS} of the other possible non-analog decays is relatively suppressed. For example, the δ_{IAS} correction for the ${}^{44}_{23}\text{V}^{(2)} \rightarrow {}^{44}_{22}\text{Ti}^{(0)}$ decay is around two-thirds the maximum one and that for the ${}^{46}_{25}\text{Mn}^{(3)} \rightarrow {}^{46}_{24}\text{Cr}^{(1)}$ decay is around one-eighth the maximum one (Table III). Such a ratio quantity exhibits a general trend of increasing δ_{IAS} isospin mixing with Z within same isospin multiplets and as well it reveals enhanced $\Delta T = 2$ decays to the ground state of even-even $N = Z$ nuclei with increasing δ_{IAS} towards the middle of the shell. Furthermore, the ratio retains its behavior for the non-analog β decays between nuclei with the same valence proton and neutron numbers as in Table III but occupying the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ major shell. Therefore, among the non-analog β decays for the $A = 60 - 64$ isobars with valence protons and neutrons in the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ shell the δ_{IAS} isospin mixing of the ${}^{64}_{33}\text{As}^{(2)} \rightarrow {}^{64}_{32}\text{Ge}^{(0)}$ decay is expected to be the largest with a tendency of a further increase towards the middle of the shell. In short, the significance of the isospin mixing caused by a charge dependent $J = 0$ pairing correlations is evident from Table III for the $1f_{7/2}$ level and continues this trend for the upper fp shell.

The small mixing of the 0^+ isospin eigenstates from different isospin multiplets yields very small but nonzero $|M_F^{NA}|^2$ matrix elements for non-analog β^\pm decay transitions as indicated by δ_{IAS} in Table III. For nuclei in the $1f_{7/2}$ shell, such non-analog transitions to energetically accessible states are shown in Figure 2 along with the ft values (where we use $K/G_V^2 = 6200s$ [60]). In the framework of the Sp(4) model, these values are symmetric with respect to the sign of T_0 [possible β^- decays are not shown in Figure 2] and of $n - 2\Omega$ (the $A = 50$ and $A = 52$ multiplets are analogous to the $A = 46$ and $A = 44$ ones, respectively). The results yield that ten of the transitions are classified as forbidden ($\log_{10} ft \geq 7$), other four are suppressed ($\log_{10} ft \approx 7$) and the four decays to the ground state of an even-even $N = Z$ nucleus appear to have comparatively larger decay rates ($\log_{10} ft \approx 6$). In Figure 2 the theoretically calculated isobaric analog 0^+ state energies are shown together with the available experimental ones. It is worth mentioning that while the energies of the lowest isobaric analog 0^+ states determined directly the parameters of the model interaction, a quite good reproduction of the experimental higher-lying isobaric analog 0^+ state energies followed without any parameter adjustment [42]. This outcome is important because the energy difference between two isobaric analog 0^+ states within a nucleus directly affects

the degree of their mixing. In summary, the theoretical Sp(4) model suggests the possible existence, albeit highly hindered, of $\Delta T = 2$ non-analog β -decay transitions.

IV. CONCLUSIONS

We employed a group-theoretical approach based on the Sp(4) dynamical symmetry to describe microscopically a possible isospin mixing induced by a short-range charge dependent nuclear interaction. The Sp(4) model interaction incorporates the main driving forces, including $J = 0$ pairing correlations and implicit quadrupole-quadrupole term, that shape the nuclear pairing-governed isobaric analog 0^+ states in the $1f_{7/2}$ level where the Sp(4) Hamiltonian correlates strongly with realistic interactions. This approach provides a reasonable reproduction not only of the energies of the lowest isobaric analog 0^+ states in total of 149 nuclei. It also reproduces the available experimental energies of the higher-lying isobaric analog 0^+ states in the $1f_{7/2}$ level and fine structure effects without any variation of the parameters of the model interaction. In this respect, as predicted by our model, the coefficient related to the isotensor part of a general non-conserving force, c , which has been recently found to be dominated by a charge dependent $J = 0$ pairing interaction [40], agrees quite well with the experimental values.

The isospin-symmetry breaking due to coupling of isobaric analog 0^+ states in nuclei was estimated to be extremely small for nuclei in the $1d_{3/2}$ and $1f_{7/2}$ orbitals with the $N = Z$ even-even nuclei being an exception. For these nuclei, strong pairing correlations, including a significant pn interaction, are responsible for the existence of comparatively larger isospin mixing, although the latter is still at least an order of a magnitude smaller than the overall isospin admixture in the ground state. The results also show that a variation of more than 60% in the F isospin mixing parameter is required to reduce the present δ_{IAS} results by an order of a magnitude.

The analysis also shows that there is a trend of increasing isospin mixing between isobaric analog 0^+ states due to a charge dependent $J = 0$ pairing interaction towards the middle of the shell and for $\Delta T = 2$ decays to the ground state of an even-even $N = Z$ daughter nucleus. Such behavior is free of the uncertainties in the strength parameters of the interaction and is adequate for larger multi- j shell domains such as $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$. For nuclei with valence protons and neutrons occupying the $1f_{7/2}$ level the strongest non-analog decay is identified to be ${}^{48}_{25}\text{Mn}^{(2)} \rightarrow {}^{48}_{24}\text{Cr}^{(0)}$, while the δ_{IAS} isospin mixing correction for the rest of the decays is $2/3$ to $1/300$ the maximum one.

In short, the $\mathfrak{sp}(4)$ algebraic model yields an estimate for the decay rates of possible non-analog β -decay transitions due to a pure strong interaction, which, though few of them may affect slightly precise calculations, are not

expected to comprise the dominant contribution to the isospin-symmetry breaking correction tested in studies of superallowed Fermi β -decay transitions.

Acknowledgments

This work was supported by the US National Science Foundation, Grant Number 0140300. The authors thank

Dr. Vesselin G. Gueorguiev for his computational MATH-EMATICA programs for non-commutative algebras.

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- [62] To avoid confusion we mention that in (1) the energies are assumed positive for bound states; V_{Coul} is also defined positive.
- [63] The factor of $\sqrt{2\Omega}$ appears due to the normalization of the basis operators adopted in the $\mathfrak{sp}(4)$ algebraic model.