10-1-2005

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Recommended Citation

Luo, Y., Bahri, C., Pan, F., Gueorguiev, V., & Draayer, J. (2005). Intruder level and deformation in the SD-pair shell model. *International Journal of Modern Physics E, 14* (7), 1023-1031. https://doi.org/10.1142/S0218301305003764

Intruder level and deformation in the SD-pair shell model

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(Dated: October 30, 2018)

The influence of the intruder level on nuclear deformation is studied within the framework of the nucleon-pair shell model truncated to an SD-pair subspace. The results suggest that the intruder level has a tendency to soften the deformation and plays an important role in determining the onset of rotational behavior.

PACS numbers: 21.60.Cs

Shell model calculations for all but light nuclei can only be carried out in highly truncated model spaces. Usually these are limited to a single major valence shell, but for heavy nuclei this is often out of reach as well even when the best of computational facilities are available. As a result, early work – for example, calculations based on the fermion dynamical symmetry model [1] and the pseudo-SU(3) model [2] – introduced additional constraints; in particular, the nucleons in unique-parity levels were typically constrained to a seniority zero configuration and as a result the dynamics was driven by nucleons in the normal-parity levels. By-and-large, theories that have used this assumption have not tested its validity.

In other venues, the importance of the intruder levels has received considerable attention: their significance for correctly reproducing available data, their role in determining the deformation of nuclei, and how to properly incorporate them into other models have been repeatedly debated. For example, studies in the single-shell asymptotic model and universal Woods-Saxon model imply that the valence nucleons in the intruder level contribute significantly to measurable quantities like B(E2) values [3]. Some mean-field theories claim that the particles in the intruder level play a dominant role in determining the deformation [4]. In Ref. [5] the authors claim that both the normal and unique parity states contribute significantly to the overall collectivity of a nuclear system.

In a nucleon-pair shell model theory truncated to a SD subspace – the SD-pair shell model (SDPM) [6, 7] – studies of the so-called O(6)-limit nuclei ^{134,132}Ba show that the effect of the intruder level is sensitive to the structure of the single-particle levels. Specifically, with non-degenerate single-particle levels and the occupancy of the intruder levels restricted to S pairs only, the collectivity of low-lying states, which is different from the O(6) symmetry limit, does not agree with the experimental results. On the other hand, if the single-particle levels are degenerate, (even though the occupancy of the intruder level is limited to S-pairs only) like those of the fermion dynamical symmetry model [1], an O(6) behavior (soft deformation) can be realized [8, 9].

The interacting boson model (IBM) reproduces rotational spectra in its SU(3) limit [10]. Since its building blocks are s and d bosons that are mapped from S and D nucleon pairs [11], it is expected that the SDPM should also be able to reproduce rotational spectra. It is the aim of this paper to study the influence of the intruder (or unique parity) level – which has the opposite parity to the other (or normal parity) levels – on deformation, and thus determine how rotational spectra can be realized within the framework of the SDPM.

The SDPM Hamiltonian is taken to be

$$H = H_0 - \frac{1}{2}\kappa(Q^2_\pi + Q^2_\nu)\cdot(Q^2_\pi + Q^2_\nu), \tag{1}$$

$$H_0 = \sum_{a\sigma}\epsilon_{a\sigma}n_{a\sigma}, \tag{2}$$

$$Q^2_\sigma = \sum_i r^2_{\sigma i}Y^2(\theta_{\sigma i},\phi_{\sigma i});\quad \sigma=\pi,\ \nu, \tag{3}$$

where Q^2_π and Q^2_ν are, respectively, the proton and neutron quadrupole operators. The $E2$ transition operator is

$$T(E2) = e_\pi Q^2_\pi + e_\nu Q^2_\nu, \tag{4}$$

where e_π and e_ν are effective charges of the proton and neutron, respectively.

The basis of this model is constructed from collective S and D pairs, which are defined as

$$S^\dagger = \sum_j \hat{j}\frac{v_j}{u_j}(C^\dagger_j \times C^\dagger_j)^0$$

$$D^\dagger = \frac{1}{2}[Q^2,\ S^\dagger], \tag{5}$$

where $\hat{j} = \sqrt{2j+1}$, and v_j, u_j are the occupied and unoccupied amplitudes for orbit j, obtained by solving the BCS equation. (Details can be found in Ref. [6, 7].)

To explore the influence of the intruder level on the deformation, for simplicity, we set the normal parity levels to be degenerate and vary the single-particle energy ϵ of the intruder level from -0.15 MeV to 0.15 MeV.

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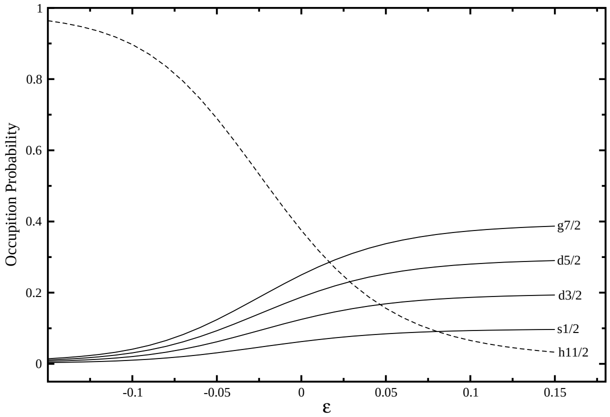

FIG. 1: Fractional occupancies of the 50-82 shell. The pairing interactional strength was fixed at 0.01 MeV in solving BCS equation.

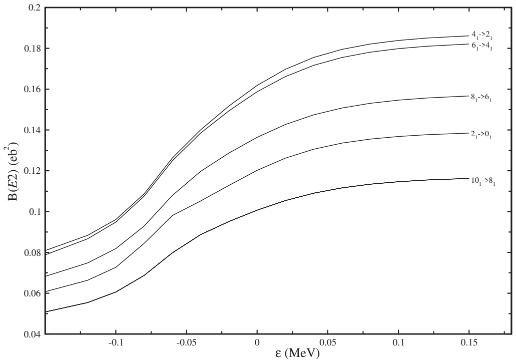

FIG. 2: Absolute B(E2) values for the 50-82 shell versus the single-particle energy of the $h_{11/2}$ intruder level. The pairing interactional strength is fixed at 0.01 MeV in solving BCS equation.

The pairing strength used in solving the BCS equation was taken to be 0.01 MeV for both the proton and neutron sectors of the space, and κ was fixed at 0.1 MeV/r_0^4 with $r_0 = \sqrt{\hbar/m\omega}$ the oscillator length. We take $N_\pi = N_\nu = 3$ for this study, with protons and neutrons occupying the same major shell except as specifically noted in the text.

The fractional occupancies of the BCS-determined S pair configuration versus the position of the intruder level is shown in Fig. 1 for the 50-82 shell. Note that for $\epsilon = -0.15$ MeV, the fractional occupancy of the intruder level is close to unity, implying that proton and neutron S pairs will primarily occupy the intruder level. As the single-particle energy of the intruder level increases, the fractional occupancy of the intruder level decreases. The higher the position of the intruder level relative to the normal parity levels, the lower its fractional occupancy, and accordingly, the less the intruder level will contribute to the overall structure of the low-lying states. (When $\epsilon = 0.0$ MeV the levels are all degenerate and in this case the fractional occupancy of each j-shell is simply the ratio $(2j+1)/\Omega$ where $\Omega = \Sigma_j(2j+1)$ is the total degeneracy of the system.) The behavior of the fractional occupancies for the normal parity levels is opposite to that of the intruder level since the sum of the fractional occupancies is normalized to unity.

As can be seen from Fig. 2, for yrast states in the 50-82 shell, the higher the relative energy of the intruder level, and hence a smaller fraction of the protons and neutrons occupying the intruder level, the stronger the E2 transition strengths and correspondingly the larger the deformation. Here, the effective charges were fixed at the usual values of $e_\pi = 1.5e$ and $e_\nu = 0.75e$. The results for the other shells are similar and therefore not shown.

The effect of the intruder level on the rotational nature of the spectrum is presented in Table I for the calculated yrast states. In the ds-shell (the $N = Z =$ 8-20 region), the spectrum exhibits a rotational structure with $E_{4_1^+}/E_{2_1^+} = 3.3$ when the position of the intruder level is at $\epsilon = 0.07$ MeV. In general, as the intruder level moves up in energy the rotational structure improves. For example, $E_{4_1^+}/E_{2_1^+} = 2.756$ when the intruder and the normal parity levels are degenerate ($\epsilon_{7/2} = 0$), while it is 3.331 when $\epsilon_{f_{7/2}} = 0.15$ MeV. This is counter to the statement made in Ref. [4] that the intruder levels play a dominant role in deformation; we find that the intruder level tends to reduce deformation in the SDPM. Specifically, the lower the intruder level, the greater the contribution from the intruder level to the low-lying states, but the smaller the deformation. From Table I one can also see that the position of the intruder levels that reproduces the rotational $E_{4_1^+}/E_{2_1^+} = 3.3$ value decreases with an increase in the number of the single-j shells and the number of single-j values in each major shell. The single-particle energies of the intruder levels that reproduce the $E_{4_1^+}/E_{2_1^+} = 3.3$ ratio is 0.6, 0.5, 0.2 and 0 MeV for the 20-50, 50-82, 82-126 and 126-184 shells, respectively. (Note, there are no physical systems with protons and neutrons occupying the same shell for the 82-126 and 126-184 shells.) We give the B(E2) ratios for each major shell in Table. II, from which one can see that the B(E2) ratio is close to the SU(3) limit of an IBM theory [10].

This analysis suggests that the presence of the intruder level tends to soften the deformation. Fig. 1 shows that if the intruder level lies low in energy relative to the normal parity levels the contribution from the intruder level to the low-lying states is large, the expected result. It also shows that this results in a decrease in the rotational nature of the spectrum and hence the overall deformation. This can be understood in two ways. First of all, the nucleons in the intruder level cannot couple directly to those in the normal parity levels to form positive-parity pairs because of the parity difference, the so-called "par-

TABLE I: The energy ratio $R_J = E_{J_1^+}/E_{2_1^+}$ for each major shell. The position of the single-particle level for the intruder level is labeled by ϵ. The boldface entries correspond to the position where the rotational structure, $R_2 = 3.3$, is realized. For protons and neutrons occupying different major shells, the shells are labelled by π (ν) for protons (neutrons) and the single-particle energies for the intruder levels are put in a pair.

shell	ϵ (MeV)	J=4	J=6	J=8	J=10	J=12
8-20 (*ds*)	-0.15	1.65	2.86	3.82	4.95	6.06
	0.00	2.76	5.23	8.34	12.00	16.07
	0.07	**3.30**	**6.85**	**11.63**	**17.60**	**24.71**
	0.15	3.33	6.99	11.97	18.25	25.84
20-50	-0.15	2.62	4.00	5.60	7.33	8.20
	0.00	3.16	6.37	10.52	15.48	21.13
	0.06	**3.30**	**6.86**	**11.62**	**17.52**	**24.52**
	0.15	3.32	6.92	11.78	17.88	25.21
50-82	-0.15	2.69	4.77	7.24	9.91	12.56
	0.00	3.17	6.37	10.51	15.44	21.05
	0.05	**3.30**	**6.85**	**11.59**	**17.45**	**24.35**
	0.15	3.32	6.92	11.78	17.86	25.14
82-126	-0.15	2.64	4.83	7.50	10.56	13.95
	0.00	3.28	6.77	11.39	17.03	23.59
	0.02	**3.31**	**6.88**	**11.65**	**17.56**	**24.53**
	0.15	3.33	6.99	11.96	18.24	25.82
126-184	-0.15	2.44	4.253	6.394	8.817	11.436
	0.00	**3.31**	**6.89**	**11.67**	**17.60**	**24.58**
	0.15	3.33	6.99	11.97	18.25	25.82
$(50\text{-}82)_\pi$	(-0.15, -0.15)	2.65	4.70	7.31	10.43	13.94
$(82\text{-}126)_\nu$	**(0.04, 0.02)**	**3.30**	**6.85**	**11.62**	**17.57**	**24.73**
	(0.15, 0.15)	3.33	6.98	11.97	18.37	26.33
$(82\text{-}126)_\pi$	(-0.15,-0.15)	2.47	4.38	6.71	9.41	12.36
$(126\text{-}184)_\nu$	**(0.00, 0.00)**	**3.30**	**6.84**	**11.56**	**17.41**	**24.38**
	(0.15, 0.15)	3.34	7.01	12.04	18.45	26.31
IBM SU(3)-limit		3.333	7.0	12.0	18.333	26.0

TABLE II: The relative B(E2) values with effective charges fixed as $e_\pi = 1.5e$ and $e_\nu = 0.75e$. Only the results for three cases are shown here: the first one is for the case with the lowest intruder level, the second one is for the case that the intruder level is degenerate with normal parity levels, while the last one is for the case with $E_{4_1^+}/E_{2_1^+} = 3.3$.

shell	ϵ	$\frac{4_1^+ \rightarrow 2_1^+}{2_1^+ \rightarrow 0_1^+}$	$\frac{2_2^+ \rightarrow 2_1^+}{2_1^+ \rightarrow 0_1^+}$	$\frac{0_2^+ \rightarrow 2_1^+}{2_1^+ \rightarrow 0_1^+}$
ds	-0.15	1.315	0.019	0.937
	0.0	1.350	0.36	0.053
	0.06	1.350	0.004	0.008
20-50	-0.15	1.219	0.004	0.019
	0.0	1.347	0.002	0.005
	0.06	1.345	0.04	0.0
50-82	-0.15	1.332	0.0	0.047
	0.0	1.346	0.0	0.005
	0.05	1.344	0.004	0.0
82-126	-0.15	1.341	0.509	0.051
	0.0	1.345	0.003	0.001
	0.02	1.345	0.004	0.0
126-184	-0.15	1.286	1.2662	0.0
	0.0	1.346	0.004	0.001
IBM SU(3)-limit		1.349	0.0	0.0

ity blocking effect". Simply stated, this means a nucleon in the intruder level and one in the normal parity sector cannot bind to one another as strongly as nucleons in the separate sectors. This coupled with the fact that the unique parity part of the space is incomplete, lacking its other unique parity partners so it cannot form optimal coherent pairs that favor deformation, means that the overall deformation is reduced when pairs reside in the unique parity sector, even though a complete unique parity shell by itself would show stronger deformation. In short, a consequence of the strong spin- orbit splitting that "isolates" the unique parity level is that as the contribution from the intruder level increases, the deformation decreases.

To check the validity of these observations, we can apply the logic to well-known experimental spectra. We know, for example, that there are some *ds* shell nuclei that show well-developed rotational bands. According to our analysis, this suggests that the intruder level should lie high above the active normal parity levels. Indeed, for those lower *ds* nuclei that show strong rotational spectra, the position of the $f_{7/2}$ level is far removed from the most active levels, Ref.[12]. But as one moves up in mass number, the role of the $f_{7/2}$ level becomes more important, and as the results indicate the collective rotational behavior of upper *ds* shell nuclei drops off. In the upper *ds* shell, the systems can be viewed as holes occupying the shells but in the reverse order. Thus, in this picture the $f_{7/2}$ intruder level lies lowest. For the 20-50 and 50-82 shells there are very few nuclei that show rotational behavior and for these nuclei the lowest high-j levels is split off from the normal parity partners and the intruder levels penetrates down among the remaining normal parity levels [12].

Matters are more interesting for systems with protons filling the 50-82 shell and neutrons the 82-126 shell, or protons filling the 82-126 shell and neutrons the 126-184 shell. In this case there are some nuclei that show rotational behavior, such as ^{160}Gd. From Table I we know that for the 50-82, 82-126 and 126-184 shells, rotational structures are predicted even though the positions of the respective intruder levels are close to the normal parity levels. This is especially true for the 126-184 shell where rotational behavior is realized when the intruder level is nearly degenerate with the normal parity single-particle levels. This means that the predictions of the theory are once again borne out by experiment.

In this study a simple hamiltonian was used to examine the effect of the intruder level on deformation. From the analysis one finds that one can give bounds on the position of the intruder level for strong rotational spectra to occur. The position of an intruder level plays a strong role in determining the rotational nature of the spectra and this agrees, on average, with what is found experimentally. The SD-pair shell model also suggests that one should not ignore intruder levels because they have a strong influence (enhancing or reducing, depending upon the relative position of the level) on the deformation of a system.

This work was supported in part by the U.S. National Science Foundation through Grant Nos. 9970769 and 0140300, and partially by Education Department of Liaoning Province (202122024).

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