2015

Degradation and Fatigue Involving Dissipated Processes

Md Liakat Ali
Louisiana State University and Agricultural and Mechanical College, mdliakatali1@gmail.com

Follow this and additional works at: https://digitalcommons.lsu.edu/gradschool_dissertations
Part of the Mechanical Engineering Commons

Recommended Citation
https://digitalcommons.lsu.edu/gradschool_dissertations/1810

This Dissertation is brought to you for free and open access by the Graduate School at LSU Digital Commons. It has been accepted for inclusion in LSU Doctoral Dissertations by an authorized graduate school editor of LSU Digital Commons. For more information, please contact gradetd@lsu.edu.
DEGRADATION AND FATIGUE INVOLVING DISSIPATED PROCESSES

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Mechanical and Industrial Engineering

by

Md Liakat Ali
B.S., Bangladesh University of Engineering and Technology, 2007
M.S., Universiti Teknologi Malaysia, 2008
May 2015
To my
father, mother, wife, brother, and sister
Acknowledgements

I would like to express my gratitude to my advisor Prof. Michael M. Khonsari for his continuous support, encouragements, and guidance throughout this research. I would also like to thank Prof. Muhammad A. Wahab, Prof. Guoqiang Li, Prof. James J. Spivey, and Dr. Joonyoung Jang, the members of my graduate advisory committee, for their time and kind help.

Thanks are also due to all of my colleagues at the Center for Rotating Machinery (CeRoM) for their help, cooperation, and support to my research and maintaining a pleasing working environment.

I must appreciate the support and inspiration of my family and my wife’s family members for continuing hard work to achieve this level of education.
# Table of Contents

Acknowledgements ........................................................................................................ iii

Abstract .......................................................................................................................... v

Chapter 1: Overview ........................................................................................................ 1

Chapter 2: Rapid Estimation of Fatigue Entropy and Toughness in Metals ................. 7

Chapter 3: Entropic Characterization of Metal Fatigue with Stress Concentration .......... 31

Chapter 4: On the Anelasticity and Fatigue Fracture Entropy in High-cycle Metal Fatigue ...... 63

Chapter 5: Nondestructive Testing and Prediction of Remaining Fatigue Life of Metals ....... 87

Chapter 6: Analysis and Life Prediction of a Composite Laminate under Cyclic Loading ...... 103

Chapter 7: An Experimental Approach to Estimate Damage and Remaining Life of Metals under Uniaxial Fatigue Loading ......................................................................................................................... 131

Chapter 8: Conclusions and Future Recommendations .................................................. 154

Appendix: Letters of Permission to Use Published Material .............................................. 156

Vita ................................................................................................................................. 160
Abstract

Irreversible material degradation due to cyclic mechanical loading is investigated utilizing the concept of thermodynamic entropy, plastic strain energy, and temperature slope measurement. Uniaxial tension-compression and fully-reversed bending fatigue tests are performed over a wide range of loading conditions with metallic and composite materials subject to both constant- and variable-amplitude loading.

A methodology is developed for the estimation of the fatigue fracture entropy (FFE) and fatigue toughness of metallic specimens in a rapid fashion. It is found that the FFE and the fatigue toughness of each material tested are within a small band. The value of FFE is found to be unique for a given type of a material, substantiating that FFE can be regarded as a material property. The concept of FFE is applied to study the effect of stress concentration on a metallic specimen. It is found that the FFE of a V-notched specimen with certain amount of stress concentration is fairly constant. A formula is derived for the prediction of the fatigue life of a V-notched specimen based on the fatigue test results of an un-notched specimen.

The concept of FFE is utilized to study the high-cycle fatigue (HCF) of carbon steel 1018. As the stress levels in HCF are substantially smaller than the yield strength of the material, a considerable amount of anelastic energy is present in the hysteresis loop along with plastic strain energy. We propose a method to calculate anelastic energy so that entropy generation can be estimated. Finite element simulations are performed to validate the assumptions made in the development of the methodology. It is found that the FFE of this material remains within a specific band both for the low- and high-cycle fatigue.

A methodology is developed for the prediction of the remaining fatigue life (RFL) of a specimen with prior history of loading in a non-destructive (NDT) fashion based on the slope of temperature rise obtained from the specimen under cyclic loading. This method which uses thermographic technique has been validated with API 5L X52, carbon steel 1018, and Glass/Epoxy composite with promising results. This approach is further extended to derive a correlation between the damage parameter and the temperature slope obtained from a fatigued specimen. This correlation and the so-called master curve of damage evolution are employed to develop a methodology for the prediction of the RFL of a metallic specimen.
Chapter 1: Overview

1.1. Introduction

Fatigue is one of the most dominant modes of failure in structures and components that are subjected to time dependent loading in cyclic manner wherein material damage tends to accumulate as a function of time. The application of cyclic load to a material causes irreversible degradation that accumulates over time and eventually leads to fracture. The process of the accumulation of degradation in a material is accompanied by the generation of irreversible plastic strain energy that converts into heat and dissipates to the surroundings. According to the second law of thermodynamics, an irreversible degradation process inherently generates thermodynamic entropy that serves as a measure of degradation [1, 2]. Bryant et al. [3] showed that degradation coefficient, $B_i$, is related to the generalized thermodynamic force $X_i^j$ and generalized degradation force, $Y_i^j$, such that

$$B_i = \frac{Y_i^j}{X_i^j} \quad (1.1)$$

Utilizing these concepts, Naderi et al. [2] developed a method for the calculation of the entropy generation in a fatigue process and showed that fatigue fracture occurs when the accumulation of entropy generated—starting with a pristine specimen without stress concentration and ending with fatigue fracture—reaches a constant value regardless of the specimen geometry and loading sequence [2, 4]. The constant value of accumulated entropy at fatigue fracture is referred to as the fatigue fracture entropy (FFE) expressed as:

$$\gamma = \int_0^{t_f} \left( \frac{\dot{W}_p}{T} \right) dt - \int_0^{t_f} \left( \frac{q}{T^2} \cdot \text{grad } T \right) dt \quad (1.2)$$

where $\gamma$ represents FFE, $t$ is the time, $T$ represents temperature, $t_f$ stands for the fatigue life, $\dot{W}_p$ denotes the time rate of plastic strain energy generation, and $q$ indicates the heat flux. Naderi et al. [2, 4] applied this approach to metals and a composite laminate subject to both constant- and variable-amplitude loading with promising results and developed a real time structural health monitoring system. The application of this approach to the specimens with stress concentration and to high-cycle fatigue (HCF) is yet to be investigated.

The presence of stress concentration in a material changes the rate of fatigue degradation that results in a shorter fatigue life. As the FFE of a material depends on both the rate of entropy generation and fatigue life, it requires further research to quantify the effect of stress concentration on the FFE. An investigation associated with the effect of stress concentration on the FFE and fatigue life of metallic specimens is presented in Chapter 3.
In the case of high-cycle fatigue (HCF), the level of applied stress is considerably less than the yield strength of a material that generates a small amount of degradation. Consequently, the amount of entropy generation and temperature evolution is less that results in a larger fatigue life. Research shows that the area of a hysteresis loop in a HCF test consists of a considerable amount of anelastic energy [4]. In order to determine the FFE in an HCF test, it is essential to estimate the amount of anelastic energy contained in a hysteresis loop. Experimental and simulation results are presented in Chapter 4 to develop a methodology for the estimation of anelastic energy and FFE in HCF.

Thermodynamic entropy based fatigue estimation requires one to monitor the entire loading history which is difficult to perform in many applications. Further, the changes in the loading and environmental conditions compared to the design conditions can reduce the fatigue life of a material considerably. Consequently, the useful remaining fatigue life (RFL) of a structure can reduce significantly. In order to maintain sufficient structural integrity, it is essential to determine the RFL of a material accurately.

The useful RFL of a material gradually decreases due to the application of cyclic load that causes irreversible degradation in the material due to the formation of slip bands, dislocations, and microcracks [5, 6]. The accumulation of degradation results in a gradual increase in the slope of temperature rise, $R_\theta$, obtained from a specimen under cyclic loading which is similar to the generation of thermodynamic entropy. According to the postulate of Bryant et al. [3], it can be stated that $R_\theta$ serves as a degradation coefficient that is measured through a thermography technique. The relevant question in this dissertation is: is there a relationship between $R_\theta$ and the RFL of a material?

Since both $R_\theta$ and FFE serve as a measure of material degradation, there must be a correlation between them. The calculation of FFE requires one to measure the amount of plastic strain energy, $\dot{W}_p$, generated in a fatigue process. To the best of our knowledge, there are two available methods to measure $\dot{W}_p$ based on empirical approach. Morrow [7] proposed a correlation between the cyclic plastic strain energy, $\Delta W_p$, and the fatigue life, $N_f$, of a material reads:

$$\Delta W_p = 2^{b+c+2}\sigma_f'\varepsilon_f'\frac{(c-b)}{(c+b)}(N_f)^{b+c}$$

(1.3)

where $\sigma_f'$ and $b$ are the fatigue strength coefficient and exponent, $\varepsilon_f'$ and $c$ stand for the fatigue ductility coefficient and exponent, respectively. These fatigue properties of many engineering materials are not available as they require rigorous experimentation. Further, this method does not consider the variation in the evolution of $\Delta W_p$ in a fatigue process due to the strain hardening or softening.
Another method is proposed by Meneghetti [8] that requires one to suddenly stop a fatigue test to measure the slope of temperature decrease, \( \frac{\partial \theta}{\partial t} \), for the assessment of the energy dissipation, \( h \), reads:

\[
-h = \rho c \frac{\partial \theta}{\partial t} \tag{1.4}
\]

where \( \rho \) is the density and \( c \) denotes the specific heat. These methods require one to measure the evolution of plastic strain energy and temperature throughout a fatigue test in order to calculate the FFE utilizing Equation (1.2). Chapter 1 presents the development of a methodology that correlates the degradation parameters FFE and \( R_\theta \) and describes a method for the estimation of FFE in a rapid fashion.

As the accumulation of entropy is a measure of material degradation, it must be correlated to the so-called damage parameter, \( D \), defined by the continuum damage mechanics (CDM). The amount of progressive degradation in the mechanical and physical properties of a material is expressed in terms of the damage parameter. Naderi and Khonsari [9, 10] showed that the damage parameter and the entropy generation rate are correlated as:

\[
D = \frac{D_c}{\ln(1-s_{ic}/s_g)} \ln(1 - s_{i}/s_g) \tag{1.5}
\]

where \( D_c \) is the critical damage and \( s_{ic}, s_{i}, \) and \( s_g \) stand for the critical, instantaneous, and total entropy generation in a fatigue process, respectively. Naderi and Khonsari extended the applicability of Equation (1.5) for variable-amplitude loading tests. Development of a methodology for the estimation of damage parameter and RFL based on the \( R_\theta \) measurement approach subject to constant- and variable-amplitude loading conditions is presented in Chapter 7.

This dissertation presents the derivation of correlations, numerical simulations of fatigue using finite element method, and an extensive range of experimental results associated with metallic and composite specimens subject to uniaxial tension-compression and completely-reversed bending fatigue loads. Both constant and variable loading conditions are considered for the characterization and validation of the procedures. Methodologies are developed for the assessment of the RFL, damage parameter, plastic strain energy, and thermodynamic entropy associated with fatigue. The effect of the presence of stress concentration on a material is studied utilizing the concept of the entropy generation and the FFE. A methodology is developed to calculate the FFE in the HCF of metallic specimens. Experimental results are presented for the validation of the models and approaches derived in this research.

1.2. Dissertation Outline

This dissertation comprised of the following sub-topics: representation of an experimental approach for the estimation of the fatigue toughness and the FFE of metallic specimens, fatigue
life characterization of metallic specimens with stress concentration using the concept of FFE, description of a procedure for the calculation of the FFE in HCF of metallic specimens, development of an NDT method for the prediction of the RFL of metallic and composite specimens using $R_\theta$ measurement, and development of a methodology to correlate the damage parameter and $R_\theta$ and its application in predicting RFL. Each sub-topic is written as a chapter in the format of a journal paper.

The development of a procedure utilizing $R_\theta$ measurement for the estimation of the fatigue toughness and FFE in metals is presented in Chapter 2. Correlations are developed for the calculation of these parameters through a rapid technique. Finite element simulations are performed as a part of the methodology development to calculate the parameters that are difficult to quantify from an experiment. The advantage of this method over existing methods is that this procedure does not require one to monitor the whole history of fatigue loading. The method is capable of predicting the evolution of plastic strain energy and entropy generation.

Chapter 3 presents the application of the concept of FFE in characterizing the fatigue behavior of metallic specimens with stress concentration. The effect of stress concentration on the plastic strain energy and entropy generation is discussed. The stress concentration is introduced by producing a V-notch in the middle of the gage section of a solid cylindrical specimen. Finite element simulations of the fatigue experiments are carried out for the calculation of the FFE. Results reveal that the presence of stress concentration decreases the FFE of a material significantly. A correlation is proposed for the prediction of the fatigue life of a V-notched specimen based on the experimental results of an un-notched specimen made of identical material. A series of validation test results are presented for the assessment of the fatigue life prediction capability of the proposed correlation.

A method for the calculation of anelastic energy in a hysteresis loop and FFE associated with an HCF test of metallic specimens is described in Chapter 4. As the level of stress in HCF is considerably less than the yield strength of a material, the area of a hysteresis loop is small that consists of non-damaging anelastic energy. It is shown that a phase lag between the stress and strain in HCF causes anelastic energy in the hysteresis loops. In order to study the HCF behavior of a material in terms of thermodynamic entropy approach, it is essential to determine the amount of plastic strain energy present in a hysteresis loop. A methodology is proposed in this work to calculate anelastic energy contained within a hysteresis loop that allows one to determine entropy generation. It is found that the FFE of carbon steel 1018 remains in a small band in HCF.

Chapter 5 presents the development of a correlation between the present material degradation and corresponding value of $R_\theta$ for the prediction of the RFL of a metallic specimen with prior history of loading in an NDT fashion. The gradual increase of $R_\theta$ evolution due to the accumulation of degradation is utilized as a fatigue parameter. It is shown that the trend of $R_\theta$ is linear with
respect to the load cycle over a wide range of loading conditions for two different materials. A series of experimental results are presented for the characterization and validation of the proposed methodology.

The application of the RFL prediction method presented in Chapter 5 to a Glass/Epoxy composite laminate is presented in Chapter 6. This work consists of both uniaxial tension-compression and completely-reversed bending fatigue tests subject to both constant- and variable-amplitude loading. As the damage evolution of a composite laminate is different compared to a metal, additional analysis is performed to assess the appropriateness of this method for this material. It is found that $R_\theta$ evolves linearly with respect to the number of load cycle performed on this material. The results of the validation tests show that this method can predict RFL of a composite laminate with reasonable accuracy.

In Chapter 7, a correlation is derived to establish a relationship between the CDM-based damage parameter, $D$, and thermography-based degradation parameter, $R_\theta$, associated with a specimen with prior fatigue damage. This work considers both constant- and variable-amplitude loading conditions. Results obtained from the proposed correlation are compared to those obtained from the pertinent formulas available in open literature. This correlation and a master curve of damage evolution are utilized to develop a methodology for the estimation of the RFL of a metallic specimen. Validation test results are presented to assess the applicability of the methodology.

A summary of all the works described in this dissertation and recommendations for future applications of the methodologies developed are presented in Chapter 8.

1.3. References


Chapter 2: Rapid Estimation of Fatigue Entropy and Toughness in Metals*

2.1. Introduction

Fatigue is the most dominant mode of failure in components that are subjected to tension-compression in cyclic manner where damage tends to accumulate as a function of time. The rate of progressive damage depends on many factors such as the stress level, \( \sigma \), load ratio, \( L_R \) (defined as the ratio of minimum to the maximum stress in a load cyclic), test frequency, \( f \), ambient temperature, \( T_a \), humidity, material properties, etc. If the stress is high enough to develop irreversible plastic deformation, then plastic strain energy is generated. Most of this energy is converted into heat and dissipates to the surroundings [1-3]. According to the second law of thermodynamics, the process of irreversible plastic strain energy generation inherently produces thermodynamic entropy, \( \gamma \), [4]. As the cyclic load continues, the plastic strain energy and the entropy accumulate monotonically. Fargione et al. [5], Risitano and Risitano [6], and Fan et al. [7-8] showed that fatigue fracture occurs when the amount of plastic strain energy generation reaches the fatigue toughness, \( W_f \), (also known as the critical energy) of the material. Research shows that fatigue fracture occurs when the accumulation of entropy generated —starting with a pristine specimen and ending with fatigue fracture— reaches a limiting value regardless of the loading sequence [9-13]. The constant value of entropy at fatigue fracture is referred to as the fatigue fracture entropy (FFE), \( \gamma_f \). This property has been successfully put to use in structural health monitoring system [10]. Development of methodologies for estimating these parameters is useful for the prediction of the present state of material damage.

Experimental results of Meyendorf et al. [14] and Walther and Eifler [15] successfully provided evidence that the level of material damage due to cyclic loading can be predicted by measuring the value of temperature increase, \( \Delta T \), with respect to the ambient temperature, \( T_a \). The level of material damage can also be suitably quantified by measuring the slope of temperature rise, \( R_\theta \), obtained from the specimen gage section through a series of short-time excitation (STE) tests that typically last 10-15 seconds [16-19]. Research shows that \( R_\theta \) increases linearly with respect to the accumulated fatigue load cycle, \( N \), which is found to be useful for predicting the remaining fatigue life of welded- and unwelded-metallic specimens [16, 18, 19].

Material damage can also be predicted by measuring the variation in the key mechanical properties —e.g., yield strength, modulus of elasticity, tensile strength, hardness, stiffness, static toughness, etc. For instance, Abraham et al. [20], Belaadi et al. [21], and Zhou et al. [22] showed that the modulus of elasticity decreases gradually with the accumulation of fatigue damage in different materials. Li et al. [23] revealed that the remaining static strength of self-piercing riveted aluminum joints can be estimated by measuring the reduction in the modulus of elasticity. Azadi

*Reprinted by permission of Materials & Design (See Appendix A)
and Shirazabad [24] demonstrated that fatigue damage affects the stress-strain response of an aluminum alloy. Azadi et al. [25] and Navaro and Gamez [26] showed that fatigue damage is represented by monotonic accumulation of the total plastic strain energy generation. Thus, researchers have found that monitoring and assessing the evolution of these material properties with time are useful for the characterization of the fatigue behavior of the materials.

In this work, a method is developed to rapidly estimate $\gamma_f$ and $W_f$ utilizing the evolution of $R_\theta$. A series of uniaxial tension-compression fatigue tests are performed with standard specimens made of low-carbon steel (LCS) 1018, medium-carbon steel (MCS) 1045, API 5L X52, and Al 6061 to assess the validity of the proposed method by comparing the results obtained from the proposed method to those measured from the experiments.

### 2.2. Experimental Details

#### 2.2.1. Materials and Equipment

Figure 2.1(a) illustrates the schematic and the dimensions of the tubular specimens made of LCS 1018 and Figure 2.1(b) and Table 2.1 present the schematic and the dimensions of the solid cylindrical specimens made of MCS 1045, API 5L X52 (a high-strength steel), and Al 6061. The specimens are produced from cold-drawn tubes and rods. The specifications of the specimens are

![Schematic illustrations and dimensions](image)

**Figure 2.1.** Schematic illustrations and dimensions of (a) tubular specimens made of LCS 1018 and (b) solid cylindrical specimens made of MCS 1045, API 5L X52, and Al 6061 (all dimensions are in millimeter)
in accordance with the ASTM: E466-07. The gage section of the specimens are polished using sand papers progressing through 600, 800, 1200, 1500, and 2000 grit sizes to reduce the surface roughness to within $R_a = 0.2 \, \mu m$.

<table>
<thead>
<tr>
<th>Material</th>
<th>$l_1$</th>
<th>$l_2$</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$r_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCS 1045</td>
<td>38.5</td>
<td>24.0</td>
<td>15.87</td>
<td>9.0</td>
<td>65.0</td>
</tr>
<tr>
<td>API 5L X52</td>
<td>38.1</td>
<td>25.4</td>
<td>14.0</td>
<td>10.0</td>
<td>80.0</td>
</tr>
<tr>
<td>Al 6061</td>
<td>38.1</td>
<td>24.0</td>
<td>15.0</td>
<td>8.0</td>
<td>62.0</td>
</tr>
</tbody>
</table>

Constant-amplitude, stress-controlled fatigue tests are carried out at the frequency of 10 Hz using an axial-torsion, servo-hydraulic fatigue tester with the capability of a maximum of 50 kN axial load and 75 Hz of frequency. An extensometer with the gauge length of 25.4 mm and travel between -10% and +50% strain is used to measure the strain in the gage section of the specimen during fatigue test. The surface temperature of the specimen gage section is recorded using a high-speed infrared (IR) camera with the resolution of 320×240 pixel, accuracy of ±2% of reading, temperature range capability between 0°C and 500°C, sensitivity of 0.08°C (at 30°C). In order to reduce IR reflection and increase thermal emissivity, a thin layer of black paint is sprayed on the gage section of the specimen. Specimen surface temperature is recorded over the entire gage section. Since the maximum temperature occurs in the middle of the gage section, average temperature over approximately 5 mm long line at that location is used in the analysis, see Figure 2.2.

Figure 2.2. A thermal image of the specimen captured by IR camera showing the temperature contour and the location of temperature measurement
2.2.2. Experimental Procedure

Figure 2.3 illustrates a specimen vertically gripped between the jaws attached to the top and the bottom grips of the fatigue tester, an extensometer mounted on the specimen gage section, and an IR camera positioned to capture the temperature contour on the specimen surface. The bottom grip oscillates vertically to apply cyclic axial load on the specimen and the top grip remains stationary. The camera is positioned to directly face the gage section of the specimen at a distance of approximately 35 cm between the specimen gage section and the lens of the camera. The fatigue tester, the extensometer, and the camera directly interface with a computer equipped with real time data acquisition capability.

Figure 2.3. Experimental set-up for the uniaxial fatigue test showing the extensometer, the IR camera, and a specimen gripped between the jaws

The experimental procedure is as follows. Beginning with a pristine specimen, first, fatigue test is performed for an arbitrary number of load cycles, $N_1$, under specific loading conditions (see Figure 2.4). The fatigue test is then stopped and the specimen is cooled down to the ambient temperature. Then, the slope of temperature rise $R_{\theta 1}$ is measured by carrying out an STE test (approximately 15 s) with identical loading conditions to those used in each fatigue test. The test is resumed for $N_2$ number of load cycles and the specimen is allowed to cool down to the ambient temperature. The slope of the temperature rise $R_{\theta 2}$ is measured by a subsequent STE test. The above procedure is repeated until fracture occurs. Figure 2.4 illustrates the test procedure in terms of the temperature evolution where three STE tests are performed in a fatigue test with API 5L X52 specimen at $\sigma = 456$ MPa, $L_R = -0.5$, and $f = 10$ Hz.

2.3. Entropy and Energy Generation in Fatigue

Following the work of Naderi et al. [9], in this section we develop a model for the prediction of thermodynamic entropy generation in the fatigue of metals. In the absence of internal heat source,
the time rate of strain energy generation, $\sigma : \dot{\varepsilon}$, is correlated to the heat flow, $q$, and the time rate of specific internal energy, $\dot{e}$, associated with a control volume by the principle of the conservation of energy as postulated by the first law of thermodynamics [4]:

$$\rho \dot{e} = \sigma : \dot{\varepsilon} - \text{div } q$$  \hspace{1cm} (2.1)

where $\rho$ is the density, $\sigma$ represents the stress tensor, and $\dot{\varepsilon}$ stands for the time rate of strain tensor. The total strain rate $\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p$ where $\dot{\varepsilon}^e$ is the time rate of elastic strain tensor that does not contribute towards the heat generation and $\dot{\varepsilon}^p$ is the time rate of plastic strain tensor that is related to the heat generation. Thus, neglecting the effect of $\dot{\varepsilon}^e$ Equation (2.1) can be expressed as follows [4]:

$$\sigma : \dot{\varepsilon}^p = \rho C T - k \nabla^2 T + A_i \dot{V}_i - T \left( \frac{\partial s}{\partial T} : \dot{\varepsilon}^e + \frac{\partial A_i}{\partial T} \dot{V}_i \right)$$  \hspace{1cm} (2.2)

where $C = T \frac{\partial s}{\partial T}$ stands for the specific heat, $T$ denotes the absolute temperature, $s$ signifies the specific entropy, $k$ is the thermal conductivity, $\nabla^2$ denotes the Laplacian operator, $V_i$ represents the internal variables associated with the microstructure of the material (where $i$ is the number of internal variables), $A_i = \rho \frac{\partial \Psi}{\partial V_i}$ represents the thermodynamic forces associated with $V_i$, and $\Psi$ is the specific free energy.

According to the second law of thermodynamics, the time rate of the volumetric entropy production, $\dot{\gamma}$, associated with a deformed body is always greater than or equal to the time rate of heating divided by the temperature expressed by the Clausius-Duhem inequality reads [4]:

$$\dot{\gamma} \geq \frac{\dot{q}}{T}$$
\[ \dot{\gamma} = \frac{\sigma \dot{\varepsilon}^p}{T} - \frac{q}{T^2} \cdot \text{grad} T - \frac{A_i \dot{V}_i}{T} \geq 0 \]  

Equation (2.3) shows that the entropy generation consists of the plastic strain energy dissipation due to the plastic deformation, \( \sigma: \dot{\varepsilon}^p \), thermal dissipation owing to the heat conduction, \( \frac{q}{T} \cdot \text{grad} T \), and the dissipation associated with the variation in the internal variables, \( A_i \dot{V}_i \). Research shows that the entropy generation due to \( A_i \dot{V}_i \) represents only 5-10% of that owing to \( \sigma: \dot{\varepsilon}^p \) and thermomechanical coupling effect, \( \frac{\partial \sigma}{\partial T} : \dot{\varepsilon}^e \), is negligible compared to the mean temperature rise [4, 9]. Therefore, it is assumed that \( A_i \dot{V}_i \approx 0 \), \( \frac{\partial \sigma}{\partial T} : \dot{\varepsilon}^e \approx 0 \), and \( \frac{\partial A_i}{\partial T} \dot{V}_i \approx 0 \). Thus, Equations (2.2) and (2.3) simplify as, respectively:

\[ k \nabla^2 T = \rho C \dot{T} - \dot{W}_p \]

\[ \dot{\gamma} = \frac{W_p}{T} - \frac{q}{T^2} \cdot \text{grad} T \]

where \( \dot{W}_p = \sigma : \dot{\varepsilon}^p \) denotes the time rate of plastic strain energy generation. The FFE, \( \gamma_f \), is estimated by integrating Equation (2.5) starting from the beginning, i.e., \( t=0 \), up to the time, \( t_f \), when fracture occurs:

\[ \gamma_f = \int_0^{t_f} \left( \frac{W_p}{T} \right) dt - \int_0^{t_f} \left( \frac{q}{T^2} \cdot \text{grad} T \right) dt \]

Equation (2.6) implies that the FFE can be determined if the amount of plastic strain energy generation, temperature evolution, and entropy flow due to heat conduction in a fatigue process are known.

2.4. Numerical Simulations

2.4.1. Computational Model

Numerical simulations are performed using a commercial code, FlexPDE, that utilizes the finite element method (FEM) to solve partial differential equations (PDE) in the 3-D model. Since both tubular and solid specimens are symmetric with respect to their longitudinal axis, only half of the specimen is simulated that contains an end section, a curved-section, and half of the gage section. In order to produce better quality of mesh, the curved-section of the specimen is modeled as a trapezoid neglecting the small amount of curvature. Figure 2.5(a and b) shows the photograph of an API 5L X52 specimen and its 3-D model meshed with 10-node quadratic tetrahedral elements.

2.4.2. Boundary Conditions

Referring to Figure 2.5(b), boundary B1 exchanges heat with the surroundings by convection and radiation. Since boundary B2 represents the plane of symmetry, heat transfer does not take place through this boundary. Therefore, this boundary is considered to be insulated. Identical
boundary conditions are used in the simulations of tubular LCS 1018, MCS 1045, and Al 6061 specimens. The expression for the boundary condition associated with the surfaces denoted by B1 is as follows:

$$k \frac{\partial T}{\partial r} = h(T - T_a) + \sigma_0 \varepsilon_0 (T^4 - T_a^4)$$

(2.7)

where \( r \) represents surface normal parameter, \( T \) and \( T_a \) denote specimen surface and ambient temperature, respectively, \( h \) stands for the heat transfer coefficient, \( \sigma_0 \) signifies the Stephan-Boltzmann constant, and \( \varepsilon_0 \) represents the surface emissivity. \( T_a \) is measured in the laboratory at the time of each experiment and \( \varepsilon_0 \) and \( \sigma_0 \) are set to be 0.93 [9] and \( 5.67 \times 10^{-8} \) Wm\(^{-2}\)K\(^{-4}\), respectively. The value of \( h \) is calculated from the following expression which is applicable for the vertical surface associated with forced convection [27]:

$$h = \frac{0.664 k_a (N_{PR})^{1/3}}{L} \sqrt{\frac{UL}{v}}$$

(2.8)
where $L$ is the effective length of the specimen gage section surface, $N_{PR}$ stands for the Prandtl number, and $U$, $k_a$, $\nu$ denote velocity, thermal conductivity, and kinematic viscosity of air, respectively. For the simplicity of analysis, it is assumed that $U = 1$ m/s around the specimen during fatigue tests. Since the ambient temperature in the laboratory is maintained around 20°C, the value of $h$ is calculated using the air properties at that temperature. Temperature evolution is determined from the simulation by solving Equation (2.4) in the entire 3-D model using hysteresis energy per cycle, $\Delta W_p$, obtained from experiment (see Figure 2.6(a, b, and c)) and the physical and the thermal properties of the given materials as shown in Table 2.2.

Figure 2.6(a). Evolution of $\Delta W_p$ in fatigue and STE test of LCS 1018 ($\sigma = 468$ MPa, $L_R = -0.7$)

Figure 2.6(b). Evolution of $\Delta W_p$ in fatigue and STE test of API 5L X52 ($\sigma = 456$ MPa, $L_R = -0.5$)
Table 2.2. Physical and thermal properties of LCS 1018, MCS 1045, API 5L X52, and Al 6061

<table>
<thead>
<tr>
<th>Material</th>
<th>Density, $\rho$ (kgm$^{-3}$)</th>
<th>Specific heat, $C$ (Jkg$^{-1}$K$^{-1}$)</th>
<th>Thermal conductivity, $k$ (Wm$^{-1}$K$^{-1}$)</th>
<th>Thermal diffusivity (mm$^2$s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCS 1018 &amp; MCS 1045</td>
<td>7870</td>
<td>486</td>
<td>51</td>
<td>13.33</td>
</tr>
<tr>
<td>API 5L X52</td>
<td>7800</td>
<td>480</td>
<td>43</td>
<td>11.48</td>
</tr>
<tr>
<td>Al 6061</td>
<td>2700</td>
<td>896</td>
<td>164</td>
<td>67.79</td>
</tr>
</tbody>
</table>

2.5. Results and Discussion

2.5.1. Effect of Heat Conduction on the Entropy Generation

Table 2.3 presents the comparison between the FFE and the entropy generation due to heat conduction for a fatigue test performed with each of the given materials. Results show that the entropy generation due to heat conduction is negligible compared to the total entropy generation in a fatigue test. Entropy generation due to heat conduction depends on the amount of heat dissipation through the ends of a specimen. Figure 2.2 shows that the maximum temperature occurs in the middle of the specimen gage section, and it drops to roughly the ambient temperature at the end of the specimen’s reduced section. This indicates that most of the heat energy generated in the gage section of a specimen is dissipated to the surroundings through convection and radiation and the amount of heat energy dissipated through the ends of a specimen via conduction is negligible.
### Table 2.3. Comparison between the FFE and the entropy generation due to heat conduction

<table>
<thead>
<tr>
<th>Material</th>
<th>Stress, $\sigma$ (MPa)</th>
<th>Load ratio, $L_R$</th>
<th>Fatigue life, $N_f$ (cycle)</th>
<th>FFE, $\gamma_f$ (MJm$^{-3}$K$^{-1}$)</th>
<th>Entropy generation due to heat conduction (MJm$^{-3}$K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCS 1018</td>
<td>492</td>
<td>-0.7</td>
<td>7843</td>
<td>20.92</td>
<td>0.26</td>
</tr>
<tr>
<td>MCS 1045</td>
<td>537</td>
<td>-0.6</td>
<td>9528</td>
<td>25.11</td>
<td>0.42</td>
</tr>
<tr>
<td>API 5L X52</td>
<td>465</td>
<td>-0.5</td>
<td>9561</td>
<td>24.47</td>
<td>0.21</td>
</tr>
<tr>
<td>Al 6061</td>
<td>215</td>
<td>-1</td>
<td>5319</td>
<td>8.15</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Thus, the irreversible plastic strain energy generation is the major source of entropy generation. Therefore, neglecting the terms associated with heat conduction from Equation (2.6) simplify as:

$$\gamma_f = \int_0^{N_f} \left( \frac{\Delta W_p}{T} \right) \, dN$$  \hspace{1cm} (2.9)

where $N_f$ stands for the fatigue life cycle.

### 2.5.2. Entropy Generation in Fatigue Test and STE Test

The evolution of temperature in a fatigue process is typically characterized by three distinct phases: an initial rapid temperature rise (Phase I), an intermediate “steady-state” temperature (Phase II), and a final sharp temperature rise (Phase III). This trend in temperature evolution has been found useful in different ways to predict the fatigue behavior of the materials. For example, some authors use the slope of initial temperature rise $R_\theta$ (e.g., [13, 17-19, 28, 29]), others rely on the steady state temperature in Phase-II (e.g., [30, 31]), and still others utilize the slope of sharp temperature rise at the onset of Phase-III (e.g., [32]) as indices for characterizing fatigue life. Results obtained from the fatigue tests performed in this work demonstrate that the value of $T$ and $\Delta W_p$ increase rapidly in Phase-I, then their values rise gradually in Phase-II followed by another sharp rise at the onset of Phase-III. These are illustrated in Figures 2.4, 2.6, and 2.7. A rapid cyclic softening is a common phenomenon in the early stage of a fatigue process for cold-drawn materials [33, 34]. Since most of the plastic strain energy generated in Phase I is consumed by the specimen to increase its temperature, a rapid rise in temperature is observed. The increase in $\Delta W_p$ evolution is a consequence of the gradual increase in the area of hysteresis loops as illustrated in Figure 2.8 for LCS 1018 subjected to constant amplitude loading. It shows the height of the hysteresis loops remains fairly constant while their width increases steadily. This occurs due to the simultaneous effects of the accumulation of irreversible microstructural damages —e.g., dislocations, slip bands, etc.— and strain softening of the material due to cyclic loading [33-36]. If a material’s microstructures are such that the application of cyclic load does not increase the density of dislocations, then gradual softening occurs [37]. It can be seen that the effect of strain softening in Al 6061 is less than those of other materials tested in this study (see Figure 2.7(c)). As the tensile
Figure 2.7. Evolution of temperature during fatigue test and STE tests performed with LCS 1018 at $\sigma = 468$ MPa ($L_R = -0.7$)

Figure 2.8. Representation of the hysteresis loops after different number of accumulated load cycles obtained from a fatigue test performed with LCS 1018 at $\sigma = 468$ MPa ($L_R = -0.7$)
and yield strength of this material is significantly lower than the other materials, smaller stress levels are applied in the fatigue tests of this material. While the stress levels associated with the fatigue tests of Al 6061 is around 200 MPa, those are between 400 and 537 MPa for the other materials tested in this work. Research shows that the effect of strain softening is less at low stress levels [36]. Phase II of the fatigue process takes up most of the fatigue life within the range of the loading conditions considered in this work. In Phase III, a macrocrack develops in the material and its propagation generates larger amount of energy which converts into heat and manifests itself by a sharp rise in the evolutions of $T$ and $\Delta W_p$. The span of this phase is substantially shorter than that of Phase II. Similar results are obtained with MCS 1045.

Figure 2.6(a, b, and c) shows that the value of $\Delta W_p$ corresponding to the STE test is smaller than that of the corresponding fatigue test. Comparison between the amount of plastic strain energy generation in an STE test and corresponding fatigue test is shown in section 2.7. The variation in $\Delta W_p$ occurs due to the effect of rapid softening at the beginning of cyclic loading. Since STE test is performed for a short period of time, the amount of plastic strain energy generated in this test is smaller than that of the gradual softening regime associated with the fatigue test (see Figure 2.6(a)). Thus, the rate of entropy generation during the fatigue test, $\dot{\gamma}$, is greater than that during an STE test, $\dot{\gamma}_{STE}$. In order to estimate $\dot{\gamma}_{STE}$, the value of $\dot{W}_p$ from Equation (2.4) and $\dot{T} = R_\theta$ are substituted into Equation (2.5) to arrive at the following relationship:

$$\dot{\gamma}_{STE} = \frac{\rho c R_\theta}{T_{STE}}$$

(2.10)

where $T_{STE}$ is the specimen temperature in STE test. Letting $\beta = \dot{\gamma}/\dot{\gamma}_{STE}$, Equation (2.10) reads:

$$\dot{\gamma} = \beta \frac{\rho c R_\theta}{T_{STE}}$$

(2.11)

$R_\theta$ is a linear function of $N$ expressed as (Liakat et al., 2013):

$$R_\theta = nN + R_{\theta0}$$

(2.12)

where $n$ and $R_{\theta0}$ stand for the slope and the intercept of $R_\theta - N$ plot, respectively. Substituting $R_\theta$ from Equation (2.12) into Equation (2.11) and integrating over the entire fatigue process (starting from $N=0$ up to $N=N_f$) yields the following expression for estimating the total fatigue fracture entropy (FFE):

$$\gamma_f = \frac{\beta \rho c}{2T_{STE} f} \left( N_f^2 + 2R_{\theta0}N_f \right)$$

(2.13)
Equation (2.13) provides a relationship for the estimation of the FFE in a fatigue process based on the evolution of $R_\theta$ obtained from STE tests if the test conditions, material properties, and $\beta$ are known.

2.5.3. Estimation of FFE Based on $\Delta W_p$ and $R_\theta$ Approaches

Figure 2.9(a and b) presents the evolutions of $R_\theta$ obtained from the STE tests performed with all four materials. Results demonstrate that $R_\theta$ increases in a linear fashion with respect to $N$ regardless of the material properties and loading conditions. Irreversible plastic deformation accumulates and microstructural changes occur monotonically in the material [38-39], thus generating progressively greater amount of heat energy during each STE test. The higher the stress level, the greater is $R_\theta$ since a higher stress produces plastic deformation and microstructural changes in the material at a greater rate compared to those associated with a smaller stress level. It can be seen that the rate of $R_\theta$ evolution associated with Al 6061 is substantially smaller than that corresponding to the other materials. Since Al 6061 has considerably different physical and thermal properties (see Table 2.2) and the applied stresses are smaller than those associated with the other materials studied in this work, smaller $R_\theta$ evolution is observed for this material.

Figures 2.10 and 2.11 present a comparison between the evolutions of $\dot{\gamma}$ obtained from experimental hysteresis area and temperature along with its predictions obtained from Equation (2.11) for all four materials. The value of $\beta$ is calculated from Equation (2.11) and found to be about 1.35 for LCS 1018, MCS 1045, and API 5L X52 and roughly 4.0 for Al 6061 within the range of loading conditions considered in this work. A greater value of thermal diffusivity yields a faster rate of diffusion of heat via conduction and a lower $R_\theta$ value in an STE test which corresponds to a greater value of $\beta$. It can be seen that $\dot{\gamma}$ obtained from both of the methods are in good agreement except at the onset of fracture. The rate of entropy generation is greater in Phase III because of the larger amount of heat generation due to the initiation and propagation of a macrocrack. Since $R_\theta$ is not measured in Phase III, Equation (2.11) does not model the greater rate in entropy generation in this phase.

Table 2.4 presents the values of the FFE obtained from $\Delta W_p$ and $R_\theta$ approaches and corresponding relative error for all the tests performed in this work. Results demonstrate that the predicted and the experimental values of the FFE are in good agreement. The FFE of LCS 1018, MCS 1045, and API 5L X52 is found to be around 25 MJm$^{-3}$K$^{-1}$ and that of Al 6061 is about 8 MJm$^{-3}$K$^{-1}$ independent of loading conditions. These results conform to the finding of Naderi et al. [9]. The comparable values of FFE for LCS 1018, MCS 1045, and API 5L X52 demonstrate that these materials have comparable resistance to fatigue load subject to similar loading and environmental conditions. The smaller value of FFE for Al 6061 indicates that this material is less resistant to fatigue load compared to the other materials tested. Thus, by knowing the FFE of a material, it is possible to determine the suitability of that material for a specific fatigue application.
Results imply that the necessary and sufficient condition for fatigue fracture corresponds to the entropy accumulation up to the FFE of that material. Thus, it is possible to develop a fatigue monitoring unit based on the concept of tallying entropy for the prevention of catastrophic fracture. Such a unit was developed by Naderi and Khonsari [10] for completely reversed bending fatigue.

Figure 2.9. Evolutions of $R_\theta$ obtained from STE tests at different stress levels (a) LCS 1018 and MCS 1045 and (b) API 5L X52 and Al 6061
Figure 2.10. Comparison between $\dot{\gamma}$ obtained from $\Delta W_p$ and $R_\theta$ approaches corresponding to LCS 1018 and MCS 1045

Figure 2.11. Comparison between $\dot{\gamma}$ obtained from $\Delta W_p$ and $R_\theta$ approaches corresponding to API 5L X52 and Al 6061
Table 2.4. Comparison between the FFES obtained from $\Delta W_p$ and $R_\theta$ approaches for LCS 1018, MCS 1045, API 5L X52, and Al 6061

<table>
<thead>
<tr>
<th>Material</th>
<th>Stress, $\sigma$ (MPa)</th>
<th>Load ratio, $L_R$</th>
<th>Fatigue life, $N_f$ (cycle)</th>
<th>FFE (MJm$^{-3}$K$^{-1}$)</th>
<th>Relative Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\Delta W_p$ approach</td>
<td>$R_\theta$ approach</td>
</tr>
<tr>
<td>LCS 1018</td>
<td>492</td>
<td>-0.7</td>
<td>7843</td>
<td>20.92</td>
<td>22.07</td>
</tr>
<tr>
<td></td>
<td>482</td>
<td>-0.7</td>
<td>9429</td>
<td>22.44</td>
<td>21.89</td>
</tr>
<tr>
<td></td>
<td>468</td>
<td>-0.7</td>
<td>16120</td>
<td>28.86</td>
<td>26.21</td>
</tr>
<tr>
<td>MCS 1045</td>
<td>537</td>
<td>-0.6</td>
<td>9528</td>
<td>25.11</td>
<td>24.36</td>
</tr>
<tr>
<td></td>
<td>527</td>
<td>-0.6</td>
<td>13804</td>
<td>27.04</td>
<td>24.73</td>
</tr>
<tr>
<td>API 5L X52</td>
<td>465</td>
<td>-0.5</td>
<td>9561</td>
<td>24.47</td>
<td>26.12</td>
</tr>
<tr>
<td></td>
<td>456</td>
<td>-0.5</td>
<td>11023</td>
<td>23.67</td>
<td>24.50</td>
</tr>
<tr>
<td></td>
<td>440</td>
<td>-0.5</td>
<td>18011</td>
<td>27.92</td>
<td>26.88</td>
</tr>
<tr>
<td>Al 6061</td>
<td>215</td>
<td>-1</td>
<td>5319</td>
<td>8.15</td>
<td>7.76</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>-0.7</td>
<td>9413</td>
<td>7.52</td>
<td>6.96</td>
</tr>
</tbody>
</table>

2.5.4. Prediction of $\Delta W_p$ and $W_f$

To obtain the entropy generation per cycle at $N^{th}$ load cycle, $\dot{\gamma}_N$, Equation (2.11) is expressed as:

$$\dot{\gamma}_N = \beta \frac{\rho C R_\theta}{f_{STE}}$$  \hspace{1cm} (2.14)

According to Equation (2.9), it can be written that $\dot{\gamma}_N = \frac{\Delta W_p}{T}$. Substituting this into Equation (2.14) $\Delta W_p$ is expressed as:

$$\Delta W_p = \beta \frac{\rho C R_\theta T}{f_{STE}}$$  \hspace{1cm} (2.15)

Thus, fatigue toughness is estimated by integrating Equation (2.15) starting from the beginning, i.e., $N=0$, up to the fracture, i.e., $N=N_f$:

$$W_f = \beta \frac{\rho C}{f_{STE}} \int_{0}^{N_f} (R_\theta T) \, dN$$  \hspace{1cm} (2.16)

Equations (2.15) and (2.16) imply that $\Delta W_p$ and $W_f$ associated with a fatigue test can be predicted if the evolution of $R_\theta$ and $T$ are known. Figure 2.12(a and b) illustrates the comparison between $\Delta W_p$ evolutions obtained from experiment and Equation (2.16) for all four materials tested. It can be seen that the predicted results are in good agreement to those obtained from the experiments.

...
Figure 2.12(a). Comparison between the evolutions of $\Delta W_p$ obtained from experiment and $R_\theta$ approach for LCS 1018 and MCS 1045

Figure 2.12(b). Comparison between the evolutions of $\Delta W_p$ obtained from experiment and $R_\theta$ approach for API 5L X52 and Al 6061
except in Phases I and III. It should be noted that the proposed model predicts $\Delta W_p$ evolution for the gradual softening and damage progression regime and does not model the initial rapid softening and macrocrack propagation regimes.

Table 2.5 presents the comparison between the values of $W_f$ obtained from experiments and $R_\theta$ approach and corresponding relative error for all the tests performed in this work. The integral part in the right hand side of Equation (2.16) is determined numerically. The results demonstrate that the fatigue toughness of each material is within a small band that conforms to the concept of the critical energy accumulation at fatigue fracture [5-8]. The comparison between the predicted and the measured values of $W_f$ for each test shows good agreement with the maximum relative error of 7.19%. This shows the usefulness of the proposed method in calculating fatigue toughness.

Table 2.5. Comparison between the fatigue toughness obtained from experiments and $R_\theta$ approach for LCS 1018, MCS 1045, API 5L X52, and Al 6061 at different stress levels

<table>
<thead>
<tr>
<th>Material</th>
<th>Stress, $\sigma$ (MPa)</th>
<th>Load ratio, $L_R$</th>
<th>Fatigue life, $N_f$ (Cycle)</th>
<th>Fatigue toughness, $W_f$ (MJ/m$^3$)</th>
<th>Relative Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\Delta W_p$ approach</td>
<td>$R_\theta$ approach</td>
</tr>
<tr>
<td>LCS 1018</td>
<td>492</td>
<td>-0.7</td>
<td>7843</td>
<td>6653.92</td>
<td>7018.15</td>
</tr>
<tr>
<td></td>
<td>482</td>
<td>-0.7</td>
<td>9429</td>
<td>7248.29</td>
<td>7108.05</td>
</tr>
<tr>
<td></td>
<td>468</td>
<td>-0.7</td>
<td>16120</td>
<td>9180.23</td>
<td>8561.33</td>
</tr>
<tr>
<td>MCS 1045</td>
<td>537</td>
<td>-0.6</td>
<td>9528</td>
<td>7104.72</td>
<td>6990.27</td>
</tr>
<tr>
<td></td>
<td>527</td>
<td>-0.6</td>
<td>13804</td>
<td>7483.97</td>
<td>7221.63</td>
</tr>
<tr>
<td>API 5L X52</td>
<td>465</td>
<td>-0.5</td>
<td>9561</td>
<td>9126.15</td>
<td>9388.54</td>
</tr>
<tr>
<td></td>
<td>456</td>
<td>-0.5</td>
<td>11023</td>
<td>8115.91</td>
<td>8700.00</td>
</tr>
<tr>
<td></td>
<td>440</td>
<td>-0.5</td>
<td>18011</td>
<td>8879.16</td>
<td>8773.94</td>
</tr>
<tr>
<td>Al 6061</td>
<td>215</td>
<td>-1</td>
<td>5319</td>
<td>2434.33</td>
<td>2265.79</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>-0.7</td>
<td>9413</td>
<td>2245.58</td>
<td>2127.82</td>
</tr>
</tbody>
</table>

Since the proposed method is developed based on the non-contact, non-destructive thermography technique, its application does not require measurement devices such as load cell and extensometer or strain gage. Instead of monitoring the whole history of loading [40-41], the proposed method can predict fatigue entropy and plastic strain energy by performing three or four STE tests.

It should be noted that the applicability of the proposed method is validated on the pristine specimens without stress concentration and prior history of loading and within the experimental
and environmental conditions stated in this work. Therefore, the application of the method to the
components with stress concentration and prior history of loading under variable experimental and
environmental conditions requires further work.

2.6. Conclusions

A procedure for the prediction of the rate and the accumulation of thermodynamic entropy and
plastic strain energy generation in a fatigue process is presented. Series of uniaxial tension-
compression fatigue tests are performed with cylindrical dogbone specimens made of tubular LCS
1018 and solid MCS 1045, API 5L X52, and Al 6061 to determine the validity of the proposed
method. The results of the numerical simulations using a finite element based commercial code show
that entropy generation due to heat conduction is negligible compared to that due to plastic
strain energy. It is shown that the proposed method can predict the rate and the accumulation of
entropy and plastic strain energy generation of metallic specimens with good accuracy within the
range of experimental and environmental conditions considered in this work. The application of
this method does not require one to monitor the entire fatigue process. Instead, assessment of the
entropy and the plastic strain energy generation can be performed by carrying out three or four
STE tests. The proposed method does not model the rapid softening and the larger amount of
entropy and plastic strain energy generation due to the propagation of a macrocrack at the onset of
the fatigue test and fracture, respectively. The validation of the developed method is performed on
the pristine specimens without stress concentration and prior history of loading under the
experimental and environmental conditions stated in the work. Hence, additional research is
required to determine the validity of this work to the components with prior history of loading and
stress concentration under variable experimental and environmental conditions.

2.7. Additional Details

To compare the amount of plastic strain energy generation in fatigue test and that in
corresponding STE test, the area of 100 hysteresis loops before each stop of the fatigue test and at
the onset of corresponding STE test are considered. Since \( R_\theta \) is measured based on the temperature
evolution for 10 s, 100 load cycles are performed over that time. Table 2.6 presents the ratio

<table>
<thead>
<tr>
<th>Material</th>
<th>Ratio of the area of 100 hysteresis loops in fatigue test to that of corresponding STE test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1\textsuperscript{st} stop</td>
</tr>
<tr>
<td>LCS 1018</td>
<td>1.56</td>
</tr>
<tr>
<td>Al 6061</td>
<td>2.08</td>
</tr>
</tbody>
</table>
between these two energy values at each stop for one test performed with LCS 1018 at \( \sigma = 468 \) MPa \((L_R = -0.7)\) and another test carried out with Al 6061 at \( \sigma = 215 \) MPa \((L_R = -1)\).

### 2.8. Nomenclature

- \( A_l \): thermodynamic forces
- \( C \): specific heat \((\text{JKg}^{-1}\text{K}^{-1})\)
- \( d \): diameter (mm)
- \( e \): specific internal energy \((\text{JKg}^{-1})\)
- \( f \): frequency (Hz)
- \( h \): heat transfer coefficient \((\text{Wm}^{-2}\text{K}^{-1})\)
- \( k \): thermal conductivity \((\text{Wm}^{-1}\text{K}^{-1})\)
- \( k_a \): thermal conductivity of air \((\text{Wm}^{-1}\text{K}^{-1})\)
- \( l \): length (mm)
- \( L \): effective length (m)
- \( L_R \): load ratio
- \( n \): slope of \( R_\theta\)–\(N\) plot \((\text{°C}s^{-1}\text{cycle}^{-1})\)
- \( N \): number of load cycle
- \( N_f \): fatigue life (cycle)
- \( N_{PR} \): Prandtl number
- \( q \): heat flux \((\text{Wm}^{-2})\)
- \( r \): surface normal parameter
- \( R, r_l \): radius (mm)
- \( R_a \): arithmetic average of surface roughness (\(\mu m\))
- \( R_\theta \): slope of temperature rise after \(N\) load cycles \((\text{°C}s^{-1})\)
- \( R_{\theta i} \): slope of temperature rise at \(i^{th}\) STE test \((\text{°C}s^{-1})\)
- \( R_{\theta 0}^c \): intercept of \(R_\theta\)–\(N\) plot \((\text{°C}s^{-1})\) on \(R_\theta\) axis
- \( s \): specific entropy \((\text{JKg}^{-1}\text{K}^{-1})\)
- \( t \): time (s)
- \( t_f \): fatigue life (s)
$T$ absolute temperature in fatigue test (K)
$T_{STE}$ absolute temperature in STE test (K)
$U$ velocity of air (ms$^{-1}$)
$V_i$ internal variables
$W_f$ fatigue toughness (MJm$^{-3}$)
$W_p$ plastic strain energy per second (MJm$^{-3}$s$^{-1}$)
$\Delta W_p$ plastic strain energy per cycle (MJm$^{-3}$cycle$^{-1}$)
$\beta$ a constant
$\gamma$ thermodynamic entropy at $N$$^{th}$ load cycle (MJm$^{-3}$K$^{-1}$)
$\gamma_f$ fatigue fracture entropy (MJm$^{-3}$K$^{-1}$)
$\gamma_N$ entropy generation per cycle at $N$$^{th}$ load cycle (MJm$^{-3}$K$^{-1}$)
$\gamma_{STE}$ thermodynamic entropy in STE test (MJm$^{-3}$K$^{-1}$)
$\epsilon$ strain
$\epsilon$ total strain tensor
$\epsilon_e$ elastic strain tensor
$\epsilon^p$ plastic strain tensor
$\epsilon_0$ surface emissivity
$\nu$ kinematic viscosity of air (m$^2$s$^{-1}$)
$\rho$ density (Kgm$^{-3}$)
$\sigma$ stress tensor
$\sigma$ stress level (MPa)
$\sigma_0$ Stephan-Boltzmann constant (Wm$^{-2}$K$^{-4}$)
$\phi$ diameter (mm)
$\psi$ specific free energy (JKg$^{-1}$)

2.9. References


Chapter 3: Entropic Characterization of Metal Fatigue with Stress Concentration*

3.1. Introduction

Many approaches are available for analyzing fatigue degradation such as strain-energy method [1-4], stress-life method [4], strain-life method [4-6], etc. Among these approaches, strain-energy method utilizes cyclic plastic strain energy (also known as the hysteresis energy) generated by the irreversible plastic deformation in a material as an index of fatigue characterization. Most of this energy is converted into heat and dissipates to the surroundings [3, 7, 8].

Fatigue is an example of dissipated process wherein the accumulation of disorder is inherently associated with generation of thermodynamic entropy in accordance with the second law of thermodynamics [9-14]. Specifically, the amount of entropy generation in a fatigue degradation process is found to be a useful parameter for the assessment of material damage in different processes such as cyclic mechanical loading [7], thermomigration, electromigration, and thermomechanical loading [15-17]. Research shows that the accumulation of thermodynamic entropy —starting with a pristine specimen without stress concentration and ending at the fatigue fracture— is a material property called fatigue fracture entropy (FFE), which is useful for estimation of fatigue life. Experiments show that within the range of operating conditions tested FFE is independent of the amplitude and the frequency of load, the geometry of the specimen, and the type of fatigue load [10, 18-21]. The concept of entropy accumulation has been successfully applied to monitor the evolution of fatigue damage in specimens without stress concentration [22] and in steam turbine rotors [23] subject to both constant and variable amplitude loading.

The thermodynamic entropy generation and temperature evolution depend on the plastic strain energy dissipation as the fatigue degradation progresses. It therefore follows that the presence of stress concentration in a component, which is known to cause significant changes in the stress-strain distribution, must affect the rate of thermodynamic entropy generation and temperature evolution. Hence, the application of the concept of FFE to characterize the effect of stress concentration in a notched specimen calls for further study.

This paper is devoted to entropic characterization of the fatigue behavior of solid cylindrical un-notched and V-notched specimens made of two different metals. The effect of stress concentration on the evolution of entropy, the shape and size of hysteresis loops, and the FFE are discussed. Numerical simulations are conducted using the finite element method (FEM) to simulate the thermal response of both types of specimens under cyclic load and to determine FFE due to cyclic stress. An empirical correlation is proposed that can predict the fatigue life of a V-notched specimen based on the hysteresis energy per cycle and FFE of an un-notched specimen subject to identical loading and environmental conditions.

*Reprinted by permission of International Journal of Fatigue (See Appendix A)
The outline of the paper is as follows. In Section 2, theoretical analysis for the calculation of the FFE is described followed by the materials, experimental procedure, instrumentation, and experimental results in Section 3. Section 4 presents the finite element model, the boundary conditions, and the validation of the FEM analysis and the predicted FFE results. The development of a correlation for the fatigue life prediction of a V-notched specimen and its validation are discussed in Section 5 followed by the conclusions.

3.2. Theory

The first law of thermodynamics postulates that in the absence of internal heat source, conservation of energy principle can be expressed as [14]:

$$\rho \dot{e} = \sigma : \dot{e} - div \mathbf{q}$$  \hspace{1cm} (3.1)

where \(\rho\) is the density, \(\dot{e} = \mathbf{\Psi} + T \dot{s} + \dot{T}\) stands for the time rate of specific internal energy, \(\mathbf{\Psi}\) is the specific free energy, \(T\) is the absolute temperature, \(s\) signifies the specific entropy, \(\sigma\) and \(\dot{e}\) denote the stress tensor and the time rate of strain tensor, respectively, and \(\mathbf{q}\) is the heat flux. \(\mathbf{\Psi}\) is a function of \(T\), elastic strain, \(\varepsilon_e\), and internal variables associated with microstructure, \(V_i\) (where \(i\) is the number of internal variables), and its derivative with respect to time yields \(\dot{\mathbf{\Psi}} = \frac{\partial \mathbf{\Psi}}{\partial \varepsilon_e} : \dot{\varepsilon}^e + \frac{\partial \mathbf{\Psi}}{\partial V_i} \dot{V}_i\).

Substituting the value of \(\dot{\mathbf{\Psi}}\) and \(\dot{\varepsilon}\) into Equation (3.1) gives:

$$\sigma : \dot{e} - div \mathbf{q} = \rho \left[\frac{\partial \mathbf{\Psi}}{\partial \varepsilon_e} : \dot{\varepsilon}^e + \frac{\partial \mathbf{\Psi}}{\partial T} \dot{T} + \frac{\partial \mathbf{\Psi}}{\partial V_i} \dot{V}_i + T \dot{s} + \dot{T}\right]$$  \hspace{1cm} (3.2)

The laws of thermoelasticity state that \(\sigma = \rho \frac{\partial \mathbf{\Psi}}{\partial \varepsilon_e}\) and \(s = -\frac{\partial \mathbf{\Psi}}{\partial T}\) and thermodynamic forces associated with internal variables \(A_i = \rho \frac{\partial \mathbf{\Psi}}{\partial V_i}\). Fourier’s law states that \(\mathbf{q} = -k \, grad \, T\) where \(k\) is the thermal conductivity. The time rate of entropy generation \(\dot{s} = -\frac{1}{\rho} \frac{\partial \sigma}{\partial T} : \dot{\varepsilon}^e + \frac{\partial s}{\partial T} \dot{T} - \frac{1}{\rho} \frac{\partial A_i}{\partial T} \dot{V}_i\), specific heat \(C = T \frac{\partial s}{\partial T}\), and total strain rate \(\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p\). Substituting these into Equation (3.2) leads to [14]:

$$k \nabla^2 T = \rho C \dot{T} - \sigma : \dot{\varepsilon}^p + A_i \dot{V}_i - T \left(\frac{\partial \sigma}{\partial T} : \dot{\varepsilon}^e + \frac{\partial A_i}{\partial T} \dot{V}_i\right)$$  \hspace{1cm} (3.3)

Since elastic strain energy, \(\sigma : \dot{\varepsilon}^e\), is reversible and does not contribute to the heat generation, this term is neglected in Equation (3.3).

The second law of thermodynamics postulates that the time rate of volumetric entropy production, \(\dot{\gamma}\), of a deformed body is always greater than or equal to the time rate of heating divided by the temperature which is expressed by the Clausius-Duhem inequality reads [14]:

[32]
\[
\dot{\gamma} = \frac{\sigma \cdot \dot{\varepsilon}^p}{T} - \frac{q}{T^2} \cdot \text{grad} \ T - \frac{A_i \dot{V}_i}{T} \geq 0
\] (3.4)

Research shows that \( A_i \dot{V}_i \) represents only 5-10% of the entropy generation due to \( \sigma \cdot \dot{\varepsilon}^p \) and thermo-mechanical coupling effect, \( \frac{\partial \sigma}{\partial T} \cdot \dot{\varepsilon}^e \), is negligible compared to the mean temperature rise \([14]\). Hence, it is assumed that \( A_i \dot{V}_i \approx 0 \), \( \frac{\partial \sigma}{\partial T} \cdot \dot{\varepsilon}^e \approx 0 \), and \( \frac{\partial A_i}{\partial T} \dot{V}_i \approx 0 \). Thus, Equations (3.3) and (3.4) simplify as, respectively, \([19]\):

\[
k \nabla^2 T = \rho C \dot{T} - \dot{W}_p
\] (3.5)

\[
\dot{\gamma} = \frac{W_p}{T} - \frac{q}{T^2} \cdot \text{grad} \ T
\] (3.6)

where \( \dot{W}_p = \sigma \cdot \dot{\varepsilon}^p \) denotes time rate of plastic strain energy generation. Let \( t = 0 \) denote the beginning of fatigue process and \( t_f \) the onset of fracture so that integration of Equation (3.6) with respect to time provides total entropy generation in a fatigue process expressed as \([19]\):

\[
\gamma = \int_0^{t_f} \left( \frac{W_p}{T} \right) dt - \int_0^{t_f} \left( \frac{q}{T^2} \cdot \text{grad} \ T \right) dt
\] (3.7)

where \( \gamma \) represents the accumulation of entropy until the number of cycles \( N \) reaches \( N_f \) when fracture occurs (i.e., \( \text{FFE}=\gamma \)). This expression can be put in terms of the number of load cycles as follows:

\[
\gamma = \int_0^{N_f} \left( \frac{\Delta W_p}{T} \right) dN + \int_0^{N_f} \left( \frac{k}{T^2} \cdot (\text{grad} \ T)^2 \right) dN
\] (3.8)

where \( \Delta W_p \) stands for the hysteresis energy per cycle. Equation (3.8) shows that FFE is the summation of entropy generation due to plastic strain energy generation and heat conduction. The FFE is calculated by solving Equation (3.5) in the FEM model of a specimen and integrating the right-hand side of Equation (3.8).

3.3. Experimental

3.3.1. Materials and specimen preparation

Figure 3.1(a and b) presents the schematics of the cylindrical un-notched and V-notched specimens produced from cold-drawn solid bars of both medium-carbon steel (MCS) 1045 and aluminum (Al) 6061. These specimens are prepared in accordance with the ASTM:E466-07 (for un-notched specimen) and ASTM:E292-01 (for V-notched specimen), with the dimensions as given in Table 3.1. It can be seen that the radius of curvature, \( r_1 \), of the un-notched specimen is larger than that of the V-notched specimen. Since an un-notched specimen has more affinity to fracture at this curvature than a V-notched specimen, a larger value of \( r_1 \) is chosen for the un-notched specimens as recommended by ASTM:E466-07. In contrast, a V-notched specimen is subjected to higher stress concentration at the notch than that at the curvature. Therefore, a smaller
value of $r_1$ is adopted for the V-notched specimens. In order to prevent the initiation of microcracks from nicks, dents, scratches, and circumferential tool marks, the gage section surface and the notch surface of the specimens are polished using sand papers to bring the surface roughness to within $R_a=0.2$-µm (ASTM:E466-07 and E292-01).

![Figure 3.1. Schematics of the solid cylindrical (a) un-notched and (b) V-notched specimens](image)

Table 3.1. Dimensions of the specimens (unit: mm)

<table>
<thead>
<tr>
<th>Material</th>
<th>Notch</th>
<th>$l_1$</th>
<th>$l_2$</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$r_1$</th>
<th>$r_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCS 1045</td>
<td>Un-notch</td>
<td>38.5</td>
<td>25.0</td>
<td>15.87</td>
<td>9.0</td>
<td>–</td>
<td>65.0</td>
<td>–</td>
</tr>
<tr>
<td>V-notch</td>
<td>38.5</td>
<td>25.0</td>
<td>15.87</td>
<td>12.0</td>
<td>9.0</td>
<td>30.0</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>Al 6061</td>
<td>Un-notch</td>
<td>38.1</td>
<td>25.4</td>
<td>15.0</td>
<td>8.0</td>
<td>–</td>
<td>62.0</td>
<td>–</td>
</tr>
<tr>
<td>V-notch</td>
<td>38.1</td>
<td>25.4</td>
<td>15.0</td>
<td>11.0</td>
<td>7.5</td>
<td>25.4</td>
<td>0.35</td>
<td></td>
</tr>
</tbody>
</table>

The presence of stress concentration in a notched component is typically characterized by the so-called stress concentration factor, $K_t$, and fatigue notch factor, $K_f$, defined as (see for example, [24-26]):

$$K_t = \frac{\sigma_{\text{max}}}{\sigma} \quad (3.9)$$
where $\sigma_{\text{max}}$ stands for the maximum stress due to stress concentration and $S_{eu}$ and $S_{ev}$ are the fatigue limits of the un-notched and V-notched specimens. Table 3.2 presents the values of $K_t$ and $K_f$ corresponding to the V-notched specimens [27, 28]. Since the notch-angle and root radius are identical for the V-notched specimens of both materials, the values of $K_t$ are equal (ASTM:E292-01). In addition to the notch geometries, $K_f$ also depends on the notch-root sensitivity, heat treatment conditions, and tensile strength of a material [27, 28].

Table 3.2. Values of $K_t$ and $K_f$ for the V-notched specimens [28]

<table>
<thead>
<tr>
<th>Notch type</th>
<th>Material</th>
<th>$K_t$</th>
<th>$K_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>MCS 1045</td>
<td>3.9</td>
<td>2.60</td>
</tr>
<tr>
<td>V</td>
<td>Al 6061</td>
<td>3.9</td>
<td>2.45</td>
</tr>
</tbody>
</table>

3.3.2. Equipment and test procedure

Constant amplitude, stress-controlled uniaxial tension-compression fatigue tests are carried out at the frequency of $f = 10$ and $5$ Hz, load ratio (defined as the ratio of minimum to the maximum stress in a load cyclic) of $L_R = -0.6$ and -1 with MCS 1045 and Al 6061, respectively. The apparatus is a servo-hydraulic fatigue tester with the capability of a maximum of 50 kN axial load and 75 Hz frequency. The gripping pressure between the specimen and the jaws is maintained to avoid any slippage between them for all the tests reported in this work. An extensometer with the gauge length of 25.4 mm and travel between -10% and +50% strain is used to measure the strain in the gage section of the un-notched specimens and the notched specimens. The measured strain across the V-notch represents the gross strain associated with the notch, as the rest of the gage section experiences stress levels smaller than the yield strength of the material. Thus, the gross strain depends on the notch size and geometry and the gage length of the extensometer. In order to eliminate these variables, identical notch geometry and size and a specific extensometer are used throughout this work. The extensometer directly interfaces with the fatigue testing machine, and the measured load and strain data are automatically recorded. Stress-strain plot is used to obtain the hysteresis loop and the area contained within its boundaries is the measure of the plastic strain energy generation in a cycle of fatigue load that dissipates as heat. The amount of plastic strain energy generated in a load cycle is known as the hysteresis energy per cycle, $\Delta W_p$. In the case of a V-notched specimen, $\Delta W_p$ represents the amount of plastic strain energy generated in the notch in a load cycle.

An infra-red (IR) camera with temperature range capability between 0°C and 500°C, resolution of 320×240 pixel, accuracy of ±2% of reading, sensitivity of 0.08°C is used to record the surface temperature of the specimens. A thin layer of black paint is sprayed on the specimens’ gage section and allowed to dry out for about 12 hours before fatigue test in order to reduce IR reflection and
increase thermal emissivity of the specimen surface. Temperature of the entire specimen gage section is recorded throughout the fatigue test. Since fatigue fracture typically occurs where temperature appears as maximum, an average temperature along an approximately 5-mm long line at that location is used for analysis. Figure 3.2(a and b) illustrates the thermographic images of MCS 1045 specimens corresponding to the consumed life $N \cong 0.55N_f$ and $0.8N_f$ where $N_f$ represents fatigue life.

![Thermographic images](image)

$N \cong 0.55N_f$

(a)

$N \cong 0.8N_f$

(b)

Figure 3.2. Thermographic images obtained from fatigue tests performed at $\sigma = 500$ MPa, $f = 10$ Hz, and $L_R = -0.6$ showing the location and the line of temperature data acquisition for analysis (a) un-notched and (b) V-notched specimen (temperature values are in °C)

3.3.3. S-N curve

Figures 3.3 and 3.4 show the S-N curves corresponding to both materials demonstrating an increase in fatigue life at lower stress levels for each type of the specimens, as expected. It can be seen that MCS 1045 corresponds to a greater stress level compared to Al 6061 subject to comparable fatigue life cycle and the specific type of the specimens. This shows that MCS 1045 is more resistant to fatigue compared to Al 6061.

3.3.4. Effect of stress concentration on hysteresis energy and temperature evolution

In this section we investigate the trend of hysteresis energy evolution obtained from un-notched specimens and how the presence of stress concentration, strain softening, and macrocrack propagation influence the hysteresis energy generation. Figures 3.5 and 3.6 present the evolution
Figure 3.3. S-N curve of un-notched MCS 1045 and Al 6061 specimens

Figure 3.4. S-N curve of V-notched MCS 1045 and Al 6061 specimens
Figure 3.5. Comparison between the evolutions of $\Delta W_p$ obtained from the experiments performed with un-notched and V-notched MCS 1045 specimens ($\sigma = 500$ MPa)

Figure 3.6. Comparison between the evolutions of $\Delta W_p$ obtained from the experiments performed with un-notched and V-notched Al 6061 specimens ($\sigma = 200$ MPa)
of $\Delta W_p$ obtained from fatigue tests performed with both specimens. It can be seen that the evolution of $\Delta W_p$ and the fatigue life of a V-notched specimen decrease significantly compared to an un-notched specimen subject to identical material and loading conditions. These occur due to the presence of stress concentration in the notched specimens. In the case of MCS 1045, $\Delta W_p$ rapidly increases at the onset of the fatigue test and then its value increases gradually, whereas for Al 6061 $\Delta W_p$ remains fairly constant over most of the fatigue life. Common to both materials is a sharp rise of $\Delta W_p$ evolution just before fracture occurs. This trend is further explained by examining the variation of the hysteresis loop area.

Figure 3.7(a and b) illustrates a rapid increase in the width of $\Delta W_p$ at the onset of fatigue test with both specimens of Al 6061 which is a manifestation of rapid strain softening. It can be seen that the increase in the area of hysteresis loops associated with an un-notched specimen is greater than the V-notched specimen. Research shows that a rapid strain softening is a common phenomenon in the early stage of a fatigue process for cold-drawn materials [29, 30]. In the case of MCS 1045, the increase in $\Delta W_p$ evolution is a consequence of the gradual increase in the area of hysteresis loops as illustrated in Figure 3.8(a and b) subject to constant amplitude loading. While the height of these hysteresis loops remains fairly constant, their width increases progressively. This is observed due to the simultaneous effects of the accumulation of irreversible microstructural damages —e.g., dislocations, slip bands, etc.— and strain softening of the material due to cyclic loading [29-33]. According to Laird and Buchinger [34], strain softening occurs in the material if the application of cyclic load does not increase the density of dislocations in the material. The presence of stress concentration decreases the evolution of $\Delta W_p$ in this phase of fatigue process. Our research shows that, within the range of the loading conditions tested, the MCS 1045 specimens tend to undergo gradual softening during most of the fatigue life whereas Al 6061 specimens remain in the steady hysteresis phase. It can be seen that the effect of softening in Al 6061 is negligible compared to that of MCS 1045. Before the specimen fractures, a macrocrack develops in the material and its propagation generates large amount of hysteresis energy as demonstrated by a sharp rise in the evolutions of $\Delta W_p$. Figure 3.9(a and b) illustrates this phenomenon in terms of the increase in the area of hysteresis loops associated with both the un-notched and V-notched Al 6061. The span of the macrocrack propagation phase is substantially shorter than the fatigue life, $N_f$, of the tested materials.

It is essential to note that there are materials that show strain hardening during fatigue instead of softening [4]. In the case of constant amplitude loading tests, strain hardening manifests itself by decreasing the width of the hysteresis loops. Fatigue response of those materials is different compared to the materials tested in this study.
Figure 3.7. Rapid increase in hysteresis loops due to rapid strain softening at the onset of fatigue test corresponding to (a) un-notched and (b) V-notched Al 6061 specimens ($\sigma = 200$ MPa)
Figure 3.8. Gradual increase in hysteresis loops due to strain softening after different number of load cycles associated with (a) un-notched and (b) V-notched MCS 1045 specimens ($\sigma = 500$ MPa)
Figure 3.9. Illustration of growth in hysteresis loops over a few number of load cycles when macrocrack tends to propagate until fracture occurs with (a) un-notched specimen at $N_f = 4421$ cycles and (b) V-notched specimen at $N_f = 2147$ cycles ($\sigma = 200$ MPa)
Figures 3.10 and 3.11 illustrate the shape and size of the hysteresis loops corresponding to un-notched and V-notched specimens of MCS 1045 approximately at 50% of consumed life, i.e., $N \equiv 0.5N_f$ and Al 6061 at $N \equiv 0.35$ and $0.75N_f$, respectively. It can be seen that the presence of stress concentration changes the shape and decreases the size of the hysteresis loops across the V-notch. The decrease in the area of hysteresis loops implies that smaller amount of heat is generated in the V-notch. The angular orientation of the hysteresis loops in Figure 3.10 is comparable as they correspond to approximately equal $N/N_f$. This trend changes as shown in Figure 3.11 due to the difference in the value of $N/N_f$. Thus, the angular orientation of the hysteresis loop is fairly independent of the presence of stress concentration. Although the plastic deformation in a V-notched specimen is a local phenomenon that takes place mostly within the notch, the results presented in Figures 3.5-3.11 reveal that the method of gross strain measurement across a V-notch provides a useful information for the comparative study of fatigue degradation between an un-notched and a V-notched specimen.

Figure 3.10. Comparison between the hysteresis loops obtained from fatigue tests performed with un-notched and V-notched MCS 1045 specimens ($\sigma = 500$ MPa)

Figure 3.12 presents the experimental evolutions of temperature corresponding to MCS 1045 specimens. Because the temperature evolution occurs due to the hysteresis energy generation, its trend is comparable to that of $\Delta W_p$ evolution as shown in Figure 3.5. Since the amount of hysteresis energy generation in a V-notched specimen is smaller compared to an un-notched specimen, temperature evolution of this specimen is lower as well.
Figure 3.11. Comparison between the hysteresis loops obtained from fatigue tests performed with un-notched and V-notched Al 6061 specimens ($\sigma = 200$ MPa)

![Hysteresis Loop Diagram](image)

Figure 3.12. Experimental temperature evolution of un-notched and V-notched MCS 1045 specimens ($\sigma = 500$ MPa)

![Temperature Evolution Diagram](image)

It should be noted that the temperature evolution in a fatigue test depends on the geometry and size of a specimen, type of fatigue load, i.e., uniaxial, torsion, bending, etc., test frequency, and
environmental conditions. However, research shows that the total amount of heat generation (or plastic strain energy generation) in a fatigue process is roughly independent of test frequency [21, 35-38].

3.3.5. Effect of stress concentration on FFE

Figures 3.13 and 3.14 present the FFE vs. \( N_f \) plots for both materials where FFE is calculated according to Equation (3.8) neglecting the amount of entropy generation due to heat conduction (see section 3.7). Results reveal that the FFE of each type of the specimens is in a small band independent of loading conditions, which validates the idea of constant entropy gain at fatigue fracture [18, 19, 22]. Reduction in the FFE of V-notched specimens occurs due to the simultaneous effects of the smaller entropy generation rates and the decrease in the fatigue life compared to that of un-notched specimens as shown in Figure 3.15. These are the consequences of the presence of stress concentration. At a higher stress level, the rate of entropy generation is greater and the fatigue life is shorter. As a result, the total entropy generation in a fatigue test remains within a narrow band for a material with a specific type of specimen. The condition for fatigue fracture corresponds to the thermodynamic entropy generation of 25±5 and 6±1 MJm\(^{-3}\)K\(^{-1}\) for un-notched and V-notched MCS 1045 and 9±2 and 2±0.5 MJm\(^{-3}\)K\(^{-1}\) for un-notched and V-notched Al 6061 specimens, respectively. These results show the usefulness of thermodynamic entropy based fatigue characterization for metallic components regardless of the presence of stress concentration.

![Figure 3.13. Experimental FFE vs. fatigue life plot of un-notched and V-notched MCS 1045 specimens](image)

---

45
Figure 3.14. Experimental FFE vs. fatigue life plot of un-notched and V-notched Al 6061 specimens

Figure 3.15. Reduction of the entropy generation rate and fatigue life of V-notched specimens compared to un-notched specimens at different stress levels.
3.4. Numerical simulations

3.4.1. Computational model

A commercial code, FlexPDE, that utilizes the FEM to solve the governing partial differential equation is used to simulate a model of both the un-notched and V-notched specimens. Since un-notched specimens are symmetric with respect to their longitudinal axis, only half of the specimen containing an end section, a curved-section, and half of the gage section is simulated. Figure 3.16(a and b) shows a photograph of the un-notched specimen mounted with the grips and its 3-D model meshed with 10-node quadratic tetrahedral elements. In the case of the V-notched specimen, the cross-sectional area of the gage section is substantially greater than that of the V-notch. Consequently, the applied stress generates plastic deformation in the V-notch due to the stress concentration [39, 40] and the rest of the gage section experiences negligible amount of plastic deformation. In contrast, the entire gage section of an un-notched specimen experiences uniform plastic deformation as the applied stress is greater than the yield strength of the material. For the specimens tested in this work, the volume of material within the notch of a V-notched specimen is significantly smaller than that of the entire gage section of an un-notched specimen. Therefore, the amount of heat generation (or hysteresis energy generation) in the V-notched specimen is smaller than the un-notched one (see Figures 3.5 and 3.6). Most of the generated heat dissipates to the surroundings from the gage section of this specimen. Therefore, the gage section of the V-notched

![Figure 3.16. Fatigue test and simulation of un-notched specimens (a) the specimen is gripped for test and (b) finite-element meshing of the specimen](image-url)
specimen is modeled for the finite element simulation. Figure 3.17(a, b, and c) illustrates a photograph, the finite-element meshing of the gage section, and a magnified view of the meshed V-notch. Finer mesh sizes are used in the V-notch compared to the rest of the model.

![Figure 3.17](image)

Figure 3.17. Fatigue test and simulation of V-notched specimens (a) photograph of the specimen, (b) finite-element meshing of the gage section, and (c) magnified view of the meshed V-notch

3.4.2. Boundary Conditions

Referring to Figures 3.16 and 3.17, boundary B1 exchanges heat with the surroundings by convection and radiation. The boundary B2 represents the plane of symmetry. Boundary B3 is the end of the gage section, assumed to be at the ambient temperature. The expression for the boundary condition as applied on the surfaces denoted by B1 is as follows:

\[
k \frac{\partial T}{\partial n} = h(T - T_a) + \sigma_0 \varepsilon_0 (T^4 - T_a^4)
\]  

(3.11)

where \( n \) represents surface normal parameter, \( T \) and \( T_a \) denote specimen surface and ambient temperature, respectively, \( h \) stands for the heat transfer coefficient, \( \sigma_0 \) represents the Stephan-
Boltzmann constant, and $\varepsilon_0$ represents the surface emissivity. $T_a$ is measured in the laboratory at the time of each fatigue test, $\varepsilon_0$ and $\sigma_0$ are set to be 0.93 [19] and $5.67 \times 10^{-8}$ Wm$^{-2}$K$^{-4}$, respectively. The value of $h$ is calculated from the following expression which is applicable for the vertical surface associated with forced convection [41]:

$$h = \frac{0.664 \ k_a (N_{PR})^{1/3}}{L} \sqrt{\frac{UL}{\nu}}$$

(3.12)

where $L$ is the effective length of the vertical surface, $N_{PR}$ stands for the Prandtl number, and $U$, $k_a$, $\nu$ denote velocity, thermal conductivity, and kinematic viscosity of air, respectively. In the case of a dogbone specimen, $L$ is the length of specimen gage section. It is assumed that $U = 1$ m/s around the specimen during the entire fatigue test. Since the ambient temperature in the laboratory is maintained at around 21°C, the value of $h$ is calculated based on the air properties at that temperature. Thermal response of the specimens are obtained from the FEM simulations by solving Equation (3.5) in the entire 3-D model at the frequency which is used in the experiment utilizing $\Delta W_p$ calculated from experimental hysteresis area and the physical and the thermal properties as shown in Table 3.3. Since $\Delta W_p$ associated with a V-notched specimen is measured based on the gross strain measurement, strain energy is applied over the entire V-notch uniformly. FFE is calculated by integrating the right-hand-side of Equation (3.8) throughout the simulation of a fatigue test.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density, $\rho$ (kg-m$^{-3}$)</th>
<th>Specific heat, $C$ (J-kg$^{-1}$-K$^{-1}$)</th>
<th>Thermal conductivity, $k$ (W-m$^{-1}$-K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCS 1045</td>
<td>7870</td>
<td>486</td>
<td>51</td>
</tr>
<tr>
<td>Al 6061</td>
<td>2700</td>
<td>896</td>
<td>164</td>
</tr>
</tbody>
</table>

### 3.4.3. Validation of the FEM model

A mesh-size dependency study is carried out to determine the effect of mesh size on the temperature evolution and the FFE associated with both types of the specimens. It is found that the temperature evolution is negligibly dependent on the mesh size. Table 3.4 presents the values of the FFE obtained from simulations and corresponding total number of mesh. In the case of the un-notched specimen, while an increase of the total number of mesh from 64325 to 81762 increases the FFE from 24.63 to 24.75, further refinement of mesh size does not have a major influence on FFE, but requires considerably more computational time. In the case of the V-notched specimen, an increase of the total number of mesh from 68432 to 82167 does not show considerable increase in the FFE. Therefore, the mesh sizes corresponding to the total number of mesh 81762 and 68432 are used in the simulations of un-notched and V-notched specimens in order to reduce the computational time, respectively.
Table 3.4. Effect of mesh size on the FFE of MCS 1045

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Stress, $\sigma$</th>
<th>Number of mesh</th>
<th>FFE, $\gamma$ (MJm$^{-3}$K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Un-notched</td>
<td>535</td>
<td>64325</td>
<td>24.63</td>
</tr>
<tr>
<td></td>
<td></td>
<td>81762</td>
<td>24.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>117511</td>
<td>24.55</td>
</tr>
<tr>
<td>V-notched</td>
<td>515</td>
<td>46955</td>
<td>6.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60223</td>
<td>6.32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>68432</td>
<td>6.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>82167</td>
<td>6.41</td>
</tr>
</tbody>
</table>

Figure 3.18(a and b) illustrates the temperature contours obtained from the FEM simulations of MCS 1045 specimens subject to identical consumed life and loading conditions associated with the thermographic images shown in Figure 3.2(a and b). In the case of the un-notched specimen, the maximum temperature occurs at the middle of the gage section, and the temperature decreases along the length such that it approaches the ambient temperature at the end of the specimen. In contrast, in the case of the V-notched specimen the temperature is maximum at the notch-root and gradually decreases to the ambient temperature at the end of the gage section. Therefore, the approach of simulating only the gage section of the V-notched specimen is pertinent for the specimens tested in this work. However, if the size of a V-notch is large such that most of the heat energy generated in the notch does not dissipate to the surroundings from the gage section, then one may need to simulate the trapezoidal and the end section of the specimen in the FEM model. It can be seen that the FEM results are in good agreement to those obtained from the experiments as shown in Figure 3.2(a and b) and conform to the assumptions made in the FEM modeling of V-notched specimen.

Figure 3.19 presents the comparison between the evolutions of temperature obtained from experiments and simulations corresponding to MCS 1045 specimens. Numerical temperature is obtained from the same locations of the FEM models of the specimens as shown in Figure 3.2(a and b). Results demonstrate that the numerical temperature is roughly 2-3°C greater than those measured experimentally. The small difference between the numerical and experimental results could be due to: (a) the assumption of constant air velocity around the specimen that may be slightly greater or smaller in the actual case; and (b) the physical and thermal properties of the given materials obtained from literature may differ by a small value for the materials tested. The agreement between these results show that the FEM models represent fatigue experiments with reasonable accuracy.
Figure 3.18. Illustrations of temperature contour obtained from FEM simulations (a) un-notched and (b) V-notched MCS 1045 specimens ($\sigma = 500$ MPa)

Figure 3.19. Comparison between experimental and simulation temperature evolutions of un-notched and V-notched MCS 1045 specimens

Material: MCS 1045

FEM model, un-notch, 500 MPa
Experiment, un-notch, 500 MPa
FEM model, V-notch, 500 MPa
Experiment, V-notch, 500 MPa
Figures 3.20 and 3.21 present the experimental and predicted FFE vs. $N_f$ plots for both materials. The predicted FFE corresponds to the simulation of a fatigue test subject to a stress level which is not considered in the experiments. Fatigue life for these simulations is determined from the S-N curve associated with the specific type of the specimen of the tested materials. Results show that the predicted FFE of each type of the specimens remains within the band as illustrated in Figures 3.13 and 3.14. This shows the usefulness of the FEM simulation of a fatigue test.

![Experimental and predicted FFE vs. fatigue life plot of un-notched and V-notched MCS 1045 specimens](image)

**3.5. Fatigue life prediction of V-notched specimen**

Referring to Figures 3.5 and 3.10, experimental results show that the ratio of the area of hysteresis loop associated with un-notched, $\Delta W_{pu}$, to the V-notched, $\Delta W_{pv}$, MCS 1045 specimens at equal fraction of consumed life is approximately equal to $K_f$. In the case of Al 6061, the ratio of $\Delta W_{pu}$ to $\Delta W_{pv}$ is roughly equal to $K_f$ except at the onset of the fatigue test and just before fracture (see Figures 3.6 and 3.11):

$$K_f \approx \frac{\Delta W_{pu}}{\Delta W_{pv}}$$  \hspace{1cm} (3.13)

In the case of sinusoidal cyclic loading, Morrow [4] showed that $\Delta W_p \propto \Delta \sigma$. Research [42, 43] shows that $K_f$ is a linear function of stress level, i.e., $K_f \propto \sigma$. Thus, it can be obtained that $K_f \propto \Delta W_p$ which agrees with Equation (3.13). Figures 3.22 and 3.23 present a comparison between the values of $K_f$ reported in Table 3.2 and obtained from Equation (3.13). Results demonstrate that the values of $K_f$ obtained from Equation (3.13) are fairly close to its theoretical value within the range of the loading conditions considered.
Figure 3.21. Experimental and predicted FFE vs. fatigue life plot of un-notched and V-notched Al 6061 specimens

Figure 3.22. Comparison between the theoretical and experimental values of $K_f$ at different stress levels
Figure 3.23. Comparison between the theoretical and experimental values of $K_f$ at different stress levels

Neglecting the heat conduction part and taking advantage of Equation (3.13), Equation (3.8) is simplified for the V-notched specimen as:

$$
y_v \approx \frac{1}{K_f} \int_0^{N_{fv}} \frac{\Delta W_{pu}}{T_v} \, dN
$$

(3.14)

where $y_v$, $N_{fv}$, and $T_v$ stand for the FFE, fatigue life, and temperature of the V-notched specimen, respectively. Since $\Delta W_{pu}$ and $T_v$ change gradually over most of the fatigue test for MCS 1045, the average values of $\Delta W_{pu}$ and $T_v$ are used for the simplification of Equation (3.14). In the case of Al 6061, since $\Delta W_{pu}$ and $T_v$ remain fairly unchanged over most of the fatigue test, they are assumed to be constants. Thus, Equation (3.14) simplifies as:

$$
y_v \approx \frac{N_{fv}}{K_f} \left( \frac{\Delta W_{pu}}{\bar{T}_v} \right) ; \quad \text{for MCS 1045}
$$

$$
y_v \approx \frac{N_{fv}}{K_f} \left( \frac{\Delta W_{pu}}{\bar{T}_v} \right) ; \quad \text{for Al 6061}
$$

(3.15)

$\bar{T}_v$ or $T_v$ of a dogbone specimen can be predicted, with the help of Equation (3.13), as [44]:

$$
\bar{T}_v = T_a + \frac{f(L_c)^2}{2k_f} \left( \frac{\Delta W_{pu}}{\bar{T}_v} \right) ; \quad \text{for MCS 1045}
$$

$$
\bar{T}_v = T_a + \frac{f(L_c)^2}{2k_f} \left( \frac{\Delta W_{pu}}{\bar{T}_v} \right) ; \quad \text{for Al 6061}
$$

(3.16)
where \( \ell_c \) denotes the characteristic length equal to the half of the reduced section length of a specimen. Table 3.5 presents the predicted and experimental values of \( T_v \) for both of the materials. It can be seen that the predicted values of \( T_v \) are in good agreement with the experimental results.

<table>
<thead>
<tr>
<th>Material</th>
<th>Specimen</th>
<th>Stress (MPa)</th>
<th>( T_v ) (K) prediction</th>
<th>( T_v ) (K) Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCS 1045</td>
<td>V-notch</td>
<td>515</td>
<td>302.3</td>
<td>300.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>500</td>
<td>299.5</td>
<td>298.1</td>
</tr>
<tr>
<td>Al 6061</td>
<td>V-notch</td>
<td>175</td>
<td>294.5</td>
<td>295.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>200</td>
<td>296.9</td>
<td>297.8</td>
</tr>
</tbody>
</table>

Referring to Figures 3.20 and 3.21, FFE vs. \( N_f \) plots show that the ratio between the range of \( y_u \) to \( y_v \) is approximately equal to \( K_t \), i.e., \( K_t \equiv \frac{y_u}{y_v} \). Substituting \( y_v \) into Equation (3.15) simplifies to:

\[
N_{f v} \cong \frac{K_f y_u}{K_t} \left( \frac{T_v}{\Delta W_{pu}} \right) \quad \text{for MCS 1045}
\]

\[
N_{f v} \cong \frac{K_f y_u}{K_t} \left( \frac{T_v}{\Delta W_{pu}} \right) \quad \text{for Al 6061}
\]

Equation (3.17) is used to predict the fatigue life of V-notched specimens. Figures 3.24 and 3.25 present the comparison between the predicted and experimental fatigue life of V-notched specimens.

![Figure 3.24](image-url)
The comparison between predicted and experimental fatigue life shows good agreement, and demonstrates the potential of thermodynamic entropy as an index of material fatigue degradation.

![Graph](image)

Figure 3.25. Predicted and experimental fatigue life of V-notched specimens at different stress levels

It is important to note that the loading frequency and environmental conditions, e.g., temperature, humidity, etc., are maintained constant during all the tests. According to Tobushi et al. [45] fatigue life is weakly dependent on the loading frequency of up to 200 Hz. Also, Morrow [4] showed that the size and the shape of hysteresis loop are only weakly dependent on the test frequency. Since the proposed methodology uses the hysteresis loop, it is expected that the method remains valid for the fatigue processes where there is measurable hysteresis area evolution.

It is also important to note that the derived correlation pertains to the specimens with V-notch and the measurement of gross strain across the notch. Therefore, the applicability of the correlation to other types of notches and to the method of local strain measurement requires further research.

### 3.6. Conclusions

Uniaxial tension-compression fatigue tests are carried out to determine the FFE of MCS 1045 and Al 6061 and the effect of stress concentration on the FFE and fatigue life using thermodynamic entropy approach. FEM simulations are carried out to study the thermal response of un-notched and V-notched specimens and to calculate their FFE. The trend of the evolution of hysteresis energy and the effect of stress concentration on that are discussed. It is shown that the angular...
orientation of a hysteresis loop has correlation to the fraction of consumed life and material degradation regardless of the presence of stress concentration. The FFE of the given materials are nearly constant for each type of the specimens and the presence of stress concentration reduces the size of the hysteresis loops, entropy generation rate, FFE, and fatigue life significantly. An empirical correlation is derived to predict the fatigue life of a V-notched specimen based on the FFE and hysteresis energy of an un-notched specimen subject of identical loading conditions. The comparison between the predicted and experimental fatigue lives of V-notched specimens show good agreement within the range of test conditions considered in this work. The results presented in this work are obtained from specific laboratory and experimental conditions where environmental conditions were maintained constant. Further research is needed to study the applicability of the method under variable environmental and loading conditions.

3.7. Supplemental

Table 3.6 presents the values of FFE and corresponding entropy generation due to heat conduction. Results show that the entropy generation due to axial heat conduction is negligible compared to the total entropy generation. This is illustrated in Figure 3.5(a and b) that shows heat dissipation through the ends of the specimen is negligibly small.

<table>
<thead>
<tr>
<th>Material</th>
<th>Specimen</th>
<th>Stress, $\sigma$ (MPa)</th>
<th>FFE, $\gamma$ (MJm$^{-3}$K$^{-1}$)</th>
<th>Entropy generation due to heat conduction (MJm$^{-3}$K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCS 1045</td>
<td>Un-notch</td>
<td>535</td>
<td>24.75</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>V-notch</td>
<td>515</td>
<td>6.32</td>
<td>0.09</td>
</tr>
<tr>
<td>Al 6061</td>
<td>Un-notch</td>
<td>200</td>
<td>7.35</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>V-notch</td>
<td>183</td>
<td>2.29</td>
<td>0.03</td>
</tr>
</tbody>
</table>

3.8. Nomenclature

$A_k$ thermodynamic forces associated with internal variables

$C$ specific heat (Wm$^{-1}$K$^{-1}$)

$e$ specific internal energy (JKg$^{-1}$)

$E$ modulus of elasticity (GPa)

$f$ frequency (Hz)

$h$ heat transfer coefficient (Wm$^{-2}$K$^{-1}$)

$k$ thermal conductivity (Wm$^{-1}$K$^{-1}$)
$k_a$ thermal conductivity of air (Wm$^{-1}$K$^{-1}$)
$K_f$ fatigue notch factor
$K_t$ stress concentration factor
$L$ effective length (m)
$L_c$ characteristic length of specimen (m)
$L_R$ load ratio
$n$ surface normal parameter
$N$ number of load cycle/consumed life
$N_f$ fatigue life (cycle)
$N_{fv}$ fatigue life of V-notched specimen (cycle)
$N_{fu}$ fatigue life of un-notched specimen (cycle)
$N_{PR}$ Prandtl number
$q$ heat flux (Wm$^{-2}$)
$R$ radius (mm)
$R_a$ arithmetic average of surface roughness (µm)
$s$ specific entropy (JKg$^{-1}$K$^{-1}$)
$S_{ev}$ fatigue limit of V-notched specimen (MPa)
$S_{eu}$ fatigue limit of un-notched specimen (MPa)
$t_f$ fatigue life (s)
$T$ temperature (K)
$T_a$ ambient temperature (K)
$T_v$ temperature of V-notched specimen (K) at the notch-root
$T_u$ temperature of un-notched specimen (K) in the middle of gage section
$U$ velocity of air (ms$^{-1}$)
$V_i$ internal variables
$W_p$ hysteresis energy per second (MJm$^{-3}$s$^{-1}$)
$\Delta W_p$ hysteresis energy per cycle (MJm$^{-3}$cycle$^{-1}$)
$\Delta W_{pv}$ hysteresis energy per cycle of V-notched specimen (MJm$^{-3}$cycle$^{-1}$)
$\Delta W_{pu}$ hysteresis energy per cycle of un-notched specimen (MJm$^{-3}$cycle$^{-1}$)
\( \gamma \)  fatigue fracture entropy (MJm\(^{-3}\)K\(^{-1}\))
\( \gamma_v \)  FFE of V-notched specimen (MJm\(^{-3}\)K\(^{-1}\))
\( \gamma_u \)  FFE of un-notched specimen (MJm\(^{-3}\)K\(^{-1}\))
\( \Delta \varepsilon \)  strain range
\( \varepsilon_e \)  elastic strain tensor
\( \dot{\varepsilon}^p \)  time rate of plastic strain tensor (s\(^{-1}\))
\( \varepsilon_0 \)  surface emissivity
\( \nu \)  kinematic viscosity of air (m\(^2\)s\(^{-1}\))
\( \rho \)  density (Kgm\(^{-3}\))
\( \sigma \)  stress tensor
\( \sigma_0 \)  nominal stress (MPa)
\( \Delta \sigma \)  stress range (MPa)
\( \sigma_{\text{max}} \)  maximum stress due to stress concentration (MPa)
\( \sigma_0 \)  Stephan-Boltzmann constant (Wm\(^{-2}\)K\(^{-4}\))
\( \Psi \)  specific free energy (JKg\(^{-1}\))

3.9. References


Chapter 4: On the Anelasticity and Fatigue Fracture Entropy in High-cycle Metal Fatigue

4.1. Introduction

There are many applications where a component undergoes cyclic mechanical loading where the stress level, $\sigma$, is substantially below the material’s yield strength, $\sigma_y$. In these applications, the number of cycles to failure is quite large and the corresponding degradation process is often referred to as the high-cycle fatigue (HCF). Although the level of stress in an HCF test remains within the elastic limit, the material experiences local micro-scale plastic deformations [1-3]. As a result, metallic materials do not exhibit truly infinite fatigue life [4-9]. Therefore, each load cycle must necessarily introduce a certain amount of irreversible degradation, albeit small, in the material regardless of the amplitude of stress [10-13]. Indeed, as demonstrated by Esin and Jones [10], at low stress levels a material experiences inhomogeneous local plastic deformations. This phenomenon, known as the microplasticity, has been observed in experiments with different materials subject to HCF [1, 2, 14-19]. Although the irreversible changes in an HCF test is small, this process generates local plastic strain energy. Most of the plastic strain energy converts into heat and dissipates to the surroundings.

Recent research shows that tallying up the accumulation of the amount of entropy generation in a low-cycle fatigue (LCF) degradation process yields a useful parameter for the assessment of the fatigue life [20-22]. Specifically, the total amount of thermodynamic entropy generation—starting with a pristine specimen and ending at the fatigue fracture—in LCF is relatively constant and remains within a narrow band for various operating condition. Naderi et al. [20] referred to this value as fatigue fracture entropy (FFE). Experimental results show that, within a wide range of operating conditions tested, the FFE is independent of the amplitude and the frequency of load, the geometry of the specimen, and the type of fatigue load [20, 23-25]. The application of this concept in HCF requires further research to develop a methodology for the estimation of the plastic strain energy generation at low stress levels.

According to the published literature [1, 26, 27], most of the area of a cyclic hysteresis loop in HCF represents a non-damaging anelastic energy generated by the magnetoelastic coupling, thermoelasticity, atomic diffusion, etc. The presence of the anelastic energy in a hysteresis loop occurs due to the effect of internal friction in the material, $Q^{-1}$, that causes a phase lag, $\phi$, between the stress and the resulting strain [26, 28, 29]. Wertz et al. [26] successfully showed that the anelastic energy from a hysteresis loop can be eliminated by gradually decreasing the loading frequency, $f$, to a value such that further decrease of the frequency does not reduce the area contained within the hysteresis loop. Application of this novel method requires one to repeat an HCF test at different frequencies with identical stress level and load ratio (defined as the ratio of minimum to the maximum stress in a load cycle), $L_R$. 

63
In order to determine the usefulness of the concept of the FFE in an HCF test, we propose a methodology to estimate the plastic strain energy generation in an HCF process that does not require one to repeat a fatigue test at lower frequencies. The methodology is applied to the results obtained from the experiments carried out with tubular specimens made of medium carbon steel (MCS) 1018 subject to uniaxial tension-compression fatigue load. The evolutions of plastic strain energy and temperature are then utilized to calculate FFE. Finite element simulations are performed with a 3-D model of the specimen to study its thermal response under cyclic load to validate the proposed methodology of the plastic strain energy measurement, and to predict the FFE of the material subject to a range of stress levels.

4.2. Analytical

4.2.1. Energy balance and entropy accumulation in fatigue

In the case of a cyclic loading test, the applied load generates plastic strain energy, \( \sigma: \dot{e}^p \), that converts into heat where \( \sigma \) and \( \dot{e}^p \) denote the stress and the time rate of plastic strain tensor, respectively. A part of the heat energy tends to increase the specimen temperature estimated as \( \rho C \dot{T} \) and dissipates to the surroundings via convection where \( T \) represents the specimen temperature, \( \rho \) is the density, and \( C \) stands for the specific heat. The rest of the heat energy conducts to the gripping jaws of the fatigue tester calculated as \( k \nabla^2 T \) where \( k \) is the thermal conductivity. Thus, the principle of the conservation of energy can be applied for a fatigue specimen as follows [20]:

\[
k \nabla^2 T = \rho C \dot{T} - \sigma: \dot{e}^p \tag{4.1}
\]

The time rate of volumetric entropy production, \( \dot{\gamma} \), associated with the plastic strain energy generation in the gage section of the specimen can be calculated according to the second law of thermodynamics postulated by the Clausius-Duhem inequality [30]. Let \( N = 0 \) denote the beginning of a fatigue test and \( N_f \) the number of load cycles required to fracture the specimen, then the total entropy generation in a fatigue test is [20]:

\[
\gamma = \int_0^{N_f} \left( \frac{\Delta W_p}{T} \right) dN + \int_0^{N_f} \left( \frac{k}{T^2} \cdot \nabla \cdot \text{grad} T \right) dN \tag{4.2}
\]

where \( \Delta W_p = \frac{\sigma \dot{e}^p}{T} \) denotes the cyclic rate of plastic strain energy generation and \( \gamma \) represents the fatigue fracture entropy, i.e., FFE=\( \gamma \) when \( N = N_f \). Thus, FFE is the summation of entropy generation due to the plastic strain energy generation and the axial heat conduction from the pristine specimen until fracture occurs.

4.2.2. Internal friction, damping, and phase lag

The application of mechanical load on a metal develops strain that tends to cause several other changes in the material [27, 31, 32]. If the stress is less than the yield strength, the material tends
to resist these changes due to the presence of internal friction according to the mechanism of damping. According to Blanter et al. [28], damping manifests itself by a phase lag between the stress and the resulting response such as the strain. The damping and the resulting internal friction transforms the mechanical energy into non-damaging internal energy which is known as the anelastic energy [1, 27]. In such a process, the area of a hysteresis loop represents both the damaging plastic strain energy and the anelastic energy. The amount of the anelastic energy depends on several factors and internal processes such as the amplitude of vibration/oscillation, temperature, frequency, thermal current, dislocations, and ferromagnetism briefly discussed as follows.

According to Zener [27], the effect of the internal friction is independent of the amplitude of oscillation/vibration if the strain amplitude is less than $10^{-5}$ and considerable at larger strain amplitudes. Foppl [33] stated that the internal friction associated with the strain amplitude is primarily a measure of the capacity of a metal to undergo plastic deformation. Usually, cold work makes a metal more resistant to plastic deformation; it decreases the internal friction at larger strain amplitudes and increases the internal friction at small strain amplitudes [34].

Forster and Koster [35] showed that the internal friction of different metals increases substantially when the temperature is greater than 200 °C. The rapid rise of the internal friction is attributed to the onset of slip between the crystals at high temperatures due to a considerable thermal expansion of the material.

The effect of frequency on the internal friction is reported to be substantial for a range of the metals. The experimental work of Wegel and Walther [31] showed that the internal friction increases at higher frequencies. Wertz et al. [26] revealed that the area of a hysteresis loop decreases by several fold as the frequency of mechanical loading is reduced from 5 Hz to 0.01 Hz. These works demonstrate that the loading frequency is a major influential factor in internal friction.

The inhomogeneous distribution of strains and modulus of elasticity in a polycrystalline metal causes thermoelasticity that manifests itself by small-amplitude temperature fluctuations [27, 36]. The local heat generation causes a small amount of heat flow from a plastically deformed crystal to other crystals within a solid. Similar inhomogeneous distribution of micro-scale strains and elastic modulus were observed in an experimental work by Esin and Jones [14, 17, 18]. Due to the presence of inhomogeneous strain distribution, the variation of the stress in a load cycle causes a periodic temperature fluctuation that contributes to increase the internal friction.

The formation of dislocations due to cold work and/or high-temperature annealing introduces local imperfections in a metal. The movement of these dislocations under mechanical loading reduces the local modulus of elasticity and relaxes the rigidity of the metal [32]. The decrease in the local modulus of elasticity and the movement of the dislocations cause internal friction which is manifested by hysteresis area [27]. Given that the movement of the dislocations and local imperfections are small at lower strain levels and the effect of internal friction is less.
Ferromagnetic metals are subjected to the internal friction due to the effect of the magnetoelastic coupling. According to Zener [27], the application of mechanical load changes the magnetization in the material due to the variation in the local strains. The rotation of the direction of magnetization and the movement of the boundary between the adjacent regions cause internal friction. The variation of the magnetization is accompanied by the electric eddy currents that contribute to the internal friction. Magnetoelastic coupling is measured by \((E_s - E_0)/E_s\), where \(E_s\) and \(E_0\) stand for the modulus of elasticity at the magnetic saturation and at the zero magnetization, respectively. Becker and Kornetzki [37] showed that the area of a hysteresis loop reduces significantly when an iron wire is magnetically saturated to avoid the effect of magnetoelastic coupling.

According to Blanter et al. [28], when a material is subjected to a time-dependent cyclic load within the elastic limit, the internal friction causes energy dissipation due to the deviation from the Hooke’s law which is manifested by a stress-strain hysteresis loop. The amount of internal friction can be measured by \(Q^{-1} = \frac{\Delta W_H}{2\pi \Delta W_{el,max}}\) where \(\Delta W_H\) and \(\Delta W_{el,max}\) stand for the total energy and maximum elastic strain energy, respectively, dissipated in a load cycle [29]. The amount of internal friction can also be measured by estimating the phase lag, \(\phi\), between the stress and the strain expressed as [28, 29]:

\[ Q^{-1} = \tan \phi \] (4.3)

If the cyclic stress and strain associated with a fatigue test are recorded, the amount of plastic strain energy in the hysteresis loops can be estimated by eliminating the phase lag between them. Since the temperature evolution in an HCF test is negligible, its influence on the internal friction can be neglected. If the load amplitude is maintained constant throughout a fatigue test, the internal friction caused by the amplitude of vibration/oscillation, thermal current, dislocation movements, and ferromagnetism can be assumed to be constant. In the case of a smaller stress level that causes low amplitude of the strain, the effect of these parameters on the internal friction must be less than that at a larger stress level. Thus, if the effect of either of these parameters increases, an increase in the internal friction and the phase lag between the stress and strain occur. A larger phase lag between the stress and strain corresponds to a greater amount of anelastic energy in a hysteresis loop.

Although the effect of vibration/oscillation, thermal current, dislocation movements, and ferromagnetism is less at a lower stress level, the influence of test frequency is considered to be the most influential parameter on the internal friction [26, 31]. In order to determine the effect of test frequency on the hysteresis area, experiments are performed at different frequencies and stress levels. The evolution of plastic strain energy is calculated by eliminating the phase lag between the stress and the strain. The FFE associated with a fatigue test is then calculated based on the
experimentally measured $\Delta W_p$ and $T$ evolution using Equation (4.2). In the corresponding simulations, Equation (4.1) is solved in the entire 3-D model of the specimen using the finite element method (FEM) to predict temperature evolution and the FFE is calculated according to Equation (4.2).

4.3. Experimental

4.3.1. Material and specimen preparation

Figure 4.1 presents a photograph of a tubular MCS 1018 specimen along with the dimensions. The specimens have a 3.0 mm diameter through-hole at the center of the cross-section and are prepared in accordance with the ASTM:E466-07. The reduced section surface of the specimens is polished longitudinally using sand papers with progressively finer grit sizes to bring the surface roughness to within $R_a$=0.2-µm (ASTM:E466-07) to reduce the possibility of the initiation of microcracks from the nicks, dents, scratches, and circumferential tool marks.

![Figure 4.1. Photograph of a tubular MCS 1018 specimen showing dimensions in millimeter](image)

4.3.2. Equipment and test procedure

Stress-controlled uniaxial tension-compression fatigue tests are performed at different stress levels with the frequency of $f = 15$, 5, and 2.5 Hz and the load ratio of $L_R = -1$. Each test is carried out at a constant stress level using a servo-hydraulic fatigue tester with the capability of a maximum of 50 kN axial load and 75 Hz frequency. The strain in the specimen gage section is measured by an extensometer with the gauge length of 25.4 mm and travel between -10% and +50% strain. The load cell mounted on the fatigue tester and the extensometer directly interface with a computer to record the load and strain data in real time. Stress-strain plot is used to obtain the hysteresis loop and the area contained within its boundaries is the measure of the hysteresis energy generation in a load cycle, $\Delta W_H$.

A static test is performed with a specimen in accordance with the ASTM:E8-04 to determine the yield strength of the material using the servo-hydraulic machine. The yield strength of MCS 1018 is found to be about 520 MPa.

An infra-red (IR) camera is used to capture the surface temperature of the specimen during each fatigue test. The IR camera has the temperature measurement range capability between 0°C and 500°C, resolution of 320×240 pixel, accuracy of ±2% of the reading, and sensitivity of 0.08°C that...
allow one to measure the small temperature rise in an HCF test. In order to reduce IR reflection and increase thermal emissivity of the specimen surface, a thin layer of black paint is sprayed on the specimens’ gage section and allowed to dry out for about 12 hours before fatigue test. Temperature of the entire specimen gage section is recorded throughout the fatigue test. As the stress levels applied on the specimen are smaller than the yield strength of the material, the temperature hot-spot is not observed over most of the fatigue life. See a typical thermographic image of the specimen shown in Figure 4.2. An average temperature along an approximately 5-mm long line at the middle of the gage section is used for the analysis.

Figure 4.2. Thermographic image corresponding to $N = 40,000$ cycle obtained from a fatigue test performed with $\sigma = 320$ MPa and $f = 15$ Hz showing the line of interest of temperature data for analysis (temperature values are in °C)

4.3.3. S-N curve

Figure 4.3 shows an S-N curve of MCS 1018 demonstrating an increase in fatigue life at lower stress levels, as expected. Each data point in the S-N curve corresponds to a fatigue test at a specific stress level and loading frequency. A fatigue test is started with a pristine specimen and the number of load cycles required to fracture the specimen is the fatigue life, $N_f$. Then, stress vs. fatigue life plot provides with an S-N curve. It can be seen that the fatigue life of the specimens at 320 MPa stress is comparable for three different frequencies (15, 5, and 2.5 Hz). This shows that the fatigue life of this material is roughly insensitive to the test frequency within the range of the frequencies considered. This finding corroborates with the published work of Liaw et al. [36] and Tobushi et al. [38]. The logarithmic correlation between the stress level and the fatigue life is also shown in this figure.
4.3.4. Effect of frequency on hysteresis energy, $\Delta W_H$

Figure 4.4 presents the evolution of $\Delta W_H$ at 15, 5, and 2.5 Hz frequency subject to 320 MPa stress level. The evolution of hysteresis energy shows a slight increase with the progress of the fatigue test over most of the fatigue life followed by a sharp rise at the onset of fracture. It can be seen that the value of $\Delta W_H$ decreases with the decrease in the test frequency. Decreasing the test frequency tends to reduce the effect of the thermal current, dislocation movements, and ferromagnetism in the material. As a result, the effect of the internal friction is decreased. The reduction of the internal friction at lower test frequencies is represented by the decrease in $\Delta W_H$ evolution. Figure 4.5 presents the results of the average hysteresis energy as a function of the test frequency. The gradual decline in the area of the hysteresis loop with the decreasing frequency indicates that the amount of anelastic energy contained within a hysteresis loop reduces with the decrease of the frequency. The trend of this plot agrees with the results reported by Wertz et al. [26] and Wegel and Walther [31]. Thus, by continuing the reduction of frequency, it is possible to arrive at a frequency when further reduction does not decrease the area of hysteresis loop. The area of such a stabilized hysteresis loop represents the amount of cyclic plastic strain energy generation at that stress level.

4.3.5. Estimation of plastic strain energy, $\Delta W_p$

Figure 4.6(a and b) presents the phase lag between the stress and strain at 50th load cycle ($N=50$) corresponding to the tests at 15 and 2.5 Hz, respectively. The 50th load cycle is selected since the fatigue tester requires several load cycles to stabilize to the desired stress level. It can be seen that
Figure 4.4. Evolution of hysteresis energy at three different frequencies subject to 320 MPa stress

Figure 4.5. Average hysteresis energy per cycle vs. test frequency plot at 320 MPa stress
the phase lag decreases with the reduction in the test frequency under the identical stress level. The phase lag between the stress and strain is reduced since the amount of anelastic energy in a hysteresis loop drops with the decrease of the frequency. This finding is in accordance with the results reported by Ozaltun et al. [39] and Wertz et al. [26]. Figure 4.7(a, b, and c) shows the phase lag between the stress and strain after different number of load cycles in a fatigue test. These results
Figure 4.7. Phase lag between the stress and strain at (a) $N = 50$, (b) $N = 150,000$, and (c) $N = 250,000$ cycles obtained from a fatigue test performed at 320 MPa stress and 5 Hz frequency.
reveal that the phase lag increases gradually with the increase of the number of load cycle that accumulates irreversible degradation in the material. This implies that the phase lag between the stress and strain in a fatigue test is observed due to the effect of the internal friction and the accumulation of the irreversible changes in the material. Since the experiments are performed with pristine specimens with no prior mechanical degradation, it can be considered that the phase lag at the onset of a fatigue test, i.e., at $N=50$ cycles, occurs due to the effect of internal friction. McGuire et al. [40] showed that the amount of internal friction during a fatigue process remains fairly constant until the onset of fracture. Thus, the gradual increase of phase lag corresponds to the accumulation of degradation. Consequently, a larger amount of degradation or plastic strain energy generation increases the phase lag between the stress and the strain. The increase in phase lag in a fatigue test is manifested by a gradual increase in the area of the hysteresis loops as shown in Figure 4.4. Since the applied stress is substantially smaller than the yield strength of the material, the area of the hysteresis loops at the onset of a fatigue test can be considered as anelastic energy and its gradual increase is attributed to the accumulation of plastic deformation. According to Equation (4.3), the subtraction of the amount of phase lag observed at the onset of a fatigue test from all the hysteresis loops in that test provides one with the evolution of the hysteresis loops that represent plastic strain energy.

Figure 4.8(a, b, and c) presents the comparison between the evolution of hysteresis and plastic strain energy corresponding to different frequencies and 320 MPa stress level. Results show that the evolution of $\Delta W_H$ is greater than $\Delta W_p$ and the trend of their evolution is fairly identical. After the beginning of these tests, the evolution of $\Delta W_p$ remains negligible for about 50,000 cycles and then its value increases slowly followed by a sudden rise at the onset of fracture. Because the applied stress is substantially less than the yield strength of the material, negligible evolution of $\Delta W_p$ is observed in the early stage of these tests. The accumulation of irreversible degradation in the material is represented by a slow increase of $\Delta W_p$ evolution followed by a sharp rise at the onset of the specimen fracture due to the initiation and propagation of a macrocrack (Liakat and Khonsari [25]). Figure 4.9(a and b) shows the effect of stress level on the evolution of $\Delta W_H$ and $\Delta W_p$ at 15 Hz frequency. It can be seen that the difference between $\Delta W_H$ and $\Delta W_p$ associated with a test increases with the increase in the stress level. At a higher stress level, a larger strain level produces a greater amount of dislocation movement, thermal current, and ferromagnetism resulting in an increase in the internal friction [27, 28, 41, 42]. The increase of the internal friction causes a larger phase lag between the stress and strain which is manifested by larger hysteresis loop evolution.

Figure 4.10 shows a correlation between the average cyclic plastic strain energy and the stress level for all the tests performed in this work. Results show that the average value of $\Delta W_p$ decreases with the decrease in the stress level, as expected. The exponential correlation between these parameters shown in Figure 4.10 can be used to predict the value of $\Delta W_p$ at other stress levels.
Figure 4.8. Evolution of $\Delta W_H$ and $\Delta W_p$ obtained from fatigue tests carried out at 320 MPa stress and (a) 15 Hz, (b) 5 Hz, and (c) 2.5 Hz frequency.
Figure 4.9. Evolution of (a) $\Delta W_H$ and (b) $\Delta W_p$ obtained from fatigue tests carried out at different stress levels and 15 Hz frequency.
4.3.6. Temperature evolution

Figure 4.11 presents the evolution of temperature at different stress levels and 15 Hz frequency. These results show that the temperature evolution at the onset of the fatigue test is negligible (see inset), remains fairly constant over most of the fatigue test, and sharply increases at the onset of the specimen fracture. Temperature evolution is smaller at lower stress levels as a smaller stress level generates less amount of plastic strain energy compared to a higher stress level as shown in Figure 4.9(b). The temperature evolutions obtained from the fatigue test performed at 255 and 245 MPa remain roughly at the ambient temperature as the evolution of plastic strain energy is smaller than the other tests shown in Figure 4.11. The small amount of plastic strain energy generated in these tests converts into heat energy and dissipates to the surroundings via convection. As a result, the temperature evolution in an HCF test shows two distinct phases, i.e., the steady evolution over most of the fatigue life followed by a sharp rise at the onset of the specimen fracture. In contrast, three distinct phases are observed in the temperature evolution in an LCF test (e.g., see [25, 43]).

4.4. Numerical simulations

4.4.1. Computational model and simulation procedure

Simulations are performed with a commercial code, FlexPDE, that solves the governing partial differential equation in the 3-D model of the specimens using FEM. Because of the symmetric geometry of the specimens with respect to their longitudinal axis, only half of the specimen
Figure 4.11. Evolution of temperature obtained from experiments at different stress levels and 15 Hz frequency. The inset shows the initial temperature rise.

containing an end section, a curved-section, and half of the gage section is simulated. Figure 4.12 illustrates the 3-D FEM model meshed with 10-node quadratic tetrahedral elements. In order to produce better quality of mesh, the curved-section of the specimen is modelled as a trapezoid neglecting the small amount of curvature. Convective boundary condition \( k \frac{\partial T}{\partial n} = h(T - T_a) \) is applied to the surfaces labelled by B1 where \( n \) represents the surface normal parameter, \( T_a \) denotes the ambient temperature, and \( h \) stands for the heat transfer coefficient. The surface normal parameter indicates a radial displacement parameter such that \( \frac{\partial T}{\partial n} \) denotes a radial temperature gradient in the specimen. The boundary B2 represents the plane of symmetry, heat transfer does not take place through this plane. The value of \( h \) is calculated according to the procedure described in [25].

Although a fatigue load cycle generates both elastic and plastic strain energy, elastic strain energy is a reversible quantity that does not cause degradation in the material. Consequently, elastic strain energy is not considered in the FEM model. In contrast, plastic strain energy is an irreversible quantity that corresponds to the irreversible degradation. Since the gage section of the specimen undergoes irreversible deformation during a fatigue test, the value of \( \Delta W_p \) is applied to the gage section. In the case of the simulation of an experiment, the value of \( \Delta W_p \) obtained from an experiment is utilized. Whereas, to estimate the FFE corresponding to a stress level that is not
considered in the experiment, a constant value of $\Delta W_p$ calculated from Figure 4.10 is utilized in the simulation. A simulation is continued for the number of load cycles predicted from Figure 4.3 for a specific stress level. Equation (4.1) is solved in the entire 3-D model at 15 Hz frequency to obtain the thermal response of the specimen using the physical and the thermal properties as shown in Table 4.1. FFE is calculated by integrating the right-hand-side of Equation (4.2) throughout a simulation.

![Figure 4.12. The FEM model of the half of the specimen meshed with 10-node quadratic tetrahedral elements](image)

**4.4.2. Effect of mesh size**

Depending on the location in the FEM model, different mesh sizes are utilized as shown in Figure 4.12. To determine the effect of mesh size on the FFE and temperature evolution, a mesh size dependency study is carried out on the FEM model following the procedure presented in [25].

<table>
<thead>
<tr>
<th>Material</th>
<th>Density, $\rho$ ($\text{kgm}^{-3}$)</th>
<th>Specific heat, $C$ ($\text{Jkg}^{-1}\text{K}^{-1}$)</th>
<th>Thermal conductivity, $k$ ($\text{Wm}^{-1}\text{K}^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCS 1018</td>
<td>7870</td>
<td>486</td>
<td>51</td>
</tr>
</tbody>
</table>

Table 4.1. Physical and thermal properties of MCS 1018
Table 4.2 presents the values of the FFE obtained from simulations, corresponding total number of mesh, and fatigue life associated with a test at $\sigma = 320$ MPa and $f = 15$ Hz. An increase in the total number of mesh from 67,232 to 94,765 increases the FFE from 26.38 to 26.51 MJm$^{-3}$K$^{-1}$. Since the experimental FFE for this test is 26.49 MJm$^{-3}$K$^{-1}$, the mesh sizes corresponding to the total number of mesh 94,765 are adopted for all the simulations. It is also found that the temperature evolution is negligibly related to the mesh size and the amount of entropy generation due to the axial heat conduction is found to be negligible compared to the total entropy generation in an HCF test.

<table>
<thead>
<tr>
<th>Test conditions</th>
<th>Fatigue life, $N_f$</th>
<th>Number of mesh</th>
<th>FFE, $\gamma$ (MJm$^{-3}$K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>320 MPa, 15 Hz</td>
<td>256549</td>
<td>67232</td>
<td>26.38</td>
</tr>
<tr>
<td></td>
<td>78651</td>
<td></td>
<td>26.44</td>
</tr>
<tr>
<td></td>
<td>94765</td>
<td></td>
<td>26.51</td>
</tr>
</tbody>
</table>

4.4.3. Validation of the plastic strain energy prediction method

Figure 4.13 illustrates a temperature contour of the 3-D model of the specimen corresponding to $N = 40,000$ cycle obtained from an FEM simulation of a fatigue test performed with $\sigma = 320$ MPa and $f = 15$ Hz. It can be seen that the heat generation in the specimen gage section increases its temperature by about 2 °C which is comparable to the temperature rise captured by the IR camera as shown in Figure 4.2.

Figure 4.13. Temperature contour corresponding to $N = 40,000$ cycle obtained from a simulation associated with $\sigma = 320$ MPa and $f = 15$ Hz (temperature values are in °C)

Figure 4.14 presents the comparison between the experimental temperature and the temperature evolutions predicted using $\Delta W_H$ and $\Delta W_p$ as the source of heat energy in separate simulations.
Temperature data is obtained from the same location of the FEM model of the specimen as shown in Figure 4.2. Results demonstrate that $\Delta W_H$- and $\Delta W_p$-based simulation temperatures are about 6-7 °C and 0.5 °C greater than the experimentally measured temperature, respectively. The larger deviation between the $\Delta W_H$-based simulation and experimental temperature shows that the area of the hysteresis loops is greater than the amount of plastic strain energy generation in HCF. The negligible error between the experimental and $\Delta W_p$-based simulation temperature indicates that the amount of heat generation in the specimen is comparable to the amount of plastic strain energy calculated based on the proposed methodology. This shows that the proposed methodology of plastic strain energy estimation is useful. The negligible difference between the simulation and the experimental results could be due to the assumption of constant air velocity around the specimen and the physical and thermal properties obtained from the literature may vary by a small value for the material tested. The agreement between the experimental and numerical temperature contours and the experimental and $\Delta W_p$-based simulation temperature evolutions show that the FEM model represents the fatigue experiments with reasonable accuracy.

![Figure 4.14](image-url)

Figure 4.14. Comparison between the experimental and $\Delta W_H$- and $\Delta W_p$-based simulation temperature evolutions at 320 MPa stress and 15 Hz frequency. The inset shows the initial temperature rise.

4.4.4. Evaluation of fatigue fracture entropy

Figure 4.15 presents the experimental and predicted FFE vs. $N_f$ plot. The experimental FFE is calculated based on the experimental $\Delta W_p$ and $T$ evolutions. The FFE of the fatigue tests performed at 15 Hz frequency are compared to that obtained from the simulation of the experiments using
experimental $\Delta W_p$ as the heat source. The value of relative error between the experimental and the numerical FFE is calculated as $\left(\frac{\text{Experimental FFE} - \text{Numerical FFE}}{\text{Experimental FFE}}\right) \times 100\%$. The range of relative error in FFE calculation is found to be between 2.35% and 6.00%. The good agreement between the values of the FFE shows that the simulation represents the fatigue test with reasonable accuracy. Then, the FEM simulations are performed corresponding to the stress levels that are not considered in the experiments to predict the FFE. Results show that the experimental and predicted FFE remains within a narrow band independent of the loading conditions, which validates the idea of constant entropy gain at fatigue fracture [20-22]. The condition for fatigue fracture corresponds to the thermodynamic entropy accumulation of $25 \pm 5 \text{MJm}^{-3}\text{K}^{-1}$ for MCS 1018 in HCF. Liakat and Khonsari [25] reported a similar band for the FFE of the given material associated with LCF.

![Figure 4.15. Experimental and predicted FFE vs. fatigue life plot of MCS 1018 specimens](image)

The concept of constant entropy accumulation at the fatigue fracture is useful for the fatigue life monitoring of a material. It implies that the necessary and sufficient condition for the fatigue fracture corresponds to the entropy accumulation up to the FFE of that material. Taking advantage of this concept, Naderi and Khonsari [21] developed a fatigue monitoring system for the prevention of the catastrophic fracture. Amiri et al. [44] showed that the evolution of fatigue damage can be estimated utilizing the concept of tallying entropy. Whereas the previous works on this subject were largely limited to LCF, the present paper demonstrates the validity of thermodynamic entropy approach in HCF.
It is important to note that the environmental conditions, e.g., temperature, humidity, etc., are maintained constant during all the tests. The proposed methodology is validated for the fatigue processes where the stress levels are substantially smaller than the yield strength of the material and with the specimens without stress concentration. Therefore, the applicability of this method for the specimens with stress concentration requires further investigation.

4.5. Conclusions

A series of uniaxial tension-compression fatigue tests are carried out with tubular MCS 1018 specimens to determine the validity of a methodology for the estimation of plastic strain energy and thermodynamic entropy generation in HCF of metallic specimens. It is shown that most of the area of a hysteresis loop is composed of non-damaging anelastic energy when the applied stress is substantially smaller than the yield strength of the material. The presence of anelastic energy in the hysteresis loops occurs due to a phase lag between the stress and the strain. A methodology is proposed to remove the anelastic energy from the hysteresis loops by subtracting the amount of phase lag between the stress and strain found at the onset of a fatigue test from all the hysteresis loops in that test. Thus, the evolution of plastic strain energy in an HCF is calculated. The evolution of plastic strain energy and the temperature in an HCF test are discussed and utilized to calculate FFE. FEM simulations are performed with a 3-D model of the specimen to determine the validity of the proposed method of plastic strain energy estimation and to predict the FFE for the loading conditions that are not considered in the experiments. Experimental and simulation results show that the FFE of MCS 1018 is nearly constant in HCF within the experimental and loading conditions considered.

4.6. Nomenclature

- $A_i$: thermodynamic forces associated with internal variables
- $C$: specific heat (Wm$^{-1}$K$^{-1}$)
- $E_s$: elasticity modulus at magnetic saturation (GPa)
- $E_0$: elasticity modulus at zero magnetism (GPa)
- $f$: frequency (Hz)
- $h$: heat transfer coefficient (Wm$^{-2}$K$^{-1}$)
- $k$: thermal conductivity (Wm$^{-1}$K$^{-1}$)
- $L_R$: load ratio
- $n$: surface normal parameter
- $N$: number of load cycle
- $N_f$: fatigue life (cycle)
- $q$: heat flux (Wm$^{-2}$)
$Q^1$  internal friction
$R_a$  arithmetic average of surface roughness ($\mu$m)
$T$  specimen temperature (K)
$T_a$  ambient temperature (K)
$\Delta W_{el,max}$ maximum elastic strain energy per cycle (MJm$^{-3}$/cycle$^{-1}$)
$\Delta W_H$  hysteresis energy per cycle (MJm$^{-3}$/cycle$^{-1}$)
$\Delta W_p$  plastic strain energy per cycle (MJm$^{-3}$/cycle$^{-1}$)
$\gamma$  fatigue fracture entropy (MJm$^{-3}$/K$^{-1}$)
$\dot{\varepsilon}^p$  time rate of plastic strain tensor (s$^{-1}$)
$\rho$  density (Kgm$^{-3}$)
$\sigma$  stress tensor
$\sigma$  nominal stress (MPa)
$\sigma_y$  yield strength (MPa)
$\phi$  phase lag (°)

4.7. References


84


[34] Quinney H, Taylor GI. The emission of the latent energy due to previous cold working when a metal is heated. Proc Roy Soc A 1937;163:157-81.


Chapter 5: Nondestructive Testing and Prediction of Remaining Fatigue Life of Metals*

5.1. Introduction

Most engineering structures and components exposed to cyclic fatigue load are subjected to time-dependent degradation that reduces their remaining fatigue life (RFL). Therefore, an accurate prediction of RFL of a specimen or component that has already been in service has long been of interest.

There are many established methods [1-4] to predict fatigue life, \( N_f \), of components in an accelerated fashion. However, they generally apply to pristine specimens without prior history of fatigue damage. There are also notable publications dealing with structural health estimation methods that describe methodologies for detecting flaws and cracks in silicon wafers [5] and metallic components [6-9], axially aligned defects in pipes [10], studies on the effects of crack properties on ultrasonic detection methods [11], and different approaches for damage prediction of metallic structures subjected to multi-stage loading [12]. However, reliable estimation of damage before crack initiation and during crack propagation through nondestructive testing (NDT) still remains a formidable challenge.

Published literature on fatigue damage characterization of specimens with prior fatigue damage is relatively scarce. Relevant contributions include the work of Sumi [13] who developed an advanced simulation method using which one can predict crack propagation behavior in marine structures and assess their RFL. Meyendorf et al. [14] reported a thermography-based fatigue damage assessment methodology in which the initial rate of temperature increase during a short-term mechanical loading is used as an index of the microstructural state and the presence of prior fatigue damage. Amura and Meo [15] demonstrated that nonlinear guided waves or the acoustic nonlinear signature at two different fatigue damage levels provide clear signs of the progressive fatigue damage, which is useful for the prediction of RFL.

More recently Amiri and Khonsari [16] and Williams et. al. [17] introduced a thermography-based NDT method to predict RFL of rotating bending and welded specimens, respectively, where they utilized the slope of temperature rise, \( R_\theta \), as a measure of fatigue degradation. The value of \( R_\theta \) is measured by inducing a series of short-time excitation (STE) tests at different intervals during a normal fatigue test (NFT) in which the number of load cycles accumulate until the specimen fractures. The evolution of \( R_\theta \) for specific loading conditions was found to be useful for the prediction of RFL.

*Reprinted by permission of Journal of Nondestructive Evaluation (See Appendix A)
In the present work, an empirical correlation is developed to characterize the evolution of $R_\theta$ of specimens produced from two different materials for a wide range of uniaxial tension-compression NFT loading. The characterization fatigue tests provide the so-called S-N (stress vs. fatigue life) curve of the materials for each NFT series. The derived correlation and the S-N curve are then utilized to develop an NDT method to predict RFL of specimens with prior fatigue damage. The description of the method, presentation of the results, and a series of validation tests comprise the content of this paper.

5.2. Experimental

5.2.1. Materials and Equipment

The materials tested for the experimental investigation are API 5L X52 (a high-strength steel) and carbon steel 1018. Specimens made of API 5L X52 are solid while the carbon steel 1018 specimens are tubular. They are manufactured according to the ASTM standard E-466-07. The test section of the specimen is polished longitudinally progressing through 0, 00, and 000 emery papers to eliminate nicks, dents, scratches, and circumferential tool marks. The polishing is performed to ensure that maximum surface roughness is within $R_a=0.2 \ \mu m$ in the longitudinal direction (ASTM E-466-07). The schematic diagrams and dimensions of the specimens are shown in Figure 5.1.

![Figure 5.1](image_url)

(a)

(b)

Figure 5.1. Schematic diagrams of (a) dogbone specimen made of API 5L X52 and (b) tubular specimen made of carbon steel 1018 (all dimensions are in millimeter unless otherwise specified)

Constant-amplitude, stress-controlled fatigue tests are carried out using an axial-torsion, servo-hydraulic fatigue tester with the capability of a maximum of 50 kN axial load, 2 kNm torsion load, and 75 Hz of frequency. An infrared (IR) camera with temperature range capability between 0°C
and 500°C, resolution of 320×240 pixel, accuracy of ±2% of reading, and sensitivity/NETD of 0.08°C (at 30°C) is used to record surface temperature of the specimen during STE test at a data acquisition (DAQ) rate of 1 Hz. In order to reduce IR reflection and increase thermal emissivity, the test section of the specimen is sprayed with black paint. The fatigue tester and the IR camera interface with a computer equipped with data acquisition capability.

5.2.2. Test Procedure

The test procedure is as follows. Referring to Figure 5.2(a), the specimen is vertically gripped between the jaws of the top and bottom grips and the IR camera is positioned to directly face the gage section of the specimen. The distance between the specimen and the lens of IR camera is around 35 cm. The top grip of fatigue tester remains stationary and the bottom grip oscillates vertically to apply cyclic axial load on the specimen, which generates heat in the gage section. The temperature evolution over the entire gage section of the specimen is captured by the IR camera in real time. The average temperature along the line of about 5 mm-long at the middle of the specimen’s gage section, where the temperature is the maximum, is used in the analysis. See Figure 5.2(b).

![Figure 5.2. (a) Experimental set-up for uniaxial fatigue test and (b) typical thermographic image of specimen showing location of temperature measurement](image)

There are two distinct types of tests: the NFT and the STE test. The NFT is performed at the load amplitude, $\sigma$, load ratio (defined as the ratio of minimum to the maximum stress of cyclic load), $L_R$, and loading frequency, $f$, that a material is expected to experience in the field during its normal operating conditions. The STE tests (typically 15-20 s) are performed at the load amplitude, load ratio, and test frequency chosen by the operator, which are maintained constant for all the
STE tests with a specific material. The STE test loading conditions chosen in the present work are: \( \sigma = 402 \text{ MPa}, L_R = -0.56, \) and \( f = 10 \text{ Hz} \) and \( \sigma = 395 \text{ MPa}, L_R = -0.6, \) and \( f = 10 \text{ Hz} \) for API 5L X52 and carbon steel 1018, respectively.

Beginning with a pristine specimen, first, the threshold slope \( R_{\theta_0} \) is measured by an STE test (See Figure 5.3(a and b)). After stopping the STE test, the NFT is performed for a specific number of load cycles. The NFT is then stopped and the specimen is allowed to cool down to the ambient temperature. Then, the slope of temperature rise \( R_{\theta_1} \) is measured by a subsequent STE test. The above procedure is repeated until the specimen fractures. Note that specimen surface temperature is only measured during the STE tests. The number of load cycle in each NFT between two subsequent STE tests should be chosen in such a way that \( R_{\theta} \) is measured at least 6-7 times if \( N_f \leq 10^4 \) cycles and around 8-10 times if \( N_f > 10^4 \) cycles.
It should be noted that the total number of load cycles corresponding to STE tests is negligible compared to the number of load cycles associated with a NFT in both LCF and HCF. Although STE tests produce negligible amount of damage, the number of the load cycles obtained from each STE test is added to the number of load cycles obtained from NFT. For simplicity, the small difference in the load amplitude and the load ratio between the STE test and the NFT is assumed to be negligible.

5.3. Analysis

Research shows that a fatigue degradation process is typically characterized by three distinct phases of temperature evolution (Figure 5.4): an initial temperature rise (Phase I), an intermediate “steady-state” temperature (Phase II), and final sharp temperature rise (Phase III). This information has been put to use by researchers in different ways to predict the fatigue behavior of the materials. For example, some rely on the slope of initial temperature rise $R_\theta$ (e.g., Amiri and Khonsari [3, 4]; Khonsari and Amiri [18]), others use the steady state temperature rise $\Delta T$ (e.g., Jiang et. al. [2]; Jiang et. al. [19]), and still others utilize the slope of sharp temperature rise $R'_\theta$ (e.g., Huang et. al. [1]) as indices for characterizing fatigue life.

![Figure 5.4](image)

Figure 5.4. Typical evolution of temperature in a fatigue test and three distinct phases

Amiri and Khonsari [3, 4] developed a correlation between the fatigue life of a specimen, $N_f$, without prior material damage, i.e., pristine material, and the slope of the initial temperature rise, $R_\theta$. Clearly the thermal response of a specimen with prior fatigue history cannot be evaluated in this fashion because of the changes in the material’s microstructures [14, 16, 20]. Meyendorf et. al. [14] revealed that as the fatigue load cycles accumulate, the magnitude of $R_\theta$ obtained from STE tests increase gradually. They postulated the following relationship between $R_\theta$ and the applied fatigue load, $\sigma$:

$$R_\theta = F \sigma^{m+1}$$

(5.1)
where \( m \) is an empirical material constant. \( F \) is a function of the material microstructure, the specimen temperature, and the STE test loading conditions expressed as:

\[
F = F \left( \text{material microstructure, specimen temperature, STE test loading conditions} \right) \tag{5.2}
\]

Since the variation in temperature during an STE test is small, its effect on the microstructure of the specimen is assumed to be negligible [14]. However, as the NFT load cycle \( N \) accumulates, material’s microstructures does change due to the nucleation of microcracks, formation of macrocracks, and eventually complete failure occurs [16]. Thus, \( F \) is expressed as follows:

\[
F = F \left( N, \text{STE test loading conditions} \right) \tag{5.3}
\]

Assume that an STE test of specific loading conditions is performed on a pristine specimen to measure the slope of temperature rise, \( R_{\theta 0} \). In case of a pristine specimen \( N=0 \), there is no change in material microstructure, and the effect of NFT load, \( \sigma \), is zero. Thus, \( R_{\theta 0} \), hereafter referred to as the “threshold slope,” is only related to the STE loading conditions. Thus, Equation (5.1) is modified as follows.

\[
R_{\theta} = F_I \sigma^{m_1} + R_{\theta 0} \tag{5.4}
\]

where \( F_I \) is a function of \( N \). Since \( F_I \) and \( R_{\theta 0} \) depend on the material’s fatigue behavior and STE test loading conditions, respectively; the value of these parameters are determined experimentally.

### 5.4. Results and Discussion

#### 5.4.1. Evolution of \( R_{\theta} \)

Figure 5.5 illustrates the gradual increase in the heat generation obtained from STE tests after different number of accumulated fatigue load cycles, \( N \), in an NFT of API 5L X52 at \( \sigma = 420 \) MPa and \( L_R = -1 \). The gradual increase in heat generation is a manifestation of increasing fatigue damage and microstructural changes in the material due to cyclic loading, which is found to be independent of the material properties and NFT loading conditions. Thus, the slope of temperature evolution, \( R_{\theta} \), obtained from STE tests in a fatigue process also tends to gradually increase. Figure 5.6(a and b) presents the evolutions of \( R_{\theta} \) of API 5L X52 obtained from two distinct series of NFTs at different load amplitudes, constant test frequency of 10 Hz, and \( L_R = -0.5 \), respectively. Note that the STE test loading conditions, \( \sigma = 402 \) MPa, \( f = 10 \) Hz, and \( L_R = -0.56 \), are identical for these two series of tests.

The evolutions of \( R_{\theta} \) of carbon steel 1018 at different NFT load amplitudes and load ratios with constant test frequency of 10 Hz are shown in Figure 5.7. The STE test loading conditions, \( \sigma = 395 \) MPa, \( f = 10 \) Hz, and \( L_R = -0.6 \), are identical for these tests.
Figure 5.5. Thermal responses of API 5L X52 specimen obtained from STE tests after different number of accumulated load cycles in a NFT at $\sigma = 420$ MPa and $L_R = -1$

Figure 5.6(a). $R_\theta - N$ plot of API 5L X52 specimens subjected to uniaxial tension-compression test at different NFT stress levels and with $L_R = -1$
Figure 5.6(b). $R_\theta$ - $N$ plot of API 5L X52 specimens subjected to uniaxial tension-compression test at different NFT stress levels with $L_R = -0.5$

Figure 5.7. $R_\theta$ – $N$ plot of carbon steel 1018 specimens subjected to uniaxial tension-compression test at different NFT stress levels and load ratios

Results presented in Figures 5.6 and 5.7 demonstrate a consistent trend of $R_\theta$ evolution in all the tests of two different materials (both tubular and solid specimens) under a wide range of NFT loading conditions. The value of $R_\theta$ of pristine material, corresponding to the first STE test, i.e., $N = 0$, is fairly constant and independent of NFT load amplitude for each series of tests. The
accumulation of fatigue load cycles induces irreversible degradation and microstructural changes in the material. The results manifest themselves in a linear increase in the evolution of $R_\theta$ (See Figure 5.8). The gradual increase in $R_\theta$ with the increase in permanent fatigue damage conforms to the postulate of Meyendorf et. al. [14] expressed by Equation (5.1). It is also observed that a larger NFT load amplitude produces greater rate of $R_\theta$ evolution, $\frac{\partial R_\theta}{\partial N}$, for both of the materials.

![Figure 5.8. $R_\theta$ evolution of API 5L X52 as a function of accumulated fatigue life obtained from uniaxial tension-compression test at 10 Hz](image)

Experimental results show that the intercept, $R_{\theta0}^c$, of the linear curve fit of $R_\theta$ evolution on the ordinate is greater than the experimental value of $R_{\theta0}$ for each NFT (See Figures 5.6-5.8). The average values of $R_{\theta0}$ and $R_{\theta0}^c$ for each series of NFT are shown in Table 5.1. Since the STE test

<table>
<thead>
<tr>
<th>Material</th>
<th>STE test loading conditions</th>
<th>NFT load ratio, $L_R$</th>
<th>Experimental $R_{\theta0}$ (°C/sec)</th>
<th>Curve fit $R_{\theta0}^c$ (°C/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>API 5L X52</td>
<td>$\sigma = 402$ MPa, $f = 10$ Hz, $L_R = -0.56$</td>
<td>-1</td>
<td>0.13</td>
<td>0.17</td>
</tr>
<tr>
<td>API 5L X52</td>
<td>$\sigma = 402$ MPa, $f = 10$ Hz, $L_R = -0.56$</td>
<td>-0.5</td>
<td>0.12</td>
<td>0.22</td>
</tr>
<tr>
<td>Carbon steel 1018</td>
<td>$\sigma = 395$ MPa, $f = 10$ Hz, $L_R = -0.6$</td>
<td>-0.53 to -0.62</td>
<td>0.09</td>
<td>0.18</td>
</tr>
</tbody>
</table>
loading conditions are maintained constant for two series of tests with API 5L X52 under different NFT loading conditions, the values of $R_{\theta 0}, 0.13$ and $0.12$, are found to be nearly constant (Table 5.1). This validates that the threshold slope is primarily dependent on the STE test loading conditions and fairly independent of NFT loading conditions.

5.4.2. Discussion

All the test results presented in Figures 5.6 and 5.7 demonstrate that $R_{\theta}$ evolutions with respect to $N$ vary in approximately linear fashion. Therefore, substituting $N$ and $R_{\theta 0}^c$ for $F_1$ and $R_{\theta 0}$, respectively, in Equation (5.4) yields:

$$R_{\theta} = \sigma^{m+1} N + R_{\theta 0}^c$$ (5.5)

Figures 5.6 and 5.7 demonstrate that the slope of $R_{\theta}-N$ plot increases as the NFT load amplitude increases. For example, a series of five fatigue test results presented in Figure 5.6(b) shows that the slope of $R_{\theta}-N$ plot increases with increasing NFT load amplitude if the fatigue test frequency and the load ratio are maintained constant. Figure 5.9(a and b) illustrates the relationship between $n$ (the slope of $R_{\theta}-N$ plot) and $\sigma$ for API 5L X52 and carbon steel 1018 at different NFT load ratios.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.9a.png}
\caption{\textit{n vs. $\sigma$} plot for API 5L X52 at different load ratios}
\end{figure}
Experimental results reveal that a change in NFT load ratio from -1 to -0.5 affects the $n$-$\sigma$ plots significantly (See Figure 5.9(a)). A correlation between $n$ and $\sigma$ is found to be: $n = Ae^{B\sigma}$ where $A$ and $B$ are constants that depend on the material properties and NFT and STE test loading conditions. Therefore, substituting $Ae^{B\sigma}$ for $\sigma^{m+1}$ into Equation (5.5) yields:

$$R_\theta = Ae^{B\sigma}N + R_\theta^C$$

Equation (5.6) is useful in predicting RFL of a specimen with prior fatigue history. Table 5.2 presents the values of $A$ and $B$ for the materials tested corresponding to the NFT load ratios considered in this work. The results of the fatigue characterization tests render the S-N curves for each material at different NFT load ratios (See Figure 5.10(a and b)). The S-N curves obtained from two series of fatigue tests with API 5L X52 demonstrate that a change in fatigue load ratio from -0.5 to -1 decreases fatigue life significantly for specific load amplitude. The results also show that the scatter in each S-N curve is small for all three series of tests.

Table 5.2. Values of $A$ and $B$ for API 5L X52 and carbon steel 1018

<table>
<thead>
<tr>
<th>Material</th>
<th>NFT condition</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>API 5L X52</td>
<td>$L_R = -0.5$</td>
<td>$9 \times 10^{-15}$</td>
<td>0.0477</td>
</tr>
<tr>
<td>API 5L X52</td>
<td>$L_R = -1$</td>
<td>$1 \times 10^{-11}$</td>
<td>0.0367</td>
</tr>
<tr>
<td>Carbon steel 1018</td>
<td>$L_R = -0.53$ to -0.62</td>
<td>$4 \times 10^{-13}$</td>
<td>0.0426</td>
</tr>
</tbody>
</table>
5.4.3. Nondestructive Prediction of Specimen’s RFL

In order to predict RFL of a specimen in terms of the number of fatigue load cycles, Equation (5.6) can be rearranged as follows:

$$N = \frac{R^\theta - R^c}{Ae^{B\sigma}}$$

(5.7)
\( N \) is referred to as the accumulated or consumed fatigue life. Equation (5.7) can be used to determine the accumulated fatigue life in a specimen that has experienced an unknown number of NFT load cycles. The value of \( R \theta \) corresponding to the present material damage can be determined by an STE test of identical loading conditions to those used in the characterization tests. The values of \( R^{\theta c} \) and \( A \) and \( B \) for that material are obtained from Table 5.1 and Table 5.2, respectively. Substituting the values of these parameters into Equation (5.7), the consumed fatigue life, \( N \), can be predicted. Then, the fatigue life, \( N_f \), of that material is obtained from respective S-N curve. Finally, the RFL of the specimen is predicted, \( RFL_{\text{pred}} \), from the following formula:

\[
RFL_{\text{pred}} = \left(1 - \frac{N}{N_f}\right) \times 100\%
\]  

(5.8)

In order to verify the prediction of RFL, a series of experiments is performed where the NFT is resumed with the fatigued specimen and continued to determine the number of load cycles required to fracture the specimen. The experimental RFL, \( RFL_{\text{exp}} \), of the specimen, at the stage of NFT where the test is stopped to predict RFL, is calculated by substituting the number of load cycles performed before the STE test and total number of load cycles required to fracture the specimen for \( N \) and \( N_f \), respectively, into Equation (5.8). Then, the difference between the experimental and the predicted RFL is calculated as \( \Delta RFL = |RFL_{\text{exp}} - RFL_{\text{pred}}| \) %. Eight validation tests are performed at different NFT loading conditions with the given materials to evaluate the RFL prediction capability of the proposed method. The results of the predicted and the experimental RFL with corresponding \( \Delta RFL \) for each validation test are shown in Table 5.3.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \sigma, L_R )</th>
<th>( N ) (cycle)</th>
<th>( R_\theta ) (°C/s)</th>
<th>Predicted consumed life, ( N ) (cycle)</th>
<th>( N_f ) from S-N curve, (cycle)</th>
<th>No. of load cycles at fracture</th>
<th>( RFL_{\text{pred}} ) (%)</th>
<th>( RFL_{\text{exp}} ) (%)</th>
<th>( \Delta RFL ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>API 5L X52</td>
<td>456, -0.5</td>
<td>12020</td>
<td>0.57</td>
<td>13518</td>
<td>19500</td>
<td>17789</td>
<td>30.68</td>
<td>32.44</td>
<td>1.76</td>
</tr>
<tr>
<td>483, -0.5</td>
<td>6178</td>
<td>0.89</td>
<td>5932</td>
<td>7500</td>
<td>8569</td>
<td>20.90</td>
<td>27.90</td>
<td>7.00</td>
<td></td>
</tr>
<tr>
<td>483, -0.5</td>
<td>3422</td>
<td>0.64</td>
<td>3700</td>
<td>7500</td>
<td>7449</td>
<td>50.66</td>
<td>54.06</td>
<td>3.40</td>
<td></td>
</tr>
<tr>
<td>400, -1</td>
<td>7012</td>
<td>0.42</td>
<td>10225</td>
<td>11500</td>
<td>8233</td>
<td>10.09</td>
<td>14.84</td>
<td>3.75</td>
<td></td>
</tr>
<tr>
<td>380, -1</td>
<td>9016</td>
<td>0.43</td>
<td>13000</td>
<td>22000</td>
<td>15107</td>
<td>40.91</td>
<td>40.30</td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td>Carbon steel 1018</td>
<td>447, -0.53</td>
<td>8983</td>
<td>0.95</td>
<td>7707</td>
<td>9000</td>
<td>11371</td>
<td>14.40</td>
<td>21.00</td>
<td>6.60</td>
</tr>
<tr>
<td>447, -0.53</td>
<td>4723</td>
<td>0.51</td>
<td>3542</td>
<td>9000</td>
<td>13455</td>
<td>60.65</td>
<td>64.89</td>
<td>4.24</td>
<td></td>
</tr>
<tr>
<td>421, -0.56</td>
<td>13550</td>
<td>0.55</td>
<td>13272</td>
<td>32000</td>
<td>39350</td>
<td>58.6</td>
<td>66.00</td>
<td>7.40</td>
<td></td>
</tr>
</tbody>
</table>

The results show that present method can predict RFL with good accuracy under different NFT loading conditions. It should be noted that the environmental conditions, e.g., temperature, humidity, etc., are kept constant during all the tests. Therefore, further research is required to
investigate the effects of the environmental conditions on the RFL estimation capability of the proposed method. Moreover, the proposed method is developed and validated for standard specimens without stress concentration. Hence, the applicability of this method to practical components, e.g., turbine blade, connecting rod, etc., and specimens with significant stress concentration requires further studies.

5.5. Conclusions

An experimental NDT study is carried out to develop and validate a methodology for evaluating RFL of mechanical components with prior fatigue damage. Uniaxial tension-compression fatigue tests are performed with dogbone specimens of API 5L X52 and tubular specimen of carbon steel 1018 to characterize their fatigue behavior. The slope of the temperature rise, $R_\theta$, obtained from STE test is utilized as an index of present microstructural state and damage in estimating RFL. A total of eight validation tests are performed under different NFT loading conditions to observe the RFL estimation capability of the proposed method. Validation test results show that $R_\theta$ is a useful fatigue parameter which is sensitive to the fatigue damage in the material. Results presented in this work are obtained from standard specimens under specific laboratory conditions where environmental conditions were maintained constant. Further research is needed to determine the applicability of the RFL prediction method on practical components and under variable environmental conditions.

5.6. Acknowledgements

The authors from Louisiana State University gratefully acknowledge funding received from Cameron International for performing this work.

5.7. Nomenclature

- $A, B$: constants
- $f$: frequency
- $F$: a function
- $F_1$: function of $N$
- $m$: empirical material constant
- $n$: slope of $R_\theta - N$ plot
- $N$: accumulated fatigue load cycle, consumed fatigue life
- $N_f$: fatigue life
- $L_R$: load ratio
- $R$: radius of fillet
\( R_a \) arithmetic average of surface roughness
\( R_\theta \) slope of temperature rise
\( R'_{\theta} \) slope of temperature rise in phase III
\( R_{\theta1}, R_{\theta2} \) slope of temperature rise
\( R_{\theta0} \) threshold slope from experiment
\( R^c_{\theta0} \) threshold slope from curve fit
\( RFL_{exp} \) experimental RFL
\( RFL_{pred} \) predicted RFL
\( \Delta RFL \) difference between \( RFL_{exp} \) and \( RFL_{pred} \)
\( \Delta T \) temperature difference
\( T \) temperature
\( \sigma \) load/stress amplitude
\( \phi \) diameter

5.8. References


Chapter 6: Analysis and Life Prediction of a Composite Laminate under Cyclic Loading

6.1. Introduction

The application of composite materials has increased substantially over the last decade. Components made of these materials have several better mechanical and physical properties, e.g., strength-to-weight ratio, low thermal conductivity, etc., compared to their metallic counterparts. Nevertheless, they are prone to irreversible damage when exposed to cyclic loading. Therefore, development of a technique for estimating the useful remaining fatigue life (RFL) of a composite structure is highly desirable.

Since composite materials are anisotropic and inhomogeneous, their underlying mechanisms of fatigue degradation are far more complicated than isotropic materials. In particular, the damage mechanism in a composite involves matrix cracking, delamination, fiber/matrix debonding, and fiber breakage [1-3]. These complexities hinder the application of damage estimation models developed for isotropic and homogeneous materials to composites [4, 5].

Several approaches have been reported for the estimation of the fatigue life, \( N_f \), of composite laminates [6-11]. However, only a few methods are available for the estimation of RFL [12, 13]. These RFL prediction methods rely on the measurement of stiffness degradation and the application of Miner’s rule to monitor the entire history of loading.

Recently, researchers developed a method to predict the RFL of metallic specimens by measuring the slope of temperature rise, \( R_\theta \), obtained from a short-time excitation (STE) test on a pre-fatigued specimen [14-16]. Liakat and Khonsari [17, 18] showed that the evolution of material damage, thermodynamic entropy, and plastic strain energy of a metallic specimen with prior history of cyclic loading can be estimated by measuring the evolution of \( R_\theta \). This approach is shown to be capable of predicting the RFL of metallic specimens in a non-destructive (NDT) fashion. However, the application of this method to composite materials has not been reported.

In this work, the gradual progression of composite fatigue degradation is related to the thermal response obtained from a series of STE tests subjected to both constant amplitude cyclic loading (CACL) and variable amplitude cyclic loading (VACL). The principle of the conservation of energy is utilized to establish a correlation between \( R_\theta \) and the time rate of hysteresis energy generation, \( \dot{W}_H \), obtained from STE tests. An extensive series of uniaxial tension-compression and fully-reversed bending fatigue tests are performed with solid cylindrical and flat specimens made of G10/FR4 composite to determine the validity of the proposed RFL prediction method.
6.2. Experimental Details

6.2.1. Material and specimen preparation

The material tested is G10/FR4, a Glass/Epoxy composite laminate. This is an unbalanced woven fabric that consists of continuous filament glass cloths laminated with an epoxy resin binder. This material has high mechanical strength, low water absorption rate, and is resistant to corrosion. It is also flame resistant and offers several superior electrical characteristics compared to metals over a wide range of temperature and humidity. These properties allow a wide variety of applications such as aerospace structures, terminal boards, washers, sleeves, and electrical insulators. Some of the properties of this material useful for this work are reported in Table 6.1.

<table>
<thead>
<tr>
<th>Table 6.1. Properties of G10/FR4 composite laminate [19]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, ( \rho ) (kgm(^{-3}))</td>
</tr>
<tr>
<td>----------------------------------------------------------</td>
</tr>
<tr>
<td>1635</td>
</tr>
</tbody>
</table>

Figure 6.1(a and b) presents the photograph of a solid cylindrical dogbone and a flat specimen made of G10/FR4 along with their dimensions. The continuous carbon fibers are aligned along the longitudinal axis of the cylindrical specimens. Cylindrical specimens are prepared from cylindrical rods using a computer numerical control (CNC) machine and flat specimens are prepared from a sheet of G10/FR4 using a water jet machine, following the procedure reported in [11, 20, 21]. The gage section surfaces produced by machining are polished with sand papers progressing through 1500, 2000, and 2500 grit sizes. Uniaxial tension-compression and fully-reversed bending fatigue tests are carried out with solid cylindrical and flat specimens, respectively. The required dimensions of the solid cylindrical specimens are determined to ensure that they do not buckle within the range of compression loads considered in this work. The flat specimens are designed with hourglass gage section so that fracture occurs in that section. Figure 6.2 shows the orientation of the warp (90°) and weft (0°) in each layer of the glass fibers in the sheet material. The fiber layers are aligned parallel to each other along the thickness of a sheet. Flat specimens are prepared at 45° off-axis stacking with respect to the weft (0°) direction. Specimen surfaces created by machining is polished longitudinally using sand papers with fine grit sizes.

The direction of the glass fibers is along the longitudinal axis of the cylindrical specimens. Therefore, its thermal conductivity in the longitudinal direction can be different than that of the radial direction. In the case of flat specimens, thermal conductivity along the thickness of the specimen is different than the directions of the fibers as shown in Figure 6.2. However, the value of thermal conductivity reported in Table 6.1 is a bulk property of this material. Since the effect of anisotropy decreases with the accumulation of fatigue degradation [22], the difference between the thermal properties due to the anisotropy becomes less pronounced.
Figure 6.1. Photograph of (a) solid cylindrical and (b) flat specimen with dimensions in millimeter.

Figure 6.2. Schematic of fiber directions in the G10/FR4 sheet and the direction of specimen preparation at 45° off-axis stacking.

Figure 6.2. Schematic of fiber directions in the G10/FR4 sheet and the direction of specimen preparation at 45° off-axis stacking.
6.2.2. Equipment and experimental procedure

6.2.2.1. Uniaxial tension-compression tests

Load-controlled uniaxial fatigue tests are carried out at different stress levels, $\sigma$, load ratios, $L_R$, (defined as the ratio of minimum to the maximum stress in a load cycle), and test frequency $f=5$, 10, and 15 Hz. In order to determine the effect of fatigue loading frequency on $R_\theta$ evolution, three tests are performed at 5, 10, and 15 Hz frequency with $\sigma = 145$ MPa and $L_R = -0.25$. The apparatus is a servo-hydraulic fatigue testing machine with the capability of a maximum of 50 kN axial load and 75 Hz frequency. The specimen is gripped between the jaws of the top and the bottom grips vertically and an extensometer is mounted on the gage section as shown in Figure 6.3. The top grip remains stationary and the bottom grip oscillates vertically to generate uniaxial tension-compression load in the specimen. The extensometer used to record the strain in the gage section of the specimen has the gauge length of 25.4 mm and travel between -10% and +50% strain. It directly interfaces with the fatigue testing machine and a data acquisition system records the load and strain data during the fatigue test. Stress vs. strain plot provides hysteresis loops associated with each load cycle and the area contained within the boundaries of these loops is the measure of hysteresis energy, $\dot{W}_H$, generated in the specimen [22-24]. Figure 6.4 presents a hysteresis loop obtained from a fatigue test performed with $\sigma = 155$ MPa, $f=10$ Hz, and $L_R = -0.25$. A static test was performed with this material and it is found that the tensile yield and ultimate strength is about 75 MPa and 208 MPa, respectively. Since the applied stress is larger than the yield strength of this material, energy generation due to the irreversible changes and viscoplasticity can be calculated by estimating the area contained within the boundaries of this hysteresis loop.

6.2.2.2. Bending tests

Displacement-controlled bending fatigue tests are performed using a compact bench-mounted machine with the capability of a maximum of 40 Hz test frequency and displacement amplitude, $\Delta$, range between 0 mm and 50 mm. One end of the specimen is firmly clamped to the holder and the other end oscillates with desired $\Delta$ and $f$ (see Figure 6.5). Three tests are performed at $f = 5$, 10, and 15 Hz with $\Delta = 35$ mm to determine the effect of fatigue loading frequency on the $R_\theta$ evolution and the rest of the tests are performed at $f = 10$ Hz. The vertical displacement at the oscillating end of the specimen develops bending stress in the specimen gage section.

6.2.3. Temperature measurement

The gage section surface temperature of both the uniaxial and the bending specimens is recorded by a high-speed, high-resolution infrared (IR) camera with the temperature range capability between 0°C and 500°C, accuracy of ±2% of reading, resolution of 320×240 pixel, and sensitivity of 0.08°C. Since the measurement of composite matrix temperature requires one to embed a sensor in the specimen, surface temperature is utilized in this work as it can be measured externally; see for example [25, 26]. In order to reduce the IR reflection and increase thermal emissivity, a thin
Figure 6.3. A close-up view of a solid cylindrical specimen gripped between the jaws and mounted with an extensometer for uniaxial tension-compression fatigue test. Cylindrical specimens are tested under uniaxial tension-compression load.

Figure 6.4. A hysteresis loop obtained from a uniaxial tension-compression fatigue test performed with $f=10$ Hz, $\sigma = 155$ MPa, and $L_R = -0.25$. Hysteresis loop associated with 3000th load cycle.
Figure 6.5. Experimental set-up for the bending fatigue test showing a flat specimen is mounted for test and the IR camera is positioned for the temperature measurement. Flat specimens are tested under fully-reversed bending load.

A layer of black paint is sprayed onto the specimen gage section surface. The entire gage section surface temperature is recorded throughout the fatigue test. An average temperature over a line of approximately 5-mm on the cylindrical and 14-mm on the flat specimen at the location where temperature appears maximum is used in the analysis (see Figure 6.6).

Figure 6.7 presents the temperature evolution obtained from a uniaxial CACL test performed at $\sigma = 170$ MPa, $L_R = -0.1$, and $f = 10$ Hz. Temperature rapidly increases during the early stage of the fatigue process owing to matrix cracking, weak fiber breakage, delamination at the weak fiber/matrix interfaces, and viscoplasticity associated with epoxy. As the applied stress is substantially greater than the yield strength of G10/FR4, the effect of viscoelasticity is negligible compared to the viscoplasticity [27]. Then the temperature increases slowly as the damage evolution rate reaches its saturation level followed by a sharp temperature rise due to the fiber breakage at the onset of the specimen fracture. Figure 6.7 also shows the fluctuation of temperature around the mean temperature rise due to the effect of thermomechanical coupling, also known as the thermoelasticity [28]. The increase of the mean temperature occurs owing to the irreversible changes in the material.
Figure 6.6. Thermographic image of (a) solid cylindrical and (b) flat specimen during fatigue test showing the location and the line of temperature data acquisition for analysis (temperature values are in °C)

Figure 6.7. Evolution of temperature obtained from a uniaxial tension-compression test performed at $\sigma = 170$ MPa and $L_R = -0.1$
6.2.4. Measurement of $R_\theta$ evolution

The evolution of $R_\theta$ is measured by performing a series of STE tests (typically about 15-20 s) applied throughout the fatigue life according to the procedure recommended by Amiri and Khonsari [14], schematically illustrated in Figure 6.8. Beginning with a pristine specimen, the slope of temperature rise, $R_{\theta 0}$, is measured by an STE test followed by the fatigue test that lasts for a specific number of load cycles, $N_i$, depending on the loading conditions (see Figure 6.9). The test is stopped, the specimen is allowed to cool down to the ambient temperature, and the value of $R_{\theta i}$ is measured by a subsequent STE test. This procedure is repeated until the specimen fractures into two pieces.

![Figure 6.8. Schematic illustration of test procedure in terms of the load cycles](image)

![Figure 6.9. Idealized illustration of the temperature evolutions obtained from STE tests performed after different number of load cycles in a fatigue test](image)
In order to obtain the evolution of \( R_\theta \), the number of load cycles in each fatigue test between two subsequent STE tests is determined in such a way that \( R_\theta \) is measured at least 6-7 times if \( N_f \leq 10^4 \) cycles and around 8-10 times if \( N_f > 10^4 \) cycles. STE tests are carried out with a specific load level, load ratio, and frequency, and maintained constant for all the STE tests performed. The STE test loading conditions used in the present work are: \( \sigma = 121 \) MPa, \( L_R = -0.7 \), and \( f = 10 \) Hz for uniaxial tests and \( \Delta = 35 \) mm, \( L_R = -1 \), and \( f = 10 \) Hz for reversed bending tests.

### 6.3. Analytical

In the absence of internal heat source, the principle of the conservation of energy can be expressed as [29]:

\[
k \nabla^2 T = \rho C \dot{T} - \boldsymbol{\sigma} : \dot{\varepsilon}^p + \mathbf{A}_i \dot{V}_i - T \left( \frac{\partial \sigma}{\partial T} : \dot{\varepsilon}^e + \frac{\partial A_i}{\partial T} \dot{V}_i \right)
\]

(6.1)

where \( k \) is the thermal conductivity, \( T \) represents the maximum absolute temperature in the specimen gage section, \( \rho \) denotes density, \( C \) stands for the specific heat, \( \sigma \) is the stress tensor, \( \dot{\varepsilon}^p \) denotes the time rate of plastic strain tensor, \( V_i \) stands for the internal variables associated with microstructure such as the matrix cracking, delamination, fiber/matrix debonding, etc. [30], \( A_i \) represents the thermodynamic forces associated with internal variables (where \( i \) is the number of internal variables), and \( \dot{\varepsilon}^e \) is the time rate of elastic strain tensor.

In composite materials, the evolution of internal variables, \( \mathbf{A}_i \dot{V}_i \), represents about 30–50% of the total mechanical dissipation [31]. The amplitude of temperature fluctuation, \( \frac{\Delta T}{2} \), due to the effect of thermomechanical coupling, \( \frac{\partial \sigma}{\partial T} : \dot{\varepsilon}^e \), is expressed as \( \frac{\Delta T}{2} = \frac{\alpha \sigma T}{\rho C} \) where \( \alpha \) represents the coefficient of linear expansion [28]. For the material and loading conditions tested in this work, the value of \( \frac{\Delta T}{2} \) is found to be negligible compared to the mean temperature rise as shown in Figure 6.7. As the thermal conductivity of this material is low (see Table 6.1), the temperature gradient, \( \nabla^2 T \), within the gage section of the specimen is negligible. Thus, conduction heat dissipation, \( k \nabla^2 T \), from the gage section to the gripping section of the specimen can be neglected compared to the total energy generation. Assuming that \( k \nabla^2 T \approx 0 \) and \( \frac{\partial \sigma}{\partial T} : \dot{\varepsilon}^e \approx 0 \), Equation (6.1) can be expressed as:

\[
\dot{T} \equiv \frac{\dot{W}_p - A_i \dot{V}_i + T \frac{\partial A_i}{\partial T} \dot{V}_i}{\rho C}
\]

(6.2)

where \( \dot{W}_p = \sigma : \dot{\varepsilon}^p \) is the time rate of energy generation due to plastic deformation, viscoelasticity, and viscoplasticity [23, 27, 32, 33]. Equation (6.2) implies that \( \dot{T} \) depends on \( \dot{W}_p \), specimen temperature, and the evolution of internal variables. If an STE test is performed on a specimen with an arbitrary amount of prior degradation, the slope of temperature rise, \( R_\theta \), is expressed as:
\[ R_\theta \cong \frac{W_p - A_1 V_1 + T \frac{\partial A_1}{\partial T} \dot{V}_1}{\rho C} \]  

(6.3)

In a fatigue experiment, the area contained within a hysteresis loop represents all the energy associated with the degradation of matrix and fiber such as the plastic deformation, creation of microcracks, molecular orientation in the matrix, and deformation induced entropy [31, 34, 35]. Therefore, the rate of hysteresis energy generation \( \dot{W}_H \cong W_p - A_1 \dot{V}_1 + T \frac{\partial A_1}{\partial T} \dot{V}_1 \). Thus, Equation (6.3) is expressed as:

\[ R_\theta \cong \frac{1}{\rho C} \dot{W}_H \]  

(6.4)

Equation (6.4) implies that the evolution of the slope of temperature rise is linearly related to the rate of hysteresis energy generation with a slope of approximately \( \frac{1}{\rho C} \).

### 6.4. Results and Discussion

6.4.1. Uniaxial constant amplitude loading tests

6.4.1.1. Correlation between \( R_\theta \) and \( \dot{W}_H \)

Figure 6.10 illustrates the S-N curve associated with uniaxial fatigue tests, showing the expected results of an increase in fatigue life with the reduction in stress. S-N curves reveal dependency of the load ratio, \( L_R \), with significantly reduced fatigue life when \( L_R \) is large. Research shows that the fatigue life of a composite material is also dependent on the angle between the fiber and the loading direction and that the fatigue life is reported to be maximum when this angle is 0° and minimum at 45° [11].

Figures 6.11 and 6.12 present the plots between \( R_\theta \) and \( \dot{W}_H \) at different load ratios obtained from the uniaxial CACL tests. Results demonstrate that the correlation between these parameters is fairly linear within the range of loading conditions considered in this work. Although the fatigue life of the material is short at high stress levels, the evolution of both \( R_\theta \) and \( \dot{W}_H \) is large compared to their evolutions at small stress levels. As a higher stress level produces greater amount of irreversible changes in the material, it generates progressively larger amount of hysteresis energy during an STE test. Most of this energy converts into heat and manifests itself by a large magnitude of \( R_\theta \) evolution. The slope of these plots is found to be between 7.05×10^{-7} and 8.6×10^{-7} °Cms²kg⁻¹. According to Equation (6.4), the slope of \( R_\theta \) vs. \( \dot{W}_H \) plot is 7.2×10^{-7} °C ms² kg⁻¹ using the value of \( \rho \) and \( C \) reported in Table 6.1. Thus, Equation (6.4) provides a reasonable prediction of \( R_\theta \) evolution with respect to \( \dot{W}_H \).
Figure 6.10. S-N curve for uniaxial tension-compression fatigue tests along the Weft (0°) direction of the fibers.

Figure 6.11. $R_\theta$ vs. $\dot{W}_H$ plot associated with uniaxial tension-compression test at $L_R = -0.25$. The equations are:

$$R_\theta = 7.05 \times 10^{-7} \dot{W}_H + 0.02$$

$$R_\theta = 8.6 \times 10^{-7} \dot{W}_H - 0.002$$
Figure 6.12. $R_\theta$ vs. $\dot{W}_H$ plot associated with uniaxial tension-compression test at $L_R = -0.1$

6.4.1.2. $\dot{W}_H$ and $R_\theta$ evolution

Figures 6.13–6.16 present the evolution of $\dot{W}_H$ and $R_\theta$ associated with uniaxial fatigue tests. It can be seen that the evolution of these parameters is fairly linear with respect to the number of load cycles. Results demonstrate that the cyclic rate of the evolution of these parameters, $\frac{d\dot{W}_H}{dN}$ and $\frac{dR_\theta}{dN}$, is greater at larger stress levels. The value of $R_\theta$ serves as an index of the level of material degradation and its cyclic rate is a measure of the fatigue life in a fatigue test. The cyclic rate of $R_\theta$ evolution, $n$, can be expressed as:

$$n = \frac{R_\theta - R_{\theta^c}}{N}$$  \hspace{1cm} (6.5)

where $R_{\theta^c}$ is the intercept of the linear curve-fit of $R_\theta$ evolution on the ordinate. The average value of $R_{\theta^c}$ is found to be approximately 0.12 °Cs$^{-1}$ for all the uniaxial CACL tests (see Figures 6.15 and 6.16). The value of $R_{\theta^c}$ corresponds to the pristine specimen without prior degradation. As the fatigue test continues, $R_\theta$ gradually evolves and becomes larger as degradation accumulates and the remaining useful life declines.
Figure 6.13. Evolution of $\dot{W}_H$ with respect to $N$ obtained from uniaxial tension-compression tests at $L_R = -0.25$

Figure 6.14. Evolution of $\dot{W}_H$ with respect to $N$ obtained from uniaxial tension-compression tests at $L_R = -0.1$
Figure 6.15. Evolution of $R_\theta$ with respect to $N$ obtained from uniaxial tension-compression tests at $L_R = -0.25$

Figure 6.16. Evolution of $R_\theta$ with respect to $N$ obtained from uniaxial tension-compression tests at $L_R = -0.1$
To determine the effect of fatigue test frequency on the $R_\theta$ evolution, the results of three fatigue tests performed at 5, 10, and 15 Hz with $\sigma = 145$ MPa and $L_R = -0.25$ are presented in Figure 6.15. It can be seen that the effect of fatigue loading frequency on the $R_\theta$ evolution and fatigue life is negligible within the range of the frequencies tested. This finding is consistent with the results of [22, 36] that also find that the fatigue life and damage evolution of a composite laminate are negligibly dependent on the loading frequency within its range considered in this work. Since the evolution of $R_\theta$ serves as a measure of damage evolution in a material, fatigue loading frequency does not have considerable effect on its evolution subject to identical STE test conditions.

Figure 6.17 illustrates the thermographic images of the specimen after different number of accumulated load cycles. These images demonstrate that the amount of temperature rise obtained from STE tests increases gradually with the increase of the number of $N$, which is observed in all the tests performed in this work. Since each load cycle accumulates irreversible damages, microstructural changes, and plastic deformation in the material [36], it generates greater amount of hysteresis energy in a subsequent STE test compared to a previous STE test. The gradual increase of the amount of hysteresis energy in a series of STE tests is manifested by a gradual rise of the specimen temperature and represented by a steady increase of $R_\theta$ evolution.

Figure 6.18 presents $n$ vs. $\sigma$ plot and the respective correlation between these parameters. These results demonstrate that the value of $n$ increases exponentially with the increase of $\sigma$. It can be seen that the value of $n$ is larger at $L_R = -0.25$ compared to its value at $L_R = -0.1$ subject to constant stress level. This occurs due to the greater compressive stress applied to the specimen at $L_R = -0.25$ compared to that at $L_R = -0.1$.

![Figure 6.17. Thermal response of a cylindrical specimen obtained from STE tests after different number of accumulated load cycles subject to uniaxial tension-compression test at $\sigma = 155$ MPa and $L_R = -0.25$ (temperature values are in °C)](image-url)
6.4.2. $R_\theta$ evolution in fully-reversed bending fatigue

Figure 6.19 shows the evolution of $R_\theta$ with respect to the number of load cycles. Results show that $R_\theta$ evolution is fairly linear when plotted against the number of load cycles in bending fatigue test and that the larger the displacement amplitude, the greater is the rate of $R_\theta$ evolution. Although the displacement amplitude remains constant throughout the bending fatigue test, the repeated cyclic deformation of the specimen generates gradual irreversible changes to the matrix, fiber, and the fiber-matrix bonding, as represented by the linear increase in $R_\theta$ evolution. Figure 6.20 illustrates this phenomenon in terms of the thermographic images of the specimens after different number of load cycles. The rate of $R_\theta$ evolution in bending fatigue can also be expressed by Equation (6.5) with the value of $R_{\theta,0} \approx 1.21 \, ^\circ\text{Cs}^{-1}$. 

The evolutions of $R_\theta$ associated with the bending test frequency of 5, 10, and 15 Hz with $\Delta = 35$ mm are shown in Figure 6.19. It can be seen that the trend of $R_\theta$ evolution is fairly independent of the loading frequency within the range of frequency considered subject to identical STE test conditions. This implies that the evolution of damage in bending fatigue is independent of the loading frequency, which also conforms to the findings of Tsai et al. [37].

Figures 6.21 and 6.22 present the $\Delta$–$N$ and $n$–$\Delta$ plots corresponding to the bending tests. Fatigue life of the specimens increases with the decrease in the displacement amplitude, and the value of $n$ increases with increasing $\Delta$. Results show that there is an exponential correlation between $n$ and
Figure 6.19. Evolution of $R_\theta$ with respect to $N$ obtained from fully-reversed bending fatigue tests at $L_R = -1$

Figure 6.20. Thermal response of a flat specimen obtained from STE tests after different number of accumulated load cycles subject to fully-reversed bending fatigue at $\Delta = 30$ mm and $L_R = -1$ (temperature values are in °C)
Figure 6.21. $\Delta$–$N$ curve for fully-reversed bending fatigue tests corresponding to CACL

Figure 6.22. $n$ vs. $\Delta$ plot corresponding to fully-reversed bending fatigue tests at $L_R = -1$

$n = 6.64 \times 10^{-7} e^{0.154\Delta}$ at $L_R = -1$
\( \Delta \), similar to the relationship between \( n \) and \( \sigma \) found for the uniaxial fatigue test results. This reveals that the relationship between \( n \) and the loading parameter such as the stress level and the displacement amplitude is similar for uniaxial tension-compression and fully-reversed bending fatigue.

6.4.3. \( R_\theta \) evolution subject to variable amplitude loading tests

Figures 6.23–6.25 illustrate the evolution of \( R_\theta \) with respect to \( N \) obtained from two-stage uniaxial and bending fatigue tests subjected to both High-to-Low (H–L) and Low-to-High (L–H) loading sequences. Two tests are performed for a specific set of load amplitudes with constant load ratio.

One test is performed at 145 MPa followed by 135 MPa and another test is performed at 135 MPa followed by 145 MPa with \( L_R = -0.25 \). Figures 6.23–6.24 show that \( R_\theta \) evolves linearly with a constant cyclic rate, \( n_1 \), in the 1st-stage, in accordance to its trend observed in CACL tests. As the stress level changes in the 2nd-stage, the rate of \( R_\theta \) evolution, \( n_2 \), changes as well. According to Turon et al. [38] and Degrieck and Paepegem [39], a variation in the load level in a sequence loading test changes the rate of microstructural damage evolution in the material. It can be seen that if the stress level in the 1st-stage is smaller than that in the 2nd-stage, \( n_1 \) is smaller than \( n_2 \) and vice versa. Results demonstrate that \( n_{1,H-L} \equiv n_{2,L-H} \) and \( n_{2,H-L} \equiv n_{1,L-H} \) where \( n_{1,H-L} \) and \( n_{2,H-L} \) represent the cyclic rates of \( R_\theta \) evolution corresponding to the 1st- and 2nd-stage in a H–L load sequence test and \( n_{1,L-H} \) and \( n_{2,L-H} \) denote those in a L–H load sequence tests. Thus, the evolution of \( R_\theta \) associated with an m-stage loading sequence test can be determined by summing up \( R_{\theta0}^c \) and the evolution of \( R_\theta \) in each load stage until the fracture of the specimen, i.e.:

\[
R_\theta = R_{\theta0}^c + \sum_{k=1}^{m} n_k N_k \tag{6.6}
\]

where \( n_k \) represents the cyclic rate of \( R_\theta \) evolution and \( N_k \) stands for the number of the load cycle in the kth-stage. Equation (6.6) implies that \( R_\theta \) evolution for an m-stage loading test of a material can be predicted by characterizing \( n_k \) for that material.

Figures 6.23–6.25 demonstrate that an H–L loading sequence test results in a shorter fatigue life compared to that of an L–H loading sequence test subject to a specific set of load amplitudes with constant load ratio. The difference in the fatigue life occurs due to the difference in the mechanism and the rate of damage progression associated with high and low loads. According to Degrieck and Paepegem [39] and Vieillevigne et al. [40], the rate of damage accumulation in a composite laminate is directly proportional to \( \sigma^a \) where \( a \) is a constant. At a low load level, a great portion of the total fatigue life is spent on the formation of micro-cracks in the matrix as the fibers are stronger than the matrix. In contrast, at a high load level the fatigue degradation process begins with the formation of delamination and fiber-matrix debonding followed by the fiber breakage at
Figure 6.23. $R_\theta - N$ plots obtained from two-stage uniaxial tension-compression fatigue tests with (a) H–L load: 145 to 135 MPa and (b) L–H load: 135 to 145 MPa and $L_R = -0.25$

Figure 6.24. $R_\theta - N$ plots obtained from two-stage uniaxial tension-compression fatigue tests with (a) H–L load: 165 to 155 MPa and (b) L–H load: 155 to 165 MPa and $L_R = -0.1$
the onset of fracture. Clearly, the rate of damage evolution is smaller in the matrix cracking phase compared to that in the subsequent phases. Consequently, fatigue life of G10/FR4 associated with an H–L load sequence is shorter than that subject to an L–H load sequence.

6.5. Prediction of remaining fatigue life

This section presents an experimental procedure along with necessary formulations for the prediction of the RFL of a specimen with prior fatigue damage subjected to CACL and 2-stage loading sequence tests. To assess the capability of the method, a series of validation test results obtained from both uniaxial tension-compression and fully-reversed bending fatigue tests are presented.

6.5.1. Constant amplitude loading tests

If a specimen has undergone $N$ number of fatigue load cycles, taking advantage of Equation (6.5) the number of remaining life cycle, $(N_f - N)$, of that specimen is expressed as:

$$N_f - N = \frac{n N_f - R_{\theta} + R_{\theta}^{\theta}}{n}$$  \hspace{1cm} (6.7)
The percentage of the RFL of the specimen is: \( \left( \frac{N_f - N}{N_f} \right) \times 100\% \). With the help of Equation (6.7), the expression of RFL becomes:

\[
RFL = \left( 1 - \frac{R_{\theta} - R_{\theta_0}}{nN_f} \right) \times 100\%
\]  

(6.8)

To determine the RFL prediction capability of Equation (6.8), a typical validation test starts with a pristine specimen. An arbitrary number of load cycles, \( N \), with specific loading conditions is performed on the specimen. The test is then stopped and the specimen is allowed to cool down to the ambient temperature. An STE test is performed on this specimen with the loading conditions chosen for the type of the fatigue test, i.e., uniaxial tension-compression or reversed bending test, to measure the value of \( R_{\theta} \) corresponding to the present material damage. The value of \( N_f \) and \( n \) corresponding to the loading conditions chosen for the validation test are obtained from Figures 6.10 and 6.18 for uniaxial tension-compression tests and Figures 6.21 and 6.22 for reversed bending tests, respectively. Substituting the value of these parameters into Equation (6.8), the RFL of the specimen, \( RFL_{\text{pred}} \), is predicted. The fatigue test is then resumed with this specimen and continued to determine the total number of load cycles required to fracture the specimen, \( N_f \). Therefore, \( N_f \) is the sum of the number of load cycles prior to the STE test, in the STE test, and after the STE test until the fracture of the specimen. The experimental RFL, \( RFL_{\text{exp}} \), at the stage of the fatigue test where the test was stopped to predict RFL, is calculated by substituting the value of \( N \) and \( N_f \) into \( \left( \frac{N_f - N}{N_f} \right) \times 100\% \). The difference between \( RFL_{\text{pred}} \) and \( RFL_{\text{exp}} \) is calculated as \( \Delta RFL = \left| RFL_{\text{exp}} - RFL_{\text{pred}} \right| \% \). Table 6.2 presents the results of the validation tests associated with CACL. Results show that the difference between the predicted and experimental RFL is reasonable and the maximum difference is found to be 9.15%.

<table>
<thead>
<tr>
<th>Fatigue load</th>
<th>( \sigma ) (MPa) or ( \Delta ) (mm)</th>
<th>Load ratio ( L_R )</th>
<th>Load cycles before STE test, ( N )</th>
<th>( R_{\theta} ) ((^\circ \text{C}s^{-1}))</th>
<th>Fatigue life, ( N_f )</th>
<th>( RFL_{\text{pred}} ) (%)</th>
<th>( RFL_{\text{exp}} ) (%)</th>
<th>( \Delta RFL ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniaxial</td>
<td>145</td>
<td>-0.25</td>
<td>4002</td>
<td>0.163</td>
<td>9231</td>
<td>53.88</td>
<td>56.64</td>
<td>2.76</td>
</tr>
<tr>
<td></td>
<td>155</td>
<td>-0.25</td>
<td>4006</td>
<td>0.221</td>
<td>6065</td>
<td>26.63</td>
<td>33.94</td>
<td>7.31</td>
</tr>
<tr>
<td></td>
<td>170</td>
<td>-0.1</td>
<td>4004</td>
<td>0.173</td>
<td>12564</td>
<td>62.95</td>
<td>68.13</td>
<td>5.18</td>
</tr>
<tr>
<td>Bending</td>
<td>39</td>
<td>-1</td>
<td>4100</td>
<td>2.420</td>
<td>8200</td>
<td>40.85</td>
<td>50.00</td>
<td>9.15</td>
</tr>
</tbody>
</table>

6.5.2. Variable amplitude loading tests

In the case of a 2-stage loading sequence test, a validation test begins with a pristine specimen and the 1\textsuperscript{st}-stage of cyclic loading with \( \sigma_1 \) stress level is performed for \( N_1 \) load cycles. The test is
then stopped and the 2nd-stage of cyclic loading with \( \sigma_2 \) stress level starts with this specimen. The test is again stopped after another \( N_u \) load cycles in this stage before the fracture of the specimen. The specimen is then allowed to cool down to the ambient temperature and the slope of temperature rise, \( R_{\theta_f} \), corresponding to the present material damage is determined by performing an STE test. The values of \( n_1 \) and \( n_2 \) corresponding to the stress levels \( \sigma_1 \) and \( \sigma_2 \) are determined following the procedure described in section 6.5.1. The cyclic rate of \( R_{\theta} \) evolution in the 1st- and 2nd-stage can be expressed as 
\[
\frac{n_1}{n_1 + n_2} = \frac{R_{\theta_f} - R_{\theta_0}}{R_{\theta_1} - R_{\theta_0}} 
\]
\[
N_1 + N_u = \frac{R_{\theta_1} - R_{\theta_0}}{n_1} + \frac{R_{\theta_2} - R_{\theta_1}}{n_2} 
\]
(6.9)

The RFL of this specimen after \( N_1 + N_u \) load cycles is expressed as:
\[
RFL = \left( \frac{N_f - (N_1 + N_u)}{N_f} \right) \times 100\% 
\]
(6.10)

where \( N_f \) is the total number of load cycles required in the 1st- and 2nd-stage to fracture the specimen. Letting \( R_{\theta_f} \) denote the value of \( R_{\theta} \) at the onset of fracture, we have: 
\[
R_{\theta_f} = R_{\theta_0} + n_1N_1 + n_2(N_f - N_1). 
\]
Substituting the value of \( N_f \) from this expression and \( N_1 + N_u \) from Equation (6.9) into Equation (6.10) yields:
\[
RFL = \left( 1 - \frac{n_1R_{\theta_f} - (n_1 + n_2)R_{\theta_1} - n_2R_{\theta_0}}{n_1(R_{\theta_f} - R_{\theta_0}) - (n_1 + n_2)N_1} \right) \times 100\% 
\]
(6.11)

Substituting the value of the parameters in the right-hand side of Equation (6.11), the RFL of the specimen is predicted, \( RFL_{pred} \). The fatigue test is resumed with \( \sigma_2 \) stress level and continued to determine the total number of load cycles, \( N_f \), required to fracture the specimen. The experimental RFL, \( RFL_{exp} \), of the specimen is determined by substituting the value of \( N_1, N_u, \) and \( N_f \) into Equation (6.10). \( \Delta RFL \) is calculated following the procedure described in the preceding section. The results of the validation tests associated with two-stage loading are presented in Table 6.3.

Table 6.3. Results of validation tests corresponding to two-stage uniaxial and bending fatigue tests

<table>
<thead>
<tr>
<th>Fatigue load</th>
<th>Load sequence (MPa) or (mm)</th>
<th>( L_R )</th>
<th>Load cycles before STE test</th>
<th>( R_{\theta} ) (°C/s)</th>
<th>Fatigue life, ( N_f )</th>
<th>( RFL_{pred} ) (%)</th>
<th>( RFL_{exp} ) (%)</th>
<th>( \Delta RFL ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniaxial</td>
<td>135 to 145</td>
<td>−0.25</td>
<td>8730</td>
<td>0.257</td>
<td>10862</td>
<td>16.05</td>
<td>19.62</td>
<td>3.57</td>
</tr>
<tr>
<td></td>
<td>165 to 155</td>
<td>−0.1</td>
<td>5004</td>
<td>0.221</td>
<td>11057</td>
<td>36.41</td>
<td>41.19</td>
<td>4.78</td>
</tr>
<tr>
<td>Bending</td>
<td>35 to 30</td>
<td>−1</td>
<td>10500</td>
<td>2.70</td>
<td>16600</td>
<td>29.89</td>
<td>36.74</td>
<td>6.85</td>
</tr>
</tbody>
</table>
These results demonstrate that the predicted and experimental RFL are in good agreement for both types of fatigue loading. The maximum difference in the RFL prediction is found to be 6.85%.

The proposed method is validated within the range of the experimental and environmental conditions considered in this work where environmental conditions were maintained constant. To assess the validity of this method in other environmental conditions such as elevated or lower temperatures, high or low humidities, etc. requires further research. The prediction of the RFL of standard specimens without stress concentration are found to be in good agreement in both CACL and VACL tests associated with uniaxial tension-compression and fully-reversed bending fatigue. The validity of the results present for other types of composites such as short-fiber composites, cross-ply composites, metal matrix composites, etc. requires further investigation.

6.6. Conclusions

Uniaxial tension-compression and fully-reversed bending fatigue tests are performed with solid cylindrical and flat specimens made of G10/FR4 subject to both CACL and VACL. It is shown that the proposed composite fatigue parameter $R_\theta$ is correlated to $\dot{W}_H$ as $R_\theta \approx \frac{1}{\rho c} \dot{W}_H$. The evolution of both $R_\theta$ and $\dot{W}_H$ is fairly linear as the material damage accumulates due to cyclic loading. It is shown that the cyclic rate of $R_\theta$ evolution is greater at large stress levels. The variation in the stress level in a two-stage loading test is demonstrated by a change in the rate of $R_\theta$ evolution. The characteristics of $R_\theta$ evolution are utilized to develop correlations for the prediction of the RFL of a specimen with prior history of fatigue damage in an NDT fashion applicable for both CACL and VACL. Results of the validation tests demonstrate that the proposed method can predict RFL with reasonable accuracy when compared to the experimental results.

6.7. Nomenclature

\begin{align*}
A_i & \quad \text{thermodynamic forces} \\
C & \quad \text{specific heat (JKg}^{-1}\text{K}^{-1}) \\
f & \quad \text{frequency (Hz)} \\
k & \quad \text{thermal conductivity (Wm}^{-1}\text{K}^{-1}) \\
L_R & \quad \text{load ratio} \\
n & \quad \text{cyclic rate of } R_\theta \text{ evolution (}^\circ\text{Cs}^{-1}\text{cycle}^{-1}) \\
n_k & \quad \text{value of } n \text{ in } k^{th}\text{-stage of loading (}^\circ\text{Cs}^{-1}\text{cycle}^{-1}) \\
n_{k,H-L} & \quad \text{value of } n \text{ in } k^{th}\text{-stage of loading in a High-to-Low load sequence test (}^\circ\text{Cs}^{-1}\text{cycle}^{-1}) \\
n_{k,L-H} & \quad \text{value of } n \text{ in } k^{th}\text{-stage of loading in a Low-to-High load sequence test (}^\circ\text{Cs}^{-1}\text{cycle}^{-1}) \\
N & \quad \text{number of load cycle}
\end{align*}
$N_f$ fatigue life (cycle)

$N_k$ number of load cycle at $k^{th}$ STE test

$N_u$ number of load cycle before STE test in the $2^{nd}$-stage of a validation test

$R$ radius (mm)

$R_{\theta}$ slope of temperature rise ($^\circ$Cs$^{-1}$)

$R_{\theta i}$ value of $R_{\theta}$ at $i^{th}$ STE test ($^\circ$Cs$^{-1}$)

$R_{\theta i}^c$ intercept of $R_{\theta}$–$N$ plot on $R_{\theta}$ axis ($^\circ$Cs$^{-1}$)

$T$ absolute temperature at the middle of specimen gage section in fatigue test (K)

$\dot{T}$ time rate of $T$ (Ks$^{-1}$)

$V_i$ internal variables

$\dot{W}_H$ hysteresis energy generation rate (MJm$^{-3}$s$^{-1}$)

$W_p$ plastic strain energy generation rate (MJm$^{-3}$s$^{-1}$)

$\Delta$ displacement amplitude (mm)

$\alpha$ coefficient of linear expansion (K$^{-1}$)

$\varepsilon^e$ elastic strain tensor

$\varepsilon^p$ plastic strain tensor

$\rho$ density (Kgm$^{-3}$)

$\sigma$ stress tensor

$\sigma$ nominal stress (MPa)

$\phi$ diameter (mm)

$\nabla$ Laplacian operator

6.8. References


[27] Beardmore P. Fatigue behavior of polymers. ASTM STP 675; 453-70.


Chapter 7: An Experimental Approach to Estimate Damage and Remaining Life of Metals under Uniaxial Fatigue Loading*

7.1. Introduction

The application of cyclic loading to a material causes irreversible and cumulative damage that diminishes its remaining fatigue life (RFL). To quantify damage, one can measure permanent changes to the key mechanical properties such as yield strength, modulus of elasticity, cross-sectional area, elongation, tensile strength, hardness, stiffness, and static toughness. For example, Belaadi et al. [1], Abraham et al. [2], and Zhou et al. [3] reported a gradual decrease in the modulus of elasticity with the accumulation of fatigue damage in different materials. Li et al. [4] showed that the reduction in the modulus of elasticity is a useful parameter for the assessment of remaining static strength of self-piercing riveted aluminum joints. Belaadi et al. [1] also showed that plastic strain energy generation per cycle decreases with the progress in the fatigue degradation of a fibrous material. Damage can also be measured by assessing the changes in the physical properties such as electric, magnetic, and thermal properties. For instance, Lemaitre and Dufailly [5] presented that the variation in the electrical potential during a fatigue process is useful for estimating present damage in the conductive materials. Basically, these changes affect the materials’ dynamic response such as stress and strain [6, 7]. Azadi et al. [8] and Navaro and Gamez [9] demonstrated that the fatigue damage is represented by monotonic accumulation of the total plastic strain energy generation. Thus, to characterize the behavior of materials under cyclic loading, researchers have resorted to monitoring and assessing the evolution of these material properties with time.

From a relevant analytical viewpoint, the continuum damage mechanics (CDM) offers a method to evaluate the progression of damage in mechanical and physical properties and expresses it in terms of a so-called damage parameter, \( D \), defined as:

\[
D = 1 - \frac{E}{\bar{E}}
\]  

(7.1)

where \( E \) is the modulus of elasticity corresponding to a specimen in its pristine condition and \( \bar{E} \) represents the modulus of the specimen’s present condition after sustaining a certain amount of damage. Among the early works on the application of CDM are Krajcinovic [10] and Chaboche [11] who demonstrated that damage parameter is an useful parameter in predicting present material degradation subjected to constant amplitude cyclic loading (CACL). Varvani-Farahani [12] demonstrated the application of damage mechanics in predicting fatigue failure of metals under variable amplitude cyclic loading (VACL).

*Reprinted by permission of Materials & Design (See Appendix A)
A pertinent development to the present study is the work of Duyi and Zhenlin [13] who developed a relationship between $D$ and the number of accumulated loading cycles, $N$, based on the exhaustion of the static toughness. The applications of the concept of thermodynamic entropy generation associated with a degradation process in predicting damage are reported by Naderi and Khonsari [14, 15], Sun and Hu [16], Amiri and Khonsari [17], Amiri et al. [18], and Khonsari and Amiri [19].

Research shows that the thermal response obtained from a short-time excitation (STE) test on a material at different stages of the fatigue is useful for predicting fatigue life, $N_f$. Meyendorf et al. [20] showed that damage associated with the progression of fatigue tends to gradually increase the temperature rise, $\Delta T$. The increase in temperature rise can be obtained from a series of STE tests performed during a normal fatigue test (NFT). The gradual increase in $\Delta T$ with the increase in fatigue damage is thus related to changes in the microstructural state and the fatigue life of a specimen [21]. In fact, as reported recently, the slope of temperature rise, $R_\theta$, obtained from STE tests can be utilized to predict the RFL of unwelded metallic specimens subjected to constant amplitude rotating-bending fatigue [22] and welded metallic specimens subjected to tension-compression tests [23].

In this paper we report the development of a useful correlation between $D$ and the thermal response of a specimen obtained from STE tests in order to predict its RFL when subjected to CACL and/or VACL. The correlation is applied to determine the evolution of $D$ in solid API 5L X52 and tubular carbon steel 1018 specimens. Further, using the relationship between the damage parameter and the normalized fatigue life, $N/N_f$, we develop a non-destructive testing (NDT) method to predict the RFL of metallic specimens with prior history of fatigue damage. A series of uniaxial tension-compression NFTs, subjected to both CACL and VACL, with API 5L X52 and carbon steel 1018 specimens are reported to assess the damage evolution and RFL prediction capability of the proposed correlation and methodology.

7.2. Experimental Details

7.2.1. Materials and Equipment

Figure 7.1(a and b) illustrates the schematic of solid, cylindrical dogbone specimens made of API 5L X52 (a high-strength steel) and tubular specimens made of carbon steel 1018 according to the ASTM: E466-07, respectively, and Table 7.1 presents the corresponding dimensions of the specimens. In order to circumvent initiation of micro-cracks from nicks, dents, scratches, and circumferential tool marks, the entire gage section of the specimens was polished longitudinally to bring the surface roughness to within 0.2-µm $R_a$ (ASTM: E466-07). Uniaxial NFTs are carried out at different stress levels, $\sigma$, and load ratios, $L_R$, (defined as the ratio of minimum to the maximum stress of cyclic loading) in accordance with the ASTM: E466-07. Monotonic static tests
are performed at a constant stroke rate of 0.02 mm/s in accordance with the ASTM: E8. A servo-hydraulic fatigue testing machine with the capability of 50 kN axial load and 75 Hz frequency (Figure 7.2) is used to perform all of the experiments.

![Schematic illustrations of (a) solid, cylindrical dogbone specimens made of API 5L X52 and (b) tubular specimens made of carbon steel 1018](image)

**Figure 7.1.** Schematic illustrations of (a) solid, cylindrical dogbone specimens made of API 5L X52 and (b) tubular specimens made of carbon steel 1018

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Material</th>
<th>a₁</th>
<th>a₂</th>
<th>a₃</th>
<th>a₄</th>
<th>a₅</th>
<th>a₆</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>API 5L X52</td>
<td>38.1</td>
<td>25.4</td>
<td>14.0</td>
<td>10.0</td>
<td>80.0</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>API 5L X52</td>
<td>38.1</td>
<td>14.0</td>
<td>13.0</td>
<td>7.0</td>
<td>56.0</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>Carbon steel 1018</td>
<td>35.0</td>
<td>22.0</td>
<td>14.3</td>
<td>11.0</td>
<td>37.0</td>
<td>8.73</td>
</tr>
</tbody>
</table>

Table 7.1. Dimensions of three different specimens (all dimensions are in millimeter)

The surface temperature of specimen gage section is recorded using a high-speed infrared (IR) camera with the resolution of 320×240 pixel, accuracy of ±2% of reading, temperature range capability between 0°C and 500°C, sensitivity of 0.08°C (at 30°C) at a data acquisition rate of 1 Hz. A thin layer of black paint was sprayed on the gage section of the specimen to reduce IR reflection and increase thermal emissivity. Specimen surface temperature is recorded over the
entire gage section. Since the maximum temperature occurs in the middle of the specimen gage section, average temperature over an approximately 5-mm long line at that location is used in the analysis.

Figure 7.2. (a) Major components of the experimental set-up for monotonic static test and uniaxial tension-compression fatigue test and (b) a close-up view of the specimen gripped between the jaws

7.2.2. Fatigue Test Procedure

Load-controlled and tension-compression NFTs are carried out subjected to CACL and VACL at the frequency of 10 Hz with API 5L X52 and carbon steel 1018 specimens at different stress levels and load ratios. Following the procedure recommended in [24], the evolution of $R_\theta$ in an NFT is determined by performing a series of STE tests (typically about 15-20 s) periodically as illustrated in Figure 7.3. The STE tests are performed at the load level, load ratio, and test frequency chosen by the operator, which are maintained constant for all the STE tests with a specific material. The STE test loading conditions chosen in the present work are: $\sigma = 402$ MPa, $L_R = -0.56$, and $f = 10$ Hz for API 5L X52 and $\sigma = 395$ MPa, $L_R = -0.6$, and $f = 10$ Hz carbon steel 1018, respectively.

The procedure is as follows. Beginning with a pristine specimen, the slope of temperature rise is measured by an STE test followed by the NFT that lasts for a specific number of load cycles depending on the loading conditions. The test is stopped, the specimen is allowed to cool down to the ambient temperature, and the value of $R_{\theta 1}$ is measured by a subsequent STE test. The above
procedure is repeated until the specimen fractures. In order to obtain the evolution of $R_\theta$, the number of load cycles in each NFT between two subsequent STE tests should be chosen in such a way that $R_\theta$ is measured at least 6-7 times if $N_f \leq 10^4$ cycles and around 8-10 times if $N_f > 10^4$ cycles.

![Figure 7.3. Illustration of typical temperature evolution during STE test and NFT](image)

**Figure 7.3. Illustration of typical temperature evolution during STE test and NFT**

### 7.3. Analytical

#### 7.3.1. Evolution of Damage Parameter

Cyclic loading causes permanent microstructural changes in the material due to irreversible plastic deformation which primarily dissipates heat to the surrounding medium. Research shows that the irreversible microstructural changes can be characterized by determining the evolution of $R_\theta$ obtained from a series of STE tests in an NFT which is linearly related to the number of accumulated fatigue load cycles, $N$, [22, 23]:

$$R_\theta = nN + \bar{R}_{\theta_0}^c$$  \hspace{1cm} (7.2)

where $n = Ae^{B\sigma}$ and $\bar{R}_{\theta_0}^c$ stand for the slope and intercept of $R_\theta - N$ plot, respectively, $A$ and $B$ are empirical constants, and $\sigma$ is the maximum stress of cyclic loading. Let us define a ‘relative slope’ $R_r = R_\theta - \bar{R}_{\theta_0}^c$ such that Equation (7.2) simply becomes:

$$R_r = nN$$  \hspace{1cm} (7.3)

Since $R_r$ is a linear function of $N$ and each cycle of fatigue load increases material damage monotonically, a general correlation between $D$ and $R_r$ is proposed as:

$$D = Cf(R_r)$$  \hspace{1cm} (7.4)
where $C$ is a constant that depends on the material and loading conditions. Since, the cyclic rate of damage evolution, $dD/dN$, is minimum at $N = 0$ and $dD/dN \to \infty$ at the onset of fracture [13, 14, 15], it can be obtained that $\frac{\partial D}{\partial R_r} = C \frac{\partial f}{\partial R_r}$ is minimum at $N = 0$ and $\frac{\partial D}{\partial R_r} \to \infty$ at the onset of fracture.

The evolution of $D$ must satisfy the two important properties of a convex function [13, 24]:

\[
\left( \frac{\partial D}{\partial R_r} \right)_{N+1} > \left( \frac{\partial D}{\partial R_r} \right)_N \quad \text{and} \quad \frac{\partial^2 D}{\partial R_r^2} > 0.
\]

All of these conditions can be satisfied if

\[
\frac{\partial f}{\partial R_r} = \frac{1}{R_{rf} - R_r}
\]

where $R_{rf}$ is the value of $R_r$ at the onset of fatigue fracture, i.e. when $N = N_f$. Integrating Equation (7.5) with respect to $R_r$ and substituting into Equation (7.4) yields:

\[
D = -C \ln \left( 1 - \frac{R}{R_{rf}} \right) \tag{7.6}
\]

Differentiating Equation (7.6) with respect to $N$ yields $C = (R_{rf} - R_r) \frac{\partial D}{\partial N}$. Duyi and Zhenlin [13] demonstrated that $\frac{\partial D}{\partial N} = \frac{D_{(N_f-1)}}{(N_f - N) \ln(N_f)}$ where

\[
D_{(N_f-1)} = 1 - \frac{\sigma_a}{2E U_{T0}} \tag{7.7}
\]

$\sigma_a$ is the load amplitude, $E$ represents the modulus of elasticity, and $U_{T0}$ stands for the static toughness of pristine material. Using Equation (7.3), $C = \frac{D_{(N_f-1)}}{\ln(N_f)}$ and Equation (7.6) becomes:

\[
D = -\frac{D_{(N_f-1)}}{\ln(R_{rf} / n)} \ln \left( 1 - \frac{R}{R_{rf}} \right) \tag{7.8}
\]

According to Equation (7.8), the evolution of damage in a degradation process can be estimated by measuring the evolution of the independent variable $R_r$. This can be accomplished by performing a series of STE tests. Equation (7.8) also offers a method to estimate the present state of damage in a material by performing an STE test subjected to CACL.

### 7.3.2. Damage Accumulation Subject to Variable Loading

When dealing with VACL, the rate of material damage accumulation depends on the stress level and the accumulation of damage in the $k$th-stage, $D_k$, which begins from the damage value at the end of the $(k-1)$th-stage, $D_{k-1}$. Thus, Equation (7.8) can be extended to assess the damage
accumulation over an $m$-stage VACL process as the sum of the damage produced in each stage of loading:

$$D = \sum_{k=1}^{m} \left\{ \frac{D_{(N_f-1)}}{\ln \left( \frac{R_{rf}}{n_k} \right)} \ln \left( 1 - \frac{R_{rk}}{R_{rf}} \right) \right\}$$  \hspace{1cm} (7.9)$$

where the suffix $k$ represents the value of the parameters in the $k^{th}$ loading stage. Material damage under the cyclic loading starts from the beginning of the 1st-stage of loading and accumulates in each loading stage until fracture occurs, corresponding to $D=1$.

7.4. Results and discussion

7.4.1. Determination of $D_{(N_f-1)}$

Table 7.2 presents the values of $E$ and $U_{T0}$ obtained from a series of monotonic static tests. $D_{(N_f-1)}$ is calculated from Equation (7.7) at three different stress levels for both materials. Results show that the value of $D_{(N_f-1)}$ is approximately equal to unity over the range of fatigue loads considered in this study for both materials. Thus, it is assumed that $D_{(N_f-1)} \cong 1$ for the materials tested.

<table>
<thead>
<tr>
<th>Material</th>
<th>$E$ (GPa)</th>
<th>$U_{T0}$ (MJ/m$^3$)</th>
<th>$\sigma_a$ (MPa)</th>
<th>$D_{(N_f-1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>API 5L X52</td>
<td>200</td>
<td>136.07</td>
<td>483</td>
<td>0.9957</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>456</td>
<td>0.9962</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>437</td>
<td>0.9965</td>
</tr>
<tr>
<td>Carbon steel</td>
<td>189</td>
<td>79.10</td>
<td>447</td>
<td>0.9933</td>
</tr>
<tr>
<td>1018</td>
<td></td>
<td></td>
<td>421</td>
<td>0.9941</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>381</td>
<td>0.9951</td>
</tr>
</tbody>
</table>

7.4.2. Evolution of Relative Slope, $R_r$

7.4.2.1. Constant Amplitude Loading

Figures 7.4 and 7.5 present the evolutions of $R_r$ obtained from the NFTs performed with API 5L X52 (Specimen-1) and carbon steel 1018, respectively. The values of $A$ and $B$ for the given materials and load ratios are presented in Table 7.3. Results shown for $R_r = 0$ corresponds to the pristine specimen without prior history of cyclic loading and microstructural damage. As the number of applied fatigue load cycle increases, the value of $R_r$ increases approximately in a linear fashion. Research shows that the application of cyclic loading causes irreversible microstructural damage in the material [25, 26]. Due to the accumulation of microstructural damage, the material
Figure 7.4. $R_r - N$ plot of API 5L X52 specimens subjected to uniaxial tension-compression tests at different stress levels and at $L_R = -0.5$ and -1.

Figure 7.5. $R_r - N$ plot of carbon steel 1018 specimens subjected to uniaxial tension-compression tests at different stress levels and $L_R = -0.6$.

generates greater amount of heat energy in an STE test compared to the prior STE test which is represented by the gradual increase in $R_r$. It can be seen that for a specific load ratio the higher the load level, the greater is the rate of $R_r$ evolution. Results presented in Figure 7.4 reveal that fatigue life of API 5L X52 decreases significantly when the specimen is subjected to $L_R = -1$ compared to
\[ L_R = -0.5 \] under comparable stress level. Similar effect of load ratio is also observed by Manigandan et al. [26] for two different high strength alloy steels. In summary, \( R_r = 0 \) for a pristine material (i.e., when \( N = 0 \)); \( R_r \) evolves linearly with respect to \( N \); and that the rate of \( R_r \) evolution is dependent on the material properties and the loading conditions.

Table 7.3. Values of \( A \) and \( B \) for API 5L X52 and carbon steel 1018

<table>
<thead>
<tr>
<th>Material</th>
<th>( L_R )</th>
<th>( A )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>API 5L X52</td>
<td>-0.5</td>
<td>( 1.5924 \times 10^{-14} )</td>
<td>0.04713</td>
</tr>
<tr>
<td>API 5L X52</td>
<td>-1</td>
<td>( 1.5025 \times 10^{-12} )</td>
<td>0.04418</td>
</tr>
<tr>
<td>Carbon steel 1018</td>
<td>-0.6</td>
<td>( 3.8643 \times 10^{-15} )</td>
<td>0.05771</td>
</tr>
</tbody>
</table>

7.4.2.2. Variable Amplitude Loading

Figures 7.6-7.8 illustrate the evolution of \( R_r \) with respect to \( N \) obtained from the two-stage loading (High-to-Low and Low-to-High load sequence) fatigue tests performed with API 5L X52 (specimen-2) and carbon steel 1018. The value of \( R^C_{\theta_0} \) is determined as the intercept of the \( R_\theta - N \) plot corresponding to the 1st-stage loading. In order to determine the effect of High-to-Low (H-L) and Low-to-High (L-H) load sequence on the evolution of \( R_r \), two tests are performed for a specific set of load amplitudes with constant load ratio. For example, one test is performed at the stress level of 483 MPa followed by 456 MPa (H-L) and another test is performed at 456 MPa followed by 483 MPa (L-H) loading pattern with API 5L X52 at \( L_R = -0.5 \). Results show that in each loading sequence \( R_r \) evolves linearly with a constant rate, \( n_1 \), in the 1st-stage of cyclic loading which is identical to its trend observed under CACL. Research shows that a change in the applied stress level demonstrates a variation in the evolution of microstructural damage [25, 27]. Present experimental results show that as the stress level changes in the 2nd-stage, the rate of \( R_r \) evolution, \( n_2 \), changes as well. This implies that the variation in the slope of \( R_r \) evolution between 1st-stage and 2nd-stage loading occurs due to the variation in the rate of microstructural evolution. If the stress level in the 1st-stage is greater than that in the 2nd-stage, \( n_1 \) is greater than \( n_2 \) and vice versa. It can be seen that \( n_{1,H-L} \equiv n_{2,L-H} \) and \( n_{2,H-L} \equiv n_{1,L-H} \) for all the results presented in Figures 7.6-7.8 where \( n_{1,H-L} \) and \( n_{2,H-L} \) are the rates of \( R_r \) evolution corresponding to the 1st- and 2nd-stage in the H-L load sequence tests and \( n_{1,L-H} \) and \( n_{2,L-H} \) stand for those in the L-H load sequence tests. In case of an \( m \)-stage load sequence \( R_{rf} \) can be determined by summing up the evolution of \( R_r \) in each loading stage until the fracture of the specimen as follows:

\[
R_{rf} = \sum_{k=1}^{m} n_k N_k
\]  
(7.10)

where \( n_1, n_2, \ldots, n_m \) represent the rates of \( R_r \) evolution and \( N_1, N_2, \ldots, N_m \) stand for the number of the load cycle in the 1st-stage, 2nd-stage, \ldots, \( m \)-th-stage, respectively, in an \( m \)-stage VACL test. Thus, if the value of \( n \) is known for a range of stress levels, the evolution of \( R_r \) can be estimated using Equation (7.10).
Figure 7.6. $R_r - N$ plot of API 5L X52 specimens subjected to a two-stage uniaxial tension-compression test at (a) H-L load: 483 to 456 MPa and (b) L-H load: 456 to 483 MPa.

Figure 7.7. $R_r - N$ plot of API 5L X52 specimens subjected to a two-stage uniaxial tension-compression test at (a) H-L load: 400 to 380 MPa and (b) L-H load: 380 to 400 MPa.
7.4.3. Evolution of Damage Parameter, $D$

7.4.3.1. Constant Amplitude Loading

To examine the validity of the damage evolution obtained from Equation (7.8) under CACL, results are compared to those obtained from the expression developed by Duyi and Zhenlin [13] that reads:

$$D = -\frac{D_{(N_f-1)}}{\ln(N_f)} \ln \left( 1 - \frac{N}{N_f} \right)$$  \hspace{1cm} (7.11)

Figures 7.9(a and b) and 7.10 illustrate the evolution of $D$ obtained from Equations (7.8) and (7.11) for each fatigue tests performed with API 5L X52 and carbon steel 1018, respectively, using $D_{(N_f-1)}=1$. The evolutions of $D$ show that at the beginning of the material degradation process it increases slowly and then the rate increases followed by a sharp increase before final fracture. At the beginning of the fatigue tests, the specimens are pristine and free of micro-cracks and defects so that $D=0$. According to [14, 25, 26], the application of cyclic loading tends to initiate degradation with the formation of dislocations and slip bands that progress with the continuation of cyclic loading. This phase of damage evolution is represented by the low rate in its accumulation. As the cyclic loading continues, the dislocations and slip bands coalesce forming micro-cracks in the plane of maximum shear [28] that increases the rate in the evolution of $D$. As soon as $D$ reaches its critical value, $D_c$, 

---

Figure 7.8. $R_r$–$N$ plot of carbon steel 1018 specimens subjected to a two-stage uniaxial tension-compression test (a) H-L load: 447 to 421 MPa and (b) L-H load: 421 to 447 MPa.
a macro-crack develops in the material and its propagation corresponds to the sharp increase in $D$ evolution that leads to the fracture of the specimen at $D=1$. Research shows that the macro-crack propagation phase is significantly shorter than the other phases of damage accumulation [29, 30]. It can be seen that the span of the sharp increase in $D$ is substantially shorter than the fatigue life for the materials tested in this research.

In the case of CACL, the damage parameter increases monotonically with the accumulation of load cycle and the value of $D$ at specific number of the fatigue load cycle induced by the larger load is greater than that induced by the smaller load, as expected. For example, Figure 7.9(b) shows that the values of $D$ at $N = 5000$ cycles are 0.183, 0.057, and 0.0268 corresponding to the load amplitudes of 400, 380, and 360 MPa, respectively. Since a greater stress level produces larger material deformation and causes more microstructural changes, the rate of $R_r$ and $D$ evolution is found to be greater for this case compared to those corresponding to the smaller stress level. This reveals that both $R_r$ and $D$ are useful fatigue parameters those are directly correlated to the present microstructural state in the material. It can be seen that the evolution of $D$ obtained from present work and Duyi and Zhenlin [13] approach are in good agreement. This shows that the evolution of damage parameter in the fatigue of metals can be estimated using the thermography method proposed in this work.

Figure 7.9(a). Comparison between $D$ evolutions in API 5L X52 from Duyi and Zhenlin [13] approach and present work at different stress levels with $L_R = -0.5$
Figure 7.9(b). Comparison between $D$ evolutions in API 5L X52 from Duyi and Zhenlin [13] approach and present work at different stress levels with $L_R = -1$.

Figure 7.10. Comparison between $D$ evolutions in carbon steel 1018 from Duyi and Zhenlin [13] approach and present work at different stress levels and $L_R = -0.6$. 
7.4.3.2. Variable Amplitude Loading

In order to determine the validity of the approach developed in the present work, Equation (7.9), under VACL degradation process, the damage evolution is compared to the predictions made using the thermodynamic entropy-based method developed by Naderi and Khonsari [15]. The method is applicable for two- and three-stage loading with the damage parameter described by the following expression:

\[
D = \sum_{k=1}^{n} \left[ D_{N_k} + \frac{D_c - D_{N_{k-1}}}{\ln \left( \frac{1 - (s_{ic} / s_g)}{1 - (s_{k-1} / s_g)} \right)} \ln \left( \frac{1 - (s / s_g)}{1 - (s_{k-1} / s_g)} \right) \right]
\]  

(7.12)

where \( D \) and \( D_c \) stand for the total and critical damage, respectively; \( D_{k-1} \) and \( s_{k-1} \) represent the accumulated damage and entropy at \((k-1)\)th-stage of loading, respectively; \( s \) denotes the entropy production, \( s_{ic} \) is the critical entropy, and \( s_g \) represents the total entropy generation in a fatigue process. Entropy production at any stage in a fatigue process is calculated as follows:

\[
s = \int_{0}^{t} \left( \frac{W_p}{T} \right) dt
\]  

(7.13)

where \( W_p \) represents the plastic strain energy generation per second, \( T \) is the absolute surface temperature of the specimen, and \( t \) stands for the time.

Figures 7.11-7.13 illustrate the evolution of the damage parameter with respect to \( N \) corresponding to the two-stage fatigue test results presented in Figures 7.6-7.8 obtained from the present work (Equation (7.9)) and the entropic approach (Equation (7.12)). In order to determine the effect of load sequence on the evolution of \( D \), two tests are performed with the H-L and L-H loading patterns for a specific set of the load amplitudes such as 483 MPa followed by 456 MPa for one test and 456 MPa followed by 483 MPa for another test with API 5L X52 at \( L_R = -0.5 \). As soon as the 1st-stage loading begins, the damage process initiates with the formation of dislocations and slip bands which is manifested by the slow increase in the evolution of \( D \) [15]. The continuation of cyclic loading tends to coalesce dislocation and slip bands to form micro-cracks that contribute to increase the rate in the evolution of \( D \). If the loading conditions remain unchanged, the rate of the evolution of \( D \) increases gradually and progresses smoothly as shown in Figures 7.9 and 7.10. According to [15], the changed loading conditions in the 2nd-stage tend to grow existing micro-cracks along the new plane of maximum shear stress associated with the change in the loading conditions. As a result, the trend of damage evolution changes in the 2nd-stage and a knee point appears in the damage evolution plots as illustrated in Figures 7.11-7.13. If the load amplitude in the 2nd-stage is greater than that in the 1st-stage, the rate of damage accumulation increases and vice versa. As the cyclic loading continues in the 2nd-stage, micro-

144
cracks combine to form a macro-crack which is typically corresponds to the critical damage, $D_c$, or critical entropy production, $s_{ic}$, state in a fatigue process. Finally, the propagation of the macro-crack leads to the final fracture [15, 28, 31, 32].

Figure 7.11. Damage evolution obtained from present work and Naderi and Khonsari [15] approach at (a) H-L load: 483 to 456 MPa and (b) L-H load: 456 to 483 MPa.

Figure 7.12. Damage evolution obtained from present work and Naderi and Khonsari [15] approach at (a) H-L load: 400 to 380 MPa and (b) L-H load: 380 to 400 MPa.
Figures 7.11-7.13 reveal that the H-L loading sequence results in a shorter fatigue life compared to that subjected to L-H loading sequence for all three cases. This occurs due to the difference in the mechanism of damage progression under high and low load amplitudes. At low-load level, major portion of the material degradation process is spent on forming dislocations and slip bands followed by micro-crack formation and propagation. In contrast, at high-load level material damage evolution starts with the initiation of micro-cracks followed by the formation of macro-cracks [33]. Therefore, the rate of material damage accumulation at high-load level is greater than that at low-load level [3, 15, 27, 28]. Consequently, fatigue life of a material subjected to H-L loading is shorter than that under L-H loading sequence. It can be seen that damage evolution determined from the proposed expression, Equation (7.9), and the entropic approach, Equation (7.12), are in good agreement for all the tests.

7.4.4. Damage Evolution Master Curve

Figures 7.14 and 7.15 present the evolution of $D$ with respect to the normalized fatigue life, $N/N_f$, for both API 5L X52 and carbon steel 1018 at different stress levels and load ratios subjected to CACL and VACL, respectively. It can be seen that the damage accumulation is independent of material properties, shape and size of the specimen, and loading conditions under both CACL and VACL. Also, the master curve of the evolution of $D$ under VACL is independent of the loading sequence and has no knee point. The master curve provides a convenient method for predicting the fraction of consumed fatigue life, $N/N_f$, in a fatigued material as the present damage level, $D$, of that material can be determined by following the procedure described in section 7.3.3 for both
CACL and VACL. If $N/N_f$ is known, the percentage of RFL of that material can be predicted from the following expression:

$$RFL = \left(1 - \frac{N}{N_f}\right) \times 100\%$$  \hspace{1cm} (7.14)

Figure 7.14. Master curve for the evolution of $D$ with respect to the normalized fatigue life for solid- and tubular-dogbone specimens under CAACL.

Figure 7.15. Master curve for the evolution of $D$ with respect to the normalized fatigue life for solid- and tubular-dogbone specimens under VACL.
7.4.5. Non-destructive Prediction of RFL and Validation

In this section, we present the results of validation experiments to test the RFL prediction capability of the method described in the preceding sections for both CACL and VACL. The CACL fatigue test is started with specific loading conditions and stopped after an unknown number of fatigue load cycles before fracture. For the case of the two-stage loading, the 1st-stage of cyclic loading is completed and the test is stopped in the 2nd-stage after an arbitrary number of total load cycles are performed before fracture. The specimen is then allowed to cool down to the ambient temperature and the value of $R_r$ corresponding to the present material damage can be determined by performing an STE test of identical loading conditions to those used in the characterization tests. The values of $A$ and $B$ corresponding to the given material and loading conditions are obtained from Table 7.3. Then, the value of $D$ is calculated from Equations (7.8) and (7.9) and the corresponding $N/N_f$ is determined from Figures 7.14 and 7.15 for CACL and VACL, respectively. The percentage of RFL of the fatigued specimen is predicted, $RFL_{pred}$, from Equation (7.14). Then, the NFT is resumed to determine the number of load cycles required to fracture of the specimen. The experimental RFL, $RFL_{exp}$, of the specimen, at the stage of NFT where the test is stopped to predict RFL, is determined by substituting the number of load cycles performed before the STE test and total number of load cycles required to fracture the specimen for $N$ and $N_f$, respectively, into Equation (7.14). Finally, the difference between the experimental and the predicted RFL is estimated as $\Delta RFL = |RFL_{exp} - RFL_{pred}| \%$.

Tables 7.4 and 7.5 present the value of $D$, $RFL_{pred}$, $RFL_{exp}$, and $\Delta RFL$ for the six validation tests performed under CACL and two validation tests performed under VACL, respectively.

<table>
<thead>
<tr>
<th>Material</th>
<th>Load (MPa)</th>
<th>$L_r$</th>
<th>Load cycles before STE test</th>
<th>$R_r$ (°C/s)</th>
<th>Material damage, $D$</th>
<th>$N_f$</th>
<th>$RFL_{pred}$ (%)</th>
<th>$RFL_{exp}$ (%)</th>
<th>$\Delta RFL$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>API 5L X52</td>
<td>483</td>
<td>−0.5</td>
<td>3422</td>
<td>0.371</td>
<td>0.090</td>
<td>7449</td>
<td>46.00</td>
<td>54.06</td>
<td>8.06</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>−1</td>
<td>7012</td>
<td>0.245</td>
<td>0.165</td>
<td>8233</td>
<td>22.00</td>
<td>14.84</td>
<td>7.16</td>
</tr>
<tr>
<td></td>
<td>380</td>
<td>−1</td>
<td>9016</td>
<td>0.233</td>
<td>0.140</td>
<td>15107</td>
<td>47.00</td>
<td>40.30</td>
<td>6.70</td>
</tr>
<tr>
<td>Carbon steel 1018</td>
<td>447</td>
<td>−0.53</td>
<td>8983</td>
<td>0.737</td>
<td>0.183</td>
<td>11371</td>
<td>18.00</td>
<td>21.00</td>
<td>3.00</td>
</tr>
<tr>
<td></td>
<td>421</td>
<td>−0.56</td>
<td>13550</td>
<td>0.326</td>
<td>0.040</td>
<td>39350</td>
<td>68.00</td>
<td>66.00</td>
<td>2.00</td>
</tr>
</tbody>
</table>
Table 7.5. Results of validation tests under two-stage fatigue loading.

<table>
<thead>
<tr>
<th>Material</th>
<th>Load Sequence (MPa)</th>
<th>$L_R$</th>
<th>No. of load cycles before STE test</th>
<th>$R_r$ (°C/s)</th>
<th>Material damage, $D$</th>
<th>$N_f$</th>
<th>$RFL_{pred}$</th>
<th>$RFL_{exp}$</th>
<th>$\Delta RFL$</th>
</tr>
</thead>
<tbody>
<tr>
<td>API 5L X52</td>
<td>483 to 456</td>
<td>−0.5</td>
<td>9400</td>
<td>0.373</td>
<td>0.168</td>
<td>11955</td>
<td>17.00</td>
<td>21.37</td>
<td>4.37</td>
</tr>
<tr>
<td></td>
<td>456 to 483</td>
<td>−0.5</td>
<td>10204</td>
<td>0.222</td>
<td>0.132</td>
<td>13967</td>
<td>30.00</td>
<td>26.94</td>
<td>3.06</td>
</tr>
</tbody>
</table>

These results demonstrate that the predicted and experimental RFL are in good agreement under different NFT loading conditions for both CACL and VACL. The maximum difference in the RFL prediction is found to be 8.06%.

The applicability of the proposed method is validated within the range of experimental, environmental, and loading conditions tested in this work where environmental conditions were maintained constant. The proposed methodology shows promising results in predicting RFL of standard specimens without stress concentration. Additional research is required to extend the applicability of the method to actual components, e.g., shaft, beam, column, etc., under variable environmental conditions and with stress concentration.

7.5. Conclusions

Uniaxial tension-compression fatigue tests and monotonic static tests are performed with both solid and tubular dogbone specimens made of API 5L X52 and carbon steel 1018, respectively, subjected to both CACL and VACL. The so-called damage parameter is correlated to the proposed fatigue damage parameter, $R_r$, that allows to estimate present material damage level by measuring its value using an IR camera from an STE test. Results reveal that $R_r$ is an effective fatigue parameter in estimating material damage accumulation. In the case of two-stage loading both $R_r$-$N$ and $D$-$N$ plots demonstrate the effect of the change in loading conditions by changing the evolution rate of $R_r$ and $D$, respectively. However, the evolution of $D$ plotted against the normalized fatigue life, $N/N_f$, is independent of material properties, shape and size of the specimen, and loading conditions in both CACL and VACL cases. Validation test results show that the proposed method can predict the RFL of metallic specimens with good accuracy in both CACL and VACL. Present research pertains to experiments conducted in the laboratory with standard specimens where environmental conditions were maintained constant. Hence, further research is required to assess the applicability of the proposed damage and RFL prediction approaches to actual components, e.g., shaft, beam, column, etc., under variable environmental conditions.
7.6. Nomenclature

$a_i$ specimen dimension (mm)
$A$ empirical constant
$B$ empirical constant
$C$ empirical constant
$D$ damage parameter
$D_c$ critical damage parameter
$D_k$ damage value in $k^{th}$-stage of loading
$D_{k-1}$ damage value in $(k-1)^{th}$-stage of loading
$D_{(N_f-1)}$ damage value at the onset of fracture
$E$ modulus of elasticity of pristine material (GPa)
$\bar{E}$ modulus of elasticity of material with prior fatigue damage (GPa)
$k$ number of load stages
$L_R$ load ratio
$m$ number of loading stages in a VACL fatigue
$n$ slope of $R_r - N$ plot (°C/s/cycle)
$n_k$ slope of $R_\theta - N$ plot in $k^{th}$-stage of loading (°C/s/cycle)
$n_{i,H-L}$ slope of $R_r - N$ plot in $i^{th}$-stage of loading in High-to-Low load sequence (°C/s/cycle)
$n_{i,L-H}$ slope of $R_r - N$ plot in $i^{th}$-stage of loading in Low-to-High load sequence (°C/s/cycle)
$N$ number of loading cycles
$N_i$ number of load cycle in $i^{th}$-stage of Loading
$N_f$ fatigue life in cycle
$R_a$ arithmetic average of surface roughness (µm)
$R_r$ relative slope of temperature rise (°C/s)
$R_{rf}$ maximum value of $R_r$ (°C/s)
$R_{rk}$ relative slope of temperature rise in $k^{th}$-stage of loading (°C/s)
$R_\theta$ slope of temperature rise (°C/s)
$R_{\theta 0}$ intercept of $R_\theta - N$ plot (°C/s)
$s$ entropy (MJ/m³/K)
$s_g$ total entropy (MJ/m$^3$/K)

$s_{ic}$ critical entropy (MJ/m$^3$/K)

$s_{k-1}$ accumulated entropy at $(k-1)^{th}$-stage of loading (MJ/m$^3$/K)

$T$ temperature (K)

$U_{T0}$ static toughness of pristine material (MJ/m$^3$)

$W_p$ plastic strain energy generation per second (MJ/m$^3$/s)

$\sigma$ maximum stress in a cycle (MPa)

### 7.7. References


Chapter 8: Conclusions and Future Recommendations

8.1. Conclusions

Research performed in this dissertation is comprised of the measurement of thermodynamic entropy and plastic strain energy in fatigue, estimation of the effect of stress concentration on the FFE and fatigue life, development of a methodology for the calculation of anelastic energy in HCF, prediction of the remaining fatigue life of metallic and composite specimens, and derivation of a correlation between the fatigue damage parameter and the slope of temperature rise obtained from a specimen. To determine the validity of the correlations and methods developed, uniaxial tension-compression and completely-reversed bending fatigue tests are carried out with metallic specimens made of API 5L X52, carbon steel 1018 and 1045 and aluminum 6061 along with a Glass/Epoxy composite laminate. Various stress levels, load ratios, and test frequencies are performed that covers both LCF and HCF with dogbone and flat specimens.

The value of the degradation parameter $R_\theta$ at a specific level of damage in a material is shown to be a promising parameter for the estimation of thermodynamic entropy and plastic strain energy. It is shown that both the rate and the evolution of entropy and plastic strain energy can be determined by adopting this method. This method shows promising agreement between the predicted and experimental results. As this method does not require one to monitor the evolution of fatigue continuously, RFL of a material can be estimate if the FFE of that material is known.

Evolution of fatigue in a specimen with certain level of stress concentration is studied utilizing the concept of the entropy accumulation. The trend of temperature and plastic strain energy evolution is comparable between a V-notched and an un-notched specimen. The method of gross strain measurement across a V-notch is found to be useful for the development of a correlation to predict fatigue life of a V-notched specimen. It is shown that the FFE of a material decreases substantially due to the presence of stress concentration and it remains in a small band for each type of the specimens tested.

The method of thermodynamic entropy based fatigue life estimation has been applied to study HCF of metallic specimens. As the hysteresis loops associated with HCF contains anelastic energy, a methodology has been developed for the calculation of the anelastic energy. It is shown that the presence of anelastic energy occurs due to a phase lag between the stress and strain. The FFE in HCF is found to be within a narrow band that conforms to the idea of constant entropy accumulation at fatigue fracture.

The remaining fatigue life prediction method developed in this dissertation correlates the value of $R_\theta$ to the number of load cycles associated with specimens made of metals and a composite laminate. Metallic specimens are tested under constant-amplitude uniaxial tension-compression loads and the Glass/Epoxy specimens are tested under both uniaxial tension-compression and completely-reversed bending loads. It is shown that the evolution of $R_\theta$ is a linear function of the
number of load cycle over a range of constant-amplitude loading conditions for both types of the materials. In the case of a two-stage loading test, the rate of $R\theta$ evolution changes with a change in the stress level for both types of the loads. Formulas are derived for the prediction of the RFL associated with both constant-amplitude and two-stage variable loading tests. The results of the validation tests show reasonable agreement when compared to the experimental results.

A correlation is developed to determine the value of damage parameter associated with a material subject to prior fatigue loading by measuring the value of $R\theta$. The evolution of damage parameter calculated based on the $R\theta$ measurement shows good agreement when compared to the methods available in literature. The concept of utilizing a master curve of damage evolution in predicting RFL is found to be a useful method for both constant- and variable-amplitude loading conditions.

**8.2 Recommendations for Future Works**

In order to extend the applicability of the methods presented in this dissertation, following works can be considered:

- In many real applications, many structures and equipment are subject to combined loading. Therefore, the applicability of the RFL prediction method should be assessed under combined loading such as axial-torsion, bending-torsion, etc.

- Since the RFL prediction method is based on the temperature measurement and the amount of temperature evolution in HCF is low, additional work is required to develop an RFL estimation method for HCF.

- The method of FFE and fatigue toughness prediction based on $R\theta$ measurement should be studied under different loading conditions, e.g., bending, torsion, etc., to develop a general method applicable for different types of fatigue loads.

- The methodology of fatigue life prediction of a V-notched specimen can be investigated for other types of notches such as circular, elliptical, etc. This approach should be studied to determine its applicability for other types of materials such as composites, polymer, rubber, etc.

- The method of calculating anelastic energy in HCF should be applied to different types of materials and loading conditions. Also, the effect of specimen type and size on the predicted results need to be investigate to extend the validity of this promising method.
Appendix: Letters of Permission to Use Published Material

The Permissions from Springer and Elsevier publishing companies are presented in the following pages:

Order Completed

Thank you very much for your order.

This is a License Agreement between Md L Ali ("You") and Elsevier ("Elsevier"). The license consists of your order details, the terms and conditions provided by Elsevier, and the payment terms and conditions.

Get the printable license.
Title: Entropic characterization of metal fatigue with stress concentration
Publication: International Journal of Fatigue
Publisher: Elsevier
Date: January 2015

Order Completed
Thank you very much for your order.

This is a License Agreement between Md L Ali ("You") and Elsevier ("Elsevier"). The license consists of your order details, the terms and conditions provided by Elsevier, and the payment terms and conditions.

Get the printable license.
Order Completed

Thank you very much for your order.

This is a License Agreement between Md L Ali ("You") and Springer ("Springer"). The license consists of your order details, the terms and conditions provided by Springer, and the payment terms and conditions.

Get the printable license.

<table>
<thead>
<tr>
<th>License Number</th>
<th>3584410985065</th>
</tr>
</thead>
<tbody>
<tr>
<td>License date</td>
<td>Mar 08, 2015</td>
</tr>
<tr>
<td>Licensed content publisher</td>
<td>Springer</td>
</tr>
<tr>
<td>Licensed content publication</td>
<td>Journal of Nondestructive Evaluation</td>
</tr>
<tr>
<td>Licensed content title</td>
<td>Nondestructive Testing and Prediction of Remaining Fatigue Life of Metals</td>
</tr>
<tr>
<td>Licensed content author</td>
<td>M. Liakat</td>
</tr>
<tr>
<td>Licensed content date</td>
<td>Jan 1, 2013</td>
</tr>
<tr>
<td>Volume number</td>
<td>33</td>
</tr>
<tr>
<td>Issue number</td>
<td>3</td>
</tr>
<tr>
<td>Type of Use</td>
<td>Thesis/Dissertation</td>
</tr>
<tr>
<td>Portion</td>
<td>Full text</td>
</tr>
<tr>
<td>Number of copies</td>
<td>1</td>
</tr>
<tr>
<td>Author of this Springer article</td>
<td>Yes and you are a contributor of the new work</td>
</tr>
<tr>
<td>Title of your thesis / dissertation</td>
<td>DEGRADATION AND FATIGUE INVOLVING DISSIPATED PROCESSES</td>
</tr>
<tr>
<td>Expected completion date</td>
<td>Apr 2015</td>
</tr>
<tr>
<td>Estimated size(pages)</td>
<td>160</td>
</tr>
<tr>
<td>Total</td>
<td>0.00 USD</td>
</tr>
</tbody>
</table>
Order Completed

Thank you very much for your order.

This is a License Agreement between Md L Ali ("You") and Elsevier ("Elsevier"). The license consists of your order details, the terms and conditions provided by Elsevier, and the payment terms and conditions.

Get the printable license.

License Number: 3584410343110
License date: Mar 08, 2015
Licensed content publisher: Elsevier
Licensed content publication: Materials & Design
Licensed content title: An experimental approach to estimate damage and remaining life of metals under uniaxial fatigue loading
Licensed content author: None
Licensed content date: May 2014
Licensed content volume number: 57
Licensed content issue number: n/a
Number of pages: 9
Type of Use: reuse in a thesis/dissertation
Portion: full article
Format: both print and electronic
Are you the author of this Elsevier article?: Yes
Will you be translating?: No
Title of your thesis/dissertation: DEGRADATION AND FATIGUE INVOLVING DISSIPATED PROCESSES
Expected completion date: Apr 2015
Estimated size (number of pages): 160
Elsevier VAT number: GB 494 6272 12
Permissions price: 0,00 USD
VAT/Local Sales Tax: 0,00 USD / 0,00 GBP
Total: 0,00 USD
Vita

Md Liakat Ali was born in Natore, Bangladesh in 1984. He received his B.S. from Bangladesh University of Engineering and Technology in 2007 and M.S. from Universiti Teknologi Malaysia in 2008 both in mechanical engineering. The focus of his M.S. thesis was on the strength analysis of a bus superstructure using finite element analysis. After receiving M.S. degree, he worked as a research officer in the department of mechanical engineering at Universiti Teknologi Malaysia for six months. He moved to the USA in January 2010 to pursue his Ph.D. in mechanical engineering emphasizing on the fatigue, strength, and life analysis of different types of materials.

Ali was awarded with Enrichment Award 2014 and Dissertation Year Fellowship 2015 for his novel contributions in research by the Department of Mechanical and Industrial Engineering and Graduate School of Louisiana State University, respectively. He has published six peer reviewed journal papers and a conference proceeding.