Trajectory and window width predictions for a cased hole sidetrack using a whipstock

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TRAJECTORY AND WINDOW WIDTH PREDICTIONS FOR A CASED HOLE SIDETRACK USING A WHIPSTOCK

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
Requirements for the degree of
Master of Science in Petroleum Engineering

in

The Department of Petroleum Engineering

by

Harshad Patil
BE. Petroleum Enginnering, Maharashtra Institute of Technology, University of Pune, 2001
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ABSTRACT

The Trackmaster system manufactured by Smith Services consists of a multi-ramped whipstock and a trimill assembly used to perform a sidetrack from a cased well.

Smith Services has observed evidence that the trimill assembly may prematurely leave the face of the whipstock and build excess inclination and dogleg severity or may fall into the original well immediately after leaving the end of the whipstock ramp thus creating a need to predict the borehole trajectory for sidetracking operations.

The goal of this project was to predict the borehole trajectory and the window profile cut in the casing by the sidetracking equipment and the curvature that would result in tubular run through the sidetracked borehole, expressed as dogleg severity.

A computer program was developed that predicts the sidetrack trajectory based on the BHA analysis method proposed by Jiazhi, and extended for calculating the side force on the mills. These side forces and a logical check on the feasibility of that force within the existing well geometry were used to predict the trajectory of each mill. A method was developed to calculate and plot the paths traversed by each mill and the width of the window subsequently cut by trimill assembly moving down the face of the whipstock.

Results obtained from the simulator, for selected cases of tool geometries, hole sizes and resistance to sidetracking, indicate an overall dropping tendency of the mill assembly and no tendency to prematurely leave the face of the whipstock. Therefore premature departure of the trimill assembly from the whipstock is unlikely to be caused by BHA design but may be related to some other factor such as the interaction of the mill profile with the casing wall.
Further, a method was developed to calculate the radius of curvature for a specific size pipe run in the predicted trajectory for a sidetracked borehole, based on pipe diameter and wellbore geometry. The curvature was expressed as dogleg severity in degrees of inclination change per 100 ft and provides a basis for determining whether the sidetracked borehole is suitable for its intended purpose.
1. INTRODUCTION

1.1 Directional Drilling

The process of directing the wellbore along some trajectory to a predetermined target is termed as Directional Drilling.

1.1.1 Reasons for Directional Drilling

Directional wells play an important role in many field development strategies. The early drilling of directional wells was clearly motivated by economics. In a number of cases, legal restrictions in developing the fields discovered beneath population centers or lakes used for drinking water purposes, prohibit the drilling of vertical wells and the only way to develop these kind of fields has been to use a drilling pad and drill directionally. Severe topographical features such as mountains can prohibit building a surface location and drilling a near vertical well. Horizontal wells have helped increase the production rates thereby increasing the recovery from existing fields. Directional drilling has increased with the increase of field developments in deep waters, remote locations, hostile environments and deeper producing zones.

1.1.2 Sidetracking

Sidetracking out of an existing wellbore is one specific application of directional drilling. Sidetracking is typically done to bypass an obstruction (fish) in the original borehole to reuse existing well or to explore for additional producing horizons in adjacent sectors of the field. Nowadays, sidetracking is also done to develop multiple wells from the existing borehole for more economically developing fields, especially in offshore environments.

1.2 Cased Hole Sidetrack Operations

Cased hole sidetracks are the specific focus of this research. These sidetracks involve
deviating the well trajectory from an existing cased wellbore at a pre-decided depth below the surface or below the sea floor in an offshore environment. The deviation or kick-off may be performed by using either a whipstock and a mill assembly or a section mill followed by a bent-sub and a mud-motor assembly.

### 1.2.1 Cased-hole Whipstocks

A typical whipstock is an inclined ramp, usually having an inclination of two to three degrees from the axis of the well that can be permanently or temporarily set inside the existing casing. A bottom hole assembly having a mill attached to its lower end rides on this inclined ramp to deviate the new well trajectory from the existing one as the whipstock forces the mill to the side, cutting through the casing. This procedure is defined as “sidetracking”. The point at which the well trajectory is deviated is called the “kick-off” point and the opening cut through the casing is called a “window”.

Smith Services manufactures a special whipstock and a milling assembly called the “Trackmaster”. It consists of a multi-ramped whipstock, having different ramp inclinations, instead of a single inclination ramp, and a trimill assembly consisting of three mills of same diameter at specified distances from each other. As shown in Fig 1.1, the first fifteen-degree ramp is at the beginning of the whipstock, followed by the straight ramp (zero-degree inclination). After the straight ramp, is the first three-degree ramp followed by the second-fifteen degree ramp and finally the last three-degree ramp, which continues until the end of the inclined face of the whipstock also referred to herein as the whipface. The trimill assembly rides on the whipface creating a lengthened hole in the casing called the ‘window’, and progressing into the surrounding cement and rock formation, resulting in a deviated well trajectory of approximately three-degrees. The lowermost mill is called the lead mill (LM).
Placed above the LM is the follow mill (FM) and the top most mill is called the dress mill (DM). All mills are designed to cut both the casing and the rock. The FM and DM are intended to help extend the window length and subsequently increase the length of the major axis of the elliptical sidetracked borehole. The mills are either dressed with high-grade tungsten carbide cutting material or polycrystalline diamond inserts as cutters. For the mills to preferentially cut the casing and not the whipstock, the whipface is made of hardened steel.

The sidetracking operation using the Trackmaster system generally consists of the trimill assembly traversing the face of the multi-ramped whipstock, milling a window in the casing and deviating the wellbore path until the DM reaches the end of the inclined ramps on the whipstock. The trimill assembly is then pulled back to surface.

Fig 1.1 Trackmaster System
1.3 Problem Statement

The Trackmaster system is designed to develop a sidetracked wellbore with an inclination of two to three degrees during the sidetracking operation explained above. In this procedure, the trimill assembly is assumed to mill a window in the casing and subsequently deviate the well trajectory as it progresses along the multi-ramped face of the whipstock. In certain sidetracking instances in the field, Smith Services found that the LM leaves the whipface and moves entirely into the formation rather than following the face of the whipstock for its entire length. In other cases, the lead mill begins cutting downward along the casing as soon as it reaches the end of the whipstock face. These scenarios raise the question of the actual trajectory and the resulting curvature of the sidetracked hole. This increased inclination or the curvature of the deviated trajectory, in the sidetracked section, increases the potential of downhole tubulars failing due to excessive stress or fatigue or sticking in this particular section.

1.4 Project Objectives

The primary objective of this research is to predict the trajectory and the resulting dogleg severity of the sidetracked borehole based on the tendency of the trimill assembly to build, hold or drop inclination, as it progresses on the multi-ramped face of the whipstock. The project also involves the calculation of the casing window width and height and the length of the major axis of the elliptical borehole created by the trimill assembly. The trajectory is to be predicted until the DM reaches the end of the inclined face of the whipstock, at which time the sidetracking process is stopped, and the tool pulled out of the borehole.
1.5 Research Plan

The plan for accomplishing the project objectives was:

1. To develop a computer program, which would act as a tool to help visualize and calculate the wellbore trajectory and geometry in 2-D and the window width milled in the casing, at discrete increments of the trimill assembly following the multi-ramped face of the whipstock. The program attempts to predict the paths traversed and the window widths cut by each mill, assuming the trimill assembly to be a rigid body.

2. To apply an analytical theory that models the behavior of the trimill assembly as if it were a two stabilizer directional drilling assembly. The model should consider the bending of the body (mandrel) of the trimill assembly and the bending of the drill string above it, as it rides on the multi-ramped whipface, cutting the casing and deviating the borehole.

3. To further develop the analytical model to predict the side forces developed on each of the three mills during the sidetrack operation.

4. To define a set of rules governing the progress of each of the mills into or away from the rock.

5. To determine the validity criteria for the calculated position of each of the mills in reference to the forces developed on each mill and its relative position with respect to the whipface and the borehole created by the preceding mill.

6. To develop a computer program, which links the geometric calculations of the trajectory and the window width cut by each of the three mills to the analytical
model, and the validity criteria to predict the position of each of the three mills at discrete increments of depths as the sidetrack operation is conducted.

7. To conduct a sensitivity analysis of the predicted wellbore geometry and trajectory, and the subsequent window width milled in the casing with consideration of the specific input parameters representing operational conditions such as the tool geometry.

8. To calculate the dogleg severity (DLS) of the sidetracked wellbore predicted by the computer program for a given size tubular to be run through the sidetrack.

1.6 Overview Of the Report

This chapter gives an introduction to sidetracking with whipstocks inside casing and an overview of the project and this report.

Chapter 2 reviews the literature on the methods used for trajectory predictions of deviated wells. It also reviews the literature related to dogleg severity calculations. It specifically describes the Jiazhi model used in the development of the semi-analytical simulator used to perform the trajectory predictions required.

Chapter 3 describes the computer program, and the calculations involved for visualization of the predicted wellbore geometry, trajectory and window width cut by each mill as the trimill assembly progresses along the multi-ramped whipface, assuming the trimill assembly to be a rigid or stiff body with no bending in the tool.

Chapter 4 explains the working of the semi-analytical simulator that was developed for predicting the trajectory and the window width cut by each mill, using the Jiazhi model. It reviews the assumptions made for the development of the Simulator. It also describes the rules for calculating the position of each of the three mills, in reference to the calculated
forces on them. Further it explains the criteria for validating the position of each mill with respect to the forces developed on it and its position relative to the whipface and the formation, in order to get an acceptable solution for the position at each incremental step as the trimill assembly progresses on the multi-ramped whipface.

Chapter 5 compares the trajectories and the window width predictions in a sensitivity analysis to changing selected input parameters of the simulator.

Chapter 6 describes the dogleg severity calculations, the assumptions necessary to make the calculations, and the implications to sidetrack usability.

Chapter 7 summarizes the overall study with conclusions and recommendations for future research.
2. LITERATURE REVIEW

2.1 Introduction

The sidetracking operation using the Trackmaster system, manufactured by Smith Services, consists of lowering the Trackmaster system in the borehole, orienting it, setting the multi-ramped whipstock and separating the trimill from the whipstock. This allows a deviation of the existing borehole using the trimill assembly, to cut through the casing into the surrounding rock\(^4,6\). The Trackmaster system is designed to enlarge the hole window created in the casing\(^5,6\).

In some of the field operations, Smith Services observed evidence of the trimill assembly prematurely leaving the face of the whipstock. This scenario raises the question of the trajectory and the resulting borehole curvature. This increased inclination and curvature of the sidetrack trajectory increases the potential of downhole tubulars failing due to excessive stress, fatigue, or sticking in this particular section.

Hence the focus of this study was to predict the trajectory path, cut by the mill assembly, by analyzing the tendency of the bottomhole assembly (BHA) to build, hold, or drop the angle of inclination. The other major task of this project was to measure the curvature or the dogleg severity (DLS) of the tubular that would be run in the predicted sidetracked section.

This chapter discusses the knowledge that currently exists in the literature regarding prediction of directional drilling trajectories and calculation of the dogleg severity.

2.2 Mathematical Models

There are four basic mathematical models used for bottomhole assembly analysis; the analytical model, finite element model, finite difference model and weighted residuals\(^8\). The
finite element method\textsuperscript{9,14} is a well-established numerical method used in mechanics and structural engineering\textsuperscript{8}. The finite difference models\textsuperscript{10} use intense numerical methods to solve differential equations that model the bottom hole assembly\textsuperscript{8}. The weighted residual method\textsuperscript{8,11} solves differential equations especially for non-linear equations\textsuperscript{8}. Whereas for the analytical models\textsuperscript{1,2,7,13}, the drill string displacements and forces are expressed in analytical form.

There are two basic analytical models used for BHA analysis for trajectory predictions; the Lubinski model\textsuperscript{3} and the Jiazhi model\textsuperscript{1}. Both of them are static models. The Jiazhi model\textsuperscript{1} was used extensively to predict the behavior of BHA’s having multiple stabilizers. As the sidetracking BHA consists of three mills, the Jiazhi model was used for the BHA analysis, assuming the mills act as stabilizers.

### 2.3 Jiazhi Model

The BHA that the Jiazhi model\textsuperscript{1} analyzes might consist of drill collars alone or a combination of drill collars and stabilizers or drill collars with a mud motor. The Jiazhi model can be used to calculate the force and its direction at the bit for a given arrangement and placement of stabilizers in the BHA. Therefore the BHA analysis, done by the model, can be used to predict the tendency of that BHA to increase, decrease, or hold the borehole inclination, based on the magnitude and direction of the side force calculated to be acting on the bit.

The model calculates the length of tangency and moments developed on the stabilizers which are used for calculating forces that act on the bit, as a function of the clearance between the stabilizers and the borehole wall, and the arrangement of stabilizers, assuming that the stabilizers contact the low side of the inclined borehole. The parameters used to predict the force and their directions are
1. Weight on bit (WOB)
2. Weight of the drill string between the stabilizers
3. The overall inclination angle of the borehole
4. The length and the moment of inertia, I, of the drill string between the stabilizers
5. The clearance between the stabilizers and the borehole wall previously cut by the bit.

2.3.1 Background (Timoshenko Approach)

The Jiazh model uses the Timoshenko approach to model an axially loaded indeterminate beam, whose supports are eccentric (supports which are not in the same horizontal line). There are two methods in the structural approach for analyzing beams, which are the consistent deformation and slope method and the deflection method. The Timoshenko approach uses the slope and deflection method for the beam analysis.

2.3.2 Indeterminate Beams

Beams for which the number of unknowns to be calculated exceeds the number of fundamental static force and moment equations are described as indeterminate beams. Fig 2.1 shows a simply supported beam having four supports and a uniformly distributed load (UDL) acting on it. There are four unknowns for the beam, F1, F2, F3 and F4, and the equations used to solve for those unknowns are ΣFx = 0, ΣFy = 0, ΣM = 0. As there would only be three equations from the conditions described above, the four unknowns are not solvable, requiring that the problem be addressed with indeterminate beam analysis.

2.3.3 Timoshenko Approach for Analyzing Axially Loaded Indeterminate Beams

The Timoshenko method calculates the internal moments developed at the intermediate supports of the beam using the slope and deflection method.

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Fig 2.1: Figure Illustrating Indeterminacy

Fig 2.2 shows the deflection of the beam due to the uniformly distributed load (UDL) on it, in absence of the intermediate supports.

Fig 2.2: Figure Illustrating Deflection due to UDL

Similarly, fig 2.3 shows the deflection of the beam due to the axial load P, in absence of the intermediate supports. The two individual deflections are summed by the ‘principle of superposition’ to determine the total deflection.

Now if two or more supports were added in the beam shown in fig 2.2 and 2.3, internal moments are developed at the supports due to the added rigid supports as shown in fig 2.4.
The resultant internal moments are then calculated from the continuity equation described below. Referring to fig 2.5, the continuity at support n, given by

$$\theta_n = -\theta_n'$$

(2.1)

Where, $\theta_n$ is the slope of the beam to the left of support n, and $\theta_n'$ is the slope of the beam to the right of support n.

**2.3.4 Working of the Jiazhi Model**

The Jiazhi model uses the Timoshenko method of beam analysis to calculate the moments developed on the stabilizers and the length of tangency as given below. The analytical
solution for calculating the length of tangency and the moments developed on the stabilizers, differ with the number of stabilizers in the bottomhole assembly, which is considered for analysis.

![Fig 2.5: Figure Illustrating Calculation of Internal Moments](image)

### 2.3.4.1 Assumptions for the Jiazhi Model

1. The borehole is straight and inclined at an angle $\alpha$ from vertical.
2. The drill string does not contact the borehole wall in-between the stabilizers.
3. There is only one contact point of the drill string with the borehole wall, above the last stabilizer, called the tangency point.
4. The stabilizers make a point contact with the wall of the casing.

### 2.3.4.2 Length of Tangency and Moments Calculations for Two-Stabilizer Bottomhole Assembly

Fig 2.6 shows a BHA with two stabilizers, $S_1$ and $S_2$ respectively. The borehole is assumed to be straight and inclined at an angle $\alpha$. The model calculates the length of tangency, which is the length of the section of the drill string from the deepest contact with the borehole wall to the top of the stabilizers.

Fig 2.7 describes the same BHA shown in Fig 2.6, with variables used for the calculating the moments and the length of tangency, $L_T$, using the Jiazhi model\(^1\).
Fig 2.6: BHA with Two Stabilizers

Fig 2.7: Analysis of BHA with Two Stabilizers
The length of tangency (\( L_T \)) is calculated as follows. There are basically three unknowns in the above system, \( M_1 \), \( M_2 \) and \( L_T \). These unknowns are calculated by the three boundary conditions given as.

The continuity equation at stabilizer 1 is,

\[
\theta_1 = -\theta_1'
\]

(2.2)

Which is expressed as,

\[
2M_1 \left( V(u_1) + \frac{l_2 I_1}{l_1 I_2} V(u_2) \right) + M_2 \frac{l_2 I_1}{l_1 I_2} W(u_2) = -\frac{q_1 l_1^2}{4} X(u_1) - \frac{q_2 l_2^2}{4} \frac{l_2 I_1}{l_1 I_2} X(u_2) + 6EI_1 \left( \frac{e_1 - e_2}{l_1^2} + \frac{e_1 - e_2}{l_2 I_2} \right)
\]

(2.2a)

The continuity at stabilizer 2 is given as,

\[
\theta_2 = -\theta_2'
\]

(2.3)

\[
M_1 W(u_2) + 2M_2 \left( V(u_2) + \frac{L_T I_2}{l_2 I_3} V(u_3) \right) = -\frac{q_2 l_2^2}{4} X(u_2) - \frac{q_3 l_2^2}{4} \frac{L_T I_2}{l_2 I_3} X(u_3) - 6EI_2 \left( \frac{e_1 - e_2}{l_2^2} + \frac{e_3 - e_2}{l_2 L_T} \right)
\]

(2.3a)

Slope of the drill string at the tangency point is zero, given as

\[
\theta_T = \frac{q_3 L_T^3}{24EI_3} X(u_3) + \frac{M_2 L_T}{6EI_3} W(u_3) - \frac{e_3 - e_2}{L_T} = 0
\]

(2.4)

Solving equations (2.2a) and (2.3a) for \( M_1 \) and \( M_2 \) we get,

\[
M_1 = -\frac{q_1 l_1^2}{2} X(u_1) - \frac{q_2 l_2^2}{2} \frac{l_1}{l_1 I_2} X(u_2) + \frac{q_2 l_2^2}{4} \frac{g}{X(u_2)} + \frac{q_3 l_2^3}{4} \frac{l_2 g}{X(u_3)} + 6EI_2 \left( \frac{e_1 - e_2}{l_2^2} + \frac{e_3 - e_2}{l_2 L_T} \right) + \frac{12EI_1 j}{l_1} \left( \frac{e_1 - e_2}{l_1} + \frac{e_1 - e_2}{l_2} \right)
\]

(2.5)
\[ M_2 = \left[ -\frac{q_1 l_1^2}{2} X(u_2) - \frac{q_2 l_2^3 I_3}{4 I_2 l_1} X(u_2) + \frac{6 E I_1}{l_1} \left( \frac{e_1}{l_1} + \frac{e_1 - e_2}{l_2} \right) - 2 M_1 h \right] / g \]  \hspace{1cm} (2.6)

Where,

\[ h = \left( V(u_1) + \frac{l_2 I_1}{l_1 I_2} V(u_2) \right) \]  \hspace{1cm} (2.7)

\[ g = \frac{l_2 I_1}{l_1 I_2} W(u_2) \]  \hspace{1cm} (2.8)

\[ j = \left( V(u_2) + \frac{L T I_2}{l_2 I_3} V(u_3) \right) \]  \hspace{1cm} (2.9)

\[ q_i = w_i * B_c * \sin(\alpha) \]  \hspace{1cm} (2.10)

\[ X(u_i) = \frac{3(\tan u_i - u_i)}{u_i^3} \]  \hspace{1cm} (2.11)

\[ V(u_i) = \frac{3}{2u_i} \left( \frac{1}{2u_i} - \frac{1}{\tan(2u_i)} \right) \]  \hspace{1cm} (2.12)

\[ W(u_i) = \frac{3}{u_i} \left( \frac{1}{\sin(2u_i)} - \frac{1}{2u_i} \right) \]  \hspace{1cm} (2.13)

\[ u_i = \frac{l_i}{2} \sqrt{\frac{P_i}{E I_i}} \]  \hspace{1cm} (2.14)

The length of tangency is calculated by substituting the \( M_2 \) in equation (2.4) given by

\[ L_T^4 = \frac{24 E I_3 (e_3 - e_2)}{q_3 X(u_3)} - \frac{4 M_2}{q_3} \left( \frac{W(u_3)}{X(u_3)} \right) L_T^2 \]  \hspace{1cm} (2.15)

The eccentricities \( e_1, e_2 \) and \( e_3 \) are the horizontal distances between the centers of the stabilizers and the center of the bit.
The length of tangency, $L_T$, is calculated using an iteration method as follows:

1. A length of tangency $L_T$ is assumed, typically with a starting value of about 45 ft, and $M_1$ is calculated using equation (2.5), $M_2$ is calculated using equation (2.6).
2. From the calculated $M_2$ value, $L_T$ is re-calculated using equation (2.15).
3. An average value of the assumed and calculated $L_T$ is taken as the next guess, and the process returns to step 1 until the assumed $L_T$ in step 1 matches with the calculated $L_T$ in step 3.

### 2.3.4.3 Length of Tangency and Moments Calculations for One-Stabilizer BHA

This solution models a BHA with one-stabilizer. There are only two unknowns for this system; the moment developed at stabilizer 1 and the length of tangency, $L_T$. The conditions of the two equations used for the calculation of the unknowns are: that the slope at the tangency point of the drill string with the casing is zero and the continuity condition at stabilizer 1. The equation for calculating the developed moment at stabilizer 1 is

$$M_1 = \left[ -\frac{g_1 I_1}{4} X(u_1) - \frac{g_2 L_T^3}{4I_2} l_1 j X(u_2) + \frac{6EI_1}{l_1} \left( \frac{e_1}{l_1} + \frac{6EI_1}{l_1} \left( \frac{e_1 - e_2}{L_T} \right) \right) \right]$$

$$2 \left( V(u_1) + \frac{L_T I_1}{l_1 I_2} V(u_2) \right)$$

(2.16)

The length of tangency is calculated from the equation

$$L_T^4 = \frac{24EI_2 (e_2 - e_1)}{q_2 X(u_2)} - \frac{4M_1}{q_2} \left( \frac{W(u_2)}{X(u_2)} \right) L_T^2$$

(2.17)

All the variables in the above equations are the same as used earlier.

Iteration procedure used for tangency length calculation:
1. A length of tangency $L_T$ is assumed, typically with a starting value of about 45 ft and $M_1$ is calculated using equation (2.16).

2. From $M_1$, $L_T$ is recalculated using equation (2.17), and the iterations return to step 1 taking an average value of the assumed and calculated $L_T$, as the next guess until the assumed $L_T$ in step 1 matches the calculated value of $L_T$ in step 2.

2.3.4.4 Length of Tangency Calculation for No-Stabilizer Bottomhole Assembly (Slick BHA)

As the length of tangency, $L_T$, is the only unknown, it is calculated as

$$L_T^4 = \frac{24El_e}{q_1X_1}$$  \hspace{1cm} (2.18)

2.4 Limitations of the Jiazhi Model

1. The values of $X(u)$, $V(u)$ and $W(u)$ (addressed as the transcendental functions) increase infinitely as $u$ approaches $\pi/2$ and $P$ approaches $P_{cr}$. $P_{cr}$ is the critical value of the applied axial load, which the beam can sustain, without failure. To understand this phenomenon, an approximate equation defining the transcendental functions would be $\frac{1}{1 - P/P_{cr}}$. That is, the model cannot calculate the tangency length for large deflections.

2. The model cannot account for the drill string making contact with a wall between the stabilizers.

3. The model was not intended to account for borehole curvatures.

2.5 Dogleg Severity Predictions

The dogleg severity (DLS) is a measure of the change in borehole direction between two survey stations expressed in degrees of change per 100 ft. The task for the current project
was to find a method to calculate the curvature of the downhole tubular, based on its size, which would be run in the predicted trajectory for a sidetracked borehole. This curvature calculation is solely dependent on the contact points of the tubular with borehole wall. These contact points, in effect, are based on the magnitude and direction of the axial loads acting on tubulars as depicted by Lubinski\textsuperscript{3}.

Lubinski’s work\textsuperscript{3} depicts calculations related to the maximum tolerable curvature or DLS that a particular size pipe can sustain without failure as a function of the tubular clearance with the borehole wall, contact or tangency points of the tubular with the borehole wall, and the magnitude of the reversible bending stress developed due the axial load acting on the tubular. He presented graphs that predict the maximum tolerable DLS, which a particular size pipe can sustain relative to the magnitude and direction of the axial load on the pipe\textsuperscript{3}.

According to Lubinski, for a given pipe curvature, pipes in tension undergoing reversible bending stresses due to rotation, are more susceptible to failure than pipes in compression. The method developed by Lubinski\textsuperscript{3} for calculating the maximum tolerable DLS considers

1. Clearance between the tubular and the borehole wall.
2. The geometry of the anticipated borehole, that is, whether the change of borehole inclination is gradual or abrupt.
3. The magnitude of the reversible bending stress, which is developed due to the rotation of the axially loaded tubular.

The method states that tubulars in tension would follow the borehole curvature, and lie on the low side of the borehole. The tolerable DLS for tubulars under tension are thus
calculated depending on the curvature calculations relative to the reversible bending stress, which the tubular can sustain under the given tensile load. But for tubulars under compression the contact or the tangency points have to be determined in order to calculate the maximum tolerable DLS.

Hence, from all the above discussion it is clear that DLS calculations require knowledge regarding the contact and tangency point of the tubular with the borehole wall. The method used for predicting the DLS of the predicted sidetracked section assumes contact points of the tubular with the casing and the borehole wall as described in chapter 6.
3. TRAJECTORY PREDICTIONS FOR TRIMILL ASSEMBLY AS A RIGID BODY

3.1 Introduction

The primary goals of this project were to determine the trajectory and the window geometry created during a casing sidetrack using a multi-ramped whipstock and trimill assembly. A necessary first step for achieving these goals was to create a method for calculating, recording and plotting the path traversed by each mill of the trimill assembly, knowing the position of the center of each mill. A simple geometry-based prediction for the trajectory of the trimill assembly was also performed to provide data for plotting a trajectory and to give a preliminary idea of what the trajectory might be and how the whipstock and the trimill geometries influence it.

This chapter describes the logic and the methods implemented in the computer program, developed to visualize the path traversed by each of the mills and the subsequent window profile cut by trimill assembly.

3.2 Overview of the Trajectory Prediction Method for a Rigid Trimill Assembly

The computer program assumes the trimill assembly to be a rigid body with the mills represented by discs of no finite lengths whose diameter is equal to the actual mill diameter. The whipstock is represented by a path that exactly matches the angles and lengths of the face of the whipstock. The program first calculates the contact point of the LM (lead mill) with the upper inside wall of the casing. For discrete increments of 0.1” of the LM traveling along the face of the multi-ramped whipstock, the program checks whether the FM (follow mill), the DM (dress mill), or top of the mill assembly contact the inside wall of the casing or the face of the whipstock. The inclination of the trimill assembly above the lead mill (LM) is then based on the points of contact on the whipstock or lower inside wall of the casing that results in the lowest
value of inclination. The position of each mill is then determined and the program plots the paths and the subsequent window widths cut by each mill, relative to the lower inside wall of the casing, as described explicitly in this chapter. The final trajectory and window geometry is then based on the maximum extent of the travel of each mill at each position along the path.

3.3 Design of the Computer Program for Rigid Body Trajectory Predictions

The following subsections describe the structure and working of the computer program. Also, they explain the calculations done by the program for plotting the paths and the window widths cut by each mill of the Trimill assembly.

3.3.1 Steps Followed by the Computer Program

Fig 3.1 shows the structure of the entire program. The steps denoted from 1 to 13 in fig 3.1 are described below.

1. START: Given the inputs to the program as described in section 3.3.8, the program first calculates the contact point of the LM with the upper inside wall of the casing, as explained in section 3.3.3.

2. After the calculation of the contact point of the LM with inside upper wall of the casing, for each increment of 0.1” of the LM on the inclined face of the whipstock, the program checks whether the FM touches the upper inside wall of the casing as described in section 3.3.4. If the program does not indicate the FM to touch the upper inside wall of the casing, the program goes to step 3, else it goes to step 4.

3. In this step, the program indicates only the LM is contacting the upper inside wall of the casing and eventually cutting it. The program calculates and plots the path traversed and the window width cut by the LM as explained in sections 3.3.5 and 3.3.7.
4. The program enters step 4 after, the FM contacts the upper inside wall of the casing. In this step, the program checks whether the distance from the lower edge of the FM to the inside lower wall of the casing, is less than the distance from the adjacent face of the whipstock to the casing wall. If the program indicates that the lower edge of the FM overlaps with of the whipstock then the position of the FM is corrected to place its edge on the face of the whipstock and the program proceeds to step 5a, otherwise the program goes to step 5a.

5. In this step the program calculates a new tool inclination angle $\gamma_1$ as described in section 3.3.6.2 and proceeds to step 6.

5a. In this step, the program checks whether the mandrel or the body of the trimill assembly contacts the face of the whipstock as explained in section 3.3.6.1. If the program indicates the mandrel contacts the face of the whipstock, it makes the necessary corrections and records a new tool inclination angle if necessary and proceeds to step 6, else it records the old tool inclination angle $\gamma$ and goes to step 6. Then a correction is applied to the position of the FM or DM as explained in section 3.3.6.2, the tool inclination would always be less when the mandrel contacts the whipface as explained in section 3.3.6.1.

6. In this step, the program checks whether the DM contacts the inside upper wall of the casing as explained in section 3.3.4. If the program indicates the DM to contact the upper inside wall of the casing, it proceeds to step 8, else the program proceeds to step 7.

7. In this step, the program calculates and plots the trajectory and window widths cut by the LM and FM as discussed in section 3.3.5 and 3.3.7. After executing this step, the program increments the position of the LM by 0.1” along the multi-ramped face of the
Start (Inputs)

Calculate contact point of the LM with the casing (1)

Increment LM by 0.1”

Plot trajectory and window width of LM (3)

No

Check if FM contacts the casing (2)

Yes

If \( [X_{DM} - OD_{DM}/2] < X_{whip} \) then correction
\( [X_{DM} - OD_{DM}/2] = X_{whip} \) (8)

No

Step 4 (9a)

Yes

Calculate ‘\( \gamma_2 \)’ (9)

Check if mandrel contacts the whipface If yes recalculate \( \gamma \) (10)

No

Calculate new ‘\( \gamma_1 \)’ (5)

Check if DM contacts the casing (6)

No

Plot trajectory and window width of LM, FM and DM (12)

Yes

Increment LM by 0.1”

No

Check if mandrel contacts the whipface (5a). If yes recalculate \( \gamma \).

Plot trajectory and window width of FM and LM (7)

Increment LM by 0.1”

If \( [X_{FM} - OD_{FM}/2] < X_{whip} \) then correction
\( [X_{FM} - OD_{FM}] = X_{whip} \) (4)

Check if mandrel contacts the whipface If yes recalculate \( \gamma \) (10)

Check if DM contacts the casing (6)

No

Plot trajectory and window width of LM, FM and DM (12)

Calculate new ‘\( \gamma_1 \)’ (11)

Fig 3.1: Structure of the Rigid Body Program
8. whipstock, in the vertical direction and returns to step 4. The program follows the same loop until it indicates the DM to touch the inside upper wall of the casing.

9. This step is executed after the program indicates that the DM contacts the inside upper wall of the casing. In this step the program checks whether the position of the DM needs to be corrected, similar to step 4. If the program does not indicate a necessary change in the DM position, the program proceeds to step 9a, else it corrects the position of the DM and goes to step 9.

10. In this step, the program calculates and records a new tool inclination $\gamma_2$ and proceeds to step 10 as explained in section 3.3.6.2.

9a. In this step, the program checks whether the follow mill position needs to be corrected as explained in step 4. If a correction for the position of the follow mill is made, the program calculates a new tool inclination $\gamma_1$.

10. This step checks whether the mandrel of the trimill assembly contacts the face of the whipstock as explained in section 3.3.6.1. If the program does indicate the mandrel to contact the face of the whipstock, it records a new tool inclination angle $\gamma$ assuming the mandrel to contact the whipface and proceeds to step 12, otherwise it proceeds to step 12 with the tool inclination angle recorded earlier.

11. In this step the program calculates a new tool inclination angle $\gamma_1$ as described in section 3.3.6.2 and proceeds to step 6.

12. In this step, the program calculates and plots the paths and the window widths cut by each mill with respect to the recorded tool inclination as described in section 3.3.5 and 3.3.7, respectively. Once the program plots the trajectories and the window widths cut by
each mill, it increments the position of the LM by 0.1” on the face of the multi-ramped whipstock and proceeds to step 8, till the end of the program.

3.3.2 Assumptions Made in the Rigid Body Program:

1. The trimill assembly is assumed to be a rigid body.

2. The follow mill and the dress mill start cutting the casing as soon as they contact the inside upper wall of the casing.

3. The upper end of the trimill assembly contacts the inside lower wall of the casing, until it is lifted off of the casing lower wall because of the FM or DM contacting the face of the whipstock.

4. The casing diameter is considered as the average of the ID and OD of the casing.

5. The mills are assumed to have no length, i.e. the mills are assumed to be represented by thin circular discs.

6. The lead mill was assumed to proceed in the direction established by the last three-degree ramp after it passes the end of the inclined face of the whipstock.

7. The well and the existing casing are vertical.

3.3.3 Calculating the Contact Point of the Lead Mill with the Casing

Referring to fig 3.2, the task is to calculate the depth of the lead mill (LM) below the top of the whipstock, where it first contacts the upper inside wall of the casing. This calculation is done assuming the pin end of the trimill assembly contacts the inside lower wall of the casing causing the trimill assembly is inclined at an angle $\gamma$, as shown in fig 3.2.

As the trimill assembly is tilted at an angle, $\gamma$, the plane of the circle representing the LM makes an angle $\gamma$ with the horizontal as shown in fig 3.2. Hence the effective diameter of the LM reduces to $[\text{OD}_{\text{mill}} \cos (\gamma)]$. Now, as the LM rides on the first fifteen-degree ramp, $\gamma$ increases. To
find the contact point at which \( \text{OD}_{\text{mill}} \cos (\gamma) \) equals the clearance between the whipface and the inside upper wall of the casing the program uses a substitution technique as described below.

Fig 3.2: Illustrating Calculation for Contact Point of Lead Mill with Inside Upper Wall of Casing
Let,

\[ k_1 = \frac{(OD_{mill} - OD_{mandrel})}{2} \]  \hspace{1cm} (3.1)

If ‘γ’ is the angle made by the trimill assembly with the casing inner wall (i.e. with the vertical), then

\[ \sin(\gamma) = \frac{(X_{whip} + k_1)}{L} \]  \hspace{1cm} (3.2)

Where ‘L’ is the length of the trimill assembly and ‘X_{whip}’ as defined in fig 1.

Therefore,

\[ X_{whip} = (L \sin(\gamma)) - k_1 \]  \hspace{1cm} (3.3)

Let, \[ X' = OD_{mill} \cos(\gamma) \] \hspace{1cm} (3.4)

Hence,

\[ X_{whip} + X' = ID_{csg} \] \hspace{1cm} (3.5)

Substituting (3.3) and (3.4) in (3.5) we get,

\[ L \sin(\gamma) + OD_{mill} \cos(\gamma) = ID_{csg} + k_1 \] \hspace{1cm} (3.6)

The solution of equation (3.6) is given as

\[ \gamma = \tan^{-1} \left( \frac{L}{OD_{mill}} \right) + \cos^{-1} \left( \frac{ID_{csg} + k_1}{\sqrt{(ID_{mill})^2 + L^2}} \right) \] \hspace{1cm} (3.7)

Knowing \[ \gamma, [OD_{mill} \cos(\gamma)] \] can be calculated and the position \[ y \] below the top of the whip at which the LM contacts the casing can be found, \[ X_{whip} \] shown in fig 3.2 can be calculated by solving equation (3.3) with \[ \gamma \], as the top of the whipstock, \[ y = 0 \], is defined by point J, where the width of the whipstock is \[ T \] as shown in fig 3.2.
As the whipstock is inclined at an angle \( \beta \) with the vertical, the contact point \( y \), below the top of the whipstock is calculated as,

\[
y = \frac{(X_{\text{whip}} - T)}{\tan(15 + \beta)}
\]

(3.7)

3.3.4 Calculation of the Contact Point of the Follow Mill and the Dress Mill with the Casing

As shown in fig 3.2, let \( l_1 \) be the length between the LM and the FM. Let \( \gamma \) be the tool (trimill assembly) inclination angle assuming the top of the trimill assembly touching the casing inner wall. At each \( y \) increment of 0.1”, of the LM, along the inclined face of the whipstock, the program calculates \( X_{LM} \), the horizontal position of the center of the LM relative to the inside upper wall of the casing assuming \( \beta \) is zero, as follows.

Assume the LM following the first fifteen-degree ramp. The thickness of the whipface, for the fifteen-degree ramp, at any \( y \) position of the LM would be given by

\[
X_{\text{whip}} = (y \times \tan(15)) + T \tag{3.8}
\]

The following logic is used to determine the tool position when the mill first contacts the upper inside wall of the casing, the tool inclination angle \( \gamma \), for the LM being on any face of the whipstock, is given by

\[
\gamma = \sin^{-1}[(X_{\text{whip}} + k1)/(A + B)]
\]

(3.9)

The tool sections A and B are shown in fig 3.6. [(A+B) is the entire tool length].

As the length \( 'l_1' \), between the LM and the FM is known, distance ‘c’, as shown in fig 3.2 is calculated by

\[
c = l_1 \sin(\gamma) \tag{3.10}
\]

Now, ‘\( X_{LM} \)’ (distance between the center of the LM and the inside lower wall of the casing) is given by,
\[ X_{LM} = X_{whip} + [(OD_{mill} / 2) \cos(\gamma)] \]  

Hence the distance between the center of the FM and the inside lower wall of the casing is given by,

\[ X_{FM} = X_{LM} - c \]  

(3.12)

As \( \gamma \) is the average inclination of mill assembly, the effective diameter of the FM, \( Eff_{FM} \) (as shown in fig 3.2), would be given by

\[ Eff_{FM} = OD_{mill} \cos(\gamma) \]  

(3.13)

Hence the clearance \( v \), between the outer-edge of the FM and the inside upper wall of the casing as shown in fig 3.2, is given by

\[ v = ID_{csg} - (X_{FM} + \frac{Eff_{FM}}{2}) \]  

(3.14)

The point at which \( v \) becomes negative is the contact point of the FM with the inside upper wall of the casing. The point of initial contact of the DM with the inside upper wall of the casing is determined in a similar manner.

3.3.5 Trajectory Calculations

This section explains the calculations and the method used by the program to plot the position of each mill at increments of 0.1” of the LM along the inclined face of the multi-ramped whipstock. At each increment, below the initial contact point of the LM with the inside upper wall of the casing, the program plots the position of the LM as described below.

The program calculates the position of the LM relative to the inside lower wall of the casing as given by (3.11) above. The input \( y \) to equation (3.8) is the vertical position of the LM relative to the top of the whipstock, which is calculated by the program for each 0.1” increment below the previous position of the LM. As the program is designed to allow the LM to follow the
inclined face of the whipstock, the lower contact point of the LM with the whipstock is at
distance $X_{\text{whip}}$ from the inside lower wall of the casing, which is plotted as an output by the
program for each new mill position, $y$.

As the trimill assembly is tilted at an angle $\gamma$ to the vertical, the upper contact point of
the LM with the casing and/or the formation is above recorded $y$ position by an amount $e_{LM}$, as
shown in fig 3 and given as
\begin{equation}
e_{LM} = OD_{\text{mill}} \times \sin(\gamma)
\end{equation}

Whereas the $x$-position of the upper contact of LM with the casing and/or formation is
given as $[X_{\text{whip}} + Eff_{LM}]$ and the $y$- position of the upper contact of the LM is plotted as, $[y - e_{LM}]$.

If the FM touches the inside upper wall of the casing, then the program starts plotting the
path traversed by the FM using the same concept as for the LM. The distance $X_{FM}$ between the
centers of the FM and the inside lower wall of the casing is calculated as given by equation
(3.12). As the trimill assembly is inclined at an angle $\gamma$ to the vertical, the FM and DM are tilted
at the same angle but with the horizontal. The lower contact point of the FM is plotted as, $[X_{FM}
- (OD_{\text{mill}}/2)\times\cos(\gamma)]$

The Y-position of the lower contact of the FM is calculated as, $[(l_{1}\times\cos(\gamma)]$
The upper contact point of the FM is plotted with the X-position given by, $X_{FM} + \{(OD_{\text{mill}}/2)\times\cos
(\gamma)\}$ and the Y-position given by, $y_{LM} - (l_{1}\times\cos(\gamma) + e_{FM})$, where $e_{LM}$ is calculated in the same
manner as $e_{FM}$. Similar calculations are done for calculating the trajectory of the DM after it
contacts the casing.
3.3.6 Complications Encountered in the Trajectory Calculations

This section describes the complications encountered in the computer program during trajectory and window width predictions.

3.3.6.1 Calculation of $X_{whip}$ on the Multi-ramped Whipstock

As $y$ increases, the program calculates the thickness of the whipface at the new $y$ increment as follows. Consider the first three-degree ramp below the straight ramp. The thickness of the face of the whipstock at the end of the first fifteen-degree ramp is

$$[\tan(15) + T]$$

(3.16)
If D was the length of the first fifteen-degree ramp and E the length of the straight ramp, then the thickness of the whipstock at a position y of the LM, when the lower contact point of the LM is in contact with the face of whipstock in the three-degree ramp, would be

\[ [(y - (D+E)) \tan (3)] \] \hspace{1cm} (3.17)

And the distance of the whipstock from the inside lower wall of the casing would be, y and the thickness of the whipstock from the inside lower wall of the casing would be

\[ X_{whip} = [(\tan (15) + T) + (y - (D+E)) \tan (3)] + y \tan (\beta) \] \hspace{1cm} (3.18)

### 3.3.6.2 Contact Between Body of Assembly and Whipstock

As the LM traverses the face of the multi-ramped whipstock, the upper end of the trimill assembly would eventually reach the top of the whipstock. The angle of inclination, \( \gamma \), of the trimill assembly may then be controlled by the mandrel or body of the trimill assembly contacting the inclined face of the whipstock.

When the mandrel of the trimill assembly touches the whipface, the program calculates a new tool (trimill assembly) inclination angle \( \gamma \) and re-defines or corrects the positions of the FM and DM relative to the newly recorded tool inclination angle.

Considering the assumption that the mills are thin circular discs, the mandrel might touch the whipface at point p shown in fig 3.4, as the LM progresses on the inclined face of the whipstock. The clearance between the mandrel and the whipface is calculated as follows.

Let y be the position of the LM below the top of the whipstock as shown in fig 3.4. The depth of the end of the first fifteen-degree ramp at point P in fig 3.4 was determined in section 3.3.2 and is defined as \( Dt \).
Fig 3.4: Describing Contact of the Mandrel of the Trimill Assembly with the Face of the Whipstock

The vertical distance \((y-D_t)\) between the center of the LM and the mandrel (body) is as shown in fig 3.4. The horizontal distance between the center of the Lm and the point \(p\), \(X_b\), is given by

\[
X_b = (y - D_t) \tan(\gamma)
\]  

(3.19)
The distance of the center of the mandrel from the casing inner wall, ‘\(X_{2b}\)’ is then defined as

\[
X_{2b} = X_{LM} - X_b
\]  

(3.20)

The horizontal distance from the OD of the mandrel to its centerline, at \(y = Dt\) is given by \(X_{3b}\)

Referring to fig 3.4,

\[
X_{3b} = \frac{OD_{mandrel}}{2} \cos(\gamma)
\]  

(3.21)

If \(X_{\text{whip}}\) is the horizontal distance of point \(p\) from the inside lower wall of the casing, then the clearance between point \(p\) and the mandrel is now given as

\[
\text{Clearance} = (X_{2b} - X_{3b}) - X_{\text{whip}}
\]  

(3.22)

When the clearance becomes negative, the mandrel touches point \(p\), and from thereon the angle ‘\(\gamma\)’ (tool inclination angle), is calculated as

\[
\gamma = \tan^{-1}\left(\frac{(X_{LM} - (X_{\text{whip}} + X_{3b}))}{(y - D_t)}\right)
\]  

(3.23)

3.3.6.3 Correction Applied for the FM/DM Contacting the Face of the Whipstock

At each depth increment of 0.1” of the LM, the program calculates the position of the FM and the DM as discussed earlier. If at any point the program calculates that the FM or the DM would be in contact with the whipstock, then the program corrects the calculated position of that particular mill to be at the face of the whipstock. Mathematically, if at \(y_{FM}\) (position of the FM with respect to the top of the Whipstock)

\[
X_{FM} > \{OD_{mill}/2\cos(\gamma)\} < X_{\text{whip}}\), then the correction applied is given by

\[
X_{FM} = \{OD_{mill}/2\cos(\gamma)\} = X_{\text{whip}}
\]

If \(l_1\) is the length between the LM and the FM, then the tool (trimill assembly) inclination angle \(\gamma\) is determined by
\[
\gamma = \sin^{-1}\left(\frac{X_{LM} - X_{FM}}{l_1}\right) = (3.24)
\]

If at a particular step, the DM and the FM simultaneously contact the whipface, then ‘\(\gamma\)’, tool inclination angle, is calculated for both the cases, and the least of the two is considered as the tool inclination angle.

### 3.3.7 Window Width Calculations

In addition to calculating the path taken by the upper and lower edges of each mill, the widths of the window cut in the casing must also be determined. The following logic was used to calculate the width versus depth relative to the top of the whipstock. If \(u\) is the distance between the center of the LM and the center of the casing as shown in fig 5, then \(u\) is defined as

\[
u = X_{LM} - (ID_{csg}/2) = (3.25)
\]

Let ‘\(R\)’ be the radius of the casing,

\[
R = (ID_{csg}/2) = (3.26)
\]

The equation of the circle made by the casing, assuming the center of the casing as the origin \((0,0)\), is

\[
x^2 + z^2 = R^2 = (3.27)
\]

As the LM is tilted at an angle \(\gamma\), the OD of the mill reduces by an amount \(\cos(\gamma)\) given by equation (3.13).

Let \(a = Eff_{LM}\)

and \(b = OD_{mill}\)

In the top view, the mill (LM) represents an ellipse, whose center lies at \([u, 0]\), given by the equation,
\[ \frac{(x-u)^2}{a^2} + \frac{z^2}{b^2} = 1 \]  \hspace{1cm} (3.28)

As the ellipse cuts the circle at two distinct z co-ordinates, with a common x co-ordinate, solving equations (3.27) and (3.208 yield,
As \( x \) cannot be negative, the positive solution in (3.29) is the valid solution for calculating the \( x \) co-ordinate for calculating the point of intersection between the ellipse (LM) and the circle (casing).

As, the width of the window is of our interest, \( z \) can be calculated from equation (3.27), by re-arranging equation (29) as, \( z = \sqrt{r^2 - x^2} \), then, the actual window width cut by the LM is given by

\[
\text{Window width} = 2z \tag{3.30}
\]

The same calculation is done for FM and DM, knowing the \( X_{FM} \) and the \( X_{DM} \) respectively.

As the LM progresses along the multi-ramped whipface, the program calculates the width of the window cut by each mill as a function of the calculated values of \( X_{LM} \), \( X_{FM} \) and \( X_{DM} \) at each incremental depth position of the LM, along the face of the whipstock.

### 3.3.8 Inputs to the Rigid Body Program

The inputs to the Rigid Body Program are as shown in fig 3.6. A list of the input data is given in Table 3.1. All dimensions of length are input in inches. The table includes the input variables used by the program to predict and plot the trajectory and window-width cut by the trimill assembly for the example case used to give the results in the following section.
Table 3.1: Inputs to the Rigid body program

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>ODcsg</td>
<td>9.625</td>
<td>Inches</td>
</tr>
<tr>
<td>IDcsg</td>
<td>8.755</td>
<td>Inches</td>
</tr>
<tr>
<td>OD mill</td>
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<td>Inches</td>
</tr>
<tr>
<td>OD whip</td>
<td>8</td>
<td>Inches</td>
</tr>
<tr>
<td>A</td>
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<td>Inches</td>
</tr>
<tr>
<td>B</td>
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<td>Inches</td>
</tr>
<tr>
<td>D (15 deg)</td>
<td>5.3</td>
<td>Inches</td>
</tr>
<tr>
<td>E (0 deg)</td>
<td>27</td>
<td>Inches</td>
</tr>
<tr>
<td>F (3 deg)</td>
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</tr>
<tr>
<td>G (15 deg)</td>
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<td>Inches</td>
</tr>
<tr>
<td>H (3 deg)</td>
<td>63.8</td>
<td>Inches</td>
</tr>
<tr>
<td>T</td>
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<td>Inches</td>
</tr>
<tr>
<td>S</td>
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</tr>
<tr>
<td>V</td>
<td>25.22</td>
<td>Inches</td>
</tr>
<tr>
<td>LdM</td>
<td>6.9</td>
<td>Inches</td>
</tr>
</tbody>
</table>

Multi-ramped whipstock

Fig 3.6: Trackmaster System
3.3.9 Outputs of the Rigid body Program.

The graphical output of the program is shown in fig 7. It shows a special window width, tapering at the bottom. The trajectory predictions show an elliptical borehole. It shows the path followed by each of the mills as the trimill assembly rides on the multi-ramped whipface in both front (trajectory) and side (window-width) views.

List of output variables are

1. $y_{LM}$: Incremental position of LM in the vertical direction parallel to the axis of the existing well
2. $y_{FM}$: Vertical distance between the LM and FM defined by $y_{LM} - L_1 \cos(\gamma)$, $L_1$ is the length between the LM and FM and $\gamma$ is the tool inclination angle
3. $y_{DM}$: Vertical distance between the LM and DM defined by $y_{LM} - L_2 \cos(\gamma)$, $L_2$ is the length between the LM and DM and $\gamma$ is the tool inclination angle
4. $X_{\text{whip}}$: Distance of the whipface from the lower inside wall of the casing.
5. $X_{LL}$: Position of lower edge of lead mill with the whipstock, equal to $X_{\text{whip}}$
6. $X_{LU}$: Position of the upper edge of the lead mill
7. $X_{FL}$: Position of lower edge of follow mill
8. $X_{FU}$: Position of the upper edge of the follow mill
9. $X_{DL}$: Position of lower edge of dress mill
10. $X_{DU}$: Position of the upper edge of the dress mill
11. $W_{LL}$: Points representing the width cut by the LM, plotted on x direction relative to the y-axis.
12. $W_{LF}$: Points representing the width cut by the FM, plotted in z direction,
13. $W_{LD}$: Positive solution of half the width cut by the DM, plotted in z direction.
3.4 Summary

1. The program calculates, records, and plots the paths traversed and the window widths cut by each mill relative to the position of the center of each mill, thus predicting the trajectory and the borehole geometry created in a casing sidetrack. The prediction is based on assuming the mill to assembly is a rigid body.

2. The program assumes the lead mill to proceed in the direction established by the last three-degree ramp after it passes over the end of the inclined face of the whipstock. The usefulness of the prediction is therefore limited by the validity of this assumption.

3. The results obtained from the program show an extended window length, above the top of the whipstock. They also show that the borehole created by the trimill assembly is elliptical with the long axis in the direction of the sidetrack which causes a less irregular borehole than expected with a single mill on a multi-ramped whipstock.

4. The plotting method developed in this chapter can also be used for trajectory predictions using directional drilling models.

5. The program does not account for the bottomhole assembly contacting the casing inside wall, which would restrict the pin end to contact the casing. Also, a certain weight on bit is applied, during the sidetracking operation, which would make the trimill assembly bend limiting the use of the rigid body program for trajectory predictions.
Note: All dimensions are in inches

Fig 3.7: Result Obtained from the Rigid Body Program
4. THE SEMI-ANALYTICAL SIMULATOR BASED ON THE JIAZHI MODEL.

4.1 Introduction

The computer software simulator created for this project models the trimill assembly and the bottom hole assembly (BHA) above it for predicting the trajectory cut as it rides on the multi-ramped whipstock. The simulator predicts this path by the trimill assembly based on the design of the trimill assembly. The prediction of the trajectory (the path cut) by the trimill assembly was intended to help analyze and understand the angle building behavior of the trimill assembly based on its design.

This chapter illustrates the structure and the logic of the developed simulator for the required trajectory predictions. It illustrates the application of the static Jiazhi model\(^1\) to predict the trajectory cut by the trimill assembly during the sidetracking operation. It explains the equations used for calculating the bit side forces, based on the parameters obtained from the Jiazhi model. This chapter elucidates the prediction of the path cut by each of the mills of the trimill assembly at every incremental depth of the trimill assembly based on the calculated side forces. This chapter also explains the validation criteria of the position of each of the mills, as a function of the forces developed on it. Lastly, it explains the inputs required to run the simulator and explains its outputs.

4.2 Overview of the Simulator

The simulator written uses the Jiazhi model to predict the trajectory cut by the trimill assembly as it rides on the multi-ramped whipface. It calculates the forces developed on the mills, which aid in calculating the angle building, dropping or holding tendency of the trimill assembly, at discrete finite increments of depths. The Jiazhi model is used to calculate the side force developed at the bit for each incremental position of the trimill assembly, as a
function of function of the eccentricity (distance between the centers) of the bit and the stabilizers, as shown in fig 4.1.

\[ e_1 = X_{LM} - X_{FM} \]
\[ e_2 = X_{LM} - X_{DM} \]
\[ e_3 = X_{LM} - (OD_{coll}/2) \]

**Fig 4.1: Eccentricity Definitions**

The number and placement of the stabilizers is one of its inputs. The simulator assumes the mills are represented by stabilizers for the side force calculations. It predicts the position (in the x-direction) of each of the mills based on the forces developed on each mill at each incremental step using a vectorial approach. The position of the mills (in x-direction) calculated at each step is then corrected with reference to the forces developed on them based on the validation criteria described later. The position of each mill is recorded at each
increment of depth and is used to determine the trajectory and the window width cut by the trimill assembly as it progresses along the multi-ramped face of the whipstock.

4.3 Design of the Semi-Analytical Simulator

The following subsections describe the concepts and the specific methods used and the assumptions made in creating the simulator for predicting the trajectory cut by the trimill assembly during the sidetracking operation using whipstocks.

4.3.1 Conceptual Application of Directional Drilling Models to Sidetracking

The simulator uses the Jiazhi model to calculate the forces developed on the lead mill, which acts as a basis for calculating the forces on the follow mill (FM) and the dress mill (DM). The moments developed at the FM and DM must be determined as described in the literature review to calculate the side force at the Lead mill (LM). The calculated moments are used to obtain the forces developed on each of the mills as illustrated in section 4.3.3.4.

The simulation begins when the program calculates the contact point of the LM with the casing on the first fifteen-degree ramp, calculating the length of tangency and the side forces at the LM. At this point, it is assumed that no other mill contacts the casing. The simulator uses the slick BHA (BHA with no stabilizers) solution, for calculating the length of tangency and the side forces on the Lead mill, until the FM touches the casing and starts cutting it. The problem now changes from a slick BHA to a one-stabilizer BHA problem. The length of tangency and the developed forces are calculated using the Jiazhi solution for one-stabilizer. As the LM rides on the multi-ramped whipface, the DM is observed to touch the casing and the problem changes from a one-stabilizer system to a two-stabilizer BHA system calculation. The literature review illustrates the slick, one-stabilizer, and two-stabilizer bottom hole assembly solutions.
4.3.2 Description of the Structure and Working of the Simulator

The flow diagram shown in fig 4.2 gives an overview of the structure and working of the simulator. The detailed calculations and the equations used at each step are given in the following sections. Steps (1) through (11) are used to describe the path and logic followed by the simulator at each incremental position of the LM along the multi-ramped face of the whipstock. The steps are as follows:

1. **START:** The program starts when the simulator calculates the contact point of the LM (lead mill) with the upper inside wall of the casing as described in section 3.3.1. At this point it is assumed that the FM (follow mill) and the DM (dress mill) do not contact the casing walls.

2. The program then checks whether the FM contacts the upper inside wall of the casing based on the equations described in section 4.3.3.3.

3a. If the calculations based on the equations described in section 4.3.3.3 do not show the FM to contact the inside upper wall of the casing, the Jiazhi solution for no-stabilizers (slick BHA) is used to calculate the length of tangency and the side forces at the bit (LM). The solution is described in the literature review, equations (2.3) – (2.5)

The simulator calculates the input to the Jiazhi solution, \( e_3 \), which is the eccentricity of the center of the LM with the center of the drill collar contacting the lower inside wall of the casing. This calculation depends on the position of the LM on the inclined ramp as described by fig 4.1. Numerically, \( e_3 = X_{LM} - OD_{coll}/2 \)
Fig 4.2: Structure and Logic of the Simulator
3b. When the simulator observes (calculates) the FM is touch the casing, the simulator uses the one-stabilizer Jiazhi solution to calculate the length of tangency ($L_T$) and the moment generated at the FM. As the simulator determines the horizontal position of the LM with respect to the inside lower wall of the casing, at each increment, the inputs to the Jiazhi solution $e_1$ and $e_3$ are calculated as follows. The eccentricity between the centers of the LM and the FM is defined as $e_1$. When the LM first contacts the casing outer wall, the horizontal distance from the center of the FM to the casing inner wall is a known quantity, given by $X_{FM}$. The input, $e_1$, to the simulator is then given by

$$e_1 = X_{LM} - X_{FM}.$$  

The input $e_3$ to the Jiazhi solution is the same as described in step 3a.

4. The side force developed on the LM, calculated by the Jiazhi model in step 3a, is used to decide the position of the LM for the next incremental position of the LM, 0.05” below the earlier position as explained in section 4.3.3.1.a. The simulator then returns to step 2, described above and follows steps 3a and 4, only if the simulator does not indicate that the FM will contact the upper inside wall of the casing, based on the equations described in step 2. If the simulator does indicate the FM will touch the upper inside wall of the casing, it goes to step 3b, instead of 3a.

5. Once the Jiazhi solution iterates for the $L_T$ and the moment created at the FM, the simulator calculates the forces developed at the LM and the FM based on the equations given in section 4.3.3.4.

6. Based on the forces developed on the FM, calculated in step 5, the position of the FM is validated and corrected if necessary as explained in section 4.3.3.5.
7a. After the validation and stabilization of the position of the FM with respect to the casing inner wall, the simulator checks whether the DM contacts the upper inside wall of the casing based on the equations explained in section 4.3.3.3. The simulator goes to step 8 only if the calculations do not predict the DM is contacting the casing, otherwise if the DM does contact the upper inside wall of the casing, the simulator goes to step 7b.

7b. The program proceeds to step 7b only if the simulator observes the DM is contacting the upper inside wall of the casing. In this step, the simulator uses the Jiazhi solution to calculate the length of tangency \( L_T \) and the moments developed on the FM and the DM as described in the literature review. The inputs, \( e_1 \) and \( e_3 \) are the same as described earlier. The new input, \( e_2 \) is the eccentricity between the centers of the LM and the DM. The simulator, at every step, calculates the horizontal position of the LM, relative to the inside lower wall of the casing, given by \( X_{LM} \). At the instant when the DM contacts the inside upper wall of the casing, the position of the DM is known and is given by, \( X_{DM} \). At every step, \( e_3 \) is calculated as, \( e_3 = X_{LM} - X_{DM} \).

8. Once the position of the FM is validated the simulator then proceeds to the \( y_{i+1} \) increment of the LM, using the rules explained in section 4.3.3.1.a. After the calculation of the new position of the LM, the simulator calculates the new assumed position of the FM based on the rules explained in section 4.3.3.5.a, which are chosen by the validation program explained in step 6. For example, if the validation criterion chooses rule “d1” to decide the new assumed position of the FM in the current step, then at the \( y_{i+1} \) step, the FM proceeds according to the \( \theta \)-rule explained in section 4.3.3.1. From here, the program returns to step 3b and follows steps 3b to step 8 until
it observes the DM to touch the casing outer wall, at which it jumps from step 7a directly to step 7b, without going to step 8.

9. Using the moments calculated on each of the mills in the earlier step, the program then calculates the forces developed on each of the mills based on the equations described in section 4.3.3.4.

10. The program then validates and if necessary, corrects the position of the FM and DM based on the validation criteria explained in section 4.3.3.5.

11. Once the positions of the FM and the DM are validated, the simulator increments the position of the LM by 0.05”, below the previous position, based on the rules described in section 4.3.3.1.a. Subsequently, the program increments the position of the FM and the DM based on the decisions made by the validation criterion. For example, if at the $y_i^{th}$ step the forces developed on the FM are above 100 lbs, then the program chooses “d1” to be the rule used for deciding the next assumed position of the FM at the $y_{i+1}^{th}$ increment of 0.05” below the $y_i^{th}$ step. The program then returns to step 7b and follows steps 7b to 11, until the DM reaches the end of the multi-ramped whipface.

At every step, for a particular validated position of the LM, FM, and DM, the trajectories and the window widths are plotted as described in sections 3.3.3 and 3.3.4.

4.3.3 Description of the Tasks Followed by the Simulator

The following section describes the various tasks performed by the simulator and the equations used for various calculations in the simulator described above.

4.3.3.1 Calculation of the New Position of a Mill (Vectorial Approach – ‘The $\theta$-rule’)

Consider fig 4.3. Let $F2$ be the side force developed on the mill. As WOB is applied in the axial direction, the angle developed due to the force $F2$ is given by (1)
\[ \theta = \tan^{-1}(F2/WOB) \]  \hspace{1cm} (4.1)

Because the mills are intentionally designed to cut in all directions, a reasonable assumption is that the direction taken by the mill is the same as that of the force applied on the mill, as implied by equation (1).

Therefore the new incremental movement of the mill in the x-direction as shown in fig 4.2 is

\[ \Delta x = \Delta y \tan(\theta) \]  \hspace{1cm} (4.2)

Equation (4.1) and (4.2) constitute the “\( \theta \) -rule”, where \( \Delta Y \) is the increment of travel in the axial direction of the trimill assembly 0.05” below the earlier position and \( \Delta X \) is the increment of travel in the lateral direction.

![Fig 4.3: \( \theta \)- rule](image)

4.3.3.1.a Rules for the Progress of the Lead Mill (LM)

The LM is assumed to follow the multi-ramped whipface as long as the calculated side force acting on the LM is negative. Conversely, the LM is expected to leave the whipface when the force developed, changes to positive. The LM is expected to build angle and move away from the whipface following the “\( \theta \) -rule” described above, when the side force is
positive and creates an angle, $\theta$, greater than the inclination angle on the adjacent face of the whipstock.

### 4.3.3.2 Length of Tangency and Moment Calculations

The position of each of the mills defines the eccentricity between the following mills and the lead mill as shown in fig 4.1. Each eccentricity is defined for the purpose of this program as the distance from the centerline of a mill perpendicular to a line through the centerline of the lead mill, parallel to the long axis of the original well. These eccentricities act as inputs to the Jiazhi model for calculating the length of tangency, as described in the literature review.

### 4.3.3.3 Determining the Contact Between the Mills and the Casing

Consider a solution of the slick BHA where no other mill is in contact with the casing except the LM. Let $L_T$ be the tangency length and ‘$l_1$’ be the length between the LM and the FM as shown in fig 4.4. As $X_{LM}$ (distance between the center of the LM and the inner side of the casing) is known, the average angle $\gamma$ (shown in fig 4.4) created by the tangent beam is given by

$$\gamma = \tan^{-1} \left( \frac{x_{LM}}{L_T} \right)$$

(4.3)

As the length ‘$l_1$’, between the LM and the FM is known, distance ‘$c$’, as shown in fig 4.4 is calculated by

$$c = l_1 \sin(\gamma)$$

(4.4)

Distance “$d$” as shown in fig 4.4, is the deflection of the beam of length ‘$L_t$’, due to the axial load and the uniformly distributed load (UDL) acting on it.
The equation for calculating the deflection due to the axial load and the UDL is given by

\[
x = \frac{qL_t^4}{16EIu^4} \left[ \frac{\cos(u - \frac{2uy}{L_t})}{\cos(u)} - 1 \right] - \frac{qL_t^2}{8EIu^2} y(L_t - y)
\]  

(4.5)

\[ q = w \cdot \sin(\alpha) \cdot Bc \]

\[ w = \text{Weight of the BHA above the LM (lb/ft)} \]

\[ \alpha = \text{Average hole inclination.} \]

\[ u = \frac{L_t}{2} \sqrt{\frac{p}{EI}} \]
P = axial load
Bc = Buoyancy factor

Substituting length ‘l₁’ for y in equation (4.5), x is calculated as the deflection ‘d’, at the FM.

Hence ‘c + d’ is the total distance between the centers of the LM and the FM. The simulator then calculates the clearance between the FM and the upper inside wall of the casing. The clearance is calculated as follows

Consider fig 4.4. Let ‘X_{FM}’ be the distance between the center of the FM and the upper inside wall of the casing.

$$X_{FM} = x_{LM} - (c + d) \quad (4.6)$$

Hence the clearance ‘v’ is given by

$$v = ID_{CSG} - (X_{FM} + (OD_{FM} \cos(\gamma)/2)) \quad (4.7)$$

The point at which ‘v’ becomes negative is the contact point of the FM with the upper inside wall of the casing.

The same calculation is done for calculating the contact point of the DM with the casing.

**4.3.3.4 Calculation of the Forces on the Mills**

The method used to solve for the forces developed on each mill is based on the structural analysis approach for calculating forces at the supports for indeterminate beams.

Consider fig 4.5. Let l₁ be the length between support A (LM) and B (FM), l₂ between supports B (LM) and C (DM), and L₄ be the length between supports C and D (Tangency point). Let q₁ be the uniformly distributed load over length l₁ acting at angle α (overall inclination of the borehole). Let M₁ be the moment created at support B (FM) and M₂ at support C (DM). Let P₁ be the average axial load acting at point B, P₂ be the average axial load acting at point C as shown in fig 4.5. Let P₃ be the average axial load acting at a point D.
that is the tangency point above the DM. Let \( e_1 \) be the lateral displacement of point B with respect to A, \( e_2 \) be the displacement of point C with respect to point A and \( e_3 \) be the displacement of point D with point A. This system represents the trimill assembly with the LM, FM and DM touching the casing and/or the rock and being tangent to the casing at point D.

**Fig 4.5: Force Calculation at a Support**

The forces are calculated as follows. Consider the support B (FM) in fig 4.5. The beam is isolated at support B into two separate beams, AB and BC. Consider beam AB. Let \( F_A \) be the reaction force at support A, and let \( F_{BA} \) be the reaction force at support B in beam AB.

Taking moments about point \( Ba \) as shown in fig 4.5, we get (\( \Sigma M_B = 0 \))

\[
F_A l_1 - \frac{q_1 l_1^2}{2} + P_1 e_1 - M_1 = 0
\]  

(4.8)

Reaction force at A is given by

\[
F_A = \frac{q_1 l_1}{2} + \frac{M_1}{l_1} - \frac{P_1 e_1}{l_1}
\]  

(4.9)

Taking the sum of lateral forces to be zero, for beam AB gives,
Now consider beam BC, taking moments about point C we get,

\[ F_{BC}l_2 = q_2l_2^2 + P_2e_2 - P_1e_1 + M_1 - M_2 = 0 \] (4.11)

As moment \( M_1 \) is the internal moment created at support B, the direction of the moment is considered to be opposite to the actual direction considered in the force calculation for beam AB.

Then, the force calculated at B considering beam BC is

\[ F_{BC} = \frac{q_2l_2^2}{2} - \frac{M_1}{l_2} - \frac{P_2e_2}{l_2} - \frac{P_1e_1}{l_2} + \frac{M_2}{l_2} \] (4.12)

Note that in the program used in this study the term \( \frac{P_1e_1}{l_2} \) was neglected. The significance of this error is small for the cases studied as explained in the following chapter.

Hence the total force calculated at support B is given as

\[ F_B = -F_{BA} - F_{BC} \] (4.13)

As this is the reaction force calculated at the support, the direction of the force exerted by the beam on support B is given as the negative value of the force calculated by equation (4.13), which is \((-F_B)\).

The above calculation is repeated for support C that represents the DM.

4.3.3.5 Validating the Position of and Force on the FM and DM

As the FM and DM progress in accordance to the “0-rule”, the lateral forces on them decrease as the eccentricity between the centers of the mills and the lead mill decreases. The fact that the mills cannot cut the casing or the rock when the forces on them are negative initiated the need for a validity criterion. To counter this problem, a minimum threshold force
of 100 lbs was assumed to be required for the FM and the DM to cut the rock and/or the casing. Further, either the FM or DM was assumed to stop cutting the casing or rock in the lateral direction, when the lateral force developed on it fell below the minimum threshold value. Likewise, the position of the FM or DM must fall between the whipstock and the previously cut rock, if the force calculated on the mill was zero. Furthermore, the FM and DM were assumed to contact the adjacent inclined face of the whipstock if the force calculated on them were negative.

The trimill assembly with the drill string above it can be visualized as being represented by the string of a bow and arrow, which requires a higher side force if the string is deflected further. The FM and DM were assumed to cut the rock and the casing if the forces on them were equal to or greater than the minimum threshold force required. If the forces on the FM and DM drop below the minimum threshold force, the simulator iterates for a valid solution by decreasing the lateral distance from the mill to the casing inner wall, at that particular y position until the force on the mill equals the minimum threshold force or otherwise gives an acceptable solution as described below.

For example, lets consider the validity criteria for the FM position, as a function of the forces developed on it. Let $X_{\text{rock}}$ be the distance between the face of the previously cut rock and the lower inside wall of the casing as shown in fig 4.6. Let $X_{\text{whip}}$ be the distance from the whipface, and “$X_{\text{FM}}$” be the distance from the center of the FM, to the casing lower inner wall, as shown in fig 4.6. Let ‘$\gamma$’ be the average tool inclination as a function of the tangency point, shown in fig 4.6, and $OD_{\text{FM}}$ be the outer diameter of the FM. Let $F_2$ be the force developed on the FM.
If a 100lbs force was assumed to be the minimum threshold force required to cut the casing and the rock, then an acceptable solution for the positioning of, and force, on a mill would be:

1. If $F_2 \geq 100$, then

   $$X_{FM} + (OD_{FM} \cos (\gamma)/2) > X_{rock} \quad \text{when the mill is cutting the rock}$$

2. If $100 > F_2 > 0$, then

   $$X_{FM} + (OD_{FM} \cos (\gamma)/2) = X_{rock} \quad \text{when the mill is touching the previously cut rock}$$

3. If $F_2 = 0$, then

---

**Fig 4.6: Figure Explaining Validity Criteria**

2. If $100 > F_2 > 0$, then

   $$X_{FM} + (OD_{FM} \cos (\gamma)/2) = X_{rock} \quad \text{when the mill is touching the previously cut rock}$$

3. If $F_2 = 0$, then
\[ X_{\text{FM}} + (OD_{\text{FM}} \cos (\gamma)/2) \leq X_{\text{rock}} \] and \[ X_{\text{FM}} - (OD_{\text{FM}} \cos (\gamma)) \geq X_{\text{whip}} \] when the mill is between the whipface and the previously cut rock without transferring load to any surface.

4. If \( F2 < 0 \), then
\[ X_{\text{FM}} - (OD_{\text{FM}} \cos (\gamma)/2) = X_{\text{whip}}, \] when the FM/DM is in contact with the whipface.

A flowchart illustrating an iterative method for finding a valid solution is shown in fig 4.7. In fig 4.7, \( x = OD_{\text{FM}} \cos (\gamma)/2 \). The rest of the symbols are as explained earlier. The functions \( \text{Incr} (X) \) and \( \text{Decr} (X) \) are the increase or decrease in \( X_{\text{FM}} \), distance from the center of the FM to the lower inner wall of the casing, required to cause the forces on the mill, which are described later. The actions \( a_1, b_1, c_1 \) and \( d_1 \) are the set of rules for predicting the positions of the FM and the DM at the \( y_{i+1} \) increment, depending on the existing conditions for a valid solution, at the \( y_i \) step. These actions are described in the following sections.

### 4.3.3.5.a Predicting Mill Positions at the Next Depth

The criteria for the mill positions at the next increment of depth, shown in fig 4.7, as \( a_1, b_1, c_1 \) and \( d_1 \) in the simulation are described as follows.

1. \( a_1 \): If at \( y_i \) increment, the program selects a valid position of the FM/DM on the whipface, \( X_{\text{FM}} - (OD_{\text{FM}} \cos (\gamma)/2) = X_{\text{whip}}, \) with negative force on it then, at the \( y_{i+1} \) increment the assumed position of the FM/DM is on the whipface.

   If at \( 'y_{i+1}' \), \( X_{\text{whip1}} \) is the new whip distance from the casing inner wall, then, \( X_{\text{FM1}} \), the new position of the FM/DM, is defined as
   \[ X_{\text{FM1}} = X_{\text{whip1}} + (OD_{\text{FM}} \cos (\gamma)/2) \]
NOTE: Force calc: At this step, the simulator uses the Jiazhi solution to solve for L_T and the moments created at the mills due to change in lateral position of the mill in the iteration (validation) process.

Fig 4.7: Program for Validation of Position Relative to the Forces Developed on the mills
2. \( b_1 \): If at \( y_i \) increment, the program selects a valid position of the FM/DM between the whipface and the previously cut rock with the mill not contacting any surface, 
\[
X_{FM} + (OD_{FM} \cos (\gamma)/2) < X_{rock} \quad \text{and} \quad X_{FM} - (OD_{FM} \cos (\gamma)/2) > X_{whip}
\]
with a zero force on it, then at the \( y_{i+1} \) increment, FM/DM are assumed to have a progress so as to keep the eccentricity between the LM and the FM/DM constant or in other words, have the same increment in the positive x-direction as that of the LM.

If \( X_{LM1} \) is the new position of the LM at \( y_{i+1} \) increment, then
\[
Dx = X_{LM1} - X_{LM}
\]
\[
X_{FM1} = X_{FM} + Dx
\]

3. \( c_1 \): If at \( y_i \) increment, the program selects a valid position of the FM/DM touching the previously cut rock, 
\[
X_{FM} + (OD_{FM} \cos (\gamma)/2) = X_{rock}
\]
with a positive force of less than 100 lbs on it then, at the \( y_{i+1} \) increment, FM/DM are assumed to continue to follow the previously cut rock surface.

If \( X_{rock1} \) is the distance of the cut rock surface from the casing lower inner wall, then
\[
X_{FM1} = X_{rock1} - (OD_{FM} \cos (\gamma)/2)
\]

4. \( d_1 \): At \( y_i \) increment, if the program selects FM/DM position as cutting the casing and/or rock, where 
\[
X_{FM} + (OD_{FM} \cos (\gamma)/2) > X_{rock}
\]
and the force on the mill is equal to or greater than 100 lbs, then at \( y_{i+1} \) increment, the progress of the FM/DM is governed by the “\( \theta \)-rule”.

**4.3.3.5.b Iterative Solutions for Valid Position**

The functions \( \text{Incr} (X) \) and \( \text{Decr} (X) \) relate to the increase or decrease of \( X_{FM} \), the distance to the center of the FM/DM from the casing lower inner wall, at a given \( y_i \) position. As described earlier, this change in the position of the FM/DM, in the x-direction, was to find
a valid position of the mill based on the forces developed on the mill. The function Incr (x) is used to increase the distance of the FM/DM from the casing inner wall, decreasing the eccentricity between the centers of the LM and the FM/DM. This decreases the positive side force developed on the FM/DM, as the deflection of the trimill BHA decreases.

Conversely the function Decr (X) decreases the distance between the center of the FM/DM and the casing lower inner wall, thus increasing the eccentricity between the centers of the LM and the FM/DM. This increases the deflection in the BHA, thus increasing the calculated positive side force on the FM/DM.

Successful implementation of this iterative process to achieve a valid position and force solution requires knowing how the force on the mill changes with the x-position of the FM/DM. This knowledge is required as an input to the program to help prevent the program from going into an infinite loop. This phenomenon is explained as follows.

Consider the 9 5/8” casing sidetrack system. It shows a change of approximately 76 lbs for a FM x-position change of 0.0005”. A minimum change in x-position of 0.0000625” was selected to give a force change of 10 lbs. This force change of 10 lbs was selected to allow a solution where F2 = 0 in fig 4.6, to terminate the iteration loops. This criterion was selected so that the program would terminate the iterations when the force falls between 0 and 10 lbs.

Consider fig 4.8. Let Xc be the distance to the outer edge of the FM/DM from the surface of the previously cut rock (X_{rock}) defined earlier. Let “Xmin” be the minimum increment of the FM/DM in the x-direction. In the case explained above, Xmin = 0.0000625”. Now if the force on the FM/DM is –5 lbs and the distance Xc < Xmin, then as the program decreases x-position of the FM/DM by 0.0000625”, the force on the FM/DM increases to a +5
lbs. At this position, position 2 shown in fig 4.8, the mill is not touching anything. Therefore the validity criteria fails as the mills have to contact a surface if the force on them is equal to or greater than a 0 lbs. As the program does not get a valid position of the FM/DM, it again increases the x-position of the FM/DM, taking the mill back to position 1, thus going into an infinite loop.

To counter this problem, the program is designed to keep track of the value of Xc, and give a solution for the force, for the case of the mill just contacting the previously cut rock face. For the case described it would always give a solution between 0 and 100 lbs for the force, which satisfies the condition that the force on the mills can be positive if the mill is touching the previously cut rock surface.

The same problem was observed to occur when the mill (FM/DM) nears the whipface and was corrected in the same manner.

As a result, the system’s sensitivity to the forces with the change in x-position is a very important consideration for calculating the correct position of the mills. This sensitivity
has to be input for three different conditions. These are the change in force for a change in the
x-position of the

1. FM for the one stabilizer solution
2. FM for two stabilizer solution, and
3. DM for the two-stabilizer solution.

The minimum allowable change in x-position and the consequent force change for each of the
above three cases has to be defined and incorporated into the simulator as one of its inputs.
For this, the static Jiazhi solution for one-stabilizer BHA and two-stabilizer BHA, with the
equations subsequently calculating the forces was programmed separately. The sensitivity of
the force to the x-position of the mills has to be checked individually by trial and error
method. Firstly, a minimum change in the force has to be decided for terminating the
iterations. In the above example we selected 10 lbs as the terminating force. This selection
was made based on the change in x-position of the mill by 0.0000625”, resulting in a force
change of 10 lbs which is an insignificant value relative to the accuracy required for the
trajectory predictions.

4.3.3.5.c Satisfying the Validation Criteria for the Trimill Assembly

The validation of the position and force of a single mill such as the FM is done for the
one-stabilizer Jiazhi solution as explained earlier. For a two-stabilizer solution with the FM
and the DM in contact with the casing or rock, the force change on one of the two mills
(FM/DM) creates a force change on the other mill. As the positions of both of the mills have
to be validated, a new function is introduced into the system, as illustrated in fig 4.9.
It was observed that the change in FM position does induce a significant force change on the DM. Fig 4.9 describes the functioning of the program for completing the validation criteria.

To understand the program illustrated in fig 4.9, let ‘e1’ be the initial assumed eccentricity between the centers of the FM and the LM and let ‘e2’ be that between the LM and the DM. The values of ‘e1’ and ‘e2’ act as the initial guess for the iterative method illustrated in fig 4.8.

First, the position of the FM is corrected using the method shown in fig 4.7. Let ‘e1a’ be the validated eccentricity calculated by the program. The program then validates the DM position with a routine as in fig 4.7, but with different inputs for the DM, like the minimum x-position change and the consequent force change as described earlier. Let e2a be the validated...
DM position. Now, if $e_{2a} = e_2$ then the program does not need to iterate any further and is allowed to go to the ‘$y_{i+1}$’ increment. If $e_{2a} = e_2$ ($e_{2a}$ is not equal to $e_2$) then the program has to recheck the validity of the FM. For the second iteration, the program uses the $e_{1a}$ value as an initial guess and validates the FM position. After calculating the new FM position, the program uses an initial value of $e_2 = e_{2a}$ from the previous iteration as an initial guess and validates the DM position. Thus, the program goes to the next increment only when $e_{2a} = e_2$ and the forces on and the positions of both mills satisfy the validity criteria.

4.3.3.6 Plotting the Window Width and the Trajectory of the Validated Positions of the FM and the DM

Once the positions of the FM and the DM are validated with respect to the above criteria, the window width and the trajectory are plotted as discussed in chapter 3.

4.4 Assumptions for Trajectory Predictions Using Semi-Analytical Simulator

The following assumptions were made when applying the simulator in this study.

1. The original borehole is straight over the interval from 50ft above the whipstock to the base of the whipstock.

2. The FM and the DM act as stabilizers for force calculations only, but cut the casing and formation depending on the forces developed on them, when predicting the well trajectory.

3. The existing borehole is tilted at an angle ‘$\alpha$’ from vertical.

4. A length-weighted average is taken as representing the moment of Inertia $I$, for the bottom hole assembly (BHA) above the trimill assembly.

5. A 5000 lb weight on bit is applied for the entire sidetracking process.
6. A minimum threshold force of a 100 lb is assumed to be required for the follow mill and the dress mill to cut the casing and the rock.

4.5 Inputs to the Program:

The inputs are:

1. \( \text{OD}_{csg} \): Outer diameter of the casing to be sidetracked (inches)
2. \( \text{ID}_{csg} \): Inner diameter of the casing to be sidetracked (inches)
3. All of the dimensions shown in fig 4.10, A, B, D, E, F to V (inches)
4. \( \text{OD}_{b2} \): Outer diameter of the mandrel of the trimill assembly above the dress mill up to the pin end (inches)
5. \( \text{ID}_{b2} \): Inner diameter of the mandrel of the trimill assembly above the dress mill up to the pin end (inches)
6. \( \text{OD}_{b1} \): Outer diameter of the mandrel of the trimill assembly between the lead mill and the follow mill (inches)
7. \( \text{ID}_{b1} \): Inner diameter of the mandrel of the trimill assembly between the lead mill and the follow mill (inches)
8. \( \text{OD (RT)} \): Outer diameter of the running tool (inches)
9. \( \text{ID (RT)} \): Inner diameter of the mandrel (inches)
10. \( \text{OD}_{coll} \): Outer diameter of the drill collar (inches)
11. \( \text{ID}_{coll} \): Inner diameter of the drill collar (inches)
12. \( \text{OD m-val} \): Outer diameter of the multi-cycle valve
13. \( \text{ID m-val} \): Inner diameter of the multi-cycle valve
14. \( \text{Mwt} \): Mud weight (ppg)
15. \( \text{Alpha} \): The average borehole inclination of the existing well (degrees)
16. WOB: Weight on bit (lbs)

17. LRT: Length of running tool (feet)

18. L (Dcoll): Length of drill collar (feet)

19. L m-val: Length of multi-cycle valve (feet)

20. ODmill: Outer diameter of the mills of the trimill assembly (inches)

21. ODwhip: Outer diameter of the whipstock (inches)

22. Ldm: Length or height of the dress mill cutting face (inches)

Multi-ramped whipstock

**Fig 4.10: Trackmaster System**
The Simulator is designed to calculate the moment of inertia ‘I’, of the drill string 50 ft above the DM by calculating a length-weighted average of the moments of inertia of each member above the DM.

Also the simulator is designed to incorporate the angle and length of each inclined ramp along the face of the whipstock as a required input for running the simulator.

4.5.1. Example of the Inputs to the Program:

Table 4.1: Inputs to the Simulator

<table>
<thead>
<tr>
<th>WOB Alpha Mwt (ppg)</th>
<th>5000</th>
<th>Lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha</td>
<td>3</td>
<td>Degrees</td>
</tr>
<tr>
<td>Mwt (ppg)</td>
<td>10</td>
<td>Ppg</td>
</tr>
<tr>
<td>ODcsg</td>
<td>9.625</td>
<td>Inches</td>
</tr>
<tr>
<td>IDcsg</td>
<td>8.755</td>
<td>Inches</td>
</tr>
<tr>
<td>OD mill</td>
<td>8.515</td>
<td>Inches</td>
</tr>
<tr>
<td>OD whip</td>
<td>8</td>
<td>Inches</td>
</tr>
<tr>
<td>A</td>
<td>114.89</td>
<td>Inches</td>
</tr>
<tr>
<td>B</td>
<td>67.83</td>
<td>Inches</td>
</tr>
<tr>
<td>D (15 deg)</td>
<td>5.3</td>
<td>Inches</td>
</tr>
<tr>
<td>E (0 deg)</td>
<td>27</td>
<td>Inches</td>
</tr>
<tr>
<td>F (3 deg)</td>
<td>41.4</td>
<td>Inches</td>
</tr>
<tr>
<td>G (15 deg)</td>
<td>3</td>
<td>Inches</td>
</tr>
<tr>
<td>H (3 deg)</td>
<td>63.8</td>
<td>Inches</td>
</tr>
<tr>
<td>T</td>
<td>0.24</td>
<td>Inches</td>
</tr>
<tr>
<td>S</td>
<td>166.5</td>
<td>Inches</td>
</tr>
<tr>
<td>V</td>
<td>25.22</td>
<td>Inches</td>
</tr>
<tr>
<td>LdM</td>
<td>6.9</td>
<td>Inches</td>
</tr>
<tr>
<td>OD b1</td>
<td>5.5</td>
<td>Inches</td>
</tr>
<tr>
<td>ID b1</td>
<td>2.25</td>
<td>Inches</td>
</tr>
<tr>
<td>OD b2</td>
<td>6.5</td>
<td>Inches</td>
</tr>
<tr>
<td>ID b2</td>
<td>2.25</td>
<td>Inches</td>
</tr>
<tr>
<td>OD (RT)</td>
<td>6</td>
<td>Inches</td>
</tr>
<tr>
<td>ID (RT)</td>
<td>3.9325</td>
<td>Inches</td>
</tr>
<tr>
<td>OD coll</td>
<td>6.5</td>
<td>Inches</td>
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<tr>
<td>ID coll</td>
<td>2.813</td>
<td>Inches</td>
</tr>
<tr>
<td>OD m-val</td>
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<td>Inches</td>
</tr>
<tr>
<td>ID m-val</td>
<td>2.9375</td>
<td>Inches</td>
</tr>
<tr>
<td>L (RT)</td>
<td>6.25</td>
<td>Feet</td>
</tr>
<tr>
<td>L (Dcoll)</td>
<td>30.66</td>
<td>Feet</td>
</tr>
<tr>
<td>L (m-valve)</td>
<td>5.3</td>
<td>Feet</td>
</tr>
</tbody>
</table>

(Table cont’d)
<table>
<thead>
<tr>
<th></th>
<th>Change in LBS</th>
<th>Change in inches</th>
<th>Minimum change (In)</th>
</tr>
</thead>
<tbody>
<tr>
<td>For one stabilizer</td>
<td>75.59213</td>
<td>0.002681</td>
<td>0.00025</td>
</tr>
<tr>
<td>For Follow mill, 2 stabs</td>
<td>75.59213</td>
<td>0.0005</td>
<td>0.0000625</td>
</tr>
<tr>
<td>For Dress mill, 2 stabs</td>
<td>59.69835</td>
<td>0.005</td>
<td>0.00067</td>
</tr>
</tbody>
</table>

4.6 Outputs of the Simulator:

Fig 4.11 shows the graphical output from a simulation. If the trajectory is defined as the side view of the path cut by the trimill components, then the front view shows the width of the window cut by the mills. Fig 4.11 shows the trajectory and the window width cut by each mill as the trimill assembly rides on the multi-ramped face of the whipstock.

Fig 4.12 shows the resultant trajectory and window width cut by the trimill assembly. These reflect the maximum extent of the borehole cut by the three mills, in the same manner as for the rigid body program described in chapter.

4.7 Summary

A simulator has been developed to predict the trajectory and width of the borehole created in performing a cased hole sidetrack using a multi-ramped whipstock and a trimill assembly. The simulator performs the following steps to generate a prediction.

1. At each incremental position of the lead mill, the simulator uses the Jiazhi solution to calculate the length of tangency the side force on the lead mill, and the moments developed on the mills as a function of the relative positions of the mills with respect to the lead mill. The simulator uses the Jiazhi solution for no-stabilizer, one-stabilizer or two-stabilizer bottom hole assemblies, depending on the number of mills contacting the casing and/or the rock.
2. The simulator then calculates the side forces developed on the mills, as a function of the moments calculated by the Jiazhi solution.

3. The simulator then validates the position of the mills relative to the calculated side forces on each mill.

4. The simulator then records and plots the position of each mill and the window width cut by the mill.

5. The position of the lead mill is increased by 0.05” in the vertical direction along the face of the multi-ramped whipstock estimates the positions for the follow mill and dress mill are made and the program returns to step 1.
Note: All Dimensions are in inches

Fig 4.11: (Simulator Results) Trajectory and Window Width Cut by Each Mill
Fig 4.12: (Simulator Results) Composite well path
5. RESULTS OF TRAJECTORY PREDICTIONS FOR SELECTED SIDETRACK OPERATIONS

5.1 Introduction

One of the objectives of this research was to predict the inclination trajectory of a sidetracked borehole based on the directional tendency of the trimill assembly. Chapter 4 described the computer program that was created to predict the directional response and trajectory of a sidetrack mill assembly.

This chapter presents the results of applying the semi-analytical simulator to selected sets of well geometries, tool configurations and resistance to sidetracking. These results allow conclusions to be drawn regarding the effects of these variables on the expected trajectory of a sidetrack.

5.2 Case Descriptions

5.2.1 Base Case

A typical application of the trimill assembly and the multi-ramped whipstock is a sidetrack from the 9 5/8” casing in a nearly vertical well. This application was selected as a base case. The following sections describe the key inputs for predicting the trajectory for this case.

5.2.1.1 Weight on Bit (WOB)

The WOB is a single number input in the program, which cannot be varied for the entire trajectory prediction run of the semi-analytical simulator. The data obtained from Smith Services indicates that an average value for WOB of about 5000 lbs used for the entire milling process for all sizes of casing.
5.2.1.2 Inclination Angle

A specific concern identified by the engineers from Smith Services, was that the mills had apparently deviated off of the whipface during casing sidetracking operations conducted on some vertical wells, whereas on others, the mills followed the whipface. Conversely, deviated wellbores are usually sidetracked from the upper side of the casing, which attempts to keep the trimill assembly against the whipface as long as possible. The cased borehole inclination \( \alpha \) was assumed to be three degrees for this study, since, a well that has an inclination angle less than three degrees is considered to be a vertical well.

5.2.1.3 Dimensions of the Casing and Sidetrack Assembly

The dimensions of the trimill assembly and the corresponding multi-ramped whipstock are dependant on the casing size. For the base case, an 8 ½” trimill assembly and whipstock were used inside a 9 5/8” casing. The sizes of the trimill assembly and multi-ramped whipstock are given in Table A.

5.2.1.4 The BHA above the Trimill Assembly

The BHA above the trimill assembly was thought, by Smith Services personnel, to play a significant role in the predictions of the trajectory or the build up angle in the sidetracked section and potentially in determining whether the lead mill left the whipface prematurely. The most common BHA used is a running tool, multi-cycle valve and drill collars above the trimill assembly. The moment of inertia of the components of the BHA can have a significant influence on the directional tendencies of the assembly. The moment of inertia \( I \) is a function of the cross sectional area, that is, the thickness of the components. Because the components may have varying dimensions, the moment of inertia, \( I \), was
calculated as the length-weighted average of the $I$, of each tubular connected 50ft above the trimill assembly.

The program was run for an assumed minimum value of 100 lbs side force required for a mill to cut into the rock and/or the casing. The selection of this value is arbitrary, but it is obvious that some side force must be applied to a mill before it will cut any material. Also, the side force is expected to be much less than the axial force, or WOB, because the assembly only cuts 9” to the side versus 250” in depth.

5.2.1.5 Other Variables

The other input variables required to describe the base case are included in Table A of Appendix I.

5.2.2 Sensitivity Analysis Cases

5.2.2.1 Casing Size

In order to understand the effects of casing size in the predicted behavior of the trimill assembly for smaller casing sizes and thickness (or cross sectional areas), a system for 7” casing was also analyzed. The trajectory and the corresponding window-widths were predicted for 7” casing and the corresponding size combination of the sidetracking tool (whipstock and trimill assembly). Table B summarizes the inputs for this case.

5.2.2.2 Bottom Hole Assembly

The magnitude of the effects that is caused by using a different stiffness BHA was also studied. In the predicted trajectory, a HWDP (heavy weight drill pipe) is sometimes used instead of a drill collar in the BHA above the trimill assembly.

This changes the moment of inertia, $I$, of the BHA above the trimill assembly, which can potentially change the directional tendencies of the trimill assembly.
5.2.2.3 Force Required to Side Cut the Rock and Casing

The intent of varying the value of this parameter was to observe the predicted angle building behavior of the trimill assembly for harder formations. This effectively allowed the simulation to account for rock strength, in terms of the force required for the mills to side cut as an input variable. Logically, the trimill assembly should experience increased bending if a larger side force is required to cause the mills to side cut. The goal was to observe the predicted trajectory and whether the trimill assembly would be more or less likely to leave the whipface, when milling harder formations. The minimum value of side force selected for this analysis was 600 lbs.

5.3 Trajectory Predictions

5.3.1 Base Case

The computer program (Semi-analytical simulator) was run for 9 5/8” casing and the corresponding sidetracking tools (Trimill assembly and Multi-ramped whipstock). Fig 5.1a shows the results of the predicted trajectory and the corresponding window-width cut by each mill for the 9 5/8” casing sidetrack. Fig 5.1b shows the composite trajectory and the window width, which is the maximum extent of the predicted trajectory and window width predicted for the 9 5/8” casing sidetracking tool.

Table C, of Appendix-I, shows the side forces calculated by the computer program on each of the mills as the LM (lead mill) progresses along the different ramps of the whipstock. The important observations for the results of the 9 5/8” casing sidetrack prediction are as follows:

1. Table C, of Appendix-I, shows that the side force developed on the FM (follow mill) have almost the same magnitude as that developed on the LM but is opposite in
direction. That is, the side force on the LM is negative (downward) and on the FM is positive (upward)

2. Table A, in Appendix -I, shows that the side forces on the FM (follow mill) develop so as to compensate for the effect of the ramp on which the LM rides. For example, as the LM rides on the first fifteen-degree ramp, the forces developed on the FM make an angle very near to fifteen degrees, so as to minimize the eccentricity between the FM and the LM. The analysis shows this phenomenon for every ramp that the LM follows. As the vertical length between the LM and the FM is very small, this phenomenon was observed on the FM. The magnitude of the forces observed on the DM does not change significantly as the LM rides on the inclined ramps.

3. The predictions also show that the DM contacts the upper inside wall of the casing while the LM rides on the first three-degree ramp and consequently increases the length of the window for almost five feet above the top of the whipstock.

5.3.2 Sensitivity Cases

5.3.2.1 7” Casing Sidetrack

The second set of predictions performed was for the 7” casing Trackmaster system. Fig 5.2a and Fig 5.2b show the trajectory and the window width cut by each mill and the maximum extent of the trajectory and the window width respectively.

Table D in Appendix-I, shows the forces predicted by the program (Semi-analytical simulator) for the 7” casing sidetracking equipment.

The results indicate behavior similar to that for the 9 5/8” casing sidetrack predictions except that the forces observed on the LM are almost 100 lbs less than observed for the 9 5/8” casing sidetrack run.
Fig 5.1a: Trajectories and Window Widths Cut by Each Mill for 9 5/8” Casing Sidetrack System

Fig 5.1b: Composite Trajectory and Window Widths Cut by the Mills for 9 5/8” Casing Sidetrack System
The results also show that the DM contacts the upper inside wall of the casing while the LM is riding on the straight ramp extending the window length above the top of the whipstock, by 4 ft.

5.3.2.2 Different BHA Above the Trimill Assembly

This case maintained the same dimensions of the sidetracking equipment except that the drill collar was replaced by a joint of HWDP (heavy weight drill pipe) to observe the change in the predicted trajectory and the corresponding window-width profile.

Fig 5.2a: Trajectories and Window Widths Cut by each Mill for 7” Casing Sidetrack System
This change affects the moment of inertia, $I$, of the bottom hole assembly above the trimill assembly.

Fig 5.3 shows an overlay of the predicted composite trajectories and the corresponding window profile for the bottom hole assemblies of the base case with a drill collar and this case, with a HWDP in the BHA above the trimill assembly.
The following observations were noted:

1. The magnitude of the forces obtained from the program for trajectory predictions, which was run using a HWDP, were less than the magnitude of forces observed for the drill collar case.

2. The major axis, length of the elliptical hole predicted by the program for the HWDP case was smaller than that predicted using a drill collar in the BHA above the trimill assembly.
3. The predicted trajectory shows that the trimill assembly should build angle slower
with a HWDP in the BHA above the trimill assembly, but the difference is
insignificant, as observed in Fig 5.3.

5.3.2.3 Higher Threshold Side Force

The sensitivity of the predicted trajectory to a change in the threshold force required
for the mills to cut the rock and the casing was performed for the 9 5/8” casing sidetrack
system assuming a drill collar in the BHA above the trimill assembly. This case was intended
to give an insight into the effects of making incorrect assumptions about the magnitude of the
threshold force or of milling a much stronger rock. The only variable changed was the side
force required for the mills to cut the rock and the casing. Fig 5.4 shows an overlay of the
different maximum trajectories predicted assuming 100 lbs and 600 lbs force required for the
mills to cut the rock and the casing.

Observations:

1. Assuming a higher threshold force of 600 lbs decreases the length of the major axis of
   the predicted trajectory as compared to the case where only a minimum threshold
   force of 100 lbs is required to cut the casing and the rock.

2. The predictions show a more rapid dropping tendency of the trimill assembly after the
   LM passes the end of the multi-ramped whipface if the 600 lbs threshold force applies.
Fig 5.4: Overlay of Composite Trajectories and Window Widths assuming 100 lbs and 600 lbs Force Required for the Mills to Cut the Rock and the Casing.
6. DOGLEG SEVERITY CALCULATIONS FOR THE PREDICTED SIDETRACK TRAJECTORY

6.1 Introduction

Dogleg severity is the most common measure of the usability of a particular hole trajectory. It is typically calculated based on the change in hole direction between two survey stations and reported in degrees of change per 100 ft as described in chapter 2. The trajectory of the hole created during a casing sidetrack changes direction rapidly and has an elliptical shape as described in the previous chapter. Consequently, the dogleg severity, affecting tools or casing run through the sidetrack, is more dependent on these abrupt angle changes than on simple survey calculations.

One of the practical applications for the trajectory predictions made in this research is to provide a basis for determining the usability of the sidetrack for passing the bottomhole assemblies, drill string, and casing that are planned to be run through it. The maximum dogleg severity in the sidetrack determines the bending or curvature required in a tool moving through the sidetrack. Lubinski described specific methods for calculating the curvature in downhole tools through an abrupt dogleg, that was dependent on tool geometry, hole geometry and whether the tool is in compression or tension. These methods are more complex than was practical in the scope of this study. However, the concepts presented by Lubinski provide a basis for calculations performed for this study.

6.2 Calculation of Dogleg Severity

The curvature that would be experienced by the downhole tubular depends on the contact points between the tubular and the sidetracked hole. These contact points are dependent on whether the tubular is in tension or in compression, and whether it is buckled.
The method applied here assumes three different contact points that were concluded to be potentially relevant, regardless of whether the tubular is in tension or compression. As three points of contact are needed to define a circle, the methods developed herein assume certain plausible contact points and tangent points in order to calculate the curvature of a given size OD downhole tubular being run in the sidetracked section. The curvature is then expressed as a dogleg severity.

6.2.1. Contact Condition 1

This set of conditions used to calculate the dogleg severity assumes the tubular to be tangent to the inside lower wall of the casing and the face of the last three-degree ramp of the whipstock, as shown in fig 6.1. It also assumes that the tubular contacts the upper inside wall of the casing at the beginning of the milled window. The dogleg severity calculated for these conditions are valid only if the tubular’s contact with the whipstock is a tangency point on the last three-degree ramp.

6.2.1.1 Assumptions

1. The tubular was assumed to contact point 2, shown in fig 6.1.
2. To be tangent to line 1, which is the inside lower wall of the casing.
3. To be tangent to line 3, which is the last three-degree ramp of the multi-ramped whipstock.

6.2.1.2 Equations to Calculate the Radius of Curvature

The equation used for calculating the radius of curvature, as explained in Appendix-II is, $r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

\[ (6.1) \]
Fig 6.1: Assumed Conditions for Tubular Curvature
Calculations for Condition 1

Where

\[ a = z_2^2 \]  \hspace{1cm} (6.2)  \\
\[ b = 2(z_1z_2 - yz_2 + 10 - x) + OD \]  \hspace{1cm} (6.3)  \\
\[ c = x^2 + y^2 + z_1^2 - 2yz_1 - 20x + 100 - OD^2 \]  \hspace{1cm} (6.4)  

Where,

\[ z_1 = \left( \tan(\varepsilon) + \frac{1}{\cos(\varepsilon)} \right), \]  \hspace{1cm} (6.5)  \\
\[ z_2 = (c_1 + 10\tan(\varepsilon)), \]  \hspace{1cm} (6.6)  \\
\[ \varepsilon = \tan^{-1}(dy/dx), \text{ and} \]  \hspace{1cm} (6.6a)
x and y are the coordinates of point 2 shown in fig 6.1.

6.2.2 Calculation of Dogleg Severity from Tubular Curvature

Because curvature \((1/r)\) is defined in radians per inch, the dogleg severity for our system in degrees per 100 ft is

\[
DLS = \frac{1}{r_{pipe}} \left(\frac{12)(100)(180)}{\pi}\right)
\]

(6.7)

Where, \(r_{pipe}\) is corrected for the centerline of the pipe as

\[
r_{pipe} = r - \text{OD}/2
\]

(6.8)

6.2.3 Contact Condition 2

This set of conditions assumes the tubular to be tangent to the lower inner wall of the casing, touching the lower end of the three-degree ramp of the whipstock, and touching the beginning of the milled window as shown in fig 6.2. This solution is valid if the path of the tubular does not overlap the face of the whipstock of the wall of the hole below the whipstock.

6.2.3.1 Assumptions

1. The tubular was assumed to contact point 2, shown in fig 6.2.
2. To be tangent to line 1, which is the casing inner wall.
3. The tubular was assumed to contact point 4 shown in fig 6.2.

6.2.3.2 Equations to Calculate the Radius of Curvature

The equation used to calculate the radius of curvature for the above set of conditions, as explained in Appendix-III is

\[
r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

(6.9)
Where,

\[ a = p_3^2 \quad (6.10) \]

\[ b = 2xp_4^2 - 20p_4^2 + 2p_2p_3 + 2yp_3p_4 + 2OD \quad (6.11) \]

\[ c = x^2p_4^2 - 20xp_4^2 + 100p_4^2 + y^2p_4^2 - 2yp_2p_4 + p_2^2 - OD^2 \quad (6.12) \]

If \((u, v)\) are the coordinates of point 4 shown in fig 6.2, then

\[ p_2 = \left( \frac{u^2 - x^2 + v^2 - y^2 + OD^2 + 20x - 20u}{2} \right) \quad (6.13) \]

\[ p_3 = 2(x - u - v) \quad (6.14) \]

\[ p_4 = 2(v - y) \quad (6.15) \]

The dogleg severity is calculated using equation (6.7).

**Fig 6.2: Assumed Conditions for Tubular Curvature Calculations for Condition 2**
6.2.4 Contact Condition 3

The conditions used for dogleg severity calculations assume the tubular to be tangent to upper inside wall of the casing and the lower side of the predicted borehole wall, below the end of the inclined ramps of the whipstock. It also assumes that the tubular contacts the casing at the beginning of the milled window in the casing as shown in fig 6.3. This solution is only valid if the tubular does make tangential contact with the low side of the predicted borehole below the whipstock.

![Fig 6.3: Assumed Conditions for Tubular Curvature Calculations for Condition 3](image)

6.2.4.1 Assumptions

1. The tubular was assumed to be in contact with point 2 as shown in fig 6.3,
be tangent to line 1, which is the lower inside wall of the casing, and be tangent to line 4, as shown in Fig 6.3.

The equations used for calculation of the radius of curvature and the dogleg severity is very similar to that used for curvature calculations using conditions for set 1.

To calculate the equation of ‘line 4’, we have to first calculate the slope of ‘line 4’, assuming any two points on it and then the constant, by substituting the value of the slope and the co-ordinates of the point in it.

6.3 Results

6.3.1 Contact Condition 1

Fig 6.4 shows a parabolic trend of the DLS versus the diameter of the downhole tubular, using the equations for condition 1.

This solution is valid only if the low side of the tubular is tangent to line 3, on the last three-degree ramp. This is true for a larger OD tubular as indicated by the solid line on the graph. The rest of the curve is not considered, because for smaller diameter, the tangency point of the arc representing the tubular curvature pipes is below the end of the last three-degree ramp.

6.3.2 Contact Condition 2

The DLS calculated as a function of the pipe diameter for condition 2 also shows a parabolic trend. The DLS is smaller than that for condition 1, for smaller pipe diameters. This is logical for pipe diameters whose tangent point with line 3 is calculated to be below the end point 4, of the whipface for condition 1. Therefore the results calculated for condition 2 are valid for intermediate pipe diameters as shown by the solid line in fig 6.4. The dotted section of the predicted DLS curve is not considered because, for tubulars with larger diameters, the
arc of a circle representing the tubular curvature would be tangent at a point above the end of the last three-degree ramp of the whipstock. The arc representing tubulars with smaller diameters would make a tangency point on the low-side of the borehole below the whipstock. The parabolic trends of the calculated DLS for conditions 1 and 2 converge and meet at the point where the pipe is both tangent to and in contact with the end of the whipface.

6.3.3 Contact Condition 3

The results for condition 3 are also shown in fig 6.4. The DLS predictions are only valid for very small pipe diameters being run in the sidetracked borehole. This happens because, for larger
pipe diameters, the low side of the tubular would be to the left of line 4, thus making it impossible for the circle representing the low side of the tubular to be tangent to line 4.

6.4 Composite Results

Fig 6.5 shows a composite graph, which is the minimum possible DLS predicted for the sidetracked borehole as a function of the tubular size being run. This composite result was based on the useable, solid line, portions of the DLS predictions in Fig 6.4 for each assumed set of conditions.

![Fig 6.5: Composite Results of DLS Predictions](image)

6.5 Conclusions

1. The predicted composite DLS curve represents the radius of curvature of the tubulars, run in the sidetracked borehole trajectory as a function of tubular diameter. The radius
of curvature expressed as a dogleg severity in degrees per 100 ft. The radius was calculated relative to the pipe diameters because of which the tubular might contact the end of the whipstock, or the tubular could become tangent to the last three-degree ramp of the whipstock, or the tubular could become tangent to the face of the borehole below the whipstock.

2. The composite DLS curve assumes the tubular is contacting the inside upper wall of the casing above where the milled-window begins. Hence, the composite DLS curve cannot be used for cases where the tubular does not contact the inside upper casing wall.

3. One of the shortcomings of the composite DLS curve is that it assumes the sidetracked borehole to be a single bend instead of a compound bend as predicted by the simulator in chapter 4. That is, the predicted trajectory does not have a single upward curvature, but instead it drops the angle below the end of the whipstock making two curvatures of different radii.

4. Nevertheless, taking into consideration the shortcomings and assumptions in developing the composite DLS curve, it can be used as a representative value of the DLS of a specific size tubular being run in the predicted trajectory for a sidetracked borehole to compare with the allowable DLS for that size tubular. The concept of an allowable DLS for a specific size tubular is introduced in chapter 2.
7. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

7.1 Summary

A computer program was developed that performs the BHA analysis necessary for predicting the trajectory cut by a mill and whipstock assembly used to perform a cased hole sidetrack. The program utilizes the Jiazhi model to calculate the forces developed on the mills, which are then used to predict the path traversed by each mill. Further, the program validates the position of each mill relative to the side forces developed on the mills and the borehole geometry. The program then calculates and plots the position and window width cut by each mill, thus making it possible to observe the borehole cross sectional geometry and length, and the window profile created during the sidetracking operation. The program was run for selected well geometries, tool configurations and resistance to sidetracking in order to understand the effects of these variables on the predicted trajectories. A method was developed to calculate the curvature, expressed as dogleg severity, for a specific pipe diameter, based on an assumed set of contact and tangency points of the pipe with the casing and the predicted borehole trajectory.

7.2 Conclusions

1. The Jiazhi model was used to predict trajectories for cased hole sidetracking operations with a mill and whipstock that satisfactorily match qualitative expectations based on physical shop test results for these sidetrack systems.
2. The simulator for all the selected cases predicts an enlarged window length and an elliptical borehole geometry, which was expected based on physical tests conducted by Smith Services.
3. The simulator predicts an overall dropping tendency for the trimill assembly during the sidetracking operation. Therefore it shows a strong tendency for the lead mill to follow the face of the whipstock and then to drop angle below the whipstock.

4. None of the cases studied show any tendency for the lead mill to prematurely leave the face of the whipstock. Hence, some other factor must be contributing to the lead mill prematurely leaving the whipface. It may be due to the interaction of the mill shape with the casing window and/or the rock or cement in the casing-hole annulus.

5. The trajectory predictions provide an appropriate basis for evaluating the actual dogleg severity associated with a sidetrack using a whipstock

6. The DLS applying to a specific size tubular run in the predicted trajectory for a sidetracked borehole was calculated based solely on geometry. This dogleg severity can be used with separate analysis to determine the feasibility of running particular size tubular through the sidetrack.

7.3 Recommendations

1. The same value of the side force was assumed to be required by the mills to cut both the rock and the casing. The program should be improved by being modified to incorporate different side forces required for the mills to cut rock and the casing.

2. Representative values of the side force required for a mill to cut a known strength rock and casing must be obtained from instrumented shop tests of the sidetracked systems for use in the improved programs.
3. The validity of the predictions using this program should be verified. The borehole geometry and window width should be predicted for the conditions in the instrumented tests and compared to the actual measured results of those tests.
REFERENCES


## APPENDIX-I

### SIMULATOR OUTPUTS FOR THE CASES STUDIED – SENSITIVITY

#### Table A
Inputs and dimensions for 9 5/8” casing sidetrack prediction

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#### Table B
Inputs and dimensions for 7” casing sidetrack prediction

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LM rides on the second fifteen-degree ramp

| -1230.59 | 1065.477 | 120.8358 |
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| -1237.56 | 1068.436 | 127.3025 |
| -1239.85 | 1069.016 | 129.4658 |
| -1242.11 | 1069.572 | 131.6104 |
| -1244.34 | 1069.126 | 133.7148 |
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| -1253.21 | 1067.242 | 141.2646 |
| -1255.42 | 1062.762 | 143.4756 |
| -1257.62 | 1053.297 | 145.7009 |
| -1259.81 | 1063.297 | 147.9364 |
| -1262.00 | 1064.312 | 150.1926 |
| -1264.19 | 1064.828 | 153.4318 |
| -1266.37 | 1065.330 | 156.6993 |
| -1268.56 | 1065.851 | 159.9743 |
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| -1275.07 | 1067.392 | 169.8616 |
| -1277.23 | 1067.891 | 173.1753 |
| -1279.59 | 1068.440 | 176.5024 |
| -1281.15 | 1068.907 | 179.8609 |
| -1283.71 | 1069.414 | 183.2386 |
| -1285.65 | 1069.912 | 186.6668 |
| -1288.56 | 1070.426 | 189.6896 |
| -1290.14 | 1070.993 | 192.1011 |
| -1292.50 | 1071.434 | 194.1213 |

LM rides on the second three-degree ramp

| -574.27 | 264.1193 | 290.8623 |
| -574.30 | 264.0959 | 290.4949 |
| -574.103 | 264.0722 | 290.7686 |
| -574.022 | 264.0492 | 290.2733 |
| -574.021 | 264.0261 | 290.1791 |
| -574.022 | 264.0035 | 290.0693 |
| -574.023 | 263.9811 | 289.9337 |
| -574.026 | 263.9569 | 289.8026 |
| -574.036 | 263.937 | 289.8123 |
| -574.054 | 263.9153 | 289.7231 |
| -574.046 | 263.8936 | 289.6348 |
| -574.334 | 263.8516 | 289.4612 |
| -574.326 | 263.8308 | 289.3768 |
| -574.192 | 263.8103 | 289.2914 |
| -574.122 | 263.7896 | 289.2078 |
| -574.054 | 263.7699 | 289.1262 |
| -574.06 | 263.7365 | 288.9435 |
| -574.919 | 263.7004 | 288.7527 |
| -574.903 | 263.6710 | 288.8827 |
| -574.788 | 263.6917 | 288.6037 |
| -574.724 | 263.6727 | 288.7265 |
| -574.861 | 263.6639 | 288.6481 |
| -574.598 | 263.6353 | 288.5716 |
| -572.537 | 263.6169 | 288.4966 |
| -572.476 | 263.5907 | 288.4212 |
| -572.416 | 263.5807 | 288.3471 |
| -572.267 | 263.5629 | 288.2739 |
| -572.241 | 263.5279 | 288.1338 |
Forces on mills for 7” casing sidetrack prediction

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LM rides below the end of the whipface
Forces shown when the FM and DM touch the whipface

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Force} & \text{Total Force} & \text{Direction} \\
\hline
-457.95 & 259.104 & 222.1706 \\
-461.103 & 259.1813 & 222.4536 \\
-464.407 & 259.2575 & 222.7386 \\
-467.079 & 259.3327 & 223.0186 \\
-470.033 & 259.4068 & 223.2907 \\
-473.256 & 259.4790 & 230.561 \\
-476.532 & 259.5516 & 233.0274 \\
-479.866 & 259.6228 & 234.039 \\
-483.146 & 259.6937 & 240.349 \\
-486.423 & 259.7617 & 246.043 \\
-489.869 & 259.8397 & 252.046 \\
-493.323 & 259.9168 & 258.289 \\
-496.911 & 260.0026 & 265.902 \\
-499.754 & 260.0822 & 273.625 \\
-502.005 & 260.1656 & 280.628 \\
-504.258 & 260.2489 & 287.639 \\
-506.504 & 260.3306 & 294.652 \\
-508.749 & 260.413 & 282.766 \\
-510.981 & 260.4961 & 289.686 \\
-513.229 & 260.5791 & 297.182 \\
-515.464 & 260.6617 & 304.202 \\
-517.697 & 260.7447 & 311.615 \\
-519.930 & 260.8283 & 319.231 \\
\hline
\end{array}
\]

LM rides below the end of the whipface
Forces shown when the FM and DM touch the whipface

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Force} & \text{Total Force} & \text{Direction} \\
\hline
-463.914 & 267.3448 & 233.5327 \\
-463.909 & 267.3434 & 233.5327 \\
-463.903 & 267.3421 & 233.5327 \\
-463.898 & 267.3407 & 233.5327 \\
-463.893 & 267.3394 & 233.5327 \\
-463.889 & 267.3382 & 233.5327 \\
-463.878 & 267.3369 & 233.5327 \\
-463.873 & 267.3357 & 233.5327 \\
-463.864 & 267.3333 & 233.5327 \\
-463.854 & 267.3311 & 233.5327 \\
-463.845 & 267.3299 & 233.5327 \\
-463.836 & 267.3287 & 233.5327 \\
-463.827 & 267.3275 & 233.5327 \\
-295.6553 & -565.82 & 242.8674 \\
-294.5229 & -564.303 & 242.6402 \\
-293.2331 & -563.875 & 242.6413 \\
-292.6111 & -563.444 & 242.6454 \\
-292.0556 & -562.986 & 242.6398 \\
-291.8343 & -562.491 & 242.6246 \\
-291.8343 & -562.326 & 242.6246 \\
-291.8161 & -561.777 & 242.6246 \\
-291.7211 & -561.431 & 242.6246 \\
-291.5758 & -561.313 & 242.6246 \\
-291.4502 & -561.202 & 242.6246 \\
-291.3567 & -561.202 & 242.6246 \\
-291.4677 & -561.202 & 242.6246 \\
\hline
\end{array}
\]

LM rides below the end of the whipface
Forces shown when the FM and DM touch the whipface

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Force} & \text{Total Force} & \text{Direction} \\
\hline
-1243.76 & 1056.618 & 253.7486 \\
-1246.13 & 1056.223 & 253.4376 \\
-1248.53 & 1056.827 & 252.9839 \\
-1250.91 & 1056.43 & 252.0705 \\
-1253.29 & 1056.032 & 252.1632 \\
-1255.66 & 1056.634 & 252.3335 \\
-1258.01 & 1056.236 & 252.524 \\
-1260.37 & 1055.835 & 252.7256 \\
-1262.73 & 1055.434 & 252.9221 \\
-1265.08 & 1055.033 & 253.5138 \\
-1267.43 & 1056.63 & 262.6116 \\
-1269.77 & 1061.227 & 279.0584 \\
-1272.11 & 1061.823 & 286.6652 \\
-1274.45 & 1062.418 & 293.641 \\
-1276.78 & 1063.012 & 299.5128 \\
-1279.11 & 1063.606 & 306.1906 \\
\hline
\end{array}
\]

LM rides on the second fifteen-degree ramp

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Force} & \text{Total Force} & \text{Direction} \\
\hline
-1246.51 & 1055.223 & 256.0712 \\
-1248.53 & 1056.827 & 258.39 \\
-1250.91 & 1056.43 & 260.705 \\
-1253.29 & 1056.032 & 263.0612 \\
-1255.66 & 1056.634 & 265.3335 \\
-1258.01 & 1056.236 & 267.6327 \\
-1260.37 & 1055.835 & 269.9265 \\
-1262.73 & 1055.434 & 272.2221 \\
-1265.08 & 1056.033 & 274.5138 \\
-1267.43 & 1060.63 & 276.8116 \\
-1269.77 & 1061.227 & 279.0584 \\
-1272.11 & 1061.823 & 286.6652 \\
-1274.45 & 1062.418 & 293.641 \\
-1276.78 & 1063.012 & 299.5128 \\
-1279.11 & 1063.606 & 306.1906 \\
\hline
\end{array}
\]
APPENDIX-II

EQUATIONS FOR CALCULATING RADIUS OF CURVATURE USING CONDITION

SET 1

Consider fig 1, the equation of line 1 is

\[ x = 10 \]  \hspace{1cm} (1)

For all the predictions the casing inner wall starts at \( x = 10 \).

Let equation of line 3 be

\[ y = m_1 x + c_1 \]  \hspace{1cm} (2)

The slope of line 3 can be approximated to

\[ m_1 = \tan(\varepsilon) \]

Hence the equation of line 3 becomes,

\[ y = x \tan(\varepsilon) + c_1 \]  \hspace{1cm} (4)

Fig 1: Diagram showing condition set 1
As the center would be at a distance $r$ from line 1 as well as line 3 (as the circle is tangent to both of them), the equation of the new ‘line 1a’, shown in fig 1, would be

$$x = r + 10$$

(5)

As ‘line 3a’ is at a perpendicular distance ‘$r$’ from ‘line 3’ the constant term in ‘line 3a’ would have $r/\cos (\varepsilon)$, in addition to $c_1$ as shown in fig 2. The angle ‘$\varepsilon$’ is considered in the anti-clockwise direction from the horizontal x-axis.

Hence the equation of ‘line 3a’ is given by

$$y = x \tan(\varepsilon) + c_1 - \frac{r}{\cos(\varepsilon)}$$

(6)

![Diagram showing development of line 3a](image)

**Fig 2: Figure illustrating development of line 3a**

The point at which these two lines meet is given by substituting equation (5) in (6) given as

$$y = r \left( \tan(\varepsilon) - \frac{1}{\cos(\varepsilon)} \right) + (c_1 + 10 \tan(\varepsilon))$$

(7)
Let, \[ z_1 = \left( \tan(\varepsilon) + \frac{1}{\cos(\varepsilon)} \right) \] and
\[ z_2 = (c_1 + 10\tan(\varepsilon)) \]

Hence equation (70 can be written as

\[ y = rz_1 + z_2 \] \hspace{1cm} (7a)

Now if \((h, k)\) is the center of the circle, then
\[ h = r + 10 \] \hspace{1cm} (8)

And
\[ k = rz_1 + z_2 \] \hspace{1cm} (9)

If \(OD\) is the outer diameter of the tubular being run in the sidetracked borehole, then the equation of circle becomes

\[ (x - h)^2 + (y - k)^2 = (r - OD)^2 \] \hspace{1cm} (10)

Now, on substituting (8) and (9) in equation (10) and solving for ‘\(r\)’ we get

\[ r^2a + rb + c = 0 \] \hspace{1cm} (11)

Which we solve for ‘\(r\)’ as

\[ r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \] \hspace{1cm} (12)

Where
\[ a = z_2^2 \] \hspace{1cm} (13)
\[ b = 2(z_1z_2 - yz_2 + 10 - x) + OD \] \hspace{1cm} (14)
\[ c = x^2 + y^2 + z_1^2 - 2yz_1 - 20x + 100 - OD^2 \] \hspace{1cm} (15)

Where \(x\) and \(y\) are the coordinates of point 2 shown in fig 1, chapter 6.
APPENDIX-III

EQUATIONS FOR CALCULATING RADIUS OF CURVATURE USING CONDITION

SET 2

Consider fig 1. Let (u, v) be the coordinates of point 4 shown in fig 2 in chapter 6. Consider the circle to pass through (x, y), which are the coordinates of point 2, in fig 2 in chapter 6.

Let, (h, k) represent the center of the circle. The equation of the circle is passing through (x, y) is

\[(x-h)^2 + (y-k)^2 = (r-OD)^2 \] \hspace{1cm} (1)

Similarly, the equation of the circle passing through point (u, v) is

\[(u-h)^2 + (v-k)^2 = (r-OD)^2 \] \hspace{1cm} (2)

Equation of line 1 is

\[x = 10 \] \hspace{1cm} (3)

Hence,

\[h = 10 + r \] \hspace{1cm} (4)

Solving (1) and (2) and substituting (4) we get

\[k = \frac{p_2 + rp_3}{p_4} \] \hspace{1cm} (5)

Where,

\[p_2 = \left( \frac{u^2 - x^2 + v^2 - y^2 + OD^2 + 20x - 20u}{2} \right) \] \hspace{1cm} (6)
Substituting equations (4) and (5) in equation (1) and solving for $r$, we get

$$r^2a + rb + c = 0$$

Which can be solved as

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where,

$$a = p_3^2$$

$$b = 2xp_4^2 - 20p_4^2 + 2p_2p_3 + 2yp_3p_4 + 2OD$$
\[ c = x^2 p_4^2 - 20x p_4^2 + 100 p_4^2 + y^2 p_4^2 - 2yp_2p_4 + p_2^2 - OD^2 \]  \hspace{1cm} (13)

In equation (10) calculates the radius of curvature of the tubular, run in the sidetracked borehole.
VITA

Harshad Patil was born to Prakash Patil and Charusheela Patil in Pune, India, on September 16, 1980. Harshad received his primary and secondary education in Pune. He subsequently graduated from the Maharashtra Institute of Technology, University of Pune, in India, with a Bachelor of Engineering degree in petroleum engineering (June, 2001).