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We study a mixture of s-bosons and like-nucleon pairs with the standard pairing interaction outside an inert core. Competition between the nucleon-pairs and s-bosons is investigated in this scenario. The robustness of the BCS-BEC coexistence and crossover phenomena are examined through an analysis of pf-shell nuclei with realistic single-particle energies, in which two configurations with Pauli blocking of nucleon-pair orbits due to the formation of the s-bosons is taken into account. When the nucleon-pair orbits are considered to be independent of the s-bosons, the BCS-BEC crossover becomes smooth, with the number of the s-bosons noticeably more than that of the nucleon-pairs near the half-shell point, a feature that is demonstrated in the pf-shell for several values of the standard pairing interaction strength. As a further test of the robustness of the BCS-BEC coexistence and crossover phenomena in nuclei, results are given for B(E2; $0_1^+ \to 2_1^+$) values of even-even $^{102-130}$Sn with ^{100}Sn taken as a core and valence neutron pairs confined within the $1d_{5/2}$, $0g_{7/2}$, $1d_{3/2}$, $2s_{1/2}$, $1h_{11/2}$ orbits in the nucleon-pair orbit and the s-boson independent approximation. The results indicate that the B(E2) values are reproduced well.

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Introduction: The interacting boson model (IBM) [1], like the collective model [2] of Bohr and Mottelson and the microscopic shell model [3, 4], has been successful in providing a logical framework for studying the structure of atomic nuclei, at least in the low-energy regime. Many studies show that the $J = 2$ d-bosons of the IBM are similar to the d-phonons that emerge naturally from a five-dimensional harmonic oscillator description of the quadrupole vibrations of the collective model. The addition of a $J = 0$ s-boson to the d-boson picture allows the IBM to accommodate couplings to low-energy monopole modes, which expands the U(5) algebra structure that underpins the five-dimensional oscillator to a six-dimensional U(6) algebra [5]. Within this expanded U(6) framework of the IBM, the total number of bosons is regarded as the number of valence particle or hole pairs [6, 7]. It is now commonly accepted that there is a close relationship between the s-bosons and $J = 0$ pairs of like-nucleons, since pairs of fermions in these systems often exhibit boson-like behavior [8, 9]. It has been shown that bosons may emerge from fermionic pairing due to spontaneous symmetry breaking of the Bardeen-Cooper-Schrieffer (BCS) type [10, 11]. However, a schematic study of the relative roles played by $J = 0$ nucleon pairs and s-bosons as employed in the IBM in nuclei is still lacking.

Generally, the pairing correlation in Fermi many-body systems can be understood in terms of attractive interactions among fermion pairs manifested by the BCS mechanism. When the attractive interaction between two fermions is strong enough, on the other hand, the two fermions may form a bosonic bound state with Bose-Einstein condensation (BEC) in the ground state of the system. As shown in [12–17], a smooth crossover from a Cooper-paired state to a Bose condensate state of tightly bound pairs takes place in the continuum model of a Fermi gas at zero temperature. In [18], the coexistence of the BCS and the BEC-like pair structures in ^{11}Li was investigated based on phase space analysis. Inspired by the above developments, we consider a mean-field plus standard pairing model including s-bosons, where the s-bosons are regarded as tightly bound nucleon pairs, which should also emerge naturally in nuclei according to the aforementioned observations.

The model: Since some nucleon-pairs in a nucleus may behave like bosons, we consider a mixture of s-bosons and like-nucleon pairs with the standard pairing interaction outside an inert core. A schematic Hamiltonian considered is given by

$$\hat{H} = \sum_j \epsilon_j \hat{N}_j - G \sum_{j,\,j'} S_j^+ S_{j'}^- + \alpha(\hat{n}_s) - r\, G\, \sqrt{\Omega} \sum_j (S_j^+ s + s^\dagger S_j^-), \quad (1)$$

where j and j' run over p distinct orbits considered, $s^\dagger$ (s) is the s-boson creation (annihilation) operator, $\{\epsilon_j\}$ is a set of single-particle energies generated from any mean-field theory, $\hat{N}_j = \sum_m a_{jm}^\dagger a_{jm}$ and $S_j^+ = \sum_{m>0} (-1)^{j-m} a_{jm}^\dagger a_{j-m}^\dagger$ ($S_j^- = (S_j^+)^\dagger$), in which $a_{jm}^\dagger$ (a_{jm}) is the creation (annihilation) operator for a nucleon with angular momentum quantum number j and that of its projection m, $G > 0$ is the overall pairing interaction strength, r is a scale factor used to describe the interaction between s-boson and nucleon pairs, $\Omega = \sum_j (j + 1/2)$ is the maximum pair-occupancy of the p-orbit system, and the pure s-boson part $\alpha(\hat{n}_s)$, which is a function of the s-boson number operator $\hat{n}_s = s^\dagger s$, is adjusted to re-

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produce the ground state energy of the Hamiltonian with the first two terms of the nucleon-pair sector for given G. With this assumption, the ground state energy of the system in either the pure BCS phase or the pure BEC phase is exactly the same. In (1), the interaction between the s-boson and the nucleon pairs is the same as that among nucleon pairs when $r = 1$. The factor $\sqrt{\Omega}$ appears in the last term of (1) because the nucleon pair operator $S^{+} \sim \sqrt{\Omega} s^{\dagger}$, where $S^{+} = \sum_j S_j^{+}$ and $S^{-} = (S^{+})^{\dagger}$, when the number of nucleon-pairs k is far smaller than Ω with $k \ll \Omega$. Here and in the following, we assume that all the nucleons in the system are paired and that there is an even number of nucleons. It is obvious that the total number of the s-bosons and that of the nucleon pairs is a conserved quantity in the model.

Since the s-bosons originate from nucleon-pairs, many nucleon-pair orbits should be blocked due to the Pauli Exclusion Principle. In a single-j case for example, when there are n_s s-bosons, the maximal nucleon-pair occupancy should become $\tilde{\Omega}_j = j + 1/2 - n_s$. Thus, for given $n = k + n_s \leq \Omega$, the eigenstates of (1) may be written as

$$|n;\zeta\rangle = \sum_{k=0}^{n} \sum_{\xi_k} C_{k,\xi_k}^{(\zeta)} |n-k;\xi_k,k\rangle |n-k\rangle_s, \tag{2}$$

where ζ labels the ζ-th excitation state of the system, $|n-k;\xi_k,k\rangle$ is the eigenstate of the mean-field plus standard pairing model of the nucleon-pair sector with $\tilde{\Omega} = \Omega - (n-k)$.

Generally, if there are p orbits, the maximum number of distinct blocking patterns is equal to the number of partitions of $n-k$ into p integers $[\mu_1, \cdots, \mu_p]$ with $\sum_{i=1}^{p} \mu_i = n - k$ and $0 \leq \mu_i \leq \Omega_{j_i}$ for $i = 1, \cdots, p$, which results in the effective maximal nucleon-pair occupancy of each orbit for a given partition $[\mu_1, \cdots, \mu_p]$ to be $\tilde{\Omega}_{j_i} = \Omega_{j_i} - \mu_i$ for $i = 1, \cdots, p$. We take the $p = 2$ case with $j_1 = 3/2$ and $j_2 = 5/2$ as an example. If there is no s-boson with $n_s = 0$, the maximal pair occupancy of each orbit is given by $\Omega_1 = 2$ and $\Omega_2 = 3$, respectively. When $n_s = 1$, there are two blocking configurations. The first has $\tilde{\Omega}_1 = 1$ and $\tilde{\Omega}_2 = \Omega_2 = 3$, for which the s-boson is formed from two nucleons in the first orbit. The second has $\tilde{\Omega}_1 = \Omega_1 = 2$ and $\tilde{\Omega}_2 = 2$, for which the s-boson is formed from two nucleons in the second orbit. Let the nucleon-pair product states be denoted as $|n_1; n_2\rangle$, where n_1 and n_2 are the number of pairs with $0 \leq n_1 \leq 2$ and $0 \leq n_2 \leq 3$, which should all be needed in diagonalizing the first two terms of (1) when $n_s = 0$. However, $|n_1 = 2; n_2 = 0\rangle$ should be ruled out in diagonalizing the first two terms of (1) for $n_s = 1$ in the first blocking configuration, while $|n_1 = 0; n_2 = 3\rangle$ should be ruled out in diagonalizing the first two terms of (1) for $n_s = 3$ in the second blocking configuration. Namely, the dimension of the subspace spanned by the nucleon-pair product states will be reduced due to Pauli blocking.

The eigenstates of the first two terms of (1), $|n-k;\xi_k,k\rangle$, appear in (2) with a given number of nucleon-pairs k for distinct blocking patterns. However, they are not linearly independent since they are all k-pair states. In order to treat this situation more precisely, we need to introduce a mixed state description if the probability of a given partition of the blocking is known. In this work, we still treat this feature within a pure state description for simplicity. Hence, only one of the blocked configurations for a given k is considered.

As a relatively simple example, we set $r = 1$ and consider a pf-shell system with the 4 orbitals $0f_{7/2}$, $1p_{3/2}$, $1p_{1/2}$, and $0f_{5/2}$ assigned the single-particle energies deduced in [19]; that is, we set $\epsilon_{7/2} = -8.624\,\mathrm{MeV}$, $\epsilon_{3/2} = -5.6793\,\mathrm{MeV}$, $\epsilon_{1/2} = -4.137\,\mathrm{MeV}$, and $\epsilon_{5/2} = -1.3829\,\mathrm{MeV}$ in the Hamiltonian (1). Then the eigen-equation used to determine the eigen-energies $E_n^{(\zeta)}$ of (1) and the corresponding expansion coefficients $C_{k,\xi_k}^{(\zeta)}$ of (2) are given by

$$\begin{gathered}\tfrac{1}{G}\left(E_k^{(\xi_k)} + \alpha(n-k) - E_n^{(\zeta)}\right) C_{k,\xi_k}^{(\zeta)} = \\ \sqrt{\Omega(n-k+1)} \textstyle\sum_{\xi_{k-1}} C_{k-1,\xi_{k-1}}^{(\zeta)} \times \\ \langle n-k;\xi_k,k|S^{+}|n-k+1;\xi_{k-1},k-1\rangle \\ +\sqrt{\Omega(n-k)} \textstyle\sum_{\xi_{k+1}} C_{k+1,\xi_{k+1}}^{(\zeta)} \times \\ \langle n-k;\xi_k,k|S^{-}|n-k-1;\xi_{k+1},k+1\rangle\end{gathered} \tag{3}$$

for $k = 0,\ 1,\ \cdots,\ n$, where $E_k^{(\xi_k)}$ is the ξ_k-th k-pair excitation energy of the original mean-field plus standard pairing Hamiltonian of the nucleon-pair sector with $\tilde{\Omega} = \Omega - n + k$ for a fixed partition $[\mu_1, \cdots, \mu_p]$ of the blocking, and the matrix elements $\langle n-k;\xi_k,k|S^{+}|n-k+1;\xi_{k-1},k-1\rangle = \langle n-k+1;\xi_{k-1},k-1|S^{-}|n-k;\xi_k,k\rangle^{*}$ can be calculated when all excited states of the original mean-field plus standard pairing model for the nucleon-pair sector are obtained. For a given number of nucleon-pairs k, we calculate all excitation energies and the corresponding eigenstates of the original mean-field plus standard pairing Hamiltonian of the nucleon-pair sector with $\tilde{\Omega} = \Omega - n + k$ for a fixed partition of the blocking, which can be done by using the the Heine-Stieltjes polynomial approach [20].

After all excitation energies $\{E_k^{(\xi_k)}\}$ and the corresponding eigenstates of the nucleon-pair part of the model are obtained for $1 \leq k \leq \Omega$ with $\Omega = 10$ for this case, we simply set $\alpha(n_s) = E_k^{(\xi_k=1)}$ with $k = n_s$ for a given value of G. The orbits are then arranged in order according to the values of the single-particle energies with $\epsilon_{j_1} < \cdots < \epsilon_{j_p}$. Specifically, we consider two blocking configurations. In the first, the blocking to the nucleon-pair orbits results in the configuration with the lowest orbits blocked, namely, we take μ_1 to be the largest possible integer, then μ_2 to be the remaining largest possible integer, and so on. In the second configuration, the

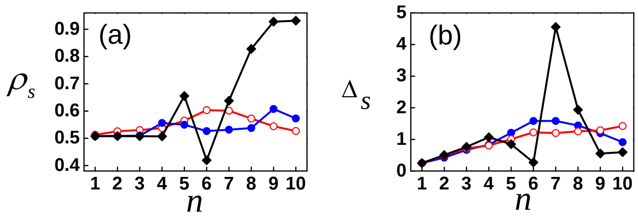

FIG. 1: (Color online) The ground-state s-boson occupation probability ρ_s (a) and the ground-state s-boson number fluctuation Δ_s (b) as functions of n for different blocking configurations considered in the pf-shell with $r = 1$ and $G = 1.4\,\text{MeV}$. The solid diamonds denote the results of the first blocking configuration, the solid dots denote the results of the second configuration, and the open circles denote the results with the nucleon-pair orbit and the s-boson independent approximation.

blocking to the nucleon-pair orbits results in the configuration with the highest orbits blocked; namely, we take μ_p to be the largest possible integer, then μ_{p-1} to be the remaining largest possible integer, and so on. More precisely, in the first blocking configuration, when $n_s \leq 4$, we assume the $j_1 = 7/2$ orbit is blocked because it is the lowest in energy. When $5 \leq n_s \leq 6$, we assume the $j_1 = 7/2$ and $j_2 = 3/2$ orbits are blocked, and so on. Similarly, in the second blocking configuration, when $n_s \leq 3$, we assume the $j_4 = 5/2$ orbit is blocked because it is the highest in energy. When $n_s = 4$, we assume the $j_4 = 5/2$ and $j_3 = 1/2$ orbits are blocked, and so on.

In the analysis, the pairing interaction strength $G = 1.4\,\text{MeV}$ is fixed. Once (3) is solved numerically, we calculate the ground-state occupation probability ρ_s of the s-bosons

$$\rho_s = \frac{1}{n}\langle n;\zeta=1|\hat{n}_s|n;\zeta=1\rangle \tag{4}$$

and the ground-state s-boson number fluctuation defined by

$$\Delta_s = \sqrt{\langle n;\zeta=1|\left(\hat{n}_s - n\rho_s\right)^2|n;\zeta=1\rangle}. \tag{5}$$

As shown in Fig. 1(a), the s-boson occupation probability ρ_s is always greater than 40%, increasing from 40% to 93% for $n = 6$ to $n = 10$ in the first blocking configuration. In the second blocking configuration it is always greater than 50%, increasing from 50% to 60% over the range of n values shown. The staggering in the occupation probability happens near the half-filling point in the first blocking configuration. In both cases, the s-boson occupation probability reaches its maximal value near the shell closure point.

The results for the nucleon-pair orbit and the s-boson independent approximation are also shown in Fig. 1 for comparison, with the eigenstates of the mean-field plus standard pairing model of the nucleon-pair sector $\{|n-k;\xi_k,k\rangle\}$ used in (2) replaced by $\{|0;\xi_k,k\rangle\}$ for any n and k with the maximal nucleon-pair occupancy of each orbit unchanged, namely, the blocking to the nucleon-pair orbits due to the formation of the s-bosons is not considered. In this case, there is a critical region of the BCS-BEC crossover, which tracks with a range of n values near the half-filling of the shell where ρ_s reaches its maximal value. Though the BCS-BEC crossover behavior in these three cases are different, the BCS-BEC coexistence seems robust. In any case, once the s-bosons emerge in the system with the strong pairing interaction shown in [14–17], which prefers to occur in a dilute fermion-pair environment [14–18], the s-boson content, in general, changes noticeably with increasing n due to the interaction between nucleon-pairs and the s-bosons. This reinforces the BCS-BEC crossover even when the number of nucleon-pairs becomes large. In addition, as further shown in Fig. 1(b), the s-boson number fluctuation also changes rapidly with n, indicating that the nucleon-pair constituent is also significant, which becomes more noticeable near the half-filling point. Therefore, once the s-boson emerges in the system, the pure BEC phase never occurs except in the first blocking configuration in the large Ω-limit. Rather, the system always seems to be in a BCS-BEC coexistence phase, which is robust for any finite Ω and n, a feature that is consistent with the conclusion made in [18], with the s-boson content greater than that of the nucleon-pairs in general. Thus, the s-boson formation induced by the pairing interaction with fewer nucleon-pairs and the further BCS-BEC crossover enhancement with increasing nucleon-pairs seems to track with the emergence of s-bosons in a nucleus.

Since we currently lack probability distribution information for different blocking configurations, the nucleon-pair orbit and the s-boson independent approximation is adopted in the following. The s-boson occupation probability ρ_s and the s-boson number fluctuation in the pf-shell for $G = 0.2\,\text{MeV}$, $0.6\,\text{MeV}$, $1.0\,\text{MeV}$, and $2.0\,\text{MeV}$ with the approximation are shown in Fig. 2, in which the scale factor $r = 1$ is still taken. It should be noted that G ranges from $0.4 \sim 0.7\,\text{MeV}$ in the pf-shell in order to reproduce the GXPF1 $J = 0$ and $T = 1$ pairing excitation spectrum [21]. Hence, the results shown in Fig. 2 not only provide information about realistic situations, but also show results for a slightly weaker pairing interaction with $G = 0.2\,\text{MeV}$ and a stronger pairing interaction with $G = 2.0\,\text{MeV}$. As shown in Fig. 2(a), similar to the case shown in Fig. 1, the ground-state s-boson occupation probability ρ_s is always greater than 50%. There is a critical region of the BCS-BEC crossover, which tracks with a range of n values near the half-filling of the shell where ρ_s reaches its maximal value. The critical region moves towards a larger n interval with $n > \Omega/2$ when the pairing interaction becomes stronger. It is also clear that the maximal value of ρ_s increases with increasing G, and is about 62% when $G = 2.0$ MeV. As can be seen in Fig. 2(b), the largest fluctuation Δ_s also occurs in the critical region, within which ρ_s reaches its maximal value. Actually, both the s-bosons and the nucleon-pairs are most non-localized in the crossover region in this case.

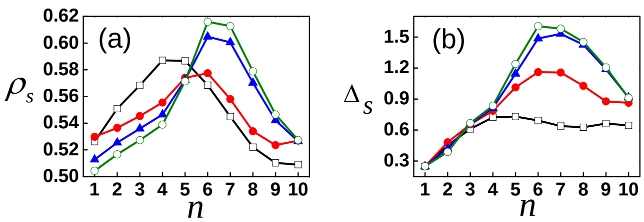

FIG. 2: (Color online) The same as Fig. 1, but for different G values in the nucleon-pair orbit and the s-boson independent approximation. The open squares denote the results with $G = 0.2$ MeV, the solid dots denote the results with $G = 0.6$ MeV, the solid triangles denote the results with $G = 1.0$ MeV, and the open circles denote the results with $G = 2.0$ MeV.

An example of application: As an example of an application of the theory with the nucleon-pair orbit and the s-boson independent approximation, we use the model to fit 2_1^+ level energies and then to calculate B(E2; $0_1^+ \rightarrow 2_1^+$) values of even-even $^{102-130}$Sn, which have attracted a lot of attention both experimentally and theoretically [22–27]. In our calculation, ^{100}Sn is considered as the core of these nuclei with valence neutron pairs confined to the $1d_{5/2}$, $0g_{7/2}$, $1d_{3/2}$, $2s_{1/2}$, and $1h_{11/2}$ orbits with single-particle energies $\epsilon_{5/2} = 0.00$ MeV, $\epsilon_{7/2} = 0.08$ MeV, $\epsilon_{3/2} = 1.66$ MeV, $\epsilon_{1/2} = 1.55$ MeV, and $\epsilon_{11/2} = 3.55$ MeV, respectively [22]. Since the low-lying states in the Sn isotopes are almost spherical, the bosonic part of the Hamiltonian of (1) in this case is chosen as the IBM U(5) type with $\alpha(\hat{n}_s, \hat{n}_d) = \alpha(\hat{n}_s) + \beta(\hat{n}_d)$, where $\beta(\hat{n}_d)$ is a function of the number of d-bosons $\hat{n}_d$, and $\alpha(n_s) = E_{n_s}^{(\xi_{n_s}=1)}$ is still assumed. The total number of bosons and nucleon pairs, $n = n_s + n_d + k$, is a conserved quantity in this case. For a given n, the ground state is given by $|n, 0_1^+\rangle = |n, \zeta = 1\rangle$ determined by (2), while the 2_1^+ state, for simplicity, is assumed to be one d-boson state, of which the d-boson is assumed to be formed from two neutrons in the lowest $1d_{5/2}$ orbit. Thus, the 2_1^+ state $|n, 2_1^+, M\rangle = d_M^\dagger |n-1, \zeta = 1\rangle$, where the Pauli blocking in the $1d_{5/2}$ orbit is considered in $|n-1, \zeta = 1\rangle$ for $|n, 2_1^+, M\rangle$. Furthermore, the 2_1^+ level energy is simply given by $E(n, 2_1^+) = \beta(1) - \beta(0) + E_{n-1}^{(1)} - E_n^{(1)}$ for a given n, where $E_{n-1}^{(1)}$ is determined by (3) with the Pauli blocking in the $1d_{5/2}$ orbit considered. In the fitting, the parameters G and $\beta(1) - \beta(0)$ in the model are fixed for all nuclei considered with $G = 0.18$ MeV and $\beta(1) - \beta(0) = -10.617$ MeV, while the scale factor r is adjusted to reproduce the experimental 2_1^+ level energy exactly, of which the values for these nuclei are given in Table I. It is clearly shown that r deceases almost linearly as the total number of bosons and nucleon pairs, n, increases.

TABLE I: The parameter r used in fitting the 2_1^+ level energies (in MeV) of even-even $^{102-130}$Sn, for which the experimental values shown in [28] are used.

Nucleus	n	r	$E(2_1^+)$	Nucleus	n	r	$E(2_1^+)$
^{102}Sn	1	3.885	1.472	^{104}Sn	2	3.544	1.260
^{106}Sn	3	3.270	1.208	^{108}Sn	4	3.025	1.206
^{110}Sn	5	2.792	1.212	^{112}Sn	6	2.579	1.257
^{114}Sn	7	2.366	1.300	^{116}Sn	8	2.127	1.294
^{118}Sn	9	1.862	1.230	^{120}Sn	10	1.585	1.171
^{122}Sn	11	1.318	1.141	^{124}Sn	12	1.063	1.132
^{126}Sn	13	0.822	1.141	^{128}Sn	14	0.591	1.169
^{130}Sn	15	0.367	1.221				

The effective E2 operator in this case is defined as

$$T_\mu(\mathrm{E2}) = q_2 \left(\frac{1-\chi}{2} (d_\mu^\dagger s + s^\dagger \tilde{d}_\mu) + \frac{1+\chi}{2\sqrt{\Omega}} (d_\mu^\dagger S^- + S^+ \tilde{d}_\mu) \right), \quad (6)$$

where $d_\mu^\dagger$ is the d-boson creation operator, $\tilde{d}_\mu = (-1)^\mu d_{-\mu}$, in which d_μ is the d-boson annihilation operator, q_2 is the effective quadrupole parameter, and $\chi \in [-1, 1]$ is used. In the fitting, $\chi = 0.44$ and $q_2 = 0.0827$ eb are chosen from the best fit for all nuclei considered. The B(E2; $0_1^+ \rightarrow 2_1^+$) obtained from this theory and the corresponding experimental values, together with the results obtained from the large-scale shell model (LSSM) with the same shell model space consideration [22], are shown in Fig. 3(a), which indicates that the experimental data are well reproduced by this theory with the parameter r determined uniquely by the 2_1^+ level energy, except that the B(E2) values of ^{116}Sn and ^{130}Sn are a little larger than the corresponding experimental values. Overall, the B(E2; $0_1^+ \rightarrow 2_1^+$) values for even-even $^{102-114}$Sn obtained from this theory are much better than those obtained from the LSSM, while those for even-even $^{116-128}$Sn obtained from the LSSM are a little better. The corresponding ground-state s-boson occupation probability ρ_s for these nuclei is shown in FIG. 3(b), which indicates that ^{118,120}Sn, according to this theory, are within the critical region of the BCS-BEC crossover with the total number of s-bosons and neutron pairs a little larger than the half-filling point value.

Conclusion: In conclusion, the BCS-BEC coexistence and crossover phenomena in the nuclear mean-field plus

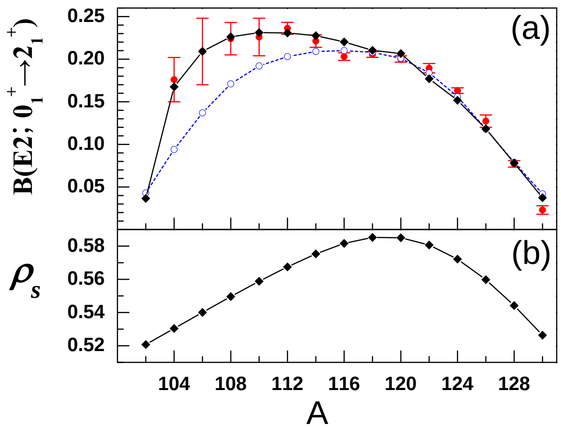

FIG. 3: (Color online) **(a)** B(E2; $0_1^+ \to 2_1^+$) values (in e^2b^2) of even-even $^{102-130}$Sn obtained in this theory and in comparison with the experimental data provided in [27] and the large-scale shell model (LSSM) results [22], where the solid dots with error-bar are the experimental values, the solid diamonds linked by a solid line are the values obtained in this theory, and the open circles linked by a dashed line are those calculated from the LSSM. **(b)** The ground-state s-boson occupation probability ρ_s in even-even $^{102-130}$Sn predicted in this model.

standard pairing interaction model involving the s-bosons is observed from the analysis of the ground state s-boson occupation probability. It is shown that, though the BCS-BEC crossover behavior may be different from one Pauli blocking configuration to another, the BCS-BEC coexistence seems robust, as demonstrated in the analysis for the pf-shell with realistic single-particle energies in two different blocking configurations or in the nucleon-pair orbit and the s-boson independent approximation. The crossover is further enhanced with increasing pairing interaction strength. In the example of the model application to even-even $^{102-130}$Sn, the B(E2) values are well reproduced by the model in the nucleon-pair orbit and the s-boson independent approximation with parameters determined by the experimental 2_1^+ level energy. A mixed state consideration related to the Pauli blocking effect in the model merits further study. Moreover, since BCS-BEC coexistence seems common in nuclei, similar to the interacting Boson-Fermion model for odd-A nuclei [29], in which, besides the s- and d-bosons, only a single nucleon degree of freedom is considered, a configuration which considers both the s- and d-bosons provided from the IBM and a few interacting valence nucleons in an effective mean-field, such as the shell model, may be a better description of the low-energy structure of nuclei. Further work along these directions is in progress.

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[1] F. Iachello and A. Arima, The Interacting Boson Model (Cambridge: Cambridge University Press, 1987).
[2] A. Bohr and B. R. Mottelson, Nuclear structure Vol. 2 (W. A. Benjamin, Reading, Mass., 1975).
[3] A. de-Shalit and I. Talmi, Nuclear Shell Theory (Dover Pub., Inc., Mineola, New York, 2004).
[4] B. A. Brown and B. H. Wildenthal, Ann. Rev. Nucl. Part. Sci. 38, 29 (1988).
[5] F. Iachello and I. Talmi, Rev. Mod. Phys. 59, 339 (1987).
[6] A. Arima, T. Otsuka, F. lachello, and I. Talmi, Phys. Lett. B 66, 205 (1977).
[7] T. Otsuka, A. Arima, F. lachello, and I. Talmi, Phys. Lett. B 76, 139 (1978).
[8] J. N. Ginocchio, Ann. Phys. (N. Y.) 126, 234 (1980).
[9] A. Klein and E. R. Marshalek, Rev. Mod. Phys. 63, 375 (1991).
[10] Y. Nambu, Physica 15D, 147 (1985).
[11] M. Mukerjee and Y. Nambu, Ann. Phys. 191, 143-162 (1989).
[12] A. J. Leggett, in Modern Trends in the Theory of Condensed Matter, edited by A. Pekalski and J. Przystawa (Springer-Verlag, Berlin, 1980).
[13] M. Randeria, J.-M. Duan, and L.-Y. Shieh, Phys. Rev. Lett. 62, 981 (1989).
[14] Y. Ohashi and A. Griffin, Phys. Rev. Lett. 89, 130402 (2002).
[15] G. B. Partridge, K. E. Strecker, R. I. Kamar, M. W. Jack, and R. G. Hulet, Phys. Rev. Lett. 95, 020404 (2005).
[16] M. A. Cazalilla, R. Citro, T. Giamarchi, E. Orignac, and M. Rigol, Rev. Mod. Phys. 83, 1405 (2011).
[17] G. Ortiz and J. Dukelsky, Phys. Rev. A 72, 043611 (2005).
[18] K. Hagino, H. Sagawa, J. Carbonell, and P. Schuck, Phys. Rev. Lett. 99, 022506 (2007).
[19] M. Honma, T. Otsuka, B. A. Brown, and T. Mizusaki, Phys. Rev. C 69 (2004) 034335.
[20] X. Guan, K. D. Launey, M. Xie, L. Bao, F. Pan, and J. P. Draayer, Phys. Rev. C **86**, 024313 (2012); Comp. Phys. Commun. **185**, 2714 (2014).
[21] L. Dai, F. Pan, and J. P. Draayer, Nucl. Phys. A 957, 51 (2017).
[22] A. Banu, J. Gerl, C. Fahlander et al., Phys. Rev. C **72**, 061305(R) (2005).
[23] J. Cederkäll, A. Ekström, C. Fahlander et al., Phys. Rev. Lett. 98, 172501 (2007).
[24] C. Vaman, C. Andreoiu, D. Bazin et al., Phys. Rev. Lett. 99, 162501 (2007).
[25] A. Ekström, J. Cederkäll, C. Fahlander et al., Phys. Rev. Lett. 101, 012502 (2008).
[26] H. Jiang, Y. Lei, G. J. Fu, Y. M. Zhao, and A. Arima, Phys. Rev. C 86, 054304 (2012) .
[27] B. Maheshwari, A. K. Jain, B. Singh, Nucl. Phys. A 952, 62 (2016).
[28] http://www.nndc.bnl.gov/ensdf/.

[29] F. Iachello and P. van Isacker, The Interacting Boson-Fermion Model (Cambridge: Cambridge University Press, 1991).