Low-order modeling of micro-flier impact with thin stationary energetic targets

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LOW-ORDER MODELING OF MICRO-FLIER IMPACT WITH THIN STATIONARY ENERGETIC TARGETS

A Thesis

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering in

The Department of Mechanical Engineering

by

Mark B. Fry
B.S.M.E., Louisiana State University and Agricultural and Mechanical College, 2010
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Thank you to my family for the support and help in guiding me through my academic career. Thank you especially to my mom, Pamela Fry, and Dad, Samuel Fry, for molding me into the man I am today. I would also like to thank my girlfriend, Sarah Navoy, for her loyalty and understanding during this time. To Dr. Keith Gonthier, thank you for your support and for pushing me to further my education and achieve goals I did not think possible. To my fellow lab partners for being there as peers. Also, I'd like to thank Dr. Mario Fajardo, Dr. Chris Molek, and Dr. George Butler at the Air Force Research Laboratory (AFRL) at Eglin AFB in Florida for their collaboration in this work. This work was funded by the AFRL/MNK under agreement number FA8651-08-1-0007.
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A.1 Flow chart for MCS process with only aleatory uncertainties [37]

A.2 Flow chart for MCS with aleatory and epistemic uncertainties [37]
Abstract

The impact of high-speed (500-1500 m/s), laser driven micro-fliers with thin energetic targets (10-100 \( \mu m \)) is being examined to characterize impact-induced heating and combustion of these materials. Aluminum fliers are propelled by a laser into a thin metallic target plate having a layer of energetic material deposited on its backside. Mass spectrometry is performed in vacuo on the energetic side to interrogate the shock-induced chemistry of the energetic material. It is important to ignite and possibly detonate the energetic material without perforation of the target plate avoiding contamination of the vacuum chamber.

To guide development of these experiments, a low-order (zero-dimensional) model is formulated to estimate ballistic performance for large dimensional parameter spaces in a computationally inexpensive manner. The imaging of post-impact target coupons gives insight into deformation and failure modes of the target plate. The model accounts for both the early-time system response with 1-D shock relations and the late-time response with quasi-static strength of flat plates. The model is validated against impact data for larger scale flier-target configurations, and gives predictions for micro-scale configurations. The post-impact target plates show that the system behavior is stochastic in nature. Thus, a method for propagating input uncertainty is presented to estimate the uncertainty in model output variables, and a sensitivity study is performed to highlight dependence on input parameters. Model output is most sensitive to the ratio of flier width to diameter. Predictions are performed for energetic materials HMX (\( \text{C}_4\text{H}_8\text{N}_8\text{O}_8 \)), TNT (\( \text{C}_7\text{H}_5\text{N}_3\text{O}_6 \)), and PETN (\( \text{C}_5\text{H}_8\text{N}_4\text{O}_{12} \)) over the initial flier velocity - flier thickness parametric plane for given target thicknesses to produce ballistic initiation maps to identify configurations for which initiation of energetic material may occur without perforation of the target plate. Because of the high critical shock initiation energy of HMX (150 \( \text{J/cm}^2 \)) and TNT (77 \( \text{J/cm}^2 \)), it is difficult to identify micro configurations resulting in initiation. Such configurations were found for PETN which has a lower critical shock energy (5.03 \( \text{J/cm}^2 \)). The area of the region for initiation increases with increasing target thickness for these configurations.
Chapter 1

Introduction

This chapter gives the background of this study including project motivation and goals. The problem statement is provided including a detailed description of the system to be studied and pertinent physical considerations. A review of the literature related to the problem is provided. Goals specific to this study are presented and discussed and the layout of the thesis is described.

1.1 Background

High-speed, laser-driven micro-flier experiments are being performed by researchers at The Air Force Research Laboratory Munitions Directorate at Eglin AFB, Florida (AFRL-MNK) to characterize impact-induced heating and combustion of energetic materials. A 20 mW fiber-coupled 532 nm YAG laser is used to propel metallic fliers produced from a thin sheet of foil at velocities between 500 m/s and 5,000 m/s into a metallic target that has a thin layer of energetic material deposited on its backside. Mass spectrometry is performed in a vacuum chamber on the backside of the target to identify chemical species associated with heating and shock induced combustion of the energetic material. The goal of the experiments is to interrogate the shock induced chemistry of small samples of energetic material for their application in insensitive munitions.

Figure 1.1 shows the experimental apparatus of the laser-driven micro-flier experiments. From right to left, the assembled target coupon is comprised of a glass substrate with a plated layer of aluminum foil, a stainless steel mask separating the flier foil from the target foil, a stainless steel target foil, another mask clamping the stainless steel flier foil, and the energetic sample. In the experiments, a laser pulse is fired through the glass substrate heating, ablating, and propelling an aluminum flier from the plated aluminum layer through the free space of the mask. The flier then impacts the stainless steel target foil, and time
of flight mass spectrometry (TOF-MS) is performed on the vacuum side of the target foil where the energetic material is located to study the chemical species associated with the energetic. Figure 1.1 also shows a schematic of the problem with representative geometries and initial values. Fliers are punched from 2024-T4 aluminum foils of thicknesses between 10 $\mu$m - 50 $\mu$m that are plated on the glass substrate. Targets are 304 stainless steel foils with thicknesses between 50 $\mu$m - 100 $\mu$m. The target plates are clamped at a diameter of 3 mm around each impact site. Figure 1.2 shows an image of a post-impact target foil and aluminum plated glass substrate and an assembled target coupon. Here each dimple represents a single experiment. As depicted, the target coupons are large enough to allow for multiple experiments to be quickly performed in succession, and the impact sites are spaced adequately and separated by the masks to limit influence between experiments.

![Figure 1.1: Apparatus for laser-driven micro-flier experiments.](image)

It is of interest to researchers to know the amount of energy and impulse to the target foil and energetic material so that the thermo-mechanical and chemical response of the energetic material can be characterized. Specifically, the researchers are interested in quantifying the amount of energy required to induce initiation of energetic materials. By initiation, we mean transition to detonation. It is also of importance to researchers that the results of the mass spectrometry be independent of the events occurring on the impact side of the target coupon. To assure that no contamination of the vacuum chamber occurs,
the target foil must not be perforated by the flier. For this reason the energy required for perforation is also an important consideration.

A computational model may aid in quantifying the energy transfer from the flier to the energetic material and provide preliminary estimates for the behavior of laser-driven flier experiments. The model must consider the initial flier kinetic energy and the transfer of this energy to the energetic target. The early-time physics associated with shock must be accounted for as well as the late-time physics associated with target strength. A simple model will allow researchers to perform computationally inexpensive predictions over a large parameter space to investigate configurations in which initiation of the energetic material may occur without perforation of the target plate. It will also provide a tool to propagate various uncertainties in input parameters of the model to quantify the uncertainty in the output of the model.

1.2 Review

There is a wealth of literature on impact problems in general [3, 12, 24, 32, 55]. These references describe many of the complexities of impact and how impact problems have been modeled historically dating back 230 years. This includes descriptions of the various modes or types of penetration, perforation, and failure associated with impact. Various
Many experimental studies have been performed to analyze the response and strength of various materials during impact for large flier-target configuration experiments \[9, 10, 21, 33\]. By large flier-target configurations, we mean flier and target thicknesses on the order of \(\sim 10\) to \(\sim 100\) mm. These experiments provide experimental results for time response, residual velocities, and the ballistic limit associated with blunt projectile impact on clamped flat plates. Correlation between quasi-static and ballistic behavior is studied experimentally in [22]. The impact of various materials is considered providing data that may be used to validate the behavior of the model presented in this analysis.

Many have modeled the impact of plates with empirical relations, analytical models, and numerical models. Analytical solutions have been presented which account for shear plugging and bending of target plates in a simple way to predict penetration depth, residual velocity, and ballistic limit [2, 15, 23, 52, 53]. Others have taken another simplistic approach and used quasi-static solutions for punch loaded plates as models for impact [50, 16]. Though these give reasonable estimates, they do not consider the shock physics associated with high-speed impact. A simple approach to modeling impact is given by Heyda, et al. [28], where they consider the early-time shock physics of impact but use a simpler strength model for the target that may not characterize target deformation as accurately as more complex constitutive models. More recently researchers including Borvik [9] have used constitutive models coupled with the combined implicit/explicit solver LS-DYNA or hydrocodes like CTH to model impact which is computationally expensive. The constitutive model and damage model employed by Borvik requires extensive material testing for parameter fitting.
To date there is little information on modeling laser-drive n micro-flier impact found in literature. There are several studies on the novelty of laser-driven flier systems by Greenaway [26, 27], Paisley [40, 41], Watson [47, 49], and Fajardo [20]. This includes experimental optical techniques for observing the thickness, planarity, and integrity of laser driven fliers during flight as well as optical measurements of the acceleration and velocity of laser driven fliers for their application in initiation of energetic materials. These studies also include experimental methods of studying flier integrity by calculating the amount of material ablated from the flier by the laser during flight. There are also papers which study the dynamic behavior of materials during impact of laser-driven fliers experimentally using velocity interferometer for any reflector (VISAR) [18]. Other applications include the study of hypervelocity impact of space debris using laser-driven flier experiments to qualitatively study the damage to polymer and glass targets performed by Verker, et al. [46] and Roybal, et al. [42, 43]. Though these studies are available, which address the novelty of various applications for laser-driven fliers and study the dynamics of laser-driven fliers experimentally, the modeling of the impact response of laser-driven fliers in particular has not been addressed in depth.

The model presented in this study considers the early-time shock physics of impact as well as the quasi-static strength of clamped plates. Impact is posed as in initial value problem (IVP) having piecewise continuous analytical solutions. The model is computationally inexpensive, and gives reasonable results in comparison to more complex analytical models and the more computationally expensive numerical methods.

1.3 Goals of the Study

The goal of this study is to provide a model which meets the following two criteria:

1. *Construct a computationally inexpensive, predictive physical model.*

   The model for this analysis must be predictive and use only well known input parameters that do not require material testing. Because of the stochastic nature of this problem, the model must be computationally inexpensive to perform large para-
metric analysis and uncertainty propagation. Without explicitly listing them here, the number of parameters in this study is in excess of 10 creating an extensive parameter space for analysis. Even if a detailed physical model could be posed and computationally solved, it would be and overkill for the purposes of this preliminary assessment.

2. Computation of ballistic initiation maps.

In this study ballistic initiation maps will be predicted which show the region in parameter space where initiation of the energetic material occurs without perforation of the target plate. These maps are generated for slices of parameter space defined by those parameters that are controlled by experimenters including flier thickness, target thickness, and initial flier velocity. The model must compute values required to produce ballistic initiation maps including the work done on the energetic material and the ballistic limit of the target plate. These maps will provide experimentalists with configurations in which detonation may occur without perforation of the target plate.

In this paper we first consider post-impact visualization of experimental target coupons to provide insight into the physical processes occurring during impact. In the following chapter, we lay out the physical and mathematical model employed in this analysis. In the fourth chapter, we outline the behavior and predictions of the current model and discuss uncertainties in model input parameters including methods of uncertainty propagation. We then illustrate how the model accounts for energetic targets to generate ballistic initiation maps for two commonly used explosives. Finally, conclusions based on the current model are made, model limitations are discussed, and suggestions for future work are provided.
Chapter 2

Post-Impact Visualization

To gain insight into the physical events occurring during micro-flier impact, SEM was performed on post-impact target coupons. These coupons were taken from experiments performed in a manner similar to that shown in Chapter 1 without energetic material. Experimenters fired laser pulses of various energies to propel aluminum fliers at stainless steel targets. The geometries associated with each configuration tested are listed in Table 2.1. The initial flier velocities are associated with various laser pulse energies. These velocities were calculated by experimenters using velocimetry techniques [20]; They are stated to have an uncertainty of ±10%.

<table>
<thead>
<tr>
<th>Table 2.1: Experiment Configurations.</th>
</tr>
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<tbody>
<tr>
<td>M-1</td>
</tr>
<tr>
<td>---------------------------------------</td>
</tr>
<tr>
<td>Flier Thickness</td>
</tr>
<tr>
<td>Target Thickness</td>
</tr>
<tr>
<td>Clamp Diameter</td>
</tr>
<tr>
<td>Laser Pulse Diameter</td>
</tr>
</tbody>
</table>

Figures 2.1 - 2.3 show sites of impact from these configurations. In all cases the stainless steel target foils were clamped at a diameter of 3 mm, and the laser pulse diameter was 1.4 mm. This laser pulse diameter is important because it is often assumed to be the diameter of the flier in the analysis presented in this thesis. Figure 2.1 shows post-impact images for Configuration M-1 corresponding to the laser energies and initial flier velocities indicated in the figure caption. In Figure 2.1 (a) the initial velocity of the flier was approximately 200 m/s. This initial velocity is a relatively low speed for the laser-driven flier experiments performed. Even at this lower velocity, and therefore lower amount of laser energy applied to the flier, the flier is seen to have broken up, possibly prior to impact. This break-up is indicated by the deformed shape of what appears to be the flier stuck to the target plate;
though we believe this to be correct, further material testing is needed to substantiate this claim. This observation suggests that some type of adhesion occurs between the flier and target. At this low velocity only slight deformation of the target plate is noticed from the view of its back side. On the front side it is noticed that a faint ring can be seen about the impact site. This ring shows the location of the clamp boundary where flier material or impact residue is restricted from moving outward away from the impact site. Figure 2.1 (b) shows an impact of the same geometrical configuration with an initial flier velocity of $\sim 800 \text{ m/s}$. We still observe very little deformation of the target plate at this velocity, but the flier has experienced much greater break-up than in the $200 \text{ m/s}$ case. A clearer representation of the clamp boundary is seen as well. Increasing the initial velocity to $\sim 2100 \text{ m/s}$, it is observed in Figure 2.1 (c) that the flier may be molten prior to impact due to the apparent splatter pattern on the target. The target plate is bent at the clamp boundary giving a bulged appearance and exhibiting local bending near the periphery of flier impact. The formation of cracks on the back side of the impact site is also apparent. In Figure 2.1 (d) the initial flier velocity was increased greatly to $\sim 3500 \text{ m/s}$. In this case the flier has completely perforated the target plate. On the impact side we can see the hole created by the flier as well as what appears to be a fine splattering of flier material. Again, this splattering suggests phase change of the flier at some point during the event. From the back side of the target it is observed that the plate has bent at the clamp boundary and that the perforation of the flier has caused petaling of the target plate.

Figure 2.2 shows images for Configuration M-2 corresponding to several initial flier velocities. Figure 2.2 (a) shows impact with an initial flier velocity of approximately $700 \text{ m/s}$. We can see immediately that the flier punched from the $33 \mu \text{m}$ thick foil is much more massive. The flier in this case appears to be mostly intact with possible spallation that is seen about its periphery. The flier appears to be welded to the target plate due to high impact induced temperature. The back side of the target plate exhibits slight bulging. Figure 2.2 (b) shows an impact with initial flier velocity of approximately $1200 \text{ m/s}$. In
Figure 2.1: Configuration 1 (a) 70 mJ, $U_{f_0} \approx 200 \, m/s$; (b) 100 mJ, $U_{f_0} \approx 800 \, m/s$; (c) 300 mJ, $U_{f_0} \approx 2100 \, m/s$; (d) 700 mJ, $U_{f_0} \approx 3500 \, m/s$. 

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this case we can see that the flier may have become molten at some point during the event and has solidified in contact with the target leaving what appears to be a tail protruding away from the target plate. At this initial velocity, the target plate is seen to experience considerable bending at the clamped boundary and additional deformation near the flier periphery. Again, cracks appear to have formed along the back side of the target plate. In Figure 2.2 (c) the initial flier velocity was approximately $1550 \, m/s$. The flier has completely perforated the target plate in this case. The impact side of the target is splattered by what appears to be flier material and the perforation hole can be seen. Interestingly, the diameter of the hole is much less than that of the laser diameter. This observation, as well as flier break-up and melting, indicate that the flier may go through changes in mass or geometry during these cases, and the flier may not have the same diameter as the laser pulse. From the back side of the target plate, bending at the clamp boundary is observed as well as petaling at the impact site with possible spallation.

Figures 2.3 shows images for Configuration M-3. Initial flier velocities of approximately $1550 \, m/s$, $1770 \, m/s$, and $1960 \, m/s$ are shown, respectively. As in the other cases, the flier appears to break-up and possibly melt during impact. It appears that in these cases most of the flier mass has either rebounded or has deformed and splattered in a manner to coat the entire clamped portion of the plate. With the thicker target plates, almost no bending of the target plate is seen in contrast to the previous cases. At these velocities, the back side of the target shows spallation directly behind the flier impact site. In Figure 2.3 (a), with an initial flier velocity of $\sim 1550 \, m/s$ the spalled target material is still attached to the back side of the target plate and has petaled outward. For $\sim 1770 \, m/s$ impact, Figure 2.3 (b), the spalled material has been ejected from the target plate leaving the spall surface completely exposed. In the highest velocity case shown, Figure 2.3 (c), the target has been perforated and exhibits a combination of what appears to be spallation and crack induced petaling. Though the target appears perforated, the extent of petaling suggests that the flier may not have traveled past the target material and may have even rebounded.
Figure 2.2: Configuration 2 (a) 200 mJ, $U_{f0} \approx 700$ m/s; (b) 400 mJ, $U_{f0} \approx 1200$ m/s; (c) 600 mJ, $U_{f0} \approx 1550$ m/s.

The post-impact visualization of these various cases illustrates the complexity of these laser-driven micro-flier impact events and the types of deformation associated with them. This complexity, would make accurate, detailed modeling difficult if not impossible to perform. Though complex, there are modeling implications that are suggested from these images.

1. An estimate of ballistic limit is provided by observing the target coupons where laser energy is sufficient for perforation. This laser energy corresponds to the initial flier
Figure 2.3: Configuration 3 (a) $600 \text{ mJ}$, $U_{f0} \approx 1550 \text{ m/s}$; (b) $750 \text{ mJ}$, $U_{f0} \approx 1770 \text{ m/s}$; (c) $900 \text{ mJ}$, $U_{f0} \approx 1960 \text{ m/s}$.

velocity for which perforation occurs and provides an upper value for the ballistic limit, $U_{BL}$, which is the minimum initial flier velocity required for perforation. An experiment in which the target is not perforated provides the lower limit. Though more experiments are needed to get a better estimate of the ballistic limit, these coupons give a range of initial flier velocities within which the ballistic limit falls.

2. *An adhesive force likely exists between the flier and target that should be considered in posing the model.* The effects of applying adhesive behavior to the model will
be examined and discussed in later chapters of this thesis.

3. *The experimental results are stochastic in nature.* It is noticed from viewing several post-impact target coupons that events occurring with the same initial experimental conditions produced varying outcomes. At a single laser energy near the ballistic limit, the flyer may or may not perforate the target plate. This may be due to variation in several parameters including, but not limited to, initial flyer velocity, flyer geometry, target geometry, and material properties. In this study, sensitivity of the modeling of these events to such parameters will be investigated and reported.
Chapter 3

Mathematical Model

The mathematical model posed in this chapter describes a cylindrical flier impacting a circular clamped target plate. A schematic of the flier and target prior to and following impact is presented in Fig. 3.1, where the geometry is axi-symmetric. The flier impacts the target with an initial velocity $U_{f0}$. The target plate is initially stationary ($U_{p0} = 0$), and both the flier and target are initially unstressed and are at ambient temperature. The target plate is assumed to deform by bending and shear in the annular region of the plate bounded between the clamp and the target plate material directly adjacent to the flier at impact. This region is shown as the shaded portion of the target plate in Fig. 3.1. The model is constructed such that the flier will either travel until it comes to rest with no perforation of the target plate or until the target plate is perforated. For perforation, the

Figure 3.1: Conceptual physical picture of flier and target used for model development. The shaded region of the target plate corresponds to the region in which deformation occurs. The non-shaded region represents the target plug material.
target plate must reach a maximum displacement specified by a failure criterion. The flier then perforates the target with a finite residual velocity. These outcomes are dependent upon the geometry of both the flier and target, their material properties, and the initial velocity of the flier. For simplicity we consider a two component system made up of the flier and the target material directly adjacent to the flier at impact bounded by the periphery of the flier. This target material will be referred to as the target plug.

In this chapter, the global form of the governing equations of the model are first presented which are based on conservation of mass, linear momentum, and total energy. Then, various assumptions are made and physical considerations applied to obtain a time-dependent dynamical system. Next, system equations are non-dimensionalized for ease in observing leading order limiting behavior. An analytical solution for the equations of motion is then presented and their leading form behavior is discussed. Finally, due to complexities in formulating the analytical solution, a numerical technique is provided for accurately integrating the equations to estimate quantities of interest to researchers such as the ballistic limit and its dependence on material properties and geometry.
3.1 Global Form of the Governing Equations

The physical system consists of the flier which occupies the spatial region \( \Omega_f(t) \), and the target plate material adjacent to the flier at impact (the non-shaded region of the target plate referred to as the target plug) which occupies the spatial region \( \Omega_p(t) \), where \( t \) is time. The impact interface is denoted by \( I \), and the lateral target shear surface between the target plug material and the outer plate material is denoted by \( II \). The labeled two-component system is shown in Fig. 3.2. The global forms of mass, linear momentum, and total energy conservation for the flier and target plug expressed in terms of the current (or deformed) configuration are given by

\[
\frac{d}{dt} \left( \int_{\Omega_f} \rho_f dV \right) = 0, \tag{3.1}
\]

\[
\frac{d}{dt} \left( \int_{\Omega_f} \rho_f v_f dV \right) = - \int_{A_I} T_I dS, \tag{3.2}
\]

\[
\frac{d}{dt} \left[ \int_{\Omega_f} \rho_f \left( e_f + \frac{v_f^2}{2} \right) dV \right] = - \int_{A_I} T_I v_I dS, \tag{3.3}
\]

\[
\frac{d}{dt} \left( \int_{\Omega_p} \rho_p dV \right) = 0, \tag{3.4}
\]

\[
\frac{d}{dt} \left( \int_{\Omega_p} \rho_p v_p dV \right) = \int_{A_I} T_I dS + \int_{A_{II}} T_{II} dS, \tag{3.5}
\]

\[
\frac{d}{dt} \left[ \int_{\Omega_p} \rho_p \left( e_p + \frac{v_p^2}{2} \right) dV \right] = \int_{A_I} T_I v_I dS + \int_{A_{II}} T_{II} v_{II} dS. \tag{3.6}
\]

Here, quantities associated with the flier are denoted by subscript \( f \) and quantities associated with the target plug are denoted by subscript \( p \). In these equations, \( \rho \) is the density, \( v \) is the velocity, \( T \) is the surface traction, \( e \) is the mass-specific internal energy, and \( A \) is the area of the interface. The above equations assume that the heat flux during the impact process is negligible across interfaces \( I \) and \( II \). Assuming the mechanical properties of the
materials are temperature independent, the equations of motion are decoupled from the energy equations and can be solved separately. Instantaneous volume average quantities for the flier and target plug can be defined by

\[ \bar{\rho}_f \equiv \frac{1}{V_f} \int_{\Omega_f} \rho_f dV, \quad \bar{\rho}_p \equiv \frac{1}{V_p} \int_{\Omega_p} \rho_p dV, \]

\[ \bar{U}_f \equiv \frac{1}{V_f} \int_{\Omega_f} \rho_f v_f dV, \quad \bar{U}_p \equiv \frac{1}{V_p} \int_{\Omega_p} \rho_p v_p dV, \]

\[ \frac{\bar{e}_f}{2} \equiv \frac{1}{V_f} \int_{\Omega_f} \rho_f v_f^2 dV, \quad \frac{\bar{e}_p}{2} \equiv \frac{1}{V_p} \int_{\Omega_p} \rho_p v_p^2 dV, \]

(3.7)

(3.8)

(3.9)

(3.10)

Here, quantities having “over-bars” denote volume average quantities or mass averaged quantities. It is noted that, in general, \( \bar{U}^2 \neq \overline{U^2} \). Over-bars for mass averaged velocities, \( \bar{U}_f \) and \( \bar{U}_p \), are dropped for the remainder of this paper for simplicity in later use (\( U = \bar{U} \)). Recognizing that \( dm = \rho dV \) and \( m = \rho V \), the above volume averages can be used to obtain mass averaged velocities, kinetic energies, and internal energies:

\[ U_f \equiv \frac{1}{m_f} \int_{\Omega_f} v_f dm, \quad U_p \equiv \frac{1}{m_p} \int_{\Omega_p} v_p dm, \]

\[ \frac{U_f^2}{2} \equiv \frac{1}{m_f} \int_{\Omega_f} \frac{v_f^2}{2} dm, \quad \frac{U_p^2}{2} \equiv \frac{1}{m_p} \int_{\Omega_p} \frac{v_p^2}{2} dm, \]

\[ \bar{e}_f \equiv \frac{1}{m_f} \int_{\Omega_f} e_f dm, \quad \bar{e}_p \equiv \frac{1}{m_p} \int_{\Omega_p} e_p dm. \]

(3.11)

(3.12)

(3.13)

Thus, the mass equations give \( m_f = \rho_f V_f = \text{cst} \) and \( m_p = \rho_p V_p = \text{cst} \), and the momentum and total energy equations for the flier and target can be expressed in terms of average quantities by

\[ m_f \frac{dU_f}{dt} = - \int_{A_t} T_d dS, \]

(3.14)
\[ m_f \frac{d}{dt} \left( \bar{e}_f + \frac{U_f^2}{2} \right) = - \int_{A_I} T_I v_I dS, \quad (3.15) \]

\[ m_p \frac{dU_p}{dt} = \int_{A_I} T_I dS + \int_{A_{II}} T_{II} dS, \quad (3.16) \]

\[ m_p \frac{d}{dt} \left( \bar{e}_p + \frac{U_p^2}{2} \right) = \int_{A_I} T_I v_I dS + \int_{A_{II}} T_{II} v_{II} dS. \quad (3.17) \]

It is noted that these equations are exact expressions for the evolution of average quantities subject to interface tractions and work, and their combined momentum and total energy only change due to the interaction force at interface II. The accuracy of the model hinges on the specification of these surface interaction terms.

### 3.2 Specification of Interaction Stresses

To solve Eqs. (3.14)-(3.17) it is necessary to specify relations for interaction stresses at interfaces I and II. In this model the interaction stresses are defined by a fast response controlled by wave mechanics and a slow response controlled by the target plate strength. At the impact interface (I), the fast response of the interaction stress is controlled by the wave mechanics. At interface II, the target plug periphery, the interaction stress is defined by the strength of the target plate representing the slow response of the system.

#### 3.2.1 Impact Interface (I)

The interaction force at the impact interface is defined by

\[ F_I = \int_{A_I} T_I dS, \quad (3.18) \]

where \( F_I \) is the force at interface I due to the impact shock pressure during the early-time response and an adhesive-like force during the later time response. The current model defines \( F_I \) as

\[ F_I(t) = \begin{cases} P_S A_f & t \leq t_r, \\ \frac{1}{\mu} (U_f - U_p) & t > t_r. \end{cases} \quad (3.19) \]
Here, $P_S$ is the shock pressure, $A_f = A_I$ is the frontal area of interface $I$ taken as the frontal area of the flyer prior to impact, $t_r$ is the time that the first release wave reaches the flyer-target interface, and $\mu$ is a relaxation constant that controls the time response of the adhesive-like force.

For time $t \leq t_r$, $F_S = P_S A_f = \text{cst}$, where $P_S$ is found by impedance matching using the Hugoniot of the flyer and target materials. These Hugoniot are relations of the shock pressure in each material as a function of particle velocity derived using conservation of mass and momentum across a shock. The conservation of mass across the shock is given by the Rankine-Hugoniot jump conditions as

$$\rho(u - D) = \rho_0(u_0 - D), \quad (3.20)$$

where $\rho_0$ and $u_0$ are the density and particle velocity ahead of the shock, $D$ is the shock speed, and $\rho$ and $u$ are the density and particle velocity behind the shock. The conservation of momentum given by the Rankine-Hugoniot jump conditions is defined by

$$P - P_0 + \rho(u^2 - D^2) = \rho_0(u_0^2 - D^2), \quad (3.21)$$

where $P_0$ is the pressure in front to the shock and $P$ is the pressure behind the shock. Substituting eq. (3.20) into eq. (3.21) we get the Hugoniot relation defined by

$$P - P_0 = \rho_0(u_0 - D)(u_0 - u). \quad (3.22)$$

This relation has the two unknowns $P$ and $u$. To mathematically close this relation, we use a linear relation for the shock speed, $D$, in terms of $u$ taken from experiments.

$$u_0 - D = C + S(u_0 - u), \quad (3.23)$$
where \( C \) and \( S \) are fitting parameters for the linear equation. Substituting into the Hugoniot relation we get the Hugoniots for the flier and target defined by

\[
P = \rho_0 [C(u_0 - u) + S(u_0 - u)^2].
\] (3.24)

Applying this equation to the flier and target at interface \( I \) we get

\[
P = \rho_f [C_f(U_{f0} - u) + S_f(U_{f0} - u)^2] \tag{3.25}
\]

and

\[
P = \rho_p (C_p u + S_p u^2) \tag{3.26}
\]

for the flier and target respectively [36]. Here, \( u_0 = U_{f0} \) is the initial flier velocity and \( \rho_f \) and \( \rho_p \) are the ambient densities of the flier and target material respectively. Equations (3.25) and (3.26) are equated knowing that the pressure and particle velocity are equal at interface \( I \). When equated, we find the interface velocity, \( U_{int} = u \), and the shock pressure, \( P_S = P \). Figure 3.3 shows a visual representation of the impedance matching for two materials upon impact. The point at which the Hugoniot curves intersect is the shock state at the interface due to impact.

The release time, \( t_r \), is given by

\[
t_r = \frac{w}{D} + \frac{w}{u \pm c}, \tag{3.27}
\]

where \( w \) is the thickness, \( D \) is the shock speed, and \( c \) is the acoustic speed associated with the flier or target. This represents the sum of the time that it takes for a shock to travel to a free surface and for a release wave to travel back to the impact interface. These values will be associated with either the flier or target based upon which release time is shorter. Figure 3.4 shows and x-t diagram representing these wave interactions. The waves traveling out toward the clamped boundary from the periphery of the flier are ignored
Figure 3.3: Impedance matching of the shock Hugoniot for an 11 \( \mu m \) thick Al-2024 flier impacting a 50 \( \mu m \) thick 304 stainless steel target at an initial velocity \( U_{f0} = 2000 \ m/s \).

because the timescale for these waves to reflect and return to the flier periphery is much larger. The speed of the release waves are estimated by the acoustic speed of the flier and

Figure 3.4: x-t diagram for the two component system.
target materials. The affect of temperature on the speed of the release wave is assumed to be defined by estimating the internal energy by $C_v T$, where $C_v$ is the volume constant specific heat and $T$ is the temperature behind the shock, we get a relation for shock speed defined by

$$c \approx c_0 \sqrt{\frac{T}{T_0}},$$

where $c_0$ is the ambient sound speed given by Hugoniot data and $T_0$ is the ambient temperature.

After time $t_r$, when a release wave from either the target or flier free surface reaches interface $I$, $F_I = \frac{1}{\mu}(U_f - U_p)$. This implies that the flier and target are coupled by an adhesive force exhibiting inelastic impact. This behavior is compatible with explosive welding of system components. Again, the use of this adhesive force is suggested by post-impact visualization of target coupons from laser-driven micro-flier experiments.

The adhesive force causes the velocities of the flier and target to approach the same value. Though the velocities approach the same value, there will always be a finite velocity difference between the flier and target in the current model due to the force applied at interface $II$ on the target plug. If the force at interface $II$ were to approach zero, only then would the velocity difference and subsequently the force, $F_I$, at interface $I$ approach zero.

In the current model, if the velocity of the target plug reaches zero, we assume that no perforation occurs. In reality, the model gives a small but finite value for flier velocity at this instant. Though the flier velocity is non-zero at this point, we assume it is sufficiently small to ignore.

In some cases, the results of applying an adhesive force differ from results of 1-D shock impedance matching. When a flier has lower shock impedance than the target, matching suggests eventual rebound of the flier; thus, at time $t_r$, $u_f < 0$ and $u_p > 0$, where $u_f$ and $u_p$ are the flier and target particle velocities found by impedance matching. Figure 3.5 shows the results of impedance matching for such a case. The black lines show the Hugonitos of
Figure 3.5: Impedance matching of the shock Hugoniots and release isentropes for an 11 $\mu m$ thick Al-2024 flier impacting a 50 $\mu m$ thick 304 stainless steel target at an initial velocity $U_{f0} = 2000$ m/s.

a 2024 aluminum flier and a 304 stainless steel target used to calculate the initial shock state at impact. The gray lines are the reflected Hugoniots passing through the initial shock state. In the absence of data, these reflected (or mirrored) Hugoniot curves are good estimates for the release isentropes of the flier and target materials. Where these gray lines intersect the horizontal axis, we find the particle velocities of the flier and target at time $t_r$, $u_f(t_r)$ and $u_p(t_r)$ respectively. In this particular case, our model gives a center of mass flier velocity $U_f(t_r) < 0$ and center of mass target plug velocity $U_p(t_r) > 0$ which is consistent with the impedance matching above. The adhesive force in the model then acts at $t = t_r$ as a tensile force between the flier and target plug to prevent rebound from occurring resulting in $U_f > 0$ and $U_p > 0$.

The relaxation constant in Eq. (3.19), $\mu$, controls how quickly the adhesive force acts to bring the velocities of the flier and target together. The value of $\mu$ is found by equating the traction integral at the surface $I$ to the interface pressure between the flier and target
and applying the initial condition $U_f(t_r) - U_p(t_r) = \Delta U_{t_r}$:

$$\int_I \tau_f dS = P_S A_f = \frac{1}{\mu} (\Delta U_{t_r}). \tag{3.29}$$

Therefore,

$$\mu = \frac{\Delta U_{t_r}}{P_S A_f}. \tag{3.30}$$

In general, $P_S A_f \gg \Delta U_{t_r}$ and $\mu \ll 1$. This means the adhesive force term is large at time $t_r$. This causes the velocity of the flier and target to approach one another quickly. This is consistent with classical inelastic impact in which the flier and target velocities equilibrate rapidly.

In cases for which the flier and target plug velocities are both positive, the flier does not rebound and the adhesive force does not act in the manner described above. The force, $F_I$, does not become tensile unless $(U_f - U_i) < 0$. For these cases the adhesive force simply acts as a compressive force at interface $I$ to keep the flier from excessively penetrating the target plug. We will still refer to this force as adhesive but will distinguish the cases for which it acts in tension or compression.

### 3.2.2 Strength Interface ($II$)

Because the model is separated into a short-term and long-term response, there exists a force that acts during the long term when the wave mechanics of the impact are no longer considered important. The interaction force at interface $II$ controls this time response and is defined by

$$F_{II} = \int_{A_{II}} \tau_{II} dS, \tag{3.31}$$

where $F_{II}$ is the force due to the material strength of the target plate described by the quasi-static behavior of a circular clamped flat plate loaded by a blunt cylindrical punch. If we consider the quasi-static limit of the current model, when the velocities and accelerations of the flier and target are close to zero, the model should be able to reproduce quasi-static
punch behavior. To do so, we must apply a force displacement relationship at interface $II$ that equals an applied punch force at interface $I$. Wen and Jones [51] give a relation for axial force versus the maximum axial displacement of the target plate as

$$ F_{II} = K_m x_p + F_c, \quad (3.32) $$

where $x_p$ is the axial displacement of the target plate center of mass, $K_m$ is the membrane stiffness of the plate, and $F_c$ is the static collapse load of the target plate. Figure 3.6 shows a schematic of the target during quasi-static deformation due to an applied external force. This relation assumes that the force must rapidly reach the static collapse load, $F_c$, before displacement of the target occurs. The membrane stiffness, $K_m$, and static collapse load, $F_c$, are defined by

$$ K_m = \frac{2\pi N_0}{\ln(R/r_f)}, \quad (3.33) $$

$$ F_c = \left( \frac{4}{\sqrt{3}} \right) \pi M_0 \left[ 1 + \frac{\left( 1 + \frac{\sqrt{3}}{2} \right)}{\ln(R/r_f)} \right]. \quad (3.34) $$

Figure 3.6: Target plate under quasi-static deformation.
Here, $N_0 = \sigma_y w_p / \sqrt{3}$ is the fully plastic membrane force per unit length, $M_0 = \sigma_y w_p^2 / 4$ is the fully plastic bending moment per unit length, $R$ is the clamp radius of the plate, $r_f$ is the radius of the flier, and $w_p$ is the thickness of the target plate. This quasi-static model also gives a failure criterion for critical target displacement, $x_{pc}$. This critical displacement is based on the maximum shear force about the target plug periphery. It is assumed that during quasi-static deformation, the force on the target plug must be equal to the shear force. Based on this assumption, the maximum shear force, $F_u$, must be equal to the quasi-static force, $F_{II}$, at failure. The maximum shear force is defined as

$$F_u = \frac{\sigma_u A_r}{\sqrt{3}},$$  \hspace{1cm} (3.35)$$

where $\sigma_u$ is the ultimate tensile strength of the target material and $A_r$ is the peripheral area of the target plug. From Eq. (3.32), the critical displacement of the target plate, $x_{pc}$, will be

$$x_{pc} = \frac{F_u - F_c}{K_m}. \hspace{1cm} (3.36)$$

The force versus displacement for an aluminum plate given by the quasi-static model is compared to experimental data in Fig. 3.7. The model with the static collapse load included, greatly over-predicts the force. The over-prediction of force is due to the assumption that the punch must overcome the static collapse load before displacement occurs. In this case, and in the cases of interest to this study, the target plates have a large clamp diameter in comparison to the thickness of the target plate. From Onat et al. [38], we know that for clamped thin plates the static collapse force required to deform the plate becomes small. Figure 3.8 shows non-dimensional force versus non-dimensional punch displacement; several curves are shown corresponding to different ratios of plate clamp radius to plate thickness, $\frac{R}{w_p}$. For small values of this ratio, as the plate gets thicker with respect to its clamped diameter, the use of a static collapse load in the force displacement relation is valid. For thin plates with respect to the clamped diameter, corresponding to high values
of $\frac{R}{w_p}$, the static collapse load goes to zero. Therefore, for our study, with $20 \leq \frac{R}{w_p} \leq 40$, it may be valid to eliminate the $F_c$ term. Doing so results in better agreement with the quasi-static data in Fig. 3.8 where $\frac{R}{w_p} \approx 50$. Though we only show comparisons to data for aluminum, the force-displacement relation was also compared to quasi-static data for steel plates achieving similar agreement. With $F_c$ eliminated, the force-displacement relation and failure criterion are then reduced to

$$F_{II} = K_mx_p,$$  \hspace{1cm} (3.37)

and

$$x_{pc} = \frac{F_u}{K_m}.$$  \hspace{1cm} (3.38)

Finally, with $F_I$ and $F_{II}$ defined, the equations of motion become

$$m_f \frac{dU_f}{dt} = \begin{cases} 
-PSAf & t \leq t_r, \\
-\frac{1}{\mu}(U_f - U_p) & t > t_r, 
\end{cases}$$  \hspace{1cm} (3.39)
Figure 3.8: Clamped mild-steel plates of various thicknesses subjected to a central load [38].

\[
\begin{align*}
    m_p \frac{dU_p}{dt} &= \begin{cases} 
        P_S A_f - K_m x_p & t \leq t_r, \\
        \frac{1}{\mu}(U_f - U_p) - K_m x_p & t > t_r, 
    \end{cases} 
    \\
    \frac{dx_f}{dt} &= U_f, \\
    \frac{dx_p}{dt} &= U_p.
\end{align*}
\]

(3.40)

(3.41)

(3.42)

Initial conditions for Eqs. (3.39)-(3.42) are given by \( U_f(0) = U_{f0}, U_p(0) = 0, x_f(0) = 0, \) and \( x_p(0) = 0. \) Failure of the target plate occurs when \( x_p \geq x_{pc}. \) Knowing that the rate of change of the impulse is equal to the rate of change in momentum, we can also calculate impulse, \( I_i, \) by integrating the following equations:

\[
\begin{align*}
    \frac{dI_f}{dt} &= \begin{cases} 
        -P_S A_f & t \leq t_r, \\
        -\frac{1}{\mu}(U_f - U_p) & t > t_r, 
    \end{cases} 
\end{align*}
\]

(3.43)
\[
\frac{dI_t}{dt} = \begin{cases} 
    P_S A_f - K_m x_p & t \leq t_r, \\
    \frac{1}{\mu} (U_f - U_p) - K_m x_p & t > t_r.
\end{cases} \tag{3.44}
\]

### 3.3 System Energetics

Though the energy equations of the system are decoupled from the momentum equations, they are useful in estimating quantities of interest including the work done on the target. Knowing that the change in total energy of the flier or target is equal to the net work done performed on them, the net power of the flier and target is defined by

\[
\frac{dW_f}{dt} = -\int_{A_I} T_I v_I dS, \tag{3.45}
\]

\[
\frac{dW_p}{dt} = \int_{A_I} T_I v_I dS + \int_{A_{II}} T_{II} v_{II} dS. \tag{3.46}
\]

During the time from \(0 \leq t \leq t_r\) we know that the pressure at the target-flier interface must be the shock pressure, \(P_S\), and the velocity must be the interface velocity, \(U_{int}\). These are both given by impedance matching of the Hugoniot of each material. Knowing this we have

\[
\int_{A_I} T_I v_I dS = P_S A_f U_{int}. \tag{3.47}
\]

To this point the equations are exact and no assumptions about the spatial velocity fields in the target or flier have been made. Since we do not track the exact velocity fields in this study, some assumptions must be made to approximate other terms. It is assumed that at interface \(II\) the total force is equal to the quasi-static force, \(F_{II} = K_m x_p\), and the velocity is equal to the center of mass velocity of the target, \(U_p\). Therefore,

\[
\int_{A_{II}} T_{II} v_{II} dS = -K_m x_p U_p, \tag{3.48}
\]

29
and the net power becomes

\[
\frac{dW_f}{dt} = -PSA_fU_{int} \quad 0 \leq t \leq t_r,
\]  
(3.49)

\[
\frac{dW_p}{dt} = PSA_fU_{int} - K_m x_p U_p \quad 0 \leq t \leq t_r.
\]  
(3.50)

For \( t > t_r \) we no longer track the wave interactions. It is assumed that the velocities at each interface will quickly approach the center of mass velocities of the target and flier. Because the center of mass velocities of the flier and target approach each other quickly, the interface velocity is then modeled as the target center of mass velocity, \( U_{int} \approx U_p \). Also, at this time, the interface force becomes, \( F_S = \frac{1}{\mu}(U_f - U_p) \). The rate of change of net work is now

\[
\frac{dW_f}{dt} = -\frac{1}{\mu}(U_f - U_p) U_p \quad t > t_r,
\]  
(3.51)

\[
\frac{dW_p}{dt} = \frac{1}{\mu}(U_f - U_p) U_p - K_m x_p U_p \quad t > t_r.
\]  
(3.52)

For the target we can also separately calculate the rate of work done at each interface

\[
\frac{dW_{pI}}{dt} = \begin{cases} 
PSA_fU_{int} & 0 \leq t \leq t_r, \\
\frac{1}{\mu}(U_f - U_p) U_p & t > t_r, 
\end{cases}
\]  
(3.53)

\[
\frac{dW_{pII}}{dt} = -K_m x_p U_p.
\]  
(3.54)

Here, \( W_{pI} \) is the work done on the target at interface \( I \) and \( W_{pII} \) is the work done on the target at interface \( II \). These equations can be integrated to obtain net work.

### 3.4 Non-Dimensionalization

The equations of motion are expressed in terms of the following non-dimensional variables:

\[
t^* = \frac{t}{t_r}, \quad U_f^* = \frac{U_f}{U_{f0}}, \quad U_p^* = \frac{U_p}{U_{f0}}, \quad x_f^* = \frac{x_f}{w_p}, \quad x_p^* = \frac{x_p}{w_p}.
\]  
(3.55)
The non-dimensional equations are given by:

\[
\frac{dU_f^*}{dt^*} = \begin{cases} 
  -\Pi_1 & t^* \leq 1, \\
  -\Pi_1(U_f^* - U_p^*) & t^* > 1.
\end{cases} \tag{3.56}
\]

\[
\frac{dU_p^*}{dt^*} = \begin{cases} 
  \Pi_2[\Pi_1 - \Pi_3x_p^*] & t^* \leq 1, \\
  \Pi_2[\Pi_1(U_f^* - U_p^*) - \Pi_3x_p^*] & t^* > 1.
\end{cases} \tag{3.57}
\]

\[
\frac{dx_f^*}{dt^*} = \Pi_4U_f^* \tag{3.58}
\]

\[
\frac{dx_p^*}{dt^*} = \Pi_4U_p^* \tag{3.59}
\]

Initial conditions for Eqs. (3.56)-(3.59) are given by \(U_f^*(0) = 1, U_p^*(0) = 0, x_f^*(0) = 0, \) and \(x_p^*(0) = 0.\) The target plate failure occurs when \(x_p^* \geq x_{pc}^*\) where \(x_{pc}^* = \frac{2}{w_p}.\) The non-dimensional parameters, \(\Pi_i,\) are defined by

\[
\Pi_1 = \begin{cases} 
  -\frac{P_SA_f t_r}{m_f U_f^0} & t^* \leq 1, \\
  \frac{t_r}{\mu m_f} & t^* > 1.
\end{cases} \tag{3.60}
\]

\[
\Pi_2 = \frac{m_f}{m_p}, \quad \Pi_3 = \frac{K_m w_p t_r}{m_f U_f^0}, \quad \Pi_4 = \frac{U_f^0 t_r}{w_p}. \tag{3.61}
\]

Each of these parameters has physical meaning. Here, \(\Pi_1\) is an effective impulse on the flier due to the shock pressure over the initial momentum of the flier, \(\Pi_2\) is a ratio of the flier mass to the target plug mass, \(\Pi_3\) is an effective impulse on the target due to the quasi-static force over the initial flier momentum, and \(\Pi_4\) is the distance of flier travel at initial velocity during time \(t_r\) over the thickness of the target plate.
3.5 Analytical Solution

Equations (3.56)-(3.59) are summarized by $\frac{d\vec{y}}{dt} = \hat{A}\vec{y}$ and can be solved analytically. During the time, $t^* \leq 1$, the solution is defined by

$$U_f^* = 1 - \Pi t^*, \quad (3.62)$$

$$U_p^* = \frac{\Pi_1}{\Pi_3} \sin(\sqrt{\Pi_2\Pi_3}t^*), \quad (3.63)$$

For the time, $t^* > 1$, the analytical solution to the governing equations takes the form:

$$\vec{y} = C_1 e^{\lambda_1 t^*} \vec{b}_1 + C_2 e^{\lambda_2 t^*} \vec{b}_2 + C_3 e^{\lambda_3 t^*} \vec{b}_3 \quad (3.64)$$

where $\vec{y}(t^*) = [U_f^*(t^*), \ U_p^*(t^*), \ x_p^*(t^*)]$, $\lambda_i$ are the eigenvalues, and $\vec{b}_i$ are the eigenvectors of the coefficient matrix, $\hat{A}$. The eigenvalues for this solution are the roots of a cubic function making the calculation of the eigenvectors difficult. In this work, the eigenvectors are calculated numerically using the well developed linear algebra package LAPACK. The constants $C_i$ are found by applying the initial conditions, $\vec{y}(1) = [1, 0, 0]$, and using LAPACK to solve the system of equations. To investigate the error associated with solving the eigenvalues using LAPACK, several well known analytical solutions were solved, each with good agreement.

3.5.1 Limiting Form Solutions

Various conditions for impact cases suggest a number of limiting form analytical solutions:

(a) **Quasi-static limit, $U_{f0} \approx 0$:**

One limiting form solution occurs in the quasi-static limit. Here we have an external force, $F^*$, pushing the flier into the target plate. The velocity and acceleration of the flier and target are zero in the quasi-static limit. This reduces the combined
momentum equations to

\[
\frac{dU^*_f}{dt^*} + \frac{dU^*_p}{dt^*} = F^* - \Pi_2 \Pi_3 x^*_p = 0. \tag{3.65}
\]

This leaves a force displacement relationship given by

\[
F^* = \Pi_2 \Pi_3 x^*_p. \tag{3.66}
\]

This is the non-dimensional form of the same relation plotted in Fig. 3.7 where the force \( F_{II} \) is defined.

\( \text{(b) High initial velocity, } U_{f0} \gg 0: \)

The opposing limiting form solution occurs when the initial velocity of the flier is high. In this case, \( \Pi_3 = \frac{K_{mwp0}}{m_f U_{f0}} \rightarrow 0 \). This leaves a system of homogeneous linear differential equations:

\[
\frac{dU^*_f}{dt^*} = \begin{cases} 
-\Pi_1 & t^* \leq 1, \\
-\Pi_1(U^*_f - U^*_p) & t^* > 1,
\end{cases} \tag{3.67}
\]

\[
\frac{dU^*_p}{dt^*} = \begin{cases} 
\Pi_2 \Pi_1 & t^* \leq 1, \\
\Pi_2 \Pi_1(U^*_f - U^*_p) & t^* > 1.
\end{cases} \tag{3.68}
\]

In this case the solution can be solved directly and becomes

\[
U^*_f = \begin{cases} 
1 - \Pi_1 t^* & t^* \leq 1, \\
C_1 - \frac{C_2}{\Pi_1} e^{-\Pi_1 t^*} & t^* > 1,
\end{cases} \tag{3.69}
\]

\[
U^*_p = \begin{cases} 
\Pi_2 \Pi_3 t^* & t^* \leq 1, \\
C_1 + C_2 e^{-\Pi_1 t^*} & t^* > 1.
\end{cases} \tag{3.70}
\]

Initial conditions at \( t^* = 1 \) are applied to compute \( C_1 \) and \( C_2 \).
(c) \(m_f \ll m_p\):

Another limiting form solution occurs as \(\Pi_2 = \frac{m_f}{m_p} \to 0\). This physically means that the target plug mass is much greater than the flier mass. In this case \(\frac{dU_f^*}{dt^*} \to 0\). Applying the initial conditions \(U_f^*(0) = 1\) and \(U_p^*(0) = 0\) we get:

\[
\frac{dU_f^*}{dt^*} = \begin{cases} 
-\Pi_1 & t^* \leq 1, \\
-\Pi_1 U_f^* & t^* > 1.
\end{cases}
\]

The solution becomes

\[
U_f^* = \begin{cases} 
1 - \Pi_1 t^* & t^* \leq 1, \\
C_1 - e^{-\Pi_1 t^*} & t^* > 1.
\end{cases}
\]

(d) \(m_f \gg m_p\):

In the opposite limit, when \(\frac{1}{\Pi_2} \to 0\), the flier mass is much greater than the target mass. In this limit \(\frac{dU_f^*}{dt^*} \to 0\). This means that the flier remains at it’s initial velocity and perforates the target bringing the target plug to the initial velocity of the flier.

The equations of motion are reduced to

\[
\frac{dU_p^*}{dt^*} = \begin{cases} 
\Pi_2[\Pi_1 - \Pi_3 x_p^*] & t^* \leq 1, \\
\Pi_2[\Pi_1(1 - U_p^*) - \Pi_3 x_p^*] & t^* > 1,
\end{cases}
\]

\[
\frac{dx_p^*}{dt^*} = \Pi_4 U_p^*.
\]

For the time, \(t^* \leq 1\), the solution for the target velocity is the same as the full analytical solution, Eq. (3.63). For the time, \(t^* > 1\), this system can be solved in the same manner as the full analytical solution and will have a solution of the form:

\[
\vec{y} = C_1 e^{\lambda_1 t^*} \vec{b}_1 + C_2 e^{\lambda_2 t^*} \vec{b}_2 + \vec{g},
\]

(3.75)
where $\vec{y} = [U_p^*, x_p^*]$, and $\lambda_i$ and $\vec{b}_i$ are the eigenvalues and eigenvectors respectively, $C_i$ are constants found in applying initial conditions, $\vec{g}(1)$, and $\vec{g}$ is the particular solution. These limiting form solutions are presented to show how the system equations may be reduced for various configurations for completeness. One of these limiting form solutions is used in the following chapter where the target mass is unknown but is much greater than the flier mass.

### 3.6 Numerical Solution

Due to the difficulty in formulating the analytical solution of this problem and for convenience in later analysis, the model will be solved using a numerical code. A fourth-order Runge-Kutta integration scheme is used to integrate the system of linear ODE’s. Figure 3.9 shows a comparison for a single case between the numerical and analytical solutions for non-dimensional flier velocity. The code is verified by checking the convergence of the discretization error using the method described by Oberkampf and Roy [37].

![Figure 3.9: Comparison numerical and analytical solution for a steel projectile 80 mm long and 20 mm in diameter impacting a steel target 12 mm thick at an initial flier velocity, $U_{f0} = 100 m/s$](image)

Figure 3.9: Comparison numerical and analytical solution for a steel projectile 80 mm long and 20 mm in diameter impacting a steel target 12 mm thick at an initial flier velocity, $U_{f0} = 100 m/s$
The discretization error used in this verification is defined by

\[ \| u - u_{ref} \|_1 = \frac{1}{N} \sum_{n=1}^{N} |u_n - u_{ref,n}|, \]  

(3.76)

where \( u \) is the numerical solution, \( u_{ref} \) is the analytical solution and \( N \) is the number of steps. For this verification we have chosen flier velocity as the dependent variable. Because our solution is piecewise continuous at the point \( t^* = 1 \), verification has been done separately for the two solutions. The observed order of accuracy, \( \hat{p} \), is defined by

\[ \hat{p} = \frac{\ln(\epsilon_{rh})}{\ln(r)}, \]  

(3.77)

where \( \epsilon_{rh} \) is the discretization error of the coarse mesh, \( \epsilon_h \) is the discretization error of the fine mesh, and \( r \) is the ratio of the number of points in the fine mesh to number of points in the coarse mesh. Table 3.1 shows the results of verification of the numerical code. The observed order of accuracy for each of the solutions is close to four as expected. In our code we will numerically integrate the initial value problem. This will introduce error with the calculation of \( \mu \) at \( t^* = 1 \) using the velocities at that numerical time step. Though there is some additional error in using this approach, it is negligibly small.

<table>
<thead>
<tr>
<th>Number of Points (coarse)</th>
<th>( t^* \leq 1 )</th>
<th>( t^* &gt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Points (fine)</td>
<td>50</td>
<td>6638</td>
</tr>
<tr>
<td>Discretization Error, ( \epsilon_{rh} ) (coarse)</td>
<td>( 1.3415 \times 10^{-12} )</td>
<td>( 4.9383 \times 10^{-11} )</td>
</tr>
<tr>
<td>Discretization Error, ( \epsilon_h ) (fine)</td>
<td>( 2.0313 \times 10^{-15} )</td>
<td>( 3.0092 \times 10^{-12} )</td>
</tr>
<tr>
<td>Observed Order of Accuracy, ( \hat{p} )</td>
<td>4.0342</td>
<td>4.0366</td>
</tr>
</tbody>
</table>
Chapter 4

Predictions

This chapter highlights the model behavior and predictions by first examining the time response for representative cases of impact. For all configurations considered in this study there are common features of transient responses that are illustrated and discussed. Cases for which impact data is available and for similar configurations to laser-driven micro-flier experiments are modeled for validation, and the effects of applying an adhesive force to the flier-target interface is investigated. Results of the model time response are then compared to available experimental data for these cases. Predictions of ballistic limit for several configurations are made and compared to available data. Model uncertainty and sensitivity are then discussed, and estimates of various quantities of interest to researchers are presented. Finally, a method of extending the model to targets having an energetic solid deposited to their backside is discussed. Representative predictions are given to illustrate the thresholds for perforation of the target plate and the initiation threshold of the energetic material where initiation implies transition to detonation. These results are given by what we will refer to as ballistic initiation maps and give researchers a range of configurations for desired experiment behavior in which initiation of the energetic occurs without perforation of the target plate.

4.1 Transient Impact Response

Two important considerations in the behavior of the model are the thickness ratio of the flier to the target, $\frac{w_f}{w_p}$, and the shock impedance ratio of the flier material to the target material, $\frac{Z_f}{Z_p}$. Here, $Z_f = \rho_f D_{sf}$ and $Z_p = \rho_p D_{sp}$ are the shock impedances of the flier and target respectively, where $\rho_f$ is the flier density, $\rho_p$ is the target density, $D_{sf}$ is the shock speed in the flier at impact, and $D_{sp}$ is the shock speed through the target at impact. The impedance and thickness ratios affect the behavior of the early-time shock
Table 4.1: Representative Configurations for Flier-Target Impact.

<table>
<thead>
<tr>
<th>#</th>
<th>Flier</th>
<th>Target</th>
<th>$r_f$ (mm)</th>
<th>$R$ (mm)</th>
<th>$w_f$ (mm)</th>
<th>$w_p$ (mm)</th>
<th>$\frac{w_f}{w_p}$</th>
<th>$Z_f$ (kg mm² s)</th>
<th>$Z_p$ (kg mm² s)</th>
<th>$\frac{Z_f}{Z_p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2024-T4 Al</td>
<td>304 SS*</td>
<td>10</td>
<td>250</td>
<td>100</td>
<td>10</td>
<td>10</td>
<td>15.2</td>
<td>36.5</td>
<td>0.4</td>
</tr>
<tr>
<td>2</td>
<td>304 SS</td>
<td>2024-T4 Al</td>
<td>10</td>
<td>250</td>
<td>100</td>
<td>10</td>
<td>10</td>
<td>36.5</td>
<td>15.2</td>
<td>2.4</td>
</tr>
<tr>
<td>3</td>
<td>304 SS</td>
<td>2024-T4 Al</td>
<td>10</td>
<td>250</td>
<td>10</td>
<td>100</td>
<td>0.1</td>
<td>36.5</td>
<td>15.2</td>
<td>2.4</td>
</tr>
<tr>
<td>4</td>
<td>2024-T4 Al</td>
<td>304 SS</td>
<td>10</td>
<td>250</td>
<td>10</td>
<td>100</td>
<td>0.1</td>
<td>15.2</td>
<td>36.5</td>
<td>0.4</td>
</tr>
</tbody>
</table>

* SS denotes stainless steel.
physics described by the model and dictate the characteristic behavior of each case. This
section will show the time response the representative configurations listed in Table 4.1
with material properties listed in Table 4.2. The time response of each configuration is
shown and discussed in detail. To this end, an initial flier velocity of \( U_{f0} = 100 \) m/s is
used to assure that the flier does not perforate the target plate allowing the time response
to be studied in detail from the time of impact until the target plug velocity is zero.

**Configuration 1** represents a case in which the impedance ratio is less than unity,
\( \frac{Z_f}{Z_p} < 1 \), and the thickness ratio is greater than unity, \( \frac{w_f}{w_p} > 1 \). Performing 1-D shock
impedance matching produces results presented in Fig. 4.1 which indicate that the flier
achieves a negative particle velocity of \( u_f \approx -50 \) m/s by the release wave from its free
surface shortly following impact whereas the target ultimately achieves a positive particle
velocity of \( u_p \approx 60 \) m/s. However, because the flier is thicker than the target, the release
wave from the free surface of the target plate arrives at the flier-target interface at time \( t_r \).
before the release wave from the flier free surface. Thus, the mass averaged velocity of the flier may remain positive because the release wave has not had time to bring the particle velocity throughout the flier to the negative value suggested by impedance matching. The velocity is spatially non-uniform with a majority of the velocity field being positive. In the model, the pressure at the interface is released at time $t_r$ when the mass averaged velocity of both the flier and target plug are positive, and no future wave interactions are tracked.

The predicted time response is shown in Fig. 4.2 where Figs. 4.2 (a) and (b) show the mass averaged velocities of the flier and target, respectively. During the early-time response, there is rapid change in the velocities of the flier and target due to the shock force at interface $I$ with the flier and target at positive velocities at time $t_r$. A significant change in the time response occurs at time $t_r$ as the force at interface $I$ becomes adhesive, $F_I = \frac{1}{\mu}(U_f - U_p)$. In this case the adhesive force does not act as a tensile force; rather, it acts in compression to prevent flier-target overlap. Figure 4.2 (a) and (b) illustrate how this force affects the velocities of the flier and target causing them to nearly equilibrate at a value of $\sim 77 \text{ m/s}$. The later-time response is controlled by the quasi-static force at interface $II$ which acts to gradually decrease the flier and target velocities until they come to rest. As discussed in Chapter 3, the flier has a small finite velocity when the target plug velocity reaches zero. The flier velocity is sufficiently small to assume $U_f \approx 0$, illustrated by Fig. 4.2 (a).

Figure 4.2 (c) and (d) show the displacement of the center of mass of the flier and target plug, respectively. Though the force at interface $I$ acts to prevent overlap of the flier and target, the small finite velocity difference between the two allows for the flier center of mass to travel slightly further than the center of mass of the target plug. Assuming that the volume of the flier and target plug, and that the frontal contact area $A_f$, are constant, this presents a non-physical overlap between them. We ignore this overlap because it is a small value in comparison to the target and flier thicknesses. The critical displacement for target failure is represented by the dashed line in Fig. 4.2 (d). In this case, the target
Figure 4.2: Time response for Configuration 1: (a) Flier velocity; (b) Target velocity; (c) Flier displacement; (d) Target displacement; (e) $F_I$; (f) $F_{II}$. 
plug displacement is not sufficient to cause failure and the target plug velocity reaches zero without perforation of the target plate.

Figures 4.2 (e) and (f) show the force at each interface illustrating that the force at interface II is very small in comparison to the shock force during the early-time response. The force at interface I is shown in Fig. 4.2 (e) as a constant during the early-time response, \( F_I = P_S A_f = \text{cst} \), until time \( t_r \). At time \( t_r \), \( F_I = \frac{1}{\mu} (U_f - U_t) \) which acts to rapidly bring the velocities of the flier and target plug together. As the velocity difference between the flier and target decreases, the force at interface I decreases. After the initial decrease in velocity difference between the flier and target, the velocity difference slowly increases due to the force acting at interface II. Subsequently, the force at interface I gradually increases. The force at interface II is a function of target displacement and is shown in Fig. 4.2 (f). This dependence on target displacement is observed in comparing Figs. 4.2 (d) and (f).

**Configuration 2** represents a case in which the thickness ratio is greater than unity, \( \frac{w_f}{w_p} < 1 \), and the shock impedance ratio is greater than unity, \( \frac{Z_f}{Z_p} > 1 \), giving the target material a lower value of shock impedance than that of the flier. Performing 1-D shock impedance matching in this case, suggests that the velocities of the target and flier are both positive at time \( t_r \) shown by Fig. 4.3 and that the target velocity has surpassed the flier velocity. This suggests that the target will separate from the flier when a release wave from the either the flier or target free surface reaches the flier-target interface. The time response of configuration 2 is shown in Fig. 4.4, with Figs. 4.4 (a) and (b) showing the velocity of the flier and target. During the early-time response, the model exhibits behavior suggested by impedance matching where the flier is decelerated and the target plug is accelerated to a velocity much higher than the flier. At time \( t_r \), the velocity difference between the flier and target is now negative, \( (U_f - U_t) < 0 \), which differs from the previous case of Configuration 1. As the impedance matching suggests, the model also shows that the target plug will physically separate from the flier if the adhesive force is not applied. At time \( t_r \), the negative velocity difference between the flier and target causes the adhesive
force at interface I to act as a tensile force preventing separation and acting to bring the velocities to nearly the same value.

During the late-time response, the force at interface II dominates, slowly reducing the velocity of the target and subsequently the flier. This represents the general behavior of the long-term response for all cases. Figure 4.4 (c) and (d) show the time response of the displacement of the flier and target for Configuration 2 which behaves similarly to Configuration 1. The target displacement does not reach the critical value for failure, with the target plug and flier coming to rest. Figure 4.4 (e) and (f) show the force at interfaces I and II respectively. Figure 4.4 (e) illustrates the behavior of the force at interface I which becomes a tensile force at time $t_r$ changing sign to prevent separation of the flier and target plug. The quasi-static force at interface II behaves no differently than in Configuration 1 as shown in Fig. 4.4 (f).

**Configuration 3** represents a case in which the thickness ratio between the flier and target is less than unity, $\frac{w_f}{w_p} < 1$, and the impedance ratio is greater than unity, $\frac{Z_f}{Z_p} > 1$. For this case, the impedance matching is given by Fig. 4.3. In the model configuration, the target is much thicker than the flier such that the release time is given by the release from the flier free surface as it reaches the flier-target interface. Because the flier release
Figure 4.4: Time response for Configuration 2: (a) Flier velocity; (b) Target velocity; (c) Flier displacement; (d) Target displacement; (e) $F_I$; (f) $F_{II}$. 
wave controls the early time response and the target is more massive than the flier, the model predicts that the target velocity does not surpass the flier giving a positive velocity difference between them. The adhesive force, in this case, will act in compression rather than tension preventing the flier from traveling further than the target plug.

Figure 4.5 shows the time response of Configuration 3. Though the trends of response quantities are similar to Configuration 1, the magnitude of these quantities including velocities, displacements, and forces are lower for Configuration 3 due to the lower thickness ratio between the flier and target. The magnitude of the force at interface $I$ during the early-time response has the same value in all configurations due to its sole dependence on material properties and initial velocity.

**Configuration 4** represents a case in which the thickness ratio and impedance ratio are both less than unity, $\frac{w_f}{w_p} < 1; Z_p < 1$. The impedance matching for this case is shown in Fig. 4.1. In this case, the model exhibits this behavior with a negative flier velocity and positive target velocity at time $t_r$. As in Configuration 2, this gives a negative velocity difference between the flier and target causing the adhesive force to become tensile at interface $I$. Figure 4.6 (a) and (b) show the velocity of the flier and target plug where the direction change of the flier is clearly seen. The target is more massive than the flier in this case which is evident in the minimal change in velocity of the target plug. Figures 4.6 (c) and (d) show the displacement of the flier and target respectively. The final displacement of the flier and target plug are small in this case with no perforation. The predicted displacement of the target plug is approximately two orders of magnitude lower than the critical displacement of $x_{pc} \approx 37$ mm. The behavior of the forces at interfaces $I$ and $II$ is similar to that for Configuration 2 and is shown in Fig. 4.6 (e) and (f). After time $t_r$ the response behaves as it does in all configurations 1-4 slowly bringing the system towards rest.

The laser-driven micro-flier cases which we model are similar to Configuration 4 in that the adhesive force acts as a tensile force between the flier and target. In addition to
Figure 4.5: Time response for configuration 3: (a) Flier velocity; (b) Target velocity; (c) Flier displacement; (d) Target displacement; (e) $F_I$; (f) $F_{II}$. 
Figure 4.6: Time response for configuration 4: (a) Flier velocity; (b) Target velocity; (c) Flier displacement; (d) Target displacement; (e) $F_I$; (f) $F_{II}$. 
the predicted response of the system that was shown above, we consider predictions of the system behavior for Configurations 4 in the absence of an adhesive force to highlight its effect on the system response. Removing the adhesive force allows for separation between the flier and target. When separation does occur, the force at interface \( I \) is zero and the flier travels at a constant velocity while the target plug is decelerated by the force at interface \( II \).

Figure 4.7 shows the time response for a system having Configuration 4 with no adhesive force. In this case the flier separates from the target plate at time \( t_r \) rebounding with a negative velocity. The target plug then decelerates due to the quasi-static force at interface \( II \) as seen in Figs. 4.7 (a) and (b). Configurations in which the flier velocity is positive at time \( t_r \) will present a non-physical result if the adhesive force is not considered. The flier, at time \( t_r \), will have no force acting upon it and will therefore travel at a constant velocity passing up the target plug. Because the laser-driven flier coupons suggest adhesion to the target, we continue to apply the adhesive force for all cases in this study.

There are several quantities that may be of interest to researchers including the work done on the flier and target and the impulse to the flier and target. Figure 4.8 shows the time response of these quantities for Configuration 4. The impulse to the flier and target are shown in Figs. 4.8 (a) and (b). The impulse to the target is negative due to the direction of the force at interface \( I \). The magnitude of the flier impulse increases rapidly during the early-time response then decreases at time \( t_r \) due to the adhesive force. The net impulse to the target plug is shown in Fig. 4.8 (b) which increases rapidly during the early-time response due to the shock force at interface \( I \) and decreases gradually during the late-time response due to the force at interface \( II \). The net work done on the flier and target are shown in Figs. 4.8 (c) and (d). Though the magnitude of the flier net work and target net work appear to be equal, they differ in magnitude by the work done at interface \( II \) which is shown in Fig. 4.8 (f). Therefore, the net work done on the system is equal to the work done at interface \( II \).
Figure 4.7: Comparison of time responses for configuration 4 with and without adhesion: (a) Flier velocity; (b) Target velocity; (c) Flier displacement; (d) Target displacement; (e) $F_I$; (f) $F_{II}$. 

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Figure 4.8: Time response for configuration 4 impulse and work: (a) Flier Impulse; (b) Target Impulse; (c) Flier Work; (d) Target Work; (e) Work done on target at interface I; (f) Work done on target at interface II.
4.1.1 Large Flier-Target Configurations

To investigate the validity of the proposed model, results are compared to available experimental data from the literature. Since there is a lack of data for micro-flier configurations, we first compare to data from larger scale impact experiments. By large we mean flier and target thicknesses on the order of $\sim 10 \text{ mm}$ to $\sim 100 \text{ mm}$. Figures 4.9 and 4.10 show comparisons to data from two large flier-target configurations listed in Table 4.3 as B-1 and B-2. Material properties taken from Borvik [10] are used in this case and are listed in Table 4.4. Values for equation of state parameters are taken as those of steel from LASL Shock Hugoniot Data [34].

Table 4.3: System configurations for large flier-target impact.

<table>
<thead>
<tr>
<th>#</th>
<th>Flier</th>
<th>Target</th>
<th>$r_f$ (mm)</th>
<th>$R$ (mm)</th>
<th>$w_f$ (mm)</th>
<th>$w_p$ (mm)</th>
<th>$U_{f0}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-1</td>
<td>Tool Steel</td>
<td>Weldox 460 E</td>
<td>10</td>
<td>250</td>
<td>80</td>
<td>10</td>
<td>296</td>
</tr>
<tr>
<td>B-2</td>
<td>Tool Steel</td>
<td>Weldox 460 E</td>
<td>10</td>
<td>250</td>
<td>80</td>
<td>12</td>
<td>179</td>
</tr>
</tbody>
</table>

Table 4.4: Material Properties: Arne Tool Steel and Weldox 460 E

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_0$ (MPa)</th>
<th>$\sigma_u$ (MPa)</th>
<th>$\rho \left( \frac{kg}{m^3} \right)$</th>
<th>$C$ ($\frac{km}{s}$)</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arne Tool Steel</td>
<td>-</td>
<td>-</td>
<td>7850</td>
<td>4.58</td>
<td>1.49</td>
</tr>
<tr>
<td>Weldox 460E</td>
<td>490</td>
<td>625</td>
<td>7850</td>
<td>4.58</td>
<td>1.49</td>
</tr>
</tbody>
</table>

Figure 4.9 shows the model compared to velocimetry measurements from experiments performed by Borvik [10] for configuration B-1. In this case, the flier completely perforates the target plate. In Fig. 4.9 (a) the model and experiments show that the velocity of the flier decreases during impact with the model and experimental data showing qualitatively similar trends. Both the model and experimental data show that the flier perforates the target, but the model predicts perforation at a later time than the experimental data suggests. In Fig. 4.9 (a) perforation has occurred when the velocity is no longer decreasing and the flier has a constant residual velocity. Though perforation does not occur at the
same time, the value of residual velocity predicted by the model is close to that shown by the experiment.

Figure 4.10 shows model comparisons to data from LS-DYNA simulations run by Borvik for Configuration B-2 from Table 4.3 [7]. Configuration B-2 is the same as Configuration B-1 with the exception that the target thickness is 12 mm and the initial velocity of the flier is 179 m/s. In this case, the model predicts that the flier comes to rest before perforation of the target while the LS-DYNA simulation shows that perforation occurs. This difference is most likely due to the difference in failure criterion for each model. Our model and the LS-DYNA simulation show different results in this regard as well as in the trends of the velocity curves shown in Fig. 4.10. The LS-DYNA suggests that there are some rate dependent influences on target due to the inclusion of thermal softening and damage in the constitutive model while model shows that the strength increases with target displacement. The force at the flier-target interface is shown in comparison to the interface force given by the LS-DYNA simulation in Fig. 4.10 (c). The initial peak of the force in both the model and the LS-DYNA simulation are comparable, but the model deviates from LS-DYNA when the force changes sign at time $t_r$. 
Figure 4.10: Model comparison to LS-DYNA simulation of a steel flier, $U_f \approx 179 \text{ m/s}$, impacting a 12 mm thick Weldox 460E steel target: (a) Flier velocity, $U_f$; (b) Flier displacement, $x_f$; (c) Flier-target interface force.

4.1.2 Small (Micro) Flier-Target Predictions

Preliminary predictions for small flier-target configurations are presented in this section. Small configurations include flier and target thicknesses on the order of 10 µm - 100µm. Figure 4.11 shows model comparisons to experimental data from micro-scale impact events of aluminum fliers on soda-lime glass and polycarbonate targets measured using a velocity interferometer system for any reflector (VISAR) [18]. Configurations are listed in Table 4.5. Material properties for glass and polycarbonate are given in Table 4.6 and aluminum in Table 4.2. Exact geometries of the target plates for these cases were not given, but the target plates are much thicker and more massive than the aluminum fliers. For this reason a limiting form solution for $\frac{m_f}{m_p} \rightarrow 0$ is used. In this case the acceleration of the target
Table 4.5: System configurations for small flier-target impact.

<table>
<thead>
<tr>
<th>#</th>
<th>Flier</th>
<th>Target</th>
<th>(r_f) (mm)</th>
<th>(w_f) (mm)</th>
<th>(U_{f0}) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-1</td>
<td>2024 Al</td>
<td>soda-lime glass</td>
<td>1.9</td>
<td>250</td>
<td>462</td>
</tr>
<tr>
<td>D-2</td>
<td>2024 Al</td>
<td>polycarbonate</td>
<td>1.9</td>
<td>100</td>
<td>1140</td>
</tr>
<tr>
<td>D-3</td>
<td>2024 Al</td>
<td>polycarbonate</td>
<td>1.9</td>
<td>250</td>
<td>558</td>
</tr>
</tbody>
</table>

Table 4.6: Material Properties: soda-lime glass and polycarbonate.

<table>
<thead>
<tr>
<th></th>
<th>(\rho) (kg/m(^3))</th>
<th>(C) (km/s)</th>
<th>(S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>soda-lime glass</td>
<td>5085</td>
<td>2.0</td>
<td>1.5</td>
</tr>
<tr>
<td>polycarbonate</td>
<td>1193</td>
<td>2.18</td>
<td>1.0</td>
</tr>
</tbody>
</table>

plug is not considered, and the force at interface \(II\) does not play a role. Material strength properties are left out of Table 4.6 for this reason. The dominating response is due to the early-time shock wave mechanics. Figure 4.11 shows the flier velocity versus time for these cases predicted by the model in comparison to VISAR measurements. In each case the model does a reasonable job at matching the time response of the flier velocity given by experiment.

CTH calculations were performed for comparison to the model [13]. CTH is a multi-material, large deformation, strong shock wave, solid mechanics code that was developed at Sandia National Laboratories which has models for multi-phase, elastic, viscoplastic, porous and explosive materials [17]. This simulation is of an 11 \(\mu\)m thick flier with a 1.4 mm diameter and an initial velocity of 1500 m/s impacting a 50 \(\mu\)m thick target clamped at a diameter of 3 mm. The diameter of 1.4 mm is the laser pulse diameter. In reality this may not be the actual flier diameter due to ablation of flier mass or changes in geometry during flight. This configuration is summarized in Table 4.7. The CTH results suggest that significant spallation occurs during the later-time response of the impact event. Figure 4.12

Table 4.7: System configuration for CTH.

<table>
<thead>
<tr>
<th>#</th>
<th>Flier</th>
<th>Target</th>
<th>(r_f) (mm)</th>
<th>(R) (mm)</th>
<th>(w_f) ((\mu)m)</th>
<th>(w_p) ((\mu)m)</th>
<th>(U_{f0}) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTH-1</td>
<td>2024-T4 Al</td>
<td>304 SS</td>
<td>0.7</td>
<td>1.5</td>
<td>11</td>
<td>50</td>
<td>1500</td>
</tr>
</tbody>
</table>
Figure 4.11: Velocity response of laser-driven aluminum fliers: (a) Configuration D-1; (b) Configuration D-2; (c) Configuration D-3.

shows the axi-symmetric flier and target prior to impact where the lighter gray represents the target and the darker gray represents the flier. The small markers (dots) on the target and flier are tracer points used to capture Lagrangian data from the calculation which are used to obtain mass averaged values. A pressure contour for the early-time CTH simulation is shown in Fig. 4.13 where the initial shock is clearly seen and spallation of the flier has begun to occur. It is noted that the shock is nearly planar with 2-D effects restricted to the periphery of the target plug region, and therefore, 1-D assumptions are reasonable for this configuration. The late-time, exhibiting extensive spallation, is shown in Fig. 4.14. This is not consistent with the post-impact visualization of coupons from this particular configuration which do not show spallation. Though this later-time spallation may not be
Figure 4.12: CTH pressure contour prior to impact.
Figure 4.13: Pressure contour from CTH early-time.
Figure 4.14: Pressure contour from CTH late-time.
physical, the early-time response of the velocity and pressure are reasonable to compare with our model.

Figure 4.15 shows the time response of the pressure and mass-averaged velocities of this CTH simulation. Mass averaged velocities are used to show an average value of velocity over the flier or target rather than showing the particle velocity at each tracer point. Material between two tracer points is assumed to travel at the average velocity between them. The mass averaged velocity for the flier or target plug is calculated by

$$\frac{1}{m} \sum_{i=0}^{N} m_i U_i,$$

(4.1)

where $m$ is the total mass, $U_i$ is the average velocity between two tracers, $m_i$ is the mass between two tracers, and $N$ is the number of sections between tracers. Figure 4.15 (a) shows the pressure at the flier-target interface. The model matches the initial pressure response relatively well, but the model deviates once the adhesive force begins acting at time $t_r$. Figure 4.15 (b) shows the mass averaged flier velocity. During the early time the model follows the same trend as the CTH simulation with the flier velocity rapidly decreasing. The CTH results indicate that the flier rebounds and experiences spallation during the time after the early response while our model imposes adhesion. Figure 4.15 (c) shows the mass averaged target velocity. The early time response of the model matches the CTH data well. After the initial increase in velocity, the target experiences spallation. The backside of the target spalls off with a much higher velocity increasing the mass-averaged velocity in the CTH calculation. At this point, comparison with our prediction is no longer meaningful.

4.2 Residual Velocity and Ballistic Limit

The previous comparisons for the small and large flier-target configurations show how the model compares qualitatively to experimental data and other simulations of the time response during impact. This section shows comparisons to data for ballistic limit and
residual velocity for various configurations. The ballistic limit, $U_{BL}$, is the minimum initial flier velocity required to perforate the target plate, and the residual velocity, $U_{r}$, is the velocity of the flier immediately after perforation of the target plate. We make these comparisons to qualitatively validate the model for non-time dependent response quantities that may give an indication to whether the strength of the target plate is modeled in a reasonable manner. Perforation is undesirable in performing laser-driven flier experiments for bench-top energetics making it an important behavior for the model to predict.

Figure 4.16 shows the model prediction in comparison to experimental data from Liss et al. [33]. The configurations for these cases are given in Table 4.8. Predictions were
Table 4.8: System configurations for large flier-target residual velocity predictions.

<table>
<thead>
<tr>
<th>#</th>
<th>Flier</th>
<th>Target</th>
<th>$r_f$ (mm)</th>
<th>$R$ (mm)</th>
<th>$w_f$ (mm)</th>
<th>$w_p$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L-1</td>
<td>hard steel</td>
<td>2024-0 Al</td>
<td>6.15</td>
<td>69.85</td>
<td>38.1</td>
<td>3.2</td>
</tr>
<tr>
<td>L-2</td>
<td>hard steel</td>
<td>2024-0 Al</td>
<td>6.15</td>
<td>69.85</td>
<td>38.1</td>
<td>6.4</td>
</tr>
</tbody>
</table>

Table 4.9: Material Properties: hard steel and 2024-0 Al.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_0$ (MPa)</th>
<th>$\sigma_u$ (MPa)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$C$ ($\text{km/s}$)</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>hard steel</td>
<td>-</td>
<td>-</td>
<td>7890</td>
<td>4.58</td>
<td>1.49</td>
</tr>
<tr>
<td>2024-0 Al</td>
<td>88</td>
<td>233</td>
<td>2770</td>
<td>5.37</td>
<td>1.29</td>
</tr>
</tbody>
</table>

made using material properties for steel and aluminum from Table 4.9. In Figs. 4.16 (a) and (b), residual velocity, $U_r$, of the projectile versus initial velocity, $U_{f0}$, of the projectile is shown. The residual velocity is zero until the initial velocity is sufficient to perforate the target plate. To highlight the importance of target plate strength in the model, we show the results of the model with no quasi-static force applied at interface $II$ represented by the dashed line in Fig. 4.16. Without strength considered, the ballistic limit is nearly zero, and the residual velocity is very close to the initial flier velocity. For these cases, with the quasi-static force applied at interface $II$, the model does reasonably well at predicting the residual velocity and ballistic limit of the flier.

Figure 4.16: Residual velocity, $U_r$, of hard steel projectiles impacting 2024-0 aluminum plates (a) Configuration L-1; (b) Configuration L-2.
Figure 4.17 shows residual flier velocity, $U_r$, versus initial flier velocity, $U_{f0}$, for cases taken from Forrestal et al. [21]. Configurations for these cases are given by Table 4.10 with material properties given by Table 4.11. Configurations F-1 and F-2 consider different materials than L-1 and L-2 to illustrate the robustness of the model. Again, reasonable agreement is seen between the model predictions and experimental data for ballistic limit and residual velocity.

Table 4.10: System Configurations.

<table>
<thead>
<tr>
<th>#</th>
<th>Flier</th>
<th>Target</th>
<th>$r_f$ (mm)</th>
<th>$R$ (mm)</th>
<th>$w_f$ (mm)</th>
<th>$w_p$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-1</td>
<td>4340 steel</td>
<td>HY-100 steel</td>
<td>15</td>
<td>152.5</td>
<td>282</td>
<td>10.5</td>
</tr>
<tr>
<td>F-2</td>
<td>4340 steel</td>
<td>HY-100 steel</td>
<td>15.4</td>
<td>152.5</td>
<td>268</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Table 4.11: Material Properties.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_0$ (MPa)</th>
<th>$\sigma_u$ (MPa)</th>
<th>$\rho$ ($Kg/m^3$)</th>
<th>$C$ ($Km/s^2$)</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4340 steel</td>
<td>-</td>
<td>-</td>
<td>7830</td>
<td>4.58</td>
<td>1.49</td>
</tr>
<tr>
<td>HY-100 steel</td>
<td>316</td>
<td>800</td>
<td>7860</td>
<td>4.58</td>
<td>1.49</td>
</tr>
</tbody>
</table>

Figure 4.17: Residual velocity, $U_r$, of 4340 steel projectiles impacting HY-100 steel plates (a) Configuration F-1; (b) Configuration F-2.
Figure 4.18: Residual velocity, $U_r$, of steel projectiles impacting 12 mm thick Weldox 460E steel plates.

Figure 4.18 shows data taken from Borvik et al. [9] for configuration B-2 given by Table 4.3. In this case, the model again does well at predicting the residual velocity of the projectile at higher velocities but deviates further from the data near the ballistic limit.

Figure 4.19 shows data from Borvik et al. [9] of the ballistic limit, $U_{BL}$, for target plates of increasing thickness. These cases are of the same configuration as B-2 with varying target plate thicknesses. Borvik performed LS-DYNA calculations for these cases which are also shown in Figure 4.19 for comparison. The simple model proposed does a reasonably predicts the ballistic limit for some cases but not in others. Further considerations of thickness dependent strength may aid the model in better predicting the correct trends in the ballistic limit for these cases. For instance, the predictions made by the model for these cases used a constant value of ultimate tensile strength. In reality, the ultimate tensile strength may increase with increasing target plate thickness. Considering this may lead to better agreement higher target thicknesses.
4.3 Uncertainty

There are several forms of uncertainty to consider when using a mathematical model for predictive purposes. There is numerical uncertainty including truncation error and discretization error; this was addressed in the verification of the numerical method in Chapter 3. In addition to numerical error, there is error due to uncertainty in model input parameters and error associated with incomplete or inaccurate descriptions of physical processes by the model referred to as model form error. These forms of error are much greater than that of the numerical uncertainty for this analysis. Where the numerical uncertainty can be considered negligible, the input uncertainty and model form error are not and should be addressed.

This section will describe these model uncertainties and will illustrate how they are propagated through the model. We perform this analysis as a preliminary strategy for obtaining model predictions that will provide experimentalists with a range for model output parameters or system response quantities (SRQs) of interest to make preliminary design decisions. We are particularly interested in the ballistic limit predicted by the model which will give researchers an upper bound for various configurations in laser-driven flier...


4.3.1 Input Uncertainty

There are two different types of model input uncertainty that must be considered in this analysis: *aleatory uncertainty* and *epistemic uncertainty*. Aleatory uncertainty is defined as uncertainty due to inherent randomness which is most commonly represented mathematically as a probability distribution. Epistemic uncertainty is uncertainty due to lack of knowledge about a given input parameter. Here, a range of input values may be known without any information about probability of values within the range. Key uncertain parameters for this analysis are listed in Table 4.12 and are designated as aleatory or epistemic. Other model parameters have either unknown or negligible uncertainty and are not considered in this study.

In the current model there are input parameters that have aleatory uncertainty including the yield strength and ultimate tensile strength of the target as well as all equation of state parameters. Each of these input parameters is assumed to have uncertainty that follows a normal distribution. The uncertainty of these parameters is defined in Table 4.12 by the mean and standard deviation. The values for uncertainty in yield strength and ultimate tensile strength are taken as average values of uncertainty for various types of aluminum and steel [1]. The uncertainty in the equation of state parameters for steel and aluminum are the standard error of the coefficients from performing a least squares linear regression of data from LASL Shock Hugoniot Data [34].

In consideration of the laser-driven micro-flier experiments, there is epistemic uncertainty in the geometry of the flier at impact. Some of the flier material may be ablated by the laser during launch decreasing the thickness of the flier [48]. Experiments also suggest that the fliers may become molten, change shape, or experience break-up during flight. Also, from observing post-impact target coupons, it is noticed that the flier may not exhibit a radius equal to that of the laser pulse. The target coupons in which the flier perforates the target suggest that the radius of the flier is in fact much smaller than
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Uncertainty Type</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield Strength</td>
<td>$\sigma_o$</td>
<td>aleatory</td>
<td>290 (MPa)</td>
<td>17.7 (MPa)</td>
</tr>
<tr>
<td>Ultimate Tensile Strength</td>
<td>$\sigma_u$</td>
<td>aleatory</td>
<td>505 (MPa)</td>
<td>15.5 (MPa)</td>
</tr>
<tr>
<td>Flier EOS parameter</td>
<td>$C_f$</td>
<td>aleatory</td>
<td>5.37 ($km/s$)</td>
<td>0.011 ($km/s$)</td>
</tr>
<tr>
<td>Flier EOS parameter</td>
<td>$S_f$</td>
<td>aleatory</td>
<td>1.29</td>
<td>0.005</td>
</tr>
<tr>
<td>Target EOS parameter</td>
<td>$C_t$</td>
<td>aleatory</td>
<td>4.58 ($km/s$)</td>
<td>0.014 ($km/s$)</td>
</tr>
<tr>
<td>Target EOS parameter</td>
<td>$S_t$</td>
<td>aleatory</td>
<td>1.49</td>
<td>0.0055</td>
</tr>
<tr>
<td>min.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>max</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Target Plate Thickness</td>
<td>$w_p$</td>
<td>epistemic</td>
<td>45 $\mu m$</td>
<td>55 $\mu m$</td>
</tr>
<tr>
<td>Flier $L/D$</td>
<td>$\frac{w_f}{\sigma_f}$</td>
<td>epistemic</td>
<td>0.0786</td>
<td>0.02156</td>
</tr>
</tbody>
</table>
that of the laser pulse. This may mean that the \( L/D \) ratio of the flier changes during flight, where \( L = w_f \) is the flier thickness and \( D = d_f \) is the flier diameter. We can use these observations from the coupons to estimate a range of values for the uncertainty in flier geometry. The maximum radius of the flier is then estimated as the laser pulse radius while its minimum radius is estimated by the observed radius of the holes in the perforated targets. This gives a range of \( L/D \) values given in Table 4.12. Since we have no information on the probability of values within this range we must treat the uncertainty as epistemic.

There is also epistemic uncertainty in the thickness of the flier and target foils from the manufacturers given by Table 4.12 [35].

### 4.3.2 Uncertainty Propagation

To illustrate how uncertainties in the input parameters affect system response quantities (SRQ’s), the various input uncertainties must be propagated through the model. Propagation of aleatory uncertainties through the model is done using Monte Carlo sampling (MCS) based on the method described by Oberkampf and Roy [37]. This technique will be briefly described in this section. For a more detailed description, the interested reader may consult [37]. If there are \( \alpha \) number of input parameters with aleatory uncertainty, \( \alpha \) strings of pseudo-random numbers are generated with values from 0 to 1. Using the first random number from the \( \alpha \) strings of random numbers, a value for each input parameter is picked from its respective cumulative distribution function (CDF). The model is then evaluated using these values of the input parameters to get a single output for each SRQ. This process is continued using the \( \alpha \) strings of pseudo-random numbers until the number of samples, \( N \), is achieved. The output then is used to produce the CDF of each SRQ. The flowchart in Fig. A.1 located in Appendix A shows this process in detail.

To illustrate the results of the MCS technique described above, we compute a CDF for ballistic limit considering aleatory uncertainty in a single input parameter, the ultimate tensile strength, \( \sigma_u \), for a small flier-target configuration given by Table 4.13. This practice will illustrate the resulting uncertainty in the ballistic limit due to the aleatory uncertainty.
Table 4.13: Small flier-target configuration for MCS example.

<table>
<thead>
<tr>
<th>#</th>
<th>Flier</th>
<th>Target</th>
<th>$r_f$ (mm)</th>
<th>$R$ (mm)</th>
<th>$w_f$ (µm)</th>
<th>$w_p$ (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCS-1</td>
<td>2024-T4 Al</td>
<td>304 SS</td>
<td>0.6</td>
<td>1.5</td>
<td>11</td>
<td>50</td>
</tr>
</tbody>
</table>

in the ultimate tensile strength. The probability density function and resulting CDF of the ultimate tensile strength is shown in Fig. 4.20 which is a normal distribution having a mean and standard deviation given in Table 4.12. For this example, $N = 1000$ samples were taken from the CDF of the ultimate tensile strength to construct the CDF of ballistic limit shown in Fig. 4.21. This simulation was performed on a system with a 2.93 GHz Quad Core Nehalem Xeon 64-bit processor, 24GB 1333MHz ram, and 160GB hard drive with a computational time of approximately 12 hours.

![Probability Density](image1)

![Cumulative Probability](image2)

Figure 4.20: Ultimate tensile strength, $\sigma_u$: (a) Probability density function (PDF); (b) Cumulative distribution function (CDF).

When epistemic uncertainty must also be considered, as in the micro-scale impact events due to the uncertainty in geometry, an additional sampling is required. Picking random values over the range of epistemic uncertainty, several CDF’s must be produced to establish the bounds of what Oberkampf calls the probability box or $p$-box. The $p$-box is a type of cumulative distribution function that gives a range of all possible values for the output parameter given input aleatory and epistemic uncertainties. This is valuable to experimentalists because it gives the full range of all possible values for an SRQ with the
inclusion of all uncertainties. The Monte Carlo method previously used must be extended to account for epistemic uncertainty and will be briefly illustrated in this section. A more detailed discussion can be found in [37]. If there are $\beta$ number of parameters with epistemic uncertainty, $\beta$ strings of random numbers must be generated. These values are used to randomly pick a value from each parameter with epistemic uncertainty. The MCS method previously described must then be used to calculate a CDF for each sample of epistemic uncertainty. This must be done a sufficient number of times to find the full range of the p-box constructed from these CDF’s. An additional flow chart showing this process is presented in Fig. A.2 located in Appendix A.

To avoid unnecessarily expensive computation, Latin Hypercube sampling (LHS) may be used to sample values from epistemic uncertainty. This method splits the range of epistemic uncertainty into subdivisions and picks a random value within each so that the full range is covered in a less expensive manner. Figure 4.22 shows an example of performing uncertainty propagation with a single aleatory and epistemic uncertainty using LHS for Configuration MCS-1. In this case, $M = 10$ samples of epistemic uncertainty values for $L/D$ of the flier were chosen, and $N = 1000$ samples of ultimate tensile strength were chosen to produce each of the $M$ CDFs shown in Fig. 4.22 (a). The calculated p-box for the ballistic
limit in this case is shown in Fig. 4.22 (b). As mentioned previously, this p-box encloses the entire range of possible values for an SRQ. In this way, a p-box can be computed for each SRQ of importance to researchers. The computational time for this simulation was approximately 106 hours. This could be reduced significantly by parallelizing the code to compute each CDF separately or even further by subdividing each CDF.

![Cumulative Probability](image)

Figure 4.22: Cumulative probability of ballistic limit (a) LHS results; (b) Constructed p-box.

### 4.3.3 Model Form Uncertainty

Model form uncertainty is a quantitative measure of how well the model describes the actual physical phenomena involved. A simple procedure for calculating the model form uncertainty using a confidence interval approach comparing the mean of experimental data to model predictions is presented [37]. The reader may skip this section without loss of continuity, as this procedure will not be performed for the current study; rather, we will simply illustrate the approach. This approach considers model uncertainty of a single SRQ, with respect to a single input parameter so that the uncertainty is assumed to be unaffected by other model input parameters. Sufficient experimental data for the SRQ over the range of input values must be available.

The estimated error in the model is given by a validation metric which is a quantitative measure of the difference in the model prediction and experimental data. For this approach
the validation metric is given by

$$\hat{E}(x) = y_m(x) - \bar{y}_e(x), \quad (4.2)$$

where $\hat{E}$ is the error, $x$ is the input parameter, $y_m$ is the SRQ given by the model, and $\bar{y}_e$ is a linear regression of the experimental data of the SRQ. A linear regression of the data is used in this procedure for simplicity in illustrating the process. However, one may choose to use a non-linear regression that may better fit the data. With adequate experimental data we can define the interval containing the true model form error with a specified confidence level. With a confidence level of $100(1 - \alpha)\%$ the interval is given by

$$(\hat{E}(x) - SCI(x), \hat{E}(x) + SCI(x)), \quad (4.3)$$

where $SCI(x)$ is the width of the Scheffé confidence interval as a function of $x$ given by

$$SCI(x) = s \sqrt{\frac{2F(2, n - 2, 1 - \alpha)}{n}} \left[ \frac{1}{n} + \frac{(x - \bar{x})^2}{(n - 1)s^2_x} \right], \quad (4.4)$$

where $s$ is the standard deviation of the residuals for the curve fit, $F(\nu_1, \nu_2, 1 - \alpha)$ is the $F$ probability distribution, $\nu_1$ and $\nu_2$ are the parameters for degrees of freedom, $1 - \alpha$ is the quantile for the confidence interval, $n$ is the number of experimental measurements, $\bar{x}$ is the mean of input values from experiments, and $s^2_x$ is the variance of the input values.

With our current model we have very little data to compare with because systematic validation experiments were not performed. Using available data from Borvik [9] taken from Fig. 4.19 for the ballistic limit as a function of target plate thickness we can get a rough estimate of the model form error of the current model. However, there is insufficient experimental data to give confidence intervals about the linear regression of the data. There is, however, epistemic uncertainty in the experimental ballistic limit which we show as the bounds surrounding the linear regression. The above describe method applied to the current
model is shown in Fig. 4.23. Figure 4.23 (a) shows a linear regression of the data from Borvik with epistemic uncertainty intervals, $\text{EUI}(x_P)$, and the model results for ballistic limit. From this image, the extent of the error in the model is unclear. The significance of the error is much clearer in Fig. 4.23 (b) which shows the model form error including the epistemic uncertainty intervals. To get a better idea of how significant this error is globally, Fig. 4.23 (c) shows the model form error as a percentage of the experimental data. Over the presented range of data, the error is less than 5%.

This exercise has only considered the model form error in calculating the ballistic limit with respect to only one input variable, target thickness. To estimate the model error with respect to all input parameters including material properties and other geometrical
parameters, a multi-dimensional model form error calculation must be performed in which
the validation space becomes an n-dimensional surface or hyper-surface. The model form
error will be an (n+1)-dimensional hyper-surface. To do so, many experiments must be
carried out over the domain of interest or as close to the domain of interest as possible. Once
done, this hyper-surface could effectively be interpolated or extrapolated to characterize
the model form error in areas of interest.

4.3.4 Total Uncertainty

To illustrate the effect of the various types of uncertainty described above we will look
at some of the micro-scale impact cases for which we have limited data. All uncertainties
listed in Table 4.12 is considered in calculating the total uncertainty of the model. The
propagation of these uncertainties was performed using the LHS sampling as described
previously. The results of the input error propagation for micro-scale impact cases are
presented in Fig. 4.24. The model results for three different configurations are presented as
well as the observed experimental data from target coupons. The configurations of these
cases are given in Table 4.14.

<table>
<thead>
<tr>
<th>#</th>
<th>Flier</th>
<th>Target</th>
<th>$r_f$ ($\text{mm}$)</th>
<th>$R$ ($\text{mm}$)</th>
<th>$w_f$ ($\mu\text{m}$)</th>
<th>$w_p$ ($\mu\text{m}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-1</td>
<td>2024-T4 Al</td>
<td>304 SS</td>
<td>0.6</td>
<td>1.5</td>
<td>11</td>
<td>50</td>
</tr>
<tr>
<td>M-2</td>
<td>2024-T4 Al</td>
<td>304 SS</td>
<td>0.6</td>
<td>1.5</td>
<td>33</td>
<td>50</td>
</tr>
<tr>
<td>M-3</td>
<td>2024-T4 Al</td>
<td>304 SS</td>
<td>0.6</td>
<td>1.5</td>
<td>33</td>
<td>75</td>
</tr>
</tbody>
</table>

As seen in Fig. 4.24 the combination of epistemic and aleatory input uncertainties pro-
duces a p-box with a very wide range of outcomes for the predicted ballistic limit. The
experimental data is also shown as a p-box with epistemic uncertainty due to a lack of data.
This epistemic uncertainty in the experimental data is due to the small number of exper-
iments performed. The ballistic limit falls between the values given by two experiments:
one which exhibits no perforation of the target coupon and one which shows perforation in
addition to this wide range given by the lack of experiments, there is also a 10% uncertainty
in initial velocity added to the experimental p-box [20]. This range could be reduced by performing more experiments to narrow the range of velocities between which the ballistic limit occurs. The p-box produced by the model can also be reduced in range by minimizing the uncertainty in the various input parameters especially the epistemic uncertainty in the flier geometry. It is noticed in going from configuration M-1 to M-3, overlap of the p-boxes for experimental results and the model predictions is not as large. This could be due to geometrically dependent target material properties which were not considered in these simulations. The thicker targets may have a higher ultimate tensile strength in reality which would move the predicted p-boxes towards the experimental resulting in better agreement.

To illustrate the effects of the aleatory and epistemic input on uncertainties on other
SRQs we calculate several p-boxes for one case using configuration M-1 with an initial flier velocity, $U_{f0} = 1500 \text{ m/s}$. These SRQs are of secondary importance but are provided for completeness. Figure 4.25 gives calculated p-boxes for several SRQs of the model including impulse, work, and target displacement. The impulse to the flier and target are affected by the epistemic uncertainty in geometry but show almost no response to the aleatory uncertainties in other parameters. The effect of the aleatory is so small that the CDFs that construct the p-boxes in Figs. 4.25 appear to be vertical lines. The impulse to the flier is negative due to the direction of the force on the flier at interface $I$. The p-box for the target impulse gives a range $-15.8 \leq I_p \leq 8.7 \text{ N} \cdot \mu\text{s}$ meaning that the sign of the total impulse is dependent on the sign of the net force acting on the target plug. Figure 4.25 (c) shows that the p-box for the work done on the flier has little dependence on the epistemic and aleatory uncertainties in the input parameters while Fig. 4.25 (d) shows that the epistemic uncertainty in geometry has a considerable effect on the net work done on the target. The p-box for maximum axial target displacement is given by Fig. 4.25 giving a large range of output with influence from both aleatory and epistemic uncertainties.

In addition the range produced by these input uncertainties, other forms of error must be added to the p-boxes including model form error and numerical error. We do not have an estimate for the model form uncertainty for these cases, but it would be added to each side of the p-box increasing the range in total uncertainty. Numerical error is so small that it is negligible for these cases, but if it was more significant it would be added to the total uncertainty in the same manner as the model form uncertainty.

4.4 Sensitivity

In addition to studying uncertainty in the model, it may also be of value to determine how sensitive SRQs are to variations in input parameters. A local sensitivity study is performed to recognize which SRQs may be greatly affected by uncertainty in input parameters. A local sensitivity study is performed on several SRQs including the ballistic limit, target displacement, impulse to the flier, net impulse to the target, work done at
Figure 4.25: Uncertainty in SRQs for M-1, $U_{f0} = 1500$ m/s: (a) Flier Impulse; (b) Target Impulse; (c) Flier Work; (d) Target Work; (e) Target Displacement.

interface $I$, work done at interface $II$, and the net work done on the target plug. The effect of input parameters including flier geometry, target yield strength, target ultimate strength, and equation of state parameters were studied. The effect of varying the release
time, $t_r$, was also considered. The model was evaluated varying each input parameter over its uncertainty range keeping all other input parameters constant. The sensitivity of each parameter is estimated by the derivative of each output parameter with respect to the input parameter which was taken as the slope of a linear regression of the output for each SRQ. To compare the sensitivity between parameters, each was normalized using the mean values of the independent and dependent variables given by $\frac{x \, dy}{y \, dx}$, where $y$ is the dependent variable and $x$ is the independent variable. The results of this study are presented in Table 4.15. Here, the sign represents an increase or decrease in the magnitude of the dependent parameter with respect to increasing the independent parameter.

<table>
<thead>
<tr>
<th>$\frac{d^2 r}{d_l}$</th>
<th>$x_p$</th>
<th>$I_f$</th>
<th>$I_l$</th>
<th>$W_I$</th>
<th>$W_{II}$</th>
<th>$W_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.35</td>
<td>-1.45</td>
<td>-1.35</td>
<td>16.1</td>
<td>0.472</td>
<td>-1.81</td>
<td>0.819</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>1.00</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>-0.50</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$t_r$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.721</td>
<td>2.81</td>
<td>0.535</td>
<td>0.0</td>
</tr>
</tbody>
</table>

From Table 4.15 it is noted that the model input parameter with the greatest affect on all output parameters is the $L/D$ ratio of the flier given by $\frac{w_l}{d_l}$. Consequently, it is important for researchers and decision makers to consider the effects of $L/D$ on the outcome of experiments. It is noted that the ultimate tensile strength only affects the ballistic limit predicted by the model because of its use exclusively in the failure criterion. The yield strength affects both the ballistic limit and the displacement of the target through the quasi-static force. As the yield strength increases, the ballistic limit decreases if all other input parameters are held constant. This is counter-intuitive, but may be caused by not considering the relationship between the ultimate tensile strength and yield strength. In reality it may not be correct to vary the yield strength without also varying the ultimate tensile strength. In addition to the aspect ratio and material strengths, the sensitivity to the shock equation of state parameters was also calculated. The dependence of the above SRQ’s on the equation of state parameters was negligibly small and therefore not included.
In the current model we calculate \( t_r \) based on time it takes for a release wave to first reach interface \( I \). In reality, a release wave has a finite thickness and there may be some variability in the time that the shock pressure is reduced at the interface due to the release wave. It is therefore of interest to see how variation in this value effects the model. Table 4.15 shows the sensitivity of the model to changes in the release time. The model prediction for the ballistic limit is unaffected by changes in the release time because it is associated with the early-time shock physics and does not affect the late-time response of the force at interface \( II \) or the failure criterion. The displacement of the target during the early-time response of the model is negligible in comparison to the displacement that occurs during the later response. As a result \( t_r \) has little effect on target displacement. The work done at interface \( II \) is also independent of \( t_r \) because only the quasi-static force is applied here. The impulse and work terms however, have some sensitivity to the specification of this release time due to the physics occurring during the early-time response.

Here we show contours of various SRQ’s over large parameter spaces to better recognize trends in the model. Figures 4.26 - 4.29 show contours of SRQ’s with varying flier thickness and initial flier velocity for configuration M-1 from Table 4.14. Figure 4.26 shows contours of the maximum axial displacement of the target plug after impact versus flier thickness and initial flier velocity. At a constant initial flier velocity, the target displacement increases with increasing flier thickness. At a constant flier thickness the target displacement increases with increasing initial velocity. This behavior continues until the initial velocity reaches the ballistic limit. In this case, the critical displacement is about 10 times the thickness of the target. The experimental coupons exhibit considerable bulging that may be within this range.

Figure 4.27 shows contours of the work done on the target plug at interface \( I \) which is equal to the work done by the flier on the target plug. For a constant flier thickness, the work done on the target plug at interface \( I \) increases with increasing initial velocity.
while for a given initial flier velocity, the work done increases with increasing flier thickness.

Figure 4.28 shows contours of the work done on the target plug at interface II. This is equivalent to the work done by the target plug on the surrounding target plate material. The magnitude of the work done at interface II increases with increasing initial flier velocity.
and increasing flier thickness. The work done at interface II plateaus at the ballistic limit because the force at interface II is dependent solely on the target displacement which is capped at the critical value.

Figure 4.29 shows the net work done on the target plug by the forces at interfaces I
and $II$. This contour shows similar trends to the work done at interface $I$. To illustrate how uncertainty may affect these ballistic performance maps, the dashed line represents uncertainty in the ballistic limit taken as an average value of the uncertainty given by the p-box of Configuration M-1 which is $\sim \pm 580 \text{ m/s}$. The uncertainty for these ballistic limit values is large due to the epistemic uncertainty in flier and target geometries. We do not show uncertainty in the ballistic performance maps in following sections but remain aware of its presence.

4.5 Energetic Targets

It is important to researchers to study impact events in which a layer of energetic material has been deposited to the backside of the target plate. Specifically, it is of interest to quantify the energy transferred from the flier to energetic material through the target plate. Figure 4.30 shows the material configuration prior to and during impact. The layer of energetic material is applied to the backside of the target plate in a very thin layer which is non-homogeneous in thickness. There may even be regions where no material has been deposited. Also, Heterogeneous energetic solids are typically particulate composites with relatively weak shear strength. For this reason we consider the energetic material strength negligible. For this reason, the model for the energetic material behavior can be described solely by 1-D shock relations which is reasonable for the $L/D$ ratios in the micro-flier configurations. Though energetic material strength will be ignored in this section, the target plate strength is still valuable for purposes of identifying the ballistic limit which is important to experimenters who do not wish to perforate the target coupons.

The three component system of the flier and two separate target layers is shown in Fig. 4.31. The target components are comprised of the material adjacent to the contact area of the flier as in the previous model. The interaction stress at interfaces $I$ and $II$ are characterized in the same way as before by the wave mechanics at the impact interface and the strength of the target plate at interface $II$. The flier and target plate plug will behave exactly is in the previously formulated model.
The interaction stress at interface III is characterized by the pressure of the shock that is transmitted from the target plug material to the energetic target material. The pressure at this interface can be calculated using shock impedance matching of the Hugoniot of the three materials. Figure 4.32 shows the impedance matching of a stainless steel target and an aluminum flier with an initial velocity of 2000 m/s to calculate the first shock state. The release isentrope of the stainless steel target, estimated by the reflected (or mirrored) Hugoniot, is then shown passing through the first shock state, \((U_{\text{int}1}, P_{S1})\). The Hugoniot of the energetic material, in this case HMX \((\text{C}_4\text{H}_8\text{N}_8\text{O}_8)\), is also shown. The point at which the shock Hugoniot of the energetic material passes through the release isentrope of the target material gives the pressure, \(P_{S2}\), and interface velocity, \(U_{\text{int}2}\), of interface III.

The shock pressure, \(P_{S2}\), is to be applied from the time the shock arrives at interface III until a release wave first reaches interface III from either the flier-target interface or the free surface of the energetic material. Figure 4.33 shows an x-t diagram for the multi-layered configuration. At impact a shock is sent back into the flier material and forward into the target plate. In the flier, the shock hits the free surface and sends back

![Figure 4.30: Conceptual picture of flier impacting target with energetic material layer.](image)
a release wave, Release 2. In the target plate the shock hits interface III and sends a release wave, Release 1, back into the target plate and transmits a shock forward into the energetic material at time $t = t_{e_1}$. This is the case if the shock impedance of the energetic material is lower than that of the target plate. The release wave travels back through the target hitting the flier-target interface and sending another release wave, Release 3, back to interface III. The second shock transmitted to the energetic material will hit the free
surface of the energetic material and send back a release wave, Release 4, which arrives at interface III. The pressure applied to the energetic material at interface III is assumed to release at time $t = t_{e2}$ when either Release 2, 3, or 4 first hits interface III. For the purposes of this study, interactions between these release waves are not considered.

The energetic material is allowed to separate from the target plate with no consideration of an adhesive force. This is consistent with preliminary experiments performed by researchers in which the energetic material was projected off of the back side of the target during impact. The equations of motion of the energetic material are therefore defined by

$$ m_e \frac{dU_e}{dt} = P_{S2}A_f \quad t_{e1} \leq t \leq t_{e2}, \quad (4.5) $$

$$ \frac{dx_e}{dt} = U_e \quad t_{e1} \leq t \leq t_{e2}, \quad (4.6) $$
where \( P_{S_2} \) is the pressure of the shock transmitted to the energetic material at interface III. An estimate for the work done on the energetic material is then

\[
W_e = P_{S_2} A_f U_{int_2} (t_{e_2} - t_{e_1}),
\]

(4.7)

where \( U_{int_2} \) is the interface velocity at interface III due to the transmitted shock. These equations can be integrated to evaluate the response of the energetic material and provide values for various SRQs of interest to researchers. To illustrate the behavior of this system, the time response as well as values of important SRQs will be calculated for the configurations given in Table 4.16 with material properties given in Table 4.17 which are similar to configurations of interest to researchers.

<table>
<thead>
<tr>
<th>#</th>
<th>Flier</th>
<th>Target</th>
<th>Energetic</th>
<th>( r_f ) (mm)</th>
<th>( R ) (mm)</th>
<th>( w_p ) (( \mu m ))</th>
<th>( w_e ) (( \mu m ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1</td>
<td>2024-T4 Al</td>
<td>304 SS</td>
<td>HMX</td>
<td>0.7</td>
<td>1.5</td>
<td>50</td>
<td>30</td>
</tr>
<tr>
<td>E-2</td>
<td>2024-T4 Al</td>
<td>304 SS</td>
<td>TNT</td>
<td>0.7</td>
<td>1.5</td>
<td>300</td>
<td>30</td>
</tr>
<tr>
<td>E-3</td>
<td>2024-T4 Al</td>
<td>304 SS</td>
<td>PETN</td>
<td>0.7</td>
<td>1.5</td>
<td>200</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 4.16: Lase-Driven Micro-Flier System Configurations.

<table>
<thead>
<tr>
<th>Energetic</th>
<th>( \rho ) (kg/m(^3))</th>
<th>( C_e ) (km/s)</th>
<th>( S_e )</th>
<th>( E_e ) (J/cm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>HMX (( \text{C}_4\text{H}_8\text{N}_8\text{O}_8 ))</td>
<td>1891</td>
<td>3.07</td>
<td>1.79</td>
<td>150</td>
</tr>
<tr>
<td>TNT (( \text{C}_7\text{H}_5\text{N}_3\text{O}_6 ))</td>
<td>1624</td>
<td>1.32</td>
<td>2.58</td>
<td>77</td>
</tr>
<tr>
<td>PETN (( \text{C}_5\text{H}_8\text{N}_4\text{O}_1\text{2} ))</td>
<td>1600</td>
<td>1.32</td>
<td>2.58</td>
<td>5.03</td>
</tr>
</tbody>
</table>

Table 4.17: Energetic Material Properties.

Figures 4.34 and 4.35 show the time response for Configuration E-1 with an initial flier velocity of \( U_{f0} = 500 \text{ m/s} \) and a flier thickness of \( w_f = 10 \mu m \). The velocity of the flier, target plug, and energetic material are shown in Fig. 4.34 to illustrate their behavior. In this case the flier is decelerated to a negative velocity while the target plate is accelerated. The energetic material remains stationary until the initial shock reaches interface III which, at this time, accelerates the energetic material until the pressure is released at time \( t_{e2} \) and the velocity remains constant. The energetic material is projected off of the target with a
velocity of $\sim 30 \, \text{m/s}$ and the target plug and flier decrease in velocity until the target plate comes to rest with no perforation. Figure 4.35 shows the displacement of the flier, target plug, and energetic material which behaves accordingly with the velocity response.

Figure 4.34: Energetic target system velocity response for Configuration E-1 (HMX).

Figure 4.35: Energetic target system displacement response for Configuration E-1 (HMX).
Figure 4.36 shows the work done on the energetic material by the shock force at interface III for the configuration in Table 4.16 for various flier thicknesses and initial flier velocities. We call these contours *ballistic initiation maps*. For a given flier thickness, the work done on the energetic material increases as initial flier velocity increases. For a given initial velocity, the work done on the energetic material increases with increasing flier thickness. It is of significant interest to know if the work done on the energetic material is enough to drive the energetic to initiation. Here, initiation implies transition to detonation. If there is no initiation, there may still be ignition of the energetic material but no detonation. HMX has a critical shock energy of $E_c \approx 150 \text{ J/cm}^2$ for detonation [54] which corresponds to a value of work done to the energetic material layer of $W_{ec} \approx 2370 \text{ mJ}$. For this configuration the threshold for initiation is well above the perforation threshold and cannot be seen in the figure. This is a configuration which may not be ideal for the purposes of the experiments if the intent is to promptly detonate the material. Here, perforation of the target plate would be necessary for detonation to occur which would contaminate the vacuum chamber on the explosive side of the apparatus. The contours of work appear hyperbolic in nature.

Figure 4.36: Contours of work done on energetic material for configuration E-1 (HMX).
For a single flier thickness there are two values of initial flier velocity that give a specific value of work done on the energetic. This occurs because the work is a function of both shock pressure and particle velocity as well as the time over which the pressure is applied. As initial velocity increases the product, $P_S U_{int_2}$, increases while the time of application, $(t_{e_2} - t_{e_1})$, decreases. These trends are shown in Figs. 4.37. The resulting product of these two values is the work done on the energetic material, $W_e$, which exhibits hyperbolic behavior shown in Fig. 4.38. The work goes to zero when the initial release from the flier free surface kills the shock in the target material before reaching the energetic material as discussed in Ch. 3. The energetic material projection velocity is shown in Fig. 4.39. The

![Figure 4.37](image1.jpg)  
**Figure 4.37:** (a) Product of shock pressure and interface velocity of energetic material; (b) Time of application of shock pressure at energetic material interface.

![Figure 4.38](image2.jpg)  
**Figure 4.38:** Work done on energetic material for configuration E-1 (HMX); $w_f = 30 \mu m$. 

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projected velocity of the energetic material increases with increasing initial flier velocity, but remains constant with respect to flier thicknesses.

Similarly with configuration E-2, where TNT is the energetic material, the target is perforated before initiation of the energetic material can occur. The maximum work done to the energetic material before perforation is around 160 $mJ$ shown by Fig. 4.40. The work required for initiation is much greater at a value of 1185.8 $mJ$. Though this is a lower critical energy than that required for HMX, there still may not be any configuration desirable to experimenters.

To illustrate a case that results in initiation, we look at Configuration E-3 given by Table 4.16 which will provide materials and geometries that produce an initiation threshold below the perforation threshold which is desirable for laser-driven flier experiments. PETN ($C_5H_8N_4O_{12}$) is used because of its much lower critical energy, $E_c \approx 5.03 \ J/cm^2$ [11] giving a critical work done on the energetic material of $W_{ec} \approx 77 \ mJ$. Thicker geometries are also required to increase the time over which the pressure is applied to the energetic material.
therefore increasing the amount of work done to the energetic material.

Three different ballistic initiation maps, Figs. 4.44 - 4.45, with configuration E-3 are shown to illustrate the effect of target thickness. Figure 4.44 shows the work done on the energetic material for Configuration E-3 with various initial flier velocities and flier thicknesses. The black area represents cases in which there is no work done on the energetic material. This occurs when a release wave from the flier free surface reaches interface III before the initial shock therefore overtaking the shock in the process. An x-t diagram of this behavior is presented in Fig. 4.41.

For cases in which the release does not overtake the initial shock, the work done on the target is a function of the flier thickness and the initial velocity. The red region in Fig. 4.44 represents the configurations in which perforation of the target plate occurs. This is undesirable behavior for experiments in which the target will not prevent contamination of the vacuum chamber used for spectroscopy of chemical species. The green region represents the response in which the work done on the energetic material is sufficient to initiate detonation while not perforating the target plate so that uninfluenced spectroscopy measurements may be taken.

If we extend the parameter space to include greater flier thicknesses, there is a point at which the flier thickness no longer has an effect on the work done to the energetic material. This is shown in Fig. 4.43 by the dashed white line. At this line, the contours become horizontal. This physically means that the wave traveling through the flier from the flier free surface is no longer the first to reach the target-energetic interface, and the work done to the energetic material is no longer dependent on flier thickness.

Figures 4.42 and 4.45 show how decreasing or increasing the target plate thickness affects the topology of ballistic initiation maps. In Fig. 4.42 the target plate thickness is decreased to $w_p = 100 \, \mu m$ which pushes the ballistic limit threshold below the initiation threshold, shown by the dashed line, exhibiting an undesirable configuration. Figure 4.45 shows the effect on the ballistic initiation map of increasing the target thickness to $w_p =$
300 $\mu m$. This moves the ballistic limit threshold further away from the initiation threshold making the initiation region larger giving more configurations with desirable outcomes.
Figure 4.40: Contours of work done on energetic material for configuration E-2 (TNT).
Figure 4.41: x-t diagram for three component system.
Figure 4.42: Ballistic initiation map for configuration E-3 (PETN).
Figure 4.43: Extended Ballistic initiation map for configuration E-3 (PETN).
Figure 4.44: Ballistic initiation map for configuration E-3 (PETN): $w_p = 100 \mu m$. 
Figure 4.45: Ballistic initiation map for configuration E-3 (PETN): $w_p = 300 \mu m$. 

- Zero Work
- Perforation
- Initiation
Chapter 5

Conclusions and Future Work

This thesis has described a simple model for simulating laser-driven micro-flier impact of thin stationary targets that is computationally inexpensive. This model characterizes the early-time physics associated with the initial shock based on 1-D shock relations as well as the late-time physics associated with the material strength of the target based on the quasi-static strength of a circular clamped flat plate. Methods for validation of the model and uncertainty propagation through the model were described in detail, and a simple method for estimating the energy transfer to energetic targets was proposed. Preliminary ballistic initiation maps were generated to illustrate configurations for which initiation of the energetic material may occur without perforation of the target plate.

In comparing the model to the time responses of various configurations, the dynamic behavior of the system including flier and target velocities, displacements, forces, work, and impulses is illustrated. The model does relatively well at characterizing the dynamic behavior of flier-target systems in comparison to experimental data. The model also does a reasonable job at predicting residual velocities and the ballistic limit for various large flier-target configurations composed of different flier and target materials. Predicting the ballistic limit is important in finding configurations for which the target plate is not perforated.

Due to the stochastic nature of the problem observed from post-impact target coupons, the uncertainty of various input parameters and their affect on system response quantities is considered. Uncertainty in input parameters is propagated through the model using Monte Carlo sampling techniques to characterize the uncertainty in key output variables including the ballistic limit to provide a range of values which experimentalists may expect. A local sensitivity study was also performed to show variations in input parameters over their uncertainty ranges affect the output parameters. It is noted that the model output
was most sensitive to the $L/D$ ratio of the flier which affects all system response quantities considered in the sensitivity study. The release time affects the values of system response associated with the early-time shock physics only because it does not affect the late-time response. The strength parameters of the target affect only the displacement of the target plug and the value of the ballistic limit due to their high dependence on the late-time response.

Extending the model to account for energetic targets, the response of three common energetic materials is considered, HMX ($C_4H_8N_8O_8$), TNT ($C_7H_5N_3O_6$), and PETN ($C_5H_8N_4O_{12}$), to identify configurations for which detonation without perforation is possible. The model gives ballistic performance maps for the $U_{f0} - w_f$ parameter space that show limits for perforation and initiation. The model predicts that for HMX and TNT with relatively high critical shock energies, there are no configurations considered in this study which provide initiation without perforation. However, considering PETN, with a much lower critical shock energy, the model was able to predict ranges in parameter space which initiation occurs without perforation. The size of this region was found to rely heavily on the thickness of the target plate.

There are certain limitations associated with the use of the model provided by this study. While the model does a reasonable job at predicting residual flier velocity and the ballistic limit for many cases, accuracy may be improved by making modifications to the current strength model. The ballistic limit predicted in some cases deviated from experimental data with thicker target plates. This again is most likely due to a deficiency in the applied quasi-static strength model. In reality, the process of impact is not a quasi-static process and may be modeled more successfully with a different constitutive model that includes rate-dependent material properties. The drawback to accounting for this behavior is that these constitutive models may require material testing to calculate parameters. Even with the current model, testing may be of value to reduce uncertainty in material properties used by experimenters.
Though values were computed for ballistic limits, and ballistic initiation maps were generated, these results must be taken with caution as they have high uncertainty due to the uncertainty in various model input parameters. Also, the actual model form error was not calculated in this study due to a lack of experimental data, though it should be calculated to indicate how well the model simulates the actual physics of the problem. To get a true estimate of the model form error for the ballistic limit and other SRQ’s of interest, validation experiments specific to each SRQ must be designed and performed by a joint effort between experimentalists and modelers. Experiments should also be performed to study the uncertainty in model input parameters to reduce these uncertainties.

Experiments should be performed across the range of parameter space of interest to researchers so model form error can be meaningfully interpolated between validation points or over the validation space. However, If validation cannot be performed over the parameter space of interest, the experiments should be performed over the nearest parameter space possible and then extrapolated to cover the domain of interest.

One experiment that could be easily performed to study model form error is to measure the final displacement of the target plate for various impact velocities. Several shots could be performed with the same impact velocity for various configurations in parameter space to give a validation domain for final displacement to calculate model form error for final displacement. A profilometer could be used to measure the final displacement of each impact. A sufficient number of experiments could be performed to give stochastic information about the displacement of the target giving its aleatory uncertainty. Another experiment could use VISAR to measure the velocity of the backside of the flier plate during impact by launching fliers at transparent targets. This would serve to validate the velocity response of the flier during impact. In addition to investigating specific SRQ’s by experiment, experiments may be performed to determine the uncertainty in input parameters. Reducing the uncertainty in the flier aspect ratio alone would greatly reduce the total model error. This could be performed simultaneously with the above VISAR experiments measuring the
flier frontal area to reduce uncertainty in the geometry. If there is a way to view the side
profile from various angles the uncertainty in the geometry could be defined.

The model presented provides a first step into modeling the impact of laser-driven
micro-flier events. The model can be used to give a reasonable estimate for the ballistic
initiation maps which give a range of configurations for initiation of energetic material
without perforation of the target plate. The model also allows for the propagation of
input uncertainty giving a range of uncertainty in each system response quantity. This
provides experimentalists with a preliminary tool for setup of bench-top energetics exper-
iments where none was previously available. This allows the experimenters to narrow the
parameter space for which they should perform experiments to achieve initiation of ener-
getic material. It also provides a basis for more sophisticated modeling which could not
consider such an extensive parameter space without great expense. These more complex
models can be used to investigate points within the parameter space region predicted by
the simple model.
References


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Appendix:  
Additional Figures
Figure A.1: Flow chart for MCS process with only aleatory uncertainties [37].
Figure A.2: Flow chart for MCS with aleatory and epistemic uncertainties [37].
Vita

Mark B. Fry was born in 1987 at Woman’s Hospital in Baton Rouge, Louisiana. He grew up in Baton Rouge attending Catholic High School and graduating in 2006. He then went on to receive his bachelor’s degree in mechanical engineering at Louisiana State University in May of 2010. As a participant in the 3-2 Accelerated Master’s program he plans to attain his Master of Science in Mechanical Engineering in May of 2012.