1970

Amplitude Effects for the Oscillating Disk in Liquid Helium-II.

Robert Patrick Roger

Louisiana State University and Agricultural & Mechanical College

Follow this and additional works at: https://digitalcommons.lsu.edu/gradschool_disstheses

Recommended Citation

Roger, Robert Patrick, "Amplitude Effects for the Oscillating Disk in Liquid Helium-II." (1970). *LSU Historical Dissertations and Theses*. 1747.

https://digitalcommons.lsu.edu/gradschool_disstheses/1747

This Dissertation is brought to you for free and open access by the Graduate School at LSU Digital Commons. It has been accepted for inclusion in LSU Historical Dissertations and Theses by an authorized administrator of LSU Digital Commons. For more information, please contact gradetd@lsu.edu.

70-18,555

ROGER, Robert Patrick, 1942-
AMPLITUDE EFFECTS FOR THE OSCILLATING DISK IN
LIQUID HeII.

The Louisiana State University and Agricultural
and Mechanical College, Ph.D., 1970
Physics, general

University Microfilms, Inc., Ann Arbor, Michigan

AMPLITUDE EFFECTS FOR THE
OSCILLATING DISK IN LIQUID HeII

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Physics and Astronomy

by
Robert Patrick Roger
B.S., Loyola University of the South, 1964
January 1970

ACKNOWLEDGEMENT

The author wishes to express his gratitude to Dr. R. G. Hussey for suggesting the topic of investigation and for his assistance throughout the course of the research. The patience and encouragement displayed by my wife and family was a decisive influence and is gratefully appreciated.

The author acknowledges the skillful help of Leslie Edelen, Charles Burlo, and Heinz Eichenseer for their help with the equipment. The financial support of this research by the National Science Foundation under Contract No. GP-7109 is acknowledged and appreciated. The author also acknowledges the financial assistance received from the Dr. Charles E. Coates Memorial Fund of the L.S.U. Foundation donated by George H. Coates for the expenses of preparation of this dissertation.

TABLE OF CONTENTS

	Page
I. INTRODUCTION	1
II. EXPERIMENT	2
III. RESULTS	5
IV. DISCUSSION	10
V. CONCLUSIONS	17
REFERENCES	20
APPENDIX I	AI-1
CALCULATION OF LOW AMPLITUDE LOGARITHMIC DECREMENT	
APPENDIX II	AII-1
COMPUTER PROGRAM USED TO CALCULATE AND PLOT LOGARITHMIC DECREMENT CURVES	
VITA	23

LIST OF TABLES

Table	Page
I. Experimental Disks	3a
II. Parameters in the Calculation of Energy Dissipation Due to Mutual Friction	16a
III. Values of Normal Fluid Viscosity and Density	A1-2
IV. Calculated and Observed Values of Logarithmic Decrement at Low Amplitude	A1-3

LIST OF FIGURES

Figure	Page
1. Experimental Cryostat	2a
2. Logarithmic Decrement vs. Amplitude for a Steep and a Flat Curve	5a
3. Flat Damping Curves for Different Temperatures	7a
4. High Amplitude Correlation for Flat Curves	7b
5. Steep Damping Curves for Different Temperatures	7c
6. High Amplitude Correlation for Steep Curves	7d
7. Critical Velocities for Flat Curves	8a
8. Critical Velocities for Steep Curves	8b
9. Amplitude Dependence of the Relative Change in I_a for Both Steep and Flat Curves for Two Different Temperatures for a 3.9 sec Period	9a
10. Amplitude Dependence of the Relative Change in I_a for Both Steep and Flat Curves at one Temperature for a 11.54 sec Period	9b
11. Depth Dependence of Low Amplitude Damping	9c
12. Comparison of Excess Damping with Zwanikken's Theory	16b

ABSTRACT

The amplitude variation of the logarithmic decrement D of a disk (radius ≈ 2.5 cm) performing torsional oscillations in liquid He II has been found to depend on the history of the helium bath. Two types of damping curves are observed: (1) The first (called "steep") occurs when the height, h , of the bath above the disk is greater than some critical height, h' , ($15.8 \leq h' \leq 16.8$ cm) and the time interval Δt between reaching the lambda temperature T_λ and the initial deflection of the disk is short ($\Delta t < 30$ min); (2) The second (called "flat") occurs when either $h < h'$, or if $h > h'$ then Δt must be long ($\Delta t > 60$ min). There is a critical velocity v_c associated with each type of curve. For the steep curves v_{c1} increases with temperature (from 1.33 to 2.15°K) and varies with period as $\tau^{-\frac{1}{2}}$ for periods between 3.9 sec and 19.6 sec. For the flat curves v_{c2} is independent of temperature (from 1.4 to 2.156°K), is larger than v_{c1} , and has a period dependence given by $v_{c2}\tau^{-\frac{1}{2}} = 1.4$ (cgs units) for periods between 3.9 sec and 45.0 sec. The difference between steep and flat curves is attributed to the presence of a long-lived motion of the superfluid (possibly an ordered array of quantized vortex lines) in the subcritical flow for the steep case and the absence of this motion for the flat case. The low amplitude logarithmic decrement D_0 is found to be the same for the steep and flat curves. The relative change in D_0 from the calculated value $((D_0 - D_{calc})/D_0)$ is found to increase as h/λ decreases (λ is the

penetration depth). The excess damping slightly above v_c cannot be explained by the effect of mutual friction. The relative change in moment of inertia $(I_a - I_{a0})/I_{a0}$ at high amplitudes (up to 2.5 rad) indicates that for the steep curves about 30% of the superfluid is obtained for a 4 sec period and 5% for a 12 sec period. No entrainment is indicated for the flat curves.

1. INTRODUCTION

When a disk oscillates in liquid helium II at very low amplitudes, the logarithmic decrement D_n is independent of amplitude and can be used to determine $\eta_n \rho_n$, the product of normal fluid viscosity and density. Hollis-Hallett¹ was the first to report that above a certain critical amplitude, φ_c , the logarithmic decrement increases with amplitude; the critical velocity is then taken to be the velocity of the edge of the disk:

$$v_c = 2\pi \varphi_c R/\tau \quad (1)$$

where R is the disk radius and τ is the period.

Experiments with oscillating disks, cylinders, and spheres have shown agreement on the following results: (1) the critical velocity varies with temperature as ρ_s^{-n} ($0.25 \leq n \leq 0.5$),^{1,2,3,4} where ρ_s is the superfluid density; (2) roughness of the oscillating boundary surface lowers the value of v_c ;^{2,3} (3) the excess decrement ($D - D_n$) above φ_c varies approximately as $\rho_s^{1,2}$. A major point of disagreement is the period dependence of the critical velocity: some investigators have found $v_c \propto \tau^{-1/2}$,^{1,3} and others have found v_c to be independent of period.^{2,4} Other results of oscillating boundary experiments are the following: (1) v_c depends on the size of the helium bath;⁴ (2) the effective density at high amplitudes (as revealed by an oscillating pile of disks) exceeds ρ_n and may approach the total density $\rho (= \rho_s + \rho_n)$ ¹;

(3) the effective kinematic viscosity at high amplitude is less than η_n/ρ_n .⁵

In 1958 Donnelly and Hollis-Hallett⁶ reviewed experiments of this type and succeeded in associating a similarity parameter with v_c . Hall⁷ has connected this similarity parameter with superfluid turbulence by assuming that v_c occurs when the average spacing between quantized vortex lines becomes of the order of the penetration depth λ [defined as $(\eta_n \tau / \pi \rho_n)^{\frac{1}{2}}$]. However, both of these results depend upon the critical velocity varying as $\tau^{-\frac{1}{2}}$.

We have measured the amplitude dependence of the logarithmic decrement of an oscillating disk over a wider range of periods (3.9 to 45.0 sec) than before. Our major results are: (1) both v_c and the shape of the damping curves are strongly affected by the previous history of the helium bath; (2) when the helium bath is undisturbed for a sufficient time (about one hour) after lowering the temperature below the lambda point, v_c is independent of temperature and varies as $\tau^{-\frac{1}{2}}$.

II. EXPERIMENT

The torsion pendulum cryostat used in this investigation is shown in Fig. 1. Both ends of the quartz torsion fiber A were glued into half inch pieces of No. 23 gauge needle with Araldite epoxy. The needle on the upper end of the fiber was then cemented into a piece of No. 8 gauge needle which was fastened to the head piece B. The head, which was used to adjust the zero, was held vacuum tight by an O-ring connection to the vertically adjustable sleeve C, which

Fig. 1

was fixed in a given position on the stainless steel tube E by the clamp D. The lower end of the quartz fiber was similarly connected to the aluminum mirror mount I. The front surface mirror G was placed so that its reflecting side lay on the axis of rotation. Just above the mirror, in the center of the mount, was placed a cylindrical piece of magnetized steel H, which was used in connection with a set of external Helmholtz coils to deflect the system to a desired amplitude. After the initial deflection the Helmholtz coils were removed. To the bottom of the mirror mount was cemented a long glass tube J of length 80 cm and outside diameter 0.31 cm. The lower end of the tube was fastened to the disk O. Three highly polished disks were used (see Table I). The disk could be pulled up to the shield N by means of the sleeve C. This was done to prevent the surface of the disk from being contaminated with frost or dust particles during the precooling and transferring processes. The shield is a piece of aluminum foil which extends to the walls of the glass dewar P and has a 13 mm diameter hole in its center. At its outer edge, the shield is attached to a brass ring which is hung by three thin-walled stainless steel tubes M (o.d. 1.5 mm) from the pumping collar L. When the disk was lowered, it was 4.6 cm from the shield and 10.5 cm from the bottom of the helium flask. The inside diameter of the helium flask was 7.0 cm.

The amplitude of oscillation was measured by reading the leading edge of the focused image of a straight-filament light source on a circular scale of radius $1/3$ m. The viewing chamber F

Table 1. Experimental Disks

Disk	Radius ^a (cm)	Thickness ^a (cm)	Moment of Inertia (g cm ²)	Material
A	2.5103	0.1566	82.576	Brass
B	2.5318	0.2976	161.164	Brass
C	2.5307	0.1584	28.428	Aluminum

^aThese are thermally contracted dimensions.

was turned down from a Plexiglas tube to a thickness of 0.0794 cm and radius of 4.85 cm. The width of the light image on the scale was 0.75 mm. A similar light, focused onto a slit placed in front of a photomultiplier tube, was used to measure the period of oscillation. The width of the light at this slit was 1.5 mm and the slit was 0.5 mm wide. The distance from the mirror to the photomultiplier was 1.5 m. The pulse from the photomultiplier was fed into an amplifier-pulse shaper and a scale-of-two circuit (to eliminate the mid-period pulse). The resulting square wave pulses were differentiated and used to activate a 1 MHz time base counter. Period accuracy at low amplitudes varied from 0.1 msec at the short periods to 1 msec at the longer periods.

The temperature was regulated with an elastic membrane regulator of the Walker type,⁸ and was measured with an oil manometer. The oil used had a density of 0.981 g/cm³.

Measurements were made of the amplitude dependence of the logarithmic decrement and period of each torsion fiber in vacuum ($\sim 2 \times 10^{-6}$ mm Hg). The vacuum decrement for all fibers used was essentially independent of amplitude and ranged from 0.00096 for the thin fibers to 0.00007 for the thicker ones. The vacuum period varied with amplitude (at 1.5 rad the change was 100 msec for a 12 sec period and less than 1 msec for a 4 sec period), but the change was reproduceable, so calibration curves were taken before and after each run and the liquid helium data were corrected for the vacuum variation. Measurements of the period in the liquid were made by measuring single periods, and at the low amplitudes several periods

were averaged and a smooth curve drawn through these points was used in analyzing the results. The damping was small enough so that the logarithmic decrement could be determined from the expression

$$D = [\ln (\varphi_i / \varphi_j)] / (j-i) \quad (2)$$

where φ_i is the i th amplitude. This expression is a factor 2π larger than the one used by Hollis Hallett.¹ For runs with the smallest damping, the number of amplitudes $(j-i)$ used in Eq. (2) ranged typically from 7 at the high amplitudes to 30 at the low amplitudes; for runs with larger damping, the range was typically 4 to 15. Values of the critical velocity determined from semi-log plots of φ_i vs. i were in very good agreement with those obtained from curves of D vs. φ . The logarithmic decrements measured in HeII ranged from 0.0022 for disk B to 0.045 for disk C.

III. RESULTS

A. Damping Curves

Figure 2 shows the amplitude dependence of the logarithmic decrement D of disk C taken on two separate days at approximately the same temperature. For convenience, we describe the upper curve as "steep" and the lower curve as "flat". As long as the temperature remained below the lambda temperature T_λ , the damping curves were all of the same type, i.e. either all steep or all flat. We have found that the difference between steep and flat curves is not due to (1) roughness of the disk surface, (2) the manner of deflecting

Fig. 2

the disk (whether by many small impulses or by a few large impulses), or (3) the position of the disk as the temperature passed through T_λ (whether lowered or held against the shield N). The difference is due to (1) the height of the bath at the time of the initial disk deflection, (2) the time interval Δt between reaching T_λ and the initial disk deflection, and possibly (3) the presence of the shield N. Using the cryostat in Fig. 1 (including shield N) we have found that in order to obtain a steep curve it is necessary that the bath height h above the disk at the initial disk deflection be greater than some critical value h' and that Δt be sufficiently short (probably less than 30 min). To obtain a flat curve it is necessary to have $h < h'$, or if $h > h'$ then Δt must be sufficiently long (about 1 hour). The limits on h' have been found to be $15.8 \leq h' \leq 16.8$ cm. A few runs taken without the shield indicate either that h' is lower in the absence of the shield (less than 10 cm) or that perhaps the presence of the shield is necessary to get a flat curve. It should be pointed out, however, that once a steep or flat curve was established the lowering of the liquid level below the shield did not change the type of curve.

Occasionally, curves that were neither steep nor flat were observed. These curves were either in between the steep and flat curves (8 out of 125 curves), exhibiting a critical velocity in between those for steep and flat, or lower than the flat curves ("ultra-flat", 7 out of 125), four of which exhibited critical velocities in agreement with the flat curves and three which had

critical velocities higher than the flat ones. However, there was no correlation between the occurrence of these odd curves and the time interval Δt ; in particular, one ultra-flat curve was obtained with a Δt of only 10 min. Of the 125 curves, 88% were either steep or flat. On three separate occasions, both steep and flat curves were obtained on the same day by warming above T_λ . It should be emphasized that the low amplitude damping was the same for both steep and flat curves.

Flat curves at different temperatures for an 11.54 sec period are shown in Fig. 3. Above the critical amplitude the logarithmic decrement increases and eventually becomes linear in the amplitude up to the highest amplitudes measured (2.5 rad). The slope ($\equiv dD/d\rho$) at the high amplitudes was found to vary as $\tau^{1/2}/I$ as shown in Fig. 4, where I is the moment of inertia of the disk and τ is the period.

Steep curves at different temperatures for an 11.54 sec period are shown in Fig. 5. At the high amplitudes these curves also are linear in the amplitude; the slope varies as $\tau/I^{1/2}d$ as shown in Fig. 6, where d is the thickness of the disk.

Rosenblat⁹ has shown that the centrifugal forces on fluid near an oscillating disk cause a radial-axial secondary flow which makes the damping of the disk change with amplitude; such changes have recently been observed in classical fluids.¹⁰ The dashed curves in Fig. 3 and Fig. 5 indicate the changes expected for secondary flow of the normal fluid only, as given by the empirical results of Folsø.¹⁰

Fig. 3

Fig. 4

Fig. 5

Fig. 6

On several occasions we studied the effect of surface roughness by placing iron filings on the disk. For the steep curves, we found that surface roughness lowers the critical velocity and makes the high amplitude part of the damping curve steeper. There was some indication that surface roughness had no effect on the flat curves, but this result is, as yet, inconclusive.

B. Critical Velocities

The critical velocities v_{c2} associated with the flat curves are shown in Fig. 7. For periods between 3.9 sec and 45.0 sec, v_{c2} is independent of temperature. The period dependence is given by $v_{c2} = 1.4 \tau^{-\frac{1}{2}}$ cm/sec, which is represented by the solid lines in Fig. 7. The critical velocities v_{c1} for the steep curves are shown in Fig. 8. For periods between 3.9 sec and 19.6 sec, we find that the period dependence at the lower temperatures is given by $v_{c1} = 0.7 \tau^{-\frac{1}{2}}$ cm/sec. The results of Gantsemlidze² ($\tau = 8.95$ sec) and Hollis Hallett¹ ($\tau = 11.0$ sec) are indicated by the broken line and the crosses respectively.

C. Period Measurements

The period of oscillation provides a measure of the added moment of inertia $I_a = \alpha(\tau^2 - \tau_v^2)$, where τ is the period of the disk oscillating in the liquid, τ_v is the period in vacuum, and α is the torsion constant. The relative change in I_a with amplitude is then given by

$$(I_a - I_{a0})/I_{a0} = (\tau^2 - \tau_0^2)/(\tau_0^2 - \tau_v^2) \quad (3)$$

where I_{a0} and τ_0 are low amplitude values. Our results for a 3.9 sec

Fig. 7

Fig. 8

period at a high and low temperature for both steep and flat curves are shown in Fig. 9. The low temperature results can be compared with those in Fig. 10 for an 11.54 sec period. In both figures the dashed curves represent what is expected from secondary flow of the normal fluid only.¹⁰

D. Low Amplitude Damping

We have compared our observed logarithmic decrement D_o at low amplitude with a calculated value D_{calc} based upon η_n from the rotating cylinder viscometer¹¹ and ρ_n from the second sound velocity.¹² The expression for D_{calc} is the same as that used by Dash and Taylor,¹⁶ but with a corner parameter of 0.30.¹³ In general, D_{calc} and D_o agree within experimental error, but at low fluid depth and large penetration depth, D_o is significantly higher. On several occasions with the helium at constant temperature, D_o was observed to increase as the fluid level fell to within 100 to 200 penetration depths of the disk surface. Fig. 11 shows that $(D_o - D_{calc})/D_{calc}$ depends only on the ratio h/λ , where h is the fluid level above the disk and λ is the penetration depth. The results in Fig. 11 are independent of whether the damping curve is steep or flat, or whether the shield was present, or which disk was used.

It appears that in superfluid helium the free surface can become a serious source of energy loss in oscillating disk experiments when the fluid level is within a few hundred penetration depths. This result may explain the high values of η_n obtained with a single disk¹⁶ (at temperatures below 1.5°K) as compared with other

Fig. 9

Fig. 10

Fig. 11

methods.^{14,11} Comparison with the results of Fols¹⁵ shows that this depth effect does not occur in classical liquids.

IV. DISCUSSION

A. Types of Damping Curves

We will assume that the excess energy dissipation at high amplitudes originates from turbulence in the superfluid. Hall⁷ and Vinen¹⁷ have shown how the concept of superfluid turbulence can be introduced in the framework of a quantized vortex line model.^{18,19}

In the present investigation we observe that the type of damping curve depends on the time interval Δt from reaching the lambda point to the initial deflection of the disk. One explanation of this behavior is that the superfluid is created originally in a state of disorganized motion and if the system is undisturbed (the disk is not deflected) this random primordial flow eventually decays, leaving the superfluid relatively motionless, so that subsequent measurements produce flat curves; but if soon after reaching T_λ the disk is deflected to high amplitudes, then the initial random motion of the superfluid is organized into a long-lived state, and a steep curve results. However, since the low amplitude damping is the same for both steep and flat curves, this long-lived motion of the superfluid must not interact with the disk at low amplitude. In the framework of the vortex line picture, we could interpret the long-lived superfluid motion in the steep case as some sort of organized array of quantized vortex lines. It should be noted however, that whatever the subcritical vortex

configuration is for the steep curves, it remains present for long periods of time (at least 8 hours for a series of curves and 70 min in between curves) and may persist indefinitely. We never observed a transition from one type of curve to another as long as the temperature remained below T_λ .

Results which depend on the previous history of the bath have been observed in a number of other liquid helium experiments, especially in those dealing with thermal counterflow in wide channels²⁰⁻²³ (diam $> 10^{-3}$ cm). All of the history effects in the thermal counterflow experiments were attributed to the presence of lingering vorticity in the superfluid. In the rotating beaker experiment of Reppy and Lane,²⁴ two different results were obtained: usually the superfluid would absorb angular momentum from the coasting beaker, occasionally it would not; these results also were attributed to the presence or absence of primordial vortex lines.

Judging from the magnitude and temperature dependence of the critical velocities and the qualitative shape of the logarithmic decrement curves for a single disk observed by both Hollis-Hallett¹ and Gamtsemlidze,² we can classify their data in the steep category. However, some of the damping curves shown for a pile of disks in the pioneering work of Hollis-Hallett are qualitatively different, and may be of the flat type.

B. Critical Velocities

The temperature independence of v_{c2} (between 1.4 and 2.15°K) reported here is in sharp disagreement with the $\rho_s^{-1/2}$ dependence found

in most previous experiments of this type. However, there have been many other types of experiment which have yielded temperature independent critical velocities. For example, Ricci and Vicentini-Missoni²⁵ have used a beam of ions to study heat flow in wide channels (0.8 cm) and have found critical velocities which were constant from 0.89 to 1.92°K; Vermeer, Van Alphen, et. al.²⁶ have performed measurements of gravitational superfluid flow in wide channels with the normal fluid held motionless by means of superleaks and have found v_c to be independent of temperature for a wide range of channel size (from 0.44 to 0.0015 cm); Keller and Hammel²⁷ have studied isothermal gravitational flow through two narrow slits (2.29×10^{-4} cm and 3.1×10^{-5} cm) and have found v_c to be independent of temperature except for a small region in the vicinity of T_λ where v_c falls rapidly to zero. Similar behavior is also found in film flow of HeII,^{28,29} although the results are complicated by uncertainties in the film thickness and the appropriate value of ρ_s .

Keller and Hammel have observed that the narrow region $T_\lambda - T$, in which v_c drops from its constant low-temperature value to zero at T_λ , decreases with increasing channel width; Clow and Reppy³⁰ have seen the same result with a superfluid gyroscope. In view of this it is understandable that we observe no decrease in v_{c2} up to the highest temperature (2.156°K) since our smallest characteristic dimension was $\lambda \approx 0.014$ cm. Langer and Fisher³¹ have developed a theory to explain this decrease of v_c near T_λ based on the idea that near T_λ there should be an intrinsic critical velocity

for the superfluid proportional to the superfluid density ρ_s .

Vinen³² distinguishes between two types of critical velocity, an "ideal" type in which the subcritical superfluid flow is frictionless and free of vortex lines, and a "non-ideal" type in which vortex lines are present in the subcritical flow. One would expect that the non-ideal critical velocity would ordinarily be lower than the ideal type because the vortex lines in the subcritical region would act as nuclei for the growth of more line. However, it is difficult to determine what types of experiment give ideal critical velocities; persistent current and film flow experiments are suggested by Vinen. Since these experiments lead to critical velocities which are probably temperature independent (except near T_λ), it is possible that temperature independence may be an identifying feature of ideal critical velocities. If these ideas are correct, it is reasonable to identify the critical velocities obtained from our flat damping curves as ideal and those obtained from the steep damping curves as non-ideal.

Wilks has correlated critical velocities in wide channels ($> 10^{-3}$ cm) by means of the expression

$$v_c = (4 h/Md) [\ln(d/3a_0)] \quad (4)$$

where d is a characteristic length, $a_0 = 1.2 \times 10^{-8}$ cm, and M is the mass of a helium atom. Since a_0 is very small, the logarithmic factor is relatively insensitive to the value of d , so eq. (4) implies $v_c d \approx \text{const.}$ Therefore, our result

$v_{c2} \tau^{\frac{1}{2}} = 1.4$ suggests that d is a penetration depth based on an effective kinematic viscosity ν which is independent of temperature. The value of ν obtained in this manner is $\nu = 1.2 \times 10^{-4} \text{ cm}^2/\text{sec}$, which is smaller than the normal fluid kinematic viscosity ν_n . We associate with v_{c2} a critical value of the Reynolds number Re . Re is defined by

$$Re = v \lambda / \nu = v(\tau / \pi \nu)^{\frac{1}{2}} \quad (5)$$

and has a critical value of 72.

Van Alphen et al.³³ have proposed that in experiments where the normal fluid velocity v_n is not negligible the critical velocities are related to classical turbulence in the normal fluid. They propose a Reynolds number $Re_{v_n} = \frac{d \rho v_n}{\eta_n}$ which is based on the normal fluid viscosity η_n and the total density ρ . Using $d = (\eta_n \tau / \pi \rho)^{\frac{1}{2}}$ and $v_n = v_{c2}$, we find that the critical value of their Reynolds number ranges from 64 to 84 between 2.15°K and 1.3°K. Our value of 72 is in good agreement, as would be expected since our effective ν is approximately equal to η_n / ρ . In spite of this agreement, we do not believe that a transition to classical turbulence takes place at this value of Re . We have made measurements in helium gas at 4.2°K and in liquid helium I at 2.4°K up to $Re = 430$ and have observed only gradual increases in damping consistent with the small increases expected from secondary flow.¹⁰ The measurements of Folsø extended up to $Re = 210$ in water; no sharp changes in damping were seen. A transition to turbulence would presumably be accompanied by sharp

increases in damping above the critical amplitude such as those observed by Benson and Hollis Hallett³ for an oscillating sphere in helium gas ($Re = 230$) and in helium I ($Re = 210$). Therefore, we conclude that although Re_{v_n} may be an important similarity parameter for liquid helium critical velocities, the critical value 72 does not correspond to classical turbulence in the normal fluid.

C. Other Results

(1) Effect of Mutual Friction

In 1950 Zwanikken³³ derived an expression for the excess energy dissipation due to the mutual friction³⁴ between the normal and superfluids in the case of the oscillating disk. Hollis-Hallett¹ applied Zwanikken's analysis to his results and found the predicted energy dissipation to be smaller than observed and to have a different temperature variation. However, there appears to be a sign error in Hollis-Hallett's expression. Furthermore, to apply Zwanikken's result to our disk requires more terms than originally included because of the large radius of our disk. Therefore, we have extended Zwanikken's analysis and have found that the logarithmic decrement due to mutual friction can be written as

$$D = A \rho_s \rho_n \lambda (\pi^2 R^4 / 8I) \epsilon [1 + \phi_1 \epsilon + \phi_2 \epsilon^2 + \phi_3 \epsilon^3 + \phi_4 \epsilon^4] \quad (6)$$

where A is the Gorter-Mellink mutual friction parameter, λ is the normal fluid penetration depth, ϵ is defined to be $\omega R^2 \varphi^2$ (φ is

the amplitude), and the ϕ_i terms depend only on temperature. Tabulated values of the major temperature dependent terms are given in Table 11. The values of A were read from a drawing in a paper by Vinen.²³ Using the table and eq. 6 to calculate the damping at an amplitude just above the critical amplitude and comparing the result with our data we find that the calculated value is several orders of magnitude too large and highly temperature dependent. For example, at $\varphi = 0.6$ rad, $\tau = 12$ sec and $T = 1.9^\circ\text{K}$, the calculated excess decrement is 1.76 and the observed value is 0.002. However, if we try to obtain better agreement with the flat curve results by assuming that instead of the amplitude φ a relative amplitude $\varphi - \varphi_0$ is the relevant quantity, we obtain the results shown in Fig. 12 where the upper curve is for a 12 sec period and the lower curve is for a 4 sec period. The points correspond to the observed damping at an amplitude of 0.6 rad for the 12 sec period and 0.3 rad for the 4 sec period. The curves were obtained by matching the data at 2.1°K , which required $\epsilon = 0.042$ for both periods. The result however is still in disagreement with observation, since the experimental points show that the excess damping is approximately independent of temperature and the theory predicts a strong temperature dependence. In summary, we find that the mutual friction force would give an excess decrement which is orders of magnitude larger than observed, and that even when the theory is adjusted to fit the data at 2.1°K , the temperature dependence is incorrect.

TABLE II

T (°K)	A (cm sec/g)	ρ_s (g/cm ³)	ϕ_1 (sec/cm ²)	ϕ_2 (sec/cm ²) ²	ϕ_3 (sec/cm ²) ³	ϕ_4 (sec/cm ²) ⁴
2.1	190	0.03617	2.062	175.4	239.4	11,340
2.0	130	0.0632	2.465	44.22	66.39	561.8
1.9	105	0.0838	2.640	14.72	22.24	49.25
1.8	88	0.0999	2.637	6.340	7.477	5.697
1.6	60	0.1218	2.192	2.372	1.359	0.4973
1.4	38	0.1344	1.532	1.229	0.535	0.1113
1.3	28	0.1383	1.162	0.746	0.264	0.0439

Fig. 12

(2) Period Measurements

The relative change in added moment of inertia I_a shown in Figs. 9 and 10 gives a measure of the amount of fluid being dragged by the disk. If the superfluid is "entrained" at high amplitudes, as suggested by Donnelly and Hollis-Hallett,⁶ then we would expect I_a to be significantly increased, particularly at the lower temperatures. Such behavior is indeed observed for the steep curves. If the effective viscosity is assumed to be approximately η_n , then the fraction of superfluid entrained can be calculated: the result for the data of Figs. 9 and 10 is 30% for the 3.9 sec period at 1.52°K but only 5% for the 11.54 sec period at 1.36°K. The flat curves, on the other hand, show no evidence of entrainment and are in qualitative agreement with the results for classical liquids. At temperatures close to T_λ the result for both steep and flat curves are consistent with the changes observed in ordinary liquids (dashed lines). Due to the fiber correction and the difficulty of determining I_{a0} precisely, the results shown in Figs. 9 and 10 are subject to large error, so only qualitative conclusions can be drawn. However, the minimum in $\Delta I_a/I_{a0}$ seen at the lower temperatures was observed repeatedly, so it is believed to be a real effect, although its origin is obscure.

V. SUMMARY AND CONCLUSIONS

Our major conclusions are: (1) that the damping curves and critical velocities observed in liquid helium II with an oscillating disk are strongly dependent on the history of the helium bath, and

(2) that when the bath is not disturbed for about one hour after reaching T_λ , the resulting critical velocities are independent of temperature and vary with period as $\tau^{-\frac{1}{2}}$.

We have conjectured that when the disk is set in motion shortly after reaching T_λ , the initial random motion of the superfluid is organized into a long-lived state of motion. This long-lived state has the following features: (1) it does not interact with the low amplitude motion of the disk, (2) it causes the critical velocities to depend upon temperature and disk roughness, (3) it causes the superfluid to be entrained at high amplitudes, and (4) it persists as long as the temperature remains below T_λ . These results suggest that the long-lived state consists of a stable array of vortex lines and that the resulting critical velocities are of the non-ideal type.

When the disk is not deflected for one hour after reaching T_λ , the initial unorganized motion of the superfluid dies out and consequent measurements reveal the following features: (1) the critical velocities are higher than previously observed, are independent of temperature, and (possibly) are independent of disk roughness, and (2) the superfluid is not entrained at high amplitudes. Since the subcritical flow should be free of vorticity in this case, the critical velocities are presumed to be of the ideal type. From these critical velocities we obtain a temperature-independent kinematic viscosity, $\nu = 1.2 \times 10^{-4} \text{ cm}^2/\text{sec}$, which is then used to calculate a critical Reynolds number, $Re = 72$. This value is in good agreement with the value calculated from the Reynolds number

proposed by Van Alphen et al.³³ However, we reject the idea that these critical velocities are due to turbulence in the normal fluid because (1) the excess damping is much larger than observed in ordinary liquids (see Fig. 3) and (2) no such critical value of the Reynolds number has been observed with oscillating disks in ordinary liquids.

The excess decrement at an amplitude slightly above critical was found to be temperature independent for the flat curves. A calculation of the excess decrement expected from mutual friction (based on a theory of Zwanikken) gave values which changed strongly with temperature and were much larger than observed.

The value of the low amplitude damping for both steep and flat curves was observed to increase when the bath height fell to within about 100 penetration depths of the disk, suggesting a long range connection between energy dissipated in the vicinity of the disk and the loss of energy at the free surface of the helium bath. This effect does not occur in classical liquids.

REFERENCES

1. A. C. Hollis Hallett, Proc. Roy. Soc. A210, 404 (1952).
2. G. A. Gamtsemlidze, Zhur. Eksp. Teor. Fiz. SSR 34, 1434 (1958) [translation: Sov. Phys. JETP 37, 678 (1960)].
3. C. B. Benson and A. C. Hollis-Hallett, Can. J. Phys. 34, 668 (1956).
4. R. Dean Taylor and J. G. Dash, in Proceedings of the Fifth International Conference on Low Temperature Physics and Chemistry, edited by J. R. Dillinger (The University of Wisconsin Press, 1958) p. 164.
5. E. L. Andronikashvili, G. A. Gamtsemlidze, and Sh. A. Dzhabaridze, J. Exp. Theo. Phys. (USSR) 50, 856 (1966) [translation: Sov. Phys. JETP, 23, 568 (1966)].
6. R. J. Donnelly and A. C. Hollis Hallett, Ann. Phys. 3, 320 (1958).
7. H. E. Hall, Adv. Phys. (Phil. Mag. Suppl.) 9, 89 (1960).
8. E. J. Walker, Rev. Sci. Ins. 30, 834 (1959).
9. S. Rosenblat, J. Fluid Mech. 6, 206 (1959).
10. R. F. Folse, R. G. Hussey, and J. S. Bosnak, Phys. Fluids, 11, 2508 (1968).
11. A. D. B. Woods and A. C. Hollis Hallett, Can. J. Phys. 41, 596 (1963).
12. R. G. Hussey, B. J. Good and J. M. Reynolds, Phys. Fluids 10, 89 (1967).
13. R. F. Folse and R. G. Hussey, Phys. Fluids 11, 2278 (1968).

14. D. F. Brewer and D. O. Edwards, Proc. Roy. Soc. (London) A251, 247 (1959).
15. R. F. Folse, Unpublished.
16. J. G. Dash and R. D. Taylor, Phys. Rev. 105, 7 (1957).
17. W. F. Vinen, in Progress in Low Temperature Physics, edited by C. J. Gorter, vol. 3 (Amsterdam, 1961) p. 1.
18. L. Onsager, Suppl. Nuovo Cimento, 6, 249 (1949).
19. R. P. Feynman, in Progress in Low Temperature Physics, edited by C. J. Gorter, vol. 1 (Amsterdam, 1955) p. 36.
20. D. F. Brewer, D. O. Edwards, and K. Mendelssohn, Phil. Mag. 1, 1130 (1958).
21. D. F. Brewer and D. O. Edwards, Phil. Mag. 6, 775 (1961).
22. K. Mendelssohn and W. A. Steele, Proc. Phys. Soc. A73, 144 (1959).
23. W. F. Vinen, Proc. Roy. Soc. A240, 128 (1957).
24. J. D. Reppy and C. T. Lane, Phys. Rev. 140, 106 (1965).
25. M. V. Ricci and M. Vicentini-Missoni, Phys. Rev. 158, 153 (1967).
26. W. Vermeer, W. M. Van Alphen, J. F. Olijhoek, K. W. Taconis, and De Bruyn Ouboter, Phys. Letters 18, 265 (1965);
W. M. Van Alphen, J. F. Olijhoek, R. De Bruyn Ouboter, and K. W. Taconis, Physica 32, 1901 (1966).
27. W. E. Kelier and E. F. Hammel, Physics 2, 221 (1966).
28. K. Mendelssohn and G. K. White, Proc. Phys. Soc. A63, 1328 (1950).

29. G. R. Hebert, K. L. Chopra, and J. B. Brown, Phys. Rev. 106, 391 (1957).
30. J. R. Clow and J. D. Reppy, Phys. Rev. Letters 19, 291 (1967).
31. J. S. Langer and M. E. Fisher, Phys. Rev. Letters 19, 560 (1967).
32. W. F. Vinen, in Liquid Helium, edited by G. Careri (Academic Press, 1963) p. 336.
33. W. M. Van Alphen, G. J. Haasteren, R. De Bruyn Ouboter, and K. W. Taconis, Phys. Letters 20, 474 (1966).
34. G. C. J. Zwanikken, Physica 16, 805 (1950).
35. C. J. Gorter and J. H. Mellink, Physica 15, 285 (1949);
C. J. Gorter, P. W. Kasteleijn, and J. H. Mellink, Physica 16, 113 (1950).

APPENDIX I

Calculation of Low Amplitude
Logarithmic Decrement

The expression used to calculate the low amplitude logarithmic decrement is that given by Dash and Taylor:¹⁶

$$D_n = \frac{R^4 \pi [\pi \tau \eta_n \rho_n (1 - D/2\pi)]^{\frac{1}{2}} k}{21(1 + \tau^2/\tau_v^2)} \quad (A1-1)$$

where $k = (1 + 2d/R + 3\lambda d/R^2 + 8\sigma\lambda/R)$ is a geometrical factor in which λ is the normal fluid penetration depth, R is the disk radius, d is the disk thickness, and σ is a corner parameter which was shown by Fols¹³ to have the value 0.30. In making the actual calculations $D/2\pi$ was neglected in comparison with unity and the ratio of the period to the vacuum period τ/τ_v was assumed approximately equal to unity. The values of normal fluid viscosity η_n and density ρ_n were taken primarily from references 11 and 12 and are given in Table III. Table IV contains (for selected temperatures and periods) the calculated value of the low amplitude logarithmic decrement $D_{calc} = (D_n + D_v)$ as well as the observed value D_o , the height of the helium bath above the disk h , and the normal fluid penetration depth λ .

TABLE III

T (°K)	η_n (μ P)	ρ_n (g/cm ³)
2.17	24.3	0.1427
2.15	22.0	0.1296
2.10	18.56	0.1097
2.05	16.36	0.0951
2.00	14.90	0.0825
1.95	13.97	0.0713
1.90	13.40	0.0617
1.85	13.12	0.0532
1.80	12.90	0.0455
1.75	12.86	0.0389
1.70	12.92	0.0333
1.65	13.07	0.0281
1.60	13.30	0.0235
1.55	13.60	0.0194
1.50	13.98	0.0160
1.45	14.41	0.0133
1.40	14.90	0.0108
1.35	15.49	0.00883
1.30	16.14	0.00690
1.25	16.96	0.00545
1.20	18.08	0.00419
1.15	19.96	0.00308
1.10	23.3	0.00220

TABLE IV

τ (°K)	T (sec)	h (cm)	λ (cm)	h/λ	D_o	D_{calc}
2.091	3.9	13.02	0.0143	910.3	0.0125	0.0125
1.846	3.9	12.38	0.0174	711.6	0.0075	0.0075
2.037	11.54	17.15	0.0251	683.1	0.01890	0.01890
2.146	8.84	14.61	0.0214	682.5	0.00460	0.00460
2.045	19.5	16.83	0.0324	519.4	0.0258	0.0257
1.824	11.54	12.7	0.0309	411.0	0.01265	0.01260
1.556	3.9	9.53	0.0290	328.4	0.0049	0.0047
1.518	11.54	9.84	0.054	182.0	0.0084	0.0079
1.408	3.9	6.67	0.0405	164.6	0.0043	0.0041
1.433	19.6	11.75	0.0851	138.04	0.0100	0.00915
1.413	11.54	8.89	0.0679	130.9	0.0076	0.0067
1.75	45.0	8.92	0.0684	130.0	0.00915	0.00845
1.825	12.4	3.81	0.0317	120.0	0.0140	0.0133
1.475	8.84	5.72	0.0520	109.9	0.0022	0.00173
1.94	45.0	5.08	0.053	95.85	0.0132	0.01115
1.582	19.1	5.25	0.0603	86.9	0.0130	0.0114
1.33	3.9	3.81	0.049	77.7	0.0019	0.0014
1.429	19.6	5.87	0.0860	68.3	0.0120	0.0091
1.346	9.33	3.81	0.0727	52.4	0.0345	0.00230
1.25	19.1	4.13	0.1409	29.3	0.0120	0.0072
1.415	19.6	2.22	0.0886	25.1	0.0130	0.0089
1.488	12.1	1.27	0.0591	21.5	0.00325	0.00183

APPENDIX II

Computer Program Used to Calculate
and Plot Logarithmic Decrement Curves

An IBM 360 Mod 65 computer was used to compute the logarithmic decrements from the data taken in the lab, and in conjunction with the computer a Calcomp 563 incremental plotter was used to plot the curves of logarithmic decrement versus amplitude. The plotter is coupled to a Calcomp 570 magnetic tape unit, and information required to drive the plotter is recorded on a magnetic tape by means of Fortran IV routines available on the 360 computer.

A typical program used in the calculation of D for disk C with a 13 second period is listed below:

```
C   LOG DECREMENT PROGRAM FOR 12.87 SEC PERIOD  7/1/69
      DIMENSION DATA(2000)
      CALLPLOTS(DATA(1),2000)
      DIMENSION A(900),W(60),CRV(60),DO(9),AO(9)
      NT=1
      READ(5,100)NB
100  FORMAT(13)
      READ(5,300)(CRV(I),I=1,NB)
300  FORMAT(18F4.1)
      READ(5,200)(W(N),N=1,NB)
      READ(5,200)(DO(N),N=1,NB)
      READ(5,200)(AO(N),N=1,NB)
200  FORMAT(12F6.3)
      N=0
```

```
5  CALLPLOT(0.0,0.0,+3)
   CALLPLOT(0.0,16.0,2)
   CALLPLOT(17.0,16.0,2)
   CALLPLOT(17.0,0.0,2)
   CALLPLOT(0.0,0.0,2)
   CALLPLOT(0.5,0.5,-3)
   CALLAXIS(0.0,0.0,9HDECREMENT,9,15.0,90.0,DO(NT),0.004,5.0)
4  CALLAXIS(0.0,0.0,9HAMPLITUDE,-9,16.0,0.0,A0(NT),0.15,5.0)
   GO TO 8
6  NT=NT+1
   IF(N.EQ.NB)GO TO 25
3  IF(CRV(NT).EQ.NT)GO TO 7
   GO TO 8
7  CALLPLOT(-0.5,-0.5,-3)
   CALLPLOT(18.0,0.0,-3)
   GO TO 5
8  N=N+1
   K=-11
   L=0
15 K=K+12
   L=L+12
   READ(5,10)(A(I),I=K,L)
   IF(A(L)-0.0)16,16,15
10  FORMAT(12F6.3)
16  J=3
   I=0
```

```
17  Y=7.00
51  Y=Y-0.50
    J=J+1
40  I=I+1
    X=J
    M=I+J
    IF(A(M).EQ.0.0)GO TO 6
    Q=A(I)-A(M)
    Z=W(N)
    A1=(A(I)-Z)*0.015
    A2=(A(M)-Z)*0.015
    IF(Q-Y)50,60,60
50  I=I-1
    GO TO 51
60  B=(A1+A2)/2.0
    D=ALOG(A1/A2)/X
20  WRITE(6,23)J,Y,B,D
23  FORMAT(1X,16,2F9.3,E16.8)
    IF(D-DO(NT))40,40,42
42  B=(B-A0(NT))*6.666666
    D=(D-DO(NT))*250.0
    IF(N.EQ.1) NJ=0
    IF(N.EQ.2) NJ=1
    IF(N.EQ.3) NJ=2
    IF(N.EQ.4) NJ=5
    IF(N.EQ.5) NJ=14
```

```
      IF(N.EQ.6) NJ=92
      IF(N.EQ.7) NJ=3
      IF(N.EQ.8) NJ=0
43  CALLSYMBOL(B,D,0.10,NJ,0.0,-1)
      GO TO 40
25  CALLPLOT(0.0,0.0,-3)
      CALLPLOT(0.0,0.0,999)
      STOP
      END
```

The number NB is the total number of curves to be analyzed. The subscripted variable CRV(I) is employed in statement 3 to determine how many curves will be plotted on a given set of axes, i.e. if after plotting 2 curves on the first set of axes I want to draw another set before plotting the third curve then CRV(3) = 003.0. The zero correction for each run is read in as W(N) and both DO(N) and AO(N) are the values of the logarithmic decrement and amplitude respectively at the origin of the axes for a given curve N. After drawing the axes (statements 5-4) the data is read in (statements 8-10). The number 00.000 is placed in the last position of the last card for each run to ensure that only one run is read in at a time. Statements 16, 17, and 51 regulate the initial choice of the number of amplitudes between those being used to calculate D and the final number to be used, which should be predetermined by the size of the damping. Each log decrement D and corresponding amplitude B is then computed, printed out, and plotted up one at a time. In plotting each point up (statement 43) NJ

refers to a code number which corresponds to the particular symbol to be used, and both B and D are converted to inches (statement 42).

VITA

Robert Patrick Roger was born in New Orleans, Louisiana on May 21, 1942. He attended elementary and high schools in New Orleans and was graduated from Jesuit High School in 1960. He entered Loyola University of the South in New Orleans, in September, 1960 and received a Bachelor of Science degree in physics in 1964. He entered the Graduate School of Louisiana State University in September, 1964, held an N.D.E.A. fellowship in the Department of Physics and Astronomy during his first three years of graduate study, and served as a graduate research assistant in the same department from September 1968 to the present. He is presently a candidate for the degree of Doctor of Philosophy in the Department of Physics and Astronomy.

EXAMINATION AND THESIS REPORT

Candidate: Robert Patrick Roger

Major Field: Physics

Title of Thesis: Amplitude Effects for the Oscillating Disk in Liquid Helium II

Approved:

R. D. Hussey
Major Professor and Chairman

Max Goodrich
Dean of the Graduate School

EXAMINING COMMITTEE:

William Young

Ray G. Goodrich

Dan K. Scholz

Charles J. Jernis

Date of Examination:

July 25, 1969