1970

Applying Feedback and Feedforward Control.

John Anthony Miller
Louisiana State University and Agricultural & Mechanical College

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A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Doctor of Philosophy

in

The Department of Chemical Engineering

by

John Anthony Miller
B.S., Louisiana State University, 1966
M.S., Louisiana State University, 1968
January, 1970
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<td>Comparison of Optimal and Suboptimal Controllers for CF1 with $r = 0.1$.</td>
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ABSTRACT

Five topics concerned with applying feedback and feedforward control are considered. The first topic provides a comparison of tuning techniques. The results showed which modeling technique and tuning relations were best. The second topic considered was how to apply feedforward control. Overall this treatment provided a procedure for developing feedforward controllers. The third topic considered the use of search techniques to determine optimal switching times. The results showed that use of a search technique was superior to other theoretical approaches particularly for higher order batch processes. The fourth topic considered a comparison of optimal and suboptimal controllers. It was shown that for a given control problem, there is definitely a point of diminishing returns when adding additional state variables to the controller. The final topic considered the dynamic modeling and control of a distillation column. Theoretical and experimental models were developed and optimal feedback control theory and feedforward control theory were applied to the simulated distillation column.
CHAPTER I
INTRODUCTION

The purpose of this dissertation is to consider the practical application of feedback and feedforward control to chemical processes. Five separate topics concerned with this area are considered.

The first topic provides a comparison of tuning techniques. This work has been published and it is presented in Appendix A. (Ref. 1) The first part of this work deals with how the first order lag plus dead time model is obtained. Two procedures for obtaining the dead time and time constant are compared and it is shown that one procedure is superior. The next part of this work compares six sets of tuning relations for proportional plus integral and proportional plus integral plus derivative controllers. It was shown that the ITAE tuning relation provided the "best" responses. Also that study indicated that good feedback control could be obtained for processes whose dead time to time constant ratio was less than 0.4. For processes with a larger ratio more advanced techniques should probably be used.

The second topic considered was how to apply feedforward control. This work has also been published and it is presented in Appendix B. (Ref. 2) The first part of this work was an extensive literature survey on feedforward control. Then it was shown how to develop feedforward controllers for a single variable and a multivariable process. Linear and nonlinear steady-state
and dynamic feedforward controllers were developed based on theoretical models. Finally feedforward controllers based on experimental models were considered. Overall this treatment provided a procedure for developing feedforward controllers.

The third topic considered the use of search techniques to determine optimal switching times. This work has also been published and it is presented in Appendix C. (Ref.3) Time optimal controllers for second and sixth order systems were obtained by using a search technique. The results showed that use of a search technique was superior to other theoretical approaches particularly for higher order batch processes.

The fourth topic considered a comparison of optimal and suboptimal controllers. Optimal controllers for a sixth order plant using all six state variables and based on solving the matrix Riccati equation were first developed. These were compared to suboptimal controllers which use only some of the state variables and based on using a search technique to obtain them. Four different control problem formulations were considered. It was shown that for a given control problem, there is definitely a point of diminishing returns when adding additional state variables to the controller. It also appears that applying the controller for one control problem to another control problem is likely to produce poor results.

The final topic considered the dynamic modeling and control of a distillation column. The control problem is first defined to be that of controlling distillate composition by manipulating
distillate rate when feed rate disturbances occur. Then a theoretical model is developed and the initial conditions specified. Based on feed and distillate rate responses a first order experimental model of the distillation column is then developed. Next the feedback control problem is formulated by specifying a quadratic performance index. Optimal control theory is used and a modified Riccati equation is used to determine the controller parameters. The responses for several feed rate disturbances are shown. Then feedforward control of the distillation column is considered and its responses to the feed rate disturbances are shown. Finally feedback-feedforward control is illustrated and shown to be the best approach.
Literature Cited


CHAPTER II
COMPARISON OF OPTIMAL AND SUBOPTIMAL CONTROLLERS

Basically there are two approaches to the design of control systems. First, one approach is to propose a given form of the control algorithm or strategy, and then determine the numerical values of the parameters that give the "best" control according to some performance criterion. This is a parameter optimization problem which yields what is commonly called a suboptimal controller.

The second approach is to determine the optimal function to be used as the control algorithm or strategy. This is a functional optimization problem which yields an optimal controller.

The purpose of this article is to compare some optimal and suboptimal controllers. There are three elements or aspects to the control problems to be considered. These are the system, the forcing function and the criterion function. There will be four problems considered.

The System

The system to be considered is given by the transfer function

\[ \frac{Y(s)}{U(s)} = \frac{1}{(s+\delta)} \]  

(2.1)

This transfer function may be written in state variable form as

\[ \dot{X}(t) = A \ X(t) + b \ u(t) \]  

(2.2)
with the output equation

\[ y(t) = c \ X(t) \]  \hspace{1cm} (2.3)

where

\[ X = \text{column} \ (X_1, X_2, X_3, X_4, X_5, X_6) \]
\[ b = \text{column} \ (0, 0, 0, 0, 0, 1) \]
\[ c = \text{row} \ (1, 0, 0, 0, 0) \]

\[ A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
-1 & -6 & -15 & -20 & -15 & -6
\end{bmatrix} \]

and \[ X(0) = \text{column} \ (1, 0, 0, 0, 0, 0, 0) \]

Typical systems which would have a transfer function of this form are serial chemical reactors, plates of a distillation column and steel rolling mills.

For this system, the following four formulations of the control problem are common.

**Problem 1**

Given the system described by equation (2.1) and forced by initial condition forcing determine the optimal and suboptimal controllers which will minimize

\[ CFI = \int_{0}^{\infty} (y^2 + ru^2) dt \]  \hspace{1cm} (2.4)
The optimal control theory for the above problem is treated in Reference 1 as the output regulator problem. The overall results of this treatment is that the optimal feedback controller is given by

\[ u(t) = - \frac{1}{r} \left[ k_1 x_1(t) + k_2 x_2(t) + k_3 x_3(t) + k_4 x_4(t) + k_5 x_5(t) + k_6 x_6(t) \right] \] (2.5)

The constants \( k_1 \) to \( k_6 \) may be obtained by solving the matrix Riccati equation for this system. Note that all of the state variables are used in the optimal controller. The matrix Riccati equation for this system is

\[ -KA - A'K + K b b' K / r - c' c = 0 \] (2.6)

where

\[
K = \begin{bmatrix}
k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\
k_{12} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\
k_{13} & k_{23} & k_{33} & k_{34} & k_{35} & k_{36} \\
k_{14} & k_{24} & k_{34} & k_{44} & k_{45} & k_{46} \\
k_{15} & k_{25} & k_{35} & k_{45} & k_{55} & k_{56} \\
k_{16} & k_{26} & k_{36} & k_{46} & k_{56} & k_{66}
\end{bmatrix}
\]

Suboptimal controllers may be composed of controllers which contain only some of the state variables. The following five suboptimal controllers were considered:  

\[ u(t) = -k_7 x_1(t) \] (2.7)
\[ u(t) = -[k_8 x_1(t) + k_9 x_2(t)] \] (2.8)
\[ u(t) = -[k_{10} x_1(t) + k_{11} x_2(t) + k_{12} x_3(t)] \] (2.9)
\[ u(t) = -[k_{13} x_1(t) + k_{14} x_2(t) + k_{15} x_3(t) + k_{16} x_4(t)] \] (2.10)
\[ u(t) = -[k_{17} x_1(t) + k_{18} x_2(t) + k_{19} x_3(t) + k_{20} x_4(t) + k_{21} x_5(t)] \] (2.11)

Table 2.1 compares the optimal controller given by (2.5) to the suboptimal controllers given by (2.7) to (2.11). The optimal constants \( k_1 \) to \( k_6 \) were obtained by solving the Riccati equation, while the suboptimal constants were obtained by Pattern Search. (Reference 2) Figure 2.1 shows the response for some of these controllers. The conclusion which can be drawn from Table 2.1 and Figure 2.1 is that a suboptimal controller containing three terms performs, for all practical purposes, as well as the optimal controller for this system, forcing function and criterion function.

**Problem 2**

Given the system described by equation (2.1) and forced by disturbance forcing determine the optimal and suboptimal controllers which will minimize

\[ CF2 = \int_0^\infty (y^2 + r \dot{u}^2) \, dt \] (2.12)

The optimal control theory for the above problem is treated in Reference 3. The first step is to include the disturbance in the state equation.
### TABLE 2.1A. Optimal versus Suboptimal Controller Costs and Parameters for Problem 1 with $r = 1.0$

<table>
<thead>
<tr>
<th>Controller Equation</th>
<th>Cost</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open Loop</td>
<td>4.64</td>
<td></td>
</tr>
<tr>
<td>(2.7)</td>
<td>4.43</td>
<td>$k_7 = 0.185$</td>
</tr>
<tr>
<td>(2.8)</td>
<td>4.39</td>
<td>$k_8 = 0.260$, $k_9 = 0.645$</td>
</tr>
<tr>
<td>(2.9)</td>
<td>4.37</td>
<td>$k_{10} = 0.330$, $k_{11} = 0.835$, $k_{12} = 1.140$</td>
</tr>
<tr>
<td>(2.10)</td>
<td>4.37</td>
<td>$k_{13} = 0.370$, $k_{14} = 1.125$, $k_{15} = 1.230$, $k_{16} = 0.940$</td>
</tr>
<tr>
<td>(2.11)</td>
<td>4.37</td>
<td>$k_{17} = 0.445$, $k_{18} = 1.455$, $k_{19} = 2.350$, $k_{20} = 1.640$, $k_{21} = 0.965$</td>
</tr>
<tr>
<td>(2.5)</td>
<td>4.37</td>
<td>$k_1 = 0.414$, $k_2 = 1.333$, $k_3 = 1.894$, $k_4 = 1.440$, $k_5 = 0.575$, $k_6 = 0.095$</td>
</tr>
</tbody>
</table>
Figure 2.1A
Comparison of Optimal and Suboptimal Controllers for CFI
With \( r = 1.0 \).
<table>
<thead>
<tr>
<th>Controller Equation</th>
<th>Cost $\int_0^2 (y^2 + 0.1u^2) dt$</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open Loop</td>
<td>4.64</td>
<td></td>
</tr>
<tr>
<td>(2.7)</td>
<td>4.10</td>
<td>$k_7 = 0.515$</td>
</tr>
<tr>
<td>(2.8)</td>
<td>3.85</td>
<td>$k_8 = 0.905$</td>
</tr>
<tr>
<td>(2.9)</td>
<td>3.72</td>
<td>$k_9 = 2.180$</td>
</tr>
<tr>
<td>(2.10)</td>
<td>3.69</td>
<td>$k_{10} = 1.450$</td>
</tr>
<tr>
<td>(2.4)</td>
<td>3.69</td>
<td>$k_{13} = 1.850$</td>
</tr>
<tr>
<td>(2.5)</td>
<td>3.69</td>
<td>$k_{17} = 2.000$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_{18} = 5.850$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_{19} = 6.745$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_{20} = 4.850$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_{21} = 0.950$</td>
</tr>
</tbody>
</table>

**TABLE 2.1B. Optimal Versus Suboptimal Controller Costs and Parameters for Problem 1 with r = 0.1**
Figure 2.18
Comparison of Optimal and Suboptimal Controllers for CFI
With $r = 0.1$. 
\[ \dot{X}(t) = A \dot{X}(t) + b u + f d \] (2.13)

where \( f = \text{column } (0,0,0,0,0,1) \)
\( d = \text{constant disturbance} = 1 \)

The next step is to define the additional state variable
\[ X_7(t) = u(t) + d \] (2.14)

and set
\[ \dot{X}_7(t) = \dot{u}(t) = v(t) \] (2.15)

Incorporating 2.14, 2.15 and 2.2 into one equation yields
\[ \ddot{X}(t) = A \dot{X}(t) + b v(t) \] (2.16)

where \( \ddot{X} = \text{column } (X_1,X_2,X_3,X_4,X_5,X_6,X_7) \)
\( \ddot{b} = \text{column } (0,0,0,0,0,0,1) \)
\( \ddot{A} = \text{skip row} \)

and
\( \ddot{c} = \text{row}(1,0,0,0,0,0) \)

Substituting 2.15 into 2.12 gives
\[ CF^2 = \int_0^\infty (y^2 + rv^2) \, dt \] (2.17)

Regular optimal control theory that was used to solve problem 1 may now be applied to 2.16 and 2.17 with the result that

\[ v(t) = \frac{-1}{r} [k_1X_1(t) + k_2X_2(t) + k_3X_3(t) + k_4X_4(t) + k_5X_5(t) + k_6X_6(t) + k_7X_7(t)] \] (2.18)

The k's are obtained by solving the modified Riccati equation
\[-\mathbf{K} \mathbf{A} = \mathbf{A}' \mathbf{K} + \mathbf{K} \mathbf{b} \mathbf{b}' \mathbf{K/r} - \mathbf{C} \mathbf{C}' = 0 \quad (2.19)\]

where

\[
\mathbf{K} = \begin{bmatrix}
  k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & k_1 \\
  k_{12} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & k_2 \\
  k_{13} & k_{23} & k_{33} & k_{34} & k_{35} & k_{36} & k_3 \\
  k_{14} & k_{24} & k_{34} & k_{44} & k_{45} & k_{46} & k_4 \\
  k_{15} & k_{25} & k_{35} & k_{45} & k_{55} & k_{56} & k_5 \\
  k_{16} & k_{26} & k_{36} & k_{46} & k_{56} & k_{66} & k_6 \\
  k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7
\end{bmatrix}
\]

Since we want \( u(t) \) and not \( v(t) \), the first part of the state equation 2.13 is solved for \( u(t) \) and substituted in place of \( x_7(t) \) into equation 2.18 which is then integrated to give

\[
u(t) = - \frac{1}{r} \left[ k_1^1 \int_0^t x_1(\tau) \, d\tau + k_2^1 x_1(t) + k_3^1 x_2(t) + k_4^1 x_3(t) + k_5^1 x_4(t) + k_6^1 x_5(t) + k_7^1 x_6(t) \right] \quad (2.20)
\]

This will be referred to as Johnson's controller. Note that the controller uses all of the state variables plus the integral of the first state variable.

Suboptimal controllers may be composed of controllers which contain only some of the state variables. For this problem the following two suboptimal controllers were considered

\[
u(t) = - [k_8^1 \int_0^t x_1(\tau) \, d\tau + k_9^1 x_1(t)] \quad (2.21)
\]

\[
u(t) = - [k_{10}^1 \int_0^t x_1(\tau) \, d\tau + k_{11}^1 x_1(t) + k_{12}^1 x_3(t)] \quad (2.22)
\]

Equation 2.21 is the familiar PI controller, and equation 2.22 is
the familiar PID controller.

Table 2.2 compares the optimal controller given by 2.20 to the suboptimal controllers given by 2.21 and 2.22. The optimal constants $k_1^o$ to $k_7^o$ were obtained by solving the modified Riccati equation while the suboptimal controllers were obtained by Pattern Search. Figure 2.2 shows the response for these controllers. The conclusion which can be drawn from this comparison is that the suboptimal controllers do fairly well for $r = 1.0$, but they do not do so well for $r = 0.1$.

Problem 3

The third problem is given the system described by equation 2.1 and forced by conventional set point forcing determine the suboptimal controller parameters which will minimize.

$$\mathcal{F}_3 = \int_0^\infty e^2 \, dt$$

The suboptimal controllers considered are the PI and PID controllers given by equations 2.21 and 2.22. The results for problem 3 is given in Table 2.3.

Problem 4

The fourth problem is given the system described by equation 2.1 and forced by a disturbance determine the PI and PID parameters which will minimize

$$\mathcal{F}_4 = \int_0^\infty y^2 \, dt$$

The results for problem 4 are given in Table 2.4.
TABLE 2.2. Optimal versus Suboptimal Controller Costs and Parameters for Problem 2

<table>
<thead>
<tr>
<th>CONTROLLER</th>
<th>COST</th>
<th>PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID 2.22</td>
<td>1.32</td>
<td>$k_1 = 1.37$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_{11} = 0.36$</td>
</tr>
<tr>
<td>PID 2.21</td>
<td>2.44</td>
<td>$k_8 = 1.34$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_9 = 0.19$</td>
</tr>
</tbody>
</table>

$r = 0.1$

<table>
<thead>
<tr>
<th>CONTROLLER</th>
<th>COST</th>
<th>PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID 2.22</td>
<td>0.70</td>
<td>$k_{10} = 1.81$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_{11} = 0.91$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_{12} = 5.68$</td>
</tr>
<tr>
<td>PID 2.21</td>
<td>1.94</td>
<td>$k_8 = 1.62$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_9 = 0.22$</td>
</tr>
</tbody>
</table>

$r = 1.0$

<table>
<thead>
<tr>
<th>CONTROLLER</th>
<th>COST</th>
<th>PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal 2.20</td>
<td>0.91</td>
<td>$k_1 = 0.53$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_2 = 8.00$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_3 = 27.45$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_4 = 28.00$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_5 = 9.60$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_6 = 1.32$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_7 = 0.14$</td>
</tr>
<tr>
<td>PID 2.22</td>
<td>1.32</td>
<td>$k_{10} = 1.37$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_{11} = 0.51$</td>
</tr>
<tr>
<td>PID 2.21</td>
<td>2.44</td>
<td>$k_8 = 1.34$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_9 = 0.19$</td>
</tr>
</tbody>
</table>

$r = 0.1$

<table>
<thead>
<tr>
<th>CONTROLLER</th>
<th>COST</th>
<th>PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID 2.22</td>
<td>0.70</td>
<td>$k_{10} = 1.81$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_{11} = 0.91$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_{12} = 5.68$</td>
</tr>
<tr>
<td>PID 2.21</td>
<td>1.94</td>
<td>$k_8 = 1.62$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_9 = 0.22$</td>
</tr>
</tbody>
</table>
Figure 2.2A
Comparison of Optimal and Suboptimal Controllers for CF2 With $r = 1.0$. 
Figure 2.2B
Comparison of Optimal and Suboptimal Controllers for CF2
With \( r = 0.1 \).
TABLE 2.3. Controller Costs and Parameters for Problem 3

<table>
<thead>
<tr>
<th>CONTROLLER</th>
<th>COST</th>
<th>PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>4.49</td>
<td>$k_1 = 1.10$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_2 = 0.17$</td>
</tr>
<tr>
<td>PID</td>
<td>3.70</td>
<td>$k_1 = 1.86$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_2 = 0.28$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_3 = 4.19$</td>
</tr>
</tbody>
</table>
### TABLE 2.4. Controller Costs and Parameters for Problem 4

<table>
<thead>
<tr>
<th>CONTROLLER</th>
<th>COST $\int y^2 dt$</th>
<th>PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>1.86</td>
<td>$k_1 = 1.68$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_2 = 0.22$</td>
</tr>
<tr>
<td>PID</td>
<td>0.48</td>
<td>$k_1 = 2.04$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_2 = 1.27$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_3 = 7.39$</td>
</tr>
</tbody>
</table>
Cross-Comparisons

In addition to the four problems considered some cross-comparisons were made. The exact form of the criterion function will depend upon the forcing function being used. Table 2.5 gives the integral to be used with each forcing function. Integrals on the diagonal of Table 2.5 are the ones customarily used in definition of the corresponding control problem. Integrals off the diagonal are modifications of respective integral applicable to the corresponding problem. Table 2.6 fills in the numbers for Table 2.5. In looking over Table 2.6 several conclusions can be made. It is seen that controllers tuned for $\int e^2 dt$ or $\int y^2 dt$ do very poorly on $\int (y^2 + ru^2) dt$. This is expected since no penalty for the control was used in the tuning. It is also seen that the optimal controllers out-perform the suboptimal controllers even on the integrals $\int e^2 dt$ and $\int y^2 dt$. This is because more state variables are used in the feedback.

Conclusions

The objective of this work was to take one system and solve various formulations of the control problem. It has been shown that for a given control problem, there is definitely a point of diminishing return when adding additional state variables to the suboptimal controller. It also appears that the controller resulting from the solution of a particular control problem is quite specific. That is, applying the controller for one control problem to another control problem is likely to produce poor results.
TABLE 2.5. Tabulation of the Integrals Associated With the Four Control Problems

<table>
<thead>
<tr>
<th>Control Problem</th>
<th>$\int (y^2 + ru^2) dt$</th>
<th>$\int (y^2 + ru^2) dt$</th>
<th>$\int e^2 dt$</th>
<th>$\int y^2 dt$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial Condition</strong></td>
<td>$\int (y^2 + ru^2) dt$</td>
<td>$\int (y^2 + ru^2) dt$</td>
<td>$\int y^2 dt$</td>
<td>$\int y^2 dt$</td>
</tr>
<tr>
<td>Johnson's Problem</td>
<td>$\int (y^2 + ru^2) dt$</td>
<td>$\int (y^2 + ru^2) dt$</td>
<td>$\int y^2 dt$</td>
<td>$\int y^2 dt$</td>
</tr>
<tr>
<td>Conventional Set Point</td>
<td>$\int (e^2 + r(u-1)^2) dt$</td>
<td>$\int (e^2 + ru^2) dt$</td>
<td>$\int e^2 dt$</td>
<td>$\int e^2 dt$</td>
</tr>
<tr>
<td>Conventional Disturbance</td>
<td>$\int (y + r(u+1)^2) dt$</td>
<td>$\int (y^2 + ru^2) dt$</td>
<td>$\int y^2 dt$</td>
<td>$\int y^2 dt$</td>
</tr>
<tr>
<td>CONTROLLER</td>
<td>ENVIRONMENT</td>
<td>Initial condition forcing, $\int (y^2 + ru^2) dt$</td>
<td>disturbance forcing $\int (y^2 + ru^2) dt$</td>
<td>Set Point forcing $\int e^2 dt$</td>
</tr>
<tr>
<td>------------</td>
<td>-------------</td>
<td>---------------------------------------------------</td>
<td>---------------------------------------------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td>Optimal Controller, solution of Problem 1, tuned to initial condition forcing using $\int (y^2 + ru^2) dt$</td>
<td>$r = .1$</td>
<td>$r = 1.$</td>
<td>$r = .1$</td>
<td>$r = 1.$</td>
</tr>
<tr>
<td>Johnson's controller, solution of Problem 2, tuned to disturbance forcing using $\int y^2 dt$</td>
<td>$r = 3.68$</td>
<td>$4.37$</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Conventional Set Point, solution to Problem 3, tuned to set point forcing using $\int e^2 dt$</td>
<td>$r = 6.93$</td>
<td>$14.87$</td>
<td>$0.14$</td>
<td>$0.91$</td>
</tr>
<tr>
<td>Conventional Disturbance, solution to Problem 4, tuned to disturbance forcing using $\int y^2 dt$</td>
<td>$r = 12.40$</td>
<td>$39.73$</td>
<td>$1.94$</td>
<td>$2.68$</td>
</tr>
</tbody>
</table>

* Integral does not converge since no integral node is present.
** Set point scaled so that final value, $c(\infty)$, equals 1.0.
Literature Cited


CHAPTER III

DYNAMIC MODELING AND CONTROL OF DISTILLATION COLUMNS

The purpose of this chapter is to consider the dynamic modeling and control of distillation columns. The first part of this chapter defines the distillation control problem. In the second part a model for a distillation column is presented. A specific system is defined and used as an example to illustrate the theoretical concepts. Then feedback control of distillation columns will be considered and illustrated by using the system defined. Next feedforward control of distillation columns will be considered and the system defined will be used for illustration.

The Control Problem for a Distillation Column

The purpose of this section is to define the distillation control problem. Overall it may be simply stated that the distillation column should maximize profit. However, it is necessary to restate this in terms of the variables associated with a distillation column. The variables for a distillation column are defined in Figure 3.1. First it should be realized that there are three types of variables. These are controlled variables, manipulated variables and uncontrolled variables. It should be noted that there are necessarily the same number of manipulated and controlled variables; however, there may be many more uncontrolled variables.

In terms of Table 3.1 the distillation control problem may be stated as follows: maximize profit by maintaining the overhead
CONTINUOUS DISTILLATION COLUMN
Taken from Reference 1
FIGURE 3.1
### TABLE 3.1. Variables for a Distillation Column

<table>
<thead>
<tr>
<th>Controlled Variables</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Overhead Composition</td>
<td></td>
</tr>
<tr>
<td>Bottoms Composition</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Manipulated Variables</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflux or Distillate Rate</td>
<td></td>
</tr>
<tr>
<td>Boilup Rate</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Uncontrolled Variables</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed Rate</td>
<td></td>
</tr>
<tr>
<td>Feed Compositions</td>
<td></td>
</tr>
<tr>
<td>Feed Enthalpy</td>
<td></td>
</tr>
<tr>
<td>Steam Pressure</td>
<td></td>
</tr>
<tr>
<td>Coolant Rate and Enthalpy</td>
<td></td>
</tr>
<tr>
<td>Ambient Conditions</td>
<td></td>
</tr>
</tbody>
</table>
and bottoms compositions at their desired values by manipulating the reflux or distillate and boilup rates when the uncontrolled variables change.

Now let's simplify this problem. First the uncontrolled variables to be considered will be limited to just feed rate. Second, it has been well established that the bottoms composition can be maintained at its desired value by using a feed rate to boilup rate ratio controller. That is, the boilup rate is equal to a constant times the feed rate where the constant is determined from initial steady-state conditions. Finally in the statement of the control problem it was not specified whether the reflux or distillate rate should be manipulated to control the overhead composition. In order to determine whether distillate or reflux rate should be used various disturbances in feed rate with the distillate or reflux rate held constant were made for the system to be described in the next section. These results are shown in Table 3.2. It can be seen that the overhead and bottoms compositions (using ratio control) and the reflux ratio varied significantly for constant reflux rate, whereas little variation resulted for constant distillate rate. The variations were very nonlinear for constant reflux rate and nearly linear for distillate rate. Since the open loop response determines the nature of the closed loop response it was concluded that distillate rate should be used as the manipulated variable.

The control problem may be restated: maximize profit by controlling overhead composition at its desired value by manipulating
### TABLE 3.2. Comparison of Distillate and Reflux Rates for Use as a Manipulated Variable

<table>
<thead>
<tr>
<th>Feed Rate (lb moles/hr)</th>
<th>Constant Distillate Rate</th>
<th>Constant Reflux Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Overhead Composition</td>
<td>Bottoms Composition</td>
</tr>
<tr>
<td></td>
<td>(mole fraction)</td>
<td>(mole fraction)</td>
</tr>
<tr>
<td>85</td>
<td>0.78</td>
<td>0.016</td>
</tr>
<tr>
<td>90</td>
<td>0.82</td>
<td>0.019</td>
</tr>
<tr>
<td>95</td>
<td>0.81</td>
<td>0.024</td>
</tr>
<tr>
<td>100</td>
<td>0.893</td>
<td>0.031</td>
</tr>
<tr>
<td>105</td>
<td>0.92</td>
<td>0.042</td>
</tr>
<tr>
<td>110</td>
<td>0.93</td>
<td>0.055</td>
</tr>
<tr>
<td>120</td>
<td>0.95</td>
<td>0.082</td>
</tr>
</tbody>
</table>
Modeling a Distillation Column

In general there are two categories of mathematical models. These are theoretical models and experimental models. In addition each of these models may be of several different types. These include the following:

- stochastic versus deterministic
- linear versus nonlinear
- time varying versus time-invariant
- lumped versus distributed parameter
- steady-state versus dynamic

In this section a deterministic, nonlinear, time-invariant, lumped parameter, dynamic theoretical model of a distillation column will be developed. Then a deterministic, linear, time-invariant, lumped parameter, dynamic experimental model of a distillation column will be developed.

**Theoretical Model Development:** A model is only an approximation to the real world. Consequently it must have certain assumptions or limitations. The assumptions for the theoretical model development are summarized in Table 3.3. The theoretical model is developed by writing the unsteady state material balances, equilibrium relations and tray efficiencies for each tray and the reboiler and condenser. Table 3.4 summarizes this step.

Having completed the theoretical model development the next step is to specify a system which can be used for illustration. Table 3.5 summarizes the initial steady state conditions and
<table>
<thead>
<tr>
<th>Assumption</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Each plate, downcomer, reboiler and condenser are perfectly mixed pools.</td>
<td>1. Each plate, downcomer, reboiler and condenser are perfectly mixed pools.</td>
</tr>
<tr>
<td>2. The feed liquid falls on the plate below the feed point, and the vapor portion enters the plate above.</td>
<td>2. The feed liquid falls on the plate below the feed point, and the vapor portion enters the plate above.</td>
</tr>
<tr>
<td>3. No vapor-liquid contact occurs in the downcomer.</td>
<td>3. No vapor-liquid contact occurs in the downcomer.</td>
</tr>
<tr>
<td>4. The vapor leaving a tray is perfectly mixed before entering the tray above.</td>
<td>4. The vapor leaving a tray is perfectly mixed before entering the tray above.</td>
</tr>
<tr>
<td>5. A total condenser is used.</td>
<td>5. A total condenser is used.</td>
</tr>
<tr>
<td>6. The pressure in constant throughout the column.</td>
<td>6. The pressure in constant throughout the column.</td>
</tr>
<tr>
<td>7. Delay times in the lines to and from the condenser and reboiler are negligible.</td>
<td>7. Delay times in the lines to and from the condenser and reboiler are negligible.</td>
</tr>
<tr>
<td>8. No heat is lost to the surroundings.</td>
<td>8. No heat is lost to the surroundings.</td>
</tr>
</tbody>
</table>
TABLE 3.4. Theoretical Model Development—Equations

1. Feed Liquid and Vapor Compositions are obtained from:
   \[ x_{f1} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \]
   \[ y_{f1} = x_{f1} \alpha/(1 - x_{f1} + \alpha x_{f1}) \]
   where
   \[ a = L_f \alpha - L_f \]
   \[ b = V_f \alpha + L_f + F \cdot Z \cdot F \cdot Z \cdot \alpha \]
   \[ c = -F \cdot Z \]

2. Unsteady state material balance describing trays in the stripping and enriching sections:
   \[ L(x_{n+1} - x_n) + V(y_{n-1} - y_n) = \frac{dx_n}{dt} \]

3. Equilibrium relationship and tray efficiency for stripping and enriching sections:
   \[ y_n^* = x_n \alpha/(1 - x_n + \alpha x_n) \]
   \[ E_{OG} = (y_n - y_{n-1})/(y_n^* - y_{n-1}) \]

4. Component material balance on downcomer
   \[ L(x_n - x_n') = H_i \frac{dx_n}{dt} \]

5. Material balance on the reboiler:
   \[ L_s(x_{2n} - x_{1n}) + V_s(x_1 - y_1) = H_r \frac{dx_{1n}}{dt} \]

6. Unsteady state material balance for feed tray:
   \[ L_e(x_{nf+1} - x_{f1}) + L_s(x_{f1} - x_{nf}) + V_s(y_{nf} - y_{ft}) = H \frac{dx_{nf}}{dt} \]

7. Unsteady-state material balance for condenser:
   \[ V_e(y_{n-1} - x_n) = H_c \frac{dx_n}{dt} \]
### Table 3.5

**Initial Steady State Conditions**

*Taken from Reference 1*

**Specifications**

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed rate</td>
<td>100.0000 lb-moles/hr</td>
</tr>
<tr>
<td>Feed composition</td>
<td>0.35000 mole fraction</td>
</tr>
<tr>
<td>Feed enthalpy</td>
<td>50% vaporized</td>
</tr>
<tr>
<td>Distillate rate</td>
<td>36.94444 lb-moles/hr</td>
</tr>
<tr>
<td>Overhead composition</td>
<td>0.8930 mole fraction</td>
</tr>
<tr>
<td>Reflux ratio</td>
<td>4.11560</td>
</tr>
<tr>
<td>Feed tray</td>
<td>Stage 11</td>
</tr>
<tr>
<td>Total stages</td>
<td>21</td>
</tr>
<tr>
<td>Tray efficiencies</td>
<td>0.70000</td>
</tr>
<tr>
<td>Reboiler efficiency</td>
<td>1.00000</td>
</tr>
<tr>
<td>Relative volatility</td>
<td>2.00000</td>
</tr>
<tr>
<td>Tray hold-up</td>
<td>0.500 lb-moles</td>
</tr>
<tr>
<td>Downcomer hold-up</td>
<td>0.250 lb-moles</td>
</tr>
<tr>
<td>Reboiler hold-up</td>
<td>5.000 lb-moles</td>
</tr>
<tr>
<td>Condenser hold-up</td>
<td>10.00 lb-moles</td>
</tr>
</tbody>
</table>
column parameters. The theoretical model and initial conditions are based on reference I. The differential equations describing the column were solved by a relaxation technique on a digital computer.

**Experimental Model Development:** Experimental procedures for model development are based on the simple concept that a component, a control system, a process unit or any general item for which a mathematical model is desired can be obtained by input-output or cause and effect measurements. For those cases where a steady-state model is desired the techniques of statistical correlation are available.

The experimental model to be developed differs from the theoretical model in several ways. The theoretical model is based on material balances and equilibrium relationships. Whereas the experimental model will be based on input-output responses. The responses used will be based on perturbing the theoretical model whereas in practice they would be based on perturbing the real system. For any real process such tests might be very difficult to run and expensive. This along with the fact that the usefulness of these responses will be dependent on the range of the variables considered, on the form of the assumed model, and on how well the model fits the noisy data, makes a statistical approach almost mandatory. These limitations also make a theoretical development of the steady-state model, followed by carefully run experiments to verify it, a much more desirable approach than extensive testing followed by curve fitting.

These same comments also apply for obtaining experimental
dynamic models; however, it should be realized that the dynamic portion of a model need not be as accurate as the steady-state portion. An exception to this is the process which practically stays in a transient state. An experimental dynamic model is based on measurement of a system response to a forcing function or input. The type of forcing function is an important element in the development of the model; however, for our purposes it will be assumed that a step input is used. Figure 3.2 shows the open-loop response based on the theoretical model of the overhead composition to several step changes in distillate rate. Similarly, figure 3.3 shows the open-loop response of the overhead composition to several step changes in feed rate.

Having obtained the step responses the next items to be considered are the form of the model and the procedure used to determine or fit the parameters in the assumed model. The form of the model is generally a simple, linear transfer function. This may be expressed in Laplace transform notation as

\[ y_D(s) = G_1(s) D(s) + G_2(s) F(s) \]  

(3.1)

\( G_1(s) \) and \( G_2(s) \) could be defined in virtually any way; however, it is common practice to define them as one of the following:

- first order lag
- first order lag with dead time
- second order lag
- second order lag with dead time
Figure 3.2
Open Loop Responses to Several Step Changes in Distillate Rate.
Figure 3.3
Open Loop Responses to Several Step Changes in Feed Rate.
By inspecting figures 3.2 and 3.3 carefully it will be noted that these are essentially first order lag responses. Therefore the form of the experimental model will be a first order lag which may be expressed as

\[
G_1(s) = \frac{k_D}{\tau_D s + 1} = \frac{\gamma_D(s)}{D(s)} \quad (3.2)
\]

\[
G_2(s) = \frac{k_F}{\tau_F s + 1} = \frac{\gamma_D(s)}{F(s)} \quad (3.3)
\]

The next step is to determine the gains and time constants for \( G_1(s) \) and \( G_2(s) \) from the step responses shown in Figures 3.2 and 3.3. The gains are determined by the following relations:

\[
k_D = \frac{\partial \gamma_D}{\partial D} = \frac{\gamma_D(t_f) - \gamma_D(0)}{D(t_f) - D(0)} \quad (3.4)
\]

\[
k_F = \frac{\partial \gamma_D}{\partial F} = \frac{\gamma_D(f_f) - \gamma_D(0)}{F(t_f) - F(0)} \quad (3.5)
\]

The time constant for a first order lag step response is the time at which the output reached 63.2% of its final value. Using this approach and the equations above, table 3.6 summarizes the gains and time constants for the responses presented. Since the theoretical model is nonlinear, the time constants and gains are a function of the distillate and feed rates. In order to obtain a linear experimental model the gains and time constants were simply averaged. The overall result is the following experimental model:

\[
\gamma_D(s) = -0.0140 \frac{D(s)}{(0.55s + 1)} + 0.0056 \frac{F(s)}{(0.55s + 1)} \quad (3.6)
\]
### TABLE 3.6. Gains and Time Constants for Experimental Model

#### Transfer Function $G_1(s)$

<table>
<thead>
<tr>
<th>Percent Change in Distillate Rate</th>
<th>Gain $k_D$ (mole fraction/lb moles/hr)</th>
<th>Time Constant $\tau_D$ (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+20</td>
<td>-0.0169</td>
<td>0.64</td>
</tr>
<tr>
<td>+5</td>
<td>-0.0163</td>
<td>0.57</td>
</tr>
<tr>
<td>-5</td>
<td>-0.0141</td>
<td>0.54</td>
</tr>
<tr>
<td>-20</td>
<td>-0.0087</td>
<td>0.43</td>
</tr>
</tbody>
</table>

#### Transfer Function $G_2(s)$

<table>
<thead>
<tr>
<th>Percent Change in Feed Rate</th>
<th>Gain $k_F$ (mole fraction/lb moles/hr)</th>
<th>Time Constant $\tau_F$ (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+20</td>
<td>0.0030</td>
<td>0.66</td>
</tr>
<tr>
<td>+5</td>
<td>0.0060</td>
<td>0.57</td>
</tr>
<tr>
<td>-5</td>
<td>0.0060</td>
<td>0.54</td>
</tr>
<tr>
<td>-20</td>
<td>0.0075</td>
<td>0.43</td>
</tr>
</tbody>
</table>
Feedback Control of a Distillation Column

In stating the control problem for a distillation column there were three elements to it. These were the cost, the desired objectives and the model (or system). Let's now look at feedback control in light of these elements. Feedback control is based upon measuring the controlled variable, comparing this to its desired value and then eliminating the error by changing the manipulated variable. The important point is that an error must occur, so that the manipulated variable can change only after the controlled variable has deviated from its desired value - i.e., we cannot have perfect control.

The first element mentioned above is the cost. It was stated earlier that it was desirable to maximize profit or minimize costs. It is generally difficult to formulate a profit criterion function, so that an alternate criterion must be developed. There are many possible criterion functions; however, the most popular one is the quadratic cost integral. A quadratic cost integral which may be used for the distillation control problem is given by

\[ J(D) = \int_{0}^{\infty} [q(y_D - y_{D_{set}})^2 + rD^2] dt \]  (3.7)

The cost \( J(D) \) consists of two parts. The first part \( (y_D - y_{D_{set}})^2 \) penalizes for deviations of the controlled variable \( y_D \) from its desired value \( y_{D_{set}} \). The second part \( \dot{D}^2 \) penalizes for changes in the manipulated variable \( D \). The reason \( \dot{D} \) is used instead of \( D \) is that it's presence will provide an integral mode in the controller. The terms \( r \) and \( q \) are weighting constants.
The next element of the control problem to be considered is the model. The model given by equation 3.6 is in Laplace transform notation; however, optimal control theory which will be used to determine the feedback controller is based on state-variable notation. To express equation 3.6 in state-variable form it is necessary to replace \( s \) by \( \frac{d}{dt} \). The result of doing this to equation 3.6 is

\[
\dot{y}_D(t) = \frac{-1}{0.55} y(t) + \frac{-0.0140}{0.55} D(t) + \frac{0.0056}{0.55} F(t) + c_1
\]  

(3.8)

where

\[
c_1 = \frac{y_{D\text{ set}}}{0.55} + \frac{0.0140}{0.55} D_{ss} - \frac{0.0056}{0.55} F_{ss} = 1.54
\]

Now we can apply optimal control theory. The problem is given the system described by equation 3.8 determine the optimal feedback controller which will minimize the performance criterion given by equation 3.7. This problem has been solved. (Ref. 2)

The first step is to define the new state variable

\[
x_1(t) = y_{D\text{ set}} - y_D(t)
\]

(3.9)

Equation 3.8 may be written in terms of \( x_1(t) \)

\[
\dot{x}_1(t) = \frac{-1}{0.55} x_1(t) + \frac{0.0140}{0.55} D(t) - \frac{0.0056}{0.55} F(t) + \frac{y_{D\text{ set}}}{0.55} + c_1
\]

(3.10)

Next the second state variable \( x_2(t) \) is defined

\[
x_2(t) = \frac{0.0140}{0.55} D(t) - \frac{0.0056}{0.55} F + \frac{y_{D\text{ set}}}{0.55} + c_1
\]

(3.11)
where it should be noted that $F$ is a constant, i.e., constant disturbances will be considered such as step changes. Taking the derivative of $x_2(t)$ yields

$$x_2(t) = \frac{0.0140}{0.55} d(t) = v(t) \quad (3.12)$$

The state equation may be rewritten in matrix form

$$\dot{x}(t) = A \, x(t) + f \, v(t) \quad (3.13)$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad f = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$A = \begin{bmatrix} -1 & 1 \\ 0.55 & 1 \\ 0 & 0 \end{bmatrix}$$

The determination of the optimal feedback controller is a functional optimization problem which may be solved by the minimum principle. The result of applying the minimum principle to this problem is

$$v(x) = \langle \gamma, x \rangle \quad (3.14)$$

where

$$\gamma = -r^{-1} M f \quad (3.15)$$

and $M$ is the unique constant positive definite symmetric matrix satisfying

$$A'M + MA - r^{-1} Mff'M + Q = 0 \quad (3.16)$$
where
\[
G = \begin{bmatrix}
1 & 0 \\
0 & 0
\end{bmatrix}
\]

This equation is called the matrix Riccati equation.

For this problem equation 3.16 reduces to the three following nonlinear algebraic equations in \(m_1, m_2\) and \(m_3\):

\[
\frac{-2}{0.55} m_1 \frac{-m_2^2}{r} + 1 = 0 \tag{3.17}
\]

\[
m_1 \frac{-m_2^2}{0.55} - \frac{m_2 m_3}{r} = 0 \tag{3.18}
\]

\[
2m_2 \frac{-m_3^2}{r} = 0 \tag{3.19}
\]

Carrying out the indicated multiplication in equation 3.14, the result is

\[
v = -m_2 x_1 - m_3 x_2 \tag{3.20}
\]

Recalling the definition of \(v\) and using equation 3.13 to eliminate \(x_2\), yields

\[
\frac{-0.0140}{0.55} \dot{D} = -m_2 x_1 - m_3 \left(\frac{x_1}{0.55} + \dot{x}_1\right) \tag{3.21}
\]

Finally collecting terms and integrating yields

\[
D = k_1 \int_{0}^{t} (y_D - y_D) d\tau + k_2 (y_D - y_D) + D_0 \tag{3.22}
\]

where

\[
k_1 = \frac{-(m_2 + m_3/0.55) \times 0.55}{0.0140 r}
\]

\[
k_2 = \frac{-m_3 \times 0.55}{0.0140 r}
\]
This is the desired result and it is seen that the optimal controller is the familiar proportional plus integral feedback controller.

The next step is to determine the constants $k_1$ and $k_2$. To do this $r$ must be selected and then $m_2$ and $m_3$ solved by equations 3.17 to 3.19. The selection of $r$ is somewhat difficult; however, suppose it is desired that errors be weighted about ten times as much as changes in distillate rate. Therefore

$$\frac{rD^2}{(y_D - y_{D_{set}})^2} = 0.1 \quad (3.23)$$

Since the magnitude of $D^2$ is approximately 1600 and the magnitude of $(y_D - y_{D_{set}})^2$ is approximately $10^{-4}$ a reasonable value of $r$ is $10^{-8}$.

The effect of various values of $r$ is shown in figure 3.4. It is seen that for small $r$ large changes in $D$ result with small errors and for large $r$ small changes in $D$ result with larger errors. It is seen that an $r$ of 1.0 is unsatisfactory. For our purposes an $r$ of $10^{-4}$ will be used. Figure 3.5 shows the overhead composition and distillate rate for various feed rate disturbances.

Before concluding this section on feedback control it should be pointed out that the optimal controller was a simple proportional plus integral controller because of the first order model. If a second order model had been used then a proportional integral derivative controller would have been optimal.

In practice a dead time will be present in the experimental model. To treat this a Smith predictor may be used to eliminate its effect. (Reference 3) The feedback controller developed here
Figure 3.4A
Closed Loop Feedback Responses for Several Values of $r$. 

Overhead Composition $\times 10^{-1}$
Figure 3.46
Distillate Rate Response for Several Values of $r$. 

$r = 10^{-8}, k_1 = -3.9 \times 10^5, k_2 = -5.5 \times 10^3$

$r = 10^{-4}, k_1 = -3918, k_2 = -488$

$r = 1, k_1 = -49.95, k_2 = -63.02$
Figure 3.5

Closed Loop Feedback Responses for Several Feed Rate Disturbances.
would be used in conjunction with the Smith predictor.

**Feedforward Control of a Distillation Column**

Feedforward control is based upon measuring an uncontrolled variable and changing the manipulated variable such that the controlled variable remains at its desired value. Unlike feedback control it is theoretically possible to obtain perfect control with feedforward action. In practice this is difficult because of modeling errors, equipment inaccuracies, etc.

With feedforward control our criterion function is that we keep the controlled variable at its desired value. We desire to find the relationship between $D$ and $F$ which will deep $y_D$ at $y_D^{\text{set}}$. The feedforward controller will be based on the experimental model given by equation 3.8. Attempts to determine the feedforward controller based on a theoretical model has been considered with the result that it is not possible. (Ref. 3) To determine the feedforward controller equation 3.8 is solved for $D$ with $\dot{y}_D(t)$ equal to 0 and $y_D(t)$ equal to $y_D^{\text{set}}$. This yields

$$D = 0.4 F - 3.3$$

(3.24)

It should be noted that this is a linear steady-state feedforward controller. A linear dynamic feedforward controller with lead-lag compensation would have resulted if the average time constants $\tau_F$ and $\tau_D$ had not been equal. Figure 3.6 shows the overhead composition for various feed rate disturbances using the feedforward controller. It will be noted that the overhead composition has a slight offset at steady state. This results from the
feedforward controller being based on an imperfect model. This is a disadvantage of feedforward controllers and consequently they are usually used in conjunction with a feedback controller. In comparing the feedback control of Figure 3.5 to the feedforward control of Figure 3.6 it is seen that the feedback is better. Figure 3.7 shows the overhead composition for combined feedback-feedforward control. This combination is best and it is what is typically used.

**Conclusions**

The purpose of this chapter has been to illustrate the dynamic modeling and control of a distillation column. First the distillation control problem was stated and a simplified version considered. Then theoretical and experimental models were developed for a particular distillation column. Next feedback control of this distillation column was considered. The optimal control theory was presented with the resulting controller being a proportional plus integral controller. The responses for various feed rate disturbances were illustrated. Finally feedforward control of this distillation column was presented and responses for various feed rate disturbances were illustrated. Straight feedback control performed better than straight feedforward control because it does not need as accurate of a model. However, combined feedback-feedforward gave the best overall response.
Figure 3.6

Feedforward Controller Responses for Several Feed Rate Disturbances.
Figure 3.7
Combination Feedforward-Feedback Controller Responses for Several Feed Rate Disturbances.
Nomenclature

A  Coefficient matrix for system
C1  Constant in system equation
D  Distillate flow rate, lb-moles/hr
D.  Derivative of distillate rate with respect to time, lb-moles/hr²
D₀  Initial steady state value of D
E_{OG}  Murphree vapor phase stage efficiency
F  Feed rate, lb moles/hr
f  Coefficient matrix for feedback controller
G₁  Transfer function for overhead composition to distillate rate
G₂  Transfer function for overhead composition to feed rate
H  Liquid hold up
H_c  Liquid hold up in condenser
H_r  Liquid hold up in reboiler
H'  Liquid hold up in downcomer
J(D)  Performance Index for feedback controller
k_D  Gain for distillate rate transfer function
k_F  Gain for feed rate transfer function
k_I  Integral mode gain
k_2  Proportional mode gain
L  Internal liquid flow rate, lb moles/hr
L_e  Enriching section liquid flow rate, lb moles/hr
L_s  Stripping section liquid flow rate, lb moles/hr
M  Riccati matrix
m₁  1, 1 element of M matrix
m₂  1, 2 and 2, 1 element of M matrix
$m_3$ 2,2 element of $M$ matrix
$Q$ Coefficient matrix in Riccati equation
$q$ Weighting constant in performance function
$r$ Weighting constant in performance function
$s$ Laplace transform variable
$V$ Internal vapor flow rate, lb moles/hr
$V_s$ Stripping section vapor flow rate, lb moles/hr
$v$ Controller function
$x_1$ First state variable
$x_2$ Second state variable
$x_{fl}$ Composition of feed liquid, mole fraction
$x_n$ Liquid composition of tray $n$, mole fraction
$x_{\text{downcomer}}$ Liquid composition of downcomer $n$, mole fraction
$y_D$ Overhead vapor composition, mole fraction
$y_D^\text{set}$ Initial steady state value of overhead
$y_{fl}$ Composition of feed vapor, mole fraction
$y_n$ Vapor composition of tray $n$, mole fraction
$y_n^*$ Equilibrium vapor composition, mole fraction
$\alpha$ Relative volatility
$\gamma$ Controller constant matrix
$\tau_D$ Distillate rate time constant, hours
$\tau_F$ Feed rate time constant, hours
Literature Cited


The purpose of this dissertation has been to consider the application of feedback and feedforward control to chemical processes. This chapter will not go over any of the results, but instead it will consider some extensions to the work presented.

The first topic dealt with the tuning of feedback controllers. Other work along these lines might include the effect of using a second order plus dead time model instead of the first order plus dead time model. Also the effect of including a term in the integral criterion functions for penalizing the control or rate of change of the control. This would have to include the effect of the weighting constant $r$.

The second topic dealt with applying feedforward control. Other work could be done on linearized feedforward controllers for reactor systems. Also the determination of optimal switching times for steady-state feedforward controllers might be considered. Finally, the effect of constraints on the manipulated variable on the feedforward control action could be investigated.

The third topic considered the use of search techniques to determine optimal switching times. Further work could consider using search techniques on other performance criteria such as minimum energy and integral criteria subject to magnitude constraints on the manipulated variable. Other work could include
the use of search techniques for determining controller parameters where no analytical theory is available such as for profit or nonlinear criterion functions and nonlinear controllers or systems. The search technique could also be used to solve for the optimal control function by considering it to consist of many points or parameters. This would be a very high dimensional search and would require a hybrid computer.

The fourth topic considered a comparison of optimal and suboptimal controllers. Much work could be done in this area as mentioned in the above paragraph. Also multivariable systems could be considered.

The final topic considered the dynamic modeling and control of a distillation column. Similar studies could be made on other types of equipment such as heat exchangers, evaporators and reactors. Much work has been done on the modeling of distillation columns, however, much work remains. The effect of dead time could be studied and the Smith predictor applied to this case.
APPENDIX A
A Comparison of Controller

J. A. MILLER, A. M. LOPEZ, C. L. SMITH, and P. W. MURRILL, Louisiana State University

Here is a clear, concise comparison of techniques for tuning controllers, and a set of conclusions that explain why one of the procedures appears superior to the others. The authors begin with an investigation of two methods for approximating the process reaction curve and demonstrate the one that is better. Using this method they develop tuning relations by applying several techniques. The resultant graphical comparisons document their choice of the best technique.

From among the many techniques for adjusting controllers based on the open loop process response, four have been chosen for purposes of comparison (Ref. 1-4). All are based on the process reaction curve discussed below. The tuning parameters are proportional gain \( K_p \), reset time \( T_r \), and rate time \( T_i \). They are derived from the assumption of an ideal controller characterized by the algorithm:

\[
\dot{n}(t) = K_p [r(t) + \int r(t) dt + T_i \frac{\dot{r}(t)}{dt}]
\]

where \( n(t) \) is the controller output signal, and \( r(t) \) is the error signal.

Process reaction curve

A process reaction curve is the process response to a unit step change in the manipulated variable (controller output). Many industrial processes can be adequately approximated by a relatively simple mathematical model consisting of a first order lag with a dead time:

\[
\text{Output} = \text{Input} \cdot K_p \exp \left( \frac{t}{T} \right) \quad (r(s) = 1)
\]

Figure 1 illustrates two approximation methods. In each method a tangent is drawn to the process reaction curve at its point of steepest ascent. Extreme care must be exercised to select this point accurately. Then reaction rate, \( R \), (tangent slope), process gain \( K \), and the time delay \( L \), are determined.

The two methods differ in their manner of obtaining first order lag time. For Fit 1 the time constant \( T_r = K_p R \), and for Fit 2 it is \( T = T_r - L \). Since \( L \) is the dead time in Figure 1, \( T_r = L \), and \( T = T_r - L \).

Figure 2 presents a comparison of these two fits where the actual process is taken to be a second order lag plus dead time. It proves to be a much better approximation.

Going from open- to closed-loop comparison of fit methods, Figure 3 confirms the superiority of Fit 2. Two sets of plots are required since different controller settings were obtained for Fits 1 and 2 because of their different first-order time constants.

It is concluded that Fit 2 should be used in comparing controller tuning techniques.

Open-loop tuning relations

The purpose of open-loop tuning techniques is to determine values of tuning parameters (\( K_p \), \( T_r \), and \( T_i \)).
Tuning Techniques

The procedure for developing tuning relations has been to establish a criterion for optimal control, then determine values of tuning parameters that will satisfy this criterion for a given combination of process parameters.

The Ziegler-Nichols criterion for optimal control (Ref. 1) was that the response of the controlled process to a unit step change in disturbance should have a 1/4 decay ratio. Figure 4. A similar criterion was developed by Cohen and Coon, Ref. 2. The difference was in the presentation of the terms \( L_n, R_n, \) and \( K. \) An index of self-regulation was introduced, defined as \( u = R_L/R. \) The resulting model was more complex than Ziegler and Nichols'.

A third set of tuning relations took advantage of the simplicity of the 1/4 decay ratio criterion, and overcame its disadvantages by adding three constraints; hence, its designation 3C. These disadvantages were the inability of 1/4 decay ratio criterion to determine unique values of tuning parameters for two and three mode controllers, and the inability to characterize an entire closed loop response. (Ref. 4).

Tuning relations designated 3C are expressed as dead time \( T_0, \) first order time constant \( T, \) and gain \( K. \) They are related by \( L_n, R_n, \) and \( u \)

\[
L_n = \frac{T_0}{K} + \frac{T}{K} = \frac{T_0}{R} + \frac{1}{R}. \quad (3)
\]

Originally presented in graphical form, 3C relations are least squares approximated very well by equations of the form

\[
K = \frac{a_1 T_0 + a_2 T + a_3}{b_1 T_t + b_2 T + b_3}
\]

Table. Equations for six sets of tuning relations

<table>
<thead>
<tr>
<th>Coher (from tuning relations)</th>
<th>( K = \frac{a_1 T_0 + a_2 T + a_3}{b_1 T_t + b_2 T + b_3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a_1 )</td>
</tr>
<tr>
<td>FC</td>
<td>1.1</td>
</tr>
<tr>
<td>FC</td>
<td>1.1</td>
</tr>
<tr>
<td>FC</td>
<td>1.1</td>
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<td>FC</td>
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<td>FC</td>
<td>1.1</td>
</tr>
<tr>
<td>FC</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Fig. 3. The superiority of Fit 2 is confirmed in a closed-loop comparison. Two sets of plots are required since different controller settings were obtained for Fits 1 and 2 because of their different time constants.
for optimum control.

to time or ratio estimating where
for limitation in the time that is available to
may be reduced by a single number which is the error integral is (Ref. 3). An entire closed-loop response can be expressed by a single number which is a direct measure of the extent to which feedback eliminates error in the controlled variable. These integral criteria may take three forms:

IAF (minimum integral of absolute error) = \int_{t_0}^{t} |e(t)| dt (5)

ISE (minimum integral of squared error) = \int_{t_0}^{t} e^2(t) dt (6)

ITA (minimum integral of absolute error multiplied by time) = \int_{t_0}^{t} |e(t)| dt (7)

Comparison based on open-loop, and closed-loop responses

The four approaches to tuning controllers actually yield six sets of tuning relations, since the fourth approach expands as indicated by equations 5, 6, and 7. Analytical comparison of the six methods for P, PI, and PID configurations is presented in the Table. Plotting these equations yield notable variation in K_c from one tuning method to another. But the variations in tuning parameters were generally not sufficient for positive indications of which tuning relation was best. For this reason the comparison was made on the basis of closed-loop response.

Computer simulation was used to calculate closed-loop response to a unit step change in disturbance for a first order lag plus dead time process with a PI or PID controller. The simulation determined values of the four criteria (decay ratio, IAE, ISE, and ITAE) for several values of normalized process parameter \(\theta_d/\tau\). This was done for each value of \(\theta_d/\tau\) with a proportional plus reset and then a three-mode controller, each being tuned by all six tuning relations. The results obtained on an x-y plotter are presented in Figures 5 through 8.

Figure 5 indicates that 1/4 decay ratio is not only a poor criterion for optimum control, but also does not indicate which tuning method is best. Figures 6 through 8 on the contrary, indicate that integral criteria are good criteria, and identify the best method. They further indicate that the best integral tuning relation, providing optimum control, corresponds to its particular integral. For example, the IAE-PID tuning relation minimizes IAE, while the ISE-PID tuning relation minimizes ISE. Since the Ziegler-Nichols, Cohen-Coon, and 3C relations do not minimize integral criteria, the question of which technique is best reduces to a choice among IAE, ISE, and ITAE.

Figures 6 through 8 also indicate important points about the controller to be used for a particular process. The curves show that for values of \(\theta_d/\tau\) less than 0.4 a PI or PID configuration gives essentially the same criterion value. Therefore a PID controller does not offer much advantage over a PI controller, except where \(\theta_d/\tau\) is greater than 0.4. In this case, however, the PID controller should be tuned with an integral tuning relation to obtain full benefit from the derivative mode. For example, a PID controller tuned with the Ziegler-Nichols, Cohen-Coon, or 3C relations results in a higher ITAE than a PI controller.
tuned with the HIAF relations. In general the full advantage of a PID controller is only realized when it is HIAF tuned and \( \theta \) is greater than 0.4.

Two specific cases are plotted in Figures 9 and 10 to illustrate the effect of \( \theta \) varying above and below the value of 0.4. Figure 9 shows that for \( \theta \geq 0.4 \), controller type and tuning is not too important, so a PID controller offers no great advantage. Figure 10 shows that for \( \theta < 0.4 \), the HIAF tuned controller provides a much better response than either PI controller. Note also that the HIAF tuned PI controller has a better response than the Zeigler-Nichols tuned PID controller.

It is concluded that a controller tuning method that utilizes integral criteria is superior to other methods that do not utilize such criteria. It is further concluded that of the three variations of integral criteria, the best one is HIAF, or the minimum integral of error - the integration taking place after the error function has been subjected to the weighting factor of time.

REFERENCES

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DECEMBER 1967
How to Apply Feedforward Control

An example shows how to formulate a feedforward control system. The advantages and disadvantages of feedforward control are compared with feedback control.

John A. Miller, Paul W. Murrell and Cecil L. Smith

Louisiana State University, Baton Rouge, La.

Feedforward control is an advanced technique which is becoming increasingly more important as the demand for better control grows. A feedforward system is one that predicts the manipulative inputs which will keep the controlled variables at their desired values when measured disturbance inputs enter the process. Compare this type of control with the conventional feedback control which is based on measuring the controlled variables, comparing these measured values with their desired values and using any difference between them to manipulate inputs that will eliminate these differences.

This feedback control is based on measurement of the controlled variables while feedforward control is based on measurement of disturbances. These two types of control differ in their use of the manipulative inputs. Feedback control uses the manipulative inputs after the controlled variables have deviated from their desired values. Feedforward control uses the manipulative inputs before

Fig. 1—A feedforward and feedback control system with solid lines indicating automatic information flow and dashed lines showing possible manual adjustment.
the controlled variables have deviated from their desired values. Fig. 1 illustrates these ideas.

It is unlikely that feedforward control will replace feedback control. The use of feedback control with feedforward control will be necessary for several reasons. First, the feedforward control will be limited by mathematical inaccuracies may cause the feedback controlled variable to have a steady-state offset which can be eliminated with conventional feedback reset control. Second, the feedforward controller cannot compensate for unmeasured disturbances while the feedback controller can. It is seen that feedforward and feedback control make a very natural combination, each compensating for the other's shortcomings.

DEVELOPING A FEEDFORWARD MODEL

The formulation of feedforward control (FFC) using a process model will be illustrated for the system shown in Fig. 2. Any hydrocarbon processing unit would be approached in exactly the same way, although the mathematics might be more complex.

Different types of feedforward controllers result from the different types of process models. For the system shown in Fig. 2, Table 1 summarizes the dynamic process model. The steady-state process model is obtained from the dynamic model by setting the time derivative, \( \dot{V} \), to zero. The steady-state material and energy balances will be used to develop nonlinear steady-state feedforward controllers and the unsteady-state models will be used to develop nonlinear dynamic feedforward controllers. Linear feedforward controllers will be obtained by linearizing the nonlinear feedforward controllers about the initial steady-state values.

Two cases with various combinations of manipulated, controlled, and disturbance variables will be considered to illustrate different aspects of feedforward control. In each case the first step will be to carefully define the controlled, manipulated, and measured disturbance variables. In practice this step might result from having tried conventional feedback control. The second step for each case will be to design four feedforward controllers. The important point to be noted is that the steady-state feedforward controllers are usually much simpler and easier to design than the dynamic feedforward controllers.

![Diagram of a flow system for feedforward control](image)

**Fig. 2—An example flow system for feedforward control.**

**Table 1—Process Model for System in Fig. 2**

<table>
<thead>
<tr>
<th>Constants</th>
<th>( C_{\text{out}} = 0.25 \text{ lb, A/gal} )</th>
<th>( c_D = 0.61 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 = 2.5 \text{ lb, A/gal} )</td>
<td>( A_{\text{in}} = 0.46 \text{ sq ft} )</td>
<td>( 1.53 \text{ sq ft} )</td>
</tr>
<tr>
<td>( T_{\text{in}} = 115^\circ \text{F} )</td>
<td>( A_{\text{out}} = 300 \text{ sq ft} )</td>
<td>( 1003 \text{ sq ft} )</td>
</tr>
<tr>
<td>( T_{\text{a}} = T_{\text{d}} = 100^\circ \text{F} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Assumptions**

Perfect mixing in Tanks I and II.

Negligible heat losses to the surroundings.

Negligible heat accumulation in tank walls.

All heat capacities are equal and independent of temperature.

Accumulation equals flow in minus flow out.

**Over-all Material Balance**

\[ Q_1 + Q_2 - Q_3 + V = 0 \]  

\[ Q_1 + Q_2 + Q_3 - Q_0 = \rho V \]  

**Component A Material Balance**

\[ Q_1 C_1 + Q_2 C_2 = Q_1 C_{\text{in}} = \rho V C \]  

**Energy Balance**

\[ Q_1 (T_1 - T_0) + Q_2 (T_2 - T_0) = \rho V (T_{\text{in}} - T_{\text{d}}) \]  

**Orifice Equations**

\[ Q = c_D A_{\text{in}} \left( \frac{2 g H_1}{\rho} \right)^{1/2} \]  

\[ Q_3 = c_D A_{\text{out}} \left( \frac{2 g H_2}{\rho} \right)^{1/2} \]  

**Literature Survey on Feedforward Control**

Most process control textbooks devote very little, if any, attention to feedforward control. More recent textbooks, however, do give much more attention to feedforward control (FFC). Most of the work done on feedforward control has appeared in journals. Discussion of all of these articles will not be undertaken since this is a classification of some representative articles is given in the accompanying table for further reference. In regard to classifications not covered in this table: FFC of heat exchangers, pH systems, absorption columns, evaporators and dryers are discussed in one reference. FFC of a cement plant is discussed in another reference and adaptive FFC is considered in another.

Although there are 42 references on FFC listed in the bibliography, only five of these describe applications of FFC to plant size equipment! It is believed that FFC will be applied more as digital computers are used for process control.
Case I: Feedforward Control of Composition C

This case will illustrate the importance of the physical location of the manipulated and disturbance inputs relative to the location of the controlled output. The variables are defined below:

- Controlled Variable: \( C_o = C_{oi} = C_{out} \)
- Manipulated Variable: \( Q_o(t) \)
- Measured Disturbances: \( Q_i(t) \) and \( C_i(t) \)
- Additional Constants: \( Q_{ii} \)

For Case I the feedforward controller must predict the value of \( Q_o(t) \) which will maintain \( C_o \) constant at \( C_{oi} \) when the measured disturbances \( Q_i(t) \) and \( C_i(t) \) enter the process. This may be expressed as:

\[
Q_o(t) = f_i [C_{oi}, Q_i(t), C_i(t)]
\]  

(9)

Let us begin by developing the nonlinear steady-state feedforward controller. The steady-state component \( A \) material balance for Tank II, Equation 4 with \( p = O \), may be solved for \( Q_o(t) \) with \( C_o = C_{oi} \):

\[
Q_o(t) = |Q_i(t)C_{ii} - Q_i(t)C_i(t) - Q_HC_{ii}/C_{oi}|
\]  

(10)

Since the term \( Q_o(t) \) is an output it must be eliminated from Equation 10 by using the Tank II overall material balance, Equation 2 with \( P = O \), to give the desired \( f_i \):

\[
Q_o(t) = Q_i(t)[C_{ii} - C_i(t)]/K_i + Q_H[C_{ii} - C_{ii}]
\]

or

\[
Q_o = Q_i(t)[C_{ii} - C_i(t)] + K_i
\]  

(11)

This controller is nonlinear because of the term \( Q_i(t)C_i(t) \); however, if \( Q_i, C_i(t) \) could be measured instead of \( Q_i(t) \) and \( C_i(t) \) separately, then it would be linear.

The corresponding linear steady-state feedforward controller can be obtained by either linearizing Equation 11 about the steady-state values, or by linearizing the steady-state component material balance and then solving for \( Q_o(t) \). The result is:

\[
Q_o(t) = Q_i(t)[C_{ii} - C_i(t)]/K_i + Q_H[C_{ii} - C_{ii}]
\]

or

\[
Q_o(t) = [Q_i(t)K_i - K_i][C_{ii} - C_{ii}] + K_i
\]  

(12)

This feedforward controller is simply a ratio controller.

The dynamic feedforward controller will be considered next. The same procedure for determining \( f_i \) may be used, so the nonlinear dynamic feedforward controller is determined by solving Equations 2 and 4 for \( Q_o(t) \) and the linear dynamic controller by linearizing the resulting controller. In doing this it should be noted \( dC_o/dt = dC_{oi}/dO = 0 \). The resulting dynamic controllers turn out to be same as the steady-state controllers, i.e., the nonlinear steady-state and dynamic are the same and the linear steady-state and dynamic are the same. The reason for this is that the manipulated input \( Q_o(t) \) and the disturbance inputs are mathematically located in the same place relative to the location of the controlled output \( C_o \).

The feedforward controllers given by Equations 11 and 12 are implemented as shown in Fig. 3 and the responses obtained with the nonlinear and linear feedforward controllers to conventional feedback control are shown in Fig. 4. It is seen that the responses for feedback and feedforward control are very different due to their different approach. Also the nonlinear feedforward controller gives perfect control because it is based on an "exact" model of the process.

For any real process this control would not apply because the process could not be modeled exactly. The linear feedforward causes a steady-state offset and the more severe the disturbances, the more offset the linear controller will exhibit. The decision to apply feedforward control should be based on how much profit each con-
controller gives in terms of $C_v$, minus the cost of designing and installing that controller. By evaluating the actual feedback system and then using simulation to check feedforward control, an accurate measure of payout could be obtained.

To apply the feedforward controller equation that has been developed, Fig. 3 shows the following steps would be used:

Step (1): The disturbances would be measured and converted to analog signals.

Step (2): The value of $Q_4(t)$ would be calculated based on Equation 11 or 12 and converted to a pneumatic signal.

Step (3): A control valve would be positioned.

The first step can be done on the order of milliseconds for a flow rate disturbance, but a composition disturbance may require anywhere from milliseconds to twenty or thirty minutes if the composition measurement takes a length of time comparable to the length of time which it takes for the composition to be passed through the process. Then the feedforward control action may be somewhat defeated. Depending on where the composition measurement is made, Step (2), the calculation and conversion to signal form of the manipulated variable can also be done on the order of milliseconds.

<table>
<thead>
<tr>
<th>Table 2—Feedforward Controllers for Case II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonlinear FFC</td>
</tr>
<tr>
<td>$Q_4: Q_4 (C_4, C_1 + T_a, T_1, K_4) / K_4$</td>
</tr>
<tr>
<td>$Q_2: Q_2 (C_2, C_{11}/K_2 + T_a - T_1, K_2) / K_2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dynamic</th>
<th>Steady-State</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_4$</td>
<td>$S_1/d_1$</td>
</tr>
<tr>
<td>$C_1$</td>
<td>$S_1/S_1$</td>
</tr>
<tr>
<td>$T_1$</td>
<td>$S_1/d_1 + T_R$</td>
</tr>
</tbody>
</table>

| Linear FFC                              |
| $Q_4: (Q_4, K_4) C_4 (C_1 - C_11 + (T_a - T_1), K_4) / K_4$ |
| $Q_2: (Q_2, K_2) C_2 (C_11, K_2 + (T_a - T_1), K_2) / K_2$ |

<table>
<thead>
<tr>
<th>Dynamic</th>
<th>Steady-State</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_4$</td>
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<td>$C_1$</td>
<td>$S_1/S_1$</td>
</tr>
<tr>
<td>$T_1$</td>
<td>$S_1/d_1 + T_R$</td>
</tr>
</tbody>
</table>

Other Equations

$S_1 = Q_4 + Q_2$
$S_1 = Q_4 + Q_1 + Q_2$
$S_1 = (C_4, C_1, T_a, T_1, K_4) / K_4$
$S_1 = (C_2, C_{11}, T_a, K_2) / K_2$
$S_1 = (C_4, C_1, T_a, T_1) / K_4$
$S_1 = (C_2, C_{11}, T_a, K_2) / K_2$
$S_1 = (C_4, C_1, T_a, T_1, K_4) / K_4$
$S_1 = (C_2, C_{11}, T_a, K_2) / K_2$
$S_1 = (C_4, C_1, T_a, T_1, K_4) / K_4$
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$S_1 = (C_2, C_{11}, T_a, K_2) / K_2$
$S_1 = (C_4, C_1, T_a, T_1, K_4) / K_4$
$S_1 = (C_2, C_{11}, T_a, K_2) / K_2$

Feedforward control it may be seconds because of the sampling time. Step (3), the valve positioning will require on the order of one to ten seconds depending mainly on valve size. Since most feedforward control applications are on processes with "time constants" in the minutes to hours range, the effect of Step (3) is usually negligible.

For the purposes of this article the time associated with all three steps will be assumed negligible.

Let us now consider the effect where the disturbance is measured and the manipulated variable is changed. Suppose that the disturbances $Q_4(t)$ and $C_1(t)$ are measured at a point $Z_1$ from the entrance of the tank and that the control valve for $Q_4(t)$ is located at a point $Z_2$ feet from the entrance as shown in Fig. 3. There is little mixing between the points $Z_1$ and $Z_2$, and the tank entrance, so the average times for a molecule of the fluid to flow through these lines are

$$t_1 = A_1 Z_1 / Q_1(t)$$

and

$$t_1 = A_2 Z_2 / Q_1(t)$$

where $A_1$ and $A_2$ are the cross sectional areas of pipes 1 and 2, respectively.

An instantaneous change in the flow rate $Q_4(t)$ at the flow meter will also result in an instantaneous change in the flow rate $Q_4(t)$ at the tank entrance since incompressible liquid flows have been assumed. For compressible fluids or for incompressible fluid where the inertia of the system is significant, the flow rate will not change instantaneously. An instantaneous change in the composition $C_1(t)$ at the analyzer will be delayed by an amount of time $t_4$ since it is an intensive variable. This may be expressed as

$$C_1(t) = C_1(t - t_4)$$

The delay time $t_4$ is not constant since it depends on $Q_4(t)$; however, for small changes in $Q_4(t)$ an average delay time $t_{4a}$ may be used.

Two other possible sources of delay time on the system shown in Fig. 3 are the delay time which would result from imperfect mixing in Tanks I and II and the delay time associated with the transport lag on the outlet of Tank II.

The effect of any of these delay times on the performance of the feedforward controller depends entirely on how well it is modeled and implemented. However, delay times or large time lags present an inherent difficulty for conventional feedback loops. If any of the delay times mentioned had been incorporated into the model for the system of Fig. 2, the feedback controller would not have performed very well.

Case II: Feedforward Control of Composition $C_1$ and Temperature $T_1$

This case will illustrate multivariable steady-state and dynamic feedforward control by considering

Controlled Variables: $T_1 = T_a, C_1 = C_{11}$

Manipulated Variables: $Q_4(t), Q_2(t)$

Measured Disturbances: $Q_4(t), T_1(t), C_1(t), Q_2(t), T_R(t), C_{11}(t)$

There will be two more important differences between Case I and Case II. First, it is assumed that the variables $Q_4(t), C_1(t)$ and $T_1(t)$ cannot be measured. There-

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fore the disturbances must be measured at a point in the process ahead of the manipulated variables. This will require dynamic feedforward control since the manipulated variables should be lagged. Second, delay time of 7.5 minutes resulting from the imperfect mixing in Tank II and transport lag in the outlet of Tank II will be incorporated into the system model. This delay time will not enter into the feedforward controllers because it has the same effect on the manipulated and disturbance variables. However, this delay time will seriously limit the performance of the feedback controllers. For any real system this delay time would probably be present.

For Case II the feedforward controller must predict the values of \( Q_1(t) \) and \( Q_2(t) \) which will hold \( T_2 \) at \( T_a \) and \( C_1 \) at \( C_a \), when the process is disturbed. This can be expressed:

\[
Q_1(t) = \frac{1}{I_m} \left[ C_m, C_m, Q_1(t), T_s(t), T_3(t), C_1(t), C_1(t), T_1(t), C_1(t) \right] \quad (16)
\]

and

\[
Q_2(t) = \frac{1}{I_m} \left[ C_m, C_m, Q_2(t), T_s(t), T_3(t), C_1(t), Q_1(t), T_1(t), C_1(t) \right] \quad (17)
\]

It will be noted that two feedforward control functions, \( f_0 \) and \( g_0 \), are required and they must be decoupled; i.e., \( f_0 \) is not a function of \( Q_1(t) \), and \( g_0 \) is not a function of \( Q_2(t) \). For complex nonlinear systems this decoupling may be a problem, however, for this example it can be done.

**Development of Feedforward Controllers for Case II...**

Nonlinear FFC: The nonlinear dynamic feedforward controllers are obtained by first solving Equation 4 for \( Q_1 \) and \( C_2 \), and Equation 6 for \( Q_2 \):  

\[
Q_1 = Q_1 - Q_1 C_2 - Q_1 C_1 + Q_2 C_2 + p(V_1 C_2) \quad (A)
\]

\[
Q_2 = Q_2 - Q_2 C_1 + Q_2 C_2 + p(V_2 C_2) \quad (B)
\]

To obtain the feedforward controllers the output terms \( Q_1 \) and \( Q_2 \) must be eliminated; the \( Q_1 \), \( C_1 \) and \( T_7 \) terms must be expressed in terms of measurable disturbances and the resulting equations solved simultaneously for \( Q_1 \) and \( Q_2 \). The output \( Q_1 \) is eliminated by Equation 2:

\[
Q_1 = Q_1 + Q_2 - Q_2 - pV_1 \quad (C)
\]

Since \( C_2 \) is constant, the term \( p(V_1 C_2) \) simplifies to \( p(V_2 C_2) \) and used to express \( Q_2 \), in terms of measurable disturbances Equation 5 is differentiated and solved for \( pV_1 \):

\[
pV_1 = \frac{2}{S_1 d_1} \cdot \frac{Q_2}{e_{D}, \mid 2 e} \quad (E)
\]

Equation 2 is substituted into Equation 1 which is then solved for \( Q_1 \):

\[
Q_1 = \frac{Q_1 + Q_2}{1 - \alpha} = \frac{S_1}{d_1} \quad (F)
\]

A similar equation is obtained for \( C_2 \) by solving Equation 3 for \( C_2 \), and using Equation 1 to eliminate \( Q_1 \) + \( pV_1 \):

\[
C_2 = \left[ \frac{Q_2 C_2 + Q_2 C_2}{Q_1 + Q_2} \right] \left[ \frac{V_1}{Q_2 + Q_1} \right] = \frac{S_1}{d_1} \quad (G)
\]

Also the equation for \( T_2 \) is obtained by solving Equation 5 for \( T_2 \), and using Equation 1 to eliminate \( Q_2 \) + \( pV_2 \):

\[
T_2 = \frac{Q_2}{Q_2 + Q_1} \left[ \frac{Q_2}{1 - \alpha} T_2 - T_2 \right] \quad (H)
\]

\[
\left[ 1 + \frac{V_1}{Q_2 + Q_1} \right] T_2 = \left[ 1 + \frac{V_1}{Q_2 + Q_1} \right] T_2 \quad (I)
\]

Equations C and D are used to simplify Equations A and B to

\[
Q_1 = \frac{[Q_1 (C_2 - C_1) + Q_2 (C_2 - C_1)]}{[C_2 (C_2 - C_1)]} \quad (1)
\]

and

\[
Q_2 = \frac{[Q_2 (T_2 - T_1) - Q_2 (T_2 - T_1)]}{[T_2 - T_1]} \quad (1)
\]

Equations I and J are solved simultaneously for \( Q_1 \) and \( Q_2 \), to give the desired nonlinear dynamic feedforward controller of Table 2:

\[
Q_2 = Q_2 (T_2 - T_1) / K_a \quad (1)
\]

\[
Q_2 = Q_2 (T_2 - T_1) / K_a \quad (1)
\]

The nonlinear steady-state controllers are obtained by setting \( p = 0 \) in the dynamic controllers, or by setting \( d_1 \) and \( d_2 \) equal to one.

**Linear FFC:** The linear feedforward controllers are obtained by simply linearizing the nonlinear controllers about the steady-state values.
HOW TO APPLY FEEDFORWARD CONTROL

The control systems for Case II are shown in Fig. 5 and the responses of the manipulated and controlled variables for the disturbances are shown in Fig. 6 and Table 3. It is seen that the nonlinear dynamic feedforward controllers give perfect control because they are based on an exact model of the process. The nonlinear steady-state controllers give perfect steady-state control, but because their manipulative inputs are not lagged, they show an offset during the transient period. The responses of the linear feedforward controllers show an offset at steady-state which would be a function of the magnitude of the disturbances. The responses of the feedback controllers first follow the no control responses because of the delay time and then settle out to the correct steady-state values because of the reset action.

This section illustrates how to develop feedforward controllers from theoretical process models. It shows the feedforward controller may be obtained simply by solving the model for the manipulated variables in terms of the known disturbances and the controlled variables. For this relatively simple system this was easy, however, the Case II multivariable feedforward controllers were fairly complex.

For more realistic systems which might be described by a larger number of more complex material and energy balances coupled with complex chemical reactions and subject to many constraints, the ideas of this section may be difficult to apply. This difficulty might occur because it may not be feasible to develop a theoretical model or because the theoretical model may not be able to be solved or implemented for use as a feedforward controller. When this situation arises the feedforward controller may be developed by the techniques discussed in the following section.

CONTROL BASED ON EXPERIMENTAL MODELS

Since experimental model development represents a large and important area of engineering, its main aspects will be considered as they apply to feedforward controller synthesis. In order to contrast this approach with the previous theoretical approach the same examples will be considered.

Experimental procedures for model development are usually based on the simple concept that a component, a control system, a process unit or any general item for which a mathematical model is desired can be obtained by input-output or cause and effect measurements. For those cases where a steady-state model is desired the techniques of statistical correlation are available.

Case I. To illustrate the experimental approach let us consider how we might experimentally arrive at the nonlinear and linear feedforward controllers for Case I. The nonlinear controller, Equation 11,

$$Q_4(t) = \frac{Q_i(t) [C_m - C_i(t)] + K_1}{K_2}$$

would be determined (or experimentally verified) by conducting a series of tests. It should be emphasized that in these tests the independent variables $Q_i(t)$ and $C_i(t)$

| Table 3—Measured Disturbances for Figs. 5 and 6 |
|-----------------|-----------------|-----------------|
| Disturbance     | Initial         | Final           |
| $Q_i$, gpm      | 4,000           | 5,000           |
| $C_i$, lb A/gal | 0.2             | 0.1             |
| $T_i$, °F       | 100             | 100             |
| $Q_i$, gpm      | 500             | 1,000           |
| $C_i$, lb A/gal | 0.2             | 0.1             |
| $T_i$, °F       | 100             | 112             |

Fig. 6—Comparison of control responses for Case II.
are measurable but not controllable and that the dependent variable \( \dot{Q}_d(t) \) is measurable and controllable. Then a test would consist of determining the values of \( \dot{Q}_d(t) \) for given values of \( Q_s(t) \) and \( C_s(t) \) which will hold \( \dot{C}_s(t) \) at \( C_s \), its desired value.

For any real process such tests might be very difficult to run and expensive. This along with the fact that the usefulness of the controller will be dependent on the range of variables considered, on the form of the assumed model, and on how well the model fits the noisy data, makes a statistical approach almost mandatory. These limitations also make a theoretical development of the steady-state model, followed by carefully run experiments to verify a much more desirable approach than extensive testing followed by curve fitting.

These same comments also apply for obtaining experimental dynamic models; however, it should be realized that the dynamic portion of a feedforward controller need not be as accurate as the steady-state portion. An exception to this is the process which practically stays in a transient state. An experimental dynamic model is based on the assumption of a system's response to a forcing function or input. The type of forcing function is an important element in the development of the model; however, for our purposes it will be assumed that a step input is used. The next important items to be considered are the form of the model and the procedure used to determine or fit the parameters in the assumed model. The form of the model is generally a simple, linear transfer function.

For chemical processes the most common dynamic model used for feedforward control is the lead-lag model. This model results from assuming first order lag dynamics for the controlled variable to disturbance variable and for the controlled variable to manipulated variable transfer functions. This may be expressed mathematically as

\[
\frac{\dot{C}(t)}{M(t)} = G_1(p), \quad \frac{\dot{C}_s(t)}{\dot{M}_s(t)} = G_2(p),
\]

The feedforward controller dynamics must provide the change in the manipulated variable required to offset a change in the disturbance variable,

\[
\text{FFC} = \frac{\dot{M}_s(t)}{\dot{D}(t)} = \frac{G_1(p)}{G_2(p)} = \left(1 + \tau_2 p\right) \frac{K_2}{1 + \tau_1 p}
\]

which is the common lead-lag transfer function. Analog and digital implementation of this commonly used element is very easy.

In practice one of the biggest shortcomings of this dynamic model is the assumption of equal dead times between the manipulated and controlled variables and between the disturbance and controlled variables. When this assumption is not valid a second order lag, a first order lag plus dead time, or a second order lag plus dead time model can be assumed for the FFC transfer function.

Case II. Let us consider briefly how the dynamic feedforward controllers for Case II might be developed by experimental techniques. First it should be noted that the nonlinear dynamic controllers would be very difficult to develop, since most techniques are based on obtaining linear dynamic models. Now the measurable, but uncontrollable disturbances for Case II were \( \dot{Q}_s, G_s, T_s, \dot{Q}_a, C_s, \)

\[
T_a. \quad \text{Then the first step might be to determine the transfer functions as follows:}
\]

\[
\begin{align*}
G_1(p) &= \frac{G_1(p)}{G_1(p)} \\
G_2(p) &= \frac{G_2(p)}{G_2(p)} \\
T_1(p) &= \frac{T_1(p)}{T_1(p)} \\
T_2(p) &= \frac{T_2(p)}{T_2(p)} \\
K_2(p) &= \frac{K_2(p)}{K_2(p)}
\end{align*}
\]

The determination of these transfer functions might be very difficult because these disturbances are uncontrollable and the input forcing function might be very irregular or random. Already such a formal approach to determine the dynamic feedforward controllers would probably not be workable.

An alternate approach would be to assume the dynamic portion of the controller to be lead-lag and then tune the lead-lag elements on-line. As more sophisticated first level controls, such as direct-digital control, becomes widespread more sophisticated dynamic models and techniques may become workable.

The costs of applying FFC result from the engineering requirements, the hardware or software required to implement it and the measurement of the disturbances. The economic incentives for applying FFC is most likely to be found where consistent quality of a valuable product is important. In particular, FFC may give considerable savings due to consistent product quality, increased recovery, reduced consumption of utilities, and reduced tankage requirements. In conclusion, FFC should not be applied to all processes; however, for many processes it should be applied and it can yield results superior to any other control technique.

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>area</td>
</tr>
<tr>
<td>( C )</td>
<td>concentration</td>
</tr>
<tr>
<td>( C^* )</td>
<td>concentration at analyzer</td>
</tr>
<tr>
<td>( C_s )</td>
<td>orifice coefficient</td>
</tr>
<tr>
<td>( D )</td>
<td>disturbance function</td>
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<tr>
<td>( d/dt )</td>
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<tr>
<td>( F )</td>
<td>mathematical function</td>
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<tr>
<td>( FFC )</td>
<td>feedforward control</td>
</tr>
<tr>
<td>( g )</td>
<td>gravitational constant</td>
</tr>
<tr>
<td>( H )</td>
<td>fluid head</td>
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<tr>
<td>( K )</td>
<td>constant</td>
</tr>
<tr>
<td>( M )</td>
<td>manipulative function</td>
</tr>
<tr>
<td>( PI )</td>
<td>proportional plus integral control</td>
</tr>
</tbody>
</table>

*Continued on next page*
HOW TO APPLY FEEDFORWARD CONTROL...

p = differential of time
d = dead time

\( \dot{x} \) = differential of variable
\( \Delta x \) = change in variable

\( \Delta u \) = change in control variable

\( \Delta y \) = change in output

\( \Delta k \) = change in throughput

\( \Delta T \) = change in temperature

\( \Delta V \) = change in volume

\( \Delta t \) = change in time

\( \Delta T \) = change in temperature

Subscripts
1, 2 = system variables

REFERENCES

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UNIVERSITY MICROFILMS.
USE OF SEARCH TECHNIQUES TO DETERMINE OPTIMAL SWITCHING TIMES

by

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An important part of most modern digital control systems in the supervisory or steady-state optimization programs. The purpose of these second level programs is to determine optimum values for the set-points of the first level control loops. Today most first level control loops are adjusted for a compromise between good set-point responses and good load responses. This paper will present a practical technique for obtaining good set-point responses, while allowing the first level controllers to be tuned for good regulator action so that load disturbances will be well-compensated.

Process Considerations

The technique to be presented for improved set-point responses should be most useful for processes that undergo the same or very nearly the same set-point changes on a routine basis. Such processes most typically fall into this category. Processes that have random changes in set-points may or may not benefit from this approach. If the random set-point changes occur over a relatively small range then this approach could be satisfactory. However, if large random set-point changes occur then this technique may not be useful.

Optimality Criterion

When considering control of commercial processes the most desirable criterion function to use is profit. However, the profit criterion function is often difficult to formulate and use so that alternate, simpler criterion functions are used. Examples of such criteria are minimum time, minimum energy, and the minimum integral of the absolute value of the error (IAE). Minimum time will be used as the criterion function here because it yields a simple implementation and it provides significant improvement over conventional controller set-point responses by any optimality criterion. However, the technique presented is directly applicable to any criterion function.

Application of Time-Optimal Control

It is obvious that the harder a system is forced, the faster it will respond. However, for any real process there is always a limit to the available energy for control. These limits must be considered in the design of a control system. In conventional regulator design it is assumed that these limits will not be violated, whereas in practice saturation often occurs.

A well-known result of optimal control theory is that the time-optimal control action for processes with the manipulated variables subject to magnitude constraints is that the manipulated variable should be at its limits throughout the transient period. This is often called bang-bang control. These comments are illustrated in Figure 1. The process is initially at its desired value $y_0$ by having the manipulated variable at $m_0$. Then at time $t_1$ the supervisory program calls for a new set-point $r$. Bang-bang control is obtained by setting the manipulated variable to its upper bound $K$ at time $t_1$ since $r$ is greater than $y_0$ (if $r$ were less than $y_0$, $m$ would be set to its lower bound $-K$ at time $t_1$).

2
Then $\alpha$ would be set to the lower bound $b$ at time $t$. Then at time $t$, the manipulated variable is switched to its lower bound $b$. It should be understood that $K$ and $h$ are not necessarily true physical limits and that they would satisfy any safety requirements. Finally, at time $T$ the manipulated variable is switched to $g$, which is given by a division of the process gain. This completes the bang-bang control action.

The true time-optimal control action would not correspond to the two-switch bang-bang control action described above unless the process is second-order with real time constants. It can be shown that the number of switches between the limits $K$ and $h$ for linear processes with real time constants is at most equal to the order of the system minus one. If the switching time to return to regulatory control is counted, then the number of switches is at most equal to the order of the system. Since real processes are higher-order, non-linear systems the actual time-optimal control action will not be simple bang-bang control, so that (as shown in Figure 1) the output will not be at its desired value at time $T$. To overcome this and to provide regulatory control the conventional controller is switched back in at time $T$. These concepts are relatively simple and the real problem in applying time-optimal control is to determine when and how often the manipulated variable should be switched.

**Summary of Previous Work**

In one method proposed to accomplish time optimal control, the first step is to obtain an overdamped second-order lag with dead-time (transport lag or delay time) model from bang-bang type response data (using non-optimum switching times) by nonlinear regression or by a search technique. This requires determination of four process parameters: two time constants, a dead time and a gain. Since a second order model is used, only two switching times are required. These two switching times are then cal-
calculated from algebraic equations based on optimal control and by knowing the model parameters, the manipulated variable bounds, and the starting and desired values of the output. It should be noted that one of these equations is implicit and must also be solved by some type of search procedure.

**Time-Optimal Control via a Search Technique**

The previous procedure for determining switching times is based on first determining a model of the process and then performing calculations based on this model. The purpose of this paper is to propose an alternate procedure which does not use a model. The procedure is to simply use an on-line, real-time search technique to determine the switching times.

If two switching times are desired, this would require a two parameter search.

The overall procedure for finding the optimum switching times would first involve supplying some initial switching times. These could be guesses or they could be obtained by the previous method. Then, control action based on these initial switching times would be taken and the criterion function evaluated. From then on the search technique would supply the switching times while trying to minimize the criterion function.

The search would never really be terminated and as the process characteristics change, the search would continuously adapt the switching times to these changes. Also, it should be understood that a single search program could be used for all of the different control loops as well as for other optimization functions.

For this application it is suggested that a pattern type search be used. This type of technique should be superior to gradient type searches because it is not as seriously affected by process noise as it does not require the calculation of partial derivatives. Another requirement of the search technique is that it must be capable of recognizing constraints, e.g., minimum time between switches, which pattern search techniques do quite well.

The criterion function which guides the search technique deserves some attention. The search is trying to find the switching times which will drive the output of the system from its initial value to its final desired value in minimum time. A criterion function which will satisfy this is

\[
C_{PFN} = \begin{cases} 
T + \text{CONST1} \cdot |r-c(T)| & \text{for } |r-c(T)| > \text{CONST2} \cdot |r-r_0| \\
T & \text{for } |r-c(T)| \leq \text{CONST2} \cdot |r-r_0| 
\end{cases}
\]

where \( T \) is the time at which the conventional controller is switched on,

\( c(T) \) is the process output at time \( T \)

\( r \) is the final desired value of the process output

\( r_0 \) is the initial value of the process output

\( \text{CONST1} \) is a constant (a typical value might be 100)

\( \text{CONST2} \) is a constant (a typical value might be 0.01)

By using a criterion function of this type, switching times which do not drive the output near its desired value will be rejected in favor of switching times which do. The calculations and logic for this criterion function are very simple. A more sophisticated criterion function might also include a term accounting for the error which results during the conventional control action such as the integral of the absolute value of the error. It is the total time or error which should really be minimized.

To illustrate that the search technique can easily determine the optimum switching times, two second-order systems were studied. The results for the second-order systems are shown in Figure 2. The switching times for the second-order system with real, distinct eigenvalues were also calculated by optimal control theory and found to agree with the results from the search technique. Figure 3 shows the time-optimal response for a sixth order system obtained using a search technique. Six switches
Figure 2(a). Time-Optimal Control of A Second-Order System with Real, Distinct Eigenvalues via a Search Technique.

Figure 2(b). Time Optimal Control of a Second-Order System with Real, Equal Eigenvalues via a Search Technique.
In the manipulated variable are required for the true time-optimal control. In practice only two or three switches would probably be used since this would give near time-optimal control. These results illustrate that a search technique can be used to determine the optimum switching times. Also for processes with dead time, the switch to conventional control should be delayed by the dead time.

Conclusions

For obtaining time optimum control for set-point changes it is felt that there are several advantages in using a search technique rather than using a process model as a basis for calculating the switching times. These are:

1. Determining the switching times from the model requires first, a four parameter search to determine the model parameters, and second, the implicit solution of an algebraic equation. Applying the search directly to the process whenever possible is certainly simpler; however, the process gain and dead time might be required.

2. Experience with using simple process models for controller tuning has indicated that the optimum settings are frequently not obtained. Similar results should be expected when determining switching times, which is really analogous to conventional controller tuning. As the search interacts directly with the process, its results should be optimal within the constraints imposed, i.e., number of switches, etc.

3. By using the search technique the control engineer has more freedom in the problem formulation. Essentially any criterion function, any number of switches or an entirely different control strategy can be easily used with a search technique. On
the other hand, the model approach may not work at all.

4. For sampled-data systems the constraint that the switching times must be multiples of the sampling time is also easily included with the search technique.

Although the instances in which this approach can be applied to process units is somewhat limited, it does offer distinct advantages in these cases.

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**Nomenclature**

\[ m(t) \quad \text{Manipulated variable} \]
\[ w_0 \quad \text{Initial value of manipulated before the set-point change} \]
\[ w_f \quad \text{Final value of the manipulated which yields the desired set-point} \]
\[ e(t) \quad \text{Process output} \]
\[ r_0 \quad \text{Initial desired value of the process output} \]
\[ r_f \quad \text{Final desired value of the process output} \]
\[ K \quad \text{Upper limit for the manipulated variable} \]
\[ k \quad \text{Lower limit for the manipulated variable} \]
\[ t_1 \quad \text{Time at which the supervisory programs calls for a set-point change} \]
\[ t_2 \quad \text{Time at which the manipulated variable is switched between K and k} \]
\[ T \quad \text{Time at which the conventional controller is switched back on} \]
\[ CFUW \quad \text{Criterion function which guides the search technique} \]
\[ \text{CONST1} \quad \text{Constant which weights the time and the deviation from the desired value in the criterion function} \]
\[ \text{CONST2} \quad \text{Constant which sets an acceptable level on the deviation from the desired value in the criterion function} \]

**Bibliography**


APPENDIX D
(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12) \times
\text{CONTINUE}
\text{rate} = 0.1 \times \text{rate}
\text{rate} = 0.2 \times \text{rate}
\text{rate} = 0.5 \times \text{rate}
\text{rate} = 0.9 \times \text{rate}
\text{rate} = 0.7 \times \text{rate}
\text{rate} = 0.3 \times \text{rate}
\text{rate} = 0.6 \times \text{rate}
\text{rate} = 0.4 \times \text{rate}
\text{rate} = 0.8 \times \text{rate}
\text{rate} = 0.2 \times \text{rate}
\text{rate} = 0.5 \times \text{rate}
\text{rate} = 0.9 \times \text{rate}
\text{rate} = 0.1 \times \text{rate}
\text{rate} = 0.2 \times \text{rate}
\text{rate} = 0.5 \times \text{rate}
\text{rate} = 0.9 \times \text{rate}
\text{rate} = 0.1 \times \text{rate}
\text{rate} = 0.2 \times \text{rate}
\text{rate} = 0.5 \times \text{rate}
\text{rate} = 0.9 \times \text{rate}
5   PUNCH  100, C(1): T1:T1
   J=0
10   CONTINUE
110  FORMAT (1:7,1,9+L)
   J=0
   K=0
   CTNT=0
   X=U, Y+1
   C=0
   CAO=C
   J0=1
   TI=U
   LJ: C1 = (X.U)
   X=1. A0Y
   E=S*T=C
   AO=A+(C+F)*DT/15
   DF=(C+F)/15
   DT=JAI•(U+O)SE+O*ATF*DF)
   XNEW=1+DC
   CN=(1. 0.0=C••)*QI/AM1+CA0
   (E/TAD*LT••.G0+1)*T02
   C(1)=CA(C••••)*NM/TAD+C(C(1)-1)
   C=C+T
20   C(1)=CA
21   CONTINUE
30   PC=C(1)=C(1-1)
50   U=1. C••••+T••+A••(CSS=C(1))+ABS(CSS=C(1-1))/2
S=U*3/4+T*•((CSS=C(1))+CSS=C(1-1))•CSS=C(1-1))/2
S=U*4/5+T*•((CSS=C(1))+CSS=C(1-1))•ABS(CSS=C(1-1))/2
S=U*5/6+T*•((CSS=C(1))+CSS=C(1-1))/2
C-------- PLOTTING PROGRAM FOR FIRST ORDER LAG PLUS DEAD
C-------- TIME RESPONSE WITH PI OR PID CONTROLLER

DIMENSION PLOT(5000)
DIMENSION G(5000), X(5000)
CALL PLOT(5000, 5000)
CALL EDT(EDT(3))

S=9

CALL I=I
READ (11), TA, TAU, OMEGA, E, F1
PRINT 11, TA, TAU, OMEGA, E1, F1

2 FORMAT(5E12.6)
112 READ 11, 'TA, TAU, OMEGA, E, F1', I, I
PRINT 11, 'TA, TAU, OMEGA, E1, F1', I

101 FORMAT(2X, 5E12.6)
S=1000./TAU
S1=100.*S
S1=1000.*S
SET=1.0/S1
RATE=1.0/S1
SET=SET/TAU

RATE=RATE/TAU
PRINT 100, 'SET, RATE'

103 FORMAT(2X, 5F10.5)
C(1)=S
C(2)=S
T=0
N=0

NLY=3*T+0.00
N(LY)=0
C(1)=S
C(2)=S

88
2000 CALL LIN('(',G,'X',10,11)
1001 CALL DUE('(',G,'X',10,11)
1002 IF (IREQ)'(',G,'X',10,11) THEN 220
GO TO 110
220 CALL DUE('(',G,'X',10,11)
STOP
END

ENTRY
1*357 0.4674 0.427 -73322 341 97501
1 0 0.1
10 0 1.2
1 0 1.3
3SYS
3SYS
C-------- A N D F E E D B A C K C O N T R O L L E R S R E S P O N S E S
D I M E N S I O N : Y 1 ( T O P ) , Y 2 ( T O P ) , Y 3 ( T O P ) , Y 4 ( T O P ) , X ( T O P )
D I M E N S I O N : Y 1 ( B O T T M ) , Y 2 ( B O T T M ) , Y 3 ( B O T T M ) , Y 4 ( B O T T M ) , X ( B O T T M )
C A L L P L O T ( L E F T , 2 0 0 )
C A L L P L O T ( R I G H T , 2 0 0 )
C O M M O N : C O N 1 , C O N 2 , C O N 3 , C O N 4 , Y 1 , Y 2 , Y 3 , Y 4 , T 1 , T 2 , D T
C O M M O N : T A B L E 1 , T A B L E 2
C O M M O N : S I N G L E
C O M M O N : Y 1 , Y 2 , Y 3 , Y 4
C = 0
979   C O N T I N U E
C-------- I N P U T L A T A-------------------------------
C 1 = 0
C 2 = 0
C 3 = 0
C 4 = 0
C 5 = 0
C 6 = 0
C 7 = 0
C 8 = 0
C 9 = 0
C 10 = 0
C 11 = 0
C 12 = 0
C 13 = 0
C 14 = 0
C 15 = 0
C 16 = 0
C 17 = 0
C 18 = 0
C 19 = 0
C 20 = 0
C 21 = 0
C 22 = 0
C 23 = 0
C 24 = 0
C 25 = 0
C 26 = 0
C 27 = 0
C 28 = 0
C 29 = 0
C 30 = 0
C 31 = 0
C 32 = 0
C 33 = 0
C 34 = 0
C 35 = 0
C 36 = 0
C 37 = 0
C 38 = 0
C 39 = 0
C 40 = 0
C 41 = 0
C 42 = 0
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C 46 = 0
C 47 = 0
C 48 = 0
C 49 = 0
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C 51 = 0
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C 60 = 0
C 61 = 0
C 62 = 0
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C 69 = 0
C 70 = 0
C 71 = 0
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C 76 = 0
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C 78 = 0
C 79 = 0
C 80 = 0
C 81 = 0
C 82 = 0
C 83 = 0
C 84 = 0
C 85 = 0
C 86 = 0
C 87 = 0
C 88 = 0
C 89 = 0
C 90 = 0
C 91 = 0
C 92 = 0
C 93 = 0
C 94 = 0
C 95 = 0
C 96 = 0
C 97 = 0
C 98 = 0
C 99 = 0
C 100 = 0
C = 0
\[ TP = 12.0 \]
\[ T0 = 115.0 \]
\[ T0 = 150.0 \]
\[ C0 = 0.60 \]
\[ h1 = 2.0 \]
\[ \nu = 0.0 \]
\[ \nabla = 0.0 \]
\[ \hat{\alpha} = \frac{1.0}{(0.75 \times 7.40)} \]
\[ \hat{\beta} = 0.05 \times 1.50 \]
\[ \hat{\gamma} = (0.75 \times 10.40) \]
\[ \hat{\delta} = 0.05 \times 1.20 \]

--- INITIAL CONDITIONS ---

\[ \psi = 0.0 \]
\[ \nu = 0.0 \]
\[ V0 = 0.0 \]
\[ V1 = 0.0 \]
\[ V2 = 0.0 \]
\[ V3 = 0.0 \]
\[ V4 = 0.0 \]
\[ V5 = 0.0 \]
\[ V6 = 0.0 \]
\[ V7 = 0.0 \]
\[ V8 = 0.0 \]
\[ V9 = 0.0 \]
\[ V10 = 0.0 \]
\[ V11 = 0.0 \]
\[ V12 = 0.0 \]
\[ V13 = 0.0 \]
\[ V14 = 0.0 \]
\[ V15 = 0.0 \]
\[ V16 = 0.0 \]

---

\[ \mu = 0.0 \]

--- 2000 ---

\[ \mu = 0.0 \]

---

\[ \mu = 0.0 \]

---

\[ \mu = 0.0 \]

---

\[ \mu = 0.0 \]
SPECIFICATION OF CONTROLLER

20 IF (Y < -1.0) GO TO 102
IF (T < -0.0) CALL C1(XA, NY)
IF (T = -0.0) CALL C2(XA, NY)
IF (T < +1.0) CALL C3(XA, NY)
IF (T < +1.0) CALL C4(XA, NY)
IF (T < +1.0) CALL C5(XA, NY)
IF (T < +1.0) CALL C6(XA, NY)
IF (T < +1.0) CALL C7(XA, NY)
IF (T < +1.0) CALL C8(XA, NY)

102 NY(1) = .A
NY(2) = 1
IF (T > +1.0) GO TO 100
M1 = 10
M2 = 1
Y3(I) = Y3(I - 46)

1005 NY(1) = Y4(1) + 46
NY(2) = Y4(2) + 30
Y3(I) = Y3(I - 30)

1006 NY(1) = Y4(1) + 30

1000 FY = PRI(1x, M = 10.0, R = 1.0, T)
X(40) = 0.
X(40) = 0.
Y1(40) = 0.
Y1(41) = 0.
Y2(4) = 1.
Y2(4) = 1.
Y3(5, 1) = 0.
Y4(5, 1) = 0.
CASE 2 LINEAR OXY FFC

\[ C^o = 6 + 2r \]
\[ C^o = 2r + 4 \]
\[ 3^o = (c - 2 + 8c) + (c + 5 - c) / 450^o \]
\[ 5^o = ((T^o - 10^o) + 50^o - (T + 10^o) - 50^o) / 450^o \]
\[ 6^o = (1 + 1 + 1) / 450 \]
\[ 7^o = (1 + 1 + 1) / 450 \]

1
\[ A1 = 5 \]
\[ A2 = 3 \]
\[ A3 = 5 \]
\[ A4 = 7 \]
\[ A5 = 11 + 1 \]

2
\[ A1 = (c^o - A1) + T / T + A1 + A1 \]
\[ A2 = (5^o - a^o) + T / T + a^o + A2 \]
\[ A3 = (3^o - a^o) + T / T + a^o + A3 \]
\[ A4 = (A1 + C^o - 2 - 2 + 3 + 0.01) / 2 + 6 \]
\[ A5 = (A1 + C^o - 2 - 2 + 3 + 0.01) / 2 + 6 \]
\[ A1 = (A1 + C^o - 2 - 2 + 3 + 0.01) / 2 + 6 \]

RETURN

PRINT

CASE 2 LINEAR OXY STATE

\[ S^o = 6 + 2r \]
\[ C^o = 2r + 4 \]
\[ S^o = (c - 2 + 8c) + (c + 5 - c) / 450^o \]

1
\[ A1 = 5 \]
\[ A2 = 3 \]
\[ A3 = 5 \]
\[ A4 = 7 \]
\[ A5 = 11 + 1 \]
\[ A6 = (A1 + C^o - A1 + (A2 + A3 + C^o)) / 3 + 6 \]
C2 = (A1 * 25 + A2 + A3 * 1.25) / 10
RETURN
END

CASE 5
SUBROUTINE SC (CA, CP)
COMMON T0, A1, A2, A3, A4, T0, T1, T2, T3, T4, T5, T6, T7, T8, CO, AV1, V1, C0, T0, DT
COMMON TAU, T1, T2, T3, T4, T5, T6, T7, T8, T9, DT
COMMON T1, T2, T3, T4, T5, T6, T7, T8, T9, DT
COMMON T0, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10
COMMON T0, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10
RETURN
END
C1 = INCP = 5.0
C2 = INCK = 1.0
FSET1 = 2.0
FSET2 = 2.0
CA = C1 + (C2 * (FSET1 + 12)) + 100
CB = C1 + (C2 * (FSET2 + 22)) + 100
ENTRY
* /SYS
!S /SYS
DO 16 I=1,NP
P(11)=P(I)
P(11)=P(I)
U(11)=STEP(I)*10.
10 T(I)=P(I)

C----STEP SIZE DETERMINATION AND REDUCTION
15 DO 20 J=1,NP
CALL MSTEP(I,J,P,Y,W,PF,CM)
COUNT=0
SN 10 J=1,NP
SN(1)=+1.0
16 T(J)=T(J)/10.
CALL MSTEP(I,J,P,Y,W,PF,CM)
IF(T(I)>T(J))PRINT(150,P(K3),K3=1,NP)
IF(T(J)>T(I))PRINT(160)
115 FORMAT(///LA,2STEP SIZES CHANGED TO ,SF15.8)
116 FORMAT(16.6)

C----REFINEMENTS ABOUT T
20 ICOUNT=0
21 DO 30 J=1,NP
IC=0
22 P(J)=T(J)-S(J)
IC=IC+1
CALL MSTEP(I,J,P,Y,W,PF,CM)
IF(150.0+P(J))PRINT(250)
IF((CM+2<CM))S(J)=S(J)
IF((CM+3<CM))S(J)=S(J)
25 IF((CM+4<CM))S(J)=S(J)
CM=CM+1
T(14)=P(14)
GOTO 35
30 P(14)=T(14)
COUNT.T = COUNT+1
35 CONTINUE
C------STILL CHECK
IF (NP=18) COUNT=COUNT+1
CALL SUBPROG(L,P,J,IP,MES)
COUNT=COUNT+1
IF (COUNT=n) RETURN
IF ((CF.FJ-CF.FJ) GT .0) ATN50
CF.FJ=CF.FJ
40 X=1 NP=1
41 IF (IP(I))=F(I)
ITER=ITER+1
CALL SUBPROG(I,IP,IP,MES)
IF (I(I)=I(I)) COUNT=COUNT+1
ETTER.CF.FJ=(NP(I),I(I),NP)
1000 FORMAT(12X,12I5,12I5,12F15.6,12F15.6)
1 1111111111111111111
C------ACCELERATION STEP
SUM=0
50 IF (M(I))=1 M(I)=1
P(I)=1(I)+2*(32(I)=8(I))
IF (T(I)) GT 0(16) T(16)=0(I)
IF (T(I)) LT 0(16) T(16)=52(I)
F(I)=G(I)
SUM=SUM+T(I)(1(I)=32(I))
45 H(16)=H(16)
IF (0(I)=G(I) AND 25 THEN 25)
GOTO 90
15 PS(110)=PS(110)
CALL CHECK(NP,LS,PS,CFNS,IA)
IF(IA.EQ.0)GO TO 10

CFNS=CFNS
IF(POINT(.T.,I1).EQ.INT4,LS,CFNS,PS(KK),KK=1,NP)
L=1-1
GO TO 5

CALL PRC(NP,P,CFNP)

CALL STORE(NP,L,P,CFNP)

CFNS=I1.GEQ(30)
GO TO 5

IF((CFP.EQ.1.EQ.0).OR.(CFP.NE.140)).OR.(CFP.NE.141).OR.(CFP.NE.142).OR.(CFP.NE.143))=L=1

FORMAT(3X,15F5.5) IFCFP.EQ.1555X,5F15.5)
FORMAT(1X,15F5.5) IFCFP.EQ.1555X,5F15.5)
RETURN
END

\*I1FTC STORE
SUBROUTINE STORE(NP,L,P,CFP1)
DIMENSION P1(10), P2(100, 10), CFN2(100), L2(100)
COMMON L, P2, CFN2
0110 K1 = 1
0:111 K2 = 1
11 = 100
L2(I2) = L2(I1)
CFN2(I2) = CFN2(I1)
0100 I1 = 1, NP
10 P2(I2, I1) = P2(I1, I1)
L2(1) = 1
CFN2(1) = CFN2(1)
0600 K2 = 1, NP
20 P2(1, I2) = 1(I2)
RETURN
END

*IPFTC CHECK

SUBROUTINE CHECK(X0, LS, PS, CFN2, IA)
COMMON L, P2, CFN2
0600 I1 = 1, NP
11 = 1
ICOUNT = 0
0600 J = 1, NP
IF(A(05(P5(J) = P2(I1, J))) GT 1.0E-30) GOTO 30
20 ICOUNT = ICOUNT + 1
IF(I(14, 33, T = 10) SF 10) GOTO 40
30 CONTINUE
IA = 0
GOTO 20
40 IA = 1
LS=LS(II)
CP=CP(II)
DE=DE
Y=I
50 P2(k)=P2(I!k)
90 RETURN
$PFC$ PROC
SUBROUTINE PFC(I,CP,CP)
DIMENSION CP(x,y),Y(y)
COMMON U.MM(100),MM(100,10),MM12(100)
COMMON FACT,T,RX
COMMON U.MM,ULU.
Y(I)=0
Y(Y)=Y(I)*
30 P2: I=3,7
25 Y(I)=Y(I)*
SUM=0.
90 10 I=1,6
10 SUM=SUM+Y(I)
IT=0
290 IT=IT+10
IF(IT.LT.T(I)) J=IT+1
IF(IT.LT.T(I)) AN(IT.LT.T(2)) U=UL0V
IF(IT.LT.T(2)) AN(IT.LT.T(3)) U=UL94
IF(IT.LT.T(3)) AN(IT.LT.T(4)) U=UL0V
IF(IT.LT.T(4)) AN(IT.LT.T(5)) U=UL94
IF(IT.LT.T(5)) AN(IT.LT.T(6)) U=UL0V
Y(I)=0
30 20 I=2,6
APPENDIX G
**--- RESPONSE PROGRAM FOR PROBLEMS 1 AND 2 ---**

```plaintext
C - OPTIMAL AND SUB-OPTIMAL CONTROLLERS

100 READ CV, IV, Q, OM, C
   READ DX, DQ, D, DC
   READ M, N, X, Y, Z, A, B, C
   READ P(1), P(2), P(3)
   READ S(1), S(2), S(3)
   READ T(1), T(2), T(3)
   READ U(1), U(2), U(3)
   READ V(1), V(2), V(3)

2000 FORMAT (1X, 10.E6)
3000 FORMAT (1X, 10.E6)
50 TO 10.
   STOP
   C - SOURCE CODE
   DO 10 I = 1, N
   C - STATE VARIABLES
   X(I) = X(I) + CV * T(I)
   Y(I) = Y(I) + IV * T(I)

10   U(I) = U(I) + D(I) + DQ * T(I)
   V(I) = V(I) + D(I) + DC * T(I)
   W(I) = W(I) + D(I) + DQ * T(I)
   Z(I) = Z(I) + 0.5 * M * T(I)
   A(I) = A(I) + 0.5 * N * T(I)
   B(I) = B(I) + 0.5 * N * T(I)
   C(I) = C(I) + 0.5 * N * T(I)
   S(I) = S(I) + 0.5 * N * T(I)
   T(I) = T(I) + 0.5 * N * T(I)
   U(I) = U(I) + D(I) + DQ * T(I)
   V(I) = V(I) + D(I) + DC * T(I)
   W(I) = W(I) + D(I) + DQ * T(I)
   Z(I) = Z(I) + 0.5 * M * T(I)
   A(I) = A(I) + 0.5 * N * T(I)
   B(I) = B(I) + 0.5 * N * T(I)
   C(I) = C(I) + 0.5 * N * T(I)
   S(I) = S(I) + 0.5 * N * T(I)
   T(I) = T(I) + 0.5 * N * T(I)
   U(I) = U(I) + D(I) + DQ * T(I)
   V(I) = V(I) + D(I) + DC * T(I)
   W(I) = W(I) + D(I) + DQ * T(I)
   Z(I) = Z(I) + 0.5 * M * T(I)
   A(I) = A(I) + 0.5 * N * T(I)
   B(I) = B(I) + 0.5 * N * T(I)
   C(I) = C(I) + 0.5 * N * T(I)
   S(I) = S(I) + 0.5 * N * T(I)
   T(I) = T(I) + 0.5 * N * T(I)
   U(I) = U(I) + D(I) + DQ * T(I)
   V(I) = V(I) + D(I) + DC * T(I)
   W(I) = W(I) + D(I) + DQ * T(I)
   Z(I) = Z(I) + 0.5 * M * T(I)
   A(I) = A(I) + 0.5 * N * T(I)
   B(I) = B(I) + 0.5 * N * T(I)
   C(I) = C(I) + 0.5 * N * T(I)
   S(I) = S(I) + 0.5 * N * T(I)
   T(I) = T(I) + 0.5 * N * T(I)
   U(I) = U(I) + D(I) + DQ * T(I)
   V(I) = V(I) + D(I) + DC * T(I)
   W(I) = W(I) + D(I) + DQ * T(I)
   Z(I) = Z(I) + 0.5 * M * T(I)
   A(I) = A(I) + 0.5 * N * T(I)
   B(I) = B(I) + 0.5 * N * T(I)
   C(I) = C(I) + 0.5 * N * T(I)
   S(I) = S(I) + 0.5 * N * T(I)
   T(I) = T(I) + 0.5 * N * T(I)
   U(I) = U(I) + D(I) + DQ * T(I)
   V(I) = V(I) + D(I) + DC * T(I)
   W(I) = W(I) + D(I) + DQ * T(I)
   Z(I) = Z(I) + 0.5 * M * T(I)
   A(I) = A(I) + 0.5 * N * T(I)
   B(I) = B(I) + 0.5 * N * T(I)
   C(I) = C(I) + 0.5 * N * T(I)
   S(I) = S(I) + 0.5 * N * T(I)
   T(I) = T(I) + 0.5 * N * T(I)
   U(I) = U(I) + D(I) + DQ * T(I)
   V(I) = V(I) + D(I) + DC * T(I)
   W(I) = W(I) + D(I) + DQ * T(I)
   Z(I) = Z(I) + 0.5 * M * T(I)
   A(I) = A(I) + 0.5 * N * T(I)
   B(I) = B(I) + 0.5 * N * T(I)
   C(I) = C(I) + 0.5 * N * T(I)
   S(I) = S(I) + 0.5 * N * T(I)
   T(I) = T(I) + 0.5 * N * T(I)
   U(I) = U(I) + D(I) + DQ * T(I)
   V(I) = V(I) + D(I) + DC * T(I)
   W(I) = W(I) + D(I) + DQ * T(I)
   Z(I) = Z(I) + 0.5 * M * T(I)
   A(I) = A(I) + 0.5 * N * T(I)
   B(I) = B(I) + 0.5 * N * T(I)
   C(I) = C(I) + 0.5 * N * T(I)
   S(I) = S(I) + 0.5 * N * T(I)
   T(I) = T(I) + 0.5 * N * T(I)
   U(I) = U(I) + D(I) + DQ * T(I)
   V(I) = V(I) + D(I) + DC * T(I)
   W(I) = W(I) + D(I) + DQ * T(I)
   Z(I) = Z(I) + 0.5 * M * T(I)
   A(I) = A(I) + 0.5 * N * T(I)
   B(I) = B(I) + 0.5 * N * T(I)
   C(I) = C(I) + 0.5 * N * T(I)
   S(I) = S(I) + 0.5 * N * T(I)
   T(I) = T(I) + 0.5 * N * T(I)
   U(I) = U(I) + D(I) + DQ * T(I)
   V(I) = V(I) + D(I) + DC * T(I)
```
C------- PROGRAM FOR SEARCHING FOR SUBOPTIMAL
C------- CONTROLLING FOR PROBLEM 2
100 READ 200, 20, 200, 20, 20, 20
PRINT 300, 20, 20, 20, 20, 20, 20
IF (20.EQ.2) 20, 100, 100
READ 300, (20(I), I = 1, 7)
PRINT 300, (20(I), I = 1, 7)
PRINT 300, (20(I), I = 1, 7)
3000 FORMAT(1X,7F9.6)
CALL PATT(20, 20, 20, 20, 20, 20, 20)
2000 FORMAT(1X, 8E8.5, 8X)
GO TO 100
101 STEP
BY
SUBROUTINE PROCS(B, COST)
DIMENSION B(7), Y(7)
DIMENSION X(7400), Z(2040)
COMMON M1, M2, M3, M4
COST = 0.
COST1 = 0.
COST2 = 0.
Y(I) = 0.
Y(P) = 0.
M1 = 1 = 7
25 Y(I) = 0.
U = 0.
U0 = 0.

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without page(s) 115

UNIVERSITY MICROFILMS.
OF COST FOR A PARTICULAR SET OF PARAMETERS, P.

DIMENSION P(NP),STEP(NP),B1(1000),B2(1000),T(1000),S(1000)

NOTE: VECTORS B1, B2, T, S NEED ONLY BE DIMENSIONED BY NUMBER
EQUAL TO THE NUMBER OF PARAMETERS, NP.

C-- STARTING POINT

NRP=NPASS
L=1
ICK=2
ITTE=0
DOS 1=1,NP
B1(I)=P(I)
B2(I)=P(I)
T(I)=P(I)
S(I)=STEP(I)*10.

C-- INITIAL BOUNDARY CHECK AND COST EVALUATION

CALL PNPASS(P,1001)
IF(INOUT.LE.0)GO TO 10

IF(INOUT.LE.0)GO TO 5
WRITE(6,1000)
WRITE(6,1001)(J,P(J),J=1,NP)

RETURN

10 CALL PROC(P,C1)

IF(INOUT.LE.1)GO TO 11
WRITE(6,1001)ITTE,C1
WRITE(6,1001)(J,P(J),J=1,NP)

C-- BEGINNING OF PATTERN SEARCH STRATEGY
11      DATA INRD*,IJ,JP
12      DO 12 I=1,JP
      S(1)=S(1)/10.
      IF(T(1)>0)GO TO 20
      WRITE(6,1002)
      WRITE(6,1000)(J,S(J),J=1,JP)
20      IFAIL=.0.
      C-----PREFERENCE ABOUT T
      IC=0
      P(I)=T(I)*S(I)
      IC=IC+1
      CALL 'MURRS(U,101)
      IF((IC<GT.0)OR.T=0)
      CALL P=RL(6,CP)
      L=I+1
      IF(L+LT.3)GO TO 22
      WRITE(6,1002)L/C2
      WRITE(6,1000)(J,P(J),J=1,JP)
      22 IF(C1=C2)23,25,27
      23 IF((IC<GT.2)OR.T=0)
      S(I)=-S(I)
      GT021
      24 IFAIL=IFAIL+1
      P(I)=T(I)
      CONTINUE
      25 T(I)=T(I)
      C1=C1
      26 CONTINUE.
      IF(1+IFAIL+1)GO TO 33
T(I)=R2(I)+S(I)*(C2(I)-B(I))

P(I)=T(I)
S(I)=S(I)+1
CALL BOUNDST(T,P)
IF (IOUT.LE.1) CONTINUE
IF (I1.LE.11) IC=1

CONTINUE

DO 47 I=1,1000
  A(I)=0.2(I)
  COST=C1

  IF (T(I).LE.0) RETURN
  WRITE(6,1001) I,C1
  WRITE(6,1000) (J,P(J),J=1,1000)

1000 FORMAT(15X,$(+15.5)$/$/)
1001 FORMAT(/$\times 15$-ITERATION NO. ,15/5X,5HCOST= F15.6,2 X/)
1  1002 FORMAT(15X,$(+15$-STEP SIZE FOR EACH PARAMETER )
1003 FORMAT(/$\times 15$-STEP SIZE FOR EACH PARAMETER )
1004 FORMAT($\times 15$-FUNCTIONAL EVALUATIONS $+/\$
1  5HCOST= F15.6,2X/$\times$OPTIMAL PARAMETERS )
1005 FORMAT(1+155-INITIAL PARAMETERS OUT OF BOUNDS

END
APPENDIX H
A FORTRAN PROGRAM TO DESCRIBE THE DYNAMIC
AND STEADY-STATE BEHAVIOR OF THE DISTILLATION COLUMN

DIMENSION X(500), Y(500), V(500)

CALL PLO(500)

CALL PLO(300)

DATA C(100), C(200), C(300), TEND/1.1, 36.04, 0.83, 1.0/

READ 10, A, B, C, D, E, F

READ 10, PLOT, ALPHA

100 CONTINUE

PLOT=PLOT+1

IF (PLOT.LT.20) GOTO 100

READ 10, PLOT, V, F, CVP, UPSET

READ 2, L

FORMAT (3F2.10)

PARAM = 0

PARAM = 0

G = 0
TOTAL MATERIAL BALANCES TO GET TOTAL FLOW RATES

IF (UPSET = (+0.0) I4 TO 163)
  151
  VLTHS = FLUX / CT
  FLUX = CT * FST * 0.0
  VARP = FLUX * CP
  FOLLOW = FST * VARP
  VARP2 = VARP / VARP
  FLUX2 = FLUX / FOLLOW
  REFUX = FLUX2
  VARP3 = VARP2
  FOLLOW3 = FOLLOW / FLUX
  IN TO 163

NEW FLOWS FROM REFUX RATIO AND/OR FEED UPSETS

163 CONTINUE
  VARP4 = FOLLOW3 / CP
  FOLLOW3 = FST / VARP4

MAINTAIN CONSTANT FILLUP TO FEED RATIO
  SOLUP = MASS - FILL
  VARP5 = SOLUP
  VARP6 = VARP5 + VARP4
  FOLLOW3 = VARP6 - MASS
  FLUX4 = VARP6 / FOLLOW3
305 CONTINUE
       DTIMSE = 0.*SMLRT
       PRINT('T',/TMLRT

SETS INITIAL RELAXATION COEFFICIENTS

CSFL = DTIMSE*FLD + TRY
CSVAR = DTIMSE*VAR + TRY
CSFL = DTIMSE*FLD + TRY
CSVAR = DTIMSE*VAR + TRY
CSFL = DTIMSE*FLD + TRY
CSVAR = DTIMSE*VAR + TRY
CSFL = DTIMSE*FLD + TRY
CSVAR = DTIMSE*VAR + TRY
CSFL = DTIMSE*FLD + TRY
CSVAR = DTIMSE*VAR + TRY

PRINT('WAVE',/CSFL,CSVAR)
PRINT('PHASE',/CSFL,CSVAR)

X AND Y COMPONENTS OF FREE LIQUID AND VAPOR

159 CONTINUE
       ALPH = XFL > 0.*ALPHA
       SQUAD = CSVAR + 2.*FLD*FED*FED*(1.-ALPHA)
       SQUAD = FLD**2
       XFL = SQUAD*(1.0-4.*SQUAD*CSVAR)-SQUAD/1./(1.+SQUAD)
       YFL = XFL > 0.*ALPHA/(1.0-XFL*FLD*ALPHA)
       GO TO 160

160 CONTINUE
       XFL = 0.*
REAL CURL COMPOSITION

CONTINUE
READ 37, XTRY, YTRY
DO 290 J=1, N
READ 34, I, XTRY(J), YTRY(J), YTRY(J), XDP(N(J))
CM*PE=1
TIME (YDP)/!ID

CONTINUE
TIME = 0
PRINT 10, TIME
PRINT 10, XTRY, YTRY
PRINT 10
DO 300 I=1, M
PRINT 17, XTRY(I), YTRY(I), I, YTRY(I), XDP(I)
CONTINUE

II=1
II=I
DIST((XY))=X1.Y((XY))
TIME = TIME + TIME
II = II + 1
RETURN
YES = Y
NO = N
314  CONTINUE
315  CONTINUE
C C
C FEEDBACK CONTROLLER
C
FEVAR = FEVAR < CD
FOL0 = FEVAR < VAR0
VAR0 = 1500.0
VAR2 = VAR0 + FEVAR
SUM = CD * (CD - YTYV(NOT = 1)) * OTIME
C C
C FEEDBACK CONTROLLER
C
FEB = FEVAR (FEVAR = CD)
C C
C FEEDBACK CONTROLLER
C
FEB = FEVAR = (CD - YTYV(NOT = 1)) / 3 + B
C C
C FEEDBACK CONTROLLER
C
FEB = FEVAR = (CD - YTYV(NOT = 1)) / 2 * OTIME
C C
C FEEDBACK CONTROLLER
\texttt{DISP} = ((\texttt{DIST} - \texttt{DIS}) / \texttt{DLINE})
\texttt{DLINE} = \texttt{DIST}
\texttt{DIST} = \texttt{DIST} + \texttt{DLINE}
\texttt{A} = \texttt{A} + \texttt{DLINE}
\texttt{B} = \texttt{B} + \texttt{DLINE}
\texttt{IF} (A < X < B) \texttt{GO TO} \texttt{D17}
\texttt{GO TO} \texttt{X} + 1
\texttt{CONTINUE}
\texttt{IF} (T < \texttt{upper}) \texttt{GO TO} \texttt{410}
\texttt{CONTINUE}
\texttt{PRINT} \texttt{11}
\texttt{PRINT} \texttt{21}
\texttt{PRINT} \texttt{22}
\texttt{PRINT} \texttt{23}
\texttt{PRINT} \texttt{24}
\texttt{PRINT} \texttt{25}
\texttt{PRINT} \texttt{26}
\texttt{PRINT} \texttt{27}
\texttt{PRINT} \texttt{28}
\texttt{PRINT} \texttt{29}
\texttt{PRINT} \texttt{30}
POINT 48
48 FORMAT(X,Y,5X,SRTME,10X,4HYTT,10X,5X,BATM,10X,HDISTR,10X
1.63X,1ST,10X,5.6,3/1)
20 47 11,1
20 7X 1ST,11,1,1011,USRT,11,1,DISR,11,1,DISR,11,1,STTM(I)
407 TESTI: C
47 FORMAT(16,10,6(LX,5.12+1))
PRINT 304
304 FORMAT(16,10,6(LX,5.12+1))
PRINT 672,304
327 FORMAT(16,10,6(LX,5.12+1))
IF(K4=K-2+1) 1 TO 999
50 CALL I=100
/(/I=150:1)
X(1)=Y(I):1
Y(I)=Y(I):1
Y(I)=Y(I):1
Y(I)=Y(I):1
Z(I)=00
Z(I)=00
Z(I)=00
Z(I)=00
IF(K4=L-1) 10 TO 100
IF(K4=L-1) 10 TO 100
999 CALL AX23C++,++: POST (HIVERS)=-12.4,0.5,X(NX+1),X(N+2))
CALL AX23C++,++: COMPOSITIONS: +20,6,90,Y(X+1),Y(NX+2)
100 CALL AX23C++,++: POST (HIVERS)=-12.4,0.5,X(NX+1),X(N+2))
CALL AX23C++,++: COMPOSITIONS: +20,6,90,Y(X+1),Y(NX+2)
100 CALL AX23C++,++: POST (HIVERS)=-12.4,0.5,X(NX+1),X(N+2))
CALL AX23C++,++: COMPOSITIONS: +20,6,90,Y(X+1),Y(NX+2)
100 CALL AX23C++,++: POST (HIVERS)=-12.4,0.5,X(NX+1),X(N+2))
CALL AX23C++,++: COMPOSITIONS: +20,6,90,Y(X+1),Y(NX+2)
100 CALL AX23C++,++: POST (HIVERS)=-12.4,0.5,X(NX+1),X(N+2))
CALL AX23C++,++: COMPOSITIONS: +20,6,90,Y(X+1),Y(NX+2)
K = ?
101 CALL FLINE \( (x, y, -nx, l, d) \)
202 \( x \neq 0 \)
207 CALL PLOT \( (x, y, -d) \)
CALL PLOT \( (x, y, 0) \)
10 FORMAT \( (AN, 4X) \)
11 FORMAT \( (P, 1X) \)
12 FORMAT \( (S, 1X) \)
13 FORMAT \( (A, 8X) \)
14 FORMAT \( (A, 8X) \)
15 FORMAT \( (L, 5X, 10, 2F2.2, 21) \)
16 FORMAT \( (L, 5X, 10, 2F2.2, 21) \)
17 FORMAT \( (L, 5X, 10, 2F2.2, 21) \)
18 FORMAT \( (L, 5X, 10, 2F2.2, 21) \)
19 FORMAT \( (L, 5X, 10, 2F2.2, 21) \)
20 FORMAT \( (L, 5X, 10, 2F2.2, 21) \)
21 FORMAT \( (L, 5X, 10, 2F2.2, 21) \)
22 FORMAT \( (L, 5X, 10, 2F2.2, 21) \)
23 FORMAT \( (L, 5X, 10, 2F2.2, 21) \)
24 FORMAT \( (L, 5X, 10, 2F2.2, 21) \)
25 FORMAT \( (L, 5X, 10, 2F2.2, 21) \)
26 FORMAT \( (L, 5X, 10, 2F2.2, 21) \)
27 FORMAT \( (L, 5X, 10, 2F2.2, 21) \)
28 FORMAT \( (L, 5X, 10, 2F2.2, 21) \)
29 FORMAT \( (L, 5X, 10, 2F2.2, 21) \)
```
| FORMAT (120, X, 5HELEN, 15X, 5HVAPS, 15X, 10HELDSP) |
| FORMAT (120, X, 5HELEN, 15X, 5HVAPS, 15X, 10HELDSP) |
| FORMAT (120, X, 5HELEN, 15X, 5HVAPS, 15X, 10HELDSP) |
| FORMAT (120, X, 5HELEN, 15X, 5HVAPS, 15X, 10HELDSP) |
| FORMAT (120, X, 5HELEN, 15X, 5HVAPS, 15X, 10HELDSP) |
| FORMAT (120, X, 5HELEN, 15X, 5HVAPS, 15X, 10HELDSP) |
| FORMAT (120, X, 5HELEN, 15X, 5HVAPS, 15X, 10HELDSP) |
| FORMAT (120, X, 5HELEN, 15X, 5HVAPS, 15X, 10HELDSP) |
| FORMAT (120, X, 5HELEN, 15X, 5HVAPS, 15X, 10HELDSP) |
| FORMAT (120, X, 5HELEN, 15X, 5HVAPS, 15X, 10HELDSP) |
| FORMAT (120, X, 5HELEN, 15X, 5HVAPS, 15X, 10HELDSP) |
```

```
// 30. PLTTAB. 0LATT, NIST, LATEC, NIST, VOLUME=SRC=SKRACH, SCR=(1,1,1)  |
// 30. SYGIN D0  |
| 0.5 | 0.8 | 5.4 | 10.6 |
| 4.11560 | 0.07444 | 0.7 | 2.1 |
| 1.0 | 0.33 | 0.6 | 1.1 |
| 0.0214561 | 0.00000235 |
```

```
```

```
| 1.0216127 | 0.01124158 | 0.06134158 | 0.0316 127 |
| 0.0520 318 |
```
--- Riccati Equation Program for
--- Distillation Column
DOUBLE PRECISION D, A1, A2, A3
REAL M, J2, Y
CP = .35

102 READ (1), Y(1), J(1)
X = 0
Y = 0
M = 0
JT = 0.107, 1
TF = 0.1

101 A1 = X**2 * 10 / (A1 + 100)**2
A2 = 2 / (Y**2 * 10)**1 + 59 + 10
A3 = Y**2 * 10**1 + 10
N = N + 1
N = N + 1
IF (T, G, E) GO TO 101

IF (T, G, E) PRINT 103, 10
105 FORMAT (7E16.6)
PRINT 103, 100
100 FORMAT (1X, 1E16.2)
PRINT 103, 100
106 FORMAT (1X, 1E16.2)
PRINT 103, 100
C1 = (12 + 10 / X) * 100 / Y
CP = (12 + 10 / X) / Y
VITA

John A. Miller is the son of Mr. and the late Mrs. W. M. Miller of Baton Rouge, Louisiana. He was born in Mobile, Alabama on August 10, 1944. He graduated from Redemptorist High School in Baton Rouge in 1962.

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Candidate: John Anthony Miller

Major Field: Chemical Engineering

Title of Thesis: Applying Feedback and Feedforward Control

Approved:

[Signatures]

Major Professor and Chairman

Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:

September 25, 1969