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Giovanni Santostasi
Louisiana State University

Warren Johnson
Louisiana State University

Juhan Frank
Louisiana State University

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Detectability of Gravitational Waves from SN 1987A

Giovanni Santostasi, Warren Johnson

and

Juhan Frank

Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803-4001

ABSTRACT

We discuss the potential for detection of gravitational waves from a rapidly spinning neutron star produced by supernova 1987A taking the parameters claimed by Middleditch *et al.* (2000) at face value. Assuming that the dominant mechanism for spin down is gravitational waves emitted by a freely precessing neutron star, it is possible to constrain the wobble angle, the effective moment of inertia of the precessing crust and the crust cracking stress limit. Our analysis suggests that, if the interpretation of the Middleditch data is correct, the compact remnant of SN 1987A may well provide a predictable source of gravitational waves well within the capabilities of LIGO II. The computational task required for the data analysis is within the capabilities of current computers if performed offline and could be accomplished online using techniques such as demodulation and decimation.

Subject headings: stars: neutron — pulsars: general — gravitational waves — supernovae: individual (SN1987A)

1. Introduction

Middleditch *et al.* (2000) have claimed the likely first detection of the compact remnant of supernova 1987A (hereafter SN 1987A). Through fast photometry of a small region around the supernova they were able to find a modulated signal with a main frequency of 467.5 Hz and a modulation period of about 1000 seconds. Assuming that this was the spin frequency of the presumed pulsar, and following the source between 1992 and 1996, they were able to determine the spindown of the pulsar and changes in the precession period. The observations were complicated by times when the pulsation or the modulation were not visible or not so evident. The pulsations seem to have disappeared completely since 1996. While astrophysically plausible explanations for the intermittency of the signal can be devised appealing to the very complex nature of the SN 1987A environment, the reality of a pulsar with the described characteristics is at best very suggestive.

In this paper we will simply assume that the pulsar interpretation is correct, adopt the parameters derived by Middleditch *et al.* at face value and derive some interesting implications for the detection of gravitational waves from this source. We base our discussion on simple free precessing neutron star models (Alpar & Pines 1985; Cutler & Jones 2000; Jones & Andersson 2001).

The general problem of emission of gravitational waves from rotating and precessing neutron stars including pulsars and low-mass X-ray binaries has been recently reviewed by Jones (2001). For the particular case of SN 1987A, while other authors (Cutler & Jones 2000; Jones & Andersson 2001; Nagataki & Sato 2001) have also examined some of the consequences of the results of Middleditch *et al.* (2000), none of these papers provides a precise calculation of the intensity and detectability of gravitational waves from this source. The main aim of this paper is to provide these estimates and to discuss the likelihood of detection of gravitational waves from the hypoyhetical pulsar in SN 1987A by LIGO I and II. In Section 5 we estimate the time required to observe the signal with different types of detectors using coherent integration techniques. We show that, within a plausible range of values of the moment of inertia I_0, the gravitational wave strain is big enough to be detectable by LIGO II within integration times ranging from days to months. Thus, if the interpretation of the periodicities in the optical observations is correct, 1987A *should* be a predictable source of gravity waves for ground based observatories. The computational requirements for the data analysis discussed in Section 6 are non-trivial but within the capabilities of modern computers.

2. Summary of the Observations

Middleditch *et al.* (2000) discuss fast photometry observations of the remnant of the supernova 1987A carried out at different times over the period 1992–1996 from several observatories. During that time interval the pulsar was detected several times at slightly different frequencies. The power in the signal faded since 1993 and was last detected in February 1996. While they found "no clear evidence of any pulsar of constant intensity and stable timing," they did find "emission with a complex period modulation near the frequency of

467.5 Hz - a 2.14 ms pulsar candidate". They also point out that: "the frequency of the signals followed a consistent and predictable spin-down ($\sim$2-3 x 10^{-10} Hz/s) over the several year timespan. They find evidence for "modulation of the 2.14 ms period with a $\sim$1,000 s period which complicates its detection."

The observed modulation of the 2.14 ms period can be interpreted as the effect of precession due to some deformation or crustal density distribution which is not symmetric about the axis of rotation, including the case in which the precesing object itself possesses axial symmetry about a body axis which is not aligned with the axis of rotation. In the absence of any external torques, this situation is termed "free precession". Classical mechanics tells us that the ratio between the precession frequency and the rotation frequency is proportional to the size of the deformation (e.g. Marion & Thornton (1995)). The size of the deformation and the frequency of rotation determine the rate of spin-down if the neutron star is assumed to lose energy mainly due to gravitational radiation.

A freely precessing neutron star emits gravitational waves (e.g. Zimmerman & Szedentis (1979), Zimmerman (1980)). Using the general relativistic energy loss equation and the classical mechanics relationship between ellipticity, rotation and precession frequency, we have that the spin down rate is proportional to the square of the precession frequency under the assumption that all the energy is lost due to gravitational back reaction. If an electromagnetic contribution to the spin down rate is also present, this term would be independent of ellipticity and would be approximately constant during the time span of the observations. The data shown on Fig. 9 of Middleditch *et al.* (2000) are consistent with a linear correlation between spin down rate and the square of the precession frequency going straight through the origin, i.e. with zero contribution from magnetic dipole emission. Thus Middleditch *et al.* (2000) conclude that the characteristics of the 2.14 ms signature and its $\sim$1,000 s modulation are consistent with precession and spindown via gravitational radiation of a neutron star with effective non-axisymmetric oblateness of $\sim 10^{-6}$. We re-examine some aspects of this problem in Section 3.

3. A Model for the Precessing Neutron Star

3.1. System Geometry

Rotating neutron stars are often mentioned as a possible continuous source of gravitational radiation. Usually what is envisioned is that the star has an axissymetric deformation perpendicular to the axis of rotation to allow for a changing mass quadrupole that will generate gravity waves. Such a prolate or oblate star, tumbling about an axis perpendicular to the axis of symmetry, will emit gravity waves at twice the rotation frequency.

If the star is deformed on a axis that is at any other angle with the rotation axis then it will precess as a spinning top, and will emit at both twice the rotation frequency and at the rotation frequency. The simplest situation is that the star is a rigid body and has just two non-equal principal moments of inertia. We have then $I_1 = I_2 = I_0 - \Delta I_{\rm d}/3$, $I_3 = I_0 + 2/3\Delta I_{\rm d}$, so that $\Delta I_{\rm d} = I_3 - I_1$. I_0 is the average value of the moment of inertia and $\Delta I_{\rm d} \ll I_0$. A more complete and realistic model is considered further below but the simplest case remains the basis for the discussion of precessing neutron stars. The main equations are the same even in the more realistic case with minor modifications. Figure 1 shows our convention in the orientation of the important vectorial quantities involved in the problem.

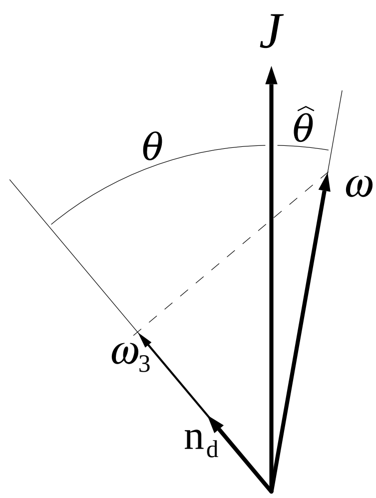

Fig. 1.— The reference plane for a freely precessing body. This diagram shows the respective orientation of the axis of deformation $\mathbf{n}_{\rm d}$, angular momentum $\mathbf{J}$, and axis of rotation ω of the star. The projection of the instantaneous angular velocity vector ω on to the symmetry axis $\mathbf{n}_{\rm d}$ is indicated by ω_3.

We can define the total moment of inertia as:

$$\mathbf{I} = I_0\boldsymbol{\delta} + \Delta I_{\rm d}\left(\mathbf{n}_{\rm d}\mathbf{n}_{\rm d} - \boldsymbol{\delta}/3\right) \tag{1}$$

where $\mathbf{n}_{\rm d}$ is a unit vector pointing along the body symmetry axis and $\boldsymbol{\delta}$ is the unit tensor. Now define the "ellipticity" as a small quantity $\epsilon = \Delta I_{\rm d}/I_0$, then classical mechanics implies

$$\epsilon = \frac{\Omega_{\rm p}}{\omega_3} = \frac{\Omega_{\rm p}}{\omega\cos\gamma} \approx \frac{\Omega_{\rm p}}{\omega\cos\theta} \tag{2}$$

where θ is the angle between the total angular momentum and the vector $\mathbf{n}_{\rm d}$, $\gamma = \theta + \widehat{\theta} \approx \theta$. The angle between the rotation axis and the angular momentum is a small quantity of order $\Delta I_{\rm d}/I_0$. The quantity $\Omega_{\rm p}$ is the precession frequency and ω is the rotation frequency and ω_3 its projection along the 3-axis that coincides in this case with the axis $\mathbf{n}_{\rm d}$. We will proceed from the assumption that we know the parameters $\Omega_{\rm p}$ and ω from the observations of SN 1987A by Middleditch *et al.* (2000).

The observed modulation or precession period varied during the span of the observations in the range from approximately 935 s to 1430 s, while ω or the spin period varied measurably but relatively little. Consequently the observed variations in $\Omega_{\rm p}$ must be attributed to variations in ϵ or θ or both. Note, however, that the correlation between $\dot{\omega}$ and $\Omega_{\rm p}$ claimed by Middleditch *et al.* requires that θ remain constant. Jones & Andersson (2001) and Jones (2001) have claimed that it is not easy to imagine how significant variations in ellipticity can occur without affecting the wobble angle. We shall return to this question in Section 3.3 and argue that it is in fact unlikely that variations in epsilon can significantly change the wobble angle.

3.2. Gravitational radiation caused by misalignment

To determine the size of deformation and consequently the strain carried by the gravitational radiation on earth we need to evaluate the wobble angle θ. This can be done assuming that the star is losing energy solely through gravitational radiation. Then we can use the general relativistic equation for the rate of energy emission by gravitational waves (Zimmerman & Szedentis 1979; Zimmerman 1980):

$$\dot{E} = -\frac{2}{5}\frac{G}{c^5}\epsilon^2 I_0^2\omega^6\sin^2\theta\left(16\sin^2\theta + \cos^2\theta\right), \tag{3}$$

where the first and second terms in parenthesis represent the contributions at 2ω and ω respectively.

If the only source of energy for this emission is the neutron star's rotational energy reservoir $E = 1/2\ I\omega^2$, we have then $\dot{E} = \omega\dot{\omega}I$, so that

$$\dot{\omega} = \frac{2}{5}\frac{G}{c^5}\epsilon^2 I_0\ \omega^5 \sin^2\theta \left(16\sin^2\theta + \cos^2\theta\right) . \tag{4}$$

Since the change in angular velocity $\dot{\omega}$ is known from observations, it is possible to solve equation (4) for the ellipticity as a function of the wobble angle for any given I_0. Figure 2 shows the relationship between the ellipticity ϵ and the wobble angle θ derived from equation (2) for the observed range of precession periods between 935 s and 1430 s (monotonically increasing curves). Figure 2 also shows the result of solving (4) (monotonically decreasing curves), for the range of observed values of $2 \times 10^{-10} < \dot{\nu} < 3 \times 10^{-10}$ Hz/s for an arbitrarily chosen representative value of $I_0 = 10^{44}$ g cm^2 .

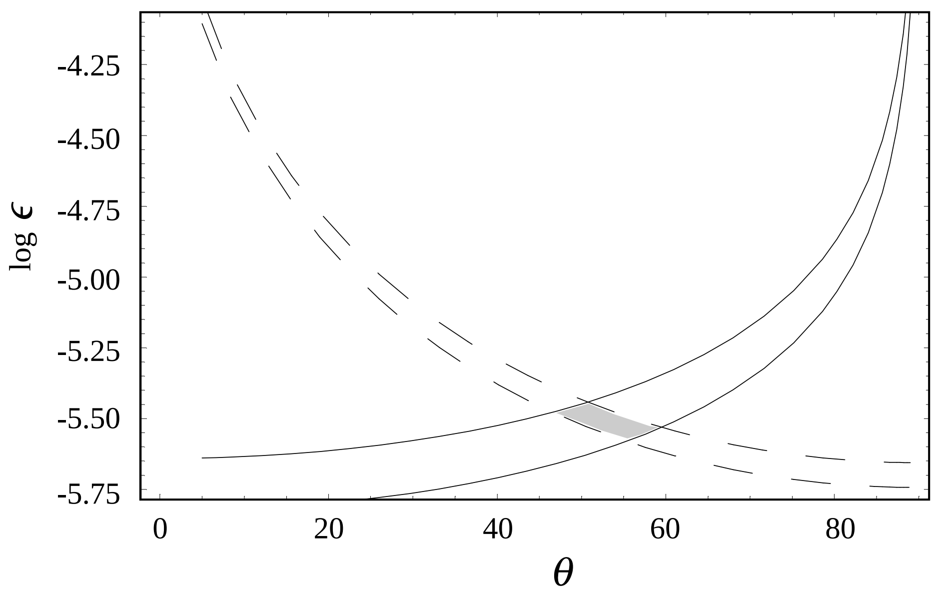

Fig. 2.— The general relativistic (dashed) and classical (solid) relationships between ϵ and θ given the ranges of observed values of spindown rate and precession frequency. The curves shown correspond to $I_0 = 10^{44}$ g cm^2, a value intermediate between the minimum (just the crust precesses) and the maximum (all the star is involved in the precession). The possible solutions for the adopted value of the moment of inertia lie in the shaded region.

The relativistic equation (4) depends on the value I_0. This value is the average moment of the inertia of the part of the star that actually participates in the precession. If the star has a crust and a liquid interior then I_0 is the crust's moment of inertia and that of any liquid coupled to the crust. In fact, part of the liquid should be stress free and not influenced by the precession. So we can take I_0 to be an arbitrary quantity equal or less than entire moment of inertia of the star $I_{\rm star} = \frac{2}{5}MR^2 = 1.12 \times 10^{45}$ g cm$^2 M_{1.4} R_6^2$, where $M_{1.4}$ is the mass of the star in units of 1.4 solar masses and R_6 the radius in units of 10^6 cm. If just the crust participates in the precession then $I_0 \approx 1/100\ I_{\rm star}$ according standard neutron star theory. Now, the classical mechanics equation (2) and the relativistic equation (4) have to be satisfied at the same time. This means that for given observed $\Omega_{\rm p}, \omega$, $\dot{\omega}$ and choice of I_0 the functions have to meet at a point in the parameter space $\epsilon - \theta$. If we consider the moment of inertia the unknown parameter of our problem we can determine which wobble angle the star should have according the value of I_0. This is illustrated in Figure 3. It seems that the 1987A remnant had some relatively big and rapid changes in precession frequency during the first years of observation. The astrophysical explanation for this could be a very active dynamic environment in the young neutron star, that can bring abrupt changes in the density of the crust, fractures and re-arrangement of surrounding material. We already mentioned that Middleditch *et al.* find a power two relationship between the observed change in $\dot{\omega}$ and $\Omega_{\rm p}$. This relationship holds exactly if we substitute equation (2) into (4), namely:

$$\dot{\omega} = \frac{2}{5}\frac{G}{c^5}\frac{\Omega_{\rm p}^2}{\cos^2\theta} I_0\ \omega^3 \sin^2\theta \left(16 \sin^2\theta + \cos^2\theta\right), \tag{5}$$

and require that θ remain constant while ϵ and hence $\Omega_{\rm p}$ vary.

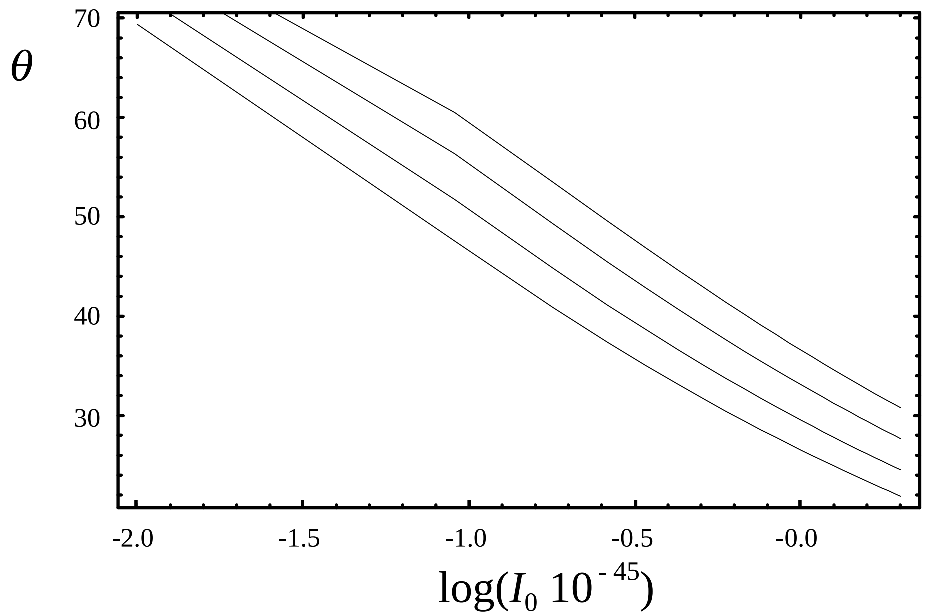

Fig. 3.— Wobble angle θ as a function of the moment of inertia involved in the precession for the four possible combinations of precession period (935 s and 1430 s) and spin down rate (2×10^{-10} Hz/s and 3×10^{-10} Hz/s). The curves shown, from bottom to top, correspond to the following pairs of parameters: (935 s, 2×10^{-10} Hz/s), (935 s, 3×10^{-10} Hz/s), (1430 s, 2×10^{-10} Hz/s), and (1430 s, 3×10^{-10} Hz/s), respectively.

3.3. The constancy of the wobble angle

Middleditch *et al.* (2000) display graphically the correlation between $\dot{\omega}$ and $\Omega_{\rm p}^2$. By reading off the values of these variables and applying the method and equations of Section 3.2, it is possible to determine the values of the wobble angle θ required for each individual pair of values $\dot{\omega}$ and $\Omega_{\rm p}$ for any assumed moment of inertia involved in the precession. This exercise reveals that despite variations of ϵ and $\Omega_{\rm p}$ exceeding a factor of 1.5, the wobble angle does not change by more than a couple of degrees and appears consistent with remaining constant within experimental errors.

In the case of a freely precessing solid body, the wobble angle is largely determined by initial conditions: taking the principal axes introduced in Section 3.1, if the associated moments of inertia remain constant, then $\omega_3 = \omega\cos\gamma$, also remains constant (see Fig. 1). It is easy to generalize Euler's equations to the case in which the principal moments of inertia change due to unspecified internal forces while the external torques vanish and the total angular momentum is conserved:

$$\frac{{\rm d}I_1\omega_1}{{\rm d}t} = \omega_2\omega_3(I_2 - I_3) \quad (6)$$

$$\frac{{\rm d}I_2\omega_1}{{\rm d}t} = \omega_3\omega_1(I_3 - I_1) \quad (7)$$

$$\frac{{\rm d}I_3\omega_1}{{\rm d}t} = \omega_1\omega_2(I_1 - I_2)\,. \quad (8)$$

When the principal moments of inertia are all of the form $I_i = I_0 + \epsilon_i$, then clearly all the time derivatives are of order $\sim \epsilon_i$ and even if the given ϵ_i were to change by factors of a few, the result would be a small wobble of the tip of ω in the body frame. Therefore we conclude that while the variations probably detected by Middleditch *et al.* (2000) in both ϵ and $\Omega_{\rm p}$ were significant, they do not imply any measurable change in θ. Referring back to Fig. 1 we see that $\widehat{\theta}$ may indeed change by amounts comparable to itself, but the wobble angle θ would change very little. This conclusion is contrary to what Jones & Andersson (2001) and Jones (2001) have claimed regarding the wobble angle, and thus it makes more plausible that the remnant of SN 1987A is indeed freely precessing while undergoing changes in ϵ and $\Omega_{\rm p}$.

3.4. A more realistic model: allowing for a elastic crust and presence of a fluid interior

The textbook discussion of a precessing body assumes that the object is perfectly rigid. A more realistic neutron star will have a more or less elastic shell, and a fluid interior. The fluid is supposed to be composed of a electron-proton plasma and a neutron superfluid. The plasma fluid interior can couple to the crust because of friction. Under these conditions the system is not simply described by the rigid body model.

Usually the approach taken to explore the properties of such more complicated systems is to understand the effect of one additional complication at the time. The paper of Cutler & Jones (2000) addresses these complications and shows how the more realistic model needs to be modified to account for these complications. In this section we summarize these results and apply them to the particular problem of the detection of the 1987A remnant.

3.4.1. The elastic crust

In the case of an elastic crust's shell we have to write the moment of inertia as:

$$\mathbf{I} = I_0 \boldsymbol{\delta} + \Delta I_{\rm d} \left(\mathbf{n}_{\rm d} \mathbf{n}_{\rm d} - \boldsymbol{\delta}/3 \right) + \Delta I_\omega \left(\mathbf{n}_\omega \mathbf{n}_\omega - \boldsymbol{\delta}/3 \right) \tag{9}$$

this is the sum of a spherical part and two small quadrupole contributions. The first term is the moment of the inertia of the undeformed shell, in the absence of rotation. The second term is a deformation due to Coulomb lattice forces and the third is the deformation due to centrifugal forces. The vector $\mathbf{n}_{\rm d}$ determines the axis of symmetry of the deformation $\Delta I_{\rm d}$. The vector $\mathbf{n}_\omega$ lies along the axis of rotation and determines the direction the axis of symmetry of the centrifugal deformation ΔI_ω.

The quantity ΔI_ω is caused by the deformation due to the centrifugal force, its value is determined by:

$$\frac{\Delta I_\omega}{I} = \frac{I_0^2 \omega^2}{4\left(A+B\right)} \tag{10}$$

where the constants A and B depend on the particular stellar equation of state. The constant A is on the order of the gravitational binding energy and the constant B is on the order of the total electrostatic binding energy of the ionic crustal lattice. The quantity B is much smaller than A so we can make the approximation:

$$\begin{aligned} \frac{\Delta I_\omega}{I} &\approx \frac{I_0^2 \omega^2}{4A} \approx \frac{\omega^2 R^3}{GM} \\ &\approx 2.1 \times 10^{-3} \left(\frac{f}{100\,{\rm Hz}} \right)^2 R_6^3 / M_{1.4} \end{aligned} \tag{11}$$

where f is simply $\omega/2\pi$.

In the general situation of non parallel $\mathbf{n}_{\rm d}$ and $\mathbf{n}_\omega$, the body will precess. As a consequence of $\mathbf{n}_\omega$ being in the direction of the rotation axis (at any given instant) the body will behave as a axysimmetric top even if the body has a triaxial shape (Cutler & Jones 2000). The angular momentum of an arbitrary body $\mathbf{J} = \mathbf{I}\boldsymbol{\omega}$, with the inertia tensor $\mathbf{I}$ given by equation (9), can be rewritten as $\mathbf{J} = \mathbf{I}_{\rm eff}\boldsymbol{\omega}$, where $\mathbf{I}_{\rm eff} = (I_0 + 2\Delta I_\omega/3)\boldsymbol{\delta} + \Delta I_{\rm d}\,(\mathbf{n}_{\rm d}\mathbf{n}_{\rm d} - \boldsymbol{\delta}/3)$. Thus in this case the three moments of inertia in the original body axes are:

$$\begin{aligned} I_1 &= I_0 - \Delta I_{\rm d}/3 + 2\Delta I_\omega/3 \\ I_2 &= I_1 \\ I_3 &= I_0 + 2\Delta I_{\rm d}/3 + 2\Delta I_\omega/3 \end{aligned} \tag{12}$$

The main implication of this is that even in the case of a elastic crust the star will still behave for what concerns precession as a biaxial rigid object. The fundamental equation (2) holds for this situation (with the appropriate inertia moments given above) and this means that the only piece of the moment of inertia that contributes to precession is $\Delta I_{\rm d} = I_3 - I_1$.

3.4.2. The presence of a fluid interior

To further improve our model we consider the effects of the presence of a fluid interior. The shape of the cavity and the viscosity of the fluid contained are important parameters. If the cavity is spherical, the presence of the fluid in absence of viscosity has no influence on the precession. If the cavity is non-spherical, then there will be a reaction force that is generated by the tendency of the fluid to assume axial symmetry around the axis of rotation. The shell will be pushed by the fluid. This problem is solved in the literature (Lamb 1932; Jones & Andersson 2001) under the assumptions of uniform fluid vorticity, small cavity ellipticity, and small wobble angle. A small wobble angle is adopted in the treatment given by Lamb (1932) only for mathematical convenience, but this assumption can be safely relaxed as long as the ellipticity remains small without altering the result. The upshot is that the usual precession equations described above are still valid. The only modification to take into account is that $\Delta I_{\rm d}$ refers to the difference in moment of inertia along the axis 1 and 3 of the whole star, and I_0 refers to the average moment of the inertia of the shell only.

In the presence of friction between the crust and a part of the interior fluid in contact with the crust we could have some coupling between the motion of the crust and the core. It can be shown that in the case of neutron stars the coupling is very weak and the core does not participate in the precession. If there are frictional forces at work in the interior of the star these will serve just to damp the free precession on time scales between 400 and 10^4 precession periods (Alpar & Pines 1985).

3.4.3. The problem of pinning

Jones & Andersson (2001) following and extending previous work by Shaham (1977) conclude that the presence of pinning of the superfluid to the crust, at least in the simplest possible configuration does not change the form of the equations that describe the precession. The main modification required is that the relevant *effective ellipticity* is generated by combination of the lattice deformation and the moment of the inertia $I_{\rm SF}$ of the pinned fluid, as in the following:

$$\epsilon_{\rm eff} = \frac{\Delta I_{\rm d}}{I_0} + \frac{I_{\rm SF}}{I_0} \tag{13}$$

The most common theories on pulsar glitches give a precise prediction on the precession behavior in the presence of pinning in a neutron star. The theories require at least a few percent of the total moment of inertia of the star to be in the pinned superfluid. Current understanding of neutron star properties indicates that the moment of inertia of the crust is a few percent of the total moment of inertia of the star. These considerations imply that:

$$\epsilon_{\rm eff} = \frac{\Omega_{\rm p}}{\omega \cos\theta} = \frac{\Delta I_{\rm d}}{I_0} + \frac{I_{SF}}{I_0} \approx 1, \tag{14}$$

in the case of small deformations $\Delta I_{\rm d}$. The precession and rotation frequency should be close in value if there is a sizable quantity of superfluid that is pinned to the crust. These predictions are not confirmed by observations of the three strong cases of precession in neutron stars: PSR B1642-03 (Jones & Andersson 2001), PSR B1828-11 (Stairs *et al.* 2000) and the SN 1987A remnant, where the precession is on a time scale much longer than the rotation. The conclusion is that if the free precession interpretation of the modulation of the signal of these pulsars is correct, then there is almost no pinned superfluid in these stars (see Link & Cutler (2002) for further discussion of this issue).

4. The wobble angle and crust fracture

Precession will cause the rotation axis of the star to change its position relatively to the body frame. This means that the centrifugal force distribution will be a function of position and time with a timescale on the order of the precession period. If the star has an elastic crust, then it will change its shape in response to variations in the centrifugal force and cause time dependent stresses in the crust. A simple order of magnitude estimate of the strain on the crust σ due to precession yields:

$$\sigma \approx (\Delta I_\omega / I) \sin\theta \approx 0.046 \sin\theta \; R_6^3 / M_{1.4}. \tag{15}$$

Experiments with crystals suggest an upper limit for the maximum possible strain sustainable by the crust before breaking, i.e. $\sigma_{\rm max} \approx 10^{-2}$. This implies that the possible maximum wobble angle for our pulsar must satisfy:

$$\theta \leq \arcsin\left(21.8\sigma_{\rm max}\right), \tag{16}$$

which would require $\theta \leq 13^\circ$ for $\sigma_{\rm max} = 0.01$. Since all of our estimated wobble angles are larger than 30°, either $\sigma_{\rm max}$ is larger for the crust or the model is too simple to account for the observations or our interpretation of the observations is incorrect.

5. The strength of the radiation at earth

Zimmerman & Szedentis (1979) and Zimmerman (1980) treat the case of a body with two distinct moments of inertia and obtain the following expressions for the strain parameter h of gravity waves from a neutron star at a distance r from Earth and average moment of inertia I_0:

$$\begin{aligned} h_+ &= \frac{G}{c^4}\frac{2I_0\omega^2\varepsilon\sin\theta}{r}\times \\ &\quad \left[(1+\cos^2 i)\sin\theta\cos 2\omega t + \cos i \sin i \cos\theta \cos\omega t\right] \\ h_\times &= \frac{G}{c^4}\frac{2I_0\omega^2\varepsilon\sin\theta}{r}\times \\ &\quad \left[2\cos i \sin\ \theta \sin\left(2\omega t\right) + \sin i \cos\theta \sin\omega t\right], \end{aligned} \tag{17}$$

where i is the unknown angle between the angular momentum vector $\mathbf{J}$ and the plane of the sky. It is important to notice that the time dependence of the wave forms is sinusoidal with two main frequencies at ω and 2ω. If the object was rotating along its symmetry axis it will emit just at a frequency 2ω (it will have also to be deformed along the axis perpendicular to the rotation axis).

We see that the frequency of rotation ω is one of the important parameters that determine the strength of the gravitational radiation on earth. We know from observation the value of the rotation frequency to be $\omega = 2\pi\ 467.5$ Hz. To determine what is the strength of the radiation we need to know also the moment of inertia I_0 involved in the precession and the wobble angle θ. We showed previously that θ depends on how much of the moment of inertia of the star is actually involved in the precession, as shown by the general relativistic energy loss equation (3). When this relationship between I_0 and θ is factored in the strain equations (17), we can determine the strength of the gravitational radiation on earth as a function of the parameter I_0. The result is shown in Figure 4.

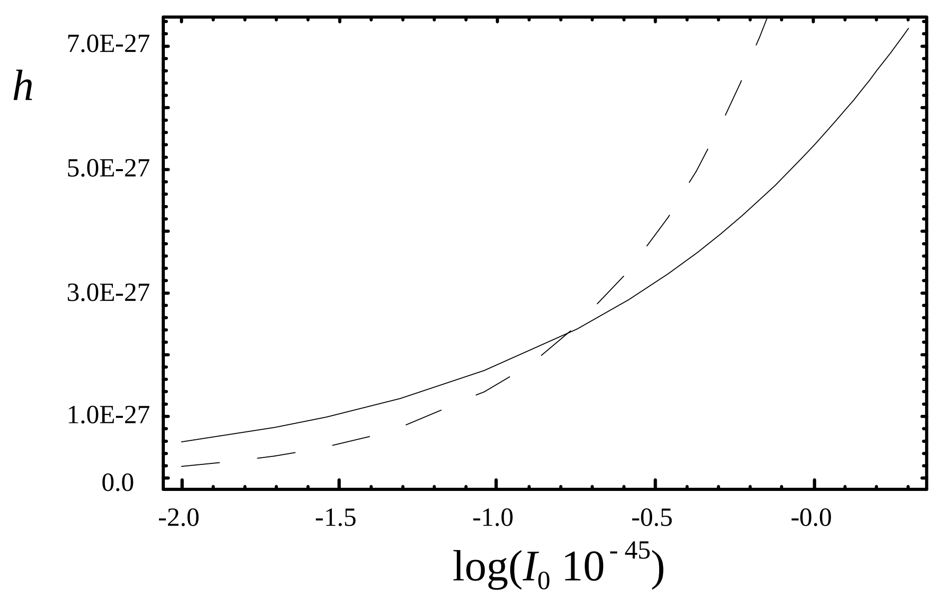

Fig. 4.— The amplitude of the strain at the Earth given by equation (18) – leaving out dependences on i and t – as a function of the moment of inertia involved in the precession, in units of 10^{45} g cm^2. The dashed curve corresponds to the contribution from the ω term and the solid curve to the 2ω term.

These small values for h may appear to require an impossible level of sensitivity from the bar detectors or interferometers existing today or soon available. It is important to notice that the source is a continuous source of radiation, of which all the fundamental parameters (besides the phase of the signal) are known. So it possible, in principle, to integrate the detector data over a long period (even years) to extract the signal from the incoherent noise. A detailed calculation of the necessary integration time τ is required. To do so we use the following equation:

$$h_n = \sqrt{S_h(f_0)}\sqrt{BW}, \tag{18}$$

This equation expresses the level of the strain h_n of the noise in the data from a detector with characteristic noise spectrum S_h. The equation evaluates the value $S_h(f_0)$ of the spectrum at the precise frequency f_0 of the looked for gravitational wave signal. The quantity BW is the bandwidth of the periodic signal. From Fourier Analysis theory in the case of a sinusoidal signal, the value of $BW = \frac{1}{\tau}$, where τ is the observation or "integration" time. So the required integration time is:

$$\tau = \left(\frac{\sqrt{S_h(f_0)}}{h_n}\right)^2 \tag{19}$$

Now we require that the noise level in the data from the detector be at least of the same size of the signal (it should less, 4 times less for a 4 σ confidence level in the statistics, for example). A typical value for the noise strain $h_{\rm d} = \sqrt{S_h(f_0)}$ in the existing bar detectors such as the Louisiana State University's ALLEGRO or first generation light interferometers as LIGO I, is currently of order 10^{-20}. The fully optimized LIGO I sensitivity is projected to attain a minimum noise strain $h_{\rm d} \approx 10^{-22}$ in a couple of years (see Fig. 5). We see from Figure 4 that a typical value for the signal amplitude strain is approximately $h_s = 5 \times 10^{-27}$. So if h_n is chosen to be $\approx 1/4 \times h_s = 1.25 \times 10^{-27}$ then we get that:

$$\begin{aligned}\tau = \left(\frac{\sqrt{S_h(f_0)}}{h_n}\right)^2 &= \left(\frac{10^{-20}}{1.25 \times 10^{-27}}\right)^2 \left(\frac{h_{\rm d}}{10^{-20}}\right)^2 \text{s} \\ &\approx 2 \times 10^6 \left(\frac{h_{\rm d}}{10^{-20}}\right)^2 \text{yr}.\end{aligned} \tag{20}$$

This is a time obviously too long to be useful, even with a fully optimized LIGO I. Thus the remnant of supernova 1987A is undetectable by the existing gravitational wave detectors, but may be detectable if the sensitivity of the detectors planned for the near future reaches the estimated levels.

In fact preliminary estimates of the noise spectrum of the second generation Laser Interferometer, LIGO II are very promising (see Fig. 5). LIGO II will be built on the experience of the first LIGO and will be a much better gravitational wave observatory. It will be on line in 4 or 5 years from now. According to Fig. 5, there is a lowest point in the total strain-noise (the sum of different kind of expected noises). This point is about $h(f_r) = 1.5\times10^{-24} / \sqrt{Hz}$ at a frequency of 350 Hz. But the LIGO II detector will be able to use narrow banding to shift this lowest point in noise level to higher frequencies. For further discussion of noise levels expected in (Advanced) LIGO II, see Abbott *et al.* (2002).

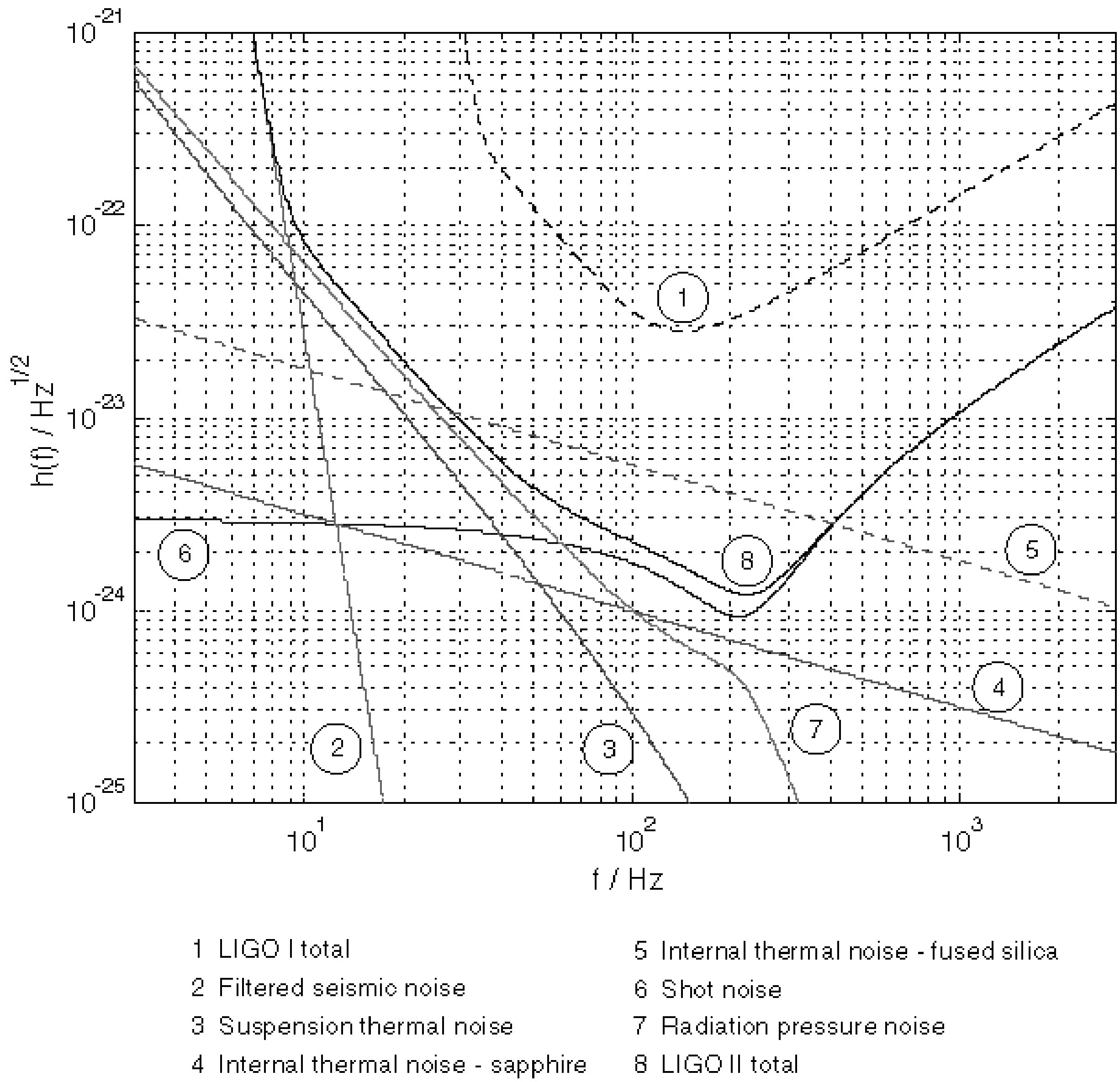

Fig. 5.— Preliminary estimates of the noise spectrum expected for LIGO II. The contributions from various sources are shown separately and added together (See http://www.ligo.caltech.edu/docs/M/M990288-A1.pdf).

So we could take this as the level of noise at the frequency of emission of 1987A. Figure 6 shows the required time of integration for LIGO II as a function of the parameter I_0. We can see that within a certain range of possible values of I_0 it will be possible to detect the signal from the 1987A remnant in reasonable time. This time is actually few days if most of the moment of the inertia participates in the precession.

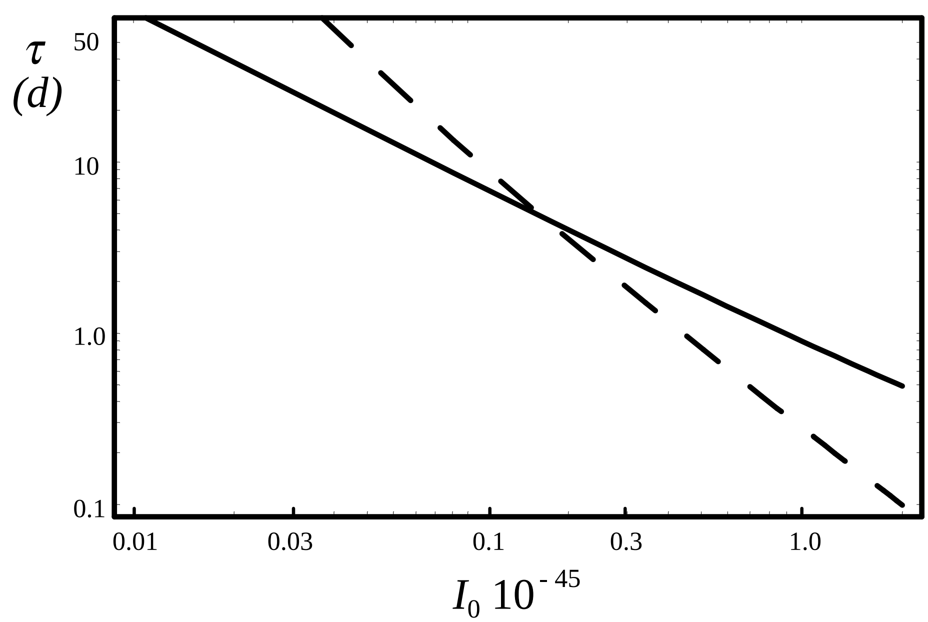

Fig. 6.— Estimated integration times for a 4 sigma detection by LIGO II for the signal at ω (dashed) and at 2ω (solid) as a function of I_0 in units of 10^{45} g cm^2. The curves shown above have been estimated using the minimum strain level for an optimized LIGO II, $h_{\rm d} = 1.5 \times 10^{-24}$. For higher noise levels, the integration times scale as $(h_{\rm d}/1.5 \times 10^{-24})^2$.

6. Data analysis and templates

The simple estimates for integration times in the previous section assume that the signal is sinusoidal with constant frequency, which clearly is not the case if our interpretation of the properties of the observed optical signal is correct. This raises the question of how detectable would be the signal if one allows for the observed changes in the spin frequency, precession frequency and spin down rate. We show here that simple templates that describe the observed behavior can be constructed, and that the required number of such templates and the total computational effort needed to adequately keep track of the phase and to detect a signal with the presumed properties is within current computational capabilities as long as there is phase stability over a time series of length comparable or longer than the integration time. Since the required integration times are on the order of 10–30 days, and the phase stability in the Middleditch data was comparable or better than that, this requirement is likely to be satisfied.

Following standard treatments we write the time-dependent frequency as a Taylor series

$$\omega(t) = \sum_{n=0}^{\infty} \frac{\omega_n(0)t^n}{n!}, \tag{21}$$

where $\omega_n(0)$ indicates the n-th derivative at some arbitrary reference time taken to be zero without loss of generality. A given choice of the parameters $\omega_n(0)$ constitutes a particular choice of template. Then the phase difference between two different templates is

$$\Delta\varphi(t) = \sum_{n=0}^{\infty} \frac{\Delta\omega_n(0)t^{n+1}}{(n+1)!} = \Delta\omega(0)\, t + \frac{1}{2}\Delta\dot{\omega}(0)\, t^2 + \cdots , \tag{22}$$

where $\Delta\omega_n(0)$ represents the difference between the n-th derivatives for a pair of templates.

During the observations a typical value for the spin-down was $\dot{f} \sim 10^{-10}\,\mathrm{s}^{-2}$ and showing a secular decreasing trend. Since the earliest opportunity for LIGO to observe this source is $T \sim 10$ years away, we take the uncertainty in the frequency to be on the order of $\dot{f}T \sim 3\times10^{-2}\,\mathrm{s}^{-1}$ or a bandwith of $BW = 3\times10^{-2}$ Hz. This is an estimate for the total range of frequencies to be explored. The standard phase stability requirement (Jaranowski & Krolak 2000) $\Delta\omega\tau \lesssim \pi/4$ over the integration time, yields an estimate of how closely spaced the frequency templates have to be. For $\tau \sim 10\,\mathrm{d} \sim 10^6\,\mathrm{s}$, this argument yields $\Delta f = \Delta\omega/2\pi \sim 10^{-7}$ Hz. Consequently the total number of frequencies to be sampled is on the order of $BW/\Delta f \sim 3\times10^5(\tau/10\mathrm{d})$.

To calculate how many frequency derivative values need to be considered, we estimate that the total range of values is comparable to $\dot{f}$ itself. The phase stability requirement then yields the spacing between spin-down values: $\Delta\dot{f} = \Delta\dot{\omega}/2\pi \lesssim \tau^{-2}/4 \sim 2.5 \times 10^{-13}$ Hz/s. And therefore the total number of values of $\dot{f}$ to be sampled is approximately $\dot{f}/\Delta\dot{f} \sim 400(\tau/10\mathrm{d})^2$.

Finally, the total number of two-parameter templates N_f we require is given by the simple product of the number of frequency values times the number of frequency derivative values: $N_f \sim 1.2 \times 10^8(\tau/10\mathrm{d})^3$. The total number of floating-point operations required to carry out the search of these templates over an integration time τ is approximately given by the formula (Brady *et al.* 1998)

$$N_{\mathrm{fpo}} = 6fN_f\tau[\log_2(2f\tau) + 1/2]\,, \tag{23}$$

where f is the maximum frequency to be searched (say 500 Hz). With the values derived above, this yields a total computational load of approximately $1.1 \times 10^{19}(\tau/10\mathrm{d})^4$ floating-point operations, which would require 3 months of calculations for a Teraflop machine. While this load is not trivial, it can be achieved by either processing the data offline or using a machine clocking at least $11(\tau/10\mathrm{d})^3$ Teraflops for online processing. However, the above estimate is an upper limit that makes little use of our prior knowledge of the expected frequency and frequency range of the signal. We need only to search over the BW of 3×10^{-2} Hz, whereas the standard argument above assumes we are searching for signals over the entire band from 0 to 500 Hz. The computational task can be significantly reduced by first 'demodulating' or filtering the signal to the bandwith BW estimated above and then 'decimating' or reducing the signal sampling rate to the bandwith. This technique cuts the processing rate essentially by a factor $BW/f \sim 6 \times 10^{-5}$ to approximately $0.7(\tau/10\mathrm{d})^3$ Gigaflops, well within the capabilities of current computers.

7. Conclusions

In this paper we discussed the implications of the observation of a precessing neutron star in the remnant of supernova 1987A for gravitational wave detection. We used the observed data on rotational velocity, spin down and precession rate to determine the value of the possible asymmetric deformation that causes the precession.

To estimate the size of deformation it is important also to determine the wobble angle between the axis perpendicular to the deformation and the rotation axis. General relativity gives us an equation of the loss of energy, trough gravity waves. Knowing the rate of spin down, the rotation frequency and the precession frequency allows us to find the wobble angle. This is possible under the assumption that the main mechanism for the loss of rotational energy is due to emission of gravitational radiation. Once we know the wobble angle, we can calculate the strength of the radiation on earth. In fact the value of the dimensionless strain parameter h depends on the value of the wobble angle quite strongly.

Our discussion shows that even with a more a realistic model of a precessing neutron star that takes in consideration the presence of a crust, with a certain elasticity and the eventual presence of a fluid interior the precessional behavior is similar to that of the simple biaxial model. The ratio of precesion frequency and spin frequency determines the order of magnitude of the ellipticity, but a complete solution requires an estimate of the wobble angle. The preceding discussion shows that it is possible to obtain such self-conssitent models as a function of essentially one parameter: the moment of inertia I_0 that is involved in the precession. Given it, the observational data allow to determine the wobble angle, the size of deformation and consequently the strength of the radiation on earth.

We saw in the previous section that to avoid crust breaking the wobble angle has to be relatively small. In fact, formally, even the smallest wobble angle among the possible range of solutions violates the maximum crustal strain. Given the uncertainties in the model and in the interpretation of the data, we conclude that even if the limits on the maximum strain $\sigma_{\rm max}$ are relaxed, any viable solution is likely to have a wobble angle near the small end of the range and consequently the moment of inertia must be near the high end of its range. In turn this means that a short integration time on the order of days is required to observe with confidence the gravity wave signal from SN1987A using advanced detectors as LIGO II. Unfortunately the presently generation of detectors such as the resonant bars and LIGO I would require observation times of the order of a million years to extract the signal from the noise. Thus if the precession interpretation is correct, the SN 1987A remnant would be among the best candidates for *a search for a* continuous source of gravitational waves. In any case, it is clear that a targeted search for gravitational waves from this source is worthwhile since both detection and absence of detection over a relatively short time will yield interesting constraints on models for precessing neutron stars.

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