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Topology and Shape Optimization of Hydrodynamically–Lubricated Bearings for Enhanced Load-Carrying Capacity

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TOPOLOGY AND SHAPE OPTIMIZATION OF HYDRODYNAMICALLY–LUBRICATED BEARINGS FOR ENHANCED LOAD-CARRYING CAPACITY

A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Doctor of Philosophy in The Department of Mechanical and Industrial Engineering

by
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ABSTRACT

Bearings are basic and essential components of nearly all machinery. They must be designed to work under different loads, speeds, and environments. Of all the performance parameters, load-carrying capacity (LCC) is often the most crucial design constraint.

The objective of this research is to investigate different design methodologies that significantly improve the LCC of liquid-lubricated bearings. This goal can be achieved by either altering the surface texture or the bearing geometrical configuration. The methodology used here is based on mathematical topological/shape optimization algorithms. These methods can effectively improve the design performance while avoiding time-consuming trial-and-error design techniques.

The first category of design studied is a micro-scale mechanical self-adaptive type which can provide “flexible surface texturing”. An accurate 3D model based on the classic plate theory and thin film lubrication is developed and a shape optimization analysis is carried out. Special attention is given to the cavitation phenomena and its numerical analysis. Also proposed is a numerical procedure to improve the convergence rate and stability of the Elrod cavitation algorithm.

The idea of using self-adaptive mechanism to improve LCC is also adopted for thrust bearings. Novel flexible-pad thrust bearing designs that provide an optimum load-responsive mechanism are presented and an accurate multi-physics model that considers the coupled mechanism between the lubricant pressure and the pad deformation is developed. The optimum shapes for different bearing geometries are given and a detailed design guideline is provided for optimum performance.

The second category of design studied focuses on bearing geometrical configuration. The optimum shape of finite width sectorial sliders, which is an open problem in the field, is determined for the first time in this research using topological optimization algorithms. Also three suboptimum solutions for special cases of 2D step profile, constant film thickness in the radial direction and constant film depth with quadrilateral shape are presented. These configurations are particularly attractive because they can be easily manufactured.

The optimum shape of bearings with periodic surface grooves is also determined in this research. It is shown that the optimum shape is dependent to the aspect ratio of the grooves and it can change from elongated “heart-like” shapes to spiral-like shapes. A series of laboratory tests to authenticate the theoretical development is carried out. Results show very good agreement with the theory validating the accuracy of the model.

Finally, the optimum geometry of spiral grooves that provide the highest LCC in liquid-lubricated parallel flat surface bearings is determined and a detailed design guideline is provided.
The thermal effects are also considered and an approximate thermo-hydrodynamic model is developed for a range of seal geometries and operating conditions.
CHAPTER 1  OVERVIEW

Bearings are basic and essential components of all machinery. To satisfy the requirements for special speeds, loads, and environments, bearings take on many different types, shapes and classifications with different design features. Optimum design of bearings with enhanced load-carrying capacity (LCC) and reduced power loss is of particular interest to the industry. For a given operating condition and lubricant, this enhanced LCC can be obtained by altering the lubricant film profile or bearing geometry. Recent approaches include surface texturing and topology/shape optimization methodology, described as follows:

1.1 Surface Texturing

Surface texturing is a method used to enhance the performance of a nominally flat active surface of a tribocomponent (i.e., mechanical face seal or thrust bearing) by adding shallow patterns or grooves. Figure 1-1 shows some typical surface patterns. Even though, in nature, surface textures have been used by living creatures for millions of years (e.g., the textures in the sharks scales can dramatically decrease the water resistance), their application in industry is in its infancy. The study of the surface texturing started in the early 1960s [1]. With the development of the new technology, such as laser technology and micro-electro-mechanical systems (MEMS) technology, the surface texturing became more practical as geometry of the surface texturing become more controllable [2,3].

Surface texturing methodology offers significant potential in improving performance. It is estimated that if used with thin film and surface chemistry technology, one can reduce the friction by 5% and increase the energy transmission efficiency by 30% [4].

Figure 1-1: Typical surface textures [3,5]
1.2 Topology/Shape Optimization

Topology optimization is a mathematical approach used at the concept level of the design process in order to find the best concept design that meets the design requirements, giving a first idea of an efficient geometry. The results of Topology optimization need to be fine-tuned for performance and manufacturability using shape optimization methods. It should be noted that topology optimization is different from shape optimization since typically in shape optimization the topological properties are fixed (i.e., having a fixed number of grooves in a bearing). Topology optimization is used to produce concept designs while shape optimization is used to fine-tune a chosen design topology (see Fig. 1-2).

Figure 1-2: Topology and shape optimization [6]

Topology and shape optimization methods can be employed to improve bearing designs. An example could be the self-adaptive bearings. Figure 1-3 shows the schematic of these bearings and the optimized designs obtained by topology/shape optimization methods. In this case using the topological optimization methods the shape of the grooves located underneath of the surface are identified. Then these shapes are polished using shape optimization methods. In this case, the final designs have more than 45% better load-carrying capacity than initial designs (see Chapter 5 for more details).

Due to mathematical complexity of topology optimization methods, they have not received attention in tribology and most researchers have preferred to use simpler shape optimization algorithms [7,8]. However, topology optimization methods can be a viable alternative to time-consuming trial and error design methods and hence reduce design development time and cost while improving the design performance.

Investigating the possibility of using topology/shape optimization methods for identifying the optimum shape of surface textures or their patterns is an interesting topic that will be considered in this research. The objective will be to find optimum pattern for surface textures
theoretically. This can be done in two different ways. In the first method the pattern shapes are assumed to be given (e.g., circular, elliptical, etc.) and using shape optimization methods the distance, depth, diameter and area ratio of the patterns are optimized for maximum load-carrying capacity. The second method is to use topology optimization methods to find optimum shapes and patterns. In the latter method, there is no assumption on the shape and it may produce shapes that are not practical or hard to manufacture. After identifying the optimum shapes and patterns, a series of experiments is carried out to evaluate the performance of the obtained pattern (see Chapter 8 for more details).

![Figure 1-3: Topology and shape optimization of self-adaptive bearings](image)

### 1.3 Research Objectives

A large variety of surface patterns have been recently developed and investigated by researchers. However, the majority of these designs have fixed geometry and cannot adapt to different loading conditions.

As the application of micro/nano technologies increases, the need grows for small-scale, self-adaptive bearings that are capable of supporting load for a wide range of operating conditions. The primary function of these bearings is to match different loading conditions by providing a “flexible surface texturing.” Developing an accurate model to predict the tribological behavior of self-adaptive bearings is one of the main objectives of this study. To this end, specifically concentrate on the recent development in the field of micro-scale mechanical self-adaptive bearings. The shape optimization of these bearings is studied in detail in this dissertation.
To accurately predict the tribological behavior of self-adaptive bearings, it is important to precisely model the lubricant cavitation. The simplified models based on Reynolds boundary condition violate mass conservation and consequently are inaccurate, especially for micro-sized applications. Thus, an accurate model based on more realistic cavitation boundary condition is necessary to capture the true mechanism. This study uses a mass conservative cavitation algorithm based on JFO boundary condition [9,10] to fulfill this requirement. However, the mass conservative cavitation algorithm is highly nonlinear and has convergence difficulties. Improving the stability and convergence speed of this algorithm is another important objective of this study.

The problem of determining the global optimum film profile in finite width sectorial sliders (i.e., thrust bearings) is an open problem in the field. Considering the wide application of these bearings, any improvement in their performance is of interest to the industry. Nevertheless, an in-depth investigation of the global optimization is needed. One of the main objectives of this research is to solve this open problem. This problem is studied in detail here and the results are presented.

Improving the performance of current thrust bearing designs and making them cost effective and easy to manufacture is another objective of this research. Novel hydrodynamic thrust bearings with flexible-pad are introduced. The pads have a slit on the side which gives them the flexibility to deform under different loading conditions. The idea of using flexible pads improves the LCC of these bearings for a wide range of loading conditions by providing an adaptive surface deformation mechanism.

Parallel flat surface bearings (i.e., thrust washers) have a wide range of industrial application. Improving the LCC of these components by determining the optimum surface groove shapes is another objective of the current research. A series of experiments to authenticate the theoretical development is also carried out.

Finally, developing a guideline for design of liquid-lubricated spiral groove bearings is also considered in this research. Thermal effects are considered for case of mechanical face seals.

1.4 Dissertation Outline

This dissertation deals with three subtopics: cavitation modeling and its numerical treatment, modeling and optimization of self-adaptive bearings, and topology/shape optimization of finite width sectorial sliders (i.e., thrust bearings). The first two chapters focus on the cavitation and the next two chapters are on the topic of self-adaptive grooves. After that, the topology/shape optimization of sectorial-shape sliders is discussed. The chapters are written in the form of a journal paper.
Chapter 2 concentrates on the cavitation phenomena and its numerical analysis. Various cavitation boundary conditions and numerical methods available to treat them are discussed.

Chapter 3 presents a fast mass-conservative cavitation algorithm. A new method is proposed that improves numerical instability and overcomes the convergence issues caused by the conventional methods. The proposed method is faster while it is less prone to numerical instabilities.

Chapter 4 presents a novel analytical model for self-adaptive surface grooves. A 2D model based on the plate theory is developed to determine the grooves surface deflection due to lubricant pressure. Using the proposed model grooves with arbitrary shape and aspect ratios can be analyzed.

Chapter 5 concentrates on the shape optimization of mechanical self-adaptive bearings. The optimization aims to improve the load-carrying capacity of these bearings by introducing novel deformable groove designs. The objective is to find the optimum groove’s thickness profile, which maximizes the load-carrying capacity. Assuming that the thickness can vary along the groove’s length, three different thickness patterns including constant, linear and spline are considered. A hybrid optimization algorithm based on the harmony search (HS) algorithm and the sequential quadratic programming (SQP) is utilized to find the optimum shape for each thickness pattern.

Chapter 6 concentrates on the problem of determining the global optimum film profile in finite-width sectorial-shape sliders. The optimum shape of hydrodynamic film that provides the greatest LCC in sectorial-shape thrust bearings is obtained using SQP. Also, three suboptimum solutions for special cases of 2D step profile, constant film thickness in the radial direction and constant film depth with quadrilateral shape are presented. These configurations are particularly attractive because they can be easily manufactured.

Chapter 7 presents a theoretical analysis of a hydrodynamic thrust bearing with a flexible-pad design. The flexibility of pads improves the LCC of the bearing for a wide range of loading conditions. To accurately predict the behavior of flexible-pad thrust bearings, an appropriate multi-physics model that considers the coupled mechanism between the lubricant pressure and the pad deformation is developed. In addition, an approximate analytical method for optimum bearing design is presented.

Chapter 8 concentrates on determining the optimum periodic surface grooves that provide the highest LCC in parallel flat surface bearings using mathematical optimization methods. Results show that the optimum grooves geometry is a function of the aspect ratio. For small aspect ratios the optimum grooves have an elongated “heart-like” shape and for high-aspect ratios they take on the shape of spirals. It is shown that optimally-designed grooves can provide
up to 36% more LCC compared to the conventional spiral grooves. Also presented are the results of a series of experiments to authenticate the theoretical development.

Chapter 9 focuses on determining the optimum geometry of spiral grooves that provide the highest LCC in liquid-lubricated parallel flat surface bearings. In addition, based on optimization results an analytical model to determine the optimum spiral angle, groove depth, number of grooves, and minimum film thickness is presented. Thermal effects are also considered and an approximate thermo-hydrodynamic model is developed for a range of seal geometries and operating conditions.

Chapter 10 summarizes the main results presented in this dissertation and gives recommendation for future works.

1.5 References


2.1 Definition

“Cavitation is the disruption of a continuous liquid phase by the presence of a gas or vapor or both” [1].

2.2 Types and Mechanism

There are the two basic forms of cavitation recognized in lubrication films: gaseous cavitation and vapor cavitation. Lubricating oil contains roughly 10% by volume of dissolved gas when saturated with air or other gases. The gaseous cavitation occurs when the lubricant pressure falls below the saturation pressure of dissolved gases. As a result, the dissolved gases come out of the solution and form cavity bubbles. This type of cavitation is relatively innocuous. Vapor cavitation occurs when the lubricant pressure drops below its vapor pressure, causing local boiling and the formation of bubbles. The bubbles will collapse when they move to the high-pressure regions, causing noise, vibration, and fatigue-type damage to bearing surfaces [2].

2.3 Governing Equations

The behavior of the hydrodynamic pressure within a lubricant film is governed by the Reynolds equation. For a Newtonian lubricant, the Reynolds equation takes on the following form.

\[
\frac{\partial \rho h}{\partial t} + \frac{\partial}{\partial x} \left[ \frac{\rho h U}{2} - \frac{\rho h^3}{12 \mu} \left( \frac{\partial p}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[ -\frac{\rho h^3}{12 \mu} \left( \frac{\partial p}{\partial y} \right) \right] = 0
\]  

(1)

where \( h \) is the film thickness, \( p \) is the local pressure, \( \mu \) is the viscosity, \( \rho \) is the density and \( U \) is the sliding speed. In the full-film region the density is constant. Consequently, equation (1) takes on the following form.

\[
\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} \left[ \frac{h U}{2} - \frac{h^3}{12 \mu} \left( \frac{\partial p}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[ -\frac{h^3}{12 \mu} \left( \frac{\partial p}{\partial y} \right) \right] = 0
\]  

(2)

In the cavitated region the pressure essentially remains constant at the cavitation pressure, \( p_c \). Thus, equation (1) is reduced to:

---

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\[
\frac{\partial \rho h}{\partial t} + \frac{\partial}{\partial x} \left( \frac{\rho h U}{2} \right) = 0
\]  

(3)

It should be noted that the unknowns are different in full-film and cavitated regions. In the full-film region the pressure is unknown while in the cavitated region the pressure is known and the density is unknown. The numerical treatment of equation (2) and equation (3) is also different. Unlike equation (2) which is an elliptic partial differential equation (PDE) requiring central differencing, equation (3) is of hyperbolic type, which requires upwind differencing.

In the presence of cavitation the treatment of the internal boundary condition in the interface of the full-film and cavitated regions (cavitation boundary condition) is the key concern in solving equation (2) and equation (3). Several numerical methods have been developed to deal with this problem.

### 2.4 Cavitation Boundary Conditions

Full-Sommerfeld and Half-Sommerfeld condition are of the oldest cavitation boundary conditions applied to the divergent section of a hydrodynamic bearing. Full-Sommerfeld retains the negative pressures predicted by solving the Reynolds equation (1). When applied to a journal bearing, the Full-Sommerfeld predicts that \( p < 0 \), in the entire region of \( \pi \leq \theta \leq 2\pi \). This treatment is unsatisfactory as it predicts an attitude angle of \( \phi = 90^\circ \), with unrealistic stability implications [2]. Half-Sommerfeld condition simply sets the negative pressures equal to zero to account for cavitation. It is easy to implement, but similar to the full-Sommerfeld condition, has a major shortcoming: it violates the principle of continuity of flow.

The so-called Swift-Stieber (Reynolds) boundary, formulated independently by Swift [3] and Stieber [4], is more realistic and has been widely used in treating lubrication problems, particularly for bearing design. Based on the Swift-Stieber boundary condition both the pressure and its normal gradient in the direction of motion vanish at the cavitation boundaries. However, the predicted film reformation boundary (i.e., where the cavitation ends and full-film begins) are not accurate; hence, it does not enforce mass-conservation.

To account for the film reformation and ensure mass continuity, Jakobsson and Floberg [5] and Olsson [6] proposed a set of self-consistent boundary conditions, commonly known as the JFO boundary conditions, which properly accounts for conservation of mass. Based on the JFO boundary condition, at the location of the film rapture (i.e., where the full-film ends and cavitation starts):

\[
\frac{\partial p}{\partial n} = 0
\]  

(4)
where $n$ represents the outward normal vector. This condition is the same as Swift-Stieber boundary condition. At the locus of the film reformation the following condition holds.

$$\frac{h^2}{12\mu} \frac{\partial p}{\partial n} = \frac{V_n}{2} \left(1 - \frac{\rho}{\rho_c}\right)$$  \hspace{1cm} (5)$$

where $V_n$ is the normal fluid velocity and $\rho_c$ is the cavitation density. Although the JFO boundary condition is realistic, its implementation in the form of computer programs is quite cumbersome since the location of cavitation boundaries is not a priori known.

Elrod [7] proposed a mass-conserving cavitation algorithm that inherently incorporates the JFO boundary condition and uses a single equation for both the full-film and cavitated regions and hence eliminates the need to distinguish the cavitation boundaries. This was achieved by introducing a variable $\phi (=\rho/\rho_c)$, which is fractional film content, and a binary switch function $g$, which is set to zero or one depending on the region. Using the new parameters the Reynolds equation can be written as follows:

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x} \left[ \frac{hU \phi}{2} - \frac{\beta h^3}{12\mu} g\frac{\partial \phi}{\partial x} \right] + \frac{\partial}{\partial y} \left[ -\frac{\beta h^3}{12\mu} g\frac{\partial \phi}{\partial y} \right] = 0 \hspace{1cm} (6)$$

where

$$\beta = \rho \frac{\partial p}{\partial \rho}$$

is the bulk modulus of the lubricant. Solving the equation (6), the pressure can be determined from the following equation.

$$p = p_c + g\beta \ln(\phi) \hspace{1cm} (7)$$

where $p_c$ is the cavitation pressure.

Using the switch function, the Elrod cavitation algorithm automatically changes the form of differencing (central or upwind) in the cavitated and full-film regions. Later, Vijayaraghavan and Keith [8], based on the Elrod’s work, proposed a systematic way to implement JFO boundary condition. Their method avoids the trial-and-error step used during the development of the Elrod algorithm. Nowadays, the Elrod algorithm and its variants are widely used for the simulation of lubrication problems involving cavitation.
2.5 Key Applications

Typically the Swift-Stiber (Reynolds) cavitation boundary condition suffices for routine design and analysis of steadily loaded bearings without misalignment. However, implementation of mass conservative cavitation algorithm is important in the treatment of dynamically-loaded bearings, misaligned bearings, and for accurate prediction of the behavior of textured surfaces involving dimples. The interested reader may refer to Jang and Khonsari [9,10] and Qiu and Khonsari [11].

2.6 Numerical Methods

There are several iterative solvers for treating equation (6). These methods include successive over-relaxation (SOR), alternating direction implicit (ADI) and multigrid method.

The SOR method, which is an explicit iterative solver, is a variant of the Gauss–Seidel method. This method introduces a relaxation factor $\omega$ which speeds up convergence. In this method the unknown value at every node is determined based on the values of the parameter from the previous iteration and the available parameter distribution on neighboring nodes. This method is also known as point SOR. There is a semi-implicit variant of this algorithm which is called successive line over-relaxation (SLOR) method where the unknown values for all nodes along a line are simultaneously determined by solving a tridiagonal matrix system. Generally, the stability and convergence speed of SLOR is better than SOR method.

The ADI method [12] has been used extensively for solving cavitation problems. This method is best suited for transient problems and it can be shown that for 2D problems it is unconditionally stable. In this method, for a 2D problem, the time step is split into two intervals. In each interval the unknown values in one direction (i.e., x or y) are implicitly solved. Alternately, the other direction is solved using the available values of the variables from the previous time interval. This method is generally faster than SOR and SLOR methods.

Multigrid methods are among the fastest solvers for elliptic PDEs. The main idea of multigrid is to accelerate the convergence of an iterative method (i.e., Gauss-Seidel method) on a fine grid by solving a series of coarser problems. This can be seen as interpolation between coarser and finer grids. The high frequency errors on a fine grid decay rapidly using an iterative method; but, the low frequency errors are difficult to eliminate. These methods overcome this problem by transferring error from the fine grid iteration process to a coarser grid configuration, iterate under this grid scale to find a correction value, and then transfer the correction value back from the coarse grid to the fine grid. In this way, the low frequency errors can be eliminated very fast. Research shows that for cavitation problems the multigrid methods are almost 10 times faster than ADI method [11].
There are other numerical schemes like approximate factorization (AF) method which has been occasionally applied to cavitation algorithms. These methods are not covered here. The interested reader is referred to Vijayaraghavan and Keith [13] for a good comparison among SOR, SLOR, ADI and AF methods for steady state and transient problems.

2.7 Numerical Instability

The main difficulty dealing with the Elrod cavitation model is its highly nonlinear nature. In some cases, oscillation and instability is encountered in the solution of equation (6). Cavitation boundaries have destabilizing effects on numerical methods and trigger instabilities [13]. Generally, the fully implicit methods like ADI are more sensitive to the movement of the boundaries and immediately sense it at every time step. As a result, an increase in the time step results in the divergence of the solution. The explicit methods like SOR are less sensitive to boundary movement because they cannot completely realize it at every step.

It is possible to decrease instabilities by modifying the switch function in equation (6). The Elrod cavitation algorithm uses a binary switch function that delineates the full-film and the cavitation zones. Abrupt changes associated with this function cause a sudden movement of the cavitation boundaries which trigger the instability and oscillations. Recently, Fesanghary and Khonsari [14] proposed an exponential switch function which changes monotonically as solution proceeds. Based on their algorithm when \( \varphi \) is less than one, \( g \) is exponentially decreased by multiplying the current value by a constant, \( g\text{Factor} \), which is equal or greater than zero and less than one \((0 \leq g\text{Factor} < 1)\). Otherwise, the value of \( g \) is increased by dividing by \( g\text{Factor} \). Their works showed that using the new switching algorithm together with SOR method, the solution can converge up to 60 percent faster while it was less prone to numerical instabilities.

Although various numerical methods have been developed to treat the cavitation in lubrication problems, further research is needed for developing more efficient and robust cavitation algorithms.

2.8 References


CHAPTER 3  A MODIFICATION OF THE SWITCH FUNCTION IN THE ELROD CAVITATION ALGORITHM

3.1 Introduction

The treatment of cavitation phenomena is a key concern in the modeling of lubrication problems. Several numerical methods have been developed to deal with this problem.

The Swift-Stieber (the so-called Reynolds) boundary condition was formulated independently by Swift [1] and Stieber [2] in the 1930s. It has been widely used because of its simplicity, ease of implementation, and superior accuracy compared to the full- and half-Sommerfeld boundary conditions. This boundary condition enforces a zero pressure and its gradient along the direction the sliding direction at the location where the film ruptures. However, this boundary condition does not treat the film reformation boundary where the cavitation ends and full-film begins; hence, it does not enforce mass-conservation. To account for the film reformation and ensure mass continuity, Jakobsson and Floberg [3] and Olsson [4] proposed an additional boundary condition, commonly known as the JFO boundary conditions. The JFO boundary condition is realistic, but its implementation is quite cumbersome.

In 1981, Elrod [5] proposed a computational algorithm which incorporates the JFO boundary condition in the form of a single equation for both the full-film and cavitated regions by the aid of a single parameter known as the switch function. Using the switch function, the Elrod algorithm automatically predicts the cavitated and full-film regions, conserves mass continuity, and predicts the hydrodynamic pressure distribution. The Elrod algorithm included a discretization method that was derived through considerable experimentation [5]. Later, Vijayaraghavan and Keith [6] modified the Elrod algorithm by introducing a procedure which automatically changes the form of differencing (central or upwind) of the shear flow terms in both regions. Their method avoids the trial-and-error step used during the development of the Elrod algorithm. Nowadays, the Elrod algorithm and its modifications are widely used for the simulation of lubrication problems involving cavitation.

The highly nonlinear nature of the Elrod algorithm with the binary switch function makes it prone to numerical instability and often causes convergence issues. This study aims to improve the stability and convergence speed of the Elrod algorithm by introducing a simple, but effective method for improving the implementation of the switch function. Instead of the conventional binary switch function, which is believed to be one of the main sources of the instabilities, a continuous switch function is proposed.

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The outline of the paper is as follows: First, the Elrod cavitation algorithm is reviewed in the next section. Then, numerical details is described in section 3 followed by the proposed switching algorithm in section 4. Next, to show the effectiveness of the proposed method, two case studies are selected and solved. Section 5 concentrates on the details of the problems and presents results and discussions. Finally section 6 summarizes the conclusions.

3.2 The Elrod Cavitation Algorithm

The behavior of the hydrodynamic pressure within a lubricant film is governed by the Reynolds equation. For a Newtonian lubricant with constant viscosity, the Reynolds equation takes on the following form.

\[
\frac{\partial}{\partial x} \left( \rho h^3 \left( \frac{\partial p}{\partial x} \right) \right) + \frac{\partial}{\partial y} \left( \rho h^3 \left( \frac{\partial p}{\partial y} \right) \right) = 6\mu U \frac{\partial \rho h}{\partial x}
\]  

(1)

where \( h \) is the film thickness, \( p \) is the local pressure, \( \mu \) is the viscosity, \( \rho \) is the density and \( U \) is the sliding speed. In the full-film region, where the density is constant equation (1) reduces to:

\[
\frac{\partial}{\partial x} \left( h^3 \left( \frac{\partial p}{\partial x} \right) \right) + \frac{\partial}{\partial y} \left( h^3 \left( \frac{\partial p}{\partial y} \right) \right) = 6\mu U \frac{\partial h}{\partial x}
\]  

(2)

In the cavitation region the pressure remains constant at the cavitation pressure. Thus, equation (1) is reduced to

\[
\frac{\partial \rho h}{\partial x} = 0
\]  

(3)

using the fractional-film content (\( \theta = \rho / \rho_c \)) introduced by Elrod [5]. It is possible to put forward a “universal” partial differential equation (PDE) that covers both the cavitated and the full-film regions. In this way the need to distinguish the boundaries of the cavitated region is eliminated. However, a switch function, \( g \), is necessary to make the resulting PDE consistent with the uniform pressure assumption within the cavitated region. The switch function suggested by Elrod [5] is defined as

\[
g = \begin{cases} 
0 & \theta < 1 \\
1 & \theta \geq 1 
\end{cases}
\]

The resulting universal PDE can be written as follows:
\[
\frac{\partial}{\partial x} \left[ \beta h^3 g(\theta) \left( \frac{\partial \theta}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[ \beta h^3 g(\theta) \left( \frac{\partial \theta}{\partial y} \right) \right] = 6\mu U \frac{\partial h}{\partial x}
\]

(4)

where

\[
\beta = \rho \frac{\partial p}{\partial \rho}
\]

is the bulk modulus of the lubricant. Solving Eq. (4) for \( \theta \), the pressure in the full-film region can be determined from the following equation.

\[
p = p_c + \beta \ln \theta
\]

(5)

where \( p_c \) is the cavitation pressure. In the following the discretization details and numerical treatment of Eq.(4) is explained in more details.

### 3.3 Numerical Method

#### 3.3.1 Discretization

The finite difference method (FDM) is used to discretize Eq. (4). The result is:

\[
\begin{align*}
\left\{ \frac{U}{2\Delta x} \left[ (1 - g_{i-1,j}) h_{i-1,j} \right] + \frac{\beta}{12\mu(\Delta x)^2} \left[ g_{i-1,j} h_{i-1/2,j}^3 \right] \theta_{i-1,j} + \frac{\beta}{12\mu(\Delta y)^2} \left[ g_{i+1,j} h_{i+1/2,j}^3 \right] \theta_{i+1,j} \\
- \left\{ - \frac{U}{2\Delta x} \left[ (1 - g_{i,j}) h_{i,j} \right] - \frac{\beta}{12\mu(\Delta x)^2} \left[ g_{i,j} h_{i+1/2,j}^3 + h_{i-1/2,j}^3 \right] \theta_{i,j} - \frac{\beta}{12\mu(\Delta y)^2} \left[ g_{i,j} h_{i,j+1/2}^3 + h_{i,j-1/2}^3 \right] \theta_{i,j} \right. \\
+ \left. \frac{\beta}{12\mu(\Delta y)^2} \left[ g_{i,j} h_{i,j-1/2}^3 \right] \theta_{i,j-1} + \frac{\beta}{12\mu(\Delta x)^2} \left[ g_{i,j} h_{i,j+1/2}^3 \right] \theta_{i,j+1} \right\} \\
+ \frac{U}{2\Delta x} \left[ \frac{g_{i-1,j}}{2} h_{i-1,j} + \frac{g_{i,j} (g_{i-1,j} - 2 + g_{i+1,j})}{2} h_{i,j} - \frac{g_{i+1,j} g_{i,j}}{2} h_{i+1,j} \right] \\
+ \frac{\beta}{12\mu(\Delta x)^2} \left[ g_{i,j} h_{i-1/2,j}^3 \right] + \frac{\beta}{12\mu(\Delta y)^2} \left[ g_{i,j} h_{i+1/2,j}^3 \right] - \frac{g_{i+1,j} h_{i,j}^3}{2} \\
+ \frac{\beta}{12\mu(\Delta y)^2} \left[ g_{i,j} h_{i,j-1/2}^3 + g_{i,j} h_{i,j+1/2}^3 \right] - g_{i,j} h_{i,j+1}^3 = 0
\end{align*}
\]

(6)

#### 3.3.2 Iterative Solver

There are several iterative solvers including successive over-relaxation (SOR), successive line- over-relaxation (SLOR), alternating direction implicit (ADI) and multigrid methods to treat
Eq. (6). The proposed switching algorithm is applicable to all of these methods. In this work the SOR method is employed. Compared to the fast solvers like multigrid, this method is very easy to implement which makes it the preferred solver for small and medium-sized problems where the computational time is not the main concern.

3.3.3 Convergence Criterion

Convergence is assumed if the calculated sums of fractional changes in the film content between two successive SOR iterations falls below the user-defined tolerance value, $\xi$. That is:

$$\sum_{i=0}^{i_{\text{max}}} \sum_{j=0}^{j_{\text{max}}} \left| \frac{\theta_{i,j}^{\text{new}} - \theta_{i,j}^{\text{old}}}{\theta_{i,j}^{\text{new}}} \right| < \xi$$

(7)

3.4 Proposed Switch Function

Several researchers [6, 7] have reported that, in some cases, oscillation and instability is encountered in the solution of equation (4). These oscillations are usually triggered at cavitation nodes due to the abrupt change in the switch function, which results in a different set of coefficients in Eq. (6). Table 1 proposes an algorithm that alleviates the difficulties with the abrupt change in the switch function. Based on this algorithm when $\theta_{i,j}$ is greater than one, $g_{i,j}$ is increased by dividing the current value by a constant, gFactor, which is equal or greater than zero and less than one ($0 \leq \text{gFactor} < 1$). Otherwise, the value of $g_{i,j}$ is exponentially decreased; in this way that $g_{i,j}$ is replaced by current value of $g_{i,j}$ times gFactor. The value of $g_{i,j}$ in the cavitated region approaches zero very fast and the last line of the algorithm ensures that this value is exactly zero at the end. The value of $10^{-6}$ was chosen based on the authors experience from different test cases. Higher values can be used and it may affect the convergence rate.

When the gFactor is zero, the algorithm is identical to the original Elrod algorithm. Based on authors’ experience a value of 0.8 for gFactor is quite satisfactory in many problems. In section 5.5 a parametric study is carried out to evaluate the effect of gFactor on benchmark problems.

3.5 Computational Results

The results of two case studies are presented to examine the efficiency of the proposed switching algorithm. The computing machine has a 2.4 GHz Pentium IV processor with 3024 MB RAM.
Table 3-1. The pseudo code of the switching algorithm

\[
\text{If } (\theta_{i,j} \geq 1) \\
\quad \text{If } (\text{gFactor} > 0) \\
\quad \quad g_{i,j} = g_{i,j}/\text{gFactor} \\
\quad \text{Else} \\
\quad \quad g_{i,j} = 1 \\
\quad \text{End If} \\
\text{Else} \\
\quad g_{i,j} = g_{i,j} \times \text{gFactor} \\
\text{End If} \\
\text{If } (g_{i,j} > 1) \quad g_{i,j} = 1 \quad \text{End If} \\
\text{If } (g_{i,j} < 10^{-4}) \quad g_{i,j} = 0 \quad \text{End If}
\]

3.5.1 Case Study 1 (journal bearing)

The correctness and accuracy of the switching algorithm is verified with a problem taken from Elrod [5]. It treats the cavitation problem in a journal bearing wherein oil is supplied through two grooves positioned to be 60 deg and 240 deg apart from the smallest film thickness location. Table 2 shows the input parameters for this problem.

A grid mesh of 72*30 was used. This grid size is fine enough to accurately capture the film rupture and reformation boundaries. The convergence tolerance, $\xi$, and gFactor were set to $10^{-4}$ and 0.8, respectively. Figure 3-1 shows the simulation results for the pressure and film content along the center of journal bearing ($y=L/2$). The results compare very well with results obtained by the Elrod algorithm [8].

Table 3-2. The input parameters for case study 1 [5]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter, D</td>
<td>62.8 mm</td>
</tr>
<tr>
<td>L/D</td>
<td>10</td>
</tr>
<tr>
<td>Eccentricity, $\varepsilon$</td>
<td>0.8</td>
</tr>
<tr>
<td>Minimum film thickness, $h_0$</td>
<td>55 $\mu$m</td>
</tr>
<tr>
<td>Viscosity, $\mu$</td>
<td>0.0035 Pa.s</td>
</tr>
<tr>
<td>Sliding speed, $U$</td>
<td>19.7 m/s</td>
</tr>
<tr>
<td>Cavitation pressure, $p_c$</td>
<td>$1 \times 10^5$ Pa</td>
</tr>
<tr>
<td>Groove pressure, $p_G$</td>
<td>$1 \times 10^5$ Pa</td>
</tr>
<tr>
<td>Bulk modulus, $\beta$</td>
<td>$1 \times 10^8$ Pa</td>
</tr>
<tr>
<td>Groove number and location</td>
<td>2 (60 deg, 240 deg)</td>
</tr>
</tbody>
</table>
A parametric study is conducted to examine the effect of gFactor and relaxation factor, \( \omega \), on the convergence speed of the algorithm.

In this problem, using a gFactor of zero (i.e., the original Elrod algorithm) the minimum required iterations is 240, which is obtained for a \( \omega \) equal to 1.176. The algorithm is not stable around this point and it diverges for larger relaxation factors. Using the proposed switching algorithm the convergence speed and algorithm stability can be enhanced. Figure 3-2 shows the effect of gFactor and \( \omega \) on the convergence speed for the first case study. With a \( \omega \) equal to 1.41 and gFactors between 0.7 and 0.8, the SOR algorithm converges after 146 iterations. This represents 64 percent improvement compared to original Elrod algorithm. Implementation of the gFactor in the SOR algorithm also tends to stabilize the solution for higher \( \omega \)s, which is not possible if the original Elrod algorithm is used.

![Graph showing pressure and film content along the center of journal bearing](image)

Figure 3-1: Pressure and film content along the center of journal bearing (y=L/2) for case study 1

An interesting result is that for \( \omega \leq 1.35 \) the algorithm is not sensitive to the value of gFactor and any value between 0.2 to 0.9 converges after the same number of iterations. After this point as \( \omega \) increases, the gFactor value has a more pronounced effect on the algorithm iterations. The algorithm eventually diverges for large values of \( \omega \) (i.e., \( \omega >1.45 \)) as shown in Fig. 3-2.
To compare the computational time of the proposed algorithm with original Elrod method a simple test was conducted. For case study 1 the average time of 200 independent simulations with a gFactor of zero were recorded for both cases. Results showed that each simulation of the proposed algorithm, in average, takes 3.765 seconds which is 2.7 percent more computationally expensive than original Elrod algorithm with an average time of 3.665 seconds. This can be easily compensated since the proposed algorithm decreases the total number of iterations by 64 percent.

3.5.2 Case Study 2 (surface-textured thrust bearing)

The second problem, taken from Qiu and Khonsari [8] pertains to a thrust bearing with surface-textured micro-dimples. A single cell containing one dimple, as shown in Fig. 3-3, is used as computational domain. The dimple is circular in shape and located at the center of the cell. A cyclic boundary condition is assumed in circumferential (X) direction and boundaries in radial (Y) direction are kept in ambient pressure. The film thickness in the domain can be mathematically described as [8]

\[ r = \sqrt{(x - x_c)^2 + (y - y_c)^2} \]  

(8a)
\[ h = \begin{cases} 
\frac{h_0}{\sqrt{\frac{r_0^2 + h_g^2}{2h_g}} - r^2 - \frac{r_0^2 + h_g^2}{2h_g} + h_g + h_0} & r < r_0 \\
r_0 & r \geq r_0 
\end{cases} \quad (8b) \]

where \( r_0 \) is the dimple radius, \( x_c \) and \( y_c \) are the center coordinate of the dimple, and \( h_0 \) and \( h_g \) are minimum film thickness and dimple depth, respectively. Table 3 shows the input parameters for this problem.

![Schematic of thrust bearing with dimples](image)

**Figure 3-3: Schematic of thrust bearing with dimples [8]**

A grid mesh of 64x64 with a gFactor = 0.8 was used in simulations. The convergence tolerance, \( \xi \), was set to \( 10^{-5} \) for this case study.

Figure 3-4 illustrates the pressure profile for the second case study. It shows an antisymmetric pressure distribution. The highest and lowest pressures occur on the rim of the dimple in the downstream and upstream positions, respectively. These results are similar to those obtained by Qiu and Khonsari [8] using the multigrid approach.

<table>
<thead>
<tr>
<th>Table 3-3 The input parameters for case study 2 [8]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>Dimple radius, ( r_0 )</td>
</tr>
<tr>
<td>Dimple depth, ( h_g )</td>
</tr>
<tr>
<td>Cell size, ( L )</td>
</tr>
<tr>
<td>Minimum film thickness, ( h_0 )</td>
</tr>
<tr>
<td>Viscosity, ( \mu )</td>
</tr>
<tr>
<td>Sliding speed, ( U )</td>
</tr>
<tr>
<td>Cavitation pressure, ( p_c )</td>
</tr>
<tr>
<td>Ambient pressure, ( p_a )</td>
</tr>
<tr>
<td>Bulk modulus, ( \beta )</td>
</tr>
</tbody>
</table>
In case study 2 the cavitation boundary is much more complex compared to case study 1. In this case using a gFactor of zero the SOR diverges for any $\omega$ greater than 0.004. For $\omega$ equal to 0.004 the algorithms needs roughly 800,000 iterations to converge which is practically unacceptable. To obtain better results a variable $\omega$ is used in a way that after certain iterations, e.g. 1000, a higher value of $\omega$ is considered. After several trial and errors the optimum value of $\omega$ found to be 0.9 for this problem. Using a variable $\omega$ the SOR algorithm converges in 10,809 iterations.

The proposed switching algorithm is applied to this problem. Figure 3-5 shows the iterations required by the SOR for different gFactors and relaxation factors. Similar to the previous case study, the algorithm is not too sensitive to the gFactor and almost all the values between 0.5 and 0.8 result in the same number of iterations. The SOR algorithm converges in 7,043 iterations with $\omega = 1.22$ and gFactor $= 0.3$, which shows 54 percent improvement compared to original Elrod algorithm when a variable $\omega$ is used.
Generally, higher values of gFactor which result in smaller change in switch function are more effective for large $\omega$ values. It is worthwhile to mention that no trial and error is needed using the new switching algorithm and the algorithm always converges for $\omega < 1.4$ and $gFactor > 0.65$.

3.6 Conclusions

A new switching algorithm for the Elrod cavitation algorithm is introduced in this paper. The conventional binary switching function is replaced with an exponentially decreasing or increasing function. A SOR algorithm was used to solve the equations. Results of case studies show that the new switching algorithm can accelerate the convergence speed of the Elrod algorithm up to 64 percent. It also improves numerical instability and overcomes the convergence issues caused by the conventional binary switch function. The examination of this method on other solvers like LSOR and ADI remain for future investigations.
3.7 Nomenclature

B Width (m)

D Diameter (m)

g Switch Function

h Local film thickness (m)

h₀ Minimum film thickness (m)

hₙ Dimple depth (m)

L Axial length (m)

p Gage Pressure (Pa)

pₐ Ambient pressure (Pa)

pₖ Cavitation Pressure (Pa)

pₖ Groove pressure (Pa)

r₀ Dimple radius (m)

U Sliding speed (m/s)

xₙ Center coordinate of the dimple (m)

yₙ Center coordinate of the dimple (m)

μ Viscosity (Pa.s)

θ Film content

ρ Density (kg/m³)

β Bulk modulus of the lubricant (Pa)

ξ Convergence tolerance

φ Angular coordinate (deg.)

ω Relaxation factor
3.8 References


CHAPTER 4 ON THE SELF-ADAPTIVE SURFACE GROOVES

4.1 Introduction

As the application of micro/nano technologies increases, the need grows for small-scale, self-adaptive bearings that are capable of supporting load for a wide range of operating conditions. The primary function of these bearings is to match different loading conditions by altering their surface texture. This mechanism is similar to the use of pivoted-pad bearings [1-4] in conventional thrust and journal bearings.

Conventional self-adaptive bearings are designed using different technologies including piezoelectric actuators and magnetic levitation [5-7]. These bearings have application in microelectronics industry for machines that require ultra-precision control such as hard drives. However, the use of computer and electronic devices needed to control the function of these bearings can be costly, and thus limit their practical applications. Recently, some efforts have been made to replace the electronic components in self-adaptive bearings with mechanical components [8,9]. The motivation to make mechanical self-adaptive bearings is that their manufacturing cost is considerably less than electronically-controlled counterparts and that they are particularly suitable in applications with tight geometrical and kinematical constraints. It should be noted, however, that in spite of their simplicity and cost effectiveness, the mechanical self-adaptive bearings may have less control capabilities compared to electronically controlled bearings [8].

In an attempt to develop a pure mechanical self-adaptive bearing, Jackson [8] theoretically showed that by placing a suitable nonlinear spring at the inset of a step bearing, the minimum film thickness can be maintained constant for different loading conditions. Although his results were promising, the design of the required springs with proper nonlinear stiffness was not practical for implementation in real-world applications. Later, Duvvuru et al. [9], further developed Jackson’s [8] ideas by replacing the springs with a thin foil that had the same functionality. The surface they used was able to adapt itself to fluid pressure by deflecting into micro grooves provided underneath the surface (see Fig. 4-1). Duvvuru et al. [9] proposed an approximate one-dimensional model that uses the beam theory to predict the foil deflection. In their analysis the grooves were assumed to be infinitely long (IL); hence, the beam theory was applicable for estimating the deformation. The fluid pressure was predicted by solving a one-dimensional Reynolds equation with the Reynolds boundary condition.

The mechanical self-adaptive surface grooves are, in a sense, similar to the static dimples [10-13] but with the advantage of providing a more “flexible surface texturing”. It is widely believed that cavitation is responsible for generating load in textured surfaces [14]. Yet recent

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research by different groups [15, 16] show that to accurately predict performance of textured surfaces, one cannot rely on the Reynolds boundary condition. For micro-textured surfaces the Reynolds boundary condition largely underestimates the cavitated area, leading to inaccuracies in the prediction of the load-carrying capacity [16]. In these applications a mass-conservative algorithm should be used.

The motivation of this study is to develop an accurate model to predict the tribological behavior of the self-adaptive surface grooves. The one-dimensional model proposed by Duvvuru et al. [9] is extended to effectively handle grooves with arbitrary length-to-width ratios. The plate theory is used to determine the grooves surface deflection due to lubricant pressure. A mass-conservative algorithm based on the Elrod and Adams p-θ model [16] is used to properly account for cavitation.

Figure 4-1: Schematic of a mechanical self-adaptive bearing

4.2 Mathematical Model

The schematic of a mechanical self-adaptive bearing is shown in Fig. 4-1. This bearing has a number of small grooves underneath the stationary surface. The surface deformation above the grooves is dependent on the lubricant pressure, while the lubricant pressure itself is dependent upon the surface deformation. The pressure and deformation are thus coupled and,
consequently, an appropriate numerical model should simultaneously solve both the governing equations for the plate deformation and the lubricant pressure.

4.2.1 Plate Deformation Equation

The groove’s top surface can be modeled by a plate which deforms within the groove as shown in Fig 4-2. The transverse displacement $\psi$ of a plate subjected to distributed load $p$ is governed by the classical plate equation [17],

$$\nabla^2 D \nabla^2 \psi = p$$  \hspace{1cm} (1)

where the differential operator $\nabla^2$ is the Laplacian operator, and $D$ is the bending or flexural rigidity of the plate defined as follows.

$$D = \frac{E t^3}{12(1-\nu^2)}$$

where $E$ is the Young’s modulus, $\nu$ is the Poisson’s ratio, and $t$ is the plate thickness. If the bending rigidity, $D$, is constant throughout the plate, the plate equation can be simplified to:

$$\nabla^4 \psi = \frac{p}{D}$$  \hspace{1cm} (2)

Figure 4-2: Schematic of groove’s top surface

The plate edges boundary conditions in the longitudinal and the transverse directions are assumed to be clamped and free, respectively. Since the grooves are equally spaced, one can take advantage of the geometrical symmetry and analyze only one single groove.
4.2.2 Fluid Pressure Equation

The mass-conserving $p - \theta$ model proposed by Elrod and Adams [18] is used to predict the pressure distribution within the lubricant film. This formulation implements the Floberg-Jakobsson-Olsson (JFO) cavitation theory [19, 20]. It corrects the lack of mass conservation that exists with Reynolds boundary condition by introducing the scalar field $\theta$. Under the steady condition, for a compressible Newtonian lubricant with constant viscosity, the Reynolds equation in $p - \theta$ form can be written as [16]

$$\nabla \cdot (\vec{h}^3 \nabla \bar{p}) = 6 \frac{\partial \bar{h}}{\partial x}$$

(3a)

$$\bar{p} \geq 0 \quad \theta = 1$$

(3b)

$$\bar{p} = 0 \quad \theta < 1$$

(3c)

where $\vec{h}$ is the film thickness, $\bar{p}$ is the local pressure of the film, $\theta$ is the film content, and the differential operator $\nabla \cdot$ is the divergence operator. The periodic boundary condition is applied in longitudinal direction because of the symmetry of the model.

$$\bar{p}(0, y) = \bar{p}(1, y)$$

(4)

4.3 Numerical Method

4.3.1 Iterative Scheme

An iterative scheme is used to solve the coupled equations. Figure 4-3 shows the flowchart of the main steps of the iterative scheme. Based on this scheme, first an initial deflection for the plate is assumed and Eq. (3) is solved. The value of $\vec{h}$ in Eq. (3) is calculated based on the initial film thickness and plate deflection as follows:

$$\vec{h} = 1 + \psi / h_0$$

(5)

where $h_0$ is the minimum film thickness.

Next, the pressure obtained from the Reynolds equation is used as a load to solve the plate deformation equation. Using the new deflection results the Reynolds equation is updated by Eq. (4) and solved again. This process is iteratively continued until the results converged, that is the difference in the pressure norms between two successive deflections is less than a specified convergence tolerance, i.e., $1 \times 10^{-3}$. 
4.3.2 Mass-Conservative Cavitation Algorithm

The finite difference method (FDM) is used to discretize Eq. (3). The discrete equation at node \((i,j)\) using central difference for special variables is as follows:

\[
\begin{align*}
& s_{i,j} \bar{p}_{i+1,j} + s_{i-1,j} \bar{p}_{i-1,j} + q^2 \left( s_{i,j+1} \bar{p}_{i,j+1} + s_{i-1,j} \bar{p}_{i-1,j} \right) \\
& - (s_{i,j} + s_{i-1,j} q^2 (s_{i,j} + s_{i,j+1})) \bar{p}_{i,j} = 6 \Delta \bar{x} (c_{i,j} - c_{i-1,j})
\end{align*}
\]  

(6)

where

\[
s_{i,j} = \frac{\bar{h}_{i,j}^3 + \bar{h}_{i+1,j}^3}{2}, \quad c_{i,j} = \theta_{i,j} \bar{h}_{i,j}, \quad q = \frac{\Delta \bar{y}}{\Delta \bar{x}}
\]

Figure 4-3: Flowchart of the iterative scheme
The resulting system of equations is solved using an iterative algorithm proposed by Ausas et al. [16, 21]. The algorithm consists of a relaxation scheme combined with a correction to enforce cavitation conditions described in Eq. (3b) and Eq. (3c). More details can be found in [16, 21]. The pressure and film content in the \( k \)-th iteration are updated from the Eq. (6) by evaluating all the unknowns on the previous iteration using the following equations.

\[
\bar{p}_{i,j}^{k} = \frac{1}{(s_{i,j} + s_{i-1,j}q^2(s_{i,j} + s_{i,j+1}))} \left\{ s_{i,j}\bar{p}_{i+1,j}^{k-1} + s_{i-1,j}\bar{p}_{i-1,j}^{k-1} + q^2(s_{i,j+1}\bar{p}_{i,j+1}^{k-1} + s_{i,j-1}\bar{p}_{i,j-1}^{k-1}) - 6\Delta \bar{c}(c_{i,j}^{k-1} - c_{i-1,j}^{k-1}) \right\} \quad (7)
\]

\[
\theta_{i,j}^{k} = \frac{1}{6\Delta \bar{c}} \left\{ s_{i,j}\bar{p}_{i+1,j}^{k-1} + s_{i-1,j}\bar{p}_{i-1,j}^{k-1} + q^2(s_{i,j+1}\bar{p}_{i,j+1}^{k-1} + s_{i,j-1}\bar{p}_{i,j-1}^{k-1}) - (s_{i,j} + s_{i-1,j}q^2(s_{i,j} + s_{i,j+1}))\bar{p}_{i,j}^{k} + 6\Delta \bar{c}c_{i-1,j}^{k} \right\} \quad (8)
\]

The pseudo code of the mass-conservative algorithm is presented in Table 1.

<table>
<thead>
<tr>
<th>Table 4-1. The pseudo code of the mass-conservative algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initiate parameters</strong></td>
</tr>
<tr>
<td>do {</td>
</tr>
<tr>
<td>for(( i = 1, j = 1 \rightarrow I, J )) {</td>
</tr>
<tr>
<td>if (( \bar{p}<em>{i,j}^{k-1} &gt; 0 ) or ( \theta</em>{i,j}^{k-1} \geq 1 )) then</td>
</tr>
<tr>
<td>( \bar{p}_{i,j}^{k} \leftarrow \text{use Eq.}(7) )</td>
</tr>
<tr>
<td>( \bar{p}<em>{i,j}^{k} = \omega \bar{p}</em>{i,j}^{k-1} + (1 - \omega) \bar{p}_{i,j}^{k-1} )</td>
</tr>
<tr>
<td>if (( \bar{p}<em>{i,j}^{k} \geq 0 )) then ( \theta</em>{i,j}^{k} = 1 ) else ( \bar{p}_{i,j}^{k} = 0 ) end if</td>
</tr>
<tr>
<td>end if</td>
</tr>
<tr>
<td>if (( \bar{p}<em>{i,j}^{k} \leq 0 ) or ( \theta</em>{i,j}^{k} &lt; 1 )) then</td>
</tr>
<tr>
<td>( \theta_{i,j}^{k} \leftarrow \text{use Eq.}(8) )</td>
</tr>
<tr>
<td>( \theta_{i,j}^{k} = \omega \theta_{i,j}^{k-1} + (1 - \omega) \theta_{i,j}^{k-1} )</td>
</tr>
<tr>
<td>if (( \theta_{i,j}^{k} &lt; 1 )) then ( \bar{p}<em>{i,j}^{k} = 0 ) else ( \theta</em>{i,j}^{k} = 1 ) end if</td>
</tr>
<tr>
<td>end if</td>
</tr>
<tr>
<td>}</td>
</tr>
<tr>
<td>residual = [ \sum_{i=1}^{l} \sum_{j=1}^{J} (\bar{p}<em>{ij}^{k} - \bar{p}</em>{ij}^{k-1})^2 ]</td>
</tr>
<tr>
<td>( k = k + 1 )</td>
</tr>
<tr>
<td>} while (residual &gt; tolerance)</td>
</tr>
</tbody>
</table>
4.3.3 Numerical Details

Although the plate equation can also be discretized using standard finite difference method (FDM), in this work a commercial finite element solver (ANSYS software) is employed to solve the plate equation. A computer code was developed which integrates the ANSYS with the Reynolds equation solver. This code transfers the obtained pressures to ANSYS as a load. It also retrieves the deformation results from ANSYS and uses them in updating the Eq. (3a).

To ensure mesh independency, various mesh sizes were examined and eventually a mesh size of 32*32 was considered for all numerical simulations. This mesh size decrease computational time while not compromising the accuracy of the results. The error of this mesh size is less than 3 percent compared to a 128*128 grid size. A relaxation factor (ω) of 0.1 was used to relax Eqs. (7) and (8). The mass-conservative algorithm was assumed to be convergent if the residual was below 10^{-8}. Typical execution time was roughly 40 minutes on a 2.4 GHz Pentium IV processor with 3024 MB RAM.

In some cases, especially when the initial deformation guess is considerably distant from the equilibrium deformation, the iterative scheme may exhibit oscillation in finding the equilibrium pressure and deformation, even though the Reynolds equation and plate equation individually converge fast. In these cases the algorithm may need 20-30 iterations to find the equilibrium instead of typical value of 4-6 iterations. To speed up the convergence rate and damp out the oscillations, the average of the last two pressure profiles is transferred to ANSYS. In this case, the iterative scheme converges quickly.

4.3.4 Load-Carrying Capacity and Friction Coefficient

Once the results converge, the program proceeds to determine the load-carrying capacity and friction coefficient. The load-carrying capacity for each groove is

\[ w = \int_0^B \int_0^L p\,dx\,dy \]  

(9)

The friction force is defined as:

\[ F = \int_A \tau_{xy} \,dA = \int_A \left( \frac{h}{2} \frac{\partial p}{\partial x} + \frac{\mu U}{h} \right) \,dA \]  

(10)

where \( \tau_{xy} \) represents the approximate shear stress evaluated on the moving surface. The friction coefficient is the ratio of the friction force to load-carrying capacity,
\[ f = \frac{F}{w} = \int_{\bar{w}} \left( \frac{h}{2} \frac{\partial \overline{p}}{\partial \overline{x}} + \frac{1}{h} \right) d\overline{A} \left( \frac{h_0}{L} \right) \]  
\[ (11) \]

where \( \bar{w} \) and \( \overline{A} \) are the dimensionless load-carrying capacity and area, respectively.

### 4.4 Results and Discussion

In this section, the performance of the self-adaptive grooves is studied for different geometrical parameters. To encompass a wide range of length-to-width ratios the results of simulations are presented for infinitely long grooves as well as finite grooves. Specifically, results are presented for both B/L=100 and B/L =1. The default simulation parameters, taken from Duvvuru et al. [9], are listed in Table 2.2. These values are used for all simulations unless otherwise specified.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, ( L )</td>
<td>200 µm</td>
</tr>
<tr>
<td>Length ratio, ( r )</td>
<td>0.5</td>
</tr>
<tr>
<td>Width, ( B )</td>
<td></td>
</tr>
<tr>
<td>infinitely long</td>
<td>2 cm</td>
</tr>
<tr>
<td>finite</td>
<td>200 µm</td>
</tr>
<tr>
<td>Foil thickness, ( t )</td>
<td>10 µm</td>
</tr>
<tr>
<td>Viscosity, ( \mu )</td>
<td>8.59×10^{-4} Pa.s</td>
</tr>
<tr>
<td>Velocity, ( U )</td>
<td>7.5 m/s</td>
</tr>
<tr>
<td>Minimum film thickness, ( h_0 )</td>
<td>5 µm</td>
</tr>
<tr>
<td>Young’s modulus, ( E )</td>
<td>0.8 MPa</td>
</tr>
</tbody>
</table>

#### 4.4.1 Effect of Foil Thickness

The effect of foil thickness on the pressure and the foil deflection for B/L=100 under a constant load of 0.0208 N is illustrated in Fig. 4-4. The plot shows that the maximum pressure occurs at the end of deformable section \( (\bar{x} = 0.5) \). The curvature of the pressure profile is strongly dependent upon the foil thickness. This is particularly noticeable in the predicted results for thinnest foil simulated \((t = 7 \mu m)\). As the foil’s thickness increases the deflection decreases and consequently the curvature reduces as well. The pressure profile in the non-deformable section \((\bar{x} > 0.5)\) is linear for all cases. This can be described by the fact that for IL grooves when \( \bar{h} \) is constant, Eq. (3) is reduced to \( \frac{\partial^2 \bar{p}}{\partial \bar{x}^2} = 0 \). As a result, the pressure profile becomes linear. Also,
it can be seen that the diverging profile of $\bar{h}$ at the beginning of the deformable section results in cavitation. In this case the cavitation only occurs in a small portion of the groove.

Generally, as shown in Fig. 4-5, the same trend is observed for B/L=1 under a constant load. Due to side leakages, the pressure is less and also unlike the IL grooves, the pressure profile in the non-deformable section ($\bar{x} > 0.5$) is nonlinear. For finite grooves when $\bar{h}$ is constant, Eq. (3) is reduced to the Laplace equation. As a result, the pressure profile becomes a 2-D curved surface. Figure 4-5 shows that compared to IL grooves, in this case, the cavitation occurs in a much wider length of the groove.

Figure 4-4: The effect of foil thickness on pressure and deflection for IL grooves

Figure 4-5: The effect of foil thickness on pressure and deflection for finite grooves
4.4.2 Load-Carrying Capacity

The effect of film thickness and foil thickness on the load-carrying capacity is illustrated in Fig. 4-6. In these simulations the foil thickness is kept constant while the minimum film thickness is varied. The results are plotted using the following non-dimensional parameter introduced by Duvvuru et al. [9]:

\[
\xi = \frac{h_0 t}{(r \cdot L)^2}
\] (12)

It can be seen that the maximum load-carrying capacity is almost the same for both cases. For IL grooves the maximum occurs at \(\xi = 0.0082\). This number is in agreement with \(\xi = 0.0084\) obtained by Duvvuru et al. [9] for IL grooves using the beam theory approximation. Using this number one can easily find the optimum foil thickness which maximizes the load-carrying capacity for a given film thickness. Similar to IL grooves, there is an optimum value for \(\xi\) for finite grooves, which maximizes the load-carrying capacity. In this case \(\xi = 0.005\) can be considered as an optimum value for finite-sized grooves.

![Figure 4-6: Load-carrying capacity for finite and IL grooves](image)

4.4.3 Effect of Length Ratio, \(r\)

Figure 4-7 shows the pressure profile for different length ratios, \(r\). It can be seen that an increase in \(r\) results in shifting the pressure peak to the right, since the maximum pressure occurs
near the end of the grooved section. Pressure profiles with smaller $r$ values have less curvature in the grooved section. As the length ratio increases the maximum pressure increases as well. However, after a certain value of $r$ the pressure starts decreasing. This trend can be attributed to the fact that for geometries with large length ratios, the average pressure in the grooved section decreases and a larger portion of the groove is under cavitation.

The effect of length ratio on the load-carrying capacity for different B/L ratios is illustrated in Fig. 4-8. It shows that grooves with larger B/L ratio have better load-carrying capacity. Although it is not shown in Fig. 4-8, all grooves with a B/L ratio equal to 10 and more have the same load-carrying capacity profile. This is due to the fact that the side leakage is negligible for large B/L ratios. Similar to previous cases, there exists an optimum $r$ that maximizes the load-carrying capacity. This optimum value is dependent on the B/L ratio, the film thickness and the foil thickness.

![Figure 4-7: The effect of length ratio on pressure profile](image)

4.4.4 Deformable vs. Rigid

In this section, the load-carrying capacity of self-adaptive grooves is compared with non-deformable (rigid) grooves. It is assumed that the static surface have the same shape as the default case (defined in Table 2) at a specified film thickness. This shape will remain constant during simulations and will not change with pressure. Two different rigid cases with the deformed shape of default case at film thicknesses of 2 µm (Case 1) and 5 µm (Case 2) were assumed. The comparison of the results is shown in Fig. 4-9. The actual (dimensional) load-
carrying capacity is also shown in Fig. 4-9. This figure shows that compared to the rigid grooves, self-adaptive grooves can produce more load support in a wide range of operating conditions. It should be noted that as the film thickness increases the load-carrying capacity of the self-adaptive groove reduces to zero, while the rigid grooves still can produce load supports.

Figure 4-8: The effect of length ratio on load-carrying capacity

Figure 4-9: Comparison between deformable and rigid grooves
4.4.5 Reynolds Boundary Condition vs. p-θ Formulation

The effect of various cavitation boundary conditions on the prediction of groove performance is examined here. Figure 4-10 shows the pressure profile predicted by the Reynolds boundary conditions and $p - \theta$ model for $h_0=7 \text{ m}$. It can be seen that the difference between two models for $B/L=100$ is less than 4 percent. This small difference is due to the fact that only a small portion of the groove is under cavitation. The difference between two models becomes unacceptably large for $B/L=1$. For small $B/L$ ratios the Reynolds model largely underestimates the cavitated area, resulting in huge errors in prediction of pressure.

The effect of the two cavitation models on load-carrying capacity of the surface is also illustrated in Fig. 4-11. The difference between the two models is not appreciable for $B/L=100$; nevertheless, due to incorrect prediction of cavitated area for small $B/L$ ratios, the Reynolds boundary condition grossly overestimates the load-carrying capacity. These results confirm the fact that only mass-conserving models should be used when dealing with micro-textured surfaces.

![Figure 4-10: Effect of cavitation model on pressure profile](image)

Figure 4-10: Effect of cavitation model on pressure profile
4.4.6 Friction Coefficient

The effect of different B/L ratios on frictional characteristics of the self-adaptive groove is studied here. Figure 4-12 shows the friction coefficient against the bearing characteristic number, $K_f$, defined as follows:

$$K_f = \frac{\mu UB \left( \frac{L}{t} \right)^2}{w} = \frac{\left( \frac{h_0}{t} \right)^2}{w}$$  \hspace{1cm} (13)

As expected, in hydrodynamic lubrication, with an increase in the bearing characteristic number, the friction coefficient increases as well. Also, it can be seen that grooves with lower B/L ratios have higher friction coefficient in comparison with grooves with high B/L ratios. This is due to smaller load-carrying-capacity of these micro-grooves.

Generally, the friction coefficient is high for all B/L ratios. This observation is in agreement with other published results [22] which show that in hydrodynamic lubrication, the values of friction coefficient (typically 0.1-1) for low-load micro-scale bearings are much higher than the values (typically 0.001-0.01) normally present in macro-scale high-load applications.
4.5 Conclusions

A two dimensional model for self-adaptive surface grooves was developed aimed to extend the current one-dimensional models for more realistic cases. The 2-D model uses plate theory to predict the surface deformation. It also uses a mass-conservative cavitation algorithm based on the Elrod-Adams p-θ model to accurately predict the cavitation. The results presented for different B/L ratios show that for B/L greater than 10 the result of the one-dimensional model proposed by Duvvuru et al. [9] is also satisfactory. A parametric study was conducted to examine the effect of groove geometry on the pressure profile and load-carrying capacity. The results show that under the conditions assumed in this study, self-adaptive grooves can maintain more load support in comparison to conventional grooved surfaces. Comparison between the mass-conservative formulation and Reynolds boundary condition for various B/L ratios shows for large B/L ratios the Reynolds formulation overestimates the load-carrying capacity by 3 percent, but for small B/L ratios it largely underestimates the cavitated area, leading to huge inaccuracies in the estimations. Therefore, only mass-conserving models should be used when dealing with grooves with a B/L ratio less than 10.
4.6 Nomenclature

A  Area (m$^2$)
\bar{A}  Dimensionless area  \[1 \times B/L\]
B  Width (m)
D  Flexural rigidity (N.m)
E  Young’s modulus (N/m$^2$)
f  Friction coefficient
F  Friction force (N)
h_o  Minimum film thickness (m)
\bar{h}  Dimensionless local film height \[h/h_0\]
K_f  Bearing characteristic number
L  Total length (m)
p  Pressure (Pa)
\bar{p}  Dimensionless pressure \[p h_o^2 / \mu U L\]
r  Length ratio (length of deformable section / total groove length)
U  Sliding Speed (m/s)
t  Foil thickness (m)
w  Load (N)
\bar{w}  Dimensionless load \[w h_o^2 / \mu U B L^2\]
\bar{x}  Dimensionless distance in x direction \[x/L\]
\bar{y}  Dimensionless distance in y direction \[y/L\]
\nu  Poisson's ratio
\psi  Plate deflection (m)
\xi  Dimensionless parameter (Eq. 9)
\omega \quad \text{Relaxation factor}

\mu \quad \text{Viscosity (Pa.s)}

\theta \quad \text{Film content}

\tau_{xy} \quad \text{Shear stress}

\textit{super scripts}

K \quad \text{Iteration index}

\section*{4.7 References}


5.1 Introduction

Over the past decade, shape optimization of bearings has captured the attention of many researchers who have tackled the problem by developing a variety of numerical algorithms [1-11]. The available methods for optimization can be broadly categorized into two main groups: stochastic methods and mathematical programming methods.

Stochastic algorithms such as simulated annealing [11], artificial life algorithm [9] and genetic algorithm [1, 3, 6] have been widely used in engineering optimization problems. In general, these methods are good at global searching, and can easily find the near-global-optimum regions. Also, the initial configuration has no effect on the solution these methods predict. However, one of their major drawbacks is that they are very slow; they do not use the gradient information to guide the search direction and are thus slow at reaching the global optimum.

The mathematical programming methods, on the other hand, are very effective in performing local search. In general, gradient-based algorithms converge faster and they can obtain solutions with higher accuracy compared to the stochastic approaches. However, these approaches rely heavily on the initial starting point. Further, depending on the problem, the gradients can be quite expensive to calculate numerically.

In recent years, some researchers [10, 12] have combined the stochastic algorithms with the gradient-based algorithms in an attempt to compensate for the deficiencies of the individual algorithms and to obtain a more efficient optimization technique. Research shows that these hybrid methods are capable of searching for an optimum solution quite efficiently [12].

The motivation of this work is to improve the load-carrying capacity (LCC) of the self-adaptive grooves by optimizing the groove shape. The primary function of these bearings is to match different loading conditions by altering their surface texture. Conventional self-adaptive bearings use magnetic levitation [13] or piezoelectric actuators [14] to provide a flexible surface texture. These bearings primarily have application in microelectronics industry for machines that require ultra-precision control such as hard drives. This work concentrates on mechanical self-adaptive bearings, a fairly recent development in the field [15-17]. A combination of the harmony search (HS) algorithm [18, 19], which is a newly developed optimization method, with the well-known sequential quadratic programming (SQP) method [20, 21] are utilized to determine the global optimum shape of self-adaptive grooves. The hybrid HS-SQP method uses the HS for finding near-global-optimum regions and the SQP method for fine-tuning the HS
solutions. Therefore, this hybrid algorithm increases the possibility of finding the global optimal point and improves the convergence speed.

The outline of this paper is as follows. Section 2 describes the optimization methodology; Section 3 presents a benchmark problem; Section 4 presents the optimization results, and Section 5 summarizes the conclusions.

5.2 Optimization Methodology Using HS-SQP

5.2.1 Objective Function

The objective of the optimization problem is to maximize the LCC of bearings that employ self-adaptive grooves (see Fig. 5-1). The LCC for the groove can be obtained from

$$
\bar{W} = \frac{L}{B} \int_{0}^{B} \int_{0}^{L} \bar{p} dx dy
$$

where $L$ is the groove length, $B$ is the groove width and $\bar{p}$ is dimensionless pressure.

To calculate the LCC one needs to obtain the pressure field in the lubricant. The lubricant pressure is dependent upon the surface deformation of the grooves, while the surface deformation itself is a function of the lubricant pressure. The pressure and deformation are thus coupled and, consequently, the governing equations for the groove deformation and the lubricant pressure should be solved simultaneously. In this work, isothermal condition is assumed and the effect of temperature gradient on the surface deformation and viscosity is neglected.

Figure 5-1: Schematic of a mechanical self-adaptive bearing with rectangular grooves
In a companion article [16], a methodology for treating this coupled mechanism is developed. A brief description of the method is as follows.

First, an initial deflection for the groove surface is assumed and the Reynolds equation is solved. Next, the pressure obtained from the Reynolds equation is used as an input for load to calculate the groove deformation. Using the new deflection results the Reynolds equation is solved again with the updated value of film thickness (h), which is the sum of minimum film thickness ($h_0$) and groove deformation ($\psi$). This process is iteratively continued until the results converge. The key equations are given in Appendix A. More details can be found in [16].

5.2.2 Design Variables

The design variables are the thickness values of the groove’s top surface along its length. In this study, the groove length and width are considered to be fixed. Three different cases are considered. In the first case the thickness of the groove’s top surface is constant (see Fig. 5-2a) and in the second case the thickness varies linearly along the length (see Fig. 5-2b). In the third case, in order to find the optimum shape, the groove is divided in eleven equally spaced points and a spline shape is fitted among these points (see Fig. 5-2c). The optimization algorithm tries to find the optimum values of these variables that maximize the LCC.

Figure 5-2: Schematic of groove’s top surface: (a) constant profile, (b) linear profile, (c) spline profile.
5.2.3 Solution Methodology

The HS starts the optimization process in the hybrid HS-SQP algorithm. First, the initial values of design variables, which define the shape of the groove, are randomly assigned by the HS. Then a simulation is done to calculate the LCC of the specified groove. Next, the HS algorithm, based on the obtained results, sets new values for design variables and another simulation is performed to evaluate the LCC of the new groove. This process is continued until a pre-specified termination criterion for the HS algorithm is satisfied. The best found solution by the HS is stored and used as a starting point for the SQP. Finally, the SQP starts its local search from the starting point and tries to find a solution with higher LCC by calculating the gradients and moving in gradient descent direction. The SQP stops its search when it cannot improve the solution by moving in any direction. More details can be found in [22, 23].

5.3 Benchmark Problem

A self-adaptive groove design problem taken from Duvvuru et al. [15] is used as a benchmark case. The groove has a rectangular shape similar to Fig. 5-1. The length of the groove is 200 µm and the length ratio \( r = 0.5 \). The aspect ratio, \( B/L \), is 100 in this problem and consequently the groove can be considered to be infinity long (IL). A lubricant with a viscosity, \( \mu = 8.59 \times 10^{-4} \text{ Pa.s} \) is used. The minimum film thickness \( h_0 = 5 \text{ µm} \) and the velocity \( U = 7.5 \text{ m/s} \). The groove’s top surface has a constant thickness of 10 µm with a Young’s modulus of \( E = 0.8 \text{ MPa} \).

Duvvuru et al. [15] applied an approximate one-dimensional model based on the beam theory for groove deformation and Reynolds boundary condition for lubricant pressure. The authors, in a companion work [16], have also analyzed this problem in detail.

In this paper, a finite flexible groove is analyzed whose aspect ratio is equal to 1. The other operational and geometrical parameters are assumed to be the same as the IL groove [15]. A mass-conservative algorithm is implemented for treating cavitation in solving the Reynolds equation.

5.4 Results and Discussion

In this section three different thickness profiles (constant, linear and spline), as explained in section 2.2, are examined and optimized. In order to investigate the effect of length ratio on the results, optimum shapes are obtained for two different length ratios (i.e., \( r = 0.5, r = 0.75 \)).
5.4.1 The HS-SQP Default Settings

The default values for the parameters used in the simulations are shown in Table 1. Appendix B provides a brief description of these parameters. More detailed parameter studies and discussions can be found in [23].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth, Δ</td>
<td>5 µm ~ 0.001</td>
</tr>
<tr>
<td>Pitch adjusting rate, P&lt;sub&gt;PAR&lt;/sub&gt;</td>
<td>0.2 ~ 0.85</td>
</tr>
<tr>
<td>Memory consideration rate, P&lt;sub&gt;HMCR&lt;/sub&gt;</td>
<td>0.6</td>
</tr>
<tr>
<td>Harmony memory size, HMS</td>
<td>10</td>
</tr>
<tr>
<td>Lower bound of variables, X&lt;sub&gt;LB&lt;/sub&gt;</td>
<td>5 µm</td>
</tr>
<tr>
<td>Upper bound of variables, X&lt;sub&gt;UB&lt;/sub&gt;</td>
<td>50 µm</td>
</tr>
</tbody>
</table>

5.4.2 Optimization Process

The evolution of the optimization process for a sample design problem with linear thickness profile (see section 4.3, IL groove with r = 0.5) is presented in this section. Figure 5-3 shows a contour plot of the LCC for this problem in which X<sub>1</sub> and X<sub>2</sub> represent the groove’s top surface thicknesses in both ends as illustrated in Fig. 5-2b. The “dots” mark the computed groove geometries by the HS algorithm after 125 iterations (objective function evaluations). After the HS algorithm finishes its search, the SQP routine starts from the best-found solution and seeks to find the optimum solution by calculating the gradients. The cross marks show the SQP search direction toward the global optimum. In this case, SQP needs 42 objective function evaluations to calculate the LCC at all the “cross” marks. A typical execution time for one objective function evaluation is roughly 8 minutes on a 2.4 GHz Pentium IV processor with 3024 MB RAM.

Figure 5-4 shows the trend of the HS search during the optimization process. It can be seen that the HS algorithm finds the near global regions fast but slowly approaches the global optimum solution. The random nature of the search can be seen in the figure. The HS algorithm chooses the value of design parameters randomly from the complete parameter domain. As the search proceeds, it concentrates more around the near-global-optimum region.

5.4.3 Pressure and Load-Carrying Capacity

The simulation results for the IL groove (B/L= 100) with a length ratio of r = 0.5 are given in Table 2. Results show that the spline profile can increase the LCC by 45 percent compared to the benchmark case. It can be seen that by increasing the thickness to 16.68 µm the LCC of the benchmark case can be improved by 30 percent. This improvement is 37 percent if a linear profile for the thickness is used.
Figure 5-3: Influence of the design variables on the load-carrying capacity for linear thickness profile

Figure 5-4: Search evolution through algorithm iterations
Table 5-2. Optimum results for \( r = 0.5 \) and B/L=100

<table>
<thead>
<tr>
<th>Thickness Profile</th>
<th>Optimum design variables (µm)</th>
<th>( \bar{W} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark [16]</td>
<td>\begin{tabular}{cccccccccc} X_1 &amp; X_2 &amp; X_3 &amp; X_4 &amp; X_5 &amp; X_6 &amp; X_7 &amp; X_8 &amp; X_9 &amp; X_{10} &amp; X_{11} \end{tabular}</td>
<td>0.0521</td>
</tr>
<tr>
<td>Constant (Case 1)</td>
<td>\begin{tabular}{cccccccccc} 16.68 &amp; - &amp; - &amp; - &amp; - &amp; - &amp; - &amp; - &amp; - &amp; - \end{tabular}</td>
<td>0.0682</td>
</tr>
<tr>
<td>Linear (Case 2)</td>
<td>\begin{tabular}{cccccccccc} 5.00 &amp; 33.89 &amp; - &amp; - &amp; - &amp; - &amp; - &amp; - &amp; - &amp; - \end{tabular}</td>
<td>0.0714</td>
</tr>
<tr>
<td>Spline (Case 3)</td>
<td>\begin{tabular}{cccccccccc} 5.00 &amp; 5.00 &amp; 20.18 &amp; 31.62 &amp; 34.76 &amp; 37.11 &amp; 34.45 &amp; 26.69 &amp; 17.83 &amp; 15.88 &amp; 15.03 \end{tabular}</td>
<td>0.0757</td>
</tr>
</tbody>
</table>

The associated pressure profile and film thickness for all cases are shown in Fig. 5-5. It shows that the maximum pressure occurs at the end of deformable section (for \( r = 0.5 \), it is \( \bar{x} = 0.5 \)). The results in Fig. 5-5 also reveal that the optimum pressure profiles are nearly triangular in shape. This pressure profile is similar to the classical solution known as the Rayleigh step problem [24]. This holds for the spline profile which is predicted to be the optimum film thickness for providing the maximum LCC.

![Figure 5-5: Film thickness and pressure profile for \( r = 0.5 \) and B/L=100 (IL groove)](image)

Table 3 shows the optimum results for \( r = 0.75 \). Similar to the previous cases the spline profile can support more loads than either the linear or the constant profile. The associated pressure profile and film thickness profiles are shown in Figure 5-6. It can be seen that an increase in \( r \) results in shifting the pressure peak to the right, since the maximum pressure occurs near the end of the grooved section (\( \bar{x} = 0.75 \)).

An interesting result is that, similar to those obtained for \( r = 0.5 \), the optimization algorithm tries to keep \( \bar{h} \) constant in the deformable section. For \( r = 0.75 \) the optimum film
thickness in the deformable section is near 1.9 which is almost the same as the analytical optimum value of 1.866 obtained for a Rayleigh step [25]. For \( r = 0.5 \) the analytical optimum value is 1.681 [25], which is again the same as those obtained in this work (see \( \bar{h} \) for spline profile in Fig. 5-5). It should be noted that analytical optimum values in Ref. [25] were obtained for the 1D case where the pressure in boundaries is zero. In this study we solve the 2D problem with periodic boundary condition. As a result the optimum pressure distribution and film thickness are not exactly the same as 1D case.

<table>
<thead>
<tr>
<th>Thickness Profile</th>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>( X_3 )</th>
<th>( X_4 )</th>
<th>( X_5 )</th>
<th>( X_6 )</th>
<th>( X_7 )</th>
<th>( X_8 )</th>
<th>( X_9 )</th>
<th>( \bar{w} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant (Case 1)</td>
<td>32.78</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0781</td>
</tr>
<tr>
<td>Linear (Case 2)</td>
<td>16.40</td>
<td>49.10</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0813</td>
</tr>
<tr>
<td>Spline (Case 3)</td>
<td>5.00</td>
<td>12.63</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
<td>0.0919</td>
</tr>
</tbody>
</table>

Table 5-3. Optimum results for \( r = 0.75 \) and B/L=100

Figure 5-6: Film thickness and pressure profile for \( r = 0.75 \) and B/L=100 (IL groove)

Comparison of the optimum thickness value for the groove’s top surface presented in Tables 2 and 3 reveals that, in general, the optimum thicknesses for \( r = 0.75 \) is more than \( r = 0.5 \). This can be attributed to the fact that, for the same pressure profile, as \( r \) increases the surface deformation increases as well. The algorithm increases the bending rigidity by increasing the optimum thicknesses to avoid large deformations and keep the deflection around the optimum value.
Table 4 shows the results of optimization algorithm for the finite groove (B/L = 1) with r = 0.5. The spline profile produces the highest LCC among the other thickness profiles. The pressure profile and film thickness for optimized finite grooves are shown in Fig. 5-7. Due to the side leakages, the pressure and LCC are less compared to those of an IL groove. It can be seen that the pressure profile in the fixed section ($\bar{x} > 0.5$) is nonlinear. This can be attributed to the fact that for finite grooves when $\bar{h}$ is constant, the Reynolds equation is reduced to the Laplace equation. As a result, the pressure profile becomes a 2-D curved surface. It is worthwhile to mention that the optimum film thickness in the finite groove is not identical to the conventional Rayleigh step, which is only valid for IL grooves.

Table 5-4. Optimum results for $r = 0.5$ and B/L=1

<table>
<thead>
<tr>
<th>Thickness Profile</th>
<th>Optimum design variables (µm)</th>
<th>$\bar{w}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant (Case 1)</td>
<td>$10.33$ $17.02$ $6.28$ $7.03$</td>
<td>$11.43$ $11.51$ $14.35$ $14.17$</td>
</tr>
<tr>
<td>Linear (Case 2)</td>
<td>$5.00$ $17.02$ $- - - - - -$</td>
<td>$- - - - - -$</td>
</tr>
<tr>
<td>Spline (Case 3)</td>
<td>$5.00$ $5.00$ $6.28$ $7.03$</td>
<td>$11.43$ $11.51$ $14.35$ $14.17$</td>
</tr>
</tbody>
</table>

Figure 5-7: Film thickness and pressure profile at $y = B/2$ for $r = 0.5$ and B/L=1 (finite groove)

5.4.4 Effect of Operating Parameters

Figure 5-8 illustrates the effect of changes in bearing number on the LCC of the optimized IL grooves. In these simulations the speed and viscosity are kept constant while the minimum film thickness is varied from 2 µm to 8 µm. It can be seen that the optimized IL grooves can support loads up to 100 percent more than the IL groove described in the benchmark
case. It is also worthwhile to mention that the spline profile provides a greater LCC in a wide range of operating conditions.

Figure 5-8: Load-carrying capacity of the optimized grooves for \( r = 0.5 \) and \( B/L=100 \) (IL Groove)

Figure 5-9: Load-carrying capacity of the optimized grooves for \( r = 0.5 \) and \( B/L=1 \) (finite groove)

The simulations are repeated for finite grooves and the results are shown in Fig. 5-9. Compared to benchmark case the LCC is less for finite grooves due to side leakages, as
expected. It can be seen that similar to previous case the spline profile has a larger LCC compared to linear and constant profile.

Figure 5-10 shows the LCC of the optimized IL grooves for \( r = 0.75 \). In this case, the spline profile has the maximum LCC among the other thickness profiles. It can be seen that the LCC of different thickness profiles in a constant \( \Lambda \) is more compared to the case of \( r = 0.5 \).

![Graph showing LCC of optimized grooves for different thickness profiles.](image)

Figure 5-10: Load-carrying capacity of the optimized grooves for \( r = 0.75 \) and B/L=100 (infinite groove)

### 5.5 Conclusions

In this study, novel self-adaptive groove designs are presented with shape optimization for producing the highest LCC. Specifically, grooves with constant thickness, linear thickness and arbitrary thickness (spline) shape are considered and optimized. A hybrid approach based on the harmony search algorithm and the sequential quadratic programming is introduced for determining the optimum shape of self-adaptive grooves. This algorithm is simple in concept, involves only a few parameters, and can be easily implemented. The algorithm’s capability is demonstrated using several test cases with comparison to the results of a benchmark case. Results reveal that optimized grooves obtained by this method, in some cases, have 45 percent more LCC than original designs. The extension of this work for non-isothermal conditions warrants further research.
### 5.6 Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>B</td>
<td>Width (m)</td>
</tr>
<tr>
<td>D</td>
<td>Flexural rigidity (N.m)</td>
</tr>
<tr>
<td>E</td>
<td>Young’s modulus (N/m²)</td>
</tr>
<tr>
<td>h</td>
<td>Film thickness (m)</td>
</tr>
<tr>
<td>$h_o$</td>
<td>Minimum film thickness (m)</td>
</tr>
<tr>
<td>$\bar{h}$</td>
<td>Dimensionless film thickness [$h/h_o$]</td>
</tr>
<tr>
<td>HM</td>
<td>Harmony memory</td>
</tr>
<tr>
<td>HMS</td>
<td>Harmony memory size</td>
</tr>
<tr>
<td>L</td>
<td>Total length (m)</td>
</tr>
<tr>
<td>NI</td>
<td>Maximum number of iterations</td>
</tr>
<tr>
<td>p</td>
<td>Gage pressure (Pa)</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>Dimensionless pressure [$p/p_a$]</td>
</tr>
<tr>
<td>$p_a$</td>
<td>Ambient pressure (Pa)</td>
</tr>
<tr>
<td>$P_{PAR}$</td>
<td>Pitch adjusting rate probability</td>
</tr>
<tr>
<td>$P_{HMCR}$</td>
<td>Memory considering rate probability</td>
</tr>
<tr>
<td>$r$</td>
<td>Length ratio (length of the flexible section /total groove length)</td>
</tr>
<tr>
<td>U</td>
<td>Sliding Speed (m/s)</td>
</tr>
<tr>
<td>t</td>
<td>thickness (m)</td>
</tr>
<tr>
<td>w</td>
<td>Load (N)</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>Dimensionless load [$w/(p_aBL)$]</td>
</tr>
<tr>
<td>$\bar{x}$</td>
<td>Dimensionless distance in x direction [$x/L$]</td>
</tr>
<tr>
<td>$X_{UB}$</td>
<td>Upper bound of decision variables</td>
</tr>
<tr>
<td>$X_{LB}$</td>
<td>Lower bound of decision variables</td>
</tr>
</tbody>
</table>
\( \bar{y} \) Dimensionless distance in y direction \([y/L]\)

\( \nu \) Poisson's ratio

\( \psi \) Plate deflection (m)

\( \mu \) Viscosity (Pa.s)

\( \theta \) Film content

\( \Lambda \) Bearing number \([6\mu UL/h_0^2/p_a]\)

\( \Delta \) Bandwidth

5.7 References


CHAPTER 6 TOPOLOGICAL AND SHAPE OPTIMIZATION OF THRUST BEARINGS FOR ENHANCED LOAD-CARRYING CAPACITY

6.1 Introduction

Lord Rayleigh [1] is credited with an ingenious solution for the film profile that yields the greatest LCC in a one-dimensional (i.e., infinitely long) slider bearing. Using the calculus of variations, he discovered that the best shape is a stepped profile, which is commonly known as the Rayleigh step. It took several decades until in 1950, Archibald [2] extended the Rayleigh’s work and presented approximate analytical solutions for 2-D stepped-shaped sliders with finite width. Interestingly, he found that compared to an inclined profile a straight step-film profile could not offer a significant improvement in LCC. Archibald suggested that LCC would be improved if it had a curved interface (i.e., a step width that gradually decreases in the sliding direction) as opposed to a constant width. Few years later, Kettleborough [3, 4] experimentally investigated the behavior of step-interface shape. After many experiments, he found out that trapezoidal pocket geometry could result in considerable improvement over the straight step sliders, and thus proved the correctness of the Archibald’s predictions. It still took another two decades until Rhode and McAllister [5], for the first time, numerically solved for the optimum film profile in rectilinear sliders using a mathematical optimization algorithm. Their findings showed that similar to Kettleborough’s experimental results, the optimum film profile is a step-like profile with a curved interface.

After the pioneering works of Rhode and McAllister [5, 6] research on step sliders has largely shifted back to one-dimensional cases considering different foci such as investigating the stability issues [7], the effect of non-Newtonian lubricant [8, 9], compressibility effects in gas bearings [10], consideration of surface roughness [11] and the study of periodic loading and boundary pressures [12]. There have been some other works dealing with the shape optimization of air bearings [13-16], which are typically used in applications that demand accurate positioning control (e.g., hard disk drives’ flying head). The main objective in these works is to improve the stability and dynamic stiffness of the bearing. Typically, in oil-lubricated bearings the dynamic stiffness is not a major concern due to higher stiffness provided by liquid lubricants compared to gases.

Despite the abovementioned researches, the determination of the optimum film profile in finite-width sectorial-shape sliders is still an open problem. The motivation of this work is to tackle the problem of finding the optimum film profile in sectorial-shape sliders using mathematical optimization methods. The optimization algorithm used here is based on the well-known sequential quadratic programming (SQP) method. This method has a superior performance over other nonlinear programming methods in terms of efficiency, accuracy, and percentage of successful solutions over a vast number of test problems [17]. The idea of the
method is as follows: it starts with an initial guess and evaluates the objective function (i.e., LCC) for that point, then it performs many simulations to calculate derivatives (in case analytical derivatives are not available) in order to determine what direction to move in the search space. After that, using the gradient information it moves to a new point. Next, the objective function as well as derivatives needs to be evaluated for the new point. This process continues until the algorithm cannot find any direction in which the objective function can be maximized. In this study, a general nonlinear constraint-handling package, DONLP2 [18] which is an implementation of the SQP method is used.

Results are presented for incompressible Newtonian lubricants under the isothermal condition. The centrifugal effects are neglected. We will first start with the optimum finite-width sectorial-shape step bearings and present a series of design charts with appropriate formulations. Then, we will calculate the global optimum film profile in finite-width sectorial-shape sliders. Three different film profiles that show different levels of manufacturing complexity are introduced and optimized.

6.2 Theoretical Backgrounds

6.2.1 Stepped Shaped Sliders

The Rayleigh step bearing has a wide range of industrial applications from high-speed turbomachinery to micro-electro-mechanical systems (MEMS) [12, 19-21]. Because of its capability to generate high LCC and the ease in manufacturing, the Rayleigh step profile is used more frequently in thrust bearings [12, 22, 23].

The analytical solutions for pressure distribution and load capacity for one-dimensional stepped sliders (see Fig. 6-1) can be easily derived. The details can be found elsewhere and is not repeated here.

![Figure 6-1: Schematic of a step bearing](image-url)
The load capacity is given by [2]:

\[ W = \frac{3\mu UBLL_1(L - L_1)(\lambda - 1)}{L_1(\lambda^2 - 1) + L}h_1^2 \]  

(1)

where \( B \) is the slider width, \( \lambda = \frac{h_2}{h_1} \), \( U \) is the sliding speed and \( \mu \) is the viscosity.

For the prescribed total length of bearing \( (L = L_1 + L_2) \) and minimum film thickness it is possible to find optimum proportions which maximize the LCC as a function of \( \lambda \) and \( L_1 \). It is maximum when \( \partial W / \partial \lambda = 0 \) and \( \partial W / \partial L_1 = 0 \). It can be shown [2] that in 1-D case, the maximum LCC, \( W_{\text{max}} \), can be achieved when \( \lambda = 1 + \sqrt{3}/2 \) and \( \xi = L_2 / L_1 = 2.549 \) and the results is:

\[ W_{\text{max}} = C_L \cdot \frac{\mu UBL^2}{h_1^2} \]  

(2)

where \( C_L \) is what we shall refer to as the load coefficient. In the case of 1-D step bearing, \( C_L = 0.206 \).

Analytical treatment of two-dimensional problems is more complicated since it requires solving a partial differential equation. The analytical solution for a finite-length stepped slider is available from the work of Archibald [2]. The result is:

\[ W = \frac{48\mu UB}{\pi^4 h_1^2} \sum_{n=1,3,5} \frac{1}{n^4} \frac{\tanh \frac{n\pi L_1}{2B} + \tanh \frac{n\pi L_2}{2B}}{\coth \frac{n\pi L_1}{B} + \lambda^2 \coth \frac{n\pi L_2}{B}} \]  

(3)

Unlike the 1-D case for a finite-width slider, it is not possible to obtain an analytical solution for the optimum LCC because of the highly nonlinear nature of Eq. (3). To obtain a numerical solution, first, the summation term in Eq. (3) is expanded for a sufficiently large number of terms (i.e., \( n=200 \)). Then similar to 1-D case the derivatives with respect to \( L_1 \) and \( \lambda \) are calculated. Next, the coupled system of equations obtained in the previous step is numerically solved using Maple®. Finally, the maximum LCC is obtained by substituting the \( L_1 \) and \( \lambda \) values in Eq. (3). Rearranging the results in the form of Eq. (2) gives:

\[ W_{\text{max}} = C_L \cdot \frac{\mu UBL^2}{h_1^2} \]  

(4)

where the load coefficients, \( C_L \), are given in Fig. 6-2 for a wide range of B/L (see Appendix C for analytical correlations).
It should be noted that $C_L$, $\lambda$, and $\xi (=L_2/L_1)$ are a function of the bearing aspect ratio, $B/L$ only. As the aspect ratio is increased, the optimum length and film-height ratios approach the values obtained for 1-D case.

For stepped sectorial-shape sliders (see Fig. 6-3) the load capacity is [2]:

$$
W = \frac{12\mu\omega R_1^4(\ln \rho)^3(\lambda - 1)}{h_1^2} \sum_{n} \left( \frac{(-1)^{n+1} \rho^2 + 1}{n^2 \pi^2 + 4(\ln \rho)^2} \right)^2 \left[ \frac{\tanh \frac{n\pi \theta_1}{\ln \rho} + \tanh \frac{n\pi \theta_2}{\ln \rho}}{\coth \frac{n\pi \theta_1}{\ln \rho} + \lambda^3 \coth \frac{n\pi \theta_2}{\ln \rho}} \right] (5)
$$

where $\rho = R_2/R_1$ is the radius ratio.

The optimum film-height ($h_2/h_1$) and length-ratio ($L_2/L_1$) for a sectorial-shape slider bearing are calculated in the same way as rectangular sliders. Here, it is assumed that 15% of the
pad area is reserved for the oil feed passages (i.e., \( k_g = 0.15 \); see [24]). The results are illustrated in Fig. 6-4. In Appendix C analytical correlations is given for these ratios. The associated maximum LCC can be obtained from

\[
W_{\text{max}} = \frac{C_L \mu \omega (\ln \rho)^3 R_1^4}{N_{\text{pad}} h_1^2}
\]  

(6)

where \( N_{\text{pad}} \) is the number of pads and \( C_L \) is given in Fig. 6-5. From the results presented in Fig. 6-5, it can be seen that for a given radius ratio LCC increases by increasing the number of pads up to a certain number. Further increase in the number of pads results in a reduction in LCC. This shows that there is an optimum for the number of pads which maximizes the LCC for a given radius ratio. The locus of peaks on all constant \( C_L \) paths (the dashed blue line in Fig. 6-5) defines the optimum number of pads. Examination of these peaks shows that they all have the same aspect ratio, \( B/L = (R_2 - R_1)/(R_{\text{ave}} \theta_0) = 1.12 \). Knowing that \( \theta_0 = 2\pi (1-k_g)/N_{\text{pad}} \) it reads:

\[
N_{\text{pad, opt}} = 3.53 (1-k_g) (\rho + 1)/(\rho - 1)
\]

(7)

Figure 6-4: Optimization chart for stepped sectorial-shape slider bearings (\( k_g = 0.15 \))
6.2.2 Optimum Film Shape

In this section, the problem of determining the global optimum film profile in finite-width sectorial-shape sliders is considered. A film profile is called global optimum if for a given set of operating conditions and minimum film thickness it can produce the highest LCC among all possible film profiles. Generally, global optimum film profiles are rather complex in their geometrical shape and often difficult to manufacture. Hence, we focus our attention to two sub-optimum special cases that can be conveniently manufactured. They are: constant film thickness in radial direction and constant film-height. The results of the optimum profiles are presented for aspect ratios of B/L=0.5, 1, 2, and 4.

The LCC for a given film profile can be obtained by integrating the hydrodynamic pressure over the entire domain:

\[ W = \int_{0}^{\theta_2} \int_{R_1}^{R_2} p r dr d\theta \]  

(9)

where the pressure, \( p \), is obtained from the Reynolds equation. For a Newtonian lubricant with constant viscosity, the Reynolds equation takes on the following form.
\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r h^3 \frac{\partial p}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left( h^3 \frac{\partial p}{\partial \theta} \right) = 6 \mu \omega \frac{\partial h}{\partial \theta}
\] 

(10)

The Reynolds equation is solved numerically using finite difference method. An iterative solver based on successive over-relaxation (SOR) is employed to solve the algebraic equations. Convergence is assumed if the calculated sum of fractional changes in the pressure between two successive SOR iterations falls below a user-defined tolerance value. That is:

\[
\sum_{i=0}^{i_{\text{max}}} \sum_{j=0}^{j_{\text{max}}} \left| \frac{p_{i,j}^{\text{new}} - p_{i,j}^{\text{old}}}{p_{i,j}^{\text{old}}} \right| < 10^{-10}
\]

(11)

A very tight tolerance value is considered here to ensure that the numerical derivatives calculated by the SQP algorithm are precise.

- Constant Film Thickness in Radial Direction (Case 1)

The objective is to find the film height along the circumferential direction in a way that it maximizes the LCC. The film thickness is assumed to be uniform in the radial direction. The Reynolds equation is solved using a computational grid of 101×81 in circumferential and radial directions, respectively. There are 101 design variables in this case. The results of the optimization are shown in Fig. 6-6.

Numerical simulations show that the optimum film profile is a function of length-to-width ratio (B/L) only. It can be seen that as the B/L ratio increases the optimum profile approaches the shape of a step. For smaller B/L ratios, the optimum profile approaches to the conventional inclined surface. It is interesting to note that all the profiles have a flat land of the same area (28% of the total pad area) in the downstream. For B/L =1 the optimum profile approximately looks like a tapered-land bearing suggesting that for this B/L ratio a tapered-land profile outperforms both the step and the tilting pad profiles; see Section 4 for more details.

The load associated with any of these profiles can be obtained from the following equation

\[
W_{\text{max}} = C_L \cdot \frac{\mu \omega R_{\text{ave}}^3 (R_2 - R_1)}{h_{i_1}^2}
\]

(12)

where

\[
\ln C_L = -22.5069678 - 1.86827339 B/L + 0.497851321 (B/L)^{1.5} + 62.66618919 / \sqrt{\rho - 47.2416853 / \rho}
\]

for \(1.1 \leq \rho = \frac{R_2}{R_1} \leq 2\) and
The range of the aspect ratio is: $0.5 \leq B/L \leq 4$. The error associated with these expressions is less than 1.0%.

Figure 6-6: Optimum film profile for constant film thickness in radial direction

- Constant Film Thickness (Case 2)

The film thickness is assumed to have a constant depth while its projection in the domain has a quadrilateral shape (see Fig. 6-7). The objective is to find the optimum location of each corner as well as the film height in a way that it maximizes the LCC. As a result, there are totally 9 design variables. To accurately capture the film shape, a grid of 101×101 is considered for computations.

Figure 6-7: Design variables
It was found that the optimum geometry is a function of B/L and ρ only. The results of simulations are summarized in Table 6-1. The optimum shapes for the special case of ρ=2 are shown in Fig 6-8. It can be seen that optimum profile is substantially influenced by the B/L ratio. For a given B/L ratio, the main difference is in the location of ψ₁ for higher ρ values. Smaller ρ values have almost the same geometry and film thickness. The optimum shape approaches the trapezoidal pocket [4] as the B/L ratio increases.

Table 6-1. Optimum geometry for constant film thickness profile (case 2)

<table>
<thead>
<tr>
<th>B/L</th>
<th>R₂/R₁</th>
<th>χ₁</th>
<th>ψ₁</th>
<th>χ₂</th>
<th>ψ₂</th>
<th>χ₃</th>
<th>ψ₃</th>
<th>χ₄</th>
<th>ψ₄</th>
<th>λ</th>
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<td>0.81</td>
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<td>0.75</td>
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<td>1.00</td>
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The load capacity associated with any of the above profiles can be obtained from the following equation.

\[ W_{\text{max}} = C_L \left( \frac{\mu\omega R_{ave}}{R_s - R_i} \right) \]

(13)

where
\[ C_L = \frac{-0.0002967 + 0.00084424 \ln \rho - 8.3586e-05(B/L)^2 + 1.82658585 (\ln \rho)^2 - 0.0031403 (B/L) \ln \rho}{1 + 3.52085112 B/L + 0.15387093 \ln \rho + 8.35278018 (B/L)^2 - 0.8226755 (\ln \rho)^2 + 4.06481208 (B/L) \ln \rho} \]

for \( 1 \leq \rho \leq 2 \) and

\[ C_L = \frac{0.01903121 - 0.02917787 \ln B/L + 0.01846002 (\ln B/L)^2 - 0.00464465 (\ln B/L)^3 + 0.000185797 \ln \rho}{1 - 0.00322096 \ln B/L - 1.25422955 \ln B/L + 0.438202862 (\ln \rho)^2} \]

Figure 6-8: Optimum film profile for constant film depth

for \( 2 \leq \rho \leq 4 \). The range of the aspect ratio is: \( 0.5 \leq B/L \leq 4 \). The error associated with these expressions is less than 1.0%.

- **Global Optimum Film Profile (Case 3)**

In this section, the objective is to find the optimum film thickness in each node in a way that it maximizes the LCC. Since the number of design variables and consequently the computational time are drastically increased by the grid size, a medium sized grid, i.e. \( 41 \times 41 \), is
considered. This grid size is accurate enough for the calculations as well as capturing the film shape. There are totally 1681 design variables in this case. A typical run-time for obtaining an optimum film profile is roughly 140 hours on a server machine equipped with an Intel Xeon processor E5630, and 16 GB of memory.

Simulations show that, similar to the previous case, the optimum profile is a function of both B/L and R₂/R₁. The optimum shapes are shown in Fig. 6-9 and Fig. 6-10. Generally, as expected, the optimum film profile is not symmetric in sectorial-shape sliders. In smaller B/L ratios (i.e., B/L=0.5) for a given R₂/R₁ the difference in the film thickness on the inner and outer radius is much greater than those correspond to higher B/L ratios (i.e., B/L=4.0). As the B/L increases, the difference in the film height on the inner and outer radius becomes smaller.

![Figure 6-9: Global optimum film profile for R₂/R₁=2](image)

Similar to the constant film-thickness profiles presented in the previous section, the step-like front expands from the central area toward the bearing inner/outer radii, as the B/L ratio...
increases; see Fig. 6-10. When B/L approaches infinity, the film profile takes on the shape of a step. Generally, the maximum height of step decreases as the B/L ratio increases.

It should be noted that in some cases there is noise in the numerical solutions. This numerical noise can perhaps occur because of the sequential linearization of the problem in the SQP. However, it can be alleviated by adding a constraint to the optimization problem, which forces SQP to find a profile that has film thickness that gradually decreases in circumferential direction.

The maximum possible LCC for a given sectorial-shape slider geometry can be obtained from the following equation

\[ W_{\text{max}} = C_L \cdot \frac{\mu \omega R_w^3 (R_2 - R_1)}{h_i^2} \]  

(14)

where

\[ C_L = \frac{-0.03477124 + 0.002223589 \cdot B/L - 0.00057184 (B/L)^2 - 0.46260613 \rho + 0.462888137 \rho^2}{1 + 5.280106396 B/L + 8.742446178 (B/L)^2 + 0.016387106 \rho} \]

for \( 1.1 \leq \rho \leq 2 \) and

\[ C_L = \frac{0.0171649 - 0.02627928 \ln B/L + 0.016412336 (\ln B/L)^2 - 0.00404219 (\ln B/L)^3 + 3.93235 \epsilon 0.05 \rho}{1 - 0.00229757 \ln B/L \cdot 0.47980744 \rho + 0.062764667 \rho^2} \]

for \( 2 \leq \rho \leq 4 \). The range of the aspect ratio is: \( 0.5 \leq B/L \leq 4 \). The error associated with these expressions is less than 1.0%.

### 6.3 Illustrative Example

An example problem taken from [24] is used here to demonstrate the utility of the design charts and formulations presented in the previous section. A thrust bearing needs to be designed for a steam turbine with a thrust load of 110 kN and speed of 3600 rpm. The inner and outer radius are 6 cm and 16 cm, respectively. ISO 32 oil with viscosity of 0.01 Pa.s is to be used. It is assumed that 15% of the area is reserved for oil feed passages (k=0.15). Determine the optimum 2D step profile and compare its minimum film thickness with the profiles presented in Section 2.2.
Figure 6-10: 2D views of global optimum film profiles for different geometries
2D Step – Radial length B for each sector is: 16 - 6 = 10 cm. The average radius \( R_{ave} = (16+6)/2 = 11 \) cm, and \( R_2/R_1 = 2.667 \). From Eq. (7) the optimum number of pads is 6.6. Using 7 pads, the circumferential length L of each pad is \( L = 2\pi R_{ave} (1-k_g)/N_{pad} = 8.39 \) cm, consequently, the B/L ratio is 1.19. The load per pad is \( W = 110000/7 = 15710 \) N. From Fig. 6-5, \( C_L = 3.8 \). Solving Eq. (6) for \( h_1 \) we have \( h_1 = 39.7 \mu m \). The optimum step dimensions can be obtained either from Fig. 6-4 or from Eq. (A2). The optimum \( \lambda \) is 1.695 and the optimum \( \theta_2/\theta_1 \) is 1.4. If the number of pads is increased or decreased, the minimum film thickness decreases as well. For example, if we use 8 pads the minimum film thickness is \( h_1 = 39.4 \mu m \) and when 5 pads are used \( h_1 = 39.1 \mu m \).

Assuming the number of pads is 7, for case of constant film thickness in radial direction (Case 1), from Eq. (12) \( h_1 = 42.7 \mu m \). Using Eq. (13), the minimum film thickness is \( h_1 = 49.3 \mu m \) for constant film thickness (Case 2). For global optimum film profile (Case 3) using Eq. (14) \( h_1 = 50.0 \mu m \). Clearly, larger minimum film thickness that satisfies the load is more desirable.

### 6.4 Discussions

In this section, the LCC of different film profiles are compared and benchmarked against the conventional tilting pad and tapered-land thrust bearings. It should be noted that to calculate the maximum LCC, the optimum tilting angle for each B/L ratio is first computed using SQP algorithm. For tapered-land bearings, it is assumed that the land area is 28% of total area as discussed in Section 2.2.1. Table 2 reports the \( C_L \) for different film profiles.

Results show that for small B/L ratios the 2D step and tilting pad profiles have the worst LCC among the other profiles. These two profiles perform the same for B/L equal to one the optimum step and tilted pad profiles have almost the same LCC. As expected, by increasing B/L ratio the 2D step outperforms the tilting pad profile. The constant film profile in radial direction (Case 1) always produces considerably higher load than the 2D and tilting pad profiles. However, its performance is not considerably better than tapered-land bearings. The quadrilateral profile (Case 2) is much better than Case 1 and tapered-land bearings. This profile can be considered as a good alternative to global optimum profile since it has a simpler geometry, yet capable of producing the closest LCC to the global optimum profile.

### 6.5 Conclusions

In this study, novel bearing designs are presented with shape optimization for producing the highest LCC. Specifically, sectorial-shape sliders with 2D step profiles, constant film thickness in radial direction and constant film depth with a quadrilateral cross section are considered and optimized. The problem of finding the global optimum film profile for sectorial-
shape sliders are also addressed in detail and the optimum profiles are given for different geometrical aspect ratios. An optimization method based on the sequential quadratic programming is used for determining the optimum shape in all cases. Results reveal that optimized shapes obtained by this method, for common bearing sizes (i.e., B/L=1, ρ=3), have more than 90 percent LCC compare to conventional tilted pad or step bearings. The extension of this work for non-isothermal conditions or non-Newtonian lubricants warrants further research.

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6.6 Nomenclature

B Width (m)

C_L Load Factor (see Eq. 5)

h Local film thickness (m)

h_1 Minimum film thickness (m)

L Axial length (m)

N_{pad} Number of Pads

R_1 Inner radius (m)

R_2 Outer radius (m)

U Sliding speed (m/s)

W Load-carrying capacity (N)

W_{max} Maximum load per pad (N)

X x-coordinate (m)

Y y-coordinate (m)

\mu Viscosity (Pa.s)

\theta Angle (rad.)

\theta_0 Total pad angle (rad.)

\lambda Film thickness ratio (H_2/H_1)

\xi Length Ratio (L_2/L_1)

\phi Angular coordinate (deg.)

\omega Angular velocity (rad/s)

\rho Radius ratio (R_2/R_1)

\psi Dimensionless radius ((R_1)/(R_2-R_1))

\chi Dimensionless (\theta/\theta_0)
6.7 References


7.1 Introduction

Bearing performance optimization in order to increase the load-carrying capacity (LCC) or reduce frictional loss has been the subject of many studies in the past decade [1-16]. The great majority of these papers have concentrated on the conventional bearing geometries with non-deformable surfaces. A new generation of self-adaptive bearings that use mechanical deformation to match different loading conditions have recently emerged [17-23]. These bearings offer several distinct advantages and research show a properly designed self-adaptive bearing can offer improved performance over a wider range of operating conditions compared to the conventional bearings.

The use of adaptive (compliant surface) mechanisms to improve the performance has been reported in many different tribo-components such as in gas foil bearings [24-26], mechanical seals [27], journal bearings [17, 28, 29] and thrust bearings [22, 30-33]. In case of the parallel flat surface thrust bearings the deformation of pads due to lubricant pressure can be on the order of the lubricant film thickness and hence can substantially contribute to the LCC of bearing. Consequently, the idea of using elastic pads has been investigated in order to improve (or control) the bearing performance [22, 30-32]. Particularly relevant to current work is that of Minculescu and Cicone [34] who proposed a simplified hydrodynamic model for parallel surface elastic sliders.

The motivation of this work is to develop a class of self-adaptive mechanical thrust bearings that utilize specially designed flexible pads. A description of the bearing concept and its numerical modeling is presented. To obtain the maximum LCC, a hybrid optimization method based on harmony search (HS) algorithm [35, 36] and sequential quadratic programming (SQP) [37, 38] is developed. Furthermore, an approximate analytical methodology is presented for the use at preliminary design stage to obtain near-optimum LCC in a thrust bearing.

The outline of this paper is as follows. First, the mathematical model is described in Section 2 followed by optimization methodology in Section 3. Next, an approximate analytical model is described in Section 4. Then, an illustrative example is presented in Section 5, and the results of shape optimization are analyzed. Finally, Section 6 summarizes the conclusions.

7.2 Mathematical Model

The schematic of a flexible-pad thrust bearing is shown in Fig. 7-1. This bearing has a split on the side of the pads. The pad surface deformation is dependent on the lubricant pressure,
while the lubricant pressure itself is dependent upon the surface deformation. The pressure and deformation are thus coupled and, consequently, an appropriate multi-physics model should be considered to simultaneously solve the governing equations for both the lubricant pressure and the pad deformation.

Figure 7-1: Schematic of a flexible-pad thrust bearing

The lubricant pressure distribution can be obtained from the Reynolds equation. Under the steady condition, for an incompressible Newtonian lubricant with constant viscosity the Reynolds equation in polar coordinate system can be written as [39]

\[
\frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} \left( \tilde{r} \frac{\partial \tilde{p}}{\partial \tilde{r}} \right) + \frac{1}{\tilde{r}^2} \frac{\partial}{\partial \tilde{\theta}} \left( \tilde{r}^2 \frac{\partial \tilde{p}}{\partial \tilde{\theta}} \right) = \Lambda \frac{\partial \tilde{\phi} \tilde{h}}{\partial \tilde{\theta}}
\]

(1)

where \( \phi \) is the fractional film content, \( \tilde{r} \) is dimensionless radius, \( \tilde{p} \) is the dimensionless pressure, \( \tilde{h} \) is the dimensionless film thickness and \( \Lambda \) is the bearing number.

The Reynolds equation is solved numerically using the finite difference method. A fast direct solver that uses banded-LU decomposition method [40] is employed to solve the algebraic equations. This method is very easy to implement. Numerical results for a moderate grid size of 60x60 shows that this method is at least 15 times faster than successive over relaxation (SOR) method, which makes it suitable for optimization process that demands a large number of simulations.

Due to the complex geometry and loading condition, it is not possible to find an analytical solution for the pad deformation. Thus, the finite element method is used to accurately predict the pad deformation. In this work a commercial finite element solver (ANSYS™) is employed to calculate the pad deformation. In the simulations it is assumed that the pads bottom surface is constrained from movement and the lubricant pressure is applied on the pad top surface. Since all the pads have the same geometry and experience an identical load (see Fig. 7-2), one can take advantage of the symmetry and analyze only one single pad.
An iterative scheme, as shown in Fig. 7-3, is used to solve the multi-physics model. Based on this scheme, first an initial minimum film thickness and an initial pressure distribution is assumed and the corresponding pad deformation is evaluated. Based on the pad deflection ($\psi$) the value of $\bar{h}$ in Eq. (1) is updated as follows

$$\bar{h} = 1 + \psi / h_0$$  \hspace{1cm} (2)

where $h_0$ is the minimum film thickness.

Next, The Reynolds equation is solved for the pressure which is used as a loading function input to the FEM analysis to calculate the pad deformation. The new pad deformation results are then used to update the film thickness and solve the Reynolds equation again. This process is iteratively continued until the results converge, i.e., the relative error in the pressure norms between two successive iterations is less than a specified convergence tolerance (here, $5 \times 10^{-4}$). Once the solution converges, the LCC is obtained by integrating the hydrodynamic pressure over the entire domain:

$$W = \int_0^{\theta_0} \int_{R_1}^{R_2} p r dr d\theta$$  \hspace{1cm} (3)

After the calculation of $W$, its value is compared with the external load on the pad; if they do not match, a new value for $h_0$ is assumed and the whole process is continued until the load balance condition is satisfied.

To ensure mesh independency in the finite element simulations, various mesh sizes were examined. Eventually a model composed of approximately 17,000 elements was considered to be satisfactory and used in all simulations. The error of this mesh size is less than 0.5 percent compared to a fine model with 67,000 elements.
Figure 7-3: Bearing simulator: iterative scheme used to calculate pressure distribution and pad deformation

Generally, the banded-LU decomposition method [40] and finite element solver are individually stable and the iterative scheme converges typically in 8-10 iterations. However, when dealing with very high loads and in cases where the pad leading edge deformation is much greater than the minimum film thickness, the iterative scheme may exhibit oscillation in finding the equilibrium pressure and deformation. To speed up the convergence rate and damp out the oscillations, the pressure needs to be under-relaxed before being transferred to ANSYS™. In some cases the deformation results also need to be under-relaxed to avoid oscillations. In this work, to ensure the convergence of the iterative scheme for any loading condition, an under-relaxation value of 0.1 was used for both the intermediate pressure and deformation solutions. To reduce the execution time, this number can be adaptively increased as the deformation becomes close to its equilibrium value. A typical execution time was roughly 20 minutes on a server computing environment equipped with an Intel Xeon processor E5630, and 16 GB of memory.
7.3 Optimization Methodology

The objective of the optimization is to maximize the LCC of a bearing by finding the optimum dimensions of the split. The design variables are the pad thickness above the split \( t_s \), and split angle ratio \( \xi = \theta_s / \theta_g \) as shown in Fig 7-4. Generally, the split height \( h_s \) is not an important factor as long as it allows the deformation of the top surface without contacting the lower surface. As a result, this parameter is considered to be fixed at 2 mm in this study.

![Figure 7-4: Geometric parameters and design variables \( t_s \) and \( \xi \)](image)

The optimization method used here is based on a hybrid of the harmony search (HS) algorithm and sequential quadratic programming (SQP) method. There is a vast literature on both algorithms so the details are not repeated here. Referring to Fig. 7-5, a brief description of the HS-SQP algorithm is as follows [35]:

First, the initial values of design variables, which define the shape of the pad, are randomly assigned by the HS. Then, the LCC of the pad with specified split is calculated. Next, the HS algorithm, based on the obtained results, sets new values for design variables and another round of simulation is performed to evaluate the LCC of the new design. This process is continued until a pre-specified maximum number of iterations (e.g., 50) for the HS algorithm is reached. After this step, the SQP begins its task and uses the best obtained solution as the starting point. In order to find an appropriate search direction, SQP calculates derivatives and uses the gradient information to move to a new point. It evaluates the LCC and its derivative with respect to the design variables associate with the new point in order to determine the search direction again. The SQP continues this process until it cannot find any direction in which the LCC can be increased. This point is considered to be the optimum solution. Typically, to find the optimum solutions, the hybrid HS-SQP algorithm needs around 100 iterations which take roughly 6-8 hours of computations.
7.4 Approximate Analytical Model

The aim of this section is to present an approximate analytical method for efficient design of flexible-pad thrust bearings. The best way to design a flexible-pad bearing for a given operating condition and geometry is to carry out a numerical optimization as discussed in the previous section. However, in the absence of numerical optimization tools, the approximate design method presented here can be used in order to achieve a near-optimum bearing design capable of providing the largest LCC.

There are three important parameters that need to be defined. They are: minimum film thickness, split angle ratio and pad thickness above the split. In order to find the optimum value of these parameters for different geometries and operating conditions, numerous numerical
simulations (approximately 150 different combinations) were carried out for stainless steel flexible-pad thrust bearings in the range of $0.5 \leq \lambda \leq 2$ and $1.2 \leq \rho \leq 4$.

It is possible to find a good approximation for the film thickness using the correlations available for global optimum LCC. The maximum possible (i.e., global-optimum) LCC ($W^*$) for a given rigid pad can be obtained from the following equation [41]:

$$W^* = C^*_L \frac{\mu \omega R^3_{av} (R_2 - R_1)}{h_0}$$  \hspace{1cm} (4a)$$

where $C^*_L$ is the load coefficient for the global-optimum film profile. This parameter is only a function of the radius ratio, $\rho = R_2 / R_1$ and the pad aspect ratio $\lambda = B / L$. The load coefficient in the range of $0.5 \leq \lambda \leq 4$ it is given by [41]:

$$C^*_L = \begin{cases}  
-0.03477 + 0.002224 \lambda - 0.0005718 \lambda^2 - 0.4626 \rho + 0.4629 \rho^2 \\ 1 + 5.28 \lambda + 8.742 \lambda^2 + 0.01639 \rho 
\end{cases} \quad 1.2 \leq \rho \leq 2$$

$$= \begin{cases}  
0.01716 - 0.02628 \ln(\lambda) + 0.01641 (\ln \lambda)^2 - 0.004042 (\ln \lambda)^3 + 3.932 \times 10^{-5} \rho \\ 1 - 0.002298 (\ln \lambda) - 0.4798 \rho + 0.06276 \rho^2 
\end{cases} \quad 2 \leq \rho \leq 4$$  \hspace{1cm} (4b)$$

The variation of the load coefficient ratio in equation (5) for different $\rho$ and $\lambda$ values in shown Fig. 7-6.

Using the above equations, it is possible to compute $h_0$ as follows:

$$h_0 = \sqrt{C_L \frac{\mu \omega R^3_{av} (R_2 - R_1)}{W}}$$  \hspace{1cm} (6)$$

The maximum error in the prediction of $h_0$ using the above equations is found to be less than 4 percent.
The split angle ratio can be obtained from the following equation:\[^1\]\
\[
\xi = \frac{\xi_{\text{max}}} {1 - (1 + 10^6 h_0)^{0.11}}
\]
(7a)

where
\[
\xi_{\text{max}} = 0.839 + 0.0585 \lambda + 0.008 \rho
\]
(7b)

\[
h_{\text{max}} = h^*_\text{max} \left(1 + \left(\frac{h_0}{h^*_\text{max}}\right)^{1.4}\right)
\]
(7c)

\[
h^*_\text{max} = \frac{3}{186} \sqrt{C_L \mu \omega R_{\text{ave}}^3 (R_2 - R_1)}
\]
(7d)

In order to obtain the optimum value for $\ell$, one needs to approximate the mean deflection of the pad leading edge, $\psi_m$. Simulation results show that for an optimum design there is an approximate relation between the $\psi_m$ and $h_0$ as follows:

\[
\frac{\psi_m}{h_0} = 1 + \frac{0.7}{\lambda}
\]
(8)

\[^1\] Note that parameters in equation (7a) are dimensional quantities in S.I. units. The maximum and average errors associated with this equation are 9.7% and 2.1%, respectively. A less accurate dimensionless equation for split angle ratio is given by $\xi/\xi_{\text{max}} = 0.228 \ln(h_0/h_{\text{max}}) + 0.945$ with an average error of 4.6% and maximum error of 22%.
To approximate \( t_s \), one can assume that \( \psi_m \) is proportional to the maximum deflection of a cantilever beam under uniformly varying load (see Fig. 7-7). Consequently,

\[
\psi_m \propto \frac{w_0 (2R_{aw} \theta_0)^4}{30EI}
\]

Assuming maximum load intensity, \( w_0 = \frac{2W}{R_{aw} \theta_0} \), \( I = \frac{1}{12} Br_s^3 \) and solving for \( t_s \) Eq. (9) reads:

\[
t_s \propto \left( \frac{W \xi^4 (\rho + 1)^3 \theta_0^3 R_i^2}{10E \psi_m (\rho - 1)} \right)^{\frac{1}{3}}
\]

(10)

Substitution of Eq. (8) in Eq.(10), yields

\[
t_s \propto \left( \frac{W \xi^4 (\rho + 1)^3 \theta_0^3 R_i^2}{E h_0 (1 + \frac{0.7}{\lambda}) (\rho - 1)} \right)^{\frac{1}{3}}
\]

(11)

Now, in order to obtain an approximation for \( t_s \), a correction factor (\( \beta \)) must be applied to Eq. (11). The corrected \( t_s \) now can be obtained as follows:

\[
t_s = \beta \left( \frac{W (\rho + 1)^3 \theta_0^3 R_i^2}{E h_0 (1 + \frac{0.7}{\lambda}) (\rho - 1)} \right)^{\frac{1}{3}}
\]

(12)

where
\[
\beta = \exp \left( -0.207 - 0.068 \lambda + 0.015 \lambda^{-2} - 0.89 \xi \right)^{-1}
\]

The correction factor is determined in a way that it minimizes the error between the predicted \( t_s \) from Eq. (12) and the actual \( t_s \) obtained from the aforementioned numerical optimization simulations. The maximum error of the above equation is less than 10 percent.

It should be noted that some of the equations developed here (i.e., Eq. (5), Eq. (7) and Eq. (12)) are only valid for stainless steel pads. Also, the use of these equations should be limited to \( 0.5 \leq \lambda \leq 2 \) and \( 1.2 \leq \rho \leq 4 \).

7.5 Results and Discussion

In this section, a flexible-pad thrust bearing is designed and its geometry is optimized for maximum LCC. The performance of the designed bearing is compared with an inclined pad thrust bearing. We begin by presenting an illustrative example to show the utility of the equations derived in Section 4.

7.5.1 Illustrative Example

A thrust bearing needs to be designed for a load of \( W=14500 \) N at the speed of 2000 rpm. The inner bearing diameter is \( D_i = 8 \) cm, and outside diameter is \( D_o = 16 \) cm, to provide a pad radial length \( B = 4 \) cm. The bearing accommodates 8 pads, with circumferential breadth \( L = 4.0 \) cm, assuming that 15% of the area [39] is reserved for oil feed grooves (\( k_g = 0.15 \)). ISO 32 oil with viscosity of \( \mu = 0.01 \) Pa.s is to be used. The bearing thickness is \( t_b = 2 \) cm and it is made from a stainless steel with a Young’s modulus of \( E = 210 \) GPa and a Poisson’s ratio of \( \nu = 0.3 \).

Making use of the approximate analytical model of Section 4, one can easily design the bearing with a side-split for near optimum LCC. The results are summarized in Table 1. The calculations show that to create sufficient flexibility in the pad a split (of thickness 2mm) should be made on the side of the pad, 4.34 mm below the pad surface. To ensure an optimum deflection mechanism the split angle ratio needs to be 0.63.

Next, for comparison purposes the full numerical optimization is carried out using HS-SQP algorithm. The optimum solution, found after 118 simulations, is shown in Table 2. In can be seen that the prediction of the analytical model is very close to the full numerical solution. The thickness obtained from analytical expressions is less than the numerical optimum resulting in higher deflections. However, the performance of the analytical design, as evidenced by \( C_L \), is the same as the numerical optimum. For comparison purpose, an optimally designed inclined pad thrust bearing which has the same geometry and works under the same condition is analyzed and reported in Table 8-2. The optimum inlet-to-outlet film thickness ratio \( (h_1/h_0) \) which results in the highest LCC in inclined-pad bearings was obtained using SQP method as follows:
\[ h / h_0 = 2.1667 + 0.164 / \lambda \quad (0.3 \leq \lambda \leq 4) \]  

Table 7-1. Results of approximate analytical design method

\begin{tabular}{l|l}
\hline
\textbf{Input data:} & \textbf{Results:} \\
\hline
\begin{align*}
\lambda &= 1, \quad \rho = 2, \quad R_{ave} = 0.06 \text{m}, \quad N_{pad} = 8 \\
W &= 14500 / 8 = 1812 N, \quad \mu \omega = 2.094 \text{Pa}
\end{align*} & \begin{align*}
\text{Eq.}(4b) & \quad C_L^* = 0.0592 \\
\text{Eq.}(5) & \quad C_L = 0.0435 \\
\text{Eq.}(6) & \quad h_0 = 20.8 \mu \text{m} \\
\text{Eq.}(7a) & \quad \xi = 0.63 \\
\text{Eq.}(8) & \quad \psi_m = 1.7h_n \\
\text{Eq.(12)} & \quad t_s = 4.34 \text{mm}
\end{align*} \\
\hline
\end{tabular}

The optimum film thickness ratio for inclined-pad can be only approximated as a function of \( \lambda \) since the effect of \( \rho \) is negligible. This value is \( h / h_0 = 2.33 \) for this case. The load coefficient for the optimum inclined-pad bearings is given by:

\[
C_L = \begin{cases} 
  \frac{-0.0349 - 0.000293 \lambda + 0.00477 \rho + 0.0243 \rho^2}{1 + 0.487 \lambda + 0.954 \lambda^2 - 0.0803 \rho} & 1.2 \leq \rho \leq 2 \\
  \frac{-0.222 - 0.00393 \lambda + 0.188 \rho - 0.0121 \rho^2}{1 + 1.18 \lambda + 1.09 \lambda^2 + 0.0136 \rho} & 2 \leq \rho \leq 4
\end{cases}
\]  

(14)

Comparison between the \( C_L \) of both bearings shows that the flexible-pad design can produce 30 percent more LCC than inclined-pad for \( h_0 = 20.2 \mu \text{m} \) (see Section 5.3 for detailed comparison).

The pressure profile and the film thickness of the optimum design are shown in Fig. 7-8. It can be seen that the maximum pressure occurs at the end of deformable section. The results in Fig. 7-8 also reveal that the optimum pressure profiles are nearly triangular. Generally, for an optimum design the mean dimensionless film thickness in the leading edge of the flexible-pad bearing, \( (h_0 + \psi_m) / h_0 \), can be approximated from Eq. (8). This value is independent of the pad’s material type. In this case, as can be seen in Fig. 7-8, this value is 2.69 which is very close to the value obtained from Eq. (8) (i.e., 1+1.7=2.7).

87
Table 7-2. Results of numerical optimization

<table>
<thead>
<tr>
<th>Type</th>
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<th></th>
<th></th>
<th></th>
<th></th>
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<td>ξ (mm)</td>
<td>t_s (µm)</td>
<td>h_0 (µm)</td>
<td>C_L</td>
<td>ψ_m h_0</td>
</tr>
<tr>
<td>Flexible-Pad (approx.)</td>
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<td>4.34</td>
<td>20.2</td>
<td>0.041</td>
<td>1.73</td>
</tr>
<tr>
<td>Flexible-Pad (optimum)</td>
<td>0.63</td>
<td>4.42</td>
<td>20.2</td>
<td>0.041</td>
<td>1.68</td>
</tr>
<tr>
<td>Inclined-Pad</td>
<td>–</td>
<td>–</td>
<td>17.7</td>
<td>0.031</td>
<td>–</td>
</tr>
</tbody>
</table>

* Numerical simulation of the analytical design for the same ξ and t_s

Figure 7-8: Optimum pressure profile and film thickness

Figure 7-9 shows a contour plot of the LCC for this example. The global optimum point and approximate analytical point have been marked in the figure. It can be observed that the analytical method provides an accurate prediction of the global optimum point.

7.5.2 Relationship between ξ and t_s

It is interesting to point out that as ξ increases, in order to maintain the optimum film thickness ratio, t_s should increase as well. Increasing the t_s increases the flexural rigidity of the pad and consequently decreases the deformation of the pad above the split. For the same reason, t_s needs to be reduced as ξ decreases. The relation between optimum ξ and t_s and their off-optimum counterparts can be approximated by a power law relation as follows:

\[
\frac{t_s}{\xi^{3/4}} \bigg|_{\text{optimum}} = \frac{t_s}{\xi^{3/4}} \bigg|_{\text{off-optimum}}
\]  

(15)
Figure 7-9: Contour plot of the LCC for illustrative example

For example, for the global optimum point we have: $\xi = 0.63$ and $t_s = 4.42$. Using the above equation to calculate the optimum $t_s$ for $\xi = 0.5$ results in $t_s = 4.42 \left( \frac{0.5}{0.63} \right)^{4/3} = 3.25 \text{ mm}$, which is the same as value observed in Fig. 7-9. In case structural integrity is a concern, especially for high $\xi$ values, it is possible to design near-optimum flexible pads with smaller $\xi$ using Eq. (15).

7.5.3 Effect of minimum film thickness

Figure 7-10 illustrates the effect of changes in minimum film thickness on the LCC of the optimized flexible-pad bearing. It also compares the performance of the flexible-pad bearing with the optimally designed inclined pad presented in Section 5.1. It can be seen that, in its nominal design point (i.e., $h_0 = 20.2 \mu\text{m}$), the flexible-pad thrust bearing can support loads up to 30 percent more than the inclined pad bearing. It maintains its superior performance for a wide range of operating conditions. However, when the film thickness is too much higher (i.e., small
loads) or too much smaller (i.e., high loads) than the nominal design point, the flexible-pad thrust bearing performance degrades because the deformation mechanism is not optimum. Consequently, for loads that are far from the design loading condition, the split parameters (i.e., $\xi$ and $t_s$) need to be recalculated to ensure an optimum deformation mechanism which results in maximum LCC.

![Graph showing the effect of film thickness on LCC](image)

**Fig. 10: The effect of film thickness on LCC**

7.6 Conclusions

A type of thrust bearing which uses flexible pads is proposed and analyzed in details. The advantage of this novel design is that it can be easily manufactured. The optimum shape for flexible-pad bearings is obtained by numerical optimization algorithms in order to maximize the LCC. Simulations show that this type of bearing can have up to 30% better LCC compared to inclined-pad thrust bearings. A detailed design guideline is presented for determining the performance parameters of the flexible-pad bearings. Further research is needed to extend the results to include thermal effects and experimentally examine the performance of this bearing and validate the proposed model.

7.7 Nomenclature

- **B**: Pad width (m)
$C_L$ Load coefficient (see Eq. 5)

$c_L^*$ Load coefficient (see Eq. 4)

$D_i$ Inner diameter (m)

$D_o$ Outer diameter (m)

$h$ Local film thickness (m) [$h_0 + \psi$]

$h_0$ Minimum film thickness (m)

$h_1$ Film thickness at the leading edge (m)

$\bar{h}$ Dimensionless film thickness [$h/h_0$]

$h_g$ Split height (m)

$E$ Young’s modulus (N/m$^2$)

$L$ Pad circumferential breadth (m) [$R_{ave}\theta_0$]

$N_{pad}$ Number of pads

$P$ Gage pressure (Pa)

$\bar{p}$ Dimensionless pressure [$\frac{P}{P_a}$]

$P_a$ Ambient pressure (Pa)

$\bar{r}$ Dimensionless radius [$r/R_2$]

$R_1$ Inner radius (m)

$R_2$ Outer radius (m)

$R_{ave}$ Mean radius (m) [(R$_1$+R$_2$)/2]

$t_s$ Pad thickness above the split (m)

$t_b$ Pad thickness (m)

$W$ Load-carrying capacity (N)

$W^*$ Global-optimum LCC (N)
$w_0$ Maximum load intensity (N/m)

$x$ x-coordinate (m)

$\bar{x}$ Dimensionless $x \ [x/R_1]$

$y$ y-coordinate (m)

$\bar{y}$ Dimensionless $y \ [y/R_1]$

$\mu$ Viscosity (Pa.s)

$\beta$ Correction factor (see Eq. (12))

$\theta$ Angle (rad.)

$\theta_0$ Total pad angle (rad.)

$\bar{\theta}$ Dimensionless angle $\ [\theta / \theta_0]$

$\theta_s$ Split angle (rad.)

$\xi$ Split angle ratio $\ [\theta_s / \theta_0]$

$\lambda$ Aspect ratio [B/L]

$\Lambda$ Bearing number $\ [\frac{\mu R_2^2 \omega}{\theta_0 h_0 p_a}]$

$\varphi$ Film content

$\omega$ Angular velocity (rad/s)

$\rho$ Radius ratio $\ [R_2/R_1]$

$\psi$ Pad displacement (m)

$\psi_m$ Mean pad displacement at the leading edge (m)

$\nu$ Poisson’s ratio
7.8 References


CHAPTER 8 ON THE OPTIMUM GROOVE SHAPES FOR LOAD-CARRYING CAPACITY ENHANCEMENT IN PARALLEL FLAT SURFACE BEARINGS: THEORY AND EXPERIMENT

8.1 Introduction

Parallel flat surface bearings characterize the configuration of many tribological systems such as mechanical face seals and thrust washers that are extensively used in diverse industrial applications [1-9]. While two parallel disks in relative sliding motion are theoretically incapable of producing hydrodynamic pressure, experiments show that in practice due to thermal and density wedge and mechanical deformation some load-carrying capacity (LCC) can be generated hydrodynamically. Generally, the LCC of parallel flat surface bearings is considerably smaller than the tapered-land bearings that take advantage of the physical wedge. Adding radial grooves to the stationary pad can also increase the LCC primarily by improving the oil feed distribution and reducing the bearing effective temperature.

Significant improvement in LCC is possible by texturing spiral grooves onto the surface of the stationary pad. It has been shown that spiral grooves outperform the radial grooves in terms of LCC [10-15]. The possibility of using other groove types like herringbone shapes have been also investigated by some researchers [16, 17]. Despite the large amount of theoretical and experimental works on spiral-shaped grooves and other common groove designs, to date the problem of determining the global-optimum groove shape in parallel flat surface bearings is still unresolved.

The motivation of this work is to tackle this problem using mathematical optimization methods. Starting from arbitrary groove shapes and solving the Reynolds equation to calculate LCC, the optimum groove geometries for given operating condition and bearing geometry are obtained using a sequential optimization process that uses SQP algorithm [18, 19]. Furthermore, the theoretical results are validated by a series of experiments.

Figure 8-1: The bearing geometry with an arbitrary periodic surface pattern
8.2 Problem Formulation

The bearing geometry pertaining to a thrust washer or a mechanical face seal is shown in Fig. 8-1. The runner surface is considered to be flat and only the stationary surface is grooved. The objective is to maximize the LCC by determining the optimum shape of the grooves. This calls for the development of a computer code to calculate the LCC for an arbitrary groove shape by solving the Reynolds equation coupled with an optimization package that performs the shape optimization and automatically determines the groove shape for maximum LCC.

8.2.1 Governing Equation

The mass-conserving \( p - \phi \) model proposed by Elrod and Adams [20] is used to predict the pressure distribution within the lubricant film. This method correctly satisfies the mass conservation law. Neglecting the centrifugal effects for an incompressible Newtonian lubricant with constant viscosity under the steady condition the Reynolds equation in \( p - \phi \) form, can be expressed in the following form [21]:

\[
\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left( \bar{r} \frac{\partial \bar{p}}{\partial \bar{r}} \right) + \frac{1}{\bar{r}^2} \frac{\partial}{\partial \bar{\theta}} \left( \bar{h} \frac{\partial \bar{p}}{\partial \bar{\theta}} \right) = \Lambda \frac{\partial \bar{h}}{\partial \bar{\theta}}
\]

where \( \bar{p} \) represents the dimensionless pressure, \( \phi \) denotes the fractional film content, \( \bar{r} \) is dimensionless radius, \( \bar{\theta} \) is dimensionless angle, \( \bar{h} \) is the dimensionless film thickness and \( \Lambda \) is the bearing number. The periodic boundary condition is applied in circumferential direction because of the symmetry of the model (see Fig. 8-1).

\[
\bar{p}(0, y) = \bar{p}(1, y)
\]

8.2.2 Numerical Method and Cavitation Boundary Condition

The Reynolds equation in \( p - \phi \) form, as shown in Eq. (1), implements the Floberg-Jakobsson-Olsson (JFO) cavitation boundary condition [22, 23]. It can be used to accurately predict the cavitation in the domain. The existence of \( \phi \) in the mass-conservative formulation of the Reynolds equation makes it highly nonlinear and prone to numerical instability. It also precludes the use of fast direct solvers for the solution of algebraic equations. Consequently, iterative solvers like successive-over-relaxation (SOR) are more desirable for this formulation. However, a careful examination of optimum periodic patterns studied in this work (see section 8.4 for more details) shows the cavitated area is very small, suggesting that even a simpler cavitation boundary condition such as half-Sommerfeld can results in accurate results. Half-Sommerfeld condition simply sets the negative pressures in the solution of the Reynolds
equation equal to zero to account for cavitation. In case of optimum groove shapes, the maximum error associated with using this cavitation boundary condition is less than 5 percent.

The use of half-Sommerfeld condition allows for the use of direct solvers. In this work a fast direct solver based on LU decomposition for banded matrixes (banded-LU) [24] is employed to solve the algebraic equations. This method is computationally stable and very fast. The stability of banded-LU makes it a favorable choice for gradient-based optimization algorithms such as SQP in which the divergence of individual solutions may affect the search direction and consequently the quality of the obtained optimum solutions. Numerical results for a moderate grid size of 100×100 shows that based on the geometry of the bearing and the shape of the patterns the banded-LU method is between 10 to 100 times faster than SOR method. This is another favorable point that makes banded-LU suitable for optimization process in which several thousand of simulation runs are needed. Using this fast solver, a typical execution time for the optimization process is roughly between 10-30 hours on a server computing environment equipped with an E5630 Xeon processor, and 16 GB of memory.

8.3 Optimization Methodology

The optimization algorithm used here is based on the sequential quadratic programming (SQP) method. This method has a superior performance over other nonlinear programming methods in terms of efficiency and accuracy over a vast number of test problems [18]. Specifically, a general nonlinear constraint-handling package, DONLP2 [19], which is an implementation of the SQP method is used.

8.3.1 Objective Function

The objective of the optimization is to maximize the LCC by determining the optimum shape of periodic patterns. The LCC of the bearing is obtained by integrating the hydrodynamic pressure over the entire domain:

\[
W = \int_0^{2\pi} \int_{R_i}^{R_o} p \, rd\theta
\]  

(3)

8.3.2 Design Variables

The groove geometry is first discretized into equally spaced sections (e.g., 4) in radial direction by a series of imaginary r = constant lines. In polar coordinate these lines are horizontal (see \(L_i\), i =1..5 in Fig. 8-2). Knowing the length of these lines and the location of their center points (see \(\theta_i\), i =1..5 in Fig. 8-2), the groove shape is formed by connecting each pair of adjacent lines together. The design variables are the length of horizontal lines and the location of their center points. The optimization algorithm seeks to find the optimum values of these variables.
that maximize the LCC. The depth of groove, \( h_g \), is assumed to be constant and its optimum value is determined by the optimization algorithm. In the simulations presented here, the computational domain is divided to 5 sections. Consequently there are 13 design variables. Generally, the higher the divisions the smoother the final groove shapes are. However, the computational time substantially increases with the number of the divisions. For example, if the number of divisions increased to 10 the computational time increased more than 100 percent in average.

![Diagram](image)

**Figure 8-2:** Design variables and computational domain for an arbitrary groove shape with 4 divisions

### 8.3.3 Solution Methodology

The optimization algorithm starts with an initial guess for the shape of the groove. The initial shape can be randomly assigned by the program or defined by the user. Typically in gradient-based algorithms the initial guess can affect the quality of the final optimum solution and the execution time; however, for this problem the optimization algorithm is not very sensitive to the initial guess. A V-shape groove shape, similar to Fig. 8-2, was found to be a good starting guess because of its lower computational time. After defining the initial guess the LCC needs to be evaluated for the specified groove shape. For this purpose, SQP calculates the gradient of LCC with respect to each design variable numerically. This is simply done by slightly (e.g., 2%) changing the value of one design variable while keeping all other variables constant. The gradients are computed by a first order finite difference approximation. This is the most time-consuming step of the algorithm as it requires numerous simulations. Based on the gradient information, SQP determines the amount of change in each design variable and creates a new groove shape. Next, another series of simulations is done to evaluate the LCC and gradients for the new groove. This process continues until SQP cannot improve the solution by moving in any direction. Additional details on the implementation of SQP optimization can be found in refs. [18, 19].
8.4 Results and Discussion

In this section, optimum groove shapes are obtained for different bearing geometries. The details of all geometries are given in Appendix D. Unless specified otherwise, the results pertain to the following specifications. The bearing outside diameter is $D_0 = 38$ mm, the rotational speed is 500 rpm, and minimum film thickness is $h_0 = 50$ µm. The lubricant is SAE 30 oil with viscosity of $\mu = 0.38$ Pa.s. The groove aspect ratio $\lambda$ of each design can be obtained as follows:

$$\lambda = \frac{N_G \cdot R_e - 1}{\pi \cdot R_e + 1}$$

(4)

where $R_e = R_z / R_i$ and $N_G$ is the number of grooves.

8.4.1 Effect of Radius ratio ($R_r$)

In this section in order to investigate the effect of $R_r$, the number of grooves is kept constant at four while the inner radius is changed gradually. The optimum groove shape for each $R_r$ is obtained using the optimization procedure described in Section 3. As shown in Fig. 8-3 the groove shape is gradually evolving as $R_r$ increases. The optimum groove geometry for $R_r = 1.2$, is an elongated “heart-like” shape. Similar to optimum groove shape for sliders with rectilinear motion the groove width gradually decreases in the sliding direction [25]. As $R_r$ increases, the curved base line advances inward until it completely separates the groove from the inner rim. For $R_r \geq 2.5$ the groove shape approaches the familiar spiral shape. The optimum geometry is more similar to spiral shape in higher radius ratios.

![Figure 8-3: Effect of radius ratio on optimum groove shape ($N_G = 4$)](image)

8.4.2 Effect of Aspect Ratio ($\lambda$)

Figure 8-4 illustrates the effect of aspect ratio on the optimum groove shape for $R_r =1.5$. It can be seen that by increasing the number of grooves in a constant radius ratio, the grooves retain their heart-like shape; however, their circumferential length decreases as the number of grooves increases. The change in groove shape by increasing $\lambda$ ratio is more pronounced for $R_r =2.5$ as shown in Fig. 8-5. Note that by increasing the number of grooves, the elongated heart-
like shaped grooves detach from inner radius and the shape becomes more spiral-like. A careful investigation of Figs. 8-3 to 8-5 shows that transition from the heart-like shapes to spiral-like shapes occurs when \( \lambda \) is in the range 0.4-0.5. For all \( \lambda \) ratios over 0.5 the optimum groove shape is spiral-like.

Figure 8-4: Effect of aspect ratio on optimum groove shape \( (R_r = 1.5) \)

Figure 8-5: Effect of aspect ratio on optimum groove shape \( (R_r = 2.5) \)

Another interesting observation is that the optimum LCC for all shapes shown in Fig. 8-4 or Fig. 8-5 is essentially constant even though the optimum groove shapes are different. The small difference between the LCC of different shapes can be attributed to numerical errors associated with the solution to the optimization problem which results in finding a near-optimum solution. These results reveal that there exists an upper limit for LCC in parallel grooved surface bearings. The results of extensive numerical simulations for different geometries and operating conditions show that this limit can be approximated from the following formula:

\[
W_{\text{max}} = C_L \cdot \frac{\mu \omega R_{\text{ave}}^3 (R_2 - R_1)}{h_0^2}
\]  

(5a)

where \( C_L \), load coefficient, is given by

\[
C_L = 0.91 \ln(R_r) + 0.0125
\]  

(5b)

It should be mentioned that this equation is only valid if both the inner and outer rims are kept in atmospheric pressure. The optimum groove depth ratio, \( h_r \) \((=h_g/h_0)\), in the range of \( 1.2 \leq R_r \leq 3 \) and \( 2 \leq N_G \leq 6 \) is given by:
The above equations can be used to predict the bearing performance for different geometries and operating conditions.

Illustrative Example – a thrust washer needs to be designed for a load of \( W = 177 \text{ kN} \) at 1000 rpm. The inner bearing diameter is \( D_i = 17.6 \text{ cm} \), and outside diameter is \( D_o = 30 \text{ cm} \), an oil with viscosity of \( \mu = 0.08 \text{ Pa.s} \) is to be used. It is desired to calculate the optimum groove depth and minimum film thickness for this bearing. Making use of Eq. (5) and Eq. (6), calculate the optimum groove depth and minimum film thickness for this thrust washer.

The results are summarized in Table 8-1 for different number of grooves. In order to validate the accuracy of these equations a numerical optimization is carried out (see Table 8-1). Figure 8-6 shows the optimum groove shapes for this problem. It can be seen that the result of the numerical simulations and the predictions of the equations are in good agreement.

<table>
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<tr>
<th>( N_G )</th>
<th>CL</th>
<th>( h_0 (\mu m) ) Eq. (5b) Simulation</th>
<th>( h_\theta (\mu m) ) Eq. (5a) Simulation</th>
<th>( h_r ) Eq. (6) Simulation</th>
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<td>0.498</td>
<td>0.492</td>
<td>49.6</td>
<td>49.3</td>
</tr>
</tbody>
</table>

Figure 8-6: optimum groove shape for the example 4.2.1 (\( R_r = 1.7 \))

8.4.3 Effect of Groove Area Ratio

The area ratio (AR) is defined as the ratio of the textured area to the total surface area. The AR effect on the optimum groove shape is given in Fig. 8-7 for \( R_r = 1.5 \). It can be seen that for small ARs the optimum geometry has the shape of a herringbone. Because of the small aspect ratio of this geometry (i.e., \( \lambda = 0.32 \)), the groove is connected to the inner rim. It is thicker in the external area compared to the inner area. With increasing AR, the groove width in the inner area increases as well and eventually both parts merge and form an elongated heart-like shape.
Figure 8-7: Effect of area ratio on optimum groove shape ($R_r = 1.5$, and $N_G = 4$)

Figure 8-8 show the optimum groove shape for $R_r = 2.5$. As expected the optimum shape is spiral-like for this geometry since its $\lambda = 0.82$ which is greater than 0.5 (see previous section). As AR increases the groove width increases as well, however, the general spiral-like shape does not change.

Figure 8-8: Effect of area ratio on optimum groove shape ($R_r = 2.5$, and $N_G = 6$)

A careful investigation of the LCC of the grooves shown in Figs. 8-7 and 8-8 reveals that there is an optimum for AR which maximizes LCC. This value is around 0.5 as shown in Fig. 8-9. Numerical simulations show that this optimum ratio remains the same for all $R_r$ values.

Figure 8-9: Comparison of LCC for different area ratios
8.4.4 Comparison with Conventional Spiral Grooves

In this section we compare the performance of these groove designs with the LCC of the conventional spiral grooves. The Logarithmic spiral shown in Fig. 8-10 is used whose shape can be described as follows [13]

\[ r = R_2 e^{\theta \tan(\alpha)} \]  

(7)

where \( \alpha \) represents the angle between the tangent line and radial line at each point on the spiral curve, and \( \Theta \) is the angular coordinate.

In order to have a meaningful comparison, first the geometry of spiral grooves is optimized using the SQP algorithm for the same bearing geometry and operating condition. The design variables are the angle \( \alpha \), groove depth ratio, \( h_r \) and the cut-off radius (i.e., the radius in which the spiral ends), \( R_c \). The results of the optimization are shown in Table 8-2.

The optimum LCC for different groove shapes is shown in Fig. 8-11. It can be seen that depending to the number of grooves used the LCC of optimum groove is between 14 percent and 36 percent greater than the conventional spiral grooves. The results showed that novel groove designs presented in this study significantly outperform the conventional spiral grooves.

![Figure 8-10: Schematic of a bearing with spiral grooves](image)

| Table 8-2: Optimum spiral groove geometry |
|---------------|----------------|-----------------|-----------|----------------|
| \( N_G \)  | \( \alpha \) (deg.) | \( (R_c-R_i)/(R_2-R_i) \) | \( h_r \) | \( W(N) \) |
| 2        | 5.50           | 0.27            | 7.08      | 131.7       |
| 3        | 8.95           | 0.27            | 5.28      | 143.1       |
| 4        | 11.65          | 0.26            | 4.60      | 150.7       |
| 5        | 11.76          | 0.27            | 4.59      | 155.4       |
| 6        | 14.17          | 0.26            | 4.18      | 157.9       |
| 10       | 19.98          | 0.28            | 3.63      | 158.7       |
8.4.5 Experimental Validation

A series of experiments on two specimens, one with spiral grooves and the other with global-optimum patterns are performed in order to validate the theoretical results obtained in previous sections. Both specimens have an outside diameter of 38mm, $R_r=2.5$, and $N_G=6$. More details about the global optimum shape and optimum spiral groove geometry can be found in Fig. 8-8 and Table 8-2, respectively. The lubricant is SAE30 with a viscosity of $\mu=0.38$ Pa.s.

8.4.5.1 Specimen Preparation

Specimens are made from stainless steel and machined to the desired dimensions. The specimens are textured using an Electrox Laser Marking machine. The working laser of this machine is 1064 nm wavelength. The texture depth is controlled by the total time that the specimen is exposed to the laser beam. After laser texturing, in order to eliminate the surface protrusions caused during texturing, specimens are lapped to a fine surface finish. A sample surface finish has been shown in Fig. 8-12.
Figure 8-12: Surface profile of specimens: (a) optimum (b) conventional spiral

8.4.5.2 Experimental Apparatus

An Anton Paar MCR 301 rheometer equipped with tribology accessory unit is used for all tests. The schematic of the machine is shown in Fig. 8-13. The instrument has the following measurement range: normal load 0.01 to 50 N and rotational speed 10-6 to 3000 rpm. Temperature can be controlled by the rheometer in the range of -40°C up to +200°C with an error of ±0.1°C. In the current experiments the temperature is set to 25°C. The specimens are attached to the inner seat of a lubricant cup and the upper measuring plate is attached to the spindle. The measuring plate is nominally flat and free of any surface textures. The gap between the measuring plate and specimen is maintained constant by the rheometer with an accuracy of ±0.2 µm.
8.4.5.3 Experimental Results

In each experiment the gap between the measuring plate and the specimen is maintained constant by the instrument and the resulting normal force is recorded. It should be noted that the maximum allowable normal force is 50 N; consequently, the speed had to be limited to 300 RPM since high speed results in normal forces higher than 50 N.

Figure 8-14 shows the results obtained for the global optimum shape and the optimum spiral groove in different rotating speeds. It is evident that in a constant minimum film thickness the global optimum groove produces considerably more load compared to spiral groove showing better performance. For validation purpose the experimental results are also compared with the simulation results in Fig. 8-14. Since the lubricant pressure in inner rim is not atmospheric the boundary condition at \( r = R_l \) is revised to \( dp/dr = 0 \) in this region, which implies that pressure essentially remains constant in the central region \( r = 0 \) to \( r = R_l \). Referring to Fig. 8-14, for small rotating speeds (i.e., 100 RPM), the theory slightly overestimates the LCC; however, for higher speeds it matches the experiment well. Generally, the simulation results show very good agreement with experimental tests, validating the theoretical model used for the simulations. The theoretical results can be used to predict the performance of optimum designs. The simulation results for different speeds are shown in Fig. 8-15. It can be seen that for different speeds the
optimum spiral-like groove has superior performance compared to conventional spiral groove. Theoretical results show that for this configuration the optimum grooves can produce around 14% more load support than spiral grooves.

### 8.5 Conclusions

In this study, novel groove geometries which produce the maximum LCC for parallel plate bearings are proposed and analyzed. A numerical optimization method based on the SQP algorithm is used for determining the optimum groove shape. It is shown that the optimum groove shapes are a function of geometrical aspect ratio. Simulations show that the proposed groove shapes can have greater LCC (e.g., 14%-36% for \( R_r = 2.5 \)) compared to conventional spiral groove shapes. The numerical model validated by a series of experiments. It is shown that the novel groove design have superior performance compared to conventional spiral grooves. Also, a series of equations are developed for determining the performance parameters of the optimum designs.

### 8.6 Nomenclature

\begin{itemize}
  \item AR Area ratio \([\text{grooved area / total area}]\)
  \item B Bearing width \([R_2 - R_1]\) (m)
  \item \(C_L\) Load Coefficient (see Eq. 5)
  \item \(D_i\) Inner diameter (m)
  \item \(D_o\) Outer diameter (m)
  \item \(h\) Local film thickness (m)
  \item \(h_0\) Minimum film thickness (m)
  \item \(h_g\) Groove depth (m)
  \item \(h_r\) Groove depth ratio \([h_g / h_0]\)
  \item \(\bar{h}\) Dimensionless film thickness \([h / h_0]\)
  \item L Sector length (see Fig. 8-1) \([R_{ave} \theta_0]\)
  \item \(N_G\) Number of grooves
  \item \(P\) Gage pressure (Pa)
\end{itemize}
\( \bar{p} \) Dimensionless pressure \( \left[ \frac{p}{p_a} \right] \)

\( p_a \) Ambient pressure (Pa)

\( p_c \) Cavitation pressure (Pa)

\( r \) radius (m)

\( \bar{r} \) Dimensionless radius \( \left[ \frac{r}{R_2} \right] \)

\( R_1 \) Inner radius (m)

\( R_2 \) Outer radius (m)

\( R_r \) Radius ratio \( \left[ \frac{R_2}{R_1} \right] \)

\( R_c \) Cut-off radius for spiral grooves (m)

\( R_{ave} \) Mean radius \( \left[ \frac{(R_1+R_2)}{2} \right] \) (m)

\( W \) Load-carrying capacity (N)

\( \mu \) Viscosity (Pa.s)

\( \theta \) Angle (rad.)

\( \theta_0 \) Groove angle \( \left[ \frac{2\pi}{N_G} \right] \) (rad.)

\( \bar{\theta} \) Dimensionless angle \( \left[ \frac{\theta}{\theta_0} \right] \)

\( \lambda \) Groove aspect ratio \( \left[ \frac{B}{R_{ave}\theta_0} \right] \)

\( \Lambda \) Bearing number \( \left[ \frac{6\mu R_2^2 \omega}{\theta_0 h_0^2 (p_a - p_c)} \right] \)

\( \phi \) Film content

\( \omega \) Angular velocity (rad/s)
Figure 8-14: Experimental results for the optimum grooves and conventional spiral grooves
8.7 References


CHAPTER 9  ON THE OPTIMUM DESIGN OF LIQUID-LUBRICATED SPIRAL GROOVE BEARINGS

9.1  Introduction

Load-bearing surfaces with spiral grooves are widely used in rotating machinery such as pumps and compressors. They offer higher load-carrying capacity and lower power loss compared to untreated surfaces, making them desirable especially for high speed applications.

Many researchers have carried out theoretical [1-8] and experimental studies [9-11] to analyze the characteristics of spiral groove bearings. Approximate analytical models were developed based on the pioneering works of Muijderman [5, 12], Vohr and Pan [13], Malanoski and Pan [4], Smalley [7], and Murata et al. [14, 15] in mid 1960s-1970s. Notable conceptual and new contributions were the Narrow-Groove theory [7, 12] and the Wing-Lattice theory [14, 15]. The former approach assumes that the number of grooves is infinite and consequently the problem is treated as a one-dimensional problem. This treatment simply ignores the complicated boundary shape and geometrical discontinuities. The latter method which is a more accurate two-dimensional model was developed based on the inviscid flow theory of circular wing lattice. With the advance of computational capabilities of computers in 1990s, numerical solution of the governing equations based on finite difference [3], finite element [16, 17] and boundary element method [18] became more prevalent since they can easily overcome the restrictive assumptions that existed in analytical models.

In recent years researchers have attempted to improve the tribological performance of spiral groove bearings using optimization methods. Most of these efforts have concentrated on gas-lubricated spiral grooves and the works on liquid-lubricated spiral grooves are scares. The motivation of this work is to determine of the optimum spiral groove shape for different operating conditions and bearing geometries using mathematical optimization methods. To this end, an optimization study is carried out that uses sequential quadratic programming (SQP) method [19] to determine the optimum geometrical parameters of the spiral groove for maximum LCC. In addition, an analytical method for optimum bearing design is presented that can be used at preliminary design stage to obtain near-optimum LCC. A semi-analytical thermohydrodynamic (THD) model is employed to study thermal effects on the performance of spiral groove bearings. The results of THD analysis are also formulated in the form of analytical expressions to help designers to avoid complex time-consuming numerical simulations.
9.2 Theoretical Model

9.2.1 Spiral Groove Geometry

A schematic of a spiral grooved bearing is shown in Fig. 9-1. The shape of the spirals is described as follows [5]

\[ r = R_2 e^{\theta \tan(\alpha)} \]  \hspace{1cm} (1)

where \( \alpha \) is the angle between the tangent and radial line at each point on the spiral curve, \( R_2 \) is the outside radius and \( \phi \) is angular coordinate. The objective is to maximize the load-carrying capacity (LCC) by finding the optimum groove geometry. The LCC, can be obtained by integrating the hydrodynamic pressure over the entire domain:

\[ W = \int_0^{\theta_2} \int_{R_1}^{R_2} p r \, dr \, d\theta \]  \hspace{1cm} (2)

where the pressure, \( p \), is obtained from the Reynolds equation.

![Figure 9-1: Schematic of a spiral groove bearing and computational domain](image)

9.2.2 Reynolds Equation

The hydrodynamic pressure profile in the case of iso-viscous Newtonian lubricant is given by the solution to the Reynolds equation, which in dimensionless form reads:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r h^3 \frac{\partial \bar{p}}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left( h^3 \frac{\partial \bar{p}}{\partial \theta} \right) = \Lambda \frac{\partial \bar{\phi} \bar{h}}{\partial \theta} \]  \hspace{1cm} (3)

where \( \phi \) is the film content and \( \bar{h} \) is the local film thickness.
The periodic boundary condition is applied in circumferential direction because of the symmetry of the model (see Fig. 9-1).

\[ \bar{p}(0, r) = \bar{p}(1, r) \] (4)

The solution of the Reynolds equation may include cavitation in a part of the groove. The existence of negative pressure areas affects the LCC of the bearing. Generally, when the number of the grooves is high the negative effect of the cavitated area is small; however, for small number of grooves especially in case of wide grooves this effect is more pronounced [2].

To accurately account for cavitation it is common to use the Jakobsson-Floberg-Olsson (JFO) cavitation boundary condition [20, 21]. However, the numerical solution of this formulation is sensitive to geometrical configuration and operating condition. This is especially undesirable for implementation in an optimization algorithm wherein thousands of arbitrary groove geometries must to be solved precisely before reaching the final solution. A careful examination of optimum spiral grooves (see section 5 for more details) shows the cavitated area is very small, suggesting that even a simpler cavitation boundary condition such as half-Sommerfeld can produce accurate results. The use of half-Sommerfeld boundary condition also allows the use of fast direct solvers such as Banded-LU method [22]. Depending on the geometry of the groove this direct solver can be between 10 to 100 times faster than an iterative solver like successive over relaxation (SOR). In this study, the Reynolds equation is discretized using finite difference method and the resulting system of algebraic equations are numerically solved using the Banded-LU algorithm.

### 9.3 Optimization Methodology

The optimization algorithm used here is based on the well-known sequential quadratic programming (SQP) method. Rich volume of literature is available on this algorithm, so the details are not repeated here. In this study, a general nonlinear constraint-handling package, DONLP2 [19] which implements the SQP method is used.

The objective of the optimization is to maximize the LCC of bearing by finding the optimum dimensions of the spiral groove. The design variables are the spiral angle (\( \alpha \)), groove depth (\( h_g \)), and dam width to face width ratio, \( \varepsilon = (R_C - R_1)/(R_2 - R_1) \). In this study, the ratio of the circumferential width of the ridges to that of the groove is kept equal to one, which is common in practice. The number of grooves (\( N_G \)) is also kept constant for each optimization simulation.
Figure 9-2 shows the flowchart of optimization methodology. Similar to other gradient-based algorithms in order to find the optimum solution the SQP starts from an initial starting point. This point can be randomly assigned or specified by the user. From the authors experience for this problem the starting point will not affect the quality of the final solution; however, a good starting point decreases the time needed for the SQP algorithm to find the optimum solution. The SQP starts its local search from the starting point and moves to a new point with higher LCC in gradient descent direction after calculating the LCC and its derivatives with respect to the design variables in the current point. The derivatives are calculated numerically using the method of finite differences. In this way the value of each design variable is increased by a small percentage (i.e., 2 percent) while all the other design variables are kept constant. Next, the change in the LCC is evaluated. Having evaluated the value of LCC before and after the small change, the derivatives are calculated using the forward difference. This process is done for all design variables. After this stage the SQP improves the current solution using the gradient information by moving in a direction in which the LCC is increased. In the new point it again evaluates the LCC and its derivatives in order to determine the search direction. The SQP stops its search when it cannot improve the solution by moving in any direction.
9.4 Model Verification

In this section an example from Qiu and Khonsari [23] is chosen in order to authenticate the results of the simulations. Qiu and Khonsari [23] conducted a series of experiments to study the tribological behavior of annular rings with spiral grooves. In one of their experiments they chose a specimen with the inner and outer diameter of 33 and 48 mm, respectively. The specimen had fifteen spiral grooves with a spiral angle of 20 degree and depth of 33 µm. The dam width to face width ratio, \( \varepsilon = (R_c - R_i) / (R_2 - R_1) \), was kept at 0.5. They conducted their experiments for different loads and speeds. Table 9-1 compares the calculated loads for different minimum film thickness with those reported in [23].

Table 9-1: Comparison between the LCC calculated in this work with those reported in [23]

<table>
<thead>
<tr>
<th>( \omega )</th>
<th>( h_0 ) (µm)</th>
<th>LCC (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>This work</td>
<td>Qiu and Khonsari [23]</td>
</tr>
<tr>
<td>100 RPM</td>
<td>48.5 µm</td>
<td>9.0 N</td>
</tr>
<tr>
<td>100 RPM</td>
<td>39.0 µm</td>
<td>18.1 N</td>
</tr>
<tr>
<td>500 RPM</td>
<td>79.0 µm</td>
<td>8.7 N</td>
</tr>
<tr>
<td>500 RPM</td>
<td>64.0 µm</td>
<td>18.0 N</td>
</tr>
</tbody>
</table>

The maximum error in the predictions is less than 3 percent, attesting to the accuracy of the load-carrying capacity predictions. The pressure profile is also shown for each case in Fig. 9-3. It can be seen that as expected the maximum pressure occurs at the end of spirals and it then decreases in the dam area. There is a small area of the cavitation in all cases.

9.5 Approximate Analytical Model

In this section an approximate analytical method for efficient design of spiral groove bearings is presented. In the absence of numerical optimization tools, the proposed design method can be used in order to predict a near-optimum groove designs capable of providing the largest LCC for a given operating condition and bearing geometry.

The most important parameters that need to be defined are minimum film thickness, spiral angle, groove depth, number of grooves and \( R_c \). In order to determine the optimum value of these parameters, several hundred optimization simulations were carried out for different geometries and operating conditions in the range of \( 2 \leq N_G \leq 100 \) and \( 1.2 \leq \rho (= R_2 / R_1) \leq 4 \). The results are curve fitted and presented in the following section.

9.5.1 Load-Carrying Capacity

In general, the LCC of the optimum groove shape, \( W^* \), can be given by the following equation [24, 25]
Figure 9-3: Pressure distribution for different operating conditions

\[ W^* = C_{Lc}^* \frac{\mu \omega R^3 \left(R_2 - R_1\right)}{h_0^2} \]  

(5)

where \( C_{Lc}^* \) is the load coefficient for the optimum spiral shape, \( \mu \) is the viscosity and \( \omega \) is the angular speed. This expression was originally developed for thrust bearing in [24] based on the pioneering work of Archibald [26] on 2-D stepped-shaped sliders. It shows that the LCC for a given geometry is proportional to \( \mu \omega / h_0^2 \). The load coefficient is only geometry dependent. In the absence of nonlinear effects, such as cavitation or mechanical deformation, \( C_{Lc}^* \) is only a function of radius ratio (\( \rho \)) and number of grooves, and the operating conditions do not influence it.

Figure 9-4 illustrates the \( C_{Lc}^* \) for different number of grooves and radius ratios. It shows that for a given number of grooves the load coefficient always increases by increasing the radius
ratio. This is expected because, for a given operating condition and an inner radius, a larger bearing supports more load compared to a smaller bearing with smaller radius ratio.

Figure 9-4: Optimum load coefficient

The value of $C_L^*$ shown in Fig. 9-4 can also be obtained from the following correlation:

$$
C_L^* = \begin{cases}
\frac{-0.016 + 0.5223 \ln \rho + 0.0068 \ln N_G + 0.3733 (\ln \rho)^2 - 0.0007 (\ln N_G)^2 - 0.0602 \ln \rho \ln N_G}{1 - 1.0824 \ln \rho - 0.2082 \ln N_G + 0.3539 (\ln \rho)^2 + 0.0159 (\ln N_G)^2 + 0.0848 \ln \rho \ln N_G} & \text{for } 1.2 \leq \rho \leq 1.5 \\
\frac{0.0162 + 0.4614 \ln \rho - 0.0138 \ln N_G + 0.0035 (\ln \rho)^2 + 0.0038 (\ln N_G)^2 - 0.0929 \ln \rho \ln N_G}{1 - 0.3074 \ln \rho - 0.3297 \ln N_G + 0.0814 (\ln \rho)^2 + 0.0345 (\ln N_G)^2 + 0.0443 \ln \rho \ln N_G} & \text{for } 1.5 \leq \rho \leq 4
\end{cases}
$$

For a given load and bearing geometry the minimum film thickness can be easily obtained by calculating $C_L^*$ from Eq. (6) and substituting its value in Eq. (5).

9.5.2 Number of Grooves

From the results presented in Fig. 9-4, it can be seen that for a given radius ratio LCC increases by increasing the number of grooves up to a certain number. Further increase in the number of grooves results in a reduction in LCC. This shows that for a given $\rho$ there exist an optimum number of grooves that maximizes the LCC. The locus of peaks on all constant $\rho$ paths
(the dashed blue line in Fig. 9-4) defines the optimum number of pads. The optimum number of grooves $N_G^{opt}$. It can be approximated as follows.

$$N_G^{opt} = 5 + 7.75 \ln \rho$$  \hspace{1cm} (7)

9.5.3 Spiral Angle

The optimum spiral angle for different number of grooves and radius ratios is shown in Fig. 9-5. It can be seen that for a given radius ratio by increasing the number of grooves the optimum spiral angle increases. The same trend is observed if the radius ratio increased for a given number of grooves. It can be also seen that the optimum spiral angle for a given number of grooves changes more rapidly for small radius ratios while in high radius ratios the changes in optimum angle is small. The optimum spiral angle shown in Fig. 9-5 can be obtained from the following equation:

$$\alpha = \exp \left( 5.779 + 0.0726 N_G^{0.5} - 1.757 N_G^{-0.5} - 14.141 \ln \rho / \rho + 38.3 \rho^{-1.5} - 34.666 \rho^{-2} - 20.154 \exp(-\rho) \right)$$  \hspace{1cm} (8)

![Figure 9-5: Optimum spiral angle.](image-url)
9.5.4 Groove Depth

The optimum groove depth is a function of the minimum film thickness, radius ratio and number of grooves. It can be easily calculated from the film thickness ratio, \( H_r = 1 + h_g/h_0 \). Figure 9-6 shows the film thickness ratio for different number of grooves and radius ratios. Results show that for a specified number of grooves as the radius ratio increases the groove depth must be reduced to have an optimum solution.

![Figure 9-6: Dimensionless film thickness ratio.](image)

It can be also seen that for a specified radius ratio the smaller number of grooves have higher optimum film thickness. However, for the case of global optimum solution where the number of grooves is optimized as well the optimum \( H_r \) is almost the same for all radius ratios. This optimum value is nearly constant (i.e., 4.12) for different radius ratios as shown in Fig. 9-6. It is possible to formulate optimum \( H_r \) shown in Fig. 9-6 using the following equation:

\[
H_r = 5.31 - 0.674 \ln N_g + 0.0553 / \ln \rho + 1.511 / \rho
\]

(9)

9.5.5 Dam Width to Face Width Ratio (\( \varepsilon \))

The optimum dam width to face width ratio, \( \varepsilon = (R_c - R_f) / (R_2 - R_f) \), is only a function of the radius ratio \( \rho \) and can be obtained as follows:

\[
\varepsilon = 0.288 - 0.0065 \rho
\]

(10)

The utility of the expressions developed is illustrated through an example presented next.
9.5.6 Example 1

A thrust washer needs to be designed for a load of 22.3 kN at 3600 rpm. The inner and outer radii are 3.33 cm and 5 cm, respectively. Oil with viscosity of 0.25 Pa.s is to be used. Determine the optimum number of spiral grooves, optimum groove geometry and minimum film thickness.

Making use of the proposed analytical expression, one can easily design a near-optimum spiral groove thrust washer. The summary of the calculations is given in Table 9-1. Results show that for the optimum performance 24 grooves needs to be created. The optimum spiral angle of these grooves is 18 degree. The depth of these grooves needs to be $3.12 h_0$. Next, for comparison purposes the full numerical optimization is carried out using SQP algorithm, which is capable of predicting the global optimum results. The optimum solution, found after 194 simulations, is also shown in Table 9-2. It can be seen that the predictions of the proposed analytical formulas are very close to the numerical solution. The $\alpha$ obtained from analytical expressions is one degree greater than the numerical optimum; however, the performance of the analytical design, as evident by $C_L$, is nearly identical to the predictions of numerical optimum.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Proposed Model</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_G$</td>
<td>24</td>
<td>[Eq. (7)]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>18 deg.</td>
<td>[Eq. (8)]</td>
</tr>
<tr>
<td>$H_r$</td>
<td>4.12</td>
<td>[Eq. (9)]</td>
</tr>
<tr>
<td>$C_L$</td>
<td>0.3137</td>
<td>[Eq. (6)]</td>
</tr>
<tr>
<td>$h_0$</td>
<td>40 $\mu$m</td>
<td>[Eq. (5)]</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.28</td>
<td>[Eq. (10)]</td>
</tr>
</tbody>
</table>

The evolution of the optimization process for this example is shown in Fig. 9-7. The cross marks show the SQP search direction toward the global optimum. In this case, SQP needs 194 objective function evaluations to calculate the LCC at all the “cross” marks.

9.6 Thermal Effects

Consideration of thermal effects requires the development of a thermohydrodynamic (THD) model. A full THD analysis is very complex and computationally expensive. For spiral groove seals, typically, it involves utilizing commercial finite element solvers [6, 27]. For the purpose of the optimization in which thousands of simulations are needed a less computationally expensive yet accurate model is desirable. Currently, a series of approximate analytical models for the THD of seals [28-34] are available. Among these models a semi-analytic THD model developed by Pascovici and Etsion [31] is adopted in this work since it produces good approximations for spiral groove bearings [35].
In the Pascoveci and Etsion model [31] the Reynolds equation and the energy equation are decoupled using the Couette flow approximation. Calculating the temperature distribution from the energy equation, the Reynolds equation is then solved using variable local viscosity that is given by:

\[
\mu = \mu_f e^{-\bar{\beta}(\bar{T}_m - 1)}
\]  

(11)

where \( \mu_f \) is the viscosity of the sealed fluid at its known temperature \( T_f \), \( \bar{\beta} \) represents the dimensionless temperature-viscosity coefficient given by \( \beta T_f \), and \( \bar{T}_m \) is the dimensionless mean local temperature given in an implicit form by

\[
\bar{T}_m = 1 + D \left[ 2 \left( \frac{1}{Nu} - K \ln \bar{r} \right) G \frac{\bar{r}^3}{h} + \frac{\bar{r}^2}{3} \right] e^{-\bar{\beta}(\bar{T}_m - 1)}
\]  

(12)

where \( D \) denotes dissipation number given by \( \mu_f \omega^2 R_z^2 / k T_f \), \( Nu \) is Nusselt number in the form \( 2R_z h_{conv} / k \), \( K \) is conductivity ratio \( k / k^* \), and \( G \) is geometric parameter \( R_z / h_0 \).

Similar to the isothermal model first a series of optimization simulations are carried out in order to find the optimum spiral geometries and then the results are curve fitted. There are many parameters involved in the THD analysis which makes it very difficult to generalize the
results; consequently, the proposed model encompasses a limited range of operating conditions and geometries. The model is applicable to applications that use SAE30 for the following range: $600 \leq G \leq 6000$, $9 \leq D \leq 600$, $2000 \leq Nu \leq 8000$, $1.1 \leq \rho \leq 1.3$.

Simulation results show that the thermal effects mainly affect the load coefficient and the groove depth. The other groove parameters such as spiral angle and $\varepsilon$ are almost the same as the isothermal model.

### 9.6.1 Load-Carrying Capacity

The load coefficient of the optimum groove shape can be obtained from the following equation for $\rho=1.1$

$$C_L^{\text{Thermal}} = 124.79N_G^{0.1686}e^{(-1.34410^{1.184}G^{2.04})} + 1.733* Nu^{0.72}D^{-0.642}G^{-3.82} + 0.01167$$ (13a)

and

$$C_L^{\text{Thermal}} = 166.59(\ln N_G)^{0.295}e^{(-0.6768219G^{0.13})} + 46.59Nu^{0.4}D^{-0.642}G^{-3.82} + 1.3717e6(\ln N_G)^{20.41} - 0.209$$ (13b)

for $\rho=1.3$. For $1.1 \leq \rho \leq 1.3$ the value of $C_L^{\text{Thermal}}$ can be obtained from linear interpolation between the values obtained from the above equation.

### 9.6.2 Film Thickness Ratio

Generally, compared to the isothermal condition the average film thickness ratio is higher when the thermal effect is considered. This helps the bearing to maintain higher loads by keeping the lubricant cooler. The optimum film thickness ratio is given by:

$$H_r = 1 + (\ln N_G)^{3.858} + 0.972(\ln \rho)^{-0.186}$$

$$1 - 0.00041N_G^{-1.306}\rho^{3.006} + 19.05(\ln G)^{-0.0235} - 18.825(\ln D)^{-0.00055}$$ (14)

An illustrate example is presented next to show how these expressions can be used.

### 9.6.3 Example 2

Consider a seal with an outer radius of 4 cm and a radius ratio $\rho=1.3$. The rotational speed is 1800 rpm. The sealed fluid is at ambient temperature $T_f=20°C$. The rotor is made of stainless steel with a thermal conductivity of $k^*= 45 \text{ W/m°C}$. A gap of 25 μm is selected and the number of spirals is set to 24. Determine the optimum shape of the spiral groves in cases where the sealed fluid is SAE 30.
Table 9-3 summarizes various properties and corresponding dimensionless parameters for sealed fluid. Using correlations developed in this section the groove parameters can be easily developed. The summary of the calculations is given in Table 9-4. It can be observed that the analytical model can predict the groove parameters with a good accuracy. The predicted spiral angle and film thickness ratio are slightly smaller than the simulation results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SAE30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_f$</td>
<td>20 °C</td>
</tr>
<tr>
<td>$\mu_f$</td>
<td>0.39 Pa.s</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.062 °C$^{-1}$</td>
</tr>
<tr>
<td>$\bar{\beta}$</td>
<td>1.24</td>
</tr>
<tr>
<td>$D$</td>
<td>9.2379</td>
</tr>
<tr>
<td>$G$</td>
<td>1600</td>
</tr>
<tr>
<td>$K$</td>
<td>0.0026</td>
</tr>
<tr>
<td>$Nu$</td>
<td>2000</td>
</tr>
</tbody>
</table>

Table 9-4: Groove parameters for example 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SAE 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>15.7 deg. [Eq. (8)] 14.5 deg.</td>
</tr>
<tr>
<td>$H_r$</td>
<td>6.47 [Eq. (14)] 6.23</td>
</tr>
<tr>
<td>$C_L$</td>
<td>0.0268 [Eq. (13)] 0.0269</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.28 [Eq. (10)] 0.28</td>
</tr>
</tbody>
</table>

9.7 Conclusions

The optimum shape of spiral grooves for liquid-lubricated parallel flat surface bearings is obtained by numerical optimization in order to maximize the LCC. The mathematical model is first validated by a series of experimental results and extensive simulations are carried out in order to develop a detailed design guideline for determining the performance parameters of liquid-lubricated spiral groove bearings. Simulation results show that there is an optimum for the number of grooves, which maximizes the LCC. This optimum is only a function of radius ratio. Further, it is found that the global-optimum film thickness ratio is about 4.12 for all radius ratios. In case the number of grooves is not optimum, the optimum film thickness ratio is only a function of radius ratio. It is also shown that the optimum spiral angle and optimum dam width to face width ratio are only a function of radius ratio and number of grooves. Thermal effects are also considered for a range of operating conditions. It is found the thermal effect mainly affects the load coefficient and optimum film thickness ratio while the other groove parameters remain almost the same as the proposed isothermal model. Further research is needed to extend the validity of the thermal model for other lubricant types.
9.8 Nomenclature

\( C_L \)  Load Coefficient (see Eq. 5)
\( D \)  Dissipation number \( \left[ \mu_f \omega^2 R_2^2 / kT_f \right] \)
\( G \)  Geometric parameter \( \left[ R_2 / h_0 \right] \)
\( h \)  Local film thickness (m)
\( h_0 \)  Minimum film thickness (m)
\( h_{conv} \)  Convection heat transfer coefficient [W/m².K]
\( h_g \)  Groove depth (m)
\( H_r \)  Film thickness ratio \( \left[ 1 + h_g / h_0 \right] \)
\( \bar{h} \)  Dimensionless film thickness \( \left[ h/h_0 \right] \)
\( k \)  Lubricant thermal conductivity [W/m.K]
\( k^* \)  Seal thermal conductivity [W/m.K]
\( K \)  Conductivity ratio \( \left[ k / k^* \right] \)
\( \text{LCC} \)  Load-carrying Capacity (N)
\( N_G \)  Number of grooves
\( Nu \)  Nusselt number \( \left[ 2R_2 h_{conv} / k \right] \)
\( p \)  Gage pressure (Pa)
\( \bar{p} \)  Dimensionless pressure \( \left[ p / p_a \right] \)
\( p_a \)  Ambient pressure (Pa)
\( p_c \)  Cavitation pressure (Pa)
\( r \)  radius (m)
\( \bar{r} \)  Dimensionless radius \( \left[ r / R_2 \right] \)
\( R_1 \)  Inner radius (m)
\( R_2 \)  Outer radius (m)
\( R_c \)  Cut-off radius for spiral grooves (m)
\( R_{ave} \)  Mean radius \( \left[ (R_1+R_2)/2 \right] \) (m)
\( T \)  Temperature [K]
\( W \)  Load-carrying capacity (N)
\( \alpha \)  
Spiral angle (rad.)

\( \beta \)  
Temperature-viscosity coefficient \([1/K]\)

\( \bar{\beta} \)  
Dimensionless \( \beta \) \([\beta T_f]\)

\( \rho \)  
Radius ratio \([R_2/R_1]\)

\( \mu \)  
Viscosity (Pa.s)

\( \theta \)  
Angle (rad.)

\( \theta_0 \)  
Groove angle \([2\pi/N_G]\) (rad.)

\( \bar{\theta} \)  
Dimensionless angle \([\theta / \theta_0]\)

\( \Lambda \)  
Bearing number \[\frac{6\mu R_f^2 \omega}{\theta_0 h_0^2 (p_s - p_c)}\]

\( \phi \)  
Film content

\( \omega \)  
Angular velocity (rad/s)

\( \varepsilon \)  
Dam width to face width ratio \[\left[(R_c - R_f)/(R_2 - R_1)\right]\]

**Subscripts:**

\( f \)  
Sealed fluid

\( m \)  
Mean

9.9 **References**


10.1 Summary

In this dissertation, optimum surface texturing and topological optimization of hydrodynamically-lubricated bearings is studied. A summary of the main results is as follows.

- The cavitation effect is modeled using mass-conservative JFO cavitation theory. The algorithm developed by Elrod is used to implement the JFO theory. A comprehensive review of numerical methods used to handle cavitation is given and a novel numerical procedure is proposed to improve the convergence rate and stability of the Elrod cavitation algorithm. The new procedure replaces the conventional binary switch function with a continuous switch function. Results of case studies show that the proposed algorithm can accelerate the convergence speed of the Elrod algorithm up to 64 percent.

- A three dimensional model for self-adaptive surface grooves is developed aimed to extend the current one-dimensional models for more realistic cases. A parametric study is conducted to examine the effect of groove geometry on the pressure profile and load-carrying capacity. The results of a series of simulations reveal that the self-adaptive groove provides a greater load-carrying capacity in comparison with conventional grooved surfaces. Comparisons between the mass-conservative formulation and Reynolds boundary condition for small length-to-width ratios show that the Reynolds boundary condition largely underestimates the cavitated area, leading to inaccuracies in the prediction of the load-carrying capacity. Therefore, only mass-conserving models should be used when dealing with grooves with a length-to-width ratio greater than 0.1.

- Novel self-adaptive groove designs are presented with shape optimization for producing the highest LCC. Specifically, grooves with constant thickness, linear thickness and arbitrary thickness shape are considered and optimized. The optimum thickness changes based on the load in a way that the dimensionless film thickness remain constant. Results show that the optimum dimensionless film thickness in the deformable section is near 1.9 which is almost the same as the analytical optimum value of 1.866 obtained for a Rayleigh step. Comparison with original designs reveals that optimized grooves have up to 45 percent more LCC.

- The problem of finding the global optimum film profile for sectorial-shape sliders is addressed in detail and the optimum profiles are given for different geometrical aspect ratios. It is found that the optimum shapes are similar to a curved trapezoidal pocket. The step-like front of the pocket expands from the central area toward the bearing inner/outer radii, as the aspect ratio increases. When aspect ratio approaches infinity, the film profile
takes on the shape of a step. Results reveal that optimized shapes for common bearing sizes (i.e., B/L=1, ρ=3), have 90 percent more LCC than conventional inclined pad or step bearings.

- A novel type of thrust bearing which uses flexible pads is proposed and analyzed. The pads have a split on the side which gives them the ability to deform under the lubricant pressure. In order to maximize the LCC, the optimum pad thickness and split angle ratio, which are the main parameters that control the pad deformation, are obtained by numerical optimization algorithms. Simulations show that the flexible-pad thrust bearing can support loads up to 30 percent more than the inclined pad bearing. It maintains its superior performance for a wide range of operating conditions. However, when the film thickness is too much higher (i.e., small loads) or too much smaller (i.e., high loads) than the nominal design point, the flexible-pad thrust bearing performance degrades because the deformation mechanism is not optimum. A detailed design guideline is also presented for determining the performance parameters of the flexible-pad bearings.

- Novel periodic groove geometries which produce the maximum LCC for parallel plate bearings are proposed and analyzed. A numerical optimization method based on the SQP algorithm is used for determining the optimum groove shape. It is shown that the optimum groove shapes are a function of geometrical aspect ratio. For small aspect ratios the optimum shape is an elongated heart-like shape. As the aspect ratio increases the optimum shape becomes more spiral-like. Simulations show that the proposed groove shapes can have greater LCC (e.g., 14%-36% for \( R_e = 2.5 \)) compared to conventional spiral groove shapes. The numerical model validated by a series of laboratory tests. Results show that the novel groove designs have superior performance compared to conventional spiral grooves.

- The optimum shape of spiral grooves for liquid-lubricated parallel flat surface bearings is obtained. The mathematical model is first validated by a series of experimental results and extensive simulations are carried out in order to develop a detailed design guideline. Simulation results show that there an optimum for the number of grooves exists that maximizes the LCC. This optimum is only a function of radius ratio. Further, it is found that the global-optimum film thickness ratio is about 4.12 for all radius ratios. It is also shown that the optimum spiral angle and optimum dam width to face width ratio are only a function of radius ratio and number of grooves.
10.2 Recommendation of Future Works

The following recommendations are made for possible future research:

- Flexible surface texturing is a relatively new topic in the tribology field and hence further research is necessary to gain an in-depth understanding of its working mechanism. The model developed for self-adaptive bearings can be further developed to consider non-isothermal conditions.

- The optimum shape of sectorial sliders is obtained under the isothermal condition. The extension of this work for non-isothermal conditions or non-Newtonian lubricants warrants further research. In addition, a series of experiments need to be done in order to validate the obtained results.

- Theoretical results show superior performance for flexible-pad thrust bearings. Further research is necessary to experimentally validate the theoretical model developed for flexible-pad thrust bearings.

- The optimum periodic groove patterns presented in this dissertation assume constant groove depth in radial direction. Future research can consider variable groove depth.

- The theoretical and experimental results on the spiral grooved annular rings presented in this dissertation are limited to the cases with groove-to-land ratio equal to one and atmospheric pressure on both sides. A more comprehensive study can be conducted to include variable groove-to-land ratio and non-atmospheric pressure boundary conditions.
APPENDIX A: SELF-ADAPTIVE GROOVE MODEL

A mathematical model for the self-adaptive grooves developed in [16]. The proposed groove model is described by the Reynolds equation, Eq. (A.1), plate deformation equation, Eq. (A.3), and total deformation equation, Eq. (A.6).

A.1 Reynolds Equation

The Reynolds equation in $p - \theta$ form for a compressible Newtonian lubricant with constant viscosity, can be written as

$$\nabla \cdot (\bar{h}^3 \nabla \bar{p}) = \Lambda \frac{\partial \bar{h}}{\partial x}$$  \hspace{1cm} (A.1a)

$$\bar{p} \geq 0 \quad \theta = 1$$  \hspace{1cm} (A.1b)

$$\bar{p} = 0 \quad \theta < 1$$  \hspace{1cm} (A.1c)

where $\theta$ is the film content, $\bar{p}$ is the dimensionless pressure, $\bar{h}$ is the dimensionless film thickness and $\Lambda$ is the bearing number. The use of $\theta$ in this formulation ensures mass conservation.

It is assumed that the grooves are equally spaced, so, one can take advantage of the geometrical symmetry and analyze only one single groove. Consequently, because of the symmetry of the model, the periodic boundary condition is applied in longitudinal direction.

$$\bar{p}(0, y) = \bar{p}(1, y)$$  \hspace{1cm} (A.2)

A.2 Plate Equation

The groove’s top surface can be modeled by a plate that deforms within the groove. The plate deformation ($\psi$) under distributed pressure $p$ is governed by classical plate equation,

$$\nabla^2 D \nabla^2 \psi = p$$  \hspace{1cm} (A.3)

where $\nabla^2$ is the Laplacian operator, and $D$ is the bending or flexural rigidity of the plate defined as,

$$D = \frac{E t^3}{12(1 - \nu^2)}$$  \hspace{1cm} (A.4)

where $E$ is the Young's modulus, $\nu$ is the Poisson's ratio, and $t$ is the plate thickness. For a constant thickness Eq. (A.3) can be further simplified as follows:
\[ D \nabla^4 \psi = p \]  \hspace{1cm} (A.5)

The plate boundary conditions in the longitudinal and the transverse directions are assumed to be clamped and free, respectively.

**A.3 Total Deformation Equation**

The lubricant pressure deforms the groove surface and consequently the film thickness \( \bar{h} \) changes. The final film thickness is the sum of minimum film thickness \( h_0 \) and groove deformation as follows:

\[ \bar{h} = 1 + \psi / h_0 \]  \hspace{1cm} (A.6)

The above equation couples the plate equation to the Reynolds equation.

**A.4 Solution Methodology**

The pressure generated in the lubricant film is obtained by iteratively solving the above equations. First, an initial deformation for the groove is assumed. Then using the Eq. (A.6) the coefficients of the Eq. (A.1) are calculated. A mass-conservative cavitation algorithm developed by Ausus et al. [26, 27] is used to solve the 2D Reynolds equation. It is important to point out that implementation of mass-conservative cavitation algorithms is crucial for small B/L ratios since non mass-conservative algorithms largely underestimate the cavitated area, leading to inaccuracies in the prediction of the LCC [16]. Using the pressure field obtained from the Reynolds equation, the plate deformation equation (Eq. (A.3)) is solved by a 3D finite element solver. This process continues iteratively until the convergence is met. More details can be found in [16].
APPENDIX B: THE HS-SQP ALGORITHM

The main steps of the HS-SQP algorithm [23] is presented in pseudo-code form in Table B1 and also explained briefly in the following subsections [23]:

B.1 Initialize parameters

The algorithm parameters such as the number of decision variables (N), the lower/upper bounds (X_{LB}/X_{UB}) for each variable, the harmony memory size (HMS), the probability of memory consideration (P_{HMCR}), the probability of pitch adjustment (P_{PAR}), and the maximum number of iterations (NI) are set in this step. An explanation of these parameters follows.

B.2 Initialize the harmony memory

The harmony memory (HM) is a memory location where all the solution vectors are stored. In Step 2, the HM matrix is filled with as many randomly generated solution vectors as the HMS.

\[
\text{HM} = \begin{bmatrix}
  x_1^1 & x_2^1 & \ldots & x_N^1 & f(x^1) \\
  x_1^2 & x_2^2 & \ldots & x_N^2 & f(x^2) \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  x_1^{HMS} & x_2^{HMS} & \ldots & x_N^{HMS} & f(x^{HMS})
\end{bmatrix} \quad (B.1)
\]

B.3 Improvise a new harmony

A new harmony vector, \( \tilde{x}'=(x_1',x_2',\ldots,x_N') \), is generated based on three rules [18]: (i) memory consideration, (ii) pitch adjustment, (iii) random selection. These rules are described in detail in the following subsections.

B.3.1 Memory consideration

In the memory consideration, the value for a decision variable is randomly chosen from the historical values stored in the HM. The P_{HMCR} determines the rate of choosing one value from the historical values stored in the HM. The larger the P_{HMCR}, the lesser is the achieved exploration; the algorithm further relies on stored values in HM and this potentially leads to the algorithm getting stuck in a local optimum [19, 23]. On the other hand, choosing too small of a
value for $P_{HMCR}$ (i.e., $P_{HMCR} < 0.4$) decreases the algorithm efficiency and the algorithm behaves like a pure random search, with less assistance from the historical memory.

Table B1. The pseudo-code of the HS-SQP algorithm

```plaintext
procedure HS-SQP
   Initiate_parameters()
   Initialize_HM()
   do{
      for (I = 1 to number of decision variables N)
         if (rand() < $P_{HMCR}$) /* (memory consideration) */
            X[I] will be randomly chosen from the HM
         If (rand() < $P_{par}$) /* (pitch adjustment) */
            X[I] = X[I] ± Δ
         end if
         else /* (random selection) */
            X[I] = X_{LB}[I] + rand() × (X_{UB}[I] - X_{LB}[I])
         end if
      end for
   /* evaluate the fitness of each vector */
   fitness_X = evaluate_fitness(X)
   /* update HM */
   update_memory(X,fitness_X) /* if applicable */
   }while(not_termination) /* if applicable */

   /* (local search using SQP) */
   Run SQP using X_{best} as starting point
   fitness_X = SQP(X_{best})
   print_final_solution()
end procedure
```

B.3.2 Pitch adjustment

Local solution refinement is undertaken by pitch adjusting. Every component obtained by the memory consideration is examined to determine whether it should be pitch-adjusted. In pitch adjustment the value of a decision variable like $x_i$ is replaced as follows [18]:

$$x_i = x_i ± rand(\times Δ$$ (B.2)
where $\Delta$ is an arbitrary bandwidth, and $\text{rand}()$ is a uniform random number. The probability of pitch adjustment ($P_{\text{PAR}}$) and $\Delta$ are important in fine-tuning of solution vectors, and can be potentially useful in adjusting convergence rate of algorithm to optimal solution. More discussions regarding the effect of these parameters on the solution quality and convergence behavior of the algorithm are given by Mahdavi et al. [19].

B.3.3 Random selection

Any variable that is not selected for memory consideration will be randomly set to a value between the lower and the upper bounds of the variable. This process helps the algorithm to maintain its diversity. A new harmony is improvised by random selection with a probability equal to $1 - P_{\text{HMCR}}$.

B.4 Update harmony memory

If the new harmony vector, $\vec{x}' = (x'_1, x'_2, ..., x'_N)$, has better fitness function than the worst harmony in the HM, the new harmony is included in the HM and the existing worst harmony is excluded from the HM.

B.5 Stopping criterion

The algorithm is terminated when there is no significant improvement in the best found solution after predetermined iterations or the maximum number of iterations is reached.

B.6 Local search

To increases the possibility of finding the global optimum and improve the convergence speed SQP method is combined with the HS algorithm. In the combination strategy used here the HS first performs a coarse search. When the HS is completed or shows a negligible trend of improvement after many iterations, the SQP begins its task and uses the best vector obtained by the HS as its starting point.
APPENDIX C: CORRELATIONS FOR THE LOAD AND OPTIMUM DIMENSIONS OF STEP PROFILES

The correlations for curves shown in Fig. 6-2 are given using the following function

\[
\psi(B/L) = \frac{a + cB/L + e(B/L)^2 + g(B/L)^3}{1 + bB/L + d(B/L)^2 + f(B/L)^3}
\]

\[B/L \geq 0.1 \quad (C1)\]

where \(\psi\) represents \(C_L\) or \(\lambda\) or \(\xi\). The coefficients are presented in Table C1.

Table C1: Coefficients of Eq. C1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(C_L)</th>
<th>(\lambda)</th>
<th>(\xi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.00030679</td>
<td>1.67610304</td>
<td>1.01353813</td>
</tr>
<tr>
<td>b</td>
<td>0.98614574</td>
<td>0.18604734</td>
<td>0.15772237</td>
</tr>
<tr>
<td>c</td>
<td>-0.00441013</td>
<td>0.33173336</td>
<td>0.02144013</td>
</tr>
<tr>
<td>d</td>
<td>1.31384823</td>
<td>0.04907858</td>
<td>0.33195296</td>
</tr>
<tr>
<td>e</td>
<td>0.12668761</td>
<td>0.01898524</td>
<td>0.52791815</td>
</tr>
<tr>
<td>f</td>
<td>0.87467291</td>
<td>0.31943289</td>
<td>0.24652221</td>
</tr>
<tr>
<td>g</td>
<td>0.18037709</td>
<td>0.59611488</td>
<td>0.62865597</td>
</tr>
</tbody>
</table>

Using the following function

\[
\phi(\rho, N_{pad}) = \frac{a + c \ln \rho + e \ln N_{pad} + g (\ln \rho)^2 + i (\ln N_{pad})^2 + k \ln \rho \ln N_{pad}}{1 + b \ln \rho + d \ln N_{pad} + f (\ln \rho)^2 + h (\ln N_{pad})^2 + j \ln \rho \ln N_{pad}} \quad 2 \leq N_{pad} \leq 30 \quad (C2)
\]

It is possible to find analytical correlations for Fig. 6-4a and Fig. 6-4b. The coefficients for each case are given in Table C2.

Table C2: Coefficients of the film profile equation for 2-D sectorial-shape step bearings

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(\lambda) (1.1&lt;(\rho&lt;6))</th>
<th>(\theta_2/\theta_1) (1.1&lt;(\rho&lt;2))</th>
<th>(\theta_2/\theta_1) (2&lt;(\rho&lt;6))</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1.682012121</td>
<td>1.027143162</td>
<td>1.342885563</td>
</tr>
<tr>
<td>b</td>
<td>-0.20065051</td>
<td>-0.61493786</td>
<td>-0.28559091</td>
</tr>
<tr>
<td>c</td>
<td>-0.34555925</td>
<td>-0.71224171</td>
<td>-0.68072858</td>
</tr>
<tr>
<td>d</td>
<td>-0.5406856</td>
<td>-0.49628777</td>
<td>-0.58030119</td>
</tr>
<tr>
<td>e</td>
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<td>-0.52178818</td>
<td>-1.03261225</td>
</tr>
<tr>
<td>f</td>
<td>0.055386037</td>
<td>0.184842073</td>
<td>0.182074518</td>
</tr>
<tr>
<td>g</td>
<td>0.095004499</td>
<td>0.235003412</td>
<td>0.28032561</td>
</tr>
<tr>
<td>h</td>
<td>0.076403585</td>
<td>0.064381788</td>
<td>0.110239951</td>
</tr>
<tr>
<td>i</td>
<td>0.129007398</td>
<td>0.069011554</td>
<td>0.225285236</td>
</tr>
<tr>
<td>j</td>
<td>0.061610694</td>
<td>0.154791661</td>
<td>0.168492823</td>
</tr>
<tr>
<td>k</td>
<td>0.109748506</td>
<td>0.213066954</td>
<td>0.481339429</td>
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</tbody>
</table>
**APPENDIX D: DETAILED GROOVE GEOMETRIES**

Table D1: Dimensionless design variables for different aspect ratios and number of grooves

<table>
<thead>
<tr>
<th>$R_t$</th>
<th>$N_G$</th>
<th>$\theta_1$</th>
<th>$L_1$</th>
<th>$\theta_2$</th>
<th>$L_2$</th>
<th>$\theta_3$</th>
<th>$L_3$</th>
<th>$\theta_4$</th>
<th>$L_4$</th>
<th>$\theta_5$</th>
<th>$L_5$</th>
<th>$\theta_6$</th>
<th>$L_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.00</td>
<td>0.04</td>
<td>0.18</td>
<td>0.43</td>
<td>0.40</td>
<td>0.85</td>
<td>0.47</td>
<td>0.85</td>
<td>0.16</td>
<td>0.37</td>
<td>0.00</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.00</td>
<td>0.00</td>
<td>0.22</td>
<td>0.52</td>
<td>0.46</td>
<td>0.85</td>
<td>0.42</td>
<td>0.85</td>
<td>0.12</td>
<td>0.27</td>
<td>0.00</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td>4</td>
<td>0.08</td>
<td>0.04</td>
<td>0.30</td>
<td>0.45</td>
<td>0.55</td>
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Author: M. Fesanghary, M. M. Khonsari
Publication: Tribology Transactions
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Date: Oct 6, 2010
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VITA

Mohammad Fesanghary was born in Sabzevar, Iran in 1981. He received his BS and MS in Mechanical Engineering from Amirkabir University of Technology (Tehran) in 2004 and 2007, respectively. After two years of work in oil/energy industries, he started his PhD research at Center for Rotating Machinery, Louisiana State University under the guidance of Prof. Michael M. Khonsari in 2009.

Mr. Fesanghary has made significant contributions in a range of interdisciplinary fields, including industrial heat transfer, energy engineering, and tribology. He has authored/published 20 first-rate scientific articles in internationally renowned, peer-reviewed journals, and 5 book chapters. Mr. Fesanghary’s works have been cited for over 1000 times worldwide and his novel findings have been featured in scientific media. Mr. Fesanghary has received 2 prestigious awards from Elsevier in recognition of his achievements along with several awards form LSU. He has served as an expert reviewer for 25 prestigious international journals.

Mohammad Fesanghary will receive his Doctor of Philosophy degree at the 2013 Summer Commencement.