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Improved bottomhole pressure control for underbalanced drilling operations

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IMPROVED BOTTOMHOLE PRESSURE CONTROL
FOR
UNDERBALANCED DRILLING OPERATIONS

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Petroleum Engineering

by
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May 2003
DEDICATION

This work is dedicated to my parents, Rafael and Eva, brothers, and sisters for providing me with education, inspiration, and confidence, specially to my wife, Virginia Morales de Perez, for her extraordinary understanding and encouragement not only during the development of this work, but also during each moment we have lived together, and to my children Carlos Rafael, Samantha Sandy and Evelin for their inspiration and for all the hours I stole from them while working in my Ph.D.
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NOMENCLATURE

\[ A = \text{area, } m^2 \text{ (in}^2\text{)} \]
\[ C_0 = \text{velocity profile coefficient, dimensionless} \]
\[ D = \text{diameter, } m \text{ (in)} \]
\[ D_e = \text{equivalent pipe diameter, } m \text{ (in)} \]
\[ f = \text{fraction, dimensionless} \]
\[ f_{Fw} = \text{homogeneous Fanning friction factor, dimensionless} \]
\[ f_F = \text{Fanning friction factor, dimensionless} \]
\[ f_i = \text{interfacial shear friction factor, dimensionless} \]
\[ f_m = \text{Moody friction factor, dimensionless} \]
\[ g = \text{gravity acceleration, } m/s^2 \text{ (ft/s}^2\text{)} \]
\[ g_c = \text{gravitational conservation constant} \]
\[ GOR = \text{gas oil ratio, } m^3/m^3 \text{ (scf/bbl)} \]
\[ h = \text{reservoir thickness, } m \text{ (ft)} \]
\[ H = \text{holdup, dimensionless} \]
\[ K = \text{diameter ratio, dimensionless} \]
\[ L = \text{length, } m \text{ (ft)} \]
\[ M = \text{gas molecular weight or increment counter} \]
\[ P = \text{pressure, Pa (psi)} \]
\[ q = \text{flow rate, } m^3/s \text{ (gpm or scfpm)} \]
\[ t = \text{time, sec} \]
\[ T = \text{temperature, } ^\circ K \text{ (} ^\circ \text{R)} \]
\[ TAI = \text{total axial increments} \]
$u = \text{velocity, m/s (ft/s)}$

$V = \text{volume, m}^3 (\text{ft}^3)$

$w = \text{weight fraction}$

$z = \text{compressibility factor, dimensionless}$

$Z = \text{axial direction}$

**Greek letters**

$\alpha = \text{gas volumetric fraction, dimensionless}$

$\beta = \text{relative bubble length parameter, dimensionless}$

$\infty = \text{discrete bubble}$

$\gamma = \text{specific gravity,}$

$\lambda = \text{no-slip holdup, dimensionless}$

$\mu = \text{viscosity, Pa.s (cp)}$

$\rho = \text{density, kg/m}^3 (\text{lbm/gal})$

$\sigma = \text{interfacial tension, N/m}$

$\delta = \text{film thickness, m (ft)}$

$\Delta = \text{increment}$

$\tau = \text{Shear stress, Pa, (psi)}$

$\tau_i = \text{interfacial shear, Pa, (psi)}$

$\varepsilon = \text{roughness, m (in)}$

$\nu = \text{specific gravity, m}^3/\text{kg (gal/lbm)}$

$\ell = \text{developing length of the bubble cap, m (ft)}$
**Subscripts**

Acc = acceleration component

bh = bottomhole

C = cap

dSU = developing slug unit

dTB = developing Taylor bubble

DF = drilling fluid

ep = equi-periphery

Fric = friction component

G = gas

h = hydraulic

Hy = gravity component

i = axial increment thickness

IC = in-situ conditions or inner casing

IT = inner tubing

L = liquid

LS = liquid slug

N = nozzle

m = mixture

N = nitrogen

NG = natural gas

oil = oil

OT = outer tubing
\( p \) = pipe

\( r \) = radius, m (in)

\( R \) = reservoir

\( s \) = surface

\( sc \) = standard conditions

\( SG \) = superficial gas

\( SL \) = superficial liquid

\( SU \) = slug unit

\( T \) = total or translational

\( TB \) = Taylor bubble

\( up \) = upstream

\( w \) = water or wall

\( wp \) = wellbore pressure

\( 1 \) = upstream condition

\( 2 \) = downstream condition

**Superscripts**

\( n \) = swarm effect exponent

\( R \) = relative liquid film

**Accents**

\( - \) = average
ABSTRACT

Maintaining underbalanced conditions from the beginning to the end of the drilling process is necessary to guarantee the success of jointed-pipe underbalanced drilling (UBD) operations by avoiding formation damage and potential hazardous drilling problems such as lost circulation and differential sticking. However, maintaining these conditions is an unmet challenge that continues motivating not only research but also technological developments.

This research proposes an UBD flow control procedure, which represents an economical method for maintaining continuous underbalanced conditions and, therefore, to increase well productivity by preventing formation damage. It is applicable to wells that can flow without artificial lift and within appropriate safety limits.

This flow control procedure is based on the results of a new comprehensive, mechanistic steady state model and on the results of a mechanistic time dependent model, which numerically combines the accurate comprehensive, mechanistic, steady-state model, the conservation equations approximated by finite differences, and a well deliverability model. The new steady state model is validated with both field data and full-scale experimental data.

Both steady state and time dependent models implemented in a FORTRAN computer program, were used to simulate drilling and pipe connection operations under reservoir flowing conditions. Actual reservoir and well geometries data from two different fields, in which the UBD technique is being employed, were used as input data to simulate simultaneous adjustments of controllable parameters such as nitrogen and drilling fluid injection flow rates and choke pressure to maintain the bottomhole pressure at a desired value. This value is selected to allow flow from the reservoir to substitute for reduction or cessation of nitrogen injection during drilling and for interruption of nitrogen and drilling fluid circulation during a pipe connection.

Finally, a specialized procedure for UBD operations is proposed to maximize the use of natural energy available from the reservoir through the proper manipulation of such controllable parameters based on the results of the computer simulations.
CHAPTER 1
INTRODUCTION

The growing number of depleted reservoirs around the world and the increasing necessity to recover hydrocarbons more efficiently has been forcing the oil and gas industry to continuously improve its drilling technology. Currently, the combination of drilling techniques that were conceptualized more than 100 years ago\(^1\), have with recent technological innovations ended up in specialized drilling techniques. These techniques, when properly designed and executed, allow drilling a well more economically, safely, and successfully in almost any given environment. One such technique is called underbalanced drilling (UBD).

Underbalanced drilling is the drilling process in which the wellbore pressure is intentionally designed to be lower than the pressure of the formation being drilled. This underbalanced pressure condition allows the reservoir fluids to enter the wellbore during drilling, thus preventing fluid loss and related causes of formation damage. As a result, special and additional equipment\(^2,4\), and procedures\(^5,6\) are required before, during, and after a UBD operation. In addition, to improving well productivity\(^7\) by preventing fluid loss and formation damage, underbalanced drilling offers several other significant benefits that are superior to conventional drilling techniques. These include increased penetration rate and bit life, reduced probability of sticking the drillstring downhole, and improved formation evaluation.

Achieving UBD conditions in subnormal pressure formations frequently requires the simultaneous injection of a mixture of liquid and gas. By this process, during which underbalanced conditions are generated artificially, the gas-liquid mixture is injected directly into the drill string at the surface, reducing the density of the entire fluid circulation system through the injection path (inside the string), and also when the returning fluid is flowing back to the surface in the annular space outside the string. However, in most normal and over pressured formations, the circulation of a liquid alone is enough to create such conditions. In either case, the flow returning to the surface consists of a compressible multiphase mixture including the formation and injected fluids, as well drilled cuttings.

1.1 Underbalanced Drilling Concepts

In underbalanced drilling, the concept of primary well control (containing the formation fluids by means of hydrostatic columns greater than the formation pressure) is replaced by the concept of flow control\(^5\). In flow control the bottomhole pressure (BHP) and influx of formation fluids must be controlled. Therefore, in UBD operations the BHP must be maintained between two pressure boundaries, which delimit the underbalanced drilling pressure window. Figure 1.1 illustrates this UBD pressure window in which the lower limit on BHP is determined by the borehole stability or the flow rate or pressure capacity of the surface equipment. Whereas, the formation pore pressure gives the upper limit on BHP.

In UBD the closed surface control system, a single barrier, multiple flow path system approach is used to maximize safety and system redundancy\(^3,5\). Conventional two barrier philosophies (overbalanced mud column plus BOP flange) are not possible during UBD and as such a single
barrier must be accepted. The conventional BOP stack configuration remains unchanged and serves the same function as for conventional overbalanced drilling. A secondary system is added for the UBD process. On top of the conventional BOP stack is a specialized devise, such as a rotating blow out preventer (RBOP) or a rotating control head (RCH), to contain annular pressure and divert returns to the surface control system. Figure 1.2 shows a sealing devise commonly used to contain the wellbore pressure during UBD operations.

In UBD, control of wellbore pressure is obtained by leading the well returns through an adjustable surface choke. Separation of drilling fluids, oil, gas, and solids is typically achieved by means of production type separation tanks. Therefore, the conventional rotary rig must be adapted for UBD with some considerable modifications. Typical required modifications include: The capacity to effect a seal around the kelly or drill pipe while concurrently rotating the drill string, equipment to separate the four phases of the well returns (gas, oil, water, cuttings), a means of regulating well flow in order to maintain the desired level of underbalanced, a method of generating and introducing gas into the fluid system, storage facilities for the produced hydrocarbons must be available, and float valves have to be incorporated to prevent well flow up the drill string.

The liquid phase of the drilling fluid system will in general have sufficient density to serve as a kill fluid in case of emergency occurs. If the liquid phase does not have the appropriate density, a separate batch of specific kill fluid has to be available at the side. Mainly, depending on the desired BHP, there is typically a choice of three basic fluid systems for UBD: single-phase liquid system, gasified liquid, and foam system. Considerations for drilling fluid design for underbalanced operations differ from the conventional overbalanced method in a number of
Filter cake materials is typically not added because underbalanced prevents fluid loss. Further, filter cake materials is generally considered to be an impairing agent, weighting materials, which also are impairing agents, are not required for the purpose of primary well control, and the addition of viscosifiers is not necessary because the annular multiphase flow system creates high friction gradients or large apparent viscosity, which provides exceptional turbulent hole cleaning characteristics. As a result, formation water, diesel, and reservoir crude are typically used as the liquid-phase of common UBD fluid systems, and nitrogen is usually injected when the formation pore pressure cannot tolerate a liquid hydrostatic head.

Figure 1.2 (RCH) for containing wellbore pressure during UBD operations.

The fact that UBD is a combined drilling and production operation, which requires drilling and production equipment as well as multidisciplinary teams for designing and executing the operations, makes that the cost of UBD be 1.3 to 2.0 times the conventional. Therefore, the expected production gains from a particular reservoir and the expected benefits in drilling performance must at least be sufficient to offset the additional costs associated with underbalanced drilling. These expected benefits are obtained when UBD improves well productivity by eliminating well impairment to a large extent. Improvements in well productivity affect UBD project profitability in the following ways: higher production rate per well, which may result in higher income by reducing the overall number of wells required for the development of a field, a higher net preset value of produced hydrocarbons due to early start of production and a faster production rate, and increase recoverable hydrocarbons because UBD
could make it possible to produce wells at economic flow rates down to lower depletion pressure\textsuperscript{10}. Consequently, the primary objective of UBD is to achieve near-zero skin damage and offer every interval of the reservoir an opportunity for production.

1.2 Problem Description

Even though the underbalanced drilling technique has proven itself to be successful in minimizing some drilling operating problems and reducing drilling time\textsuperscript{7}, it has been recognized by the petroleum industry that its greatest advantage is to increase well productivity through the formation damage prevention during the drilling process. It is also being accepted that the success of an underbalanced drilling operation is function of the ability to maintain underbalanced conditions during the entire drilling process. Unfortunately, during jointed-pipe drilling, the surface injection must be interrupted every time a connection or trip is needed. This stopping of injection causes the disruption of steady state conditions.

Additionally, during a connection, when injection is stopped, the bottomhole pressure initially decreases due to the frictional pressure loss. Then, during the connection time, due to buoyancy and inertial forces the gas phase continues moving upwards while the liquid phase flows backwards. This fluid separation forms liquid slugs in the annulus and inside the drillstring. Upon restarting injection and regaining circulation, frictional pressure is exerted on the bottom hole and the liquid slugs in the drillstring are pumped into the annulus thus increasing the hydrostatic pressure. Consequently, during a pipe connection a pressure spike is observed with a short period of sustaining higher bottomhole pressure that usually exposes the formation to overbalanced conditions.

Since this phenomenon occurs each time a connection takes place, and the time between drilling and connections is insufficient to regain steady state conditions, UBD pipe connection operations trigger a bottomhole pressure fluctuation. Figure 1.3 shows actual annular bottomhole pressure fluctuations recorded while drilling the Mexican well Muspac 52. This well was jointed-pipe drilled from 2610 m (8563 ft) to 2779 m (9118 ft) simultaneously injecting nitrogen and drilling fluid. This figure illustrates that after the very first pipe connection, the bottomhole pressure fluctuates and that the initial pseudosteady state conditions were never regained. Figure 1.3 also shows that the bottomhole pressure fluctuates within a pressure window greater than 6.89 MPa (1000 psi) and that the pressure spikes each connection.

This is the typical bottomhole pressure behavior observed in jointed-pipe UBD operations\textsuperscript{7,8}. Therefore, if these BHP fluctuations are not properly maintained below the formation pressure, the formation will be exposed to an overbalanced condition every time a connection or trip takes place. These periods of overbalanced can ruin or reduce the advantages obtained after making the efforts and expenses to drill the well underbalanced\textsuperscript{7-13}.

The major issue here is that, since the formation pressure is greater than the borehole pressure in a truly underbalanced operation, there is no impetus for the formation of any type of classic sealing filter cake on the surface of the rock. Evidently, this is advantageous with respect to prevention of formation damage and differential sticking, which may be associated with the influx of potentially damaging filtrate or mud solids into the formation, but it also means that the protective ability and presence of this filter cake as a barrier to fluid and solid invasion is
negated. Then, if the formation is abruptly or gradually (e.g. during a trip or during connections) exposed to a condition of periodic pulses of overbalanced pressure caused by the BHP fluctuations, very rapid and severe invasion of filtrate and associated solids may occur, causing even greater formation damage than that occurring when using a well-designed conventional overbalanced drilling program\textsuperscript{9,11}. Figure 1.4 schematically shows these conditions\textsuperscript{9,12}. This problem is often compounded by the fact that very thin, low viscosity base fluid systems are usually used in most UBD operations\textsuperscript{11}.

![Figure 1.3 Typical BHP fluctuations observed during UBD.](image)

Although in practices, borehole stability turns out not to be a major limitation\textsuperscript{10}, borehole instability can also be caused by the bottomhole pressure fluctuations\textsuperscript{7} because such BHP fluctuation can mechanically destabilize the formation. Additionally, these BHP fluctuations cause that the returning rates of liquid and gas and wellhead pressures are unstable, too. In practice, these unstable wellhead conditions are adverse for the good performance of the rotating head rubbers\textsuperscript{7}.

### 1.3 Attempted Solutions

The use of different drilling systems, such as snubbing and coiled tubing units, has been attempted as potential solutions to achieve 100% underbalanced conditions; however, their success has been limited to specific conditions. For example, the snubbing unit, which increases drilling time and cost when used, allows tripping with pressure but does not eliminate the bottomhole pressure fluctuation during connections\textsuperscript{13}. The coiled tubing unit eliminates both connection and tripping problems but cannot be used as a general underbalanced drilling rig because of its mechanical limitations and high cost\textsuperscript{4}. Different gas injection techniques (parasite tubing string or parasite casing string) have only partially reduced the bottomhole pressure.
fluctuation, but at a very high cost (additional gas injection, extra casing or tubing string, etc.)\textsuperscript{13}. Also, when using parasite string configurations, the full hydrostatic column of fluid causes bit jetting and flushing effects\textsuperscript{12}.

![Figure 1.4 Schematic representations of fluids and solids loss in overbalanced and underbalanced conditions (after Bennion et al.\textsuperscript{9,12}).](image)

There are also some new and emerging technologies that could be used to better manage the wellbore pressure\textsuperscript{14}. For example, a technology called the Closed Loop Continuous Circulation System\textsuperscript{15} enables a rig to make a drill pipe connection while maintaining continuous circulation. This prevents the drilling mud from developing gel strength and fluid separation, thereby reducing BHP fluctuations and potential reservoir damage upon pump restart after a connection. The Equivalent Circulation Density Reduction Tool\textsuperscript{15} utilizes a pump positioned in the drilling string, so that the pump remains in the cased section of the wellbore. This technology achieves a reduction in effective equivalent circulation density (ECD) across the open hole section and increases the ECD in the cased hole section where higher pressure can be more easily managed. These partially proven technologies are expensive, limiting their application to high productivity wells where the use of such technology is judged profitable.

On the other hand, the use of new designs of UBD fluids, which use elastic fiber-shaped additives to temporarily plug the formation pores when instantaneous positive pressure
difference is formed during UBD, has only experimentally shown that the formation damage can be reduced by 20% to 40% compared to conventional clay drilling fluids\textsuperscript{16,17}.

For all these reasons, the loss of underbalanced conditions during connections and trips is still a concern to be addressed by future technological developments in the petroleum industry. A downhole valve system\textsuperscript{18} (deployment valve) that is run as an integral part of the well’s casing string will allow isolating the open hole from the cased hole during tripping and completion operations. Although this valve has not yet completely been accepted by the industry, it seems to be a practical solution for maintaining at least balanced condition during tripping and completion operations. A solution to ensure that wells are maintained underbalanced during pipe connections is to reduce the target bottomhole pressure low enough to accommodate any pressure fluctuation that may occur. Unfortunately, this would require higher gas injection rates and additional surface equipment to adequately handle the flow of formation fluids. Therefore, a preferable approach is to more fully understand the dynamics of gas and liquid flow behavior during UBD operations and use this information to more effectively control the bottomhole pressure.

1.4 Research Goals

The necessity of maintaining 100% underbalanced conditions and controlling BHP fluctuation within a desirable UBD pressure window motivated the present research. Therefore, its main focus is to improve bottomhole pressure control for UBD operations to maintain underbalanced conditions and avoid formation damage during both routine drilling and drill pipe connections. Two-phase flow behavior predictions using a time dependent model coupled with a reservoir inflow performance equation will be developed to allow the interactive effect of changing drilling fluid/nitrogen flow rates, choke pressure, and reservoir inflow versus time to be studied. The well geometry, fluid properties, formation pressure, and gas and oil flow rates corresponding to a wellbore flowing pressure are used as the model’s inputs to predict variations in wellbore pressure, gas and liquid in-situ velocities, gas and liquid fractions, mixture densities, reservoir influxes and other two-phase flow parameters as a function of position and time caused by changes in surface gas and liquid injection flow rates and choke pressures.

A specific flow control concept that will then be studied is based on finding the best combination of controllable parameters such as gas and liquid injection and choke pressure, so that the bottomhole pressure can be maintained so that the reservoir influx substitutes for the interrupted surface injection during a pipe connection. These conditions should allow preservation of underbalanced conditions and consequently avoid formation damage during such operations.

The time dependent model should be composed of a method for flow pattern prediction and a set of independent models for calculating wellbore pressure and two-phase flow parameters as a function of position and time. For that, the time dependent model should rely on mechanistic models, which have shown significant progress in multiphase flow predictions, rather than empirical correlations, which are the most common among the current commercial UBD simulator and have been shown to over predict or fail to predict bottom hole pressures.
CHAPTER 2
LITERATURE REVIEW

Maintaining 100% underbalanced conditions and bottomhole pressures within a desirable UBD pressure window have mostly been attempted by designing the UBD hydraulic system using computer program outputs. Since in this work a computer program was also developed to propose from its outputs the best combination of parameters such as gas and liquid injection flow rates and choke pressure, which can be controlled during UBD operations to achieve proper UBD conditions, the first part of this review includes a summary of previous computer programs developed to predict such controllable parameters. It shows how these computer programs have evolved by describing, first the steady state computer programs that neglect slip between phases by assuming that aerated mud can be treated as a homogeneous mixture, second the steady state computer programs that used empirical correlations to take into account slip between phases and recognize different flow patterns, third the steady state computer programs that are based on mechanistic models rather than empirical correlations to take into account slip between phases and predict different flow patterns, and fourth the few time dependent models that claim to predict dynamic effects like drillstring tripping and starting/stopping of circulation or bottomhole pressure fluctuations during UBD pipe connections.

Finally, considering that the primary interest of this study is about flow control procedures to improve bottomhole pressure control for UBD operations to maintain underbalanced conditions and avoid formation damage during drill pipe connections, the little information about UBD flow control procedures available in the literature is summarized in the second and last part of this review.

2.1 Steady State Computer Simulators

2.1.1 Homogeneous Approach

Guo et al.\textsuperscript{19} developed a computer program to predict the optimum air injection rate that ensures a maximum penetration rate and cuttings transport capacity. Although they recognize that four principal flow patterns can be distinguished in multiphase flow (bubbly, slug, churn, and annular), based on experiences gained from well control, they assumed that the aerated mud can be treated as a homogeneous mixture of liquid, gas and solids, provided that it is flowing in the bubbly regime. Based upon this assumption, the program’s mathematical model, formed by the mechanical energy equation, the real gas law equation, and the rate-weighted average density, allows prediction of an air injection rate which gives the lowest flowing annular pressure for a particular given well geometry and mud rate. Only low gas injection rates that vary from 0 to 19.3 m$^3$/min (0 to 680 scfpm) and high liquid injection rates that vary from 0.68 to 1.14 m$^3$/min (180 to 300 gpm) were considered. These gas and liquid injection flow rates, which greatly favor the occurrence of homogeneous flow conditions, were used to validate the program’s output against observed field standpipe pressures of three specific wells. Although their computer program outputs were not validated for non-homogeneous flow conditions, they used the computer program to predict wellbore pressures from gas and liquid injection flow rates that greatly differ from the gas and liquid injection rates utilized in its validation.
Sharma et al\textsuperscript{20} developed a steady state model to study the simultaneous flow of two-phase flow mixtures in conduits. Their complex mathematical model is composed of a set of six equations that express conservation of mass (without considering mass accumulation) and one equation for conservation of momentum. The closure of the system of conservation equations is achieved by two drift flux equations. However, arguing that very little work had been done related to the phase drift velocities, they assumed that a homogeneous mixture flows in all sections of the drillstring and the annulus. As Guo et al.’s model\textsuperscript{19}, this assumption presented their model as being much less complex, but inaccurate to predict wellbore pressure and two-phase flow parameters for UBD hydraulic systems where slip between phases occurs.

2.1.2 Empirical Correlation Approach
Liu et al\textsuperscript{21} developed a computer model to analyze UBD foam operations. They also considered that foam can be treated as a homogeneous fluid and used the mechanical energy equation in which the frictional pressure drop depends on the foam rheology and the equation of state. They validated their model against Chevron’s Foamup program and full-scale test data gathered from a shallow experimental well. The validation results showed an 11.2\% margin of error. Although further comparison of the model results with two-field observed standpipe pressures, gathered from a well in which high gas-foam solution ratios were used, showed that the model accuracy ranged from +3.1 to –4.1\%, this foam program is based upon theoretical assumptions that are only valid for true foams.

This mathematical model was coupled with the Beggs and Brill\textsuperscript{22} empirical correlation and used to develop the UBD commercial computer program called MUDLITE\textsuperscript{23,24}. In addition to the wellbore pressure predictions, this computer program allows the prediction of flow patterns, liquid holdup, and in-situ gas and liquid velocities. However, it has been shown that the Beggs and Brill\textsuperscript{22} correlation over predicts or fails to predict bottom hole pressures\textsuperscript{25,26}.

Tian et al\textsuperscript{27,28} developed another UBD commercial computer program named the Hydraulic UnderBalanced Simulator (HUBS) to assist in designing underbalanced operations, especially for the process of optimizing underbalanced circulation rates. Although the mathematical model is not well described, they also considered that a two-phase empirical correlation is valid for predicting the UBD hydraulic system. Similar to MUDLITE\textsuperscript{23,24}, the Beggs and Brill\textsuperscript{22} empirical correlation was also incorporated to the model in order to predict flow patterns and liquid holdup inside the drillstring, as well as in the wellbore annulus. They show through simulation examples of drillstring injection how the model can predict the optimum circulation rate for a liquid and gas mixture; however, they did not present validation results for their model.

2.1.3 Phenomenological or Mechanistic Approach
Since the mid 1970’s, significant progress has been made in understanding the physics of two-phase flow in pipes and production systems. This progress has resulted in several two-phase flow mechanistic models to simulate pipelines and wells under steady state as well as transient conditions. The mechanistic or phenomenological approach postulates the existence of different flow configurations and formulates separate models for each one of these flow patterns to predict the main parameters, such as gas fraction and wellbore pressure. Consequently, mechanistic models, rather than empirical correlations, are being used with increasing frequency for the design of multiphase production systems. Nevertheless, most of the calculation approaches in
current practice of UBD are based on empirical correlations, which frequently fail to accurately predict the wellbore pressure.

Bijleveld et al.\textsuperscript{29} developed the first steady state UBD computer program using the mechanistic approach. To calculate wellbore pressure and two-phase flow parameters, initially stratified flow is assumed and from the stratified flow model the liquid holdup is calculated. With these data, the existence of this flow pattern is thus checked. If this flow type cannot exist under these conditions, annular dispersed flow pattern is assumed to be taking place. The same method is applied with its matching model and, providing this flow type cannot exist, bubble flow is assumed. If none of the calculated flow patterns can exist, intermittent flow is selected as the flow pattern. Although there is no further information about these mechanistic models, and how they were implemented in this trial and error procedure, the validation results against field and experimental data showed that the accuracy of this model (average absolute error less than 10\%) is better than that shown by the Beggs and Brill\textsuperscript{22} empirical correlation (average absolute error equal to 12\%)\textsuperscript{30}.

Hasan and Kabir\textsuperscript{31} developed a mechanistic model to estimate void fraction during upward cocurrent two-phase flow in annuli, and Hasan\textsuperscript{32} developed a mechanistic model to estimate void fraction during downward cocurrent two-phase flow in pipes. They utilized the drift-flux approach to predict the gas void fraction in bubble and slug flow. However, for slug flow, this represents a simplification that does not rigorously consider the difference in the drift-flux between the liquid slug and the Taylor bubble. Caetano\textsuperscript{33}, from experimental and analytical work, stated that two possible conditions must be considered to accurately predict slug flow parameters. The first is fully developed Taylor bubble, which occurs when the bubble cap length is negligible as compared to the total Taylor bubble length. Under this condition, the film thickness can be assumed constant for the entire film zone. The other is developing Taylor bubble, which consists only of a cap bubble. For this case, the film thickness varies continuously along the field zone, and cannot be assumed as constant. Thus, inaccurate predictions may be expected from a model that strictly used the Hasan and Kabir approach.

Lage et al.\textsuperscript{30,34} and Lage\textsuperscript{35} developed a mechanistic model based on a comprehensive experimental and theoretical investigation of upward two-phase flow in a concentric annulus. The model, which requires the input of the geometry, fluid properties and surface velocities, is composed of a procedure for flow pattern prediction and a set of independent mechanistic models for calculating gas volumetric fraction and pressure drop in bubble, dispersed bubble, slug, churn, and annular flow. Although the model performance (average absolute error less than 7\%) was extensively validated against small and full-scale experimental data gathered from annular geometries, they recommended evaluating the model in other annular configurations. Moreover, they did not consider mechanistic models to predict drillstring pressures and two-phase flow parameters for downward two-phase flow in pipes, neither they considered a model to calculate the pressure drop through the nozzles. Although Lage\textsuperscript{35} performed a lot of downward two-phase flow, small-scale experiments in a U-tube, the extensive experimental data gathered was mainly used to identify transitions between different flow patterns and to analyze the pressure oscillations in full-scale tests.
Perez-Tellez et al. developed an improved, comprehensive, mechanistic model for pressure predictions throughout a well during UBD operations. The comprehensive model is composed of a set of state-of-the-art mechanistic steady-state models for predicting flow patterns and calculating pressure and two-phase flow parameters in bubble, dispersed bubble, and slug flow. This model takes into account the entire flow path including downward two-phase flow through the drill string, two-phase flow through the bit nozzles, and upward two-phase flow through the annulus. Additionally, more rigorous analytical modifications to the previous mechanistic models for UBD give improved wellbore pressure predictions for steady state flow conditions. The results of using the new, comprehensive model were validated against full-scale experimental data obtained by Lopes from two experiments performed in a full-scale well located at Louisiana State University and field data from a Mexican well. These validations showed that the model performance is very good (absolute average error of less than 3%). Additionally, a comparison of the model results with two commercial UBD computer programs that rely on empirical correlations confirmed the expectation that mechanistic models perform better in predicting two-phase flow parameters in UBD operations.

2.2 Time Dependent Computer Simulators

As explained in chapter 1, the bottomhole pressure variation caused by the disruption of steady state conditions during jointed-pipe UBD operations is a very complex phenomenon that is not completely understood. Fluid segregation, backflow, liquid slug formation at the bottom, void spaces at surface, and gas expansion and/or compression occur during the time interval comprised between the time at which the circulation is interrupted and the time at which the circulation is regained.

There are four available dynamic UBD computer programs, which have shown from validated results, their capability of predicting the wellbore pressure variation during well unloading processes. However, only two of them have partially demonstrated their limited capability for predicting the complex bottomhole pressure fluctuation caused by the injection interruption during an UBD pipe connection.

One of these two dynamic computer programs is RF-Rogaland Research’s DynaFloDrill, which has been described in a series of papers, reports, and publications. The main features of this transient, 1-D model includes reservoir-wellbore interaction, alternative geometries for gas injection and rheology of different fluids. The numerically solved mathematical model consists of seven mass conservation equations (for free produced gas, free injected gas, mud, dissolved gas, formation oil, formation water, and drill cuttings), one overall momentum conservation equation, and a number of submodels (gas and liquid density, gas solubility, cuttings velocity, drilling fluid rheology, and frictional pressure losses) to close the system of equations. The model has been extensively validated with full-scale experimental data using both parasite and drillstring injection. Although the validation results have demonstrated the capability of the model to accurately predict well unloading processes, changes in liquid and/or gas flow rates, and changes in choke pressure during coiled tubing and parasite injection operations, the model predictions for the case of a pipe connection when both gas and liquid are injected through the drillstring are not accurate (relative errors greater than 100%).
Rommetveit et al.\textsuperscript{45} carried out a validation of the DynaFloDrill model results against full-scale experimental data gathered from a 1300 m (4265 ft) vertical well. The experiment consisted of simultaneously injecting liquid and nitrogen through an 88.9 mm (3-1/2 in) pipe until steady state conditions were reached. Then, interrupting the injection of both liquid and gas during approximately 10 minutes simulated a pipe connection. Although wellbore pressure was recorded with memory sensors placed at bottom (1262 m or 4140 ft), 998 m (3274 ft), 605 m (1985 ft), and 185 m (607 ft), the model validation, shown in a graphic given by Rommetveit et al.\textsuperscript{45}, was only performed against pressure data gathered at 998 m (3274 ft). This validation shows that the model cannot predict the actual wellbore pressure variation recorded by the memory gauge and that the wellbore pressure predictions during the simulated pipe connection considerably differ from the actual ones. Rommetveit et al.\textsuperscript{45} concluded that some development efforts are still necessary to improve the predictions of DynaFloDrill.

Lorentzen et al.\textsuperscript{47,48} recently implemented a statistical approach as a pressure filter into a numerical solution of a drift-flux formulation of the two-phase flow conservation equations to calculate wellbore pressure fluctuations during UBD pipe connections. Although this approximation gave very good results when compared with experimental data acquired during a pipe connection simulation, this model needed as inputs data wellbore pressure measurements gathered from four pressure gauges placed along the annulus of the full-scale well. Therefore, currently this approach is very limited. First, it makes several statistical assumptions that require several annular wellbore pressure measurements along the wellbore, which are typically only available in experimental facilities, and second, conventional survey techniques are ineffective when drilling with a compressible fluid, and electromagnetic tools cannot be used to simultaneously measure wellbore pressure at different depths along the wellbore.

Jun et al.\textsuperscript{38} developed the second dynamic UBD computer program, whose capability for predicting the complex hydraulic system behavior during an UBD pipe connection has been reported in the literature. Similar to DynaFloDrill, this computer program considers co-current flow of two-phase drilling fluid, water, gas, oil, and solid particles in one direction along the flow path, and its governing equations are those expressing conservation of mass (mud, water, cuttings, oil, and gas) and conservation of mixture momentum. Some other sub models and equations are also needed to close the system. A finite difference method is also employed as the solution procedure for this theoretical model. Even though several important factors affecting UBD operations seem to be taken into account in the model (reservoir influx, physical properties and mass transfer behavior of fluids, flow regime and phase migration features, geometry and deviation of wellbore as well as different operating modes), the validation of the model is carried out through a hypothetical example of a jointed pipe drillstring injection, which only displays the models response to a different UBD operations. In this hypothetical simulation, during the pipe connection simulation the bottomhole pressure decreases, this BHP decrement should have caused and increase in reservoir influx. However, in the graphic results they presented, the simulator response is opposed to what should actually happen. That is, the reservoir influx decreases. On the other hand, the bottomhole pressure stabilized long before choke pressure, and oil and gas flow rates became constant, which is also an unlikely result.
2.3 Flow Control Procedures during UBD Pipe Connections

Deis et al\textsuperscript{13} and Mullane et al\textsuperscript{49} describes the development of an underbalanced drilling process in Canada. They report having had success in the reduction of bottomhole pressure after modifying their operational procedures by trial and error. In the early phases of the project, fluid slugging, enhanced by pipe connection operations, made it difficult to maintain BHP below reservoir pressure and the BHP fluctuations were as high as 8.20 MPa (1190 psi). After that, pipe connections were made after pumping only a sufficient amount of drilling fluid to displace the drill pipe to the first float valve from surface. Float valves were inserted into the drillstring approximately every 300 m (984 ft). After implementing this new procedure for making pipe connections, the BHP was almost always maintained below the reservoir pressure and the BHP fluctuations were as high as 3.45 MPa (600 psi). Later changes included displacing the drill pipe to the nearest float valve with nitrogen rather than drilling fluid prior to breaking a connection. This additional change in procedure further decreased the variance of BHP. They described that in oil wells, in an attempt to slow the fluid fall back in the annulus, thus limiting the liquid loading at the bottom of the hole, the wells were shut in during connections. However, in gas wells, without further explanation, the wells were not shut in during connection, but allowed to flow. Ultimately, they had success in reducing the bottomhole pressure below the reservoir pressure, but not the fluctuations.

Negrao and Lage\textsuperscript{50} report that achieving a steady state ECD has been a concern while dealing with UBD technology in Brazil. Again, trial and error procedures were used to improve bottomhole pressure fluctuations. First, due to the procedures adopted for connecting a new pipe, drilling from 860 to 884 m (2822 to 2900 ft) they showed that the time interval required to drill the length of one joint was not sufficient to let the bottomhole pressure reach the steady-state regime. Then, following the recommendations of Saponja\textsuperscript{8} (shut in the well and pre-charging the annulus), they claim to have mitigated the bottomhole pressure fluctuation, giving an example in which the ECD behavior is almost a flat line while drilling from 190 to 199 m (623 to 653 ft). However, unfortunately from a global point of view, this cannot be considered as a success in reducing bottomhole pressure fluctuations during connections because normally most of the wells drilled underbalanced are much deeper than 199 m (653 ft).

Bennion et al\textsuperscript{12} stated that the major factor in the disappointing results from many UBD operations conducted in the past is the fact that the underbalanced condition is not maintained 100\% of the time during drilling. Also, they stated that if a rotary rig is used, the underbalanced condition is potentially compromised each time gas injection must be terminated to make a pipe connection because a pressure spikes higher than reservoir pressure are generated during pipe connections. Then, they showed, with a bottomhole pressure surveys without scale data, that circulating out to pure gas prior to each pipe connection tends to minimize the effect of overbalanced pulses. However, they concluded saying that fluctuations in BHP is still common in some UBD operations. In addition to not mitigate BHP fluctuation, this technique is limited to very shallow wells with very low productivity in which circulating out to pure gas can be made in a short period of time, with low gas volumes, and in safe conditions.

Similar to Bennion et al\textsuperscript{12}, Saponja\textsuperscript{8} determined that UBD has been unsuccessful in some reservoirs because wells believed to be drilled underbalanced were found to have formation
damage or positive skin. He explained that after reviewing operating procedures and circulating systems, the results revealed that overbalance pressure occurred during drillstring connections and incompatible drilling fluids were used. Therefore, he concluded that drillstring connections influence BHP and that the annular and frictional effects of a multiphase circulation system must be controlled in order to maintain proper underbalanced conditions. Then, using BHP surveys while drilling underbalanced and outputs from steady state computer programs, Saponja\textsuperscript{8} defined UBD concepts that are still very useful for UBD operations.

Saponja\textsuperscript{8} stated that in UBD hydraulic systems, annular frictional effects are not linear and at low gas injection rates the effects of friction are small and do not significantly influence the BHP. As the gas rate increases, friction becomes more substantial and the rate of change of BHP decreases. He called this portion of the curve as being hydrostatically dominated. Ultimately, an optimal circulating point is reached when reduced hydrostatic pressure is balanced by increase annular friction. Thus, he concluded that this point is the minimum achievable BHP for a given liquid rate, and that an increase in gas rate beyond this point increases the BHP and the system becomes friction dominated. From this analysis, Saponja\textsuperscript{8} defined that circulating systems operating on the hydrostatic-dominated side are unstable. Whereas, circulating systems operating on the friction-dominated side are stable. Therefore, he recommended that during underbalanced drilling it must be determined if the circulating system is operating on the hydrostatic or friction dominated side so that the BHP can be controlled and proper underbalanced conditions can be maintained during drilling.

Saponja\textsuperscript{8} also stated that pressure spikes produced during a drillstring connection must be minimized, controlled, and quantified to avoid losing underbalanced conditions during such operations. He suggested that the decision to use an open or close annulus during connections is dependent on the type of underbalanced well being drilled. For a well that is capable of flowing freely under its own energy, he recommended that the annulus should remain open to avoid high shut in surface pressure and unnecessary increases in BHP. On the other hand, Saponja\textsuperscript{8} recommended that the annulus should be shut in for wells with insufficient energy to maintain flow during connections and under pressured wells that produce significant volume of liquid. This reduces annular fluid separation and stores the annular gas phase energy. In addition, Saponja\textsuperscript{8} described the annular pre-charging technique to make it easier to regain circulation after a connection. This technique, which allows increasing the annular pressure and gas to liquid ratio prior to a connection, reduces drawdown on the formation, liquid inflow, total volume of liquid in the wellbore, and formation of liquid slugs. Execution of the annular pre-charging technique requires precise timing of annulus closure followed by a period of continued gas injection in order to avoid overbalanced BHP. This technique is also limited to very shallow wells with very low productivity.

Finally, Saponja\textsuperscript{8} established that connections and tripping procedures must be specialized for UBD and underlined that to minimize bleed back time during a connection, as Deis et al\textsuperscript{13} and Mullane et al\textsuperscript{49} suggested, gas can be displaced from the drillstring to the first float with liquid. However, this pipe connection procedure introduces a liquid slug into the circulating system creating a pressure spike and possible system instabilities. On the other hand, if the drillstring is displaced to gas each connection, the bleed down period can be 5 to 15 minutes and the pressure spikes are not eliminated.
Taking into account the complexity of multiphase flow, the non-steady state nature of UBD hydraulic systems caused by the injection interruption during pipe connections, the lack of accuracy of existing dynamic UBD computer programs to predict such complex UBD hydraulic systems, and the necessity of better field procedures to improve BHP control for UBD operations so that proper underbalanced conditions can be maintained during the entire drilling process, in this work, instead of trying to rigorously predict mathematically the bottomhole pressure fluctuations occurring during UBD pipe connections, a procedure for avoiding or reducing them using the reservoir energy through the liquid and gas injection rates and the choke pressure manipulation is alternatively proposed in Chapter 6.
CHAPTER 3
COMPREHENSIVE, MECHANISTIC STEADY STATE MODEL

It is generally accepted that the success of UBD operations is dependent on maintaining the wellbore pressure between the boundaries defined by the designed UBD pressure window. Therefore, the ability to accurately predict wellbore pressure is critically important for both designing the UBD operation and predicting the effect of changes in the actual operation. As shown in Chapter 2, most of the pressure prediction approaches used in current practice for UBD are based on empirical correlations, which frequently fail to accurately predict the wellbore pressure. Consequently, the current trend is toward increasing the use of prediction methods based on phenomenological or mechanistic models.

This chapter describes in detail the improved, comprehensive, mechanistic model for UBD operations developed in this research. The comprehensive model is composed of a set of state-of-the-art mechanistic steady state models for estimating flow patterns and calculating pressure and two-phase flow parameters in bubble, dispersed bubble, and slug flow. The model takes into account the entire flowpath including downward two-phase flow through the drill string, two-phase flow through the bit nozzles, and upward two-phase flow through the annulus. The model’s validation results show that the model improves wellbore pressure predictions.

First, the most important model assumptions and a brief introduction of necessary two-phase flow terms are given. Second, UBD flow patterns, which occur in downward two-phase flow in the drillstring and upward two-phase flow in the annulus during normal UBD operations, are defined. Third, the mechanistic models used to determine the transitions between such flow patterns will be developed. Fourth, the particular steady state mechanistic model used to predict wellbore pressure and two-phase flow parameters for each flow pattern previously predicted are presented. Fifth, the implementation of the model in a computer program is described. Finally, the validation results and a comparison of the model performance are given.

3.1 Model Assumptions and Key Two-phase Flow Concepts

3.1.1 Basic Model Assumptions
During ordinary UBD operations with conventional rigs (jointed pipe drilling), drilling fluids (liquid or gasified liquid) are pumped down through the drillstring, through the bit, and then up the annulus. Within the annulus, drilling fluids are mixed with rock cuttings and production fluids (gas, oil, or water). Therefore, underbalanced hydraulic circulating systems are typically characterized by the complex flow of two or more phases (liquid mixture, gas mixture, and solid cuttings). Considering that hydraulic properties between the injected and produced gases are relatively close compared to those of solid or liquid phases, it is assumed that injection gas and formation gas flow at the same speed. For the same reason, injection liquid and formation liquids also are assumed to flow at the same speed in the wellbore annulus. Moreover, taking into account that multiphase flow creates high friction gradients or large apparent viscosities, which provide exceptional turbulent hole cleaning characteristics, in UBD instead of using conventional mud rheology hole cleaning methods, annular velocity and apparent multiphase viscosity are used. Hence, it is considered that the liquid portion of the multiphase fluid provides
the cuttings lifting capacity and that the cuttings travel at the liquid velocity. Bearing in mind these assumptions, the multiphase underbalanced hydraulic circulation system may be simplified to a two-phase flow system in which only a mixture of liquid and gas flows.

### 3.1.2 Key Two-phase Flow Concepts

When liquid and gas flow simultaneously in the wellbore, they tend to separate because of differences in density and flow at different velocities. Expansion of the highly compressible gas phase with decreasing pressure increases the in-situ volumetric flow rate of the gas. As a result, the gas and liquid phases normally do not travel at the same velocity. This variation in the physical distribution of the phases in the fluid conduit causes the occurrence of a wide range of flow patterns. This section defines some of the more important concepts unique to two-phase flow that must be understood before describing the comprehensive mechanistic steady state model.

**Superficial velocity** is the velocity that a phase would exhibit if it flowed through the total cross-sectional area available for flow alone. The superficial velocities of the liquid and gas phases are

\[
 u_{SL} = \frac{q_L}{A_p} 
\]

\[
 u_{SG} = \frac{q_G \cdot P_e \cdot Z_T}{A_p T_p \cdot p} 
\]

A total or mixture velocity then can be defined as

\[
 u_m = u_{SL} + u_{SG} 
\]

**Slip flow** occurs when the liquid and gas phases travel at different velocities. For upward flow, the less dense, more compressible, less viscous gas phase tends to flow at a higher velocity than the liquid phase. However, for downward flow, the liquid often flows faster than the gas phase.

**Liquid holdup** is defined as the fraction of a pipe cross-section or volume increment that is occupied by the liquid phase. The value of liquid holdup varies from zero for single-phase gas flow to one for single-phase liquid flow. It is function of gas and liquid properties, flow pattern, and well geometry

\[
 H_L = \frac{A_L}{A_p} 
\]

**No-slip flow** occurs when the liquid and gas phases travel at the same velocity. Thus, the **No-slip liquid holdup** is the fraction of pipe cross-section area that the liquid phase would occupy if the liquid and gas phases traveled at the same velocity. The no-slip liquid holdup, \( \lambda_L \), is defined by
\[ \lambda_L = \frac{q_L}{q_L + q_G} \quad (3.5) \]

**Actual or in-situ velocity** is the velocity a phase exhibits when it flows along with the other phase. Therefore, the actual area through which the phase flows is reduced by the presence of the other phase. Thus

\[ u_L = \frac{q_L}{H_L A_p} = \frac{u_{SL}}{H_L} \quad (3.6) \]

\[ u_G = \frac{q_G}{(1 - H_L) A_p} = \frac{u_{SG}}{(1 - H_L)} \quad (3.7) \]

**Weighting factors** are used when drilling fluid, oil, and water flow simultaneously, with or without gas. It is possible for slippage to occur between the oil and drilling fluid or water phase. This type of slippage is normally very small compared to the slippage that can occur between gas and any liquid\(^5\). Assuming there is no slippage among liquid phases, the drilling fluid fraction in the liquid phase is calculated from

\[ f_{DF} = \frac{q_{DF}}{q_{DF} + q_{oil} + q_w} \quad (3.8) \]

Similarly, the fraction of gas produced or injected in a gas mixture can be determined.

**Two-phase flow patterns.** Whenever two fluids with different flow properties flow simultaneously in a conduit, there is a wide range of possible flow patterns. The flow pattern that exists depends on the relative magnitudes of the forces that act on the fluids. Buoyancy, turbulence, inertia, and surface tension forces vary significantly with flow rates, wellbore geometry, and fluid properties of the phases. Consequently, several different two-phase flow patterns can exist in a given well as a result of the large pressure and temperature changes that occur along the flow path. Published work on flow patterns suggest that the most accepted flow patterns are: dispersed bubble, bubble, slug, churn, and annular\(^3,35,53\).

**Dispersed bubble flow (DB):** The gas is distributed as small discrete bubbles within a continuous liquid phase. The spherical shaped bubbles are the only ones observed in this flow pattern. Due to the high liquid velocities encountered in this flow pattern, the mixture flows at the same velocity with no slippage between the phases.

**Bubble flow (B):** The gas is distributed as small discrete bubbles within a continuous liquid phase, but in this case the discrete bubbles occurred in two different shapes; namely spherical and cap bubbles\(^3\). The spherical bubbles are very small as compared to the cap bubbles that are relatively larger. These cap bubbles move faster than the liquid phase because of slippage.

**Slug flow (SL):** Slug flow is characterized by a series of slug units. Each unit is composed of a gas pocket called a Taylor Bubble\(^5\), a plug of liquid called a slug and a film of liquid around the
Taylor bubble flowing downward relative to the Taylor bubble. The Taylor bubble is an axisymmetric, bullet-shaped gas pocket that occupies almost the entire cross section area of the pipe or annulus. The liquid slug, carrying distributed small gas bubbles, bridges the conduit and separates two consecutive Taylor bubbles.

**Churn flow (CH):** Churn flow exists in upward flow only. It is a chaotic flow of gas and liquid in which the shape of both the Taylor bubbles and the liquid slugs are distorted. The continuity of the liquid in the liquid slug between successive Taylor bubbles is repeatedly destroyed by a high local gas phase concentration. As this happens, the liquid in the slug falls backward, accumulates, forms a temporary bridge and is again lifted upward by the gas. An alternating direction of motion in the liquid phase is typical of churn flow.

**Annular flow (AN):** The gas is a continuous phase flowing in the core of the pipe or annulus cross-section area. The liquid flows upward, both as a thin film along the walls and as dispersed droplets entrained in the core. Annular flow can exist throughout the entire range of inclinations.

Based on the flow pattern definition, investigators determine experimentally and/or theoretically the region of existence for each of the flow patterns. Once these regions are known, they are normally presented in a two-dimensional plot, in terms of superficial phase velocities, called the flow pattern map. Figures 3.1 and 3.2 show the most widely accepted flow pattern maps for downward two-phase flow in pipes and upward two-phase flow in annuli, respectively.

![Flow pattern map](image)

**Figure 3.1 Flow pattern map for downward two-phase flow in pipes** (After Barnea et al. and Lage)

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3.2 Dominant UBD Flow Patterns

3.2.1 Annular Geometries

Particular flow patterns depend on flow rates, fluid properties, and well geometry. Typical injection gas and liquid flow rates used in UBD operations vary from 10 to 50 m³/min (353 to 1766 scfpm) and 0.189 to 1.325 m³/min (40 to 350 gpm), respectively. On the other hand, common annular cross section areas available to flow near the surface vary between 0.016 to 0.033 m², which correspond to 168.3 mm x 88.9 mm (6.625 in x 3.5 in) and 222.4 mm x 88.9 mm (8.755 in x 3.5 in) annuli. Substituting these values into the superficial velocity definitions given by equations (3.1) and (3.2) and superimposing the results on a common annular flow pattern map, superficial velocities for UBD gas and liquid injection flow rates and flow areas near the surface are presented in Figure 3.3.

Figure 3.3 shows that very high superficial velocities would be observed, even for low gas flow rates, when flow is at atmospheric pressure. However, a small increase in choke pressure would be enough to drastically decrease such superficial gas velocities shifting from annular to churn or slug flow conditions. In UBD, due to well control safety and surface fluid handling considerations, if high gas superficial velocities are expected at the surface, the return line must be choked to increase the pressure and consequently reduce the gas velocity as shown in Figure 3.3. Additionally, if we consider the changes in pressure and temperature along the wellbore of a typical UBD well, we would observe that churn flow may occur only at conditions close to the surface while at wellbore conditions dispersed bubble, bubble, and slug flow predominate. This can be seen in the flow pattern map shown in Figure 3.4, in which the horizontal straight
lines that stand for the flow through the annulus link the surface and bottom hole conditions. Based on this analysis, it is possible to conclude that the window of occurrence of annular flow in UBD operations is quite limited. Also, the possibility that churn flow occurs is small and since there is not a well defined churn flow model, it is usually treated as slug flow. Therefore, in annular geometries, UBD operations deal mostly with dispersed bubble, bubble, and slug flow. This agrees with the experimental results of Sunthankar et al who identified mainly bubble and slug flow during their experiments with aerated mud in annular geometries.

![Flow pattern map for the annulus near the top of the well.](image)

Figure 3.3 Flow pattern map for the annulus near the top of the well.

### 3.2.2 Drillstring Geometries

Regarding downward two-phase flow, Barnea et al and recently Lage, in small-scale experiments conducted at nearly atmospheric conditions, observed that only annular, slug, and bubbly flow regimes occur in vertical downward flow. They also observed that the system has a tendency to arrange more spontaneously in annular flow, which takes the form of falling film at low superficial gas velocities and normal annular flow for high superficial gas velocities. However, during common UBD operations, gas and liquid are simultaneously injected through a drillstring cross-section area of 0.00387 m² (6 in²), which corresponds to a 88.9 mm (3 ½ in) pipe, at high injection pressure, normally greater than 6.9 MPa (1000 psi). This high pressure generates turbulent forces and compressible effects high enough to maintain the gas phase dispersed in the continuous liquid phase and therefore, limits the occurrence of annular flow.

Similar to the upward flow in annular geometries, using the superficial velocity definitions considering the changes in pressure and temperature that may occur along the drillstring of a typical UBD well and the gas and liquid flow rates and drillstring cross-section areas mentioned above, one can conclude that dispersed bubble, bubble, and slug flow, as suggested by
Hasan, are also the dominant flow patterns in the downward flow through the drill string. Figure 3.5 shows the drillstring flow pattern map for typical UBD conditions. In this figure, the sets of three red circles in horizontal line, from left to right, represent the conditions at the bottom, middle, and surface for different combinations of gas and liquid injection flow rates.

Figure 3.4 Dominant UBD flow patterns for annular geometries.

Figure 3.5 Dominant UBD flow patterns for drillstring geometries.
3.3 Flow Pattern Prediction Models

3.3.1 Upward Flow in Annuli

Caetano, Hasan and Kabir, Kelessidis et al, and recently Lage et al agree that flow patterns observed in vertical concentric annuli are similar to those seen in pipes. Also, they agree in using the framework developed by Taitel et al to predict the flow pattern transitions adapting annular geometrical parameters such as diameter ratio, hydraulic diameter, and equi-periphery diameter, defined by equations (3.9) to (3.11), respectively.

\[ K = \frac{D_{ot}}{D_{ic}} \quad (3.9) \]
\[ D_n = D_{ic} - D_{ot} \quad (3.10) \]
\[ D_{ep} = D_{ic} + D_{ot} \quad (3.11) \]

Based on these different works, flow patterns can be predicted by defining transition boundaries between them (Figure 3.2). Although these authors consider five different flow patterns (dispersed bubble, bubble, slug, churn, and annular), for the reason explained above, this work considers only dispersed bubble, bubble, and slug flow. However, to avoid convergence problems during the calculations, a transition to churn and annular flow are considered. If churn flow occurs, it is treated as slug flow. For the annular flow occurrence, a simplified annular flow model proposed by Taitel and Barnea was implemented. In UBD operations, these simplistic assumptions have a negligible effect in the overall calculations because when churn or annular flow occurs, they occur relatively close to the surface.

Bubble to slug transition. During bubble flow, discrete bubbles rise with the occasional appearance of a Taylor bubble. The discrete bubble rise velocity after Harmathy is given by

\[ u_w = 1.53 \left( \frac{\rho_l - \rho_g}{\rho_l^2} \right) \sqrt{\frac{g\sigma}{L}} \quad (3.12) \]

The rise velocity of the Taylor bubbles on the other hand is given by

\[ u_{TB} = 0.345 \sqrt{gD_{ep}} \quad (3.13) \]

Taitel et al. suggested that whenever the discrete bubble rise velocity is greater than the rise velocity of the Taylor bubbles, the discrete bubble approaches the back of the Taylor bubble and coalescence occurs. Under these conditions bubble flow cannot prevail. On the other hand, when the rise velocity of the Taylor bubbles is grater than the discrete bubble rise velocity, the Taylor bubble rises through an array of discrete bubbles and the relative motion of the liquid at the nose of the Taylor bubble sweeps the small bubbles around the larger one, and coalescence does not take place. This phenomenon allows the existence of the bubble flow pattern. Therefore, combining Equations (3.12) and (3.13), the bubble flow pattern in annuli takes place when

\[ \frac{D_{ot}}{D_{ic}} = \frac{D_{ic} - D_{ot}}{D_{ic} + D_{ot}} \]
When \( D_{ep} \) is greater than the right hand side of Equation (3.14), bubble flow can take place. Therefore, the agglomeration or coalescence of small gas bubbles into large Taylor bubbles, which occurs when the in-situ gas rate increases (void fraction increases), is the basic transition mechanism from bubble to slug flow. Except for Caetano\textsuperscript{33}, who suggested that the bubble to slug transition occurs at a void fraction of about 0.20, other investigators\textsuperscript{53,63} and recently Lage et al\textsuperscript{30} agree that such transition occurs at a void fraction of about 0.25. Although there is a wide agreement in the value of gas void fraction at which bubble to slug transition occurs, there is an inconsistency in the criterion used to express this transition in terms of measurable variables, such as superficial phase velocities.

Caetano\textsuperscript{33}, using his experimentally determined void fraction value of \( \alpha = 0.2 \), equates the slip between phases with the terminal rise velocity of a discrete bubble \( u_G - u_L = u_w \) and represent the boundary between bubble and slug flow by

\[
 u_{SL} = 4.0u_G - 0.8u_w \tag{3.15}
\]

Hasan and Kabir\textsuperscript{31} taking into account that the gas void fraction depends on the in-situ velocity of the gas phase relative to the mixture and the gas bubbles tend to flow to the central portion of the channel where the local mixture velocity is higher than the average velocity, proposed Equation (3.16) to predict the boundary between bubble and slug flow.

\[
 u_G = 1.2u_m + u_w \tag{3.16}
\]

Expressed in terms of superficial velocities, with a gas fraction value equal to 0.25, Equation (3.16) becomes

\[
 u_{SL} = 2.332u_G - 0.833u_w \tag{3.17}
\]

Kelessidis et al\textsuperscript{53} and Lage et al\textsuperscript{30,34} took into account the effect of a single bubble rising in a swarm of bubbles \( H_L^n \) proposed by Wallis\textsuperscript{64} and equated the slip between phases with the discrete bubble terminal rise velocity affected by this effect

\[
 u_G - u_L = u_m H_L^n \tag{3.18}
\]

Then, using the most common value \( n = 0.5 \) and a gas void fraction of 0.25, they defined the bubble-slug transition as follows

\[
 u_{SL} = 3.0u_G - 0.866u_w \tag{3.19}
\]
Zuber and Findlay\textsuperscript{65} stated that the effect of the non-uniform flow and concentration distribution across the pipe and the effect of the local relative velocity between the two phases affect two-phase flow systems and defined a velocity profile coefficient, $C_0$.

In view of the fact that the swarm effect coefficient $n$ and the velocity profile coefficient $C_0$ affect two-phase flow systems, differently from the authors mentioned above, who separately considered these effects, in the present model both are taken into account. Thus, the bubble-slug transition is defined by

$$u_G - C_0 u_m = u_m H_L^n$$  \hspace{1cm} (3.20)

To fit experimental with analytical data, different authors\textsuperscript{30,33,34} used different values for the velocity profile coefficient $C_0$, but all of them agree in using 0.5 for the swarm effect exponent. In the present model, the most widely used values ($C_0 = 1.2$)\textsuperscript{31,33} and ($n = 0.5$)\textsuperscript{66} are used.

Although most authors agree that the bubble-slug transition occurs at a void fraction of about 0.25, in the present work better results were obtained when considering a void fraction of 0.20 as suggested by Caetano\textsuperscript{33} and Lage et al\textsuperscript{34}. This probably happens because the studies reported above were carried out with Newtonian fluids, rather than with non-Newtonian fluids, as those used during the model validation. Thus, using a value of gas void fraction of 0.20, Equation (3.20) may be reduced to

$$u_{SG} = 3.167 u_{SG} - 0.745 u_m$$  \hspace{1cm} (3.21)

This equation defines transition A in Figure 3.2.

**Bubble or slug to dispersed bubble transition.** Considering the maximum stable diameter of the dispersed phase under highly turbulent conditions and the critical diameter at which the turbulent breakup process causes the gas bubbles to remain spherical regardless of whether the gas void fraction exceeds the value of 0.25, Taitel et al\textsuperscript{60} developed an equation to predict the bubble or slug to dispersed bubble transition for two-phase flow in pipes. As reported by Caetano\textsuperscript{33}, this equation was later improved by considering the relatively small effect of the gas void fraction on the process of coalescence and breakup, and on the resulting bubble size. Then, using the hydraulic diameter concept, in the improved equation, Caetano\textsuperscript{33} proposed Equation (3.22) for the bubble or slug to dispersed bubble flow transition, which is shown in Figure 3.2 as transition B.

$$\left[\frac{1.6\sigma}{(\rho_L - \rho_G) g}\right]^{0.5} \left(\frac{\rho_L}{\sigma}\right)^{0.6} \left(\frac{2 f_{mu}}{D_h}\right)^{0.4} u_m^{1.2} = 0.725 + 4.15 \left(\frac{u_{SG}}{u_m}\right)^{0.5}$$  \hspace{1cm} (3.22)
Since the bubble rise velocity in dispersed bubble flow is very small compared to the local velocity values, a non-slip homogeneous mixture flow description represents the flow parameters relatively well\textsuperscript{33}. Therefore, the homogeneous Fanning friction factor, $f_{HF}$, in Equation (3.22) is calculated using the no-slip liquid holdup concept defined by

$$H_L = \frac{u_{SL}}{u_m}$$

\textbf{Dispersed bubble to slug flow transition.} Taitel et al\textsuperscript{60} stated that regardless how much turbulent energy is available to disperse the mixture, bubbly flow can no longer exist at in-situ gas rates so high that bubbles are packed close enough to be in contact. Assuming that bubbles are spherical and arranged in a cubic lattice, they determined that the maximum allowable gas void fraction under bubbly conditions is 0.52. Higher values of void fraction will cause the transition to slug flow. Thus, considering this gas void fraction limit and the dispersed bubble flow homogeneous conditions, Equation (3.24) gives the transition boundary between dispersed bubble and slug flow. This is shown as transition C in Figure 3.2.

$$u_{SL} = 0.923u_{SG}$$

\textbf{Slug to churn transition.} Equation (3.25), proposed by Tengesdal et al\textsuperscript{67}, is used to predict this transition. At a gas void fraction equal to 0.78 the slug structure is completely destroyed and the two distinct regions, liquid slug and Taylor bubble, no longer exist causing the transition to churn flow, which is represented as transition D in Figure 3.2.

$$u_{SL} = 0.0684u_{SG} - 0.292gD_p$$

\textbf{Churn to annular transition.} Based on the minimum gas velocity required to prevent the entrained liquid droplets from falling back into the gas stream that would originate churn flow, Taitel et al\textsuperscript{60} proposed the following Equation to predict the transition to annular flow. This is shown as transition E in Figure 3.2.

$$u_{SG} = 3.1 \left[ \frac{(\rho_L - \rho_G)g\sigma}{\rho_G^2} \right]^{0.25}$$

\textbf{3.3.2 Downward Flow in the Drillstring}

In contrast to the extensive research in upward two-phase flow in pipes, there are only few investigations of gas liquid mixtures in downward flow in pipes. Moreover, these investigations have been carried out at nearly atmospheric conditions, which greatly differ from those occurring during jointed-pipe UBD operations where liquid and gas are simultaneously injected at high injection pressure. With these limitations, the approach of Hasan\textsuperscript{32}, which was extensively validated with available data from the literature, was implemented in the present study to predict downward two-phase flow behavior in the drillstring.
Bubble to slug transition. Hasan\textsuperscript{32} proved that in downward flow the effect of buoyancy, expressed by the terminal bubble rise velocity, Equation (3.12), has the same magnitude as in the case of upward flow but in the opposite direction. He stated that the transition from bubble to slug flow occurs because of bubble agglomeration at high in-situ gas flow rates and assumed that this transition occurs at a void fraction of about 0.25, similar to that predicted in upward flow\textsuperscript{60}. Thus, modifying Equation (3.20) for downward flow conditions, the bubble-slug transition in the present model is given by

\[ u_{SG} = 2.332u_{SG} + 0.7214u_w \]  \hspace{1cm} (3.27)

Bubble or slug to dispersed bubble transition. Because of the high velocities associated with dispersed bubble flow, Hasan\textsuperscript{32} concluded that the flow direction is unlikely to have a significant effect on this flow pattern. Then, Equation (3.22) can also be used to estimate the bubble or slug to dispersed bubble flow transition for downward flow.

Similarly, as stated by Taitel et al\textsuperscript{60}, Hasan\textsuperscript{32} considers that regardless of the existing turbulence forces, the gas void fraction cannot exceed 0.52 without causing transition to slug flow. Therefore, Equation (3.24) also may be used to predict the transition boundary between dispersed bubble and slug flow for downward flow conditions.

3.4 Flow Behavior Prediction Models

Models that allow accurate prediction of pressure and phase concentration are required for each particular flow pattern previously predicted. Considering that the three dominant flow regimes in UBD operations are dispersed bubble, bubble, and slug, six independent models are required to handle both downward two-phase flow through the drillstring and upward two-phase flow through the annulus. Additionally, a two-phase flow bit model is required to predict pressure drop through the nozzles and an annular flow model for annular geometries to avoid model convergence.

3.4.1 Bubble Flow Model for Annular Geometries

This model is based on the drift-flux approach, which considers the velocity difference between the phases or between a phase and the average volumetric velocity of the mixture\textsuperscript{68}. Therefore, as in the bubble-slug transition model (Equation 3.20), the implemented model takes into account both the velocity profile coefficient \( C_0 \), and the bubble swarm effect \( H_L^p \). Thus, expressed in superficial velocities, the bubble drift-flux model used to predict the liquid holdup is given by

\[ H_L^p u_w = \frac{u_{SG}}{(1-H_L)} - C_0 u_m \]  \hspace{1cm} (3.28)

This equation is solved using an iterative procedure due to its implicit nature. Considering the experimental work of Caetano\textsuperscript{33}, who determined that bubble flow occurs for a liquid holdup between 0.8 and 1, an initial value required for the numerical solution of Equation (3.28) may be any within this range. In this work, the Newton-Raphson method was used to solve for \( H_L \).
After the liquid holdup is calculated from Equation (3.28), the total pressure gradient can be estimated. For steady state flow, it is composed of gravity, friction, and convective acceleration losses and is given by

$$ \left( \frac{dp}{dZ} \right)_T = \left( \frac{dp}{dZ} \right)_{Hy} + \left( \frac{dp}{dZ} \right)_{Fric} + \left( \frac{dp}{dZ} \right)_{Acc} $$

(3.29)

Modeling of two-phase pressure gradient requires analyzing each component as a function of the existing flow pattern. Thus, the gravity component is given by

$$ \left( \frac{dp}{dZ} \right)_{Hy} = g \rho_m $$

(3.30)

where

$$ \rho_m = \rho_f H_L + \rho_G (1 - H_L) $$

(3.31)

The friction component is given by

$$ \left( \frac{dp}{dZ} \right)_{Fric} = \frac{2 f_F \rho_m u_m^2}{D_h} $$

(3.32)

As suggested by Caetano\textsuperscript{33}, the Fanning friction factor $f_F$ is calculated with the Gunn and Darling\textsuperscript{69} approach for turbulent flow

$$ \left\{ f_F \left( \frac{F_F}{F_{CA}} \right)^{0.45 \exp \left[ - (N_{Re} - 3000) / 10^4 \right]} \right\}^{-0.5} = 4 \log \left[ N_{Re} \left\{ f_F \left( \frac{F_F}{F_{CA}} \right)^{0.45 \exp \left[ - (N_{Re} - 3000) / 10^4 \right]} \right\}^{0.5} \right] - 0.4 $$

(3.33)

which is a function of the diameter ratio $K$ given by Equation (3.9) and the mixture Reynolds number defined by

$$ N_{Re} = \frac{\rho_m u_m D_h}{\mu_L H_L + \mu_G (1 - H_L)} $$

(3.34)

where $F_F$ and $F_{CA}$ are geometry parameters defined by Equations (3.35) and (3.36)

$$ F_F = \frac{16}{N_{Re}} $$

(3.35)
\[ F_{CA} = \frac{16(1-K)^2}{1-K^4\left(1-K^2\right)\ln(1/K)} \]  

(3.36)

For estimating the Reynolds number, the gas viscosity is calculated by the most widely used method presented by Lee et al. On the other hand, considering the fact that some of the physical models like the non-Newtonian flow behavior in a two-phase flow environment remain untested and that in UBD operations most of the times Newtonian fluids are used, Newtonian behavior is modeled for the liquid phase and for the gas-liquid mixture. Therefore, the apparent viscosity of the liquid phase is used as the liquid viscosity in equation (3.34).

Using the Beggs and Brill approach, the acceleration component is given by

\[
\left(\frac{dp}{dZ}\right)_{Acc} = \frac{\rho_u u_m u_{SG}}{p} \frac{dp}{dZ} 
\]

(3.37)

3.4.2 Dispersed Bubble Flow Model for Annular Geometries

Due to the high turbulent forces during dispersed bubble flow, the dispersed gas bubbles do not exhibit significant slippage through the liquid phase and the velocity profile remains approximately flat. Therefore, the slip velocity is negligible, and the velocity profile coefficient is approximately one. Thus, liquid holdup can be calculated by using Equation (3.23).

After calculating the corresponding dispersed bubble liquid holdup, the pressure gradient components are calculated as those in bubble flow.

3.4.3 Slug Flow Model for Annular Geometries

As mentioned above, slug flow, shown in Figure 3.6, is characterized by the alternate flow of gas and liquid. The gas phase appears in two different forms: large bullet shaped bubbles (Taylor bubbles) and small spherical bubbles dispersed in the liquid phase. The Taylor bubbles occupying almost the whole configuration cross-section move uniformly upward. The liquid phase appears both in the form of liquid slugs which bridge the pipe cross section and as falling liquid films which flow downward between the Taylor bubble and pipe walls. The liquid slugs, which separate successive Taylor bubbles, contain the small spherical gas bubbles as a discrete distributed phase.

Fernandes et al. developed the first mechanistic model for slug flow in vertical pipes. Then, Caetano implemented this model for vertical annuli. These works are adopted and modified in this study. Different from these works, the present model considers variable liquid holdup in the liquid slug and incorporates the bubble drift-flux model represented by Equation (3.28) to predict the in-situ gas velocity in the liquid slug zone. Moreover, the model takes into account two possible situations. First, fully developed Taylor bubble slug flow can occur where the bubble cap length, \(L_c\), is negligible as compared to the total Taylor bubble length (Figure 3.6a). Under these conditions, the film thickness reaches a constant terminal value that can be used as the
average thickness for the entire film zone. Second, developing Taylor bubble slug flow can occur, which consists only of a cap bubble (Figure 3.6b). For this particular case, the film thickness cannot be considered as constant because it varies continuously along the Taylor bubble zone, and therefore, must be numerically calculated.

Figure 3.6 Slug flow unit, fully developed (FDTB) and in developing stage (DTB).

The complex distribution of the two phases and the intermittent nature of the flow make slug flow one of the most difficult flow patterns to model. Consequently, the hydrodynamic parameters that describe this flow behavior are required to calculate pressure drops in slug flow. These hydrodynamic parameters are illustrated and deduced in Appendices A and B for both fully developed Taylor bubble (FDTB) and developing Taylor bubble (DTB) slug flow.

First, assuming that for the in-situ flow conditions at one point in the annuls fully developed Taylor bubble exist, determine the length of the Taylor bubble and Taylor bubble cap following the approaches described in Appendix A and B, respectively. Second, if the Taylor bubble cap length is less than the Taylor bubble length, the assumption of fully developed Taylor bubble slug flow is correct and the hydrodynamic parameters for pressure drop predictions are calculated as mentioned in Appendix A. However, if the Taylor bubble cap length is greater than the Taylor bubble length, developing Taylor bubble slug flow must be considered and the hydrodynamic parameters for pressure drop predictions are calculated as mentioned in Appendix B.

After defining the slug flow conditions and calculating the hydrodynamic parameters that describe this flow behavior, the total pressure gradient can be estimated, using Equation (3.29).
Thus, for slug flow the gravitation component is given by

\[
\frac{dp}{dZ}_{\text{g}} = [(1 - \beta)\rho_{m_{LS}} + \beta \rho_{m_{TB}}]g
\] (3.38),

the friction component by

\[
\frac{dp}{dZ}_{\text{Fric}} = \frac{2 f_{f_{ic}} \rho_{m_{LS}} u_m^2}{D_h}(1 - \beta)
\] (3.39),

and the pressure drop due to acceleration across the mixing zone at the front of the liquid slug by

\[
\frac{dp}{dZ}_{\text{Acc}} = \frac{H_{k_{LS}} \rho_{L} u_{t_{LS}} + |u_{f_{ic}}| u_{f_{ic}}}{L_{SU}}(u_f - u_{t_{LS}})
\] (3.40)

where \( \rho_{m_{LS}} \) is the mixture density in the liquid slug zone defined by

\[
\rho_{m_{LS}} = \rho_{L} H_{k_{LS}} + \rho_{G}(1 - H_{k_{LS}})
\] (3.41)

and the friction factor is calculated as described above with a Reynolds number defined by

\[
N_{Re} = \frac{\rho_{m_{LS}} u_m D_h}{\mu_{L} H_{k_{LS}} + \mu_{G}(1 - H_{k_{LS}})}
\] (3.42)

In Equations (3.38) through (3.42), \( \beta \) is the relative bubble length parameter, \( \rho_{m_{TB}} \) is the mixture density in the Taylor bubble zone, and \( u_{f_{ic}} \) is the in-situ liquid velocity in the Taylor bubble zone, which are function of the slug flow conditions.

For fully developed Taylor bubble slug flow

\[
\beta = \frac{L_{TB}}{L_{SU}} \quad \text{and} \quad \rho_{m_{TB}} = \rho_{G}
\] (3.43)

and for developing Taylor bubble slug flow

\[
\beta = \frac{L_{dfB}}{L_{SU}} \quad \rho_{m_{TB}} = \rho_{L} H_{t_{ Boca}} + \rho_{G}(1 - H_{t_{ Boca}}), \quad \text{and} \quad u_{t_{TB}} = u_{t_{Boca}}
\] (3.44)

### 3.4.4 Annular Flow Model for Annular Geometries

As explained above, in common UBD operations, the window of occurrence of annular flow is quite limited and when it occurs, it takes place in the annulus at a few meters close to the surface.
Therefore, in this work, the simplified annular flow model proposed by Taitel and Barnea\cite{61} was implemented only to avoid convergence problems during the computations.

Assuming that in annular flow the liquid film thickness $\delta$ is much less than the inner pipe diameter and fairly constant (Figure 3.2), Taitel and Barnea\cite{61} from a momentum balance on the gas core, proposed that the pressure drop in annular flow can be calculated with

$$\left(\frac{dp}{dZ}\right)_G = \frac{4\tau_i}{D_e - 2\delta} \left[\rho_L H_L + \rho_G (1 - H_L)\right] g$$

\begin{equation}
(3.45)
\end{equation}

where $D_e$ is the equivalent pipe diameter defined by

$$D_e = \sqrt{D_{ic}^2 - D_{ov}^2}$$

\begin{equation}
(3.46)
\end{equation}

and the liquid film thickness $\delta$ can be calculated with its value for the case of free falling film as suggested by Wallis\cite{64}. Thus for turbulent flow

$$\delta = 0.115 \left(\frac{\mu_L^2}{g (\rho_L - \rho_G) \rho_L}\right)^{1/3} \left(\frac{\rho_L \mu_{sl} D_e}{\mu_L}\right)^{0.6}$$

\begin{equation}
(3.47)
\end{equation}

The interfacial shear $\tau_i$ is given by

$$\tau_i = \frac{0.5 f_i \rho_G \mu_{SG}^2}{\left(1 - 2 \frac{\delta}{D_e}\right)^4}$$

\begin{equation}
(3.48)
\end{equation}

For the interfacial shear friction factor $f_i$, the Wallis correlation is used

$$f_i = 0.005 + 1.5 \left(\frac{\delta}{D_e}\right)$$

\begin{equation}
(3.49)
\end{equation}

Considering that the liquid film thickness $\delta$ is constant, the liquid holdup can be estimated by

$$H_L = \frac{\delta}{D_e} - \left(\frac{\delta}{D_e}\right)^2$$

\begin{equation}
(3.50)
\end{equation}

Because of the limited occurrence of this flow pattern, additional efforts to implement a complex annular flow model would have a negligible effect on the overall calculations.
3.4.5 Bubble Flow Model for Drillstring Geometries
Based on the fact that in downward flow, buoyancy opposes the flow of the gas phase, Hasan proposed a model for estimating the liquid holdup based on the drift-flux approach for modeling the slippages between phases. Thus, according to Hasan, Equation (3.28) may be rearranged to calculate liquid holdup in downward bubble flow

$$H_L^m u_m = C_g u_m - \frac{\mu_{SG}}{(1 - H_L)} \quad (3.51)$$

For estimating the pressure gradient components, similar equations as those used in the annular-bubble flow model can be used with the friction factor calculated with the Colebrook function given by

$$\frac{1}{\sqrt{f_m}} = -4 \log \left( \frac{0.269 \varepsilon}{D} + \frac{1.255}{N_{Re} \sqrt{f_m}} \right) \quad (3.52)$$

considering that Moody friction factor $f_m$ is four times larger than the Fanning friction factor $f_F$ and that the hydraulic diameter becomes the inner pipe diameter for drillstring geometries.

Similarly, taking into account the friction factor and hydraulic diameter adjustments, the pressure gradient components for the drillstring dispersed bubble flow model are evaluated using the approach suggested for the annulus.

3.4.6 Slug Flow Model for Drillstring Geometries
Considering that the gas phase in the liquid slug is usually a small fraction of the total gas phase in a slug unit, and the difference in the drift velocities in the liquid slug and in the Taylor bubble usually is also small, Hasan and Kabir and Hasan developed an approach to predict the hydrodynamic parameters of a slug unit needed to calculate pressure drop in downward slug flow in pipes. These works are the basis for the development of a model for downward slug flow in the drillstring.

Assuming that the liquid and gas phases in the liquid slug behave analogously to fully developed bubble flow and that the bubble swarm effect in downward flow is negligible ($n = 0$), the liquid holdup in the liquid slug can be calculated by solving Equation (3.51) as follows

$$H_{rez} = 1 - \frac{\mu_{SG}}{C_g u_m - u_m} \quad (3.53)$$

Using the velocity defined by Equation (3.13) as the rise velocity of a Taylor bubble in downward flow, the liquid holdup in the Taylor bubble may be calculated by

$$H_{rez} = 1 - \frac{\mu_{SG}}{(C_g u_m - u_{TB})} \quad (3.54)$$
After extensive validations, Hasan\textsuperscript{32} recommended using $C_0 = 1.2$ and $C_1 = 1.12$.

Considering a slug unit formed by a Taylor bubble and a liquid slug regions (Figure 3.6a), the liquid holdup in the slug unit may be approximated to\textsuperscript{31}

$$H_{LS} = 1 - \left[ \frac{L_{TB}}{L_{SU}} (1 - H_{LS}) + \frac{L_{LS}}{L_{SU}} (1 - H_{LS}) \right]$$

(3.55)

Akagawa and Sakaguchi\textsuperscript{75} showed that the average volume fraction of gas in the liquid slug ($\alpha_{LS}$) is approximately equal to 0.1 when $u_{SG} > 0.4\text{ m/sec}$ ($u_{SG} > 1.3\text{ ft/sec}$) and equal to 0.25$u_{sg}$ for lower superficial gas velocities. Hasan\textsuperscript{32} validated that this approximation can be extended to downward slug flow in pipes. Thus, rearranging Equation (3.53) for the gas void fraction in the liquid slug $\alpha = (1 - H_{LS})$ and knowing that $L_{LS} = 16D_{IT}$\textsuperscript{60}, an equation for the slug unit length may be obtained. Therefore

$$L_{SU} = \frac{160D_{IT}u_{SG}}{C_d u_m - u_w} \quad \text{for} \quad u_{SG} > 0.4\text{ m/sec}$$

(3.56a)

$$L_{SU} = \frac{64D_{IT}}{C_d u_m - u_w} \quad \text{for} \quad u_{SG} \leq 0.4\text{ m/sec}$$

(3.56b)

Considering a fully developed Taylor bubble flow (Equation 3.43) and knowing that $L_{LS} / L_{SU} + \beta = 1$, the gravitation and friction components can be calculated using Equations (3.38) and (3.39) in which the friction factor is calculated with the Colebrook\textsuperscript{51} function as mentioned above.

Since in UBD, the most common flow patterns in downward flow are dispersed bubble and bubble (Figure 3.5), the acceleration component in drillstring geometries is relatively small and may be either neglected or calculated using the approach suggested for bubble flow, Equation (3.37).

### 3.5 Two-phase Flow Bit Model

Applying conservation of mechanical energy\textsuperscript{76,77} to the flow through bit nozzles, the governing equation for pressure loss through the bit can be computed. For downward flow, Bourgoyne et al\textsuperscript{78} defines the differential mechanical energy equation as

$$\frac{udu}{g_c} - \frac{g dZ}{g_c} + wpd + \frac{2u^2 f_d dZ}{g_c Z} = 0$$

(3.57)

where $\nu$ is the specific volume
Similar to single phase liquid flow, to calculate the pressure drop caused by the passage of a
liquid-gas mixture through the nozzles, one can assume that the change in elevation is negligible,
the velocity upstream of the nozzles is negligible compared to the nozzle velocity, and the
frictional pressure drop across the nozzles is also negligible. Therefore, Equation (3.57) can be
reduced to

\[
\frac{udu}{g_c} + wdp = 0
\]  (3.58)

Assuming that the gas-liquid mixture passing through the nozzles is homogeneous, the specific
volume may be defined as follows

\[
\nu = \frac{w_G}{\rho_G} + \frac{(1-w_G)}{\rho_L}
\]  (3.59)

where the weight fraction of gas \( w_G \) is defined by

\[
w_G = \frac{q_G \rho_G}{q_G \rho_G + q_L \rho_L}
\]  (3.60)

and the gas density given by

\[
\rho_G = \frac{M_G p}{zRT}
\]  (3.61)

Integrating along the flow path, Equation (3.58) can be written as

\[
\frac{1}{g_c} \int_0^u u du + \int_{p_{bh}}^{p_{up}} \nu dp = 0
\]  (3.62)

where \( u_n \) is the nozzle velocity, \( p_{bh} \) is the bottom hole pressure or nozzle downstream pressure,
and \( p_{up} \) is the nozzle upstream pressure.

Substituting Equations (3.59) and (3.61) into equation (3.62) and performing the integrations
results in

\[
\frac{u_n^2}{g_c} + \frac{(1-w_G)}{\rho_L} (p_{bh} - p_{up}) + \frac{w_G zRT}{M_G} \ln \left( \frac{p_{bh}}{p_{up}} \right) = 0
\]  (3.63)

For steady state flow conditions the continuity equation for a gas-liquid mixture may be
expressed as\(^{51}\)
\[
\frac{\partial (\rho_u u_n A_n)}{\partial Z} = 0
\]  

Therefore,

\[
\rho_u u_n A_n = q_l \rho_l + q_g \rho_g = \text{constant} \tag{3.65}
\]

Consequently, the nozzle velocity may be expressed by

\[
u_n = \frac{q_g \rho_g + q_l \rho_l}{A_n} \tag{3.66}\]

Then, Equation (3.63) and (3.66) can be solved numerically to obtain the pressure upstream of the nozzles \(p_{up}\), knowing the corresponding bottom hole pressure \(p_{bhp}\).

3.6 Computer Program Description

All the models described above were implemented into a FORTRAN 90 computer program that performs an iterative two-phase flow analysis on a discretized wellbore. The well is divided into many axial increments and each increment is treated separately. Any increment length may be used, but 6 to 15 m (20 to 50 ft) segments provide the best results when compared to real data. Both drillstring and annulus may have sections of different cross-sectional area as desired. Calculations start from the surface, based on the pressure and temperature at the wellhead, proceeds down the annulus to the bottom hole, then up through the bit nozzles, and finishes on the drillstring surface. Figure 3.7 illustrates a discretized wellbore and the calculation path implemented in the computer program.

The pressure gradient predictions use a marching algorithm which allows calculating the flow parameters along the flow path (wellbore) after dividing it into cells. After dividing the wellbore into axial increments or cells, the initial conditions of pressure and temperature existing at the wellhead (top of the first cell in Figure 3.7), the gas and liquid injection flow rates, and a guessed total pressure drop across the axial increment are used to solve the set of equations to determine the flow pattern, the liquid holdup, and the total pressure gradient along the axial increment. After that, the pressure and temperature at the bottom of this cell can be estimated. These pressures and temperatures represent the initial conditions at the top of the next axial increment (cell 2 in Figure 3.7), which similarly are utilized to calculate the corresponding pressure and temperature at the bottom of this new cell. Following this procedure, flow pattern, liquid holdup, two-phase flow parameters, and wellbore pressure can be calculated along the wellbore flow path for a specific point in time.

3.6.1 Algorithm Steps

Appendix C presents the computer flow diagram for the comprehensive, mechanistic steady state model to calculate flow patterns, two-phase flow parameters, and wellbore pressure along the flow path following the algorithm steps described below.
1. Input gas and liquid flow rates, fluid properties, and well geometry.

2. Select the length of the axial increments (Figure 3.7). If the wellbore has more than one cross sectional area (e.g. annular geometry \( AG \) or drillstring geometry \( DSG \) greater than zero), initialize a geometry counter variable \( GC \).

3. Guess the total pressure drop corresponding to the length increment. Since the hydrostatic pressure drop accounts for approximately 80% of the total pressure drop, a good guess is the hydrostatic pressure caused by a column of the corresponding drilling fluid being used.

4. Using the surface temperature and geothermal gradient, estimate the downstream temperature of the first axial increment, \( T_1 \).

5. Similarly, using the casing choke pressure and the guessed total pressure drop from step 3, estimate the downstream pressure of the first axial increment, \( P_1 \).

6. Using the surface pressure and temperature and the downstream pressure and temperature previously estimated in steps 4 and 5, calculate the average pressure and temperature corresponding to the axial increment.

7. Estimate surface liquid and gas velocities and fluid properties at average conditions.
8. Program the flow pattern prediction models given in section 3.3 and with the superficial velocities estimated in step 7, identify the flow pattern at the in-situ flow conditions.

9. After identifying the existing flow pattern, use the corresponding flow behavior prediction model (Section 3.4) to calculate liquid holdup, mixture density, mixture viscosity, and friction factor. If slug flow is the existing flow pattern, the hydrodynamic parameters must be calculated as well.

10. Calculate the gravity, friction, and acceleration pressure gradients, and then the total pressure gradient for the axial increment selected in step 2.

11. Compare the total pressure gradient calculated in step 10 against that guessed in step 3. If the difference between them is less than a tolerance (0.01 psi) continue with the next step. Otherwise, substitute the total pressure gradient guessed in step 3 for that calculated in step 10 and repeat steps 3 through 11 until convergence. When that happens, the cell downstream pressure $P_2$ will be the actual wellbore pressure occurring at the end of the first axial increment for the existing flow conditions.

12. Increase the depth by one axial increment and compare the current depth to the total depth of the first section with constant cross-section area ($D_T$). If the current depth is less than $D_T$, print depth, wellbore pressure, and any two-phase flow parameter and then go on to the next step. On the other hand, if the current depth is greater than $D_T$ adjust the axial increment and repeat steps 3 through 12.

13. Compare the current depth against $D_T$. If the current depth is not equal to $D_T$, repeat steps 3 through 13. If they are equal, continue the process.

14. When the geometry counter is greater than zero ($GC>0$) and the geometry counter is less than the annular geometries ($GC<AG$), the computations are adjusted to the corresponding cross-sectional area change and repeated as many times as different cross-section areas happen within the annulus.

When any of these conditions is not met, the geometry counter and the axial increment are returned to their initial value or new values are assigned, if desired for the drillstring calculations.

15. Using the bottom hole pressure calculated, calculate pressure drop through the bit nozzles and the nozzle upstream pressure.

16. Considering drillstring flow pattern prediction and flow behavior models, nozzle upstream pressure and temperature, and downward pipe flow instead of upward flow in an annulus (Figure 3.7), the same flow diagram can be used for drillstring computations.

This algorithm implemented in a FORTRAN 90 computer program, allows calculating the wellbore pressure and flow parameters at any position along the flow path in few seconds. Afterward, the data generated is brought to an Excel work sheet to manipulate it as we require.
3.7 Steady State Model Validation

The steady state model was validated with two different sets of data. First, the predicted wellbore pressures were compared against field data measured while drilling two Mexican wells using nitrified mud, and second with full-scale experimental data obtained from the literature.

3.7.1 Field Data Validation

To validate the computer program, two sets of measured field data were obtained from two Mexican wells: Agave 301 and Muspac 53. Both of these wells were drilled using rotary jointed-pipe drilling with drillstring injection.

3.7.1.1 Well Agave 301. Figure 3.8 describes the well geometry and the computer program input parameters of the well Agave 301.

The well Agave 301 was drilled from 3895 m (12779 ft) to 3984 m (13071 ft) with the simultaneous injection of a constant nitrogen rate of 10 m³/min (353 scf/min) and a constant mud rate of 0.45 m³/min (119 gal/min). During drilling, a pressure/temperature recorder, placed at 1645 m (5397 ft) above the bit, measured the drillstring and annular wellbore pressures and temperatures. Figure 3.9 presents the annular and drillstring pressures recorded while drilling the well Agave 301 from 3895 m (12779 ft) to 3984 m (13071 ft).

Mechanistic model outputs are shown in Figure 3.10. The figure shows a comparison between the measured pressures (black circles) with the predicted wellbore pressures considering that only fully developed Taylor bubble (FDTB) slug flow occurs and that both fully developed and/or developing Taylor bubble (FDTB-DTB) slug flow take place as implemented in the comprehensive, mechanistic steady state model. The predicted annular (black line) and drillstring (gray line) flow patterns are also shown. The absolute value percentage error, given by Equation (3.67), is used for the evaluation of the model. Table 3.1 shows the error between the predictions and field measurements.

\[
\epsilon_a(\%) = ABS\left(\frac{\text{Calculated} - \text{Measured}}{\text{Measured}}\right)\times 100 \tag{3.67}
\]

The model evaluation results show that when both fully developed and developing Taylor bubble (FDTB-DTB) slug flow conditions are taken into account, the model predictions are very good (absolute percent error is less than 5%). On the other hand, when only the simplified fully developed Taylor bubble (FDTB) slug flow model is used, the absolute percent error is near 15%. This causes a difference in bottomhole pressure predictions of 238 psi. Consequently, if near-balanced drilling is the objective or slightly underbalanced conditions are required, lack of accuracy in the wellbore pressure predictions jeopardizes the success of UBD operations.

The flow patterns predicted are schematically shown in the vertical lines at the very right of Figure 3.10. In the annulus, churn flow (CH) occurred at the surface, slug flow (SL) extends to 730 m (2395 ft) from the surface, bubble flow (B) occurred in almost 80% of the annulus, and dispersed bubble flow (DB) occurred at the bottom between the casing and drill collars annulus. On the other hand, only dispersed bubble flow was predicted to occur in the drillstring. In the
slug flow two conditions may occur. First, after the transition from bubble to slug flow takes place, developing Taylor bubble slug flow can occur which consist only of a cap bubble. Then, after more gas volume exists due to the gas expansion, fully developed Taylor bubble slug flow may occur where the bubble cap length is negligible as compared to the total Taylor bubble length.

**Figure 3.8 Agave 301 well geometry and computer input data.**

**Drillstring**

<table>
<thead>
<tr>
<th>Depth</th>
<th>Inner diameter</th>
<th>Outer diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-2259 m (0-7411 ft)</td>
<td>66.1 mm (2.602 in)</td>
<td>88.9 mm (3.5 in)</td>
</tr>
<tr>
<td>2259-3803 m (7411-12477 ft)</td>
<td>54.6 mm (2.151 in)</td>
<td>73.0 mm (2.875 in)</td>
</tr>
<tr>
<td>3803-3904 m (12477-12808 ft)</td>
<td>31.8 mm (1.250 in)</td>
<td>79.4 mm (3.125 in)</td>
</tr>
</tbody>
</table>

**Annulus**

<table>
<thead>
<tr>
<th>Depth</th>
<th>Inner diameter</th>
<th>Outer diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-2578 m (0-8458 ft)</td>
<td>168.3 mm (6.625 in)</td>
<td>193.7 mm (7.625 in)</td>
</tr>
<tr>
<td>2578-3895 m (8458-12779 ft)</td>
<td>108.6 mm (4.276 in)</td>
<td>127 mm (5.0 in)</td>
</tr>
<tr>
<td>3895-3904 m (12779-12808 ft)</td>
<td>104.7 mm (4.125 in)</td>
<td></td>
</tr>
</tbody>
</table>

Pressure recorder depth (PR)=2259 m (7411 ft)

**Computer program input data**

- Simulation depths: 3904 m (12808 ft)
- Surface temperature: 294 °K (530 °R)
- Temperature gradient: 1.745 °K/100 m (0.954 °R/100 ft)
- Back pressure: 0.069 MPa (10 psi)
- Mud density: 949 kg/m³ (7.91 lbm/gal)
- Mud viscosity: 10 MPa.s (10 cp)

**3.7.1.2 Well Muspac 53.** The second field data set used to validate the computer simulator program is from the Mexican well Muspac 53. This well was drilled from 2597 m (8520 ft) to 2686 m (8812 ft) with the simultaneous injection of nitrogen and drilling fluid. During drilling, a pressure/temperature recorder, whose installation was authorized to improve the UBD process, was placed at 5 m (16.4 ft) above the bit. It measured the annular wellbore pressure and temperature. Figure 3.11 presents the annular wellbore pressure and the nitrogen and mud flow rates measured during the first 24 hours. In chapter five, this field example will be described in detail. Only the bottomhole pressure measurements are used here to validate the steady state part of the model.

Figure 3.12 describes the well geometry and the computer program input parameters of the well Muspac 53, and Figures 3.13 and 3.14 show the pressure traverse curves and the flow patterns computed by the computer simulator program versus the actual field data at two different depths at which pseudosteady state conditions were observed. On the other hand, Table 3.2 gives the nitrogen and mud injection flow rates at the two different depths at which pseudosteady state conditions were observed as indicated in Figure 3.11. Also, it gives the bottomhole pressure
measurements and those predicted by the comprehensive, mechanistic steady state model as well as the absolute percent error between measurements and predictions.

Figure 3.9  Pressure recorded while drilling well Agave 301.

Figure 3.10 Field measurements versus mechanistic model outputs.
As one can see in table 3.2 and figures 3.13 and 3.14, the proposed model predicts the bottomhole pressure very well (absolute error equal or less than 2.0%). On the other hand, the injection pressure predictions were less accurate (absolute error less than 10%). This lack of accuracy may be due to the fact that the injection pressure measurements, which were recorded manually from a hard-used manometer installed in the standpipe, are much less accurate than the bottomhole pressure ones. Additionally, as mentioned in section 3.3.2, different from the bottomhole pressure predictions, which relies on mechanistic models developed from extensive research in upward two-phase flow in pipes and annulus, the injection pressure predictions are based on mechanistic models developed from few investigations of gas liquid mixtures in downward flow in pipes, which have mainly been carried out nearly atmospheric conditions.

### 3.7.2 Full-scale Experimental Data Validation

The computer simulator program was further validated with data obtained by Lopes from two experiments performed in a full-scale well located at Louisiana State University. This is a vertical well with 1793 m (5884 ft) of depth and 244 mm (9 5/8 in) casing of different inner diameters. Its completion includes a 32 mm (1 ¼ in) gas injection line that runs inside an 89 mm (3 ½ in) drilling fluid injection line. A 60 mm (2 3/8 in) perforated tubing runs outside the
drilling fluid injection line. Figure 3.15 shows the LSU well No. 2 and Table 3.3 describes the LSU No. 2 well geometry.

**Drillstring**

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Inner Diameter (mm)</th>
<th>Outer Diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-2343</td>
<td>66.1 (2.602)</td>
<td>88.9 (3.5)</td>
</tr>
<tr>
<td>2343-2555</td>
<td>52.4 (2.063)</td>
<td>88.9 (3.5)</td>
</tr>
<tr>
<td>2555-2597</td>
<td>55.6 (2.271)</td>
<td>120.7 (4.75)</td>
</tr>
</tbody>
</table>

**Annulus**

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Inner Diameter (mm)</th>
<th>Outer Diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-2597</td>
<td>152.5 (6.004)</td>
<td>177.8 (7.0)</td>
</tr>
<tr>
<td>2597-2614</td>
<td>149.2 (5.875)</td>
<td></td>
</tr>
</tbody>
</table>

Pressure recorder depth (PR)=2600 m (8530 ft) and 2609 m (8560 ft)

**Computer program input data**

- Simulation depths: 2605 m (8547 ft) and 2614 m (8576 ft)
- Surface temperature: 301.15 °K (542.4 °R)
- Temperature gradient: 2.83 °K/100 m (1.56 °R/100 ft)
- Back pressure: 0.310 MPa (45 psi) and 0.345 MPa (50 psi)
- Mud density: 940 kg/m³ (7.84 lbm/gal)
- Mud viscosity: 10 MPa.s (10 cp)

Figure 3.12 Muspac 53’s well geometry and computer input data.

![Drillstring Diagram]

---

**Wellbore pressure (psi)**

![Wellbore Pressure Graph]

Figure 3.13 Model’s pressure predictions vs. actual measured data at 2605 m.
Figure 3.14- Model’s pressure predictions vs. actual measured data at 2614 m.

Table 3.2 Model validation results using field data of well Muspac 53.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Depth 2605 m</th>
<th>Depth 2614 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mud flow rate (gpm)</td>
<td>133</td>
<td>135</td>
</tr>
<tr>
<td>Nitrogen flow rate (scfpm)</td>
<td>530</td>
<td>706</td>
</tr>
<tr>
<td>Wellhead pressure (psi)</td>
<td>45</td>
<td>50</td>
</tr>
<tr>
<td><strong>Bottomhole pressure measured (psi)</strong></td>
<td><strong>3376</strong></td>
<td><strong>3221</strong></td>
</tr>
<tr>
<td>Injection pressure measured (psi)</td>
<td><strong>834</strong></td>
<td><strong>916</strong></td>
</tr>
<tr>
<td>Bottomhole pressure predicted (psi)</td>
<td>3306</td>
<td>3281</td>
</tr>
<tr>
<td>Injection pressure predicted (psi)</td>
<td>903</td>
<td>1003</td>
</tr>
<tr>
<td>BHP absolute percent error (%)</td>
<td>2.0</td>
<td>1.8</td>
</tr>
<tr>
<td>Injection pressure absolute percent error (%)</td>
<td>8.3</td>
<td>9.5</td>
</tr>
</tbody>
</table>

The experiments consisted of injecting nitrogen through the gas injection line and mud through the drilling fluid injection line while measuring annular wellbore pressure with pressure recorders lowered through the perforated tubing, until steady state conditions were reached. During the first experiment, nitrogen was injected at 32 m³/min (1120 scf/min), mud of 1.12 specific gravity with a plastic viscosity of 6 MPa·sec (6 cp) was injected at 0.58 m³/min (152 gpm), annular wellbore pressure was measured at 1768 m (5800 ft) and 1186 m (3890 ft), the choke pressure was maintained at 0.972 MPa (141 psi), and the surface and bottomhole temperature were 297 and 318 °K (530 and 572 °R), respectively. During the second test, nitrogen was injected at 26 m³/min (923 scf/min), mud of 1.12 specific gravity with a plastic viscosity of 24 MPa·sec (24 cp) was injected at 0.53 m³/min (140 gpm), annular wellbore pressure was measured at 1768 m (5800 ft), the choke pressure was maintained at 1.586 MPa.
(230 psi), and in this case, the surface and bottomhole temperature were 2850 and 316 °K (513 and 569 °R), respectively.

Figure 3.15 LSU No. 2 Well Geometry

Table 3.3 LSU No. 2 Well Tubulars.

<table>
<thead>
<tr>
<th>OD (mm)</th>
<th>ID (mm)</th>
<th>Depth (m)</th>
<th>Weight (kg/m)</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>244.5</td>
<td>216.8</td>
<td>0-966</td>
<td>79.7</td>
<td></td>
</tr>
<tr>
<td>244.5</td>
<td>220.5</td>
<td>966-1191</td>
<td>70.0</td>
<td></td>
</tr>
<tr>
<td>244.5</td>
<td>222.4</td>
<td>1191-1693</td>
<td>64.8</td>
<td></td>
</tr>
<tr>
<td>244.5</td>
<td>216.8</td>
<td>1693-1791</td>
<td>79.7</td>
<td></td>
</tr>
<tr>
<td>244.5</td>
<td>220.5</td>
<td>1791-1793</td>
<td>70.0</td>
<td></td>
</tr>
<tr>
<td>88.9</td>
<td>76</td>
<td>0-1775</td>
<td>13.9</td>
<td>J55, EUE</td>
</tr>
<tr>
<td>60.3</td>
<td>50.7</td>
<td>0-1775</td>
<td>7.0</td>
<td>94x0.5holes/jt</td>
</tr>
<tr>
<td>42.2</td>
<td>32.5</td>
<td>0-1775</td>
<td>4.5</td>
<td></td>
</tr>
</tbody>
</table>

Figures 3.16 and 3.17 show the comparison between the measured pressures (black circles) with the calculated wellbore pressures, the annular and drillstring flow patterns predicted, and the absolute percent error between the predictions and measurements for each experiment. Again, the model performance is very satisfactory (absolute percent error less than 3.5%). In the two experiments, slug and bubble flow patterns were predicted in both annulus and drillstring. Only single-phase flow was presented in the injection lines, so measurements were not available to validate the drillstring models.
Figure 3.16 Model validation with first full-scale experimental data.

Figure 3.17 Model validation with second full-scale experimental data.
3.8 Model Comparison

Using the actual field data measured while drilling the Mexican wells Agave 301 and Muspac 53 shown above, the performance of the proposed comprehensive, mechanistic steady state model was compared against the performance of four different steady state commercial UBD programs: The Hydraulic UnderBalanced Simulator HUBS of the company Signa Engineering Corp., the MUDLITE Version 2 and MUDLITE Version 3 Air/Mist/Foam Hydraulic Model of the company Maurer Technology Inc., and the UBD software Neotec WELLFLO 7 of the company Neotechnology Consultants LTD.

The Mexican oil company PEMEX, owner of a license of MUDLITE V-2 and MUDLITE V-3 allowed the used of these two commercial UBD programs during the time this work lasted. First, MUDLITE V-2, and then an improved version of it, MUDLITE V-3, which was available until the end of 2001. The commercial UBD program HUBS was only available for four hours through the demo application that Signa Engineering Corp. authorize through the internet. Finally, the UBD software Neotec, used while drilling Muspac 53 well, was only available through the service company that supplies the UBD equipment to PEMEX.

3.8.1 Well Agave 301
In August 2000, the field data of Agave 301 well was first available, at that time only MUDLITE V-2 and HUBS could be used for the comparison. Figure 3.18 shows the wellbore pressures calculated by MUDLITE V-2 and HUBS and those computed by the proposed LSU model using inputs data from well Agave 301, in which the pressure recorder allowed measurements of annular and drillstring pressures. As one can see, the proposed model and program HUBS predict the annular wellbore pressure very well, but only the proposed model gives a very good approximation of both the drillstring wellbore pressure and injection pressure. On the other hand, MUDLITE V-2 gives a good approximation of the drillstring wellbore pressure, but its pressure predictions for the injection and annular wellbore pressures are very poor. Table 3.4 presents the absolute percent error as a comparison parameter for the measurements gather from well Agave 301.

3.8.2 Well Muspac 53
Using the actual field data recently gathered while drilling the well Muspac 53 (May 2002), whose well geometry is shown in figure 3.12, the performance of the proposed comprehensive, mechanistic steady state model was further compared against MUDLITE V-2, the improved version MUDLITE V-3 and Neotec. The basic difference between MUDLITE V-2 and MUDLITE V-3 is that the improved version MUDLITE V-3 allows predicting wellbore pressure and two-phase flow parameters with the Hasan and Kabir\textsuperscript{31,32} mechanistic model.

Figure 3.19 shows the wellbore pressures calculated by programs Neotec, MUDLITE V-2, the improved version MUDLITE V-3, and those computed by the proposed LSU model using input data from well Muspac 53 at the depth of 2614m (8576 ft). In this well, the pressure recorder, installed in the drillstring above the bit, only measured the bottomhole annular wellbore pressure. Table 3.5 presents the absolute percent error as a comparison parameter for the measurements gathered from well Muspac 53.
Figure 3.18 Wellbore pressure comparison using Agave 301’s field data.

Table 3.4 Model comparison against UBD programs HUBS and MUDLITE V-2.

<table>
<thead>
<tr>
<th></th>
<th>Absolute percent error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field data</td>
<td>LSU</td>
</tr>
<tr>
<td>Annular measurement</td>
<td>1.9</td>
</tr>
<tr>
<td>Drillstring measurement</td>
<td>1.4</td>
</tr>
<tr>
<td>Injection pressure</td>
<td>4.6</td>
</tr>
</tbody>
</table>

The model comparison against four different UBD programs clearly shows that the comprehensive, mechanistic steady state model, proposed in this work, performs much better than any of these UBD programs. Despite a general lack of technical details of these commercial UBD programs, it is possible to speculate that for handling two-phase flow HUBS and MUDLITE V-2 use the Beggs and Brill empirical correlation, the improved version MUDLITE V-3 used either the Beggs and Brill empirical correlation or the simplistic mechanistic model of Hasan and Kabir, and program Neotec WELLFLO 7 is based on the multiphase flow program OLGAS. As mentioned in the literature, these correlations and simple models frequently fail to accurately predict the wellbore pressure. Consequently, one can conclude that the very good performance of the proposed model is due to the fact that it is composed of a more complete set of state-of-the-art mechanistic steady state models.

In summary, a comprehensive mechanistic model, which allows more precise predictions of wellbore pressure, with an average absolute error less than 2.5%, and two-phase flow parameters for UBD operation, is proposed. The model incorporates the effects of fluid properties and pipe sizes and thus is largely free of the limitations of empirically based correlations.
Although the proposed model gives highly accurate wellbore pressure predictions, additional work is necessary to improve the injection pressure calculations that currently are in error on the order of 7.5%. More complete mechanistic models such as those developed by Caetano\textsuperscript{33} for upward flow, which takes into account the actual differences in the drift-flux between the liquid slug and the Taylor bubble in the slug flow model, should ideally replace the simplistic mechanistic model of Hasan and Kabir\textsuperscript{31,32}, used in the drillstring predictions of the proposed model. However, additional investigation of gas-liquid mixtures in downward flow in pipes are needed for this improvement because currently this issue has not received enough attention.

Although mechanistic models seem to be superior to empirical correlations, the value of flow variables such as velocity profile coefficient $C_0$, swarm effect exponent $n$, transition gas void fraction, and liquid holdup in the liquid slug must be adjusted for each specific condition being predicted. Therefore, additional work should also be performed to develop a unique method for predicting these parameters explicitly.
CHAPTER 4

MECHANISTIC TIME DEPENDENT MODEL

The unsteady state or transient conditions occurring during an UBD pipe connection are extremely complex to estimate. Yet, there are a few time-dependent models and dynamic computer codes that claim to be capable of predicting such complex conditions. However, despite the apparent gain in acceptance of those dynamic computer codes, the literature contains little information about the evaluation of these programs, and states that dynamic models that handle transient conditions are already available but not yet validated. Besides, those computer code developers that have carried out validations and reported their results, only used experimental data gathered from a pressure recorder placed at some depth above the bottomhole in spite of having the experimental data recorded from a pressure gauge placed at the bottomhole. Moreover, the validation results show that the transient model predictions for the case of a pipe connection when both gas and liquid are injected through the drillstring are not accurate. Consequently, dynamic program developers conclude that most of the current transient two phase flow models available in the industry would hardly reproduce the oscillatory behavior observed during an UBD pipe connection, and that the definition of proper operational procedures for bottomhole pressure maintenance while connecting a new pipe still requires additional effort.

As stated in Chapter 1, instead of trying to rigorously predict the bottomhole pressure fluctuations occurring during an UBD pipe connection, a procedure for reducing them using the reservoir energy through the gas and drilling fluid injection flow rates and casing choke pressure manipulation is alternatively proposed. Therefore, a less complex time dependent computer program is developed to achieve this goal. Thus, in this section, a mechanistic time dependent model is described in detail. This mechanistic time dependent model consists of numerically implementing the comprehensive, mechanistic steady state model, described in Chapter 3, into a one-dimensional drift-flux formulation of the two-phase flow conservation equations, which coupled with a reservoir inflow performance equation, allows estimating reservoir influxes, wellbore pressure and two-phase flow parameters as a function of the bottom hole pressure variation caused by changes in surface gas and liquid injection flow rates and choke pressure.

The implementation of the comprehensive, mechanistic steady state model has three different purposes: first, to furnish the initial and boundary conditions required to solve the partial differential equations of the drift-flux formulation; second, to determine the actual flow pattern occurring at the existing in-situ flow conditions at a point in the wellbore; and third, to calculate the pressure drop components to be compared to those calculated with the mixture momentum equation. This new implementation, recently proposed by Lage, substitutes the simplistic assumptions made in previous time dependent models developed at LSU, which was first proposed by Nickens. This simplistic assumption basically consists of calculating the liquid holdup with the equation of mass conservation of liquid and comparing it to ranges of liquid holdup values, obtained from the experimental work carried out by Caetano to determine the corresponding in-situ flow pattern.
As in the steady state model, a discretization of the wellbore into finite cells or axial increments was also used to solve the flow equations for pressure and two-phase flow parameters as functions of spatial location along the flow path. Also, similar to the steady state model, the time dependent model can consider annular sections of different cross-sectional areas as desired. This time dependent model is validated against both field and experimental data\textsuperscript{37}. Moreover, the performance of the time dependent model coupled with the reservoir inflow performance equation is evaluated by comparison to data reported in the literature\textsuperscript{38}.

4.1 Model Assumptions and Considerations

Due to the nature of the UBD process with conventional rigs (jointed pipe drilling), drillstring injection was the first and currently is the most common injection technique used in UBD operations. Therefore, the model will be limited to the case of drillstring injection. Additionally, based on the fact that the underbalanced drilling process starts after having reduced the bottomhole pressure to the target pressure, the model does not take into account the unloading process. Instead, it begins considering that the initial conditions are those reached after the simultaneous injection of liquid and gas has already achieved the bottomhole steady state conditions desired.

Gases such as nitrogen, air, and natural gas may be used as a means to lower the average density of fluids while drilling underbalanced\textsuperscript{7,13}. Nitrogen currently is the most common gas utilized both in drilling underbalanced and in making full-scale experiments related to this technology\textsuperscript{34}. Its negligible solubility in liquids, availability, and not flammable make nitrogen the most suitable gas to be used in UBD operations. For those reasons, the model will only consider the use of nitrogen as a means to lower the density of drilling fluids.

Strictly speaking, the flow in a conduit varies both along the direction of flow and over the cross section area of the pipe. However, due to the complexity of the equations that result from handling two-dimensional flow, in the development of the present model a one-dimensional approximation is considered.

Underbalanced drilling wells are usually planned in reservoirs, which have been produced for a considerable time through previously drilled wells. Therefore, surface temperature and geothermal gradient are usually known. Thus, the assumption of a known temperature distribution will be used. Additionally, for the time the reservoir has been exploited, it may be considered as saturated and, hence, the Vogel method\textsuperscript{52} can be used to predict the inflow performance of the well during drilling.

The basis for virtually all computations involving fluid flow in pipes is the conservation of mass, momentum, and energy\textsuperscript{51}. Considering that the temperature is known, only the principles of conservation of mass and momentum will be applied to the gas and liquid phases. Since the bottomhole pressure fluctuations are mainly caused by time dependent conditions, both position and time will be considered in the application of these principles to the UBD circulation system. This allows determining the flow variables as a function of time and position along the flow path.
As stated in Section 3.1, weighting factors are used to calculate a unique gas density, gas viscosity, liquid density, and liquid viscosity. Thus, the time dependent model can consider a two-phase flow system in which only a mixture of liquid and gas flows.

In this time dependent code only vertical annular geometries with different cross-section flow areas are considered, and mass transfer between the gas and liquid phases and compressibility of the liquid phase are considered negligible.

4.2 Mechanistic Time Dependent Model Formulation

4.2.1 Governing Equations

Application of mass, momentum, and energy conservation permits the calculation of pressure, temperature, and flow parameters as a function of position and time. Considering that in UBD operations the surface temperature and the geothermal gradient are usually known, it is not necessary to solve the equation of conservation of energy. Thus, in the case of two-phase flow, partial differential equations for conservation of mass and momentum should be written for each phase, and constitutive relationships for the fluid properties to specify the interaction between the two phases are needed. This would lead to a complex model with four conservation equations (two for each phase) and a number of problematic interfacial relationships. Since the specification of the interfacial conditions between liquid and gas remains a significant problem in a two-fluid model, a major simplification in the two-fluid model can be made. Instead of writing two momentum conservation equations (one for each phase), a single momentum equation can be written for the mixture as a whole resulting in a drift-flux model. Based on the fact that the motion of two phases in vertical conduits is strongly coupled, the idea of the drift-flux model is to concentrate on the mixture as a whole rather than the individual phase. Thus, neglecting mass transfer between phases, the one-dimensional form of the three-equation drift-flux model is given by

Mass conservation of liquid

\[
\frac{\partial (\rho_L H_L)}{\partial t} + \frac{\partial (\rho_L u_L)}{\partial Z} = 0
\] (4.1)

Mass conservation of gas

\[
\frac{\partial [\rho_G (1 - H_L)]}{\partial t} + \frac{\partial [\rho_G u_G (1 - H_L)]}{\partial Z} = 0
\] (4.2)

Considering that the axial direction \(Z\) is positive along the upward direction, the conservation of mixture momentum

\[
\frac{\partial}{\partial t} [\rho_L H_L u_L + \rho_G (1 - H_L) u_G] + \frac{\partial}{\partial Z} \left[ \rho_L H_L u_L^2 + \rho_G (1 - H_L) u_G^2 \right] = -\frac{\partial \rho}{\partial Z} \frac{2 f_f \rho u_m^2}{D_h} - \rho u g
\] (4.3)
Although Equations (4.1), (4.2), and (4.3) are derived in the literature⁶⁸,⁷⁶,⁸¹,⁸⁴, for easy reference Appendix D shows a simple and practical derivation of them.

The solution to the one-dimensional form of the three-equation drift flux model gives information as a function of position and time of pressure \( p \), liquid holdup \( H_L \), in-situ gas velocity \( u_G \), in-situ liquid velocity \( u_L \), gas density \( \rho_G \), and liquid density \( \rho_L \), which are the six unknowns of the model formulation. Assuming that the liquid compressibility is negligible (i.e. liquid density constant), only two additional equations would be required to close the system; equal number of unknowns equal number of equations. Since these unknowns depend on the existing flow pattern, a procedure to determine the in-situ flow pattern is also required. Moreover, it is necessary to provide an appropriate model to calculate the frictional pressure losses. Therefore, following the new approach proposed by Lage³⁵, the closure of the system will be fulfilled by the comprehensive, mechanistic steady state model described in Chapter 3. This mechanistic model, which is composed of a procedure for flow pattern prediction and a set of independent models for calculating liquid holdup, in-situ velocities and pressure drops for dispersed bubble, bubble, slug, and annular flow configurations, is responsible for providing the additional information required to solve the one-dimensional form of the three-equation drift-flux model.

4.2.2 Well Deliverability Model

The UBD technique is most frequently used when drilling wells in reservoirs that have been producing for several years. Therefore, in most cases, the reservoir in which a well is drilled underbalanced is at saturated conditions and very well characterized. Consequently, the Vogel equation⁵² is implemented to predict the influxes of formation fluids while drilling.

\[
\frac{q_o}{q_{o,max}} = 1 - 0.2 \frac{p_{bh}}{p_R} - 0.8 \left( \frac{p_{bh}}{p_R} \right)^2
\]  

(4.4)

In this equation, \( p_{bh} \) is the bottomhole pressure, \( p_R \) is the average reservoir pressure, \( q_{o,max} \) is the oil flow rate that would result from a zero bottomhole pressure, and \( q_o \) is the oil flow rate that would result for the value of bottomhole pressure being considered. Using this equation to approximate the well deliverability during drilling, only one stabilized flow test from a correlation well is needed. However, since Vogel’s equation predicts the inflow performance relationship considering the whole pay zone open to flow. The estimated reservoir thickness is divided into axial segments of thickness \( h \) and then the actual oil flow rate \( q_{o,i} \) is estimated as a function of the sum of the axial segments drilled at the simulation depth. Expressed mathematically

\[
q_{o,i} = \frac{q_{o,max}}{h} \sum h \left[ 1 - 0.2 \frac{p_{bh}}{p_R} - 0.8 \left( \frac{p_{bh}}{p_R} \right)^2 \right]
\]  

(4.5)
4.2.3 Initial and Boundary Conditions

The solution of a time-dependent partial differential equation (PDE) must be computed by marching outward on an open domain, from initial conditions while satisfying a set of boundary conditions. Figure 4.1 illustrates the domain and marching direction of the one-dimensional form of the three-equation drift-flux model.

Considering that \( U(Z,t) \) is a dependent variable of position and time, the initial conditions \( U(Z,0) \) for the model developed in this study are selected at the time the injected mixture of gas and liquid has reached steady-state conditions prior to drilling underbalanced. That is, the unloading process, from the time at which the nitrogen injection starts in the drillstring at the surface to the time it reaches the surface in the annulus and steady-state conditions are achieved, is not considered. Therefore, the steady-state mechanistic model, described in Chapter 3, supplies these initial conditions along the flow path at time equal zero. On the other hand, the surface boundary conditions \( U(Z = 0,t) \), which are assumed to be known, are those occurring at the casing choke, and the bottomhole boundary conditions \( U(Z = \text{total depth},t) \) are calculated by an iterative procedure, described later, which makes the known surface casing choke pressure (surface boundary condition) equal to the casing choke pressure calculated by such iterative procedure.

4.3 Numerical Solution

The one-dimensional form of the three-equation drift-flux model coupled with the mechanistic steady-state and well deliverability models represent a system of non-linear partial differential
equations. Upon solution of this system, any dependent variable is known as a function of time and position along the flow path. Due to the complexity of the system there is no analytical solution available and thus a numerical solution remains the preferable solution approach. Furthermore, the finite-difference numerical method is commonly used for solving systems of equations that describe fluid dynamics.

### 4.3.1 Finite Difference Approximation

The idea of the finite-difference approach is to achieve an algebraic representation of the PDE’s by substituting finite differences obtained by the use of the Taylor-series expansion approach for the derivatives in the governing equations. There are two finite difference methods to solve PDE’s, the explicit finite difference method and the implicit one. Although both methods have been used by the petroleum industry to predict the flow dynamics occurring in multiphase wellbore systems, the explicit method is the most popular among the time dependent simulators developed for drilling because its implementation is less complex than the implicit one and its results are good enough for the purposes for which it is intended. Therefore, the explicit finite difference method of Wendroff is implemented in this work.

In the explicit finite difference method, the basic principle is that after a PDE has been replaced by its finite-difference approximation, we can solve for the solution explicitly at one value of time in terms of the solution at earlier values of time. That is, the solution must be computed by marching outward from the initial data while satisfying the boundary conditions. In this way, an initial-boundary value problem can be solved by consecutively finding the solution at larger and larger values of time. Consequently, the marching algorithm process employed for solving the mechanistic model in Chapter 3 is adapted to also consider time increments to march in time. Therefore, a finite-difference mesh or grid is formed allowing any flow variable to be known at every position along the wellbore flow path and at the current time step.

Figure 4.2 illustrates a grid or cell formed by marching one step in position and time. Nodes A and D denote the flow properties at the previous time step \((t=0)\) at the lower and upper boundaries of an axial increment, while nodes B and C correspond to flow properties at the present time step \((t=1)\) for the same axial positions of the previous time step. Nodes E and F stand for arithmetic averaging between the previous and present time steps, while nodes G and H represent arithmetic averaging between the previous and present positions. As mentioned above, the flow properties at the nodes of the previous time step (A and D) are predicted by the mechanistic steady state model and correspond to the initial conditions needed to estimate the corresponding flow properties at the nodes of the present time step (B and C), which are numerically calculated by an iterative process in which the finite-difference representation of the PDE’s are solved.

### 4.3.2 Finite Difference Approximation Formulation

The theory of a finite-difference representation for a derivative can be introduced by recalling the definition of the derivative for the function

\[
\frac{\partial U(Z,t)}{\partial Z} = \lim_{\Delta Z \to 0} \frac{U(Z+\Delta Z,t) - U(Z,t)}{\Delta Z}
\]  

(4.6)
Thus, it is expected that the right hand side of the equation will be a reasonable approximation to
the derivative for a sufficiently small but finite $\Delta Z$.

Figure 4.2 Grid or cell formed by marching one step in position and time.

![Grid diagram](image)

Applying the Taylor series expansion\(^{91}\) in two variables ($Z$ and $t$) and truncating these series after
two terms, one can extend the finite-difference approximation to partial derivatives. Then, at the
previous time step ($t=0$)

$$U_D(Z + \Delta Z,t) = U_G(Z + \Delta Z/2,t) + U_G'(Z + \Delta Z/2,t)[Z + \Delta Z - (Z + \Delta Z/2)] \quad (4.7)$$

$$U_A(Z,t) = U_G(Z + \Delta Z/2,t) + U_G'(Z + \Delta Z/2,t)[Z - (Z + \Delta Z/2)] \quad (4.8)$$

Subtracting Equation (4.8) from Equation (4.7) and using simplified notation (Figure 4.2), the
partial derivative of $U$ with respect to $Z$ evaluated at node G may be approximated by

$$\left( \frac{\partial U}{\partial Z} \right)_G = \frac{U_D - U_A}{\Delta Z} \quad (4.9)$$

Similarly at the present time step ($t=1$), the partial derivative of $U$ with respect to $Z$ evaluated at
node H may be approximated by
\[
\frac{\partial U}{\partial Z}_H = \frac{U_C - U_B}{\Delta Z}
\] (4.10)

Proceeding in similar manner but in the time direction at the axial increment lower boundary \((Z=\text{total depth})\), the partial derivative of \(U\) with respect to \(t\) evaluated at node \(F\) can be expressed by

\[
\left(\frac{\partial U}{\partial t}\right)_F = \frac{U_B - U_A}{\Delta t}
\] (4.11)

And at the axial increment upper boundary \((Z=\text{total depth}-\Delta Z)\), the partial derivative of \(U\) with respect to \(t\) evaluated at node \(E\) is

\[
\left(\frac{\partial U}{\partial t}\right)_E = \frac{U_C - U_D}{\Delta t}
\] (4.12)

Applying the Wendroff's explicit approach\(^9\) the space derivative may be expressed by

\[
\frac{\partial U}{\partial Z} = \frac{1}{2}\left\{\left(\frac{\partial U}{\partial Z}\right)_G + \left(\frac{\partial U}{\partial Z}\right)_H\right\} = \frac{U_C + U_B - U_A - U_B}{2\Delta Z}
\] (4.13)

and the time derivative by

\[
\frac{\partial U}{\partial t} = \frac{1}{2}\left\{\left(\frac{\partial U}{\partial t}\right)_E + \left(\frac{\partial U}{\partial t}\right)_F\right\} = \frac{U_B + U_C - U_A - U_D}{2\Delta t}
\] (4.14)

Bearing in mind that \(U(Z,t)\) is a dependent variable of position and time, upon substituting these approximations into the conservation equations, the finite difference formulation of the equation of conservation of mass of liquid can be expressed by

\[
\frac{(\rho_l H_l)_B + (\rho_l H_l)_C - (\rho_l H_l)_A - (\rho_l H_l)_D}{\Delta t} + 
\frac{(\rho_l H_l u_l)_C + (\rho_l H_l u_l)_D - (\rho_l H_l u_l)_A - (\rho_l H_l u_l)_B}{\Delta Z} = 0
\] (4.15)

the finite difference formulation of the equation of conservation of mass of gas by

\[
\frac{[\rho_g (1-H_l)]_B + [\rho_g (1-H_l)]_C - [\rho_g (1-H_l)]_A - [\rho_g (1-H_l)]_D}{\Delta t} + 
\frac{[\rho_g (1-H_l) u_g]_C + [\rho_g (1-H_l) u_g]_D - [\rho_g (1-H_l) u_g]_A - [\rho_g (1-H_l) u_g]_B}{\Delta Z} = 0
\] (4.16).

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and the finite difference formulation for the conservation of mixture momentum equation is

\[
\frac{1}{2\Delta t}\{[\rho_L H_L u_L + \rho_G (1- H_L) u_G]_A + [\rho_L H_L u_L + \rho_G (1- H_L) u_G]_C
\]

\[-[\rho_L H_L u_L + \rho_G (1- H_L) u_G]_A - [\rho_L H_L u_L + \rho_G (1- H_L) u_G]_D \}

\[+ \frac{1}{2\Delta Z}\{[\rho_L H_L u_L^2 + \rho_G (1- H_L) u_G^2]_C + [\rho_L H_L u_L^2 + \rho_G (1- H_L) u_G^2]_D \}
\]

\[-[\rho_L H_L u_L^2 + \rho_G (1- H_L) u_G^2]_A - [\rho_L H_L u_L^2 + \rho_G (1- H_L) u_G^2]_D \}

\[+ \frac{0.25}{\Delta Z}\left( \frac{\Delta p}{\Delta Z} \right)_A + \left( \frac{\Delta p}{\Delta Z} \right)_B + \left( \frac{\Delta p}{\Delta Z} \right)_C + \left( \frac{\Delta p}{\Delta Z} \right)_{p_{fric}}
\]

\[+ \frac{0.25}{\Delta Z}\left( \frac{\Delta p}{\Delta Z} \right)_A + \left( \frac{\Delta p}{\Delta Z} \right)_B + \left( \frac{\Delta p}{\Delta Z} \right)_C + \left( \frac{\Delta p}{\Delta Z} \right)_{p_{jale}} = 0 \tag{4.17} \]

This second order finite difference approximation\(^{85,90}\) of the one-dimensional form of the three-equation drift-flux model is a widely used scheme for solving fluid flow equations because it removes the necessity of computing unknowns at the grid midpoints. The numerical stability requirement for this explicit numerical method is the Courant, Friedrichs, and Lewy (CFL) condition\(^85\), which is

\[
\frac{\Delta t}{\Delta Z} \leq 1 \tag{4.18} \]

The best results for hyperbolic systems using explicit methods are obtained with Courant number near unity.

### 4.4 Time Dependent Computer Program Description

Equations (4.15) to (4.18), the well deliverability Equation (4.5) and the comprehensive, mechanistic steady state model described in Chapter 3 are combined and numerically implemented into a FORTRAN 90 time dependent computer program. Similar to the steady state code, this time dependent computer program also performs an iterative two-phase flow analysis on a discretized wellbore. The time dependent computer program additionally performs another, much more complex, iterative two-phase flow analysis to attain convergence between the casing choke pressure calculated and that previously stated as a surface boundary condition. Furthermore, after accomplishing this second convergence, the time dependent computer program allows the change of flow conditions (new gas and liquid flow rates and choke pressure) to then march in time repeating the previous two iterative two-phase flow analysis at each time step. This change in flow conditions may be caused either by varying manually the previous gas and liquid injection flow rates and choke pressure or by underbalanced conditions, which cause reservoir influxes into the wellbore through the deliverability model. Different from the steady state computer program, which permits computations in both drillstring and annulus, the time dependent program only allows computations of wellbore pressure and two-phase flow
parameters along the annulus. However, it may also have annular wellbore sections of different cross-section areas as desired. On the other hand, in the time dependent computer program, based on the initial conditions (IC) and bottomhole boundary condition (BBC), the computations start at the bottomhole and then performing an iterative two-phase flow analysis at each axial increment proceed up the annulus until convergence is achieved between the calculated casing choke pressure and the known casing choke pressure or surface boundary condition (SBC). Figure 4.3 pictorially shows the calculation path in a discretized wellbore and the two iterative analyses required at each time step.

![Figure 4.3 Calculation path in a discretized wellbore and iterative analyses.](image)

### 4.4.1 Time Dependent Algorithm Steps
Appendix E presents the flowchart used for constructing the time dependent code to compute the wellbore pressure and two-phase flow parameters as a function of the bottomhole pressure variation caused by changes in gas and liquid flow rates (injected and/or produced) and choke pressure. The logical flow of the algorithm is described below.

1. In addition to the data entered in the mechanistic steady state model (MSSM), input the produced fluid properties, average reservoir pressure, oil flow rate corresponding to a stabilized bottomhole flowing pressure, gas oil ratio, reservoir thickness, and reservoir thickness drilled for the simulation depth.
2. Select a time step $\Delta t$, state the total axial increments ($TAI$) into which the well was divided to perform the iterative two-phase flow analysis.

3. Initialize a time loop value $t=0$.

4. Initialize an axial iteration counter $M=0$. This axial iteration counter serves to identify when a time step occurs. A guess of the bottomhole pressure is made every time $M=0$.

5. Initialize an axial decrement loop value $Z=TAI$.

6. At the previous time step ($t=0$), with the initial gas and liquid flow rates and choke pressure, use the MSSM to calculate the initial conditions (pressure and flow properties) at the lower (node A) and upper (node D) boundaries of each axial increment along the discretized wellbore, Figure 4.4. As mentioned in Section 4.2.3, the initial gas and liquid flow rates and choke pressure correspond to the steady state flow conditions desired prior to start drilling underbalanced.

7. When $Z$ is equal to the $TAI$ (lower boundary of the first axial increment) and $M = 0$, guess the bottomhole pressure that corresponds to the present time step ($t=1$) (node B of Figure 4.4). A good guess is the bottomhole pressure of the previous time step (node A).

   When $Z$ is equal to the $TAI$ but $M \neq 0$, convergence was not achieved with the first guessed bottomhole pressure. Therefore, the bottomhole pressure required for the next iteration will be equal to the previously guessed bottomhole pressure plus or minus a pressure increment so that convergence between the known casing choke pressure or surface boundary condition (SBC) and the calculated casing choke pressure can be accomplished. The Newton-Raphson and secant root finding numerical methods were implemented to speed up the convergence process.

8. With the bottomhole pressure defined in step 7 compute the corresponding two-phase flow properties (flow pattern, liquid holdup, in-situ gas and liquid velocities, and gas density) at the present time step (node B of Figure 4.4).

9. Guess a pressure for the upper boundary of the first axial increment at the present time step (node C). As in step 7, a good guess is the pressure of the previous time step (node D).

10. With the pressure defined in step 9, compute the corresponding two-phase flow properties at the present time step (node C).

11. Use the flow properties calculated in step 10 to compute the pressure at node C using the finite difference representation of the time dependent equations given by Equations (4.15) through (4.17) and the frictional and elevation pressure gradients using the same models as in the MSSM.
12. Compare the wellbore pressure calculated in step 11 with that guessed in step 9. If the difference between them is less than a tolerance (1.0 psi), continue with the next step. Otherwise, substitute the wellbore pressure calculated in step 11 for that guessed in step 9 and repeat steps 10 through 12 until the condition stated in step 12 is met. As soon as convergence is obtained at node C, pressure and two-phase properties at the lower and upper boundaries of the first axial increment at the present time step (nodes B and C) are known as a function of the guessed bottomhole pressure defined in step 7.

13. Prepare the computer program to march in position and time. To march upward in position, the pressure and two-phase flow properties corresponding to the upper boundary of the preceding axial increment at the previous and present time steps (nodes D and C of Figures 4.4) become the pressure and two-phase flow properties of the lower boundary of the present axial increment at the previous and present time steps (nodes A and B of Figure 4.5). That is, \( A(TAI-1,0) = D(TAI-1,0) \) and \( B(TAI-1,1) = C(TAI-1,1) \). Figure 4.5 schematically shows the cell formed by marching one step in position.

On the other hand, to march in time, pressure and two-phase flow properties calculated at the present time step \( (t=1) \), nodes B and C of Figure 4.4, become the initial conditions for the next time step \( (t=2) \), nodes A and D of Figure 4.6. Figure 4.6 schematically shows the cell formed by marching one step in time. That is, \( A(TAI,1) = B(TAI,1) \) and \( D(TAI-1,1) = C(TAI-1,1) \). The computer program automatically does these adjustments by equalizing node information as illustrated in the flowchart given in Appendix E.
14. When the axial loop value $Z$ is greater than one, decrease $Z$ by one and repeat steps 6 through 14 until this loop reaches its limit ($Z=1$), which corresponds to the last axial increment. When this happens, the choke pressure computed by the computer program will be known as a function of the guessed bottomhole pressure defined in step 7. However, since $Z \neq TAI$, steps 7 and 8 will not be executed as illustrated in the flowchart given in Appendix E.

15. When $Z$ is equal to one, increase the axial iteration counter $M$.

16. Compare the calculated casing choke pressure with the known casing choke pressure or surface boundary condition (SBC). If the difference between them is less than a tolerance (1.0 psi), continue with the next step. If this condition is not met, reset $Z = TAI$ and repeat steps 6 through 16 until accomplishing convergence. After this convergence is achieved, wellbore pressure, liquid holdup, in-situ gas velocity, in-situ liquid velocity, gas density and any other two-phase flow variable such as superficial velocities, friction factor, mixture density, mixture velocity, and mixture viscosity will be known at any position along the wellbore and at the previous ($t=0$) and present ($t=1$) time steps. This information may be printed if desired.

As illustrated in the flowchart given in Appendix E, when convergence is not achieve at the first iteration, in step 7, the value of $M$ will be different from zero and, therefore, no additional pressure guess is necessary. Instead, the bottomhole pressure required for the next iteration will be equal to the previously guessed bottomhole pressure plus or minus a pressure increment or decrement.
17. Compared the bottomhole pressure, which corresponds to the surface casing choke pressure calculated above, with the average reservoir pressure inputted. If the bottomhole pressure calculated is less than the reservoir pressure, calculate the oil and gas flow rates flowing into the wellbore, print the results, and continue. If the bottomhole pressure calculated is greater than or equal to the average reservoir pressure inputted, print the result and continue.

![Figure 4.6 Cell formed by marching one step in time.](image)

18. Input the new gas and liquid injection flow rates and choke pressure.

When the bottomhole pressure is less than the average reservoir pressure inputted, reservoir fluids enter into the wellbore. Consequently, as mentioned in Section 3.1.2 in chapter 3, weighting factors are used to calculate the mixture fluid properties. Appendix F gives the adjustments and additional equations required to handle reservoir influxes.

19. When the time loop $t$ is less than its limit ($t=t_{\text{desired}}$), increase a time step and repeat steps 4 through 19 until the loop $t$ reaches its limit.

As illustrated in the flowchart given in Appendix E, step 6 is not executed after $t$ is different from zero.

These algorithm steps were implemented into the FORTRAN 90 time dependent computer program, which numerically combines the accurate comprehensive, mechanistic, steady-state model, the conservation equations approximated by finite differences of second order, and a well
deliverability model. Program simulations allows predicting variations in wellbore pressure, gas and liquid in-situ velocities, gas and liquid fractions, mixture densities, reservoir influxes, and other two-phase flow parameters as a function of position and time caused by changes in surface gas and liquid injection flow rates and choke pressures. Therefore, the best combination of these controllable parameters should be able to be determined to maintain the bottomhole pressure at a value at which the reservoir influxes substitute the interrupted surface injection during a pipe connection. This would allow developing a flow control procedure for maintaining underbalanced conditions and avoiding formation damage during drill pipe connections using the reservoir influxes.

4.5 Time Dependent Model Validation

Successful jointed-pipe UBD operations require maintaining control of the annular bottomhole pressure. Therefore, it is a common practice to use a bottomhole pressure recorder while drilling underbalanced so that operational adjustments can be made. As a consequence, there are multiple examples in the open literature that provide actual bottomhole pressure measurements showing how the bottomhole pressure fluctuates during underbalanced drilling pipe connections\(^7\),\(^8\),\(^35\),\(^96\).

Unfortunately, this information typically cannot be used for validating time dependent models mainly because it only shows graphics of bottomhole pressure variations versus time without providing the well geometry and the gas and liquid injection flow rates and choke pressure used during drilling\(^7\),\(^8\) or is for flow patterns not modeled in this study\(^35\). Real bottomhole pressure measurements, in which reservoir fluids were flowing while drilling underbalanced, is very limited\(^12\). Consequently, the validation of the time dependent model was carried out with the field data available from the Mexican well Agave 301, the full-scale experimental data gathered by Lopes\(^37\) in a full-scale well of the Blowout Prevention research Well Facility at Louisiana State University and the one set of complete data from another simulator\(^38\).

4.5.1 Field Data Validation

To evaluate the functionality of the time dependent model for predicting wellbore pressure and two-phase flow parameters as a function of position and time, the program was first evaluated by maintaining steady state conditions during 160 minutes. This simulation was carried out using the field data of the Mexican well Agave 301, described in Section 3.6.1.1, and the simulation results were compared with the wellbore pressure measured while drilling such well at 3904 m (12808 ft). At this depth, pseudosteady state conditions were observed (Figure 3.9). Figure 4.7 shows the simulation results comparing the wellbore pressure measured while drilling such well at 3904 m (12808 ft). At this depth, pseudosteady state conditions were observed (Figure 3.9). Figure 4.7 shows the simulation results comparing the wellbore pressure measured while drilling with that predicted by the time dependent computer program. In addition, it shows the wellbore pressure calculated by the steady state commercial UBD simulators SIGNA and MUDLITE, which can only predict the wellbore pressure at a specific point in time. As one can observe in this figure, due to the fact that gas and drilling fluid injection flow rates and choke pressure were maintained constant during this simulation, the wellbore pressure predicted by the time dependent computer program remains also constant during the period of time considered.

Like in the steady state model validation, the absolute percent error and absolute average percent error were used to evaluate the time dependent code. Figure 4.8 shows that due to the actual bottomhole pressure variation the absolute percent errors are smaller than 8.5 percent, while the
absolute average percent error is 4.2 percent. This indicates a very good agreement between predictions and measurements and corroborates that the time dependent computer program can predict wellbore pressure and two-phase flow parameters as a function of position and time.

![Wellbore pressure measured vs. calculated](image)

Figure 4.7 Time dependent model simulation results, first evaluation.

### 4.5.2 Full-scale Experimental Data Validation

The second validation was carried out using the full-scale experimental data obtained by Lopes from an experiment performed at LSU and shown in Figure 3.15. With the well full of drilling fluid of 1.12 specific gravity, the experiment consisted of injecting nitrogen through the gas injection line and mud through the drilling fluid injection line while measuring annular wellbore pressure with pressure recorders lowered through the perforated tubing, until approximately steady state conditions were reached.

Figure 4.9 shows the annular bottomhole pressure recorded by a sensor located at 1768 m (5800 ft) from the time the nitrogen injection started (minute 158) to the time the nitrogen injection ended (minute 340). Although the nitrogen reached the surface approximately 48 minutes after nitrogen injection started (minute 205), the nitrogen injection flow rate and choke pressure were not maintained constant. Instead, they were varying for about 60 minutes after the nitrogen arrived at the surface and then maintained approximately constant until steady state conditions were achieved. These injection flow rate and choke pressure variations, which were used as program inputs in this simulation, are shown in Figure 4.10. This figure shows that both nitrogen injection flow rate (orange circles) and choke pressure (purple asterisks) were gradually increased for approximately 60 minutes from the time the nitrogen reached the surface. Whereas, the drilling fluid injection flow rate (black triangles) was maintained approximately constant.
Thus, starting from the time the nitrogen reach the surface, the second simulation to validate the model was carried out using the LSU No.2 well geometry shown in Figure 3.15 and the input data illustrated in Figure 4.10. Figure 4.11 shows a comparison of the simulation results with the measurements gathered during the experiment. Figure 4.12 shows the absolute and absolute average percent errors calculated from comparing the bottomhole pressure measurements with the bottomhole pressure predictions. As one can observe, the absolute percent errors are smaller than 8.0 percent along the 140 minutes of simulation, while the absolute average percent error is 3.5 percent. Once again, this model evaluation indicates reasonable agreement between predictions and measurements overall, but poor responsiveness to changing conditions over time.

As one can observe in Figures 4.10 and 4.11, the effect of increasing the choke pressure causes bottomhole pressure to increase in the simulation and overcomes the effect of increasing the nitrogen injection flow rate, which should cause a reduction in bottomhole pressure. Conversely in the experiment, the effect of increasing the nitrogen injection flow rate overcomes the effect of increasing the choke pressure. Consequently, during the first 80 minutes of simulation the bottomhole pressure trend predicted by the simulator is opposite from the actual trend, as shown in Figure 4.11.

Considering that the present time dependent model does not consider the unloading process, ideally the simulations should start after steady state conditions have been reached. In this simulation, the flow conditions from which the time dependent model initiated the computations correspond to those maintained at the time the nitrogen reached the surface (minute 205). From Figure 4.10, these initial flow conditions are nitrogen injection flow rate of 18 m$^3$/min (625 scfpm), drilling fluid injection flow rate of 0.53 m$^3$/min (140 gpm), and choke pressure of 0.903...
MPa (131 psi). However, as one can observe in Figure 4.11, steady state conditions were only achieved approximately 80 minutes after the nitrogen reached the surface. This may explain the opposite bottomhole pressure trends observed in Figure 4.11. Therefore, this validation is inconclusive whether the time dependent computer program accurately predicts wellbore pressure and two-phase flow parameters as a function of position and time considering variations in nitrogen and drilling fluid injection flow rates and choke pressure.

![Figure 4.9 Annular bottomhole pressure recorded (after Lopes37).](image)

### 4.5.3 Literature Data Validation

The third and last validation was carried out using data from a different simulator published by Jun et al38. The simulation was run by Jun et al38 to show their time dependent UBD model performance. Although this is a hypothetical simulation and its results were not validated against real field data, it does take into account reservoir influxes. The well geometry, fluid properties, and well temperatures used in this simulation are summarized in Figure 4.13. The flow rate and choke pressure variations, which were used as program inputs for this simulation are shown in Figure 4.14.

As shown in Figure 4.14, during the total period of simulation (160 minutes), the drilling fluid injection flow rate was maintained constant at 1.8 m$^3$/min (475 gpm), the casing choke pressure was first kept constant at 0.207 MPa (30 psi) during 44 minutes, then gradually increased during the next 97 minutes to a maximum value of 0.614 MPa (89 psi), and finally maintained constant at this value for the rest of the simulation period. On the other hand, formation fluids were gradually introduced into wellbore while drilling underbalanced into the hydrocarbon-bearing formation. The oil and natural gas flow rates were started simultaneously at the minute 53. After that, the oil flow rate was increased gradually from 0.0074 m$^3$/min (1.95 gpm) to a maximum
value of 0.07 m³/min (17.6 gpm) and then maintained constant for the last 20 minutes, and the natural gas flow rate was increased gradually from 0.4 m³/min (14 scfpm) to a maximum value of 10.80 m³/min (379 scfpm) and then maintained constant for the last 15 minutes of the simulation period.

Figure 4.10 Full-scale experimental data used as computer inputs.

Figure 4.15 shows a comparison of the simulation results attained by using the time dependent computer program developed during this work with those obtained by Jun et al. with their time dependent UBD computer program. Figure 4.16 shows the absolute and absolute average percent errors calculated from comparing both simulation results. Figure 4.16 shows that the absolute percent errors are very small (less than 2%) and the absolute average percent error is 0.63 percent. Therefore, this evaluation shows that these two independently developed simulators perform very similarly for production of oil and natural gas into the wellbore while drilling.

As one can observe in Figure 4.15, the bottomhole pressure curves almost match each other for about 100 minutes from the time the simulation starts. After that, the bottomhole pressure curve predicted by the proposed computer program slightly separates from that calculated by Jun et al’s computer program. However, from a close view of the input data and bottomhole pressure curves predicted by both computer programs, one can realize that the bottomhole pressure predicted by the proposed program decreases gradually because of the natural gas and oil flow rates increase. Then, it reaches a steady state condition after all the input data are constant. On the other hand, the bottomhole pressure curve predicted by Jun et al’s computer program also decreases gradually because the natural gas and oil flow rates increase, but in this case the bottomhole pressure stabilizes long before all the input data are constant, which is an unlikely outcome.
Figure 4.11 Time dependent model simulation results, second evaluation.

Figure 4.12 Time dependent computer program second evaluation.
Figure 4.13 Well geometry and computer input data, third evaluation.

<table>
<thead>
<tr>
<th>Depth</th>
<th>Inner diameter</th>
<th>Outer diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-4172 m (0-13688 ft)</td>
<td>100.0 mm (3.937 in)</td>
<td>127.0 mm (5.0 in)</td>
</tr>
<tr>
<td>4172-4340 m (13688-14239 ft)</td>
<td>74.0 mm (2.91 in)</td>
<td>158.8 mm (6.25 in)</td>
</tr>
</tbody>
</table>

**Annulus**

<table>
<thead>
<tr>
<th>Depth</th>
<th>Inner diameter</th>
<th>Outer diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-4240 m (0-13911 ft)</td>
<td>218.4 mm (8.66 in)</td>
<td>244.5 mm (9.625 in)</td>
</tr>
<tr>
<td>4240-4340 m (13688-14239 ft)</td>
<td>220.0 (8.66 in)</td>
<td>244.5 mm (9.625 in)</td>
</tr>
</tbody>
</table>

**Simulation depths 4340 m (14239 ft)**

- **Surface temperature**: 293.2 °K (528.0 °R)
- **Temperature gradient**: 2.84 °K/100 m (1.56 °R/100 ft)
- **Mud density**: 1000 kg/m³ (8.33 ppg)
- **Oil density**: 739 kg/m³ (6.16 ppg)
- **Mud viscosity**: 1 MPa.s (1 cp)
- **Oil viscosity**: 8 MPa.s (8 cp)
- **Gas specific gravity**: 0.65
- **Reservoir pressure**: 44.8 MPa (6498 psi)
- **Thickness of reservoir**: 8 m (26.2 ft)
- **Pembelity of formation**: 50 mD

**Program Input data from Jun et al**

<table>
<thead>
<tr>
<th>q oil (gpm)</th>
<th>Pchoke (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-4172 m (0-13688 ft)</td>
<td>100.0 mm (3.937 in)</td>
</tr>
<tr>
<td>4172-4340 m (13688-14239 ft)</td>
<td>74.0 mm (2.91 in)</td>
</tr>
<tr>
<td>4240-4340 m (13688-14239 ft)</td>
<td>218.4 mm (8.66 in)</td>
</tr>
</tbody>
</table>

Figure 4.14 Jun et al’s literature data used as computer inputs.
Figure 4.15 Comparison of the simulation results, third evaluation.

Figure 4.16 Time dependent computer program third evaluation.
Although the simulation results given by Jun et al. do not represent an actual field example, this last evaluation shows that the time dependent computer program can predict wellbore pressure and two-phase flow parameters as a function of position and time, taking into account variations in oil and natural gas production and choke pressure.

In summary, the time dependent model validation results show an excellent agreement between model predictions and data from three different sources steady state conditions and gradual changes in operating conditions. Therefore, this less complex, mechanistic, time dependent model implemented into a computer program can potentially be used to develop the procedure for avoiding or reducing the bottomhole pressure fluctuations occurring during UBD pipe connections. During the simulations, the three most effective controllable parameters gas and drilling fluid injection flow rates and casing choke pressure will be manipulated in order to maintain a desirable bottomhole pressure by means of the reservoir energy and the time dependent response of the model to longer changes in operating conditions will be evaluated.
CHAPTER 5

NUMERICAL SIMULATION EXAMPLES

In this Chapter, the functionality of the mechanistic time dependent model to simulate bottomhole pressure control during both drilling and UBD pipe connection operations is shown by simulation examples. One of the purposes of this work is to develop a model to predict variations of wellbore pressure and two-phase flow parameters as a function of position and time, caused by manipulating nitrogen and drilling fluid injection flow rates and casing choke pressures. Therefore, actual field descriptions of two different Mexican wells, in which the UBD technique is being employed, are used as model inputs. Also in this Chapter, other possible model uses are analyzed and potential for improvements to overcome the model’s limitations are described. As explained later in Section 5.5, the simulator predicts much faster reactions than can be achieved in practice.

5.1 Oil and Gas Well Simulation Example

A simulation was carried out using information from the Mexican well, Iride 1166, which was drilled in the Samaria-Iride oil and gas field. The Iride reservoir, composed of limestone and naturally dolomite fractured, was discovered in 1973. Since then, more than 183 wells have been drilled and produced causing the reservoir pressure to decline from its initial value of 51.4 MPa (7455 psi) to its current overall average reservoir pressure of 21.1 MPa (3055 psi)57. Since 1995, this reservoir pressure depletion has forced the Mexican oil company to drill most of the wells in this field applying the jointed-pipe nitrogen injection UBD technique.

In this reservoir, the UBD technique is mainly applied for avoiding formation damage and reducing operational problems such as loss of circulation and drillstring differential sticking. Table 5.1 describes the well geometry, fluid properties, formation pressure, and gas and oil production flow rates corresponding to a specific, representative bottomhole flowing pressure, which were used as time dependent computer program inputs for these simulations.

The simulation was performed assuming that the whole pay zone 56 m (184 ft) had already been drilled. Thus, using the well deliverability model implemented into the computer program (Equation 4.5), the oil and natural gas flow rates that will enter into the wellbore during drilling, as the bottomhole pressure becomes less than the average reservoir pressure of 27.1 MPa (3930 psi) in the Iride 1166, are shown in Figure 5.1. As one can see in this figure, low underbalance pressure drawdown conditions would cause natural gas flow rates higher than the common maximum nitrogen injection flow rate of 30 m³/min (1060 scfpm). Therefore, the underbalanced conditions must be maintained within a narrow, safe pressure window so that reservoir influx, flow through the UBD surface equipment, and surface pressure can be maintained within desirable operating limits.

Since the lowest pressure rating equipment in the UBD system is the rotating head (RH) or rotating blowout preventer (RBOP), a common practice in Mexico is to restrict the casing choke pressure to a maximum value of 4.83 MPa (700 psi).
Table 5.1 Iride 1166 oil and gas well simulation input data.

<table>
<thead>
<tr>
<th>Annular well geometry</th>
<th>Depth</th>
<th>Inner casing diameter (ICD)</th>
<th>Outer tubing diameter (OTD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0-3764 m (0-12349 ft)</td>
<td>168.3 mm (6.625 in)</td>
<td>88.9 mm (3.5 in)</td>
</tr>
<tr>
<td></td>
<td>3764-3901 m (12349-12798 ft)</td>
<td>168.3 mm (6.625 in)</td>
<td>120.7 mm (4.75 in)</td>
</tr>
<tr>
<td>Pipe roughness</td>
<td>0.2286 mm (0.009 in)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drilling fluid density</td>
<td>949 kg/m³ (7.91 lbm/gal)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drilling fluid viscosity</td>
<td>5 MPa (5 cp)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface temperature</td>
<td>302.4 °K (544.5 °R)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geothermal gradient</td>
<td>3.06 °K/100 m (1.68 °R/100ft)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nitrogen specific gravity</td>
<td>0.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nitrogen molecular weight</td>
<td>28.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial nitrogen injection flow rate</td>
<td>10 m³/min (353 scfpm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial drilling fluid injection flow rate</td>
<td>0.4542 m³/min (120 gpm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial casing choke pressure</td>
<td>0.207 MPa (30 psi)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial increments</td>
<td>15 m (50 ft)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time steps</td>
<td>30 seconds</td>
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<tr>
<td><strong>Flow test (May 2001)</strong></td>
<td></td>
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<tr>
<td>Average reservoir pressure</td>
<td>27.1 MPa (3930 psi)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottomhole flowing pressure</td>
<td>20.7 MPa (3026 psi)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bubble point pressure</td>
<td>31.8 MPa (4615 psi)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oil flow rate</td>
<td>474 m³/day (2981 bbl/day)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum oil flow rate</td>
<td>1275.2 m³/day (8020 bbl/day)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas oil ratio</td>
<td>287.3 m³/m³ (1613 scf/bbl)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>API gravity of the oil</td>
<td>44</td>
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<td></td>
</tr>
<tr>
<td>Oil density</td>
<td>805.6 kg/m³ (6.72 lbm/gal)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Natural gas specific gravity</td>
<td>0.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Natural gas molecular weight</td>
<td>18.83</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Taking into account this restriction, the present simulations had two purposes: 1) drilling in underbalanced conditions using the reservoir energy (gas and oil influxes), and 2) making the UBD pipe connection without losing the underbalanced conditions.

Due to the fact that underbalanced conditions are almost always lost during UBD pipe connections, which disrupt steady state conditions due to the interruption of drillstring injection, maintaining continuous circulation or steady state conditions during such operations would be the ideal solution to avoid losing underbalanced conditions and causing formation damage and BHP fluctuation during UBD operations. However, as explained in Chapter 1, the available technology that could make this continuous circulation possible is too expensive and only partially proven. Therefore, the approach followed by the present research, which combines the UBD hydraulic system with the reservoir inflow performance through a time dependent model, will focus on maintaining the effect of continuous circulation or steady state conditions although the drillstring injection has been interrupted.
Figure 5.1 Potential reservoir influxes during UBD operations in the oil-gas well.

Since bottomhole pressure is a result of the hydrostatic pressure of the annular fluids, plus pressure drops created by friction, plus the inertial pressure of fluid acceleration, plus choke pressure, this hypothetical condition is found by running the time-dependent model, which allows adjusting these pressure components through the two-phase flow effects imposed on the UBD hydraulic system by changes in surface injection rates and choke pressure.

During a simulation, the computer program outputs, such as bottomhole pressure and oil and gas flow rates, can be known at each time step. Therefore, the nitrogen and/or drilling fluid injection flow rate and/or the choke pressure can correspondingly be adjusted to maintain the desired bottomhole and surface conditions. That is, during a simulation, the desired BHP control is achieved by maintaining equilibrium among the hydrostatic, friction, and acceleration pressure drops as well as the choke pressure, which are all functions of the gas and liquid flowing along the wellbore.

Since steady state conditions are wanted, one must first determine if the circulation system is operating on the hydrostatic or the friction dominated side. If the circulation system is operating on the hydrostatic dominated side, it is unstable because the BHP rapidly decreases due to reduction in the hydrostatic pressure caused by gas injection. On the other hand, if the circulation system is operating on the friction dominated side, it is stable, but an increase in gas rate increases the BHP. Figure 5.2 shows that for the Iride 1166 well’s conditions, the friction dominated side are achieved after gas injections are greater than approximately 40 m³/min (1400 scfpm). Then, to gain good understanding of how the bottomhole pressure and reservoir influx
behave after a change on nitrogen and/or drilling fluid injection flow rate and/or choke pressure, several computer program runs were needed before achieving the purposes of the simulations. Figures 5.3 and 5.4 show the simulation results.

Figure 5.3 shows how the nitrogen injection flow rate (orange circles, left vertical scale), drilling fluid injection flow rate (black triangles, left vertical scale) and casing choke pressure (purple asterisks, right vertical scale) were manipulated during drilling and pipe connections to maintain the bottomhole pressure below the average reservoir pressure using the reservoir influxes. On the other hand, Figure 5.4 shows the bottomhole pressure (blue dotted line, left vertical scale), natural gas flow rate (pink circles, left vertical scale) and oil flow rate (brown squares, right vertical scale) responses to the changes of nitrogen and drilling fluid injection flow rates and choke pressure.

5.1.1 Oil and Gas Well Simulation Example for hi/h = 1.0

The initial conditions for the simulation were considered when steady state conditions were attained with the initial nitrogen and drilling fluid injection flow rates and choke pressure given in Table 5.1 and shown in Figures 5.3 and 5.4. Since for these initial conditions the bottomhole pressure was above the average reservoir pressure, at minute 1 the nitrogen injection was increased from 10 m$^3$/min (353 scfpm) to 17 m$^3$/min (600 scfpm) keeping the drilling fluid injection and choke pressure constant. These new conditions were kept constant until minute 11 at which the computer program again predicted steady state conditions. Because underbalanced conditions were still not reached, once again only the nitrogen injection was increased from 17 m$^3$/min (600 scfpm) to 23 m$^3$/min (800 scfpm) and maintained constant until underbalanced conditions were achieved at minute 16, at which reservoir influxes began to enter into the wellbore.
Figure 5.3 Manipulation of controllable parameters during the simulation for hi/h = 1.

Figure 5.4 BHP and reservoir influxes response during the simulation for hi/h = 1.
Since the circulation system was on the hydrostatic dominated side, one minute later, to control the bottomhole pressure reduction and reservoir influxes, the nitrogen injection flow rate and choke pressure were simultaneously and gradually decreased, the nitrogen injection from 23 m$^3$/min (800 scfpm) to zero and the choke pressure from 0.207 MPa (30 psi) to 0.069 MPa (10 psi). This action stopped the decreasing trend in bottomhole pressure, which was causing high increases in natural gas and oil flow rates. At this time, the flow of natural gas and oil effectively replaced the nitrogen injection as a means for reducing the hydrostatic pressure in the annulus and the hydraulic system became friction dominated. After that, for about 21 minutes, the casing choke pressure was manipulated between 0.069 MPa (10 psi) and 0.483 MPa (70 psi) to maintain the variation in bottomhole pressure and reservoir influxes under pseudo steady state condition. Also, during these 21 minutes, at minute 27 the drilling fluid injection flow rate was decreased from 0.4542 m$^3$/min (120 gpm) to 0.4164 m$^3$/min (110 gpm) allowing more natural gas to enter into the wellbore, see Figures 5.3 and 5.4.

Since the connection was about to take place (minute 37), before it had effect, the choke pressure was increased to compensate for the reduction of drilling fluid injection. Then, from minute 37 to 39 the drilling fluid injection was gradually interrupted. This caused a bottomhole pressure decrease and an increase in reservoir influx, which required an increase in the choke pressure from 0.414 MPa (60 psi) to 2.55 MPa (370 psi) to continue maintaining control of the bottomhole pressure and reservoir influxes during the pipe connection without losing the underbalanced conditions. During the connection time (from minute 39 to 49), the choke pressure was manipulated taking care of not exceeding the casing choke pressure restriction (4.83 MPa or 700 psi) and of maintaining the reservoir production within safe handling limits.

To continue drilling underbalanced, although oil and gas were flowing in the annulus, the drilling fluid injection had to be restarted to ensure hole cleaning and bit lubrication. As shown in Figure 5.2, a sudden increase in drilling fluid injection would cause a high increase in bottomhole pressure and risk the maintaining of underbalanced conditions. Therefore, the nitrogen injection was restarted together with the drilling fluid injection at minute 49; simultaneously the choke pressure was decreased gradually from 2.31 MPa (335 psi) to 0.069 (10 psi) and the drilling fluid increased from zero to 0.3407 m$^3$/min (90 gpm). After that, the nitrogen and drilling fluid injection were manipulated trying to maintain the bottomhole pressure and reservoir influxes under pseudosteady state conditions keeping the choke pressure constant and at a minimum value.

Since it is common that the rotating head rubber leaks after having been exposed to severe conditions such as those occurring during the UBD pipe connection, keeping the choke pressure at a constant and minimum value helps to increase the rotating head rubber life span and avoid the necessity of controlling the well to replace it. Thus, in this occasion the nitrogen injection was shut down (minute 64) after being sure of maintaining the choke pressure within desirable limits mentioned above. After that, to compensate the frictional pressure drop caused by the nitrogen injection, which allowed maintaining low choke pressure, the drilling fluid injection was gradually reduced to 0.265 m$^3$/min (70 gpm) letting the natural gas and oil production increase. The nitrogen injection at minute 69 was stopped to stop the increase in bottomhole pressure caused by the previous sudden interruption of nitrogen at minute 64. Finally, under similar conditions, a second UBD pipe connection was simulated from minute 85 to 98 and
drilling in underbalanced conditions was achieved until minute 102, at which time the simulation was concluded.

5.1.2 Oil and Gas Well Simulation Example for hi/h = 0.18
The previous simulation was made considering that the whole pay zone of 56 m (184 ft) had already been drilled. However, assuming negligible vertical permeability, as mentioned in chapter 4, the well deliverability is function of the interval thickness open up to flow. Therefore, to achieve the desirable underbalanced conditions shown in Figure 5.4, they must be induced from the very first UBD pipe connection and maintained till the last one. Thus, additional computer program runs were needed to simulate an UBD pipe connection considering that only 10 m (33 ft) had been drilled. Figures 5.5 and 5.6 show the simulation results based on the experience gained from previously discussed simulation one, in order to reach underbalanced conditions, the nitrogen injection was directly increased to 23 m³/min (800 scfpm) at minute 1, keeping drilling fluid injection and choke pressure constant. At these conditions, 9 minutes after the nitrogen injection increment, reservoir influxes started. To speed up the gas and oil production, at minute 12 the drilling fluid injection and choke pressure were simultaneously decreased. The drilling fluid from 0.4542 m³/min (120 gpm) to 0.3785 m³/min (100 gpm), and the choke pressure from 0.207 MPa (30 psi) to 0.103 MPa (15 psi). After enough natural gas had been produced, the nitrogen injection was interrupted. Afterward, only the choke pressure was manipulated to maintain pseudosteady state conditions.

![Figure 5.5 Manipulation of controllable parameters during the simulation for hi/h = 0.18.](image)

Figure 5.5 Manipulation of controllable parameters during the simulation for hi/h = 0.18.
Figure 5.6 BHP and reservoir influxes response during the simulation for $hi/h = 0.18$.

Although, in this case pseudosteady state conditions were easier to achieve, the pipe connection was much more complicated because the system was very sensitive to choke pressure changes. At minute 49, the connection was planned. Thus, before stopping the drilling fluid injection, the choke pressure was suddenly increased from 0.345 MPa (50 psi) to 0.621 MPa (90 psi), but this caused a bottomhole pressure increment that could not be controlled and the underbalanced conditions were lost, even though the choke pressure and drilling fluid injection was immediately decreased. Afterwards, nitrogen injection was again required and the drilling fluid injection was further decreased to 0.3028 m$^3$/min (80 gpm) to regain underbalanced conditions. On this occasion, higher underbalanced conditions were allowed so that more natural gas and oil would enter into the wellbore to permit the pipe connection. This allowed increasing the choke pressure gradually until the moment the drilling fluid injection was also gradually interrupted. Subsequently, during the connection time (from minute 70 to 82) the choke pressure was manipulated to maintain under control the bottomhole pressure and reservoir influxes. After the connection had been secured, the nitrogen injection was restarted before the drilling fluid injection, and the choke pressure decreased. Finally, after regaining pseudosteady state conditions the nitrogen was interrupted an the drilling continues on as before the UBD pipe connection until minute 95, at which time this simulation ended.

5.2 Gas and Condensate Well Simulation Example

The third simulation example was performed using information from the Mexican well Agave 303, which was drilled in the Agave gas and condensate field. This field, formed by carbonate
rocks slightly fractured in a dolomite sequence\textsuperscript{57}, was discovered in 1976. Since then, more than 44 wells have been drilled and produced causing its original reservoir pressure to decline from its initial value of 52.3 MPa (7583 psi) to its current average reservoir pressure of 22.5 MPa (3266 psi). Similar to the Samaria-Iride oil and gas field, since 1996 most of the wells drilled in the Agave gas and condensate field are drilled employing the jointed-pipe nitrogen injection UBD technique. Since the well geometry and drilling conditions used to drill wells in both of the above fields are very similar\textsuperscript{56}, the same well geometry and initial conditions used in the above simulation example were considered for the present one. However, the flow test information corresponds to the gas and condensate Agave 303 well. Table 5.2 describes the time dependent computer program inputs used in this simulation.

<table>
<thead>
<tr>
<th>Table 5.2 Agave 303 gas and condensate well simulation input data.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Annular well geometry</strong></td>
</tr>
<tr>
<td><strong>Depth</strong></td>
</tr>
<tr>
<td>0-3764 m (0-12349 ft)</td>
</tr>
<tr>
<td>3764-3901 m (12349-12798 ft)</td>
</tr>
</tbody>
</table>

| **Drilling fluid density** | 949 kg/m\(^3\) (7.91 lbm/gal) |
| **Drilling fluid viscosity** | 5 MPa (5 cp) |
| **Surface temperature** | 302.4 °K (544.5 °R) |
| **Geothermal gradient** | 3.06 °K/100 m (1.68 °R/100 ft) |
| **Nitrogen molecular weight** | 28.02 |
| **Initial nitrogen injection flow rate** | 10 m\(^3\)/min (353 scfpm) |
| **Initial drilling fluid injection flow rate** | 0.4542 m\(^3\)/min (120 gpm) |
| **Initial casing choke pressure** | 207 kPa (30 psi) |
| **Axial increments** | 15 m (50 ft) |
| **Time steps** | 30 seconds |

<table>
<thead>
<tr>
<th><strong>Flow test results (October 2001)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average reservoir pressure</strong></td>
</tr>
<tr>
<td><strong>Bottomhole flowing pressure</strong></td>
</tr>
<tr>
<td><strong>Bubble point pressure</strong></td>
</tr>
<tr>
<td><strong>Oil flow rate</strong></td>
</tr>
<tr>
<td><strong>Maximum oil flow rate</strong></td>
</tr>
<tr>
<td><strong>Gas oil ratio</strong></td>
</tr>
<tr>
<td><strong>API gravity of the oil</strong></td>
</tr>
<tr>
<td><strong>Oil density</strong></td>
</tr>
<tr>
<td><strong>Natural gas specific gravity</strong></td>
</tr>
<tr>
<td><strong>Natural gas molecular weight</strong></td>
</tr>
</tbody>
</table>

Like the first simulation for the oil and gas reservoir, the current simulation was performed considering that the whole pay zone 66 m (216 ft) had already been drilled. As before, using the well deliverability model implemented into the computer program (Equation 4.5), the natural gas and condensate that will enter into the wellbore as the bottomhole pressure becomes less than the average reservoir pressure during drilling, are shown in Figure 5.7. As one can observe in this figure, for any underbalanced conditions that can be induced during drilling, the gas and condensate well would produce almost the same amount of natural gas as that produced by the
oil and gas well. However, the condensate production is almost negligible compared with the oil production of the oil and gas well and drilling fluid flow rates commonly used while drilling underbalanced.

Figure 5.7 Potential reservoir influxes during UBD operations, gas-condensate well.

Since high natural gas flow rates can be achieved by a small pressure drawdown, drilling underbalanced in these kind of reservoirs could be very dangerous if the bottomhole pressure while drilling is reduced beyond a safe margin. Besides, in gas wells, the choke pressure restriction is more rigorous and therefore a common practice in Mexico is to restrict such pressure to a maximum value of 3.45 MPa (500 psi). Therefore, pursuing the same simulation purposes mentioned above, the current simulation was carried out trying to maintain the least possible underbalanced conditions both while drilling and making the connection and to limit the choke pressure to its maximum allowable value. Figures 5.8 and 5.9 show the simulation results.

Similarly, several computer program runs were performed trying to achieve the simulation purposes. However, on this occasion, making the UBD pipe connection maintaining the choke pressure below its maximum allowable value and without losing underbalanced condition, was not possible even though in this simulation was easier to achieve pseudosteady state conditions and maintaining them during simulating underbalanced drilling using the reservoir inflow. Figures 5.8 and 5.9 show the most representative simulation results obtained after several abortive computer program runs attempting to simulate the UBD pipe connection.

The simulation started when the computer program predicted steady sate conditions for the initial conditions, nitrogen injection flow rate of 10 m³/min (353 scfpm), drilling fluid injection flow
rate of 0.4542 m³/min (120 gpm) and choke pressure of 0.207 MPa (30 psi). Since for these conditions the bottomhole pressure was still greater than the Agave 303 average reservoir pressure (29.8 MPa or 4325 psi), considering, the suggestions of Saponja⁸, who reports that for high drilling fluid flow rates and low gas flow rates the circulating system becomes stable, at minute 2 the nitrogen and drilling fluid injections were simultaneously increased to 20 m³/min (706 scfpm) and 0.5299 m³/min (140 gpm), respectively. At the same time, the choke pressure was decreased to 0.1379 MPa (20 psi). The combined effect of these three changes caused the underbalanced conditions to be reached 6 minutes after the adjustments had been made. As soon as underbalanced conditions were induced, keeping the drilling fluid injection constant, the nitrogen injection was gradually decreased from 20 m³/min (706 scfpm) to 10 m³/min (353 scfpm) and simultaneously the choke pressure gradually increased from 0.1379 MPa (20 psi) to 0.3448 MPa (50 psi). At these conditions, the bottomhole pressure and reservoir influxes could be maintained at stable and relatively low values of drawdown and gas flow rate.

Figure 5.8 Manipulation of controllable parameters during the gas-condensate well simulation.

To save nitrogen, during the simulation the nitrogen injection was gradually interrupted at minute 25 and simultaneously the choke pressure increased to 0.5171 MPa (75 psi) and then maintained constant during 14 minutes. Although the underbalanced conditions and natural gas production were slightly increased, the circulating system (bottomhole pressure) was stabler than before the nitrogen interruption. As explained by Saponja⁸, this occurred when high nitrogen injection or natural gas production is taking place because the circulation system operates on the friction-dominated side (Figure 5.2). Therefore, if no interruptions are required during underbalanced drilling, the nitrogen, which can be one of the highest additional costs in a typical UBD operation, can be saved without losing the underbalanced conditions.
At minute 40, the connection was finally attempted. During this operation, before interrupting the drilling fluid injection, the choke pressure was gradually increased to 0.827 MPa (120 psi ). Subsequently, from minute 43 to 54 the drilling fluid injection was decreased by steps from 0.5299 m³/min (140 gpm) to 0.1514 m³/min (40 gpm) trying at the same time to maintain under control the bottomhole pressure and reservoir influxes through incremental choke pressure changes. However, it was not possible to simulate the pipe connection because the choke pressure had to be increased to values much higher than the maximum allowable one. As one can see in Figure 5.9, when the drilling fluid injection flow rate was 0.1514 m³/min (40 gpm), the choke pressure was raised to 5.378 MPa (780 psi). Nevertheless, the bottomhole pressure continued falling causing natural gas productions as high as 162.06 m³/min (5723 scfpm), equivalent to 233363 m³/day (8.24x10⁶ scfpd). Thus, the choke pressure was further increased to 8.619 MPa (1250 psi), but this sudden choke pressure increment caused the bottomhole pressure to increase to a value higher than the Agave 303 average reservoir pressure even though the choke pressure was immediately decreased.

As mentioned above, varying the controllable parameters (nitrogen and drilling fluid injection flow rates and choke pressure), several computer program runs were performed trying to simulate the UBD pipe connection. However, all the simulation results were similar to those shown in Figures 5.8 and 5.9.

Trying to make a pipe connection in a gas well, like the one described here, controlling the choke pressure and allowing reservoir influxes, would be very dangerous and difficult. Therefore, as
the simulation results illustrate, in gas wells an UBD pipe connection using the reservoir energy may not be possible because the UBD safety boundaries such as maximum allowable casing choke pressure and UBD surface equipment capacity cannot be respected. It must be noted that the need for a rapid increase in choke pressure predicted by the time dependent model is exaggerated. The cause is explained in detail in Section 5.5. Consequently, control may not be as difficult as implied by this simulations. An additional important conclusion is that reservoir liquid-phase helps to perform the UBD pipe connection using the reservoir energy. Mexican oil-gas fields such as Luna, Puerto Ceiba, and Sen$^{36}$ whose gas-oil ratios are lower than 285 m$^3$/m$^3$ (1600 scf/bbl) with relatively high average reservoir pressure are good candidates to practice UBD pipe connections maintaining underbalanced conditions by manipulating the reservoir influxes.

5.3 Underbalance Drawdown Considerations

As mentioned by Pérez-Téllez et al$^{36}$, successful UBD operations depend not only upon maintaining the bottomhole pressure below the average reservoir pressure, but also in keeping the differential pressure drawdown, the annular velocity, and reservoir influx between safe boundaries. These boundaries are a function of the borehole stability, minimum annular velocity required to provide adequate hole cleaning, and UBD surface equipment capacity. Therefore, the knowledge of additional two-phase flow parameters is needed to properly design the circulation system and the UBD surface equipment capacity.

The time dependent computer program can give detailed output in addition to the bottomhole pressure and natural gas and oil flow rates outputs shown in Figures 5.4, 5.6 and 5.9. Hence during the simulation, any other two-phase flow parameter such as hydrostatic and frictional pressure drops, liquid holdup, in-situ liquid and gas velocities, liquid and gas superficial velocities, mixture density, etc. can be monitored at any depth along the flow path. These parameters may then be used in determining whether the hydraulic system is operating within acceptable boundaries.

5.3.1 Borehole Stability

The allowable magnitude of the drawdown differential between the borehole pressure and the formation pore fluid pressure is dependent on considerations for connections, borehole instability, and reservoir inflow performance$^8$.

As explained in Chapter 1, UBD pipe connection operations trigger a bottomhole pressure fluctuation. This unstable cycling of the bottomhole pressure can cause borehole instability. Therefore, as proposed in the present work and demonstrated with the simulation results, maintaining continuous annular flow during drilling and during UBD pipe connections, the bottomhole pressure fluctuation can be drastically attenuated because the fluid segregation during the connection time is mitigated.

Although borehole stability is not a primary concern during UBD operations$^{12,57}$, underbalanced drilling borehole instabilities may also be expected due to high annular velocities caused by high underbalance drawdown conditions. Figure 5.10 shows the underbalance pressure drawdown conditions resulting from the above simulations. As one can note in this figure a high
underbalanced drawdown was required to simulate the connection when partial penetration was considered in the oil-gas well, but this condition was significantly improved when the entire pay zone was open to flow. On the other hand, during drilling, the pressure drawdown could vary between 6.89 MPa (1000 psi) when partial penetration is considered and 3.45 MPa (500 psi) when the total pay zone is open to flow. In the case of the gas-condensate well, the pressure drawdown was maintained at less than 4.83 MPa (700 psi).

Deis et al.\textsuperscript{13} shows that the average underbalance pressure drawdown induced to drill Canadian oil and gas wells is greater than 3.45 MPa (500 psi) and that underbalance pressure drawdown as high as 7.58 MPa (1100 psi) were necessary to drill some of those wells.

Additional computer program runs could be carried out trying to reduce the underbalanced pressure drawdown during drilling, by increasing the choke pressure or keeping the nitrogen injection to a minimum value. However, there is no more literature information showing actual field examples of underbalance pressure drawdown to compare with.

5.3.2 Hole Cleaning
In UBD operations the liquid portion of the two-phase flow provides the lifting capacity\textsuperscript{7,8}. Therefore, to ensure vertical transport of cuttings, the annular liquid velocity must be maintained at the minimum allowable value, especially in annular regions in which the cross-sectional area increased. In these simulations the in-situ liquid velocity occurring above the drill collars (3764 m or 12349 ft) was monitored and restricted to the minimum annular liquid velocity of 0.508 m/sec (100 ft/min), required to guarantee vertical transport of cuttings. Figure 5.11 illustrates the in-situ liquid velocity predicted by the computer program during the simulations. As one can see in this figure, while drilling, the annular liquid velocity was maintained above the limit stated
above and only during the UBD pipe connections the annular liquid velocity became less than that required to lift the cuttings. However, considering that before the connection takes place, most of the cuttings must have been circulated out of the hole, the annular liquid velocity during the connection should not be a concern.

Figure 5.11 Annular in-situ liquid velocities at 3764 m (12349 ft)

5.3.3 Reservoir Inflow
In UBD operations the decision of using the complete set of UBD surface equipment, which basically consist of a rotating blowout preventer, separator, cryogenic nitrogen pump, flow meters, and acquisition data system, is based on expected reservoir influxes and surface flowing pressures. Additionally, sufficient surface storage capacity and ability to transfer fluids from the drilling side are required. Figure 5.12 illustrates the underbalanced surface equipment commonly used by PEMEX97.

In addition to the significant advantages to underbalanced drilling, optimizing the UBD surface equipment would result in a huge saving of money. Therefore, another very important information, which can be used to design and plan the underbalanced surface equipment, storage capacity, and fluid transfer requirement, can be obtained from the simulations by calculating the area under the curve of the natural gas and oil influx curves. Figures 5.13 and 5.14 give the cumulative production of oil and natural gas for the simulations described above. Considering the well characteristics used in the simulations, from this information one can realize that considerable oil storage capacity and oil fluid transfer must be considered for the oil-gas well, but it should not be a major concern in the gas-condensate well. On the other hand, since it is not commonly possible to route gas from the separator to a production line because there are not local production facilities, substantial amounts of gas would be flared from any of the wells and,
therefore, the flare pit must be designed large enough to handle the maximum anticipated gas rate, which can be also predicted from the simulation.

Figure 5.12 Underbalanced surface equipment used by PEMEX (courtesy of Precision Drilling service company).

Figure 5.13 Cumulative oil production.
5.4 Field Example Simulation

Although it is reported in the literature that the UBD technology has widely been used in the oil and gas industry for multiple purposes, including minimizing formation damage and controlling lost circulation\(^9\), detailed information regarding geology, reservoir characteristics, operational problems, and specially field measurements is unfortunately withheld by the operator as confidential information\(^10\). Besides, except for the unloading process, which is accurately predicted by some time-dependent models such as DynaFlowDrill\(^11\), there is no literature report of an actual jointed pipe UBD well that had been designed by using a time dependent model. Therefore, no data was available from a real field example to compare the performance of the proposed model to predict the bottomhole pressure and reservoir influx behavior under the conditions simulated.

On the other hand, in May 2002 PEMEX authorized the use of a bottomhole pressure/temperature memory gauge installed on the drill string immediately above the bit to measure the bottomhole pressure while drilling, injecting through the drillstring nitrogen and oil-based drilling fluid, in the Muspac 53 well. Since reservoir influxes were expected while drilling this well, the field measurements were going to be used for validating the proposed model. Unfortunately, the well conditions were not appropriate to induce underbalanced conditions so that reservoir influxes could be observed during drilling. Nevertheless, this information was very useful to expose, through real field data, the limitations of the time-dependent model observed from the above simulations, as well as to further validate the mechanistic steady state model, which is the heart of the time dependent model.
5.4.1 Field Measurements from Mexican Well Muspac 53
The Muspac 53 well, strategically sited in the best part of the Muspac geological structure and in the middle of three producer wells, was drilled to increase the Mexican gas production.

After cementing the 177.8 mm (7 in) production casing at 2597 m (8520 ft), the complete set of UBD surface equipment was installed. Afterward, with a 149.2 mm (5-7/8 in) bit, it was drilled from 2597 m (8520 ft) to 2686 m (8812 ft) injecting nitrogen and oil drilling fluid of 0.94 specific gravity. During drilling, partial loss of circulation prevailed from 2618 m (8589 ft) to 2686 m (8812 ft), and a gas kick was observed at 2665 m (8743 ft) while waiting for a mechanical breakdown to be repaired. Figure 5.15 illustrates the overall view of the drilling process. In this figure, the bottomhole pressure measured (left vertical scale) is represented by the blue dotted line, the reservoir pressure (left vertical scale) by the green horizontal line, the nitrogen injection flow rate (right vertical scale) by the orange circles, the drilling fluid injection flow rate (right vertical scale) by the black triangles, and the casing choke pressure (right vertical scale) by the purple crosses. The circled numbers on the graph point to events that are relevant for the present study:

![Figure 5.15 Field measurements from Mexican well Muspac 53](image)

Figure 5.15 Field measurements from Mexican well Muspac 53.

1. At 2600 m (8530 ft) the nitrogen injection started at minute 282 with an injection rate of 15 m$^3$/min (530 scfpm). After 160 minutes of nitrogen injection, pseudosteady state conditions were reached.

2. Under pseudosteady state conditions, at minute 442 while drilling at 2607 m (8553 ft) the nitrogen injection flow rate was increased from 15 m$^3$/min (530 scfpm) to 20 m$^3$/min (706 scfpm). After 129 minutes, pseudosteady state conditions were also achieved.
3. While drilling at 2618 m (8589 ft) partial loss of circulations started at minute 679.

4. Choke pressure effect under pseudosteady state conditions.

5. At 2665 m (8743 ft) during a mechanical breakdown a gas kick was taken. Waiting for the failure to be repaired, the well was shut in from minute 4181 to 4815 (10.6 hours). This allowed the determination of the average reservoir pressure, which was equal to 16.64 MPa (2414 psi).

6. Partial underbalanced drilling conditions achieved during drilling.

Even though during drilling, underbalanced conditions were not achieved and partial loss of circulation were observed in almost the whole interval drilled, the Muspac 53 well was completed in open hole and put into production. A flow test with a 22.2 mm (7/8 in) choke setting resulted in gas production of 548,780 m$^3$/day (19.4 MMscf/day) and oil production of 42 m$^3$/day (264 bbl/day) with a wellhead pressure of 8.81 MPa (1278 psi), and a bottomhole pressure of 12.23 MPa (1774 psi). However, the gas and oil production expected were of 991,090 m$^3$/day (35.0 MMscf/day) and of 44.5 m$^3$/day (280 bbl/day), respectively.

5.4.2 Time Dependent Model Predictions vs. Field Data

Because it was planned to validate the time-dependent model predictions with the bottomhole pressure measurements of Muspac 53 well, before drilling; a computer program run was performed using as input data the Muspac 53 well geometry, fluid properties, and possible drilling fluid and nitrogen injection flow rates. Table 5.3 gives the computer program input data.

<table>
<thead>
<tr>
<th>Annular well geometry</th>
<th>Inner casing diameter (ICD)</th>
<th>Outer tubing diameter (OTD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-2572 m (0-8438 ft)</td>
<td>152.5 mm (6.004 in)</td>
<td>88.9 mm (3.5 in)</td>
</tr>
<tr>
<td>2572-2614 m (8438-8576 ft)</td>
<td>152.5 mm (6.004 in)</td>
<td>120.7 mm (4.75 in)</td>
</tr>
<tr>
<td>Pipe roughness</td>
<td>0.2286 mm (0.009 in)</td>
<td></td>
</tr>
<tr>
<td>Drilling fluid density</td>
<td>919 kg/m$^3$ (7.66 lbm/gal)</td>
<td></td>
</tr>
<tr>
<td>Drilling fluid viscosity</td>
<td>5 MPa (5 cp)</td>
<td></td>
</tr>
<tr>
<td>Surface temperature</td>
<td>301.15 °K (542.4 °R)</td>
<td></td>
</tr>
<tr>
<td>Geothermal gradient</td>
<td>2.83 °K/100 m (1.56 °R/100 ft)</td>
<td></td>
</tr>
<tr>
<td>Nitrogen specific gravity</td>
<td>0.97</td>
<td></td>
</tr>
<tr>
<td>Drilling fluid injection flow rates</td>
<td>0.606, 0.530, 0.454 m$^3$/min (160, 140, 120 gpm)</td>
<td></td>
</tr>
<tr>
<td>Nitrogen injection flow rates</td>
<td>0.37 m$^3$/min (0-1300 scfpm)</td>
<td></td>
</tr>
<tr>
<td>Casing choke pressure</td>
<td>0.689 MPa (100 psi)</td>
<td></td>
</tr>
<tr>
<td>Reservoir pressure (Estimated from offset wells)</td>
<td>17.24 MPa (2500 psi)</td>
<td></td>
</tr>
<tr>
<td>Time step</td>
<td>49 second</td>
<td></td>
</tr>
<tr>
<td>Axial increment</td>
<td>15.24 m (50 ft)</td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.16 shows the simulation results obtained by using the time-dependent model symbolized by LSU, those obtained by using the steady state commercial UBD simulator NEOTEC symbolized by NT, and the field measurements obtained from the pseudosteady state flow conditions pointed to with the circled numbers 1 and 2 in Figure 5.15 and above described. The horizontal green line, in Figure 5.16, corresponds to the reservoir pressure estimated from offset wells (17.24 MPa or 2500 psi).

![Figure 5.16 Bottomhole pressure vs. Nitrogen injection flow rate](image)

Figure 5.16 Bottomhole pressure vs. N₂ and drilling fluid injection flow rates.

To clearly observe the measured pseudosteady state flow conditions, Figure 5.17 shows a scale expansion of Figure 5.15 from minute 280 to 580. These field measurements were obtained after recovering the pressure/temperature memory gauge.

As noted in Figure 5.16, considering the time dependent model results, underbalanced conditions would never be achieved with the prevailing well conditions. That is, the equivalent circulation density caused by the hydrostatic and frictional pressure drops is too high to be lowered to a value less than the reservoir pressure equivalent density, because the mud density was too high for such reservoir pressure. Therefore, additional simulations to predict bottomhole pressure under the influence of reservoir influxes, such as those performed in Sections 5.1 and 5.2, were not possible. On the other hand, taking into account the NEOTEC results, for any drilling fluid flow rate considered, underbalanced conditions would be achieved after 25.5 m³/min (900 scfpm) of nitrogen injection, and only 16 m³/min (565 scfpm) would be required when the drilling fluid injection were 0.454 m³/min (120 gpm).

Without any change the drilling continued to the programmed depth. Then, after recovering the pressure/temperature memory gauge, the bottom hole pressure measurements proved that
underbalanced conditions could not be reached just as predicted by the proposed model and that the model predictions were very accurate as shown by the black circular spots in Figure 5.16.

![Figure 5.17 Measured BHP under pseudosteady state conditions.](image)

### 5.5 Model Limitations

Using the Muspac 53 well field data given in Table 5.3, the following additional computer program run was performed and the results, shown in Figure 5.18, compared against the well field measurements to point out this model shortcoming.

Figure 5.18 illustrates the actual bottomhole pressure response (blue circles) caused by a nitrogen injection change, which was previously explained by the event number 2 in Section 5.4.1, and shown in Figure 5.17. Taking into account that the time dependent model considers the nitrogen injection at the bottom of the well, Figure 5.18 also illustrates the bottomhole pressure response (green squares) predicted by the proposed model considering a drilling fluid injection flow rate of 0.522 m³/min (138 gpm) and a choke pressure of 45 psi as measured during the drilling process (Figure 5.15).

It was observed from the above simulations that even though the time dependent model predictions are quite good for the initial and final steady state before and after a gas injection change, they are less accurate during the transition in between the two steady states. That is because the predicted time delay calculated by the model between the steady state before and the steady state after the gas injection change is too short compared to the actual one.

Table 5.4 gives the absolute percent errors calculated by comparing field measurements against model predictions, before and after the nitrogen injection change and during the transient one.
Table 5.4 Absolute percent errors.

<table>
<thead>
<tr>
<th>Field measurement</th>
<th>Model prediction</th>
<th>Absolute error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BHP before MPa (psi)</td>
<td>23.3 (3376)</td>
<td>23.0 (3334)</td>
</tr>
<tr>
<td>BHP after MPa (psi)</td>
<td>22.2 (3221)</td>
<td>21.7 (3148)</td>
</tr>
<tr>
<td>BHP during the transient MPa (psi)</td>
<td>23.3 (3369)</td>
<td>21.7 (3148)</td>
</tr>
<tr>
<td>Time delay (minutes)</td>
<td>50</td>
<td>23</td>
</tr>
</tbody>
</table>

As noticed in Figure 5.18 and Table 5.4, the predictions before and after the nitrogen injection change are very accurate. On the other hand, the prediction during the transient shown in table 5.4, which corresponds to the maximum possible error, is not bad. However, the proposed model predicts a time period from the steady state previous to the nitrogen injection change to the steady state after it, less than fifty percent of the actual one. Figures 5.19 and 5.20 show the simulated liquid holdup and mixture density at the bottom hole during the transient. Additional two-phase flow parameters such as in-situ liquid and gas velocities were also analyzed. All of them showed the same behavior as that illustrated in Figures 5.19 and 5.20. As one can observe in these figures, during the transient the two-phase flow parameters suddenly change during the first time step after the gas injection adjustment. Subsequently, the flow parameters gradually continue changing until steady state conditions for the new gas injection is achieved. Figure 5.21 illustrates the liquid holdup along the wellbore, for the steady state previous to the nitrogen injection change (blue circles), for the steady state after the nitrogen injection change (green triangles), and for the first time step (black rhomboids).
Figure 5.19 Simulated liquid holdup at the bottom hole during the transient.

Figure 5.20 Simulated mixture density at the bottom hole during the transient.
As noticed from Figures 5.19 through 5.21, the mass transport phenomenon up the annulus between cells occurring during the transient caused by a gas flow rate change is not well predicted by the proposed model. This causes the simulated time delays for reaching a new steady state condition after a gas injection change, to be 50% shorter than the actual ones. However, as shown in the above simulations, to control the bottomhole pressure under desirable conditions, the UBD hydraulic system is much more dependent on choke pressure adjustments, which are more accurately predicted by the proposed model.

### 5.6 Necessary Future Development Efforts

As shown by Figures 5.19 through 5.21, the present numerical solution of the time dependent computer program forces a nearly steady state set of conditions through the entire well after every time step. This happens because the current numerical solution uses the same gas and liquid flow rates to compute two-phase flow parameters at node C (Figure 4.2) of each axial increment along the wellbore and at each time step. Consequently, a change in flow rate or gas fraction is affected all along the well in the first time step after the change. Consequently, a change in flow rate or gas fraction is affected all along the well in the first time step after the change. As a result, the mass transport rates throughout the well change completely within a few time steps after a change in inputs.

Therefore, the proposed future work to improve the model predictions is to implement the following new iterative two-phase flow analysis to compute pressure and two-phase flow properties at node C of each axial increment dependent on the mass transfer from the adjacent
cell. For that, the time dependent algorithm steps, given in Section 4.4.1 in Chapter 4, should be modified as follows:

Steps 1 through 8 should not be modified. After that, the following new algorithm steps may be implemented so that the computations apply the new gas or liquid injection flow rate occurring at the bottom hole only at node B of the first axial increment (Figure 4.4) not throughout the entire system.

9. Guess an initial in-situ liquid velocity for the upper boundary of the first axial increment at the present time step (node C). A good guess is the in-situ liquid velocity of the previous time step (node D).

10. Calculate the liquid holdup at node C through the finite difference formulation of the equation of conservation of mass of liquid, Equation 4.15.

11. Assume bubble flow, and calculate the slip velocity between the gas and liquid in-situ velocities using equations 3.18 and 3.12. Use the gas density value calculated at the previous time step.

12. Calculate the in-situ gas velocity at node C using the slip relation between the gas and liquid in-situ velocities given by

\[
u_G = \frac{\alpha \nu_L H_L + \nu_s}{1 - \alpha \nu_G H_L}
\]

13. Calculate superficial gas and liquid velocities and determine the flow regime. If the flow regime corresponds to that assumed in step 11, continue with the next step. Otherwise, calculate the slip velocity between the gas and liquid in-situ velocities using the equation defined for the existing flow pattern and repeat steps 12 and 13 until the guessed and predicted flow pattern agrees.

If slug flow is predicted, calculate the slip velocity between the gas and liquid in-situ velocities using Equation 3.13. On the other hand, if dispersed bubble or annular flow is predicted, the slip velocity between the gas and liquid in-situ velocities will be equal to zero.

14. Calculate the gas density at node C through the finite difference formulation of the equation of conservation of mass of gas, Equation 4.16.

15. Calculate the pressure at node C using the equation of state given by

\[
\rho_G = \frac{pM}{zRT}
\]

16. Use the flow properties calculated at node C and the assumed in-situ velocity to compute the wellbore pressure at node C using the finite difference formulation for the
conservation of mixture momentum, Equation 4.17, with the frictional and elevation pressure gradients determined using the same models as in the MSSM.

17. Compare the wellbore pressures calculated in steps 15 and 16. If the difference between them is less than a specified tolerance (1.0 psi), continue with the next step. Otherwise, assume another in-situ liquid velocity and repeat steps 9 through 17 until this condition is met. If the choke pressure calculated is greater than the choke pressure stated as boundary condition, the liquid velocity should be increased. On the other hand, if the choke pressure calculated is less than the choke pressure stated as boundary condition, the liquid velocity should be decreased. As soon as convergence is obtained at node C, pressure and two-phase flow properties at the lower and upper boundaries of the first axial increment at the present time step (nodes B, and C) are known as a function of the guessed bottomhole pressure defined in step 7 of the original time dependent algorithm steps given in Chapter 4.

18. Continue at step 13 of the original time dependent algorithm given in Chapter 4.

The new algorithm steps above must be substituted for steps 9 through 12 of the original time dependent algorithm given in Chapter 4 for the solution of each axial element.

As mentioned above, this new iterative analysis should be implemented so that the computations apply the new gas or liquid injection flow rate occurring at the bottom hole only at node B of the first axial increment and not throughout the entire system. The mass transport will be a function of the mixture velocity calculated at the in-situ flow conditions, which is used to calculate the size of the next time step.

Since the finite-difference method implemented in this work is restricted by the condition that the ratio between time and space discretization must be less than one but close to one, and jointed-pipe UBD hydraulic systems have proved to be a complex phenomenon to simulate, another different finite-difference method should also be considered because convergence problems were experienced with the current program when too small time steps were used.

During UBD pipe connections, maintaining the bottomhole pressure under control and lower than the average reservoir pressure is an issue that causes some technology developers\(^\text{14,15,100}\) to try a way of holding continuous circulation during UBD pipe connections so that the underbalanced condition can be conserved during the connection.

Even though some development efforts are still necessary to improve the model predictions during the transients, the time-dependent model validation results showed that reasonably accurate predictions could be obtained when the flow condition adjustments are gradually made. Therefore, for reservoir conditions similar to those of these Mexican oil-gas fields, the simulations show that it is not necessary to maintain continuous circulation if the nitrogen and drilling fluid injection and choke pressure are appropriately manipulated so that the reservoir energy can substitute for the surface injection during UBD pipe connections. Also, they show that maintaining continuous annular flow during drilling and UBD pipe connections, the problematic bottomhole pressure fluctuation observed during jointed-pipe UBD operations
(Figure 1.1) can probably be attenuated significantly. Additionally, the simulation results show that the manipulation of the controllable parameters allows drilling underbalanced using the reservoir energy and therefore saving the use of nitrogen.
CHAPTER 6

UNDERBALANCED DRILLING FLOW CONTROL PROCEDURE

In jointed-pipe UBD operations, maintaining underbalanced conditions during the entire drilling process is critical to achieve the intended advantages of drilling the well underbalanced. However, it has been shown by actual field measurements that fluctuations in bottomhole pressure, triggered mainly by the interruption of surface injection during pipe connections, often produce short-period overbalanced conditions each time a pipe connection takes place. It also has been shown by the petroleum industry that the negative effects of these short-period overbalanced conditions and pressure fluctuations are detrimental to the success of UBD operations\textsuperscript{8,11,12}. Consequently, most of the effort and expense of drilling underbalanced may be wasted.

This work was aimed at improving bottomhole pressure control for UBD operations so that continuous underbalanced conditions can be maintained not only during drilling but also during pipe connections. Ultimately, this improved bottomhole pressure control will also allow maintaining a reservoir influx rate, flow rate through the UBD surface equipment, and surface pressure within safe operating limits. In this chapter, a proposed underbalanced drilling flow control procedure is described. It is based on the computer program results obtained from both comprehensive, mechanistic steady state model, described in Chapter 3, and mechanistic time dependent model, presented in Chapter 4, and on the field experience gained during field trips made throughout the development of this work, and on the Muspac 53 well.

6.1 UBD Flow Control Design Stage

A successful UBD application must be scrupulously designed prior to execution. This will allow optimizing the UBD surface equipment, the nitrogen and drilling fluid consumption, the storage capacity, and most important maintaining continuous UBD conditions during the whole drilling process. Additionally, this will allow the rig personnel to be prepared for chronologically executing every single UBD operation so that all unnecessary interruptions to circulation can be avoided. Therefore, after deciding to drill a well underbalanced, the following design steps should be followed.

1. Determine reservoir pore pressure, reservoir fracture pressure, wellbore stability pressure, formation properties such as rock composition, permeability, and porosity, reservoir fluid properties, representative flow test gas and oil data (production flow rates, flowing pressure and temperature), reservoir depth and thickness, well geometry, representative surface and formation temperatures, and maximum flow rate and pressure capacity of the available surface equipment. Successful UBD operations rely on the knowledge of all these parameters. Therefore, all of them must be known with a reasonable degree of certainty.

Since the UBD technique is almost always employed to drill development wells (up to now only Negrao and Lage\textsuperscript{50} have reported the use of this technique to drill a shallow low pressure and high permeability exploratory well in Brazil), fairly accurate information may typically be obtained from offset wells.
For example, in Chapter 5 section 5.4.1, the estimated reservoir pressure from offset wells was of 17.24 MPa (2500 psi), while the actual reservoir pressure recorded while drilling was of 16.64 MPa (2414 psi). On the other hand, the estimated gas and oil production rates from offset wells were 991,090 m³/day (35.0 MMscf/day) and 44.5 m³/day (280 bbl/day), respectively. Whereas, the actual measurements given for a flow test after well completion with a 22.2 mm (7/8 in) choke setting resulted in gas production of 548,780 m³/day (19.4 MMscf/day) and oil production of 42 m³/day (264 bbl/day) with a wellhead pressure of 8.81 MPa (1278 psi), and a bottomhole flowing pressure of 12.23 MPa (1774 psi). The failure to consistently achieve underbalanced conditions and the partial loss of circulation during drilling (Figure 5.15) apparently caused the actual productivity, especially of gas, to be different than estimated.

2. Use a well deliverability model to estimate potential reservoir influx during drilling, such as those shown in figures 5.1 and 5.7. In this work, the model of Vogel, Equation 4.5, was used. However, the deliverability model that best describes the reservoir inflow performance must be used.

3. Using the above information, determine the maximum UBD pressure window as described in the following paragraph and plot it in a wellbore-reservoir interaction graphic (Figure 1.1). This graphic, formed by the reservoir inflow performance relationship (IPR) curve and the upper and lower pressure boundaries of the UBD pressure window, gives the maximum underbalanced pressure drawdown that could be induced under safe conditions to maintain 100% underbalanced conditions during the pipe connections and during drilling.

The reservoir pore pressure is the UBD pressure window upper boundary. On the other hand, the UBD pressure window lower boundary may be controlled either by the wellbore stability pressure or by the BHP at which the maximum flow rate capacity of the available surface equipment would be reached. However, some wells do not have significant stability problems and the maximum flow rate capacity of the available surface equipment is high enough to handle a large reservoir influx rate. For those wells, the maximum underbalanced pressure drawdown or UBD pressure window lower boundary will be controlled by the BHP at which the hydrostatic pressure due to annulus fluids would be so small that choke pressure might exceed the maximum allowable wellhead pressure especially if flow from the well were reduced on shut in. Therefore, whichever of this limits gives the greatest BHP must be chosen as the lower pressure boundary.

4. The UBD hydraulic circulation system, which combines injection fluids with produced reservoir fluids, must be designed for a variety of possible conditions to determine the optimal circulation system. Therefore, use well geometry, possible drilling fluid properties, formation temperature, maximum allowable surface pressure, surface equipment capacity and the above information as computer program inputs to determine:

4.1 The proper drilling fluid density and viscosity that will allow achieving the desired underbalanced conditions through the nitrogen injection before flow from the reservoir is observed.
This is achieved by predicting hydrostatic-friction curves, such as those calculated in Figures 5.2 and 5.16. These curves must be predicted at a variety of conditions to determine when the circulating system is on the hydrostatic dominated side (unstable), when it is on the friction dominated side (stable), and the appropriate conditions to induce underbalanced conditions.

Maintaining constant drilling fluid density and viscosity, drilling fluid injection flow rate, and choke pressure and varying gas injection flow rate, will allow determining a hydrostatic-friction curve for the assumed constant conditions. Points over the IPR curve within the UBD pressure window give production fluid rates and BHP at which they will flow. Nitrogen and drilling fluid flow rates and choke pressure that correspond to a BHP chosen can be calculated by trial and error. These predictions are made to know the hydraulic system response to possible change in flow conditions. They will help to make flow conditions adjustments during the time dependent computer program runs.

After determining the appropriate drilling fluid properties and choke pressure at which underbalanced conditions can be induced, varying gas flow rate (produced plus injected gas), liquid (drilling fluid plus oil) flow rate, and choke pressure will allow computing possible combinations of these controllable parameters within the UBD pressure window. This is possible by overlaying the wellbore-reservoir interaction curve and the hydrostatic-friction curves for different liquid flow rates and choke pressures. Figure 6.1 shows a wellbore-reservoir interaction curve calculated for the oil-gas well Iride 1166 [hydrostatic-friction curves were calculated maintaining constant drilling fluid density and viscosity at 949 kg/m³ (7.91 lbm/gal), 5 MPa (5 cp), respectively, and choke pressure at 0.345 MPa (50 psi)].

It can be extremely important that the selected fluid properties and injection rates are calculated with an accurate model such as the one developed in this research, which allows wellbore pressure predictions with an average absolute error less than 2.5%. A model that under predicts or over predicts by more than 10% of the actual wellbore pressure, as is the case of some current commercial UBD computer programs, may risk the UBD operation success. For example, using a model that predicts BHP that is significantly less than the actual BHP can result in not having adequate nitrogen injection capacity to achieve underbalanced conditions.

The actual field example described in Chapter 5, Section 5.4, proves this lamentable situation. In Figure 6.1 it can be observed that underbalanced conditions could not be induced when the drilling fluid injection was of 0.530 m³/min (140 gpm). Therefore, it is very important to design a drilling fluid density as close as possible to the equivalent pore pressure density so that it can be lowered to the desired equivalent circulation density by the nitrogen injection. This will guarantee that underbalanced conditions will be achieved by the nitrogen injection before reservoir flow is initiated and minimize nitrogen consumption.
4.2 As soon as the UBD pressure window, the drilling fluid properties, and the possible combinations of gas flow rate, liquid flow rate, and choke pressure have been established, determine the optimum combination sets of nitrogen and drilling fluid injection flow rates and choke pressures that will allow keeping the bottomhole pressures within the designed UBD pressure window. These bottomhole pressures should be selected to maintain the reservoir influx and wellhead pressure within safe limits and guarantee continuous underbalanced conditions during drilling and pipe connection operations.

This is achieved by running the time dependent model that predicts changes in the hydrostatic, friction, and acceleration pressure gradients caused by adjustments in gas and/or liquid flow rate and/or choke pressure. Although the time dependent model developed in this research fails to appropriately predict such conditions, because it cannot correctly predict the mass transport phenomenon that occurs when a change of gas flow rate takes place, this limitation is partially overcome by controlling the system with choke pressure adjustments. These are more accurately predicted by the model than gas flow rate changes.

Controlling the system with the choke is desirable because bottomhole pressure is much more responsive to choke pressure adjustments than to liquid or gas flow rate changes. This can be observed in the actual field data from the Muspac 53 well, described in Chapter 5, Section 5.4.1. As seen in Figure 6.2, when a gas flow rate change occurs, the time required for the BHP to stabilize at a new level was 69 minutes. From the time the injection rate changed at the standpipe. Even after the increased gas rate begins to enter

Figure 6.1 Wellbore-reservoir interaction curve for the oil-gas well Iride 1166.
the annulus, 50 minutes are required before the BHP stabilizes. On the other hand, when a choke pressure adjustment occurs, the BHP response is as short as seven minutes, as seen in Figure 6.3. Additionally, during actual UBD operations, changes in liquid and gas flow rates are typically made slowly so, actual adjustments in BHP may be even slower. However, as the time dependent model validations showed, if changes on gas or liquid flow rates are made gradually, more accurate predictions from the time dependent program may be expected.

Therefore, the following steps wise procedure is recommended for determining initial injection rates once underbalanced conditions are achieved.

![Figure 6.2 Actual BHP responses to a gas flow rate change.](image)

4.2.1 Determine a gas injection rate, which ensures that the circulation system operates on the friction-dominated side, so that adjustments in gas flow rates do not cause high BHP changes, using the program

As shown in the simulation examples of Chapter 5, at the previously designed injection flow rates, nitrogen and drilling fluid are simultaneously injected until underbalanced conditions are induced. This will occur after the well has been completely unloaded and two-phase steady state flow is observed at surface.

4.2.2 Immediately after reservoir influx is identified, choke pressure and injection flow rates adjustments are necessary to control the bottomhole pressure reduction and reservoir influx, especially in the typical cases in Mexico where the reservoir gas-oil-ratio (GOR) is higher than the injected gas-liquid-ratio.
4.2.3 Determine the optimal combination of nitrogen and drilling fluid injection flow rates and choke pressure that will maintain the bottomhole pressure and reservoir influx within the UBD operating window under pseudo steady state conditions. Since at this time the UBD hydraulic circulation system should be operating on the friction dominated side, gradually reduce or cease the nitrogen injection to the extent that reservoir gas replaces nitrogen in reducing the average fluid density in the annulus.

4.2.4 Gradually increase the choke pressure to compensate for the reduced frictional pressure loses that will occur as the circulation is also gradually interrupted.

4.2.5 During the connection time, maintain a controlled, continuous annular flow by using the reservoir energy through choke pressure adjustments. The choke pressure must be maintained lower than the maximum allowable wellhead pressure. For the typical high GOR well in Mexico, this will require limiting the formation flow rate by gradually increasing the choke pressure to offset the loss of hydrostatic as reservoir fluids replace the drilling fluid.

4.2.6 To restart surface injection, reduce the choke pressure while progressively increasing drilling fluid injection until regaining previous underbalanced pseudosteady state flow conditions.
4.3 The annular liquid velocity that ensures vertical transport of cuttings should be maintained during the simulation. This condition should especially be monitored at annular regions at which the cross-section area increased (e.g. above drill collars and change in casing diameter). Therefore, gas and liquid flow rates should be a function of the minimum allowable annular liquid velocity, which should be stated during the calculation of the wellbore-reservoir interaction curves.

Considering that most of the time the circulating system should be operating on the friction dominated side and that two-phase flow is almost always turbulent, in UBD operations hole cleaning is normally not a concern7,8.

4.4 An estimation of the total volume of reservoir fluids that would be expected during drilling, as shown in Figures 5.13 and 5.14, can be determined by calculating the area under the curve of the natural gas and oil influx curves.

5. Additionally, the computer program outputs can be wed to:

5.1 Select the proper UBD surface equipment pressure and capacity ratings.

5.2 Calculate the rate and volume of nitrogen required to ensure continuous circulation.

5.3 Determine the surface oil storage capacity required.

5.4 Plan the logistics for fluid transfer.

5.5 Design the flare pit sizes.

6. Implement the designed UBD flow control.

6.2 UBD Flow Control Execution Stage

Due to the complexity of multiphase flow and to the non-steady state nature of UBD hydraulic systems caused by the injection interruption during pipe connections, modeling of UBD bottomhole pressure fluctuations has only been published for one pipe connection experiment in a full-scale well following steady state conditions45. The drilling time between connections is usually insufficient to achieve steady state conditions, as shown by the actual field measurements illustrated in Figure 1.1 and stated by several authors such as Saponja8, Wang43, Rommetveit45 and Negrao and Lage50. Therefore, even if the modeling would have been accurate, predictions must account for the subsequent pipe connections and are therefore even more complex. Consequently, in this work the concept of modeling UBD bottomhole pressure fluctuations was avoided by attempting to maintain steady state conditions from the beginning to the end of the drilling process. This requires maximizing the use of natural energy available from the reservoir through the choke pressure manipulation, so that reservoir influxes can substitute for the interrupted surface nitrogen and drilling fluid injection during a pipe connection.
The proposed steps that should be followed during the execution of UBD operations to improve bottomhole pressure control during UBD pipe connections are given below. The specific rate and choke pressure adjustments would be based on the results from the underbalanced drilling flow control design stages above.

1. The philosophy of this flow control procedure is mainly based on the ability of preserving steady state conditions throughout the entire drilling process. As stated by Bourgoyne\(^2\), rig personnel who perform UBD operations must have very good drilling and well control skills. Therefore, it is very important to properly train the people that will carry out UBD operations. This will help not only to improve bottomhole pressure control during UBD pipe connections, but also to achieve successful UBD operations.

2. Set the intermediate casing as close as possible to the top of the hydrocarbon-bearing formation. This is a common practice in current UBD operations. Pipe used in casing and drill strings must be designed to work under multiphase flow conditions.

3. Before drilling out the float collar, cement, and casing shoe, install the rotating blowout preventer or rotating head and the additional surface equipment to drill underbalanced. Also, displace the drilling fluid used in the previous stage for the one that will be used for drilling underbalanced.

4. Install the rotating head rubber and the nitrogen unit. If the well conditions allow, the rotating head rubber should be installed after drilling the casing shoe.

Maintaining steady state conditions rigorously requires that the rotating head rubber last during the entire drilling process. Since this is the equipment in the UBD system that is most likely to require replacement due to wear, we should wait to use it until it is strictly necessary. Therefore, it is recommended that the casing shoe be drilled before installing the rotating head rubber. This will be possible when the ECD is greater than the reservoir pressure equivalent density, but less than the fracture pressure equivalent density.

5. Divert the circulation path toward the UBD surface equipment and unload the well by simultaneously injecting nitrogen and drilling fluid until the designed two-phase steady state flow condition is reached. As shown in figures 5.3 and 5.4, these are the designed initial conditions from which the time dependent computer program starts its calculations. These two-phase steady state flow conditions should be induced before starting drilling to understand the two-phase circulation system, to identify any possible adjustments required, and also to calibrate the computer program predictions for the actual system.

Since the flow goes through the UBD surface equipment, pseudosteady state conditions can be identified as nitrogen flow rate outputs become stable. At first, this operation will require additional nitrogen consumption, but later on it will be compensated by the gas production if proper two-phase steady state flow conditions are established from the very beginning.
Figure 6.4, in which bottomhole pressure (left vertical scale) is represented by the blue dotted line, the reservoir pressure (left vertical scale) by the green horizontal line, the nitrogen injection flow rate (right vertical scale) by the orange circles, the drilling fluid injection flow rate (right vertical scale) by the black triangles, and the casing choke pressure (right vertical scale) by the purple crosses, shows the measured field data from Muspac 52 well in Mexico. This well was drilled after the Muspac 53 well with almost the same conditions as those described in Section 5.4.1. Underbalanced conditions were not reached in either well during drilling.

![Figure 6.4 Measured field data from Mexican well Muspac 52](image)

Figure 6.4 Measured field data from Mexican well Muspac 52.

Figure 6.4 illustrates the two-phase pseudosteady state flow conditions achieved during about two hours (from 22.5 hour to 24.5 hour) after the simultaneous injection of nitrogen and drilling fluid started at the standpipe at 21.4 hour. The figure also shows that after the first pipe connection at 25 hour the pseudosteady state condition was lost and never regained. Additionally, the figure shows that for a 2608 m (8556 ft) well, the time it takes for achieving two-phase flow pseudosteady state flow conditions after the simultaneous injection of nitrogen and drilling fluid started is about one hour.

As explained in Chapter 4, Section 4.1, this unloading process is not predicted by the numerical solution implemented to solve the conservative equations. However, using the in-situ mixture velocity at each axial increment could make a good estimation of the time it will last.

6. After unloading the well, increase the nitrogen injection flow rate, so that underbalanced conditions can be induced sometime after such nitrogen flow rate increment.
Considering the current limitation of the proposed model, described in Chapter 5, Section 5.5, the real time for reaching underbalanced conditions after the nitrogen injection change will approximately be twice as much than that estimated by the computer program.

7. Meticulously monitor injection pressure, choke pressure, injection flow rates, drillstring weight, pit volumes, and especially well flow returns to identify the time at which reservoir influx occurs.

8. After inducing underbalanced condition, start manipulating the controllable parameters (choke pressure, drilling fluid and nitrogen injection flow rates) as designed.

As soon as underbalanced conditions are induced, the reservoir influx will cause continued decrease in the bottomhole pressure. Therefore, it is necessary to control the reservoir influx rate and preventing bottomhole pressure from decreasing below operating limits. In the simulations described in Chapter 5, Sections 5.1.1 and 5.1.2, this was done by decreasing first the choke pressure, which causes further bottomhole pressure reduction and therefore gas production. Then, to compensate this, the nitrogen injection was gradually interrupted allowing the gas produced to replace it. After that, only the choke pressure is manipulated to maintain the bottomhole pressure and reservoir influx at the designed pseudosteady state conditions.

9. After drilling the kelly down, preserve underbalanced conditions and maintain pseudosteady state flow conditions by making the connection as follows:

9.1 While circulating the drilled cuttings out, slowly lift and lower the length of the kelly to ensure that the open hole is in gauge.

9.2 Lift the length of the kelly and increase the choke pressure as designed while gradually interrupting the drilling fluid injection. The choke pressure manipulation should allow reservoir fluid flow increases by a rate approximately equal to the normal standpipe rate.

9.3 Make the connection. Since only drilling fluid was being injected before the connection, a conventional pipe connection procedure may be implemented. During the connection the bottomhole pressure and reservoir influx must be maintained under control by manipulating the choke pressure as designed.

9.4 To restart surface injection, choke pressure and drilling fluid injection must be carefully manipulated so that continuous underbalanced conditions and low bottomhole pressure variation can be maintained. Reduce the choke pressure while progressively increasing drilling fluid injection until regaining previous underbalanced pseudosteady state flow conditions. The choke pressure manipulation should allow reservoir fluid flow decreases by a rate approximately equal to the normal standpipe rate.

With this UBD flow control procedure, continuous reservoir, and therefore annulus flow will hopefully be maintained during both drilling and pipe connection operations.
Consequently, fluid segregation which is the main cause of pressure spikes\textsuperscript{8,43}, will no longer occur.

As only drilling fluid was being injected before breaking the connection, continuous drilling fluid flow from the drillstring into the annulus is expected during the connection time due to the U-tube effect. Since a conventional pipe connection can be made, which takes less than two minutes when well coordinated, the bottomhole pressure fluctuation effects associated with connections due to segregation can furthered be mitigated.

9.5 Repeating this drilling-connection procedure, continue drilling to the desired depth.

Although additional work is necessary to improve the time dependent computer program predictions, this UBD flow control procedure represents an economical alternative to achieve increasing well productivity through the formation damage prevention during the drilling process.

In this UBD flow control procedure, specialized execution of properly designed UBD operations is proposed to maximize the use of natural energy available from the reservoir through the proper manipulation of controllable parameters such as nitrogen and drilling fluid injection flow rates and choke pressures. The following improvements to the UBD technique should be achieved by implementing this flow control procedure in UBD operations.

1. Continuous reservoir, and therefore annulus flow can be maintained during both drilling and pipe connection operations. Therefore, fluid separation in the wellbore is eliminated.

2. Bottomhole pressure fluctuation is mitigated and maintained within a designed UBD pressure window. Therefore, formation damage is avoided from the beginning to the end of the drilling process.

3. No expensive additional tools, such as the Closed Loop Continuous Circulation System\textsuperscript{15} to attempt continuous circulation during a connection or the use of an additional parasite casing or tubing string, are required.

4. Reservoir influx, flow through the UBD surface equipment, and surface pressure can be maintained within desirable safe conditions.

5. Nitrogen consumption is only required to unload the well and induce underbalanced conditions.

6. Conventional pipe connections can be practiced.

7. Differential sticking problems are minimized.
8. Reliance on drillstring float valves to minimize fluid separation can be eliminated for most operations because nitrogen injection is only required during the initial unloading of the well.

This UBD flow control procedure is based on the philosophy of maintaining continuous annular flow by using reservoir fluids to substitute for injection fluids during both drilling and connection operations. Therefore, this method will only apply to wells that will flow without artificial lift and within safe limits. It has been evaluated favorably in this study for oil-gas wells whose reservoir energy will flow oil and natural gas at rates similar to common nitrogen and drilling fluid injection flow rates within the operating window limits on bottomhole pressure. Although some development efforts are still necessary to improve the time dependent computer program, the gas-condensate well simulation would lead to the conclusion that due to the wellhead pressure restriction and safety, this UBD flow control procedure would be much more difficult to apply successfully in gas, high pressure, and very high deliverability wells.
CHAPTER 7
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

This chapter summarizes the overall results and conclusions obtained throughout the development of this work and presents important recommendations to improve the ultimate result of this investigation and the computer programs developed to support it.

7.1 Summary

In jointed-pipe UBD operations, the bottomhole pressure must be maintained between two specific pressure boundaries. Therefore, the UBD hydraulic system, which consists of a compressible multiphase mixture including the formation and injected fluids, has to be accurately designed so that bottomhole pressure fluctuations triggered by the interruptions of steady state operating conditions to make a pipe connections can be mitigated, and short-period overbalanced conditions avoided. This requires that controllable parameters such as nitrogen and drilling fluid injection and choke pressure be analyzed in conjunction with production rates of reservoir fluids at a variety of conditions to determine the proper operating limits and sequences.

Consequently, in this research to design UBD hydraulic systems, a new comprehensive, mechanistic steady state model for pressure predictions throughout a well during UBD operations was developed. It was based on mechanistic models that generally perform better than empirical correlations. Then, a mechanistic, time dependent model, which still requires some development to improve its predictions, was developed. After that, both steady state and time dependent models were utilized to predict gas and liquid flow rates, choke pressures, and two-phase flow parameters during UBD pipe connection simulations for representative field conditions. Finally, based on the simulation results and on the field experience gained during field trips made throughout the development of this work, a UBD flow control procedure was proposed.

7.1.1 Comprehensive, Mechanistic Steady State Model
The ability to accurately predict wellbore pressure and two-phase flow parameters is critical for the development of a UBD flow control procedure. Since most of the pressure prediction approaches used in current practice for UBD are based on empirical correlations, which frequently fail to accurately predict the wellbore pressure, a new comprehensive, mechanistic steady state model for pressure predictions throughout a well during UBD operations was developed in this research. The comprehensive model is composed of a set of state-of-the-art mechanistic steady-state models for predicting flow patterns and calculating pressure and two-phase flow parameters in bubble, dispersed bubble, and slug flow. This model takes into account the entire flow path including downward two-phase flow through the drill string, two-phase flow through the bit nozzles, and upward two-phase flow through the annulus. More rigorous, analytical modifications to the previous mechanistic models for UBD give improved wellbore pressure predictions for steady state flow conditions.

The model was implemented into a FORTRAN computer program that performs an iterative two-phase flow analysis on a discretized wellbore. The well is divided into many axial
increments and each increment is treated separately. Both drillstring and annulus may have sections of as many different cross-sectional area as desired.

The results of using this new, comprehensive model were extensively validated with two different sets of data. First, the predicted wellbore pressures were compared against field data measured while drilling two Mexican wells using nitrified mud, and second, compared with full-scale experimental data obtained from the literature. These validations showed that the model allows wellbore pressure predictions with an average absolute error less than 2.5% of the actual wellbore pressure. Additionally, a comparison of the model results against four different commercial UBD computer programs, which primarily rely on empirical correlations, confirmed the expectation that mechanistic models perform better in predicting two phase flow parameters in UBD operations for actual field conditions.

### 7.1.2 Mechanistic Time Dependent Model

The unsteady state or transient conditions occurring during UBD pipe connections are extremely complex to estimate. Besides, literature reports state that dynamic models that claim to handle such transient conditions are already available but not yet fully validated\textsuperscript{80}. Therefore, in this research, instead of trying to rigorously predict the bottomhole pressure fluctuations occurring during an UBD pipe connection, a procedure for reducing them using the reservoir energy through manipulation of the gas and drilling fluid injection flow rates and choke pressure is alternatively proposed. Thus, a less complex time dependent computer program was proposed to achieve this goal.

This mechanistic, time dependent model consists of numerically implementing the comprehensive, mechanistic steady state model into a one-dimensional drift-flux formulation of the two-phase flow conservation equations, which coupled with a reservoir inflow performance equation, should allow estimating reservoir influxes, wellbore pressures and two-phase flow parameters as a function of the bottom hole pressure variation caused by changes in nitrogen and drilling fluid injection flow rates and choke pressures. An explicit finite-difference numerical method was implemented to simultaneously solve the one-dimensional form of the three-equation drift-flux model, the mechanistic steady state pressure drop model, and the well deliverability models. This numerical solution was also implemented into a FORTRAN computer program that finds the solution marching outward on an open domain, from initial conditions while satisfying a set of boundary conditions.

No actual field data was available to compare the performance of the proposed time dependent model to predict the bottomhole pressure variation and reservoir influx under simultaneous adjustments of nitrogen and drilling fluid injection and choke pressure, such as those required during drilling and pipe connection operations to maintain underbalanced conditions. Therefore, the validation of the time dependent model was carried out with the field data available from the Agave 301 well in Mexico, the experimental data gathered by Lopes\textsuperscript{37} in a full-scale well of the Blowout Prevention research Well Facility at Louisiana State University and the one set of complete data from another simulator\textsuperscript{38}. This validation showed that this less complex, mechanistic, time dependent model can potentially be used to help develop a procedure for avoiding or reducing the bottomhole pressure fluctuations occurring during UBD pipe connections. However, it was determined that when a gas flow rate change occurs, the time
dependent model computes higher pressures changes than would really be experienced as explained in the following section.

7.1.3 Simulation Scenarios
Both steady state and time dependent models were used to simulate drilling and pipe connection operations under flowing reservoir conditions. Actual reservoir data from two different Mexican wells, in which the UBD technique was being employed, were used as input data, to simulate conditions at which simultaneous adjustments of nitrogen and drilling fluid injection flow rates and choke pressures allowed maintaining the bottomhole pressure at a desired value. The concept applied was for the reservoir influx to substitute for the interrupted nitrogen injection during drilling and nitrogen and drilling fluid during a pipe connection.

First, the steady state model was used to determine hydrostatic-friction curves to identify if the circulating system was operating on the hydrostatic or friction dominated side and to determine the magnitude of the hydrostatic, friction, and acceleration pressure drops under different two-phase flow conditions to select conditions to operate on the friction-dominated side. Then, the time dependent model was run to simulate drilling and pipe connection operations. First, an oil-gas well, with a gas-oil ratio of 287 m³/m³ or 1613 scf/bbl, was considered at the conditions for the very first connection to be made and, then, the conditions at which the very last connection had to be made to ensure that the procedure can potentially be implemented from the beginning to the end of the drilling process. And second, a gas-condensate well, with a gas-oil ratio of 3789 m³/m³ or 21277 scf/bbl, was considered. In this simulation, surface nitrogen injection was easily substituted with reservoir influx during drilling. However, the choke pressure increase required during the pipe connection simulation, to offset the loss of friction when injection is interrupted, was much higher than the maximum allowable wellhead pressure. This caused the simulation to terminate. However, problems with the time dependent model appear to cause the rapidity and maybe the amplitude of this pressure change to be greatly overstated.

From the time dependent model results, it was also possible to determine: whether proper hole cleaning requirements are met, total volume of reservoir fluids that would be expected during the drilling process, appropriate UBD surface equipment (pressure and capacity ratings), volume of nitrogen required for the entire operation, surface storage capacity required, and fluid transfer necessity.

Using representative field conditions, an additional simulation was carried out to study the model limitation to appropriately predict flow parameters after a gas flow change. From this simulation, it was determined that even though the time-dependent model predictions are quite good for the initial and final steady state, before and after a gas flow change, they are less accurate during the transition in between the two steady states and do not properly predict the transport of fluids in the well during the transition period.

The present numerical solution of the time dependent computer program essentially forces a new steady state through the entire well after every time step such that the mass transport, after a gas flow change, occurs in few time steps and that most of it takes place during the first one. This happens because the current numerical solution, uses the same gas and liquid flow rates to compute two-phase flow parameters at node C (Figure 4.2) of each axial increment along the
wellbore and at each time step. This causes a new volume fraction of gas all along the well for the first time step after a change in a control condition.

Therefore, additional development efforts are still necessary to improve the predictions of the proposed model. A new, iterative procedure for determining the two-phase flow condition in each cell dependent on fluid transfer from the upstream cell over a given time step is proposed to solve this problem.

7.1.4 UBD Flow Control Procedure
Improving bottomhole pressure control for UBD operations so that underbalanced conditions can be maintained not only during drilling but also during pipe connections was the main focus of this research. Therefore, an underbalanced flow control procedure was proposed based on the computer program results obtained from comprehensive, mechanistic, steady state model and mechanistic, time dependent models and the field experience gained during field trips made throughout the development of this work.

In this UBD flow control procedure, a design stage is first proposed. In this stage, the UBD hydraulic system is designed by first determining the maximum UBD pressure window. Then, the potential reservoir influx during drilling, and the magnitudes of the hydrostatic, friction, and acceleration pressure gradients are predicted for a variety of possible two-phase flow conditions to define a range of conditions that fit within the designed UBD pressure window. After that, the optimum combination sets of nitrogen and drilling fluid injection flow rates and choke pressures that will allow keeping the bottomhole pressures within the designed UBD pressure window are determined. These bottomhole pressures are selected to maintain the reservoir influx and wellhead pressure within safe limits and guarantee underbalanced conditions during drilling and pipe connection operations. After that, the flow control execution stage must follow. It basically consists in properly maintaining the previously designed UBD hydraulic system by specialized manipulation of controllable parameters such as nitrogen and drilling fluid injection flow rates and choke pressure, so that reservoir influxes can substitute the interrupted surface nitrogen and drilling fluid injection during both drilling and pipe connection operations.

7.2 Conclusions

1. Defining the operating conditions required to maintain continuous underbalanced conditions and bottomhole pressures within a desirable UBD pressure window has typically relied on designing the UBD hydraulic system using computer programs. In this research, a new and very accurate, two-phase, steady state model and computer program were developed. Extensive validation with actual field and full-scale experimental data showed that the model allows wellbore pressure predictions with an average absolute error less than 2.5%.

2. A two-phase flow, steady state model comparison against four different UBD computer programs showed that inaccurate wellbore pressure predictions might be expected from a model that relies on an empirical, two-phase flow method or on simplified, mechanistic models Consequently, this comparison concluded that the very good performance of the proposed two-phase flow steady state model is due to the fact that it is composed of a
more complete set of state-of-the-art, mechanistic steady state models. This comparison also showed that even the simplified mechanistic models perform better than empirical correlations.

3. The simultaneous solution of the accurate, two-phase flow, steady state model, the one-dimensional form of the three-equation drift-flux model approximated by finite differences, and a well deliverability model numerically implemented into a time dependent model represents a potential solution for designing the complex UBD hydraulic system. However, detailed information, such as geology and reservoir characteristics, operational problems, and especially field measurements, required to validate this numerical implementation under actual reservoir flow conditions is, unfortunately, usually withheld by the operators as confidential information. Thus, the time dependent model was validated versus available literature data for two-phase flow conditions from a full-scale experiment without reservoir flow and from a separate computer simulator.

4. Although development is necessary to improve the time dependent model predictions, the time dependent model validation results showed that if changes in gas or liquid flow rates are made gradually, reasonably accurate predictions from this model may be expected. Also, the convenience of controlling the hydraulic system with gradual choke adjustments, which are better predicted by the model, instead of gas or liquid flow rate changes, makes this time dependent model a potential tool to design UBD hydraulic systems.

Therefore, for reservoir conditions similar to those of these oil-gas Mexican fields, the simulation results show that it is not necessary to maintain continuous circulation if the nitrogen and drilling fluid injection flow rates and choke pressures are appropriately manipulated so that the reservoir energy can substitute for the surface injection during UBD pipe connections.

5. Both the steady state and time dependent models primarily rely on comprehensive mechanistic models to predict flow pattern and two-phase parameters. Thus, they are largely free of additional limitations introduced by the use of empirical correlations.

6. Pressure spikes caused by the interruption of steady state conditions during UBD pipe connections represent unsteady state or transient conditions that are extremely complex to model and predict. Indeed, the current transient two-phase flow models available in the industry have only shown partial success in reproducing the wellbore pressure for the only published pipe connection experiment in a full-scale well following steady state conditions. Considering that the drilling time between connections is usually insufficient to achieve steady state conditions, predictions must account for the subsequent pipe connections and are therefore even more complex.

In this work, the concept of modeling large UBD bottomhole pressure fluctuations was avoided by attempting to maintain steady state conditions from the beginning to the end of the drilling process. Simulation results showed that maintaining continuous annular
flow during drilling connections attenuated bottomhole pressure fluctuations. Logically, this should occur because of continuous flow minimizing the fluid segregation in the annulus. However, the problems with the current time dependent program prevent it from giving conclusive predictions.

7. The goal of this research was to improve bottomhole pressure control for UBD operations. Thus, combining the UBD hydraulic system with the reservoir inflow performance through steady state and time dependent models implemented into this computer programs allowed proposing an underbalanced drilling flow control procedure. This procedure potentially represents an economical solution to maintain continuous underbalanced conditions during both drilling and pipe connection operations. Thus, several improvements to the UBD technique should be achieved by implementing this flow control procedure where applicable, specifically in wells that can flow without artificial lift and within appropriate safety limits. One of these improvements is that bottomhole pressure fluctuation is mitigated and maintained within a designed UBD pressure window. Therefore, formation damage is avoided from the beginning to the end of the drilling process.

7.3 Recommendations

1. Due to the necessity of accurate two-phase flow predictions to properly maintain the UBD hydraulic system within a designed UBD pressure window and to the fact that mechanistic models perform better than empirical correlations, it is recommended that prediction methods based on phenomenological or mechanistic models be used for planning UBD operations.

2. Although the proposed two-phase flow mechanistic model gives highly accurate wellbore pressure predictions, additional work to improve the injection pressure calculations through the drillstring are recommended. These calculations currently have an error on the order of 7.5%. More comprehensive, mechanistic downward flow, models, such as those developed by Caetano33 for upward flow, should be substituted for the current simplified model used in the drillstring predictions of the proposed model. Additional investigations of gas liquid mixtures in downward flow in pipes would contribute to the improvement because currently this issue has not received enough attention.

It was identified that in the current UBD operations in Mexico, the dominant two-phase flow patterns are dispersed bubble, bubble, and slug flow and that churn and annular flow could only exist at conditions very close to the surface. Therefore, a simplified annular flow model was implemented. However, in high-pressure gas wells, annular flow could occur in a considerable well section. Therefore, it is also recommended to implement a comprehensive mechanistic model to predict annular flow so that accurate predictions can also be expected when high gas and very low liquid flow rates are expected.

3. The time dependent model performance showed that the simulator predicts much faster reactions than can be achieved in practice. As mentioned above, this happens because the current numerical solution relies on the new gas and/or liquid flow rates entered at node
B (Figure 4.2) at each time step to predict flow parameters at node C of each axial increment along the wellbore.

Therefore, the future work recommended for improving the model is to implement a new iterative two-phase flow analysis to compute pressure and two-phase flow properties at node C of each axial increment. This new iterative analysis should calculate the mass transport from cell to cell as a function of the mixture velocity governed by the in-situ flow conditions.

4. The time dependent model does not perform drillstring predictions. Therefore, it is also recommended that drillstring effects be included in the program, so that surface nitrogen injection before and/or after a connection can be analyzed to further improve the UBD flow control procedure proposed in this work.
REFERENCES


97. Department of Technology Development: Summary of Underbalanced Drilling Activities in Mexico.


APPENDIX A

FULLY DEVELOPED TAYLOR BUBBLE PARAMETERS

The hydrodynamic model developed for slug flow in annuli considers steady state, one dimensional, axisymmetric flow. The physical model of gas bubbles in a fully developed slug flow is shown schematically in Figure A-1.

![Figure A-1 Physical model and hydrodynamic parameters of fully developed slug flow.](image-url)
Since the flow characteristics at any cross-sectional plane vary with time due to the intermittent nature of the slug pattern, a conventional modeling strategy is to consider a unit cell consisting of one Taylor bubble and its surrounding liquid film, plus one adjacent liquid slug. Therefore, without any relative velocity, Taylor bubbles and liquid slugs rise steadily at a translational velocity given by

\[ u_T = 1.2u_m + 0.345 \left( \frac{gD_{ep}(\rho_L - \rho_G)}{\rho_L} \right)^{0.5} \]  

(A.1)

For a coordinate system traveling at the translational velocity, slug variables are independent of time and vary only with respect to space. Thus, taking upward flow as positive, a liquid mass balance from liquid slug to Taylor bubble gives

\[ (u_T - u_{ls})H_{ls} = (u_T + u_{ls})H_{lb} \]  

(A.2)

Considering the flow of a slug unit through a fixed plane in a region where fully developed flow exists (plane \(A\) in Figure A.1a), the passing time of a Taylor bubble is

\[ \Delta t_{TB} = \frac{L_{TB}}{u_T} \]  

(A.3)

Thus, the liquid volume in the Taylor bubble zone may be calculated by

\[ V_{lb} = u_{ls} A_{ls} \frac{L_{TB}}{u_T} \]  

(A.4)

Similarly, the passing time of a liquid slug is

\[ \Delta t_{LS} = \frac{L_{LS}}{u_T} \]  

(A.5)

and hence the liquid volume in the liquid slug zone can be calculated by

\[ V_{ls} = u_{ls} A_{ls} \frac{L_{LS}}{u_T} \]  

(A.6)

Since the total time for a slug unit to pass through plane \(A\) is \(\Delta t_{SU} = \Delta t_{TB} + \Delta t_{LS}\), the total volume of liquid in the slug unit can be estimated by

\[ V_{su} = u_{sl} A_p \Delta t_{SU} \]  

(A.7)
Expressing the areas in equations (A.4) and (A.6) as a function of liquid holdup, substituting equations (A.3) and (A.5) into equations (A.7), and then equating it to the sum of equations (A.4) and (A.6). The overall liquid balance gives

\[ u_{SL} = u_{ls} H_{ls} \frac{L_{LS}}{L_{SU}} - u_{ls} H_{ls} \frac{L_{TB}}{L_{SU}} \]  

(A.8)

Considering that the liquid and gas phases in the liquid slug behave analogous to fully developed bubble flow conditions, from Equation (3.28) the in-situ gas velocity in the liquid slug is

\[ u_{Gls} = C_0 u_m + H_{ls} u_m \]  

(A.9)

Then, instead of assuming that the liquid holdup in the liquid slug is constant, different methods\textsuperscript{70-73} were implemented to calculate it as the slug flow progresses. For the actual well conditions used to validate the model better performance was obtained while using the approach followed by Ansari et al\textsuperscript{70}, which is given by

\[ H_{ls} = \frac{u_{SG}}{0.425 + 2.65u_m} \]  

(A.10)

Considering that the free falling film thickness in the Taylor bubble reaches a terminal constant value, Fernandes et al\textsuperscript{66} proved that the film thickness equation proposed by Wallis\textsuperscript{64} can be used to estimate its value. Thus, for turbulent flow

\[ \delta = 0.0682 \left( \frac{\mu_L^2}{g(\rho_L - \rho_G)\rho_L} \right)^{1/3} \left( \frac{4\rho_L u_{ls} \mu_{ls} \delta}{\mu_L} \right)^{2/3} \]  

(A.11)

Based on the annular slug flow geometry, Caetano\textsuperscript{33} gave the following expression for the liquid holdup in the Taylor bubble zone.

\[ H_{ls} = \frac{4\delta(D_{ic} - \delta)}{D_{ic}^2 - D_{of}^2} \]  

(A.12)

Fernandes et al\textsuperscript{66} and Taitel et al\textsuperscript{60} have shown that over a wide range of flow conditions the slug length in upward two-phase flow in pipes has a fairly constant value equal to 16 pipe diameters. Later, using the hydraulic diameter concept, Caetano\textsuperscript{33} confirmed this for annuli. Hence, the liquid slug length is given by

\[ L_{LS} = 16D_h \]  

(A.13)
The numerical solution of equations (A.1) through (A.13) gives the necessary hydrodynamic parameters \( \left( L_{ls}, L_{tb}, u_F, u_{ls}, u_{lb}, H_{ls}, H_{lb} \right) \) for pressure drop calculations if fully developed slug flow is considered.

First, knowing the liquid and gas flow rates and fluid properties at in-situ conditions, the liquid holdup in the liquid slug, the in-situ gas velocity in the liquid slug, the translational velocity, and the length of the liquid slug can be calculated with Equations (A.10), (A.9), (A.1) and (A.13), respectively. Then, the simultaneous solutions of equations (A.2), (A.8), (A.11), and (A.12) give the length of the Taylor bubble, the in-situ liquid velocity in the Taylor bubble, the in-situ liquid velocity in the liquid slug, and the liquid holdup in the Taylor bubble.
APPENDIX B

DEVELOPING TAYLOR BUBBLE PARAMETERS

In appendix A, the Taylor bubble length is considered to be sufficiently long for the existing curvature in the cap bubble region to have a negligible influence on the pressure drop predictions. However, substantial error can be introduced if the Taylor bubble is in its developing stage because the film thickness varies continuously along the developing Taylor bubble zone rather than reaching a constant terminal value as in fully developed Taylor bubble. Figure B.1 schematically shows the physical model and the hydrodynamic parameters of developing slug flow.

![Physical model and hydrodynamic parameters of developing slug flow.](image)

- $u_{G,ra}$ - In-situ gas velocity in the developing Taylor bubble
- $u_{L,ra}$ - In-situ liquid velocity in the developing Taylor bubble
- $H_{l,ra}$ - Liquid holdup in the developing Taylor bubble
- $\ell$ - Developing length of the bubble cap
- $L_{DTB}$ - Developing length of the Taylor bubble
- $L_{DSU}$ - Developing length of the slug unit

Figure B.1 Physical model and hydrodynamic parameters of developing slug flow.

If the Taylor bubble consists only of a cap bubble, the film thickness varies continuously along the film zone. Then, it is necessary to determine the cap length $L_C$ and compare it with the fully
developed Taylor bubble length $L_{TB}$ to determine the basis for the flow parameters to be used in calculating pressure losses, which are dependent on which flow condition exists.

Caetano\textsuperscript{33} stated that the Taylor bubble cap length starts at the bubble nose and ends when the film thickness decreases to the Nusselt film thickness value, which corresponds to the thickness expected for a free falling film under laminar flow, and can be predicted as\textsuperscript{74}

$$\delta_n = \left[ \frac{3u_{t_\text{eq}}A_{t_\text{eq}} \mu_L}{\pi D_{ic} g (\rho_L - \rho_G)} \right]^{1/3} \quad (B.1)$$

Once the film thickness decreases to the Nusselt film thickness value, the resulting Taylor bubble area in an annulus is\textsuperscript{33}

$$A_{g_{eq}} = 0.25\pi \left[ (D_{ic} - 2\delta_n)^2 - D_{G_{eq}}^2 \right] \quad (B.2)$$

Similarly to Equation (A.2), a gas mass balance expressed as a function of areas ($A_L=H_L A_P$) gives

$$(u_f - u_{G_{eq}}) A_{g_{eq}} = (u_f - u_{G_{eq}}) A_{g_{eq}} \quad (B.3)$$

A net volumetric flow rate across the plan B-B in Figure B.1 gives

$$u_m A_p = u_{G_{eq}} A_{g_{eq}} - \left| u_{t_\text{eq}} \right| A_{t_\text{eq}} \quad (B.4)$$

Then, knowing that $A_{t_\text{eq}} = A_p - A_{g_{eq}}$ the simultaneous solution of Equations (B.1) through (B.4) together with Equations (A.9) and (A.10) are used to calculate the Nusselt film thickness, $\delta_n$.

Applying the Bernoulli’s theorem to the top region of the Taylor bubble, McQuillan and Whalley\textsuperscript{74} defined the velocity of the liquid film relative to the nose of the bubble as

$$u_{t_\text{eq}}^L = u_f + \left| u_{t_\text{eq}} \right| = \sqrt{2g\ell} \quad (B.5)$$

where $\ell$ is the distant from the nose of the bubble to the point of interest. Combining Equations (B-4) and (B-5) and taking $\ell = L_C$, the length of the Taylor bubble cap can be calculated by

$$L_C = \left[ \frac{u_f}{g} + \frac{u_{G_{eq}} A_{g_{eq}}}{A_{t_\text{eq}}} - \frac{u_m A_p}{A_{t_\text{eq}}} \right]^2 \quad (B.6)$$

From the comparison of the bubble cap length $L_C$ calculated with equation (B.6) and the Taylor bubble length $L_{TB}$ estimated with equation (A.8), if $L_C > L_{TB}$, the slug flow is in its developing
stage. Therefore, different hydrodynamic parameter values \( (L_{\text{dfB}}, u_{\text{tgb}}, H_{\text{tgb}}, L_{\text{dSU}}) \) are required for pressure drop estimations.

The gas volume in the developing Taylor bubble is given by

\[
V_{G_{\text{d}}} = \int_{0}^{L_{\text{dfB}}} A_{G_{\text{d}}} d\ell
\]  

As liquid holdup can be expressed as a function of areas, combining Equations (A.2) and (B.5) the area of gas in the developing Taylor bubble can be estimated by

\[
A_{G_{\text{d}}} = \left(1 - \frac{(u_T - u_{tgs})H_{tgs}}{\sqrt{2g\ell}}\right)A_p
\]  

Since the gas volume in a developing slug unit is equal to the gas volume in the developing Taylor bubble plus the gas volume in the liquid slug, the gas volume in the developing Taylor bubble can be also expressed as

\[
V_{G_{\text{d}}} = V_{G_{\text{dsv}}} - V_{G_{lS}} \tag{B.9}
\]

As in the case of fully developed Taylor bubble, considering the passing time of a developing slug unit through a fixed plane in a region where developing slug flow exist, equation (B.9) can be written as

\[
V_{G_{\text{d}}} = u_{SG}A_p \left(\frac{L_{\text{dfB}} + L_{lS}}{u_T}\right) - u_{G_{lS}}A_p (1 - H_{lS}) \frac{L_{lS}}{u_T} \tag{B.10}
\]

Substituting Equations (B.8) and (B.10) into (B.7), and performing the integration, an implicit equation for the developing length of the Taylor bubble is obtained

\[
\left(1 - \frac{u_{SG}}{u_T}\right) L_{\text{dfB}} - \frac{2(u_T - u_{tgs})H_{tgs} L_{\text{dfB}}^{3/2}}{\sqrt{2g}} - \left[\frac{u_{SG} - u_{G_{lS}} (1 - H_{lS})}{u_T}\right] L_{lS} = 0 \tag{B-11}
\]

Using the Newton-Raphson root-finding method to solve Equation (B.11), rather than the quadratic solution proposed by Caetan33 and used by Ansari70, facilitates the calculation of the developing length of the Taylor bubble, \( L_{\text{dfB}} \).

After calculating the developing length of the Taylor bubble \( L_{\text{dfB}} \), the in-situ liquid velocity in the developing Taylor bubble can be calculated rearranging Equation (B.5) as follows

\[
u_{tgb} = \sqrt{2gL_{\text{dfB}}} - u_T \tag{B-12}
\]
Since the gas volume in the developing Taylor bubble can also be expressed as \( V_{G,tb} = A_{G,tb} L_{dTB} \), solving Equation (B.7) after substituting Equation (B.8) the average liquid holdup in the developing Taylor bubble is

\[
H_{t,av} = \frac{2(u_f - u_{l,av})H_{t,ss}}{\sqrt{2gL_{dTB}}} \quad (B.13)
\]

The developing slug unit length is defined by

\[
L_{dTB} + L_{LS} = L_{dSU} \quad (B.14)
\]

Thus, after defining that developing Taylor bubbleslug flow exists, the hydrodynamic parameters for pressure drop predictions are obtained by the solution of equations (B.11) through (B.14).
APPENDIX C

COMPUTER FLOW DIAGRAM FOR THE COMPREHENSIVE, MECHANISTIC STEADY STATE MODEL

Input data
\( q_{Gic}, q_L, T_s, P, \rho_L, \mu_L, \gamma_G, D_{IC}, D_{OT}, \varepsilon, G_g, D_T, D_{N1}, D_{N2}, D_{N3}, AG, DSG \)

Select \( \Delta Z, GC = 1, Depth = 0 \)

Guess the total pressure drop corresponding to the length increment
\( \Delta p_{\text{guess}} = g \rho_1 \Delta Z \)

\( p_2 = p_1 + \Delta p \)

\( T_2 = T_1 + \Delta T \)

Calculate \( T, \bar{p} \)

Calculate
\( z, \rho_G, B_G, q_{Gs}, A_L, u_{SL}, u_{SG}, u_m, A_p, \sigma, u_m, \mu_G \)

Program the flow pattern prediction models given in section 3.3 and with the superficial liquid and gas velocities defined the existing flow pattern.

Use the corresponding flow behavior prediction model (section 3.4).

Dispersed bubble flow pattern

Bubble flow pattern

Slug or Churn flow pattern

Annular flow pattern (for annular Geometry)
Calculate liquid holdup, mixture density, mixture viscosity, and friction factor. If slug flow is the existing flow pattern, the hydrodynamic parameters must be calculated as well.

Calculate $\Delta p$

$\Delta p_{\text{guess}} = \Delta p$

$\text{ABS}(\Delta p-\Delta p_{\text{guess}}) \leq 0.001$

Yes

$\text{Depth} = \text{Depth} + \Delta Z$

$\text{Depth} > D_f$

Yes

$\text{Depth} = \text{Depth} - \Delta Z$

$\Delta Z_1 = D_f - \text{Depth}$

$\Delta Z = \Delta Z_1$

Print depth, wellbore pressure, and any flow parameter.

$p_1 = p_2$

$T_1 = T_2$

B

A

C

D
Calculate pressure drop through the nozzles and the nozzle upstream pressure

Considering drillstring flow pattern prediction and flow behavior models, nozzles upstream pressure and temperature, and decrements instead of increments, the same flow diagram can be used for drillstring computations.
APPENDIX D
DERIVATION OF THE ONE-DIMENSIONAL DRIFT-FUX MODEL EQUATIONS

Conservation of Mass
Considering the volume element \( \pi^2 \Delta Z \) shown in the Figure 1-D in which mass is allow to flow into and out of the volume element during a period \( \Delta t \).

![Figure 1-D Volume element \( \pi^2 \Delta Z \).]

Using the mass conservation principle, which can be stated as

\[
\text{Mass entering volume element} - \text{Mass leaving volume element} = \text{Rate at which mass accumulates during interval} \, \Delta t
\]

The mass entering the volume element during \( \Delta t \) is given by

\[
(q\rho)_Z = \pi^2 (\rho u)_Z \quad (D.1)
\]

The mass leaving the volume element during \( \Delta t \) is given by

\[
(q\rho)_{Z+\Delta Z} = \pi^2 (\rho u)_{Z+\Delta Z} \quad (D.2)
\]

The rate at which mass accumulates during the interval \( \Delta t \) is given by
Combining Equations (D.1) through (D.3) as stated by the conservation of mass principle written above

\[
\frac{\pi^2 \Delta \varepsilon (\rho_{z,\text{M}} - \rho_i)}{\Delta t}
\]

Dividing both sides of Equation (D.4) by the volume of the element \( \pi \Delta Z \)

\[
\frac{-[(\rho u)_{z,\Delta z} - (\rho u)_z]}{\Delta Z} = \frac{(\rho_{z,\text{M}} - \rho_i)}{\Delta t}
\]

Taking the limits in each side of Equation (D.5) as \( \Delta Z \) and \( \Delta t \) approach zero, the resulting equation, which takes into account the mass accumulation, is the continuity or mass conservation equation.

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial Z} = 0
\]

For two-phase flow, the liquid holdup or liquid fraction in the system is defined by \( H_L = (1 - \alpha) \). Then, the continuity Equation (D-6) may be applied to each phase. Thus

For the liquid phase

\[
\frac{\partial (\rho_L H_L)}{\partial t} + \frac{\partial (\rho_L H_L u_L)}{\partial Z} = 0
\]

and for the gas phase

\[
\frac{\partial [\rho_G (1 - H_L)]}{\partial t} + \frac{\partial [\rho_G u_G (1 - H_L)]}{\partial Z} = 0
\]

**Conservation of Momentum**

It is necessary to introduce the principle of conservation of momentum so that friction pressure losses and flow resistance (friction factor) can be taking into account in fluid flow in pipes.

Applying the Newton’s second law that states that the sum of all external forces acting on the system (fluid weight, shear forces, and pressure forces) is equal to the time rate of change of linear momentum of the system and the concept of total derivative (local acceleration plus convective acceleration), the conservation of momentum can be defined by
\[
\frac{\partial}{\partial t} (\rho u) + \frac{\partial}{\partial Z} (\rho u^2) = -\frac{\partial p}{\partial Z} \frac{2\tau_w}{r} - \rho g \tag{D.9}
\]

where the first and second terms of the left hand side of equation (D.9) represent the local and convective acceleration, respectively, whereas, the first, second, and third terms of the right hand side correspond to the pressure forces, shear forces, and fluid weight, in that order.

Defining the friction factor (Fanning friction factor) as the ratio of the wall shear stress to the kinetic energy of the fluid per unit volume

\[
f_f = \frac{2\tau_w}{\rho u^2} \tag{D.10}
\]

Solving Equation (D.10) for the wall shear stress and substituting it into equation (D.9), the momentum equation becomes

\[
\frac{\partial}{\partial t} (\rho u) + \frac{\partial}{\partial Z} (\rho u^2) = -\frac{\partial p}{\partial Z} \frac{2f_f \rho u^2}{d} - \rho g \tag{D.11}
\]

In two-phase flow the density and velocity are considered as the density and velocity of the mixture. Therefore, the momentum equation can be written for the mixture as a whole as

\[
\frac{\partial}{\partial t} [\rho_L H_L u_L + \rho_G (1 - H_L) u_G] + \frac{\partial}{\partial Z} [\rho_L H_L u_L^2 + \rho_G (1 - H_L) u_G^2] = -\frac{\partial p}{\partial Z} \frac{2f_f \rho u^2}{D_h} - \rho_m g \tag{D.12}
\]
APPENDIX E

COMPUTER FLOWCHART FOR THE MECHANISTIC TIME DEPENDENT MODEL

Input data
\[ M_N, M_{NG}, T_{NG}, P_R, q_{oil}, \rho_{oil}, API_{oil}, GOR, h, \sum h \]

\[ \Delta t, TAI \]

\[ t = 0 \]

\[ M = 0 \]

\[ Z = TAI \]

\[ t = 0 \]

Yes

From MSSM get flow parameters at initial conditions \( A(Z,t) \) and \( D(Z\cdot1,t) \)

No

\[ Z = TAI \]

Yes

\[ (p_{bh})_{B(Z,t+1)} = (p_{bh})_{B(Z,t+1)\text{guessed}} \pm 1 \]

No

\[ M = 0 \]

Yes

\[ (p_{bh})_{B(Z,t+1)\text{guessed}} = (p_{bh})_{A(Z,t)} \]

With the MSSM get flow parameters at node \( B(Z,t+1) \)

No

\[ (p_{bh})_{B(Z,t+1)\text{guessed}} \]

No

\[ (p_{bh})_{B(Z,t+1)\text{guessed}} = (p_{bh})_{A(Z,t)} \]

Yes

With the MSSM get flow parameters at node \( B(Z,t+1) \)

I
With the MSSM compute flow parameters at \( C(Z-1,t+1) \)

Using the finite difference representation of the time dependent equations, calculate the wellbore pressure at \( C(Z-1,t+1) \)

\[
(p_{wp})_{C(Z-1,t+1)} \approx (p_{wp})_{C(Z-1,t+1)_{previous}}
\]

\[
A(Z,t+1) = B(Z,t+1)
\]
\[
B(Z-1,t+1) = C(Z-1,t+1)
\]
\[
D(Z-1,t+1) = C(Z-1,t+1)
\]

**A**

\( Z = Z - 1 \)

**No**

\( Z = 1 \)

**Yes**

\( M = M + 1 \)

**B**

\( p_{wp} \) at \( C(t+1) \) \( \approx p_{choke} \)

**No**

**Yes**
Calculate the oil and gas flow rates flowing into the wellbore

Yes

No

Print results

Input the new gas and liquid injection flow rates and choke pressure

$t = t + 1$

$t = t_{desired}$

Yes

STOP

No

Print results

$\left( p_{bh} \right)_{B(2,t+1)} < p_R$
APPENDIX F

ADJUSTMENTS FOR GASES/LIQUIDS MIXTURES

While drilling underbalanced, when reservoir fluids enter into the wellbore and mix with the nitrogen and the injected drilling fluid, a multiphase mixture is consequently formed. However, taking into consideration the assumptions stated in section 3.1.1 and followed during the development of both steady state and time dependent models, the multiphase hydraulic circulation system occurring during UBD operations may be theoretically simplified to a two-phase flow system in which only a mixture of liquid and gas flows. Therefore, for the time dependent computer program, which takes into account reservoir influxes, weighting factors are used to calculate a unique gas density, gas viscosity, liquid density, and liquid viscosity.

In the computer code additional equations must be implemented to handle reservoir influx when the bottomhole pressure is less than the average reservoir pressure. Therefore, the following additional computations are required.

1. Estimate the nitrogen and natural gas compressibility factors at the in-situ pressure and temperature. Several equations or algorithms are available to calculate the compressibility factor, but the most accurate ones are trial and error or iterative processes. Therefore, the iterative Dranchuk and Abou-Kassem equation of state was chosen to estimate this fluid property.

2. Using the engineering equation of state for a gas, given by equations \( F.1 \), calculate the nitrogen and natural gas densities at the in-situ pressure and temperature

\[
\rho_{\text{Gas}} = \frac{2.7 \gamma_{\text{Gas}} p}{zT}
\] \( (F.1) \)

3. Estimate the nitrogen and natural gas formation volume factors at the in-situ pressure and temperature with

\[
B_{\text{Gas}} = \frac{0.0283 zT}{p}
\] \( (F.2) \)

4. Calculate the nitrogen and natural gas flow rates at the in-situ pressure and temperature with

\[
q_{\text{Gas,c}} = q_{\text{Gas,c}} B_{\text{Gas}}
\] \( (F.3) \)

5. Thus, the total gas flow rate used by the time dependent computer program is

\[
q_G = q_{N_c} + q_{NG_c}
\] \( (F.4) \)

6. Calculate the fraction of nitrogen or natural gas in the mixture of gases as follows
7. Calculate the density of the mixture of gases, which is the total gas density used by the time dependent computer program

\[ \rho_G = f_N \rho_N + f_{NG} \rho_{NG} \]  \hspace{1cm} (F.6)

8. Considering that the mixture of liquids is incompressible, the total liquid flow rate used by the time dependent computer program is

\[ q_L = q_{DF} + q_{oil} \]  \hspace{1cm} (F.7)

9. Calculate the fraction of drilling fluid or oil in the mixture of liquids with

\[ f_{DF} = \frac{q_{DF}}{q_{DF} + q_{oil}} \]  \hspace{1cm} (F.8)

10. Calculate the density of the mixture of liquids, which is the total liquid density used by the time dependent computer program

\[ \rho_L = f_{DF} \rho_{DF} + f_{oil} \rho_{oil} \]  \hspace{1cm} (F.9)

11. The most widely used method to estimate gas viscosity presented by Lee et al.\textsuperscript{94} was chosen to calculate the nitrogen or natural gas viscosity as a function of the in-situ temperature

\[ \mu_{Gas} = A \times 10^{-4} \exp\left(1000B \rho_{Gas}^C\right) \]  \hspace{1cm} (F.10a)

\[ A = (9.4 + 0.02M_{Gas})T^{1.5}/209 + 19M_{Gas} + T \]  \hspace{1cm} (F.10b)

\[ B = 3.5 + 0.01M + 986/T \]  \hspace{1cm} (F.10c)

\[ C = 2.4 - 0.2B \]  \hspace{1cm} (F.10d)

12. Thus, the viscosity of the mixture of gases used by the time dependent computer program is

\[ \mu_G = f_N \mu_N + f_{NG} \mu_{NG} \]  \hspace{1cm} (F.11)

13. The approach proposed by Beggs and Robinson\textsuperscript{95} was used to compute the oil viscosity changes caused by temperature

\[ \mu_{oil} = A_{oil} \left(10^x - 1.0\right)^{\gamma_{oil}} \]  \hspace{1cm} (F.12a)

\[ x = YT^{-0.163} \]  \hspace{1cm} (F.12b)
\[ Y = 10^{2.14} \]  
\[ Z_{oil} = 3.0324 - 0.0203(\text{API}) \]  
\[ A_{oil} = 10.715(150)^{0.515} \]  
\[ B_{oil} = 5.44(150)^{-0.338} \]

14. Thus, the viscosity of the mixture of liquids used by the time dependent computer program is

\[ \mu_t = f_D\mu_D + f_{oil}\mu_{oil} \]  

The mechanistic steady state computer program at the step 7 of the algorithm steps described in section 3.5.1 performs these additional computations.
VITA

Carlos Perez-Tellez was born in Mexico City. He received a bachelor of Science degree in petroleum engineering from the National Polytechnic Institute in 1987.

In June 1987, he joined the National Oil Company called PEMEX. He worked as a field engineer mainly in completion, work over and cementing operations in the onshore fields of Chiapas and Tabasco, Mexico.

In August 1993, he entered the National University of Mexico from which he received a master of science degree in petroleum engineering in 1995.

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In August 1997, he entered Louisiana State University to pursue a doctoral degree in petroleum engineering.

In March 2002, he returned to PEMEX where he became a member of the Technology Development Department in Villahermosa, Tabasco, Mexico.