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Writing in the Geometry Classroom

Amy Lynn Rome
Louisiana State University and Agricultural and Mechanical College

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Amy L. Rome
B.A., Winthrop University, 1996
J.D., University of Tennessee College of Law, 2004
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ABSTRACT

This study sought a time-efficient way to implement writing in ninth-grade Geometry. Students wrote responses to five expository writing prompts spread out over the spring semester of the 2014-2015 school year. Students' first attempts were graded and returned to them along with feedback in the form of a teacher-written exemplar. Students rewrote assignments to improve their grades. All first and second attempts were collected and evaluated. We found that students were more successful after seeing the exemplar. Moreover, on assignments occurring later in the semester, more students were able to score in the top categories of the writing assignments on their first attempts. This suggests that students were not only able to improve their performance within attempts of the same assignment, but they were also able to improve their performance between assignments.

CHAPTER 1. INTRODUCTION

The purpose of this thesis is to describe specific effective ways to implement writing in a mathematics classroom. The literature suggests that there are numerous benefits to incorporating writing in a math curriculum. I introduced writing in three sections of Geometry in order to improve students' conceptual knowledge of mathematics and their ability to communicate that knowledge effectively, with the hope that it would lead to higher End of Course (EOC) test scores for the students and a higher School Performance Score for WFHS.

I gave my ninth grade geometry students five expository writing assignments spread out over the duration of the spring semester of the 2014-2015 school year. Students were asked to solve a math problem and to explain and justify their solution processes. Each assignment required that certain criteria be satisfied to receive full credit. Students' responses were graded and returned to them. For four of the five writing assignments, I also provided students with a prototype or exemplar, which was a teacher-written response to a problem similar to the particular writing assignment.

The collection of student writing served two purposes. Students were graded on the assignments, and the responses also had to be sorted in a way that would allow me to determine whether providing the students feedback in the form of an exemplar was beneficial. Because the writing assignments were graded, I had already decided the criteria that students would need to satisfy in order to receive full credit. I used these criteria to create categories into which I sorted the responses. The categories were based on the number of criteria satisfied for the particular writing assignment. If students satisfied all of the criteria for an assignment, the writing was placed in Category 1. If students satisfied all but one of the criteria for an assignment, the writing was placed in Category 2. The number of criteria depended on the number of steps required to fully work the problem and provide an explanation using correct mathematical

vocabulary. The number of categories in each writing assignment ranged from three categories to five categories. All first and second attempts of the writing assignments were sorted into categories to help me determine the impact that expository writing coupled with feedback in the form of an exemplar had on student performance.

Giving students the exemplar was a time-efficient way for me to provide meaningful feedback to all of my students at one time. Students were then offered an opportunity to rewrite the assignment so that they could receive a higher grade. All of the first and second attempts of the writing assignments were collected and sorted into the categories described in Chapter 5. By reviewing all of the responses, I was able to comment on common errors made by students and see the improvement in the students' ability to clearly and effectively communicate their mathematical knowledge. Based on my review of the students' writing, I was able to conclude that providing students with feedback in the form of an exemplar was an effective way to improve students' abilities to communicate their mathematical knowledge.

Chapter 2 presents a literature review of studies by researchers who implemented writing in their mathematics classrooms. I include studies illustrating the benefits of using reflective journal writing and expository writing into varying levels of mathematics classrooms. Chapter 3 describes the population and setting of my research. I also explain my motivation for wanting to include expository writing in my classroom rather than a blend of journal writing and expository writing. I also explain why I opted to provide feedback to students in the form of an exemplar rather than providing feedback in another way. Chapter 4 details the process that I used to assign the expository writing tasks as well as the types of assignments given. I explain the procedure that I followed when giving the assignments, the type of feedback that I provided to the students after their first attempts, and the process used for students to rewrite the assignments if desired.

Chapter 5 sets forth the findings of my study. I break this section down by writing assignment and provide various examples of student writing that fell into the different categories after their first attempts and their second attempts. I also provide the exemplars that I gave to the students. Finally, Chapter 6 provides a conclusion to my thesis and includes suggestions for teachers who would like to implement writing in their mathematics classrooms in the future.

CHAPTER 2. LITERATURE REVIEW

Research suggests that there is a benefit to integrating writing in the mathematics classroom. There are multiple ways to do this. Many teachers use journal writing and expository writing. Writing assignments may also be more formal and extended, such as a research paper on an influential mathematician or the history of pi. The purpose of my thesis is to describe specific effective ways to implement writing in a mathematics classroom. This review of the literature reports what is known about the varying levels of effectiveness of incorporating journal writing and expository writing in mathematics classes.

2.1 Reflective Journal Writing

Reflective journal writing has a number of distinctive features. Reflective journal writing is typically assigned to students in the form of a short writing prompt. Students are given a short amount of time, such as five minutes in class, to respond to the prompt in writing. The journal assignments are sometimes given daily and sometimes weekly depending on the preference of the teacher. Through reflective journal writing, students produce a chronological record of their learning experiences, often analyzing and commenting on their feelings or concerns about what the class has been learning or their recent performance on an assessment. Teachers read the journal entries and comment back to students typically within a short period of time such as one or two school days. The term "journal writing" is used in different ways; however, only studies incorporating reflective journal writing are reviewed in this section.

In a study conducted on disadvantaged adult students, Powell (1989) used freewriting and journal writing in his college mathematics classes in order to empower students through writing in two ways: "(i) to promote students' awareness of and facility in the use of writing as a vehicle of learning, and (ii) to put students at the center and in control of their own learning by engaging

them in reflection and critical reflection on mathematical experiences" (Powell and Lopez, 1989, p. 162). Students engaged in freewriting for five minutes at the beginning of each class meeting. Freewriting was used in a meditative sense and gave students the opportunity to "clear the mind of preoccupations" (Powell and Lopez, 1989, p. 165). Students wrote about anything they wanted, related or not related to mathematics. Students completed journal writing assignments during every class meeting, and the assignments were collected weekly and returned with comments from the instructor. The journal writing assignments were designed to retain an expressive function and provided "a substantive account of how and what one was learning and feeling" (Powell and Lopez, 1989, p. 167). Powell concluded that journal writing was a "powerful medium for dialogue between instructor and students" (Powell and Lopez, 1989, p. 167; see also, Ishii, 2002, pp.18-19).

Having students create a permanent record of their thoughts and experiences is a benefit of reflective journal writing. Burns (2004) herself is a veteran elementary and middle school teacher, and she reports using a variety of writing formats in her mathematics classes. Of journal writing Burns says "[w]hen students create ongoing records about what they're doing and learning in math class, they have a chronological record of their learning experiences" (2004, p. 30; see also Spalding and Wilson, 2004).

In addition to the benefit of students being able to make a record of their experiences, Spalding and Wilson found three other benefits when students engaged in reflective journal writing, including that students were able to create a relationship with their teacher, able to safely express frustrations and concerns about math, and able to facilitate internal dialogue (Spalding and Wilson, 2004; see also Borasi and Rose, 1989; Freitag, 1997; and Miller, 2006).

In her action research study, Streeks (2007) found similarly that reflective journal writing can help establish a rapport between students and teacher. Streeks implemented a combination of reflective journal writing and expository writing in her eighth grade math class, although she refers to all assignments generally as journal writing. Every Friday, students wrote reflective journal entries summing up their thoughts about the week. Students communicated their thoughts and concerns about what they had been learning in class that week. She read the journal writings and commented back in the journals, which opened the lines of communication between the students and teacher (Streeks, 2007, p. 8). Students appreciated that they could ask questions to the teacher that they did not feel comfortable asking in front of the whole class (Streeks, 2007, p. 10).

Another benefit to journal writing is that students can become more fluent with mathematical vocabulary. In her action research project, Lefler (2006) studied the impact of journal writing on the performance of her mathematics students. Although Lefler refers to the assignments as journal writing, the writing completed in this study can be categorized as a mixture of journal writing and expository writing. From the reflective journal writing prompts, she concluded that journal writing helped students become “more adept at using correct terminology in both writing and speaking” (Lefler, 2006, p. 12). For example, a student self-corrected when giving a verbal explanation about fractions, revising her language from “top number” and “bottom number” to “numerator” and “denominator” (Lefler, 2006, p. 12). Lefler noted that she will continue to use writing in her math classes, but she intends to decrease the frequency of journal writing assignments from daily to weekly because she believes the students burned out on writing during the school year (2006, p. 22).

Matt Coaty, a specialist in educating the gifted at an elementary school in Illinois, uses reflection journals in his elementary classes. He originally used these journals with English assignments but realized how beneficial they could be when used with mathematics instruction. After each unit assessment, he asks students to reflect on how they feel about their performance on the assessment and allows them to include a drawing to aid in communication. He comments back on these journal entries. Students say that the journals let them reflect on how they are doing in class, and some students say that writing journal responses helps them to set math goals (Coaty, 2012, para. 3).

Reflective journal writing can also result in an improvement in students' attitudes toward writing about mathematics in general. McAllister (2013) studied the effects of journal writing and expository writing on proof-writing ability in a high school geometry class. McAllister's students completed journal writing assignments requiring them to reflect on learning processes and expository writing assignments requiring them to write explanations of solutions to word problems using details and rules. All writing was kept in a file for the duration of the year. "The journal writings were used for students to reflect on their learning processes and to comment on features of the course that they felt beneficial to them. The journals gave students an opportunity to become comfortable when writing in a mathematics class and an opportunity to write without the stress of getting a correct answer" (McAllister, 2013, p. 14). At the end of the first year of the study, McAllister concluded that all students showed growth in their ability to communicate conceptual knowledge of mathematics and to use vocabulary and reasoning skills (2013, p. 39). Survey results indicate an improvement in students' attitudes and beliefs about whether writing assignments were beneficial to their ability to write geometric proofs (McAllister, 2013, p. 52).

Based on the above literature, reflective journal writing offers a number of benefits to students when incorporated in a mathematics classroom. Students are able to voice concerns about difficulties they are having in class and are able to ask questions they do not feel comfortable asking in front of the entire class. When teachers respond to the journal entries, a rapport is established between the teacher and students and the lines of communication open among them. An increase in the use of correct mathematical terminology could be a benefit. Journal writing can also lead to an improvement in students' attitudes and beliefs about writing in mathematics class in general.

2.2 Expository Writing

Expository writing has a number of distinctive features. Expository writing can be completed in class or outside of class and can take as little as ten minutes or as much as forty-five minutes or more. Expository writing requires students to provide explanations, reasons, or procedural steps in a logical order. Students might describe their thought processes while solving a mathematical problem or construct knowledge by connecting new ideas with prior knowledge. The teacher analyzes and assesses the writing, often using a rubric.

"Expository writing incorporates many of the same ideas that journal writing does, but is used more in an explanatory capacity as opposed to a reflection on a particular experience. This type of writing allows students to catch mistakes in addition to remembering and understanding problems better" (Ishii, 2001, para. 3; see also, Johanning, 2000, para. 20). "Expository writing is writing that is intended to describe and explain mathematical ideas" (Quealy, 2014, p. 20). Like reflective journal writing, incorporating expository writing assignments in a mathematics classroom has been shown to have benefits to students.

A benefit to introducing expository writing in the mathematics classroom is that students begin to connect new ideas to prior knowledge and to construct mathematics on their own. Clarke et. al. (1993) reported on a four-year study of journal writing in mathematics, in which students in grades 7 through 11 completed journal writing under the headings of “what we did,” “what we learned,” and “examples and questions.” It should be noted that the journal writing was not reflective in nature but instead was more characteristic of expository writing (Shield and Galbraith, 1998, p. 31). The researchers categorized the writing into three main categories, specifically “recount,” in which students described mathematical concepts, “summary,” in which students summarized connections between concepts, and “dialogue,” in which students constructed mathematical knowledge. Writing in this last category indicated that students were showing “their ability to connect new ideas with what they already know” and “that they are actively constructing mathematics” (Clarke, 1993, p. 248). The result of implementing expository writing was that students interpreted math in a way personal to them, which allowed them to develop meaning and make connections.

In one study, when students were provided the opportunity to respond to open-ended expository writing assignments rather than simply solve problems that were only computational in nature, students were able to develop their mathematical understanding. Johanning (2000) found that students benefitted from independently responding to expository writing prompts before working in collaborative groups. In her study of forty-eight gifted seventh graders and advanced eighth graders in a pre-algebra class, students were regularly asked to respond to conceptual and problem-solving writing prompts and explain the reasoning for their solutions. At the beginning of the school year, Johanning described her expectations, explained the format of the responses, provided a grading rubric, and shared examples with the students (2000, pp. 3-

4). Students were given time to write a response to a prompt and then brought their responses to a group to be discussed and used for collaboration. In groups, students were able to see alternative methods for solving the problem and to increase their understanding of the problem. Students described three main benefits from writing: "[w]riting helped students find their mistakes, helped them remember the problem better, and helped them understand the problem better" (Johanning, 2000, p. 8). Johanning noted that the group environment is very important because "[t]hese ideas need to be valued and respected in order for the students to be comfortable with writing and group collaboration" (2000, p. 11). She states that "[i]f writing is to benefit student understanding in mathematics, it has to be used often and as a regular part of instruction" (Johanning, 2000, p. 13).

Another benefit to students is that when students use expository writing to explain their thought processes while solving problems, evidence of metacognitive behaviors is apparent. Pugalee (2001) studied the writing of twenty ninth-grade algebra students to investigate whether writing about their mathematical problem-solving processes showed evidence of metacognitive behaviors. Students generally wrote weekly about the processes that they employed while solving problems. Then, students wrote daily during a two-week enrichment period, in which they were "instructed to record every though that came to mind while solving the problem" (Pugalee, 2001, p. 238). Teachers responded daily to the students' writing and returned comments to the students during the next class period. "Students were encouraged to read the teacher's responses and questions in order to improve the detail of their responses on subsequent assignments" (Pugalee, 2001, p. 238). For data collection, students wrote responses to one problem per day for six consecutive school days and were instructed to write anything that came to mind while working the problem (Pugalee, 2001, p. 238). Pugalee found that "[a]

metacognitive framework was evident in the students' writings about their problem solving processes" (2001, p. 242) and stated that there is potential for writing to support metacognitive behaviors that are important in mathematics classes.

Another benefit to students who participated in expository writing in math class is improved performance in their courses. Pugalee (2004) looked at the writing of ninth grade algebra students to investigate the impact of writing during mathematical problem solving. He analyzed the students' written and verbal descriptions of their mathematical problem solving processes. Students solved six problems, two easy, two medium, and two difficult. Students wrote out every thought that occurred to them as they solved one of each type and were videotaped providing oral explanations while solving. The video tapes were transcribed and analyzed. There were some notable differences between students' written responses and their think-aloud responses. "Students who wrote about their problem solving processes produced correct solutions at a statistically higher rate than when using think-aloud processes" (Pugalee, 2004, p. 43). "This study demonstrates that writing can be a tool for supporting a metacognitive framework and that this process is more effective than the use of think-aloud processes" (Pugalee, 2004, p. 44).

In three studies conducted by LaMSTI graduates, teachers provided support to students engaged in expository writing. In these studies, students were able to improve their writing abilities. Chimwaza (2012) studied the impact of journal writing on her students' ability to solve geometry problems and explain their solution steps. The students were asked to solve problems and explain their solutions, which is a traditional function of expository writing. Writing assignments were given on Wednesdays and took the entire class period to complete. She created a template for her students to use when responding to the writing prompts. The template

contained eight questions that students answered when solving the given math problem; these questions were based on the Common Core State Standards for Mathematical Practice. Students were scored on a rubric that measured students' use of correct vocabulary, diagrams, and real-world connections as well as mathematical and grammatical correctness. Students wrote their entries on the right side of their journals, reserving the left side for later reflection (Chimwaza, 2012, p. 16). The teacher did not answer students' questions during the writing time period, which prompted students to engage in discussions with each other. Student writing was analyzed and classified into three categories. A student from each category was selected, and the writing of those students was included in the study as representative of each category. Chimwaza concluded that the work of all students, as represented by the three, showed "change and growth both in how they explained their reasoning and how correct it was" (2012, p. 45). She noted that the students in the study scored 14% higher than her previous year's students on the Geometry End of Course test, the state's standardized assessment for geometry students (Chimwaza, 2012, p. 45). All students also "improved their mathematical writing and reasoning abilities" (Chimwaza, 2009, p. 45).

Hargrave (2013) studied the impact of feedback on students' proof-writing ability. The feedback was in the form of a checklist and one-on-one teacher conferences. In his study, which took place at the end of the school year, students were required to write five proofs. Students were given a checklist for reference. Students were given a week to complete their proofs in two column format, during which time they were permitted to consult resources and use the checklist. After students turned in their proofs, each student received written feedback from the teacher and had a five to ten minute conference with the teacher. Students were given another week to write the proofs in paragraph format. Hargrave noted that students' proof writing performances

increased after the consultations with the teacher in which the rubric and checklist were discussed (Hargrave, 2013, p. 48).

As mentioned previously, McAllister (2013) studied the effect of writing assignments on proof-writing ability in a high school geometry class. McAllister's journal-writing assignments were discussed earlier in this thesis. The expository writing assignments required students to explain solutions to word problems in detail, following prescribed rules. All writing was kept in a file for the duration of the year. Students were tested for growth by completing the same test[1] at the beginning of the course and at the end of each quarter. Students were graded using a rubric measuring the use of vocabulary, correct use of vocabulary, grammatical correctness, and mathematical correctness. McAllister concluded that all students showed growth in their ability to communicate conceptual knowledge of mathematics and to use vocabulary and reasoning skills (2013, p.39).

As previously mentioned, McAllister continued her study for a second year. During the second year, students completed all writing assignments in composition notebooks. Students felt this was helpful because it allowed them to have all of their writing together and organized in one place where they could refer back to it. McAllister adapted Chimwaza's template (discussed previously) and referred to it as a framework. Students used the framework to evaluate and critique the writing of the class, and they discussed possible improvements and alternate ways to solve the problems. Students were also given a copy of the teacher's solution, and the class discussed its strengths and weaknesses. At the end of the year, McAllister noted an increase in the proof-writing ability of the students as well as an increase in their inclusion of mathematical vocabulary, postulates, and theorems in their responses. Students' attitudes and beliefs about writing in a mathematics class also improved (McAllister, 2013, p. 48).

[1] The test contained one algebra problem, one geometric proof of a theorem, and geometry problems.

Based on the above literature, when expository writing is implemented under the right conditions in a mathematics classroom, it offers a number of benefits. Students are able to connect new ideas with prior knowledge and actively construct mathematics. Students are able to find their own mistakes and remember the problem better. Expository writing can result in an increase in the performance of students on assessments, as compared to students not engaged in expository writing. Students also improve their communication of mathematical concepts, including the use of mathematical vocabulary and reasoning skills.

2.3 Limitations of Writing in the Mathematics Classroom

Some studies showed the limitations of implementing writing assignments in a mathematics class. Shield and Galbraith (1998) based their study on the three categories from Clarke et. al.'s 1993 research described previously in this thesis. In this study, students from three Grade 8 classes completed various expository writing tasks. Two types of expository writing tasks formed the data for the study. For the first task, students had to write a letter explaining a concept for an absent student. Students were specifically instructed to "explain all about…" rather than "how to do…" (Shield and Galbraith, 1998, p. 36). For the second task, students were asked to write an explanation of a mathematical concept to a person who was having difficulty understanding the concept (Shield and Galbraith, 1998, p. 37). The researchers analyzed the student writing and developed a coding scheme to describe the writing. This developed into a general model of the writing. The researchers discovered that "there was great consistency in the style of the writing examples collected throughout the study" (Shield and Galbraith, 1998, p. 41). The researchers noted that "[t]here are a number of parallels between the writing of the student's textbook and the student writing examples" (Shield and Galbraith, 1998, p. 44). Researchers observed "over the course of the study that there was little development in

the elaborateness of the students' writing, despite some attempts by the teachers to stimulate further elaboration through discussion. The students simply became more skilled at presenting their algorithmic style of writing" (Shield and Galbraith, 1998, p. 44). The researchers concluded that student writing is influenced by the format of the textbook, which also impacts the writing style of the teacher.

> The writing products of students in schools appear to be constrained by the models of mathematical presentation to which they have become accustomed. It will be a long-term task for teachers of mathematics to increase the meaningfulness of their students' mathematical writing in a way which promotes a higher level of thinking about the ideas. Such a change implies major shifts in the teaching practices and textbooks to which students are exposed throughout their school lives (Shield and Galbraith, 1998, p. 45-46).

In the previously described 2007 action research study, Streeks implemented a combination of reflective journal writing and expository writing in her eighth grade mathematics class. In the beginning of the study, students completed daily expository writing assignments requiring them to write out a solution to a problem; later this changed to a weekly schedule. Daily for two weeks, Streeks wrote out a response to a problem with the class so that they could see the writing process. Streeks noted, however, that despite engaging in journal writing and expository writing, most students continued to have difficulty explaining their solution processes in writing, even in situations when the students could work the problems completely and correctly (Streeks, 2007, p. 10).

In seeming contradiction to Pugalee's 2001 study, Porter (2000) found no significant benefit for her students who engaged in expository writing assignments. Porter used writing-to-learn-mathematics (WTLM) in her college calculus classes. When the students in the WTLM group were assigned questions, they were required to write out answers to be submitted in class and graded. The students who were not in the WTLM group were assigned the same questions but were not required to write out responses. Instead, they were asked to prepare for verbal

discussion in class, and they were graded on the discussion. An example of a problem given to each of the groups is shown below as Figure 1 (W denotes the WTLM group, and N denotes the Non-WTLM group). Porter found no significant difference between the number of procedural or conceptual errors made by students in the WTLM group and those who were not in the WTLM group. She proposed that it is the act of articulating ideas, in writing or verbally, that is essential to learning (Porter, 2000, p. 175).

W: (Students were encouraged to work in groups on this, and they were told that they should be prepared to discuss their answers next time in class.)

(1) Write in words, in terms of *distance* (not absolute value) what $|\ x - 8\ | < \partial$ means.
(2) Explain in words what it means to *solve* an inequality.
(3) Explain your steps and reasoning as you solve: $|\ 3x - 2\ | < 4$.
(4) Write $2 < x < 7$ in the form $|\ x - x_0\ | < \partial$, and explain how you did it.

N: (Students were encouraged to work in groups on this, and they were told they did not need to write anything down but to be prepared to discuss their answers next time in class.)

(1) Discuss with your group: What does $|\ x - 8\ | < \partial$ mean, in terms of *distance* (not absolute value)?
(2) Discuss with your group and define what it means to *solve* an inequality.
(3) *Show your steps* as you solve: $|\ 3x - 2\ | < 4$.
(4) Write $2 < x < 7$ in the form $|\ x - x_0\ | < \partial$, and show your work.

Figure 1. Sample Problem from Porter (2000) Study.

As the literature shows, not every attempt to implement writing in a mathematics classroom produces desired results. In some cases, students do not improve their mathematical writing even if they are able to solve the math problems correctly. In some cases, even when students are able to write out the solution steps to problems, the writing is often in the format of a textbook. This begs the question of the style in which we would like students to write. The studies in this literature review do not evaluate students' writing style; the assessments generally focus on mathematical correctness, use of vocabulary, and incorporation of a diagram. This focus can produce writing that is very textbook like. The writing is not incorrect, but style does not appear to be a concern for most researchers or students.

2.4 Takeaways from the Literature Review

The literature shows strong evidence of a benefit for students when writing is implemented in a mathematics classroom. Reflective journal writing impacts the attitudes and feelings of the students. Participating in reflective journal writing provides students with a voice to have a dialogue with the teacher. Students can ask questions or express frustrations to the teacher, which establishes a rapport between student and teacher. Expository writing impacts the academic performance of students. The literature shows expository writing to be an effective tool in assisting students to connect new ideas with prior knowledge, to deepen their knowledge of mathematical concepts, increase performance on assessments, and to communicate mathematically.

CHAPTER 3. SETTING AND MOTIVATION

In Chapter 3, I discuss the population and setting of my study, including specific information about the students at the school and the students who participated in the study. I also discuss my motivation for completing this study, including its contribution to the education profession.

3.1 Population and Setting

The data collected for this research came from students at Walker Freshman High School (WFHS) in Walker, Louisiana. This school is a ninth grade only school in a rural/suburban location that operates on a 4 by 4 block schedule, with courses lasting ninety minutes per day for one semester. During the 2014-2015 school year, 466 students were enrolled at WFHS. The data in this study came from eighty-eight (88) ninth grade students enrolled in my 2014-2015 spring semester Geometry classes. These three sections of Geometry were the only sections offered on our campus during the 2014-2015 school year with the exception of one section of gifted Geometry, which was also taught during the spring semester by a different teacher. Seventy-nine of the eighty-eight students (approximately 90%) took Algebra I in eighth grade. Seven of the eighty-eight students (approximately 8%) took Algebra I in the fall of 2014 and were recommended by their teachers to take Geometry in the spring semester. One of the eighty-eight students had been homeschooled for Algebra I instruction, and one of the eighty-eight students moved to Louisiana from a state with traditional year-long classes where he had already been enrolled in Geometry for the fall 2014 semester.

Of the eighty-eight students, fifty were male, and thirty-eight were female. Seventy-nine were white non-Hispanic, four were African American, and five were Asian. A prerequisite of the Geometry class was that the students had taken Algebra I and passed the Algebra I End of

Course (EOC) exam with a score of Good or Excellent. The average grade point average (GPA) for the school population at the end of the 2014-2015 school year was 2.56. The average GPA for my three Geometry classes were as follows: second block 3.51, third block 3.61, and fourth block 3.50. Eighty-seven of the eighty-eight students received a passing grade in the Geometry course, and eighty-one of the eighty-eight students (approximately 92%) scored Good or Excellent on the Geometry EOC, while seven of the eighty-eight students (approximately 8%) scored Fair.

3.2 Motivation

The literature review illustrated various ways that reflective journal writing and expository writing have been incorporated into all levels of mathematics classes and suggested that writing was an effective strategy that teachers could use to establish a good rapport with students and to increase the academic performance, reasoning, and vocabulary usage of students. Therefore, I decided to implement writing in my ninth grade geometry classes. My experience and results show mathematics teachers one method of incorporating writing in a geometry classroom that was successful.

I wanted to implement writing in my classes for two main reasons. First, writing across the curriculum has shown to be beneficial to students, and participation in this endeavor was encouraged by the administrators at WFHS. All teachers attended an in-service designed to help teachers integrate reading and writing techniques into their curriculum regardless of the subject being taught.

Second, I wanted to help my students improve their EOC scores. At the end of the spring semester, the students are required to take the Geometry EOC, which typically contains two or three constructed response questions, each with multiple parts. Often, students are required to

justify the steps in their solutions or to explain a mathematical concept. Therefore, helping students improve their expository writing was important for the students' success.

The success of the students' directly correlates to that of the school. Each year, the Louisiana Department of Education assigns each public school a numerical score (School Performance Score, or SPS) to reflect its level of success during the school year. There are four scoring categories. Because WFHS is a freshman only school, three of the four scoring categories do not apply to the school; therefore, WFHS receives all of its points for its SPS from the remaining scoring category, student performance on EOC tests, including those for Algebra I, Geometry, and Biology[2]. WFHS receives points if students score Good or Excellent on an EOC but receives no points if a student scores Fair or Needs Improvement. Improving the students' conceptual knowledge of mathematics and their ability to effectively communicate that knowledge would lead to higher EOC scores for the students and a higher SPS for WFHS.

Students typically take their Geometry EOC tests two to three weeks before the end of the semester. I often have difficulty covering the curriculum before the EOC test date. Therefore, I had to make good use of my class time when incorporating writing assignments into my lessons. I could not have students write simply for the sake of writing; I had to have a purpose for asking them to write. I decided to create writing assignments that were relevant to the curriculum, manageable in length, and fast to grade. I wanted the writing to be less about grammar and more about mathematical concepts. There were several options to choose from including a focus on proof writing, reflective journal writing, expository writing, and formal writing.

[2] WFHS also receives points for its SPS based on how our former students perform on EOC tests at Walker High School.

One of my strengths as a teacher is that I find it easy to establish a good rapport with my students where they feel comfortable asking questions and voicing their concerns, so I decided not to implement reflective journal writing in my classroom. I could not justify the time spent having students write about their feelings and perceptions about mathematical concepts, when the benefit from this writing is something I routinely achieve without assigning the writing. I decided to implement expository writing using prompts similar to those the students might see on the constructed response portion of the Geometry EOC. I originally intended to assign some of the writing as homework assignments and some as in-class assignments depending on how quickly I was able to progress through the curriculum during the semester; however, I decided to make four of the five assignments in-class assignments after I had a lower than expected completion rate for the first writing assignment, which was a homework assignment. Having students complete the four in-class writing assignments and subsequent attempts took approximately four hours of class time, which equates to approximately three days of class.

I decided to offer students feedback on their writing in the form of an exemplar. With the exception of the first writing assignment, I wrote out a prototype or exemplar problem to give to the students after the first attempt of each writing assignment. This exemplar showed students my response to a problem similar to the one in the particular writing assignment. Creating the exemplar took very little time, requiring only that I alter the problem slightly and write a response that fully answered the problem using proper mathematical vocabulary. By providing students the exemplars, I was able to show them what a full-credit response should look like.

Implementing writing in my classes answered all of the questions I had asked myself. I had considered whether the students would benefit from completing the writing assignments in general and whether they would be able to increase their EOC scores specifically. The literature

review answered these questions in the affirmative. My main concern was whether I would have the time to implement writing without compromising my ability to finish teaching the entire Geometry curriculum, and based on my calculations of the writing only taking three days of class, I felt the benefit outweighed the cost. I believe that the method that I used can be easily replicated by other mathematics teachers and that they will see similar results that I did.

CHAPTER 4. PROCESS

In Chapter 4, I discuss the process used to assign the expository writing tasks. I discuss the types of assignments given as well as the procedure that I followed when giving the writing assignments. Finally, I discuss the type of feedback that I provided to the students after their first attempts at the writing assignments and the process used for students to rewrite the assignments if desired.

Students were given five expository writing assignments that were spaced out approximately one month apart during the spring semester. The first writing assignment was given as a homework assignment. The remaining four assignments were completed in class. Students were given thirty minutes of class time to complete their responses. Students who requested more time were given more time. Most students did not take the entire thirty minutes to complete the writing assignments. The length of each assignment varied depending on the assignment but could typically be completed in three to five sentences.

Each assignment required students to communicate effectively in different ways. Writing Assignment 1 asked students to provide instructions to complete a task. Writing Assignment 2 asked students to provide algebraic and geometric justifications for conjectures as well as definitions for terms and variables. Writing Assignment 3 asked students to use precise terminology such as parallel, perpendicular, and congruent and to discuss the properties of quadrilaterals. Writing Assignment 4 asked students to find missing angle measurements and to explain the properties of angles underlying their solution steps. Writing Assignment 5 asked students to perform the dilation of a triangle when the center of dilation was not the origin and to use precise terminology such as translation, dilation, center of dilation, and scale factor.

The first writing assignment was completed during the first week of the spring semester before I had begun instruction on mathematical content. For this assignment, students were asked to write instructions telling an alien how to make a peanut butter and jelly sandwich. This assignment was given so that I could see what type of basic organization structure students used and the level of clarity and amount of detail students employed when given no specific guidance other than to use clear and effective communication. Students were given specific instructions for each writing assignment, which are detailed in Chapter 5, but all writing assignments asked students to use clear and effective communication. With the exception of the first writing assignment, each of the writing assignments was formatted similarly to a problem students had recently completed in class. On the day of a writing assignment, I reviewed the relevant problem with the students immediately before handing out the writing assignment. Students were encouraged to refer to their textbooks and notes when completing the writing assignments. After students completed the assignment, they turned it in to me for grading. I typically returned graded writing assignments within three school days. The grades were included in the weighted category "Graded Assignments/Projects," which makes up 25% of a student's grade.

Eighty-eight students were enrolled in my three sections of Geometry, but not all students turned in every assignment. All assignments were graded for mathematical correctness and whether the solution steps has been communicated effectively, with a focus on the use of vocabulary. With the exception of the first writing assignment, students were provided minimal teacher-written comments and an exemplar after their first attempt at each writing assignment. The exemplars were similar to the writing assignments that the students completed, often with only one element of the assignment changed. Students were then offered an opportunity to rewrite the assignments to increase their scores if they were not satisfied with the grade for their

first attempts. Students who opted to rewrite the assignments were given another thirty minutes to do so. Students were again permitted to refer to their textbooks and notes, along with the exemplar, to rewrite the assignments. Students turned in their responses, and I graded them and returned them typically within one week. Students who opted to keep their original scores occupied their time in other ways during the thirty minutes, including writing definitions for their unit vocabulary terms, completing makeup work, and copying class notes missed due to absence. The goal of these writing assignments was for students to communicate clearly and effectively when explaining the answers to the various mathematics problems and, thereby, deepen their understanding of the particular concept.

CHAPTER 5. ANALYSIS OF STUDENT WORK

In this chapter, I provide a descriptive analysis of the writing that I collected from students for each of the five writing assignments completed. In the sections that follow, I provide the text of each assignment that was given to the students and the context in which the assignment was given. Because the writing assignments were graded, I predetermined the criteria that students would need to satisfy in order to receive full credit. I used these criteria to create categories into which I could sort the students' writing. The categories were based on the number of criteria satisfied for the particular writing assignment. For example, a student's response would fall into Category 1 if the student satisfied all possible criteria for the assignment and would fall into Category 2 if the student satisfied all but one of the criteria for the assignment. In the sections that follow, I provide specific information about the categories for each writing assignment.

For each writing assignment detailed below, I provide samples of the students' first attempts of the writing assignment, organized by the categories into which the responses fell. I provide commentary on each of the samples and an overall summary of the first attempt responses, including a discussion of the common mistakes made by students. For Writing Assignments 2 through 5, I provide the exemplar that I gave to the students, which illustrates the specific feedback that the students received before they rewrote the writing assignment. Additionally, I provide samples of the students' second attempts of the writing assignment, organized by the same categories used to classify the first attempts. I include commentary on each of these samples and an overall summary of the second attempts of the writing assignment.

5.1 Writing Assignment 1

> Write directions that instruct an alien to our planet how to make a peanut butter and jelly sandwich. This assignment will be graded as follows: (1) Effective communication of

all relevant steps in the process (logical organization, effective word choice, proper tone, etc.) and (2) Proper use of writing conventions such as sentence formation, capitalization, spelling, grammar, and punctuation[3]. All writing must be inside the box.

I gave the students this writing assignment during the first week of January. I handed out the assignment and read the writing prompt aloud. The students were asked to complete the assignment for homework. Several students did not return the assignment the next day, which prompted me to give the remainder of the writing assignments as in-class assignments.

5.1.1 First Attempt of Writing Assignment 1

Examining and comparing papers, I found that most students identified five basic steps in the process of making a peanut butter and jelly sandwich. The steps that the students included are (1) collect all materials or ingredients, including bread, peanut butter, jelly, and a utensil; (2) take out the bread; (3) spread the peanut butter onto the bread; (4) spread the jelly onto the bread; and (5) put the two pieces of bread together (referred to as "the basic steps").

The majority of the students wrote only the basic steps for making a peanut butter and jelly sandwich. Several students attempted to be more specific than just the basic steps, and only a small percentage omitted basic steps. Based on my review of the students' responses, the writing fell into one of three categories.

5.1.1.1 Writing Assignment 1, First Attempt, Category 1

Responses that fell into Category 1 include more than just the basic steps for making a peanut butter and jelly sandwich. Each of these responses include extra instructions for the alien such as, what to do with the ingredients, where ingredients are located, or descriptions of the ingredients. Twenty-four of the seventy-seven students (approximately 31%) who completed this assignment had writing that fell into this category.

[3] After this assignment was given but before it was graded, I decided not to base students' grades on the proper use of writing conventions.

Example. In addition to stating the basic steps, the writer of Sample 1A first tells the alien where to find each of the ingredients and then tells the alien what to do with each ingredient. This student's response is representative of the other student writing in this category.

First you need a plate to rest it on, go and
get a plate from the cabinet. Now you need to
place it onthe counter. We need to grabthe
bread, go and get it from the food storage, or the
pantry. Now we need the peanut butter,
You can find it in the pantry. Its on the
second shelf next to where the bread was.
The jelly is in a purple container in the
refrigarator door. Now take two small slices
of bread and place them onthe plate. Using
a small butter knife, take the peanut butter and
spread it evenly on the bread. Now using a spoon
take some of the jelly and spread it on the
other slice. now join the two halves
and enjoy

Figure 2. Writing Assignment 1, First Attempt, Category 1, Sample 1A.

Example. The writer of Sample 1B tells the alien a list of materials to acquire and also provides extra information, beyond the basic steps, such as the direction to turn the lid of the peanut butter jar, that the alien should put down an item before picking up the next item, and the benefits of putting the two pieces of bread together slowly.

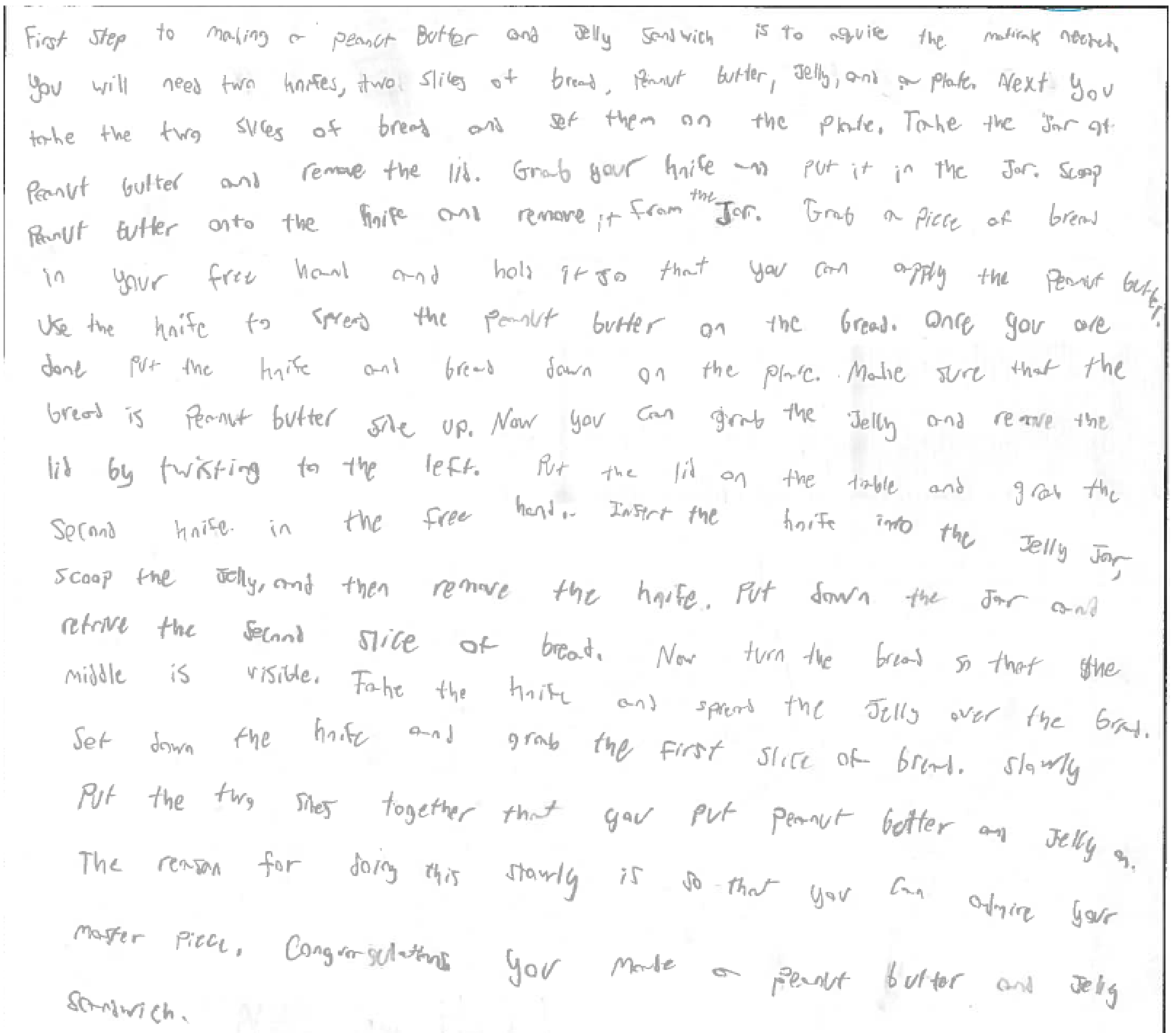
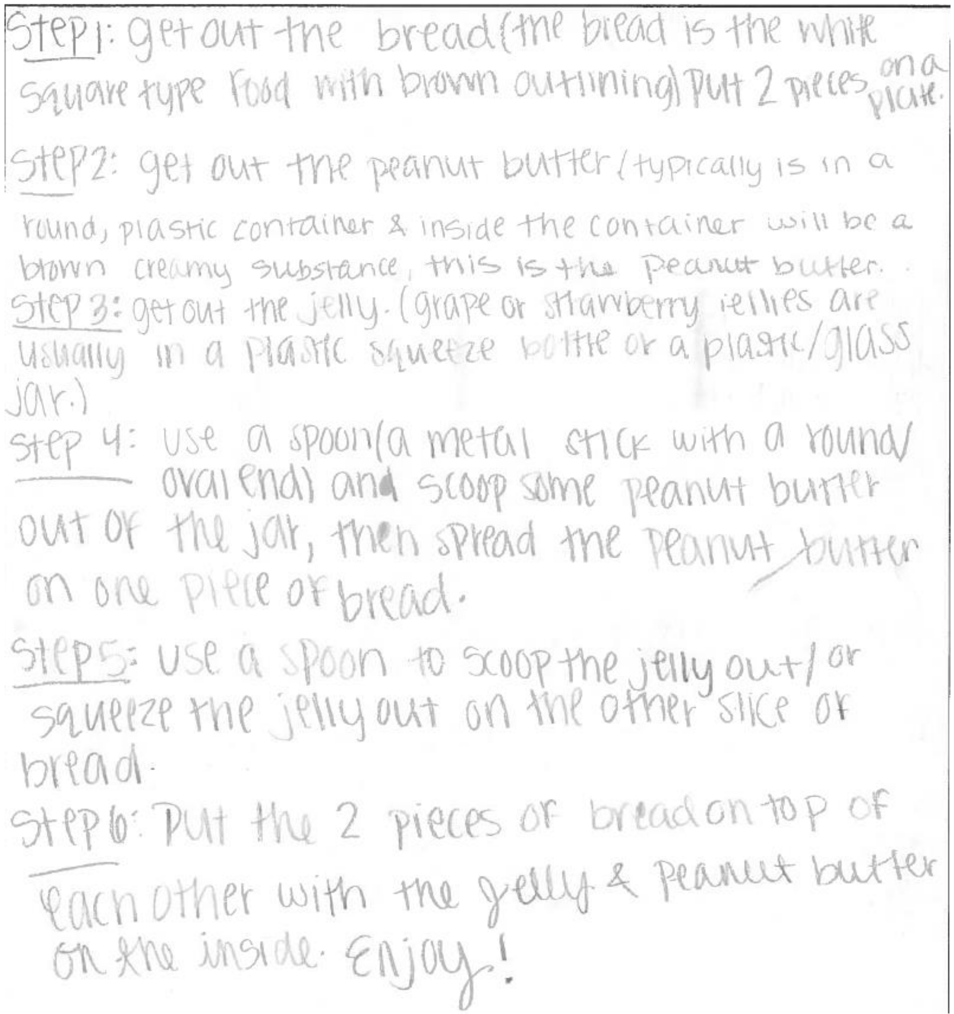

First step to making a peanut Butter and Jelly Sandwich is to aquire the materials needed. You will need two knifes, two slices of bread, peanut butter, Jelly, and a plate. Next you take the two slices of bread and set them on the plate. Take the Jar of peanut butter and remove the lid. Grab your knife and put it in the Jar. Scoop peanut butter onto the knife and remove it from the Jar. Grab a piece of bread in your free hand and hold it so that you can apply the peanut butter. Use the knife to spread the peanut butter on the bread. Once you are done put the knife and bread down on the plate. Make sure that the bread is peanut butter side up. Now you can grab the Jelly and remove the lid by twisting to the left. Put the lid on the table and grab the second knife in the free hand. Insert the knife into the Jelly Jar, scoop the Jelly, and then remove the knife. Put down the Jar and retrive the second slice of bread. Now turn the bread so that the middle is visible. Take the knife and spread the Jelly over the bread. Set down the knife and grab the first slice of bread. Slowly put the two slices together that you put peanut butter and Jelly on. The reason for doing this slowly is so that you can admire your master piece. Congratulations you made a peanut butter and Jelly sandwich.

Figure 3. Writing Assignment 1, First Attempt, Category 1, Sample 1B.

Example. The writer of Sample 1C enumerates each of the basic steps and also describes each of the materials as she instructs the alien to get them. Instead of telling the alien where to find each item, she gives a general description of each item so that the alien can locate it by sight. For example, this student describes bread as “the white square type food with brown outlining.”



Step1: get out the bread(the bread is the white square type food with brown outlining) Put 2 pieces on a plate.

Step2: get out the peanut butter (typically is in a round, plastic container & inside the container will be a brown creamy substance, this is the peanut butter.

Step 3: get out the jelly. (grape or starberry jellies are usually in a plastic squeeze bottle or a plastic/glass jar.)

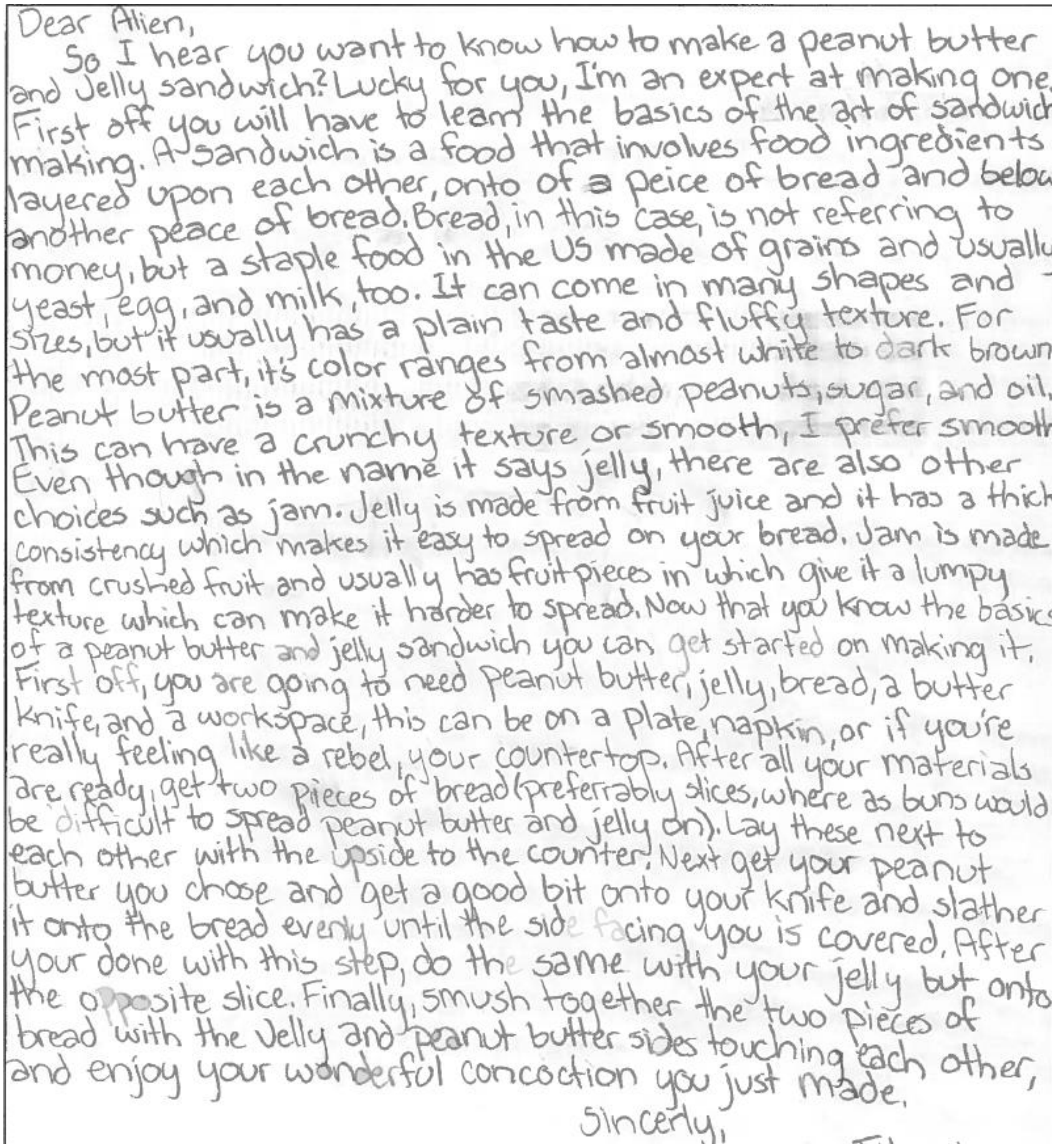
Step 4: Use a spoon(a metal stick with a round/oval end) and scoop some peanut butter out of the jar, then spread the peanut butter on one piece of bread.

Step5: Use a spoon to scoop the jelly out/ or squeeze the jelly out on the other slice of bread.

Step6: Put the 2 pieces of bread on top of each other with the jelly & peanut butter on the inside. Enjoy!

Figure 4. Writing Assignment 1, First Attempt, Category 1, Sample 1C.

<u>Example.</u> The writer of Sample 1D writes far more than the basic steps. She writes the instructions in letter format, including a personal introduction and information on what a sandwich is. She also describes what each ingredient is made of and its appearance. For example, bread is a U.S. staple food made of grains and other ingredients such as yeast, egg, and milk. She distinguishes between jelly and jam and suggests that jam might be harder to spread since it has fruit pieces in it. She also gives specific instructions regarding which side of the bread to put the peanut butter and the jelly.



Dear Alien,

So I hear you want to know how to make a peanut butter and Jelly sandwich? Lucky for you, I'm an expert at making one. First off you will have to learn the basics of the art of sandwich making. A sandwich is a food that involves food ingredients layered upon each other, onto of a peice of bread and below another peace of bread. Bread, in this case, is not referring to money, but a staple food in the US made of grains and usually yeast, egg, and milk, too. It can come in many shapes and sizes, but it usually has a plain taste and fluffy texture. For the most part, it's color ranges from almost white to dark brown. Peanut butter is a mixture of smashed peanuts, sugar, and oil. This can have a crunchy texture or smooth, I prefer smooth. Even though in the name it says jelly, there are also other choices such as jam. Jelly is made from fruit juice and it has a thick consistency which makes it easy to spread on your bread. Jam is made from crushed fruit and usually has fruit pieces in which give it a lumpy texture which can make it harder to spread. Now that you know the basics of a peanut butter and jelly sandwich you can get started on making it. First off, you are going to need peanut butter, jelly, bread, a butter knife, and a workspace, this can be on a plate, napkin, or if you're really feeling like a rebel, your countertop. After all your materials are ready, get two pieces of bread (preferrably slices, where as buns would be difficult to spread peanut butter and jelly on). Lay these next to each other with the upside to the counter. Next get your peanut butter you chose and get a good bit onto your knife and slather it onto the bread evenly until the side facing you is covered. After your done with this step, do the same with your jelly but onto the opposite slice. Finally, smush together the two pieces of bread with the Jelly and peanut butter sides touching each other, and enjoy your wonderful concoction you just made.

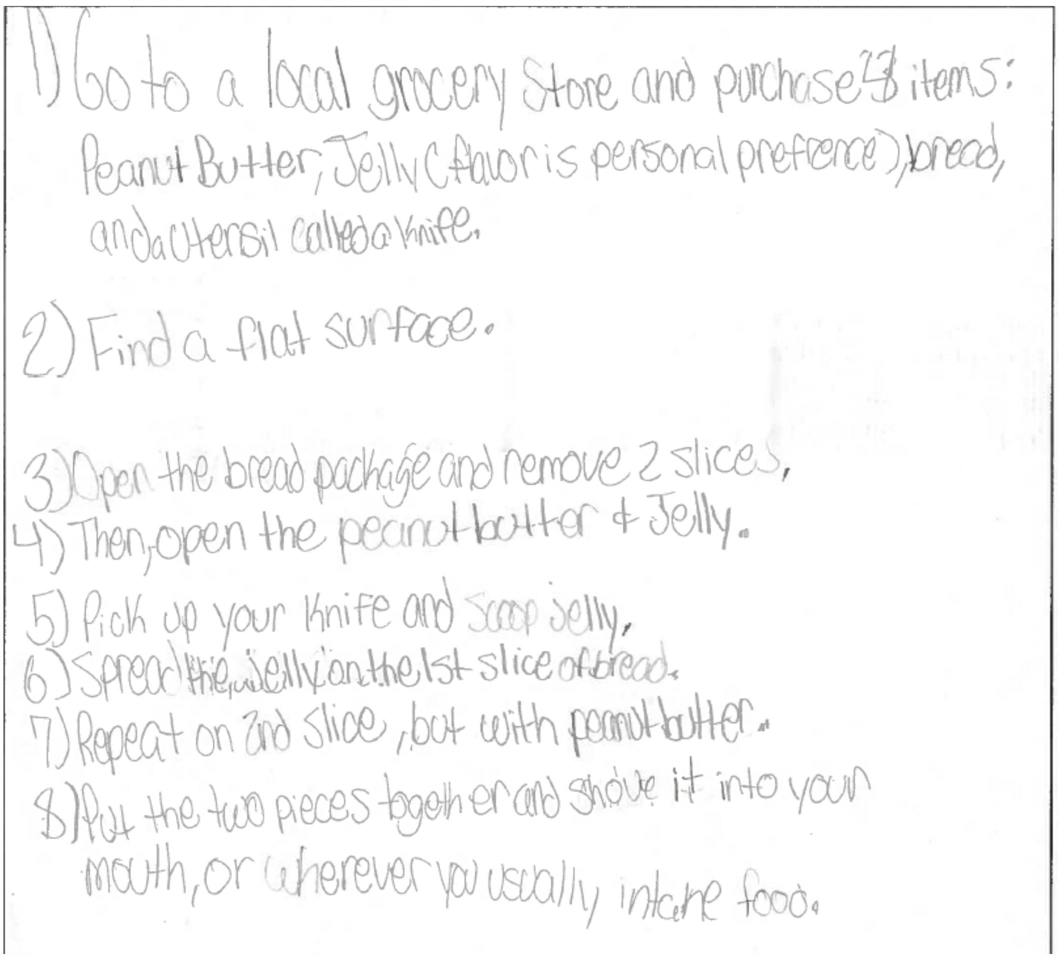
Sincerly,

Figure 5. Writing Assignment 1, First Attempt, Category 1, Sample 1D.

5.1.1.2 Writing Assignment 1, First Attempt, Category 2

The student responses in Category 2 include only the basic steps for making a peanut butter and jelly sandwich. Forty-eight of the seventy-seven students (approximately 62%) who completed this assignment wrote responses that fell into this category.

Example. The writer of Sample 1E instructs the alien on the basic steps. He tells the alien to get the ingredients from the grocery store but does not attempt to provide specific information about how to find or select the items while at the grocery store. He then gives general instructions for scooping and spreading peanut butter and jelly onto the bread and putting the two pieces of bread together.


1) Go to a local grocery Store and purchase 3 items: Peanut Butter, Jelly (flavor is personal preference), bread, and a utersil called a knife.

2) Find a flat surface.

3) Open the bread package and remove 2 slices,
4) Then, open the peanut butter & Jelly.
5) Pick up your knife and scoop jelly,
6) Spread the jelly on the 1st slice of bread.
7) Repeat on 2nd slice, but with peanut butter.
8) Put the two pieces together and shove it into your mouth, or wherever you usually intake food.

Figure 6. Writing Assignment 1, First Attempt, Category 2, Sample 1E.

Example. Sample 1F is fairly typical of the assignments that fell into this category. This student writes all of the basic steps without providing more information like where to find the ingredients and which direction to turn the jar lids and what the act of spreading looks like.

First of all, you must acquire a jar of peanut butter, a jar of jelly, a loaf of bread, a plate, and a butter Knife.

Next, you open the loaf of bread, take out two slices of bread, and put the two slices of bread on the plate.

Then, open the jars of peanut butter and jelly. Grab the Knife and put it in the peanut butter shoveling the peanut butter onto the Knife. Then you use the Knife to spread the peanut butter onto a slice of bread.

Now, you use the same Knife to put in the jelly and then shovel the jelly onto the Knife. After that you spread the jelly using your Knife.

Finally, you put the slices of bread together with the peanut butter and jelly on the inside. Now put your sandwhich on a plate and eat up.

Figure 7. Writing Assignment 1, First Attempt, Category 2, Sample 1F.

<u>5.1.1.3 Writing Assignment 1, First Attempt, Category 3</u>:

The student responses in Category 3 were incomplete, omitting either a step in the process or an ingredient needed to make a peanut butter and jelly sandwich. Five of the seventy-seven students (approximately 6%) who attempted this assignment wrote responses that fell into this category.

<u>Example.</u> The writer of Sample 1G uses language such as first, second, and third but omits an ingredient and a step in the construction of the sandwich. He never tells the alien to get a utensil such as a knife or spoon and never instructs the alien to put the two pieces of bread together to complete the sandwich. Two of the five students whose writing is in this category neglected to include the step of putting the two pieces of bread together.

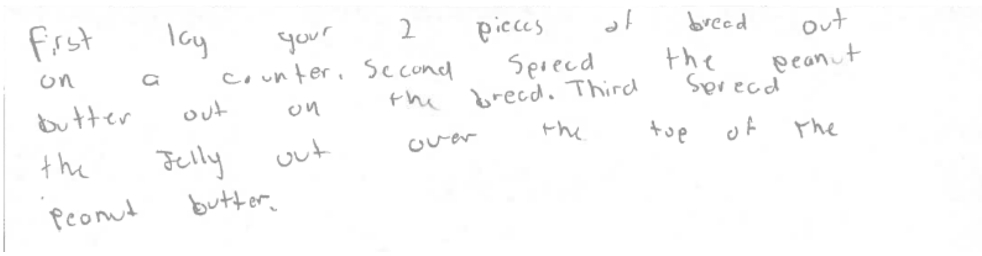

First lay your 2 pieccs of bread out on a counter. Second Spreed the peanut butter out on the breed. Third Spreed the Jelly out over the top of the peanut butter.

Figure 8. Writing Assignment 1, First Attempt, Category 3, Sample 1G.

Example. The writer of Sample 1H did not finish the assignment. He turned the assignment in on time, but he was writing at the beginning of class while I was walking around collecting assignments. He attempted to complete as much of the assignment as possible before I collected it from him. If he had completed the assignment, it is anticipated that his writing would have fallen into Category 1 since he had begun providing information beyond the basic steps of instruction like untying and retying the tie on the bread bag and pulling the knife out of the peanut butter jar with peanut butter on it. Two of the five students in this category were actively writing when I collected the assignment from them and, therefore, did not complete the assignment. After reading what the two students had been able to complete, it is clear that both responses would have fallen into Category 1 if the students had completed the assignment.

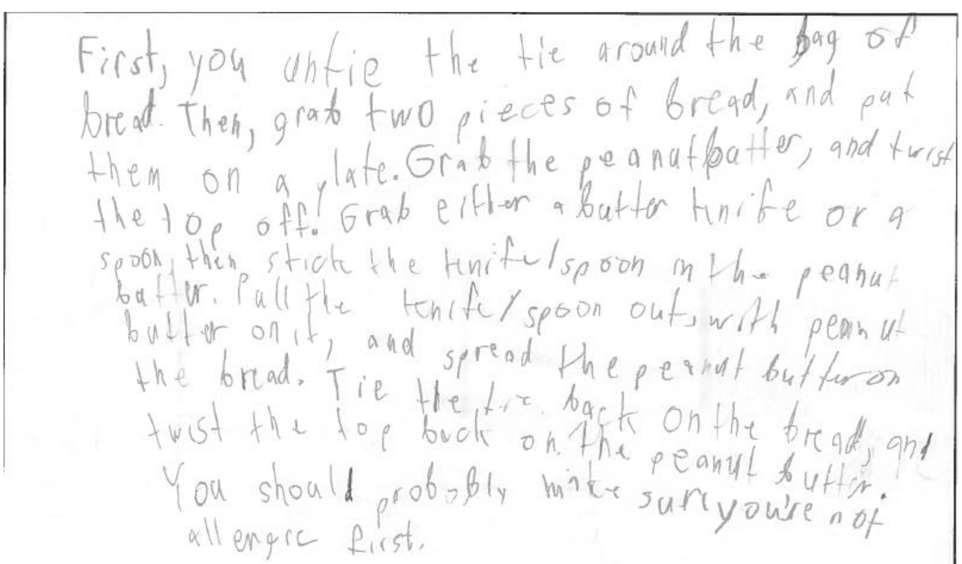

First, you untie the tie around the bag of bread. Then, grab two pieces of bread, and put them on a plate. Grab the peanut butter, and twist the top off! Grab either a butter knife or a spoon, then stick the knife/spoon in the peanut butter. Pull the knife/spoon out with peanut butter on it, and spread the peanut butter on the bread. Tie the tie back on the bread, and twist the top back on the peanut butter. You should probably make sure you're not allergic first.

Figure 9. Writing Assignment 1, First Attempt, Category 3, Sample 1H.

5.1.2 Second Attempt of Writing Assignment 1

I did not offer an opportunity for students to revise this assignment.

To summarize Writing Assignment 1, seventy-two out of seventy-seven students (approximately 94%), include at least the basic steps for making a peanut butter and jelly sandwich. The responses in Category 3 do not contain the basic steps for making a peanut butter and jelly sandwich. Two of the students in Category 3 fell into the category because the students turned the assignment in despite not having completed it. Looking at the data in Category 2, this tells me that the majority of my students (approximately 62%) write only what is necessary to complete an assignment.

Table 1. Summary of Responses from Writing Assignment 1

	Category			
	1	2	3	Total
Number of students in each category after attempt	24	48	5	77

5.2 Writing Assignment 2

> Write conjectures explaining what happens when you do the following: (a) subtract two even integers, (b) subtract two odd integers, and (c) subtract and even integer from an odd integer. For each of your three conjectures, provide a geometric AND algebraic justification. Define all abbreviations and variables that you use. Your audience has not seen your SpringBoard book but knows basic algebra. All writing must be inside the box.

This writing assignment asked students to create conjectures stating what type of integer results when an even integer is subtracted from an even integer, when an odd integer is subtracted from an odd integer, and when an even integer is subtracted from an odd integer. Students were required to justify each of their conjectures algebraically and geometrically and to define all abbreviations and variables.

The day prior to the writing assignment, students had worked in groups of four to complete Activity 2-2, a discovery learning lesson, in their SpringBoard textbooks (a consumable workbook)[4]. In this lesson, students were asked to write and justify similar conjectures involving addition rather than subtraction. Students were provided definitions of even and odd integers and were asked to create examples to help them develop conjectures about the result from adding two even integers, two odd integers, and an even integer and an odd integer. In groups, students created several examples such as $2 + 4 = 6$, which helped them to form the conjecture that the sum of an even integer and an even integer is an even integer. While I walked around the classroom, I overheard students speculate that the sum of an odd integer and an odd integer would be an odd integer, and a few students verbally expressed surprise that the result was an even integer.

After students wrote their conjectures, I guided the students through the remainder of the lesson in the textbook, which instructed students how to prove their conjectures. The textbook provided examples of algebraic justifications and geometric justifications of the conjectures.

First, students were asked to provide geometric justifications for the three conjectures. I provided students with geometric images that would represent even and odd integers (shown in Appendix A as Figures A and C, respectively). Students discussed why each shape could represent an even or odd integer. Then, students combined these images in a variety of ways to prove the conjectures that they had developed. Students were comfortable using images to depict even and odd integers. This portion of the lesson made sense to them.

With respect to the algebraic justifications, the textbook instructed students how to write an expression for an even integer (e.g., $2p$ or $2m$) and an odd integer (e.g., $2t + 1$ or $2n + 1$) and explained why those expressions fit the definitions of even and odd integers. The textbook

[4] The relevant portions of the lesson from the teacher's edition of the textbook are included as Appendix A.

also instructed the students to include a statement that the variables chosen represented integers (e.g., where p, m, t, and n are integers). I guided students through the algebraic justifications of the three conjectures. Some students were unsure why the algebraic expressions represented even and odd integers even after we had completed the lesson.

On the day of the writing assignment, I engaged the students in a discussion of the prior day's lesson, which included reviewing the notes on how to write a conjecture and how to provide geometric and algebraic justifications for those conjectures. Then I handed out the writing assignment, read the instructions to the students, and reminded them to refer to their textbooks and notes. I gave students thirty minutes in class to complete this assignment. Students who requested more time were permitted to keep working after the elapse of the thirty minutes.

5.2.1 First Attempt of Writing Assignment 2

Eighty-six students completed this assignment. In order to receive full credit for the writing assignment, student responses had to include (1) three correct conjectures stating the results for the various subtraction problems, (2) definitions for even and odd integers and a statement defining the chosen variables as integers, (3) a geometric justification for each conjecture, (4) an algebraic justification for each conjecture, using correct expressions for even and odd integers, and (5) correct simplification of the algebraic justifications. After reviewing the student responses, I divided the student writing into one of five categories based on how many of the five criteria they satisfied in their responses.

You will notice that many of the students wrote outside of the boxes on their first attempts. That is because I was unable to copy the assignment before class, and students were instructed to write their responses on a piece of computer paper using a three-inch margin at the

top of the paper and one-inch margins on the sides. I printed the assignment onto their papers so that they would have the prompt and their responses on one page. Some students had a difficult time maintaining the correct margins, and their writing overlaps the boxes in some places.

5.2.1.1 Writing Assignment 2, First Attempt, Category 1

No students satisfied all five criteria for full credit on their first attempt at this assignment.

5.2.1.2 Writing Assignment 2, First Attempt, Category 2

Nine of the eighty-six students (approximately 10%) satisfied four of the five criteria of the assignment in their responses. The most commonly omitted criteria was the correct simplification of the algebraic justifications.

Example. The writer of Sample 2A provides all of the correct conjectures, geometric justifications, and properly simplified algebraic justifications; however, he neglects to include the definitions of odd and even integers and a statement defining his selected variables as integers. Unlike the other students whose responses fall into this category, this student correctly simplifies all of the algebraic justifications.

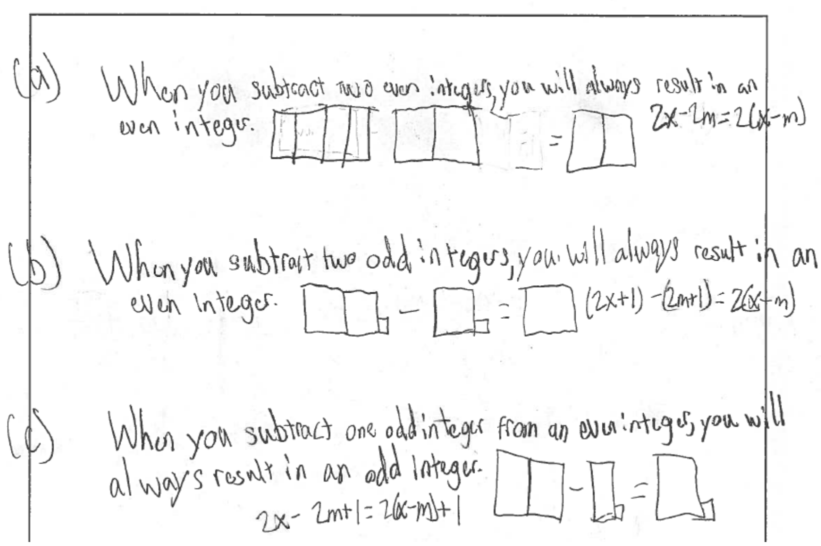
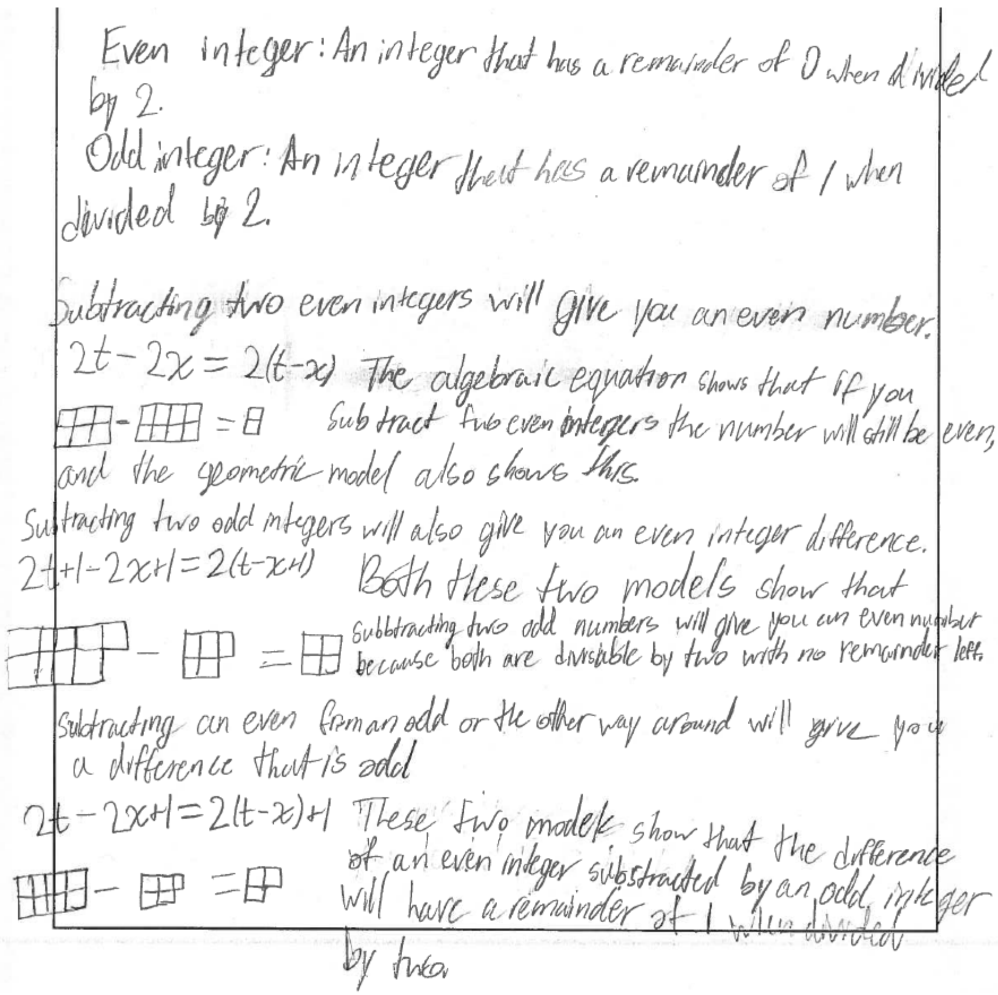


Figure 10. Writing Assignment 2, First Attempt, Category 2, Sample 2A.

Example. Sample 2B represents the writing of the other eight students whose responses fell into this category. The student provides all of the correct conjectures and algebraic and geometric justifications but makes a mistake when simplifying one of the algebraic justifications (i.e., subtracting an odd integer from another odd integer). This student uses correct expressions to represent odd integers but does not correctly simplify the expressions. All eight of the students include something very similar to what this student writes, which is the following: $2t + 1 - 2x + 1 = 2t - 2x + 2 = 2(t - x + 1)$. Even though the student simplifies the algebraic expression incorrectly, the resulting expression does represent an even integer, which would explain why students does not realize that he had simplified incorrectly.



Even integer: An integer that has a remainder of 0 when divided by 2.

Odd integer: An integer that has a remainder of 1 when divided by 2.

Subtracting two even integers will give you an even number.

$2t - 2x = 2(t-x)$ The algebraic equation shows that if you subtract two even integers the number will still be even, and the geometric model also shows this.

Subtracting two odd integers will also give you an even integer difference.

$2t+1-2x+1 = 2(t-x+1)$ Both these two models show that subbtracting two odd numbers will give you an even number because both are divisible by two with no remainder left.

Subtracting an even from an odd or the other way around will give you a difference that is odd

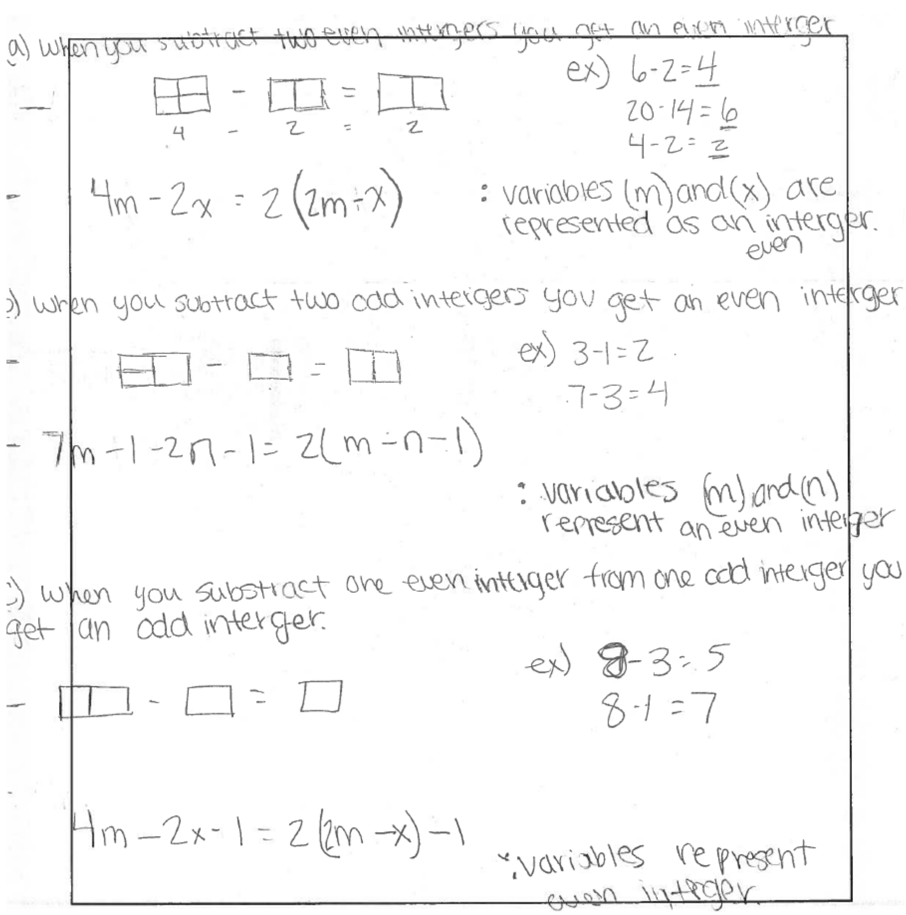
$2t-2x+1 = 2(t-x)+1$ These two models show that the difference of an even integer substracted by an odd integer will have a remainder of 1 when divided by two.

Figure 11. Writing Assignment 2, First Attempt, Category 2, Sample 2B.

5.2.1.3 Writing Assignment 2, First Attempt, Category 3

Thirty-four of the eighty-six responses (approximately 40%) satisfied three of the five criteria for the writing assignment. Several of these students had difficulty expressing even and odd integers with algebraic expressions. Others had difficulties simplifying the expressions that they had created. Almost all of these responses omitted a definition of even and odd integers and/or a statement defining the variables chosen as integers.

Example. The writer of Sample 2C omits definitions for even and odd integers and does not correctly set up the algebraic expressions for even and odd integers. The student uses $4m$ and $2x$ to represent even integers, which is acceptable even though only $2m$ and $2x$ would suffice, but she mistakenly writes $7m - 1$ as an expression for an odd integer.


a) when you subtract two even intergers you get an even interger

4 - 2 = 2

ex) 6-2=4
20-14=6
4-2=2

4m - 2x = 2(2m - x) : variables (m) and (x) are represented as an even interger.

b) when you subtract two odd intergers you get an even interger

ex) 3-1=2
7-3=4

7m-1-2n-1= 2(m-n-1) : variables (m) and (n) represent an even interger

c) when you substract one even intiger from one odd interger you get an odd interger.

ex) 8-3=5
8-1=7

4m-2x-1 = 2(2m-x)-1 : variables represent even interger

Figure 12. Writing Assignment 2, First Attempt, Category 3, Sample 2C.

Example. The writer of Sample 2D writes correct algebraic expressions for the conjectures, using $2p$ and $2m$ for even integers and $2t+1$, $2n+1$, and $2x+1$ for odd integers. The student does not, however, simplify these algebraic expressions correctly. Her attempts at simplifying the algebraic expressions indicate a weak foundation in algebra. She does not attempt to combine like terms. She does not provide definitions for even and odd integers.

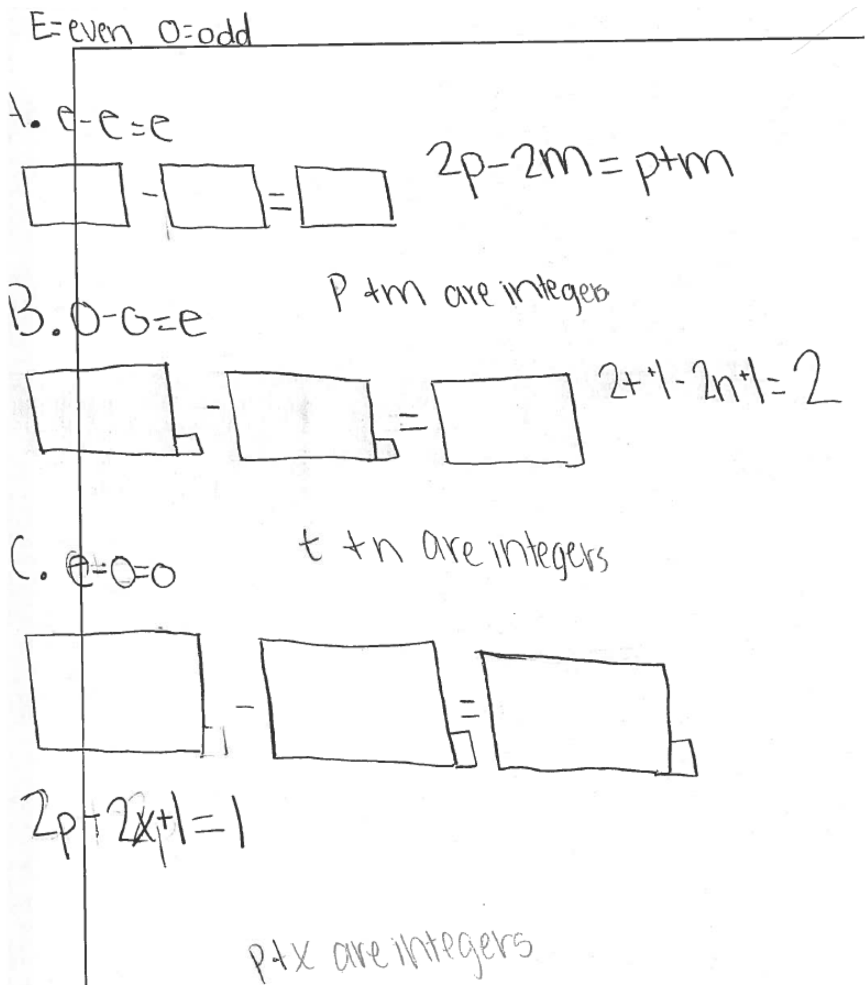


Figure 13. Writing Assignment 2, First Attempt, Category 3, Sample 2D.

5.2.1.4 Writing Assignment 2, First Attempt, Category 4

Thirty-one of the eighty-six responses (approximately 36%) satisfy two of the five criteria for the writing assignment. Many of these students either do not attempt to include an algebraic justification or instead include an example such as $6 - 2 = 4$ or $\mathrm{E} - \mathrm{E} = \mathrm{E}$ as an

algebraic justification. Almost all of the students state the conjectures correctly but do not provide definitions for even and odd integers.

Example. The writer of Sample 2E provides examples such as $8 - 6 = 2$ and $9 - 3 = 6$ as algebraic support for his conjectures. He provides no algebraic expressions or definitions for even or odd integers. He does, however, correctly state all of the conjectures and provides suitable geometric justifications for his conjectures.

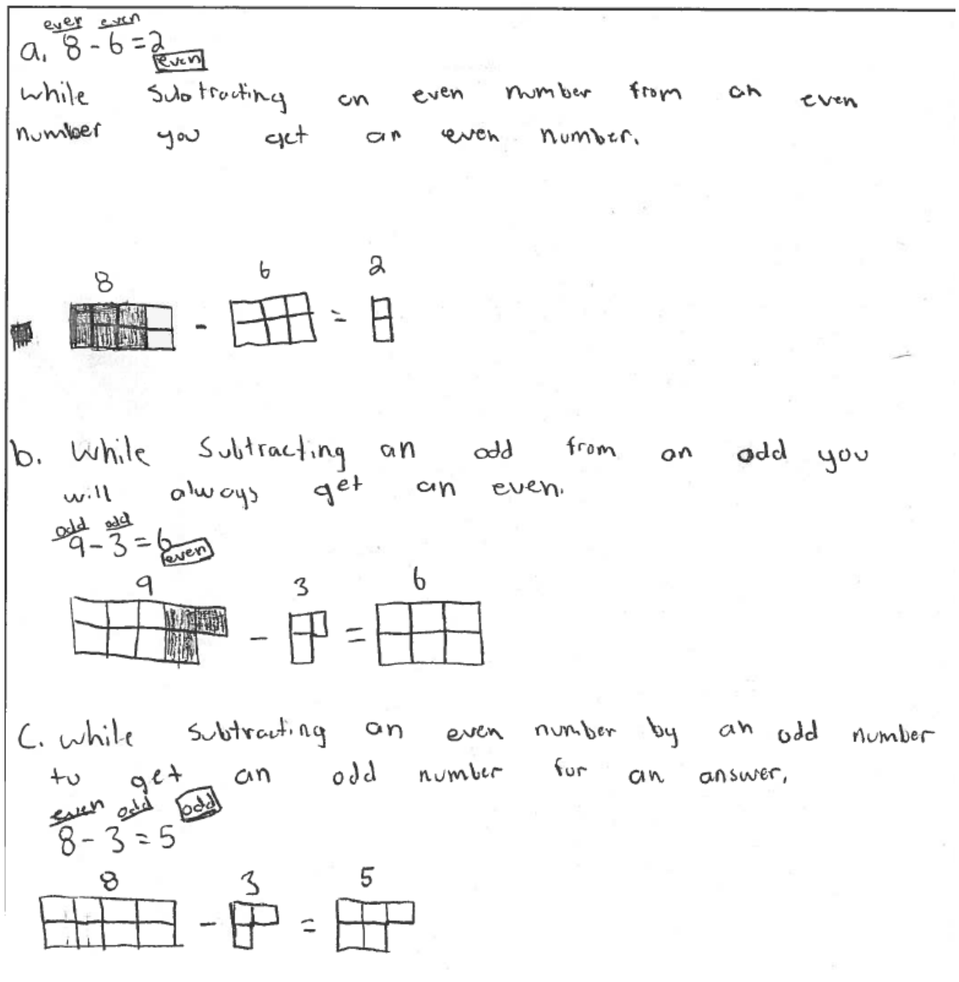
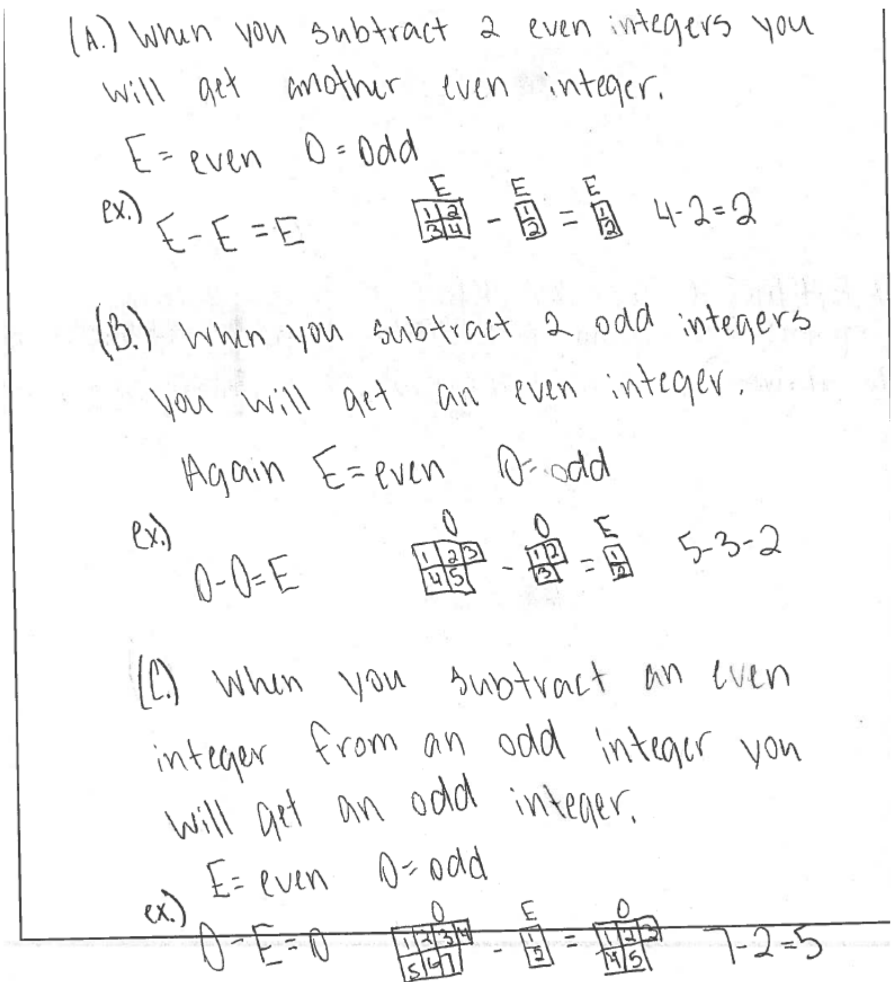


Figure 14. Writing Assignment 2, First Attempt, Category 4, Sample 2E.

Example. The writer of Sample 2F provides equations such as $E - E = E$ and $O - O = E$ and examples such as $4 - 2 = 2$ and $5 - 3 = 2$ as algebraic support for his conjectures. He

provides no algebraic expressions or definitions for even or odd integers. He does state that E represents “an even” and O represents “an odd” and also states the conjectures correctly.


(A.) When you subtract 2 even integers you will get another even integer.

E = even O = Odd

ex.) E - E = E 4-2=2

(B.) When you subtract 2 odd integers you will get an even integer.

Again E = even O = odd

ex.) O - O = E 5-3-2

(C.) When you subtract an even integer from an odd integer you will get an odd integer.

E = even O = odd

ex.) O - E = O 7-2=5

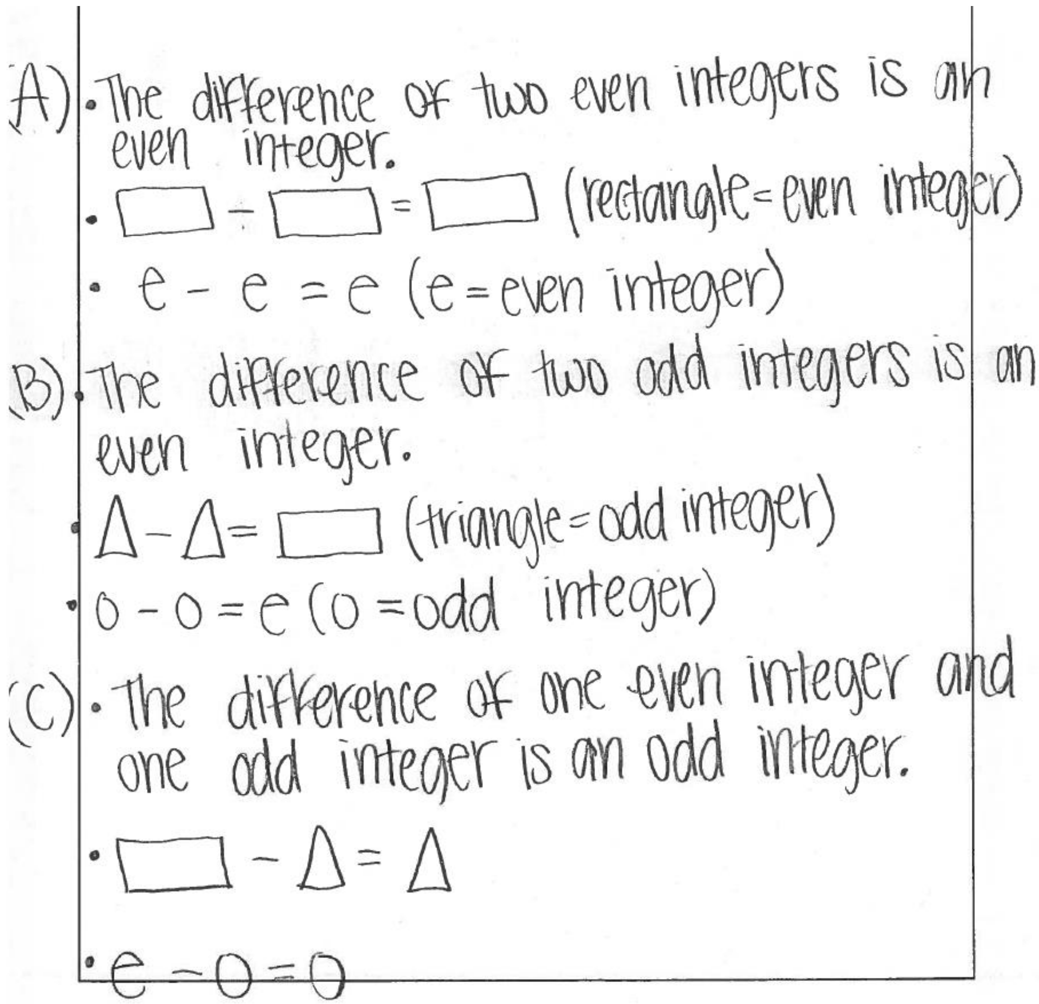
Figure 15. Writing Assignment 2, First Attempt, Category 4, Sample 2F.

<u>5.2.1.5 Writing Assignment 2, First Attempt, Category 5</u>

Twelve of the eighty-six responses (approximately 14%) satisfied only one of the five criteria for the writing assignment. Students in this category typically are able to state all of the conjectures correctly and are able to provide either correct geometric justifications or properly simplified algebraic expressions.

<u>Example.</u> The writer of Sample 2G is able to state the conjectures correctly but nothing further. She does not provide an acceptable geometric justification for any of the conjectures.

She uses geometric shapes to represent even and odd integers such as a rectangle for even integers and a triangle for odd integers, which leads to statements such as rectangle – triangle = triangle. She also incorrectly uses statements such as e – e = e and e – o = o as algebraic justifications for the conjectures. She does not provide algebraic expressions or definitions for even and odd integers.



A) • The difference of two even integers is an even integer.

• $\square - \square = \square$ (rectangle = even integer)

• e − e = e (e = even integer)

B) • The difference of two odd integers is an even integer.

• $\Delta - \Delta = \square$ (triangle = odd integer)

• o − o = e (o = odd integer)

(C) • The difference of one even integer and one odd integer is an odd integer.

• $\square - \Delta = \Delta$

• e − o = o

Figure 16. Writing Assignment 2, First Attempt, Category 5, Sample 2G.

<u>Example.</u> The writer of Sample 2H provides correct conjectures but nothing further. He does not attempt to provide geometric justifications for any of the conjectures. For his algebraic justifications, he simply assigns A to be an even number and B to be an odd number and writes

equations such as A – B = A and B – B = A. He does not provide algebraic expressions or definitions for even and odd integers.

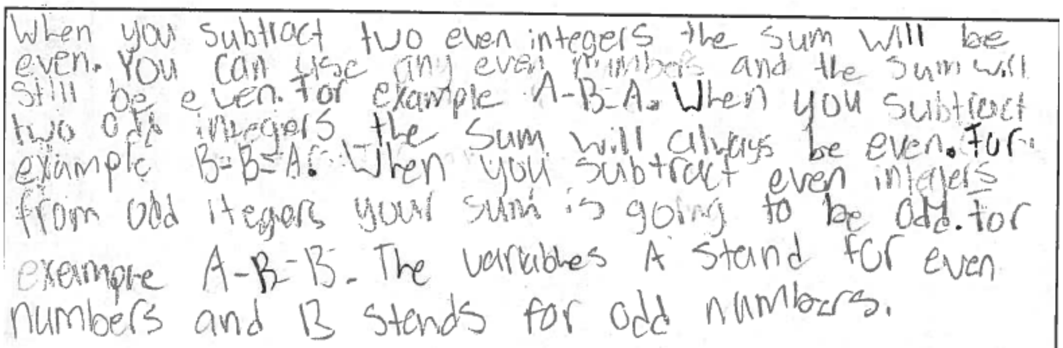

When you subtract two even integers the sum will be even. You can use any even numbers and the sum will still be even. for example A-B=A. When you subtract two odd integers the sum will always be even. For example B-B=A. When you subtract even integers from odd itegers your sum is going to be odd. for example A-B=B. The varibles A stand for even numbers and B stends for odd numbers.

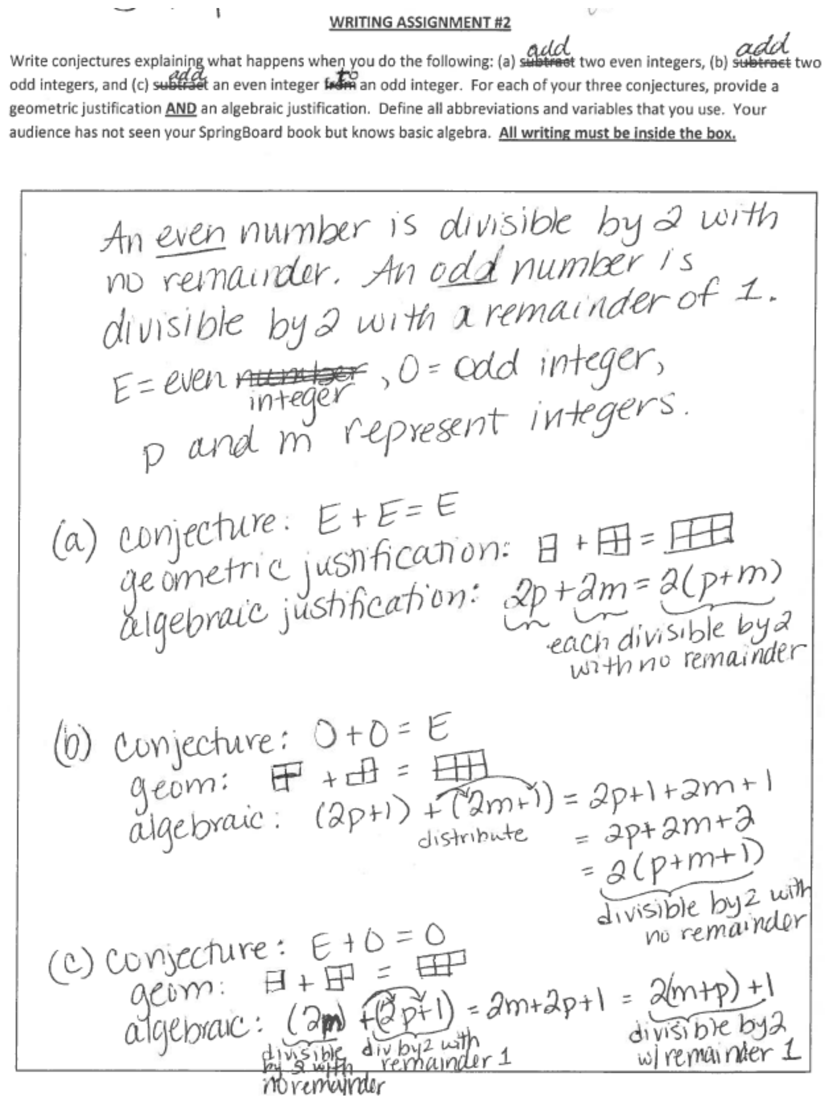
Figure 17. Writing Assignment 2, First Attempt, Category 5, Sample 2H.

5.2.2 Second Attempt of Writing Assignment 2

After completion of the first attempts of Writing Assignment 2, student responses were collected, graded, and reviewed. Students were given their first attempts, now containing grades and minimal comments from the teacher, and the prototype answer below, which shows the level of specificity I was looking for in an answer to this type of problem. Students were offered an opportunity to rewrite the assignment. In the prototype writing sample, I condensed all of the information that was included in the students' textbook and the students' notes related to the addition of even and odd integers. Because the exemplar repackaged the notes from the textbook, it contained conjectures related to addition rather than subtraction. Students had to apply the concepts from the prototype in the context of subtraction, as required by the writing assignment.

To create the prototype, I simply changed the wording of the assignment to indicate the shift in focus from subtraction to addition. I did this to help the students see that responses would be similar to what they had written in their textbooks and notes only a few days prior. After being provided the prototype, eighty-three of the eighty-six students (approximately 97%)

opted to revise their original writing assignments. The students' writing was classified into the same five categories.


WRITING ASSIGNMENT #2

Write conjectures explaining what happens when you do the following: (a) ~~subtract~~ add two even integers, (b) ~~subtract~~ add two odd integers, and (c) ~~subtract~~ add an even integer ~~from~~ to an odd integer. For each of your three conjectures, provide a geometric justification **AND** an algebraic justification. Define all abbreviations and variables that you use. Your audience has not seen your SpringBoard book but knows basic algebra. **All writing must be inside the box.**

An even number is divisible by 2 with no remainder. An odd number is divisible by 2 with a remainder of 1.

E = even ~~number~~ integer, O = odd integer,

p and m represent integers.

(a) conjecture: $E + E = E$

geometric justification:

algebraic justification: $\underbrace{2p}_{}+\underbrace{2m}_{} = \underbrace{2(p+m)}_{}$

each divisible by 2 with no remainder

(b) Conjecture: $O + O = E$

geom:

algebraic: $(2p+1) + (2m+1) = 2p+1+2m+1$

distribute

$= 2p+2m+2$

$= \underbrace{2(p+m+1)}_{}$

divisible by 2 with no remainder

(c) conjecture: $E + O = O$

geom:

algebraic: $(2m) + (2p+1) = 2m+2p+1 = \underbrace{2(m+p)+1}_{}$

divisible by 2 with no remainder

div by 2 with remainder 1

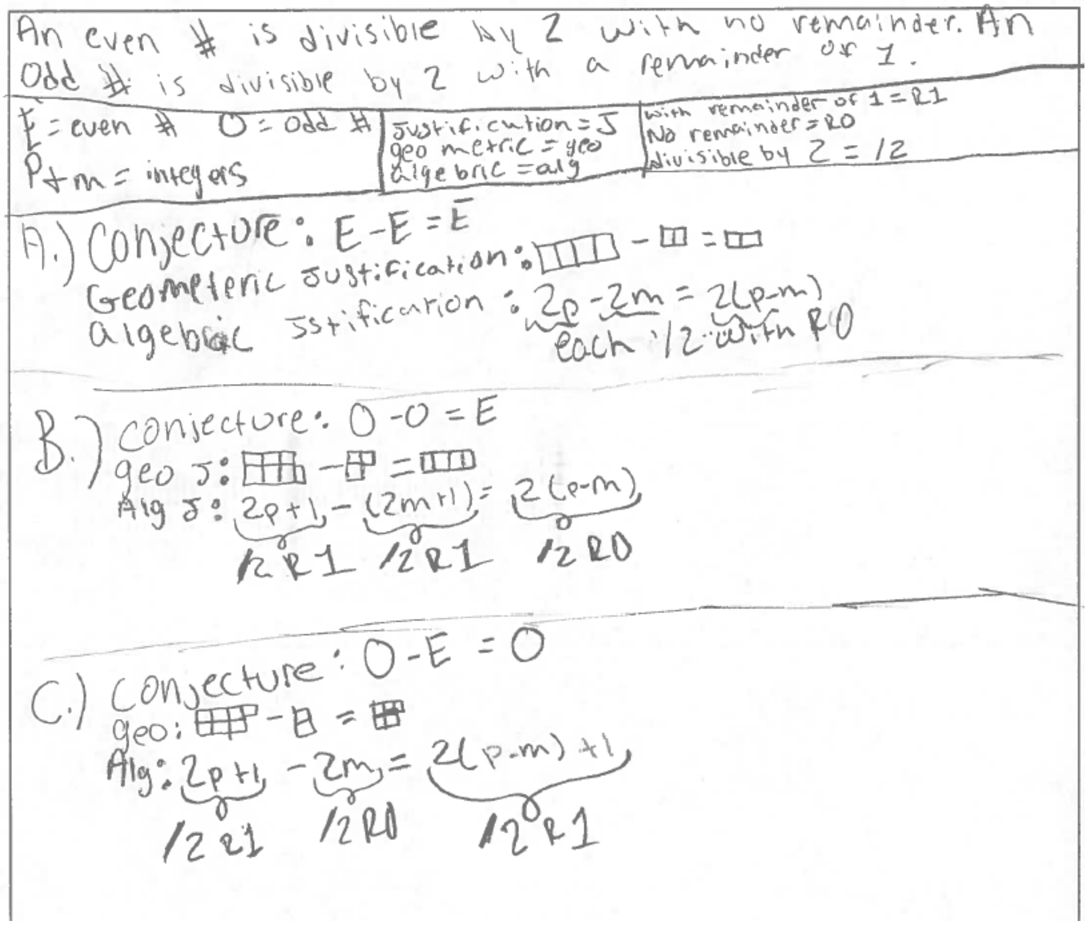
divisible by 2 w/ remainder 1

Figure 18. Exemplar for Writing Assignment 2.

<u>5.2.2.1 Writing Assignment 2, Second Attempt, Category 1</u>

Thirty-two of the eighty-three students (approximately 39%) who revised this assignment satisfied all five criteria on the second attempt of this writing assignment. No student writing originally fell into Category 1; thus, thirty-two represents the total number of students out of eighty-six (approximately 37%) whose writing fell into Category 1 on either their first or second attempt.

Example. Sample 2I is typical of other responses in this category. The student clearly defines even and odd integers, states that the variables represent integers, and provides three correct conjectures, all with geometric and algebraic justification.



An even # is divisible by 2 with no remainder. An Odd # is divisible by 2 with a remainder of 1.

E = even # O = odd # P + m = integers	Justification = J geo metric = geo algebric = alg	with remainder of 1 = R1 No remainder = R0 divisible by 2 = /2

A.) Conjecture: E - E = E
Geometeric Justification:
algebraic Jstification: $\underbrace{2p - 2m}_{\text{each } /2 \text{ with } R0} = 2(p-m)$

B.) conjecture: O - O = E
geo J:
Alg J: $\underbrace{2p+1}_{/2\,R1} - \underbrace{(2m+1)}_{/2\,R1} = \underbrace{2(p-m)}_{/2\,R0}$

C.) Conjecture: O - E = O
geo:
Alg: $\underbrace{2p+1}_{/2\,R1} - \underbrace{2m}_{/2\,R0} = \underbrace{2(p-m)+1}_{/2\,R1}$

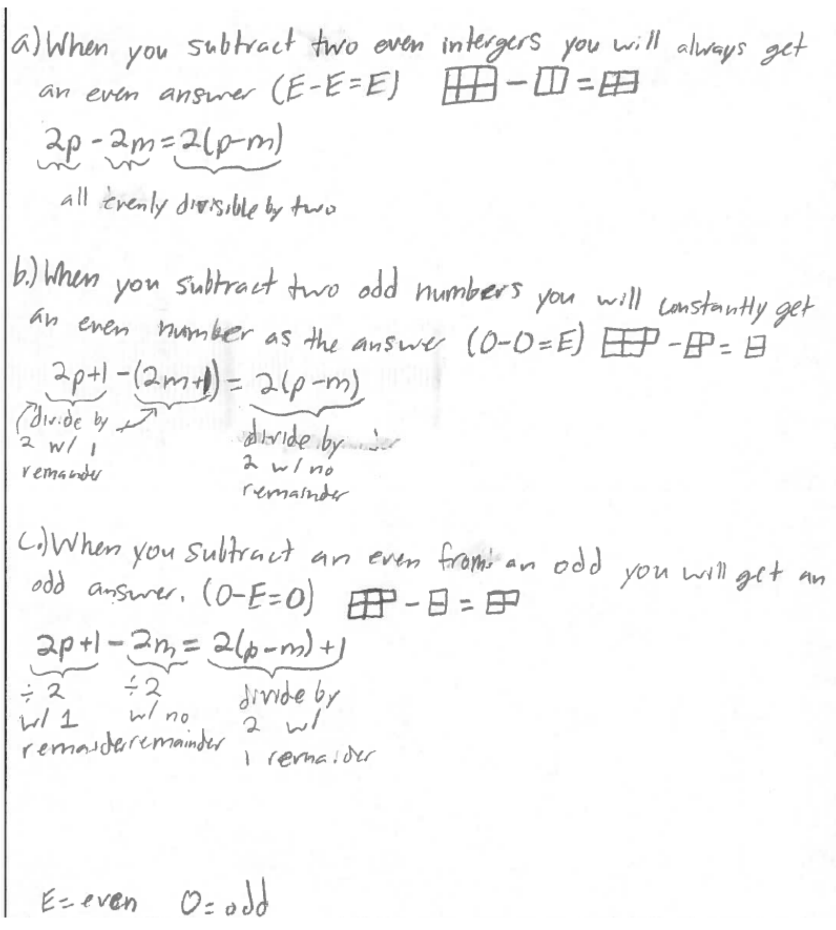
Figure 19. Writing Assignment 2, Second Attempt, Category 1, Sample 2I.

5.2.2.2 Writing Assignment 2, Second Attempt, Category 2

Thirty-three of the eighty-three students (approximately 40%) who revised this assignment have responses that fell into this category. Thirty-two of these thirty-three students omit the same criterion: they neglect to include a definition component of the writing assignment, either the definition of even and odd integers or a statement defining the chosen variables as integers. No students whose writing originally fell into Category 2 opted to keep

their scores; thus, thirty-three out of eighty-six students (approximately 38%) wrote responses that satisfied four of the five criteria for this assignment on either their first or second attempt.

Example. The writer of Sample 2J defines even and odd integers, correctly states the three conjectures, and provides correct geometric and algebraic justifications. She neglects, however, to include a statement indicating that the variables chosen represent integers. This is a minor error, but because of the type of classification system that I chose, this response fell into Category 2.


a) When you subtract two even intergers you will always get an even answer (E-E=E)

$\underbrace{2p}-\underbrace{2m}=\underbrace{2(p-m)}$

all evenly divisible by two

b.) When you subtract two odd numbers you will constantly get an even number as the answer (O-O=E)

$\underbrace{2p+1}-\underbrace{(2m+1)}=\underbrace{2(p-m)}$

divide by 2 w/ 1 remainder

divide by 2 w/ no remainder

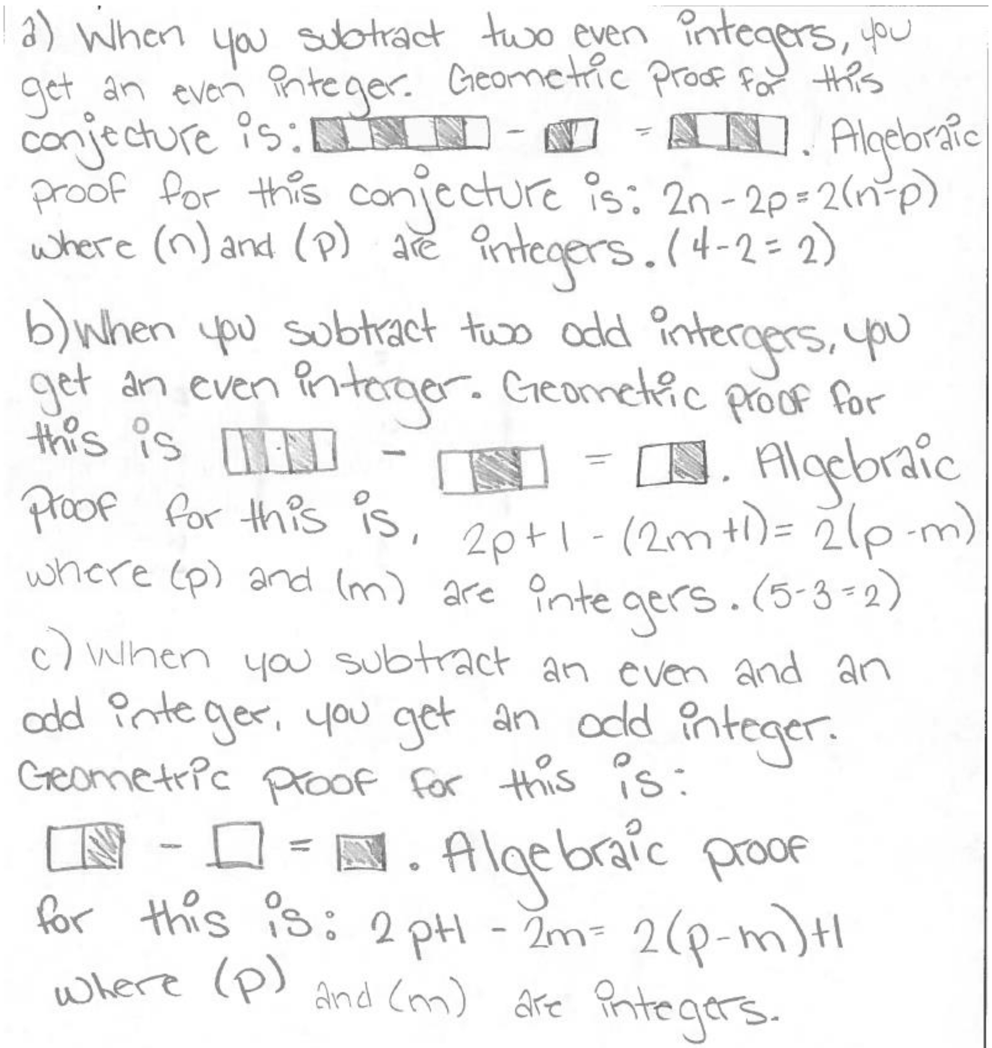
c.) When you subtract an even from an odd you will get an odd answer. (O-E=O)

$\underbrace{2p+1}_{\div 2 \text{ w/ } 1 \text{ remainder}}-\underbrace{2m}_{\div 2 \text{ w/ no remainder}}=\underbrace{2(p-m)+1}_{\text{divide by } 2 \text{ w/ } 1 \text{ remainder}}$

E= even O= odd

Figure 20. Writing Assignment 2, Second Attempt, Category 2, Sample 2J.

Example. The writer of Sample 2K satisfies all of the criteria except one. She includes a statement indicating that the variables chosen represent integers but never provides a definition for even and odd integers.


a) When you subtract two even integers, you get an even integer. Geometric proof for this conjecture is: - = . Algebraic proof for this conjecture is: $2n - 2p = 2(n-p)$ where (n) and (p) are integers. $(4-2=2)$

b) When you subtract two odd intergers, you get an even interger. Geometric proof for this is - = . Algebraic proof for this is, $2p+1 - (2m+1) = 2(p-m)$ where (p) and (m) are integers. $(5-3=2)$

c) When you subtract an even and an odd integer, you get an odd integer. Geometric proof for this is: - = . Algebraic proof for this is: $2p+1 - 2m = 2(p-m)+1$ where (p) and (m) are integers.

Figure 21. Writing Assignment 2, Second Attempt, Category 2, Sample 2K.

Example. The writer of Sample 2L omits a different criteria than the other students in this category. He provides definitions for even and odd integers and a statement indicating that the chosen variables represent integers. He also provides three correct conjectures and correct algebraic justifications. He does not attempt to provide geometric justifications. Even though neglecting to provide one of the two types of justification seems like a major omission, because I did not weight any particular criteria more than another, this student satisfied four of the five criteria and, therefore, fell into Category 2.

An even number is divisible by 2 with no remainder. An odd is divisible by 2 with a remainder of 1. E= even number, O= odd number, p and m represent integers.

(A) conjecture: E-E=E

$\underline{2p}-\underline{2m}=2(p-m)$

each divisible by 2 with no remainder.

(B) conjecture: O-O=E

$2p+1-(2m+1)=2(p-m)$

divide by 2 with 1 remainder	divide by 2 with 1 remainder	divide by 2 with no remainder

(C) conjecture: O-E=O

$2p+1-2m=2(p-m)+1$

divide by 2 with 1 remainder	divide by 2 with no remainder	divide by 2 with 1 remainder.

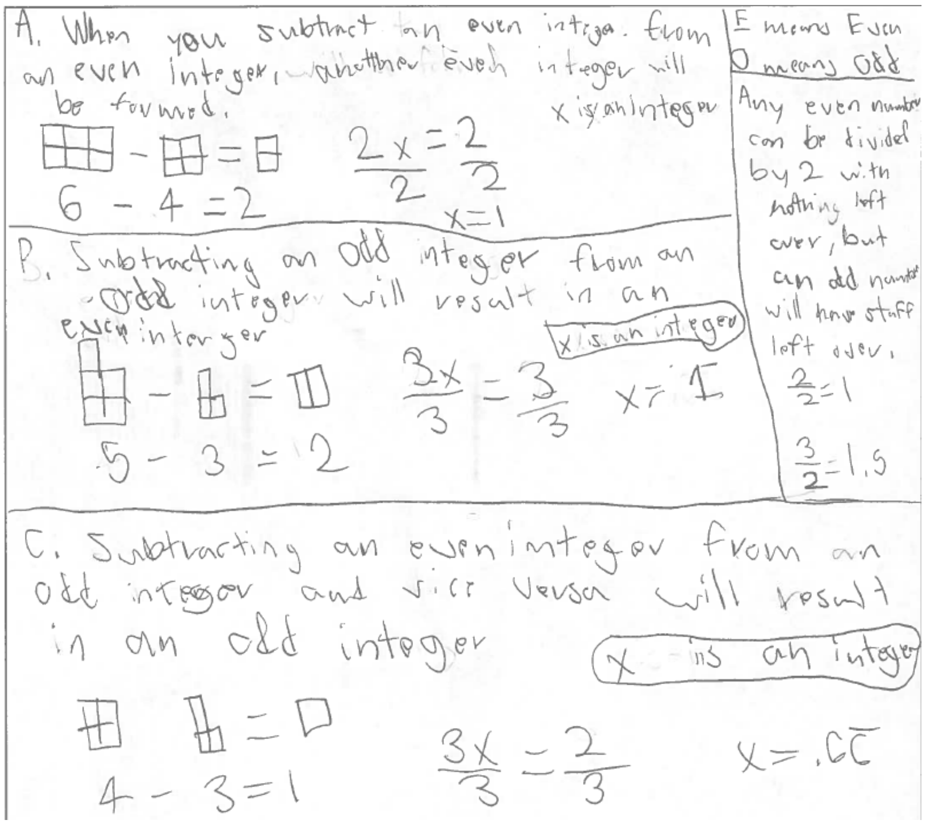
Figure 22. Writing Assignment 2, Second Attempt, Category 2, Sample 2L.

5.2.2.3 Writing Assignment 2, Second Attempt, Category 3

Four of the eighty-three students who revised this assignment (approximately 5%) wrote responses that fell into this category. These students satisfied three of the five criteria for the assignment. One student whose first attempt fell into Category 3 opted to keep his score. Thus, after considering both first and second attempts of this writing assignment, five out of eighty-six students (approximately 6%) had responses that fell into Category 3.

Example. The writer of Sample 2M provides definitions for even and odd integers and a statement indicating that the chosen variables represent integers. He also provides three correct conjectures and correct geometric justifications. He attempts to provide algebraic justifications

but does not write correct algebraic expressions for even and odd integers. The remaining three students in this category differ from this student only in that they do not provide correct geometric justifications but do provide correct algebraic justifications, although not all of the algebraic justifications are properly simplified.


A. When you subtract an even integer from an even integer, another even integer will be formed.

x is an integer

$6 - 4 = 2$

$\frac{2x}{2} = \frac{2}{2}$

$x = 1$

E means Even
O means Odd

Any even number can be divided by 2 with nothing left over, but an odd number will have stuff left over.

$\frac{2}{2} = 1$

$\frac{3}{2} = 1.5$

B. Subtracting an odd integer from an odd integer will result in an even integer

x is an integer

$5 - 3 = 2$

$\frac{3x}{3} = \frac{3}{3}$ $x = 1$

C. Subtracting an even integer from an odd integer and vice versa will result in an odd integer

x is an integer

$4 - 3 = 1$

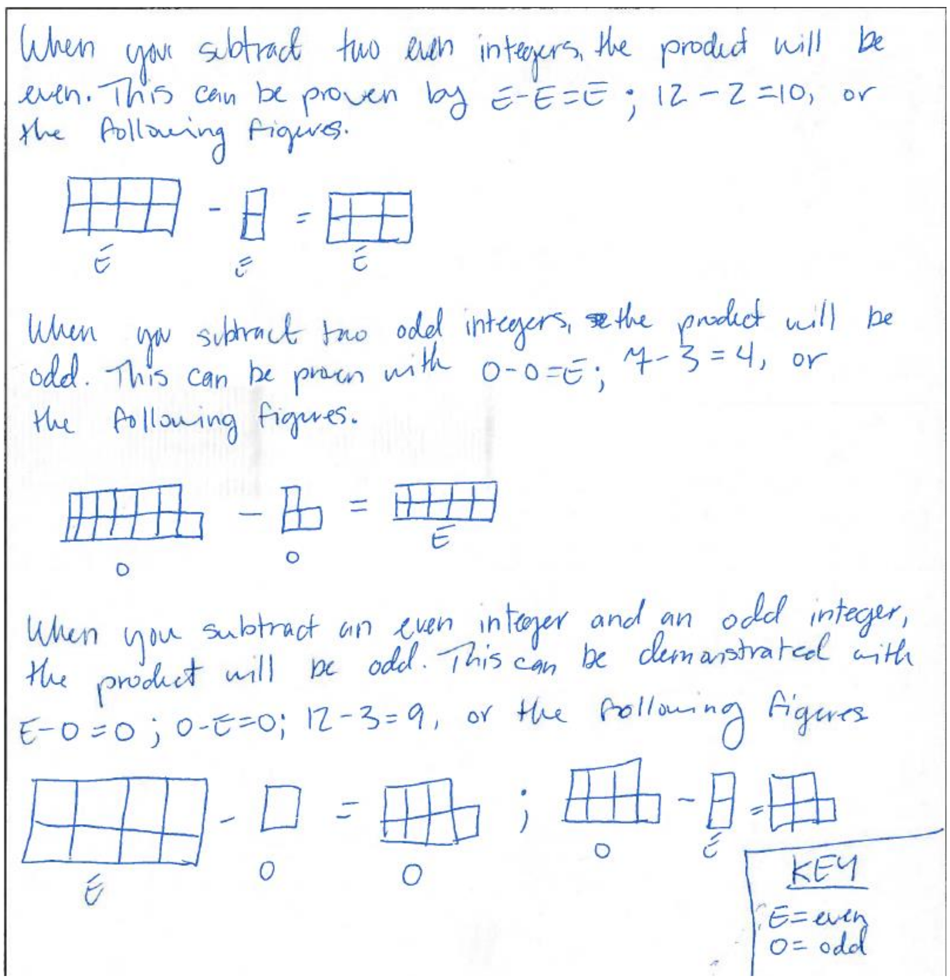
$\frac{3x}{3} = \frac{2}{3}$ $x = .\overline{66}$

Figure 23. Writing Assignment 2, Second Attempt, Category 3, Sample 2M.

<u>5.2.2.4 Writing Assignment 2, Second Attempt, Category 4</u>

Ten of the eighty-three students (approximately 12%) who revised this assignment wrote responses that fell into Category 4. These students satisfied two of the five criteria for the assignment. Two students whose writing originally fell into Category 4 opted to keep their scores. Therefore, twelve out of eighty-six students (approximately 14%) wrote responses that satisfied two of the five criteria for this writing assignment on either their first or second attempt.

Example. Sample 2N represents all of the students in this category. This student makes correct conjectures and provides geometric justifications. She neglects to provide a statement that the chosen variables are integers, and she does not provide algebraic expressions for even and odd integers. She provides examples such as $12 - 3 = 9$, $7 - 3 = 4$, and E – O = O instead of algebraic expressions suggesting that she does not discern the difference between examples illustrating the conjectures and proofs of the conjectures.


When you subtract two even integers, the product will be even. This can be proven by E-E=E; 12-2=10, or the following figures.

When you subtract two odd integers, ~~se~~ the product will be odd. This can be proven with O-O=E; 7-3=4, or the following figures.

When you subtract an even integer and an odd integer, the product will be odd. This can be demonstrated with E-O=O; O-E=O; 12-3=9, or the following figures

KEY
E= even
O= odd

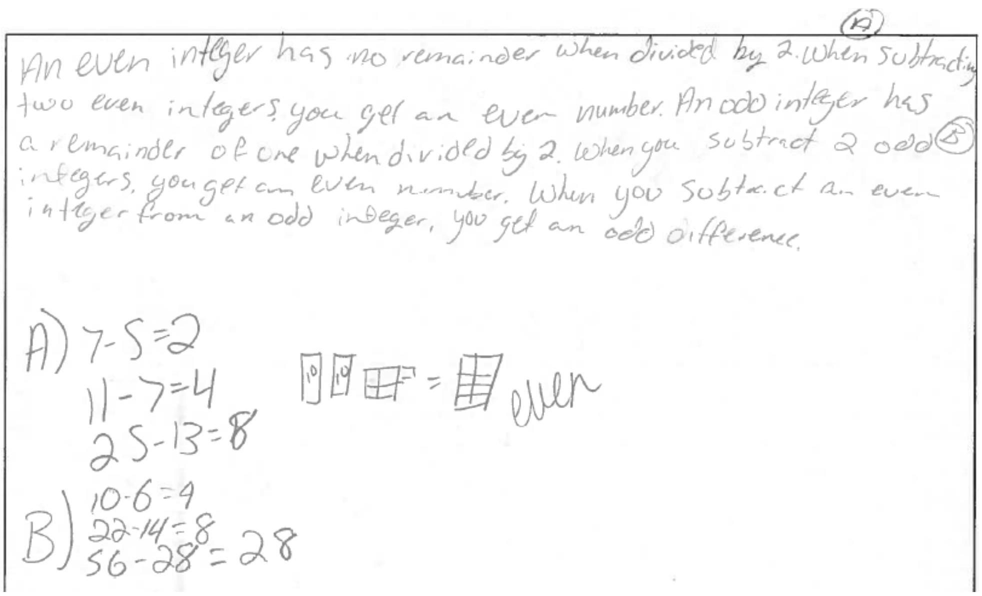
Figure 24. Writing Assignment 2, Second Attempt, Category 4, Sample 2N.

5.2.2.5 Writing Assignment 2, Second Attempt, Category 5

Four of the eighty-three students (approximately 5%) who revised this assignment remained in this category. These students satisfied one of the five criteria for the writing

assignment. Two students turned in incomplete second attempts, and one student rewrote the entire assignment but referenced addition instead of subtraction. The remaining student struggled to communicate his thoughts in writing.

Example. The writer of Sample 2O does not complete her second attempt of this writing assignment. She writes the three conjectures correctly and provides definitions for even and odd integers, but she does not provide a statement that any chosen variables represent integers. She does not write any algebraic expressions for even and odd integers, and she provides one unclear attempt at a geometric justification for the first conjecture. Like other students, she provides examples illustrating the conjectures and might mistakenly believe that the examples are the same as algebraic proofs.



(A)

An even integer has no remainder when divided by 2. When subtracting two even integers you get an even number. An odd integer has a remainder of one when divided by 2. When you subtract 2 odd (B) integers, you get an even number. When you subtract an even integer from an odd integer, you get an odd difference.

A) 7-5=2
11-7=4
25-13=8

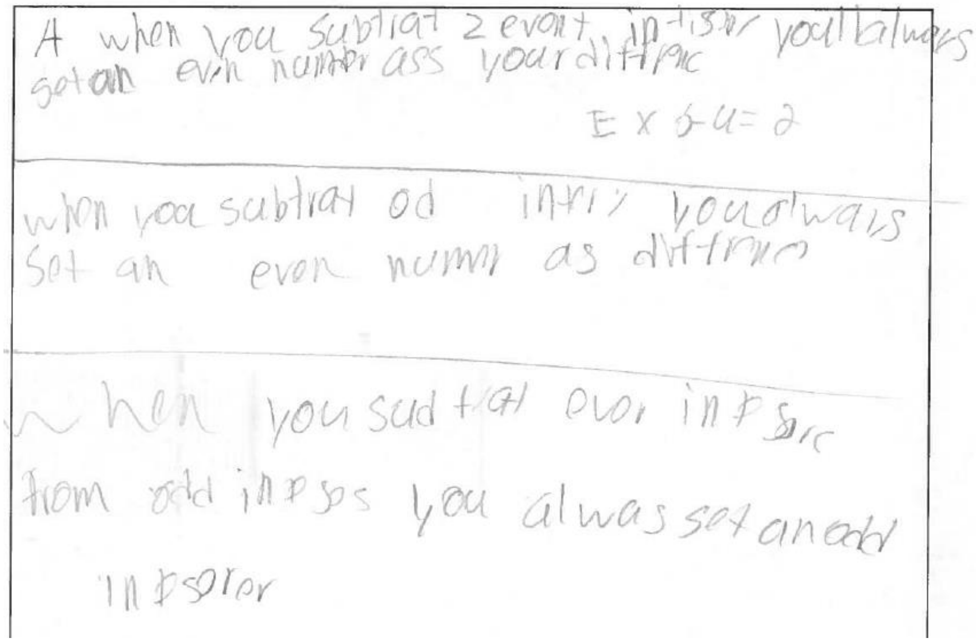
= even

B) 10-6=4
22-14=8
56-28=28

Figure 25. Writing Assignment 2, Second Attempt, Category 5, Sample 2O.

Example. The writer of Sample 2P writes the three conjectures correctly but has great difficulty expressing himself in writing. He provides one example (i.e., $6 - 4 = 2$) for the first conjecture but otherwise makes no attempt to provide geometric or algebraic justifications for

the conjectures. He makes no attempt to provide definitions or algebraic expressions for even and odd integers.


A when you subtrat 2 evont intiser youl always got an evn numbr ass your diffrnc

Ex 6-4=2

when you subtrat od intri you alwais got an even numm as diffrnc

when you sudtrat evon intsrs from odd intsrs you alwas get an odd intsrer

Figure 26. Writing Assignment 2, Second Attempt, Category 5, Sample 2P.

Below is a summary of the overall performance of students on Writing Assignment 2.

Table 2. Summary of Responses for Writing Assignment 2.

		Category					
		1	2	3	4	5	Total
Cumulative number of students in each category after each attempt	First	0	9	34	31	12	86
	Second	32	33	5	12	4	86
Net change in category counts		+32	+24	–29	–19	–8	

Even though students were permitted to refer to their notes both in their SpringBoard books and their notebooks, many of them did not use resources on the first attempt of this writing assignment. Several students gave a cursory glance to the material, enough to remember that we had used E to represent even integers and O to represent odd integers when we were developing

conjectures for a similar problem. Most students did not look further to see that we had used $2p$ and $2m$ to represent even integers and $2t + 1$ and $2n + 1$ to represent odd integers. The majority of students (60 out of 86, or 70%) know that algebraic justifications involve variables but do not necessarily know how to set up the expressions to show subtraction and then correctly simplify. Most students neglect to define terms or define variables.

Students received their graded responses along with an exemplar showing a response that met all criteria for a similar assignment related to addition rather than subtraction, which is the problem that they had completed the day before. In the exemplar, I condense the relevant information from the activity in the textbook onto one page and label some of the criteria (e.g., conjecture, geometric justification, algebraic justification). Even though students were permitted to refer to the exemplar and their notes both in their textbooks and in their notebooks prior to writing their second attempts, fourteen of the eighty-three students (approximately 17%) were able to satisfy only one or two criteria for this writing assignment.

My conclusion is that the students believed that the importance of this writing assignment was to have them formulate the three conjectures, which is something that almost all students were able to do. The format of the lesson in the textbook required students to write examples to help them create conjectures that are arguably common sense for even elementary students. Most students include examples in their responses although none need to be included. Students likely do not understand that the inclusion of the examples serves the purpose of generating the conjectures not proving the conjectures. If the textbook had presented the conjectures about addition as statements of truth rather than having students formulate them, students might have focused on the proof aspect of the problem, which would have been limited to providing definitions and algebraic and geometric justifications for the conjectures.

5.3 Writing Assignment 3

> Please answer the question fully using good communication skills. Please make sure all of your writing is within the box.
> Given quadrilateral $A(-1,4)$, $W(2,2)$, $S(0,-1)$, $M(-3,1)$. Classify the quadrilateral as the most specific type it can be. Show your work, and explain what the math tells you about the quadrilateral you are classifying.

This assignment required students to take four given coordinates and determine the type of quadrilateral that was formed. This assignment was given to students after instruction on classifying quadrilaterals. Students were not provided a blank graph but were permitted to graph the coordinates by hand with the understanding that properties of the quadrilateral would need to be discussed.

I gave this writing assignment in the second half of a unit on quadrilaterals. The first half of the unit was spent discussing the properties of various types of quadrilaterals (e.g., parallelogram, rectangle, rhombus, square, trapezoid, kite) and classifying them as the most specific quadrilateral possible. A typical problem in the first half of this unit would involve giving students figures labeled with side lengths, angle measurements, hash marks, or some combination thereof. Students would analyze the figures and determine which type of quadrilateral was shown.

The next part of the unit required students to graph four points on a coordinate plane and then classify the quadrilateral. As a class, we calculated the slopes of the segments from the graph (using the rise over run technique) to determine whether opposite sides were parallel and whether the slopes of consecutive sides were perpendicular. We substituted the rise over run numbers from the slope calculations into the distance formula to determine that when the absolute values of the slope numbers were the same, the distances would be the same, indicating when sides were congruent. Although this was not the only method discussed in class, it was the

method that appealed to most students. The students preferred to determine first whether the quadrilateral was a parallelogram and, then, if it was, to determine what type of parallelogram it was by calculating side lengths and slopes of consecutive sides. If the quadrilateral was not a parallelogram, the students would then check to see whether it was a trapezoid or a kite or whether it was not any particular type of quadrilateral. The students stated that they preferred this method because they only had to perform four slope calculations, and then they could use those numbers in the distance formula to check for congruence. They could tell with little effort whether the slopes were negative reciprocals indicating right angles.

Finally, at the end of the unit, students worked with four points and performed the same slope calculations without the aid of a coordinate plane, using the slope formula $\left(\frac{y2-y1}{x2-x1}\right)$ instead of the rise over run method. Most students preferred this purely calculation method to graphing the quadrilateral and even came up with the phrase, "You can graph it or math it." Some students had difficulties knowing which sides were opposite sides, and I recommended that those students continue to draw quadrilaterals until they became comfortable identifying opposite sides from the name of the quadrilateral.

On the day of the writing assignment, I reviewed a problem similar to the writing assignment that the class had completed on the prior day. Then I handed out the assignment in class, read the instructions at the top of the paper, and informed students that they could use their notes during the completion of the writing assignment. I reiterated that the goal of the writing assignment was clear and effective communication of the properties of quadrilaterals. I told the students that they could draw their own coordinate planes and graph the quadrilateral if they needed the extra support. Students were given thirty minutes of class time to complete the

writing assignment although most students did not take the entire thirty minutes. Any student who requested additional time was permitted to finish writing after the elapse of thirty minutes.

The given quadrilateral in the writing assignment was a square. In order to receive full credit for this assignment, students had to include the following statements: (1) all sides are congruent, (2) adjacent or consecutive sides are perpendicular, and (3) the name of the specific type of quadrilateral based on their calculations. Students were not penalized for incorrectly naming the quadrilateral so long as the quadrilateral selected was based on their other statements. However, students did not receive credit for criteria 3 if they named the correct quadrilateral but did not base the decision on a discussion of mathematics.

While the majority of students had no difficulty naming the shape, they did not clearly communicate why they knew the quadrilateral was a square. Most students performed the slope calculations correctly, but they often neglected to state that the shape had right angles or that the sides were congruent. Approximately one quarter of the class struggled to perform calculations that would allow them to classify the quadrilateral or neglected to discuss mathematics at all.

5.3.1.1 Writing Assignment 3, First Attempt, Category 1

Thirty-five out of eighty-seven students (approximately 40%) satisfied all three criteria on their first attempt at this writing assignment.

Example. The writer of Sample 3A follows the process that we discussed in class, which was first to determine whether the quadrilateral is a parallelogram and if so, what type. She even crosses out the other types of parallelograms as she eliminates them from contention. She calculates the slopes of the opposite sides and states that they are parallel. She also states that all sides are the same length and that the sides are perpendicular to each other.

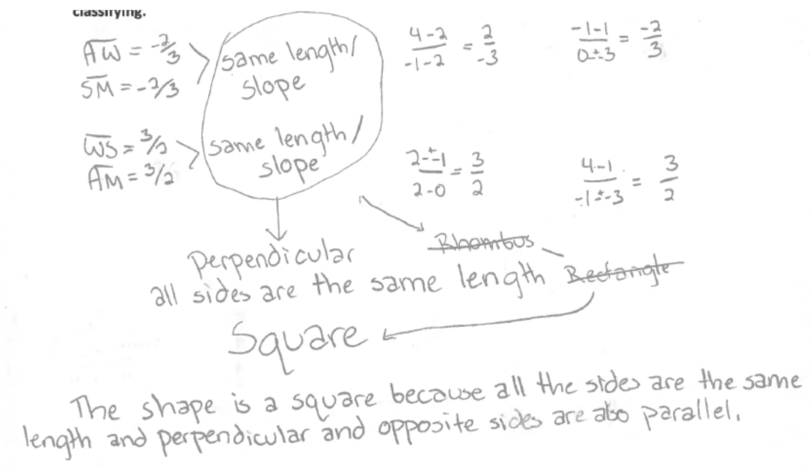


Figure 27. Writing Assignment 3, First Attempt, Category 1, Sample 3A.

Example. The writer of Sample 3B states that segments AW and SM are opposite sides and that they are parallel. He also states that segments AM and WS are opposite sides and parallel. He states that segments AM and WS are perpendicular to segments AW and SM because their slopes are negative reciprocals. This student also follows the procedure that we used in class in order to classify the quadrilateral as a square.

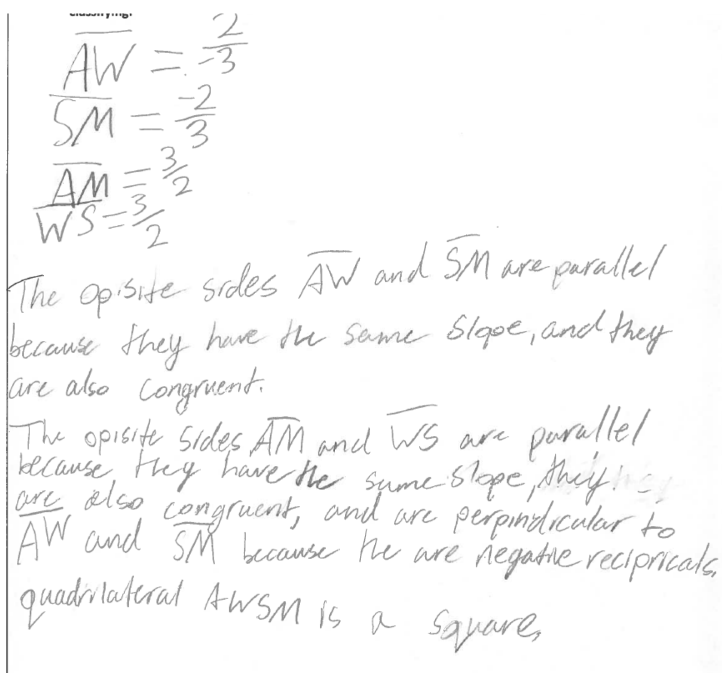


Figure 28. Writing Assignment 3, First Attempt, Category 1, Sample 3B.

Example. The writer of Sample 3C does not use the process that we followed in class but still satisfies all of the criteria for the problem. He states that all sides are congruent and that the angles are perpendicular. He specifically shows that the slopes he calculated, $-\frac{2}{3}$ and $\frac{3}{2}$, are perpendicular to each other. Based on his calculations and analysis, he is able to identify the quadrilateral as a square.

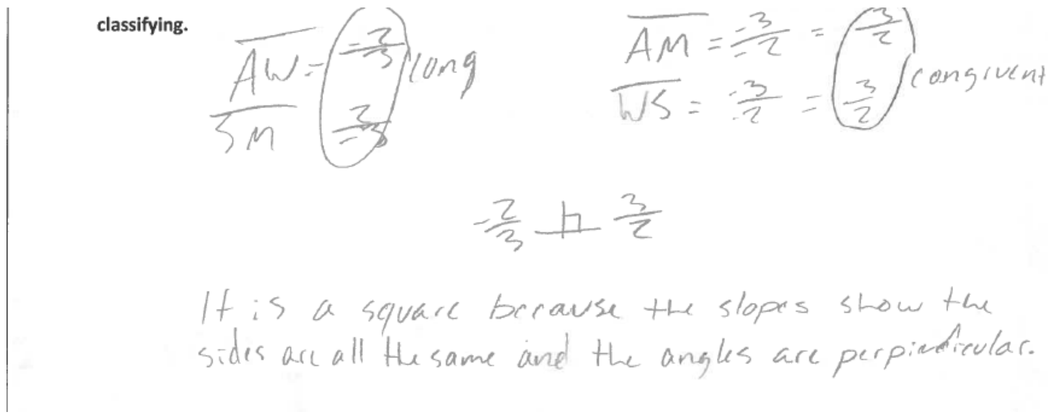


Figure 29. Writing Assignment 3, First Attempt, Category 1, Sample 3C.

5.3.1.2 Writing Assignment 3, First Attempt, Category 2

Nineteen of the eighty-seven students (approximately 22%) satisfied two of the three criteria on their first attempt at this writing assignment. Students either neglected to point out that sides were congruent of that consecutive sides were perpendicular.

Example. The writer of Sample 3D explains how he knows the figure is a quadrilateral and how he knows it is, more specifically, a parallelogram. He states that the figure is a square because the figure has congruent side lengths. He does not, however, complete the analysis of the quadrilateral by considering the slopes of the consecutive sides in order to discount the rhombus, which also has congruent side lengths. He should have calculated the slopes to be perpendicular, which would have given him the last element of a square—90° angles.

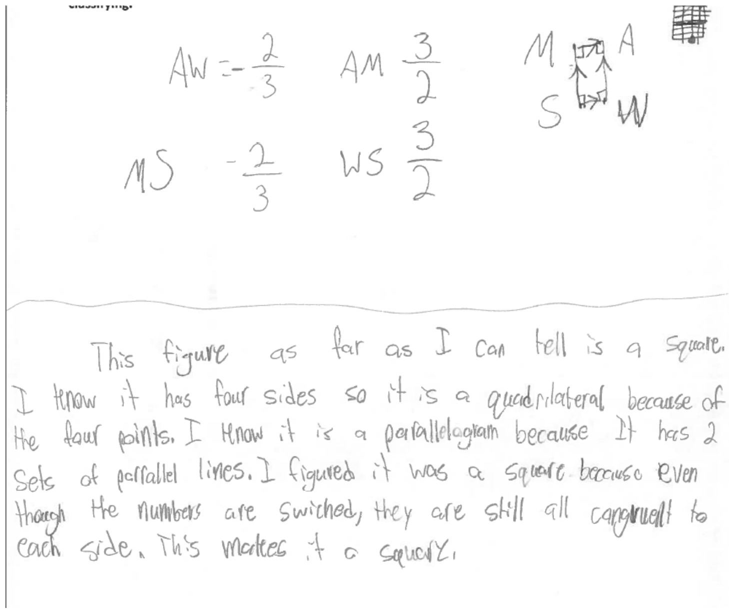


Figure 30. Writing Assignment 3, First Attempt, Category 2, Sample 3D.

Example. The writer of Sample 3E notes that opposite sides are parallel and that the angles are all 90°. He concludes that the figure is a square without discounting a rectangle as an option. He should have stated that all four sides were congruent to distinguish between a rectangle and a square.

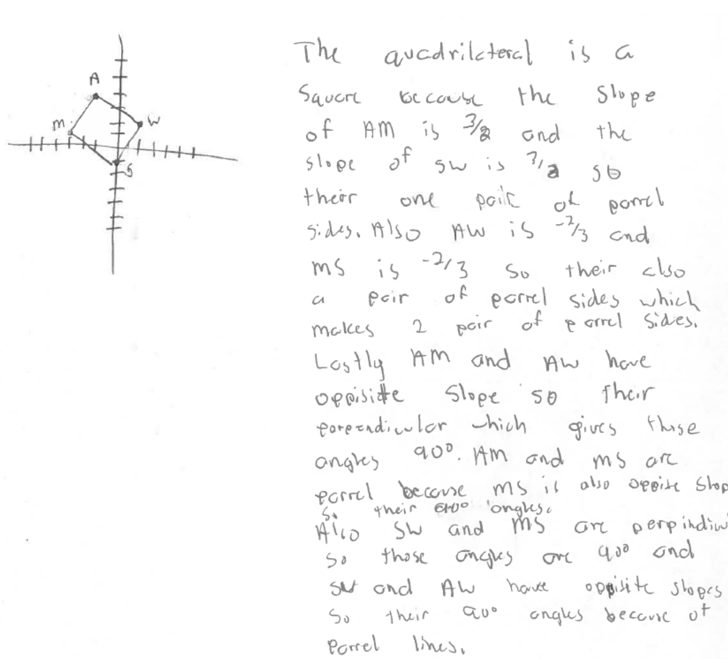


Figure 31. Writing Assignment 3, First Attempt, Category 2, Sample 3E.

5.3.1.3 Writing Assignment 3, First Attempt, Category 3

Thirteen of the eighty-seven students (approximately 15%) satisfied one of the three criteria on their first attempt at this writing assignment.

Example. The writer of Sample 3F satisfies only one of the three criteria in his response. He classifies the quadrilateral correctly. He does state that opposite sides of the quadrilateral are congruent, but he never clearly states that all four sides of the quadrilateral are congruent. Although his slope calculations show that there are sides that are perpendicular to other sides, he never indicates which sides are consecutive and that they are perpendicular.

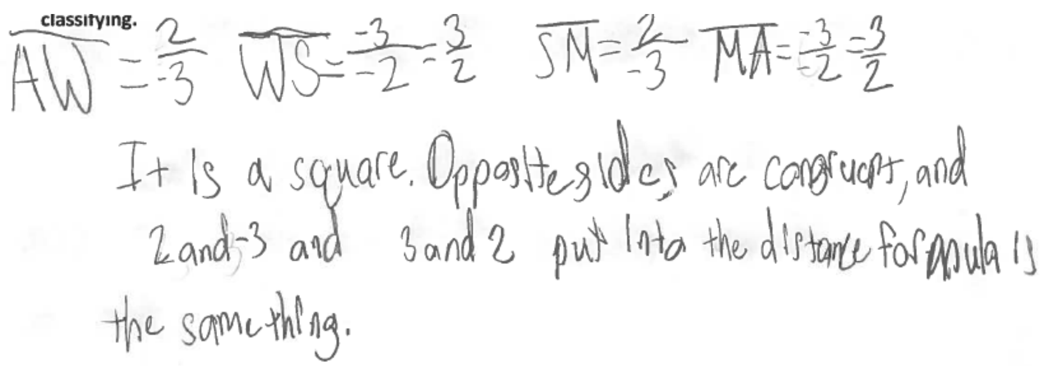


Figure 32. Writing Assignment 3, First Attempt, Category 3, Sample 3F.

5.3.1.4 Writing Assignment 3, First Attempt, Category 4

Twenty of the eighty-six students (approximately 23%) satisfied none of the three criteria on their first attempt at this writing assignment.

Example. The writer of Sample 3G does not classify the quadrilateral correctly. Even though she states that all sides have the same length, the math that she performs does not support her conclusion. She calculates the slope of only one segment and cannot, therefore, claim to know that all four side lengths are congruent.

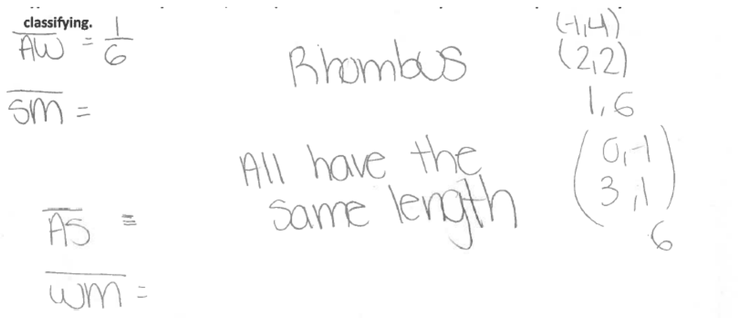


Figure 33. Writing Assignment 3, First Attempt, Sample 3G.

<u>Example.</u> The writer of Sample 3H, sketches the coordinates on a coordinate plane and concludes that the shape is a square. He does not provide any mathematical support for this classification, so he did not receive any credit for this assignment. He made a guess from his graph, and the directions clearly state that students must explain what the math tells them about classifying the quadrilateral.

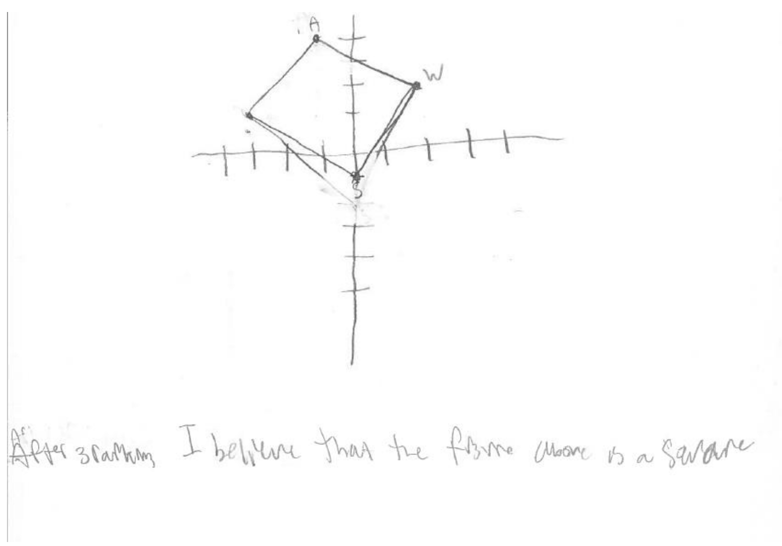
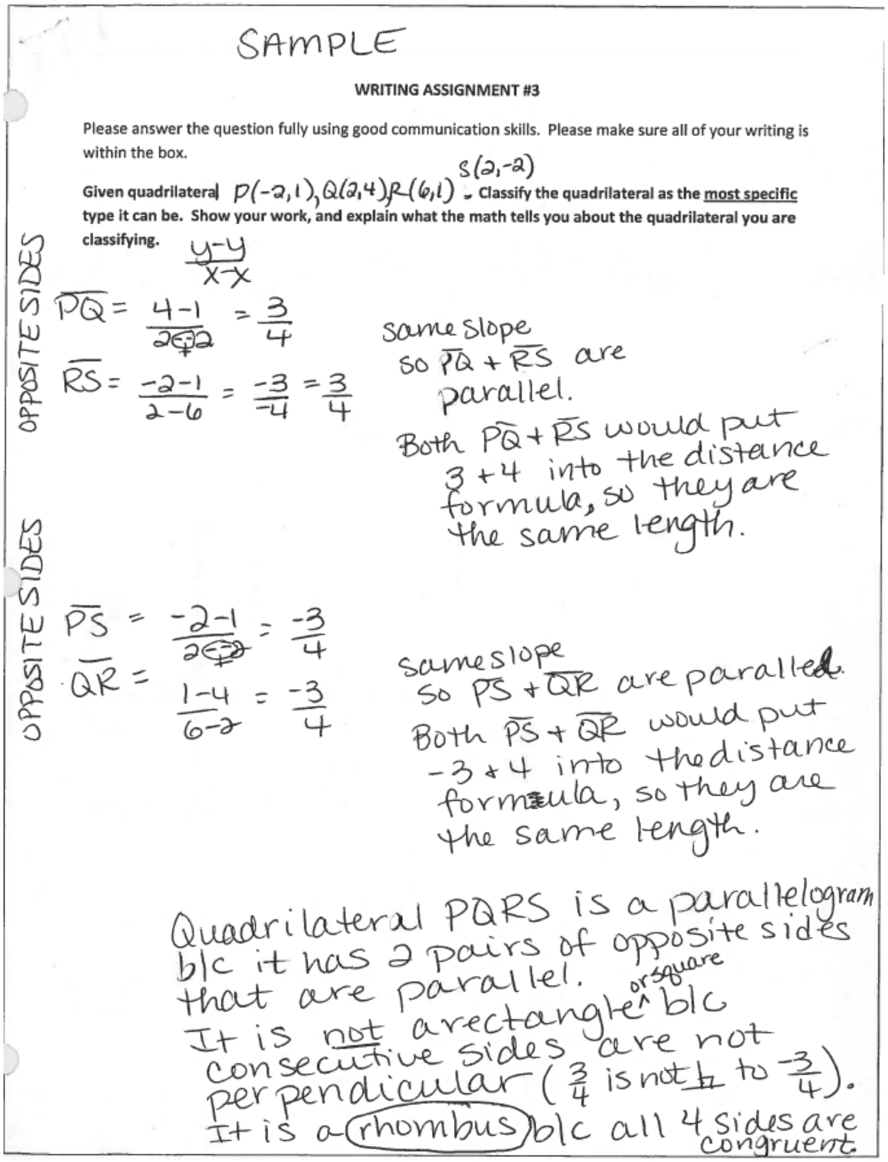


Figure 34. Writing Assignment 3, First Attempt, Category 4, Sample 3H.

I gave the following exemplar to the students along with their graded first attempts at the writing assignment. I changed the four coordinates and provided the solution with a level of specificity that satisfied all three criteria for the assignment.



SAMPLE

WRITING ASSIGNMENT #3

Please answer the question fully using good communication skills. Please make sure all of your writing is within the box.

Given quadrilateral $P(-2,1), Q(2,4), R(6,1), S(2,-2)$. Classify the quadrilateral as the <u>most specific</u> type it can be. Show your work, and explain what the math tells you about the quadrilateral you are classifying.

OPPOSITE SIDES

$\frac{y-y}{x-x}$

$\overline{PQ} = \frac{4-1}{2-(-2)} = \frac{3}{4}$

$\overline{RS} = \frac{-2-1}{2-6} = \frac{-3}{-4} = \frac{3}{4}$

same slope so $\overline{PQ} + \overline{RS}$ are parallel.

Both $\overline{PQ} + \overline{RS}$ would put 3 + 4 into the distance formula, so they are the same length.

OPPOSITE SIDES

$\overline{PS} = \frac{-2-1}{2-(-2)} = \frac{-3}{4}$

$\overline{QR} = \frac{1-4}{6-2} = \frac{-3}{4}$

same slope So $\overline{PS} + \overline{QR}$ are parallel.

Both $\overline{PS} + \overline{QR}$ would put $-3 + 4$ into the distance formula, so they are the same length.

Quadrilateral PQRS is a parallelogram b/c it has 2 pairs of opposite sides that are parallel.

It is not a rectangle or square b/c consecutive sides are not perpendicular ($\frac{3}{4}$ is not ⊥ to $-\frac{3}{4}$).

It is a rhombus b/c all 4 sides are congruent.

Figure 35. Exemplar for Writing Assignment 3.

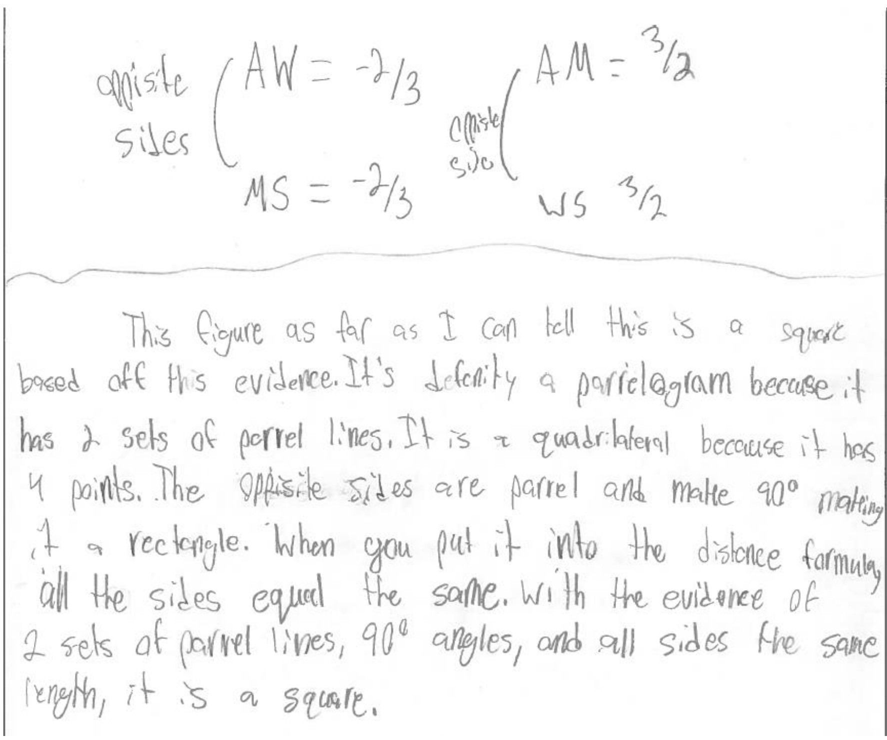
5.3.2 Second Attempt of Writing Assignment 3

Twenty-four out of a possible fifty-two students (approximately 46%) opted to rewrite this assignment. Twenty-two of the students who rewrote the assignment scored in higher categories. Two students did improve and remained in Category 2.

5.3.2.1 Writing Assignment 3, Second Attempt, Category 1

Eighteen of the twenty-four students (approximately 75%) who rewrote this assignment had responses that fell into Category 1. Thirty-five students originally had responses that fell into Category 1. Therefore, when considering first and second attempts, a total of fifty-three out of the eighty-seven students (approximately 61%) had responses that fell into Category 1.

Example. Sample 3I represents the writing in Category 1. The student shows that the quadrilateral is a parallelogram because it has two sets of parallel lines. He also states that the quadrilateral has 90° angles and congruent sides, prompting him to conclude that the quadrilateral is a square.



oppisite sides (AW = -2/3, MS = -2/3

oppisite side (AM = 3/2, WS 3/2

This figure as far as I can tell this is a square based off this evidence. It's defenity a parrelagram because it has 2 sets of parrel lines. It is a quadrilateral because it has 4 points. The oppisite sides are parrel and make 90° making it a rectangle. When you put it into the distance formula, all the sides equal the same. With the evidence of 2 sets of parrel lines, 90° angles, and all sides the same length, it is a square.

Figure 36. Writing Assignment 3, Second Attempt, Category 1, Sample 3I.

5.3.2.2 Writing Assignment 3, Second Attempt, Category 2

Four of the twenty-four students (approximately 17%) who rewrote this assignment had responses that fell into Category 2. Two of these students improved their scores from their first

attempts, and two of these students scored in this category for the second time. Eleven students who originally scored in Category 2 on their first attempts, opted to keep their original scores. When considering first and second attempts, fifteen out of eighty-seven students (approximately 17%) had responses that fell into Category 2.

Example. The writer of Sample 3J shows the correct slope calculations for the pairs of opposite sides and concludes that they are parallel. She indicates that the sides are congruent to each other and that the quadrilateral is a square. She never states that the consecutive sides are perpendicular.

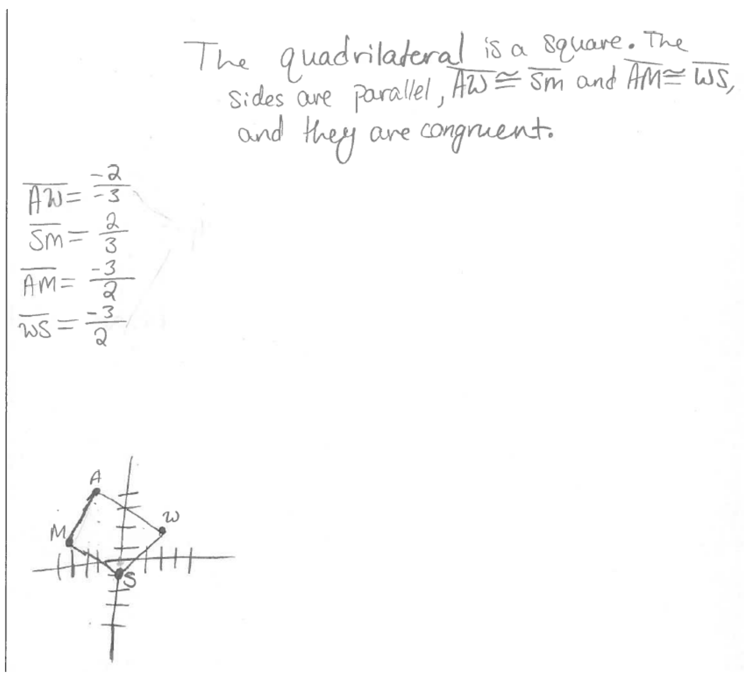
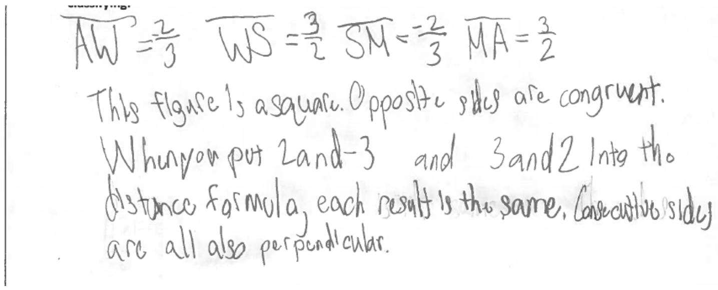


Figure 37. Writing Assignment 3, Second Attempt, Category 2, Sample 3J.

Example. The writer of Sample 3K states that the quadrilateral is a square and that consecutive sides are perpendicular. He states that opposite sides are congruent but never states that all four sides are congruent.



$\overline{AW} = \frac{2}{3}$ $\overline{WS} = \frac{3}{2}$ $\overline{SM} = \frac{-2}{3}$ $\overline{MA} = \frac{3}{2}$

This figure Is asquare. Opposite sides are congruent. When you put 2and-3 and 3and2 Into the distance formula, each result is the same. Consecutive sides are all also perpendicular.

Figure 38. Writing Assignment 3, Second Attempt, Category 2, Sample 3K.

5.3.2.3 Writing Assignment 3, Second Attempt, Category 3

Two of the twenty-four students (approximately 8%) who rewrote this assignment had responses that fell into Category 3. Three students who originally scored in Category 3 on their first attempts opted to keep their scores; thus, five out of eighty-seven students (approximately 6%) had responses that fell into Category 3 when considering both first and second attempts.

Example. The writer of Sample 3L performs four slope calculations and states that the quadrilateral is a square. His discussion of the math that he performed did not support his conclusion that the quadrilateral was a square. Without a discussion about the side lengths and angle measures, he is unable to do more than guess that the quadrilateral is a square.

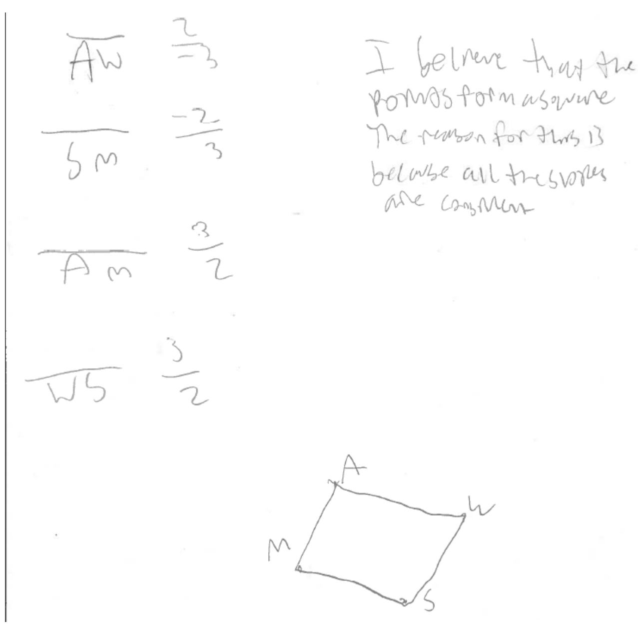


Figure 39. Writing Assignment 3, Second Attempt, Category 3, Sample 3L.

5.3.2.4 Writing Assignment 3, Second Attempt, Category 4

No students who rewrote this assignment had writing that fell into this category. Fourteen students who originally scored in Category 4 opted to keep their scores. Thus, fourteen out of eighty-seven students (approximately 16%) had writing that fell into Category 4. Samples of this work were provided in the discussion of the first attempts of this writing assignment.

To summarize the responses for Writing Assignment 3, a large percentage of students (approximately 62%) were able to satisfy at least two of the three criteria for this assignment on their first attempts. This is an increase from the percentage of students who were able to score in the top two categories for Writing Assignment 2 on their first attempts, which was only 10%. After reviewing the exemplar, twenty-four students rewrote the assignment. Eighteen of the twenty-four students (75%) improved their writing so as to fall in Category 1. Overall, 61% of students wrote responses that satisfied all three criteria for Writing Assignment 3.

Table 3. Summary of Responses for Writing Assignment 3.

		Category				
		1	2	3	4	Total
Cumulative number of students in each category after each attempt	First	35	19	13	20	87
	Second	53	15	5	14	87
Net change in category counts		+18	–4	–8	–6	

5.4 Writing Assignment 4

(Please see diagram in sample assignments.)

Set up an equation to solve for x, and explain what facts about angles you used in doing so. Then solve for x and fill in the angle measures for each angle. In the picture, *AD* and *CG* are lines meeting at *O* (not labeled) and all other segments have *O* as an endpoint. Remember to use clear and effective communication when you explain what facts about angles you are using to set up your equation.

This assignment required students to review a diagram consisting of several intersecting lines and segments that created angles. Students were provided with information about the degree measurements, some specifically stated and others written as variable expressions. Students were required to write an equation useful in determining the measure of each angle and to provide a reasoning for the steps taken. This assignment occurred during the final exam review in May. It was meant to be a review of the angle addition postulate, vertical angles, supplementary angles, and other concepts.

I handed out the assignments in class, read the instructions at the top of the paper, and informed students that they could use their notes during the completion of the writing assignment. I reiterated that the goal of the writing assignment was clear and effective communication of geometric principles, specifically properties of angles. Students were given thirty minutes to complete the writing assignment during class.

A large percentage of students (approximately 78%) wrote responses that fell into the top two categories on their first attempts. This indicates that students are becoming accustomed to writing clearly and including the requisite vocabulary in their responses without being prompted.

5.4.1 First Attempt of Writing Assignment 4

Eighty-six students completed this assignment. In order to receive full credit for the writing assignment, students had to include (1) a correct equation that could be used to solve for x, (2) a correct solution to the equation that they had written, (3) a statement indicating that a straight line contains $180°$ or that supplementary angles sum to $180°$, (4) a statement indicating that vertical angles are congruent or equal in measure, and (5) correct measurements for all four missing angles. I divided the student writing into one of five categories depending on how many of the five criteria the response satisfied.

5.4.1.1 Writing Assignment 4, First Attempt, Category 1

Forty-eight of the eighty-six students (approximately 56%) satisfied all five criteria for this writing assignment on their first attempts.

Example. Sample 4A represents the majority of the responses in this category. The student first defines a line as having $180°$. She then finds $m\angle COD$ to be $48°$ using her knowledge of supplementary angles. She then determines that $m\angle AOG$ is also $48°$ because $\angle COD$ and $\angle AOG$ are vertical angles, which are congruent. She then sets up an equation to solve for x, calculates x to be 11, and substitutes that value in for x to find the remaining missing angles.

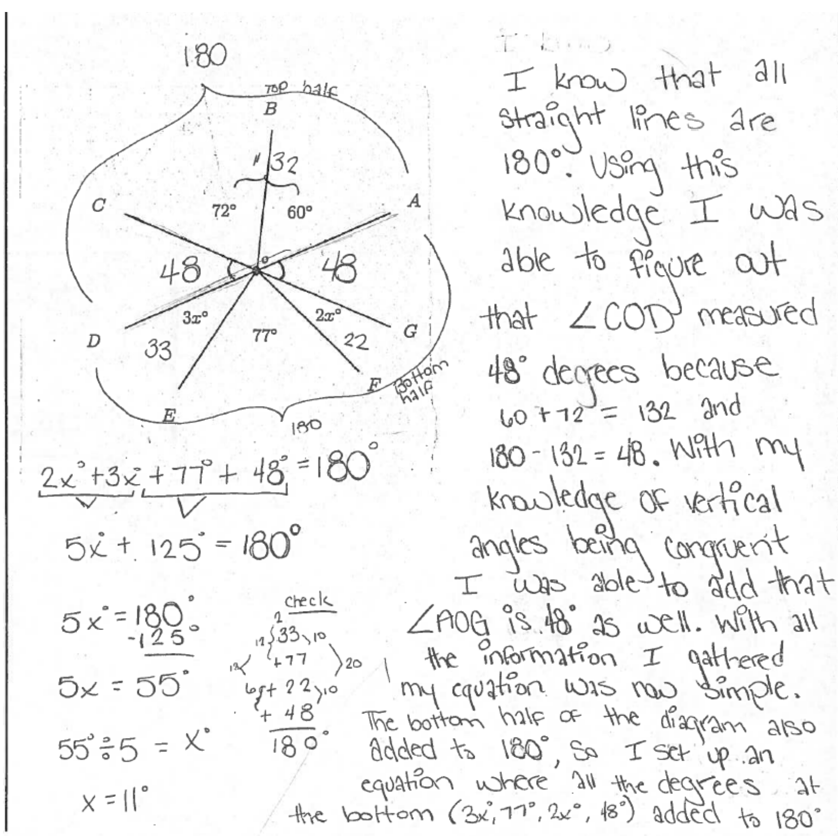


Figure 40. Writing Assignment 4, First Attempt, Category 1, Sample 4A.

Example. Sample 4B represents the minority of the responses in this category. The student first identifies a pair of vertical angles, $\angle COA$ and $\angle DOG$, and indicates that they are

equal to each other. Using this knowledge, he sets up an equation to solve for x. He substitutes this value in for x to find the remaining missing angles. He then determines that $m\angle AOG$ is 48° based on his knowledge of straight lines having 180°. Then, because $\angle AOG$ and $\angle COD$ are vertical angles, $m\angle COD$ is also 48°.

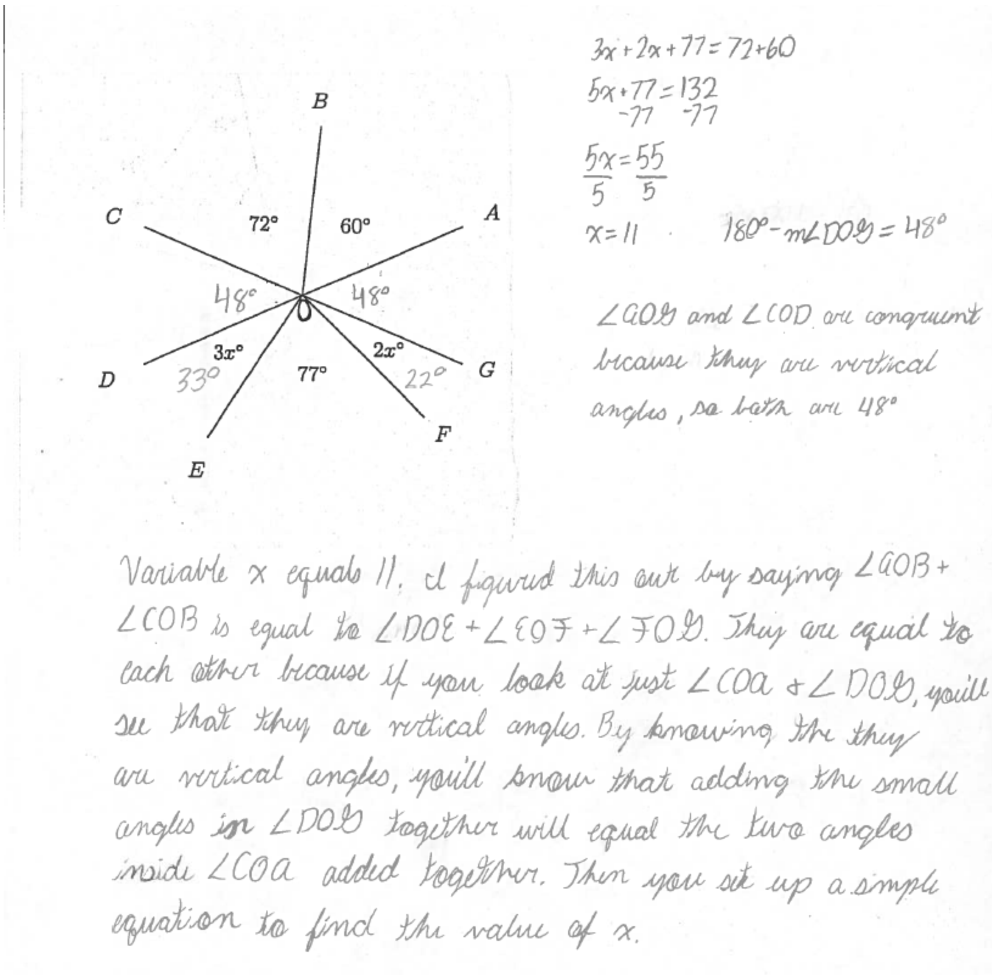


Figure 41. Writing Assignment 4, First Attempt, Category 1, Sample 4B.

<u>5.4.1.2 Writing Assignment 4, First Attempt, Category 2</u>

Nineteen of the eighty-six students (approximately 22%) omitted one of the five criteria for the writing assignment. Generally, students performed the math correctly to find the missing angles but neglected to discuss why they were able to fill in certain angles without performing

more calculations (e.g., in the case of vertical angles) or neglected to explain why they were using 180 in their calculations.

Example. Sample 4C is typical of half of the writings in this category. This student determines every angle correctly and sets up an equation to solve for x based on a straight line having 180°. He shows the calculation to find that $m\angle COD = 48°$, but he never says why he knows that $m\angle AOG = 48°$. He adds the measurement of $\angle AOG$ to his diagram but never discusses that $\angle COD$ and $\angle AOG$ are vertical angles and, therefore, congruent.

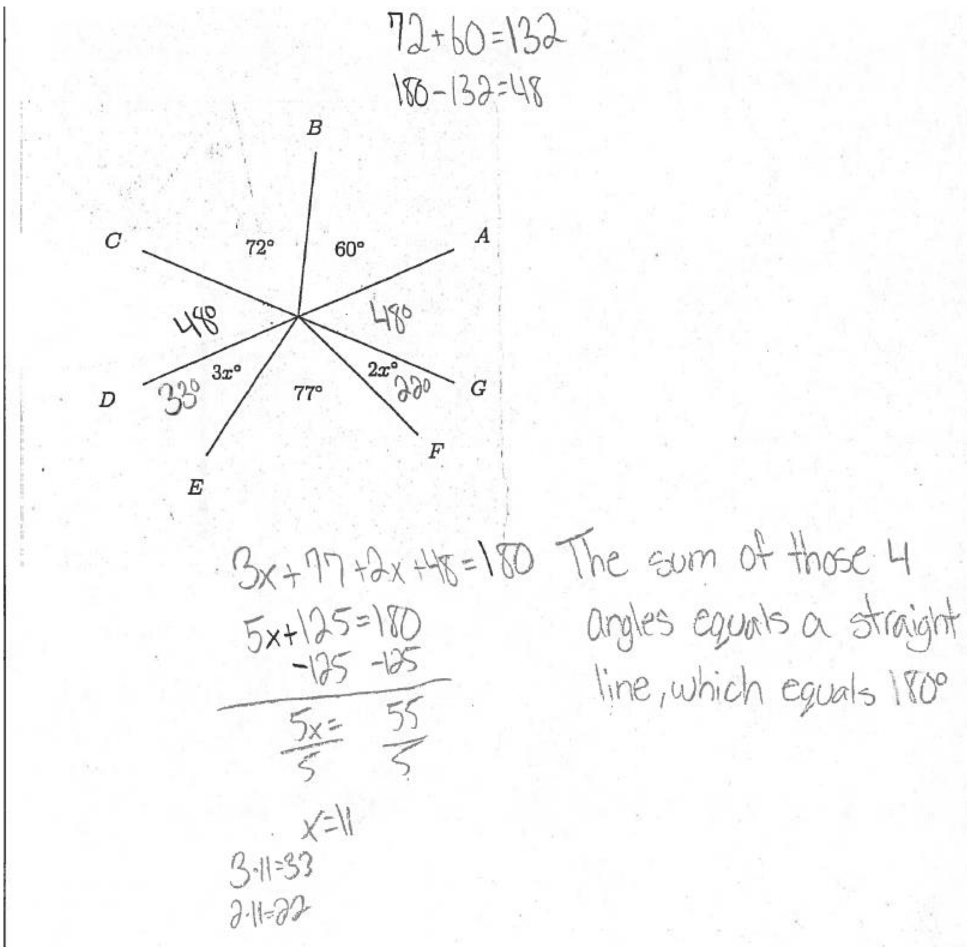


Figure 42. Writing Assignment 4, First Attempt, Category 2, Sample 4C.

Example. Sample 4D represents approximately one quarter of the responses in this category. Although the student determines every angle correctly and discusses the concept of

vertical angles, he never discusses supplementary angles or the angle addition postulate. At a minimum, he should have stated that a straight line has 180°. He uses 180 in several calculations but never explains why he is using that particular number.

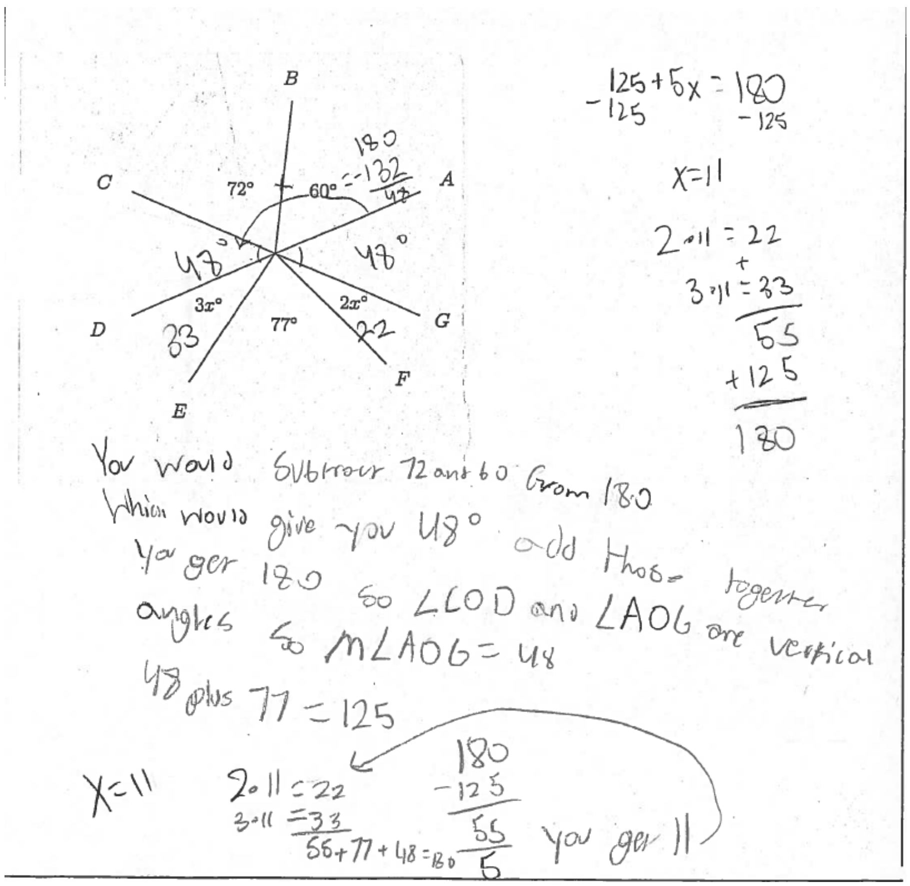


Figure 43. Writing Assignment 4, First Attempt, Category 2, Sample 4D.

5.4.1.3 Writing Assignment 4, First Attempt, Category 3

Five of the eighty-six students (approximately 6%) satisfied three of the five criteria for this writing assignment. Most students whose writing fell into this category made a combination of the same errors made by students whose writing fell into Category 2. That is, they stated the measure of an angle without explaining that they knew the measurement because two angles were vertical angles, and they never explained why they used 180 in their equations.

Example. The writer of Sample 4E finds the measurements of all four of the missing angles. He sets up his equation for x correctly and substitutes the value into the expressions to find the missing angles. He does all of this without discussing any properties of angles. He never explains why he chooses to subtract from 180 to find $m\angle COD$, and he never explains why he uses 180 in his equation to solve for x. He also never explains how he knew that $m\angle AOG = 48°$. This student's writing tells me that he knows the processes involved in finding missing angle measurements, but the writing does not show that he can discuss the properties of angles underlying his calculations.

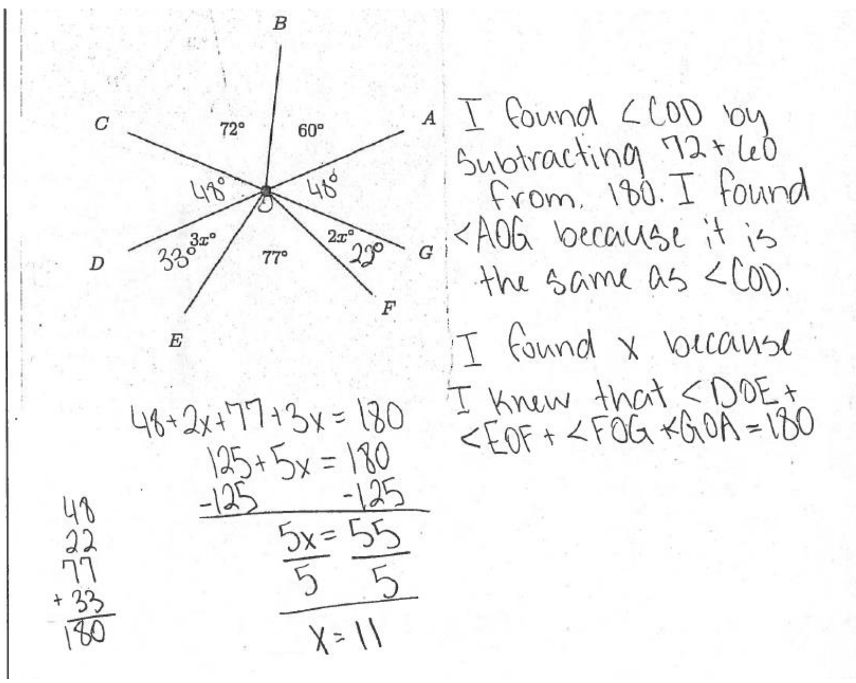


Figure 44. Writing Assignment 4, First Attempt, Category 3, Sample 4E.

5.4.1.4 Writing Assignment 4, First Attempt, Category 4

Three of the eighty-six students (approximately 3%) satisfied two of the five criteria for this writing assignment on their first attempts.

Example. Sample 4F represents all of the responses in this category. The student sets up a correct equation to solve for x and solves for x correctly. He never discusses why he chooses

to include 180 in his equation and does not discuss the concept of vertical angles. He does not determine all of the missing angle measurements. Based on the written explanations of the other two students in this category, it is likely that those two students copied their equations from other students but did not understand what the equations meant.

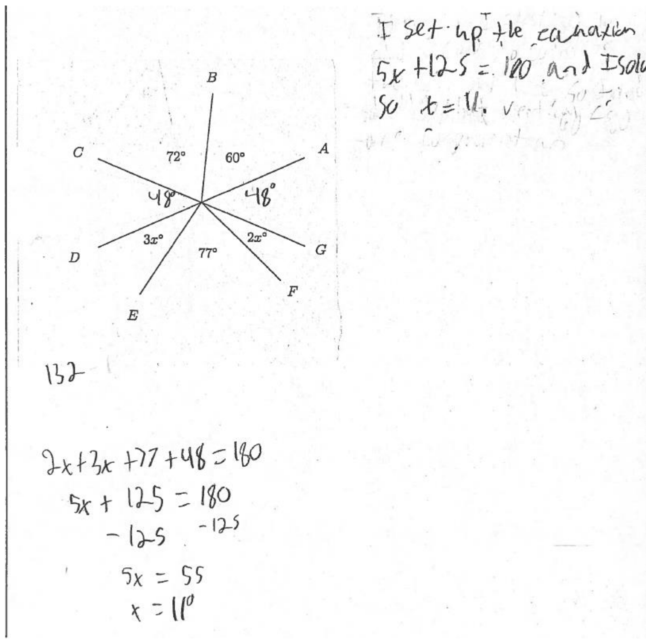


Figure 45. Writing Assignment 4, First Attempt, Category 4, Sample 4F.

<u>5.4.1.5 Writing Assignment 4, First Attempt, Category 5</u>

Eleven of the eighty-six students (approximately 13%) satisfied one of the five criteria for this writing assignment. The responses in this category have little in common with each other. Students were able to satisfy one criteria in their writings (e.g., stating that a straight line has 180°, a circle has $360°$, $\angle COA$ and $\angle DOG$ are vertical angles, or $5x + 77 = 132$). Students who set up a correct equation for x were not able to solve for x correctly. Four students set up incorrect equations for x but solved them correctly, getting answers such as $x = 12$ and $x = 15.4$.

Example. The writer of Sample 4G sets up a correct equation to solve for x but never solves it, and she justifies her equation by saying that the angles are alternate exterior angles rather than vertical angles. She does not attempt to solve for x or determine the measurements of any of the missing angles.

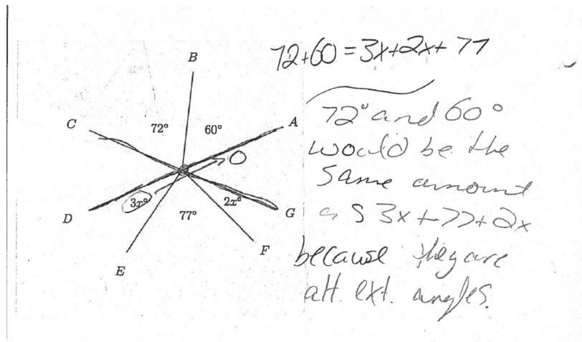


Figure 46. Writing Assignment 4, First Attempt, Category 5, Sample 4G.

Example. Sample 4H represents four of the students whose writing fell in this category. This student sets up an equation for x but does not appear to have a basis for writing his equation. He does, however, solve his own equation correctly, getting $x = 15.4$. He does not attempt to fill in any of the missing angles. He does not discuss supplementary angles or vertical angles.

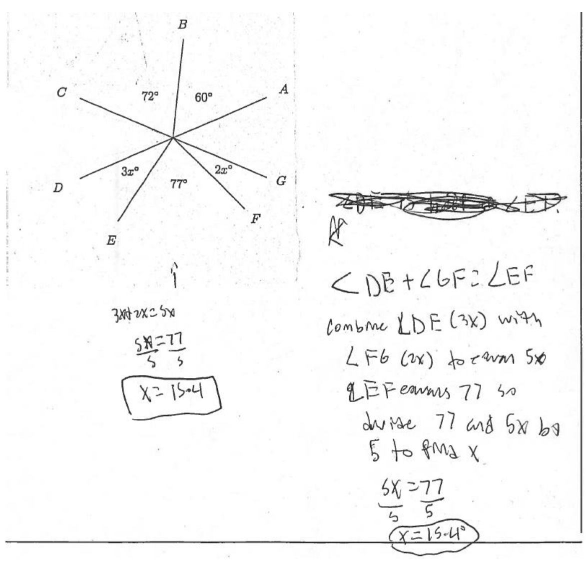


Figure 47. Writing Assignment 4, First Attempt, Category 5, Sample 4H.

5.4.2 Second Attempt of Writing Assignment 4

Students were provided the exemplar below, which used the same diagram but different angle measurements and angle expressions. The concepts of supplementary angles, angle addition postulate, and vertical angles were discussed in the context of the angle measurements and angle expressions in the diagram. An equation for x was provided as well as the solution steps for the solution process. The substitution for x was shown, and each angle was labeled with its correct measurement. The students were offered the chance to rewrite their answers to the original writing assignment. The forty-eight students whose writing satisfied all five criteria on the first attempt did not have to rewrite their assignments. Thirty-eight students were offered the opportunity to rewrite the assignment, and thirty-seven students attempted the writing assignment for a second time. The writing responses were classified into the same five categories as before.

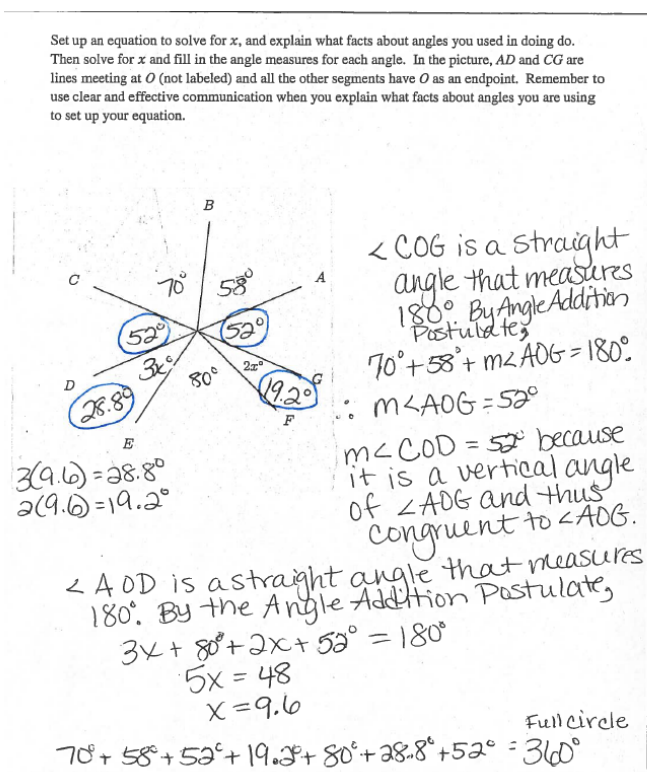


Figure 48. Exemplar for Writing Assignment 4.

5.4.2.1 Writing Assignment 4, Second Attempt, Category 1

Forty-eight students satisfied all five criteria for the writing assignment on their first attempt. Additionally, twenty-three of the thirty-seven students (approximately 62%) who rewrote the assignment had responses that fell into Category 1 on their second attempts. Therefore, seventy-one out of eighty-six students (approximately 83%) had writing that fell into Category 1 when considering both first and second attempts.

Example. The writer of Sample 4I sets up an equation showing that vertical angles are equal in measure ($\angle COA$ and $\angle DOG$) and uses that knowledge to solve for x. He knows that a straight line has 180° and uses that knowledge to find $m\angle COD$ and $m\angle AOG$, which he also names as vertical angles.

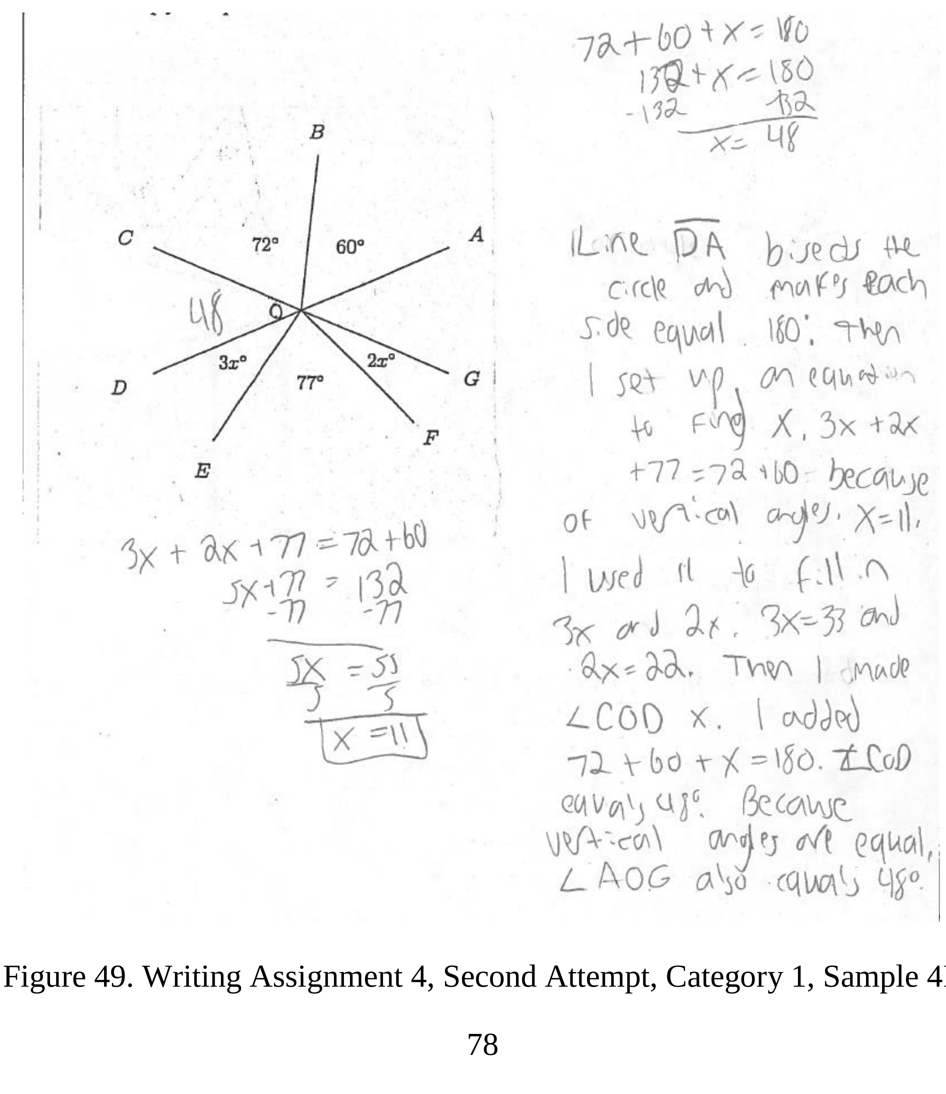


Figure 49. Writing Assignment 4, Second Attempt, Category 1, Sample 4I.

Example. The writer of Sample 4J finds $m\angle COD$ based on supplementary angles and then states that $\angle AOG$ is congruent to $\angle COD$ because they are vertical angles. She then sets up her equation to solve for x, based on her knowledge of supplementary angles. She fills in the missing angle measurements and checks her work by adding them, $48 + 33 + 77 + 22 = 180$, using her original equation, $77 + 48 + 3x + 2x = 180$.

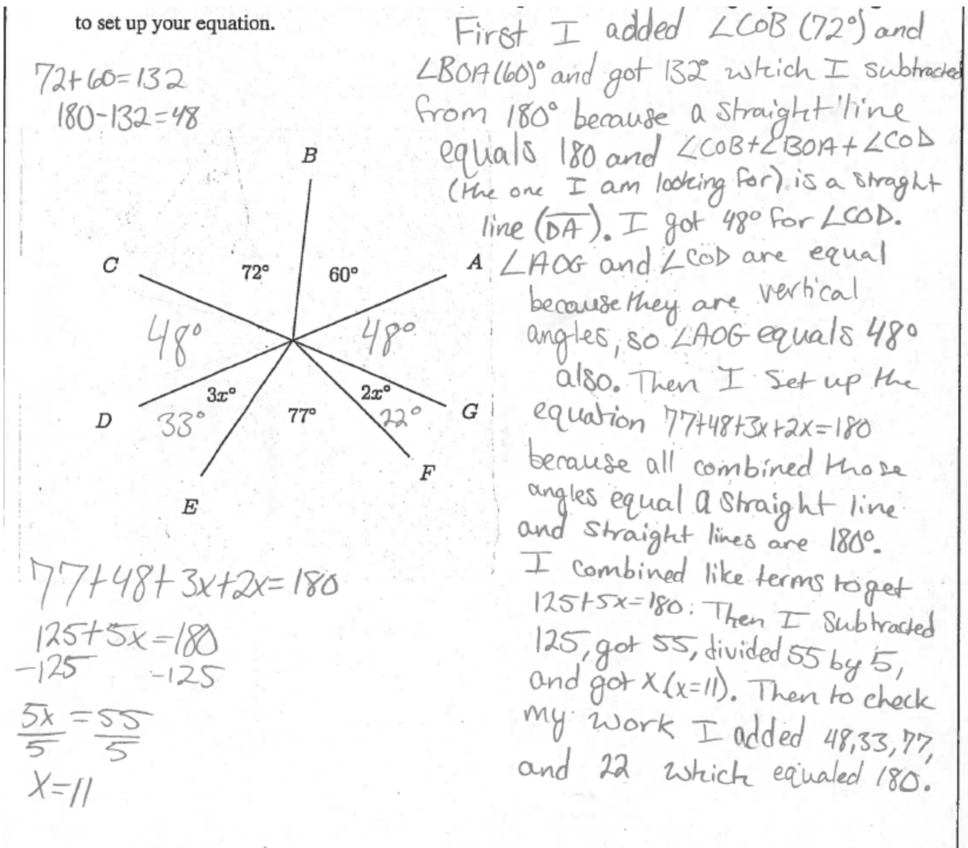


Figure 50. Writing Assignment 4, Second Attempt, Category 1, Sample 4J.

5.4.2.2 Writing Assignment 4, Second Attempt, Category 2

One student who originally had a response that fell into Category 2 opted to keep his score. Additionally, seven of the twenty-four students (approximately 29%) who rewrote this assignment satisfied four of the five criteria for the writing assignment. Four of the students did not discuss the concept of vertical angles, and the remaining three students did not provide the

measurements for all of the missing angles. Overall, eight out of eighty-six students (approximately 9%) had responses that fell into Category 2 when considering both first and second attempts.

Example. The writer of Sample 4K states that lines have 180°, sets up an equation to solve for x, and correctly solves the equation. The student provides all of the measurements for the missing angles and performs all calculations correctly, but he never explains how he knows that both $m\angle COD$ and $m\angle AOG$ are 48°.

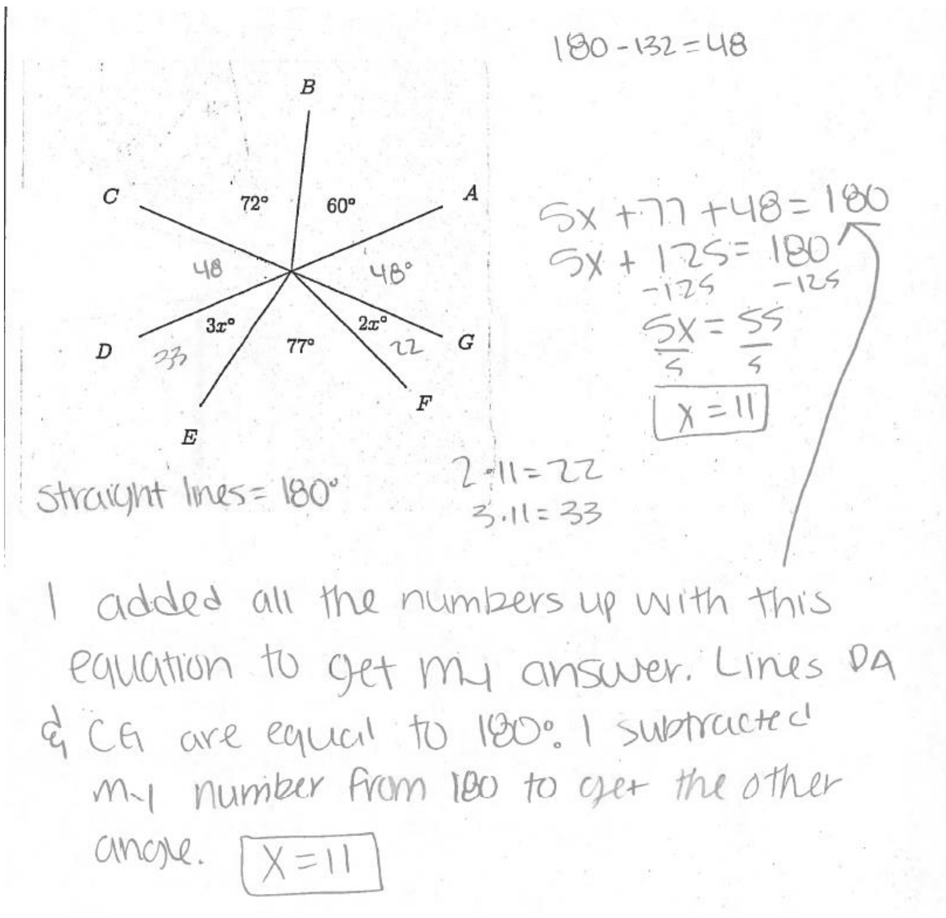


Figure 51. Writing Assignment 4, Second Attempt, Category 2, Sample 4K.

Example. The writer of Sample 4L discusses straight lines and vertical angles. She states that vertical angles are congruent, but she never states which angles are vertical angles.

She sets up and correctly solves an equation for x. She even states two ways to check her work, but she never actually provides the measurement of any of the missing angles.

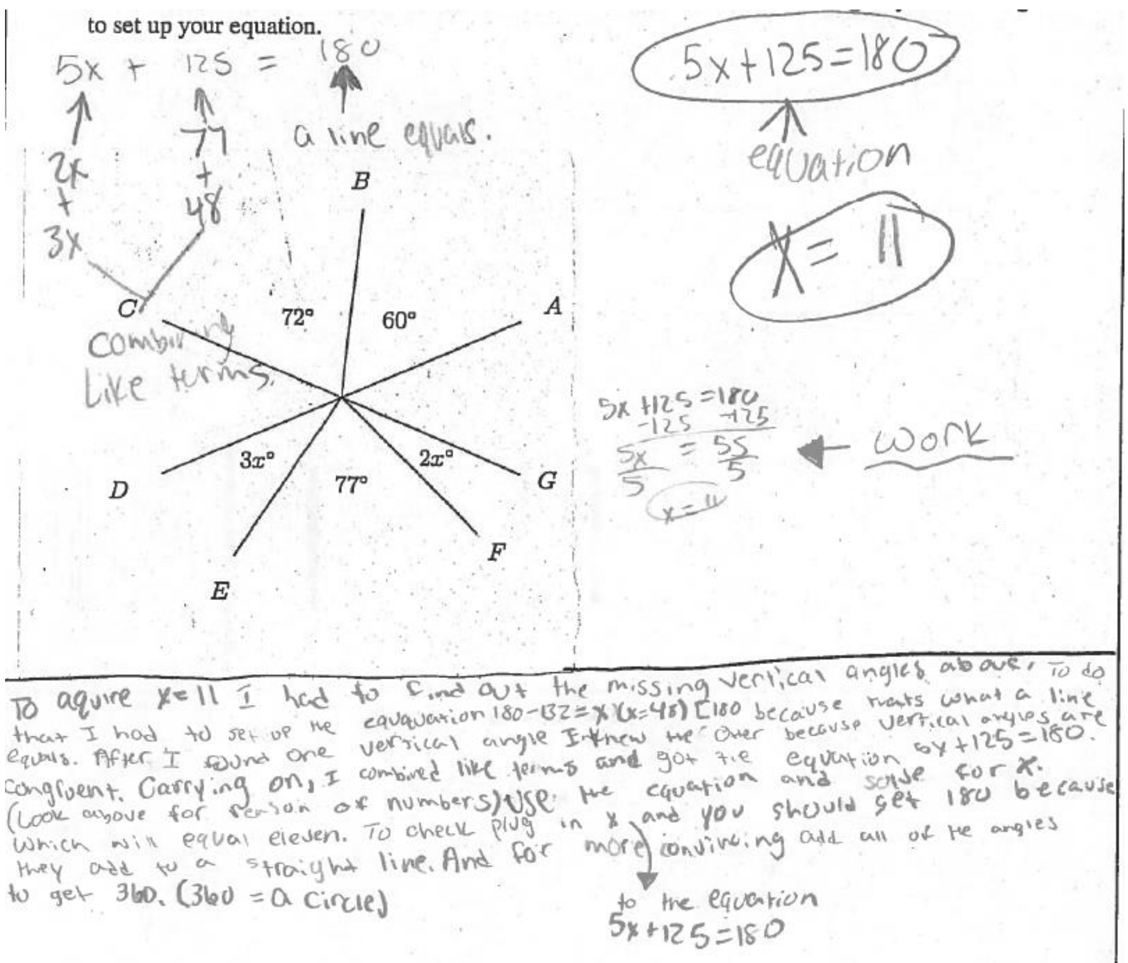


Figure 52. Writing Assignment 4, Second Attempt, Category 2, Sample 4L.

5.4.2.3 Writing Assignment 4, Second Attempt, Category 3

Five of the twenty-four students (approximately 21%) who rewrote this assignment satisfied three of the five criteria for the writing assignment. Four of the students improved their scores from their first attempts, and one student's response fell into this category for a second time. No students who originally had responses that fell into Category 3 opted to keep their original scores. Thus, five out of eighty-six students (approximately 6%) had responses that fell into Category 3 when considering both first and second attempts.

Example. Sample 4M represents four of the students whose writing fell in this category. These students make the same errors: they do not state that vertical angles are congruent, and they do not provide the measurements for all four missing angles.

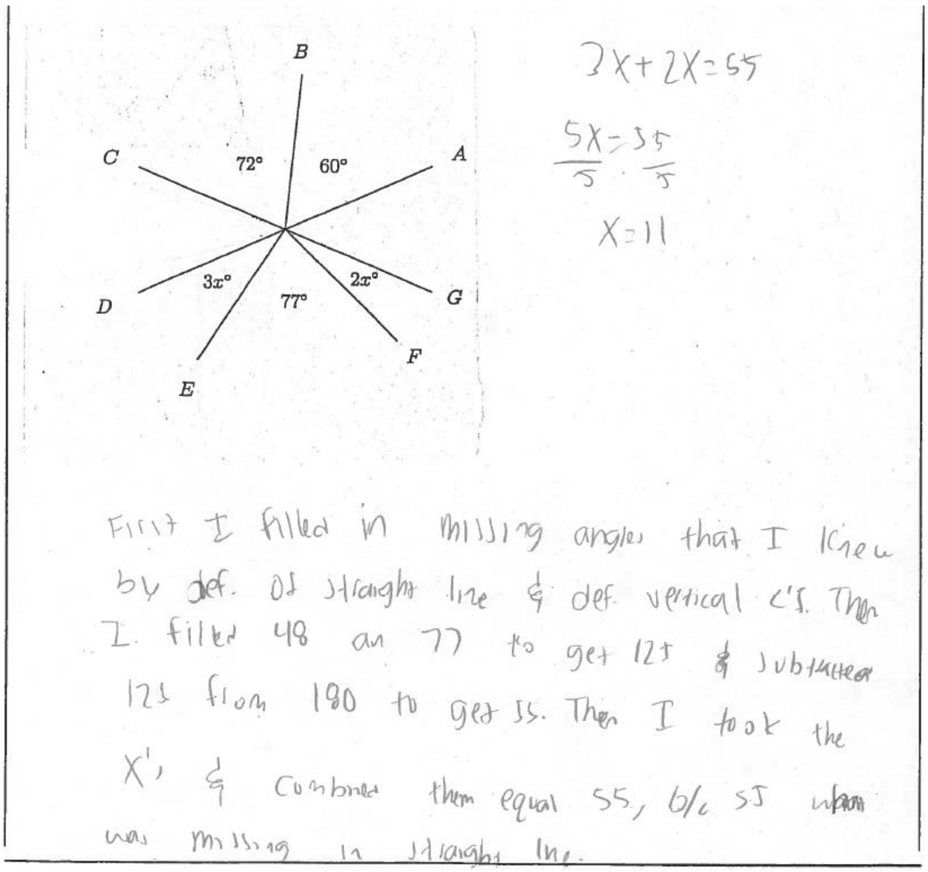


Figure 53. Writing Assignment 4, Second Attempt, Category 3, Sample 4M.

Example. The writer of Sample 4N explains that $\angle AOG$ and $\angle COD$ are vertical angles and are therefore congruent but does not discuss supplementary angles. He provides a correct equation for x but does not solve it correctly. He does fill in all angle measurements, which would have been correct if he had solved for x correctly.

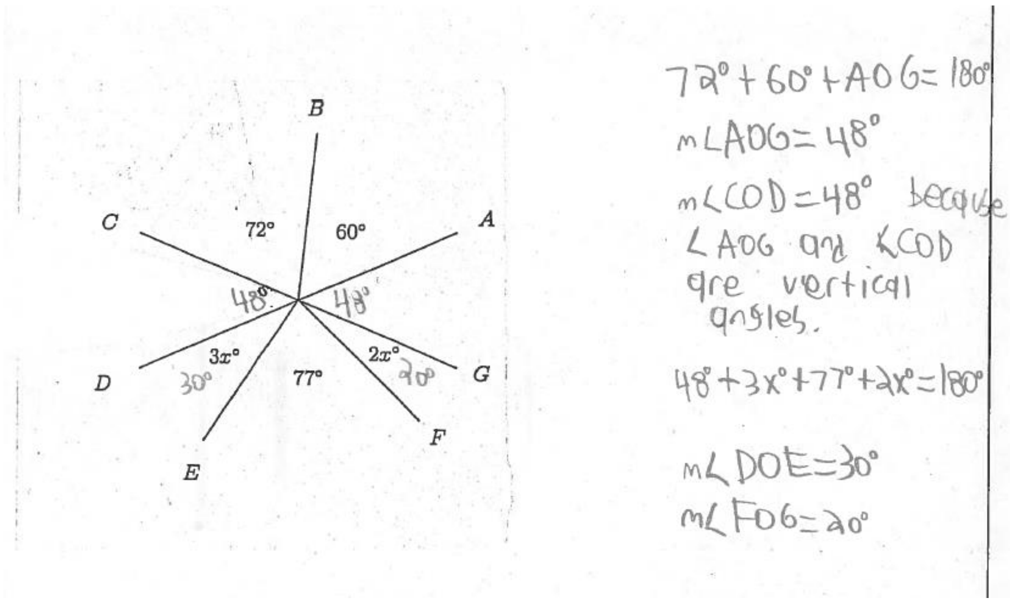


Figure 54. Writing Assignment 4, Second Attempt, Category 3, Sample 4N.

<u>5.4.2.4 Writing Assignment 4, Second Attempt, Category 4</u>

One of the twenty-four students (approximately 4%) who rewrote this assignment satisfied two of the five criteria for this writing assignment. This student moved up one level from his first attempt. No students who originally had responses in Category 4 opted to keep their scores. Therefore, one out of eighty-six students (approximately 1%) satisfied two of the five criteria for this writing assignment when considering both first and second attempts.

<u>Example.</u> The writer of Sample 4O mentions that a straight line has 180°, and although he sets up an incorrect equation to solve for x, he does correctly solve the equation that he does set up. Therefore, he does get credit for correctly solving an equation. But he does not substitute the value that he found for x into either of the angle expressions. In fact, he makes no attempt to determine the measurements of any of the missing angles. He seems to advocate for a guess and check method to find the angle measures.

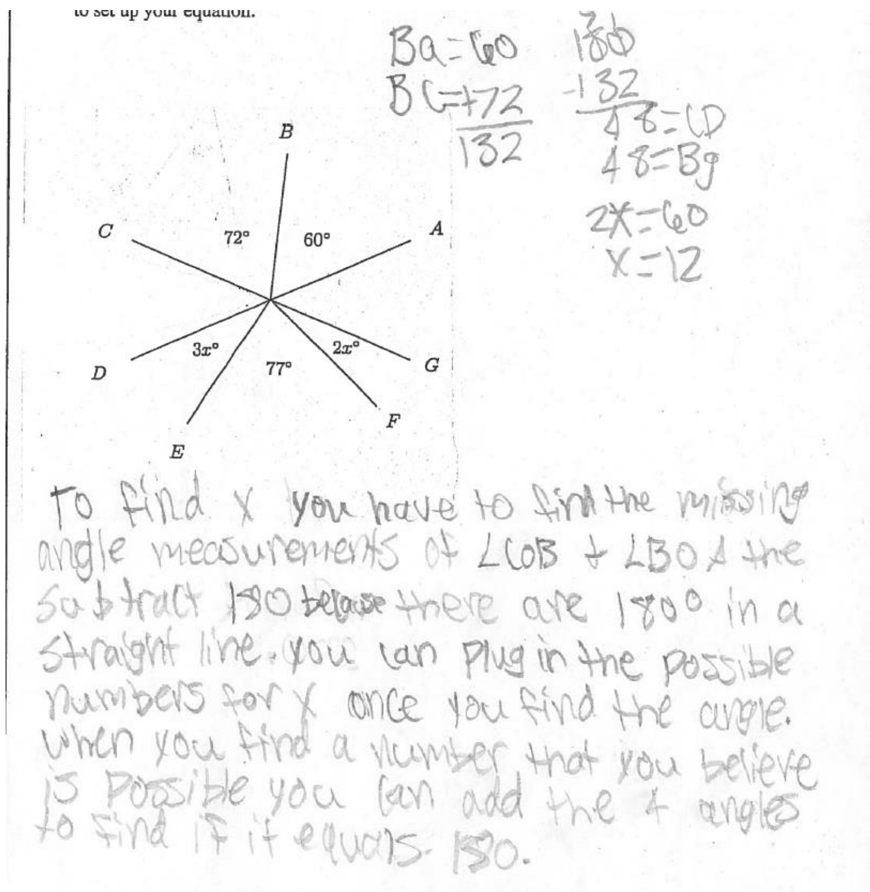


Figure 55. Writing Assignment 4, Second Attempt, Sample 4O.

5.4.2.5 Writing Assignment 4, Second Attempt, Category 5

One student out of the twenty-four students (approximately 4%) who rewrote this assignment satisfied one of the five criteria for this writing assignment. This student's first attempt was originally in Category 4, and his second attempt satisfied fewer criteria than his first attempt. No students who originally scored in Category 4 on their first attempts opted to keep their scores. Thus, one student out of eighty-six students (approximately 1%) had a response that fell into Category 5.

Example. The writer of Sample 4P has a difficult time expressing his thoughts in writing, and he shows a lack of understanding of angle relationships. He states that a straight line has 180°, but he does not attempt to set up an equation to solve for x or make an attempt to find any

angle measures. He does not discuss vertical angles. Looking back at his first attempt, I can see that he set up an equation and solved it, but his rationale did not make sense. Reading his second attempt tells me that he likely copied the equation from a classmate during his first attempt, which explains why he could not explain how he set up the equation.

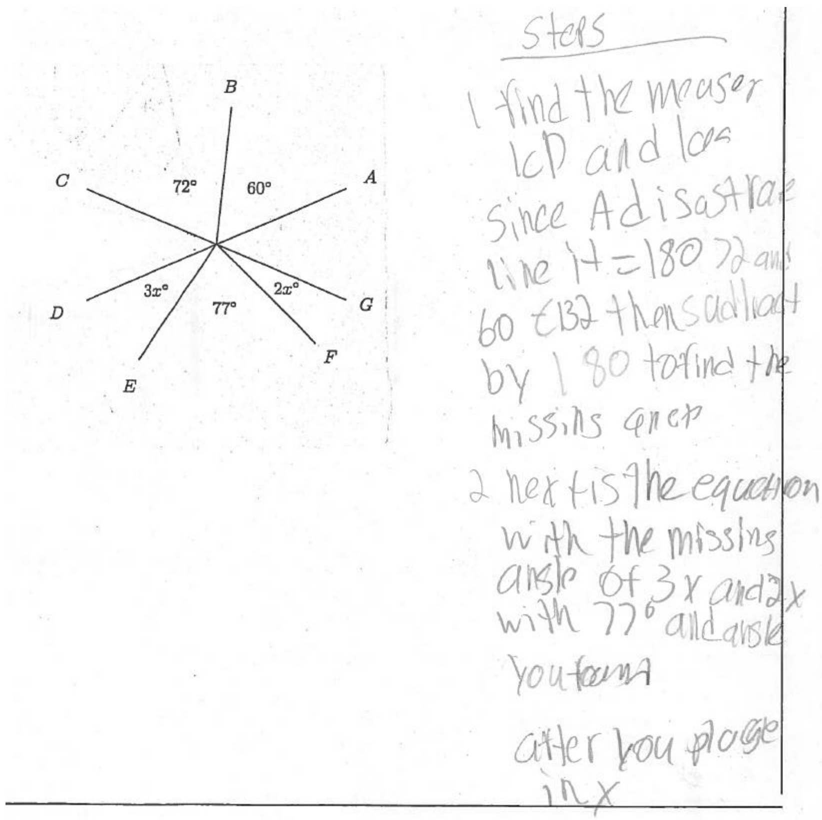


Figure 56. Writing Assignment 4, Second Attempt, Sample 4P.

To summarize the responses for Writing Assignment 4, the majority of students did not have difficulty writing an equation to help them solve for x. However, many neglected to discuss in their first attempts some basic principles of angles such as a straight line has 180° and vertical angles are congruent. Reviewing the exemplar reminded them that they needed to explain why they were using 180 in their calculations and why, after determining that one angle measured 48°, they knew that another angle had the same measurement. After considering both first and second attempts, 83% of students were able to satisfy all five criteria for the assignment.

Table 4. Summary of Responses for Writing Assignment 4.

		Category					
		1	2	3	4	5	Total
Cumulative number of students in each category after each attempt	First	48	19	5	3	11	86
	Second	71	8	5	1	1	86
Net change in category counts		+23	–11	0	–2	–10	

5.5 Writing Assignment 5

> Given the points $A(2{,}4)$, $B(-1{,}2)$, $C(4{,}1)$. What are the new points if the scale factor of dilation is ½ and the center of dilation is $(2,-2)$? Use the space to the right of the graph to explain your steps. Remember to use clear and effective communication. Use of the graph is optional but could aid in your communication.

This assignment required students to take three given coordinates of a figure and a coordinate of a center of dilation that was not the origin and to determine the image points after dilating the figure by a scale factor of one half.

Students were given this writing assignment during the final exam review in May. It had originally been a problem on their Unit 3 assessment given at the end of March. On the assessment, the students were not required to explain their steps, only perform the required math to find the image coordinates. The writing assignment was given during the final exam review and was meant to be a review of dilations, scale factors, translations, and other concepts.

During Unit 2 (Transformations, Triangles, and Quadrilaterals), students studied congruence and were taught how to perform rotations of figures when the center of rotation was at the origin, both on a coordinate plane and off the coordinate plane. Then students learned how to perform rotations of figures when the center of rotation was not at the origin, both on the coordinate plane and off the coordinate plane. For rotations around a center of rotation not the

origin, the students used a method that required them to translate the center of rotation to the origin, apply the rotation rule specified in the problem, and then translate the center of rotation back to its original position.

Later, in Unit 3 (Similarity and Trigonometry), students learned to perform dilations with a center of dilation at the origin both on a coordinate plane and off the coordinate plane. Students were also taught how to perform dilations with a center of dilation not at the origin, both on a coordinate plane and off the coordinate plane. When working these types of problems off the coordinate plane, students used the same process that they had used with rotations in Unit 2. They translated the center of dilation to the origin, applied the scale factor specified in the problem, and then translated the center of dilation back to its original position.

I handed out the writing assignment in class, read the instructions at the top of the paper, and informed students that they could use their notes during the completion of the writing assignment. I reiterated that the goal of the writing assignment was clear and effective communication of principles of transformations. Students were given thirty minutes of class time to complete the writing assignment. Students were allowed more time if needed.

5.5.1 First Attempt of Writing Assignment 5

In order to receive full credit for this assignment, students had two options. Option 1: students could have done the following: (1) translate the center of dilation to the origin and apply the translation rule to the other coordinates of the figure, (2) perform a dilation by applying the given scale factor to the coordinates of the figure, and (3) translate the center of dilation back to its original position and apply the translation rule to the other coordinates of the figure.

Option 2: students could have done the following: (1) perform a calculation of some type to represent the distances from the preimage points to the center of dilation, (2) multiply the

distance calculations by the scale factor, and (3) add the scaled distances to the center of dilation to create the image points. After reviewing the students' writings, the responses fell into the following categories: Category 1: all mathematical steps were performed correctly and proper vocabulary was used in the explanations; Category 2: all mathematical steps were performed correctly, but proper vocabulary was not consistently used in the explanations; and Category 3: mathematical steps were performed incorrectly or vocabulary was not used in the explanations.

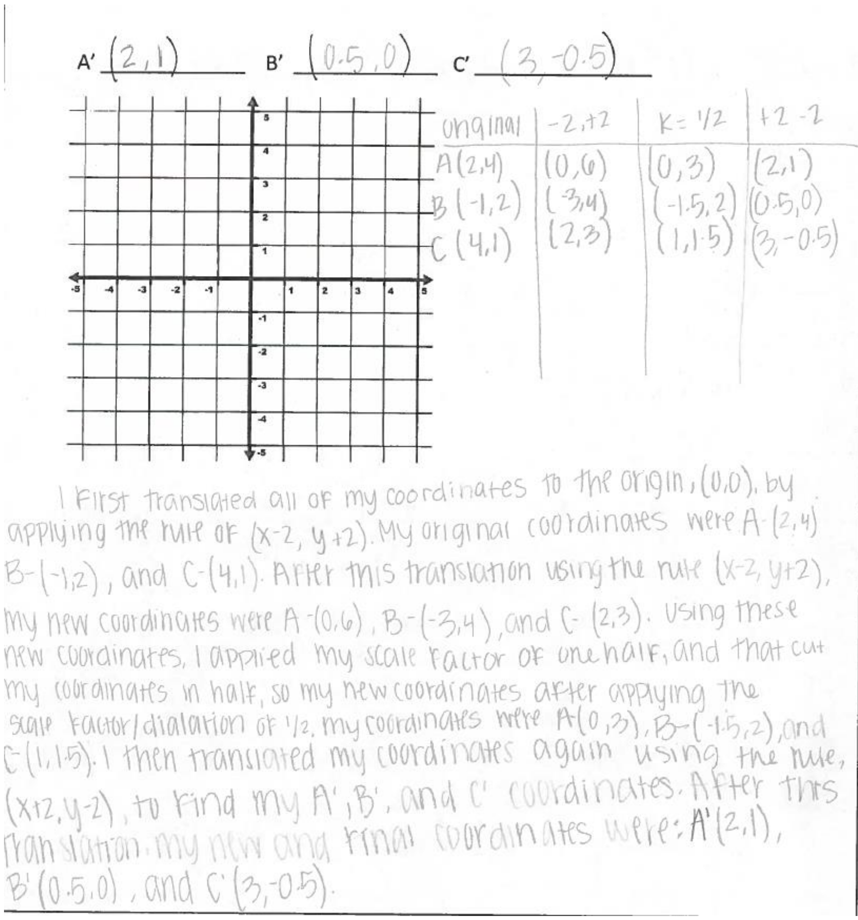
Eighty-six students completed this assignment. On their first attempt, thirty-four of the eighty-six students (approximately 40%) fell into Category 1 while thirty-seven out of the eighty-six students (approximately 45%) fell into Category 2. Students whose writing fell into Category 2 used terms like "moved" or "get to" instead of "translation." Fifteen of the eighty-three students (approximately 18%) fell into Category 3 on their first attempt. These students did not perform all of the mathematical steps correctly. Some students performed the dilation without performing the translations, and some students attempted to perform translations but did not apply the correct mathematical rules. Others did not attempt to perform any mathematics all.

5.5.1.1 Writing Assignment 5, First Attempt, Category 1

Thirty-four of the eighty-six students (approximately 40%) wrote responses that fell into this category on their first attempts. Each of these students performed the mathematical steps necessary to find the image points of the figure after performing a dilation when the center of dilation was not the origin and used proper vocabulary when explaining their mathematical steps.

Example. The writer of Sample 5A correctly calculates the image points of the figure. She uses correct vocabulary to indicate that her steps involve a translation, a dilation, and another translation. She specifies the translation rules that she applies to the coordinates during

her translations (e.g., $(x+2, y-2)$ and $(x-2, y+2)$). This student is able to communicate her thoughts clearly both in mathematics and in writing.



A' (2,1) B' (0.5,0) C' (3,-0.5)

original	-2,+2	K= 1/2	+2 -2
A(2,4)	(0,6)	(0,3)	(2,1)
B(-1,2)	(-3,4)	(-1.5,2)	(0.5,0)
C(4,1)	(2,3)	(1,1.5)	(3,-0.5)

I First translated all of my coordinates to the origin, (0,0), by applying the rule of (x-2, y+2). My original coordinates were A-(2,4) B-(-1,2), and C-(4,1). After this translation using the rule (x-2, y+2), my new coordinates were A-(0,6), B-(-3,4), and C-(2,3). Using these new coordinates, I applied my scale factor of one half, and that cut my coordinates in half, so my new coordinates after applying the scale factor/dialation of 1/2, my coordinates were A(0,3), B-(-1.5,2), and C(1,1.5). I then translated my coordinates again using the rule, (x+2, y-2), to find my A', B', and C' coordinates. After this translation, my new and final coordinates were: A'(2,1), B'(0.5,0), and C'(3,-0.5).

Figure 57. Writing Assignment 5, First Attempt, Category 1, Sample 5A.

<u>Example.</u> The writer of Sample 5B calculates the distance that each preimage point is from the center of dilation by finding the changes in the x-values and the changes in the y-values. He then applies the scale factor to those distance calculations, which divides them in half in this case. He then adds the scaled distance calculations to the center of dilation to find the image points of the figure. His writing clearly communicates all steps that he takes to find the image points.

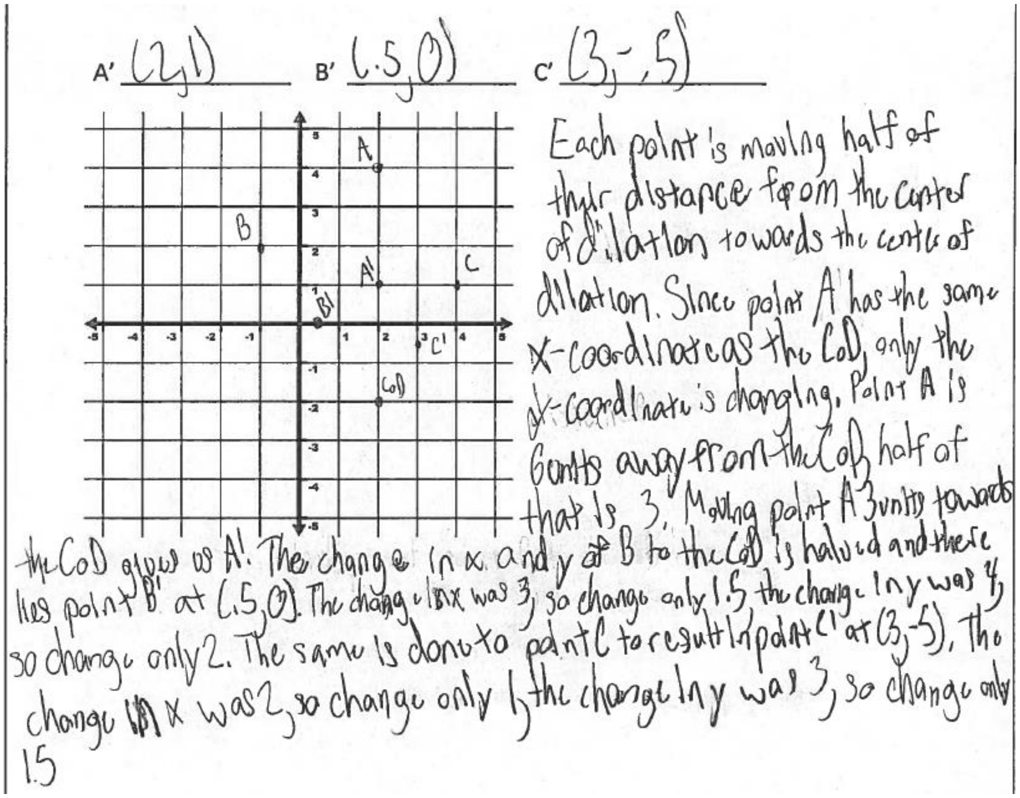


Figure 58. Writing Assignment 5, First Attempt, Category 1, Sample 5B.

5.5.1.2 Writing Assignment 5, First Attempt, Category 2

Thirty-seven out of eighty-six students (approximately 43%) fell into this category. The writing that is in this category is here because of incorrect use of vocabulary. All of the mathematical steps have been correctly performed to find the image points of the figure.

Example. Sample 5C is representative of the majority of the responses in this category. Although this student correctly performs all of the mathematical steps to get the correct image points, she does not use the correct vocabulary when describing her steps. Instead of using the term “translation,” she talks about “moving” the points. This is a minor error, but the proper use of vocabulary aids in the clear communication of ideas.

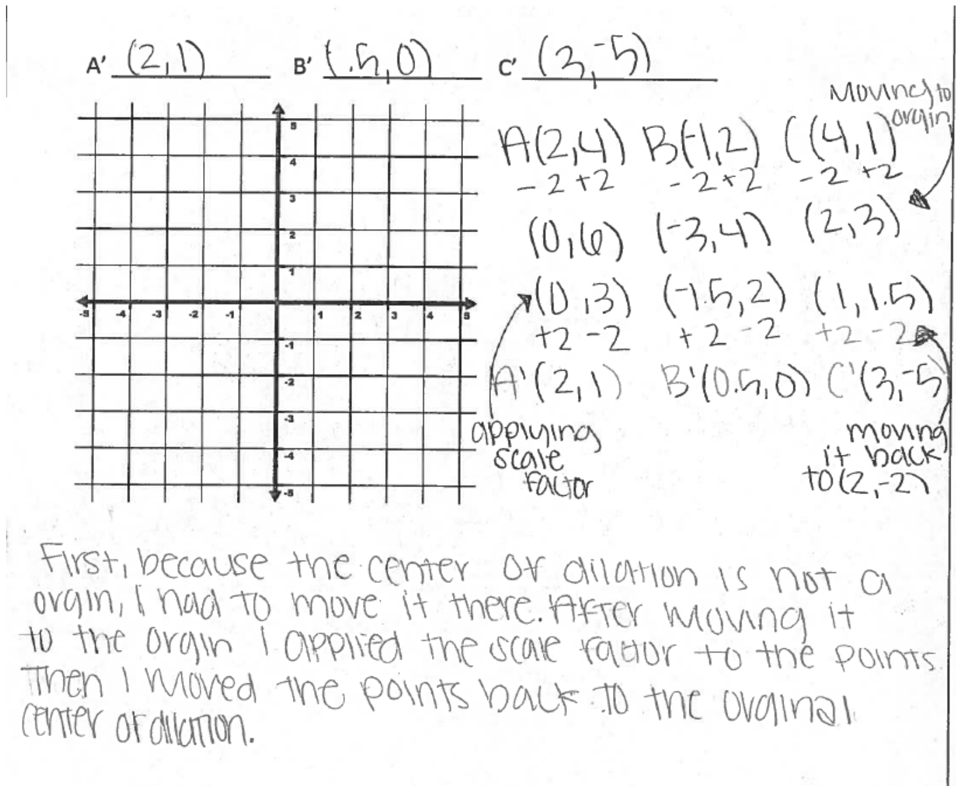


Figure 59. Writing Assignment 5, First Attempt, Category 2, Sample 5C.

5.5.1.3 Writing Assignment 5, First Attempt, Category 3

Fifteen out of eighty-six students (approximately 17%) wrote responses that fell into this category. Some students performed the dilation without performing the translations, and some students attempted to perform translations but did not apply the correct mathematical rules. Others did not attempt to perform any mathematics at all.

Example. The writer of Sample 5D does not translate the center of dilation to the origin before performing the dilation. This suggests that she does not distinguish between dilations that occur at the origin and those for which the center of dilation is not the origin. She plots the preimage points, creating the larger triangle on the graph. The outline of her first attempt at drawing the dilated triangle has been erased. It is unclear why she redraws the x-axis 2 units

lower than its original location and then used it to position the dilated triangle. She does not attempt to reposition the original triangle to compensate for the redrawing of the x-axis.

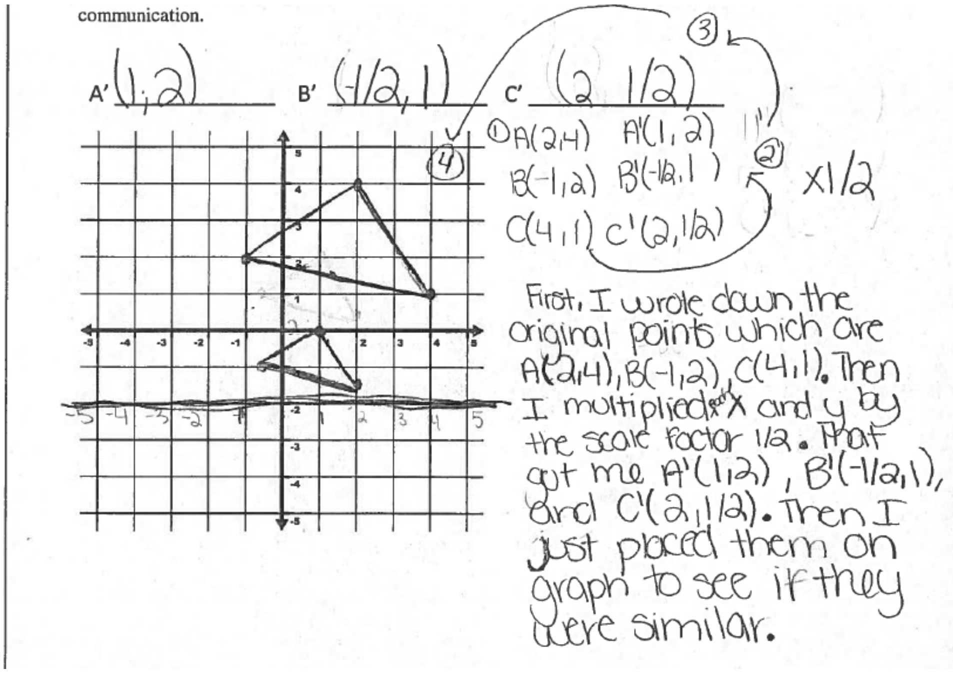


Figure 60. Writing Assignment 5, First Attempt, Category 3, Sample 5D.

<u>Example.</u> The writer of Sample 5E writes the correct image points, but his math work does not support his answers. If this student had applied the translation rule that he claims he uses, $(x + 2, y - 2)$, he would have found the image points to be $\mathrm{A}'(4, 2)$, $\mathrm{B}'(1, 0)$, and $\mathrm{C}(6, -1)$. He uses the graph but does not mark the center of dilation, which indicates that he does not use a graphing method to determine the image points. In addition, this student does not attempt to discuss any mathematics in any way. This is a good indicator that the student does not know how to perform the steps necessary to find the image points. He knows that he should apply a translation rule but is not sure which to use. He applies the translation rule that should be applied after the first translation and after the dilation.

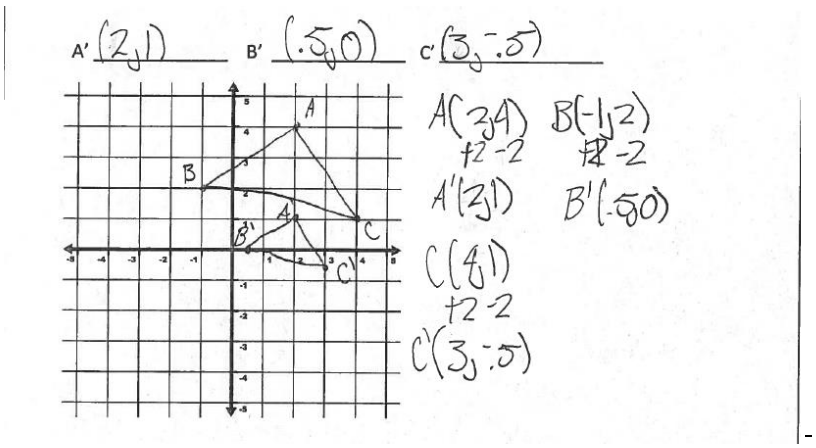


Figure 61. Writing Assignment 5, First Attempt, Category 3, Sample 5E.

Example. The writer of Sample 5F writes the correct image points but does not provide an explanation that justifies his final answers. His first step is to perform a dilation, which indicates that he does not distinguish between a dilation occurring at the origin and a dilation occurring elsewhere on the coordinate plane. The dilation is mathematically correct, but then he says that he multiplies those points by the center of dilation, which means that he suggests applying the rule $(2x, -2y)$. The points that he lists as A", B", and C" do not mathematically result from the step he suggests taking.

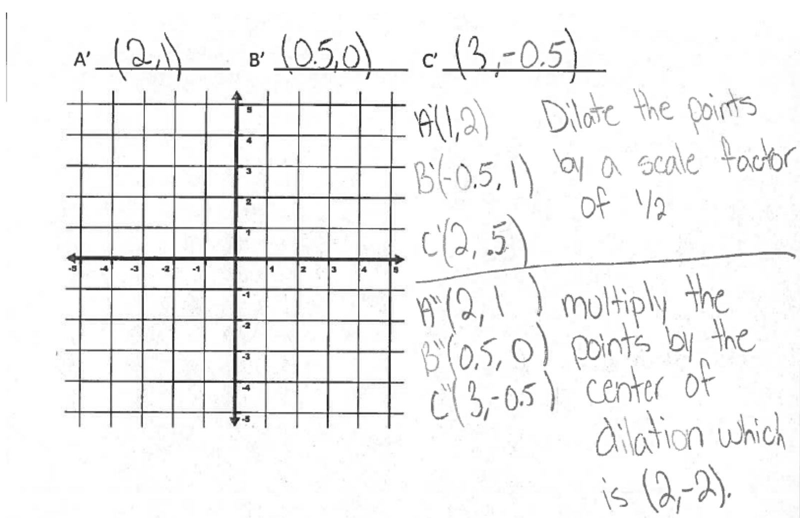
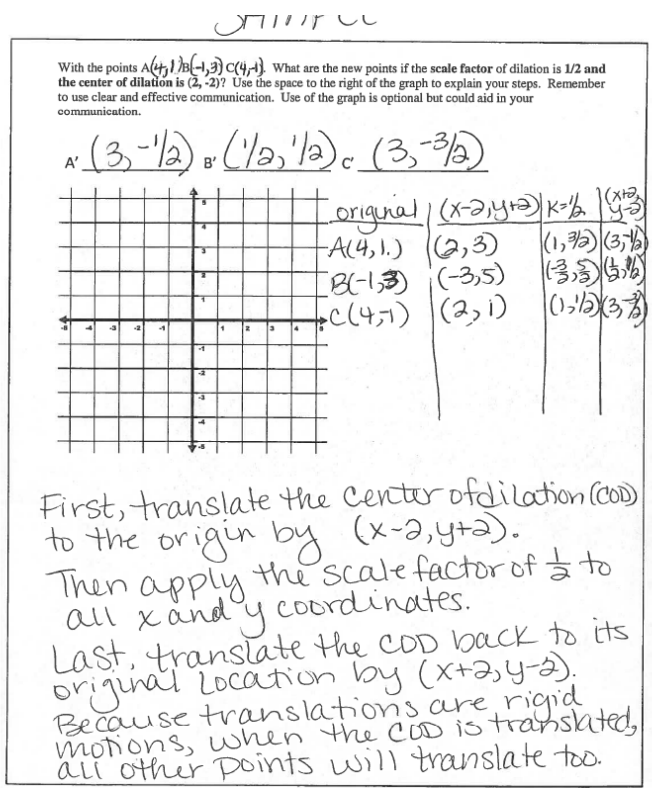


Figure 62. Writing Assignment 5, First Attempt, Category 3, Sample 5F.

Below is the exemplar that I provided to the students when handing back their scored writing assignments. I changed the preimage points but left the scale factor and center of dilation the same. Most students did not have difficulty performing the calculations to find the image points; they just neglected to use the correct vocabulary in their explanations. In the exemplar, I am careful to use proper vocabulary such as "translate" and "scale factor." I specify the rules that I use to perform the translations both in my mathematical work to the right of the graph and in my writing. I specify that translations are rigid motions, so when I translate the original center of dilation, all of the other coordinates translate similarly. I included this statement because many students wrote in their first attempts that their first step was to translate all of the points to the origin, which is incorrect. In reviewing their math, it is clear that they know that the points are not all translating to the origin, so I included the statement to assist them in their use of precise language.



With the points A(4, 1) B(-1, 3) C(4, -1). What are the new points if the **scale factor** of dilation is **1/2 and the center of dilation is** (2, -2)? Use the space to the right of the graph to explain your steps. Remember to use clear and effective communication. Use of the graph is optional but could aid in your communication.

A' $(3, -1/2)$ B' $(1/2, 1/2)$ C' $(3, -3/2)$

original	$(x-2, y+2)$	$K = 1/2$	$(x+2, y-2)$
A(4, 1)	(2, 3)	(1, 3/2)	(3, -1/2)
B(-1, 3)	(-3, 5)	(-3/2, 5/2)	(1/2, 1/2)
C(4, -1)	(2, 1)	(1, 1/2)	(3, -3/2)

First, translate the center of dilation (COD) to the origin by $(x-2, y+2)$.
Then apply the scale factor of $\frac{1}{2}$ to all x and y coordinates.
Last, translate the COD back to its original location by $(x+2, y-2)$.
Because translations are rigid motions, when the COD is translated, all other points will translate too.

Figure 63. Exemplar for Writing Assignment 5.

5.5.2 Second Attempt of Writing Assignment 5

Fifty-two students were given the opportunity to rewrite this assignment, and fifty students took advantage of that opportunity. The majority of student writing fell into Category 1 after taking into account students' first and second attempts at this assignment. This indicates that the majority of students know how to perform a dilation when the center of dilation is not the origin and can use the proper vocabulary when describing the steps.

5.5.2.1 Writing Assignment 5, Second Attempt, Category 1

The thirty-four students whose writing originally fell into Category 1 were not asked to rewrite this assignment since they had already earned full credit. Thirty-four of the fifty students (68%) who rewrote the assignment had writing that fell into Category 1. Therefore, when considering both first and second attempts, a total of sixty-eight out of eighty-six students (approximately 79%) satisfied all three criteria for this writing assignment.

Example. The writer of Sample 5G correctly performs a translation, a dilation, and another translation to find the correct image points of the figure and uses correct vocabulary.

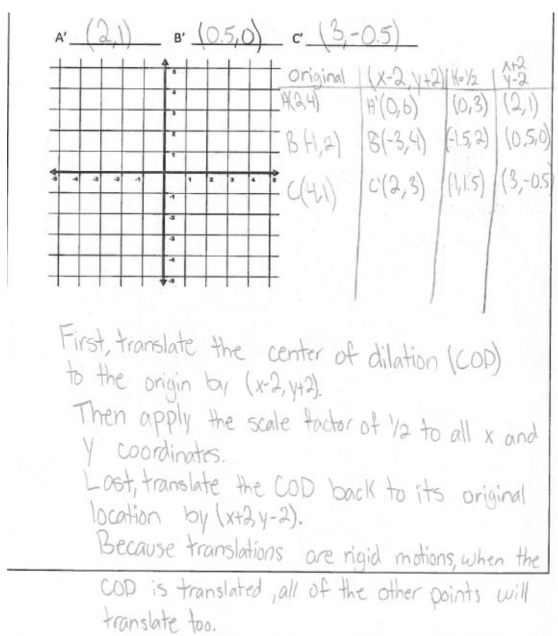


Figure 64. Writing Assignment 5, Second Attempt, Category 1, Sample 5G.

5.5.2.2 Writing Assignment 5, Second Attempt, Category 2

Student writing in Category 2 included responses that showed correct mathematical steps but neglected to use the correct vocabulary. Twelve of the fifty students (approximately 24%) who rewrote the assignment fell into Category 2. Two students originally had writing classified in Category 2 and opted not to rewrite the assignment. Overall, fourteen of the 86 students (approximately 16%) fell into Category 2.

Example. Sample 5H is representative of the majority of students who rewrote this assignment. This student produces the correct solution and performs all of the math correctly, but he does not use the proper vocabulary. He say "get to 0" to indicate that the center of dilation should be translated to the origin and "move them back" to indicate that the center of dilation should move back to its original location after the dilation. He says "use the scale factor" to indicate that the coordinates should be multiplied by the scale factor. In general, the student can perform the process but has a difficult time or is unwilling to explain the steps using the proper terminology.

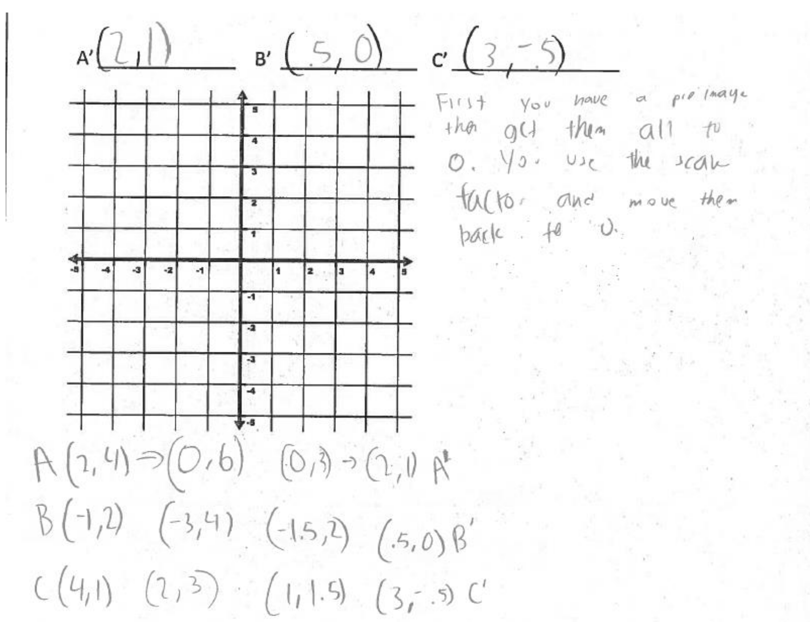


Figure 65. Writing Assignment 5, Second Attempt, Category 2, Sample 5H.

Example. Sample 5I is like that in Sample 5C in the sense that vocabulary is not used correctly. In this student's case, it is the use of one word that causes her response to be in Category 2 rather than Category 1. In the last step, she says to "dilate the points back" instead of saying to translate the center of dilation back to its original location. This is a very minor error. In this case, I know that the student understands the concept of dilating a figure at a center of dilation that is not the origin, but because of the way I chose to classify the responses, her writing fell into Category 2 instead of Category 1.

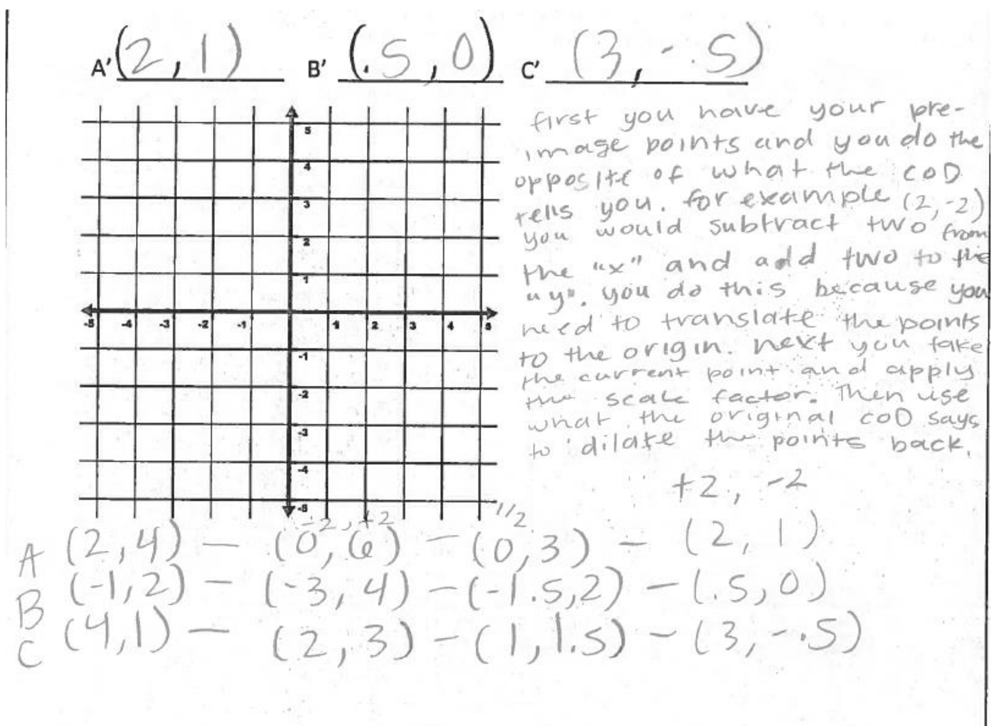


Figure 66. Writing Assignment 5, Second Attempt, Category 2, Sample 5I.

5.5.2.3 Writing Assignment 5, Second Attempt, Category 3

Four out of the fifty students (approximately 8%) who opted to rewrite this assignment had responses that fell into Category 3. These four students' writing originally fell into Category 3 on their first attempts, and their writing did not improve on their second attempts. Thus, four

out of eighty-six students (approximately 5%) satisfied only one of the three criteria for this writing assignment.

Example. The writer of Sample 5J writes the correct image points, but his math work does not support his answers. His explanation mimics the exemplar, but the math he performs is incorrect. Specifically, he does not apply the scale factor correctly, and he does not state the translation rule that he uses for the second translation. The image points resulting from his calculations are listed to the right of his explanation: $A'(2,1)$, $B'(-4,0)$, and $C'(6,4)$. He does not write his results as the image points but instead manages to write the correct image points even though he does not actually calculate them.

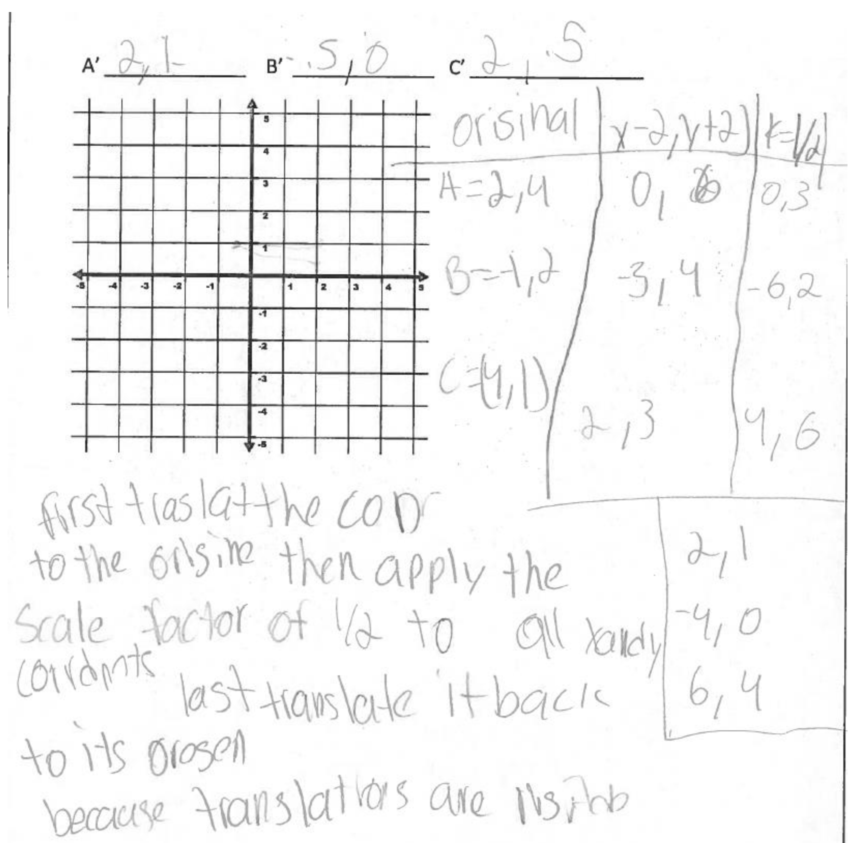


Figure 67. Writing Assignment 5, Second Attempt, Category 3, Sample 5J.

Overall, a large number of students (approximately 79%) were able to not only perform a dilation of a figure whose center of dilation is not the origin but could also explain the steps

using proper vocabulary. After reviewing an exemplar, only four students were unable to satisfactorily indicate that they could perform this transformation. These four students did not improve their writing responses from their first attempts and made the same mistakes in their second attempts of the assignments.

Table 5. Summary of Responses for Writing Assignment 5.

		Category			
		1	2	3	Total
Cumulative number of students in each category after each attempt	First	34	37	15	86
	Second	68	14	4	86
Net change in category counts		+34	–23	–11	

5.6 Discussion of Findings

The number of categories used per writing assignment ranged from three categories to five categories. In order to more easily compare the results from each writing assignment, I created Tables 6 and 7 below. In these tables, student results from each writing assignment are organized into three categories. Category 1 represents the top third of student responses, Category 2 represents the middle third of student responses, and Category 3 represents the bottom third of student responses. Table 6 shows the percentage of students whose writing fell into each category on their first attempts. Table 7 shows the cumulative percentage of students whose writing fell into each category after the students were provided with an exemplar and rewrote the assignments. Because students were not offered an opportunity to rewrite Writing Assignment 1, it has been omitted from Table 7.

Based on the evidence, providing students with a teacher-written exemplar after their first attempts at expository writing assignments helps students improve their ability to effectively

communicate their conceptual knowledge of mathematics. It was anticipated that students' responses would improve on subsequent attempts of the same writing assignment after they were provided an exemplar. The data in Tables 6 and 7 below show that, on each writing assignment, a greater percentage of students were in the top category after their second attempts at that writing assignment. For example, on Writing Assignment 2, 7% of students wrote responses that fell in the top category on their first attempts, while 50% of students had writing that fell in the top category on their second attempts. For Writing Assignment 5, 40% of students wrote responses that fell in the top category on their first attempts, while 79% of students had writing that fell in the top category on their second attempts.

In addition to improving on subsequent attempts of the same assignments, students also improved their ability to communicate mathematics over the duration of the semester. As shown in Table 6, at the beginning of the semester (Writing Assignments 1 and 2), few students wrote responses that met all or most of the criteria required by the writing assignment. Gradually, over the course of the semester, students began improving their writing and produced more clearly written responses that included more vocabulary than they had in prior assignments. For Writing Assignment 3, which was given in mid-March, and Writing Assignments 4 and 5, which were both given during the same week of EOC review in May, high percentages of students wrote responses that fell into the top categories on their first attempts, which indicates that the students were communicating their knowledge effectively as a matter of course rather than because they had been reminded.

The evidence suggests that a time-efficient way to implement writing in a mathematics classroom is by providing an exemplar to students after their first attempt at an expository writing assignment. Not only will their subsequent attempts on the same writing assignment be

more clearly written, but their first attempts on future writing assignments will improve as well. If I were to incorporate writing in future math classes, I would continue to provide my students with exemplars. Because of students tended to not use correct mathematical vocabulary in their first attempt responses, I would consider adding to my verbal instructions a reminder to students that they should use proper vocabulary. I would also consider asking students individually when they turned the assignment in whether they had used proper vocabulary, which might prompt some students to go back to their desks to revise their writing before ever seeing an exemplar.

Table 6. Summary of Responses for First Attempt All Assignments.

		Category		
		Top	Middle	Low
Percentage of students in each category after first attempt of each assignment	1	31%	62%	6%
	2	7%	55%	38%
	3	51%	19%	30%
	4	63%	22%	15%
	5	40%	43%	17%

Table 7. Summary of Responses for Second Attempt of Writing Assignments 2 through 5.

		Category		
		Top	Middle	Low
Percentage of students in each category after second attempt of each assignment	2	50%	37%	13%
	3	70%	11%	19%
	4	86%	12%	2%
	5	79%	16%	5%

It must be acknowledged that the writing assignments could have been graded and classified in a number of different ways. I opted to list the individual criteria required to answer all parts of each problem. For example, Writing Assignment 2 had five criteria and five categories into which student responses were sorted, while Writing Assignment 5 had three criteria and three categories into which student responses were sorted. As stated previously, the different categories were created based on the number of required criteria that were satisfied; the relative importance of each criteria was not considered.

Because the writing responses were categorized in this way, it was possible for an assignment to provide evidence of mastery but not meet all of the criteria. For example, in Writing Assignment 2, students were instructed to include all definitions and to define all variables, so that was one criterion for the writing assignment. Students who did not define even and odd integers but otherwise demonstrated understanding of the concept were nevertheless placed in Category 2 for not meeting all criteria. This might be perceived as penalizing answers for simply not following directions, but it might also be viewed as a way of taking into account attention to details, which is important when solving mathematical problems.

It was also possible for a student to meet all of the criteria for a writing assignment but not truly understand the concept being assessed. These students could mimic the exemplar without completely understanding what they are writing. I tried to minimize this outcome by providing exemplars for problems similar to those being graded, which would require students to at least apply the requisite mathematics to a different problem. Another way that a teacher could minimize this outcome would be to ask students to write their second attempts for different problems. That way the students would write responses to two different but similar problems, and the exemplar viewed between the attempts would be of a third different yet similar problem.

CHAPTER 6. CONCLUSION

The purpose of this thesis is to describe specific effective ways to implement writing in a mathematics classroom. I introduced writing in three sections of geometry in order to improve students' conceptual knowledge of mathematics and their ability to effectively communicate that knowledge with the hope that it would lead to higher EOC scores for the students and a higher School Performance Score for WFHS.

In order to find a time-efficient way to incorporate writing in my classroom, I referred to several studies. They described effective methods of including writing in geometry classes, but the time requirements were considerable. With approximately thirty students per class, I did not have time to hold five to ten minute one-on-one conferences with my students to discuss their writing assignments, and after the poor return rate on the first writing assignment, I did not want to have students complete their first attempts at the writing assignments at home. I also did not have time to require the students to answer five or more questions for every writing assignment, which can often take forty-five minutes or longer to complete. Because of this, I needed to modify the strategy.

The use of an exemplar allowed me to minimize the amount of time that I spent administering the writing assignments in class and still see an improvement in my students' ability to communicate their knowledge of math. I allowed students to take thirty minutes of class time for four of the assignments. I allowed students more time if they needed it, but typically, only one or two students needed extra time. I handed back the scored responses along with the exemplar that I had prepared. I provided students with an additional thirty minutes to make optional revisions. This time spent on revisions could be further minimized by having students complete the revisions at home rather than in class. In addition, to reduce the number of

students who needed to make second attempts at the writing assignments, I could specifically remind students about the importance of using proper vocabulary in their responses either before they began the writing assignment or when they turned their responses in.

The evidence indicates that providing students with feedback in the form of an exemplar benefits students in two ways. First, when students were provided an exemplar after their first attempts at the writing assignments, in almost every instance their scores improved in their subsequent attempts at that writing assignment. Overall, there were only seventeen instances when students did not improve their scores on a second attempt. This occurred four times on Writing Assignment 2, twice on Writing Assignment 3, once on Writing Assignment 4, and ten times on Writing Assignment 5. Second, as the semester progressed, the number of students who scored in the top category or categories on their first attempts increased. For example, for Writing Assignment 2, which had 5 categories, no student made it into Category 1 on the first attempt and only 10% scored in the second highest. In contrast, for the first attempts at Writing Assignment 5, which had 3 categories, 40% scored in Category 1, 43% scored in Category 2, and 17% scored in Category 3.

The literature review shows strong evidence that incorporating writing in a mathematics classroom can have many benefits for students, both emotionally and academically. Teachers have many options when determining how to do this. Teachers who have a difficult time building a rapport with students might consider implementing journal writing in their classes. Teachers who want to help students improve their conceptual knowledge of mathematics and their ability to communicate that knowledge might consider incorporating expository writing. The literature review suggests that providing feedback to students is important, so teachers must find the time to give thoughtful commentary to the students. This work gives evidence that

providing an exemplar problem to the students after their first attempts at expository writing assignments is a time-efficient way to provide feedback to students and that using this method may support growth in students' knowledge of mathematics and their ability to communicate it effectively.

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APPENDIX A: EXCERPTS FROM SPRINGBOARD TEXTBOOK ACTIVITY 2-2

In mathematics, there are certain methods and rules of ***argument*** that mathematicians use to convince someone that a conjecture is true, even for cases that extend beyond the observed data set. These rules are called rules of logical reasoning or rules of ***deductive reasoning***. An argument that follows such rules is called a ***proof***. A statement or conjecture that has been proven, that is, established as true without a doubt, is called a ***theorem***. A proof transforms a conjecture into a theorem.

Below are some definitions from arithmetic.

Even integer: An integer that has a remainder of 0 when it is divided by 2.

Odd integer: An integer that has a remainder of 1 when it is divided by 2.

Express regularity in repeated reasoning. In the following items, you will make some conjectures about the sums of even and odd integers.

1. Calculate the sum of some pairs of even integers. Show the examples you use and make a conjecture about the sum of two even integers.

   Sample conjecture: The sum of two even integers is always an even integer.

2. Calculate the sum of some pairs of odd integers. Show the examples you use and make a conjecture about the sum of two odd integers.

   Sample conjecture: The sum of two odd integers is always an even integer.

3. Calculate the sum of pairs of integers consisting of one even integer and one odd integer. Show the examples you use and make a conjecture about the sum of an even integer and an odd integer.

   Sample conjecture: The sum of an even integer and an odd integer is always an odd integer.

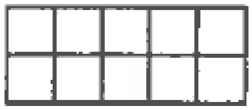


Figure A

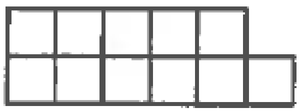


Figure C

The following items will help you write a convincing argument (a proof) that supports each of the conjectures you made in Items 1–3.

4. Figures A, B, C, and D are puzzle pieces. Each figure represents an integer determined by counting the square pieces in the figure. Use these figures to answer the following questions.

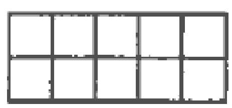


Figure A

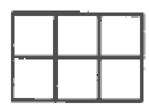


Figure B

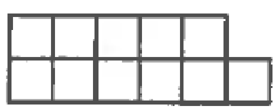


Figure C

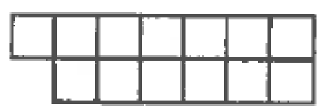


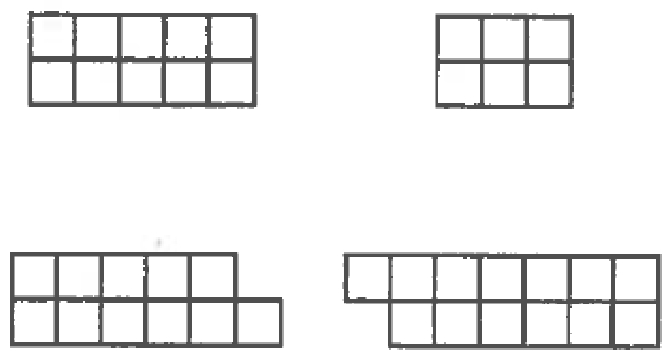
Figure D

a. Which of the figures can be used to model an even integer?
Figure A and Figure B

b. Which of the figures can be used to model an odd integer?
Figure C and Figure D

c. Compare and contrast the models of even and odd integers.
Sample answer: A rectangle built with columns of two squares models even integers, while odd integers are modeled by a two-rowed rectangular shape with an extra square attached or jutting out.

5. **Model with mathematics.** Which pairs of puzzle pieces can fit together to form rectangles? Make sketches to show how they fit.
Figure A and Figure B can fit together to make a rectangle. Figure C and Figure D can also fit together to make a rectangle.



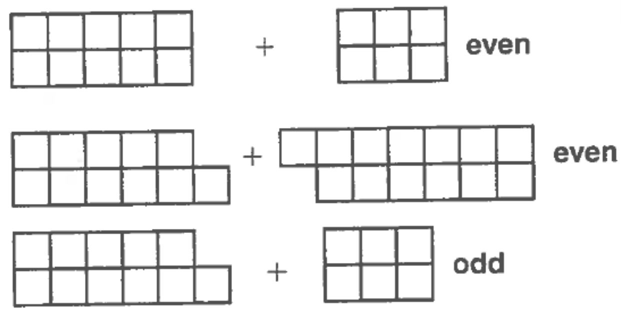
6. Explain how the figures (when used as puzzle pieces) can be used to show that each of the conjectures in Items 1 through 3 is true.
Answers may vary. See below for possible sketches.

Answers

6. The sum of two even integers is an even integer. Two rectangles representing even integers fit together to form another rectangle.

   The sum of two odd integers is an even integer. Two figures representing odd integers fit together to form a rectangle, which represents an even integer.

   The sum of an even integer and an odd integer is an odd integer. Figures representing an odd integer and an even integer do not fit together to form a rectangle.



In Items 5 and 6, you proved the conjectures in Items 1 through 3 geometrically. In Items 7 through 9, you will prove the same conjectures algebraically.

This is an algebraic definition of even integer: An integer is *even* if and only if it can be written in the form $2p$, where p is an integer. (You can use other variables, such as $2m$, to represent an even integer, where m is an integer.)

7. **Reason abstractly.** Use the expressions $2p$ and $2m$, where p and m are integers, to confirm the conjecture that the sum of two even integers is an even integer.
   $2p + 2m = 2(p + m)$, which has the form of an even integer. So the sum of two even integers is even.

This is an algebraic definition of odd integer: An integer is *odd* if and only if it can be written in the form $2t + 1$, where t is an integer. (Again, you do not have to use t as the variable.)

8. Use expressions for odd integers to confirm the conjecture that the sum of two odd integers is an even integer.
   $(2t + 1) + (2n + 1) = 2t + 2n + 2 = 2(t + n + 1)$, which has the form of an even integer. So the sum of two odd integers is even.

9. Use expressions for even and odd integers to confirm the conjecture that the sum of an even integer and an odd integer is an odd integer.

$2p + (2t + 1) = 2p + 2t + 1 = 2(p + t) + 1$, which has the form of an odd integer. So the sum of an even integer and an odd integer is odd.

APPENDIX B: IRB APPROVAL

ACTION ON EXEMPTION APPROVAL REQUEST

Institutional Review Board
Dr. Dennis Landin, Chair
130 David Boyd Hall
Baton Rouge, LA 70803
P: 225.578.8692
F: 225.578.5983
irb@lsu.edu | lsu.edu/irb

TO: Amy Rome
Natural Science

FROM: Dennis Landin
Chair, Institutional Review Board

DATE: May 21, 2015

RE: **IRB#** E9365

TITLE: Improving Written Communication in the High School Geometry Classroom

New Protocol/Modification/Continuation: New Protocol

Review Date: 5/21/2015

Approved X **Disapproved** ______

Approval Date: 5/21/2015 **Approval Expiration Date:** 5/20/2018

Exemption Category/Paragraph: 1

Signed Consent Waived?: No. Parental consent and child assent need signing.

Re-review frequency: (three years unless otherwise stated)

LSU Proposal Number (if applicable):

Protocol Matches Scope of Work in Grant proposal: (if applicable)

By: Dennis Landin, Chairman DLandin

PRINCIPAL INVESTIGATOR: PLEASE READ THE FOLLOWING –
Continuing approval is CONDITIONAL on:

1. Adherence to the approved protocol, familiarity with, and adherence to the ethical standards of the Belmont Report, and LSU's Assurance of Compliance with DHHS regulations for the protection of human subjects*
2. Prior approval of a change in protocol, including revision of the consent documents or an increase in the number of subjects over that approved.
3. Obtaining renewed approval (or submittal of a termination report), prior to the approval expiration date, upon request by the IRB office (irrespective of when the project actually begins); notification of project termination.
4. Retention of documentation of informed consent and study records for at least 3 years after the study ends.
5. Continuing attention to the physical and psychological well-being and informed consent of the individual participants, including notification of new information that might affect consent.
6. A prompt report to the IRB of any adverse event affecting a participant potentially arising from the study.
7. Notification of the IRB of a serious compliance failure.
8. **SPECIAL NOTE:**

**All investigators and support staff have access to copies of the Belmont Report, LSU's Assurance with DHHS, DHHS (45 CFR 46) and FDA regulations governing use of human subjects, and other relevant documents in print in this office or on our World Wide Web site at http://www.lsu.edu/irb*

VITA

Amy Lynn Rome is a native of Goose Creek, South Carolina. She attended Winthrop University and received Bachelor of Arts degrees in English and Psychology in 1996. Amy taught high school English for four years at Chester Senior High School in Chester, South Carolina, and then decided to further her education. She taught adult education classes in Greeneville, Tennessee, for one year and then attended and graduated from The University of Tennessee College of Law in 2004. She practiced commercial lending and real estate law as an associate attorney with Bass, Berry & Sims PLC in Nashville, Tennessee, from 2004 to 2008. Amy then taught English at Breaux Bridge Senior High School in Breaux Bridge, Louisiana, during the 2008–2009 school year. She relocated to Livingston Parish and taught at Walker Freshman High school from 2010 to 2015. While there, Amy taught a variety of subjects, including Algebra I, Geometry, World Geography, Law Studies, and Speech. Amy is certified to teach secondary Mathematics, English, Social Studies, Physical Education, and ESL and prides herself on being a life-long learner. She joined the LAMSTI Program at the urging of her friend and colleague Tiah Alphonso.