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Project-based high school geometry

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PROJECT-BASED HIGH SCHOOL GEOMETRY

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ABSTRACT

Project-based learning (PBL) is an instructional strategy that allows students the autonomy to learn, explore and investigate throughout the learning process by means of projects. Many educators have seen the need for such a strategy in the classroom as a remedy for motivating students, showing relevance of student's education to everyday life, preparing students for college and the work force, and the dire need for students to develop critical thinking skills to encourage future success.

In my thesis I will define project-based learning, discuss its characteristics, compare PBL to traditional teaching methods and reflect on my experiences with project-based learning in the classroom. I will also show how a traditional math problem can become more interesting and applicable to students if project-based elements are incorporated.

CHAPTER 1. PROJECT-BASED LEARNING (PBL)

1.1 What is Project-Based Learning (PBL)?

The Buck Institute for Education defines project-based learning as "*a systematic teaching method that engages students in learning essential knowledge and life-enhancing skills through an extended, student-influenced inquiry process structured around complex, authentic questions and carefully designed products and tasks"* (The Buck Institute for Education and Boise State University 2005). In PBL, the students engage in relevant, rigorous, and authentic projects to learn different subject matter. The projects are *relevant* in that they address real-world problems and require students to complete tasks that are useful and beneficial, *rigorous* in that they encourage students to learn concepts in depth and develop critical thinking and other skills, and *authentic* in that the students create and present tangible, explicit products to obtain feedback and make necessary revisions (Houghton Mifflin 2009; The Buck Institute for Education and Boise State University 2005).

There are five main elements that guide the implementation of project-based instruction:

- a driving question that is interdisciplinary and embedded in a real-world scenario,
- student inquiry and extensive investigation,
- cooperative learning and meaningful collaboration between the teacher and students,
- use of technology and other resources to aid students in illustrating their ideas,
- creation of a product that represents their gained knowledge (Houghton Mifflin 2009; The Buck Institute for Education and Boise State University 2005)

This framework gives students the opportunity to master concepts by reflecting on what they've previously learned and getting meaningful feedback not only from teachers, but peers as well. Project-based learning allows students to be active participants in the learning process and to

take ownership of their learning. Projects require a lengthy amount of time, usually weeks, and critical thinking skills (Evertson 2006).

1.2 Project-Based Learning versus Traditional Teaching Methods

There are distinct differences between PBL and traditional teaching methods. “Project-based learning is a model which [is distinguished] from traditional teaching since the focus is put on the learner and his project. Learners have the opportunity to work more autonomously and build their knowledge” (Schneider 2005). A traditional classroom setting is teacher-centered, with lecture and note taking as key components. A project-based setting is student-centered with student inquiry and exploration as key elements.

In PBL, student’s complete contextualized tasks as opposed to isolated lessons. In this manner, students can see the relevance of the task to their everyday lives. “Learning from projects rather than from isolated problems is, in part, so that students can face the task of formulating their own problems, guided on the one hand by the general goals they set, and on the other hand by the 'interesting' phenomena and difficulties they discover through their interaction with the environment” (Collins, Brown and Newman, 1989). Unlike traditional teaching methods, projects are designed to “reflect the learning and work people do outside of the classroom.” For that reason, students are “assessed in a manner that reflects how quality is judged in the real world” (Evertson 2006).

Project-based instruction is an engaging way to teach state required standards. The state’s content standards are indeed taught, but they are joined with other content and skills to make a meaningful, rigorous and interesting learning experience. With traditional teaching methods, it is very difficult to keep students engaged in the learning process. In project-based learning, students can become self-motivated learners through creating products “valuable in their own right” and collaborating with other students (Evertson 2006).

The main difference between traditional and project-based methods is the student's acquisition of procedural versus conceptual knowledge. Through projects, students can not only learn concepts, they are provoked and encouraged to investigate, ask questions and develop new knowledge. It's not that the previous could not happen in a traditional *lecture/note-taking* classroom setting, but PBL is designed around *student-centeredness* to allow each individual student to draw on previous knowledge, from any level, and develop new knowledge.

New Technology Foundation characterizes a traditional classroom setting as a *teacher-centered* environment where, for the most part, students work individually with little feedback from the teacher. There are not many opportunities for students to connect their math knowledge to real life scenarios or other content areas. Students are often required to memorize information as opposed to understanding concepts through investigation, inquiry and exploration.

New Technology Foundation considers a PBL classroom a "21st century" classroom. This student-centered environment focuses on preparing students for college and their future careers. In a PBL classroom, learning can come from multiple sources. The students are required to work collaboratively in groups. The teacher provides meaningful feedback to the students as a means of improvement for the students and to reduce recurring errors. The content of the projects is interdisciplinary and connected to issues relevant to everyday life. In this manner, students can become life-long learners.

Differing from traditional teaching methods, PBL gives students the right amount of choice and autonomy. In PBL, the teacher is not the sole contributor to the learning that occurs in the classroom. The teacher's role is a guide and facilitator. The teacher creates the project and many of the scaffolding activities, but the students do the exploration and discovery. The teacher's role is not just a *transmitter* of knowledge, rather an *advisor* of learning (Newell 2003).

In traditional teaching methods, the majority of the curriculum comes from designated textbooks. Moreover, assessment of student learning comes from traditional paper/pencil tests. In PBL, the students may use a designated text, but this supplements many other resources. In PBL the students are assessed traditionally with quizzes and tests, but they are also assessed in other ways. Rubrics play a major role in PBL assessment. Because the students are assessed in a variety of ways more than one rubric may be used to grade a project. For example, the content of the project may have a rubric, the oral presentations may have a separate rubric, and the student's ability to collaborate well with other group members may be assessed in another rubric.

1.3 Project-Based Learning at Algiers Technology Academy

I teach Geometry at Algiers Technology Academy. We are a PBL school associated with New Technology Foundation (NTF). "NTF was established in 1999 … working to achieve national education reform with schools that desire to model the Napa New Technology High School" (www.newtechnologyfoundation.org). NTF follows a PBL model that we implement at our school. The core principles of NTF are:

1) a professional culture of trust, respect and responsibility,
2) a focus on 21st century skills as well as state content standards,
3) implementing student centered, project and problem-based learning methodology to increase relevance and rigor,
4) courses and curriculum designed to connect learning to other subject areas and to the post-high school world,
5) infusion of technology as a tool for communicating, collaborating and learning, and
6) partnerships with community, higher education and business.

At Algiers Technology Academy we model these core principles by following New Technology Foundation's hierarchy of needs creating an atmosphere that begins with an emphasis on positive school culture. According to New Technology Foundation: "The hierarchy of needs for school change informs us that without addressing issues of culture, purpose and structure, our attempts to improve instructional practice are very difficult to sustain."

In our school's culture, healthy student-teacher relationships are important. Through these relationships a good rapport with the students is built creating an atmosphere where learning can take place. The students are valued and treated with respect. Our school setting is not only a caring environment, but also a professional environment that encourages our students to develop various 21st century skills that are necessary for them to be successful in the future. Some of these skills include written communication, oral communication, collaboration, work ethic, critical and innovative thinking and technology literacy. The students are assessed on these skills and given the opportunity to improve in these areas. Also, in keeping with a professional atmosphere, to model a real work environment, and to foster responsibility among the students in our school, we do not have bells. When dismissed by the teacher the students are required to manage their time wisely and report to their next class.

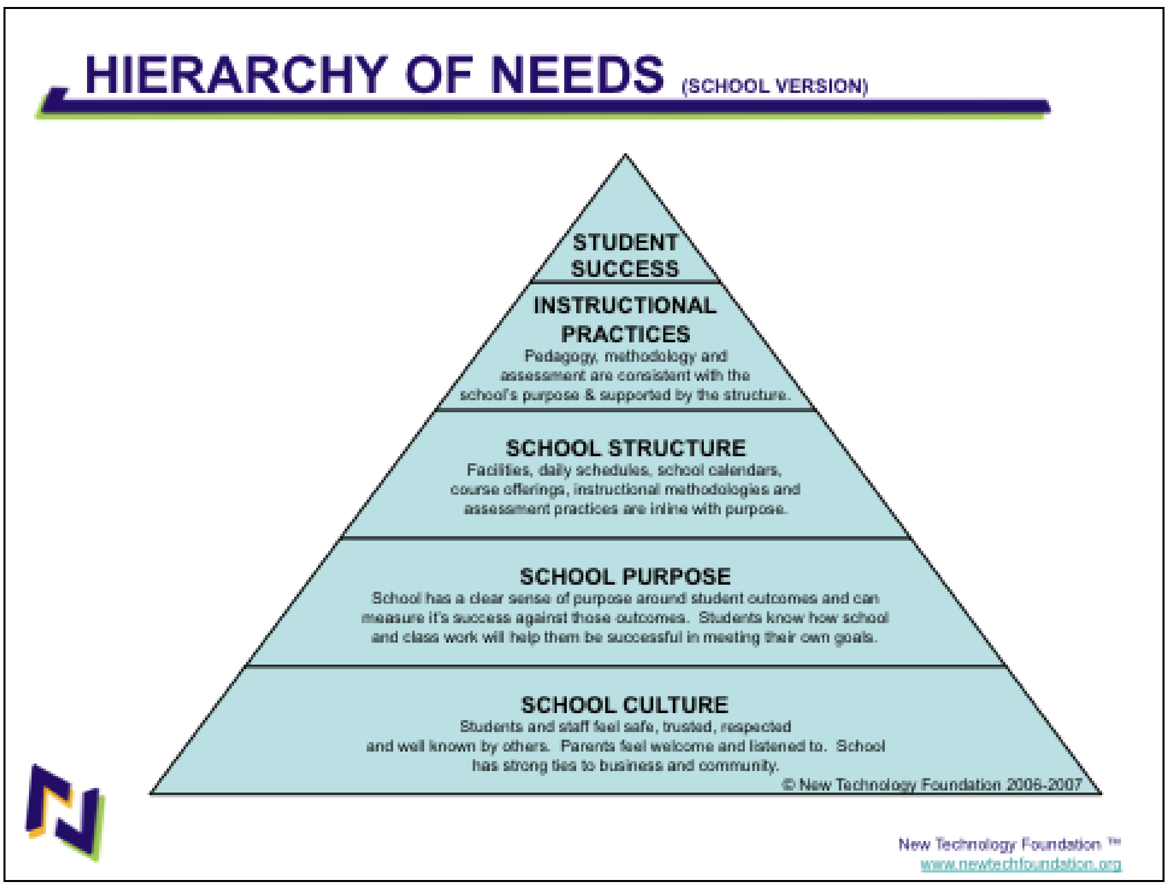


Figure 1.1 New Technology Foundation's Hierarchy of Needs

In a world that is technologically sophisticated with constant changes and advancements in science and technology, our students need to be prepared for these challenges and changes. At our school, technology is a very important tool. We have a one-to-one student-computer ratio. The students and teachers use technology as a means of collaboration among teachers and students and evaluation of our students, school, and colleagues. We also use technology for research, development and implementation of projects.

CHAPTER 2. SAMPLE HIGH SCHOOL GEOMETRY PROJECTS

2.1 "FuN.Ed Toy's Inc."

2.1.1 The Assignment

This project was designed for students to investigate proportionality in the real world. Students learned the meaning of drawing or constructing something to scale through the creation of scale drawings and scale models of their own bodies. The real world context of the project gave students the opportunity to develop an assortment of additional skills. TexTeams Institute on Proportionality inspired many of the documents used for this project. TexTeams is a series of professional development courses for teachers that provide many thought-provoking hands-on activities written under the direction of the Charles A. Dana Center, UT Austin.

This project had many parts. Not only were the students required to create a scale drawing and scale model of their own bodies, they also completed a portfolio where they discussed and displayed characteristics of proportional relationships and how to represent them. In the portfolio, the students recorded their measurements in a chart, used a scale to calculate new measurements, examined the chart for patterns, wrote necessary equations, graphed the results, and answered critical thinking and application questions regarding proportional relationships. The students were required to present their final products (scale model, scale drawing, and portfolio) to a panel of educators and other professionals. The entire project took place within a 3-week period.

The project had three phases: 1) constructing a scale drawing, 2) constructing a scale model and 3) an interview (the presentation of their final products). Each phase lasted approximately one week. To begin the first phase, the students and I read the entry document (Figure 2.1) aloud and we discussed it.

FuN.Ed Toys Inc.....

The best toys in the World!

FuN.Ed is a toy company dedicated to designing toys for children that serve both educational and entertainment purposes. FuN.Ed's staff not only consists of toy designers, but teachers, parents, and young, creative Geometry students like you. FuN.Ed is currently looking for more talented and creative Geometry students to join their staff for Summer 2009 as interns. FuN.Ed is a rather competitive company so they are looking for the brightest and best students to serve as "Junior" toy designers to some of the most notable toy designers in the country. The application process is rather lengthy and difficult. Only 5 junior toy designers will be chosen.

As part of the application process you will have to turn in an application signed by your parent or guardian, a brief essay of why you should be chosen as a junior toy designer, a scale drawing of yourself for a Barbie/Ken line called "High School Students," as well as a scale model of this drawing using everyday, household items to prove your creativity. You will also have an interview where you will present yourself as well as your scale drawing and scale model to the CEO of FuN.Ed and other FuN.Ed staff. Thanks for your interest in our company. If chosen, you will be working for the best toy company in the world!

Danica Robinson, Founder and CEO

Figure 2.1 "FuN.Ed Toy's Inc." Entry Document

Then, based on the entry document, the students made a list of what they knew and what they needed to know in order to complete the project (see Appendix A). During this time, the students and I engaged in dialogue and in the interpretation of the entry document, making sure that the task and its requirements were understood. Following the completion of the know/need-to-know lists, the students were ready to begin phase one. I put the students in pairs to begin measuring. In centimeters, they measured certain parts of their body and recorded their measurements in a chart. This all occurred on day one.

The following day the students were placed in groups of three or four. First, they completed a group contract (see Appendix B) to establish group norms and guidelines. Then they were required to complete their measurement chart (Figure 2.2).

Scale drawing Guidelines: *Must use the scale: every 6 cm of actual length represents 1 cm on the scale drawing *Must complete your scale drawing on legal sized paper *Must be accurately drawn using centimeters		
Your Actual Measurements (cm)	Process Scale Drawing's Measurements	Scale Drawing's Measurements (cm)
Example: height 175.3 cm	175 ÷ 6	29.2 cm
Height:		
Length of torso:		
Length of arms:		
Length of legs:		
Length of foot:		
Length of hand:		
Length *x*	Write an equation:	*Y*=

Figure 2.2 Scale Drawing Measurement Chart (modified from Texteams Proportionality Professional Development)

They were asked to use a scale in which 6cm of actual length would represent 1cm on the scale drawing. They then used this relationship to calculate the measurements for their scale drawing. They were assigned to think about the general rule for going from actual measurements to scaled measurements by writing an equation. Following the measurement chart, the students had to use their acquired knowledge of proportional reasoning to answer the questions in Figure 2.3.

Proportionality Questions:

1) What is the ratio of your actual measurements to the scale drawing's measurements for each row (Round to the nearest whole number.)?

2) Does the ratio above represent a variable or a constant for your table of measurements? Justify your answer.

3) Using the scale above, if Jared's arm length were 63.5 cm, what would be the length of his arm for the scale drawing?

4) Using the scale above, if the length of Jasmine's leg in her scale drawing is 16 cm, what is the actual length of her leg?

5) If you wanted to make another scale drawing using a different scale where the actual height of 140 cm becomes 28 cm, how would you determine the other measurements (lengths of legs, arms, torso, etc.) needed to make the scale drawing?

6) What equation could be used to determine the other measurements needed to make the scale drawing in question 5? Be sure to label each part of your equation!!

7) If Alexandra's torso has a length of 43.2 cm and you wanted to reduce it to 6.4 cm for a scale drawing, how would you determine the other measurements (lengths of legs, arms, height etc.) needed to make the scale drawing?

8) If you wanted to make a life-sized doll from a scale drawing where each measurement of the life-sized doll is 5 times as long as each measurement of your scale drawing, what equation could you use to determine the measurements of the life sized doll? Be sure to label each part of your equation!!

Figure 2.3 Proportionality Questions

The next day, the students were required to start drawing their scale drawing using the measurements from their chart. They were also required to graph the results from their measurement chart on a coordinate plane using actual measurements as x-coordinates and scale drawing measurements as y-coordinates (Figure 2.4).

Name:____
Date:________________________

Proportionality Graph

Directions: Graph your measurement table on the grid below. Use your actual measurements as your x-coordinates and your scale drawing measurements as your y-coordinates. Be sure to label your axes and give your graph a title. When you've completed the graph, answer the questions that follow.

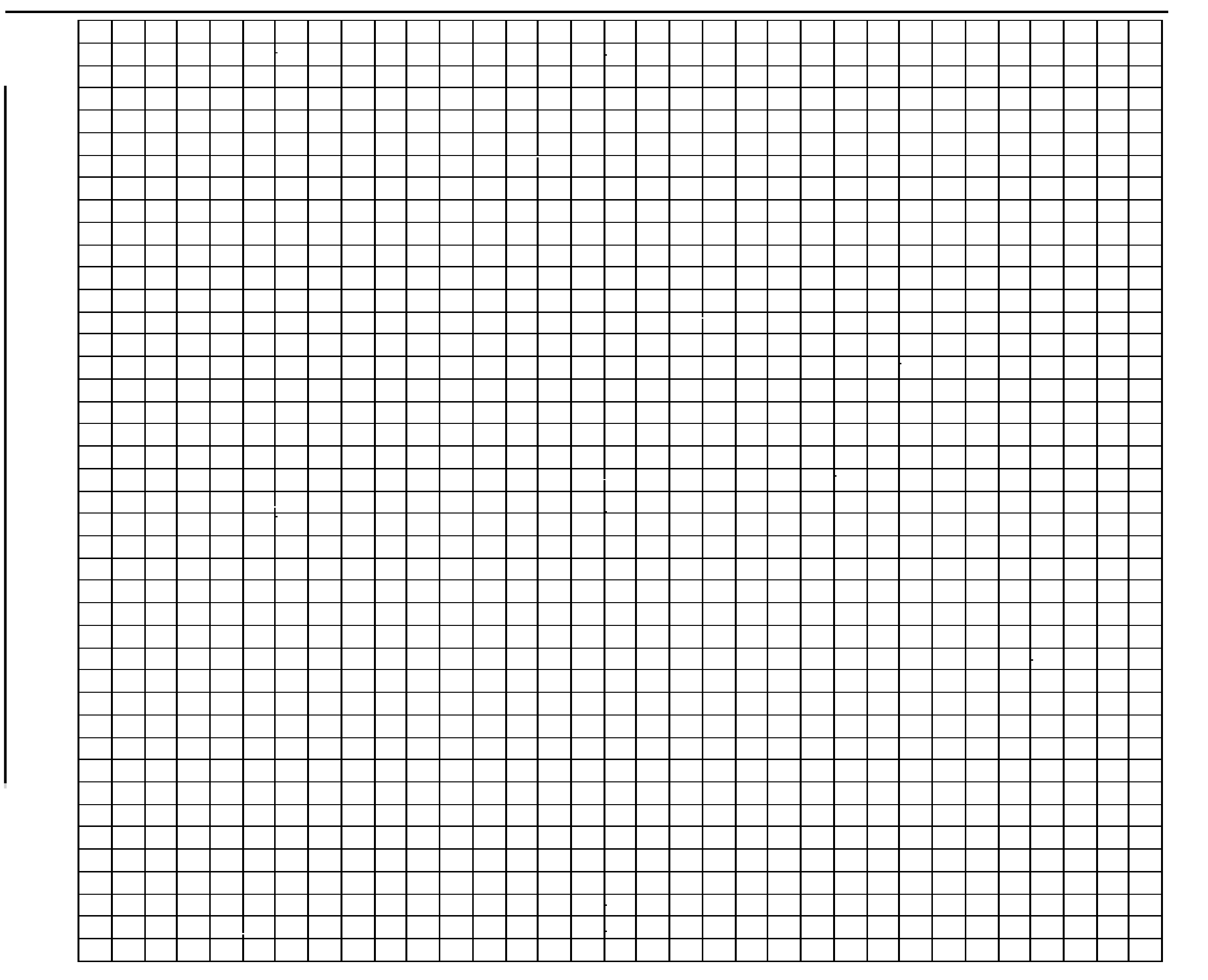


Figure 2.4 Proportionality Graph

Proportionality Graph Questions

1) Describe the graph (slope, y-intercept, etc.)Name an ordered pair from your graph.

2) What does each coordinate of the ordered pair above mean in this problem?

3) If Phillicia's torso is 45 cm and we are looking for y in the ordered pair (45 cm, y) on her graph, state what this means in words (or what does y represent)?

4) State and compare the ratios of y/x for each ordered pair graphed. What is significant about these ratios? Explain.

5) What is another name for the ratio you found in #5?

Figure 2.5 Proportionality Graph Questions

Following that, the students were asked to answer questions concerning their graph (Figure 2.5). Throughout the duration of the project, the students and I engaged in classroom as well as individual discussions to process what was occurring.

On the final day of phase one, the students completed their scale drawings and all supporting documents and were ready to begin phase two: constructing a scale model. We began with a discussion of phase one (constructing a scale drawing) to recap what was learned, and we made a list of acceptable household items to use to construct scale models. The students could only use items found around their house or other art and craft items. The students were given approximately 1 week to complete this phase.

I then handed out a second measurement chart with a different set of guidelines for the scale model (Figure 2.6). The guidelines were, for the most part, the same. The students were required to measure different parts of their body and scale them down to construct a three-dimensional model of their own bodies. They were required to choose a scale other than the one

<u>Scale Model Guidelines</u>: *Must tell the scale you used: every ___________cm of actual length represents __________ cm on the scale model *Must complete your scale model with the list of acceptable household items *Must be accurately constructed to scale using centimeters		
Your Actual Measurements (cm)	Calculate Scale Models Measurements	Scale Drawing's Measurements (cm)
<u>Example:</u> height 175.3 cm	175.3 ÷ your scale from above	??? cm
Height:		
Length of torso:		
Length of arms:		
Length of legs:		
Length of neck:		
Length of top of head to the chin:		
Length x	Write an equation:	Y=

Figure 2.6 Scale Model Measurement Chart

they used for the scale drawing, continue to use centimeters, and stick with household items for the creation of their models. Next, they were given three more critical thinking questions related to scale models (Figure 2.7).

Scale Model Proportionality Questions:

1) What scale did you use for your scale model and tell why you chose that scale?

2) How can you use your table to determine if your actual measurements and your scale model measurements have a proportional relationship?

3) **Leonardo da Vinci** Exploration: "Vitruvius, a Roman engineer of the first century B.C., influenced Leonardo da Vinci's work in architecture and also his drawing of the human figure. One of Leonardo's drawings is called the **Vitruvian Man**. It is based on a model of ideal proportions which Vitruvius established."
(http://mathforum.org/alejandre/frisbie/math/leonardo.html).
Based on the video we watched about Leonardo and scale models, is the ratio of your arm span to your height really equal to 1? Justify your answer.

Figure 2.7 Scale Model Proportionality Questions

In the final week of the project, the students were given time to work on their interview. In keeping with the real world scenario and context of the project, the students were required to complete a job application, write an essay, and get a teacher's recommendation for the internship. Some also used this final week to tie up loose ends on their portfolios, drawings and models. They then presented their final products to a panel of educators and other professionals in interview form. They were required to answer, in detail, two of the following conceptual questions:

1) If something is not "drawn to scale," what does this mean?

2) In your own words, what is a scale drawing?

3) Give three real-world examples of where scales are used.

4) In your own words, what is a scale model?

5) State three mathematical things you learned while completing this project.

2.1.2 Report on What the Students Learned and Need More Work On

At first, the students were accustomed to solving proportions not knowing why they worked and not understanding the characteristics, uses, or reasoning behind them. Research has shown that students often overuse proportions in "missing-value" problems. They tend to

become masters of the art of solving and, maybe even, setting up proportions, but many students have not developed a conceptual understanding of when the use of a proportion is appropriate. Many teachers throughout elementary and secondary education tend to focus more on the process of solving proportions rather than their applicability (Dooren, et al. 2009).

I believe that through the completion of this project, the students were enlightened. The definition of a proportion finally began to make sense to them. At the beginning, they knew that a proportion consists of two equivalent ratios, but they were not able to explain what that means or why it’s important.

The phrase “drawn to scale” is often seen in the real world and often misunderstood. After interviewing the students, I believe some students finally realized that when they see maps that say, “not drawn to scale,” the distances on the map are not proportional to the actual distances. There was no consistent scale used throughout the construction of the map to show the relationship among the actual and scaled distances. From this map, you cannot really get a clear picture of how far a place is from another place.

Through discussion and much individual collaboration with the students, I think that the students finally began to understand what it meant for things to be proportional. They realized they could use a proportion to solve problems where things are being compared.

During the project, I showed a video about proportionality, and they were able to see many other real world uses of proportional reasoning. After the video, we discussed what was presented in the video and many students verbalized that they learned that proportions and scales are used in many of the things they enjoy today such as toys and taking photographs.

The student’s portfolios showed that they understood how to write equations based on observed patterns in a table. Many of them understood what each portion of the equation meant and why it was written as such. They also realized that there was a constant ratio involved in

finding the measurements needed for their scale drawing. By completing the measurement chart (Figure 2.2), the students had to use a constant ratio of 6 to 1 to get their scaled measurements. As a follow-up, question 1 in Figure 2.3 asks: “What is the ratio of your actual measurements to the scale drawing’s measurements for each row?” Question 2 asks: “Does the ratio above represent a variable or a constant for your table of measurements? Justify your answer.” Most students gave a correct answer of 6 to 1 for question 1, and they answered “constant” for question 2. Also, many students said that the ratio is constant because it does not change from measurement to measurement.

Yet, there were also things that the students still had trouble understanding. They were required to graph the results of their measurement chart using their actual measurements as x-coordinates and their scaled measurements as y-coordinates (Figure 2.4). The students were fine with graphing, but they had trouble with the questions that accompanied the graph (Figure 2.5). The students had difficulty describing the graph (Figure 2.5 Question 1). Many of them realized that the graph was a line, but they still had trouble remembering and understanding the attributes of a line (slope and y-intercept) and the equation of a line. When they see a graph they do not automatically think that graphs have equations, and those equations describe the characteristics of graphs.

The students knew that actual and scaled measurements were proportional, with a ratio of 6cm to 1cm nonetheless, many students gave an incorrect value for the slope of the line. Some said the slope was 6 rather than 1/6. They knew that the scale-factor was related to slope, but they got the details muffled. They did not consider the conceptual meaning of slope. In very general terms, slope is the “steepness” of a line and it can be determined by calculating the change in y over the change in x. The units in which x and y are measured provide the units for the slope. For example if the units used on the x-axis are seconds and the units on the y-axis are

meters, the units for the slope would be meters per second. Using different scales on the two axes will make the apparent slope different from the actual slope. Most students used different scales on their x and y-axes creating confusion and a misunderstanding of how to calculate slope. In addition, some students said their line had no y-intercept instead of saying that the y-intercept was 0. I believe this can be attributed to the fact that students did not remember that a line extends infinitely in both directions. In their mind, their graph was a line segment, not a line, so they did not extend it to see that it would pass through zero on the y-axis.

2.1.3 Lessons I Learned about Project-Based Instruction

Working on this project helped me understand the importance of the student's voice, reasoning, and opinion to the learning process. Many students seemed to benefit by speaking aloud their thought processes in solving a problem or understanding a concept. I also learned the role that each portion of the New Tech model plays in the implementation of a project. It is important to introduce the project in a way that is meaningfully and explicitly structured. The student's benefit by being able to construct an overall view of the project at first, with not many details so as not to confuse. This helps the students understand the goals and objectives of the project. They determine what they know and what they need to know to complete the task, encouraging them to think about and care about their learning.

I also learned that students have mathematical misconceptions that can be addressed through project-based learning, if implemented correctly. Some misconceptions include using proportions to solve all *missing-value* problems and calculating the area of any two-dimensional shape by multiplying its length and width. A concept may be presented in a certain way, but that does not mean that students will interpret it the way it was presented. Student learning is built upon prior knowledge, misconceptions and all. Therefore, the way students process a concept can sometimes misconstrue what they are currently learning. Project-based learning addresses

this issue in that it seeks to weed out misconceptions through a strong emphasis on communication via frequent teacher feedback and peer collaboration. The students are given the freedom to (and are required to) verbalize their learning through participation in a peer evaluation process and through final presentations. In this manner, many misunderstandings are verbalized, discussed, and alleviated.

2.2 "Crescent City Parks and Recreation Mural Design"

2.2.1 The Assignment

This project was designed for students to examine characteristics of triangles. The focus of this project was classifying triangles by their sides and by their angles, understanding and applying the Angle Sum Theorem, constructing triangles that are congruent by SSS, SAS, ASA, and AAS, and identifying corresponding parts of congruent triangles. The goal of the project is for the students to incorporate these concepts into a mural design for Crescent City Parks and Recreation based on artwork by Lois Maliou Jones.

The students were required to create a mural in keeping with specified geometric guidelines, complete a written portfolio, and present their work. The project took place in a time period of approximately two weeks. Some necessary materials for the project included bulletin board paper or poster board, ruler, protractor, compass, and 7 pieces of artwork by Lois Jones.

We began the project by reading and discussing the entry document (Figure 2.8). The students were given the opportunity to read the entry document silently first. Then, I read it aloud to them. We discussed the contents of the entry document to make sure the students understood the overall view of the project. After reading the entry document we made our know/need-to-know list (Appendix A). This would determine what topics or content I would need to include as mini-lessons and scaffolding activities for the students. Then I put the students in groups of 3 or 4, and a leader for each group was chosen.

Crescent City
Parks and Recreation

Crescent City Parks and Recreation (CCPR) playgrounds are currently undergoing renovations. Being commissioned and funded by Mayor Jay Cajun, CCPR has hired different people to make playgrounds more educational and culturally relevant. To accomplish this task, CCPR is teaming up with students and teachers of the Crescent City Public School System (CCPSS) to include murals that display geometric ideas. CCPR has decided to make this a competition among CCPSS students. The most creative and geometrically interesting murals will be chosen. Other prizes will be given. CCPR has identified Lois Maliou Jones as the feature exhibitionist because of her use of triangles and her urban, eclectic style.

To begin this project, you will need to become very familiar with the works of Lois Jones so you will provide an art and mathematical analysis of Jones work, analyzing and locating triangles in Jones art, and identifying other geometric elements of this piece. Based on this analysis, you will develop a similar artwork that can be displayed as a mural in a Crescent City playground. We want you to replicate Jones's style and use of triangles, but create your own original work. You will be required to present your work on the final day of the project.
We hope you have great success on this project!

Crescent City Parks and Recreation Council

Figure 2.8 Crescent City Parks and Recreation Entry Document

The students were then given ample time to complete their group contracts (Appendix B). I monitored the students, assisted them with their contracts, and answered their questions.

Following the group contract, we began the project with an analysis of Lois Maliou Jones artwork. Each group chose an artwork to analyze from the 7 artworks I pre-selected. Their instructions were to work together to complete the CCPR Mural Project Art Analysis assignment, Figure 2.9a.

NORD Mural Project: Jones Art Analysis

Group Names:____ **Date:**__________
____
____

Directions: Answer each question in the space provided. To receive full credit, you must show all work. Use separate paper if necessary.

Materials: painting by Lois Jones, protractor, centimeter ruler, pencil, paper

1) What are the three ways we can classify a triangle by its sides?
2) What are the four ways we can classify a triangle by its angles?
3) Use a protractor and a ruler to classify three triangles from your artwork by its sides and angles. Please indicate which triangles you chose by writing its description in the first column. An example is shown in the first row.

Triangle Description	**Length of each side(cm)**	**Classification by sides**	**Angle measurements**	**Classification by angles**
Example: purple triangle with white dots	6 cm, 6 cm, 7 cm	isosceles	90 degrees, 45 degrees, 45 degrees	right

4) What is the sum of the angles in a triangle?
5) What is the Angle Sum Theorem?
6) If two angles of a triangle are 35 and 25 degrees, what is the measure of the third angle?

Figure 2.9a CCPR Mural Project Art Analysis

In this assignment the students were to locate triangles in Jones art, measure the sides and the angles and classify them by both sides and angles. After that, the students were asked to state the

Angle Sum Theorem and then use it to predict the measure of the remaining angle when two angles are known.

The next day, the students were given an assortment of missing angle problems involving the Angle Sum Theorem, supplements, complements and vertical angles. Some of the problems were multi-step problems in which several of these principles needed to be used together.

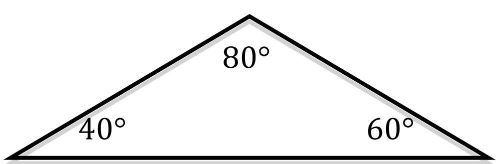
Reading the Lesson

1. Supply the correct numbers to complete each sentence.
  a. In an obtuse triangle, there are ______acute angle(s), ______right angle(s), and _____obtuse angle(s).
  b. In an acute triangle, there are ______acute angle(s), _______right angle(s), and _______obtuse angle(s).
  c. In a right triangle, there are _________acute angle(s), ______right angle(s), and ________obtuse angle(s).

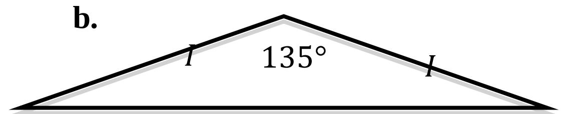
2. Determine whether each statement is *always*, *sometimes*, or *never* true.
  a. A right triangle is scalene.
  b. An obtuse triangle is isosceles.
  c. An equilateral triangle is a right triangle.
  d. An equilateral triangle is isosceles.
  e. An acute triangle is isosceles.
  f. A scalene triangle is obtuse.

3. Describe each triangle by as many of the following words as apply: *acute*, *obtuse*, *right*, *scalene*, *isosceles*, or *equilateral.*

a.



b.



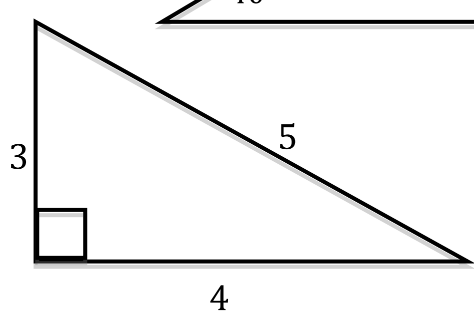


Figure 2.9b CCPR Mural Project Art Analysis Additional Questions from Glencoe Practice Workbook

Then the students moved on to interpreting the artwork they selected, naming things that stood out to them in the artwork, interpreting what Jones could have meant in creating the particular piece, and thinking about how the artwork inspired them as a group. This would lead them to brainstorming ideas about the mural they would create.

Throughout the project, I taught mini-lessons on various topics that the students identified as need-to-knows. Some of the topics included the Angle Sum Theorem, constructing congruent triangles and identifying corresponding parts of congruent triangles. I often held meetings with the group leaders to get updates on the progress of their groups or to teach them how to solve certain problems and send them back to their group to teach the other group members. This had its advantages and disadvantages.

The following day the students were given an assignment where they had to identify the corresponding parts of congruent triangles. Then they were given assignments where they had to construct two triangles congruent by SSS postulate, two congruent by SAS, two congruent by ASA, and two congruent by AAS. They then were required to incorporate these exact 8 triangles into their murals. On the final day of the project, the students presented a brief synopsis of the life and artworks of Lois Jones, and their murals. The students were not only assessed on content knowledge, they were also assessed on collaboration and oral communication.

2.2.2 Report on What the Students Learned and What They Still Need Work On

The student's portfolio's displayed their understanding of how to classify triangles by sides and by angles. The students also understood that the sum of the angles in a triangle is 180 degrees. They learned that if two angle measurements of a triangle are known, then the third angle measurement can be found. They also learned how to apply previously gained knowledge to new situations. For example, the students were first asked to apply the Angle Sum Theorem to individual triangles. Then they were asked to use this theorem in more complicated settings such

as with 2 or more conjoined triangles. Many students remembered to consider the complements and supplements of known angles to find adjacent angle measurements. Many students also remembered to look for vertical angles knowing that the measurements of vertical angles are congruent.

The students also became more comfortable using a protractor and a compass, yet they need additional practice. The students also need reinforcement on the triangle congruence postulates: SSS, SAS, ASA, and AAS. Many students had difficulty distinguishing between them. Somehow they could not understand that triangles are congruent according to the measurement of their sides and their angles. The students also had problems with identifying corresponding parts of congruent triangles.

2.2.3 Lessons I Learned about Project-Based Instruction

One major eye-opener for me is being extremely careful not to create a project with a product where the students get totally lost or caught up in the product alone or in unimportant details of the project. I need to create projects where the focus is on the mathematics involved, not the creation of a product. The product should only be a reflection of what the students learned throughout the duration of the project.

I also learned that for this particular project, I should have chosen an artist with more prominent geometry. The artwork of Lois Jones was not necessarily geometric or mathematical in nature; it only displayed geometric ideas. I could have incorporated perspective drawing. This would not only accomplish the task of applying geometric principles in a more meaningful way, it could also add a new dimension to the students drawing ability. I found that the mural did not reflect all that the students learned in 3 weeks. I feel that the students drew beautiful pictures and added the required triangles later as opposed to letting the creation of the mural be a mathematical process itself.

I also learned that I should have used various methods to teach triangle congruence, not just teaching the students to construct congruent triangles. Students need multiple representations of a concept to fully understand. Many students did not do well on the constructions. They had trouble using a compass. This was evident in their portfolios.

2.3 Conclusions of Chapter 2. Principles of Project Design

Teacher planning and preparation are important to project-based instruction. One must begin planning a project by looking at the state mandated standards to make sure students learn what they are supposed to at a particular grade level. Following the standards, it is important for a teacher to carefully think a project through and determine the desired outcomes. The teacher should also carefully plan various scaffolding activities throughout the project to bring structure to the learning process.

Concerning math projects, the teacher should start by choosing a mathematically rich and interesting problem as the foundation for a project. In this manner students would be able to begin solving the problem through examining extreme cases and allowing this to guide them to "a more detailed quantitative analysis" (Stanley 2001).

In conclusion, communication is pertinent to successful project-based instruction. It is important that the lines of communication between the teacher and the students are always open. The students need to verbalize what they are learning. The teacher's voice should not be the only one that counts. The students should feel comfortable and play an active role in both the learning and the teaching of concepts.

CHAPTER 3. A PROJECT-BASED LESSON ON OPTIMIZATION

3.1 Introduction

I first encountered the optimization problem discussed in this chapter in a graduate-level calculus course. It appeared in a TexTeams Institute. The graduate course explored this problem in depth and looked at its many extensions and applications. I also worked on this problem with three high school students from the summer 2008 Math Circle at LSU.

I began to ponder how I could present such a mathematically sophisticated problem to my geometry students as a preview of what they would see in a calculus course. As a result of the discussion concerning this problem, and with my professor's encouragement, I presented this problem as professional development to a group of middle school and high school teachers with the assistance of two other students. Following the presentation, I decided to contextualize the original problem to incorporate elements of project-based instruction.

This problem could be presented to students at various grade levels, and it could be extended as a mathematical discussion with graduate level students. It could be given to middle school students as a lesson to explore and compare areas of various two-dimensional shapes such as rectangles, triangles, and trapezoids. This problem could be extended further to enrich Geometry students on understanding areas of triangles, rectangles, and trapezoids, comparing slopes of lines, informal proofs, geometric proof and algebraic proof.

3. 2 The Initial Problem

In this section we state and solve the original TexTeams problem.

> A point (5,2) is given in the first quadrant. Of all the lines that pass through (5,2), which line cuts off the smallest area?

Analytic solution. The optimal line must cross both the positive x-axis and the positive y-axis.

Therefore, if the line has equation $y = mx + b$, we must have $b > 2$ and $m < 0$ (negative slope). Notice that the line is completely determined by b. Now, since the line passes through $(0,b)$ and $(5,2)$, the slope is $m = \frac{2-b}{5}$. Its x-intercept, $c = c(b)$ is found by solving $0 = m\,x + b$:

$$
\begin{aligned}
&0 = \frac{2-b}{5}x + b \\
&-5b = (2-b)x \\
&\frac{-5b}{2-b} = \frac{2-b}{2-b}x \\
&c(b) := \frac{5b}{b-2}
\end{aligned}
$$

The area under the line is a function of b:

$A(b)$:= (area under the line with y-intercept b)

$$= \frac{1}{2}bh$$

$$= \frac{(x\text{-intercept})(y\text{-intercept})}{2}$$

$$= \frac{5b^2}{2(b-2)}$$

Now, we can take the derivative of $A(b)$ to find the critical points:

Using the Quotient Rule,

$$\frac{dA}{db} = \frac{10b(2(b-2)) - 5b^2(2)}{(2(b-2))^2} = \frac{20b^2 - 40b - 10b^2}{(2(b-2))^2} = \frac{10b(b-4)}{(2(b-2))^2} = \frac{5b(b-4)}{2(b-2)^2}$$

From this, we see that A is decreasing on $(2, 4]$ and increasing on $[4, \infty)$. Thus, there is a minimum at $b = 4$. We can conclude that the best choice for the boundary is the line $y = -0.4x + 4$. This line cuts out an area of 20 square units.

Geometric solution: Draw rectangle (0,0), (10,0), (10, 4), (0,4). The point (5,2) is at the center. It is a fact that any line through the center of a rectangle cuts it into two congruent figures, either triangles or trapezoids. Thus, any line through (5,2) divides this small rectangle into two equal areas, so no matter what line is chosen, half of this small rectangle *must* be given up.

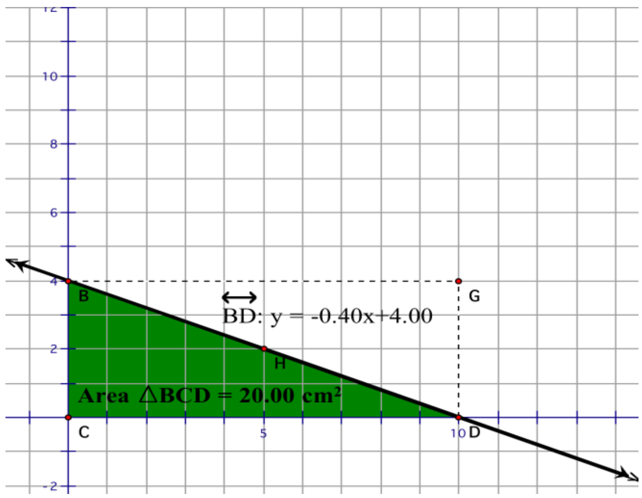
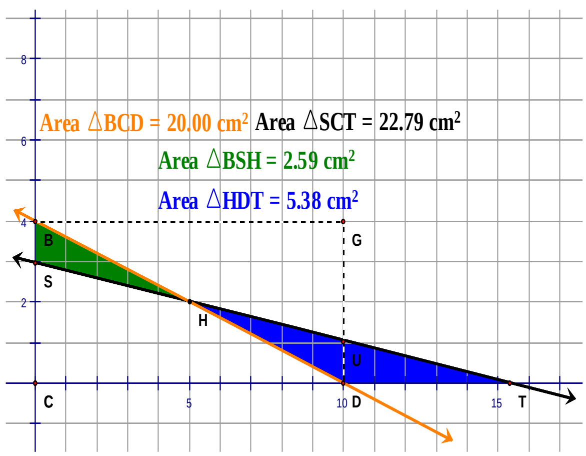


Figure 3.1 Illustration of Geometric Solution to Initial Problem

Now observe that any line that is not the diagonal of this smaller rectangle gives up more area; see Figures 3.2a and 3.2b. Therefore, the best line is the diagonal.



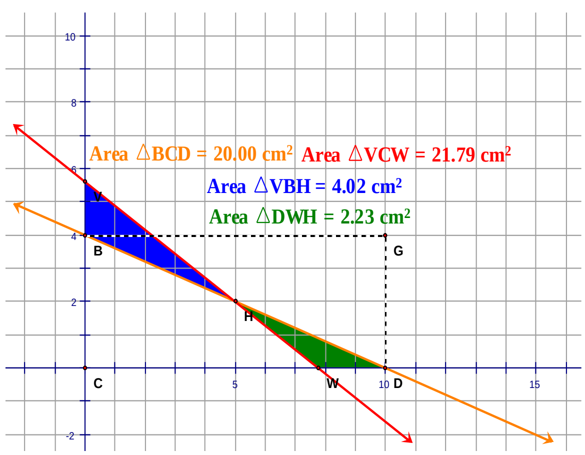


Figure 3.2a Comparison of the Optimal Line (m = -2/5) to an Arbitrary Line with m > -2/5

Figure 3.2b Comparison of the Optimal Line (m = -2/5) to an Arbitrary Line with m < -2/5

The TexTeams Institute considered extensions of this problem that involved cutting off areas when the axes were tilted, cutting off areas of two-dimensional shapes such as discs and arbitrary

shapes such as *blobs*, and slicing three-dimensional solids to minimize the volume of the portion cut off. I chose a specific extension to develop as a basis for a PBL project.

3.3 An Optimization-Based PBL Project

This section describes and solves a contextualized problem that is based on the concepts from the previous section. Mathematically, this problem is more complex, but it relates to a concrete situation that is easy to appreciate because of its real-world meaning. We have posed the problem so that it is easy to model with classroom materials. A standard piece of 8.5 by 11 inch paper can serve as a scale model.

Problem. A logging company owns a rectangular piece of property that measures 8.5 miles by 11 miles. At a point 2 miles from the long edge and 5 miles from the short edge, there's a well. In order to settle a land dispute, the logging company agrees to give up a piece of the land to the government for a nature preserve. The land must be cut off from the existing property by a straight line and have access to the well. How should the boundary be drawn so the company gives up the least amount of land?

In solving this problem, it is useful to start by selecting a coordinate system. Set up a coordinate system so that the land occupies the rectangle with vertices as (0,0), (11, 0), (11, 8.5), (0, 8.5) and the well is at (5,2). Note that the current problem is different from the problem in the previous section because it is set in a rectangle and not in the (unbounded) first quadrant. Thus, any line that passes through the rectangle could be a candidate for the optimal cut, whereas, in the previous problem the slope of the line had to be negative in order to cut off a finite area.

We will show that the best choice for the boundary is the line from (0, 4) to (10,0). The land given up is the right triangle with a vertex at (0,0). There are many ways to see this. First, we make an essential observation that they all rely on: the boundary that gives up the least

amount of area *must pass through the position of the well,* (5,2). Indeed, any line not drawn through this point would result in excess area for the government's nature preserve, for if the line does not pass through the well, we can decrease the amount of land given up by moving the line to a new position, parallel to the old one but passing through the well (5,2). Then the company retains the land in the strip between the old and new position.

I. Quick geometric solution. Draw rectangle (0,0), (10,0), (10, 4), (0,4). The well (5,2) is at the center. Any line through the well divides this small rectangle into two equal areas, so half of this small rectangle *must* be given up. Furthermore, any line that is not the diagonal of this smaller rectangle gives up more land as illustrated in Figure 3.2. (Please note that the scale 1cm = 1mi. is used in all illustrations.)

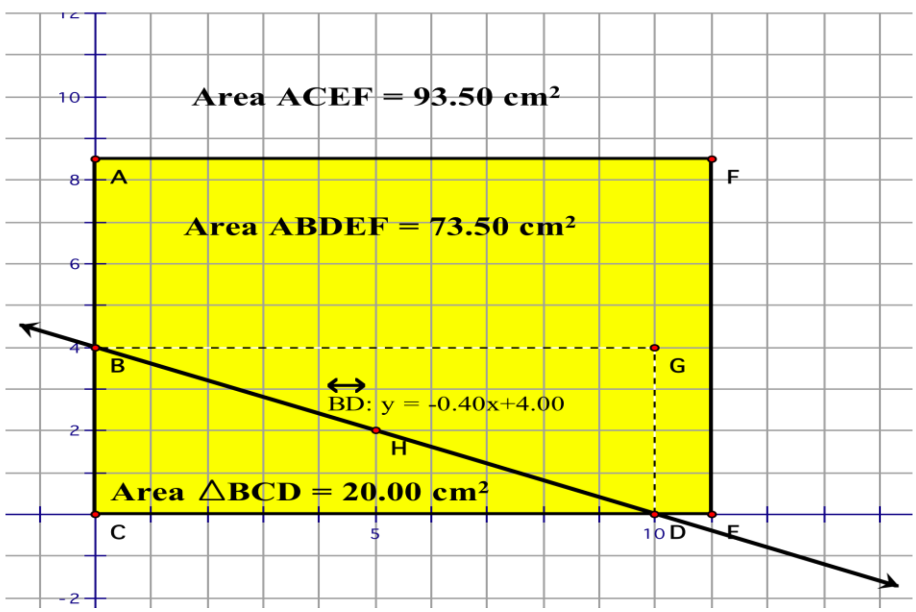


Figure 3.3 Illustration of Geometric Solution to Contextualized Problem

This solution gives very quick insight into the problem, but it is unlikely that a 10^{th} grade student would come up with this idea immediately. Since it is likely that many approaches to the problem will be discussed in class, we will present various approaches. Also, this solution presents limitations because it would not work if the logging companies land were not a rectangle.

II. A longer geometric solution using Geometer's Sketchpad.

In searching for the solution, we considered many things. There are an infinite number of possible lines that could divide the land. The line that cuts off the smallest area will intersect two of the sides of the rectangular piece of land. Since there are four sides and the line must pass through two of them, there are six possible combinations of sides. Two of the six combinations are impossible for a line passing through (5,2): the top and the left (i.e., the side on the line y = 8.5 and the side on the y-axis) and the top and the right (i.e., the side on the line y = 8.5 and the side on the line x = 11). From now on we will refer to the sides by the line that they are on. The possible combinations for a line passing through the well are a) the y-axis and the line x = 11, b) the x-axis and the line y = 8.5, c) the x-axis and the line x = 11, and d) the x-axis and the y-axis.

Within these cases we looked at some specific lines to gain an intuitive sense of what was occurring. Drawing the boundary from the origin to (11, 8.5) or from (11, 0) to (0, 8.5) would divide the land in half, so we did not need to look closely at these cases because the logging company wanted to minimize the amount of area they would give up.

We examined the problem in Geometer's Sketchpad and created a display that would calculate the cut off area under several lines. Let's start by looking at the red line x = 5 in Figure 3.4. This line intersects the x-axis and the line y = 8.5. If the land were divided in this manner, the logging company would lose 42.5 square miles of land. If we rotate the line clockwise approximately 45 degrees, the line still crosses the x-axis and the line y = 8.5 and the area given up decreases to 36.13 square miles. Keeping in mind that the smaller area will be given up by the logging company, as we keep rotating the line in the clockwise direction, the area given up decreases until a certain point. By inspection, we discover that the smallest areas occur when the lines intersect the x-axis and y-axis near (0,4) and (10,0). As we increase our y-coordinate past 4 and decrease our x-coordinate accordingly, the areas begin to increase again. So we can

conjecture that the optimal cut crosses both the *x*-axis and the *y*-axis at some point near (0,4) and (10,0).

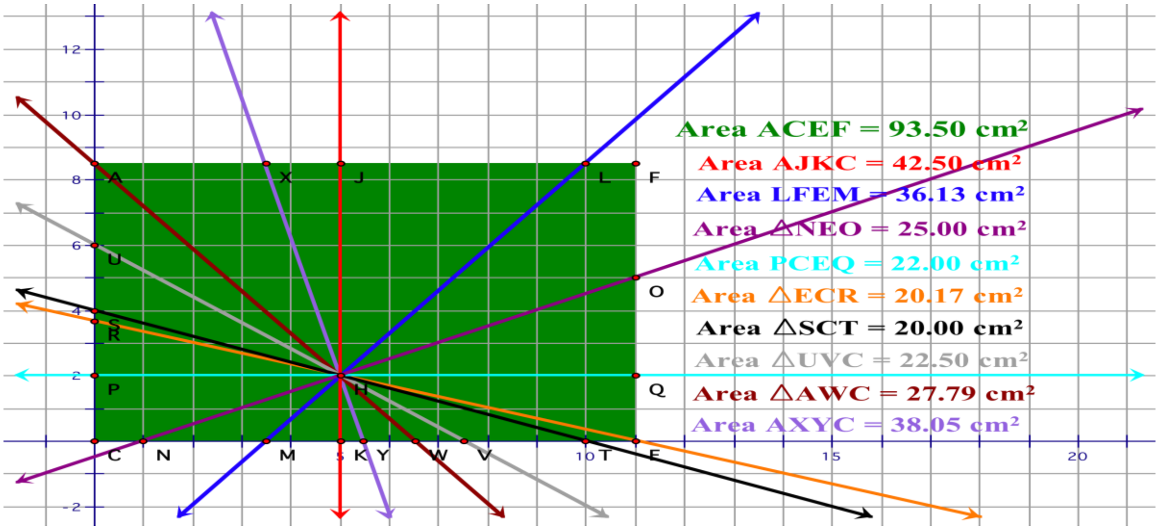


Figure 3.4 Comparison of Areas that Arbitrary Lines Create to Area Created by the Optimal Cut

To further determine exactly which line would cut out the smallest area, let's look at an area under our "potential" optimal lines that will remain constant. As illustrated in Figure 3.5, we can draw the rectangle with vertices (0,0), (0,2), (5,0), and (5,2). This rectangular area (10 square miles) will always be given up no matter how we rotate the line as long as the line intersects the sides on the *x* and *y*-axes.

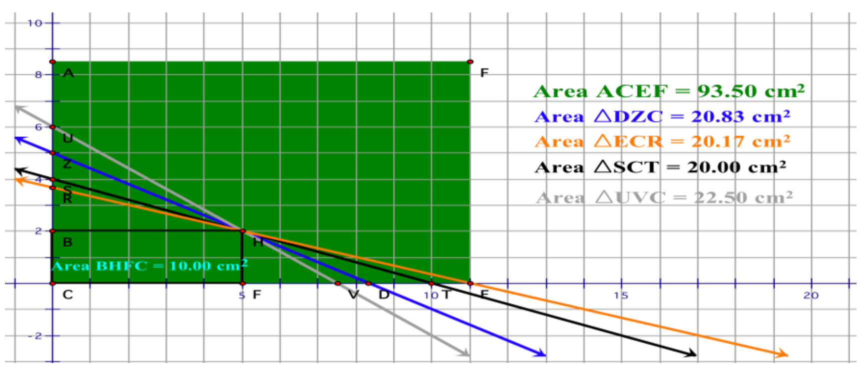


Figure 3.5 "Potential" Optimal Lines

Keeping in mind that the slope must be *close to* – 2/5, let's look at a geometric argument to determine the optimal line that gives up the least amount of land. We will see that the slope of

the optimal line is actually *exactly* -2/5. See the following sequence of figures. Note that the argument below holds for any numbers (p, q) in the first quadrant.

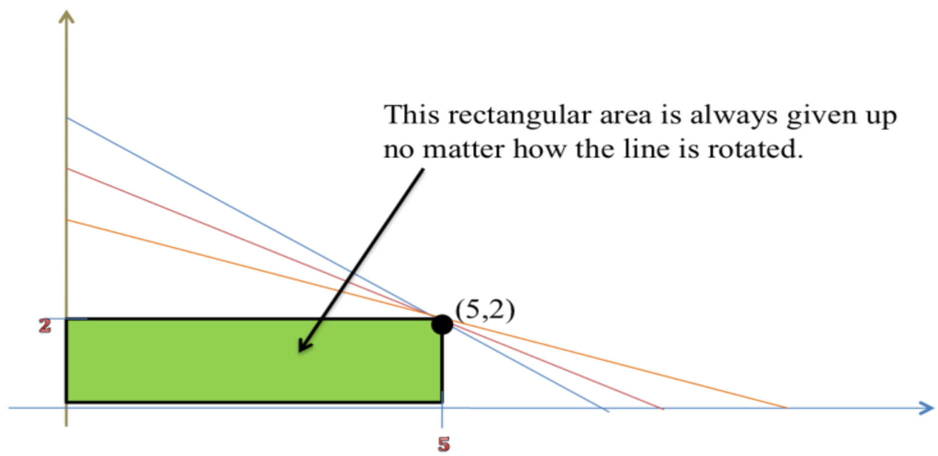


Figure 3.6a Geometric Proof Part 1

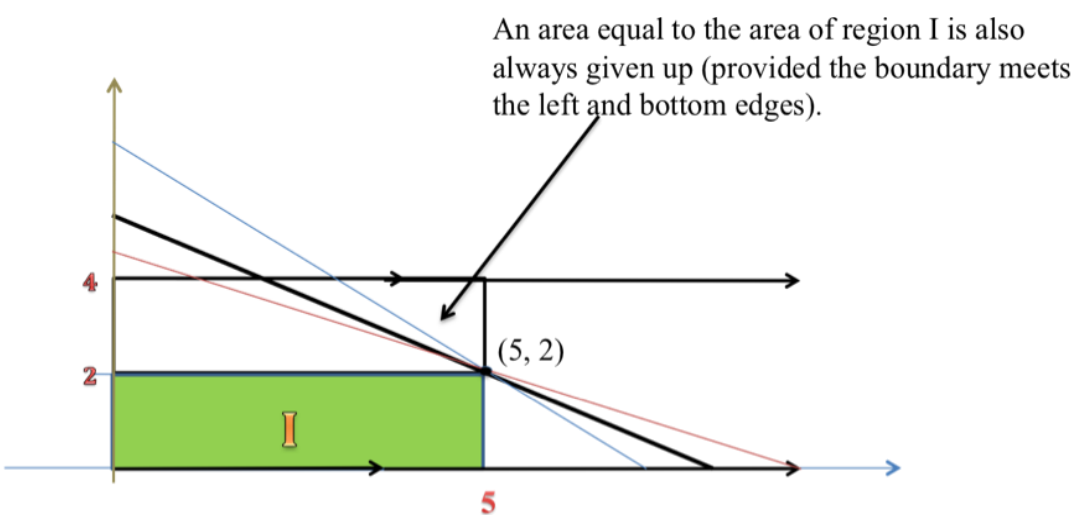


Figure 3.6b Geometric Proof Part 1 Continued

We can see that this is the case by the diagram at the left. Region I, II and IV are given up (plus a little more above region II). But region V is congruent to region IV by ASA.

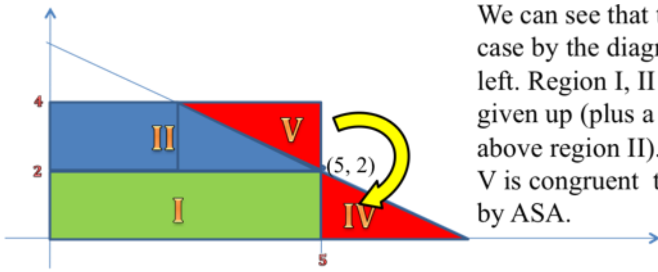


Figure 3.7a Geometric Proof Part 2

We'll call the remaining region, III. We can see that the area given up equals I +II+III+IV and we know that II+IV = I so the area given up equals I + I + III which equals 2 I + III. We can minimize the area under the line by eliminating triangle III so the least amount of area to give up is 2I + 0 = 2 I .

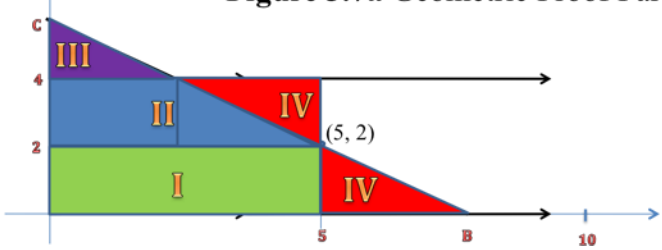


Figure 3.7b Geometric Proof Part 2 Continued

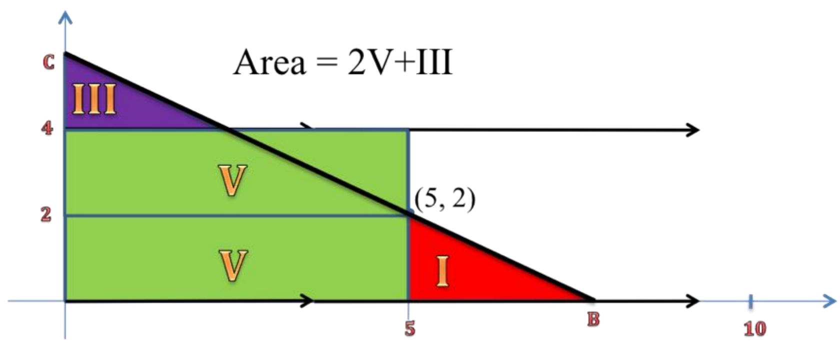


Figure 3.8 Arbitrary Line

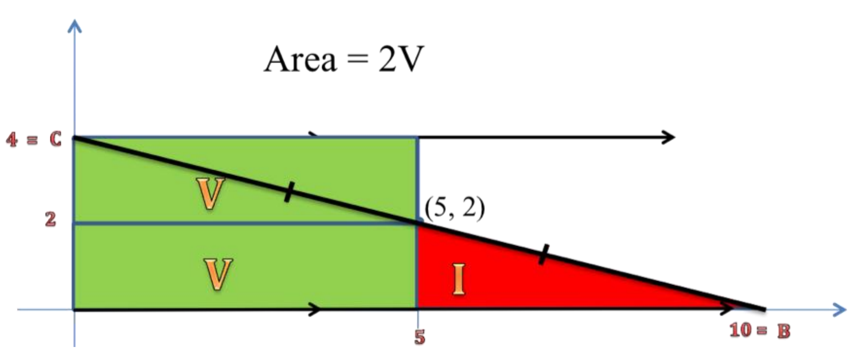


Figure 3.9 Optimal Line

Conclusion: Figure 3.8 shows an arbitrary line. The area under the line is 2V+III. This area can certainly be minimized when triangle III is removed by sliding the line down the *y*-axis so that C equals 4 and B equals 10. Figure 3.9 displays the optimal cut that minimizes the area. In this problem, when this cut is made, C will always equal 4 and B will always equal 10. Furthermore, (5, 2) will always be the midpoint of the line. Therefore, to minimize area, find the line such that (p, q) is the midpoint. Moreover, to settle the land dispute and to ensure that the logging company retains the majority of it's land, the boundary line that cuts out the minimal area of land is $y = -0.4x+4$ where (5, 2) is the midpoint. This proves the case where $m < -2/5$. Let's take a look at the case when $m > -2/5$. As illustrated in Figure 3.10, the argument is the same with a slight change in details. The excess area lies along the *x*-axis instead of the *y*-axis.

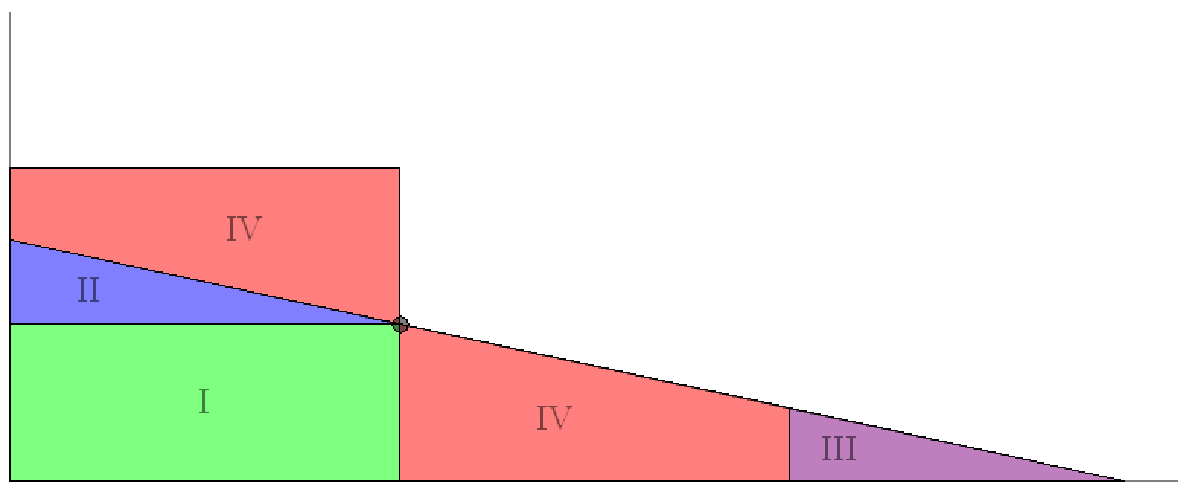


Figure 3.10 Geometric Proof when $m > -2/5$

Summary. Solution *I* showed that half of the smaller rectangle with vertices (0, 0), (10, 0), (10, 4), (0, 4) must be given up. Solution *II* provides a longer geometric argument using Geometer's Sketchpad. It showed that the line is not optimal if (5, 2) is not the midpoint of the line.

Solution *III* (see Appendix C) provides an analytic solution. Coming up with this solution required very complicated algebra with piecewise rational functions. By algebra we eliminated lines with slope not in [-13/10, -1/3]. Then we used calculus. When compared with the other

solutions, this approach of using algebra without thinking appears very time consuming, wasteful, and error prone.

By reflecting on the solutions we have found and the various approaches we have used, we can make the following general observation. Suppose we have a point P inside of any figure (a *blob*) with a continuous boundary. Suppose further that if we travel along any ray drawn from P, we remain inside the blob until crossing the boundary and then we never re-enter the blob. Finally, let us suppose that we must cut the blob in two by a line through P and give up the smaller half. How do we draw the line so that we loose the least amount of area?

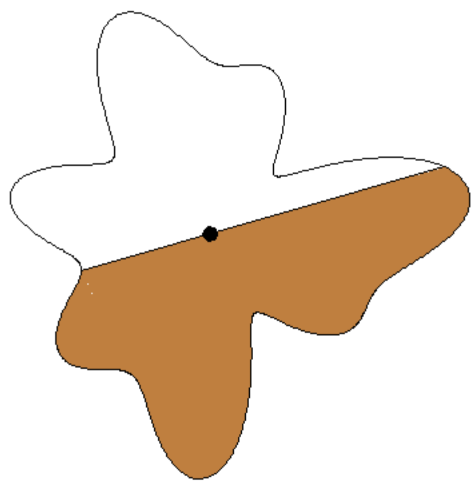


Figure 3.11a The *Blob*

Having drawn any line through the P let AB be the segment that lies inside the blob. We make the following claim. If the line is optimal, the pivot point is the midpoint of AB. To see this, let’s look at a picture. In the picture we are giving up the brown region. Suppose we have chosen a line, and we find that PA and PB are not of equal length. Then rotate the line so that the longer side moves into the brown region while the shorter side moves into the white region making sure that the long side never gets shorter than the average of PA and PB and the short

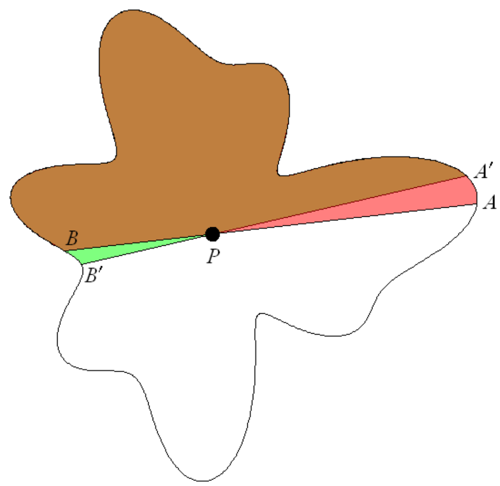


Figure 3.11b The *Blob* Continued

side never gets longer than this average. In the picture above I save the red region but give up the green region, but the radius of the red region is always larger than the radius of the green region. This shows that if the segments are not of equal length then I can rotate the line a little and give up less area.

3.4 Conclusions of Chapter 3

This is a very interesting math problem that has many mathematical implications as well as real world applications. Investigating this problem on a high school level allows students to use and apply their math knowledge to solve a real world dilemma, accurately dividing land to maximize what is kept and minimize what must be given away.

Solving a problem of this sort is not an easy feat. It takes time, thought, energy and the use of various approaches to find the correct solution. "One can infer that… students learn best by first struggling to solve mathematics problems, then by participating in discussions about how to solve them, and then hearing about the pros and cons of different methods and the relationships between them" (Stigler and Hiebert 1999). Often in traditional teaching methods, students are not encouraged to grapple with problems to come up with logical solutions because of the emphasis on lecture and practice exercises as opposed to inquiry, discovery and in-depth

understanding of concepts. Most students are taught to seek immediate results: the answer to a problem. The product of this mentality is that students give up easily when asked to solve open-ended questions. This optimization problem requires a great deal of what Paul Zeitz calls "mental toughness." He describes this as confidence and concentration (Zietz 2007). Students need to understand that it may take several incorrect attempts to solve a problem, but with each attempt, slightly changing the approach may lead to further insight of the correct solution.
As can be seen, there is not only one correct approach to solving this problem. I took an algebraic, geometric and an analytic approach.

In order to present this problem to students, teachers must carefully map out the objectives and standards they want their students to learn. This takes careful planning and preparation, and it takes the teacher working through the entire problem to gain a breadth and depth of understanding of the problem to become aware of its many implications.

CHAPTER 4. CONCLUDING THOUGHTS

Finding correct solutions is not the primary goal of PBL. Project-based learning seeks to develop, for students, through exploration and investigation, *habits of mind* that will further improve their problem-solving skills. As it relates to mathematics, it is desired that students develop a "profound understanding of fundamental mathematics"(Ma 1999). It's important for students to not only learn how to solve one problem, they must learn how to think so they can solve many problems.

Some teachers only feel accountable for "shaping the task into pieces that are manageable for most students, providing all the information needed to complete the task and assigning plenty of practice" (Stigler and Hiebert 1999). Classrooms where there is only lecture and practice lack critical thinking and creativity. Many students become proficient at solving some problems through rote memorization when several examples of a particular problem have been presented. If one aspect of the problem changes, students become confused and they do not see the connection between the question they could solve and a slightly different question.

In conclusion, a PBL classroom is a student-centered environment where teachers serve as facilitators. Much emphasis is placed on cooperative learning and collaboration among the teachers and students. The students work in groups to complete projects facilitated and primarily designed by the teacher. The teacher consistently provides feedback throughout the duration of the project to aid the students in understanding concepts rather than memorizing facts. The students not only learn a specific content area, they learn 21^{st} century skills that prepare them for college and work.

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APPENDIX A: SAMPLE KNOW/NEED-TO-KNOW FROM STUDENT EMPOWERMENT ACADEMY AT JEFFERSON HIGH SCHOOL IN LOS ANGELES, CA

What do you *"Know"* and what do you *"Need to know?"*

Please add your group members below:

1.	**3.**	**5.**
2.	**4.**	**6.**

Know List everything you know about this project	Need to Know Need-to-Knows are a list of questions that you need to ask me in order to obtain the information you need for this project.		
	Skills What do you need to learn to do Physically? (This section will mostly begin with "HOW")	Content What content knowledge do you need to ***learn***? (This section will mostly begin with "What" or "Why")	Logistics What items/knowledge do you need to ***complete the project***? What do you need to know about the time-frame for completing the project? (This section will mostly begin with "How" or "When" or "What")

APPENDIX B: SAMPLE GROUP CONTRACT FROM STUDENT EMPOWERMENT ACADEMY AT JEFFERSON HIGH SCHOOL IN LOS ANGELES, CA

Group Contract

Name of the Project

PERIOD / BLOCK

DATES (Beginning / Due Date)

SECTION 1: Contact Info

Name	Email Address	Phone#	Best Time to Call

SECTION 2: Strengths and Weaknesses

Name	Strengths	Weaknesses	How will they compensate?

SECTION 3:

Group Goals

1.

2.

3.

Roles and Responsibilities

1. Who will check final documents against Criteria Chart and/or **RUBRIC**?
2. Who will check to see that all images, slides, and paragraphs have **CITATION**?
3. Who will **PROOFREAD** all work (grammar & spelling) and/or **CHECK** calculations?
4. Who will be in charge of making sure your group is ready for the **ORAL PRESENTATION**?
5. Who will be in charge of **SUBMITTING** any or all group documents in the discussion board or through email?
6. Who will be in charge of checking off completed assignments listed on the **PROJECT PACING CHART**?

Name of group member	Primary ROLE in the group	Responsibilities for the assigned ROLE	Responsibilities for the PROJECT

Group rules (Please be <u>specific</u> and write complete sentences)

Absence Policy – What is your group's absence policy? Example: Will points be deducted for missing days of school? If so, how many points per day and where are the points taken from? Final Collaboration Evaluation? Mid-Project Evaluations? By when should work be made up?
Work Policy – What is your group's work policy? Example: What happens when a group member does not finish their share of the work and/or the work is late? How many points will be deducted? Where are the points taken from? Does someone get automatic warnings? Fired?
Project Points – How can a member receive all project points? How can a member lose points or not receive the project points? Project points can be points from the final grade for the final product at the end of the project or points from whole group assignments.
Workshops – How many mandatory workshops will you have as a group? When and what time will you meet? What happens when a member does not go to a mandatory workshop?

Individual Rules

What is one or two rules that each person will set in order to make the group more productive and efficient?

Group member name	Individual Rule(s)

SECTION 4: Group Member Dismissal

1. First written warning

What leads to a first warning? (list with bullet points)	What **consequences/plan/strategy** is going to be used to help the group member **avoid** a **second** warning?
•	•

2. Second written warning

What leads to a second warning? (list with bullet points)	What **consequences/plan/strategy** is going to be used to help the group member **avoid** a **third** warning?
•	•

3. Third warning – Meeting with teacher

Who was warned and what was the outcome of the meeting? (Write down any recommendations or warnings for that individual as result of that meeting)	

4. **Dismissal from group** – Upon dismissal group member is entitled to group products leading up to dismissal date, but all future assignments completed as an individual. Individuals dismissed from the group may not form or join another group.

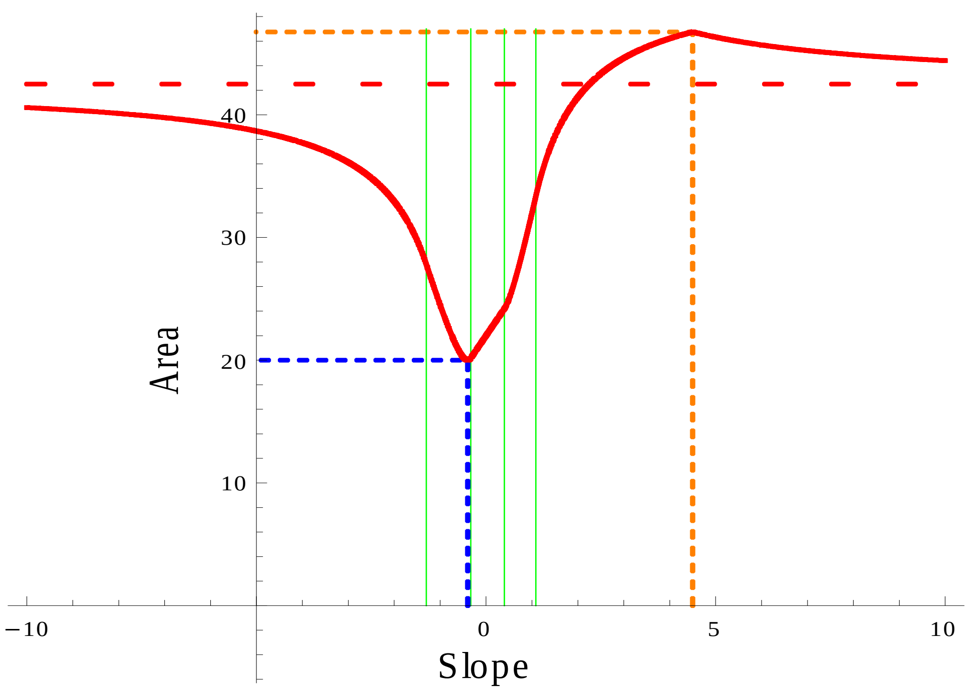
SECTION 5: Group signatures

Names	**Signatures**	**Date of Signing**
1.		**Date:**
2.		**Date:**
3.		**Date:**

APPENDIX C: ANALYTIC SOLUTION

Analytic solution. This is made very complex by the fact that we don't have a single algebraic formula that gives the area of the land in terms of parameters of the line. (The table and the diagram were prepared jointly by Dr. James Madden and myself. Dr. Madden did the Mathematica programming to produce the table and the graph.)

Slope m belongs to:	Left/lower endpoint	Right/upper endpoint	Area of smaller piece
$(-\infty, -13/10]$	$(2/m - 5, 0)$	$(13/(2m) + 5, 17/2)$	$42.5 + (19.125/m)$
$(-13/10, -1/3]$	**$(2/m - 5, 0)$**	**$(0, -5m + 2)$**	**$-(5m - 2)^2/(2m)$**
$(-1/3, 2/5]$	$(0, -5m + 2)$	$(11, 6m + 2)$	$11(4 + m)/2$
$(2/5, 13/12]$	$(2/m - 5, 0)$	$(11, 6m + 2)$	$2(1 + 3m)^2/m$
$(13/12, 9/2]$	$(2/m - 5, 0)$	$(13/(2m) + 5, 17/2)$	$(51m - 19.125)/m$
$(13/12, \infty)$	$(2/m - 5, 0)$	$(13/(2m) + 5, 17/2)$	$42.5 + (19.125/m)$



Minimize Area = $-(5m - 2)^2/(2m) = x^2/(5+x) = (5/2)\, y^2/(y-2)$, where m is in $(-13/10, -1/3]$

Vita

Danica Le'Trice Robinson was born in New Orleans, LA to Donald and Kristi Robinson. She is the second of four siblings. She graduated from Louisiana State University with a Bachelor of Science in mathematics, a concentration in secondary education and a minor in Spanish in May 2006. She taught Pre-Algebra for two years at Sherwood Middle Magnet School in East Baton Rouge Parish and is currently teaching Geometry at Algiers Technology Academy in New Orleans.