1969

The Transport of Oxygen and Carbon-Dioxide in Blood Flowing in a Permeable Tube.

Claude Glendon Bradley
Louisiana State University and Agricultural & Mechanical College

Follow this and additional works at: https://digitalcommons.lsu.edu/gradschool_disstheses

Recommended Citation
https://digitalcommons.lsu.edu/gradschool_disstheses/1531
BRADLEY, Claude Glendon, 1942-
THE TRANSPORT OF OXYGEN AND CARBON DIOXIDE
IN BLOOD FLOWING IN A PERMEABLE TUBE.

Louisiana State University and Agricultural and
Mechanical College, Ph.D., 1969
Engineering, biomedical

University Microfilms, Inc., Ann Arbor, Michigan
THE TRANSPORT OF OXYGEN AND CARBON DIOXIDE
IN BLOOD FLOWING IN A PERMEABLE TUBE

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Chemical Engineering

by

Claude Glendon Bradley
B.S., Mississippi State University, 1964
M.S., Louisiana State University, 1967
January, 1969
ACKNOWLEDGMENTS

This research project was performed under the direction of Dr. Ralph W. Pike. His counsel and encouragement is gratefully acknowledged. The author considered it a singular honor to have been Dr. Pike's first Ph.D. candidate.

Appreciation is expressed to the National Aeronautics and Space Administration for their fellowship support during the course of this research. The funds necessary to provide special analytical equipment necessary for the research were jointly funded by the Louisiana Heart Association and the Baton Rouge Tuberculosis and Respiratory Disease Association.

The publication and typing cost of the dissertation was provided by the Charles E. Coates Memorial Fund of the L.S.U. Foundation, donated by George H. Coates.

Grateful appreciation is expressed to Dr. C. L. Seger of the L.S.U. Veterinary Science Department who provided cattle blood and to the Baton Rouge General Hospital who drew blood from student volunteers, for their unselfish assistance to the experimental portion of the research.

Special thanks is given to Leonard Neumann who aided in the experimental portion of the research and to Mrs. Ruth Albright who typed the manuscript.
# Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgment</td>
<td>ii</td>
</tr>
<tr>
<td>List of Tables</td>
<td>vi</td>
</tr>
<tr>
<td>List of Figures</td>
<td>viii</td>
</tr>
<tr>
<td>Abstract</td>
<td>xiii</td>
</tr>
<tr>
<td>Chapter I: Introduction</td>
<td>1</td>
</tr>
<tr>
<td>A. General Discussion</td>
<td>1</td>
</tr>
<tr>
<td>B. The Lungs</td>
<td>4</td>
</tr>
<tr>
<td>C. The Blood</td>
<td>10</td>
</tr>
<tr>
<td>Plasma</td>
<td>10</td>
</tr>
<tr>
<td>The Formed Elements</td>
<td>11</td>
</tr>
<tr>
<td>Oxygen Blood Chemistry</td>
<td>15</td>
</tr>
<tr>
<td>Carbon Dioxide Blood Chemistry</td>
<td>16</td>
</tr>
<tr>
<td>D. Transport of Blood Gases</td>
<td>19</td>
</tr>
<tr>
<td>E. Anticoagulants</td>
<td>25</td>
</tr>
<tr>
<td>Chapter II: Previous Related Research</td>
<td>30</td>
</tr>
<tr>
<td>A. Oxygenator</td>
<td>30</td>
</tr>
<tr>
<td>B. Membranes</td>
<td>36</td>
</tr>
<tr>
<td>C. Literature Review of Mathematical Models of Permeable Tube Oxygenators</td>
<td>40</td>
</tr>
<tr>
<td>Chapter III: Theoretical Analysis of Mass Transfer in a Tubular Membrane Oxygenator</td>
<td>45</td>
</tr>
<tr>
<td>Oxygen Absorption Model</td>
<td>45</td>
</tr>
<tr>
<td>Numerical Solution to Equation (III-17)</td>
<td>55</td>
</tr>
<tr>
<td>Carbon Dioxide Desorption Model</td>
<td>62</td>
</tr>
<tr>
<td>CHAPTER</td>
<td>PAGE</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>Numerical Solution To Equation (III-46)</td>
<td>66</td>
</tr>
<tr>
<td>Summary of Theory</td>
<td>69</td>
</tr>
<tr>
<td>IV EXPERIMENTAL APPARATUS, FLUIDS AND PROCEDURE</td>
<td>70</td>
</tr>
<tr>
<td>Apparatus</td>
<td>70</td>
</tr>
<tr>
<td>Fluids</td>
<td>80</td>
</tr>
<tr>
<td>Procedure</td>
<td>81</td>
</tr>
<tr>
<td>V DISCUSSION OF THEORETICAL AND EXPERIMENTAL RESULTS</td>
<td>82</td>
</tr>
<tr>
<td>A. Physical Properties</td>
<td>83</td>
</tr>
<tr>
<td>B. Respiratory Quotient</td>
<td>92</td>
</tr>
<tr>
<td>C. Discussion of Theoretical and Experimental Results for Distilled Water</td>
<td>94</td>
</tr>
<tr>
<td>D. Discussion of Theoretical and Experimental Results for Human and Cattle Blood</td>
<td>104</td>
</tr>
<tr>
<td>Parameter Study</td>
<td>104</td>
</tr>
<tr>
<td>The Effect of pH on Oxygen and Carbon Dioxide Transport</td>
<td>104</td>
</tr>
<tr>
<td>The Effect of Hemoglobin Concentration on Oxygen and Carbon Dioxide Transport</td>
<td>110</td>
</tr>
<tr>
<td>The Effect of Tube Wall Thickness on Mass Transfer</td>
<td>114</td>
</tr>
<tr>
<td>The Effect of Hematocrit on Mass Transfer</td>
<td>117</td>
</tr>
<tr>
<td>Inlet Oxygen Partial Pressure Effect on Mass Transfer</td>
<td>117</td>
</tr>
<tr>
<td>Radial and Longitudinal Partial Pressure Profiles in Blood</td>
<td>122</td>
</tr>
<tr>
<td>Axial Saturation Profile</td>
<td>129</td>
</tr>
<tr>
<td>Comparison of Theoretical and Experimental Results</td>
<td>129</td>
</tr>
<tr>
<td>Summary</td>
<td>137</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE NUMBER</th>
<th>CHAPTER I</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-1</td>
<td>Composition of Dry Inspired, Expired and Alveolar Air in Man at Rest, at Sea Level, in Mols Per Cent or Volumes Per Cent</td>
<td>8</td>
</tr>
<tr>
<td>I-2</td>
<td>Partial Pressures of Respiratory Gases at Various Sites in Respiratory Circuit of Man at Rest at Sea Level</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE NUMBER</th>
<th>CHAPTER II</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>II-1</td>
<td>Gas Permeabilities in Membrane Polymers at 25°C</td>
<td>37</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE NUMBER</th>
<th>CHAPTER V</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-1</td>
<td>Oxygen Diffusivities and Solubilities in Water at 38°C</td>
<td>83</td>
</tr>
<tr>
<td>V-2</td>
<td>Carbon Dioxide Diffusivities and Solubilities in Water at 38°C</td>
<td>84</td>
</tr>
<tr>
<td>V-3</td>
<td>Diffusivities and Solubilities of O₂ in Silastic at 25°C</td>
<td>85</td>
</tr>
<tr>
<td>V-4</td>
<td>Diffusivities and Solubilities of CO₂ in Silastic at 25°C</td>
<td>86</td>
</tr>
<tr>
<td>V-5</td>
<td>Constants for Determining the Diffusivities and Solubilities of CO₂ and O₂ in Silastic at Any Temperature</td>
<td>87</td>
</tr>
<tr>
<td>V-6</td>
<td>Solubilities and Diffusivities of CO₂ and O₂ in Silastic at 38°C</td>
<td>88</td>
</tr>
<tr>
<td>TABLE NUMBER</td>
<td>PAGE</td>
<td></td>
</tr>
<tr>
<td>--------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>A-1</td>
<td>154</td>
<td></td>
</tr>
<tr>
<td>A-2</td>
<td>156</td>
<td></td>
</tr>
<tr>
<td>A-3</td>
<td>158</td>
<td></td>
</tr>
<tr>
<td>A-4</td>
<td>159</td>
<td></td>
</tr>
<tr>
<td>A-5</td>
<td>160</td>
<td></td>
</tr>
<tr>
<td>A-6</td>
<td>161</td>
<td></td>
</tr>
<tr>
<td>A-7</td>
<td>162</td>
<td></td>
</tr>
</tbody>
</table>

A-1: Experimental Data for Oxygenation of Distilled Water Flowing in a Permeable Silicone Tube

A-2: Experimental Data for Carbon Dioxide Removal From Water Flowing in a Permeable Silicone Tube

A-3: Experimental Data for Oxygen Absorption by Cattle Blood with Simultaneous Carbon Dioxide Desorption

A-4: Experimental Data for Oxygen Absorption by Human Blood with Simultaneous Carbon Dioxide Desorption, Donor M-1

A-5: Experimental Data for Oxygen Absorption by Human Blood with Simultaneous Carbon Dioxide Desorption, Donor K-1

A-6: Experimental Data for Carbon Dioxide Desorption from Blood with Simultaneous Oxygen Absorption, Donor M-1

A-7: Experimental Data for Carbon Dioxide Desorption from Blood with Simultaneous Oxygen Absorption, Donor K-1
LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE NUMBER</th>
<th>FIGURE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-1</td>
<td>Heme Group</td>
<td>14</td>
</tr>
<tr>
<td>I-2</td>
<td>Blood Chemistry</td>
<td>18</td>
</tr>
<tr>
<td>I-3</td>
<td>Oxygen Dissociation Curves (Bohr Effect)</td>
<td>20</td>
</tr>
<tr>
<td>I-4</td>
<td>Carbon Dioxide Dissociation Curve (Haldane Effect)</td>
<td>23</td>
</tr>
<tr>
<td>III-1</td>
<td>Permeable Tubular Membrane Showing Blood Control</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>Volume Over Which Specie Material Balances Were Made</td>
<td></td>
</tr>
<tr>
<td>III-2</td>
<td>Numerical Grid and Nomenclature</td>
<td>57</td>
</tr>
<tr>
<td>IV-1</td>
<td>Complete Experimental Apparatus</td>
<td>71</td>
</tr>
<tr>
<td>IV-2</td>
<td>Blood Oxygenator Experimental Apparatus</td>
<td>72</td>
</tr>
<tr>
<td>IV-3</td>
<td>Jacketed Reservoir Containing Blood</td>
<td>73</td>
</tr>
<tr>
<td>IV-4</td>
<td>Internal View of Blood Pump</td>
<td>74</td>
</tr>
<tr>
<td>IV-5</td>
<td>Experimental Tube and Water Jacket Assembly</td>
<td>75</td>
</tr>
<tr>
<td>IV-6</td>
<td>Analytical Apparatus for $P_{O_2}$, $P_{CO_2}$ and pH Measurement</td>
<td>76</td>
</tr>
<tr>
<td>IV-7</td>
<td>Experimental Tubular Membrane</td>
<td>78</td>
</tr>
<tr>
<td>V-1</td>
<td>Carbon Dioxide Desorption from Distilled Water Flowing in a Permeable Tube</td>
<td>96</td>
</tr>
<tr>
<td>V-2</td>
<td>Oxygen Absorption by Distilled Water Flowing in a Permeable Silicone Tube</td>
<td>97</td>
</tr>
<tr>
<td>FIGURE NUMBER</td>
<td>Description</td>
<td>PAGE</td>
</tr>
<tr>
<td>---------------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>V-3</td>
<td>Comparison of Experimental Data with Theoretical Prediction of Equation (III-17) for Oxygen Absorption into Water Flowing in a Permeable Silicone Tube</td>
<td>98</td>
</tr>
<tr>
<td>V-4</td>
<td>Comparison of Experimental Data with Theoretical Prediction of Equation (III-17) for Carbon Dioxide Desorption from Water Flowing in a Permeable Silicone Tube</td>
<td>100</td>
</tr>
<tr>
<td>V-5</td>
<td>Radial Profiles for Oxygen Absorption by Water</td>
<td>101</td>
</tr>
<tr>
<td>V-6</td>
<td>Radial Profiles for Carbon Dioxide Desorption from Water</td>
<td>102</td>
</tr>
<tr>
<td>V-7</td>
<td>The Effect of pH on Oxygen Absorption into Blood Flowing in a Permeable Tube</td>
<td>105</td>
</tr>
<tr>
<td>V-8</td>
<td>The Effect of pH on Carbon Dioxide Desorption from Blood Flowing in a Permeable Tube</td>
<td>107</td>
</tr>
<tr>
<td>V-9</td>
<td>The Effect of pH on Respiratory Quotient</td>
<td>109</td>
</tr>
<tr>
<td>V-10</td>
<td>The Effect of Hemoglobin Concentration on Oxygen Absorption into Blood Flowing in a Permeable Tube</td>
<td>111</td>
</tr>
<tr>
<td>V-11</td>
<td>The Effect of Hemoglobin Concentration on Carbon Dioxide Desorption from Blood Flowing in a Permeable Tube</td>
<td>112</td>
</tr>
<tr>
<td>V-12</td>
<td>The Effect of Hemoglobin Concentration on Respiratory Quotient</td>
<td>113</td>
</tr>
<tr>
<td>V-13</td>
<td>The Effect of Membrane Thickness on Oxygen Absorption into Blood Flowing in a Permeable Tube</td>
<td>115</td>
</tr>
<tr>
<td>V-14</td>
<td>The Effect of Membrane Thickness on Carbon Dioxide Desorption from Blood Flowing in a Permeable Silicone Tube</td>
<td>116</td>
</tr>
<tr>
<td>V-15</td>
<td>The Effect of Silicone Membrane Thickness on Respiratory Quotient</td>
<td>118</td>
</tr>
<tr>
<td>V-16</td>
<td>The Effect of Hematocrit on Oxygen Absorption into Blood Flowing in a Permeable Tube</td>
<td>119</td>
</tr>
<tr>
<td>FIGURE NUMBER</td>
<td>Title</td>
<td>PAGE</td>
</tr>
<tr>
<td>--------------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>V-17</td>
<td>The Effect of Hematocrit on Carbon Dioxide Desorption from Blood Flowing in a Permeable Tube</td>
<td>120</td>
</tr>
<tr>
<td>V-18</td>
<td>The Effect of Hematocrit on Respiratory Quotient</td>
<td>121</td>
</tr>
<tr>
<td>V-19</td>
<td>The Effect of Initial Oxygen Partial Pressure on the Oxygenation of Blood Flowing in a Permeable Tube</td>
<td>123</td>
</tr>
<tr>
<td>V-20</td>
<td>The Effect of Initial Oxygen Partial Pressure on Carbon Dioxide Removal from Blood Flowing in a Permeable Tube</td>
<td>124</td>
</tr>
<tr>
<td>V-21</td>
<td>The Effect of Initial Oxygen Partial Pressure on the Respiratory Quotient</td>
<td>125</td>
</tr>
<tr>
<td>V-22</td>
<td>Radial Profiles for Oxygen Absorption by Blood</td>
<td>126</td>
</tr>
<tr>
<td>V-23</td>
<td>Radial Profiles for Carbon Dioxide Desorption from Blood</td>
<td>127</td>
</tr>
<tr>
<td>V-24</td>
<td>Theoretical Longitudinal Partial Pressure Profiles in Blood Flowing in a Permeable Silicone Tube 73.5 cm in Length</td>
<td>128</td>
</tr>
<tr>
<td>V-25</td>
<td>Theoretical Saturation Profile for Blood Flowing in a Permeable Silicone Tube</td>
<td>130</td>
</tr>
<tr>
<td>V-26</td>
<td>A Comparison of Experimental Data and the Theoretical Prediction of Equation (III-17) for Oxygen Absorption by Human Blood - Donor K-1</td>
<td>132</td>
</tr>
<tr>
<td>V-27</td>
<td>The Comparison of the Theoretical Prediction of Equation (III-17) with Experimental Data for Human Blood - Donor M-1</td>
<td>133</td>
</tr>
<tr>
<td>V-28</td>
<td>Comparison of Experimental Data with Theoretical Prediction of Equation (III-17) for Oxygen Absorption into Cattle Blood Flowing in a Permeable Silicone Tube</td>
<td>134</td>
</tr>
<tr>
<td>V-29</td>
<td>Comparison of Experimental Data with the Theoretical Prediction of Equation (III-46) for Carbon Dioxide Desorption from Human Blood - Donor K-1</td>
<td>135</td>
</tr>
<tr>
<td>FIGURE NUMBER</td>
<td>PAGE</td>
<td></td>
</tr>
<tr>
<td>--------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>V-30</td>
<td>136</td>
<td></td>
</tr>
<tr>
<td>B- 1 Oxygen Absorption by Distilled Water Flowing in a Permeable Silicone Tube</td>
<td>164</td>
<td></td>
</tr>
<tr>
<td>B- 2 Oxygen Absorption by Distilled Water Flowing in a Permeable Silicone Tube</td>
<td>165</td>
<td></td>
</tr>
<tr>
<td>B- 3 Oxygen Absorption by Distilled Water Flowing in a Permeable Silicone Tube</td>
<td>166</td>
<td></td>
</tr>
<tr>
<td>B- 4 Carbon Dioxide Desorption from Distilled Water Flowing in a Permeable Silicone Tube</td>
<td>167</td>
<td></td>
</tr>
<tr>
<td>B- 5 Carbon Dioxide Desorption from Distilled Water Flowing in a Permeable Silicone Tube</td>
<td>168</td>
<td></td>
</tr>
<tr>
<td>B- 6 Carbon Dioxide Desorption from Distilled Water Flowing in a Permeable Silicone Tube</td>
<td>169</td>
<td></td>
</tr>
<tr>
<td>B- 7 Oxygen Absorption in the Presence of Simultaneous Carbon Dioxide Desorption in Cattle Blood Flowing in a Permeable Silicone Tube</td>
<td>170</td>
<td></td>
</tr>
<tr>
<td>B- 8 Oxygen Absorption in the Presence of Simultaneous Carbon Dioxide Desorption in Human Blood Flowing in a Permeable Silicone Tube</td>
<td>171</td>
<td></td>
</tr>
<tr>
<td>B- 9 Carbon Dioxide Desorption in the Presence of Simultaneous Oxygen Absorption in Human Blood Flowing in a Permeable Silicone Tube</td>
<td>172</td>
<td></td>
</tr>
<tr>
<td>B-10 Oxygen Absorption in the Presence of Simultaneous Carbon Dioxide Desorption in Human Blood Flowing in a Permeable Silicone Tube</td>
<td>173</td>
<td></td>
</tr>
<tr>
<td>B-11 Carbon Dioxide Desorption in the Presence of Oxygen Absorption in Human Blood Flowing in a Permeable Silicone Tube</td>
<td>174</td>
<td></td>
</tr>
<tr>
<td>FIGURE NUMBER</td>
<td>PAGE</td>
<td></td>
</tr>
<tr>
<td>---------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>C-1 Program Flow Chart</td>
<td>176</td>
<td></td>
</tr>
</tbody>
</table>
The simultaneous transport of oxygen and carbon dioxide in blood flowing in a permeable silicone tube was studied experimentally and theoretically. It was found that venous blood could be arterialized while flowing through a tubular silicone membrane surrounded by pure oxygen at 38°C. Carbon dioxide was found to be the limiting component for adequate transfer. A mathematical model was derived to describe the transport of each gas. The model used to describe the oxygen absorption by blood flowing in a tubular membrane was assumed to be independent of the simultaneous carbon dioxide desorption. The carbon dioxide desorption was assumed to be dependent upon the oxygen absorption. A comparison of the theoretical models with the experimental data indicated that the oxygen absorption model was optimistic in its prediction of the amount of oxygen transferred and the carbon dioxide model was found to be slightly conservative in its prediction of the amount of carbon dioxide transferred. The net result was a conservative prediction of the length of tubing needed to arterialize venous blood. It should be noted that both mathematical models were developed from theoretical concepts and no attempt was made to fit the experimental data with pseudo-diffusivities.

An experimental study was also made to determine if pulse rate, for a given average flow rate, affected the mass transfer of oxygen and carbon dioxide in blood flowing in a permeable tube. It was concluded that pulse rate had no appreciable effect on the transfer of either oxygen or carbon dioxide in the range studied.
A study of the variables which affect the transfer of oxygen and carbon dioxide in blood flowing in a permeable tube showed that the effect of the variables on oxygen and carbon dioxide were not the same. For a given tube length and flow rate, the tube exit dimensionless partial pressure of oxygen was significantly increased by increasing the pH, decreasing the hemoglobin concentration or raising the tube inlet oxygen partial pressure. Oxygen absorption was a weaker function of tube wall thickness and hematocrit. Carbon dioxide desorption was greatly decreased by increased wall thickness, but it was only a very weak function of pH, hematocrit, and inlet oxygen partial pressure. The carbon dioxide desorption was independent of tube inlet carbon dioxide partial pressure and the hemoglobin concentration.
Chapter I

INTRODUCTION

A. General Discussion

Each year in the United States approximately 10.6% of the death rate is attributed to diseases and disorders of the lung (90). This statistic has not gone unnoticed. In recent years, several attempts have been made to replace a diseased or defective lung with a healthy one. Unfortunately, the body rejection reaction is so poorly understood that no patient has yet survived this operation. Even the most recent transplant (65), which appeared promising, failed because only one of the injured lungs was transplanted and before the transplant could take; the remaining injured lung collapsed.

It is obvious, in the light of this information, that there is a pressing need for an artificial lung. Such an artificial lung could take the form of an implantable device or an extracorporeal device. As an implantable device it would actually take the place of the human lung. It would therefore need to be extremely sophisticated in design, structure, and chemical properties. It would be faced with the same rejection problems of a transplant in addition to many problems associated with its own makeup. Fewer problems would be associated with an extracorporeal device. It would serve a similar function as do present oxygenators used in cardiac surgery, but it would be capable of performing its function adequately for days instead of hours. Such a device could be a "temporary lung"
while transplants overcame initial rejection, or perhaps it could be used to remove a majority of the work load from an unhealthy lung while the body repairs its defective part. Finally, an artificial lung could be used to replace present blood oxygenators used in the extracorporeal circulation of heart-lung bypass operations. Presently, these type of operations result in an extremely high mortality rate which can be either directly or indirectly attributed to the oxygenator used (86).

It becomes obvious then, that in order to develop an artificial lung, first a prototype must be developed which can serve as an improvement to functional blood oxygenators. Secondly, the design of such a blood oxygenator must be made in the perspective that its design might conceivably be altered in some reasonable manner to ultimately become a prosthetic lung.

It is the primary purpose of this dissertation to determine if blood can be sufficiently oxygenated and sufficiently depleted of carbon dioxide while flowing in a permeable tube. If the answer to the problem is affirmative, then the exact nature of the effects of various parameters on this system must be determined.

Since the objective of this research is to design a more effective blood oxygenator and project its possibilities to an artificial lung, there are six background areas which should be thoroughly covered. First, the exact function and construction of the human lung is important since the proposed device must, in some manner, duplicate its function. Second, the nature and chemistry of the blood must be known in order to formulate an effective mathematical model
for the device. Third, the relationships for determining the concentrations of gases within the blood must be known. Fourth, methods must be available for circumventing the problem of blood clotting during experimentation. Fifth, the blood oxygenators presently used in cardiac-surgery should be reviewed and examined for their relative merits. Sixth, information on the compatibility of various membrane materials should be known. Finally, previous investigations on flow of blood through permeable tubes should be reviewed.
B. The Lungs

The lungs are one constituent of the pulmonary system. The pulmonary system functions to provide enough arterialized blood to satisfy the tissues of the body. The lungs accomplish these functions with the aid of the remainder of the cardiovascular and respiratory system. First, venous blood which contains a relatively low oxygen concentration and a relatively high carbon dioxide concentration enters the right atrium and passes to the right ventricle of the heart. The right ventricle pumps blood through the lungs where carbon dioxide is removed and oxygen is absorbed. The blood flows from the lung capillaries back to the left atrium. From the left atrium blood flows to the left ventricle and is pumped to the tissues of the body. In the tissues, carbon dioxide is exchanged for oxygen and the cycle repeats itself.

The lungs are located within the chest or thoracic cavity roughly symmetrical about the heart. Surrounding each lung is a pleural cavity. It should be noted that the pleural cavity is not a true cavity because no actual space exists between the two pleura walls. Instead, a thin film of fluid lies between the two membranes and serves to lubricate them during respiratory movements. This cavity consists of two layers of pleura which are continuous with each other around and below the root of the lung. The lung is held to the surface of the pleura membrane by surface tension (7). The root of the lung consists of the bronchus, pulmonary artery, pulmonary
veins, bronchial arteries and veins, pulmonary nerves, lymphatic vessels and bronchial lymph nodes. Each lung is divided into several distinct sections or lobes. The left lung consists of two lobes while the right lung contains three lobes. The right lung is shorter than the left lung by about 2 to 3 cm due to the diaphragm rising higher on the right side to accommodate the liver. The right lung, however, is broader than the left due to the inclination of the heart to the left side. Thus, the total weight and capacity of the right lung is somewhat greater than the left lung.

The diaphragm is located directly below the lungs and is dome-shaped. The diaphragm falls during inspiration and rises during expiration. The chest expands during inspiration and falls during expiration. These movements of the chest and diaphragm are totally responsible for the movement of air into and out of the lungs. The lungs themselves are a passive element conforming to the movement of the thoracic cavity. Air moves in and out of the lungs strictly due to the difference in atmospheric pressure and the pressure within the lung. As the diaphragm moves down and the chest out, the pressure within the lung falls below atmospheric pressure and air rushes into the lung via the trachea. When the diaphragm rises the pressure in the lung becomes greater than atmospheric pressure and air moves out. The amount of air which moves into or out of the lungs varies with need. It can be as little as 500 ml or as much as 2000 ml per cycle and is continually mixed with 1500 ml of reserve air. Reserve air is that which normally remains in the lungs at all times but can be
expelled upon demand. Another 1500 ml remain in reserve in the lungs but cannot be expelled upon demand. The normal breathing rate is about 10-14 breaths per minute.

Air enters the nasal openings where it is warmed and filtered of large impurities by fine hair within the nostrils and mucus secretions. From the nasal cavity air passes to the pharynx which is a common tract for air and food. Air enters the laryngeal opening which is closed by reflex when food enters the pharynx. Air then passes the vocal chords and enters the trachea. The trachea branches into the bronchial tubes for each lung. The trachea and upper bronchial tubes contain cells called cilia. These cells propel mucus and waste material to the mouth by means of a wave motion. The individual cilium moves with a whip-like motion upward and then slowly returns to its original position.

The large bronchial tubes subdivide in each lung in a manner similar to a tree branch. The ends of these branches are called the terminal bronchioles. All parts lying after the terminal bronchiole are intimately involved in the exchange of gases between the lung and the blood. These parts are the respiratory bronchioles, alveolar ducts, alveolar sacs and pulmonary alveoli.

The respiratory bronchioles have the same diameter as the terminal bronchioles and are actually an extension of the branch. Attached to the respiratory bronchioles are about five or six alveolar ducts. The terminal bronchiole is about 0.24 mm in diameter while the inner diameter of the alveolar duct is about 0.19 mm in diameter (76). The alveolar sacs are attached to the alveolar ducts and
number three to six per duct. The pulmonary alveoli are hemispherical protrusions from the wall of the alveolar sacs. They measure from 0.075 to 0.125 mm in diameter and their total number is estimated to be 750 million (7). The pulmonary alveoli are lined with but a single layer of epithelial cells. The walls of the alveoli contain elastic fiber and a massive network of capillaries.

The average distance through which a gas must pass to enter the red blood cell is between 1.0 and 3.0μ (44). This distance is not composed of a homogeneous resistance, but rather a composite of six resistances. These resistances are the alveolar wall, the interstitial fluid between the capillary and alveolar walls which is composed primarily of water, the capillary wall, the plasma, the red cell wall, and the red cell itself. The average residence time of blood in the capillaries is approximately 0.75 sec. The rate of diffusion across the alveolar wall then depends upon the following:

1. Difference in partial pressure between the alveolar gas and the dissolved gases in the plasma
2. Thickness of the tissues and interstitial fluid
3. Total alveolar capillary surface area available
4. Residence time of blood in the capillaries
5. Resistance of the plasma and red blood cell.

In Tables I-1 and I-2, the compositions and pressures O₂, CO₂, H₂O and N₂ are given at various places within the respiratory system (76). Any artificial lung must at least meet the minimum functional requirement of a human at rest.
### Table I-1

**Composition of Dry Inspired, Expired and Alveolar Air in Man at Rest, at Sea Level, In Mole Per Cent or Volume Per Cent (76)**

<table>
<thead>
<tr>
<th></th>
<th>(N_2) Mols %</th>
<th>(O_2) Mols %</th>
<th>(CO_2) Mols %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inspired Air</td>
<td>79.02</td>
<td>20.94</td>
<td>0.04</td>
</tr>
<tr>
<td>Expired Air</td>
<td>79.20</td>
<td>16.30</td>
<td>4.50</td>
</tr>
<tr>
<td>Alveolar Air</td>
<td>80.40</td>
<td>14.00</td>
<td>5.60</td>
</tr>
</tbody>
</table>

### Table I-2

**Partial Pressures of Respiratory Gases at Various Sites in Respiratory Circuit of Man at Rest at Sea Level (76)**

<table>
<thead>
<tr>
<th></th>
<th>(O_2) mm Hg</th>
<th>(CO_2) mm Hg</th>
<th>(N_2) mm Hg</th>
<th>(H_2O) mm Hg</th>
<th>Total mm Hg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inspired Air</td>
<td>158</td>
<td>0.3</td>
<td>596</td>
<td>5.7</td>
<td>760</td>
</tr>
<tr>
<td>Expired Air</td>
<td>116</td>
<td>32</td>
<td>565</td>
<td>47</td>
<td>760</td>
</tr>
<tr>
<td>Alveolar Air</td>
<td>100</td>
<td>40</td>
<td>573</td>
<td>47</td>
<td>760</td>
</tr>
<tr>
<td>Arterial Blood</td>
<td>100</td>
<td>40</td>
<td>573</td>
<td>47</td>
<td>760</td>
</tr>
<tr>
<td>Venous Blood</td>
<td>40</td>
<td>46</td>
<td>573</td>
<td>47</td>
<td>706</td>
</tr>
<tr>
<td>Tissues</td>
<td>30 or less</td>
<td>50 or more</td>
<td>573</td>
<td>47</td>
<td>700</td>
</tr>
</tbody>
</table>
In Table I-1 it is shown that the amount of $O_2$ entering the blood is not balanced by the amount of $CO_2$ removed. This is because not all the $O_2$ is used to oxidize carbon. Some is used to oxidize $H_2$ and is eliminated from the system as water.

In Table I-2 the average partial pressure difference for $CO_2$ across the alveolar-capillary wall is shown as only 3 mm Hg while for $O_2$ it is 30 mm Hg; yet, essentially equal volumes of $CO_2$ and $O_2$ cross the membranes. This is due to the high solubility of $CO_2$ in the body tissues and fluids. $CO_2$ can diffuse through the tissue 20 to 30 times as fast as $O_2$; therefore, there is no problem for the lung to remove an equal volume of $CO_2$.

The total surface area of the lungs available for gas transfer to the capillaries has been variously estimated between 50 and 125 sq meters (86, 7, 70). The total amount of $O_2$ intake is 250 cc (STP)/min for a blood flow rate of 5000 cc/min. The amount of $CO_2$ removed is about 240 cc (STP)/min. It should be emphasized that these conditions are for a resting subject only. During heavy exercise these values could vary more than a factor of ten (18).
C. The Blood

Due to the fact that the blood circulates through every organ, it participates in every major process which is carried out in the body. Its two primary functions are nutrition and excretion. The blood carries both food and oxygen to the cells and moves carbon dioxide and other waste products to the lungs and kidneys. During the metabolic process taking place within the cells, heat is constantly being produced. Since the cells can only function within a very narrow temperature range, the blood also plays a very important role in heat removal. The majority of the heat transported by the blood is given off by evaporation and radiation through the skin. At least 10% of the heat loss from the body is given up in the lungs by evaporation.

Blood can be separated into two visible portions. One portion is a liquid called the plasma; the other is a settled portion sometimes referred to as the formed elements. Plasma is a complex watery fluid which contains colloids, proteins, electrolytes, buffers, and other substances. The formed elements consist of the red and white blood corpuscles, and the platelets.

Plasma

The plasma is 10% solution of solutes in water. Of these solutes, 7% are plasma protein, 0.9% are inorganic salts and the remainder are nonprotein organic compounds. Coagulation is primarily a plasmatic process, although the platelets and red cells modify it.
When the blood encounters a break in a vessel wall, a foreign object, or a rough surface, fibrinogen, a plasmic protein, precipitates from the plasma into a jelly-like web called fibrin. Corpuscles and platelets become entangled in the fibrin web and disintegrate forming a blood clot. The clot presents a barrier to invading germs from a wound or smooths rough surfaces. Foreign objects which are placed in the blood stream are sometimes more than the blood can cope with. Normally, a foreign object will be dissolved in the metabolic process; however, if this does not occur in a reasonable period, the blood will clot so severely around the foreign object that it may block the flow of blood completely. Although there are many theories (26) as to how and why clotting takes place, its exact nature is not known.

The process of coagulation is probably the most disturbing of all the blood functions from an engineering viewpoint because it severely limits the materials which can be used to build any artificial device which must come into contact with the blood. Very few synthetic materials have been found which will not cause a reaction by the body. Certain stainless steels and chrome-cobalt alloys have been used successfully in bone repair, but the only materials which have shown promise in soft tissues and blood are certain medical grade silicone rubbers (12).

The Formed Elements

The Red Blood Cell: The human red blood cell is a biconcave disk having a mean diameter of 7.2 microns and a thickness of 2.2 microns
at its thickest portion and about 1 micron or less at the center. As a result of the shift in acid-base balance of the blood toward the acid side in venous blood, water diffuses into the red blood cell making the cell slightly larger in venous than arterial blood. The average surface area of a red cell is 125 square microns. Its average volume is 85 cubic microns. The average number of cells in a healthy individual is about 5,500,000 per cubic millimeter. The number of red blood cells will vary about 5% over a 24 hour period. They deform easily, like a bag filled with water (13).

The red blood cell is enclosed by a thin fragile membrane made of protein in association with lipids and steroid materials. The thickness of this membrane is probably no more than a few molecules (200 to 300Å). It is not sticky but rather hydrophobic. The body of the cell possesses a spongelike stroma made of the same or similar materials most probably in the form of a gel. Bound in the mesh of the stroma are hemoglobin molecules. Hemoglobin makes up about 34% by weight of the red blood cell. Water accounts for 60% and lipids, proteins and salts make up the remaining weight. Potassium is the principal base of the red cell in humans. The specific gravity of the red cell is 1.091.

The chief function of the red blood cell is the transportation of $O_2$ and $CO_2$. This is accomplished for the most part by the hemoglobin molecule. The hemoglobin of all mammals has a molecular weight of approximately 67,000 and is essentially tetramers consisting of four peptide chains, to each of which is bound a heme group. The heme group is a ferroprotoporphyrin. The structure of a ferro-
protoporphyrin is shown in Figure I-1. The iron in the molecule is divalent but the group has no net charge.

The heme group lies in a valley surrounded by the coiling peptide chain. The hydrophobic vinyl groups of the porphyrin are surrounded by hydrophobic amino acids side chains of the polypeptide. The two propionate side chains of each heme lie in juxta position to the positively charged nitrogens of lysine and arginine. The iron atom of the heme is closely coordinated to an imidazole side chain of a histidine residue. The iron in the heme group is loosely bound, if at all, to the histidine residue. A water molecule may lie between the iron atom and the histidine, and oxygen binding occurs at this site.

In the complete structure of hemoglobin the four chains mesh closely together with little free space between them. The forces bonding the chains together are secondary forces such as hydrogen bonding, salt links, or hydrophobic bonds. The hydrophobic interior of the molecule results in low dielectric properties. The exterior residues, however, give way to hydrophilic properties which account for its high solubility.

**White Cells:** The major function of the white cells is protection of the body from infection. They provide this protection by accumulating in large numbers around a place of infection and ingesting foreign materials. This ingestion is aided by the sticky composition of the outer surface of the cell which attracts the foreign material. The elastic nature of the cell membrane helps with the ingestion by allowing the cell to expand. The white cell appears in a variety of
Figure I-1. Heme Group
shapes, but its average diameter is 8 to 15 microns (46). There is an average of one white cell for every thousand red blood cells in a healthy adult.

The Platelets: The platelets are the smallest of the formed elements with a diameter of from 2 to 4 microns. They may occur in the form of rods, disks or cones and do not change shape. They number between 250,000 and 500,000 per cubic millimeter (10). The best known function of the platelets is their role in blood-clotting. They form the first line of defense against blood loss by collecting in the opening of a wound and closing it by fastening the clot to the vascular wall. Besides this function, they have a pronounced tendency to agglomerate upon roughened surfaces or foreign materials. To this end they may play a role in the body's defense against infection.

Oxygen Blood Chemistry

The important fact to remember about hemoglobin is not that it binds oxygen, but the manner in which it is bound. Instead of oxidizing the ferrous ion to ferric ion the oxygen forms a stable complex in which the iron remains in the ferrous state. In so doing the oxygen maintains its identity as a molecule. This special behavior is thought to be due to the cover of the iron by hydrophobic groups. Thus each heme group can bind one oxygen molecule. The bond between iron and oxygen is weak and is extremely sensitive to a change in pH. This sensitivity is due to the nature of the complex bond which is made by loose electron association. Both $O_2$ and Hb have unpaired electrons and are therefore paramagnetic. Oxyhemoglobin, however,
is diamagnetic—it has no magnetic moment. This indicates that the Fe and \( O_2 \) do combine. The effect of the electrophilic \( O_2 \) extends through the iron to the dissociable hydrogen of the imidazole group.

\[
\text{HHb}^+ + O_2 \rightleftharpoons \text{HbO}_2 + H^+
\]

With the withdrawal of electrons the hydrogen is released as an ion. Thus, the loose bonding of oxygen makes the imidazole group more acidic, lowering its pH from 7.9 to 6.7. The reaction is highly reversible. In arterial blood, the \( CO_2 \) and carbonic acid content are lower than in venous blood. The slight shift in pH which occurs due to \( CO_2 \) absorption from the tissue is enough to reverse this reaction and release the \( O_2 \) to the tissue. Likewise the removal of \( CO_2 \) by the lungs shifts the reaction to the right.

**Carbon Dioxide Blood Chemistry**

Carbon dioxide is produced by the tissues and moves into the blood by diffusion. Once in the blood however, the effect of carbon dioxide as an acid is diminished by a unique buffering system. Carbon dioxide, then, is found in many states. The majority of the \( CO_2 \) is in the form of the bicarbonate ion, both in the plasma and red cell. The majority of \( CO_2 \) entering the plasma, however, is in the form of the dissolved molecule with little hydration to \( H_2CO_3 \) because the reaction is slow. The \( CO_2 \) which diffuses into the red cell on the other hand is catalyzed by a specific enzyme, carbonic anhydrase, and practically all of the \( CO_2 \) is hydrated to \( H_2CO_3 \) which is carried as the \( HCO_3^- \) ion. When \( H_2CO_3 \) dissociates, it releases a free
hydrogen ion which attacks the negative nitrogen on the imidazole group which in turn releases the $O_2$ from the iron. By this buffering action the pH of the blood stays relatively constant.

Bicarbonate ions within the red cells are in equilibrium with bicarbonate ions in the plasma. As a result of the increase in $HCO_3^-$ ions in the red cell due to $CO_2$ absorption from tissue, diffusion of bicarbonate to the plasma occurs. When the diffusion takes place the red cell is left with an unbalanced electrical charge on the positive side. To counteract this charge, chloride ions diffuse into the cell. The entire sequence of reactions is shown in the Figure I-2.

The plasma has some buffering characteristics but this effect on the transport of $O_2$ and $CO_2$ is negligible. The carbamino reaction of the dissolved $CO_2$ in the red cell does include a significant, though small, portion of the total $CO_2$. 
Figure I-2. Blood Chemistry (22)
D. Transport of Blood Gases

Oxygen and carbon dioxide are transported by the blood in both physical solution and chemical combination. Figure I-2 shows schematically how each of these processes takes place but does not indicate the quantity carried in each mode.

The quantity of oxygen or carbon dioxide carried in physical solution may be described by Henry's Law:

\[
C = \alpha P
\]  

(I-1)

where \( P \) is partial pressure in mm Hg, \( \alpha \) is Henry's Law constant in cc gas/cc blood mm Hg, and \( C \) is concentration in cc gas (STP)/cc blood.

The mechanism of chemical combination of oxygen with hemoglobin is not simple. In Figure I-3 the equilibrium relationship between the partial pressure of oxygen and saturation is shown. This relationship is known as the Bohr effect. It is known that each gram of hemoglobin can physically bind 1.34 cc of oxygen at standard conditions. The total amount of chemically bound oxygen then is

\[
C = 1.34 \times C_{Hb} \times S
\]  

(I-2)

where \( C \) is concentration in cc gas (STP)/cc blood, \( C_{Hb} \) is concentration of hemoglobin per cc blood, and \( S \) is fraction saturation.

In 1925 Adair (1) proposed a four parameter model for fitting the Bohr curve.
Figure I-3. Oxygen Dissociation Curves (Bohr Effect)
where \( K_1 \) --- \( K_4 \) are the equilibrium constants for the following equations

\[
\begin{align*}
\text{Hb}_4^0 + 2O_2 & = \text{Hb}_4 + 2O_2 \\
\text{Hb}_4^0 + 4O_2 & = \text{Hb}_4^2 + 2O_2 \\
\text{Hb}_4^2 + 6O_2 & = \text{Hb}_4^4 + 2O_2 \\
\text{Hb}_4^4 + 8O_2 & = \text{Hb}_4^6 + 2O_2
\end{align*}
\]

These equations, in effect, assume a four step oxygenation of the hemoglobin tetramer. Adair's equation has limited use because it does not incorporate a factor for determining the change of the \( K \) value for a variety of pH's.

Pauling (72) in 1935 introduced an equation similar to Adair's but with a modification for pH alterations.

\[
S = \frac{K'P + 2K_1K_2K_3P^2 + 3K_1K_2K_3P^3 + 4K_1K_2K_3K_4P^4}{4(1 + K_1P + K_1K_2P^2 + K_1K_2K_3P^3 + K_1K_2K_3K_4P^4)}
\]

where \( K' \) is a constant and \( \alpha \) is the oxyheme interaction constant.

Pauling qualified the constant \( K' \):

\[
K' = \frac{K(1 + BA/[H^+])^2}{(1 + A/[H^+])^2}
\]
where \( K = 0.0035 \)
\[
B = 4, \text{ the acid interaction constant}
\]
\[
-\log A = 7.94, \text{ the acid ionization constant}
\]
\[
[H^+] = \text{hydrogen ion concentration}
\]

Pauling's equation reduced the number of system parameters to two, but it did little to provide for the effect of carbon dioxide on saturation.

Margaria (61) in 1963 offered an alternative to previous models by combining several of Pauling's constants and produced a one parameter model which could predict the saturation for a given oxygen partial pressure and a given pH.

The constant \( m \) was found to be independent of pH and equal to 125.0. By knowing one point on the curve the value of \( K \) may be computed. With this value, the entire curve may be generated. This is an important feature for mathematical modeling, and this is the equation chosen to represent the Bohr effect in the mathematical model which is developed in Chapter III.

The Haldane effect is the term given to the effect analogous to the Bohr effect for carbon-dioxide. In Figure I-4 the relationship is shown between the total amount of carbon dioxide in the blood at
Figure I-4. Carbon Dioxide Dissociation Curve (Haldane Effect)
various partial pressures of carbon dioxide. In this respect it is
different from the Bohr curve for it includes the physically dissolved
oxygen. The parameter here is saturation. Only a portion of the en-
tire effect is shown, specifically, only over the range of carbon
dioxide partial pressures which are normally found in the blood. If
the curves were extrapolated to zero they would meet. Fortunately,
however, in the area of interest these curves are parallel and lend
themselves easily to mathematical reduction. If this set of curves
is collapsed into a two independent variable model the result is

\[ C = 0.373 - 0.0748S + 0.00456 P_{CO_2} \]  

(I-11)

where \( C \) is the total concentration of \( CO_2 \) in the blood in cc(STP)/cc
blood, \( S \) is the fraction saturation, and \( P_{CO_2} \) is the partial pressure
of \( CO_2 \) in the blood in mm Hg.

That Equation (I-11) is not a function of hemoglobin concentration
may, at first, seem to be erroneous. The exact amount of carbon di-
oxide carried in the blood by hemoglobin carbamate has been in dis-
pute for some time (23). It is known however that less than 7% of
the total carbon dioxide is carried in this manner and there does not
seem to be a correlation between the amount of hemoglobin present and
the amount of carbon dioxide bound by the hemoglobin (22).
E. Anticoagulation

In any experiment involving blood as a working fluid the problem of clotting must be surmounted. In an artificial lung this could be accomplished in one of two ways. Either an anticoagulant may be added to the blood which will prevent it from clotting, or the lung can be made from materials which do not cause clotting.

There are several chemicals which may be added to the blood to prevent clotting. The most promising of these are ACD (Acid, Citrate, Dextrose), ACDI (Acid, Citrate, Dextrose, Inosine), and heparin. ACD is attractive from the point of view that it can preserve blood up to 33 days. It has the disadvantage of being toxic to the muscle tissue of the heart (58). ACDI extends the useful life of preserved blood and may offer additional protection to the red blood cell during the storage period (37). Heparinized blood has the advantage of being nontoxic to the body but it can only be stored for a maximum of 24 hours.

It would be advantageous if citrated blood could be used because of its long shelf life. Unfortunately, the addition of ACD or ACDI markedly changes the pH of the blood due to the acidic nature of the anticoagulant itself. This shift of pH also shifts the dissociation curves for oxygen and carbon dioxide. These shifts produce conditions which are not normally found in the body, and thus measurements on saturation in these regions cannot be directly related to normal physiological conditions. Heparin on the other hand does not alter the blood properties in any measurable manner and it does not contain
acids. It is therefore the preferred anticoagulant for experiments of this type.

A truly antithrombogenic surface has not yet been produced but many researchers are searching for the answer. In this research silicone tubes were used as the membrane material. Silicones have a certain amount of inertness when placed in blood (29), but the experiments of Sharp, et. al. (82) place grave doubts to any long term antithrombogenic effects of the material. Sharp, et. al. (82) replaced dog arteries with 4 mm diameter tubing and found that flow was completely stopped by clotting in 42 minutes or less for all 8 animal experiments. Fry, et. al. (36), on the other hand used glass tubes of 2 mm diameter and found no clotting after 20 hours. After 20 hours, clotting became a problem. Glass, then, could be used for intermediate profusion as auxiliary tubing but because of its impermeable nature would not do as a membrane.

Lambert (59) observed that the coagulation time was inversely proportional to the wettability of the material surface. Lyman, et. al. (59), showed that the best property to correlate with coagulation time was the critical surface tension of the polymer. The equations Lyman used for surface tension was:

\[ \gamma_s = \gamma_l (1 + \cos \theta)^2 \]  

where \( \gamma_s \) is the surface tension of polymer in dynes/cm\(^2\), \( \gamma_l \) is the surface tension of liquid in dynes/cm\(^2\) and \( \theta \) is the contact angle.

Using this relationship, Lyman et. al. (59) were able to obtain
a semi-log straight line correlation with coagulation times which fit their data. The experiments showed silicones and tetrafluoroethylenes to be the least thrombogenic of the common synthetic polymers.

Some of the more promising work directed toward development of a compatible surface was performed by Whiffen and Gott (93, 94). They prepared and tested a laminated surface prepared by impregnating a plastic surface with 0.1 to 0.5μ graphite particles. This was done by suspending the graphite particles in a weak solvent for the plastic and then evaporating the solvent. Absorbed on the graphite layer was a surfactant. The most frequently mentioned is benzalkonium chloride. As a final layer, heparin is absorbed on the benzalkonium chloride. The resulting surface is commonly referred to as GBH. The compatibility can probably be contributed to the similarity between this surface and the high concentration of heparin or heparin-like mucopolysaccharides in the endothelium of blood vessels (51). Unfortunately, the graphite sublayer is relatively impermeable to blood gases. This, alone, would make it undesirable for semi-permeable membranes. In addition, Gott, et. al. (42) reported that a satisfactory GBH coating could not be imparted to polyethylene, silicone, or tetrofluoroethylene surfaces. Nevertheless, GBH coatings offer real possibilities for auxiliary or associated parts such as valves and headers for artificial lungs.

Lipps (57), Merrill, Lipps, et. al. (68) extended the work of Gott by producing a CIH (Cellulose-Imine-Heparin) membrane. This membrane was designed for artificial kidneys, but offers further insights into the mechanisms of impregnated surface phenomena. The
authors claim this surface to be more compatible with blood than GBH surfaces in addition to being fairly permeable. Proof of these claims have not yet been corroborated.

Leninger, et. al. (56), impressed by the success of the GBH heparinized surfaces felt that a heparinized surface would lead to nonthrombogenicity. Since GBH surfaces could not be satisfactorily imparted to many plastics, they looked for a chemical method for bonding heparin to flexible and inert surfaces. Since heparin contains three sulfate groups per unit, it will react with positively charged quaternary ammonium salts to form complexes. In some cases, as the GBH system, these complexes are very insoluble.

As an example of their techniques, polystyrene is chloromethylated and aminated as follows:

\[
\text{Polystyrene} \xrightarrow{\text{Cl-CH}_2\text{-O-CH}_3} \text{H} \quad \text{H} \quad \text{C} - \text{N} - \text{Cl}^+ \quad \text{CH}_3 \quad \text{CH}_3
\]

This reaction scheme may be applied to polyolefins, tetrafluroethylene and to silicone rubber by grafting styrene to the surface, chloromethylating, quaternizing, and finally contacting the surface with aqueous sodium heparinate. The surfaces thus created have significantly better compatibilities and, curiously enough, a less hydrophobic surface indicating that the empirical correlation of
Lambert (59) may not be of value as an absolute indication of non-thrombogenicity.

In conclusion, it may be said that the optimal solution for a truly nonthrombogenic surface has yet to be found. Many researchers are working in the area, and the solution would appear to be one of time. In deference to their research, no attempt was made in this research to develop a compatible surface. Instead, silicone tubes were used which, though only poorly compatible with blood, are extremely permeable to blood gases. In addition, it appears that a solution to the problem of imparting a nonthrombogenic surface to silicones is not too far in the future. Until that time, anticoagulants can be used in the blood to prevent clotting. Heparin was used exclusively in these experiments.
Chapter II

PREVIOUS RELATED RESEARCH

A. Oxygenators

Although there are a considerable number of ingenious designs for artificial oxygenators, each may be placed in one of three basic categories: bubble, film, or membrane.

The bubble oxygenator consists primarily of three components: a bubbler, a defoamer, and a settling chamber. Blood and oxygen bubbles flow co-currently up a vertical column where the blood flows over a weir into a settling chamber through a mesh of polyurethane or stainless steel sponges. The sponges are coated with a defoaming agent and are used to filter as well as debubble the blood. Oxygenation takes place as the bubbles and blood rise up the bubbling chamber. Excess oxygen is removed from the top of the device and arterialized blood is returned to the body from the bottom of the settling chamber. The flow of oxygen should be at least three times the flow of the blood. Notable researchers in this area were Gott, et. al. (43), Cooley, et. al. (20), DeWall, et. al. (25), Goetz (40), and Williams, et. al. (96).

There are several disadvantages associated with the bubble oxygenator. If the bubbles are too small, a severe foaming problem results which is difficult to handle even when defoaming agents are applied. Larger bubbles, though easier to dispose of, still produce hemolysis due to their shearing action on the red cells. The necessity of the defoaming agent is in itself a disadvantage (19). Lee
et al. (55) and Dobell, et. al. (27) also note that considerable protein denaturation takes place when blood is exposed to a direct gas interface. This is apparently confined to the plasma. As a result of denaturation and hemolysis, operation times must be short.

For all of its disadvantages, the bubble oxygenator is probably the most frequently used in open-heart surgery because it is efficient and simply designed. In addition, this type oxygenator can be made of inexpensive disposable materials and discarded after each operation.

The film oxygenator also employs the direct exposure of blood to gas. In this case, however, bubbles are not introduced into the blood. Instead, blood flows in very thin films through an oxygen rich atmosphere. An elementary example of such an oxygenator is a wetted wall column. To insure that a large area of blood is exposed to oxygen several types of surfaces have been used. Bjork suggested the use of a rotating disc contactor whereby many thin discs dip into a pool of blood and continually expose fresh surface area to an oxygen rich gas. Other investigators have suggested various arrangements of this basic device as well as an array of stationary screens and sponges (66, 19). Recent investigators of this type oxygenator include Hirose et. al. (49), Esmond, et. al. (31), and Pemberton (73).

The advantages of the film oxygenator are that foaming is normally not a problem and turbulence responsible for red cell damage can normally be kept to a minimum. Unfortunately, the disc oxygenator does not eliminate the direct blood-gas interface and denaturation is still a significant problem. In addition, the disc
oxygenator is complicated to assemble and clean. It is normally not a disposable piece of apparatus.

The majority of reviewers of oxygenators (19, 39, 66) agree that the ideal oxygenator is a membrane oxygenator. It is not surprising that they do because the membrane oxygenator virtually eliminates the principal disadvantages of the bubble and film oxygenators, i.e. denaturation, foaming, and hemolysis. In addition, it is the only oxygenator which can operate equally well in any plane. Film and bubble oxygenators are normally confined to a vertical or horizontal position. This versatility is a necessity if an oxygenator is to ever be developed into a prosthetic lung. Unfortunately, the membrane lung suffers from several disadvantages. First, the membranes used in conventional models are expensive. Secondly, the plate and frame design of present membrane oxygenators is bulky and difficult to assemble.

Clowes, et. al. (17), described the first practical membrane oxygenator. It consisted of two plastic membranes supported by plates. Blood flowed between the membranes and oxygen was introduced into grooves cut in the plates. Pierce (74) later improved this oxygenator by a better support design. The major problem which early investigators attempted to overcome was that of blood distribution. Streaming and stagnation seemed to be a problem. To eliminate these problems researchers have tried many ingenious, and often complicated, ideas. Bramson, et. al. (14), attempted to provide a better distribution of blood along with a more compact design. The oxygenator consisted of a circular sandwich arrangement consisting of approxi-
mately 14 cells. Each cell contains two circular discs, clamped together at the edges. A silicone membrane supported by an open-weave fiber glass is placed on either side of a screen which defines the blood channel. Blood enters from the center of each cell and leaves through four equi-spaced exit passages on the periphery of each disc. The radius of this oxygenator is only 12 inches and, therefore, the blood flow path is short. This factor helps keep the pressure drop to about 50 mm Hg for a total perfusion of 5 L/min. Integral heat exchange was incorporated into the device by thin polyvinyl disc bags above and below each cell. A thermostated fluid passed through the bags. The authors projected that the device could be used for long perfusions though none was attempted at the time of the writing.

Crystal et al. (21) attempted to improve on the design of the original Clowes unit by obtaining better blood distribution by rocking the membranes back and forth. This obviously would increase the distribution; however, it is questionable as to whether the increase is worth the sacrifice of simplicity of design.

Kolobow (60, 53) designed a pulsating membrane oxygenator. The membrane is formed from two sheets of silicone rubber supported by a nylon mesh. A plastic spacer is used to separate the sheets. Sealed and inserted along the edges of the spacer plate are alternate tubes connected to oxygen supply and vacuum. Oxygen flow and suction are applied in such a manner as to produce a pulsating effect on the membranes. This agitates the blood flow and produces better oxygenation. The pulsation is gentle enough that cells are not damaged. The
membrane sandwich is wrapped around a plastic core to provide a compact unit.

From a theoretical viewpoint, a bank of small tubular membranes should be a more efficient method of oxygenation—eliminating the particularly disturbing problem of nonuniformity in blood distribution in present plate and frame models. It would also eliminate the preoccupation of the literature with searching for ideal membrane supports and imaginatively idealized effective blood film thicknesses (9), because the film thickness is fixed by the diameter of the tube. A tubular membrane offers the additional advantage, due to its geometry, of considerably greater strength than a flat sheet of equal thickness.

Although membrane oxygenators have been experimented with for more than ten years, few attempts have been made to design a workable multi-tubular membrane oxygenator. Kolff and Balzer (50) attempted to oxygenate blood in two long polyethylene tubes about 15 mm in diameter and 0.025 mm in thickness. They were able to oxygenate 75 ml of blood per minute but the priming volume was excessive. Bodell, et al. (11), made a similar attempt with silicone tubes, but they submerged the tubes in blood and passed oxygen through the tubes. In 1965, Wilson, et al. (97), described a multi-tubular membrane oxygenator consisting of fine silicone rubber tubes suspended in a plastic box. Blood flowed through the tubes and oxygen was passed around the outside of the tubes. In practice they found it impossible to maintain a uniform flow of blood through the fine tubing which resulted in progressive deterioration of the oxygenator's
performance. In 1967 Zingg (99) revived Bodell's idea of reversing the flow patterns in a multi-tubular membrane oxygenator. Zingg put 250 pieces of 0.037"O.D., 0.020"I.D. silicone rubber tubes into a 3/4"I.D. casing. He then passed blood over the tubes and oxygen through the tubes. Zingg's major conclusions were that the resistance to blood flow was excessive in a single pass arrangement, oxygen saturation preceeded carbon dioxide depletion to the point that bubble formation was observed before adequate carbon dioxide removal could be obtained, and hemolysis was low. In addition, he noted that the oxygenator did not have a reserve capacity for handling higher than normal flow rates.

It would seem from previous attempts at oxygenation of blood with permeable tubes that the effect on mass transfer of the various parameters is not known. The performance of a series of properly modelled experiments could shed light upon the true nature of these variables. It is the primary purpose of this dissertation to investigate those variables and determine if a feasible multi-tubular membrane oxygenator can be designed.
B. Membranes

The search for the ideal membrane for perfusion purposes has been the subject of many investigations. The proper membrane, of course, must have an anti-thrombogenic surface if it is to be used for long durations. If the lung is used only for short term bypasses with an anticoagulant added to the blood then almost any permeable substance can be considered as a membrane.

The fundamental theory of diffusion in membranes may be found in standard texts (88, 2). In general, it may be said that diffusion of gases through semi-permeable membranes follows Fick's law (39) at least in the ranges of thickness which have been investigated as oxygenator surfaces. The use of Fick's law was applied in the theoretical section where the influence of the wall thickness was considered. A further look into more advanced techniques for considering diffusion across membranes in liquids as well as gaseous systems was presented in a series of articles by Friedlander and Rickles (35, 79). These authors also reviewed the general methods of making membranes.

Probably the first membrane to be investigated was cellophane. Kolff and Beck (52) suggested its use for artificial lungs in 1944. The idea never gained wide acceptance, however, because the permeability of cellophane is much less than other materials as shown in Table II-1.

Clowes (17) used an ethyl cellulose membrane in his first experiments with an artificial lung, but later went to Teflon for addi-
Table II-1

Gas Permeabilities in Membrane Polymers at 25°C

<table>
<thead>
<tr>
<th>Membrane Polymer</th>
<th>Permeability x 10^9 (cc cm/sec cm^2cm Hg)</th>
<th>CO₂</th>
<th>Ref</th>
<th>O₂</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicone Rubber</td>
<td></td>
<td>320</td>
<td>144</td>
<td>60</td>
<td>144</td>
</tr>
<tr>
<td></td>
<td></td>
<td>288</td>
<td>151</td>
<td>54</td>
<td>151</td>
</tr>
<tr>
<td>Flurosilicone</td>
<td></td>
<td>64</td>
<td>151</td>
<td>11</td>
<td>151</td>
</tr>
<tr>
<td>Natural Rubber</td>
<td></td>
<td>13.1</td>
<td>151</td>
<td>2.4</td>
<td>151</td>
</tr>
<tr>
<td>Teflon</td>
<td></td>
<td>4.3</td>
<td>144</td>
<td>1.65</td>
<td>144</td>
</tr>
<tr>
<td>Polyethylene (low density)</td>
<td></td>
<td>0.5</td>
<td>151</td>
<td>0.8</td>
<td>151</td>
</tr>
<tr>
<td>Polypropylene</td>
<td></td>
<td>0.38</td>
<td>152</td>
<td>0.112</td>
<td>152</td>
</tr>
<tr>
<td>Mylar</td>
<td></td>
<td>0.009</td>
<td>151</td>
<td>0.002</td>
<td>151</td>
</tr>
</tbody>
</table>
tional strength and permeability. Berman (6) tried a hydrophobic Millipore membrane. This membrane was essentially a sheet of Millipore filter paper treated to make its surface non-wettable. Berman reported permeabilities of this surface to be approximately 70 times greater than Teflon for carbon dioxide. Berman (6) later felt this membrane to be too fragile in its present form for further evaluation.

The more recent membrane lungs have confined themselves to silicone and polytetrafluoroethylene membranes (38). The first polymer is preferred from its anti-thrombogenic properties and its extremely high permeability to blood gases. The second is often used because it has better strength properties in thin sheets. Pierce (74) and Snider (84) felt the thickness of the membrane was a price to be paid for protection of the blood. They also felt that Teflon and silicone rubber caused an imbalance in the amounts of CO₂ and O₂ transferred. Respiratory quotient is the term applied to the ratio of carbon dioxide to oxygen transferred. They implied that the thinner the membrane, and with due regard for strength, the more effective the oxygenation. This point could be questioned. The relative permeabilities of the blood layer and membrane layer may be such that by proper balancing of these thicknesses an acceptable respiratory quotient could be maintained. This could half or even totally eliminate the acid build-up problem associated with insufficient depletion of carbon dioxide from the blood. A theoretical analysis of this problem is discussed in Chapter V.

The best membrane to use for studies of laminar flow of blood
in a tube is a dimethyl silicone polymer such as a medical grade Silastic produced by the Dow Corning Company. This membrane was chosen primarily for its high permeability to blood gases. In addition, the polymer can be repeatedly autoclaved with no change in physical properties (32, 4), and is commercially available in tubular form in various diameters and wall thicknesses (83).

Although Dow Corning would give the impression that Silastic has a low incidence of blood clotting (83), Sharp, (82) showed very conclusively that dimethyl silicones will never be used for long term perfusion unless anticoagulants are added to the blood.
C. Literature Review of Mathematical Treatments of Permeable Tube Oxygenators

To date, there have only been two published accounts for the steady state oxygenation of blood in permeable tubes. Both of these accounts involved the use of permeable silicone tubes. The first, by Buckles (15) in 1966 was primarily a study of the permeabilities of membranes to respiratory gases ($O_2$, $CO_2$, and $N_2$). As an addition to this work, Buckles presented the following model for the oxygenation of the blood flowing in a permeable silicone tube.

$$v_Z \left(1 + \frac{1.34}{\alpha} C_{Hb} \frac{\partial S}{\partial P}\right) \frac{\partial P}{\partial Z} = D \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial P}{\partial r}\right)$$  \hspace{1cm} (II-1)

Boundary Conditions:

- $P = P_0$ \hspace{1cm} any $r$ \hspace{1cm} at $Z = 0$  \hspace{1cm} (II-2)
- $P = P_m$ \hspace{1cm} any $Z$ \hspace{1cm} at $r = R$  \hspace{1cm} (II-3)
- $\frac{\partial P}{\partial r} = 0$ \hspace{1cm} any $Z$ \hspace{1cm} at $r = 0$  \hspace{1cm} (II-4)

Buckles tested his model with both flat and parabolic velocity profiles, but decided that Benis' (67) profile was more appropriate since it included any non-Newtonian effects attributable to the heterogeneous nature of blood. A diffusivity for whole human blood was calculated from Fricke's heterogeneous media theory (34). Buckles performed his experimentation under constant $P_{CO_2}$ conditions. $P_{CO_2}$ of the oxygen rich atmosphere surrounding his experimental tube was
kept at the same partial pressure as the blood \((P_{CO_2} = 38 \text{ mm Hg})\). To this end, Buckles experimented with a true oxygenator, and his results were not affected by simultaneous counter diffusion of carbon dioxide. Even so, he concluded that his oxygenator had 33% excess area for carbon dioxide removal -- although he did not offer any explanation for this statement.

As far as the accuracy of his model was concerned, Buckles concluded that it was "slightly conservative". The experimental justification of this statement was, however, scarce since he only reported 18 data points over a range of hematocrits from 9.1% to 38.5%.

Weissman and Mockros (91) in 1967 independently derived an equation similar to Equation (II-1). This equation was:

\[
2v_{\text{avg}} V \left(1 + f(C)\right) \frac{\partial C}{\partial Z} = D \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C}{\partial r} \right) \tag{II-5}
\]

Boundary Conditions:

\[
C(r,Z) = C_0 \quad \text{at} \quad Z = 0 \tag{II-6}
\]

\[
C(R,Z) = C_\infty \quad \text{at} \quad Z > 0 \tag{II-7}
\]

\[
\frac{\partial C}{\partial r} (0,Z) = 0 \quad \text{at} \quad Z > 0 \tag{II-8}
\]

where \(f(C)\) related the oxyhemoglobin to the physically dissolved oxygen and was called a nonlinear oxygen sink term. The term \(f(C)\) was modeled by an exponential fit to the dissociation curve.
Unlike Buckles development, their model neglected the resistance of the membrane. Weissman and Mockros found no change in the average concentration of the blood leaving their experimental tubes with the same flow rate for wall thickness to inner diameter ratios as high as 0.563. They felt, therefore, their assumptions of a negligible wall resistance to be valid. This assumption would, however, not be valid for a less permeable membrane at these thickness to diameter ratios.

Weissman and Mockros did not predict a priori an oxygen diffusivity for blood. Instead, for each experiment they calculated the appropriate diffusivity which would bring the solution to their equation into close agreement with the experimental finding. By this technique they found the average diffusivity for all experiments to be $0.885 \times 10^{-5} \text{ cm}^2/\text{sec}$. They did not consider the effect of hematocrit (Chapter V) on diffusivity. In addition, Weissman and Mockros used cattle blood and observed that the shape of the oxygen dissociation curve had no appreciable affect on the solution of their equation.

As in Buckles experiments, a constant $P_{CO_2}$ was maintained to prevent simultaneous diffusion of carbon dioxide, and in effect the device which they experimented with was also an oxygenator alone and did not simulate the lung's function. Weissman and Mockros did, however, offer a model for carbon dioxide desorption but they did not allow for the influence of oxygen saturation of the carbon dioxide dissociation curve (Haldane Effect). Their equation for carbon dioxide removal was


\[
2v^2 \frac{\partial C}{\partial z} = \frac{D}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C}{\partial r} \right) \quad \text{(II-9)}
\]

Boundary Conditions:

\[
C(r, z) = C_0 \quad \text{at } z = 0 \quad \text{(II-10)}
\]

\[
C(R, z) = C_\infty = 0 \quad \text{at } z > 0 \quad \text{(II-11)}
\]

\[
\frac{\partial C}{\partial r} = 0 \quad \text{at } z > 0 \quad \text{(II-12)}
\]

Their solution to Equation (II-9) indicated the quantity of carbon dioxide removed for adequate oxygenation to be triple the physiological requirement. They offered no experimental corroboration for this result.

Weissman and Mockros (92) have recently completed a study on the effects of secondary flow patterns, introduced into tubular membranes by coiling, on the mass transfer of oxygen and carbon dioxide. Their theoretical findings indicate significant improvement over straight tubes, but they have been unable to confirm the expected improvement experimentally. In addition, it appears that the theoretically improved mass transfer may be masked by the tube wall except when extremely thin wall tubing is used.

In summary, the previous work only partially describes the mass transfer processes which take place while blood is flowing in a permeable tube. From the above discussion it is obvious that several important areas have not been thoroughly investigated. As a result, vital questions remain unanswered on the design of tubular membrane
oxygenators. These questions are:

1. Does the simultaneous removal of carbon dioxide from blood alter the oxygen absorption in a tubular membrane oxygenator?

2. Does the simultaneous absorption of oxygen affect the carbon dioxide removal from a tubular membrane oxygenator?

3. What are the effects of alterations in hemoglobin concentration, hematocrit, pH, tube wall thickness, inlet concentration of the blood, tube length, tube diameter and flow rate on the simultaneous absorption of oxygen and desorption of carbon dioxide in a tubular membrane oxygenator?

4. What is the effect of a pulsating flow on the oxygen absorption and carbon dioxide desorption in a tubular membrane oxygenator?

The objectives of this dissertation will be to answer these questions by theoretical and/or experimental methods. In so doing, this research will extend the work of Buckles (15) and Weissman and Mockros (91) to a more thorough understanding of the problems which must be solved in order to design an acceptable tubular membrane oxygenator.
THEORETICAL ANALYSIS OF MASS TRANSFER IN A TUBULAR MEMBRANE OXYGENATOR

Mathematical models for describing the mass transfer of oxygen and carbon dioxide in a tubular membrane oxygenator can be derived from a differential material balance which includes convection, diffusion, and chemical reaction of the species in the blood. Two models are developed. The first model considers steady state oxygenation as independent of carbon dioxide removal. The second model considers carbon dioxide removal as dependent on the simultaneous oxygen uptake.

Figure III-1 shows a schematic representation of the physical situations. An oxygen rich atmosphere surrounds a permeable tube in which blood is flowing. Within the tube, a finite control volume is shown with the effective driving forces on mass transfer of oxygen indicated. The reaction and diffusion arrows would be reversed for carbon dioxide removal.

**Oxygen Absorption Model**

A model for the oxygen absorption in a tubular membrane can be derived from an oxygen mass balance on the control volume shown in Figure III-1. The general equation for such a balance is:

\[
\text{Rate of Mass Accumulation} = \text{Net Rate of Mass Addition by Convection} + \text{Net Rate of Mass Addition by Diffusion}
\]

\[
\text{Rate of Mass} + \text{Addition by Chemical Reaction}
\]

\[\text{(III-1)}\]
Figure III-1. Permeable Tubular Membrane Showing Blood Control Volume Over Which Specie Material Balances Were Made.
The values of these terms are given below

Net rate of mass addition by convection
\[ 2\pi r \Delta r v Z C_{02}^Z - 2\pi r \Delta r v Z C_{02}^{Z+\Delta Z} \]

Net rate of mass addition by axial diffusion \(2\pi r \Delta r N_{02,Z}^Z - 2\pi r \Delta r N_{02,Z}^{Z+\Delta Z}\)

Net rate of mass addition by radial diffusion \(2\pi r \Delta r N_{02,r}^r - 2\pi r \Delta r N_{02,r}^{r+\Delta r}\)

Rate of mass appearance by chemical reaction \(2\pi r \Delta r \Delta Z R_{O2}\)

Rate of mass accumulation \(2\pi r \Delta r \Delta Z \frac{\partial C_{O2}}{\partial t}\)

Where \(C_{O2}\) is the concentration of oxygen, \(N_{02,Z}\) is the oxygen mass flux in the Z direction, \(N_{02,r}\) is the oxygen mass flux in the r direction, \(v_Z\) is the velocity in the Z direction (a function of r), and \(R_{O2}\) is the rate of disappearance of oxygen due to reaction with hemoglobin.

Incorporation of these terms into Equation (III-1) with division by \(2\pi r \Delta r \Delta Z\) yields:

\[
\frac{\partial C_{O2}}{\partial t} = -\left( \frac{r N_{O2,r}^{r+\Delta r} - r N_{O2,r}^r}{r \Delta r} \right) - \left( \frac{N_{O2,Z}^{Z+\Delta Z} - N_{O2,Z}^{Z}}{\Delta Z} \right) - \left( \frac{C_{O2}^{Z+\Delta Z} - C_{O2}^Z}{\Delta Z} \right) + R_{O2}
\]  

(III-2)

Taking the limit of Equation (III-2) as \(\Delta r\) and \(\Delta Z\) approach zero gives a partial differential equation.
\[
\frac{\partial C_{O_2}}{\partial t} = - \frac{\partial (rN_{O_2}, r)}{\partial r} - \frac{\partial N_{O_2}, Z}{\partial z} - v_z \frac{\partial C_{O_2}}{\partial z} + R_{O_2} \quad (III-3)
\]

Fick's First Law defines the mass fluxes

\[
N_{O_2}, r = - D_{O_2}, b \frac{\partial C_{O_2}}{\partial r} \quad (III-4)
\]

\[
N_{O_2}, Z = - D_{O_2}, b \frac{\partial C_{O_2}}{\partial z} \quad (III-5)
\]

Substitution of Fick's Law into Equation (III-3) gives:

\[
\frac{\partial C_{O_2}}{\partial t} + v_z \frac{\partial C_{O_2}}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left( D_{O_2}, b r \frac{\partial C_{O_2}}{\partial r} \right) + \frac{\partial}{\partial z} \left( D_{O_2}, b \frac{\partial C_{O_2}}{\partial z} \right) + R_{O_2} \quad (III-6)
\]

The system is considered to be at steady state and therefore the time derivative in Equation (III-6) will equal zero. In addition, for this situation axial diffusion is insignificant when compared to the convective term. Incorporation of these two restrictions into Equation (III-6) yields:

\[
v_z \frac{\partial C_{O_2}}{\partial z} = \frac{D_{O_2}}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C_{O_2}}{\partial r} \right) + R_{O_2} \quad (III-7)
\]

where the diffusivity of oxygen in blood is assumed constant.

Whether this diffusivity should be considered a constant is subject
to controversy. Some arguments for and against this assumption will be given in the Physical Property Section of Chapter V where the validity of the constant diffusivity assumption is justified.

Reneau et. al. (78) derived equations similar to Equation (III-7) to describe the oxygen diffusion from blood capillaries into brain tissue. They compared the solution to their equation containing an axial diffusion term with the solution to their equation which did not, and concluded the axial diffusion term was insignificant. The assumption of negligible axial diffusion, then, seems justified.

Equation (III-7) when solved would describe the dissolved oxygen concentration profile within the flowing blood stream. A similar equation for the concentration of oxygen bound as oxyhemoglobin may also be derived using the same restrictions employed to develop Equation (III-7).

\[

v_Z \frac{\partial C_{HbO_2}}{\partial Z} = \frac{D_{HbO_2}}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C_{HbO_2}}{\partial r} \right) + R_{HbO_2} \quad (III-8)

\]

Addition of Equation (III-7) and (III-8) yields an expression which describes the total oxygen concentration in all forms in the blood stream:

\[

v_Z \left( \frac{\partial}{\partial Z} \left( C_{O_2} + C_{HbO_2} \right) \right) = D_{O_2} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C_{O_2}}{\partial r} \right) \right) + \ldots .

\]
Intuitively, one would believe the diffusivity of oxyhemoglobin to be small relative to the diffusivity of oxygen since the molecular weight of oxyhemoglobin is greater than 66,000. LaForce and Fatt (48) measured both diffusivities and found that the oxyhemoglobin diffusivity was from 1/30 to 1/50 of the oxygen diffusivity. In addition to its small diffusivity, oxyhemoglobin does not possess as steep a gradient as oxygen across the radius of the tube. It is for these reasons that the diffusion term for oxyhemoglobin in Equation (III-9) is neglected. Thus Equation (III-9) becomes:

\[
\nu Z \left( \frac{\partial}{\partial Z} \left( C_{O_2} + C_{HbO_2} \right) \right) = D_{\text{O}_2} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C_{O_2}}{\partial r} \right) \right) \quad (\text{III-10})
\]

The concentration \( C_{O_2} \) and \( C_{HbO_2} \) can both be expressed in terms of oxygen partial pressure. This is preferable because it relates the theoretical equation directly to the experimentally measured variable. The concentration of dissolved oxygen is related to the partial pressure by Henry's Law.
where $\alpha_{O_2}$ is the solubility of oxygen in blood. The concentration of oxyhemoglobin is related to the partial pressure of oxygen in the following manner. As shown in the Introduction, each gram of hemoglobin can bind 1.34 cc (STP) of oxygen at 100% saturation. The concentration of oxygen bound by oxyhemoglobin for any saturation is:

$$C_{HbO_2} = 1.34 C_{Hb} S \quad (\text{III-12})$$

where $C_{Hb}$ is the hemoglobin concentration in grams per cc of blood and $S$ is the fractional saturation. The fractional saturation is in turn related to the partial pressure of oxygen as shown in the oxygen dissociation curves in Figure I-3. As previously shown, the relationship between saturation and oxygen partial pressure has been modeled by many investigators. The best model, however, seems to be that of Margaria (61) since it contains but two parameters.

$$S = \frac{(1 + KP_{O_2})^3}{(1 + KP_{O_2})^4} + m - 1 \quad (\text{III-13})$$

Margaria found the parameter $m$ to be a constant equal to 125.0 for human blood. The parameter $K$ is the pH parameter. If the cor-
responding saturation is known for a single $P_{O_2}$, $K$ may be calculated from Equation (III-13). Once $K$ is known, an entire iso-$pH$ dissociation curve may be constructed from Equation (III-13). If the $pH$ is constant along the length of the tube, as was the case for the previous studies (201, 200), then Equation (III-13) describes the corresponding saturation for a given $P_{O_2}$ along the length of the tube. If the $pH$ of the blood changes as it moves through the tube, and this is the case for the simultaneous diffusion of carbon dioxide, then each point along the tube would require a new $K$ value for Equation (III-13). The approach taken in this research is that an average $K$ value determined from the inlet and outlet partial pressures and $pH$'s will be a sufficient approximation for the phenomena. This assumption is justified on the basis of Weissman and Mockros's (91) finding that the shape of the dissociation curve had practically no effect on their solution.

Since the oxygen dissociation curve is for equilibrium, the relationship of the oxyhemoglobin to the partial pressure of oxygen shown in Equation (III-11) and used in the theoretical development, implies that chemical equilibrium exists at all points within the blood stream. There is general agreement in the literature that this is a good assumption because the oxyhemoglobin reaction is extremely fast (85,15). Nakamura and Staub (80) further noted that the oxyhemoglobin reaction was ten times faster under the influence of a simultaneous diffusion of carbon dioxide. Thus, the assumption of local chemical equilibrium seems justified.
Equation (III-10) is now expanded by substitution of the values of $C_0$ and $C_{\text{HbO}_2}$ defined in Equations (III-11) and (III-12).

$$v_Z \frac{\partial}{\partial Z} \left( \alpha_0 P_{O_2} + 1.34 C_{\text{HbS}} \right) = \frac{\alpha_0 D_{O_2}}{r} \left( \frac{\partial}{\partial r} \left( r \frac{\partial P_{O_2}}{\partial r} \right) \right) \tag{III-14}$$

The right hand term in Equation (III-14) is expanded into two terms by performing the implied differentiation.

$$v_Z \frac{\partial}{\partial Z} \left( \alpha_0 P_{O_2} + 1.34 C_{\text{HbS}} \right) = \frac{\alpha_0 D_{O_2}}{r} \left( \frac{1}{r} \frac{\partial P_{O_2}}{\partial r} + \frac{\partial^2 P_{O_2}}{\partial r^2} \right) \tag{III-15}$$

Equation (III-15) is now non-dimensionalized to confine the variables to a range of unity. The dependent variable was not normalized in this development.

$$\frac{V_Z}{V_0} = V; \quad \frac{Z}{L} = A; \quad \frac{R}{r} = Y$$

Substitution of these non-dimensionalized variables into Equation (III-15) yields:

$$V \frac{\partial}{\partial A} \left( \alpha_0 P_{O_2} + 1.34 C_{\text{HbS}} \right) = \frac{\alpha_0 D_{O_2}}{R^2 V_0} \left( \frac{1}{Y} \frac{\partial P_{O_2}}{\partial Y} + \frac{\partial^2 P_{O_2}}{\partial Y^2} \right) \tag{III-16}$$
Applying the chain rule to the left side of Equation (III-16) produces the final form of the differential equation which must be solved in order to predict the two dimensional oxygen partial pressure profile for blood flowing a permeable tube.

\[
V\left(1 + \frac{1.34}{\alpha_{O_2}} C_{Hb} \frac{\partial S}{\partial P_{O_2}}\right) \frac{\partial P_{O_2}}{\partial A} = \frac{D_{O_2} L}{R^2 V_o} \left(\frac{1}{Y} \frac{\partial P_{O_2}}{\partial Y} + \frac{\partial^2 P_{O_2}}{\partial Y^2}\right)
\]

(III-17)

Equation (III-17) is a two dimensional, second order, parabolic, partial differential equation with non-linear coefficients.

The boundary conditions for Equation (III-17) are:

B.C. 1 \( P = P_{O_2,0} \) at \( A = 0 \) for all \( Y \)

B.C. 2 \( \frac{\partial P}{\partial Y} = 0 \) at \( Y = 0 \) for all \( A \)

B.C. 3 \( N_{O_2,b} = N_{O_2,m} \) at \( Y = 1 \) for all \( A \)

Boundary condition one is the initial partial pressure of the blood coming into the tube. The blood is assumed to be completely mixed, i.e. the concentration profile is flat. Boundary condition two is the symmetry relationship for a tube. The third boundary condition states that at the blood-membrane interface, the molal fluxes of oxygen are equal. These fluxes are not constant over the length \( L \). Indeed, it is this changing boundary condition, along with the peculiar nature of the variable coefficients, which prevents the equation from having an analytical solution!
The term $\frac{\partial S}{\partial P_{O_2}}$ is evaluated from Equation (III-13) and found to be:

$$\frac{\partial S}{\partial P_{O_2}} = \frac{[(Z)^4 + 124.0](3)(Z)^2 - [(Z)^3 + 124.0](4)[Z]^{3}}{[(Z)^4 + 124.0]^{2}} \times \ldots$$

$$\ldots \times \left( \frac{K^2_{P_{O_2}} - (1 + K_{P_{O_2}})K}{KP_{O_2}} \right)$$

Equation (III-18)

where $Z = \left( \frac{1 + K_{P_{O_2}}}{KP_{O_2}} \right)$

Equation (III-17) is essentially the same equation which Buckles (15) used to describe simple oxygenation of blood in a permeable tube, (Equation II-1). The only differences between Equation (III-17) and Equation (II-1) are in the derivation and application to simultaneous oxygen and carbon dioxide transport. In Chapter V, Equation (III-17) is used to model the oxygenation of blood flowing in a permeable tube with simultaneous counter diffusion of carbon dioxide. As previously explained, the influence of the simultaneous carbon dioxide removal on oxygen absorption is considered small if not negligible.

**Numerical Solution to Equation (III-17)**

In order to integrate Equation (III-17) a numerical technique is required. A modified Crank-Nicholson technique was used. This
The method is convergent and stable for all $\lambda > 0$ where $\lambda$ is the ratio of the $A$ increment to the square of the $Y$ increment (64). The following is a list of the finite difference approximations which were used for the solution of Equation (III-17).

\[
\frac{\partial P}{\partial A} = \frac{P_{j+1,i+1} - P_{j,i}}{\Delta A} \quad \text{(III-19)}
\]

\[
\frac{\partial^2 P}{\partial Y^2} = \frac{1}{2(\Delta Y)^2} \left[ (P_{j+1,i+1} - 2P_{j,i+1} + P_{j-1,i+1}) + \ldots \right]
\]

\[
\ldots + (P_{j+1,i} - 2P_{j,i} + P_{j-1,i}) \quad \text{(III-20)}
\]

\[
\frac{1}{Y} \frac{\partial P}{\partial Y} = \frac{1}{2j\Delta Y} \left[ (P_{j,i} - P_{j-1,i} - P_{j-1,i+1} + P_{j,i+1}) \right] \quad \text{(III-21)}
\]

Figure III-2 is helpful in understanding the physical significance of the subscripting. Substitution of Equations (III-19), (III-20), and (III-21) into Equation (III-17) yields:

\[
\frac{P_{j+1,i+1} - P_{j,i}}{\Delta A} = \theta \frac{1}{2(\Delta Y)^2} \left[ (P_{j+1,i+1} - 2P_{j,i+1} + P_{j-1,i+1}) + \ldots \right]
\]

\[
\ldots + (P_{j+1,i} - 2P_{j,i} + P_{j-1,i}) + \frac{1}{2(\Delta Y)^2} \quad \text{x} \ldots
\]

\[
\ldots \times (P_{j,i} - P_{j-1,i} - P_{j-1,i+1} + P_{j,i+1}) \quad \text{(III-22)}
\]
Figure III-2. Numerical Grid and Nomenclature.
where

\[
\theta = \frac{D_{O_2}L}{R_v v_o} \left( 1 + \frac{1.34 C_{Hb} \Delta S/\Delta P_{O_2}}{\alpha_{O_2}} \right) (III-23)
\]

and

\[
\frac{1}{Y} = \frac{1}{j\Delta Y} (III-24)
\]

Rearranging Equation (III-22) to solve for \(P_{j,i+1}\) gives:

\[
P_{j,i+1} \left( 1 + \frac{\Delta\Theta}{(\Delta Y)^2} - \frac{\Delta\Theta}{2(\Delta Y)^2} \right) = \frac{\Delta\Theta}{2(\Delta Y)^2} \frac{P_{j+1,i+1}}{P_{j-1,i} + P_{j+1,i} + P_{j-1,i} (1-1/j) - \ldots
\]

\[
\ldots + P_{j-1,i+1} (1-1/j) + P_{j+1,i} + P_{j-1,i} (1-1/j) - \ldots
\]

\[
\ldots - P_{j,i} (2-1/j) + P_{j,i}
\]

(III-25)

The following variables are defined:

\[
G = \left( 1 + \frac{\Delta\Theta}{(\Delta Y)^2} - \frac{\Delta\Theta}{2(\Delta Y)^2} \right) (III-26)
\]

\[
F = \frac{\Delta\Theta}{2(\Delta Y)^2} (III-27)
\]

Substitution of Equations (III-26) and (III-27) into Equation (III-25)
with suitable rearrangement gives the finite difference equation
which was solved with the computer program described in Appendix C.

\[ P_{j,i+1} = \frac{F}{G} P_{j+1,i+1} + P_{j-1,i+1} (1-1/j) + P_{j+1,i} + \cdots \]

\[ \cdots \cdots + P_{j-1,i} (1-1/j) - P_{j,i} (2-1/j) + \frac{P_{i+1,i}}{G} \quad \text{(III-28)} \]

\[ j = 1, 2 \ldots k - 1 \]
\[ i = 0, 1, 2 \ldots i \]

where the subscript 0 represents the entrance of the tube, \( k \) repre­
ts the membrane-fluid interface, and \( J \) represents the end of the

Equation (III-28) cannot predict values at \( P_{0,i+1} \) nor at
\( P_{k,i+1} \). It cannot predict the point \( P_{0,i+1} \) because the equation be­
comes indeterminate. The special case of \( P_{0,i+1} \) is now considered.

Recall the second boundary condition in Equation (III-17), i.e.

\[ \frac{\partial P}{\partial Y} = 0 \quad \text{at } Y = 0, \; j = 0 \quad \text{for all } \Lambda \]

In Equation (III-17) an indeterminate form is found in the right

hand member -- specifically

\[ \frac{1}{Y} \frac{\partial P}{\partial Y} \quad \text{(III-29)} \]

Application of L'Hospital's Rule eliminates the indeterminate form.
If Equation (III-30) is substituted into Equation (III-17) and the
same rearrangements performed as for Equation (III-25) the value of
\( P_{0,i+1} \) becomes:

\[
P_{0,i+1} = \frac{P_{0,i}}{(1+4F)} + \frac{4F}{(1+4F)} (P_{1,i+1} + P_{1,i} - P_{0,i})
\]

where the terms \( P_{j+1,i+1} \), \( P_{j+1,i} \), \( P_{j-1,i+1} \), \( P_{j-1,i} \)
respectively at the center of the tube. \( P_{k,i+1} \) cannot be determined
from Equation (III-28) because it requires knowledge of the \((k+1)\)th
point which would be located in the membrane and the slope of the
concentration profile may change drastically as it crosses the in­
terface. To determine \( P_{k,i+1} \) the third boundary condition is used.
It relates the concentration profiles at the interface as follows:

\[
N_{O_2,b} = N_{O_2,m} \text{ at } Y = 1.0, j = k \text{ for all } A
\]

By Fick's First Law

\[
\frac{D_{O_2,b} \alpha_b}{R} \left( \frac{\partial P_{O_2}}{\partial Y} \right)_b^{k,i} = \frac{D_{O_2,m} \alpha_m}{R} \left( \frac{\partial P_{O_2}}{\partial Y} \right)_m^{k,i}
\]

By Fick's Second Law at steady state,

\[
\frac{\partial}{\partial Y} \left( Y \left( \frac{\partial P_{O_2}}{\partial Y} \right)_m^{k,i} \right) = 0
\]
Integration of Equation (III-33) yields:

\[ P_{j,i} = K_1 \ln Y + K_2 \quad \text{(III-34)} \]

where the boundary conditions are:

**B.C. 1** \[ P_{j,i} = \frac{P_{g,o2}}{g} \text{ at } Y = \frac{R + TM}{R} \]

**B.C. 2** \[ P_{j,i} = P_{k,i} \text{ at } Y = 1.0 \]

Solving for the constants of Equation (III-34) gives

\[ K_2 = P_{g,o2} - K_1 \ln \frac{R + TM}{R} \]

and from Equations (III-32) and (III-34)

\[ K_1 = \frac{D_{O_2,b} \alpha_b}{D_{O_2,m} \alpha_m} Y \left( \frac{\partial P}{\partial Y} \right)_{Y=1.0} = \frac{D_{O_2,b} \alpha_b}{D_{O_2,m} \alpha_m} \left( \frac{\partial P}{\partial Y} \right)_{Y=1.0} \]

Substituting \( K_1 \) and \( K_2 \) into Equation (III-34) and solving for \( P_{k,i} \) gives

\[ P_{k,i} = \frac{D_{O_2,b} \alpha_b}{D_{O_2,m} \alpha_m} \left( \frac{\partial P}{\partial Y} \right)_{b @ Y=1.0} \ln \frac{R}{R+TM} + P_{g,o2} \quad \text{(III-35)} \]

which can be further expanded by letting

\[ \left( \frac{\partial P}{\partial Y} \right)_{b @ Y=1.0} = \frac{P_{k,i} - P_{k-1,i}}{\Delta Y} \quad \text{(III-36)} \]
Substituting Equation (III-36) into Equation (III-35) and solving for \( P_{k,i} \) gives

\[
P_{k,i} = \frac{P_{g,O_2} - P_{k-1,i}}{\frac{D_{O_2,b}}{D_{O_2,m}} \left( \ln \frac{R}{R+TM} \right) \frac{1}{\Delta Y}} \cdot R + TM \cdot A Y
\]

Equation (III-28), (III-31), and (III-37) are the basis for the computer program found in Appendix C.

Carbon Dioxide Desorption Model

A model for the desorption of carbon dioxide in the presence of simultaneous adsorption of oxygen is derived by the same techniques employed in the oxygen absorption model.

Employing the same restrictions as in the oxygen analysis:

1. Laminar flow
2. Fully developed velocity profile
3. Steady state, isothermal
4. Diffusion is described by Fick's Law
5. Mass flow rate into or out of the blood is small when compared with the total mass flow rate of the blood.
6. Constant physical properties
7. Chemical equilibrium at all points along the tube.
A set of equations analogous to the oxygen Equations (III-7) and (III-8) are derived for the various forms of carbon dioxide in the blood -- specifically:

\[
\frac{\partial C_{CO_2}}{\partial z} = \frac{D_{CO_2}}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C_{CO_2}}{\partial r} \right) + R_{CO_2} \quad \text{(III-38)}
\]

\[
\frac{\partial C_{HCO_3}}{\partial z} = \frac{D_{HCO_3}}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C_{HCO_3}}{\partial r} \right) + R_{HCO_3} \quad \text{(III-39)}
\]

\[
\frac{\partial C_{HbCO_2}}{\partial z} = \frac{D_{HbCO_2}}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C_{HbCO_2}}{\partial r} \right) + R_{HbCO_2} \quad \text{(III-40)}
\]

Addition of Equations (III-38), (III-39) and (III-40) yields:

\[
\frac{\partial}{\partial z} \left( C_{CO_2} + C_{HCO_3} + C_{HbCO_2} \right) = \frac{D_{CO_2}}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C_{CO_2}}{\partial r} \right) + \ldots
\]

\[
\ldots + \frac{D_{HCO_3}}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C_{HCO_3}}{\partial r} \right) + \frac{D_{HbCO_2}}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C_{HbCO_2}}{\partial r} \right)
\]

\[
(III-41)
\]

where \( R_{CO_2} = -(R_{HCO_3} + R_{HbCO_2}) \)

At this point, each term on the left of Equation (III-41) has to be analyzed for its significance. The diffusivity of carbamino hemoglobin is assumed to be much smaller than the diffusivity of carbon dioxide. As in the case of oxyhemoglobin in the oxygen
analysis, therefore, the diffusion term for carbamino hemoglobin is considered negligible compared to the transfer of dissolved carbon dioxide. The bicarbonate diffusion term is not as easily dismissed. Little is known about the mechanism of bicarbonate transport even though the majority of carbon dioxide in the blood is carried in the form of the bicarbonate ion. Due to the lack of information on the quantitative aspect of this term, it was necessary to neglect it. The affect of this arbitrary assumption is considered in further detail in Chapter V. Neglecting the bicarbonate and carbamino hemoglobin terms reduces Equation (III-41) to:

\[ v \frac{\partial}{\partial z} \left( C_{\text{CO}_2} + C_{\text{HCO}_3} + C_{\text{HbCO}_2} \right) = \frac{D}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C_{\text{CO}_2}}{\partial r} \right) \] (III-42)

The dissolved oxygen concentration is related to partial pressure by Henry's Law

\[ C_{\text{CO}_2} = \alpha_{\text{CO}_2} P_{\text{CO}_2} \]

and

\[ C_{\text{CO}_2} + C_{\text{HCO}_3} + C_{\text{HbCO}_2} = 0.373 - 0.0748S + 0.00456 P_{\text{CO}_2} \] (III-43)

relates the total carbon dioxide concentration in all forms to the oxygen saturation fraction and the carbon dioxide partial pressure. Equation (III-43) and Henry's Law were previously described in Chapter I.
Substituting Equations (III-43) and Henry's Law into Equation (III-42) with suitable rearrangement yields:

\[ v_Z \frac{\partial}{\partial Z} \left( \left( \frac{0.00456}{\alpha_{CO_2}} \right) P_{CO_2} - 0.0748 \right) = \frac{D_{CO_2}}{r} \frac{\partial}{\partial r} \left( r \frac{\partial P_{CO_2}}{\partial r} \right) \]  

(III-44)

which upon separation of terms becomes

\[ v_Z \left( \frac{0.00456}{\alpha_{CO_2}} \right) \frac{\partial P_{CO_2}}{\partial Z} - 0.0748 \frac{\partial S}{\partial Z} = \frac{D_{CO_2}}{r} \frac{\partial}{\partial r} \left( r \frac{\partial P_{CO_2}}{\partial r} \right) \]  

(III-45)

The same non-dimensionalization are made on Equation (III-45) as were made on Equation (III-15), thus the non-dimensionalized form of Equation (III-45) is:

\[ V \left( \frac{0.00456}{\alpha_{CO_2}} \right) \frac{\partial P_{CO_2}}{\partial A} - 0.0748 \frac{\partial S}{\partial A} = \frac{D_{CO_2}}{R^2 \nu_0} \left( \frac{1}{Y} \frac{\partial P_{CO_2}}{\partial Y} + \ldots \right) \]

\[ \ldots + \frac{\partial^2 P_{CO_2}}{\partial Y^2} \]  

(III-46)

The boundary conditions are:

B.C. 1 \hspace{1cm} P = P_0 \hspace{1cm} at \hspace{0.5cm} A = 0 \hspace{1cm} for \hspace{0.5cm} all \hspace{0.5cm} Y

B.C. 2 \hspace{1cm} \frac{\partial P}{\partial Y} = 0 \hspace{1cm} at \hspace{0.5cm} Y = 0 \hspace{1cm} for \hspace{0.5cm} all \hspace{0.5cm} A
Equation (III-46) differs from Weissman and Mockros' Equation (II-9) in that an allowance is made for the influence of a changing oxygen saturation on the carbon dioxide removal.

Numerical Solution to Equation (III-46)

Using the same finite difference technique as previously defined by Equations (III-19), (III-20), and (III-21) and by defining the new term,

\[ \theta_2 = \frac{\alpha_{CO_2} L D_{CO_2}}{0.00456 V_{y_o} R^2} \]  

(III-47)

an expansion into a finite difference formula is made for Equation (III-46).

\[
\frac{P_{j,i+1} - P_{j,i}}{\Delta A} = \theta_2 \left\{ \frac{1}{2(\Delta Y)^2} \left[ (P_{j+1,i+1} - 2P_{j,i+1} + P_{j-1,i+1}) + \ldots \right.ight.
\]

\[
\ldots + (P_{j+1,i} - 2P_{j,i} + P_{j-1,i}) + \frac{1}{2(\Delta Y)^2} \times \ldots
\]

\[
\ldots \times \left[ (P_{j,i} - P_{j-1,i} - P_{j-1,i+1} + P_{j,i+1}) \right] + \ldots
\]

\[
\ldots + \left( \frac{0.0748}{0.00456} \right) \frac{\alpha_{CO_2}}{V} \frac{\partial S_i}{\partial A}
\]  

(III-48)
Separation of variables and collection of like terms further reduces

Equation (III-48) to:

\[
P_{j,i+1} \left( 1 + \frac{\Delta A \theta_2}{(\Delta Y)^2} - \frac{\Delta A \theta_2}{2(\Delta Y)^2} \right) = \frac{\Delta A \theta_2}{2(\Delta Y)^2} \left( P_{j+1,i+1} + P_{j-1,i+1} \right) \times \ldots
\]

\[\times (1-1/j) + P_{j+1,i} + P_{j-1,i}(1-1/j) - P_{j,i}(2-1/j) + P_{j,i} - \ldots\]

\[
... - \Delta A \left( 16.40 \right) \frac{\partial S_{j,i}}{\partial A}
\]

Redefining the following quantities,

\[
G_2 = \left( 1 + \frac{\Delta A \theta_2}{(\Delta Y)^2} - \frac{\Delta A \theta_2}{2(\Delta Y)^2} \right)
\]

\[
F_2 = \frac{\Delta A \theta_2}{2(\Delta Y)^2}
\]

an equation for \( P_{j,i+1} \) may be formulated.

\[
P_{j,i+1} = \frac{F_2}{G_2} \left[ P_{j+1,i+1} + P_{j-1,i+1}(1-1/j) + P_{j+1,i} + P_{j-1,i} \times \ldots
\]

\[\left( 1-1/j \right) - P_{j,i}(2-1/j) \right] + P_{j,i}/G_2 - \frac{16.40 \alpha_{CO_2}}{G_2 V} \Delta S_{j,i}
\]

(III-52)
for \( j = 1, 2, \ldots, k-1 \)
\[ i = 0, 1, \ldots, \lambda \]

For the center of the tube the finite difference equation is derived analogous to Equation (III-31),

\[
P_{0,i+1} = \frac{P_{0,i}}{1 + 4F_2} + \frac{4F_2}{1 + 4F_2} (P_{1,i+1} + P_{1,i} - P_{0,i}) - \ldots
\]

\[
\frac{16.40\alpha_{CO_2}}{(1+4F)^2} \Delta S_{0,i}
\]

and finally, at the wall position \( P_{k,i+1} \) is derived analogous to Equation (III-37) and becomes

\[
P_{k,i+1} = \frac{D_{b,CO_2} \alpha_{b,CO_2}}{D_{m,CO_2} \alpha_{m,CO_2}} \left( \ln \frac{R}{R+TM} \right) \frac{1}{\Delta Y}
\]

From Equation (III-52), (III-53), and (III-54) both radial and longitudinal partial pressure profiles for carbon dioxide were generated using the computer program found in Appendix C.

The inside diameter of the tubes under investigation was 0.025". It was practically impossible, therefore, to experimentally measure the radial profile for either oxygen or carbon dioxide. In
view of the physical situation, an average partial pressure was calculated at each longitudinal point along the tube. The mixing cup concentration (8) was used to calculate the average partial pressure at the exit of the tube. It was calculated as follows:

\[ P_{avg,i} = 4 \int_{0}^{1} P(Y) V(Y) Y \, dY \]  

(III-55)

**Summary of Theory**

The partial differential equations were presented for modeling the simultaneous adsorption of oxygen and desorption of carbon dioxide in blood flowing through a permeable tube. The first equation (III-17) assumed that oxygen adsorption was independent of carbon dioxide desorption. The second equation (III-46) incorporates the influences of a changing hemoglobin saturation fraction on the carbon dioxide depletion. Both equations were integrated using a modified Crank-Nicholson technique. The solution to these equations will be discussed in Chapter V. Also in Chapter V the solution will be compared with experimental data taken with human blood, cattle blood and water.
Chapter IV

EXPERIMENTAL APPARATUS, FLUIDS AND PROCEDURE

Apparatus

The experimental apparatus used for this study is shown in Figure IV-1, and again in schematic form in Figure IV-2. It consists of six basic pieces of equipment: a jacketed fluid reservoir (Figure IV-3), a pump (Figure IV-4), a reactor containing a permeable tubular membrane (Figure IV-5), a temperature control system (Figure IV-1), equilibration gases, and an analytical section (Figure IV-6) for measuring $P_{CO_2}$, $P_{O_2}$, pH, hemoglobin concentration and hematocrit. Each one of these pieces will be described in detail.

Fluid Reservoir: The reservoir was constructed from a 300 ml, three-necked, round bottom flask. This flask was jacketed with a cylindrical glass vessel which was sealed to the round bottom flask by epoxy resin. One neck of the flask was used for the suction to the blood pump and for the return of recycled blood. The second neck was used for the return of blood to the reservoir after it has passed through the permeable tube. The amount of recycle to the reservoir via this route was usually less than one cc per minute. The third and central neck was fitted with two extension pieces of glass. Passing through the center of these extension pieces and extending to the bottom of the blood reservoir was a piece of 9 mm glass tubing. The gaseous mixture of carbon dioxide, oxygen, and nitrogen necessary to bring the blood to a venous composition passed through this tube into the
Figure IV-1. Complete Experimental Apparatus.
Figure IV-2. Blood Oxygenator Experimental Apparatus
Figure IV-3. Jacketed Reservoir Containing Blood.
Figure IV-4. Internal View of Blood Pump.
Figure IV-5. Experimental Tube and Water Jacket Assembly
Figure IV-6. Analytical Apparatus for $P_{O_2}$, $P_{CO_2}$ and pH Measurement.
blood. The extension glass pieces served as a nonchemical defoaming apparatus. Blood bubbles climbed the wall of the glass extensions and grew in size until they collapsed. Experiments on this apparatus have shown a constant $P_{O_2}$ and $P_{CO_2}$ after one and one-half hours of bubbling at a rate of one bubble per second. Hematocrit and hemoglobin values were not detectably altered by this procedure.

**Pump:** A multiple finger pump was used in the experimentation. It was a Sigmamotor Model TM4. This particular model contained a variable speed drive and, depending on the size of tubing used, could produce flow rates from essentially zero to over a liter per minute. The pump tubing used in the experimentation was 1/4" I.D., 1/16", wall thickness Tygon tubing. The discharge of the pump tubing was fitted with a 1/4" glass tee through which part of the blood could be recycled to the reservoir and part of it passed through the permeable tube in the reactor section. The purpose of the recycle was to determine the effect of pulse rate, if any, on mass transfer. This was accomplished by measuring the $P_{O_2}$ or $P_{CO_2}$ for equal volumetric flows at different pulse rates. The excess fluid at higher pulse rates was recycled to the reservoir. In this manner, flow rates from 0.1 cc/min. to 30.0 cc/min. through the permeable tube were obtained.

**Reactor:** The reactor was constructed from a laboratory condenser and a single 0.047" O.D., 0.025" I.D. Silastic tube (Figure IV-7). The permeable tube, in initial experiments, was supported by a porous mesh of polyurethane. Later, it was found that the tube could support itself if a small amount of tension was applied. A check of the exit concentration under both conditions yielded identical results.
Figure 1D-7. Experimental Tubular Membrane
indicating that an insignificant amount of transfer area was lost due to the contact with the support. The entrance to the tube was sealed into one of the exits of the glass tee used for the recycle line. The discharge end was sealed into a 0.055" I.D., 0.075" O.D. polyethylene tube which transported the blood to the analytical section. A General Electric type RTV-102 white silicone rubber cement was used to provide the seals at each end of the permeable tube. In addition, a heavy coat of the rubber cement was used to coat the polyethylene tube to prevent any loss of oxygen after exiting the permeable section.

A 1/4" Tygon tube was inserted into the entrance of the condenser along with the glass tee. This tube brought a pure oxygen stream into contact with the permeable tube. The glass tee and oxygen tube were sealed with modeling clay around the entrance to the condenser to insure that the oxygen would flow along the length of the permeable tube.

Temperature Control Systems: The temperature control system consisted of a water reservoir and a circulation pump. The temperature of the water was regulated with a Chemical Rubber Company Circu-Temp combined heating element and thermostat. The controlled temperature was 38°C ± 1.0°C. The circulation pump was an Eastern Model B-1 type 100.

Experimental Gases: Two gas stream were utilized during the course of the experimentation. One gas was pure oxygen and was passed over the outside of the permeable tube to provide a constant oxygen concentration. The second gas was a mixture of CO₂, O₂, and N₂. This
mixture was used to equilibrate the blood gases to approximately venous conditions, i.e. 46 mm Hg $P_{CO_2}$, and 40 mm Hg $P_{O_2}$. In practice these mixtures were not always exact and slightly higher or lower partial pressures resulted as indicated in the tables of Appendix A. The $CO_2$ mixture was jacketed in a water bath at 42°C to prevent sublimation of the $CO_2$.

**Analytical Equipment:** Partial pressures of oxygen and carbon dioxide as well as pH were measured by a Radiometer Gas Monitor Type PHA 9276 with thermostated micro-electrodes type E5036, E5046, and G297/G2. The $P_{CO_2}$ electrode was calibrated with known gaseous standards. The $P_{O_2}$ electrode was calibrated with an oxygen free solution and a water sample equilibrated with the atmosphere. The pH electrode was calibrated with standard buffer solutions provided by Radiometer specifically for that purpose.

An International Equipment Company Micro-Capillary Centrifuge Model MB, and a Micro-Capillary Reader Model 2201 were used to determine hematocrits. A Hellige Hemometer Type 303 was used to determine hemoglobin concentrations. The accuracy of this analysis is $\pm 0.0025$ g/cc. The accuracy of the reader is $\pm 0.5$ Hematocrit units.

**Fluids:** Three fluids were used in the experimentation: distilled water, cattle blood, and human blood. Cattle blood was provided by the Louisiana State University Animal Science Department. It was taken in 500 ml quantities in Abbo-Vac Pamheprin bottles to which 10,000 U.S.P. units of heparin sodium dissolved in 40 ml of saline were added. Experiments were run within 24 hours of collection. Human blood was obtained from university volunteers. It was taken
in 500 ml quantities by the Baton Rouge General Hospital Blood Bank in heparinized polypropylene bags. Additional heparin was added to the blood before experimentation to bring the total number of U.S.P. units to 10,000. Again, the experiments were run in a 24 hour period. Specific values for the physical properties of each fluid are given in Appendix A.

**Procedure**

Three hundred ml of either distilled water or blood were placed in the reservoir. The $\text{CO}_2$, $\text{O}_2$, $\text{N}_2$ mixture was bubbled into this fluid for approximately one hour. After one hour, samples were taken every fifteen minutes and checked for consecutively equal readings. Once an equilibrium was established, the pump was activated at a preset pulse rate. The amount of flow through the permeable tube was adjusted by means of a screw clamp on the recycle line to the reservoir.

Flow rates were measured with graduated cylinders whose accuracy was ± 0.1 cc. Each rate was measured three times and the average of these readings was recorded. Once a flow rate was established, the polyethylene tube was connected to one of the electrodes for measurement. $P_{\text{CO}_2}$, $P_{\text{O}_2}$, and pH were measured in this manner. A total of eighteen separate experiments were performed during the course of the research. The combined experiments yielded 230 data points.
Chapter V

DISCUSSION OF THEORETICAL AND EXPERIMENTAL RESULTS

In this chapter, the answer to the questions posed at the end of Chapter II are answered based on the solutions to the theoretical mass transfer equations and experimental results. The chapter is broken into four sections: a physical property section, a respiratory quotient section, a section comparing theoretical and experimental data on water and a section comparing theoretical and experimental data on human and cattle blood.

The physical property section discusses the various values reported in the literature for diffusivities and solubilities of water and blood. It also explains why the particular values chosen were used in this research. The respiratory quotient section explains the meaning of that term and its importance in a blood oxygenator. The water section shows the theoretical predictions of Equation (III-17) for oxygen absorption and carbon dioxide desorption from water flowing in a permeable tube as compared to experimental results. In addition, experiments were made on water to determine if a pulsating flow rate markedly altered the mass transfer. The final section, the blood section, compares the theoretical prediction of Equations (III-17) and (III-48) with experimental observations on human and cattle blood.
A. Physical Properties

In the previous section, general equations were derived to describe the transport of oxygen and carbon dioxide across a permeable wall into a laminar blood stream or an inert fluid. These equations were derived in terms of physical properties -- principally diffusivities and solubilities. These properties must be accurately known in order for the theoretical predictions to be of value. Unfortunately, the literature does not report consistent values for these properties. Accordingly, it was necessary to investigate these values in some detail and chose the most accurate values from the available data.

**Water:** The solubilities and diffusivities of oxygen and carbon dioxide are well reported in the literature. Table V-1 and Table V-2 give representative values of these data.

**Table V-1**

Oxygen Diffusivities and Solubilities in Water at 38°C

<table>
<thead>
<tr>
<th>Diffusivity $D \times 10^5$ (cm²/sec)</th>
<th>Solubility $\alpha$ (cc(STP)/cc-atm)</th>
<th>Reference</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.33</td>
<td>0.0233</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>3.58</td>
<td>0.0236</td>
<td>77</td>
<td>47</td>
</tr>
<tr>
<td>2.92</td>
<td>0.0234</td>
<td>Value Selected</td>
<td>Value Selected</td>
</tr>
<tr>
<td>2.92</td>
<td>Value Selected</td>
<td>Value Selected</td>
<td>Value Selected</td>
</tr>
</tbody>
</table>
Literature values for the diffusivity of oxygen in water at 38°C vary about 20%. The data obtained by Yoshida and Oshima (98) was chosen for use in the present research since their data was carefully taken in a wetted wall column apparatus and modeled with the Higbie penetration theory. In addition, the Yoshida and Oshima study was the most recently available data, and it was taken specifically with the idea that this value must be known in order to accurately predict the diffusivity of blood. This statement will be elaborated on in further detail in the blood section of this chapter.

The solubility of oxygen in water at 38°C was taken as a simple average of the available data for the present study since there was less than 1% deviation between literature values.

<table>
<thead>
<tr>
<th>Diffusivity</th>
<th>Solubility</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D \times 10^5$ (cm$^2$/sec)</td>
<td>Reference</td>
</tr>
<tr>
<td>2.61</td>
<td>77</td>
</tr>
<tr>
<td>2.61</td>
<td>16</td>
</tr>
<tr>
<td>2.72</td>
<td>87</td>
</tr>
<tr>
<td>2.82*</td>
<td>85</td>
</tr>
<tr>
<td>2.61</td>
<td>Value Selected</td>
</tr>
</tbody>
</table>

*37.5°C
Carbon dioxide literature diffusivities varied approximately 10% and its solubility varied less than 1%. Data taken for carbon dioxide was, however, subject to some error due to the slight effect of hydrolysis. The values chosen for use in the present study, as shown in the table, were chosen because they were most frequently reported. No recent data has been taken on carbon dioxide.

If diffusivities were not available at 38°C, the Stokes-Einstein equations (87) was used to scale literature data to the desired temperature of 38°C.

Membranes: Little data were available in the literature for the solubility and diffusivity of oxygen and carbon dioxide in dimethyl silicone polymers at 38°C. Some was available; however, at 25°C. Tables V-3 and V-4 show a sampling of these data from two references at 25°C to indicate the relative scatter of the literature values.

<table>
<thead>
<tr>
<th>Diffusivity</th>
<th>Solubility</th>
</tr>
</thead>
<tbody>
<tr>
<td>D x 10^6 (cm^2/sec)</td>
<td>Reference</td>
</tr>
<tr>
<td>16.0</td>
<td>84</td>
</tr>
<tr>
<td>22.1</td>
<td>15</td>
</tr>
</tbody>
</table>
Table V-4

Diffusivities and Solubilities of CO₂ in Silastic at 25°C

<table>
<thead>
<tr>
<th>Diffusivity</th>
<th>Solubility</th>
</tr>
</thead>
<tbody>
<tr>
<td>D x 10⁶ (cm²/sec)</td>
<td>α (cc(STP)/cc-atm)</td>
</tr>
<tr>
<td>12.0</td>
<td>2.300</td>
</tr>
<tr>
<td>18.7</td>
<td>1.625</td>
</tr>
</tbody>
</table>

Solubilities and diffusivities are exponential functions of temperature in polymers (2). The relationship is similar to a reaction rate constant.

\[ D_i = D_{0,i} e^{-E_i/RT} \]  \hspace{1cm} (V-1)

and

\[ \alpha_i = \alpha_{0,i} e^{-H_i/RT} \]  \hspace{1cm} (V-2)

where \( D_{0,i} \) and \( \alpha_{0,i} \) are constants peculiar to the polymer and diffusing gas, in cm²/sec and cc(STP)/cc-atm, respectively. \( E_i \) and \( H_i \) represents the temperature coefficients in cal/g-mole, \( R \) is the universal gas constant, and \( T \) is the absolute temperature. For Silastic, the constants for these equations are given in Table V-5.
### Table V-5

**Constants for Determining the Diffusivities and Solubilities of CO₂ and O₂ in Silastic at Any Temperature**

<table>
<thead>
<tr>
<th>Constant</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{O_2}$</td>
<td>$9.65 \times 10^{-4}$ cm²/sec</td>
</tr>
<tr>
<td>$D_{CO_2}$</td>
<td>$8.43 \times 10^{-4}$ cm²/sec</td>
</tr>
<tr>
<td>$\alpha_{O_2}$</td>
<td>0.1295 cc(STP)/cc-atm</td>
</tr>
<tr>
<td>$\alpha_{CO_2}$</td>
<td>0.0210 cc(STP)/cc-atm</td>
</tr>
<tr>
<td>$E_{O_2}$</td>
<td>2232 cal/g-mole</td>
</tr>
<tr>
<td>$E_{CO_2}$</td>
<td>2247 cal/g-mole</td>
</tr>
<tr>
<td>$H_{O_2}$</td>
<td>-387 cal/g-mole</td>
</tr>
<tr>
<td>$H_{CO_2}$</td>
<td>-2566 cal/g-mole</td>
</tr>
</tbody>
</table>
Table V-6

Solubilities and Diffusivities of CO₂ and O₂ in Silastic at 38°C

<table>
<thead>
<tr>
<th>Gas</th>
<th>D x 10^5 cm²/sec</th>
<th>α cc(STP)/cc-atm</th>
</tr>
</thead>
<tbody>
<tr>
<td>O₂</td>
<td>2.64</td>
<td>0.243</td>
</tr>
<tr>
<td>CO₂</td>
<td>2.21</td>
<td>1.350</td>
</tr>
</tbody>
</table>

Solubility of Gases in Blood: Since blood is not a homogenous fluid, the solubility must be considered a combination of the solubility of its various constituents. In 1934, Sendroy et al. (81) proposed a simple relationship for determining the solubility of human blood for oxygen.

\[
\alpha = \alpha_{rc}(H) + \alpha_{p}(1-H)
\]

(V-3)

where H represents the hematocrit fraction.

For oxygen in blood at 38°C, \( \alpha_{O₂,rc} \) was measured as 0.0258 cc(STP)/cc-atm and \( \alpha_{O₂,p} \) was measured as 0.0209 cc(STP)/cc-atm. The same relationship was assumed to hold for carbon dioxide at 38°C where the value of \( \alpha_{CO₂,rc} \) was 0.423 cc(STP)/cc-atm and \( \alpha_{CO₂,p} \) was 0.509 cc(STP)/cc-atm as reported by Davenport (22).

The solubility of a gas in plasma is approximately 10% lower than the corresponding solubility of that gas in water because salts
in the blood act to depress the solvent power of water and because proteins which displace water in the plasma are inert as solvents to oxygen and carbon dioxide (81).

**Diffusivity of Gases in Blood:** Oxygen diffusivities in blood have been frequently reported in the literature (62, 85, 33, 15, 91). Regretfully, these values are not in general agreement. Reported values ranged from $7.0 \times 10^{-6}$ cm$^2$/sec (83) to $1.3 \times 10^{-5}$ cm$^2$/sec for concentrated hemoglobin solutions (62). Diffusivities for plasma ranged from $1.63 \times 10^{-5}$ cm$^2$/sec (15) to $3.0 \times 10^{-5}$ cm$^2$/sec (33).

Fricke (34) developed an equation to describe the electrical conductivity of a suspension of noninteracting solids. He applied this equation to the electrical conductivity of blood and found close agreement. Since diffusivities are mass conductivities, Fricke's heterogeneous media theory should also hold for predicting the diffusivity of a suspension. Substituting diffusivities for electrical conductivities into Fricke's equation yields:

\[
\frac{(D_b/D_p) - 1}{(D_b/D_p) + x} = H \frac{(D_{rc}/D_p) - 1}{(D_{rc}/D_p) + x} \tag{V-4}
\]

where the factor $x$ is dependent upon the shape of the solid and the ratio of the diffusivity of the suspended medium to the suspending medium. $H$ is the volume fraction solids or, in the case of blood, the hematocrit.

Spaeth (85) using the data of Goldstick (41) established the ratio of $D_{O_2,p}/D_{O_2}$, water at 0.58 and the ratio $D_{O_2,rc}/D_{O_2,p}$ at 0.63.
for all temperatures. Spaeth used Equation (V-4) to predict the diffusivity of blood at various hematocrits, but in so doing he chose the value of $x$ in Equation (V-4) to be 2.0 which, in effect, reduces the solid particles to spheres. In reality, a red cell may best be described as an oblate spheroid having a minor to major axis ratio of $1/4$. This was the value which Fricke (34) arrived at for the dimension of the red cells of a dog and those of a human are not appreciably different. Using the value of $1/4$ for the axial ratio and the value of 0.63 for $D_{D_2,rc}/D_{O_2,p}$ the value of $x$ becomes 1.7.

It should be noted at this point that the true value of $D_{O_2,p}$ is still in dispute (85, 33, 54, 98). Since Goldstick's data (85) was well substantiated by Yoshida and Oshima's data it was chosen for the present research. Substitution of the values of $x$ and $D_{D_2,rc}/D_{O_2,p}$ into Equation (V-4) gives a general expression for the prediction of blood and oxygen diffusivities at various hematocrits.

$$\frac{D_b}{D_p} = \frac{1 - H(0.65)}{1 + 0.49H}$$

(V-5)

Yoshida and Oshima (98) found that the Stokes-Einstein viscosity correction predicted their experimental results for oxygen diffusivity in plasma, as well as the Higbie penetration theory. It was reasonable, therefore, to assume that the same relationship held for carbon dioxide. With such an assumption Equation (V-5) holds equally well for carbon dioxide.

The diffusivity predicted by Equation (V-5) is known as the non-facilitated diffusivity -- so called because it does not take into
account a simultaneous diffusion of oxyhemoglobin. Such a simultaneous diffusion would, of course, raise the effective value of the diffusivity. Facilitated diffusion has been studied in detail with both a term included and excluded for resistance due to the red cell membrane (33, 85), but the strongest facilitation effect is at oxygen partial pressures of 30 mm Hg or less. Blood oxygenator partial pressures of oxygen are rarely lower than 40 mm Hg. At higher partial pressures the effective diffusivity is relatively constant. Spaeth (85), and Buckles (15) found the facilitated diffusivity theory of little value in predicting oxygen absorption and carbon dioxide desorption from blood.
B. Respiratory Quotient

Respiratory quotient (RQ) is the term used to define the volumetric ratio of carbon dioxide to oxygen exchanged with blood through the alveolar walls in the lung. As previously noted, this ratio is slightly less than one. For all practical purposes, however, it may be considered to be unity. Unfortunately, in artificial oxygenators this term may not be unity. In fact, a respiratory quotient of unity is not claimed for any device now used except for cases where experimental manipulation of the contacting gases has been adjusted to obtain the unity figure.

The problems which can arise from the lack of a proper balance of oxygen and carbon dioxide are serious. If the instrument fails to oxygenate the blood properly the body will fall into a state of anoxia. Every cell in the body suffers to one degree or another from the various grades of anoxia (89). Assume that the artificial lung is designed for the saturation of the blood with oxygen. This is its minimal function. What happens now to the CO₂/O₂ exchange due to the design of the lung? If the case of unity is not obtained there will be either an excess or a deficit of CO₂ when the blood leaves the oxygenator. If there is a serious excess of CO₂ the blood becomes more acid due to the build-up of CO₂, and the pH falls. This condition is known as respiratory acidosis. The opposite of respiratory acidosis is respiratory alkalosis. This occurs when the CO₂/O₂ ratio is less than one. The pH of the blood rises and the CO₂ content is slowly depleted. Both of these conditions can cause serious damage
to the cells and, if allowed to exist unchecked for long periods of
time, may lead to death. The varying stages of these two conditions
are discussed in almost all standard references on general physi­
ology of the respiratory system (80, 22, 18), and they will not be
discussed further here. In general, it may be said that the oxygen­
ator which may be operated within a range of respiratory quotients
of from 0.7 to 1.0 will not need external chemical compensation.

The respiratory quotient was calculated in the computer program
given in Appendix C. It was computed from the differences in inlet
and outlet mixing cup concentrations of oxygen and carbon dioxide.
C. Discussion of Experimental and Theoretical Results for Distilled Water

Before experiments were performed on blood, experiments were made on water flowing in a permeable tube. These water experiments were made for two reasons. First, water could be used to determine if pulse rate appreciably altered the mass transport of oxygen or carbon dioxide. The mathematical models were developed assuming a steady state velocity profile. If pulse rate affected the mass transfer in water, a study would have to be performed on blood to determine the exact nature of the parameter. On the other hand, if pulse rate did not affect the mass transfer of oxygen and carbon dioxide in water then it would be reasonable to assume that it would not affect the mass transfer in blood. Secondly, a test of the validity of Equation (III-17) could be made with water by setting the hemoglobin concentration to zero and substituting in the appropriate physical properties for oxygen or carbon dioxide in water. If good agreement was obtained using an inert fluid such as water, the only remaining tasks for an adequate description of mass transfer to and from blood would be the proper identification of the amount of gas incorporated in chemically combined forms and the correct physical properties of blood.

Figure V-1 and V-2 were used to determine the influence of pulse rate on mass transfer to a fluid flowing in a permeable tube. Referring to Figure V-1, the dimensionless partial pressure appeared to decrease slightly as the pulse rate increased for a given flow
rate. Referring to Figure V-2 the mass transfer seemed to decrease then increase as the pulse rate was increased, but this effect is questionable due to the scatter in the data. Although Figure V-1 and Figure V-2 represent different diffusing species, the phenomena was essentially the same. If pulse rate truly affected the mass transfer then the two effects should have been the same in Figures V-1 and V-2. Since the effects were not the same, it was concluded that the pulse rate did not appreciably affect the quantity of mass transferred at a given flow rate. The apparent change in mass transfer due to pulse rate was attributed to experimental error principally in the calibration of the analytical equipment. This was not to say that pulse rate does not affect mass transfer but rather to say that the effect is small for this range of pulse rates, and it was hidden by random experimental fluxuation. Theoretical treatments of pulsating flows associated with biological functions involving mass transfer have been proposed (71, 24), but no experimental confirmation of the effect has yet been made.

Figure V-3 is a comparison of the theoretical mass transfer predicted by Equation (III-17) and experimental results for oxygen diffusion into water through a permeable tube. The initial theoretical inlet partial pressure was taken as 64.1 mm Hg since this was the average inlet partial pressure to the tube based on the experimental data. From Figure V-3 it can be seen that the theoretical prediction of Equation (III-17) is an accurate description of the experimental findings.

A log-log graph was used to describe the comparison of theoreti-
Figure V-1. Carbon Dioxide Desorption from Distilled Water Flowing in a Permeable Tube.

- $T = 38^\circ C$
- $L = 73.5$ cm
- □ = 4 pulse/min
- ○ = 46 pulse/min
- △ = 80 pulse/min
Figure V-2. Oxygen Absorption by Distilled Water Flowing in a Permeable Silicone Tube.

- $T = 38^\circ C$
- $L = 73.5$ cm
- $\square = 4$ pulse/min
- $\bigcirc = 46$ pulse/min
- $\triangle = 80$ pulse/min
Figure V-3. Comparison of Experimental Data with Theoretical Prediction of Equation (III-17) for Oxygen Absorption into Water Flowing in a Permeable Silicone Tube.

Constants

\[ T = 38^\circ C \]
\[ L = 73.5 \text{ cm} \]
\[ D_{O_2} = 2.92 \times 10^{-5} \text{ cm}^2/\text{sec} \]
\[ \alpha_{O_2} = 0.0234 \text{ cc(STP)/cc-atm} \]

\[ \square = 4 \text{ pulse/min} \]
\[ \circ = 46 \text{ pulse/min} \]
\[ \triangle = 80 \text{ pulse/min} \]
cal prediction to experimental data because the experimental points varied over such a wide range. In addition, the logarithmic plot of dimensionless partial pressure versus L/Q is a modified form of the classical Graetz solution applied to mass transfer (3). Since the physical properties were constant for each particular experiment, the term L/Q was felt to be a less complicated and a more descriptive independent variable for the present research. In the Graetz nomenclature L/Q is equivalent to 2L/(\pi RD_i k ReSc). Buckles (15) and Weissman and Mockros (91) had shown earlier that the solution of the Graetz problem for mass transfer was unaffected by changing the radius for a given flow rate. This was not surprising since for a given flow rate the residence time increases with the radius, but the longer diffusion time is offset by the increasing thickness through which the gas must diffuse. Experimental results by Buckles (15) confirmed the theoretical conclusion.

Figure V-4 compares experimental data with the prediction of Equation (III-17) for carbon dioxide desorption from water flowing in a permeable tube. Simultaneous oxygen absorption was not present. The theoretical solution and experimental data agree quite well at large L/Q values, but deviate significantly at low L/Q values. The large deviations in experimental data at low L/Q values was due to the division of a small difference in large numbers. Any slight instrument deviation was therefore greatly magnified.

The dimensionless partial pressure plotted as the ordinate in Figure V-1, V-2, V-3, and V-4 refers to the dimensionless mixing cup partial pressure. Figures V-5 and V-6 show the theoretical radial partial pressure profiles for oxygen absorption and carbon dioxide
Figure V-4. Comparison of Experimental Data with Theoretical Prediction of Equation (III-17) for Carbon Dioxide Desorption from Water Flowing in a Permeable Silicone Tube.

Constants

\[ T = 38^\circ C \]
\[ L = 73.5 \text{ cm} \]
\[ D_{CO_2} = 2.61 \times 10^{-5} \text{ cm}^2/\text{sec} \]
\[ \alpha_{CO_2} = 0.555 \text{ cc(STP)/cc-atm} \]

\[ \square = 4 \text{ pulse/min} \]
\[ \bigcirc = 46 \text{ pulse/min} \]
\[ \triangle = 80 \text{ pulse/min} \]
Figure V-5. Radial Profiles for Oxygen Absorption by Water.

Constants

\[ L = 73.5 \text{ cm} \]
\[ T = 38^\circ \text{C} \]
\[ Q = 0.01 \text{ cc/sec} \]
\[ TM = 0.0279 \text{ cm} \]
\[ P_{g, O_2} = 710.3 \text{ mm Hg} \]
\[ R = 0.0317 \text{ cm} \]
Figure V-6. Radial Profiles for Carbon Dioxide Desorption from Water.

Constants

\[ L = 73.5 \text{ cm} \]
\[ T = 37^\circ \text{C} \]
\[ Q = 0.01 \text{ cc/sec} \]
\[ P_{g, CO_2} = 0.0 \text{ mm Hg} \]
\[ T_M = 0.0279 \text{ cm} \]
\[ R = 0.0317 \text{ cm} \]
\[ H = 43.8\% \]
desorption from distilled water at various points along the permeable tube. From Figures V-5 and V-6 it can be seen that the resistance to mass transfer presented by the tube wall was relatively small considering its thickness. In general, the fractional resistance of the wall is:

\[ \text{Wall Resistance} = \frac{D_{H_2O^\alpha}T}{D_{H_2O^\alpha}T + D_{\alpha}R} \quad (V-6) \]

For the experimental tube used to generate the profiles of Figures V-5 and V-6, the resistance due to the wall for oxygen was 8.6% of the total resistance whereas for CO\textsubscript{2} it was 30% of the total.
D. Discussion of Theoretical and Experimental Results for Human and Cattle Blood

In Chapter III two equations were developed to model the simultaneous counter diffusion of oxygen and carbon dioxide in blood flowing through a permeable tube. In the oxygen transfer model, Equation (III-17), the oxygen absorption was taken to be independent of carbon dioxide desorption. In the carbon dioxide transfer model, Equation (III-46), carbon dioxide transport was considered to be dependent upon the oxygen absorption as described by the oxygen absorption model. From the solutions to these two equations a respiratory quotient was calculated. In this section these models will be compared with experimental data and shown to be an adequate description of the oxygen and carbon dioxide transfer in blood flowing in a permeable tube.

Before presenting the comparison between experimental and theoretical results, it is necessary to understand what variables affect the oxygen and carbon dioxide transport in blood flowing through a permeable tube. Each of the variables, pH, hemoglobin concentration, tube wall thickness, hematocrit, and inlet dissolved oxygen partial pressure influence the mass transfer to a different degree. The following parameter study was made to determine the exact degree to which each variable affected the mass transfer.

Parameter Study

The Effect of pH on Oxygen and Carbon Dioxide Transport: In Figure V-7 a comparison was made between the dimensionless mixing cup
Figure V-7. The Effect of pH on Oxygen Absorption into Blood Flowing in a Permeable Tube.

Constants

\[ T = 38^\circ C \]
\[ C_{Hb} = 0.15 \text{ g/cc} \]
\[ H = 43.8\% \]
\[ R = 0.0317 \text{ cm} \]
\[ T \text{M} = 0.0279 \text{ cm} \]
\[ P_{O_2} = 40 \text{ mm Hg} \]
oxygen partial pressure and the independent variable which is the length to volumetric flow ratio, L/Q, with the pH as a parameter. As the pH increases the dimensionless oxygen mixing cup partial pressure increases. The increased partial pressure effect is greatest at high pH's. The rise in partial pressure with a rise in pH could have been predicted qualitatively from the oxygen dissociation curves (Figure I-3). In Figure I-3, as the pH increases, the dissociation curve shifts to the left. For a given partial pressure, then, the saturation becomes greater with this shift. For a constant tube entrance partial pressure of oxygen in blood, the saturation is greatest at high pH's; and, therefore, less additional oxygen is needed to fulfill the oxyhemoglobin requirement. Thus, a smaller mass of oxygen is required to bring the saturation to an acceptable value. The oxygen partial pressure will therefore increase as pH increases.

The nonlinearity of the dimensionless partial pressure increase, shown in Figure V-7, is due to the nonlinear nature of the oxygen dissociation curve. As a result of this nonlinear pH effect, the use of Margaria's model (85) for the dissociation curve is definitely preferable to an exponential fit of average blood pH values such as was employed by Weissman and Mockros (91).

The carbon dioxide model, Equation (III-46), predicted that the degree of oxygen saturation influenced the carbon dioxide desorption from blood flowing in a permeable tube. In Figure V-8, the influence of the pH parameter on the dimensionless carbon dioxide partial pressure is shown for values of the dependent variable, L/Q. Figure V-8
Figure V-8. The Effect of pH on Carbon Dioxide Desorption from Blood Flowing in a Permeable Tube.

Constants

T = 38°C
C_{Hb} = 0.15 g/cc
H = 43.8%
R = 0.0317 cm
TM = 0.0279 cm
P_{O_2} = 40 mm Hg
indicates that the saturation influence, although small, is significant. The oxygen saturation effect on carbon dioxide transfer is greatest at low flow rates.

Figure V-9 represents the relationship between pH and the respiratory quotient. As the pH increases, the respiratory quotient increases in a non-linear manner. Although the dimensionless partial pressure of oxygen has increased in the blood while that of the carbon dioxide has remained essentially the same for a given flow rate and pH, the difference between the inlet and outlet concentration of oxygen has actually decreases, and therefore the respiratory quotient rises. This statement may be verified by appropriate substitutions in Equations (I-1) and (I-2) for the total oxygen and carbon dioxide concentrations in blood, respectively. No theoretical curve is shown below an L/Q value of 1000 because as L/Q approaches zero the respiratory quotient becomes indeterminate.

A respiratory quotient may be calculated for any tube length at a constant volumetric flow as indicated in Figure V-9. Only one value of the respiratory quotient is of immediate interest; however, and that is the value which just satisfies the physiological requirement of the body. If the tube entrance partial pressures of oxygen and carbon dioxide in blood are taken to be $P_{O_2} = 46$ mm Hg and $P_{CO_2} = 40$ mm Hg with a pure oxygen stream surrounding the permeable tube, then the dimensionless carbon dioxide partial pressure, $\bar{P}_{CO_2}$, should reach 0.130 and the dimensionless oxygen partial pressure, $\bar{P}_{O_2}$, should reach 0.090 at the exit of the tube in order for the physiological requirements to be met (see Table I-2). Figures (V-7) and (V-8)
Figure V-9. The Effect of pH on Respiratory Quotient.

Constants

T = 38°C
C_{Hb} = 0.15 g/cc
H = 43.8%
R = 0.0317 cm
TM = 0.0279 cm
P_{O_2} = 40 mm Hg
indicate that carbon dioxide is the limiting component. That is, it takes longer for the carbon dioxide removal then it takes for adequate oxygen absorption. For a pH of 7.40, the respiratory quotient has reached an acceptable level of 0.90 when the carbon dioxide is adequately removed for a radius of 0.0317 cm and a wall thickness of 0.0279 cm. Values of the respiratory quotient between 0.7 and 1.0 are acceptable.

The Effect of Hemoglobin Concentration on Oxygen and Carbon Dioxide Transport: Figure V-10 shows the effect of hemoglobin concentration on the dimensionless oxygen partial pressure. As the hemoglobin concentration increased the dimensionless partial pressure decreased because more oxygen was absorbed by the chemical reaction. The hemoglobin range from 0.10 g/cc to 0.20 g/cc encompasses 99.99% of all reported hemoglobin concentration for humans (26).

Figure V-11 indicates that hemoglobin concentration does not affect the carbon dioxide desorption curve. This result arises from the assumption of hemoglobin independence for the carbon dioxide desorption model, Equation (III-46). In truth, the carbon dioxide desorption is probably dependent upon the hemoglobin concentration but the exact nature and degree of this dependency is unknown.

Figure V-12 shows the respiratory quotient increasing for decreasing values of the hemoglobin concentration in the blood. The respiratory quotient rises because the volume concentration of oxygen has become smaller for a given L/Q at the lower hemoglobin values while the carbon dioxide volume concentration and difference has remained unchanged. Again, this effect can be reproduced by appropriate
Figure V-10. The Effect of Hemoglobin Concentration on Oxygen Absorption into Blood Flowing in a Permeable Tube.

Constants

\( T = 38^\circ C \)
\( H = 43.8\% \)
\( R = 0.0317 \text{ cm} \)
\( TM = 0.0279 \text{ cm} \)
\( pH = 7.40 \)
\( P_{o_2} = 40 \text{ mm Hg} \)
Figure V-11. The Effect of Hemoglobin Concentration on Carbon Dioxide Desorption from Blood Flowing in a Permeable Tube.

Constants

\[ T = 38^\circ C \]
\[ H = 43.8\% \]
\[ R = 0.0317 \text{ cm} \]
\[ TM = 0.0279 \text{ cm} \]
\[ pH = 7.40 \]
\[ P_{O_2} = 40 \text{ mm Hg} \]
Figure V-12. The Effect of Hemoglobin Concentration on Respiratory Quotient.
The Effect of Tube Wall Thickness on Mass Transfer: Figures V-13 and V-14 represent the influence of tube wall thickness on the mass transfer of oxygen and carbon dioxide. Comparison of Figure V-13 and V-14 reveals that the tube wall represents a greater portion of the total resistance to mass transfer for carbon dioxide than it does for oxygen.

Ordinarily, the added resistance of the membrane is considered a price to be paid for the removal of direct air-blood contact. As previously discussed (Chapter II), efforts have been made to eliminate as much resistance to mass transfer as possible by reducing the thickness of the membrane. It was questioned whether the improved mass transfer of a thin wall tube was worth the sacrifice of strength and fixed shape. There is no doubt that increasing the wall thickness reduces the effectiveness of a single tube oxygenator, but this may not be the case for a full scale multi-tubular oxygenator. For example, the difficulties in blood distribution which Wilson et.al. (97) encountered were probably due in part to the irregular shape of the thin walled tubing employed in their experiments. These difficulties might have been overcome with thicker walled tubes. The theoretical area required to produce sufficient oxygenation and adequate carbon dioxide removal from blood would increase, but the overall efficiency of the unit could be greatly improved. Additional research is needed in the area of blood distribution in a bundle of flexible tubes.

Figures V-13 and V-14 indicate that as the tube wall thickness
Figure V-13. The Effect of Membrane Thickness on Oxygen Absorption into Blood Flowing in a Permeable Tube.

Constants

\( T = 38^\circ \text{C} \)

\( C_{\text{Hb}} = 0.15 \text{ g/cc} \)

\( H = 43.8\% \)

\( R = 0.0317 \text{ cm} \)

\( \text{pH} = 7.40 \)
Figure V-14. The Effect of Membrane Thickness on Carbon Dioxide Desorption from Blood Flowing in a Permeable Silicone Tube.
is increased, the additional area required for adequate CO₂ elimination above that required for oxygenation increases. With no tube wall, there is 27% excess area above that required for oxygenation, and with a wall to radius ratio of 1.0, there is 89% excess area.

Figure V-15 gives the respiratory quotients for various wall thickness.

The Effect of Hematocrit on Mass Transfer: The hematocrit of the blood determines the diffusivity and solubility of gases in the blood. As the hematocrit increases, the diffusivity and solubility of both oxygen and carbon dioxide in blood decrease. These decreases are due to the lower diffusivities and solubilities of blood gases in the red cell than in the plasma. The higher blood gas diffusivities at lower hematocrits produced higher partial pressures for a given flow rate. Figures V-16 and V-17 indicate that the magnitude of the increased partial pressure is not large over the range of hematocrits shown. The 30% to 50% hematocrit range represents 99% of the hematocrits recorded for human beings (26). Since the magnitude of partial pressure change is not large, the respiratory quotient curves do not vary greatly as shown in Figure V-18.

Inlet Oxygen Partial Pressure Effect on Mass Transfer: The magnitude of the initial partial pressure of oxygen entering the experimental tube produced the strongest effect on the outlet dimensionless partial pressure. Normally, in a non-reacting system, the magnitude of the inlet conditions will not affect the exit dimensionless partial pressure. When mass is removed from or added to the system by chemical reaction this is not the case. No previous author has investigated
Figure V-15. The Effect of Silicone Membrane Thickness on Respiratory Quotient.

Constants
- T = 38°C
- C_{Hb} = 0.15 g/cc
- H = 43.8%
- R = 0.0317 cm
- pH = 7.40
- p_{O_2} = 40 mm Hg
Figure V-16. The Effect of Hematocrit on Oxygen Absorption into Blood Flowing in a Permeable Tube.

Constants:
\[ T = 38^\circ C \]
\[ C_{Hb} = 0.15 \text{ g/cc} \]
\[ p\text{H} = 7.40 \]
\[ R = 0.0317 \text{ cm} \]
\[ TM = 0.0275 \text{ cm} \]
Figure V-17. The Effect of Hematocrit on Carbon Dioxide Desorption from Blood Flowing in a Permeable Tube.
Figure V-18. The Effect of Hematocrit on Respiratory Quotient.

Constants

\[ T = 38 \, ^\circ \text{C} \]
\[ C_{\text{Hb}} = 0.15 \, \text{g/cc} \]
\[ \text{pH} = 7.40 \]
\[ R = 0.0317 \, \text{cm} \]
\[ T_M = 0.0279 \, \text{cm} \]
this theoretical concept for blood. Figure V-19 indicates the magnitude of the dimensionless partial pressure change for blood leaving the tube for varying inlet conditions. The dimensionless oxygen partial pressure displacement produces a corresponding effect in the carbon dioxide exit partial pressure (Figure V-20) where the effect is greatest at low partial pressures of oxygen and gradually diminishes to zero at values of initial oxygen partial pressure greater than 60 mm Hg. There is little or no effect on the carbon dioxide desorption above 80 mm Hg inlet oxygen partial pressure because at a pH of 7.40, saturation is for all practical purposes complete.

Figure V-21 reflects the effect of inlet oxygen partial pressure on the respiratory quotient. The respiratory quotient increases with inlet oxygen partial pressure because the volume of oxygen absorbed from the tube inlet to exit has decreased whereas the volume of carbon dioxide transferred has remained essentially the same.

Radial and Longitudinal Partial Pressure Profiles in Blood: The dimensionless longitudinal partial pressures referred to in Figures V-7 through V-21 represented an integration of theoretical radial profiles. Typical radial oxygen and carbon dioxide partial pressure profiles for blood flowing in a permeable tube are shown in Figures V-22 and V-23, respectively. Comparing Figures V-22 and V-23 for blood with Figures V-5 and V-6 for water demonstrates the difference between a reacting fluid and an inert fluid. The partial pressure profiles break much sharper near the wall for blood, and the concentration gradient is greater near the wall for blood than for water.

Figure V-24 gives the theoretical mixing cup partial pressures
Figure V-19. The Effect of Initial Oxygen Partial Pressure on the Oxygenation of Blood Flowing in a Permeable Tube.

Constants

- $T = 38^\circ C$
- $C_{Hb} = 0.15 \text{ g/cc}$
- $H = 43.8\%$
- $R = 0.0317 \text{ cm}$
- $TM = 0.0279 \text{ cm}$
- $pH = 7.40$
Figure V-20. The Effect of Initial Oxygen Partial Pressure on Carbon Dioxide Removal from Blood Flowing in a Permeable Tube.

Constants

- $T = 38^\circ C$
- $C_{Hb} = 0.15 \text{ g/cc}$
- $H = 43.8\%$
- $R = 0.0317 \text{ cm}$
- $TM = 0.0279 \text{ cm}$
- $P_{o,CO_2} = 46 \text{ mm Hg}$
- $\text{pH} = 7.40$
Figure V-21. The Effect of Initial Oxygen Partial Pressure on the Respiratory Quotient.

Constants
- $T = 38^\circ C$
- $C_{Hb} = 0.15 \text{ g/cc}$
- $H = 43.8\%$
- $R = 0.0317 \text{ cm}$
- $TM = 0.0279 \text{ cm}$
- $P_{o_2,CO_2} = 46 \text{ mm Hg}$
- $pH = 7.40$
Figure V-22. Radial Profile for Oxygen Absorption by Blood.

Constants

$C_{Hb} = 0.143 \text{ g/cc}$
$H = 43.8\%$
$L = 73.5 \text{ cm}$
$Q = 0.01 \text{ cc/sec}$
$TM = 0.0279 \text{ cm}$
$R = 0.0317 \text{ cm}$
$P_{g,O_2} = 710.3 \text{ mm Hg}$
Figure V-23. Radial Profiles for Carbon Dioxide Desorption from Blood.

Constants

- $C_{Hb} = 0.143$ g/cc
- $H = 43.8\%$
- $L = 73.5$ cm
- $Q = 0.01$ cc/sec
- $TM = 0.0269$ cm
- $R = 0.0317$ cm
- $P_{g,CO_2} = 0.0$ mm Hg
Figure V-24. Theoretical longitudinal Partial Pressure Profiles in Blood Flowing in a Permeable Silicone Tube 73.5 cm in Length.
of oxygen and carbon dioxide along the length of a 73.5 cm tube with a radius of 0.0317 cm and a wall thickness of 0.0279 cm. These dimensions were typical of an experimental tube.

Axial Saturation Profile: In reference to Figure V-25, the majority of the saturation of blood theoretically takes place in the first few centimeters of the experimental tube. The remainder of the tube completes the saturation, but it operates at an extremely low efficiency. The dashed lines represent a theoretical method of raising the exit saturation with a shorter tube. This idea was first proposed by Buckles (15). If after an initial steep rise in saturation is obtained, the blood is completely mixed, i.e. a flat concentration profile, advantage can be taken of a second steep rise in saturation. This same procedure may be repeated several times, but the improvement diminishes with each subsequent remixing. A multipass "mass exchanger" may be able to provide enough mixing by expansion and contraction within its headers to produce this effect, if not, mechanical mixing could augment the natural mixing and determine if the effect could be verified experimentally. This would represent an area where future research would be useful.

Comparison of Theoretical and Experimental Results

In Figures V-26, V-27, and V-28, the theoretical prediction of the oxygen absorption model, Equation (III-17), is compared with the experimental results for oxygen absorption into blood flowing in a permeable tube with simultaneous carbon dioxide desorption. The theoretical curve is shown by the solid line. In both cases for human blood and cattle blood the theoretical oxygen absorption pre-
Figure V-25. Theoretical Saturation Profile for Blood Flowing in a Permeable Silicone Tube.

Constants

\[ \begin{align*}
T &= 38^\circ C \\
C_{\text{Eb}} &= 0.15 \text{ g/cc} \\
pH &= 7.4 \\
R &= 0.0317 \text{ cm} \\
TM &= 0.0279 \text{ cm} \\
Q &= 0.007 \text{ cc/sec} 
\end{align*} \]
diction is slightly optimistic. This discrepancy is undoubtedly partially due to the experimental procedure and partially due to portions of the theory.

The scatter in the experimental data was attributed to several causes. First, for low L/Q values the partial pressure will not change greatly from the entrance to the exit of the tube. As a result, the dimensionless partial pressure is calculated by dividing a small difference by a large number. Errors are therefore magnified. Second, the electrodes used to measure partial pressure are subject to a build up of protein deposits over the membrane protecting the electrode. Reported partial pressures will therefore tend to be biased toward lower values.

The shape and position of the oxygen dissociation curve was of primary importance in determining the solution to Equation (III-17). Unfortunately, the literature did not agree on the exact placement of this curve for a given pH. Displacement, even by a small amount, can lead to rather large errors as was demonstrated in Figure V-19. The only recourse for accuracy is to determine the saturation curve for each sample of blood used. This was not done in this research because the analytical equipment was not available. Future studies should include such experiments.

Figures V-29 and V-30 compare the theoretical solution of Equation (III-46) with experimental results for carbon dioxide desorption in the presence of simultaneous oxygen absorption. In Figure V-29, the experimental and theoretical results compared quite favorably; however, the theoretical prediction is conservative in Figure V-30.
Figure V-26. A Comparison of Experimental Data and the Theoretical Prediction of Equation (III-17) for Oxygen Absorption by Human Blood - Donor K-1.

Constants

\[
\begin{align*}
T & = 38^\circ C \\
R & = 0.0317 \text{ cm } \\
TM & = 0.0279 \text{ cm } \\
L & = 33.02 \text{ cm } \\
H & = 43.8\% \\
C_{Hb} & = 0.143 \text{ g/cc } \\
pH & = 7.00
\end{align*}
\]
Figure V-27. The Comparison of the Theoretical Prediction of Equation (III-17) with Experimental Data for Human Blood - Donor M-1.

Constants

$T = 38^\circ C$
$L = 32.86 \text{ cm}$
$R = 0.0315 \text{ cm}$
$TM = 0.0279 \text{ cm}$
$H = 35.0\%$
$C_{Hb} = 0.140 \text{ g/cc}$
$pH = 7.11$
Figure V-28. Comparison of Experimental Data with Theoretical Prediction of Equation (III-17) for Oxygen Absorption into Cattle Blood Flowing in a Permeable Silicone Tube.

Constants

\[ T = 38^\circ C \]
\[ C_{Hb} = 0.123 \text{ g/cc} \]
\[ H = 40.0\% \]
\[ L = 73.5 \text{ cm} \]
\[ R = 0.0315 \text{ cm} \]
\[ TM = 0.0279 \text{ cm} \]
\[ P_{O2} = 63.0 \text{ mm Hg} \]
\[ pH = 7.20 \]
Figure V-29. Comparison of Experimental Data with the Theoretical Prediction of Equation (III-46 for Carbon Dioxide Desorption from Human Blood - Donor K-1.

Constants

\[ T = 38^\circ C \]
\[ R = 0.0317 \text{ cm} \]
\[ TM = 0.0279 \text{ cm} \]
\[ L = 33.02 \text{ cm} \]
\[ H = 43.8\% \]
\[ C_{Hb} = 0.143 \text{ g/cc} \]
\[ pH = 7.00 \]
Figure V-30. A comparison of Experimental Data with the Theoretical Prediction of Equation (III-46) for Carbon Dioxide Desorption from Human Blood - Donor M-1.

Constants

\[ T = 38^\circ C \]
\[ C_{Hb} = 0.140 \, \text{g/cc} \]
\[ H = 35.0\% \]
\[ L = 32.06 \, \text{cm} \]
\[ R = 0.0315 \, \text{cm} \]
\[ TM = 0.0279 \, \text{cm} \]
\[ pH = 7.11 \]
With due regard to experimental difficulties and errors, the majority of any offset between theory and experiment for carbon dioxide is unquestionably due to the limited nature of the model. The carbon dioxide model was independent of hemoglobin concentration (Figure V-11) and practically independent of pH (Figure V-8), but carbon dioxide desorption is definitely a function of pH via the bicarbonate ion as discussed in Chapter I, and also it is at least a weak function of hemoglobin (22). The mathematical description of these relationships is not presently available in the literature and future research is needed in this area.

In the parameter study of the theoretical models, it was pointed out that carbon dioxide removal determines the total area required of the oxygenator. The experimental data does not justify this conclusion. Investigation of Figure pairs V-26, V-29, and V-27, V-30 indicates that the physiological requirements were met approximately simultaneously (i.e. $P_{O2} = 0.09$ and $P_{CO2} = 0.13$). This experimental result means that the tubular oxygenator is more efficient than had been expected.

**Summary**

The mathematical model proposed for oxygen absorption in Chapter III was found to be slightly optimistic in its prediction when compared with experimental data. The mathematical model proposed for carbon dioxide desorption was found to be conservative when compared with the experimental data. Since carbon dioxide was the theoretically limiting component for satisfying the physiological needs of the
human body; the overall prediction of oxygenator area was conserva-
tive.

The mathematical model for oxygen and carbon dioxide was based
on theoretical concepts and no curve fitting techniques were employed.
The deviations between the experimental data and the mathematical
models in general represented shortcomings in the present theory.
Chapter VI

CONCLUSIONS

Based on the theoretical and experimental results of this re-
search, the following conclusions were drawn:

1. For a given volumetric flow rate, pulse rate did not
   appreciably affect the mass transfer in blood flowing
   in a permeable silicone tube.
2. Carbon dioxide desorption in blood flowing in a per-
   meable tube is dependent upon the oxygen absorption.
3. Oxygen absorption is independent of carbon dioxide
   desorption for blood flowing in a permeable tube.
4. The dimensionless exit partial pressure of oxygen de-
   pended strongly upon the initial partial pressure of
   oxygen entering the tube.
5. Oxygen absorption depended moderately upon pH and
   hemoglobin concentration and to a lesser degree upon
   the hematocrit.
6. Carbon dioxide desorption was a moderate function of
   oxygen absorption.
7. Carbon dioxide desorption was theoretically independent
   of pH, hemoglobin concentration, and inlet carbon di-
   oxide partial pressure.
8. The tube wall represented a significant portion of the
   total resistance to mass transfer for carbon dioxide.
9. The tube wall did not represent a significant portion of the total resistance to mass transfer for oxygen.

10. Experimental results indicated that the physiological requirement of oxygen absorption and carbon dioxide desorption were met approximately simultaneously for blood flowing through a permeable Silastic silicone tube.

11. The mathematical models proposed in this study conservatively predict the length of permeable tube needed to maintain the physiological requirement at a given flow rate.

12. A tubular membrane oxygenator can satisfy the physiological requirements of the body which are normally provided by the lung. The one exception to this conclusion is in regard to capacity. A simple tubular oxygenator is designed for a fixed flow rate and may only be operated within a narrow range of that flow rate.
<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DEFINITION</th>
<th>DIMENSIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Dimensionless length</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Concentration of gas in a liquid</td>
<td>cc(STP)/cc</td>
</tr>
<tr>
<td>C_{Hb}</td>
<td>Hemoglobin concentration in blood</td>
<td>g/cc</td>
</tr>
<tr>
<td>D_{i,k}</td>
<td>Diffusivity of component $i$ in medium $k$</td>
<td>cm$^2$/sec</td>
</tr>
<tr>
<td>D_{o,i}</td>
<td>Ground state constant for solubility-temperature relationship in a polymer, gas $i$</td>
<td>cc(STP)/cc-atm</td>
</tr>
<tr>
<td>E_{i}</td>
<td>Temperature coefficient for solubility-temperature relationship in a polymer, gas $i$</td>
<td>cal/g-mole</td>
</tr>
<tr>
<td>F</td>
<td>A variable defined by Equation (III-27)</td>
<td></td>
</tr>
<tr>
<td>F_{2}</td>
<td>A variable defined by Equation (III-51)</td>
<td></td>
</tr>
<tr>
<td>f(c)</td>
<td>A nonlinear oxygen sink term used to denote the effect of chemical combination in Weissman and Mockros' Equation (II-5)</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>A variable defined by Equation (III-26)</td>
<td></td>
</tr>
<tr>
<td>G_{2}</td>
<td>A variable defined by Equation (III-50)</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>Hematocrit</td>
<td>fraction</td>
</tr>
<tr>
<td>H_{i}</td>
<td>Temperature coefficient for the diffusivity-temperature relationship in a polymer, gas $i$</td>
<td>cal/g-mole</td>
</tr>
<tr>
<td>K</td>
<td>Constant in Margaria's Equation (I-10)</td>
<td>(mm Hg)$^{-1}$</td>
</tr>
<tr>
<td>K'</td>
<td>Constant in Pauling's Equation (I-8)</td>
<td>(mm Hg)$^{-1}$</td>
</tr>
<tr>
<td>K_{1,K_{2}}</td>
<td>Constant of integration</td>
<td></td>
</tr>
<tr>
<td>SYMBOL</td>
<td>DEFINITION</td>
<td>DIMENSIONS</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>$K_1, K_2, K_3, K_4$</td>
<td>Constants in Adair Equation</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>Tube length</td>
<td></td>
</tr>
<tr>
<td>$N_{i,k}$</td>
<td>Mass flux of species $i$ in direction $k$</td>
<td>cc(STP)/cm$^2$-sec</td>
</tr>
<tr>
<td>$P_{avg,i}$</td>
<td>Mixing cup partial pressure of species $i$</td>
<td>mm Hg</td>
</tr>
<tr>
<td>$P_{BAR}$</td>
<td>Barometric pressure</td>
<td>mm Hg</td>
</tr>
<tr>
<td>$P_{g,i}$</td>
<td>Partial pressure of species $i$ in the gas surrounding the permeable tube</td>
<td>mm Hg</td>
</tr>
<tr>
<td>$P_i$</td>
<td>Partial pressure of gas $i$</td>
<td>mm Hg</td>
</tr>
<tr>
<td>$P_m$</td>
<td>Partial pressure of gas in liquid at membrane-liquid interface</td>
<td>mm Hg</td>
</tr>
<tr>
<td>R</td>
<td>Tube radius</td>
<td>cm</td>
</tr>
<tr>
<td>r</td>
<td>Radial distance</td>
<td>cm</td>
</tr>
<tr>
<td>Re</td>
<td>Reynold's number</td>
<td></td>
</tr>
<tr>
<td>$R_i$</td>
<td>Rate of appearance of component $i$ by chemical reaction</td>
<td>cc(STP)/cc-sec</td>
</tr>
<tr>
<td>RQ</td>
<td>Respiratory quotient</td>
<td>$\frac{cc(STP)CO_2}{cc(STP)O_2}$</td>
</tr>
<tr>
<td>S</td>
<td>Saturation fraction</td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>time</td>
<td>sec</td>
</tr>
<tr>
<td>TM</td>
<td>Membrane thickness</td>
<td>cm</td>
</tr>
<tr>
<td>Sc</td>
<td>Schmidt number</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>Dimensionless velocity, $v_z/v_o$</td>
<td></td>
</tr>
<tr>
<td>SYMBOL</td>
<td>DEFINITION</td>
<td>DIMENSIONS</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>(v_{\text{avg}})</td>
<td>Average axial velocity</td>
<td>cm/sec</td>
</tr>
<tr>
<td>(v_o)</td>
<td>Maximum velocity in a tube</td>
<td>cm/sec</td>
</tr>
<tr>
<td>(v_Z)</td>
<td>velocity as a function of radius</td>
<td>cm/sec</td>
</tr>
<tr>
<td>(Y)</td>
<td>Dimensionless radius</td>
<td></td>
</tr>
<tr>
<td>(Z)</td>
<td>Axial distance</td>
<td>cm</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Solubility or Henry's Law constant</td>
<td>cc(STP)/cc-atm</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Oxyheme interaction constant in Pauling's Equation</td>
<td>cc(STP)/cc-atm</td>
</tr>
<tr>
<td>(\alpha_{o,i})</td>
<td>Ground state constant for solubility-temperature relationship in a polymer, gas, i</td>
<td>cc(STP)/cc-atm</td>
</tr>
<tr>
<td>(\gamma_1)</td>
<td>Surface tension of a liquid</td>
<td>dyne/cm²</td>
</tr>
<tr>
<td>(\gamma_S)</td>
<td>Surface tension of a polymer</td>
<td>dyne/cm²</td>
</tr>
<tr>
<td>(\Theta)</td>
<td>Constant angle</td>
<td>degrees</td>
</tr>
<tr>
<td>(\Theta)</td>
<td>A variable defined by Equation (III-23)</td>
<td></td>
</tr>
<tr>
<td>(\Theta_2)</td>
<td>A variable defined by Equation (III-47)</td>
<td></td>
</tr>
</tbody>
</table>

**Subscripts**

- \(b\) Blood
- \(\text{CO}_2\) Carbon dioxide
- \(i,j,k,l\) General subscripts
- \(m\) Membrane
- \(o\) Initial
- \(O_2\) Oxygen
<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>Plasma</td>
</tr>
<tr>
<td>rc</td>
<td>Red cell</td>
</tr>
</tbody>
</table>


70. Motley, Hurley L., "Oxygen Transport or Transfer in the Lung", Lecture: 6th Postgraduate Course on Recent Advances in the Diagnosis and Treatment of Diseases of the Heart and Lungs, Los Angeles, California, (Dec. 4-8, 1961).


APPENDIX A

EXPERIMENTAL DATA
Table A-1

EXPERIMENTAL DATA FOR OXYGENATION OF DISTILLED WATER FLOWING IN A PERMEABLE SILICONE TUBE

L = 73.5 cm
R = 0.0317 cm
T = 38°C
Tm = 0.0279 cm
P = Outlet Partial Pressure O₂
P₀ = Inlet Partial Pressure O₂

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>P</th>
<th>P₀</th>
<th>P_{BAR}</th>
<th>P̅</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mm Hg</td>
<td>mm Hg</td>
<td>mm Hg</td>
<td>P</td>
<td>cc/min</td>
</tr>
<tr>
<td>1</td>
<td>95.0</td>
<td>29.1</td>
<td>774.0</td>
<td>0.095</td>
<td>35.29</td>
</tr>
<tr>
<td></td>
<td>145.0</td>
<td>29.1</td>
<td>774.0</td>
<td>0.167</td>
<td>11.59</td>
</tr>
<tr>
<td>2</td>
<td>260.0</td>
<td>29.2</td>
<td>776.0</td>
<td>0.331</td>
<td>2.63</td>
</tr>
<tr>
<td></td>
<td>340.0</td>
<td>29.2</td>
<td>776.0</td>
<td>0.446</td>
<td>2.61</td>
</tr>
<tr>
<td>3</td>
<td>146.5</td>
<td>26.5</td>
<td>776.0</td>
<td>0.171</td>
<td>14.80</td>
</tr>
<tr>
<td></td>
<td>355.0</td>
<td>26.5</td>
<td>776.0</td>
<td>0.469</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>518.5</td>
<td>26.5</td>
<td>776.0</td>
<td>0.703</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>480.0</td>
<td>26.5</td>
<td>776.0</td>
<td>0.648</td>
<td>0.85</td>
</tr>
<tr>
<td>4</td>
<td>176.0</td>
<td>38.0</td>
<td>773.0</td>
<td>0.201</td>
<td>12.00</td>
</tr>
<tr>
<td></td>
<td>235.0</td>
<td>38.0</td>
<td>773.0</td>
<td>0.287</td>
<td>5.20</td>
</tr>
<tr>
<td></td>
<td>120.0</td>
<td>38.0</td>
<td>773.0</td>
<td>0.120</td>
<td>30.00</td>
</tr>
<tr>
<td></td>
<td>170.0</td>
<td>38.0</td>
<td>773.0</td>
<td>0.193</td>
<td>24.00</td>
</tr>
<tr>
<td></td>
<td>154.0</td>
<td>38.0</td>
<td>773.0</td>
<td>0.169</td>
<td>17.65</td>
</tr>
<tr>
<td></td>
<td>210.0</td>
<td>38.0</td>
<td>773.0</td>
<td>0.251</td>
<td>8.96</td>
</tr>
<tr>
<td></td>
<td>307.5</td>
<td>38.0</td>
<td>773.0</td>
<td>0.393</td>
<td>3.20</td>
</tr>
<tr>
<td></td>
<td>522.5</td>
<td>38.0</td>
<td>773.0</td>
<td>0.706</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>350.0</td>
<td>38.0</td>
<td>773.0</td>
<td>0.455</td>
<td>2.27</td>
</tr>
<tr>
<td></td>
<td>220.0</td>
<td>38.0</td>
<td>773.0</td>
<td>0.266</td>
<td>6.98</td>
</tr>
<tr>
<td>5</td>
<td>176.0</td>
<td>38.0</td>
<td>778.0</td>
<td>0.200</td>
<td>15.00</td>
</tr>
<tr>
<td>10</td>
<td>231.0</td>
<td>78.0</td>
<td>777.5</td>
<td>0.235</td>
<td>7.53</td>
</tr>
<tr>
<td></td>
<td>270.0</td>
<td>78.0</td>
<td>777.5</td>
<td>0.295</td>
<td>4.74</td>
</tr>
<tr>
<td></td>
<td>355.6</td>
<td>78.0</td>
<td>775.5</td>
<td>0.428</td>
<td>2.33</td>
</tr>
<tr>
<td></td>
<td>388.0</td>
<td>78.0</td>
<td>775.5</td>
<td>0.478</td>
<td>1.86</td>
</tr>
<tr>
<td>14</td>
<td>491.0</td>
<td>78.0</td>
<td>775.5</td>
<td>0.637</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>311.0</td>
<td>78.0</td>
<td>775.5</td>
<td>0.360</td>
<td>3.31</td>
</tr>
<tr>
<td></td>
<td>381.0</td>
<td>78.0</td>
<td>775.5</td>
<td>0.468</td>
<td>2.04</td>
</tr>
<tr>
<td></td>
<td>240.0</td>
<td>83.0</td>
<td>780.5</td>
<td>0.242</td>
<td>5.73</td>
</tr>
<tr>
<td></td>
<td>415.0</td>
<td>83.0</td>
<td>780.5</td>
<td>0.512</td>
<td>1.50</td>
</tr>
<tr>
<td></td>
<td>556.0</td>
<td>83.0</td>
<td>780.5</td>
<td>0.730</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>432.5</td>
<td>83.0</td>
<td>780.5</td>
<td>0.540</td>
<td>1.72</td>
</tr>
<tr>
<td></td>
<td>346.0</td>
<td>83.0</td>
<td>780.5</td>
<td>0.406</td>
<td>3.05</td>
</tr>
</tbody>
</table>

P = (P-P₀)/(P_{G}-P₀)

Water Vapor Pres. = 49.7 mm Hg

(Pulse Rate = 46 pulse/min)
<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>P  (mm Hg)</th>
<th>P₀ (mm Hg)</th>
<th>P_BAR (mm Hg)</th>
<th>P̅</th>
<th>Q (cc/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>291.0</td>
<td>83.0</td>
<td>780.5</td>
<td>0.321</td>
<td></td>
<td>5.18</td>
</tr>
<tr>
<td>212.5</td>
<td>83.0</td>
<td>780.5</td>
<td>0.200</td>
<td></td>
<td>13.44</td>
</tr>
<tr>
<td>(Pulse Rate = 80 pulse/min)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>480.0</td>
<td>89.0</td>
<td>774.8</td>
<td>0.615</td>
<td>1.76</td>
</tr>
<tr>
<td></td>
<td>325.0</td>
<td>89.0</td>
<td>774.8</td>
<td>0.371</td>
<td>4.62</td>
</tr>
<tr>
<td></td>
<td>275.0</td>
<td>89.0</td>
<td>774.8</td>
<td>0.292</td>
<td>10.17</td>
</tr>
<tr>
<td></td>
<td>400.0</td>
<td>89.0</td>
<td>774.8</td>
<td>0.489</td>
<td>2.47</td>
</tr>
<tr>
<td></td>
<td>375.0</td>
<td>89.0</td>
<td>774.8</td>
<td>0.450</td>
<td>3.19</td>
</tr>
<tr>
<td></td>
<td>340.0</td>
<td>89.0</td>
<td>774.8</td>
<td>0.394</td>
<td>5.45</td>
</tr>
<tr>
<td></td>
<td>300.0</td>
<td>89.0</td>
<td>774.8</td>
<td>0.332</td>
<td>7.14</td>
</tr>
<tr>
<td></td>
<td>370.0</td>
<td>89.0</td>
<td>774.8</td>
<td>0.442</td>
<td>3.33</td>
</tr>
<tr>
<td>(Pulse Rate = 4 Pulse/min)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>440.0</td>
<td>80.5</td>
<td>774.0</td>
<td>0.558</td>
<td>2.52</td>
</tr>
<tr>
<td></td>
<td>350.0</td>
<td>80.5</td>
<td>774.0</td>
<td>0.419</td>
<td>4.27</td>
</tr>
<tr>
<td></td>
<td>690.0</td>
<td>80.5</td>
<td>774.0</td>
<td>0.947</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>365.0</td>
<td>80.5</td>
<td>774.0</td>
<td>0.442</td>
<td>3.97</td>
</tr>
<tr>
<td></td>
<td>320.0</td>
<td>80.5</td>
<td>774.0</td>
<td>0.372</td>
<td>5.33</td>
</tr>
<tr>
<td></td>
<td>325.0</td>
<td>80.5</td>
<td>774.0</td>
<td>0.380</td>
<td>5.02</td>
</tr>
<tr>
<td></td>
<td>390.0</td>
<td>80.5</td>
<td>774.0</td>
<td>0.481</td>
<td>3.36</td>
</tr>
<tr>
<td></td>
<td>330.0</td>
<td>80.5</td>
<td>774.0</td>
<td>0.388</td>
<td>5.61</td>
</tr>
<tr>
<td></td>
<td>538.0</td>
<td>80.5</td>
<td>774.0</td>
<td>0.712</td>
<td>1.87</td>
</tr>
</tbody>
</table>
Table A-2

EXPERIMENTAL DATA FOR CARBON DIOXIDE REMOVAL FROM WATER FLOWING IN A PERMEABLE SILICONE TUBE

\[ L = 73.5 \text{ cm} \]
\[ R = 0.0317 \text{ cm} \]
\[ TM = 0.0279 \text{ cm} \]
\[ T = 38^\circ \text{C} \]
\[ P = \frac{(P-P_0)}{(P_g-P_0)} \]
\[ P_0 = \text{Inlet Partial Pressure CO}_2 \]
\[ P = \text{Outlet Partial Pressure CO}_2 \]
\[ \text{Water Vapor Press.} = 49.7 \text{mm Hg} \]

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Exp. P</th>
<th>Exp. P₀</th>
<th>\bar{P}</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mm Hg</td>
<td>mm Hg</td>
<td></td>
<td>cc/min</td>
</tr>
<tr>
<td>6</td>
<td>130.0</td>
<td>722.3</td>
<td>0.820</td>
<td>0.44</td>
</tr>
<tr>
<td>7</td>
<td>63.0</td>
<td>134.0</td>
<td>0.530</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>40.0</td>
<td>134.0</td>
<td>0.701</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>25.0</td>
<td>134.0</td>
<td>0.813</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>14.0</td>
<td>134.0</td>
<td>0.896</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>39.0</td>
<td>134.0</td>
<td>0.709</td>
<td>0.48</td>
</tr>
<tr>
<td>8</td>
<td>55.4</td>
<td>145.0</td>
<td>0.618</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>65.3</td>
<td>145.0</td>
<td>0.550</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>52.8</td>
<td>145.0</td>
<td>0.636</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>102.0</td>
<td>145.0</td>
<td>0.296</td>
<td>1.77</td>
</tr>
<tr>
<td></td>
<td>109.3</td>
<td>145.0</td>
<td>0.246</td>
<td>2.22</td>
</tr>
<tr>
<td></td>
<td>123.0</td>
<td>145.0</td>
<td>0.152</td>
<td>4.28</td>
</tr>
<tr>
<td></td>
<td>130.3</td>
<td>145.0</td>
<td>0.101</td>
<td>5.94</td>
</tr>
<tr>
<td></td>
<td>136.3</td>
<td>145.0</td>
<td>0.060</td>
<td>7.84</td>
</tr>
<tr>
<td></td>
<td>138.5</td>
<td>145.0</td>
<td>0.045</td>
<td>9.27</td>
</tr>
<tr>
<td></td>
<td>148.0</td>
<td>145.0</td>
<td>0.000</td>
<td>22.22</td>
</tr>
<tr>
<td></td>
<td>35.9</td>
<td>145.0</td>
<td>0.753</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>60.4</td>
<td>145.0</td>
<td>0.583</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>72.5</td>
<td>145.0</td>
<td>0.500</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>83.5</td>
<td>145.0</td>
<td>0.424</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td>110.0</td>
<td>145.0</td>
<td>0.241</td>
<td>3.00</td>
</tr>
<tr>
<td>10</td>
<td>48.5</td>
<td>60.5</td>
<td>0.198</td>
<td>3.82</td>
</tr>
<tr>
<td></td>
<td>51.0</td>
<td>60.5</td>
<td>0.157</td>
<td>4.74</td>
</tr>
<tr>
<td></td>
<td>46.0</td>
<td>60.5</td>
<td>0.240</td>
<td>2.33</td>
</tr>
<tr>
<td></td>
<td>43.9</td>
<td>60.5</td>
<td>0.274</td>
<td>1.86</td>
</tr>
<tr>
<td></td>
<td>35.6</td>
<td>60.5</td>
<td>0.412</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>52.3</td>
<td>60.5</td>
<td>0.135</td>
<td>3.31</td>
</tr>
<tr>
<td></td>
<td>48.0</td>
<td>60.5</td>
<td>0.207</td>
<td>2.04</td>
</tr>
<tr>
<td>11</td>
<td>45.3</td>
<td>50.0</td>
<td>0.093</td>
<td>5.73</td>
</tr>
<tr>
<td></td>
<td>33.7</td>
<td>50.0</td>
<td>0.325</td>
<td>1.50</td>
</tr>
<tr>
<td></td>
<td>24.1</td>
<td>50.0</td>
<td>0.518</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>36.6</td>
<td>50.0</td>
<td>0.268</td>
<td>1.72</td>
</tr>
</tbody>
</table>
Table A-2 (Contd.)

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>P mm Hg</th>
<th>P₀ mm Hg</th>
<th>P</th>
<th>Q cc/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>43.8</td>
<td>50.0</td>
<td>0.125</td>
<td>3.05</td>
<td></td>
</tr>
<tr>
<td>47.3</td>
<td>50.0</td>
<td>0.054</td>
<td>5.18</td>
<td></td>
</tr>
</tbody>
</table>

(Pulse Rate = 80 pulse/min)

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>P mm Hg</th>
<th>P₀ mm Hg</th>
<th>P</th>
<th>Q cc/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>135.0</td>
<td>142.0</td>
<td>0.049</td>
<td>13.44</td>
</tr>
<tr>
<td>12</td>
<td>38.0</td>
<td>57.0</td>
<td>0.333</td>
<td>1.71</td>
</tr>
<tr>
<td></td>
<td>50.0</td>
<td>57.0</td>
<td>0.123</td>
<td>4.48</td>
</tr>
<tr>
<td></td>
<td>56.0</td>
<td>57.0</td>
<td>0.018</td>
<td>9.70</td>
</tr>
<tr>
<td></td>
<td>50.0</td>
<td>57.0</td>
<td>0.123</td>
<td>3.70</td>
</tr>
<tr>
<td></td>
<td>46.0</td>
<td>57.0</td>
<td>0.193</td>
<td>2.24</td>
</tr>
<tr>
<td></td>
<td>48.5</td>
<td>57.0</td>
<td>0.149</td>
<td>3.11</td>
</tr>
<tr>
<td></td>
<td>54.0</td>
<td>57.0</td>
<td>0.053</td>
<td>5.31</td>
</tr>
<tr>
<td></td>
<td>47.0</td>
<td>57.0</td>
<td>0.000</td>
<td>5.66</td>
</tr>
<tr>
<td></td>
<td>51.0</td>
<td>57.0</td>
<td>0.105</td>
<td>3.26</td>
</tr>
</tbody>
</table>

(Pulse Rate = 4 pulse/min)

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>P mm Hg</th>
<th>P₀ mm Hg</th>
<th>P</th>
<th>Q cc/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>44.3</td>
<td>50.8</td>
<td>0.126</td>
<td>5.81</td>
</tr>
<tr>
<td></td>
<td>41.8</td>
<td>50.8</td>
<td>0.176</td>
<td>5.42</td>
</tr>
<tr>
<td></td>
<td>40.0</td>
<td>50.8</td>
<td>0.212</td>
<td>3.08</td>
</tr>
<tr>
<td></td>
<td>37.2</td>
<td>50.8</td>
<td>0.268</td>
<td>1.91</td>
</tr>
<tr>
<td></td>
<td>31.7</td>
<td>50.8</td>
<td>0.376</td>
<td>1.33</td>
</tr>
<tr>
<td></td>
<td>24.3</td>
<td>50.8</td>
<td>0.522</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>12.6</td>
<td>50.8</td>
<td>0.752</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>16.2</td>
<td>50.8</td>
<td>0.681</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>47.0</td>
<td>50.8</td>
<td>0.074</td>
<td>5.04</td>
</tr>
<tr>
<td></td>
<td>37.7</td>
<td>50.8</td>
<td>0.258</td>
<td>3.28</td>
</tr>
<tr>
<td></td>
<td>17.1</td>
<td>50.8</td>
<td>0.663</td>
<td>0.65</td>
</tr>
<tr>
<td>15</td>
<td>45.0</td>
<td>54.3</td>
<td>0.171</td>
<td>5.34</td>
</tr>
<tr>
<td></td>
<td>38.2</td>
<td>54.3</td>
<td>0.296</td>
<td>2.42</td>
</tr>
<tr>
<td></td>
<td>19.9</td>
<td>54.3</td>
<td>0.633</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>35.9</td>
<td>54.3</td>
<td>0.339</td>
<td>1.94</td>
</tr>
<tr>
<td></td>
<td>17.3</td>
<td>54.3</td>
<td>0.681</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>24.4</td>
<td>54.3</td>
<td>0.551</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>36.6</td>
<td>54.3</td>
<td>0.326</td>
<td>2.81</td>
</tr>
<tr>
<td></td>
<td>15.4</td>
<td>54.3</td>
<td>0.717</td>
<td>0.64</td>
</tr>
</tbody>
</table>
### Table A-3

**Experimental Data for Oxygen Absorption by Cattle Blood with Simultaneous Carbon Dioxide Desorption**

- \( L = 73.5 \text{cm} \)
- \( R = 0.0315 \text{cm} \)
- \( TM = 0.0279 \text{cm} \)
- \( T = 38^\circ \text{C} \)
- \( H = 40.0\% \)
- \( C_{\text{Hb}} = 0.123 \text{ g/cc} \)
- \( P_{\text{BAR}} = 771.8 \text{ mm Hg} \)
- Water Vapor Press. = 49.7 mm Hg
- Pulse/min = 46
- \( pH = 7.20 \)

\[ P = (P-P_0)/(P_{g}-P_0) \]

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>( P ) (mm Hg)</th>
<th>( P_0 ) (mm Hg)</th>
<th>( \bar{P} )</th>
<th>( Q ) (cc/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>82.0</td>
<td>58.5</td>
<td>0.035</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>150.0</td>
<td>58.5</td>
<td>0.138</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>90.0</td>
<td>58.5</td>
<td>0.048</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td>81.0</td>
<td>58.5</td>
<td>0.034</td>
<td>2.66</td>
</tr>
<tr>
<td></td>
<td>72.5</td>
<td>58.5</td>
<td>0.021</td>
<td>4.40</td>
</tr>
<tr>
<td></td>
<td>67.3</td>
<td>58.5</td>
<td>0.013</td>
<td>9.19</td>
</tr>
<tr>
<td></td>
<td>179.0</td>
<td>58.5</td>
<td>0.182</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>88.0</td>
<td>68.0</td>
<td>0.031</td>
<td>6.47</td>
</tr>
<tr>
<td></td>
<td>93.0</td>
<td>68.0</td>
<td>0.038</td>
<td>5.70</td>
</tr>
<tr>
<td></td>
<td>112.0</td>
<td>68.0</td>
<td>0.067</td>
<td>2.36</td>
</tr>
<tr>
<td></td>
<td>153.0</td>
<td>68.0</td>
<td>0.130</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>113.0</td>
<td>68.0</td>
<td>0.069</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td>107.0</td>
<td>68.0</td>
<td>0.060</td>
<td>2.49</td>
</tr>
</tbody>
</table>
Table A-4

EXPERIMENTAL DATA FOR OXYGEN ABSORPTION BY HUMAN BLOOD
WITH SIMULTANEOUS CARBON DIOXIDE DESORPTION, DONOR M-1

$L = 32.86\text{cm}$
$R = 0.0315\text{cm}$
$TM = 0.0279\text{cm}$
$P = $ Outlet Partial Pressure $O_2$
$P_o = $ Inlet Partial Pressure $O_2$
$\bar{P} = (P-P_o)/(P_g-P_o)$
$T = 38^\circ C$
$H = 35.0\%$
$C_{Hb} = 0.140 \text{ g/cc}$
$P_{BAR} = 774.1 \text{ mm Hg}$
$\text{Water Vapor Press.} = 49.7 \text{ mm Hg}$
$\text{Pulse/min} = 46$
$pH = 7.11$

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>$P$ (mm Hg)</th>
<th>$P_o$ (mm Hg)</th>
<th>$\bar{P}$</th>
<th>$Q$ (cc/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>52.0</td>
<td>39.0</td>
<td>0.019</td>
<td>1.81</td>
</tr>
<tr>
<td></td>
<td>53.0</td>
<td>39.0</td>
<td>0.020</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>54.0</td>
<td>39.0</td>
<td>0.022</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>51.0</td>
<td>39.0</td>
<td>0.018</td>
<td>1.68</td>
</tr>
<tr>
<td></td>
<td>54.0</td>
<td>39.0</td>
<td>0.022</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>64.0</td>
<td>39.0</td>
<td>0.036</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>161.0</td>
<td>39.0</td>
<td>0.178</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>163.5</td>
<td>39.0</td>
<td>0.182</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>174.0</td>
<td>39.0</td>
<td>0.197</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>54.0</td>
<td>39.0</td>
<td>0.089</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>100.0</td>
<td>39.0</td>
<td>0.133</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>130.0</td>
<td>39.0</td>
<td>0.139</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>134.5</td>
<td>39.0</td>
<td>0.088</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>99.5</td>
<td>39.0</td>
<td>0.039</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>66.0</td>
<td>39.0</td>
<td>0.005</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>42.5</td>
<td>39.0</td>
<td>0.026</td>
<td>1.37</td>
</tr>
<tr>
<td></td>
<td>57.0</td>
<td>39.0</td>
<td>0.007</td>
<td>3.54</td>
</tr>
<tr>
<td></td>
<td>44.0</td>
<td>39.0</td>
<td>0.010</td>
<td>2.79</td>
</tr>
<tr>
<td></td>
<td>46.0</td>
<td>39.0</td>
<td>0.009</td>
<td>4.29</td>
</tr>
<tr>
<td></td>
<td>45.0</td>
<td>39.0</td>
<td>0.007</td>
<td>5.26</td>
</tr>
<tr>
<td></td>
<td>44.0</td>
<td>39.0</td>
<td>0.018</td>
<td>2.06</td>
</tr>
<tr>
<td></td>
<td>51.0</td>
<td>39.0</td>
<td>0.054</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>76.0</td>
<td>39.0</td>
<td>0.018</td>
<td>1.37</td>
</tr>
<tr>
<td></td>
<td>51.3</td>
<td>39.0</td>
<td>0.040</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>66.5</td>
<td>39.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exp. No.</td>
<td>P</td>
<td>P₀</td>
<td>㏉</td>
<td>Q</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
<td>------</td>
<td>----</td>
<td>------</td>
</tr>
<tr>
<td></td>
<td>mm Hg</td>
<td>mm Hg</td>
<td></td>
<td>cc/min</td>
</tr>
<tr>
<td>18</td>
<td>62.0</td>
<td>47.0</td>
<td>0.022</td>
<td>1.39</td>
</tr>
<tr>
<td></td>
<td>54.5</td>
<td>47.0</td>
<td>0.011</td>
<td>2.91</td>
</tr>
<tr>
<td></td>
<td>47.0</td>
<td>47.0</td>
<td>0.000</td>
<td>6.82</td>
</tr>
<tr>
<td></td>
<td>158.0</td>
<td>47.0</td>
<td>0.163</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>49.0</td>
<td>47.0</td>
<td>0.033</td>
<td>4.07</td>
</tr>
<tr>
<td></td>
<td>47.8</td>
<td>47.0</td>
<td>0.001</td>
<td>4.62</td>
</tr>
<tr>
<td></td>
<td>58.8</td>
<td>47.0</td>
<td>0.017</td>
<td>2.58</td>
</tr>
<tr>
<td></td>
<td>56.0</td>
<td>47.0</td>
<td>0.013</td>
<td>1.42</td>
</tr>
<tr>
<td></td>
<td>48.2</td>
<td>47.0</td>
<td>0.020</td>
<td>3.40</td>
</tr>
<tr>
<td></td>
<td>46.0</td>
<td>47.0</td>
<td>0.000</td>
<td>2.32</td>
</tr>
<tr>
<td></td>
<td>63.5</td>
<td>47.0</td>
<td>0.024</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td>78.0</td>
<td>47.0</td>
<td>0.046</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>62.0</td>
<td>47.0</td>
<td>0.022</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td>91.5</td>
<td>47.0</td>
<td>0.065</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>140.5</td>
<td>47.0</td>
<td>0.107</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>80.0</td>
<td>47.0</td>
<td>0.048</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>36.5</td>
<td>36.5</td>
<td>0.000</td>
<td>7.20</td>
</tr>
<tr>
<td></td>
<td>40.0</td>
<td>36.5</td>
<td>0.005</td>
<td>2.71</td>
</tr>
<tr>
<td></td>
<td>58.6</td>
<td>36.5</td>
<td>0.032</td>
<td>1.49</td>
</tr>
<tr>
<td></td>
<td>45.0</td>
<td>36.5</td>
<td>0.012</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>78.0</td>
<td>36.5</td>
<td>0.060</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>42.5</td>
<td>36.5</td>
<td>0.010</td>
<td>6.87</td>
</tr>
<tr>
<td></td>
<td>47.5</td>
<td>36.5</td>
<td>0.016</td>
<td>2.12</td>
</tr>
<tr>
<td></td>
<td>56.0</td>
<td>36.5</td>
<td>0.028</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>290.0</td>
<td>36.5</td>
<td>0.367</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>68.0</td>
<td>36.5</td>
<td>0.047</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>69.0</td>
<td>28.5</td>
<td>0.058</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>43.0</td>
<td>28.5</td>
<td>0.021</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>50.0</td>
<td>28.5</td>
<td>0.031</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td>38.3</td>
<td>28.5</td>
<td>0.014</td>
<td>3.90</td>
</tr>
<tr>
<td></td>
<td>42.0</td>
<td>28.5</td>
<td>0.019</td>
<td>1.68</td>
</tr>
</tbody>
</table>
Table A-6

EXPERIMENTAL DATA FOR CARBON DIOXIDE DESORPTION FROM BLOOD WITH SIMULTANEOUS OXYGEN ABSORPTION, DONOR M-1

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>P (mm Hg)</th>
<th>P₀ (mm Hg)</th>
<th>( \bar{P} )</th>
<th>Q (cc/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>49.5</td>
<td>59.72</td>
<td>0.171</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>50.5</td>
<td>59.72</td>
<td>0.154</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>52.5</td>
<td>59.72</td>
<td>0.121</td>
<td>1.37</td>
</tr>
<tr>
<td></td>
<td>58.0</td>
<td>59.72</td>
<td>0.029</td>
<td>3.45</td>
</tr>
<tr>
<td></td>
<td>52.0</td>
<td>59.72</td>
<td>0.129</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>54.5</td>
<td>59.72</td>
<td>0.087</td>
<td>1.37</td>
</tr>
<tr>
<td></td>
<td>50.0</td>
<td>59.72</td>
<td>0.163</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>57.0</td>
<td>59.72</td>
<td>0.046</td>
<td>1.81</td>
</tr>
<tr>
<td></td>
<td>56.0</td>
<td>59.72</td>
<td>0.062</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>55.0</td>
<td>59.72</td>
<td>0.079</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>56.0</td>
<td>59.72</td>
<td>0.062</td>
<td>1.68</td>
</tr>
<tr>
<td></td>
<td>52.5</td>
<td>59.72</td>
<td>0.121</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>50.5</td>
<td>59.72</td>
<td>0.154</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>47.8</td>
<td>59.72</td>
<td>0.200</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>47.0</td>
<td>59.72</td>
<td>0.213</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>55.8</td>
<td>59.72</td>
<td>0.066</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>52.0</td>
<td>59.72</td>
<td>0.129</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>51.0</td>
<td>59.72</td>
<td>0.146</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>50.3</td>
<td>59.72</td>
<td>0.158</td>
<td>0.42</td>
</tr>
</tbody>
</table>
Table A-7

EXPERIMENTAL DATA FOR CARBON DIOXIDE DESORPTION FROM BLOOD WITH SIMULTANEOUS OXYGEN ABSORPTION, DONOR K-1

\[ L = 33.02 \text{cm} \]
\[ R = 0.0317 \text{cm} \]
\[ TM = 0.0279 \text{cm} \]
\[ T = 38^\circ \text{C} \]
\[ H = 43.8\% \]
\[ C = 0.143 \text{ g/cc} \]
\[ P_{\text{BAR}} = 776.6 \text{ mm Hg} \]
\[ \text{Water Vapor Press.} = 49.7 \text{ mm Hg} \]
\[ \text{Pulse/min} = 46 \]
\[ pH = 7.00 \]

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>(P) mm Hg</th>
<th>(P_0) mm Hg</th>
<th>(\bar{P})</th>
<th>Q cc/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>69.0</td>
<td>69.0</td>
<td>0.000</td>
<td>7.20</td>
</tr>
<tr>
<td></td>
<td>69.0</td>
<td>69.0</td>
<td>0.000</td>
<td>2.71</td>
</tr>
<tr>
<td></td>
<td>67.8</td>
<td>68.0</td>
<td>0.003</td>
<td>1.49</td>
</tr>
<tr>
<td></td>
<td>60.0</td>
<td>68.0</td>
<td>0.118</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>61.3</td>
<td>68.0</td>
<td>0.099</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>68.0</td>
<td>68.0</td>
<td>0.000</td>
<td>6.87</td>
</tr>
<tr>
<td></td>
<td>68.0</td>
<td>68.0</td>
<td>0.000</td>
<td>2.12</td>
</tr>
<tr>
<td></td>
<td>63.0</td>
<td>68.0</td>
<td>0.074</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>36.0</td>
<td>68.0</td>
<td>0.471</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>63.5</td>
<td>68.0</td>
<td>0.066</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>68.0</td>
<td>68.0</td>
<td>0.000</td>
<td>4.07</td>
</tr>
<tr>
<td></td>
<td>62.0</td>
<td>62.0</td>
<td>0.000</td>
<td>4.62</td>
</tr>
<tr>
<td></td>
<td>62.0</td>
<td>62.0</td>
<td>0.000</td>
<td>2.58</td>
</tr>
<tr>
<td></td>
<td>60.0</td>
<td>62.0</td>
<td>0.032</td>
<td>1.42</td>
</tr>
<tr>
<td></td>
<td>61.0</td>
<td>62.0</td>
<td>0.016</td>
<td>2.32</td>
</tr>
<tr>
<td></td>
<td>60.0</td>
<td>62.0</td>
<td>0.032</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td>59.5</td>
<td>62.0</td>
<td>0.040</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>59.5</td>
<td>62.0</td>
<td>0.040</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td>51.0</td>
<td>62.0</td>
<td>0.177</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>48.0</td>
<td>62.0</td>
<td>0.226</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>55.9</td>
<td>62.0</td>
<td>0.098</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>57.0</td>
<td>62.0</td>
<td>0.081</td>
<td>1.39</td>
</tr>
<tr>
<td></td>
<td>61.0</td>
<td>62.0</td>
<td>0.016</td>
<td>2.91</td>
</tr>
<tr>
<td></td>
<td>62.0</td>
<td>62.0</td>
<td>0.000</td>
<td>6.82</td>
</tr>
<tr>
<td></td>
<td>46.0</td>
<td>62.0</td>
<td>0.258</td>
<td>0.20</td>
</tr>
</tbody>
</table>
APPENDIX B

LINEAR GRAPHS OF EXPERIMENTAL DATA
Figure B-1. Oxygen Absorption by Distilled Water Flowing in a Permeable Silicone Tube.

Constants

\[ T = 38^\circ C \]
\[ L = 73.5 \text{ cm} \]
\[ R = 0.0317 \text{ cm} \]
\[ TM = 0.0279 \text{ cm} \]
\[ \text{Pulse/min} = 46 \]
Figure B-2. Oxygen Absorption by Distilled Water Flowing in a Permeable Silicone Tube.

Constants

$T = 38^\circ C$
$L = 73.5 \text{ cm}$
$R = 0.0317 \text{ cm}$
$TM = 0.0279 \text{ cm}$
Pulse/min = 80
Figure B-3. Oxygen Absorption by Distilled Water Flowing in a Permeable Silicone Tube.

Constants

\( T = 38^\circ C \)
\( L = 73.5 \text{ cm} \)
\( R = 0.0317 \text{ cm} \)
\( TM = 0.0279 \text{ cm} \)
\( \text{Pulse/min} = 4 \)
Figure B-4. Carbon Dioxide Desorption From Distilled Water Flowing in a Permeable Silicone Tube.

Constants

\[ T = 38^\circ C \]
\[ L = 73.5 \text{ cm} \]
\[ R = 0.0317 \text{ cm} \]
\[ TM = 0.0279 \text{ cm} \]
\[ \text{Pulse/min} = 46 \]
Figure B-5. Carbon Dioxide Desorption From Distilled Water Flowing in a Permeable Silicone Tube.

Constants

\[ T = 38^\circ C \]
\[ L = 73.5 \text{ cm} \]
\[ R = 0.0317 \text{ cm} \]
\[ TM = 0.0279 \text{ cm} \]
\[ \text{Pulse/min} = 80 \]
Figure B-6. Carbon Dioxide Desorption From Distilled Water Flowing in a Permeable Silicone Tube.

Constants

$T = 38^\circ C$
$L = 73.5 \text{ cm}$
$R = 0.0317 \text{ cm}$
$TM = 0.0279 \text{ cm}$
$Pulse/min = 4$
Figure B-7. Oxygen Absorption in the Presence of Simultaneous Carbon Dioxide Desorption in Cattle Blood Flowing in a Permeable Silicone Tube.

Constants

\[ T = 38^\circ C \]
\[ L = 73.5 \text{ cm} \]
\[ R = 0.0317 \text{ cm} \]
\[ TM = 0.0279 \text{ cm} \]
\[ H = 40.0\% \]
\[ C_{Hb} = 0.123 \text{ g/cc} \]
Figure B-8. Oxygen Absorption in the Presence of Simultaneous Carbon Dioxide Desorption in Human Blood Flowing in a Permeable Silicone Tube.

Constants

\[ T = 38^\circ C \]
\[ L = 32.86 \text{ cm} \]
\[ R = 0.0315 \text{ cm} \]
\[ TM = 0.0279 \text{ cm} \]
\[ H = 35.0\% \]
\[ C_{Hb} = 0.140 \text{ g/cc} \]

Donor M -1
Figure B-9. Carbon Dioxide Desorption in the Presence of Simultaneous Oxygen Absorption in Human Blood Flowing in a Permeable Silicone Tube.

Constants

\[ T = 38^\circ C \]
\[ L = 32.86 \text{ cm} \]
\[ R = 0.0317 \text{ cm} \]
\[ TM = 0.0279 \text{ cm} \]
\[ H = 35.0\% \]
\[ C_{Hb} = 0.140 \text{ g/cc} \]
Figure B-10. Oxygen Absorption in the Presence of Simultaneous Carbon Dioxide Desorption in Human Blood Flowing in a Permeable Silicone Tube.

Constants

\[ T = 38^\circ C \]
\[ L = 33.02 \text{ cm} \]
\[ R = 0.0317 \text{ cm} \]
\[ TM = 0.0279 \text{ cm} \]
\[ H = 43.8\% \]
\[ C_{\text{Hb}} = 0.143 \text{ g/cc} \]

Donor K-1
Figure B-11. Carbon Dioxide Desorption in the Presence of Oxygen Absorption in Human Blood Flowing in a Permeable Silicone Tube.

Constants

\begin{align*}
T &= 38^\circ\text{C} \\
L &= 33.02 \text{ cm} \\
R &= 0.0317 \text{ cm} \\
TM &= 0.0279 \text{ cm} \\
H &= 43.8\% \\
C_{Hb} &= 0.143 \text{ g/cc} \\
\text{Donor K-1} &
\end{align*}
APPENDIX C

COMPUTER PROGRAM FOR THE CALCULATION OF
OXYGEN AND CARBON DIOXIDE PARTIAL PRESSURE:
PROFILES IN BLOOD FLOWING IN A PERMEABLE TUBE.

In this appendix the flow diagram and computer program are given
for the numerical solution to Equation (III-17) and Equation (III-46).
In addition to the flow diagram, comment cards are used throughout
the program for additional clarity.
START

READ BOUNDARY CONDITIONS AND PHYSICAL DATA

PRINT INPUT DATA

ASSUME RADIAL PROFILE I

CALCULATE RADIAL PROFILE FOR LONGITUDINAL STEP I BASED ON I-1 STEP AND ASSUMED I PROFILE

IS ASSUMED PROFILE EQUAL TO CALCULATED?

YES

HAS LONGITUDINAL PROFILE REACHED THE END OF TUBE?

YES

PRINT
1. PARTIAL PRESSURE PROFILE
2. CONCENTRATION
3. RESPIRATORY QUOTIENT

RETURN

STOP

NO

USE CALCULATED PROFILE FOR SECOND ASSUMPTION

NO

I = I + 1

Figure C-1. Program Flow Chart
BRADLEY-CRANK-NICHOLSON NUMERICAL SOLUTIONS FOR THE LONGITUDINAL AND RADIAL PROFILES OF OXYGEN AND CARBON DIOXIDE FOR BLOOD FLOWING IN A PERMEABLE SILASTIC SILICONE RUBBER TUBE. THE SOLUTION IS ALSO VALID FOR WATER WHEN THE CHEMICAL COMBINATION TERMS ARE SET TO ZERO.

SYMBOL DEFINITION

- **A** - NORMALIZED LENGTH
- **ALPB** - SOLUBILITY OF GAS IN THE BLOOD
- **ALPM** - SOLUBILITY OF GAS IN THE MEMBRANE
- **BAR** - BARCHETRIC PRESSURE
- **B(JI)** - GAS PARTIAL PRESSURE IN BLOOD
- **CHB** - HEMOGLOBIN CONCENTRATION
- **DELA** - LENGTH INCREMENT
- **DELS** - CHANGE IN S BETWEEN INTERVALS
- **DELY** - RADIAL INCREMENT
- **DO2B** - DIFFUSIVITY OF GAS IN THE BLOOD
- **DO2M** - DIFFUSIVITY OF GAS IN THE MEMBRANE
- **DSDP** - PARTIAL OF S WITH RESPECT TO P
- **I** - SUBSCRIPT FOR A DIRECTION
- **IIN** - A COUNTER
- **J** - SUBSCRIPT FOR Y DIRECTION
- **KKK** - A COUNTER
- **KTF** - A COUNTER
- **MC** - NUMBER OF GRID POINTS ON Y
- **NC** - NUMBER OF GRID POINTS ON A
- **NTF** - A COUNTER
- **NTT** - A COUNTER
- **P(JI)** - DIMENSIONLESS PARTIAL PRESSURE
- **PINT** - PARTIAL PRESSURE BLOOD ENTERING TUBE
- **PTOT** - GAS PARTIAL PRESSURE IN GAS PHASE
- **R** - INSIDE RADIUS OF TUBE
- **S** - SATURATION

DIMENSIONS

- **NCNE**
- **CC GAS/CCB*ATM**
- **CC GAS/CCM*ATM**
- **MM HG**
- **MM HG**
- **GRAM/CC BLCCD**
- **NCNE**
- **FRACTION**
- **NONE**
- **CM**
- **CM**
- **MM HG**
- **NONE**
- **FRACTION**
CALL FPTRAP(-3)
DIMENSION XTC(80), XTCC(EC)
DIMENSION SI(80)
DIMENSION NB(40,80)
DIMENSION PAVG(80), CELS(40,80)
DIMENSION P(80,80), PNEW(8C), ZP(40,80)
DIMENSION XM(80), SOLC(8C)

44 IIN=0
555 KKK=0
YK=0.0
READ86, TM, PIOT, TEMP
READ87, NC, MC
READ88, FIAT, CHB, BAR
READ90, ALPM, ALPB, D02M, CC2B
READ91, XL, R, XMK
READ89, VCL, VEL, WATV
86 FORMAT(3E2C.8)
87 FORMAT(ZI6)
88 FORMAT(3E2C.8)
89 FORMAT(3E2C.8)
90 FORMAT(4E15.5)
91 FORMAT(3E20.8)
   IF(IIN.EQ.1) GO TO 45
   IF(KKK-1)52,53,53
C
C CORRECT GAS PHASE PARTIAL PRESSURE FOR WATER VAPOR PRESSURE
C
52 PTOT=PTCT-WATV
C
C PUT SOLUBILITIES IN TERMS CF MM MERCURY
C
45 ALPM=ALFP/760.0
   ALPB=ALFB/760.C
53 KKK=KKK+1
C
C PRINT CUT INPUT DATA
C
   PRINT101,NC,MC
1C1 FORMAT(1X,2CHLONGITUCINAL POINTS=,I3,1X,14HRADIAL POINTS=,I3/)
   IF(IIN.EQ.1) GC TO 112
   PRINT 1C3,PRINT,TEMP,PHAT,Vol,XMK
103 FORMAT(1X,33HINITIAL BLOOD OXYGEN PRESSURE =,F2C,8,2X,5HM+ HG///
11X,33HTEMPERATURE =,F2O.8,2X,5HDEG C///
21X,33HWATER VAPOR PRESSURE =,F2O.8,2X,5HM+ MG///
31X,33HVOLUMETRIC FLOW RATE =,F2O.8,2X,6HCC/SEC///
41X,33HARGARIA EQUATION CONSTANT =,F2O.8///)
   PRINT1C2,BAR,PTOT,CHB,R,TM,DO2B,DO2M,ALPB,ALPM,XL
1C2 FORMAT(1X,33HBAROMETRIC PRESSURE =,F2O.8,2X,5HM+ HG///
11X,33HGAS PHASE P.P. CF OXYGEN =,F2O.8,2X,5HM+ HG///
21X,33HHEMOCYLOBIN CONCENTRATION =,F2O.8,2X,7HGRAM/CC///
31X,33HINSIDE RADIUS =,F2O.8,2X,2HCM///
41X,33HMEMBRANE THICKNESS =,F2O.8,2X,2HCM///
51X,33HCXYGEN DIFFUSIVITY IN BLOOD =,F2O.8,2X,9HCM**2/SEC///
61X,33HCXYGEN DIFFUSIVITY IN MEMBRANE =,F2O.8,2X,9HCM**2/SEC///
Non Non Non

71X, 33H SCLABILITY OF OXYGEN IN BLOOD =, F20.8, 2X, 9HCC/CC*MHG///
81X, 33H SCLABILITY OF OXYGEN IN MEMBRANE =, F20.8, 2X, 9HCC/CC*MHG///
91X, 33H TLBE LENGTH =, F20.8, 2X, 2HCM///)

112 IF(INC=0) GO TO 113
PRINT 110, PINT, TEMP, WATV, VOL, XMK
110 FORMAT(1X, 33H INITIAL BLCC CO2 PRESSLRE =, F20.8, 2X, 5HMM HG///
11X, 33H TEMPERATURE =, F20.8, 2X, 5HDEG C///
21X, 33H WATER VAPOR PRESSLRE =, F20.8, 2X, 5HM MG///
31X, 33H VOLUMETRIC FLOW RATE =, F20.8, 2X, 6HCC/SEC///
41X, 33H MARGARIA EQUATION CONSTANT =, F20.8///)
PRINT111, BAR, PCTC, CHE, R, TM, DO2B, DO2M, ALPB, ALPM, XL
111 FORMAT(1X, 33H BAROMETRIC PRESSLRE =, F20.8, 2X, 5HMM HG///
11X, 33H GAS P.P. OF CO2 =, F20.8, 2X, 5HM MG///
21X, 33H HEMOGLOBIN CONCENTRATION =, F20.8, 2X, 7HGRAM/CC///
31X, 33H TLBE INSIDE RADIUS =, F20.8, 2X, 2HCM///
41X, 33H MEMBRANE THICKNESS =, F20.8, 2X, 2HCM///
51X, 33H CO2 DIFFUSIVITY IN BLCCD =, F20.8, 2X, 9HCM**2/SEC///
61X, 33H CO2 DIFFUSIVITY IN MEMBRANE =, F20.8, 2X, 9HCM**2/SEC///
71X, 33H SCLABILITY OF CO2 IN BLCCD =, F20.8, 2X, 9HCC/CC*MHG///
81X, 33H SCLABILITY OF CO2 IN MEMBRANE =, F20.8, 2X, 9HCC/CC*MHG///
91X, 33H TLBE LENGTH =, F20.8, 2X, 2HCM///)

113 CONTINUE
C
C SET FIXED FCINT VARIABLES TO FLOATING POINT VARIABLES
C
XC=NC
NCC=NC-1
XCC=NCC
MCC=MC-1
C
C CALCULATE THE ACNDIMENSIONALIZED LENGTH INCREMENT
C
DELA=1.C/XCC
GC=MCC
CALCULATE THE NONDIMENSIONALIZED RADIAL INCREMENT

\[ DLY = 1.0 / \sqrt{C} \]

GENERATE A FLAT CONCENTRATION PROFILE FOR THE ENTRANCE CONDITIONS

\[ PBP = PCT - PINT \]
\[ \text{DO 5 } N = 1, MC \]
\[ P(N,1) = C \cdot \text{CO200000} \]
\[ B(N,1) = F(N,1) \cdot PBP + PINT \]
\[ ZZZ = \frac{(1.0 + XMK \cdot B(N,1))}{(XMK \cdot B(N,1))} \]
\[ ZZZZ = \frac{(1.0 + XMK \cdot B(N,1))}{(XMK \cdot B(N,1))} \]
\[ \text{SOLD}(N) = \frac{(ZZZ + 124.0)}{(ZZZZ + 124.0)} \]
\[ 5 \text{ CONTINUE} \]

NTF = 0

THE VARIABLE VAL REPRESENTS THE CHANGE FROM PLUG TO PARABOLIC FLOW

\[ \text{VAL} = VCL \cdot 2.0 \]

THE OUTER LCOP GENERATES THE LONGITUDINAL PROFILE

THE INNER LCOP GENERATES THE RADIAL PROFILE

KTF = 0

1CC DO 2 I = 1, NCC

\[ NIT = 0 \]

25 DO 3 J = 2, NCC

XJ IS INTRODUCED TO PREVENT A ZERO SUBSCRIPT FROM FORMING

\[ XJ = J - 1 \]
\[ \text{IF(IIN.EQ.1) GO TO 3C} \]
C ZZZZ ZZZ ZZ ZPZ ARE USED TO CALCULATE THE SATURATION
C

B(J,I)=F(J,I)*PB*PIAT
ZZZZ=((1.0*XM*BM(J,I))/(XM*BM(J,I)))*4
ZZ=((1.0*XM*BM(J,I))/(XM*BM(J,I)))*3
ZZ=((1.0*XM*BM(J,I))/(XM*BM(J,I)))*2
ZPZ=((1.0*XM*BM(J,I))/(XM*BM(J,I)))*2
S=(ZZZ+124.0)/(ZZZ+124.0)
DELS(J,I+1)=S-SOLD(J-1)
SOLD(J)=S
CSDP=((ZZZ+124.0)*3.0*(ZPZ-(ZZZ+124.0)*4.0*(ZZZ)*ZPZ)/{1
ZZZZ+124.0})*2
C
C VEL REPRESENTS THE NEWTONIAN PROFILE
C
VEL=(1.0-(CELY*XJ)**2)
THETA=CC2B*XL*3.1417/(VAL*VEL*(1.0+1.34*CHB*DSDP/ALPB))
30 IF(II IN .EQ. .C) GO TO 31
VEL=(1.0-(CELY*XJ)**2)
THETA=(ALPB*XL*D02B*3.14)/(0.0456*VAL*VEL)
31 CONTINUE
C
C AND F ARE PART OF THE GENERAL NUMERICAL SOLUTION FOUND IN THE THEORY
C
G=(1.0+(CELA*THETA/DELY**2)-(CELA*THETA/(2.0*DELY**2*XJ)))
F=(CELA*THETA/(2.0*DELY**2))
NTF=1+NTF
IF(NTF-1)17,17,21
C
C AS A FIRST ASSUMPTION LET THE I+1 VALUE EQUAL THE I VALUE
C FOR ALL J
C
17 CC18 A=1,MC
P(N,I+1)=P(A,I)
THE VALUE OF P(N,I+1) AS A ZP VARIABLE SO THAT IT MAY BE USED LATER

ZP(N,I+1)=P(N,I+1)

CONTINUE

NTT=NTT+1

IF(NTT-1)72,72,73

CONTINUE

CALCULATE THE VALUE FOR THE CENTER LINE ONLY WHEN I IN Equals ZERO THE O2 PROFILE IS CALCULATED
WHEN I IN BECOMES ONE THE PROGRAM CALCULATES THE CO2 PROFILES

P(1,I+1)=P(1,I)/(1.0+4.0*C*F)+4.0*F*(P(2,I+1)+P(2,I)-P(1,I))/(1.0+
1.0*F)

IF(IIN.EQ.C) GO TO 32
P(1,I+1)=P(1,I)/(1.0+4.0*C*F)+4.0*F*(P(2,I+1)+P(2,I)-P(1,I))/(1.0+
1.0*F)-(16.4*ALPB*DELS(1,I+1))/(1.0+4.0*F)*VEL)

CONTINUE

CONTINUE

IF(IIN.EQ.1) GO TO 34

GENERATE THE INTERNAL POINTS FOR THE Racial PROFILE

PNEW(J)=F/G*(P(J+1,I+1)+P(J-1,I+1)*(1.0-1.0/XJ)+P(J-1,I)*(1.0-
1.0/XJ)-P(J,I)*(2.0-1.0/XJ)+P(J+1,I))/G

IF(IIN.EQ.C) GO TO 35
PNEW(J)=F/G*(P(J+1,I+1)+P(J-1,I+1)*(1.0-1.0/XJ)+P(J-1,I)*(1.0-
1.0/XJ)-P(J,I)*(2.0-1.0/XJ)+P(J+1,I))/G-
2*(16.4*ALPB*DELS(J,I+1))/(G*VEL)

CONTINUE
SET THESE NEW POINTS EQUAL TO THEIR PROPER I VALUES

\[ P(J, I+1) = P_{\text{NEW}}(J) \]

CALCULATE THE WALL BOUNCERY PARTIAL PRESSURE

\[ P(MC, I+1) = \frac{(1.00 - (P(MCC, I+1) \cdot DC2B \cdot ALPB \cdot \frac{ALG(R)}{R + TM}) / (DC2M \cdot ALPM \cdot \text{DELY})]}{1.00 - (DC2B \cdot ALFB \cdot \frac{ALG(R)}{R + TM}) / (DC2M \cdot ALPM \cdot \text{DELY})} \]

AFTER GENERATION OF THE ENTIRE RADIAL PROFILE FOR A GIVEN I, TEST EACH POINT TO SEE IF IT FALLS WITHIN THE SET ERROR LIMIT. SHOULD ANY POINT FAIL REJECT ALL POINTS

\[ DC 4 \text{ J=1,MC} \]

\[ \text{IF(ABS(ZP(J, I+1) - P(J, I+1)) > 0.0005C00C)4,4,20} \]

SHOULD THE TEST FAIL GO TO STATEMENT 20 AND USE THE VALUES CALCULATED PREVIOUSLY AS YOUR SECOND TRIAL

\[ DC 4 \text{ CONTINUE} \]

\[ \text{IF(I-NCC)6,22,22} \]

PREPARE A GCCC ESTIMATE OF THE PROBABLE VALUE OF THE NEXT PROFILE BY PROJECTING FROM KNOWN VALUES OF THE PREVIOUS PROFILE

\[ DC68 \text{ J=1,MC} \]

\[ XM(J) = (P(J, I+1) - P(J, I)) / CELA \]

\[ P(J, I+2) = P(J, I+1) + XM(J) \cdot CELA / 2.0 \]

\[ ZP(J, I+2) = P(J, I+2) \]

\[ CC \text{CONTINUE} \]
GO TO 1C
C
C STORE THE F VALUES THUS CALCULATED FOR LATER COMPARISON
C
20    DO 58 JT=1,MC
58    ZP(JT,I+1)=P(JT,I+1)
37    CONTINUE
      GO TO 25
23    I=I+1
10    CONTINUE
      IF(I.EQ.1)GO TO 11
      IF(I.EQ.10)GO TO 11
      IF(I.EQ.20)GO TO 11
      IF(I.EQ.30)GO TO 11
      IF(I.EQ.40)GO TO 11
      IF(I.EQ.50)GO TO 11
      IF(I.EQ.60)GO TO 11
      IF(I.EQ.70)GO TO 11
      IF(I.EQ.80)GO TO 11
      GO TO 728
11    CONTINUE
C
C CALCULATE THE MASS TRANSFER COEFFICIENT AT THE WALL
C
   ZK=D02M*ALPM/TM
C
C CALCULATE THE VOLUME FLUX AT THE WALL
C
   YK=ZK*(FTCT-P(MC,I))*DELA*2.0*3.14*R*X+YK
   PRINT20,
   202  FORMAT(5X,2H1M MASS TRANSFER COEFFICIENT = ,E2C.8,2H1M CCC2/CM**2*SEC*1MM H2/9X,13M VOLUME FLUX = ,E2.8,15H CCC2/CM**2*SEC)
   PAVG(I)=C.C
C
MIXING CUP PARTIAL PRESSURE

DO109 JJ=1,10
   XJXJ=JJ
   XJJJ=JJ-1
   JJJ=JJ+1
   ABCD=(1.0-(CELY*XJXJ)**2)
   ADCCD=(1.0-(CELY*XJJJ)**2)
   PAVG(I)=PAVG(I)+((P(JJ,I)*ADCC*DELY*XJJJ)+(P(JJJ,I)*ABCD*DELY*1XJXJ))*CELY/2.0
   PAVG(I)=4.0*PAVG(I)*PBP+PINT
   PRINT905,PAVG(I),I
9C9 FORMAT(9X,9FPAVG(I) =,F20.8,AX,9HWHERE I = ,IA)
   IF(IIN.EC.l) GO TO 116
   ZZZZ=((1.0*XMK*PAVG(I))/(XMK*PAVG(I)))*12A
   ZZZ=((1.0*XMK*PAVG(I))/(XMK*PAVG(I)))*12A
   CALL THE AVERAGE SATURATION
   SI(I)=(ZZZ+124.0)/(ZZZZ+124.0)
   CALL THE TOTAL CONCENTRATION OF GAS IN THE LIQUID
   C=PAVG(I)*ALPB+1.34*PB*SI(I)
   XCT0(I)=C
116 IF(IIN.EC.C) GO TO 117
   C =0.373-C.0748*SI(I)+C.30456*PAVG(I)
   XCTCC(I)=C
117 CONTINUE
   PRINT203,C
2C3 FORMAT(9X,22H GAS CONCENTRATION =,F20.8,13H CCG2/CCBLCCC)
   II=I+1
   XI=I
   A=DELA*X1
\[
XXL = (A \cdot XL) - XL/XCC \\
XLL = XXL/VCL
\]

CALCULATE THE ABCISSA AND ORDI NATE FOR LOG-LOG PLOTS

\[
DLPC = CC28 \cdot XXL \cdot 3.1417/VAL \\
PU = (PAVG(I) - PINT)/(FTOT - PINT) \\
PRINT302, PL, DLPC, SI(I), XXL, XLL
\]

302 FORMAT(SX, 21HPAVG-PINT/PTCT-PINT =, F20.8/ \\
19X, 10HCLPI/VAL =, F20.8/ \\
29X, 21HSATURATION FRACTION =, F20.8/ \\
39X, 13HTLBE LENGTH =, F20.8/ \\
49X, 13HL/C =, F20.8 /////)

992 RQ = (XTCC(1) - XTCO(I))/(XTO(I) - XTO(1)) \\
PRINT99C, RQ

990 FORMAT(2X, 22HRESPIRATORY QUOTIENT =, F20.8 /////)

728 CONTINUE \\
IF(I - NC) 726, 727, 727

726 CONTINUE \\
IF(I - NCC) 2, 23, 23

2 CONTINUE

727 CONTINUE \\
IIN = 1 + IIN \\
IF(IIN.EQ.2) GO TO 44 \\
GO TO 555

14 STOP

END
BEGIN DATA

<table>
<thead>
<tr>
<th>80</th>
<th>40</th>
<th>0.750 E-02</th>
<th>760.0 E+00</th>
<th>38.0 E+00</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>46.0 E00</td>
<td>0.150 E00</td>
<td>760.0 E00</td>
</tr>
<tr>
<td>0.243 E00</td>
<td>0.023 E00</td>
<td>2.64E-05</td>
<td>1.40E-05</td>
<td></td>
</tr>
<tr>
<td>72.50E00</td>
<td>3.17E-02</td>
<td>1.00 E00</td>
<td>49.7 E00</td>
<td></td>
</tr>
<tr>
<td>0.7 E-02</td>
<td>0.0000E+00</td>
<td>38.0 E00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>80</th>
<th>40</th>
<th>4.0 E00</th>
<th>0.150 E00</th>
<th>760.0 E00</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>4.0 E00</td>
<td>0.150 E00</td>
<td>760.0 E00</td>
</tr>
<tr>
<td>1.350 E00</td>
<td>0.471 E00</td>
<td>2.21E-05</td>
<td>1.25E-05</td>
<td></td>
</tr>
<tr>
<td>72.50E00</td>
<td>3.17E-02</td>
<td>1.00 E00</td>
<td>49.7 E00</td>
<td></td>
</tr>
<tr>
<td>0.7 E-02</td>
<td>1.00 E00</td>
<td>49.7 E00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
VITA

Claude Glendon Bradley was born in Rome, Georgia, on November 14, 1942. He completed his secondary education in that community in June of 1960. He then attended Mississippi State University where he received his B. S. degree in Chemical Engineering in June of 1964. After graduation he worked for the Research Institute of the University of Oklahoma as a Research Associate until June of 1965.

In September of 1965, he enrolled at Louisiana State University where he is presently pursuing the degree of Doctor of Philosophy in Chemical Engineering.
EXAMINATION AND THESIS REPORT

Candidate: Claude Glendon Bradley

Major Field: Chemical Engineering

Title of Thesis: The Transport of Oxygen and Carbon Dioxide in Blood Flowing in a Permeable Tube

Approved:

[Signatures]

Major Professor and Chairman

Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:

July 24, 1968