A Study of Selected Probabilistic Applications to Common Accounting Decision Areas.

Macil Caldwell Wilkie Jr
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A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Doctor of Philosophy in

The Department of Accounting

by

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B.S., Louisiana State University, 1963
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August, 1968
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TABLE OF CONTENTS

ACKNOWLEDGMENTS ................................................ ii

LIST OF TABLES ................................................ vii

LIST OF FIGURES ................................................ ix

ABSTRACT .................................................... x

CHAPTER

I. INTRODUCTION ............................................... 1

Need for Probability in Accounting
Decision Analysis. ........................................ 1
Obsolescence and Weakness of
Current Accounting Decision
Aids ............................................................. 1
Statistics and Accounting Decision
Aids .......................................................... 2
Probability and Accounting
Decision Aids. ............................................ 4

Some General Comments on
Probability. ............................................... 5
Nature of Probability. .................................... 5
Three Important Theorems of
Probability. ............................................... 7

A Survey of the Study. ..................................... 8
Purpose of the Study ..................................... 8
Scope of the Study ....................................... 9
Organization of the Study ............................... 10
Summary. .................................................. 12

II. PROBABILITY IN DATA ESTIMATION ........................ 14

Need for Probability ....................................... 14
Probability and Managers' Beliefs. .................... 15
Beliefs of a Single Manager ............................ 17
Averaging Single Manager Beliefs ..................... 21
Combining Single Manager Beliefs .................... 23
Importance of Probabilistic
Belief Concept ............................................ 25
<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability Signals for Budget Revision</td>
<td>29</td>
</tr>
<tr>
<td>Probability and Period by Period Comparison of Projections</td>
<td>30</td>
</tr>
<tr>
<td>Probability, Projections, and Cumulative Comparisons</td>
<td>35</td>
</tr>
<tr>
<td>Summary</td>
<td>36</td>
</tr>
<tr>
<td>III. PROBABILITY IN COST-VOLUME-PROFIT ANALYSIS . . . 39</td>
<td></td>
</tr>
<tr>
<td>Need for Probability</td>
<td>39</td>
</tr>
<tr>
<td>A Simple Probabilistic Cost-Volume-Profit Application</td>
<td>41</td>
</tr>
<tr>
<td>Traditional Cost-Volume-Profit Analysis</td>
<td>41</td>
</tr>
<tr>
<td>Probabilistic Cost-Volume-Profit Analysis</td>
<td>43</td>
</tr>
<tr>
<td>Risk and Probabilistic Cost-Volume-Profit Analysis</td>
<td>46</td>
</tr>
<tr>
<td>Limitations and Improvements of this Simple Probabilistic Technique</td>
<td>51</td>
</tr>
<tr>
<td>Deriving Required Probability Distributions</td>
<td>51</td>
</tr>
<tr>
<td>Use of Discrete Probability Distributions</td>
<td>53</td>
</tr>
<tr>
<td>Replacing of Discrete with Continuous Probability Distributions</td>
<td>54</td>
</tr>
<tr>
<td>Illustration of Continuous Distribution Application</td>
<td>57</td>
</tr>
<tr>
<td>A Comprehensive Probabilistic Cost-Volume-Profit Application</td>
<td>62</td>
</tr>
<tr>
<td>Comprehensive Probabilistic Cost-Volume-Profit Analysis</td>
<td>62</td>
</tr>
<tr>
<td>Risk and Comprehensive Probabilistic Cost-Volume-Profit Analysis</td>
<td>63</td>
</tr>
<tr>
<td>General Equations for the Comprehensive Probabilistic Analysis</td>
<td>66</td>
</tr>
<tr>
<td>The Utility Concept in Cost-Volume-Profit Analysis</td>
<td>68</td>
</tr>
<tr>
<td>Limitation of Expected Value as a Guide to Action</td>
<td>68</td>
</tr>
<tr>
<td>Utility in Cost-Volume-Profit Analysis</td>
<td>69</td>
</tr>
<tr>
<td>Summary</td>
<td>70</td>
</tr>
<tr>
<td>IV. PROBABILITY IN CAPITAL INVESTMENT ANALYSIS . . . 73</td>
<td></td>
</tr>
<tr>
<td>Need for Probability</td>
<td>73</td>
</tr>
<tr>
<td>Probability and the Time Adjusted Rate of Return Technique</td>
<td>74</td>
</tr>
</tbody>
</table>


V. PROBABILITY IN COST CONTROL ANALYSIS ........... 102

Need for Probability .................................. 102
Cost Control and the Shewhart Control
Chart............................................. 103
The Shewhart Control Chart .................... 103
Application of the Shewhart Control
Chart to Cost Control.......................... 104
Limitations of the Control Chart
for Cost Control.................................. 113
Probability and Cost Variance Reports........... 114
Variance Reports.................................. 114
Application of Probability to
Variance Reports................................ 115
Probabilistic Analysis of Variable
Costs--An Extension......................... 122
Limitations of Probabilistic Analysis
of Cost Variance Reports..................... 124
Application of Probability to the Control
of Similar Branch Costs........................ 126
Binomial Probabilities and Similar
Branch Costs................................. 126
Tests of the Difference of Two Means
and Similar Branch Costs.................... 130
Summary............................................ 135
## LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Territorial Sales Estimates and Related Dispersion Measures.</td>
<td>24</td>
</tr>
<tr>
<td>II. Common Normal Curve Limits</td>
<td>29</td>
</tr>
<tr>
<td>III. Monthly Sales With Their Mean and Standard Deviation</td>
<td>32</td>
</tr>
<tr>
<td>IV. Net Income of Machines at Projected Average Demand</td>
<td>43</td>
</tr>
<tr>
<td>V. Probability Distribution of Demand for Product</td>
<td>44</td>
</tr>
<tr>
<td>VI. Expected Value of Demand</td>
<td>45</td>
</tr>
<tr>
<td>VII. Net Income of Machines at Expected Value of Demand Level</td>
<td>45</td>
</tr>
<tr>
<td>VIII. Probability Distribution of Demand and Expected Value of Demand</td>
<td>48</td>
</tr>
<tr>
<td>IX. Net Income of Machines at Expected Value of Demand Level</td>
<td>49</td>
</tr>
<tr>
<td>X. Probability Distribution of Inflows and Expected Value of Inflows</td>
<td>75</td>
</tr>
<tr>
<td>XI. Project A: Probability Distribution of Inflows and Expected Value of Inflows</td>
<td>77</td>
</tr>
<tr>
<td>XII. Project B: Probability Distribution of Inflows and Expected Value of Inflows</td>
<td>77</td>
</tr>
<tr>
<td>XIII. Data on Possible Investment Projects</td>
<td>86</td>
</tr>
<tr>
<td>XIV. Estimated Range and Probabilities of Initial Investment</td>
<td>97</td>
</tr>
<tr>
<td>TABLE</td>
<td>PAGE</td>
</tr>
<tr>
<td>-------</td>
<td>------</td>
</tr>
<tr>
<td>XV. Estimated Range and Probabilities of Useful Life</td>
<td>97</td>
</tr>
<tr>
<td>XVI. Estimated Range and Probabilities of Cash Inflows</td>
<td>97</td>
</tr>
<tr>
<td>XVII. Hourly Material Production Cost for Some Product for a Two-Week Interval</td>
<td>108</td>
</tr>
<tr>
<td>XVIII. Results of Similar Branch Cost</td>
<td>129</td>
</tr>
<tr>
<td>XIX. Results of Similar Cost Samples From Branches</td>
<td>133</td>
</tr>
<tr>
<td>XX. Probability Distribution of Demand</td>
<td>141</td>
</tr>
<tr>
<td>XXI. Expected Profit Under Certainty</td>
<td>144</td>
</tr>
<tr>
<td>XXII. Expected Profit of Most Favorable Alternative</td>
<td>145</td>
</tr>
<tr>
<td>XXIII. Defect Record of Standby Machine</td>
<td>154</td>
</tr>
<tr>
<td>XXIV. Expected Cost of Production with Standby Machine</td>
<td>155</td>
</tr>
<tr>
<td>XXV. Expected Cost Under Certainty</td>
<td>156</td>
</tr>
<tr>
<td>XXVI. Calculation of Posterior Probabilities</td>
<td>157</td>
</tr>
<tr>
<td>XXVII. Expected Cost of Production with Standby Machine</td>
<td>158</td>
</tr>
<tr>
<td>XXVIII. Sample Based Projected Hourly Savings</td>
<td>162</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Control Chart for Material Production Cost for Some Product</td>
<td>110</td>
</tr>
<tr>
<td>2. Cost Control Decision Chart--Unfavorable Variance</td>
<td>121</td>
</tr>
</tbody>
</table>
The accountant in the performance of his duties has to face daily the problem of deciding among alternative courses of action. Most of the analytical techniques currently being utilized by him in making such decisions, however, cannot adequately cope with the numerous uncertainties so common in today's dynamic business environment. Fortunately, probability theory can be used to remedy this weakness and restore the accountant's techniques to maximum efficiency and realism.

One of the accounting areas in which probability can prove extremely useful is data estimation. Present procedures in this area yield only a measure of central tendency, but a good forecasting technique should also yield one of estimate dispersion since knowing both permits the probabilities of various results to be computed and considered in overall planning. Probability theory, the beliefs of individuals within the firm, and the normal curve can be combined to provide an estimation procedure which yields these two desired measures.

A second accounting area where probability theory can prove helpful is cost-volume-profit analysis. In those situations where some of the factors involved in the analysis
are variable, probability distributions for the possible values of the variable items can be derived, their expected values computed, and these figures used with any fixed quantities in the traditional analytical approach. Such a procedure makes the cost-volume-profit analysis more realistic and allows the computation of probabilities for various possible values of the variable factors, which should prove invaluable in evaluating any relevant risk considerations.

Capital investment analysis is another area which can be improved by the application of probability theory. Several techniques are available here. One approach bases the computation of time adjusted rates of return or excess present value indexes on probability distributions and expected values of the possible cash inflows of investment opportunities. A second approach involves the determination of all possible investment combinations given the current capital budget and the calculation of their respective means and variances. The optimum combination is naturally the one with the highest mean and a dispersion acceptable to management. The final approach is to simulate, using any fixed factors, probability distributions for variable factors, and random numbers, each opportunity until a probability distribution for its possible rates of return has been defined. These distributions can then be used to allocate available capital funds. Under all three approaches derived probability distributions can be used to compute any probabilities desired for risk analysis.
Cost control analysis is still another area where probability theory can prove beneficial. Traditional cost control procedures do not tell management whether a particular cost variation is most likely the result of normal or assignable causes, and as a result needless investigations are often undertaken by a firm. A Shewhart control chart prepared from past cost data or a realistic standard cost system with the standards defined in terms of normal probability distributions can be used to provide such information. Thus, management can be given some quantitative basis for deciding when an investigation seems to be warranted for a cost variation and when the situation most probably should simply be left as is.

Several refinements can make the above probabilistic procedures even more valuable as decision tools. One possible refinement involves use of the statistical measure, the expected value of perfect information, to help management determine if a decision should be made on the basis of current information or delayed until additional data can be gathered. When a delay is indicated, a second refinement based on Bayesian statistics can be utilized to combine additional data and original information in the form of a posterior probability distribution.
CHAPTER I

INTRODUCTION

NEED FOR PROBABILITY IN ACCOUNTING DECISION ANALYSIS

Obsolescence and Weakness of Current Accounting Decision Aids

The complexities of the modern business world have outmoded many of the analytical concepts presently being used by accountants as aids in the area of decision making. Some of these concepts themselves have only been accepted as valid and useful in the last decade or two, but like their predecessors they have become victims of the rapid scientific, mathematical, and educational advances of twentieth-century man. Especially important to the accountant have been those advances in the area of statistics. This fact was apparent to some foresighted individuals as early as 1964 as evidenced by the following quotation from an article by Joseph Mauriello, a professor of accounting at New York University, concerning the area of cost accounting. "In statistical science lie innovations of thinking which may well render obsolete, or at the least radically modify, the cost methods
In general, the analytical decision concepts currently being used by accountants have one major weakness. They are not comprehensive enough to cope successfully with the varied, multitudinous, and extremely significant uncertainties which are an integral part of today's dynamic business environment. As a result, many managerial decisions are predicated on a false assumption—that the figures involved in the analysis are fixed, known with certainty. The problem of variability is considered subjectively, but often erroneously slighted, because quantitative factors are much easier to evaluate, interpret, and rely upon than qualitative ones. In the past this situation, while not desirable, has at least been acceptable. Even with it the business could still be operated at or near maximum efficiency. Now, however, competition has become so intense and the business environment so dynamic that intelligent, thoughtful, and well formulated planning is a necessity for such operation. Maximum efficiency cannot be truly achieved as long as management continues to ignore or only superficially consider uncertainty and chance.

Statistics and Accounting Decision Aids

Fortunately, the previously mentioned advances in the field of statistics have provided certain techniques which

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can be used in conjunction with current accounting decision aids to rectify the above weakness and enable them to cope successfully with uncertainty. Many noted accountants have become aware of these statistical techniques and their extreme importance to the continued growth and development of the accounting profession. Prolific author Harold Bierman noted in a recent article:

There has recently been a revolution in statistics which is soon going to affect the decisions which accountants have traditionally considered their own reserve (in fact it has already affected the area of inventory control), and it might well affect accounting theory itself. This revolution has been given the title, statistical decision theory.²

In yet another article Professors Richard Cyert and John Wheeler said, "In recent years it has become increasingly evident that many managerial problems could best be handled by an integrated application of accounting and statistical knowledge."³ The importance of statistical techniques to the accountant is also evidenced by the increasing number of articles advocating increased emphasis upon them in the accounting curriculums of colleges and universities. Concerning this R. Gene Brown has said:

In the past few years the use of mathematical and statistical tools in the business world has increased significantly. Hand and hand with this increase has

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been the trend in universities toward more quantitative methods requirements for business graduates. Since most accounting graduates are products of the business curriculum, it would be well to consider the value of increasing the mathematics and statistics requirements for accountants.\textsuperscript{4}

\textbf{Probability and Accounting Decision Aids}

The various statistical-mathematical techniques or concepts which can be used by the accountant to introduce uncertainty into his decision analyses are fairly numerous. Some examples are correlation analysis, linear programming, analysis of variance, queuing theory, simulation, regression analysis, and probability. Of these the one which this study considers, probability, is without doubt the most important given the present educational training and background of the accounting profession. It is the most important because it is a concept which the profession is acquainted with to some extent and whose comprehension and effective application does not require knowledge of advanced theoretical mathematics. In other words, probability represents a statistical decision tool that the average accountant of today is capable of understanding and utilizing.

Current accounting literature is very cognizant of this situation. Even some business-oriented government publications have argued the merits of probability. According to a pamphlet issued by the Cotton Board Productivity

Centre of Great Britain, "... if in a management situation we can estimate the real chances of events occurring, we can, in fact, capitalise on uncertainty and use it to our advantage." Occasionally writers have become somewhat adamant and dictatorial in their articles about the merits of probability and the failure of the accounting profession as a whole to undertake its utilization immediately. This style of writing is well demonstrated by the following quotation.

In the course of his evolution from cost accountant the modern management accountant has developed many new techniques and concepts. However, despite these advances there is still a vital omission in his range of professional skills. This omission relates to his lack of techniques involving probability theory. Probability theory is so fundamental to management accounting that accountants who neglect to incorporate it into their figures could be failing in their task and presenting to management advice which, in view of current developments, is substandard.6

SOME GENERAL COMMENTS ON PROBABILITY

Nature of Probability

Even though probability is a concept which everybody lives with and knows something of it is extremely difficult to define generally in a truly descriptive and universally meaningful manner. There are two definitions which are most commonly given for it. One of these is termed the relative frequency definition and states:

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5Cotton Board Productivity Centre, "Playing the Odds," Management Accounting, 44 (May, 1966), 185.

Assume that if a large number of trials be made under the same essential conditions, the ratio of the number of trials in which a certain event happens to the total number of trials will approach a limit as the total number of trials is indefinitely increased. This limit is called the probability that the event will happen under these conditions.\(^7\)

The other is known as the classical definition and says:

If an event may happen in \(a\) ways and fail to happen in \(b\) ways, and all of these ways are mutually exclusive and equally likely to occur, the probability of the event happening is \(a/(a+b)\), the ratio of the number of ways favorable to the event to the total number of ways.\(^8\)

Even the above definitions do not set forth all that is meant by the term probability, but fortunately this failure is not really important for purposes of this study. For purposes of this study all that is really important is the fact that the accountant has to deal constantly with various business problem areas involving chance and uncertainty and that probability represents a concept which can without doubt facilitate and improve his handling of such areas.

In general, probabilities can be divided into two types—objective and subjective. The former are based on definitive empirical evidence or complete knowledge of the possible events and their environment. Naturally they are extremely reliable. Examples would be the .5 probability of obtaining a head on the toss of a coin or the .7 probability of drawing a red bead from an urn containing seven red and


\(^8\)Loc. cit.
three blue beads.

The latter are based in part on intuition and guesswork as there is not sufficient empirical evidence or complete knowledge as above. Examples are the assignment of probabilities to possible product demands for a new item or to possible future cash inflows from an investment opportunity. Unfortunately, it is obviously this latter type with which the accountant has most contact. However, probability theory does provide quantitative techniques for manipulating and analyzing both types of likelihoods, and certainly their use should be preferred to only qualitative consideration or perhaps even the ignoring of important and vital subjective business probabilities.

Three Important Theorems of Probability

There are three important theorems of probability which should be mentioned here since they will be utilized from time to time in following chapters. The addition theorem or theorem of total probabilities states that, "If we have mutually exclusive events (no two events can occur at the same time), the probabilities of these events can be added to obtain the probability that more than one of the events will occur."9 The multiplication theorem or theorem of compound probabilities says, "When two (or more)

events are independent, the probability of both events (or more than two events) occurring is equal to the product of the probabilities of the individual events."\(^{10}\) Finally, the theorem of conditional probabilities relates that, "The probability that both of two dependent events will occur is the probability of the first multiplied by the probability that if the first has occurred the second will also occur."\(^{11}\) Using these three theorems the probability of any grouping of events possible in a particular situation can easily be determined as long as the probabilities of the individual events and their relationship to one another are known.

A SURVEY OF THE STUDY

Purpose of the Study

The basic purpose of this study is to discuss the rather broad and somewhat unutilized area of probabilistic applications to certain accounting decision analyses. Its objectives are three in number—first, to acquaint the reader with those fairly common situations involving accounting decision problems where the application of probability theory will prove to be beneficial; second, to consider why it will prove to be so; and, third, to explain and illustrate the procedures, techniques, and computations necessary for applying it in each particular case. It is further hoped

\(^{10}\text{Ibid.},\ p.\ 11.\)

\(^{11}\text{Grant, op. cit.},\ p.\ 201.\)
that the reader will become so convinced of the potential value of these applications that he will not only utilize them himself where possible but also educate and encourage others to do likewise.

Scope of the Study

As previously indicated this study deals only with the application of probability theory to certain accounting decision analyses. It does not attempt to consider the use of probability in other areas—such as financial statement preparation, analysis of financial statements, and cost allocation—although other applications are undoubtedly well worthy of consideration. A corporate executive even proposed recently that probability be used to develop some authoritative accounting principles. His remarks were as follows:

Neither deductive analysis, reasoning by analogy, nor inductive reasoning can be utilized by anyone to establish the truth or falsity of accounting generalizations, although strong beliefs can exist as to the validity of a particular assumption. However, beliefs as to the truth or falsity of a given proposition can be developed by a pattern of rationalization which utilizes the theory of probability although whether or not this pattern is satisfactory as a base for authoritative accounting principles remains a question.12

Furthermore, this paper does not consider in any detail the most difficult problem to be faced in establishing probability theory as a legitimate and beneficial tool in accounting decision analyses—the education and motivation

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of the accountant in its use. It has already been stated that one of the objectives of this study is education and motivation through self interest, and perhaps the increasing number of writings concerning such applications of probability will help to solve, or at least cause some decisive action to be taken regarding this problem.

Finally, this study does not attempt to explain completely the highly theoretical derivations involved in some of the probabilistic applications considered. Comprehension of such derivations requires more mathematical knowledge than the average accountant possesses and is really not necessary for the effective utilization of their end results. For practical purposes all that the accountant need be able to do is recognize when each of the various probabilistic applications is appropriate, know what mechanical steps are necessary to compute any required or desired figures, and know what types of thought processes are necessary to interpret these results. Therefore, guided by this philosophy this paper simply presents certain derivations giving only a brief narrative description of how they were obtained. However, references are given for the benefit of those readers who would like to investigate further the actual manipulations and reasonings involved.

**Organization of the Study**

This study is divided into seven chapters. The first chapter introduces the subject to be considered, indicates
why it needs such extensive consideration, examines several theorems fundamental to its application in various situations, and finally discusses briefly the purpose, scope, and organization of the entire paper.

Chapter II discusses the use of probability theory in the area of data estimation. Methods for deriving estimate distributions with known measures of central tendency and estimate dispersion and the advantages of their effective utilization by the accountant in decision analyses are explained and illustrated.

The use of probability theory to improve cost-volume-profit analysis is considered in Chapter III. Probabilistic techniques for introducing uncertainty into situations where such analysis is appropriate are presented and explained along with the improved risk factor evaluation which their utilization makes possible.

Probability and capital investment analysis are examined in Chapter IV of the study. The utilization of probabilistic concepts to introduce chance into both the time adjusted rate of return and the excess present value techniques for resolving capital budgeting decisions are discussed and illustrated.

Chapter V considers the use of probability theory to improve and facilitate certain types of cost control. Primarily examined is the utilization of probabilistic techniques which enable the decision maker to determine when a cost variation is most likely due to assignable rather than
normal or chance causes. Obviously, the former merit management's investigation and the latter do not.

Two refinements applicable to some of the probabilistic concepts discussed in previous chapters are examined in Chapter VI. The first of these aids in determining whether a decision should be made on the basis of information currently available or delayed until additional data can be gathered, and the second represents a method of combining any additional information with the original information so that maximum benefits are obtained from all data available.

The final chapter, Chapter VII, contains primarily a summary of the study. Also included in it, however, are several conclusions which can be drawn concerning probability in accounting decision analyses and several comments on what the future is likely to hold for this area.

SUMMARY

Many of the techniques utilized by the accounting profession in various types of decision analyses are outmoded and inadequate. They do not represent the best means possible for handling such situations. One important example concerns the subject of this paper, probability theory, which can be used to improve and facilitate numerous decision analyses commonly relied upon by the accountant.

This chapter comments in general on the major weakness of most of the present accounting analytical techniques—their inability to cope with uncertainty—and considers how
probability can be used to partially eliminate this inade­quacy. It also includes comments on the nature of prob­ability, examines three theorems which must be an integral part of any discussion on this subject, and presents the purpose, scope, and organization of the study.
CHAPTER II

PROBABILITY IN DATA ESTIMATION

NEED FOR PROBABILITY

Most modern businesses could not be successfully operated without the use of estimated data. Without doubt, this situation is well demonstrated by the importance attached today to budgets and budgeting techniques. Many of the estimates that result from the application of such techniques, however, are erroneously looked upon by management as being virtually definite. In other words, management seems to ignore or forget the fact that the figures they are manipulating and analyzing are only predictions of the outcomes of future events and not by any stretch of the imagination certainties.

Depending upon the individual situation, there are three types of information which may be helpful in data estimation--point estimates, the entire distribution of possibilities, and central tendency estimates. As management is usually most concerned with the latter type in business oriented problems, it is the problems and possible improvements associated with such that are considered in the following paragraphs.
A good forecasting technique should inform its user about two aspects of the item or items being forecasted—central tendency and estimate dispersion; or, as Thompson and Kemper say, "An effective estimating procedure should provide: (1) an expected value and (2) a measure of variability of the estimate."¹ Traditional forecasting techniques usually provide only the former, and as a result the decision maker who must use the estimate in some analysis cannot consider all of the possible contingencies. He cannot even determine them from the incomplete information that he has been given. This chapter examines and illustrates several probabilistic-based estimating techniques which provide management with the measurements necessary to adequately and effectively use estimated data in the planning and controlling of business operations. Naturally, their utilization cannot assure the modern manager that his decisions will always be proven correct by subsequent events, but it can assure him that they will be more informed, and more informed decisions are always desirable.

PROBABILITY AND MANAGERS' BELIEFS

Most estimates or predictions are obtained from the beliefs of an individual or individuals who, through experience, research, and intuition, have become familiar with the

pattern of the item in question. The estimate itself provides the first requirement of a good forecasting technique, a measure of central tendency, but the second requirement, a measure of the dispersion associated with the distribution of possible estimates, necessitates additional thought and consideration. In certain specific cases the variability will most likely be zero. For example, the forecasting of $100,000 for advertising purposes in the next planning period may well mean that this is the exact amount that will be spent unless actual conditions prove to be extremely different from those anticipated at the time the forecast was made. With many estimates, however, the variability will most certainly not be zero. Examples in this area are demand for products, useful life of facilities, and cash inflows of potential investments. In such cases the probabilistic technique discussed and illustrated in the following paragraphs can be used to provide management the needed measure of the dispersion of the distribution of estimates.

Before explaining the use of probability in deriving this measure, it should be pointed out that one other source may exist. This source is past records of the business and is applicable where there have been previous transactions dealing with the item, or items similar to the one being forecasted or estimated.
Beliefs of a Single Manager

To be able to use probability as a means of providing a measure of estimate dispersion, the individual making the estimate must be capable of expressing some idea of its reliability. This is accomplished by having him state what chance he thinks there is that his guess is within plus and minus some specific number of units of the true value of the item's central tendency. Furthermore, the forecaster must be willing to use a normal curve to express the distribution or pattern of his beliefs, since only with it can his ideas as to reliability be converted into a measure of variability. In deciding whether this curve is appropriate in a given situation, its properties must be considered. According to Yamane they are as follows:

1. It is symmetrical and bell shaped.

2. As a result, the mean is in the middle and divides the area in half, and the mean, median, and mode are identical.

3. Theoretically, the curve extends in both directions, gradually coming closer to the horizontal axis. It extends out to infinity, but never reaches the horizontal axis.²

If these properties seem to indicate a distribution that either represents or approximates the estimator's beliefs, which are in turn hopefully similar to the actual distribution of the item in question, then the probabilistic analysis can be applied.

The measure of estimate dispersion that is ultimately to be obtained here is the standard deviation of the distribution of beliefs. Its calculation can best be explained perhaps through the use of an example. Assume that a firm is trying to estimate the useful life of a piece of equipment for depreciation and planning purposes. The firm's mechanical expert believes that the machine's expected life will be 10 years, but that there is only a 50-50 chance that his guess is within plus or minus 1 year of the true mean life. As indicated previously, the estimate itself serves as a measure of the mean or expected value of the distribution of beliefs. Its standard deviation is calculated in the following manner. Since a symmetrical normal curve is being required and the forecaster believes that there is a 50-50 chance that his guess is within plus and minus 1 year of the true mean, then 50 per cent of the area under the distribution of beliefs must be included in the interval from the mean of the estimate minus 1 year to the mean plus 1 year— from 9 to 11 years. Therefore, and once more due to the symmetrical property of the normal curve, 25 per cent of the area of the distribution must be included in the range from minus infinity to the mean minus 1 year, or 9.

The computation must now make use of the normal deviate concept and a table of areas under the normal curve.

---

3 This is true since the estimate represents the forecaster's idea of the most likely life for the machine and the latter is the mean or expected value of a normal distribution.
The normal deviate is defined as the difference between some value of a normal distribution and its mean divided by its standard deviation, and is used in conjunction with a table of areas under the normal curve for various normal deviates to determine the portion of the curve occupied by the interval from minus infinity to that "some value" or from that "some value" to plus infinity. In equation terms it would look as below.

\[
\text{Normal deviate} = \frac{X - M}{\text{STD}}
\]

\[
X = \text{Some value of a normal distribution}
\]

\[
M = \text{Mean of that distribution}
\]

\[
\text{STD} = \text{Standard deviation of that distribution}
\]

Substituting from the useful life example the mean of 10 and an assigned value of 9 for \(X\) in this equation gives the following:

\[
\text{Normal Deviate} = \frac{9 - 10}{\text{STD}}
\]

However, it has previously been established that 25 per cent of the area of the distribution must be included in the interval from minus infinity to 9, so a table of normal curve area content can be used to determine what particular value the normal deviate must have for this condition to be met, and it is approximately .67. After substituting .67 for the normal deviate in the above equation the standard deviation of the distribution of beliefs can easily be computed since it is the only remaining unknown. Solving of the equation...
yields a standard deviation for this example of 1.49, and is illustrated below.

\[ .67 = \frac{9 - 10}{STD} \]

\[ .67STD = 1 \]

\[ STD = 1.49 \text{ years} \]

It should be noticed that the minus sign in the numerator of the formula can be ignored. This is possible since the sign of a normal deviate refers only to the tails of the distribution in question. In other words, the minus sign indicates that it is the left tail of the distribution which is being considered here, and nothing more.

Whatever the nature of the item being predicted, an analysis similar to the one used in the above useful life example can be utilized to provide a measure of central tendency and one of estimate dispersion. The only factors that will change from situation to situation are the specific values assigned to the terms of the normal deviate equation, and these can be determined by the estimate itself, the estimator's beliefs as to its reliability, and a table of areas under the normal curve.

Before leaving this section, it should be pointed out that it is not the actual or true distribution of the item under study that is being described by a normal curve with some mean and standard deviation, but rather it is the estimator's beliefs concerning its pattern of variation. In other words, this probabilistic technique treats the
estimator's mind as a random process which in situations like the one currently under analysis yields predictions that are correct on the average but are erroneous by some amount on some percentage of all individual occasions.

**Averaging Single Manager Beliefs**

In those situations where the experience and research of the estimator are not quite as extensive as may be desired, it is undoubtedly a good idea to base the standard deviation of the distribution of beliefs on several reliability estimates rather than just one. The utilization of this procedure in such a situation will very likely result in a reduction of any bias in the measure that may otherwise be present. As an illustration of its application, assume that in the useful life example that besides believing there is a 50 per cent chance that his guess is within plus and minus 1 year of the true mean the estimator also believes that there is a 60 per cent chance that it is within plus and minus 1½ years of the true mean and an 80 per cent chance that it is within plus and minus 2 years of the true mean. The first new reliability belief means that 20 per cent of the distribution of beliefs must be included in the interval from minus infinity to 8.5 and the second that 10 per cent must be included in the interval from minus infinity to 8. A table of areas under the normal curve indicates that the normal deviates must be .84 and 1.28, respectively, for each of these conditions to be satisfied. Substituting the
appropriate data in the normal deviate formula yields the two additional standard deviation estimates shown below.

\[ .84 = \frac{8.5 - 10}{\text{STD}} \quad 1.28 = \frac{8 - 10}{\text{STD}} \]

\[ .84 \times \text{STD} = 1.5 \quad 1.28 \times \text{STD} = 2 \]

\[ \text{STD} = 1.78 \text{ years} \quad \text{STD} = 1.57 \text{ years} \]

The next step naturally is to use these two estimates plus the one previously determined to compute an average standard deviation to serve as the measure of dispersion for the distribution of beliefs. However, since the standard deviation is not additive, that is the standard deviation of two or more independent random variables is not equal to the sum of their individual standard deviations, this manipulation must be accomplished using variances (the variance is the square of the standard deviation). It is illustrated below.

\[ \text{Average STD} = \sqrt{\frac{1.49^2 + (1.78)^2 + (1.57)^2}{3}} \]

\[ \text{Average STD} = \sqrt{\frac{2.22 + 3.17 + 2.46}{3}} \]

\[ \text{Average STD} = \sqrt{\frac{7.85}{3}} = \sqrt{2.62} = 1.62 \text{ years} \]

Thus, the useful life of the piece of equipment in question has a projected mean or expected value of 10 years and the standard deviation to be associated with this projection is 1.62 years.
Combining Single Manager Beliefs

There may be situations in a business where the estimates of two or more managerial personnel must be combined in order to get a total projection for some item for planning and/or control purposes. Fortunately, this combination presents no real difficulties whatsoever if the individual estimates were derived using the probabilistic belief concept just considered and illustrated. The reason for this is the following three mathematical theorems:

**THEOREM 1:** Given distributions of random variables \( X_1 \) and \( X_2 \), the expected (mean) value of the distribution of the variable \( X_1 + X_2 \) equals the expected value of \( X_1 \) plus the expected value of \( X_2 \).

\[
EV(X_1 + X_2) = EV(X_1) + EV(X_2)
\]

**THEOREM 2:** Given distributions of independent random variables \( X_1 \) and \( X_2 \), the variance of the variable \( X_1 + X_2 \) equals the variance of \( X_1 \) plus the variance of \( X_2 \).

\[
VAR(X_1 + X_2) = VAR(X_1) + VAR(X_2)
\]

**THEOREM 3:** If distributions of the variables \( X_1 \) and \( X_2 \) are both normal, the distribution of the sum \( X_1 + X_2 \) is also normal.\(^4\)

Since the distribution of beliefs of a manager is composed of random variables and in most cases is independent of those of other managers, the above three theorems are applicable to the area under consideration. Thus, as a result of the first theorem the expected value beliefs of one or more individuals can simply be added together if an estimate of

the expected value of their total distribution is desired. As a result of the second one, the variance, which is equal to the square of the standard deviation, of this combination can be computed by summing the variances of the individual belief distributions. The standard deviation of the total distribution can then be determined by taking the square root of its variance. Such an indirect procedure is necessary here because, as explained previously, the standard deviation is not additive. Finally, the third theorem means that the total distribution will be normal and that the soon to be discussed possibilities of such a pattern in estimation problems are appropriate in this area.

As an illustration of the previously described technique assume that a company with six marketing territories wants to combine the sales estimates of each individual territory in order to obtain a total sales projection for the firm. Each of the six estimates was made by the appropriate territorial head salesman using the probabilistic belief concept, and are shown in Table I along with the related standard deviations and variances.

TABLE I

<table>
<thead>
<tr>
<th>Territory</th>
<th>Mean</th>
<th>Std.</th>
<th>(Std.²) Var.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,000</td>
<td>60</td>
<td>3,600</td>
</tr>
<tr>
<td>2</td>
<td>1,500</td>
<td>85</td>
<td>7,225</td>
</tr>
<tr>
<td>3</td>
<td>800</td>
<td>40</td>
<td>1,600</td>
</tr>
<tr>
<td>4</td>
<td>1,200</td>
<td>70</td>
<td>4,900</td>
</tr>
<tr>
<td>5</td>
<td>900</td>
<td>45</td>
<td>2,025</td>
</tr>
<tr>
<td>6</td>
<td>1,100</td>
<td>60</td>
<td>3,600</td>
</tr>
<tr>
<td></td>
<td>6,500</td>
<td></td>
<td>22,950</td>
</tr>
</tbody>
</table>
The mean of the combined distribution can be read directly from Table I since it is the sum of the individual estimates. The standard deviation can be calculated by extracting the square root of 22,950, the sum of the individual estimate variances and the variance of the combined distribution. Thus, the total sales distribution for the firm is a normal one with a mean or expected value of 6,500 units and a standard deviation of approximately 151 units. Naturally, the same general procedure used in this example would be appropriate regardless of the number of estimates that had to be combined in any given situation.

Importance of Probabilistic Belief Concept

At this point it might be wise to consider the question of why this probabilistic belief estimation technique that results in a measure of estimate dispersion as well as one of central tendency is so worthwhile. One direct advantage is that the standard deviation gives the estimate user some idea of its reliability or dependability. In general, the smaller this figure the more reliable the estimate to which it relates. Absolute size and the relative importance of the projection must also be taken into consideration here. The major advantage of the technique, however, is that it makes possible the determination of various probability statements about the item being forecasted, and this should result in more informed planning and decision making. The probability of the projected item being
more or less than some specific figure can be computed along with the probability of it being contained within some definite range of values. Certainly few persons would argue that such knowledge has little or no merit to the management of a progressive modern firm.

This computation of probability statements concerning the item being forecasted is possible because the distribution of beliefs is normal, and a normal curve is fully defined when only two measures, its mean and standard deviation, are known. The actual computation is accomplished using the previously considered formula for the normal deviate. By determining the number of normal deviates between the mean or expected value and some other value the area content and thus the probability of more than or less than that value can easily be read from a table of areas under the normal curve. A similar procedure with still another value can be utilized to determine the percentage of the curve's area contained in the interval between the two values if such a probability is desired.

To illustrate the calculation of various probabilities from belief distributions assume the situation encountered in the example concerning the combination of territorial sales estimates. It should be recalled that the end result of this example was a distribution of projected sales for the entire firm with a mean of 6,500 units and a standard deviation of 151 units. Now, assume that management wants to determine the probability of yearly sales being 6,300
units or more since this figure represents the firm's break-even point. The first step is to substitute the appropriate amounts in the normal deviate formula, and this is done and the equation solved below.

\[
\text{Normal Deviate} = \frac{6,300 - 6,500}{151}
\]

\[
\text{Normal Deviate} = \frac{-200}{151} = -1.32
\]

From a table of areas under the normal curve it can be determined that a normal deviate of -1.32 means that 9.34 per cent of the area under such a distribution is contained in the interval from minus infinity to the mean minus 1.32 standard deviations. Thus, the probability of yearly sales being 6,300 units or more is \(1 - .0934 \) or .9066 since the area not contained in the interval from minus infinity to the mean minus 1.32 standard deviations must be contained in the interval from the mean minus 1.32 standard deviations to plus infinity.

Another area that management may be interested in is the probability of at least some specific amount of profit. Assume that in the situation under consideration the firm will make a profit of at least $50,000 if yearly sales are 6,600 units or more. To compute the probability of this occurrence the following substitutions are required in the normal deviate formula.
Normal Deviate = \( \frac{6,600 - 6,500}{151} \)

Normal Deviate = \( \frac{100}{151} \) = .66

Once more a table of areas under the normal curve can be used to determine that the area content of a normal distribution from the mean plus .66 standard deviations to plus infinity is 25.46 per cent. The probability of sales of 6,600 units or more and a profit of at least $50,000 is therefore .2546. Combining this result with knowledge gained from the previous computation the probability of yearly sales being within a certain interval can be derived. Since the likelihood of sales of 6,300 units or less is .0934 and of 6,600 units or more is .2546, then the probability of their being 6,300 units to 6,600 units must be 1.0000 minus .0934 minus .2546, or .6520.

Standard deviation based limits used frequently by a firm can be set up generally in advance. Table II illustrates four of the most common of these. Management need only add and subtract the designated number of standard deviations from the mean of the distribution of beliefs in question, and the probability of the item's actual value being contained in that interval is as indicated. For example, the individuals involved in the territorial sales situation can be 99.73 per cent certain that yearly sales

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will be within the interval 6,500 minus 3(151) to 6,500 plus 3(151), or 6,047 units to 6,953 units. Certainly, the availability of the types of probability statements discussed in this and the preceding paragraphs makes the probabilistic belief estimation technique an extremely valuable managerial tool—one that will in most cases result in more informed decision making.

TABLE II

COMMON NORMAL CURVE LIMITS

<table>
<thead>
<tr>
<th>Limits</th>
<th>Per Cent of Total Area Within Specified Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>EV + 0.6745SD</td>
<td>50.00</td>
</tr>
<tr>
<td>EV ± 1SD</td>
<td>68.26</td>
</tr>
<tr>
<td>EV ± 2SD</td>
<td>95.46</td>
</tr>
<tr>
<td>EV ± 3SD</td>
<td>99.73</td>
</tr>
</tbody>
</table>

EV = Expected Value
SD = Standard Deviation

PROBABILITY SIGNALS FOR BUDGET REVISION

In those situations where the budgeted amounts for a particular item are based primarily on the company's past experience with the item as indicated by various records, probability can be used to help determine when and if a revision of them is needed. The following paragraphs explain and illustrate two such probabilistic applications.
Probability and Period by Period

Comparison of Projections

The logic behind the application of probability to determine if budgeted figures based primarily on past records need revision is very simple. Basically, what is done is to discover if actual amounts appear to be reasonable given the hypothesis that they were generated by the particular distribution that past experience seems to indicate for the item in question. The first step in the analysis is to compute the mean and standard deviation of this past distribution, and is accomplished using the results of some specific historical period and the two formulas given below.

\[
\text{Mean} = \frac{\sum X}{n}
\]

\[
\text{Standard Deviation} = \sqrt{\frac{\sum(X - \text{Mean})^2}{n - 1}}
\]

\[X = \text{Values of the individual observations in the period}\]

\[n = \text{Number of observations in the period}\]

The second step, granted that management is willing to assume the distribution is or can be approximated by a normal one, is to use these two measurements to compute the likelihood of results similar to the actual ones using the normal deviate concept discussed earlier. This probability can
then be analyzed by the appropriate decision makers to see if it appears to support the hypothesis that the actual results were generated by a distribution such as the one that has been derived from past company records.

As an illustration of this technique assume that a firm bases its monthly sales budgets primarily on sales of similar periods in the preceding year. Some adjustments are occasionally necessary due to changes in environmental or operating conditions, but they are usually of a very minor nature and can be ignored for practical analytical purposes. Table III gives the monthly sales of the firm for the past year as well as the mean and standard deviation of their distribution.

Now, assume further that actual sales for January of the budgeted period are 950 units. To compute the likelihood of such a figure the normal deviate formula is used as below.

$$\text{Normal Deviate} = \frac{950 - 850}{117}$$

$$= \frac{100}{117} = .85$$

Perusal of a table of areas under the normal curve indicates that a normal deviate of .85 means an area content of 19.77 per cent in the interval from the mean plus .85 standard deviations to plus infinity. Thus, the probability of sales being 950 units or more given the hypothesis that they were generated by the past distribution is .1977. The analysis becomes more meaningful, however, if the more general
TABLE III
MONTHLY SALES WITH THEIR MEAN AND STANDARD DEVIATION
(Sales in Units)

<table>
<thead>
<tr>
<th>Month</th>
<th>X Amounta</th>
<th>Mean - X</th>
<th>(Mean - X)^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>850</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>February</td>
<td>975</td>
<td>-125</td>
<td>15,625</td>
</tr>
<tr>
<td>March</td>
<td>700</td>
<td>150</td>
<td>22,500</td>
</tr>
<tr>
<td>April</td>
<td>900</td>
<td>-50</td>
<td>2,500</td>
</tr>
<tr>
<td>May</td>
<td>850</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>June</td>
<td>1,050</td>
<td>-200</td>
<td>40,000</td>
</tr>
<tr>
<td>July</td>
<td>775</td>
<td>75</td>
<td>5,625</td>
</tr>
<tr>
<td>August</td>
<td>875</td>
<td>-25</td>
<td>625</td>
</tr>
<tr>
<td>September</td>
<td>725</td>
<td>125</td>
<td>15,625</td>
</tr>
<tr>
<td>October</td>
<td>800</td>
<td>50</td>
<td>2,500</td>
</tr>
<tr>
<td>November</td>
<td>700</td>
<td>150</td>
<td>22,500</td>
</tr>
<tr>
<td>December</td>
<td>1,000</td>
<td>-150</td>
<td>22,500</td>
</tr>
<tr>
<td></td>
<td>10,200</td>
<td></td>
<td>150,900</td>
</tr>
</tbody>
</table>

Mean = 10,200/12 = 850 units

\[
\text{Standard Deviation} = \sqrt{\frac{150,900}{12 - 1}}
\]

\[
= \sqrt{13,718} = 117 \text{ units}
\]

These monthly sales figures are not adjusted for seasonal variation though such would probably result in an improved analysis.
question "What is the likelihood of a deviation of 100 units or more?" is answered. This brings into the picture the area from minus infinity to the mean minus .85 standard deviations as well as the above area from the positive tail of the normal curve, and the probability associated with it is likewise .1977. The probability of a deviation of 100 units or more is therefore .3954 (.1977 + .1977).

Perhaps the most difficult aspect of the technique is the determination by management of how small the probability of the deviation has to be before the past distribution hypothesis is rejected. Naturally, its rejection means that the budget figures for future periods must be revised since some other universe now appears to be generating the monthly sales projections. The cutoff point for each individual situation must be based on the experience and intuition of the personnel involved plus the cost of a budget revision. One possible initial course of action is to use the three standard deviation limits that have proved effective for quality control purposes and adjust them as deemed necessary by subsequent happenings. The use of such limits means that the probability of at least some particular deviation must be .0027 or below before the past distribution hypothesis is rejected. In the above example the probability was .3954 so a budget revision is definitely not indicated if the firm's

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6One possible source for their revision is the probabilistic belief concept discussed earlier in this chapter.
management is willing to accept the three standard deviation limits.

An alternative method of handling the analysis is simply to construct the range which has been deemed acceptable. For example, in the above situation where three standard deviation limits were considered appropriate this means that if actual results fall within the mean plus and minus three standard deviations the hypothesis is accepted. Thus, as long as monthly sales are within the range 499 \((850 - 3(117))\) to 1,201 \((850 + 3(117))\) no budget revision is indicated, and such would be the conclusion for January’s sales of 950 units.

It should be pointed out here that there may be other indications of the need for a budget revision even though actual results are within the range considered acceptable by management. The most useful of these indications for the technique under consideration is the occurrence of seven straight figures on one side of the mean or expected value of the past distribution. The reason for this is that the probability of such happening due to natural causes is extremely low, .0156 to be exact. As the normal curve is symmetrical and its mean divides the distribution in half then the likelihood of a value on either side of this central measure must be .5. The likelihood of seven straight values on one side is .5\(^7\) or .0078, and of seven straight on either side .0078 + .0078 or the above-mentioned figure of .0156. Thus, management can now have two indications of those
situations where the basis for a budget may no longer be appropriate and a revision of the latter should perhaps be considered.

**Probability, Projections, and Cumulative Results**

It is possible to apply this probabilistic technique for determining the need of a budget revision in a slightly different manner. Instead of testing the reasonableness of each period's actual results, their cumulative amount can be tested. This application, concerning which Paul C. Taylor has done some outstanding exploratory work, is accomplished in the following manner. First and as with the period by period application, management must determine the limits, be they three standard deviation limits or otherwise, within which actual results can fluctuate and still be accepted as supporting the past distribution hypothesis. The results of the first period are then compared with these limits and for each period checked afterwards the mean of the past distribution is added to both the lower and upper limits and the cumulative results compared with this adjusted range.

For illustration purposes assume the same situation previously used concerning monthly sales budgets. It should be remembered that the past distribution used to derive the budgets had a mean of 850 units and a standard deviation of

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117 units, and that actual sales for the first month were
950 units, well within the three standard deviation limits
decided upon by management of 499 to 1,201. The cumulative
technique utilizes the same procedure as above for the first
period and would therefore give identical results and the
same conclusion—the budgets do not appear to need revision.
To test the second period, however, cumulative sales would be
compared with limits of 1,349 (499 + 850) and 2,051 (1,201 +
850), the first period's limits increased by the mean of the
past distribution. Thus, if sales for the second month are
1,150 units then 2,100 (950 + 1,150) would be checked against
the above limits. Since it exceeds the upper one the hypoth­
esis that current sales are being generated by a distribution
similar to that of past sales should be rejected and budgets
for future months should probably be revised.

SUMMARY

The successful planning, operating, and controlling
of a modern business depends to a large degree upon estimates
of the outcomes of future actions. Most estimation techniques
currently being used give management only one figure—a mea­
sure of central tendency. This chapter considers and illus­
trates a probabilistic forecasting technique that gives an
additional measure—one of estimate dispersion, and also dis­
cusses the importance and advantage of utilizing such a
technique.

Utilizing the beliefs of a single manager for
probabilistic projections is considered first. The individual involved must estimate the mean of the item in question and also decide how reliable he considers this estimate. Using this last information and assuming a normal curve the standard deviation of the distribution of beliefs is then derived. As a result a measure of central tendency and a measure of estimate dispersion is available for management's disposition.

The next area discussed concerns the combining of the belief distributions of two or more managers. Several mathematical theorems are cited that indicate that the mean of the combined distribution will be equal to the sum of the individual means, that its standard deviation will be equal to the square root of the sum of the individual variances, and that it will be normally distributed.

At this point the importance of the probabilistic estimation technique is considered. The ability its use gives management to compute various probabilities concerning the projected item is introduced and the mechanics involved in doing so explained. The benefits that can be derived from knowing such probabilities are undoubtedly self evident, and if not will become evident in subsequent chapters.

The chapter concludes with a discussion of several probabilistic techniques that enable a firm to decide when budgets based primarily on past actions need revision. The general approach followed is to make the hypothesis that current results will be generated by a normal distribution
similar to the one found in the past, and then to determine if they do actually support this hypothesis. Naturally, if such support is not indicated the hypothesis should be rejected and a revision of the budget undertaken.
CHAPTER III

PROBABILITY IN COST-VOLUME-PROFIT ANALYSIS

NEED FOR PROBABILITY

One of the more important accounting areas open to application of probabilistic techniques is cost-volume-profit analysis. This valuable decision tool, most frequently used by management to help choose among alternative courses of action, does not usually include any provision for the quantitative consideration of uncertainty and risk. There is a definite emphasis here on quantitative consideration since most decision makers will adamantly argue that they do consider risk and uncertainty problems in their qualitative analysis.

Without doubt, there is nothing wrong with such an approach to the situation. It is better for management to give qualitative consideration to these two areas than no consideration at all. Unfortunately, many decision makers tend to give more, and in most cases unjustified, weight and intelligent deliberation to the quantitative factors involved. They slight the qualitative ones as being too subjective and inexplicable. Therefore, it would be advantageous if uncertainty and risk could be considered in some
meaningful quantitative manner in conjunction with cost-volume-profit analysis.

Robert Jaedicke and Alexander Robichek, who have done some outstanding exploratory work in this area, use an oversimplified example of the following type to illustrate why uncertainty should be a major consideration of cost-volume-profit analysis.¹ Suppose a company is evaluating the introduction of one of two newly developed products. Both of the products would necessitate a similar increase in fixed costs and both would have the same variable cost per unit. The planned selling prices of the two items would also be identical. Now, if the expected sales volumes of the two products are estimated as being equal, present cost-volume-profit analytical techniques would indicate that the two alternatives should be treated as perfect substitutes—that is equally desirable if expected sales volume is greater than the breakeven point or undesirable if the reverse is true. Obviously, however, management would not really be indifferent between the two products unless their equal expected sales volumes were definite or known with certainty. As long as there were varying degrees of risk associated with each of the sales figures management would definitely or should definitely prefer one to the other. The use of probability in cost-volume-profit analysis gives the

decision maker one quantitative technique for evaluating risk and uncertainty. This combined and improved analytical procedure, in conjunction with any qualitative considerations deemed advisable, can then be used as a basis for the final decision.

A SIMPLE PROBABILISTIC COST-VOLUME-PROFIT APPLICATION

Traditional Cost-Volume-Profit Analysis

Traditional cost-volume-profit analysis is simply what its name implies, a means of analyzing the relationship between sales price, volume, and costs. It is based on the concept of the contribution margin, sales price less variable costs, and "... provides helpful information for decisions as to pricing, cost alternatives, sales mix, channels of distribution, possible sales promotion, addition or deletion of product lines, acceptance of special orders, entering foreign markets, and changing plant layout."\(^2\) An important part of cost-volume-profit analysis is breakeven analysis. This latter technique is used to determine the point of activity at which a firm has exactly zero profits, a point that managerial personnel most certainly need to have cognizance of in many of the decision situations confronting them. Since breakeven analysis will be used to help explain the introduction of probability to cost-volume-profit analysis,

an illustration of it will now be given.

Assume the following simple situation. A manufacturing company has just been incorporated and plans to produce, among others, a product which will be sold for $10 per unit. Average demand for the product has been projected to be 4,500 units per year. There are two machines available that are capable of producing this particular product, each of which has a capacity of 8,000 units per year. Machine 1 would have fixed costs of $30,000 per year and the variable costs associated with it would be $4 per unit. The same figures for Machine 2 would be $16,000 and $6, respectively.

Traditional cost-volume-profit analysis would yield breakeven points of 5,000 units for Machine 1 and 4,000 units for Machine 2. These figures are easily determined by dividing the fixed costs required by each machine by their appropriate contribution margins. For Machine 1 this would be $30,000 divided by $6 ($10 - $4) and for Machine 2 $16,000 divided by $4 ($10 - $6). Naturally, since the projected average demand, 4,500 units, is more than Machine 2's breakeven point but less than Machine 1's, the decision that seems to be indicated is to purchase Machine 2. The same decision could also have been reached by comparing the net income under each machine at the projected demand level. Table IV does this and it shows Machine 2 definitely to be the more profitable. In fact, as already indicated by the comparison of its breakeven point with the projected average demand, Machine 1 involves a net loss at this demand level.
TABLE IV

NET INCOME OF MACHINES AT PROJECTED AVERAGE DEMAND

<table>
<thead>
<tr>
<th></th>
<th>Machine 1</th>
<th>Machine 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales (4,500 units X $10)</td>
<td>$45,000</td>
<td>$45,000</td>
</tr>
<tr>
<td>Less variable costs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4,500 units X $4)</td>
<td>18,000</td>
<td>27,000</td>
</tr>
<tr>
<td>(4,500 units X $6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contribution to fixed costs</td>
<td>27,000</td>
<td>17,800</td>
</tr>
<tr>
<td>Less fixed costs</td>
<td>30,000</td>
<td>16,000</td>
</tr>
<tr>
<td>Net income or loss</td>
<td>($ 3,000)</td>
<td>$ 1,800</td>
</tr>
</tbody>
</table>

Probabilistic Cost-Volume-Profit Analysis

The first step in probabilistic cost-volume-profit analysis is to determine whether each factor involved is fixed or variable. Fixed factors, of course, present no problem. For each of the variable factors, however, a probability distribution must be derived. Such a distribution is simply a table showing all possible values which a particular variable can assume along with their corresponding likelihood or probability. Next, these distributions are used to compute the expected value of each variable. This is accomplished by multiplying each possible value in a distribution by its probability and adding the results. Thus, the expected value of a variable is nothing but a weighted average of all possible values where the weight assigned to a possibility is equal to its probability. The final step is to use these expected values and any fixed amounts to complete the analysis utilizing the same
procedures as the traditional cost-volume-profit approach.

To illustrate probabilistic cost-volume-profit analysis assume the same situation previously used to illustrate the traditional technique and the following additional facts. The selling prices and various costs involved are known with certainty, but the demand for the hypothetical product to be produced is variable. Also, from sources to be discussed in subsequent paragraphs management has derived a probability distribution of demand for the product as shown in Table V.

### TABLE V

<table>
<thead>
<tr>
<th>Demand</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,000</td>
<td>.10</td>
</tr>
<tr>
<td>3,000</td>
<td>.20</td>
</tr>
<tr>
<td>4,000</td>
<td>.25</td>
</tr>
<tr>
<td>5,000</td>
<td>.30</td>
</tr>
<tr>
<td>6,000</td>
<td>.15</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

As explained earlier, the next step in the analysis is to compute the expected value of demand. This is done and the results shown in Table VI. This weighted average of 4,200 units can now be used to determine which of the two machines will result in the most profitable operations. As with the traditional cost-volume-profit analysis, Machine 2
appears to be the one that this company should purchase since its breakeven point is higher than the expected value of demand and Machine 1's is not. Table VII utilizes the income approach to reach the same decision.

**TABLE VI**

**EXPECTED VALUE OF DEMAND**

<table>
<thead>
<tr>
<th>Demand</th>
<th>Probability of Demand</th>
<th>(1 X 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,000</td>
<td>0.10</td>
<td>200</td>
</tr>
<tr>
<td>3,000</td>
<td>0.20</td>
<td>600</td>
</tr>
<tr>
<td>4,000</td>
<td>0.25</td>
<td>1,000</td>
</tr>
<tr>
<td>5,000</td>
<td>0.30</td>
<td>1,500</td>
</tr>
<tr>
<td>6,000</td>
<td>0.15</td>
<td>900</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>4,200</td>
</tr>
</tbody>
</table>

**TABLE VII**

**NET INCOME OF MACHINES AT EXPECTED VALUE OF DEMAND LEVEL**

<table>
<thead>
<tr>
<th></th>
<th>Machine 1</th>
<th>Machine 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales (4,200 units X $10)</td>
<td>$42,000</td>
<td>$42,000</td>
</tr>
<tr>
<td>Less variable costs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4,200 units X $4)</td>
<td>16,800</td>
<td>25,200</td>
</tr>
<tr>
<td>(4,200 units X $6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contribution to fixed costs</td>
<td>25,200</td>
<td>16,800</td>
</tr>
<tr>
<td>Less fixed costs</td>
<td>30,000</td>
<td>16,000</td>
</tr>
<tr>
<td>Net income or loss</td>
<td>($ 5,000)</td>
<td>$ 800</td>
</tr>
</tbody>
</table>
The machine decision made under either the traditional or probabilistic cost-volume-profit analysis, however, should not be considered final until concrete weight and thought have been given to the risk factor. One of the main advantages of the latter approach is that it encourages and aids management to do this. Using the probability distributions required for its application, much more objective conclusions can be drawn about various risk considerations than would otherwise be possible. The following paragraphs will hopefully demonstrate this advantage of probabilistic cost-volume-profit analysis.

Assuming the same example previously used, consideration of risk will actually not affect the choice between the two possible machines. If demand should be at the lowest level indicated possible by Table V the loss will be less with Machine 2, and if it should be at the highest level possible the net income will be more with Machine 2. Thus, over the demand range given, 2,000 to 6,000 units, Machine 2 is preferable to Machine 1 at all levels. A study of Table V does bring out some interesting observations as to whether or not this particular product should be introduced at all, however. The probability of demand being 3,000 units or less per year is .30, and this means that the company has almost one chance in three of losing at least $4,000 on the product (fixed costs of $6,000 less 3,000 times the $4
contribution margin). The likelihood of breaking even or making a profit is naturally 1.00 - .30 or .70, but if the probability of exactly breaking even of .25 (if demand is for the breakeven point amount of 4,000) is subtracted from this figure, it can be seen that the firm has only a .45 likelihood of making a profit on the product. Therefore, if the financial position of this company is somewhat precarious, as it usually is for most new businesses, the 30 per cent chance of losing $4,000 or more will undoubtedly outweigh the 45 per cent chance of making a profit and the product should be dropped from the proposed product line. On the other hand, if the company's financial position is strong the product might well be produced as there is a 70 per cent chance of at least breaking even, a 45 per cent chance of making a profit of at least $4,000 (if demand is for 5,000 units or more), and a 15 per cent chance of profits of $8,000 (if demand is for the maximum possible of 6,000 units).

Obviously, then, a decision cannot be reached in this case with the data given. Additional information on the current and expected financial condition of the firm must be obtained and studied, and a guideline established on the amount of risk allowable, before a final decision as to the product can be determined.

As a further example of risk consideration under the

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\(^3\)Naturally, these and the following profit figures are based on the utilization of Machine 2 for the product's production.
probabilistic cost-volume-profit approach assume that the probability distribution of demand and the expected value of demand for the above product had looked as in Table VIII rather than Tables V and VI. Since the expected value of demand is now in excess of both machines' breakeven points, the decision must be based on the net income analysis as shown previously in Tables VI and VII. Table IX does this for the new expected value of 6,990 units. According to its results Machine 2 is still preferable to Machine 1, but now only by $20.

TABLE VIII
PROBABILITY DISTRIBUTION OF DEMAND AND EXPECTED VALUE OF DEMAND

<table>
<thead>
<tr>
<th>Demand</th>
<th>Probability of Demand</th>
<th>(1 X 2) Expected Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>.01</td>
<td>10</td>
</tr>
<tr>
<td>2,000</td>
<td>.03</td>
<td>60</td>
</tr>
<tr>
<td>3,000</td>
<td>.05</td>
<td>150</td>
</tr>
<tr>
<td>4,000</td>
<td>.07</td>
<td>280</td>
</tr>
<tr>
<td>5,000</td>
<td>.09</td>
<td>450</td>
</tr>
<tr>
<td>6,000</td>
<td>.12</td>
<td>720</td>
</tr>
<tr>
<td>7,000</td>
<td>.14</td>
<td>980</td>
</tr>
<tr>
<td>8,000</td>
<td>.17</td>
<td>1,360</td>
</tr>
<tr>
<td>9,000</td>
<td>.22</td>
<td>1,980</td>
</tr>
<tr>
<td>10,000</td>
<td>.10</td>
<td>1,000</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>6,990</td>
</tr>
</tbody>
</table>
TABLE IX

NET INCOME OF MACHINES AT EXPECTED VALUE OF DEMAND LEVEL

<table>
<thead>
<tr>
<th></th>
<th>Machine 1</th>
<th>Machine 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales (6,990 units X $10)</td>
<td>$69,990</td>
<td>$69,900</td>
</tr>
<tr>
<td>Less variable costs</td>
<td>27,960</td>
<td>41,940</td>
</tr>
<tr>
<td>(6,990 units X $4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6,990 units X $6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contribution to fixed costs</td>
<td>41,940</td>
<td>27,960</td>
</tr>
<tr>
<td>Less fixed costs</td>
<td>30,000</td>
<td>16,000</td>
</tr>
<tr>
<td>Net income</td>
<td>$11,940</td>
<td>$11,960</td>
</tr>
</tbody>
</table>

This choice may be changed, however, if the risk factor is taken into consideration. Should demand reach 7,000 units, only 10 units more than the expected value, profits under Machine 1 will equal profits under Machine 2 and should demand surpass this figure then the former machine will be more profitable than the latter. Table VIII shows that there is a 63 per cent change of demand for the product being 7,000 units or more and a 49 per cent chance of it being 8,000 or more. Thus, if Machine 1 is selected instead of Machine 2 the probability is .63 that profits for the firm will be the same or more and .49 that they will be more. The converse of the above says that the likelihood of profits being less under Machine 1 is 1.00 minus .63 or .37. Further examination of this last area reveals that there is only one chance in four that the results will be more than $4,000 worse if the decision is reversed. This is true
since such can happen only if demand is for 5,000 units or less, and the probability of this is .25. The largest difference in outcomes would occur if demand were for the minimum of 1,000 units. Machine 2 would involve a $12,000 loss and Machine 1 a $24,000 one. Of course, the probability of this happening is almost negligible—.01 in fact.

Many more outcomes and their respective probabilities could be considered, but this would hopefully be an unproductive effort. It is hoped that by now the significance of the use of probability for risk consideration in cost-volume-profit analysis has been adequately demonstrated. Once management has decided what chances it is willing to take, what results must be how likely to offset various risk factors, then probability distributions can be used to help choose between available alternatives in a decision situation. Unfortunately, no general rules can be established to aid management in determining these risk rules. They must be based on the financial situation of the particular firm involved, the alternatives under consideration, the reliability placed upon the probability distributions used in the analysis, and the beliefs and attitudes of the individuals responsible for making the analysis and the final decision.
LIMITATIONS AND IMPROVEMENTS OF THIS SIMPLE PROBABILISTIC TECHNIQUE

Deriving Required Probability Distributions

Probably the major limitation of the simple probabilistic cost-volume-profit technique is the difficulty involved in deriving the variable factor probability distributions. According to Robert Schlaifer, "Reasonable men base the probabilities which they assign to events in the real world on their experience with events in the real world." Thus, one of the best sources for such distributions may well be the past records of the firm. For example, demand probability distributions can be based on historical sales data in those cases where the product or products in question are existing ones. These data must be adjusted, of course, for any known conditions which management believes will alter demand in the future periods under consideration. The best qualified persons to make these adjustments would be the market research personnel. If a company does not have such a department, the individual in charge of marketing activities would be the most logical substitute. The same type of approach could be used for any variable factor with which the firm has had previous similar experiences.

The derivation of the required probability distributions presents a much more difficult problem where the use

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of past records is not appropriate. One source to use in such a case is the probability beliefs of the managerial personnel involved in the analysis. This concept was examined in detail in Chapter II and will not be examined further here. Other sources of information on which to base the distributions are trade associations, equipment manufacturers, and market research studies. It is extremely important that management realize, however, that a probability distribution based on data from such sources is very likely to be less reliable than one based on fairly extensive past experiences. Thus, it will undoubtedly be necessary for an adjustment to be made in risk attitudes--except less risk--to take this condition into account.

Regardless of the source of the data used to derive the required probability distributions, there may be times when the only information available is somewhat skimpy and appears incomplete. In such cases there is a method for estimating the complete behavior of the distribution. This method involves three basic steps. First, graph the distribution that is indicated by the available data. Second, fit by eye a smooth curve to this graph that has the right general shape. Third, adjust the curve so that the probabilities read from it add up exactly to one. This last step is accomplished by reading from the curve the probability of each possible value of the variable, summing these figures, and increasing or decreasing each probability by the proportional amount necessary to make their total add to
Even though a distribution derived in this manner is not precise, it is certainly better than no distribution at all or one that is obviously incomplete. As Jaedicke and Robichek say, "An estimate . . . is necessary to make a decision. Hence, the question is not whether an estimate must be made, but simply a question of the best way to make and express the estimate."

Use of Discrete Probability Distributions

A second limitation of the simple probabilistic cost-volume-profit technique is that so far all the distributions used have been discrete ones. In other words, based on the information presented in Table V a demand for 2,005 units per year is absolutely impossible. In fact, Table V says that demand can never be less than 2,000 units, more than 6,000 units, or anything in between those two figures except for the three values given. This is obviously an oversimplification and not a very realistic picture of the demand patterns confronting most businesses. Usually they are composed of an almost infinite number of possible values. Using discrete probability distributions that were more realistic, however, would require that every one of these possible values be tabulated along with its appropriate

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5Ibid., p. 99.

likelihood. This would be extremely expensive, tedious, and time consuming, and unless computers or other automated equipment were available would make the computation of the expected values of the distributions an almost endless task. Furthermore, the abundance of possible values would be so confusing that the drawing of meaningful risk conclusions from them would be a virtual impossibility. Without doubt then, some acceptable alternative procedure must be found and utilized by management as a means of overcoming this limitation.

Replacing of Discrete with Continuous Probability Distributions

One alternative procedure for obtaining a more realistic picture of the business environment is to replace discrete probability distributions with continuous ones. This is true since all values between the two tails or extremes of a distribution are possible with the latter. There are many such distributions that can occur and may be appropriate in various business situations, but only one of these is perhaps worthy of examination in this paper. That one is the normal probability distribution. When it is considered applicable to a cost-volume-profit decision area and used certain important advantages result. The major one of these is that the necessary calculations for its expected value and for the various probabilities required for a proper risk analysis are greatly simplified. This is the case
since, as explained in Chapter II, the normal distribution is completely defined when only two values, its mean and standard deviation, are known.

How frequently will the normal curve be applicable to business situations, however? Fortunately, it is found quite often in such surroundings. Concerning this aspect Eugene Grant says, "Its (the normal curve's) general pattern with a concentration of frequencies about a mid-point and with small numbers of occurrences at the extreme values, repeats itself again and again."\(^7\) But, Grant also goes on to say at the same time, "Nevertheless, the normal curve is frequently misused; it is not safe to assume that unknown distributions are necessarily normal."\(^8\) Unfortunately this last statement is very apropos. Numerous other distributions decline continuously from their midpoint to their tails, like the normal, but they are not symmetrical. In statistical terms these are known as skewed curves.

In summary, then, the normal distribution is not appropriate for all business situations. The decision maker involved with a particular problem must determine if the distribution in question seems to have the characteristics of the normal curve as enumerated in Chapter II. If it does, the analytical techniques discussed and illustrated in


\(^8\)Loc. cit.
subsequent paragraphs can be utilized. In some cases even though a distribution does not completely satisfy all the requirements of a normal one, it may come close enough to allow it to be treated as one for practical analytical purposes. As Schlaifer says, "... the Normal distribution is an excellent approximation to a wide variety of real distributions of great practical importance. ..."9 Before a normal distribution is used to approximate distributions which do not extend out to infinity, however, management must decide if the slight probabilities which it will give to these extreme tail values are of a critical nature. If they are, the normal approximation procedure should naturally not be used. In such a case the actual distribution of the item in question, or some approximation that does accurately reflect the actual one, should be used in the analysis.

In a preceding paragraph the probability beliefs of the managerial personnel involved in the cost-volume-profit analysis were stated to be one source of required variable factor probability distributions. This source has perhaps one major advantage over all others. Its use automatically results in a distribution that is normal. Furthermore, there are very few situations where it is not applicable. For almost every variable factor that may be involved in a cost-volume-profit problem facing a firm there is very likely some individual whose background and experience qualifies

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9Schlaifer, op. cit., p. 274.
him to make the required predictions concerning its outcome.

**Illustration of Continuous Distribution Application**

To illustrate in general the use of a continuous distribution in cost-volume-profit analysis and more specifically the application of the normal one in this area assume the following situation. A company which manufactures and sells smoking pipes and related items is presently considering the addition of a windproof pipe lighter to its product line. The lighter is expected to sell at a price of $10 and will involve variable costs of $6 per unit and fixed costs of $20,000 for equipment and other items. Sales for the first year are expected to average approximately 7,000 units.

Under present cost-volume-profit analytical techniques this information would be used to determine the breakeven point of 5,000 units for the lighters (fixed costs of $20,000 divided by the contribution margin of $4). This figure would then be compared with the sales expectation of 7,000 units and the product deemed desirable since the latter exceeds the former. What about the risk involved if the lighter is added to the product line, however? What is the chance of demand being less than the breakeven point and the firm sustaining a loss, or of profits being at least some minimum desired figure? Unfortunately, these questions cannot be objectively answered by management with the information given above. Only qualitative consideration can be rendered them.

Assume now, however, that as a result of experience,
preliminary surveys, and other sources that some individual in the marketing department of the firm has been able to make certain predictions concerning the demand for the pipe lighters. He believes that it will have an average value of 7,000 units per year and that there is a 50-50 chance of his estimate being within plus or minus 1,500 units of the true universe average. As explained in the previous chapter, such predictions give directly the mean of a normal distribution of beliefs and can be used to determine its standard deviation, thus completely defining it. Naturally, this latter aspect means that the probabilities of various values of the distribution can easily be computed.

Calculation of the standard deviation from the above estimates has also been considered earlier, but as a review proceeds generally as follows. Since it is thought that there is a 50 per cent chance of the estimate of average demand being within plus or minus 1,500 units of the true population mean and since the distribution being defined is a normal one, then 50 per cent of the curve's area must be included in the interval from 5,500 units (the estimated mean minus 1,500 units) to 8,500 units (the estimated mean plus 1,500 units). From this it follows that 25 per cent of the area under the distribution must be included in the interval from minus infinity to 5,500 units and 25 per cent in the interval from 8,500 units to plus infinity. For these conditions to be satisfied the curve's standard deviation must have one particular value, and the normal deviate
formula plus a table of areas under the normal curve can be used to determine this value. In this pipe lighter example the standard deviation of the distribution of beliefs for their demand must be 2,240 units.\(^\text{10}\)

Various probabilities for this normal belief distribution can now be computed and the risk questions posed earlier answered. Computation of probabilities from a normal curve has also been discussed previously and involves simply the normal deviate formula and a table of areas under the normal curve. Thus, the probability of at least breaking even on the pipe lighters can be found by subtracting the breakeven point from the mean, dividing this result by the standard deviation, and looking up the resulting normal deviate in the area table. Calculation of the normal deviate is illustrated below.

\[
\text{Normal Deviate} = \frac{5,000 - 7,000}{2,240}
\]

\[
\text{Normal Deviate} = \frac{-2,000}{2,240} = -.89
\]

Based on this figure the table indicates that 81.33 per cent of the area under a normal curve is included in the interval from the mean minus .89 standard deviations to plus infinity, or in this particular example from 5,000 units to plus infinity. This means that the probability of breaking even

\(^{10}\)This number has been rounded off for ease of calculation. A more exact figure would be 2,238.8 units.
or making a profit is also .8133. Naturally, the complement of this condition says that the probability of incurring a loss of some amount on the lighters is .1867 (1.0000 minus .8133).

If the management of this firm believes that loss of $8,000 or more will place them in a virtual state of bankruptcy they will undoubtedly want to know the likelihood of such an event happening. A loss of $8,000 or more can occur only if demand is for 3,000 units or less. This figure can be obtained by dividing the contribution margin of $4 into $12,000, the fixed costs of $20,000 that will be necessitated less the $8,000 loss. The normal deviate for such a demand is 1.79 (7,000 minus 3,000 divided by 2,240), and a table of areas under the normal curve yields an area content of 3.67 per cent for this figure. In other words, there are only approximately four chances in one hundred that demand for the lighters will go so low that the company will sustain a loss of $8,000 or more. In most cases this degree of risk would probably not be sufficient to cause rejection of a proposal that was otherwise favorable.

As a further illustration of the potential of this probabilistic technique assume that the company has decided it does not want to undertake production and distribution of the pipe lighters unless there is a 50 per cent chance that they will yield an annual profit of $10,000 or more. The demand required to satisfy this condition can be calculated by dividing $30,000, the sum of the fixed costs of $20,000
and the desired profit of $10,000, by the contribution margin of $4, and the resulting answer is 7,500 units. Computing the normal deviate of .22 for this situation (7,500 minus 7,000 divided by 2,240) and looking it up in the table indicates that only 41.29 per cent of the area under a normal distribution is included in the interval from the mean plus .22 standard deviations to plus infinity. Therefore, the likelihood of the firm making at least a $10,000 profit on the lighters, of demand being 7,500 units or more, is only .4129. The .50 limitation desired by management is definitely not satisfied and the item should not be added to the company's product line.

Incidentally, at the beginning of the lighter example traditional cost-volume-profit analysis was used to show that the product appeared favorable, but not the probabilistic analysis. This latter method would yield the same conclusion, however, since the expected value of the demand distribution, 7,000 units, would be compared with the break-even point of 5,000 units and found to exceed it. Both techniques involve identical analyses in this area of the application. The discussed and illustrated risk evaluation area of the continuous normal probabilistic technique, though, would have been impossible if only the informational needs of the traditional approach had been fulfilled, and hopefully its usefulness and importance is now beyond question. Where appropriate it can be utilized to answer practically any questions concerning risk considerations that a thorough
and thoughtful decision maker might formulate, and certainly such a procedure represents a tool that can truly result in more informed managerial decisions.

A COMPREHENSIVE PROBABILISTIC COST-VOLUME-PROFIT APPLICATION

Comprehensive Probabilistic Cost-Volume-Profit Analysis

As explained earlier in this chapter, probabilistic cost-volume-profit analysis requires that a probability distribution be derived for each variable factor involved. These distributions, along with those factors that are fixed, are then used to determine whether or not the situation appears favorable and to analyze any appropriate risk considerations. In all the examples used so far, however, only one factor—demand—has been treated as variable. All others have been considered as being known with certainty. To see what the analysis would look like with this restriction dropped assume the same basic facts as in the previous lighter example, except that fixed costs are now also uncertain or variable. Production and financial experts at the company believe that they will average around $20,000 and that there is a two-thirds chance that actual fixed costs will be within plus or minus $500 of this figure. Assuming that the company is willing to fill in this distribution using the normal curve, its standard deviation must be calculated so that it will be completely defined. This can be done by the same procedures described in the case of the pipe lighter
demand distribution, and the resulting answer will be $500. Now the analysis can be completed with not one, but two curves with known means and standard deviations.

The procedure to determine if the lighters appear favorable and perhaps should be added to the company's product line is unchanged by the addition of this second variable factor. The expected value of the fixed costs of $20,000 would be divided by the contribution margin of $4 to yield a breakeven point of 5,000 units. This is naturally the same breakeven point obtained when the fixed costs were a definite $20,000. Comparison of this figure with the expected value of demand of 7,000 units would also give the same result as before—indication of the pipe lighters as being an advantageous undertaking.

Risk and Comprehensive Probabilistic Cost-Volume-Profit Analysis

To handle risk consideration under comprehensive probabilistic cost-volume-profit analysis the variable factor probability distributions along with any fixed data must be converted into a single distribution, a distribution of profits. Since the information from which it is derived is either definite or normally distributed, this profit distribution will be normal with a mean and standard deviation that can be calculated. In the present pipe lighter example the mean will be equal to the expected value of demand times the contribution margin, less the expected value of the
fixed costs, or $8,000 (7,000 times $4 less $20,000). The
standard deviation can be determined in the following manner.
First, square the contribution margin. Second, multiply this
figure by the square of the standard deviation of the demand
distribution. Third, add to this result the square of the
standard deviation of the fixed costs distribution.\(^\text{11}\)
Fourth, take the square root of this last figure. In equa-
tion form this procedure would appear as follows:\(^\text{12}\)

\[
SDpr = \sqrt{(CM^2 \times SDq^2) + SDfc^2}
\]

\(SDpr\) = Standard deviation of profit distribution
\(CM\) = Contribution margin
\(SDq\) = Standard deviation of demand distribution
\(SDfc\) = Standard deviation of fixed cost distribution

Substituting the information from the lighter example in
this equation yields a standard deviation for the profit
curve of $8,974. Probabilities can now be computed for this
derived normal distribution just as they were previously for
the normal demand distribution. Of course, the probabili-
ties are now directly in terms of profits and not in terms

\(^{11}\)The square of the standard deviation is known as
the variance. Because the variance is additive (i.e., the
variance of a sum of independent random variables is equal
to the sum of their individual variances) while the standard
deviation is not, the former measurement must be used in
these calculations.

\(^{12}\)Robert K. Jaedicke and Alexander A. Robichek, "Cost-
Volume-Profit Analysis under Conditions of Uncertainty,"
Accounting Review, 39 (October, 1964), 925.
of demands that must be converted into the respective profits which they would yield.

Assume that management wants to know the same three probabilities as before— at least breaking even, a loss of $8,000 or more, and a profit of $10,000 or more. The chance of at least breaking even is equal to the likelihood of zero profits or more under the profit distribution, and a normal deviate must be determined for such an event and looked up in a table of areas under the normal curve. The normal deviate is calculated by subtracting zero from the mean and dividing this result by the standard deviation. The data of this example yields a normal deviate of .89 (8,000 minus 0 divided by 8,974), which can be found through the table as representing an area content of .8133. Thus, the probability of at least breaking even in the new situation is .8133, the same as before. The addition of uncertain fixed costs has not caused any significant change in the odds. Such a result can be understood when the formula for the standard deviation of the profit distribution is examined closely. This shows that the demand data is much more heavily weighted than the fixed cost data, and changes in the latter therefore have only a very slight effect on profit probabilities.

The probabilities of a loss of $8,000 or more and of a profit of $10,000 or more can be calculated from the profit distribution in a manner similar to the probability of at least breaking even. The former can be found to
involve a normal deviate of 1.78 and a likelihood of .0375 and the latter .22 and .4129, respectively. Thus, a loss of $8,000 or more is just a fraction more likely now (.0375 compared with .0367 previously) and the chance of a $10,000 or more profit is the same as before. The reason for these minimal or zero changes is once more the heavier weight given to the demand distribution in the profit distribution standard deviation formula. Furthermore, it should have been obvious that the probabilities obtained from the profit distribution had to be approximately the same or more unfavorable. In other words, the probability of a loss on the lighters had to be greater and the probability of a profit smaller since added variability had been brought into the situation by the addition of a second uncertain factor, fixed costs.

General Equations for the Comprehensive Probabilistic Analysis

The illustration given above allowed only two factors, demand and fixed costs, to be uncertain. The other factors involved, selling price and variable costs, were still assumed to be fixed or known with certainty. Even if these latter items had also been variable, though, the general concept of the analysis would have been the same. In such a case there would have been four normal distributions instead of two that had to be converted into a single distribution of profits. The mean and standard deviation of this combined
normal distribution would be as follows:\(^1\)

\[
\begin{align*}
\text{Mpr} &= \text{Mq}(\text{Mp} - \text{Mv}) - \text{Mfc} \\
\text{SDpr} &= \sqrt{\left(\text{SDq}^2 (\text{SDp}^2 + \text{SDv}^2) + \frac{\text{Mq}^2 (\text{SDp}^2 + \text{SDv}^2) + \text{SDq}^2 (\text{Mp} - \text{Mv})^2 + \text{SDfc}^2}{\text{SDq}^2 + \text{SDp}^2 + \text{SDv}^2}}
\end{align*}
\]

\(\text{Mpr} = \text{Mean of profit distribution}\)

\(\text{Mq} = \text{Mean of demand distribution}\)

\(\text{Mp} = \text{Mean of selling price distribution}\)

\(\text{Mv} = \text{Mean of variable costs distribution}\)

\(\text{Mfc} = \text{Mean of fixed costs distribution}\)

\(\text{SDpr} = \text{Standard deviation of profit distribution}\)

\(\text{SDq} = \text{Standard deviation of demand distribution}\)

\(\text{SDp} = \text{Standard deviation of variable price distribution}\)

\(\text{SDv} = \text{Standard deviation of variable costs distribution}\)

\(\text{SDfc} = \text{Standard deviation of fixed costs distribution}\)

These general formulas for the mean and standard deviation of the distribution of profits when all factors are variable can also be utilized when one or more of the factors are not variable. In those cases where a particular factor is fixed rather than uncertain its definite amount can be substituted in the above formulas as its mean and its standard deviation is naturally zero. For example, the formula given previously for those situations where demand and fixed costs were the

\(^1\text{Loc. cit.}\)
only variable factors was derived by substituting zeroes in
the general formula for SDp and SDv since price and variable
costs were known with certainty, and by substituting also
their definite amounts for Mp and Mv and calling the differ­
ence between them by its synonym, contribution margin. The
initial substitutions caused the first two terms of the
general equation to drop out and the second substitutions
yielded the more restrictive formula.

THE UTILITY CONCEPT IN COST-VOLUME-PROFIT ANALYSIS

Limitation of Expected Value as a Guide to Action

The probabilistic cost-volume-profit technique dis­
cussed and illustrated in the preceding paragraphs really
requires two separate analyses before a final decision can
be reached in any given situation. One analysis is necessary
to determine if the alternative in question is favorable in
the light of expected value considerations, and another is
necessary to determine what effect risk factors have on the
decision that seems to be indicated by the results of the
first analysis. Such a procedure is necessitated because,
as Schlaifer says, expected value is a valid guide to action
only when the worst possible consequences of an action are
not too bad and the best possible consequences not too good.14

14Robert Schlaifer, Probability and Statistics for
Business Decisions: An Introduction to Managerial Economics
1959), p. 28.
As a further explanation of this point Schlaifer uses the following example.

A businessman with net assets of $500,000 who must choose between a deal which is certain to result in a profit of $40 and another which in his eyes is equally likely to result in a profit of $0 or a profit of $110 is likely to choose the latter act in accordance with the fact that its expected monetary value is $55; but if this same businessman is given the happy opportunity to choose between a deal which is certain to net him $5 million and another which has equal chances of yielding $0 and $11 million, he is very likely to take the $5 million.\(^{15}\)

In other words, risk factors can be ignored in a decision situation only when the amounts involved in the alternatives are extremely small in relation to the company's total financial structure, and this is more often than not the exception rather than the rule.

**Utility in Cost-Volume-Profit Analysis**

There is a technique available that allows the two analyses required in probabilistic cost-volume-profit analysis to be merged into a single one. It involves the assigning of utility values to the various possible outcomes of the alternatives in a decision situation. The decision maker bases these utility values upon monetary expectations adjusted for any risk considerations. Naturally, the influence of this latter factor upon the values assigned depends on the individual assigning them and the present and predicted economic environment of the company. Different persons

\(^{15}\text{loc. cit.}\)
may, and in most cases probably will, assign different utility values to similar possible results. The expected utility of each available alternative is then calculated by weighting the utility of its possible outcomes by their respective probabilities and adding the results, and the alternative with the highest expected utility is the one that should be undertaken. No additional risk analysis is necessary because such factor was considered in the assigning of utility values to the possible results of each alternative. For a more comprehensive explanation of this technique the reader is referred to Chapter Two of Schlaifer's book *Probability and Statistics for Business Decisions: An Introduction to Managerial Economics under Uncertainty*.

This concept of a single analysis is not being explored very extensively in this study because utility is too refined a technique for the present state of affairs in the accounting profession in this area. Decision makers must first be taught to use probability itself as a decision tool, and to properly do this its concept must be kept at a fairly elementary level. Once they have been accustomed to it and become familiar with its use, then they can be introduced to such refinements as the idea of utility.

**SUMMARY**

Traditional cost-volume-profit analysis does not usually include any provision for the quantitative considera-
tion of uncertainty and risk. In fact, the common practice is to treat the factors involved in such a decision situa-
tion as fixed, and to consider uncertainty and risk qualitatively subsequently. This chapter examines proba-
bilistic cost-volume-profit analysis, a technique that permits some or all of these factors to be variable.

The addition of a single variable factor to traditional cost-volume-profit analysis is discussed and illustrated first. A probability distribution must be derived for the factor and its expected value computed. This latter figure is then used in conjunction with the fixed factors to determine which alternative appears to be the most favorable. Risk conditions can then be evaluated using the probabilities of various outcomes as indicated by the probability distribution.

The use of the continuous normal curve rather than discrete probability distributions is considered next as a means of introducing even more reality into the analysis. Still assuming only a single variable factor, computation of the expected value, determination of the most favorable alternative, and risk analysis under this technique is discussed and illustrated. A similar approach is taken for comprehensive probabilistic cost-volume-profit analysis which removes the restriction of only a single variable factor among the alternatives. General formulas which can be used regardless of the number of variables involved are given at this point.
The chapter concludes with a few comments on the concept of utility. This advanced technique allows the two studies required under probabilistic cost-volume-profit analysis—the most favorable alternative and the effect of risk considerations on it—to be combined into a single study. This is possible because utility values are assigned to the possible outcomes of an alternative according to expected monetary value and degrees of risk.
CHAPTER IV

PROBABILITY IN CAPITAL INVESTMENT ANALYSIS

NEED FOR PROBABILITY

Another important accounting area open to probability applications is capital investment analysis. In recent years much attention has been focused on several techniques developed for allocating capital expenditures in such a manner that optimum results are obtained from a given capital budget. Most of these techniques involve uncertainties which are usually treated as being known with certainty under present practice. Naturally, this erroneous treatment of various factors mitigates much of the effectiveness of the techniques. Probability statements can and should be used in capital investment analysis to describe those conditions or assumptions that are subject to fluctuation. This will result in a more realistic picture of the situation and help assure that the various allocation procedures truly yield optimum results.

In addition, the use of probability in capital investment analysis, as in cost-volume-profit analysis, allows a more meaningful consideration of the risk factor. By using
probability management has some quantitative data available which can be used as a basis for risk decisions and does not have to rely mainly on intuition. The following paragraphs will explain and discuss selected applications of probabilistic techniques to this very important area of business decisions.

PROBABILITY AND THE TIME ADJUSTED RATE OF RETURN TECHNIQUE

The Time Adjusted Rate of Return

One of the more common methods of allocating funds to capital investment opportunities involves the determination of the time adjusted rate of return for each prospective investment. The opportunities are then ranked by rates and available funds are allocated to the ones indicated as being the most profitable. The rate of return for each prospect is found by determining the interest rate which equates the present value of the future cash inflows from it with the required initial investment. For example, assume that a company has the opportunity to invest $20,000 and receive from that investment cash inflows of $4,000 per year for six years. A table of present values for various interest rates can be used to determine that a rate of approximately 6 per cent equates the present value of these inflows with the initial outlay of $20,000. Thus, the time adjusted rate of return from this investment will be about 6 per cent.
Probability and the Time Adjusted Rate of Return

The above analysis is valid, however, only if these annual cash inflows are certain or are the expected value of the distribution of possible cash inflows. If management simply believes that $4,000 is the most probable figure, then the approach followed has not been realistic. In other words, in those situations where the inflows are not definite, are not known with certainty, the decision maker must establish a distribution of possible inflows, calculate the expected value of this distribution, and then use it in determining the time adjusted rate of return for the project. For example, assume that management upon reinvestigation discovers that the cash inflows are not certain to be $4,000 yearly, but instead have a distribution as shown in Table X.

TABLE X

<table>
<thead>
<tr>
<th>Inflows</th>
<th>Probability</th>
<th>(1 X 2)</th>
<th>Expected Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2,000</td>
<td>.20</td>
<td>$400</td>
<td></td>
</tr>
<tr>
<td>3,000</td>
<td>.30</td>
<td>900</td>
<td></td>
</tr>
<tr>
<td>4,000</td>
<td>.40</td>
<td>1600</td>
<td></td>
</tr>
<tr>
<td>5,000</td>
<td>.10</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>$3400</td>
<td></td>
</tr>
</tbody>
</table>
In determining the time adjusted rate of return for the project now, it is not $4,000, the inflow with the highest probability, that should be used, but rather the expected value of the distribution of inflows of $3,400. The $600 difference causes significant changes in this case. The return on the investment drops from about 6 per cent to less than 1 per cent. Thus, using the most likely inflows made the project appear somewhat favorable while using the expected value of the distribution of possible inflows--definitely the more appropriate technique--undoubtedly makes the project undesirable.

Risk and the Probabilistic Approach

As long as the distribution of possible inflows is available it can be used to help evaluate the risk factor. According to Table X there is a 50 per cent chance that the time adjusted rate of return will prove to be negative (if inflows are $3,000 or less), a gamble which few firms would probably be willing to accept. This, of course, would depend primarily upon the financial position of the firm. There is also a 50 per cent chance that the return will be 6 per cent or more (if inflows are $4,000 or more) and a 10 per cent chance of it being as high as 13 per cent (if inflows are the maximum of $5,000).

Unfortunately, this information as to risk has little or no importance in this example, since the probabilistically determined time adjusted rate of return is so low as to
eliminate the project from consideration. To illustrate more realistically the use of probability in investment risk analysis assume that there are two investment opportunities each requiring an initial outlay of $20,000 and involving inflows for six years as shown in Tables XI and XII.

TABLE XI

PROJECT A
PROBABILITY DISTRIBUTION OF INFLOWS AND EXPECTED VALUE OF INFLOWS

<table>
<thead>
<tr>
<th>(1) Inflow</th>
<th>(2) Probability</th>
<th>(1 X 2 Expected Value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3,000</td>
<td>.10</td>
<td>$300</td>
</tr>
<tr>
<td>4,000</td>
<td>.20</td>
<td>800</td>
</tr>
<tr>
<td>5,000</td>
<td>.30</td>
<td>1500</td>
</tr>
<tr>
<td>6,000</td>
<td>.40</td>
<td>2400</td>
</tr>
<tr>
<td>1.00</td>
<td></td>
<td>$5000</td>
</tr>
</tbody>
</table>

TABLE XII

PROJECT B
PROBABILITY DISTRIBUTION OF INFLOWS AND EXPECTED VALUE OF INFLOWS

<table>
<thead>
<tr>
<th>(1) Inflows</th>
<th>(2) Probability</th>
<th>(1 X 2 Expected Value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4,000</td>
<td>.40</td>
<td>$1600</td>
</tr>
<tr>
<td>5,000</td>
<td>.30</td>
<td>1500</td>
</tr>
<tr>
<td>6,000</td>
<td>.20</td>
<td>1200</td>
</tr>
<tr>
<td>7,000</td>
<td>.10</td>
<td>700</td>
</tr>
<tr>
<td>1.00</td>
<td></td>
<td>$5000</td>
</tr>
</tbody>
</table>
Project A and Project B both have expected values for the cash inflows of $5,000, and a table of present values would indicate that an interest rate of approximately 13 per cent is necessary to equate six-year inflows of such an amount to the required initial outlay of $20,000. Thus, if the company has $40,000 available for investment opportunities, probabilistic time adjusted rate of return analysis suggests that both of the projects be undertaken as they both undoubtedly offer a satisfactory yield.

Suppose, however, that the company has sufficient funds to finance only one of the projects. Which one should be undertaken? The previous analysis indicates that they are equally acceptable since they have identical time adjusted rates of return of about 13 per cent. In actuality, though, they are not equally desirable because the degree of risk associated with each of them is different. Fortunately, the distributions of possible cash inflows can be used to determine which of the two projects involves the lesser risk and is therefore the more preferable. In this example Project B is obviously the one that should be undertaken by the company. This is the case since the lowest yield it can return is 6 per cent while there is a 10 per cent chance that Project A will yield a negative return (if the inflows of both projects are at their minimum levels). In addition, there is a 10 per cent chance that the time adjusted rate of return will be as high as 26 per cent under Project B, but the maximum return under Project A can only be 20 per cent.
(if the inflows of the two projects are at their maximum levels). It is true that the probability of a 13 per cent or more yield under Project A is .70 as compared with .60 for Project B, but most decision makers would probably agree that the other advantages more than offset this one disadvantage and would, therefore, select Project B.

Limitations of this Probabilistic Approach

In the preceding illustrations each series of possible cash inflows for an investment opportunity was constant over the opportunity's life. The inflows were expected to be $4,000 per year for six years, $5,000 per year for six years, and so forth. Frequently this may not be the case. Many projects often have cash inflows that vary from year to year. Fortunately, this condition does not present a very difficult problem. An appropriate distribution of inflows is determined for each year of the project's life, an expected cash inflow value computed for each of the years, and then these figures used to determine the time adjusted rate of return in the same manner as explained earlier. A similar adjustment in the analysis would be necessary if the inflows were constant but their probabilities varied from year to year, or if both varied.

The preceding illustrations also assumed that the distributions of possible cash inflows were discrete ones. In other words, the inflows could take on only certain isolated values on an interval rather than all values on it.
It may be that the pattern of inflows is best described by a
distribution of this latter type, a continuous one. Where
such is the case and the continuous distribution is a normal
one, or can be approximated by a normal one, its mean can be
used to determine the time adjusted rate of return and its
mean and standard deviation to determine the various prob-
abilities required for risk factor analysis in accordance
with the procedures discussed in Chapter II. If a continuous
distribution is required but the normal curve is not appro-
priate, then the actual distribution or some approximation
that does realistically reflect the situation under consider-
ation should be used in the analysis.

PROBABILITY AND THE EXCESS PRESENT VALUE TECHNIQUE

The Excess Present Value Technique

A second technique currently used in allocating
capital funds is the excess present value method. Under this
approach a minimum desired or acceptable rate of return,
usually at least equal to the cost of capital for the firm,
is established and the expected cash flows from available
investment opportunities discounted at this rate. If their
present values exceed the respective required initial invest-
ments then the projects return this minimum rate or more and
are thus considered desirable. For purposes of ranking
desirable projects an excess present value index, defined as
the present value of the inflows divided by the initial
investment, is computed for each of them. Naturally, the
projects with the highest indices are considered the more desirable.\(^1\) For example, assume that a company has the opportunity to invest $20,000 and receive from that investment cash inflows of $5,000 per year for six years, and that they desire at least a 6 per cent return on all investments. The present value of $5,000 per year for 6 years discounted at 6 per cent is $24,585. Since this figure exceeds the initial required investment of $20,000 the project will return more than the minimum rate and should be considered desirable. Its excess present value index for ranking purposes would be $24,585 divided by $20,000 or approximately 123 per cent.

**Probability and the Excess Present Value Approach**

With the aid of probability and certain other selected statistical procedures this concept of excess present value can be adapted to yield a more meaningful capital budgeting technique. To illustrate this adaptation assume the following situation. A company finds itself confronted with more investment opportunities than it has funds available and, therefore, must choose some combination of investments from

\(^1\)The simple probabilistic approach to the allocation of capital funds discussed in the first section of this chapter was illustrated using the time adjusted rate of return method of analysis. The excess present value method could easily have been used instead. If this were the approach desired, the expected value of the cash inflows would simply be discounted at the minimum acceptable rate of return and the analysis proceed in the manner explained above.
among those possible. Furthermore, management believes that the excess present value of each available opportunity is an independent random variable with some mean and standard deviation. According to Schlaifer, a random variable is any quantity which has a definite value corresponding to every possible event.\(^2\) Thus, management simply believes that the excess present value of each project has a number of possible outcomes and that to each outcome there is associated a definite value. If management can compute values for the means and standard deviations of these random variables then the technique described in the following paragraphs can be used to allocate the capital budget funds. Unfortunately, computation of these values presents one of the most difficult problems of the analysis, difficult in the sense that there is usually little information readily available upon which the computations can be based. For most firms there are two major sources of such information. One is the probability beliefs of the firm such as were discussed and illustrated in Chapter II, and the other is historical data or past records. Where projects similar to those presently being considered have been undertaken previously, then means and standard deviations can be computed for the potential projects using data gathered from the past projects and adjusted as necessary.

Granted that means and standard deviations have somehow been determined for the random variable excess present values of the investment opportunities, the next step in the analysis is to enumerate all reasonable investment combinations. The main constraint in determining the possible combinations will of course be the amount of available funds, though physical facilities and personnel are two other factors that will undoubtedly have to be considered also. The expected value or mean and standard deviation of each of the combinations of excess present values must now be computed. Since management believes that the excess present value of each available opportunity is an independent random variable, there is a mathematical theorem which can be used to compute the mean. This theorem states that, "The expectation of a sum of random variables is the sum of their individual expectations."^3 Thus, the mean of a combination will be equal to the sum of the means of the individual excess present value distributions of which it is composed. A second mathematical theorem states that, "The variance of a sum of independent random variables is the sum of their individual variances,"^4 and it can be used to compute the required standard deviations. Their actual computations are accomplished by summing the squares of the standard deviations of the individual excess present value distributions composing a combination and taking the square root of the

^3Ibid., p. 263.  
^4Ibid., p. 364.
result. The variance, which is the standard deviation squared, must be used in the calculation since it is additive and the standard deviation is not.

Using the information determined above management can now decide which combination of investments they should select. What is desired is a combination with a high expected value and a low standard deviation. It should be noted that the combination with the highest expected value is not automatically chosen. Some consideration must be given to variability or risk, as evidenced by the standard deviation. Obviously, if all the groups under consideration have the same mean then the one with the lowest standard deviation is the most desirable. However, the decision is not as easy when there are some groups with high expected values and high standard deviations to be compared with others that have lower expected values but also lower standard deviations. In such a situation the choice will depend upon the financial condition of the firm and the risk attitudes of the individuals responsible for conducting the analysis and making the final selection. Where conservative risk attitudes are indicated as appropriate management may be willing to choose one of the combinations with a lower mean in order to get the reduced uncertainty--standard deviation--that would accompany it. On the other hand, when liberal risk attitudes are indicated one of the combinations with a higher expected value and larger standard deviation may be selected instead.
The main advantage of the probabilistic excess present value approach is that it permits the inclusion of some fairly risky investments when there are an adequate number of more reliable ones to offset them. This advantage results from the fact that the investment decision concerns not a single opportunity but rather a combination of them, and is handled routinely simply as an integral part of the analysis. Neil Paine adequately summarized this situation, stating, "The importance of a proper investment 'mix' is emphasized in the probability approach\(^5\) to capital budgeting."\(^6\)

To illustrate this use of probability in allocating capital budget funds assume the following facts. A company has a capital investment budget for the current year of $100,000. Management has discovered five projects which it would like to undertake, but must instead choose some combination of them because the required initial investment for all five far exceeds the $100,000 available. Assume also, that management believes that the excess present value of each of the projects is a random variable with mean, standard deviation, and required initial investment as shown in Table XIII.

Considering the current capital budget of $100,000, there are three possible investment combinations for this

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\(^5\)By "the probability approach to capital budgeting" Paine is referring in this quotation to an analytical technique similar to the probabilistic excess present value approach being considered here.

firm. They are projects A, B, and D, projects B and C, and projects D and E. Each of these combinations has a total required initial outlay of $100,000. The excess present value means and standard deviations of the three possibilities are computed and shown below.

**TABLE XIII**

**DATA ON POSSIBLE INVESTMENT PROJECTS**

<table>
<thead>
<tr>
<th>Identification</th>
<th>Initial Investment</th>
<th>Excess Present Value</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$30,000</td>
<td>$10,000</td>
<td>$500</td>
</tr>
<tr>
<td>B</td>
<td>60,000</td>
<td>20,000</td>
<td>2,000</td>
</tr>
<tr>
<td>C</td>
<td>40,000</td>
<td>15,000</td>
<td>1,000</td>
</tr>
<tr>
<td>D</td>
<td>10,000</td>
<td>5,000</td>
<td>200</td>
</tr>
<tr>
<td>E</td>
<td>90,000</td>
<td>30,000</td>
<td>5,000</td>
</tr>
</tbody>
</table>

Combination A-B-D

Mean = Mean A + Mean B + Mean D
= 10,000 + 20,000 + 5,000
= 35,000

Standard Deviation = \( \sqrt{(\text{Std. Dev. A})^2 + (\text{Std. Dev. B})^2 + (\text{Std. Dev. D})^2} \)

= \( \sqrt{(500)^2 + (2000)^2 + (200)^2} \)
= \( \sqrt{4,290,000} \)
= 2,071
Combination B-C
Mean = Mean B + Mean C
= 30,000 + 15,000
= 35,000
Standard Deviation = \( \sqrt{(\text{Std. Dev. B})^2 + (\text{Std. Dev. C})^2} \)
= \( \sqrt{(2000)^2 + (1000)^2} \)
= \( \sqrt{5,000,000} \)
= 2,236

Combination D-E
Mean = Mean D + Mean E
= 5,000 + 30,000
= 35,000
Standard Deviation = \( \sqrt{(\text{Std. Dev. D})^2 + (\text{Std. Dev. E})^2} \)
= \( \sqrt{(200)^2 + (5000)^2} \)
= \( \sqrt{25,040,000} \)
= 5,004

In this particular example all possible combinations have identical excess present value means of $35,000. Therefore, the one that should be selected is the one with the lowest standard deviation. The first combination's standard deviation of $2,071 is the smallest so projects A, B, and D are the ones that the company should undertake.

Risk and the Probabilistic Excess Present Value Technique

Probability concepts can also be utilized with this adaptation of the excess present value technique to provide certain information about the risk associated with the various possible combinations. The quantity and quality of this information depends upon what is known about the distributions of the combinations. Even if nothing at all is known about the distributions except that they exist, some
probabilistic risk conclusions can still be drawn. Tchebycheff's inequality, a mathematical theorem, states that more than \(1 - (1/t^2)\) of any set of finite numbers must fall within the closed range of \(X\) plus or minus \(ts\) (\(t\) equals any desired value, \(X\) equals the arithmetic mean of the set of finite numbers, and \(s\) equals the standard deviation of the numbers). For example, if \(t\) is given the value 4 then more than \(1 - (1/4^2)\) or 15/16 of any set of numbers must be included in the limits \(X\) plus or minus 4s. The distribution of a combination certainly meets the requirement of being a set of finite numbers so this theorem is applicable to the probabilistic excess present value technique at all times.

A more informative or limiting mathematical theorem can be used to provide risk data when it is known that the distribution of a combination possesses certain characteristics. These characteristics are one mode, an arithmetic mean equal to the mode, and continuously declining frequencies on both sides of the mode. The inequality appropriate when the above restrictions are met by a distribution states that more than \(1 - (1/2.25t^2)\) of the distribution will fall within the closed interval \(X\) plus or minus \(ts\).\(^7\) For example, if \(t\) is given the value 4 then more than \(1 - (1/2.25(4^2))\) or 35/36 of any distribution meeting the requirements will be included in the range \(X\) plus or minus 4s. The more exactness

\(^7\)The symbols \(t\), \(X\), and \(s\) have the same meaning for this inequality as they did for the Tchebycheff inequality discussed in the preceding paragraph.
of this inequality is obvious since only $15/16$ of the distribution would have been included in the same range if the Tchebycheff inequality had been used instead. This theorem can also be used frequently to approximate patterns not quite satisfying its required characteristics. According to Grant, "Many distributions . . . come close enough to meeting these conditions for the Camp-Meidell inequality\(^8\) to be applied with confidence."\(^9\) Obviously, whether or not it should be used as an approximator in a given situation depends upon the degree of precision desired by management. Unfortunately, there is one major disadvantage of this theorem that should be mentioned. In most cases it will probably be extremely difficult to determine if the properties required for its valid application are present in the distribution under study.

The best information concerning risk can be provided when it is known that the distribution of a combination is normal. When such is the case the probabilities of various excess present values for the combination can be computed using the normal deviate concept and a table of areas under the normal curve. The steps involved in these computations have been explained and illustrated in Chapter II, and require only that the mean and standard deviation of the normal distribution be known. These two requirements

\(^8\)The inequality discussed in this paragraph is an adaptation of the Camp-Meidell inequality.

definitely present no problem here since the mean and standard deviation of each combination must be computed so that they can be used in the determination of the most preferable alternative. Thus, if management can reach the decision that a combination has a normal distribution then all the information needed to calculate any desired probabilities is known.

How is this decision to be reached, however? The best way to determine if a combination's distribution is a normal one is to examine the patterns of the individual projects of which it is composed. If their distributions are normal then the excess present value distribution of the combination must also be normal. Even where such is not the case, however, the normal curve, like the Camp-Meidell inequality, can frequently be used to approximate other somewhat similar distributions.

To illustrate the use of these three methods for obtaining risk data the same situation previously used to illustrate the probabilistic excess present value approach will be further analyzed. In this example there were three investment combinations possible, all having identical means. As a result, combination A-B-D, the one with the lowest standard deviation, was chosen as the one to be undertaken. Now if all management knows about the A-B-D combination is simply that it has some unknown distribution then the Tchebycheff inequality must be used to compute various ranges and their related probabilities. For example, assigning a value of 3 to $t$, this theorem would state that more than
1 - (1/3^2) or .89 of the distribution of the combination falls within the range $35,000 plus or minus 3($2,071). In other words, the probability is .89 that the actual excess present value will be contained in the interval from $28,787 to $41,213. Such information certainly should be helpful to management since it indicates that there is little possibility of a net present value for the combination of less than $28,787 or more than $41,213. Naturally, by changing the value assigned to t additional probabilities can be computed for either smaller or larger ranges.

Management can apply the adaptation of the Camp-Meidell inequality if it can be determined that the distribution of the combination meets either completely or practically the required conditions. Assigning still a value of 3 to t, this theorem would indicate that more than 1 - (1/2.25(3^2)) or .95 of the combination's distribution must be included in the interval $35,000 plus or minus 3($2,071). The advantage of this inequality over the previous one should be obvious. The probability that the actual excess present value will be contained in the range from $28,787 to $41,213 has been increased from .89 to .95. Thus, management can be even more confident now that the net present value will not be less than $28,787 or more than $41,213. As with Tchebycheff's inequality the range can be increased or decreased and a new appropriate probability determined by changing the value assigned to t.

The most limiting probabilistic conclusions can be
drawn here if management can determine that the investment combination is normally distributed. Assuming such is the case then probabilities can be computed using the normal deviate concept and a table of areas under the normal curve. Application of this procedure here will yield a probability of .9973 that the actual excess present value of combination A-B-D will be included in the interval $35,000 plus or minus 3($2,071). In other words, if it can be concluded that the distribution of this combination is normal, management can be virtually certain that its net present value will be contained in the range from $28,787 to $41,213.

Many other questions concerning risk that cannot be answered using either of the two inequalities can be answered under the normal distribution. For example, the probability of the net present value being $32,000 or less, $38,000 or more, or any figure plus more or less that management would like to know can easily be determined. Thus, under the probabilistic excess present value approach to capital budgeting the decision maker can isolate the optimum combination of all investment opportunities and, especially where the combination is normally distributed, learn something about the uncertainty that will be associated with it.

\(^{10}\)There are actually no computations necessary to determine this probability. Since the interval desired is the mean plus and minus 3 standard deviations, then 3 is the normal deviate to be looked up in a table of areas under the normal curve to locate the answer.
Simulation and the Computer

Simulation in its general sense refers simply to the representation of some real world system through a model. Its purpose is to discover more about the operation of the system and how changes in various factors affect it. Only since the advent of the computer, however, has simulation truly become a valuable decision making tool in business administration. Prior to this there was just no feasible means of manipulating and solving the complex, highly mathematical models needed to simulate realistically comprehensive real world activities.

Computerized Simulation and Capital Budgeting

The actual rate of return on an investment is determined by some unique combination of specific values for a large number of variable factors. Traditional analytical techniques simply cannot cope with all of these variables and instead treat them as being fixed or known with certainty. The result is that management is informed of the location of only one point on a continuous curve of possible rates of return, a curve composed of the return for every different combination of the variable factors involved. The probabilistic capital budgeting applications discussed in previous sections of this chapter have permitted some uncertainty to be introduced into the analysis, but even they are
not completely realistic. The technique discussed in the following paragraphs does not give a single answer for the return on an investment opportunity but rather informs management of all possible rates of return along with the probability of each. The decision maker is thus given a complete picture of the possible outcomes of an opportunity and is enabled to make a more informed and realistic selection.

To be able to utilize this computerized simulation approach management must first estimate the range of possible values for every variable factor affecting an investment project. Next they must determine the probability of each value's occurrence within this range. Some of the more common variables likely to be pertinent are selling price, sales growth rate, initial investment required, residual value of the investment, operating costs, useful life of any facilities involved, and sales demand. The best source for determining ranges and probabilities for these factors will probably be company records showing the results of similar projects undertaken in the past. Other possible sources are trade association records and the experience and probability beliefs of the managerial personnel involved in the analysis.

With this information in hand the next step in the procedure is to write a computer program that will randomly select from each range of variable factors one specific value, and then use these figures to compute a rate of return for the investment opportunity. The program should be written
so that this series of steps, this random combination of variables to determine a single point on a continuous curve, is repeated over and over until it has adequately defined the entire distribution of possible rates of return. This definition is accomplished simply by having the computer record the number of times the series of steps is repeated and the number of times each individual or different rate of return occurs. This recorded data will not only show the range of possible rates of return but can also be used to set up a frequency distribution that will show the probabilities associated with each of the rates. Naturally, the number of repetitions needed will depend upon the nature of the individual investment project being simulated. In general, though, the more variable factors the project involves and the more values they can take on the larger the number of computer runs required. Incidentally, the program could also be written so that it would compute the variance of the rate of return distribution. This figure might be useful in helping management reach a decision between several similar mutually exclusive investment opportunities, since they would undoubtedly want to choose the one with the least variation.

To illustrate how this investment simulation would actually be accomplished assume the following facts. A firm finds itself confronted with the opportunity to buy a piece of manufacturing equipment. The cost of this equipment, its useful life, and the cash inflows it will provide are all
independent variables with ranges and appropriate probabilities as given in Tables XIV, XV, and XVI. The mechanics of the computer program necessary to simulate this project are fairly simple. First, it would generate a two digit random number, or a table of such numbers would be stored in the computer's memory and one of these values selected. This random number would then be tested. If it were between 00 and 29, $9,000 would be used as the required initial investment. A number between 30 and 84 would result in $10,000 being used and one between 85 and 99, $11,000. In other words, what is being done here is to choose, over the long run, an initial investment of $9,000 30 per cent of the time, $10,000 55 per cent of the time, and $11,000 15 per cent of the time. A specific value would be determined in a similar manner for the useful life and the cash inflows. These three values would then be combined and used to determine one possible outcome and rate of return for the investment project. Any method desired could be used to calculate the return, but the time adjusted rate of return technique would undoubtedly be the best as it does consider the time value of money. This procedure would be repeated a sufficient number of times, using new random numbers with each repetition, to allow all possible rates of return an opportunity to occur. Furthermore, by recording the number of times each particular rate occurs and comparing this with the total number of repetitions some idea of each rate's probability of occurrence can be determined.
### TABLE XIV

**ESTIMATED RANGE AND PROBABILITIES OF INITIAL INVESTMENT**

<table>
<thead>
<tr>
<th>Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$9,000</td>
<td>.30</td>
</tr>
<tr>
<td>10,000</td>
<td>.55</td>
</tr>
<tr>
<td>11,000</td>
<td>.15</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

### TABLE XV

**ESTIMATED RANGE AND PROBABILITIES OF USEFUL LIFE**

<table>
<thead>
<tr>
<th>Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 years</td>
<td>.15</td>
</tr>
<tr>
<td>6 years</td>
<td>.30</td>
</tr>
<tr>
<td>7 years</td>
<td>.45</td>
</tr>
<tr>
<td>8 years</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

### TABLE XVI

**ESTIMATED RANGE AND PROBABILITIES OF CASH INFLOWS**

<table>
<thead>
<tr>
<th>Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2,000</td>
<td>.20</td>
</tr>
<tr>
<td>2,500</td>
<td>.50</td>
</tr>
<tr>
<td>3,000</td>
<td>.30</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>
A rate of return probability distribution would naturally be prepared for each investment opportunity being considered. From these management can determine a project's most likely return, its expected value of return, and all kind of information necessary to analyze any risks associated with it. Certainly this computerized simulation technique offers the decision maker the most and best information possible for allocating capital budget funds.

Two Unique Advantages of Capital Budgeting Simulation

Computerized capital budgeting simulation offers two advantages that are not available under traditional techniques. One of these is that management can determine the sensitivity of the outcome of an investment project to changes in one or more of the input variables. This is done simply by changing or altering the probability distribution of an input factor and running the simulation program using this new data. The results of this second run are then compared with those of the prior run using the old data and any differences noted. If it is found that small changes in a variable significantly affect the return earned on the investment, the decision maker might well consider studying this variable once more in order to get additional or improved information on its probability distribution. In other words, it is very important that such a variable's distribution be as realistic as possible. On the other hand, the review of an input factor is probably not economically
justified if fairly large changes in it result in only minor alterations to the project's rate of return distribution.

The second advantage that this technique offers which most others do not is that it can handle those situations where the input variables are not independent of one another. This occurs when one or more of the variables has several probability distributions and the appropriate one to use in a simulation depends upon the particular value selected for some other variable. Such a situation is handled with this technique by choosing a particular value for the independent variable first and using this value to determine which probability distribution should be used for the dependent variable. For example, assume that two of the input factors of an investment project are sales price and demand, and that there is a distinct distribution of demand for each possible sales price. The procedure here would be to have the computer program choose a specific value for the sales price to be used in the simulation and then utilize this figure to determine which of the demand probability distributions should be used in selecting the demand value.

SUMMARY

Traditional methods for allocating capital budget funds cannot adequately handle the numerous uncertainties that usually accompany investment opportunities. In fact, most of the techniques currently in use treat all factors involved as being definite or known with certainty. This
chapter examines three probabilistic capital budgeting methods that permit some or all of these factors to be variable.

The first technique considered involves the addition of uncertain cash inflows to the time adjusted rate of return concept. The expected value of these inflows is computed and this figure used in determining the interest rate which equates the present value of the inflows to the required initial investment.

A technique based on the excess present value concept and two additive statistical measurements—the mean and variance—is the second one considered. This approach requires that management look upon the net present value of every investment project as a random variable with known mean and standard deviation. Using the amount of capital funds available as the main limiting factor, all possible investment combinations are enumerated. The mean and standard deviation of each of these combinations is next computed. Management then chooses the optimum combination by determining the one with the highest mean that has an acceptable standard deviation or dispersion.

The third and final probabilistic capital budgeting technique considered is the most comprehensive. It permits all factors influencing an investment opportunity to be variable. Under it a distribution for every variable showing the values it may assume along with the probability of each is prepared. Using random numbers, any fixed factors, and these probability distributions the investment project is
simulated over and over until the probability distribution of the possible rates of return from it is defined. This same procedure is followed for each project and the resulting distributions used as a basis for management's allocation decisions.
CHAPTER V

PROBABILITY IN COST CONTROL ANALYSIS

NEED FOR PROBABILITY

One of the most rewarding applications of probability theory to accounting in terms of potential financial savings is in the very important area of cost control. The utilization of probability in this area can enable management to obtain a greater and more effective degree of control over costs than is possible with most of the techniques and procedures currently in use. Furthermore, not only are expenses more subject to minimization as a result of this increased control, but, due to the tremendous efficiency of the probabilistic techniques, the increased control is obtained at a lower cost. It is true that in some cases such applications do require the gathering of special and somewhat rare data, but not always. This chapter explains, discusses, and illustrates a number of probabilistic cost control techniques that are possible with certain types of commonly recorded information.

Some variation in the cost of most factors or items must always be expected. Part of this variation will be the result of chance or normal causes and part of it will very
likely be due to assignable outside causes. The former type is not controllable—cannot be entirely eliminated—but the latter one is and it is these that the accountant-manager should be most interested in for cost control purposes. Unfortunately, many individuals are not aware of the existence of these two kinds of variations and as a result they strive for a perfection of operating performance that is absolutely impossible to attain. In their efforts they naturally expend and waste both valuable time and money. Fortunately, certain techniques based on probabilistic concepts exist which can be used to minimize this needless waste by helping to distinguish between assignable and natural causes of cost variation. Management can thus know which variations they should investigate and try to correct and which ones they should simply leave alone. This last aspect is itself extremely important since it prevents un-called for adjustments or corrections that many times have a tendency to increase rather than decrease variability.

COST CONTROL AND THE SHEWHART CONTROL CHART

The Shewhart Control Chart

The Shewhart control chart, which has achieved its major success as a means of controlling and improving the quality of the output of manufacturing processes, is one probabilistic technique that can be used to distinguish between natural and assignable causes of variation. As a result, it can tell management both when to, and when not to,
initiate an investigation into a particular cost's behavior. In addition, it can provide information on the consistency of performance and average level of the cost in question. Undoubtedly, proper use of the Shewhart control chart can help lead to optimum cost control.

Application of the Shewhart Control Chart to Cost Control

The Shewhart control chart is a fairly simple cost control technique to apply. The only data required for its use is a record of the cost's behavior. This information is first divided into some logical arrangement of subgroups. Time, such as hours, days, or shifts, is usually the basis for this division, and the period covered depends primarily upon the degree of control desired by management. In general, the shorter the time allowed to generate a subgroup the greater the possible degree of control that can be exercised. The next step is to calculate the arithmetic average and range of each of the subgroups. From these two statistical measures a grand arithmetic average (an average of the subgroup averages) and an average range are computed. Using the results of these two sets of calculations, the control chart can then be prepared.

The control chart itself is simply a graph on which are plotted the arithmetic averages of the subgroups. The vertical axis commonly represents dollars of cost and the horizontal axis the basis of subgrouping. Control limits are also plotted on the graph. For subgroups of similar
size these limits are straight lines placed at such a distance from the grand arithmetic average that it is very unlikely that the average of a subgroup will fall outside them as the result of normal or chance variation. Thus, any averages that do fall outside them are said to be out of control--that is very likely due to a cost variation that is the result of a controllable assignable cause. In effect, then, the control chart tries to help answer the question: Is the variation among the subgroups the result of a stable pattern of variation? Or as Grant says, "Is there one universe from which these samples appear to come?"¹ Naturally, it will sometimes lead to the drawing of incorrect conclusions. An assignable cause of variation may be indicated when in truth there is only normal variation or vice versa. The control limits plotted on the chart are designed to balance the costs associated with these two types of errors. Production personnel have generally found that limits placed at plus and minus three standard deviations of the distribution of sample means from the grand average strike this balance, and such limits are assumed in this chapter as also striking a balance in the case of control

purposes. This is an area, however, in which there has not been adequate research to date and which the accounting profession needs to investigate further.

Calculation of the three standard deviation control limits for a particular situation is accomplished as follows. First, estimate the population standard deviation of the subgroups. This estimation can be achieved by dividing their average range by a correction factor that is "a function of \( n \) (subgroup size) and expresses the ratio between the expected value of \( R \) (average range) from a long series of samples from a normal universe and the known standard deviation of that universe." Tables of this correction factor for various subgroup sizes are available in most statistical quality control texts. Second, divide this estimated figure by the square root of the subgroup size to determine an estimate of the standard deviation of the distribution of subgroup averages or sample means. Third, multiply this last figure by three and add the resulting amount to the grand arithmetic average of the subgroups to obtain the upper control limit and subtract it to obtain the lower control limit. This procedure is shown in equation form below.

\[ \text{Upper Control Limit} = \text{Grand Average} + 3 \times \text{Estimated Standard Deviation} \]
\[ \text{Lower Control Limit} = \text{Grand Average} - 3 \times \text{Estimated Standard Deviation} \]

Approximately 99.7 per cent of the area of a normal distribution is included in the interval bounded by its mean plus and minus three standard deviations. Thus, when the average of a subgroup falls outside of three sigma limits management can be fairly certain that an assignable cause of variation does exist.

Grant's book Statistical Quality Control is one text that contains such a table.
SD-P = \frac{R}{CF} \\
SD-SA = \frac{SD-P}{\sqrt{n}} \\
UCL = GAA + 3(SD-SA) \\
LCL = GAA - 3(SD-SA) \\
SD-P = \text{Standard deviation of the subgroup population} \\
R = \text{Average range of subgroups} \\
CF = \text{Correction factor} \\
SD-SA = \text{Standard deviation of subgroup averages} \\
n = \text{Subgroup size} \\
UCL = \text{Upper control limit} \\
GAA = \text{Grand arithmetic average of subgroups} \\
LCL = \text{Lower control limit} \\
As an illustration of the application of the control chart to the area of cost control assume that the data in Table XVII is the average hourly material cost for some manufactured product for a two-week period. Each day's information is to be treated as a subgroup for control chart purposes, and Table XVII also gives the average and range of each of the resulting subgroups along with their grand arithmetic average and average range. Calculation of the standard deviation of the population, the standard deviation of the distribution of sample averages, and the control limits from this data is shown below.
<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>.77</td>
<td>.74</td>
<td>.80</td>
<td>.78</td>
<td>.79</td>
<td>.75</td>
<td>.75</td>
<td>.79</td>
<td>.76</td>
<td>.71</td>
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<td></td>
<td>.80</td>
<td>.78</td>
<td>.73</td>
<td>.81</td>
<td>.75</td>
<td>.77</td>
<td>.76</td>
<td>.79</td>
<td>.79</td>
<td>.73</td>
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<tr>
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<td>.78</td>
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<td>.79</td>
<td>.78</td>
<td>.75</td>
<td>.78</td>
<td>.81</td>
<td>.73</td>
<td>.71</td>
</tr>
<tr>
<td></td>
<td>.72</td>
<td>.76</td>
<td>.76</td>
<td>.76</td>
<td>.77</td>
<td>.76</td>
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<td>.76</td>
<td>.76</td>
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<td>.77</td>
<td>.78</td>
<td>.73</td>
<td>.73</td>
</tr>
<tr>
<td></td>
<td>.75</td>
<td>.77</td>
<td>.74</td>
<td>.74</td>
<td>.74</td>
<td>.79</td>
<td>.79</td>
<td>.79</td>
<td>.75</td>
<td>.75</td>
</tr>
<tr>
<td>Total</td>
<td>4.60</td>
<td>4.56</td>
<td>4.52</td>
<td>4.64</td>
<td>4.59</td>
<td>4.59</td>
<td>4.64</td>
<td>4.76</td>
<td>4.50</td>
<td>4.33</td>
</tr>
<tr>
<td>Average</td>
<td>.767</td>
<td>.760</td>
<td>.753</td>
<td>.773</td>
<td>.765</td>
<td>.765</td>
<td>.773</td>
<td>.793</td>
<td>.750</td>
<td>.722</td>
</tr>
<tr>
<td>Range</td>
<td>.08</td>
<td>.04</td>
<td>.07</td>
<td>.05</td>
<td>.04</td>
<td>.04</td>
<td>.03</td>
<td>.06</td>
<td>.05</td>
<td></td>
</tr>
</tbody>
</table>

Grand Average = 7.621/10 = .762
Average Range = .53/10 = .053
\[
SD-P = \frac{.053}{2.534} = .021
\]
\[
SD-SA = \frac{.021}{\sqrt{6}} = .0086
\]
\[
UCL = .762 + 3(.0086) = .788
\]
\[
LCL = .762 - 3(.0086) = .736
\]

Figure 1 illustrates the control chart that would be prepared for this situation.

An examination of the control chart shown in Figure 1 reveals that the material cost for the product was out of control on two days, 8 and 10. On the former day the sub-group average was above the upper control limit and on the latter day it was below the lower control limit. Thus, the chart tells management that assignable causes of cost variation were most likely present on these days and they should be investigated. The investigation will hopefully result in the discovery and elimination of the assignable causes, and if no new ones appear the costs should be under control in the future. It should undoubtedly be pointed out here that management might not want to eliminate the cause of the out of control point on day number 10. On this day average material cost was below the lower control limit, and if the reason for it being so was not an undesirable one, such as a reduction in the average quality level of the product below acceptability, then future costs may be decreased by encouraging it. This pinpointing of possible cost saving
Upper control limit = .762 + 3(.0086) = .762 + .026 = .788

Lower control limit = .762 - 3(.0086) = .762 - .026 = .736

FIGURE 1

CONTROL CHART FOR MATERIAL PRODUCTION COST FOR SOME PRODUCT
procedures is a definite added advantage of the use of the Shewhart control chart for cost control purposes.

If the control chart in the preceding example had shown no points out of control, management's conclusion would have been that there was a stable pattern of chance-caused variability in operation over the time period covered by it. This simply means that material costs are in control with known mean and standard deviation, and that variations from this mean were probably due to normal chance causes and very likely cannot be completely eliminated. Therefore, since any investigation and subsequent action is unlikely to result in reduced variation, it can be said in general that the situation should be left as is. It has already been mentioned, but deserves repeating, that this ability to indicate to management when a process should be left alone is also a major benefit of the control chart as numerous unneeded adjustments can actually lead to an increase in the process' variability.

There may be some cases, however, where action will still have to be taken even though the control chart indicates that a cost is in control and no assignable causes of variation are present. This is possible since costs may be in control but not at a reasonable, desired, or acceptable level and management has to take action in an attempt to
remedy the problem. To uncover such situations standard costs should be utilized with the control chart. In fact, to truly achieve the maximum benefits that it offers the use of standard costs in conjunction with the chart is almost a necessity. By considering them in light of the information gathered from the control chart concerning actual cost results, management can determine if they are obtainable or if they are unrealistic under the present operating conditions and environment. Without doubt, once realistic standards have been assured they can provide management a basis for comparison by which actual costs can be minimized with the control chart to the fullest extent possible.

The control chart discussed and illustrated in the preceding paragraphs is a control chart for the arithmetic mean. In other words, its primary purpose is to control the average value of the cost behavior in question. From the same data used to prepare it one for the range could also have been prepared. This latter chart could be used to try and hold constant or reduce the dispersion of the cost as it indicates whether variations in the dispersion were due to normal or assignable causes. For a complete explanation of the techniques involved in its preparation and use the reader

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5 The action undertaken here may involve 100 per cent inspection, an extensive overhaul of the process in an attempt to reduce its natural variability, raise its average, or lower its average, or the adoption of a completely new process which is capable of yielding results that are satisfactory.
is referred to Chapter Six of Eugene L. Grant's book *Statistical Quality Control*.

Limitations of the Control Chart for Cost Control

The Shewhart control chart as a cost control tool has one major limitation. It assumes that the distribution of the cost being studied is a normal one. This assumption is necessary for two reasons. First, the correction factor used to estimate the population standard deviation from the average range is based on the normal distribution, and second, so is the location of the control limits. Concerning this second aspect, the basic foundation of the control chart is the fact that practically all, 99.73 per cent to be exact, of the area under a normal distribution is included in the interval from the mean minus three standard deviations to the mean plus three standard deviations. Thus, assuming a normal curve, the probability of a point falling outside of control limits placed at the mean plus and minus three standard deviations is extremely small, and when one does it is usually safe to assume that it is the result of an assignable cause of variation. Naturally, if the cost under consideration has a distribution that is not normal, or approximately normal, then the use of the control chart can lead to numerous false conclusions as to the reason for variations. For example, a cost distribution that is positively skewed will have more of its area outside of the mean plus three standard deviations than the normal curve indicates and, therefore,
chance-caused points above the upper control limit would be more likely. Thus, management must realize that there are certain situations where the control chart is not appropriate and must consider the behavior pattern of each cost carefully before deciding to apply this tool to its control.

PROBABILITY AND COST VARIANCE REPORTS

Variance Reports

According to a research publication of the National Association of Cost Accountants:

Cost control has as its objective production of the required quality at the lowest cost attainable under existing conditions. It pre-supposes a plan or program which is embodied in a set of standards specifying how each job is to be done and what it ought to cost to do it. Operation under the standards then proceeds by comparing actual costs with the standard costs as the work is being done and by taking appropriate action to correct unfavorable deviations from the standard costs as such deviations occur.6

These deviations are usually brought to management's attention through variance reports. The techniques discussed and illustrated in the following paragraphs, like the Shewhart control chart, are designed to help a manager distinguish between those reported variances that are the result of non-controllable natural causes and those that are the result of controllable assignable causes. With this information it is much more likely that only the appropriate deviations from

Application of Probability to Variance Reports

For every product, process, or situation subject to cost control there is a distribution of possible costs. If it can be assumed that this distribution is a normal one and if a standard cost system is in operation, then the probabilities of various deviations can possibly be computed and used to help management decide whether a particular variance is due to assignable or chance causes. It was pointed out in a previous chapter that a normal curve is completely defined when its mean and standard deviation are known. Thus, if these two measurements can be determined for the distribution of possible costs in question then calculation of the required various probabilities presents no problem whatsoever. Determination of the mean requires no computation. It is equal to the standard cost since the latter is simply an estimate of, or should be an estimate of, expected cost for the period in light of the existing operating conditions. The standard deviation will have to be based on past records of similar costs or estimated using the probability beliefs of some managerial member as explained and illustrated in Chapter II of this paper. The normal deviate concept for computing probabilities from the normal curve using these two measurements was also explained in this chapter and therefore will not be considered once again here.

The next area which must be considered is how
management can use the probability of a particular variance to aid them in their cost control decisions concerning it. The probabilities computed using the normal cost distribution derived above represent the likelihood of a variance of some amount or more resulting from normal or chance causes. Certainly this information should give management an improved basis for deciding whether or not the variance should be investigated rather than some arbitrary amount rule or simply intuition. Unfortunately, the probability of a variance being due to an assignable cause cannot be determined from the cost distribution. Some business writers have tried to argue that one minus the probability of it being due to natural causes gives this figure. This type of reasoning is completely invalid. All such an arithmetic operation gives is the probability of a variance of less than the amount in question resulting from normal causes.

One difficult question still lies ahead of the manager who utilizes this cost control technique. What probability is required to deem a variance as being not due to chance causes and thereby necessitate an investigation of it? Is .2 a small enough probability or should the probability of a variation being due to normal causes be as low as .05 before an investigation is required? There is no single correct answer to this dilemma. Each individual cost situation must be analyzed to determine the appropriate cut-off point. Its location will hopefully strike an economic balance between the costs associated with two possible types of errors. One
of these errors is not looking for an assignable cause of variation when in actuality one does exist and the other is looking for an assignable cause when in reality one does not exist. Unfortunately, striking this balance is frequently very difficult to accomplish, especially when the technique is being applied by management for the first time and is somewhat strange and unfamiliar. In such a case it may be necessary to pick some arbitrary cut-off figure and as experience is acquired over time adjust it as deemed advisable.

Other factors besides its probability will very likely be important in deciding whether or not a particular deviation from standard should be investigated. Some of these are the absolute size of the variance, the relative size of the variance, and the time and personnel currently available. For example, a variance might not be unreasonable in terms of probability of occurrence but so large in dollar amount that management may decide that it should be investigated anyway; or, probability may deem a variance as being worthy of investigation but such cannot be done simply because there is no time or personnel available to do it.

It should be pointed out here that there has been no differentiation made in the preceding comments between favorable and unfavorable variations from standard due to assignable causes. This is because both types should be subject to investigation. Unfavorable variances should be subject to investigation so that their causes may be
discovered and eliminated where possible, and favorable ones so that either budget overestimations can be located and adjusted, undesired reductions in quality levels discovered and eliminated, or new cost saving operational techniques located and utilized in the future.

As an illustration of how probability, standard costs, and variance reports can be used together assume the following situation. The December budget for materials for the grinding department of some manufacturing company shows a standard cost of $4,500. After the month has expired records indicate that the actual material cost for this period was $4,900 with no alteration in the expected volume of production. Under typical existing procedures this information would be used to determine if the variance exceeds some percentage of standard. If it does an investigation is initiated and if it does not no action is undertaken. This approach is basically correct since it does follow the principle of management by exception, but it is not the best one that management could make use of as a decision tool in this area. It does not take advantage of all the potential available information concerning the deviation from standard. The technique most likely to result in the most informed decision is the probabilistic one described and explained in the preceding paragraphs.

To use this probabilistic technique in the situation just described management must first compute the standard deviation of the distribution of possible material costs.
If some responsible individual in the company believes that there is a 50-50 chance of the budgeted standard material cost being within plus or minus $300 of the actual cost for the period, then this information can be used to calculate the standard deviation of the distribution. Since it is thought that there is a 50 per cent chance that the standard will be within plus or minus $300 of the true mean material cost, 50 per cent of the area under the cost distribution must be included in the interval $4,200 ($4,500 - $300) to $4,800 ($4,500 + $300). Furthermore, since the normal curve is symmetrical, 25 per cent of the area under the cost distribution must be included in the interval from minus infinity to $4,200 and 25 per cent in the interval from $4,800 to plus infinity. A table of areas under the normal curve can be used to determine that for the above conditions to be met the standard deviation has to have a value of approximately $448.

With the mean and standard deviation of the distribution of possible costs known, the probability of the unfavorable variance of $400 that resulted, or a larger one, being due to normal or chance causes can now be determined. Subtracting the mean of $4,500 from $4,900 and dividing this result by the standard deviation of $448 gives a normal deviate of .89 for this example. This figure when looked up in a table of areas under the normal curve indicates that the probability of an unfavorable deviation from standard of
$400 or more as a result of normal causes is .1867.\footnote{Since the normal distribution is symmetrical this is also the probability of a favorable variance of $400 or more as a result of assignable causes. In other words, it is also the probability that the material cost for the month of December will be $4,100 or less due to chance.}

Management must now decide whether or not this probability is sufficient to deem the variance of $400 as being due to chance causes. Such a conclusion would result in the decision that no action be taken on the process. On the other hand, if it is concluded that this probability is too low to completely justify such a decision then an investigation into the cost process should be initiated. This is the case since management is in effect saying that in their opinion the unknown probability of the deviation from standard being due to an assignable cause is most likely sufficient to warrant its consideration as a definite possibility.

Naturally, it is hoped that the investigation will find and eliminate the assignable cause if one does in fact exist. Some of the factors which have to be considered in determining the location of the cut-off point have already been discussed. Assuming that management has decided in this case that the probability of a chance-caused variance has to be at least .15 before such a hypothesis becomes acceptable, an investigation is not required for December's material variance. Had the cut-off probability been set at .2, however, then obviously an investigation would have been necessitated.
Some proponents of the use of probability in this area of cost control have developed what they call a cost control decision chart. Once devised, it can be used to tell management what action should be taken for a particular variation from standard. An example of such a chart that might have been prepared for the preceding illustration is shown in Figure 2, and the decision that it indicates is an investigation of the material variance since an unfavorable variation of $400 or more with a probability of .1867 is well inside the investigate area of the chart. Several aspects of the figure undoubtedly merit additional explanatory comment.

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First, as it should be, the dividing line between the two areas 'Investigate' and 'Do Not Investigate' is based upon both probabilistic and economic considerations. In fact, its location is dependent upon the same factors as was the cut-off probability rate considered previously. Second, the chart illustrated is only for unfavorable deviations from standard. Usually the economic considerations are not constant between favorable and unfavorable variances and thus management may well have to construct two district charts. Third, the dividing line between the two areas does not intersect the horizontal axis at zero, but rather at some positive amount. The reason for this is that the amount of an unfavorable variance should at least exceed the cost of an investigation before the latter is deemed necessary. The manner in which the cost control decision chart in Figure 2 is drawn implies that the cost of the particular type of investigation required for the situation of the example is $100 on the average, since the amounts recorded on the horizontal axis commence at this figure.

**Probabilistic Analysis of Variable Costs--An Extension**

In the preceding example of probabilistic analysis of cost variance reports the statement was made that actual production equaled expected production. This statement was necessary because the cost under consideration was a variable one. For such a cost there is a distribution of possible costs for each and every level of production. The means of
these distributions will definitely be different but their
determination presents no real difficulties. This is true
since they will simply be equal to the standard costs of the
item for the particular levels of production under analysis.
The standard deviations, however, may or may not be different
depending upon the nature and characteristics of the cost
involved. In some cases distribution variation will be
independent of volume, but in others increased variation may
well be likely as volume increases. One possible method of
handling these latter type situations is to define two of the
possible cost distributions and, assuming lineation, determine
the relationship between the standard deviation and volume in
equation form.⁹

As an example of the above method assume that the
material cost mean of $4,500 and standard deviation of $448
previously used was for a volume of 10,000 units. Assume
further that for a volume of 15,000 units management deter­
mines using the probabilistic belief concept that the
distribution of possible costs has a mean of $6,750 (1.5(4.500))
and a standard deviation of $548. Letting 10,000 units be the
base volume and dividing the difference between the two stan­
dard deviations of $100 by the difference between the two
volumes of 5,000 units gives the linear relationship indicated
below.

⁹This technique requiring the assumption of a linear
relationship between the standard deviation and production
volume may not yield precise answers, but it definitely will
yield practical usable estimates.
Standard Deviation = $448 + .02(Volume - 10,000)

Thus, the appropriate standard deviation for the appropriate cost distribution can now easily be determined by management regardless of the actual level of production, and the probabilistic analysis can proceed as usual.

Limitations of Probabilistic Analysis of Cost Variance Reports

Probabilistic analysis of cost variance reports is subject to the same limitations as the use of the Shewhart control chart for cost control purposes. One of these is the assumption that the distribution of possible costs will be normally distributed. Adequate studies have not been made yet to determine how realistic or unrealistic this assumption really is. However, Henderson and Copeland do state, "Experience with statistical control of the quality characteristics of manufactured products would tend to imply that the normal curve assumption would hold for all direct costs of manufacturing." Thus, as with the control chart, until further studies have been made management must be cognizant of the fact that this probabilistic technique may not be appropriate for some costs, especially those of a non-manufacturing nature.

A second similar limitation is that the use of this

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technique will sometimes lead to a wrong decision. There are two types of errors possible, assuming that there is an assignable cause of variation when there is not one and assuming that there is not an assignable cause when there is one. In statistical terms the first error is known as a Type I error and the second as a Type II error. The probability of the former type can be calculated and management made aware of just what degree of risk they are taking concerning it. For example, if the decision is made that the probability of a chance-caused deviation from standard must be as low as .15 before this hypothesis can be accepted, then the probability of a Type I error is .15. Fifteen per cent of the time in the long run a decision will be made to investigate a variance that is in fact the result of chance causes and therefore non-controllable. Unfortunately, the likelihood of a Type II error cannot be computed with the information that is usually available about a distribution. This consequence is not as bad as it seems, though, since the end result of a Type II error is not as crucial or detrimental as that of a Type I error. Even when the former occurs the cost deviation is within some acceptable range that was determined by the location of the cut-off probability.

In summary, then, probabilistic analysis of variance reports for cost control purposes will result in occasional erroneous decisions. However, it is a technique that permits management to take maximum advantage of the information available and, without doubt, will result in better cost control
and more correct decisions in the long run than most of the methods currently being applied.

APPLICATION OF PROBABILITY TO THE CONTROL OF SIMILAR BRANCH COSTS

One cost control problem that frequently confronts the managerial personnel of those companies with one or more branches concerns similar costs at the various locations. The problem is: Are differences in these costs significant enough to require investigation? This question can still be important even in those situations where the similar costs appear to be under control—deviating within acceptable limits—at each of the branches. Costs at one of the locations could be under control at a lower level because of improved procedures or techniques that have not been communicated and encouraged at the others. The use of probability to test for significant difference is one method of minimizing the possibility of such undesired circumstances.\(^{11}\)

Binomial Probabilities and Similar Branch Costs

The first probabilistic technique to be considered in this area is a very simple one. All that is required for its application is a record of the amounts of the similar costs for each branch for some period of time. From this data it can be determined how many times during the period one

\(^{11}\)Another method is the use of the Shewhart control chart as a test for homogeneity.
branch's cost exceeded the other's. Then, using the binomial distribution the probability of this particular combination occurring can be computed and used by management as a basis for deciding if the costs do appear to be significantly different.

The binomial distribution is applicable in those cases where there are only two possible outcomes for independent trials and the likelihood of each of the outcomes is constant from trial to trial. If the number of branches being considered in the cost analysis is restricted to two then the first condition is satisfied; and, in most cases, the assumption that the probability of the outcomes does remain constant from trial to trial can undoubtedly be made without the loss of too much reality. The only remaining step before the technique can be applied is the determination of this constant probability. The hypothesis being tested here is that the similar costs at the two branches are essentially equivalent, and assuming this to be true means that the probability of either of the two possible outcomes is .5. In other words, the probability of Branch 1 performing better than Branch 2 is .5 and so is the probability of Branch 2 performing better than Branch 1. Using this information the likelihood of one branch performing better than the other some number of times out of so many trials can be calculated. This is done using the binomial probability
formula shown below, or better yet by utilizing a table of binomial probabilities. The latter can be found in most statistics texts and in virtually every book of commonly used statistical tables.

\[ P(r) = \frac{n!}{r!(n - r)!} p^r q^{n-r} \]

\( r = \) number of successes (number of times one branch's performance is better than the other's)
\( n = \) number of trials
\( p = \) probability of a success
\( q = 1 - p \)

To illustrate the use of this technique assume that the data of Table XVIII represents the average daily amounts of some similar cost for a period of 10 days at two branch locations. From the table it can be determined that Branch 2 performed better than Branch 1 7 out of the 10 days covered. Using the previously discussed probability of .5 and a table of binomial probabilities the likelihood of such an occurrence can be found to be .1172. A more meaningful likelihood, however, would be that of 7 or more superior performances by Branch 2 and this figure is .1719.

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Thus, management now has some quantitative basis for deciding if the two costs do appear to be significantly different. A cut-off probability based upon experience, expected savings, and the cost of an investigation will have to be determined and used in making the final decision. If it is decided after consideration of the above likelihood that whenever one branch out-performs the other 7 or more times out of 10 an investigation will be made, then management will be wrong 34.38 per cent of the time. This is true since the probability of Branch 2 performing thusly due to chance is .1719 and so is the probability of Branch 1. Naturally, this situation could be improved by requiring more superior performances or by increasing the number of trials being considered, but in doing so the degree of control that can be exercised over the costs is undoubtedly lessened.

<table>
<thead>
<tr>
<th>Day</th>
<th>Branch 1</th>
<th>Branch 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3.60</td>
<td>$3.25</td>
</tr>
<tr>
<td>2</td>
<td>4.00</td>
<td>3.85</td>
</tr>
<tr>
<td>3</td>
<td>3.75</td>
<td>4.00</td>
</tr>
<tr>
<td>4</td>
<td>3.75</td>
<td>3.50</td>
</tr>
<tr>
<td>5</td>
<td>4.20</td>
<td>3.75</td>
</tr>
<tr>
<td>6</td>
<td>3.95</td>
<td>4.00</td>
</tr>
<tr>
<td>7</td>
<td>3.85</td>
<td>3.50</td>
</tr>
<tr>
<td>8</td>
<td>3.50</td>
<td>3.75</td>
</tr>
<tr>
<td>9</td>
<td>3.75</td>
<td>3.50</td>
</tr>
<tr>
<td>10</td>
<td>4.00</td>
<td>3.85</td>
</tr>
</tbody>
</table>
Tests of the Difference of Two Means
and Similar Branch Costs

The probabilistic technique described above for analyzing similar branch costs has one major weakness. It does not consider the absolute size of the costs at the branches but only which one was lower. There is an alternative technique available based upon the statistical concept of tests concerning the difference between two means which does not have this disadvantage.

This alternative procedure involves first the taking of a sample of the similar cost at each of the branches and the calculation of the sample means. Then, a distribution of differences between sample means is derived. Assuming that the cost distributions are normal, this derived distribution will also be normal with mean and standard deviation as indicated below.

\[
M_d = M_1 - M_2
\]

\[
STD_d = \sqrt{\left(\frac{STD_1^2}{n_1} + \frac{STD_2^2}{n_2}\right)}
\]

- \(M_d\) = Mean of difference distribution
- \(M_1\) = Mean of cost distribution at Branch 1
- \(M_2\) = Mean of cost distribution at Branch 2
- \(STD_d\) = Standard deviation of difference distribution
- \(STD_1\) = Standard deviation of cost distribution at Branch 1
- \(STD_2\) = Standard deviation of cost distribution at Branch 2
- \(n_1\) = Sample size at Branch 1
- \(n_2\) = Sample size at Branch 2
Furthermore, it can be used to calculate the probability of a difference equal to the one found between the two sample means as a result of chance causes given the hypothesis that the two cost distributions are identical. This figure, of course, will then provide management a basis for deciding if the similar branch costs do appear to be significantly different.

The actual probability computation is accomplished by means of the previously considered normal deviate concept and a table of areas under the normal curve. For the type of derived distribution being discussed here the formula for the normal deviation is as shown below. One of its terms is the standard deviation of the distribution of sample average differences which is dependent upon the standard deviations of each of the cost distributions.

\[
\text{Normal Deviate} = \frac{(SM_1 - SM_2) - M_d}{STD_d}
\]

Where:
- \(SM_1\) = Mean of sample from Branch 1
- \(SM_2\) = Mean of sample from Branch 2
- \(STD_d\) = Standard deviation of sample average differences
- \(M_d\) = Mean of sample average differences

These latter measurements are very seldom known and so must usually be estimated using the standard deviations of the samples which can be calculated as below.

\[
\text{SSTD} = \sqrt{\frac{\sum (X - SM)^2}{n - 1}}
\]

Where:
- \(SSTD\) = Standard deviation of sample
- \(X\) = Individual cost measurements in sample
Substituting this term in the formula for the standard deviation of the differences and the resulting one then in the normal deviate equation shown above yields the formula for the latter that will most likely have to be used in the probability computation. It appears below.

\[
\text{Normal Deviate} = \frac{\text{SM}_1 - \text{SM}_2}{\sqrt{\left(\frac{\text{SSTD}_1^2}{n_1}\right) + \left(\frac{\text{SSTD}_2^2}{n_2}\right)}}
\]

It should be noticed that the term \( M_d \) has dropped out of the formula. This is the case since it equals the difference between the means of the cost distributions at the branches and this figure is zero assuming the two distributions are identical— not significantly different. Naturally, the final step is to look up the calculated normal deviate in a table of areas under the normal curve and determine the probability of a difference equal to the one found between the two sample means as a result of chance.

To illustrate the application of this technique assume that the data given previously in Table XVIII represents the results of two cost samples of 10 each taken at the indicated branch. Table XIX shows the computation of the sample means and standard deviations using the formulas given above. With this information the required normal deviate can be calculated, and for this particular example it appears to be 1.40. Substitution of the required amounts into the normal deviate formula to obtain this figure is illustrated below.
### TABLE XIX

RESULTS OF SIMILAR COST SAMPLES FROM BRANCHES

<table>
<thead>
<tr>
<th>Branch 1</th>
<th>Branch 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>M₁ - X</td>
</tr>
<tr>
<td>$3.60</td>
<td>.24</td>
</tr>
<tr>
<td>4.00</td>
<td>-.16</td>
</tr>
<tr>
<td>3.75</td>
<td>.09</td>
</tr>
<tr>
<td>3.75</td>
<td>.09</td>
</tr>
<tr>
<td>4.20</td>
<td>-.36</td>
</tr>
<tr>
<td>3.95</td>
<td>-.11</td>
</tr>
<tr>
<td>3.85</td>
<td>-.01</td>
</tr>
<tr>
<td>3.75</td>
<td>.34</td>
</tr>
<tr>
<td>3.75</td>
<td>.09</td>
</tr>
<tr>
<td>4.00</td>
<td>-.16</td>
</tr>
<tr>
<td>$38.35</td>
<td>.3905</td>
</tr>
</tbody>
</table>

SM₁ = 38.35/10 = $3.84  
SSTD₁ = .3905/(10 - 1) = .208  

Normal Deviate = \[
\frac{3.84 - 3.70}{.208^2/10 + .248^2/10}
\]

= \[
\frac{.14}{.0043 + .0062} = \frac{.14}{.10} = 1.4
\]

Looking up 1.4 in a table of areas under the normal curve yields an area content of 8.08 per cent. Thus, the probability of the mean of sample one exceeding the mean of sample two by $0.14 or more as the result of chance given the hypothesis that the two cost distributions are essentially equivalent is .0808. Since .0808 is also the
probability of sample mean two exceeding sample mean one by $0.14 or more, it can be said in general that the probability of a difference in the sample means of $0.14 or more is .1616 (.0808 + .0808). As with the binomial application to similar branch costs management will have to determine a cut-off chance probability based on experience, expected savings, and the expense of an investigation. For example, if they had decided for the situation under consideration that this probability had to be .20 or more then a figure of .1616 would indicate that the costs do appear to be significantly different and should be investigated. Naturally, the opposite would be true if the cut-off point had been .16 or less.

This probabilistic technique would seem to have two major limitations. One is the number and lengthiness of the calculations required, and unfortunately nothing can be done about this. However, it can most likely be said that without a doubt the benefits which can be derived from this technique's application will more than offset the expense of the time and effort necessary for the calculations.

A second limitation might seem to be the fact that the cost distributions at the branches were assumed to be normal. This assumption had to be made so that the two distributions of sample means would be normal and thereby also the distribution of sample mean differences. Fortunately, in certain situations this limitation can be eliminated. If the cost samples are sufficiently large (as a general rule
greater than 30)\(^{13}\) then the central limit theorem can be invoked and the distributions of sample means deemed to be asymptotically normal. The central limit theorem is defined by Schlaifer as follows:

If \( z \) is the sum of \( n \) independent random variables all having the same probability distribution, then as \( n \) increases the distribution of \( z \) is more and more closely approximated by a Normal distribution . . . ; and this is true regardless of the nature of the distribution of the individual variables.\(^{14}\)

Thus, in many cases the only limitation on the application of tests of the difference between two means to similar branch costs is the calculation involved, and, as previously pointed out, this is usually not a truly valid argument.

**SUMMARY**

Many traditional cost control procedures do not adequately take advantage of all the information that is or can be available for management's disposition in such areas. One of the most important and prevalent examples of this concerns variations in cost behavior. Traditional analytical methods do not provide management any quantitative basis for deciding if the variations are due to uncontrollable chance causes or controllable assignable causes. Decisions must be based solely on experience and intuition. This chapter

\(^{13}\)Actually, even with samples of only 4 the distributions of sample means would probably be sufficiently normal for them to be approximated as such.

\(^{14}\)Schlaifer, *op. cit.*, p. 284.
examines several probabilistic based techniques that do give management a quantitative basis for distinguishing between these two types of cost variations.

The first technique considered involves the use of the Shewhart control chart to distinguish between natural and assignable causes of cost variation. For its application cost data must be divided into rational subgroups and the average and range of each computed. These figures are graphed and also used to calculate a grand average and an average range. Three standard deviation control limits for the chart are determined from the latter measurements and graphed. Any subgroups falling outside of these limits are assumed to be varying due to assignable causes since the probability of such a variation as a result of natural causes is extremely low.

Causal analysis of the variances in cost reports is considered next. When a normal distribution can be derived for the cost in question from its standard and probability beliefs of the individuals involved, the chance likelihood of a variance equal to or more than the one which actually resulted can be determined. This figure is then available to help management in deciding if the cause of the variance appears to be an assignable or normal one.

Two techniques for analyzing similar branch costs are the final ones discussed. The first of these involves the use of the binomial probability distribution to determine if the number of times one branch out performed the other over
some period of time appears reasonable. The second involves the use of the normal curve or a normal approximation to determine if the difference between the means of samples from each branch appears reasonable given the hypothesis that the two cost distributions are essentially equivalent. In both cases an answer of no to this test of reasonableness indicates that the similar branch costs are most likely significantly different and should be investigated.
CHAPTER VI

REFINEMENT OF SELECTED PROBABILISTIC PROCEDURES

NEED FOR REFINEMENT

Preceding chapters have introduced various probabilistic applications to the areas of data estimation, cost-volume-profit analysis, capital budgeting, and cost control. Certain of these applications can be refined so that they accomplish their desired objective—facilitating and improving managerial decision making—more efficiently, more thoroughly. Unfortunately, many of these refinements involve highly mathematical and theoretical concepts whose effective comprehension and utilization is impeded by the now inadequate educational background formerly considered sufficient for the accounting profession. This situation is currently being remedied by conscientious practitioners and educators, but it is a remedy that takes time. Fortunately, however, other refinements are not so complicated and this chapter discusses and illustrates several refinements of this latter type.
EXPECTED VALUE OF PERFECT INFORMATION

Determining Feasibility of Additional Information

In many cases the managerial personnel of a firm are confronted with the problem of whether they should act on the basis of information currently available or gather additional information first in order to make possible a more informed decision. The answer to this dilemma in any given situation naturally depends upon the relationship between the cost of obtaining additional information and its potential value. If the former exceeds the latter then the decision should not be delayed and if the reverse is true then additional information is probably desirable.

Thus, the real problem in such a situation is the determination of these two figures. The cost of obtaining additional information is usually fairly easy to derive. It depends primarily upon the cost of the time required to make the estimate by the individuals assigned to do so and upon the cost of whatever devices, methods, or procedures that they employ in accomplishing their task. Determining the potential value of additional information presents a much more difficult problem. There is a statistical technique which can help in this area, however, since its utilization results in a measure of the value of perfect information in a given situation—the amount which management should be willing to pay for an infallible predictor for the situation under study. Naturally, it is not very likely that any
additional information which can be derived will lead to perfect variable factor estimates, but some idea of its maximum potential value should certainly be helpful in deciding among the two alternatives available, act now or delay for further investigation. The following paragraphs explain and illustrate the procedures for determining the expected value of perfect information first for discrete probability distributions and then for continuous ones.

**Expected Value of Perfect Information and Discrete Distributions**

To determine the expected value of perfect information in those situations where the probability distribution involved is a discrete one two measures must be known to management. These are the expected profit under certainty for all of the possible acts under consideration and the expected profit of the act indicated as most favorable by expected value probabilistic analysis. Subtracting the latter figure from the former gives the amount by which the company could increase its expected profit if it had an infallible predictor for the variable factor, and as mentioned previously this is the expected value of perfect information for that particular situation. It should be noted that the relevant figure here is not the entire amount of the expected profit under certainty, but rather this amount reduced by the expected profit which the firm can most likely obtain from its use of expected value probabilistic analysis.
Computation of these two required measures is relatively easy. The expected profit under certainty can be calculated by determining for each possible event in the probability distribution the profit which would result if the best act for that particular event were the one selected by management, weighting each of these profits by the related event's probability, and summing these latter weighted figures. Calculation of the expected profit of the most favorable act is accomplished by determining for each event the profit which would result given selection of this act, weighting each of these profits by the related event's probability, and then summing these weighted amounts.

As an illustration of this probabilistic refinement assume that a manufacturer of leather goods is considering the production of a new style attache case. Its contribution margin will be $10 per unit and its production will require expenditures for fixed costs in the amount of $500,000. The company has been able to make certain predictions about the yearly demand of the attache case and these are shown in Table XX below.

<table>
<thead>
<tr>
<th>Demand</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>30,000</td>
<td>.05</td>
</tr>
<tr>
<td>40,000</td>
<td>.10</td>
</tr>
<tr>
<td>50,000</td>
<td>.20</td>
</tr>
<tr>
<td>60,000</td>
<td>.35</td>
</tr>
<tr>
<td>70,000</td>
<td>.30</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>
The first step would be to use probabilistic cost-volume-profit analysis to determine if production of the attache cases is desirable. Naturally, this is accomplished by comparing the expected value of demand for the case, 57,500 units, with its breakeven point of 50,000 units, and since the latter is less than the former production of the product does appear to be a profitable project for the company to undertake. In conjunction with this step in the analysis the probability distribution of demand would also be used to evaluate any risk factor questions considered pertinent and relevant to the situation.

At this point management must determine either implicitly or explicitly whether the final decision should be delayed until additional information can be obtained. By implicitly is meant that this matter is really given no consideration at all and definitely it is not the best approach to rely upon. Optimum decisions are not made by ignoring some of the alternatives involved. By explicitly is meant the approach described in previous paragraphs--determination of the cost of the additional information, its potential value, and based on these two figures the most advantageous action to follow--and it is the one that should lead to better decisions.

Using this second approach, assume that qualified managerial personnel estimate that the cost of obtaining additional information will be approximately $7,500. The amount involved is fairly sizable since the only productive
method of obtaining the information appears to be a rather extensive market research study. As indicated previously, the expected value of perfect information, the measure of the additional information's potential value, depends upon the expected profit under certainty and the expected profit of the most favorable alternative under consideration. Calculation of the former is shown in Table XXI. Column one of this table contains the possible demands for the attache case and column two their related probabilities. The profit that would result if the best act were chosen for each possible demand is shown in column three—in other words, the profit that would result if the company had a perfect predictor, perfect information. For example, if demand is for the breakeven point of 50,000 units or less the best action would be not to produce the attache cases, and thus the conditional profit for demands of 30,000, 40,000, and 50,000 units is zero. However, if demand is for 60,000 units or more the best action would be production of the cases and the conditional profit for demands of 60,000 and 70,000 units is determined by multiplying the contribution margin of $10 by the individual demands and then subtracting the fixed costs of $500,000. The answers are naturally $100,000 and $200,000, respectively. The fourth and final column of Table XXI contains the conditional profits for each demand weighted by the appropriate probability, and its sum is the expected profit under certainty for this example. Therefore,
if the management of this company had perfect information their expected profit would be $95,000.

TABLE XXI
EXPECTED PROFIT UNDER CERTAINTY

<table>
<thead>
<tr>
<th>Demand</th>
<th>Probability</th>
<th>Conditional Profit</th>
<th>Expected Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>30,000</td>
<td>.05</td>
<td>$ 0</td>
<td>$ 0</td>
</tr>
<tr>
<td>40,000</td>
<td>.10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>50,000</td>
<td>.20</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>60,000</td>
<td>.35</td>
<td>100,000</td>
<td>35,000</td>
</tr>
<tr>
<td>70,000</td>
<td>.30</td>
<td>200,000</td>
<td>60,000</td>
</tr>
<tr>
<td>1.00</td>
<td></td>
<td></td>
<td>$95,000</td>
</tr>
</tbody>
</table>

Table XXII illustrates the determination of the expected profit of the most favorable alternative under consideration, in this situation production of the attache cases. Its first two columns are identical to those of Table XXI, but its third column gives the conditional profits of the possible demands assuming production of the cases has been undertaken rather than the best possible act selected in each case. Thus, in this table all of the conditional profits are determined by multiplying the contribution margin of $10 by the appropriate demand and then subtracting the fixed costs of $500,000. This results in negative profits or losses for demands of 30,000 and 40,000 units since they are less than the breakeven point of 50,000 units. The last column once more contains the conditional profits for each demand weighted by the related probability and adds
up to the expected profit of the particular act under consideration, in this case $75,000 for production of the product.

TABLE XXII

EXPECTED PROFIT OF MOST FAVORABLE ALTERNATIVE

<table>
<thead>
<tr>
<th>Demand</th>
<th>Probability</th>
<th>Conditional Profit</th>
<th>Expected Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>30,000</td>
<td>.05</td>
<td>$-200,000</td>
<td>$-10,000</td>
</tr>
<tr>
<td>40,000</td>
<td>.10</td>
<td>-100,000</td>
<td>-10,000</td>
</tr>
<tr>
<td>50,000</td>
<td>.20</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>60,000</td>
<td>.35</td>
<td>100,000</td>
<td>35,000</td>
</tr>
<tr>
<td>70,000</td>
<td>.30</td>
<td>200,000</td>
<td>60,000</td>
</tr>
<tr>
<td>1.00</td>
<td></td>
<td></td>
<td>$ 75,000</td>
</tr>
</tbody>
</table>

Management can now determine the expected value of perfect information for this example. Subtracting $75,000, the expected profit of producing the attache cases, from $95,000, the expected profit under certainty, yields a value of $20,000 for this measure. Thus, based on probabilistic expected value analysis the most the company should be willing to pay for perfect information is $20,000, since this is the maximum amount by which they can increase their expected profit over the level established for it by selection of the most favorable act. This figure is next of course compared with the cost of obtaining additional information about demand for the cases which management has previously estimated to be approximately $7,500. There does appear to be
in this situation then some basis for delaying the decision on addition of the item to the company's product line until additional information can be gathered, as its potential value far exceeds its estimated cost. It is true that the additional information which can be gathered here will most certainly not be perfect, but based on the relative sizes of the above two measures it appears very likely that it will lead to more informed decisions of such a nature that the value obtained from it by the company will more than offset its cost.

**Expected Value of Perfect Information and Continuous Distributions**

In previous chapters one continuous distribution, the normal curve, has been introduced and extensively utilized in solving various accounting problems. Therefore, it is this continuous distribution for which calculation of the expected value of perfect information will now be explained and illustrated.

In the previous paragraphs it was learned that for a discrete distribution the expected value of perfect information for a given situation is determined in part by calculating the conditional profit for each possible event, weighting these figures by the events' probabilities, and summing the results. When the distribution involved is a normal one the analysis proceeds in basically the same manner although the continuity factor may make it seem
different. Two possible situations must be considered, those where the mean of the derived normal distribution is less than the breakeven point and those where it is more. These two cases must be distinguished and analyzed separately because the conditional profits are calculated differently for each.

For the first situation the conditional profits are zero if the true universe mean is less than the breakeven point since probabilistic expected value analysis would have already indicated the best decision, and are equal to the contribution margin times the true mean minus the breakeven point if the true mean is greater than the breakeven point. In other words, conditional profits are possible in the case where the mean or expected value of the derived normal distribution is less than the breakeven point only when additional information would lead to a change in the decision indicated by expected value analysis—when it reveals that the true mean is in fact greater than the breakeven point. Naturally, this would mean acceptance of the project rather than rejection of it.

With the above knowledge of how to calculate conditional profits, a formula can now be derived which weights and sums these profits and thus yields the expected value of perfect information for the case where the mean of the derived normal distribution is less than the breakeven point. Such a formula is shown below (this formula is applicable even if the curve under consideration is not normal).
\[
EVPI = \sum_{TM=BEP}^{\infty} CM(TM - BEP) P(TM) \quad EV < BEP
\]

**EVPI** = Expected value of perfect information  
**CM** = Contribution margin  
**TM** = True or universe mean  
**BEP** = Breakeven point  
**EV** = Expected value of derived distribution  
**P(TM)** = Probability of particular true mean

It should be noticed that only the terms of the equation from the breakeven point to infinity are summed since the conditional profits of those terms where the true mean is less than the breakeven point are zero. Through a series of rather complex manipulations and substitutions which will not be considered in this study the above formula can be rewritten as below.\(^1\)

\[
EVPI = CM(STD)[P'(BEP) - \frac{BEP - EV}{STD} P(TM > BEP)] \quad EV < BEP
\]

**STD** = Standard deviation of derived normal distribution  
**P'(BEP)** = Probability per unit width at breakeven point

Fortunately, Schlaifer has prepared a table\(^2\) which can be

\(^1\)For a complete explanation of these manipulations and substitutions the reader is referred to pages 450-55 of the book cited in Footnote 2.

used to evaluate the bracketed expression in this second equation and, utilizing it, the expected value of perfect information formula can be written as follows:

\[ D = \frac{\text{BEP} - \text{EV}}{\text{STD}} \quad \text{EV} < \text{BEP} \]

\[ \text{EVPI} = \text{CM(STD)}G(D) \]

\[ G(D) = \text{Unit Normal loss integral from Schlaifer's table} \]

Now that a formula has been derived for the expected value of perfect information when the expected value of the normal distribution in question is less than the breakeven point, the next step would appear to be derivation of one for the case where the expected value exceeds the breakeven point. This is not really necessary, however, since the symmetry of the normal curve means that the formula just derived for one situation is actually applicable to both possible cases. As Schlaifer says, "... the expected value of perfect information under a Normal distribution depends on the absolute magnitude of the difference between (Eu) (EV) and \( u_b \) (BEP) but not on its direction or sign."^4 Thus, general equations for the calculation of the expected value of perfect information for those problem areas where the distribution involved is a normal one are:

---

^3This bracketed expression is termed the unit normal loss integral by Schlaifer.

^4Schlaifer, op. cit., p. 454.
\[
D = \frac{|\text{BEP} - \text{EV}|}{\text{STD}}
\]

\[
\text{EVPI} = \text{CM} (\text{STD}) G (D)
\]

To illustrate the calculation of the expected value of perfect information for normal distributions, assume the same basic situation previously used for discrete distributions concerning possible production of attache cases. This time, however, the probabilistic belief concept is utilized to derive a probability distribution of demand, and it results in a normal curve with a mean or expected value of 57,500 units and a standard deviation of 5,000 units. With this information, the contribution margin of the cases of $10 and their breakeven point of 50,000 units, and the above formulas the expected value of perfect information can now be determined. This is done below.

\[
D = \frac{|50,000 - 57,500|}{5,000}
\]

\[
D = \frac{7,500}{5,000} = 1.5
\]

\[
\text{EVPI} = 10(5,000)G(1.5)
\]

\[
\text{EVPI} = 10(5,000)(.0293) = $1,465
\]

Thus, the expected value of perfect information for this problem is $1,465, and since it is far less than the estimated cost of obtaining additional information of $7,500 the decision as to addition of the attache cases to the company's
product line should undoubtedly be made now on the basis of the information already in management's possession. Delaying this decision until additional information can be gathered does not appear to be economically feasible.

UTILIZING ADDITIONAL INFORMATION

Given that a company has utilized one of the preceding techniques to determine that additional information should be gathered before a particular decision is made, and given that it has been gathered, the next question which confronts management is how this information can be most effectively used to facilitate and improve their decision making. The following paragraphs consider one possible answer to this question—Bayesian statistics.

Bayesian Statistics

Generally, Bayesian statistics represents a procedure which can be used to combine the evidence obtained from a sample with whatever information was available about the situation in question before the sampling was undertaken. Its potential merit for the manager who has decided to delay a particular decision until additional information can be gathered is therefore obvious. More specifically, Bayesian statistics is a means of combining a priori probabilities or hypotheses about a universe (probabilities derived before sampling) with sampling probabilities to yield posterior probabilities based upon all available knowledge, both...
subjective and objective, concerning the universe under study. The technique by which the actual combination is accomplished is based on Bayes' theorem which states "... the probability that any one hypothesis is correct is a fraction whose numerator is the conditional probability obtained from the product of the a priori probability and the sampling probability for that hypothesis and whose denominator is the sum of such conditional probabilities for all the possible hypotheses." The following paragraphs discuss the application of this field of statistics to business problems involving first discrete probability distributions and then continuous ones.

**Bayesian Statistics and Discrete Distributions**

When the management of a company has formulated certain hypotheses about a business universe and expressed the probabilities of these in a discrete distribution, and when they have sought additional information on this situation by means of sampling, Bayesian statistics can be used to combine all the available data concerning the universe in the form of posterior probabilities for the hypotheses in the following manner. First, determine for each hypothesis about the universe the probability of the sampling results if that hypothesis were in fact the true one. The exact technique for determining this probability depends upon the type of

---

discrete distribution that is involved in the analysis. Second, multiply the above sampling probabilities by the related hypothesis's probability as originally formulated, in other words the a priori probability, to obtain conditional probabilities for the hypotheses. Third, sum the conditional probabilities and compute the posterior probability of each hypothesis concerning the universe by dividing its conditional probability by this total. Naturally, the next step is for management to redetermine the best act to undertake using these posterior probabilities and expected value analysis.

As an illustration of when and how to use sampling and Bayesian statistics in solving business problems involving discrete probability distributions, assume that a manufacturing company which has to produce a special part for use in subsequent processing is faced with the following situation. Five thousand of the parts are needed immediately and the machine regularly used for their production is not in operating condition and cannot be made so far several weeks. There is a standby machine which can be used to manufacture the parts and which in the past after initial set-up adjustments has had the defect record shown in Table XXIII. The incremental cost of production for the special part using this machine is $11.20 and all defective parts must be discarded as a complete loss. They do not even have any scrap value. The company does have one other course of action available, however, and that is to subcontract
production of the parts at a cost of $12.00 per unit.

TABLE XXIII

DEFECT RECORD OF STANDBY MACHINE

<table>
<thead>
<tr>
<th>Fraction Defective</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>.01</td>
<td>10</td>
</tr>
<tr>
<td>.05</td>
<td>30</td>
</tr>
<tr>
<td>.10</td>
<td>60</td>
</tr>
</tbody>
</table>

The first step that management should undertake to help analyze this situation is to use expected value analysis to determine which of the two possible actions appears to be the most desirable and whether or not the decision should be delayed until additional information can be gathered. The expected cost of subcontracting the special parts is naturally $60,000, determined by multiplying the 5,000 needed by the quoted unit cost of $12.00. The expected cost of producing the parts with the standby machine is computed in Table XXIV by utilizing the defect record of the machine as a probability distribution for the fraction defective. The conditional cost of each of the possible fractions defective (column three in the table) is calculated by multiplying 5,000 divided by the fraction good times the incremental cost of production per unit of $11.20. Division of 5,000 by the fraction good is necessary to determine how many parts must be produced in order to obtain 5,000 acceptable ones since defective pieces must be discarded at a
complete loss. For example, if the fraction defective is .01 then 5,051 (5,000 divided by .99) parts must be manufactured if 5,000 good ones are to be obtained, and 5,051 times $11.20 gives a conditional cost for this event of — $56,571. The expected costs of the fractions defective compose the final column of the table, are calculated by multiplying each event's probability by its conditional cost, and their total of course represents the expected cost of the act production with the standby machine.

TABLE XXIV

EXPECTED COST OF PRODUCTION WITH STANDBY MACHINE

<table>
<thead>
<tr>
<th>Fraction Defective</th>
<th>Probability</th>
<th>Conditional Cost(^a)</th>
<th>Expected Cost(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.01</td>
<td>.1</td>
<td>$56,571</td>
<td>$ 5,657</td>
</tr>
<tr>
<td>.05</td>
<td>.3</td>
<td>58,957</td>
<td>17,687</td>
</tr>
<tr>
<td>.10</td>
<td>.6</td>
<td>62,227</td>
<td>37,336</td>
</tr>
<tr>
<td>1.0</td>
<td></td>
<td></td>
<td>$60,680</td>
</tr>
</tbody>
</table>

\(^a\)The conditional and expected costs in this table have been rounded to the nearest dollar.

Thus, since the expected cost of subcontracting is $60,000 and of company production $60,680, the manufacture of the special parts should be subcontracted if the final decision is to be made on the basis of information currently available. However, assume that first management wishes to determine if the gathering of some additional information appears feasible. Sample pieces can obviously be manufactured on the standby machine for $11.20 each, but what is the
potential value of additional information? Table XXV illustrates the calculation of the expected cost under certainty so that the expected value of perfect information can be determined and some basis provided for answering this question. The expected cost under certainty is computed in a similar manner—as the expected profit under certainty, by weighting the conditional cost of the best act for each possible event by the event's probability and summing the results. In other words, it represents the minimum expected cost for the firm if it were operating with perfect knowledge. Subtracting $59,344, the expected cost under certainty, from $60,000, the expected cost of the more favorable act—subcontracting, yields a figure of $656 for the expected value of perfect information in this situation.

TABLE XXV

EXPECTED COST UNDER CERTAINTY

<table>
<thead>
<tr>
<th>Fraction Defective</th>
<th>Probability</th>
<th>Conditional Cost of Best Acta</th>
<th>Expected Costa</th>
</tr>
</thead>
<tbody>
<tr>
<td>.01</td>
<td>.1</td>
<td>$56,571</td>
<td>$ 5,675</td>
</tr>
<tr>
<td>.05</td>
<td>.3</td>
<td>58,957</td>
<td>17,687</td>
</tr>
<tr>
<td>.10</td>
<td>.6</td>
<td>60,000</td>
<td>36,000</td>
</tr>
<tr>
<td>1.0</td>
<td></td>
<td>60,000</td>
<td>$59,344</td>
</tr>
</tbody>
</table>

aThe conditional and expected costs in this table have been rounded to the nearest dollar.
Thus, since the expected value of perfect information is $656 and sampling can be conducted at only $11.20 per piece, it does appear that the gathering of some additional information may well be worthwhile here. For illustrative purposes, assume that management decides to produce fifteen special parts on the standby machine as a sample; does so, and upon inspection finds no defectives. If the process by which the part is produced can be assumed to represent a binomial distribution as explained in Chapter V, then the sampling results can be used to adjust the original probabilities assigned to the possible fractions defective.\textsuperscript{6} This is accomplished using Bayesian statistics and is illustrated in Table XXVI.

\begin{table}[h]
\centering
\caption{Calculation of posterior probabilities}
\label{tab:posterior_prob}
\begin{tabular}{cccc}
\hline
\hline
.01 & 1 & .8601 & .0860 & .25 \\
.05 & 3 & .4633 & .1390 & .40 \\
.10 & 6 & .2059 & .1235 & .35 \\
1.0 & & & .3485 & 1.00 \\
\hline
\end{tabular}
\end{table}

The a priori probabilities shown in column two of the table are naturally the original probabilities derived from

\textsuperscript{6}For a discussion and illustration of the application of the Pascal distribution to this type of problem the reader is referred to Chapter Eight of Schlaifer's book \textit{Probability and Statistics for Business Decisions}. 
the defect record of the standby machine. The sampling probabilities were obtained from a table of binomial probabilities and represent for each particular fraction defective the likelihood of obtaining a sample of fifteen items in which there are no defectives. The conditional probability of each event reflects the product of its a priori and sampling probability and when divided by the sum of the conditional probabilities for all events gives its posterior probability. Using these more knowledgeable figures the expected cost of production of the special parts on the company's standby machine can now be recomputed. This is done in Table XXVII. The conditional cost of each of the possible fractions defective is the same as previously used in Table XXIV, but they are now multiplied by the posterior probability rather than the a priori probability to obtain the event's expected cost.

**TABLE XXVII**

EXPECTED COST OF PRODUCTION WITH STANDBY MACHINE

<table>
<thead>
<tr>
<th>Fraction Defective</th>
<th>Posterior Probability</th>
<th>Conditional Cost</th>
<th>Expected Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>.01</td>
<td>.25</td>
<td>$56,571</td>
<td>$14,143</td>
</tr>
<tr>
<td>.05</td>
<td>.40</td>
<td>58,957</td>
<td>23,583</td>
</tr>
<tr>
<td>.10</td>
<td>.35</td>
<td>62,227</td>
<td>21,779</td>
</tr>
<tr>
<td>1.00</td>
<td></td>
<td></td>
<td>$59,505</td>
</tr>
</tbody>
</table>

\( \text{aThe conditional and expected costs in this table have been rounded to the nearest dollar.} \)
Examination of Table XXVII indicates that the expected cost of production with the standby machine has been reduced to $59,505 with the utilization of the sampling information. This reduction makes company production the more desirable act now since the expected cost of subcontracting is the same as before, $60,000.\(^7\) Thus, the sample of fifteen pieces with no defectives has deemed the a priori probability assigned to the fraction defective .10 unreasonable, and reduced it almost in half allocating the difference to the other fractions defective. As .10 was the event with the highest conditional cost, this allocation has decreased the expected cost of the act company production; in fact, decreased it to the extent that it is now less than the expected cost of the act subcontracting. Incidentally, the final decision does not necessarily have to be made at this point and on the basis of the information currently available. Management can redetermine the expected value of perfect information using the posterior probabilities, and if it once more exceeds the cost of sampling a second sample may well be justified. This would of course mean new posterior probabilities to be computed and used by the company in its expected value analysis of the problem.

\(^7\)The fact that the sample has produced 15 of the 5,000 parts required is ignored here for the sake of simplicity. Its consideration would not alter the decision.
Bayesian statistics involves the same theoretical concepts for continuous distributions as for discrete ones, but the actual computational techniques necessary with the former are much more technical and confusing. Obviously, this is a result of the infinite number of values which are possible when dealing with a continuous distribution. Fortunately, however, when the original or prior and sampling information are described by normal probability distributions, the posterior distribution will also be normal with mean and variance\(^8\) as indicated below.

\[
EV_P = \frac{EVO(1/SDO^2) + SM(1/SDS^2)}{1/SDO^2 + 1/SDS^2}
\]

\[
1/SDP^2 = 1/SDO^2 + 1/SDS^2
\]

\[
EV_P = \text{Expected value or mean of posterior distribution}
\]
\[
EVO = \text{Expected value or mean of original distribution}
\]
\[
SDO = \text{Standard deviation of original distribution}
\]
\[
SM = \text{Sample average or mean}
\]
\[
SDS = \text{Sample standard deviation}
\]
\[
SDP = \text{Standard deviation of posterior distribution}
\]

These formulas were derived by Schlaifer\(^9\) and as should be expected are based on Bayes theorem. Their actual derivation

\(^8\)It should be remembered that the variance is the square of the standard deviation.

\(^9\)Schlaifer, op. cit., p. 441.
will not be considered in this paper because of its highly mathematical nature.

As an illustration of Bayesian statistics and the normal distribution assume the following situation. A company produces an item which requires a large amount of manual labor at $5.00 an hour. An industrial equipment supplier has offered to supply them with a new machine that should mean some savings in labor costs. It will cost $10,000 and have a life of only one year. The firm's shop foreman using the probabilistic belief concept estimates that the machine will result in mean savings of 2,100 hours and have a standard deviation of 400 hours.

Expected value analysis at this point would indicate that the machine should be purchased since mean savings of 2,100 hours would result in a cost savings of $10,500 ($5.00 times 2,100) and this exceeds the purchase price of $10,000. The company computes the expected value of perfect information using the formulas derived earlier in this chapter as below, and based on its amount decides that perhaps additional information should be gathered before a final decision is made, however. The breakeven point of 2,000 hours is naturally determined by dividing the machine's price by the hourly labor cost and the 'contribution margin' of $5 here is simply the hourly labor cost.

---

10 The general pattern for this situation is based on one utilized by Bierman, Fouraker, and Jaedicke in their book *Quantitative Analysis for Business Decisions*. 
The additional information is gathered in the form of ten timed trial runs with the machine, the results projected in yearly terms. Such projections are shown in Table XXVIII along with the computation of the sample average and the estimated standard deviation of sample averages.

**TABLE XXVIII**

**SAMPLE BASED PROJECTED HOURLY SAVINGS**

<table>
<thead>
<tr>
<th>Projected Hours Saved</th>
<th>(X - (\bar{X}))</th>
<th>((X - \bar{X})^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,300</td>
<td>100</td>
<td>10,000</td>
</tr>
<tr>
<td>2,300</td>
<td>100</td>
<td>10,000</td>
</tr>
<tr>
<td>1,800</td>
<td>-400</td>
<td>160,000</td>
</tr>
<tr>
<td>2,500</td>
<td>300</td>
<td>90,000</td>
</tr>
<tr>
<td>2,200</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2,100</td>
<td>-100</td>
<td>10,000</td>
</tr>
<tr>
<td>2,400</td>
<td>200</td>
<td>40,000</td>
</tr>
<tr>
<td>2,300</td>
<td>100</td>
<td>10,000</td>
</tr>
<tr>
<td>2,100</td>
<td>-100</td>
<td>10,000</td>
</tr>
<tr>
<td>2,000</td>
<td>-200</td>
<td>40,000</td>
</tr>
<tr>
<td>22,000</td>
<td></td>
<td>380,000</td>
</tr>
</tbody>
</table>

\(\bar{X} = \frac{\sum X}{n} = \frac{22,000}{10} = 2,200\) hours

\[SDS = \sqrt{\frac{\sum(X - \bar{X})^2}{n(n - 1)}} = \sqrt{\frac{380,000}{10(9)}} = \sqrt{\frac{380,000}{90}} = \sqrt{4,222} = 65\) hours
The actual standard deviation of sample averages is described by the formula:

\[ SDS = \frac{SDP}{\sqrt{n}} \]

\[ SDP = \text{Standard deviation of population} \]

However, since the standard deviation of the population is not known it must be estimated using the sample standard deviation as follows:

\[ SDP = \sqrt{\frac{\sum(X - \bar{X})^2}{n - 1}} \]

Substituting the latter expression in the former equation gives the one utilized in Table XXVIII.

Using the formulas derived earlier in the chapter the original and sampling distributions can now be combined. This is done below.

\[ EVP = \frac{EVO(1/SDO^2) + SM(1/SDS^2)}{1/SDO^2 + 1/SDS^2} \]

\[ EVP = \frac{2100(1/(400)^2) + 2200(1/(65)^2)}{1/(400)^2 + 1/(65)^2} \]

\[ EVP = \frac{2100(1/160,000) + 2200(1/4225)}{1/160,000 + 1/4225} \]

\[ EVP = \frac{2100(.00000625) + 2200(.00024)}{.00000625 + .00024} \]
\[ EVP = \frac{.013 + .528}{.00024625} = \frac{.541}{.00024625} = 2,197 \text{ hours}^{11} \]

\[ \frac{1}{SDP^2} = \frac{1}{SDO^2} + \frac{1}{SDS^2} \]
\[ \frac{1}{SDP^2} = \frac{1}{(400)^2} + \frac{1}{(65)^2} \]
\[ \frac{1}{SDP^2} = \frac{1}{160,000} + \frac{1}{4225} \]
\[ \frac{1}{SDP^2} = .00000625 + .00024 = .00024625 \]
\[ SDP^2 = \frac{1}{.00024625} = 4060 \]
\[ SDP = \sqrt{4060} = 64 \text{ hours}^{11} \]

Thus, the combined posterior distribution has a mean of 2,197 hours saved and a standard deviation of 64 hours. These figures are very close to those of the sampling distribution since its standard deviation is so small relative to the standard deviation of the original data pattern. In other words, the distribution with the smallest variance is given more weight in determining the posterior measures of central tendency and dispersion. The repeated use of the reciprocals of the variances in the above formulas demonstrates this fact quite clearly.

Expected value analysis based on these adjusted measures supports even more strongly now the purchase of the machine. The savings expected to result from its use is $10,985 as compared to $10,500 previously, and of course its cost is still $10,000. Management can make the decision

\[ ^{11}\text{Some of the results of the intermediate steps involved in this calculation as well as the end result itself have been rounded off.} \]
now, or they can recompute the expected value of perfect information and see if further sampling is feasible. In any case, Bayesian statistics can be used to assure a company that maximum value will be obtained from any additional information which it has decided to gather.

Limitations of Bayesian Statistics

There are probably no major limitations connected with the use of Bayesian statistics in situations where the distributions involved are discrete ones. The only problem which may be encountered in such cases is the determination of the appropriate method for computing the sampling probabilities, and this depends upon the nature of the particular discrete distribution that is applicable, be it binomial, hypergeometric, Poisson, or other. In some cases the normal curve can even be used to approximate the above distributions. For a discussion of when this is possible and the techniques involved the reader is referred to Chapters Seventeen and Eighteen of Schlaifer's book Probability and Statistics for Business Decisions: An Introduction to Managerial Economics under Uncertainty.

The use of Bayesian statistics in situations which involve continuous distributions would seem to have two major limitations. One of these is the requirement that the original and sampling distributions be normal in order for the posterior formulas given in this chapter to be appropriate. What is to be done when the distribution involved is not a normal one? Actually, upon closer examination it
can be discovered that this limitation is not as binding as it would seem. It has just been mentioned that the normal curve is a good approximator for some discrete distributions, and this also holds true for some continuous ones. Concerning this Schlaifer has said:

If the variance of the decision maker's true prior distribution is large compared with the sampling variance . . . he can simplify his calculations with no material loss of accuracy by substituting the mean and variance of his true prior distribution into the formulas which apply to a Normal prior distribution.12

Thus, in many cases the original distribution can be treated as normal even though in actuality it is not. Furthermore, when the sample is sufficiently large the normal curve can be used to approximate the sampling distribution. As regards this Schlaifer says, "... the mean of 100 observations on almost any population will have a very nearly Normal distribution. . . ."13 Definitely, then, the normal assumption does not really represent a major limitation to the use of Bayesian statistics in situations which involve continuous distributions.

The second limitation stems from the fact that the posterior formulas given in this chapter assume two action problems involving linear costs or profits. Unfortunately, there is really nothing which can be done about this limitation and it definitely does represent a very restrictive

12Schlaifer, op. cit., p. 448.

13Ibid., p. 442.
condition. Management must be aware of the fact that Bayesian statistics cannot be used to combine original and sampling continuous distributions where the problem involves more than two possible courses of action or costs and profits which behave in a non-linear fashion, at least not utilizing the posterior formulas given in this chapter.

SUMMARY

In previous chapters of this paper various probabilistic techniques have been discussed as aids in data estimation, cost-volume-profit analysis, capital investment analysis, and cost control. This chapter considers and illustrates several refinements of these techniques—refinements that make them even more valuable to management as one possible analytical decision tool.

The first refinement discussed is one which helps management to determine if a decision should be made on the basis of information currently available or postponed until additional information can be gathered. The relevant factors in such a problem are the cost of obtaining the additional information and its potential value. The former can usually be estimated fairly easily but the latter may be difficult to determine. One possibility is to use the statistical measure the expected value of perfect information. Obviously any additional information gathered will not lead to perfect decisions, but at least some quantitative basis for solving the problem is provided. In general the
expected value of perfect information can be expressed as the difference between the profit or cost that would result if the best act were chosen for each possible event and the cost or profit that would result if the act indicated as most favorable by expected value analysis were chosen.

The second and final refinement considered involves the use of Bayesian statistics to combine an original information distribution with the distribution of any additional information which may be gathered. Its application provides management with a basis for making a decision that is influenced or predicated on all pertinent information currently available. The actual basis is in the form of a posterior probability distribution based on Bayes theorem which states that the posterior probability of an event is equal to the ratio of its conditional probability (its original probability times its sampling probability) to the sum of the conditional probabilities of all events involved.
CHAPTER VII
SUMMARY, CONCLUSION, AND FUTURE DEVELOPMENTS

SUMMARY

Introduction

The accountant in the performance of his duties has to face daily the problem of deciding among alternative courses of action. Unfortunately, most of the analytical techniques utilized by him in making such decisions have a major weakness—they cannot adequately cope with the numerous uncertainties which have become so prevalent and important in America's modern and dynamic business environment. As a result, this traditional accounting area is in danger of being gradually usurped by other professions. Fortunately, however, probability theory can be used to remedy this weakness and restore the accountant's analytical techniques to maximum efficiency and realism.

Probability in Data Estimation

One of the accounting areas in which probability can prove extremely useful is data estimation. This is a very important area since most business firms plan for the future in the form of budgets, and these are naturally based to a
large extent on forecasts. Utilizing the beliefs of a single manager or group of managers and assuming a normal curve, probability theory can be used to derive for an item which must be estimated a complete distribution of possible results with known measures of central tendency and estimate dispersion. This latter figure is one that most present techniques do not yield, but one that is vital since it gives an indication of the reliability of an estimate. Furthermore, it allows the probabilities of various results to be computed and considered in overall planning coordination.

Probability in Cost-Volume-Profit Analysis

Cost-volume-profit analysis is a second area in which probability theory can prove helpful. Traditional cost-volume-profit techniques do not allow for uncertainty. All factors involved are treated as fixed regardless of how they were obtained. In those situations where some of the factors are in fact variable, a probabilistic procedure should be used. One can derive a probability distribution for the possible values of each of the variable factors, compute from these distributions the expected value of each factor, and then, using these central tendency measures in conjunction with any fixed quantities and traditional analytical methods, determine the most desirable course of action. In addition to making the cost-volume-profit analysis more realistic, utilization of the probability distribution approach allows the computation of the probabilities of
various possible values of the variable factors, and these likelihoods will prove invaluable in evaluating any relevant risk questions or conditions.

**Probability in Capital Investment Analysis**

Another area which can be improved by the application of probability theory is capital investment analysis. Present methods of allocating capital budget funds, like those utilized in cost-volume-profit analysis, cannot adequately cope with uncertainty. Several probabilistic techniques are available here. One approach is to determine probability distributions and expected values for the cash inflows of possible investment opportunities and use the latter to compute time adjusted rates of return or excess present value indexes. A second approach involves the determination of all possible investment combinations given the current capital budget and the calculation of their respective means and variances. The optimum combination is naturally the one with the highest mean that has a dispersion which management considers acceptable. The final probabilistic approach available is usually feasible only with a computer. It requires the determination of a probability distribution for each variable involved in an investment opportunity. Using any fixed factors, these probability distributions, and random numbers the project is simulated again and again until sufficient findings are available to define a probability distribution for the possible rates of
return on it. This procedure is followed for each opportunity under consideration and the resulting distributions used to allocate the capital funds. All three approaches do have one common characteristic, the probability distributions derived under them can be utilized to compute any probabilities which may prove helpful in evaluating the risk associated with a particular investment opportunity or a combination of opportunities.

**Probability in Cost Control Analysis**

Cost control analysis is the final area considered in which probability can prove beneficial. Traditional cost control procedures do not take advantage of all the information commonly available. The most important example of this concerns the causes of cost variations. They are due either to normal or assignable causes, and the latter are controllable but the former are not. Thus, if management can determine the most likely cause of a cost variation needless investigations can often be eliminated. One technique for accomplishing this utilizes the Shewhart control chart. Under it cost data is divided into subgroups and the average and range of each computed. This information is then used to determine control limits and any subgroup falling outside of the limits is assumed to have an assignable cause for its variation. A second technique is possible when a company uses a realistic standard cost system and the standards can be defined in terms of normal
probability distributions with known means and standard deviations. These statistical measures can be used to compute the probabilities of various variations from a particular standard as a result of normal or chance causes. As a result management has some basis for deciding when an investigation seems to be warranted and when the situation should simply be left alone. The same general approach can also be utilized to determine if the difference between similar branch costs appears to be reasonable or not.

Refinement of Probabilistic Procedures

The preceding probabilistic procedures can be made even more helpful to the accountant through certain refinements. One such refinement involves the statistical measure the expected value of perfect information. Management is constantly faced with the problem of deciding whether to make a decision on the basis of information currently available or to delay it until additional data can be gathered. By computing the expected value of perfect information in a situation, defined as the expected profit under certainty minus the expected profit of the most favorable alternative in the situation, and comparing it with the cost of obtaining additional information, some quantitative basis can be provided for solving this dilemma. A second refinement permits management to combine original and additional information in the form of a posterior probability distribution for possible hypotheses. This distribution incorporates all
available data into one unit and is based on Bayes theorem which states that the probability of a hypothesis being correct is a fraction whose numerator is the conditional probability obtained from the product of the a priori probability and the sampling probability for that hypothesis and whose denominator is the sum of such conditional probabilities for all the possible hypotheses.

CONCLUSION

The importance of accounting decision analyses has definitely increased in recent years, and will continue to do so as long as the successful operation of a business becomes increasingly complex. In order to ensure that these analyses result in the most optimum decisions possible based on available information, the accountant must constantly be on the alert for means of improving them. Without doubt, one such means is probability theory. This division of statistics can be used to improve and facilitate virtually all of the common accounting decision analyses. Especially meritorious is the increased and more realistic consideration of uncertainty which it permits, since uncertainty, an integral part of the modern dynamic business environment, is one of the major reasons business management is becoming increasingly complex. To the extent that the accountant fails to take full advantage of the potential benefits offered by probability theory in his decision analyses he is shirking part of his professional responsibility, and he
must expect other more progressive professionals to move into the managerial advisement area which he has surrendered by his neglect.

FUTURE DEVELOPMENTS

Several comments can most likely be safely made concerning the future of probability theory in accounting decision analyses. First, through the efforts of either the accountant or some other professional man probability theory will soon be an integral part of many of the traditionally-accounting decision analyses. Second, as computers become more and more common in the operation of firms of all sizes they will be programmed to handle many of the computations involved in such analyses. In fact, they will make feasible probabilistic techniques considerably more sophisticated than those discussed and illustrated in this paper. Finally, there will be increased emphasis placed on probability theory, as well as other mathematical and statistical concepts, in the accounting curriculums of business schools as the accountant realizes and acts upon the crucial problem which he faces in keeping pace with the technological and educational advances of twentieth century man.
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VITA

Macil Caldwell Wilkie, Jr., son of Mr. and Mrs. Macil Caldwell Wilkie, was born in Alexandria, Louisiana, on September 26, 1941.

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