2013

Lost in translation : algebraic modeling in the middle school classroom

Danielle Denise Ricks

Louisiana State University and Agricultural and Mechanical College

LOST IN TRANSLATION:
ALGEBRAIC MODELING IN THE MIDDLE SCHOOL CLASSROOM

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Master of Natural Sciences

in

The Interdepartmental Program in Natural Sciences

by
Danielle Denise Ricks
B.S., Louisiana State University, 2005
August 2013

ACKNOWLEDGEMENTS

First, I would like to thank God for allowing me to be in a position to complete this phase of my life. I would also like to thank my son, Chaz, for being my reason to strive harder to accomplish my life goals. I would like to express complete love and gratitude to my support system, my best friends, who have stood by my side through this entire chapter of my life.

I would like to thank Dr. James Madden, the LaMSTI coordinator and my thesis advisor, for taking a chance on a stranger and having faith in my abilities as a math teacher and a student. In addition, I would like to acknowledge and thank Dr. Ameziane Harhad and Dr. P. Sundar for being members of my thesis committee. A special thanks goes out to Leslie Blanchard, Robyn Carlin, and Kimberly De'Souza for being such valuable contributors to the LaMSTI program.

Finally, I would like to thank my Westdale Middle School colleagues and the members of my cohort for their individual contributions in my transformation as teacher over the past three years. The voyage and path would not have been the same without each of you traveling right beside me. I wish each of you much success in the next phases of your lives.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

This study was conducted to investigate the disparity that seems to exist between students' abilities to solve equations, solve word problems, and model word problems with algebraic equations. Over the course of fourteen weeks, students enrolled in an advanced seventh grade mathematics course were given a series of algebra assignments, tasks, and surveys that focused on their abilities to solve and write algebraic equations. The results show that students are more competent in determining solutions for equations and simple word problems than modeling word problems with algebraic equations. Students were willing to exert substantial effort to use arithmetic procedures to find solutions, but were not as invested in writing the equations. Students have difficulty stating the relationships between known and unknown quantities using the language of algebra. Our results suggest that classroom instruction should be focused on bridging the conceptual gaps that exist between solving equations, solving word problems, and modeling with algebraic equations.

CHAPTER 1: INTRODUCTION

One major topic for students in the middle school grades is linear equations. The Common Core State Standards (CCSS) provide teaching standards for linear equations within the Expressions and Equations domain. According to the Common Core Progressions Document for Grades 6-8 Expressions and Equations, “students should be able to 1) connect abstract symbols to their numerical referents, 2) be precise in defining variables, 3) look for structure in expressions by putting them into a sequence of operations, 4) make use of the structure to interpret the expression’s meaning in terms of the quantities represented by the variables, and 5) look for regularity in a repeated calculation and express it with a general formula” (Expressions and Equations, 2011). Middle school proficiency of linear equations means that students are able to solve single- and multi-step equations for an unknown variable and model real-word or mathematical situations with an equation to determine an unknown quantity. “While many students can solve equations successfully, the degree of efficiency and sophistication of those solutions vary” (Capraro, 2006). More importantly, “many students have difficulty formulating algebraic equations from information presented in words” (MacGregor, 1993). This leads to a situation in which algebra becomes “a gatekeeper and barrier for students” (Capraro, 2006 and Lott, 2000).

My own observations are that middle school students are often able to find numerical solutions to word problems. This suggests that these students possess good number sense and a strong arithmetic foundation. However, these same children have difficulty in translating word problems into algebraic equations. Given a specific word problem, the students can successfully determine the solution, but are not able to model the scenario algebraically with an equation. Previous research supports these observations, declaring “middle school students often

demonstrate much stronger skills in solving formal and informal problems that require algebraic reasoning than in symbolizing equations" (Capraro, 2006).

To understand these strengths in solving equations and weaknesses in formulating algebraic equations, this study measured students' abilities to:

1) Translate verbal phrases and real-world scenarios into algebraic expressions

2) Solve simple (one-variable) linear equations,

3) Translate real-world scenarios into algebraic equations

4) Analyze word problems to generate algebraic equations, and

5) Synthesize knowledge of solving and writing algebraic equations.

These five areas address a key part of the algebra expectations for students in grades 6-8. These areas also measure students' proficiency of concepts and skills associated with linear equations.

In an effort to compare and contrast students' abilities to solve equations, solve word problems, and write algebraic equations, students enrolled in an Advanced Seventh Grade Mathematics course were given a series of assignments and tasks on a weekly basis during units within the curriculum that focused on expression and equation concepts. Each assignment was completed during regularly scheduled class time, but specific times for each assignment varied depending on the length and level of difficulty of each assignment. Each assignment addressed and measured students' abilities to solve equations, to solve word problems, and/or write equations for real-world scenarios. Each assignment was scored based on the number of correct solutions and/or written equations. Each assignment received further analysis on the number of incorrect answers, the common student errors for incorrect answers, and the relationship between each student's individual ability to determine the solutions and model scenarios with algebraic equations. At the conclusion of the study, a survey was conducted to allow students to judge and

critique their own ability to solve equations, solve word problems, and write algebraic equations from real-world mathematical problems.

The overall objective of this study is to investigate the disparity that seems to exist in students' abilities to solve equations, to solve word problems, and to write equations from word problems. I will compare students' abilities to solve equations or solve word problems with their ability to model real-world scenarios with equations. I will identify some specific misconceptions and weaknesses in student thinking that lead to difficulties in solving equations, solving word problems, and writing equations. Acknowledging and addressing student misconceptions allows teachers to correct faulty thinking patterns, thus allowing students to be proficient at both arithmetically solving equations and algebraically modeling equations. My findings will equip teachers with useful knowledge of student misconceptions in algebraic problem solving that can be incorporated within the framework of instruction.

This document is structured as follows: Chapter 2 reviews the literature concerning where students' algebra education begins and how it relates to the algebraic requirements of equations in middle school. I will also examine the results and discussed the findings of similar studies that have investigated the difficulties students have with solving algebraic equations and translating algebraic equations from real-world mathematical problems. In Chapter 3, I will review the national and state educational standards for algebraic equations to show the requirements for teacher instruction and student learning. In the subsequent chapters, the design and results of this study will be described, examined, and used to provide recommendations for future instruction and further investigation.

CHAPTER 2: LITERATURE REVIEW

Algebra is a very broad domain of understanding, and the foundations for learning algebra are built from the moment a student learns to count, recognize similarities and differences, manipulate quantities, and determine solutions for unknown quantities. Operational and algebraic thinking begins as early as kindergarten. Kindergarten students "represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations" (Operational and Algebraic Thinking, 2011). These students are learning to use mathematical and non-mathematical language at an early age. First grade students begin to "represent and solve for unknowns" in subtraction (Operational and Algebraic Thinking, 2011). This foreshadows the introduction of variables in middle school. At the second grade level, students "use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem" (Operational and Algebraic Thinking, 2011). This represents a student's first introduction to writing and solving equations that contain unknown quantities. The fostering of these particular skills begins in kindergarten and is carried throughout mathematics in the primary grades. With these experiences, students are prepared for more rigorous algebra training as they move into secondary schools.

In the middle school grades, students are learning elementary algebra. "Elementary algebra is the most basic form of algebra. It is typically taught to secondary school students and builds on their understanding of arithmetic" (Algebra, 2012). In elementary algebra, students transition from performing operations with numbers and are introduced to quantities with fixed values represented as variables. Stanford University professor and mathematician Keith Devlin

stresses, “algebra is not arithmetic with letters.” Instead he states, “at the most fundamental level, arithmetic and algebra are two different forms of thinking about numerical issues” (Devlin, 2011). It is these two types of thinking in elementary algebra that allows for the “formulation of equations and the study of how to solve these” (Algebra, 2012).

Before students begin solving and writing algebraic equations, they must develop an intuitive understanding of an equation. The Common Core defines an equation as “a statement that two expression are equal” (Expressions and Equations, 2011). An important aspect of equations is “that the two expressions on either side of the equals sign might not actually always be equal” (Expressions and Equations, 2011). The equation may be true for some values and false for others. Another element of equations is their solutions. “An equation may have one solution, no solution, or an infinite number of solutions” (Expressions and Equations, 2011). However, the instruction for middle school students, generally, isolates the meaning of an equation to represent scenarios in which the two expressions are always equal. This leads to solving procedures that result in only one solution, which, for middle school students, becomes the focus when solving linear equations.

Nicholas Herscovics and Liora Linchevski determined that “a cognitive gap exists between arithmetic and algebra.” Generally, textbooks and teachers organize the two skills consecutively, where solving equations is a building block for the more rigorous task of modeling with equations. Solving equations is arithmetic in nature, while modeling real-world scenarios with an equation requires a student to express the relationship between two or more quantities. This suggests a student’s ability to solve an equation may be independent of their ability to write an equation. According to Kenneth Koedinger and Mitchell Nathan, external representations can affect performance and learning. “A student’s success in algebraic problem

solving may be better when one representation is easier to comprehend than another" (Koedinger, 2004).

Students are generally taught to solve equations by performing inverse operations, or operations that undo each other. "Students will use primitive processes on equations they find very easy and use their more sophisticated procedures when they feel warranted" (Herscovics, 1994). The primitive processes refer to a student's tendency to use systematic substitution, also called guess-and-check, to determine the value of the variable and successfully solve the equation. This often occurs when the values included in the equation are smaller values, with which students can easily use mental math. As the values within the equation get larger or contain different forms of rational numbers (ie. fractions, decimals, or integers), students will transition to using the *sophisticated procedure* of inverse operations. In either case, students tend to "solve equations by working around the unknown (variable) at a purely numerical level" (Herscovics, 1994). In this regard, the success that students experience when solving equations for an unknown primarily comes from their foundations in arithmetic. "Students who understand only an algorithmic method of solving equations will experience difficulty when they encounter equations in different forms, solving for different variables, and working with non-linear equations later in their mathematics career" (Capraro, 2006).

Students' abilities to solve equations "are affected by the misconceptions they have about key problem features" of equations (Booth, 2007). This suggests that a student's success in solving equations is not only related to the numbers provided in the equation, but also the structure of the equation. "Students who hold misconceptions about equations tend to learn less from instruction on how to solve equations" (Booth, 2007). The following problem features have been identified as factors that may influence the ability of students to solve equations:

1. Meaning of the equals sign
2. Presence of rational numbers (fractions, decimals, or integers)
3. Size of the numbers
4. Number of operations
5. Number of occurrences of the variable (Herscovics, 1994 and Booth, 2007).

Tailoring instruction so that it addresses the key problem features allows teachers to understand student difficulties with equations and create opportunities for students to be successful in solving more rigorous and challenging equations.

Understanding the equals sign has previously been shown to be crucial for algebraic problem solving. "Equality and the meaning of the equals sign in particular are difficult for students who are in the process of transitioning from arithmetic and algebraic thinking" (Knuth, 2006). Due to the training in the primary grades, students "view the equals sign operationally in the sense of indicating the need to perform the required operation" (Herscovics, 1994). In addition to this, "students often think of the equals sign as an indicator of the answer to the problem rather than the equivalence of two phrases" or expressions (Baroody, 1983). Students also have the "common perception of the equals sign as a separator symbol" (Kieran, 1990). Students are conditioned to following arithmetic procedures and they view the equals sign as a signal requiring them to find an answer.

In connection with the equals sign, students experience difficulty when encountering decomposition problems, or problems in which the *answer* to the problem appears to the left of the equals sign, as opposed to the right. For example, if given the equation $42 = 4x + 6$, "many students would refuse to accept it and claim that it is written backwards" (Herscovics, 1994). Students having a limited understanding of the meaning of the equals sign may ignore the

arithmetic equivalence presented by the equation and may have difficulty in successfully solving the equation.

"Another problem feature that seems important for algebraic equations is the negative signs" (Booth, 2007). "Due to their abstract nature, working with negative numbers is inherently difficult for students, especially for those who are transitioning from arithmetic to algebraic thinking" (Linchevski, 1999). "In algebra, students have to understand not only the magnitude, but the direction of numbers or terms in order to fully comprehend the problem" (Moses, 1989). Some students have "a tendency to ignore the negative sign" and this "detachment of a number from the preceding minus (or negative) sign may explain the very different answers" students obtain when solving equations with integers (Herscovics, 1994). Students also sometimes possess "the misconception that negatives can enter and exit phrases without consequence and that their locations (and connections to numbers or variables in the problem) are not significant" (Booth, 2007).

Numerical problem features that affect a student's ability to solve equations are the size and types of numbers involved. When values included in the equation are small enough, students will rely on known numbers facts and arithmetic education from primary grades to solve the equation. "Students will overwhelmingly revert to solving equations using inverse operations when the numbers in the equation are large enough to go beyond known number facts" (Herscovics, 1994). "The foundation for understanding algebra is laid in the understanding of arithmetic that students encounter before they reach an algebra course, such as arithmetic with rational numbers" (Rotman, 1991). This suggests that student difficulties in solving equations, in which fractions, decimals, or integers are included, may originate from an arithmetic standpoint

as opposed to an algebraic one. Student difficulties with operating with rational numbers, such as fractions and decimals, are separate from their ability to solve algebraic equations.

In addition to having difficulties solving equations with various types of number, “some students have difficulty solving equations involving several arithmetic operations” (Herscovics, 1994). This is another instance in which difficulties with solving an equation may be the result of an arithmetic deficiency because students must be able to apply the order of operations or the distributive law. Studies have shown that “in general, students will have no problems spontaneously simplifying the given equation by performing the indicated operations” (Herscovics, 1994). The study highlighted the difficulties that arose in equations when the variable was positioned between the numerical terms. For equations in which the variable separated numerical additions, students were successful in solving the equations. Yet, for equations in which the variable separated numerical additions and subtractions, “a major shift in the choice of solution procedures” was noticed and only half of the students were able to successfully solve the equation (Herscovics, 1994). Students ignored the subtraction symbols that preceded numerical values, focused on the posterior addition signs, and performed subtraction as the last operation. This highlights the detachment that students experience with the negative (or minus) sign and demonstrates the need for a conceptual understanding of the minus sign.

The number of occurrences of the variable is a problem feature that may deter a student from successfully solving an equation. When the unknown appears more than once on the same side of the equation, “the frequency of substitution procedures increases dramatically” (Herscovics, 1994). This solution method was also used for equations where the unknown appeared on both sides of the equation. This shows that, unlike numerical situations, “grouping

of the unknowns is not a procedure that students acquire spontaneously" (Herscovics, 1994). These results show that students have an "inability to operate with or on the unknown" which leads to a reliance on numerical solution processes (Herscovics, 1994).

Possessing a "conceptual understanding of the features of a problem is key to learning in terms of algebraic problem solving" (Booth, 2007). The literature shows that when students do not possess the necessary conceptual understanding associated with solving equations, they rely on previously learned arithmetic skills. "These students focus on the computational procedure rather than the conceptual" to solve equations (Thompson, 1994). "Students need a balance of conceptual (comprehension) and procedural (vocabulary) skills as they begin to develop algebraic understanding" (Carpraro, 2006). "Mathematics students must possess conceptual understanding so that, once words have mathematical meanings, they can accurately translate those words into mathematical symbols called linear equations" (Capraro, 2006).

"There are two ways for students to formulate equations from verbal data: either by direct translation of keywords to symbols or by trying to express the meaning of the problem" (MacGregor, 1993). These two methods are referred to as syntactic translation and semantic translation. Syntactic translation is "a sequential left-to-right method of translating without regard to meaning" (Mestre, 1988). "Semantic translation refers to the process of using the relative meaning between words" to translate the relationship into an equation (Herscovics, 1989). In education, syntactic translation has become the norm and it is a very common practice for students (Mestre, 1988). "Textbooks teach students to form number sentences or algebraic equations by matching specific verbal cues (or keywords) to mathematical symbols from left-to-right" (MacGregor, 1991). This results in a "direct sequential word-for-symbol mapping" where students write the equations based on the order of words (Cocking, 1988). "One major cause of

error is believed to be the attempt to translate words into equations using only syntactic translation" (Mestre, 1988). However this conclusion has been contested. "In test items designed so that syntactic translation would produce a correct equation, most students did not translate words to symbols sequentially from left –to-right, but tried to express the meaning of the statement and wrote incorrect equations" (MacGregor, 1993). This suggests that errors in equations are not due primarily to syntactic translation. "In order to understand a statement before representing it algebraically, a combination of several processes- including the application of syntactic, semantic, and pragmatic rules- is involved" (MacGregor, 1993).

Regardless of the method chosen, studies show that "many students have difficulty formulating algebraic equations from information presented in words" (MacGregor, 1993). In a study conducted with middle school students, it was determined that "students were not procedurally or conceptually ready even at the seventh and eighth grade level to translate from written words to mathematical equations" (Capraro, 2006). These students are not able to write equations to express what they know about a quantity based on the indirect information given. Subsequently, "the difficulties students experience in translating word problems into algebraic equations indicate another major cognitive gap" in algebra (Herscovics, 1994).

CHAPTER 3: EDUCATIONAL VISION

During the study, the curriculum guidelines for the seventh grade mathematics course required the use of the newly developed Common Core State Standards along with specific Louisiana Grade Level Expectations. Both sets of educational standards formed the framework for algebra instruction delivered to the students in this study. In this chapter, the standards are described to show the algebra skills to be taught to students, which include evaluating expressions, solving equations, and translating verbal models into expressions and equations. Following this, the curriculum for the course and the algebra unit within the course are described to show the constraints under which these skills are to be taught and the other skills that must also be included in the unit.

3.1 Common Core State Standards

The Common Core State Standards (CCSS) were designed to “provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world” by reflecting knowledge and skills needed for students to be successful in future math experiences (Mission, 2013). In terms of mathematics education, the CCSS introduces the domain “Expressions and Equations” beginning in sixth grade to create a foundation for algebra and working with variables. The standards associated with this domain expand through seventh grade and, ultimately, conclude in the eighth grade. Students are expected to apply the skills learned from these standards to more elaborate goals within various domains at the high school level.

In sixth grade, the algebra domain (Expressions and Equations) is separated into three clusters, where each cluster highlights the abilities a student should attain by the end of sixth

grade. These clusters are: 6.EE.A “Apply and extend previous understandings of arithmetic to algebraic expressions,” 6.EE.B “Reason about and solve one-variable equations and inequalities,” and 6.EE.C “Represent and analyze quantitative relationships between dependent and independent variables” (Mathematics Grade 6, 2013). The standards that relate directly to algebraic modeling and solving equations fall within clusters one and two. The first reference of algebraic modeling amongst these standards is located within the first cluster and relates to Standard 6.EE.A.2a, which states, students will “write expressions that record operations with numbers and with letters standing for numbers” (Mathematics Grade 6, 2013). For example, a student should possess the ability to represent the phrase *the sum of h and 7* as the expression h + 7 or 7 + h. The succeeding standard within this cluster expands on a student’s ability to write and translate expressions to being able to create equivalent expressions.

The Common Core State Standards define equivalent expressions as “two expressions that name the same number regardless of which value is substituted into them” (Mathematics Grade 6, 2013). Standard 6.EE.A.3 states that students will “apply the properties of operations to generate equivalent expressions” (Mathematics Grade 6, 2013). More simply stated, this standard requires a student to be able to use basic operations to rewrite or simplify expressions. For example, a student should understand 3g + 2g is equivalent to 5g. The student has simply added the two terms and performed a skill that is commonly referred to as *combining like terms*. In a different scenario, a student should be able to use basic multiplication to translate the expression 7(3x + 5) into the expression 21x + 35. In the same sense, a student can use division (or factoring) to reverse the process by translating the expression 21x + 35 into the expression 7(3x + 5). All three of these scenarios can be performed using basic operations, but the students are simply applying the distributive law in all three examples.

In order for students to reason about and solve one-variable equations, the CCSS uses standard 6.EE.B.5 to define and describe, conceptually, what it means to solve an equation. This standard highlights that “solving an equation is a process of determining what value makes the equation true” (Mathematics Grade 6, 2013). Once students grasp this concept, the CCSS transitions into writing equations from real-world scenarios. Standard 6.EE.B.6 states that students will “use variables to represent numbers and write expressions when solving a real-world or mathematical problem” (Mathematics Grade 6, 2013). It continues by emphasizing that students will “understand that a variable can represent an unknown number or any number in a specified set” (Mathematics Grade 6, 2013). This particular standard revisits Standard 6.EE.A.2a of writing expressions and applies the skill in a real-world context. Students must “solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p q, and x are all nonnegative rational numbers” (Mathematics Grade 6, 2013). This standard allows sixth grade students to create a foundation for solving equations, to understand the grammar of equations and expressions, and to understand that equations can be used to represent known data about an unknown quantity.

Unlike sixth grade, the CCSS for the algebra domain (Expressions and Equations) in seventh grade mathematics contains only two clusters: 1) “Use properties of operations to generate equivalent expressions” and 2) “Solve real-life mathematical problems using numerical and algebraic expressions and equations” (Mathematics Grade 7, 2013). Cluster one focuses on extending students’ abilities to combine like terms, factor expressions and equations, and their understanding of equations. The standards that directly relate to algebraic modeling and solving equations fall within cluster two. Cluster two builds on the sixth grade understanding of using variables within real-world mathematical problems to write and solve equations.

Standard 7.EE.B.3 states that students will "solve multi-step real-life and mathematical problems posed with positive and negative numbers rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically" (Mathematics Grade 7, 2013). Algebraically, this standard progresses students to solving problems that require a minimum of two steps. However, in order for students to be successful in achieving the algebraic requirements of this standard, they must possess the fundamental skills for working with fractions, decimals, and integers. The standard adds that students must be able to "apply operations to calculate with numbers in any form and convert between forms as appropriate" (Mathematics Grade 7, 2013). These particular skills are developed and fostered in the sixth and seventh grades within The Number System domain of the CCSS. Adequate understanding of these skills will allow students to master standard 7.EE.B.3 and apply it to real-world mathematical problems.

Standard 7.EE.B.4 states that students will "use variables to represent quantities in a real-world or mathematical problem and construct simple equations to solve problems by reasoning about the quantities" (Mathematics Grade 7, 2013). Substandard 7.EE.B.4a specifies that students will "solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers" (Mathematics Grade 7, 2013). In the middle school classroom, an equation of the form $px + q = r$ is typically referred to as a two-step equation and an equation of the form $p(x + q) = r$ is would be considered a multi-step equation. Both types of these equations can be solved in a minimum of two steps (or two operations). The process, or steps, taken by a student to solve these types of equations will depend upon the student's conceptual understanding of solving equations, ability to perform operations with rational numbers, and ability to generate equivalent expressions. The students must be able to

integrate the knowledge obtained in The Number System domain in sixth and seventh grade (i.e. operations with rational numbers) with the knowledge obtained in the Expressions and Equations domain in sixth grade to successfully master the standards within the Expressions and Equations domain in the seventh grade.

3.2 Louisiana Transitional Comprehensive Curriculum

The Louisiana Transitional Comprehensive Curriculum was developed by the Louisiana Department of Education as a tool to assist schools and teachers with the transition from using Louisiana's educational standards (formerly referred to as Grade Level Expectations, or GLEs) to the Common Core State Standards (CCSS). One goal of the Louisiana Transitional Comprehensive Curriculum is to connect previously taught GLEs with corresponding CCSS to prepare teachers and students for the more meaningful requirements fostered by the CCSS. For seventh grade students, the Louisiana Transitional Comprehensive Curriculum introduces algebraic problem solving in Unit 3: Expressions and Equations. The recommended timeframe for this unit is six weeks and is described as a "unit that ties numerical problem solving to algebraic problem solving" (Grade 7 Mathematics). The description continues by explaining that the unit begins "with computations using the distributive property and the unit moves into using properties of operations to generate equivalent expressions to solving real-life and mathematical problems using numerical and algebraic expressions, equations, and inequalities" (Grade 7 Mathematics). Table 3.1 and 3.2 lists the Grade Level Expectation and Common Core State Standards addressed in Unit 3.

By analyzing the contents within these tables, the algebra component of this unit is mainly covered within the CCSS. The GLEs focus on pre-requisite algebra skills (order of

operations and evaluation with exponents and roots), algebraic modeling within patterns, and a supplementary skill (coordinate graphing) that is incidental to the unit.

Table 3.1 Grade 7, Unit 3 Grade Level Expectations

Grade-Level Expectations	
GLE #	**GLE Text and Benchmarks**
Number and Number Relations	
3.	Solve order of operation problems involving grouping symbols and multiple operations. (N-4-M)
Algebra	
12.	Evaluate algebraic expressions containing exponents (especially 2 and 3) and square roots, using substitution (A-1-M)
18.	Describe linear, multiplicative, or changing growth relationships (e.g., 1, 3, 6, 10, 15, 21, …) verbally and algebraically (A-3-M) (A-4-M) (P-1-M)
Geometry	
29.	Plot points on a coordinate grid in all 4 quadrants and locate the coordinates of a missing vertex in a parallelogram (G-6-M) (A-5-M)

Table 3.2 Grade 7, Unit 3 Common Core State Standards

CCSS for Mathematical Content	
CCSS#	**CCSS Text**
Expressions and Equations	
7.EE.A.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related
7.EE.B.3	Solve multi-step real-life and mathematical problems posed with positive and negative numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms when appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

The Louisiana Transitional Comprehensive Curriculum for Unit 3 discusses the level of student understanding by stating the following:

> In this unit, students gain an understanding of exponents of 2 and 3 and the evaluation of expressions containing these exponents. They should be able to use mental math to match algebraic inequalities with the situations they model, particularly in using inequalities to approximate the values of square roots that are not perfect. Students should be able to apply the distributive property of multiplication over addition to solve problems. Students will also gain an understanding that rewriting an expression in different forms in a

> problem context can shed light on how the quantities in it are related. They should be able to use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

This excerpt reinforces the concept that students need to build a strong mathematical base in order to accurately use variables and equations to model real-world mathematical problems.

For advanced, mixed math courses, in this case a seventh grade math course that combines sixth and seventh grade students, the teacher should make adjustments within instruction to accommodate the younger students who have not been exposed to the prerequisite material. Although the seventh grade curriculum prescribes six weeks to cover the information listed in tables 3.1 and 3.2, the teacher must also include the pertinent GLEs and CCSS recommended in Unit 8, titled Patterns and Algebra, of the sixth grade transitional curriculum to address the needs of sixth grade students. Table 3.3 identifies the necessary GLEs and CCSS listed in this unit that would promote student learning of the seventh grade GLEs and CCSS.

Table 3.3 Grade 6, Unit 8 Grade Level Expectations and Common Core State Standards

Algebra Grade-Level Expectations and Expressions and Equations CCSS	
GLE #	**GLE Text and Benchmarks**
15.	Match algebraic equations and expressions with verbal statements and vice versa
16.	Evaluate simple algebraic expressions using substitution
CCSS #	**CCSS Text**
6.EE.B.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6.EE.B.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

The Common Core State Standards (CCSS) provide a thorough explanation and guide of the skills necessary for students to be successful at solving equations, solving word problems, and translating verbal models into algebraic equations. The difficulty students have mastering

these skills may be linked to the curriculum that teachers are required to follow. The curriculum bottles these standards together with supplementary skills into six weeks, with the expectation students will exit the unit with a foundational understanding of equations that can be applied to later units and succeeding math courses. However, sufficient time is not allotted for students to develop a true conceptual understanding of equations. Teachers will attempt to distribute time according to what they deem is important or necessary, and this often leads to more emphasis and time spent on solving equations. The time–limited situation allows students to connect algebra to arithmetic, but it conditions students to think one-dimensionally about equations, not recognizing them as statements while believing that they have no other purpose than to be solved. Once a student grasps the concept of solving equations, teachers then progress to modeling word problems with equations and addressing the remaining skills in the unit. The curriculum seems to create a rushed environment for teachers and students, which may aggravate the disparity that seems to exist in students' abilities to solve equations, solve word problems, and write algebraic equations.

CHAPTER 4: MATERIALS AND METHODS

4.1 Participants

Participating in this study were middle school students enrolled in an advanced seventh grade mathematics course. Each class consisted of a mixture of sixth and seventh grade students and was receiving instruction in the algebra unit of the course. All students in the participating classes had previously received instruction on variables, expressions, and solving single and multi-step algebraic equations. However, students had not yet begun to translate verbal models and real-world scenarios into equations. Instruction on this topic occurred during the initial phases of the study, when students were assessed on their ability to complete the previously taught material.

Over a period of fourteen weeks, these students were given a series of basic algebra assignments, algebra tasks, and a survey. The researcher designed these assessment tools to monitor student progress, create a student-investigator feedback system, guide subsequent instruction, and examine the thought processes of students. The algebra assignments (see Appendix A) ranged in difficulty based on the skills required and the standards (or Grade Level Expectations) they addressed. The researcher created algebra tasks (see Appendix B) focused on students' ability to recall, identify, write and apply strategies for translating and solving real-world or mathematical problems. The survey (see Appendix C) was designed to gather students' opinions of their own mathematical capabilities and for students to demonstrate those capabilities.

4.2 Algebra Assignments

Students begin the series of algebra assignments by addressing CCSS 6.EE.A.2a, 6.EE.B.6 and Grade 6 Algebra GLE 15, which require students to translate verbal and real-world

models into algebraic expressions. The scenarios used in this assignment (Appendix A, Assignment 1) were selected from a series of expressions provided by IXL, an interactive mathematics tutoring website. The assignment, titled Writing Expressions, contained twenty questions that were separated into two groups. The first group (questions one through ten) required students to write algebraic expressions from direct verbal models. For example, question 4 asks students to write an expression for the phrase *u more than 22*. The second group (questions eleven through twenty) required students to write algebraic expressions to describe real-world models that contain characters and situations. Within each group, some expressions require one operation, but students are also challenged with questions in which two operations are needed. This assignment is designed to probe and analyze students' prerequisite understanding of operational vocabulary (keywords that indicate a specific operation) and writing expressions that include variables.

After students have demonstrated the ability to write one- and two-step algebraic expressions, the second assignment (Appendix A, Assignment 2) in the series focused on students being able to write one-step algebraic equations from real-word mathematical problems. A one-step equation can be defined as an equation in which only one operation is being performed on an unknown quantity. This assignment addresses CCSS 6.EE.B.5 and 6.EE.B.7 and builds on the previous assignment by compelling students to continue writing algebraic expressions, but now these expressions must be set equal to specified value. The assignment included ten real-world scenarios modeled after word problems, from the IXL website, practiced at the third grade level. Directing students to use a specific variable to write an equation was the only alteration applied to each third grade word problem. Students are instructed to a) select five of the ten scenarios, b) determine the solution to each scenario, and c) to model each scenario

with an equation. Students were allowed to select five scenarios as a motivational tool and to accommodate class time. There were no guidelines on whether the solution or equation should be determined first; this allowed the investigator to analyze strengths or weaknesses in solving basic word problems and writing one-step equations.

Understandably, after writing one-step equations from real-world scenarios, the third assignment (Appendix A, Assignment 3) in the series focused on students being able to write two-step (or multi-step) algebraic equations from real-word mathematical problems. A multi-step equation can be defined as an equation in which two or more operations are being performed on an unknown quantity. This assignment addresses CCSS 7.EE.B.3 and 7.EE.B.4 and extends the second assignment by asking students to determine how multiple (three or more) quantities relate. The assignment included six real-world scenarios modeled after word problems, from the XL website, practiced at the sixth and seventh grade levels. Students are instructed to a) select three of the six scenarios, b) determine the solution to each scenario, and c) to model each scenario with an equation. The students were allowed to select three scenarios as a motivational tool and to accommodate class time. There were no guidelines on whether the solution or equation should be determined first; this allows the investigator to analyze strengths and/or weaknesses in solving more intricate word problems and establishing arithmetic relationships with multiple quantities by writing multi-step algebraic equations.

The fourth and final assignment (Appendix A, Assignment 4) in the series of algebra assignments focused on students' ability to solve single and multi-step equations. In assignments two and three, students had to use given information to construct appropriate equations to model the scenarios and then solve the proposed equations. The final assignment required students to solve sixteen equations, ten single-step and six multi-step equations. All sixteen equations have

been translated from the real-world scenarios administered in assignments two and three. Students were not informed of this detail and are directed to solve each equation for the specified variable. The structure of this assignment allowed students to demonstrate their ability to satisfy CCSS 6.EE.B.5, 6.EE.B.7, and 7.EE.B.3 by solving simple linear equations.

4.3 Algebra Tasks

Students were given five algebra tasks during the fourteen-week study. Students had four focused algebra tasks (Appendix B, Tasks 1-4) and one extended-response algebra task (Appendix B, Task 5). The focused algebra tasks were designed to solely address CCSS 6.EE.B.7, 7.EE.B.3, and 7.EE.B.4 and had students translate real-world word problems into solvable single and multi-step equations. All scenarios included in the focused algebra tasks and the extended-response task were created by the investigator. The focused algebra tasks differed from the algebra assignments because they provided students with a guide for writing an equation from a real-life word problem. The extended-response task differed from both the algebra assignments and the focused algebra tasks because it required students to write equations from real-world scenarios in two-variables. The students were also required to use the equations to perform other algebraic processes, such as substitution and evaluation. The extended- response task reinforces the CCSS practiced in the focused algebra tasks and includes skills that address CCSS 6.EE.A.2, 7.EE.A.2 and sixth grade Algebra GLE 16, where students substitute for variables within equations and then solve those equations.

In each of the four focused algebra tasks, students were given one real-world mathematical problem, where each problem is numbered in correspondence to the task. To accommodate for seventh grade requirements, the four scenarios included all forms of positive rational numbers (whole numbers, fractions, and decimals). During instruction, the students

were given a guide for writing equations, included in Table 4.1, which was applied to the focused algebra tasks. Of the problems given to students, scenarios one and three (Appendix B, Tasks 1 and 3) result in one-step equations that take on the form $x + p = q$. Scenario two (Appendix B, Task 2) is designed to create a one-step equation in the form of $xp = q$. Scenario 4 (Appendix B, Task 4) results in a multi-step equation in the form of $px + q = r$.

Table 4.1 Four-Step Process for Writing Equations

STEP	INVESTIGATOR EXPLANATION
1. Identify the unknown/variable	Read the problem and identify the unknown information. Label the unknown with a variable.
2. Identify the important details	Identify all information/details necessary to determine the unknown
3. Identify the mathematical relationship	Illustrate how the important details (#2) can be arithmetically used to determine the value of the unknown (#1) (Example: $f = 45 \div 5 + 7$)
4. Write and solve an equation	Using inverse operations, construct an equation that can be solved using the mathematical relationship (#3)

The extended-response task (Appendix B, Task 5) required students to use one multifaceted real-world situation to generate two different multi-step equations of the form $px + q = y$, where x and y are variables and p and q are constants. Students go through a series of questions that oblige them to work with these equations and observe the relationship that exists between the quantities. In the series of questions, students are given values for one of the unknowns and must substitute these values into the equations. After students have substituted for one unknown, the result is a two-step (or multi-step) equation that must be solved to determine the value of the remaining unknown. At the conclusion of the extended-response assignment, students had to answer an open-ended question with justification reflecting an understanding of the work they have performed on the previous sections of the assignment.

4.4 Student Survey

At the end of the fourteen-week study, students completed a two-part survey (Appendix C) that sought to gather information about students' opinions of their abilities to complete tasks associated with solving word problems that involve single- and multi-step equations. The first section of the survey consisted of eleven questions concerning students' perceptions of their ability to: a) analyze and solve word problems, b) solve problems involving all forms of rational numbers (integers, fractions, decimals, and whole numbers), and c) solve equations with varying levels of difficulty. Students were asked to rate their opinions as: 1) Strongly Disagree, 2) Disagree, 3) Neutral, 4) Agree, or 5) Strongly Agree. The second section consisted of two real-world mathematical problems that were used to gather students' perceptions of their ability to translate word problems into algebraic equations and determine a solution. The first real-world mathematical problem was designed to translate into a single-step equation and the second real-world mathematical problem was designed to translate into a two-step equation. Each word problem had three survey questions requiring students to rate their opinion (agree, disagree, or neutral) of their abilities to solve the word problem with or without an equation. After completing the survey questions, students had to demonstrate their ability to solve the word problem with or without an equation. The survey opinions were then compared to their actual ability to solve each word problem with and without an equation.

CHAPTER 5: RESULTS AND DISCUSSION

5.1 Writing Expressions

In the Writing Expressions algebra assignment (Appendix A, Assignment 1), student answers were marked correct if they contained an expression that correctly represented the verbal phrase or real-world scenario and incorrect if the algebraic expression did not represent the verbal phrase or scenario. Students' answers were tabulated and scored in two areas: 1) total number of correct responses for problems 1 through 10 (direct verbal phrases) and 2) total number of correct responses in problems 11 through 20 (real-world scenarios). A sum of the two scores was taken to give each student a total score. Of the sixty-six students completing the assignment, sixteen were sixth graders and fifty were seventh graders. Table 5.1 displays a summary of the results from the Writing Expressions assignment.

Table 5.1 Summary of Results for Writing Expressions Assignment

Students	**Average**	**Percent**	**Problems (1-10) Average**	**Problems 1-10 Percent**	**Problems (11- 20) Average**	**Problems 11-20 Percent**
6th Grade (n=16)	13.8	69.1%	7.25	72.5%	6.56	65.6%
7th Grade (n=50)	12.48	62.4%	6.46	64.6%	6.02	60.2%
Group (n = 66)	12.8	64%	6.65	66.5%	6.15	61.5%

The table shows the average number of correct translations (and corresponding percentage) for the twenty verbal models. The table includes a breakdown of the average number of correct translations (and corresponding percentages) for problems 1 through 10 (direct verbal phrases) and problems 11 through 20 (real-world scenarios). Overall, the students were able to correctly translate at least 60% of the expressions correctly. The sixth-grade group of students received higher percentages in all three categories and performed slightly better than the seventh-grade group. When comparing problems 1 through 10 to problems 11through 20, students

(overall and grade-specific) were more competent in writing expressions from direct verbal phrases than from real-world scenarios.

Figure 5.1 illustrates the correlation between each student's individual ability to write expressions from verbal phrases and to write expressions from real-world scenarios. Each value within the grid denotes the number of students who had that specific correspondence of correct translations between direct verbal phrases and real-world scenarios. With more data clustered to the right side of the graph, we can see that students were more competent in writing expressions from direct verbal phrases than from real-world scenarios. The upward slope of the data illustrates that a student who translated more verbal phrases correctly was likely to be capable of translating more real world-scenarios correctly.

Number of Real–World Correct										
10								1		
9							1	3	3	
8						1	2	5	1	
7			1		1	2	1	7	1	
6				2	4	3	2	2		
5			2		3		5	1		
4				1	1	2	2	1		
3				1						
2						1	1			
1						2				
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
	Number of Verbal Phrases Correct									

Figure 5.1 Comparison of Verbal Phrases and Real-World Scenarios

These data suggest that the majority of the students included in this study are equipped with some of the skills (connecting keywords to operations, properly relating the unknown and known quantities) necessary for writing algebraic equations from information presented in

words. In the Writing Expressions assignment (see Appendix A, Assignment 1), the three most commonly missed items were scenarios 10, 12, and 19. These items had success rates of 3%, 36%, and 8%, respectively. Item 10 required students to perform an operation on a quantity that already contained an operation. During the administration of this assignment, the most frequent request from students was to have the meaning of the word *quantity* explained. This suggests that students' academic mathematics vocabulary did not include the word *quantity*. Item 12 required students to use the keyword *split among* to construct an expression containing division. The low success rate for this item can be attributed to students not understanding which value, the variable or the constant, was performing the division. Item 19 required students to write an expression that included two operations, where one operation was performed with a fraction. Students exhibited confusion in how to express the relationship with the variable and the fraction. For example, as opposed to writing (1/3)h students frequently expressed this element as (1/3) $\div$ h. This shows a misunderstanding in the rules of reciprocals, where multiplying by the fraction 1/3 is synonymous with dividing by 3. For those students considered scoring below group averages in this assignment, their errors can possibly be attributed to a misunderstanding of the commutative property in mathematics. Addition and multiplication expressions are correct regardless of the order the variable and constant(s), subtraction and division are not commutative. 56 (or 85%) of the students transposed the variable and the constants within the expression. For items that included addition or multiplication, the item was correct, but of course, the subtraction and division items were not. Another error observed in this assignment were students using exponents for tuples (i.e. double, triple, etc.) instead of multiplication. For example, in the expression, *triple j*, students expressed this quantity as j^3 instead of 3j. Other minor errors noted were students writing expressions with the equals sign or an inequality

symbol. The errors with the division symbol, equals sign, inequality symbols, and exponents also show that students may have a basic misunderstanding what these mathematical symbols mean and how to use them.

5.2 One-Step Word Problems

In the One-Step Word Problems algebra assignment (Appendix A, Assignment 2), students selected five scenarios from a group of ten. This option was given to students as a motivation tool and to accommodate for time restraints. Each real-word scenario required two answers, a solution to satisfy the word problem and a modeled equation representative of the situation. Sixty-five students completed the assignment and results were tabulated to show: 1) the frequency at which each question was selected, 2) the number of students who were able to correctly solve each word problem, 3) the number of students who were able to correctly model each scenario with an equation, and 4) the number of students who were able to correctly solve and write an equation for each scenario. Table 5.2 displays the results from the One-Step Word Problems assignment.

Table 5.2 Summary of Results for One-Step Word Problems Assignment

Problem	Number of Students Selecting this Problem	Number of Students with Correct Solution	Number of Students with Correct Equation	Number of Students with Correct Equation & Solution
1	41	28	19	19
2	27	25	11	10
3	40	25	6	6
4	39	38	21	20
5	30	29	15	15
6	35	33	14	14
7	18	18	14	14
8	30	26	17	17
9	11	10	3	3
10	37	34	20	20

The five most frequently selected items were 1, 3, 4, 6, and 10. These five questions resulted in equations that modeled subtraction, multiplication and division. This shows that students did not have a preference by selecting scenarios that focused on a particular operation. For each scenario, it is numerically evident that more students were able to solve for the unknown than write the appropriate equation. The information expressed in table 5.2 can be visually represented using the bar graph in Figure 5.2.

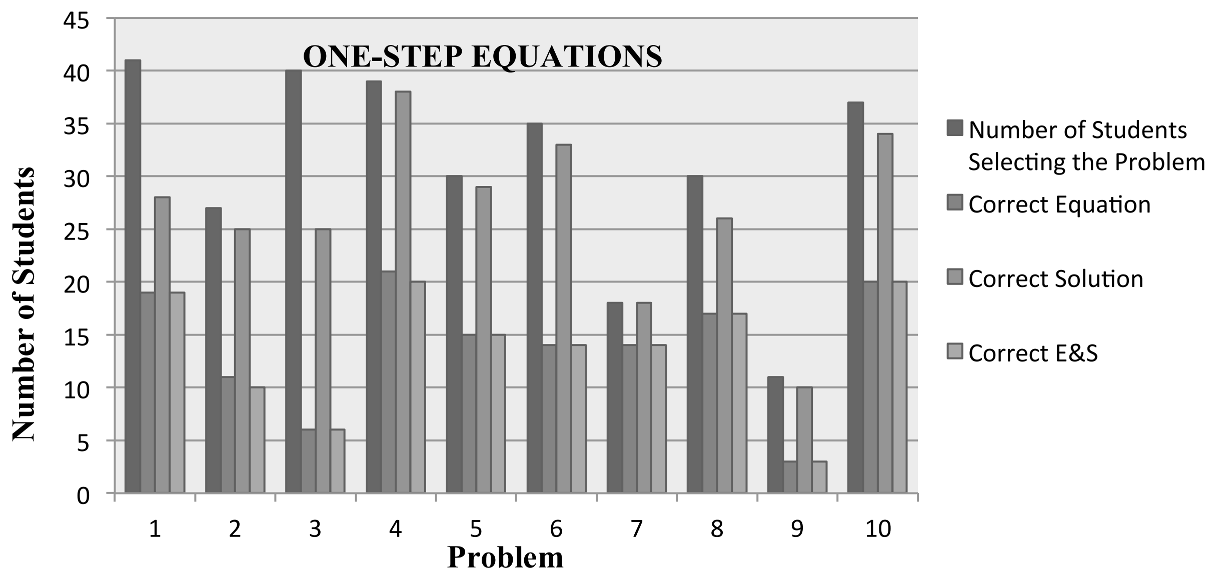

Figure 5.2 Graph of Results for One-Step Equation Assignment

Using Table 5.2 and Figure 5.2, it is evident that those students who could write an appropriate equation for each scenario could also solve for the unknown. With the exception of questions two and four, the correlation between the students who could write the equation and those who could find both the equation and solution is 100%. Questions two and four had a correlation of 90.9% and 95.2%, respectively.

Upon analysis of common errors observed in the One-Step Word Problems assignment, students were successful in determining the appropriate operation required for the scenario, but were unsuccessful in applying the operation appropriately for the equation and solution. For

example, item 3 gave the following scenario: *The stuffed animals at the toy store are split among 4 shelves. There are 12 stuffed animals on each shelf. Write and solve an equation to determine the number of stuffed animals, a, there are in all.* Students showed an understanding that division was required, but they assumed this division would be necessary for the solution. These students calculated the solution as $12 \div 4$ and stated there were 3 stuffed animals on each shelf. These students then represented the scenario with the equation $4a = 12$, which results in a solution of $a = 3$. This error demonstrates a misinterpretation of the relationship of the quantities in the scenario, students not understanding the reasonableness of answers, or students overlooking context clues in the wording of the scenario.

Another frequent error noted in the One-Step Word Problems assignment was the format in which students opted to write their equations. The equations for this assignment were marked correct if they were written in terms of an operation being performed with the variable. The common error (which accounts for the low correct equation rates) was students writing their equations as a string of arithmetic operations that lead to the variable. Students simply wrote their equations as the method in which they solved the problem. For item 3, the most common equation given for students with an incorrect solution was $a = 12 \div 4$ and for students with a correct solution was $a = 12 \times 4$. The last error observed, in terms of the equation, was students including the solution in the equation. Examples of incorrect equations for item 3 with this error took on the following forms: $a \div 3 = 4$, $a \div 4 = 3$, $48 \div a = 12$, and $48 \div a = 4$

5.3 Two-Step Word Problems

In the Two-Step Word Problems algebra assignment (Appendix A, Assignment 3), students selected three scenarios from a group of six. This option was given to students as a motivation tool and to accommodate for time restraints. Each real-word scenario required two

answers, a solution to satisfy the word problem and an equation that models the scenario. Sixty-nine students completed this assignment and results were tabulated to show: 1) the frequency at which each question was selected, 2) the number of students who were able to correctly solve each word problem, 3) the number of students who were able to correctly model each scenario with an equation, and 4) the number of students who were able to correctly solve and write an equation for each scenario. Table 5.3 displays the results from the Two-Step Word Problems assignment.

Table 5.3 Summary of Results for Two-Step Word Problems Assignment

Problem	**Number of Students Selecting the problem**	**Number of Students with Correct Solution**	**Number of Students with Correct Equation**	**Number of Students with Correct Equation & Solution**
1	58	31	13	12
2	51	33	20	18
3	43	32	10	9
4	13	8	5	4
5	14	5	5	3
6	8	1	1	0

The three most frequently selected items were questions 1, 2, and 3. The least selected items were questions 4, 5, and 6. All six items resulted in the equations of the form $px+ q = y$, as addressed in CCSS 7.EE.B.4a, so scenarios only varied in the quantities presented in each scenario. Problems 1 through 4 show that more students were able to solve for the unknown than write the equation and problems 5 and 6 show that these two categories were equal. The information expressed in table 5.3 can be visually analyzed using the bar graph in Figure 5.3. There is some evidence that those students who could write an appropriate equation for each scenario were more likely to correctly solve for the unknown. With the exception of questions five and six, the correlation between the students who could write the equation and those who

could find both the equation and solution was at least 80%. Questions 5 and 6 had correct solution rates of 60% and 0%, respectively.

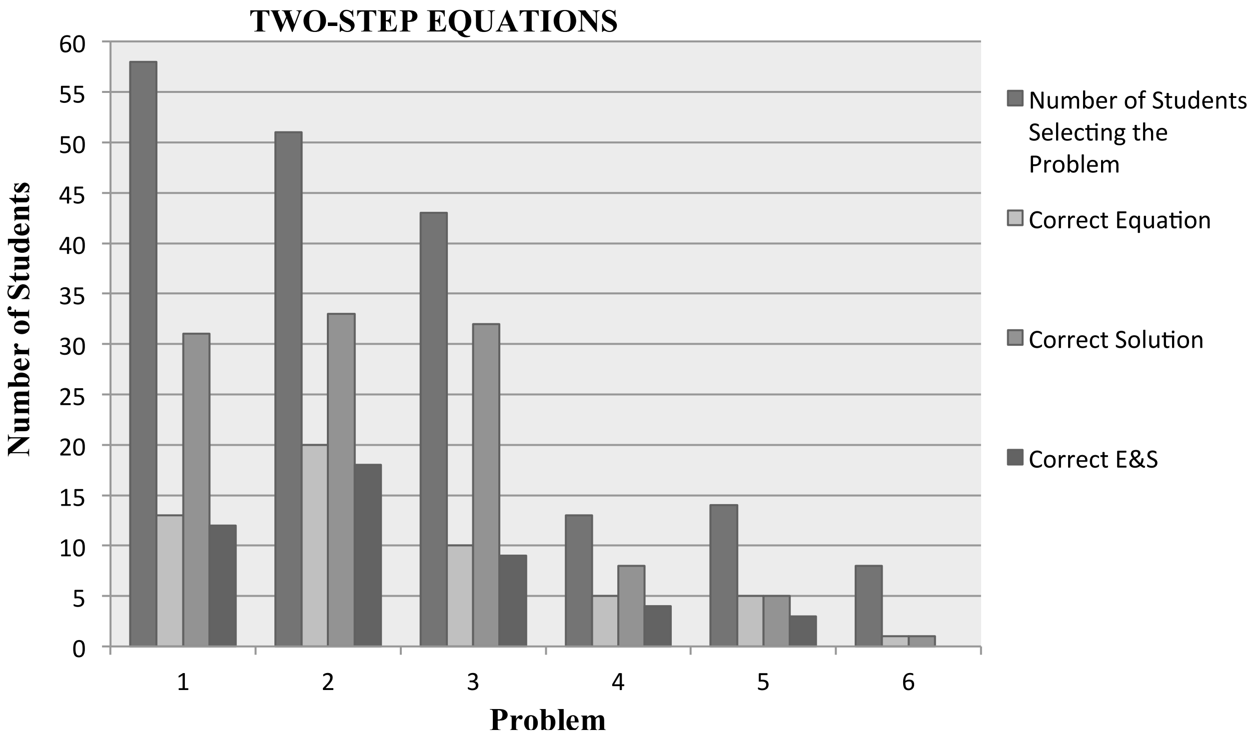

Figure 5.3 Graph of Results for Two-Step Equation Assignment

Upon analysis of the data retrieved from this assignment, students' did not perform as well with two-step word problems as one-step word problems. In question selection, there is a significant gap in the number of students choosing questions 1, 2, or 3 and those that chose questions 4, 5, or 6. Questions 4, 5, and 6 contained numerical values that were large or contained decimals. It appears these values were deterrents for student selection and indicate students would prefer to avoid working with rational numbers that are too large or appear in different formats (i.e. decimals). In determining solutions, students frequently attempted to solve each problem using only one operation. These students were only able to recognize or account for one of the operations needed in the problem. Other students simply made careless mathematical errors, such as borrowing errors and decimal operation errors. In writing

equations, students made similar mistakes. Several students, including those who could solve the problem correctly, modeled the scenarios with a one-step equation instead of a two-step equation. Similar to the one-step word problem assignment, a common error was students writing their equations as a string of arithmetic operations that lead to the variable. Students simply wrote their equations as the method in which they determined the solution. Another common error, which also appeared during the one-step equations, was students including the solution in the equation.

5.4 Solving Equations

In the Solving Equations algebra assignment (Appendix A, Assignment 4), students solved sixteen simple linear equations, ten single-step equations and six multi-step equations. Student answers were marked *correct* for determining the right solution and *incorrect* for any other solution obtained. The results for this assignment were tabulated to show: 1) the total number of correct single-step solutions and 2) the total number of correct multi-step solutions. The sum of these two groups was taken to give students an overall score as well. A comparison was also performed for each equation, where students' performance in solving the equation was compared to their performance in solving for the same solution within a word problem. Of the sixty-two students completing this assignment, seventeen were sixth-graders and forty-five were seventh graders. Tables 5.4 and 5.5 display summaries of the results for the Solving Equations assignment.

Table 5.4 Summary of Results for Solving Equations Assignment

Students	**Overall Average (Out of 16)**	**One-Step Equation Average (Out of 10)**	**Two-Step Equation Average (Out of 6)**
Sixth Grade (n=17)	14.2	9.4	4.8
Seventh Grade (n=45)	13.3	8.9	4.3
Entire Group (n=62)	13.6	9.1	4.5

Table 5.5 Summary of Percentages for Solving Equations Assignment

Students	Overall Percent Correct	One-Step Equation Percent Correct	Two-Step Equation Percent Correct
Sixth Grade (n=17)	89.0	94.1	80.4
Seventh Grade (n=45)	83.2	89.8	72.2
Entire Group (n=62)	84.8	91.0	74.5

According to the results listed in Tables 5.4 and 5.5, the students, as a group and grade-specific, were more competent at solving one-step (or single-step) equations than two-step (or multi-step equations). As with the Writing Expressions assignment, the sixth-grade group received higher overall averages and percentages than their seventh-grade peers. Table 5.6 compares students' performance on solving each equation with their performance on determining the same solution within the context of a word problem. Eleven of the sixteen scenarios had

Table 5.6 Comparative Analysis for Equations and Word Problems

Problem	Word Problem Correspondence	Percent of Students Correct as Word Problem*	Percent of Students Correct as Equation+
1	One-Step 1	68.3	91.9
2	One-Step 2	92.6	77.4
3	One-Step 3	62.5	83.9
4	One-Step 4	97.4	93.5
5	One-Step 5	96.7	96.8
6	One-Step 6	94.3	93.5
7	One-Step 7	100	98.4
8	One-Step 8	86.7	88.7
9	One-Step 9	90.9	85.5
10	One-Step 10	91.9	96.8
11	Two-Step 1	53.4	90.3
12	Two-Step 2	64.7	88.7
13	Two-Step 3	74.4	83.9
14	Two-Step 4	61.5	80.6
15	Two-Step 5	35.7	64.5
16	Two-Step 6	12.5	37.1

*Percent based on the number of students selecting each word problem
+ Percent based on all 66 students selecting each word problem

higher solving percentages when the information was presented as a direct equation. Within these eleven problems were all six two-step equations. This suggests that students have difficulty formulating the algebraic relationship for information presented in words for two-step (or multi-step) algebraic equations. Five of the one-step scenarios (questions 2, 4, 6, 7, and 9) had occurrences where the solving percentages were higher when the information was presented in words. Note that students completed only select word problems, but were required to solve all sixteen equations.

5.5 Focused Algebra Tasks

In the four focused algebra tasks (Appendix B, Tasks 1-4), students followed a four-step writing guide to translate real-world mathematical scenarios into algebraic equations. The real-world scenarios were mathematical story problems written by the investigator. The results for each focused algebra task were tabulated to show: 1) the number of students identifying the unknown and relating it to the variable, 2) the number of students able to recognize the important details and their relationship to the problem, 3) the numbers of students able to determine the arithmetic relationship between the variable and important details, and 4) the number of students able to write and solve an algebraic equation for the scenario given. Observations were also made of students who could not complete the four skills addressed and the methods and rationale they employed. Errors or misconceptions in writing or solving the equation were also analyzed. Tables 5.7 and 5.8 display summaries of the results for the four focused algebra tasks.

Upon analysis of each individual task, students were more successful in identifying the unknown and important details than establishing the arithmetic or algebraic relationship. With the exception of task four, students were able to identify the important details more often than identifying the unknown quantity. In each scenario, students were the least successful in being

able to write *and* solve the equation. Task one appears to be the task in which students performed the best. The percentages for task one were higher than the remaining tasks in three out of the four categories. Figure 5.4 is a visual, graphic representation of the information presented in Table 5.8.

Table 5.7 Summary of Focused Algebra Tasks

Tasks	**Type of Equation**	**Number of Students**			
		Identify Unknown/ Variable	**Identify Important Details**	**Describe Arithmetic Relationship**	**Write *and* Solve Equation**
Task One (n=62)	One-Step Addition	32	47	31	29
Task Two (n=62)	One-Step Multiplication	23	29	9	3
Task Three (n=60)	One-Step Subtraction	33	42	27	23
Task Four (n=65)	Two-Step Addition/Multiplication	29	26	26	14

Table 5.8 Summary of Percentages for Focused Algebra Tasks

Tasks	**Type of Equation**	**Percent of Students**			
		Identify Unknown/ Variable	**Identify Important Details**	**Describe Arithmetic Relationship**	**Write *and* Solve Equation**
Task One (n=62)	One-Step Addition	51.6%	75.8%	50%	46.8%
Task Two (n=62)	One-Step Multiplication	37.1%	46.8%	14.5%	4.8%
Task Three (n=60)	One-Step Subtraction	55%	70%	45%	38.3%
Task Four (n=65)	Two-Step Addition/Multiplication	44.6%	40%	40%	21.5%

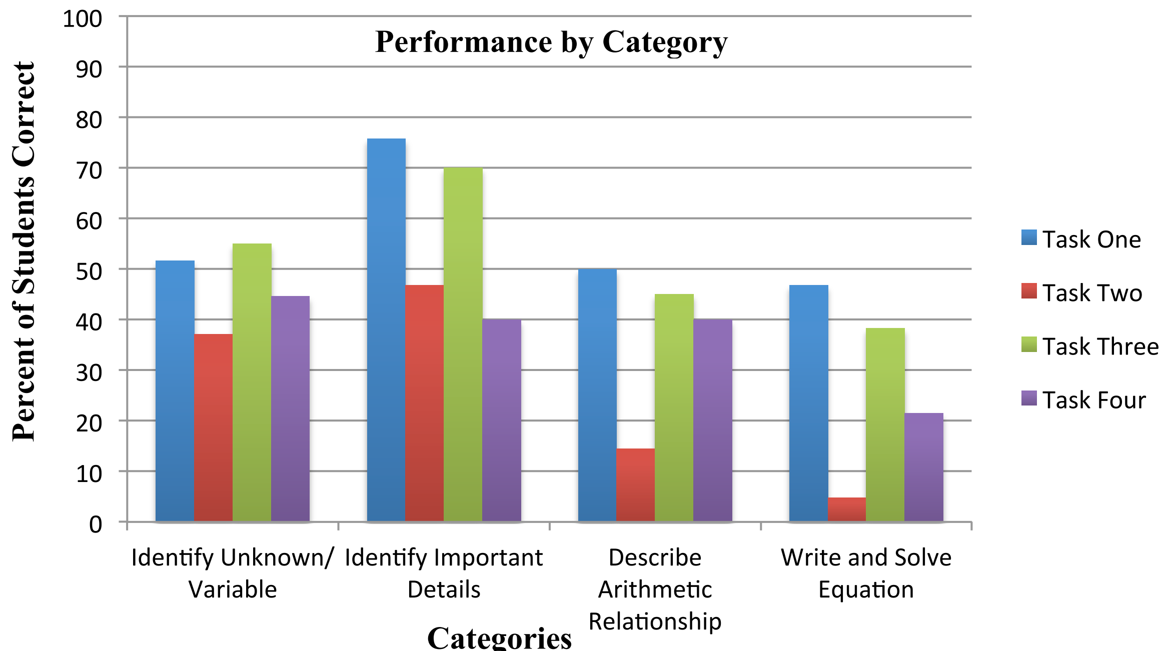

Figure 5.4 Graph of Performance by Category for Focused Algebra Tasks

In terms of completing the entire task correctly, Task One (Appendix B1) had nine (14.5%) students, Task Two (Appendix B2) had three (4.8%) students, Task Three (Appendix B3) had four (6.7%) students, and Task Four (Appendix B4) had two (3.1%) students. These low completion rates suggest further scrutiny of the results is required. Upon individual student analysis, the focused algebra tasks highlighted two new misconceptions held by students that led to incorrect answers. The first misconception held by students is that the variable and the unknown quantity are two separate entities. For example, in Task One, the word problem requires students to let h represent the unknown height of a structure. Student responses indicated that h represented a variable, but the unknown was the height of the structure. Instead of representing the unknown quantity *with* a variable, students listed the unknown quantity separately from the variable and did not see the connection between the variable and the quantity it represented. The second misconception held by students is that the important details are just numbers or values in the problem. For example, in Task Four, students stated that the important

details were $4, $17, and $45. In their written answers, students did not consider that the $4 was the cost per raffle ticket, the $17 was the cost to attend the banquet, and the $45 was the amount of money they were allowed to spend. Students did not acknowledge the meaning of the quantities or how they relate to the information presented in the problem; instead they focused on their numerical values given in the scenario. Table 5.9 displays the presence of these specific misconceptions within each task.

Table 5.9 Summary of Misconceptions within Focused Algebra Tasks

Tasks	**Number of Students**		**Percent of Students**	
	Separate Unknown and Variable	**Details as Numbers**	**Separate Unknown and Variable**	**Details as Numbers**
Task One (n=62)	28	7	45.2%	11.3%
Task Two (n=62)	26	13	41.9%	21%
Task Three (n=60)	25	14	41.7%	23.3%
Task Four (n=65)	31	5	47.7%	7.7%

Almost 50% of students completing each task did not connect the unknown quantity to the variable. Roughly 10-20% of students completing each tasks failed to acknowledge the details connected to the numerical values within the word problem. As presented with the one-step and two-step word problem algebra assignments, several students were able to determine the solution of the problem or the equation, but not both.

Task Four (Appendix B4), the only task that required a two-step equation, highlighted errors not present in the other tasks, which required one-step equations. One error observed is that 31 (or 47.7%) students could not identify all the necessary details presented in the problem. For this task, there were three main details students were to select; these students were only able to identify one or two of the necessary details. The second error was students identifying only one of the two necessary operations for solving the problem. 25 (or 38.5%) of students could not

identify the second operation required to solve for the unknown quantity. Both of these errors suggest students failed to comprehend the additional relationships present in the problem.

Equation and solution errors seem to mirror those found in the algebra assignments (Appendix A). Students who did not correctly solve or write the equation presented errors in the following: structure of the equation, confusion with operations, and mathematical errors. Students were still writing the equation as a string of arithmetic operations equal to the variable. This particular occurrence was the intended response for category 3 – establishing the arithmetical relationship. Students were able to recognize the operation necessary for the equation, but misinterpreted it as the operation needed to solve for the unknown. Students also made arithmetical errors during the solving process- incorrect borrowing, incorrectly operating with fractions, and incorrect division.

5.6 Extended-Response Algebra Task

In the extended-response algebra task (Appendix B5), students were to use their understandings of algebraic equations to complete a multi-layered (five part) scenario written in words. This task was analyzed based on students' abilities to: a) model a written scenario with an equation, b) substitute and evaluate an equation, c) solve an equation for an unknown quantity, d) solve equivalent expressions containing variables on both side of an equation, and e) analyze the relationship between two equations. The five parts of the extended-response task are labeled A through E. Of the forty-six students completing this assignment, 15 were sixth-graders and 31 were seventh graders. The scenario for the extended response appears in Appendix B, Task Five.

For extended-response task item A, Figure 5.5, there were 36 (or 78.3%) students to correctly model both equations correctly. Ten students were unsuccessful in modeling either equation. The two most common errors noted were: a) variables transposed and b) addition of the

coefficient and the constant term creating a one-step equation. Some errors were: C = 75d or C = 75+d for Enterprise and C = 63d or C = 63+d for Hertz. Other errors noticed for this item were: a) not representing both quantities (insurance and per day rate), b) representing incorrect operations, and c) elimination of the variables completely to create an arithmetic equation (i.e. Enterprise C = 20 +55 or Hertz C = 23 +40).

A) Write an equation for each company that can be used to determine the total cost, C, for the car rental after d number of days.

Solutions → Enterprise: C=20d+55 Hertz: C= 23d+40

Figure 5.5 Extended-Response Task Item A

For extended-response task item B, Figure 5.6, there were 30 (or 65.2%) students who were able to substitute for the variable, d=7, and correctly evaluate each expression to determine the total cost, C. There were 2 (or 4.3%) students who were able to evaluate at least one of the expressions to determine the total cost, C. Fourteen (or 30.4%) students were unsuccessful in calculating the total cost for either company. Within this group of fourteen, there were 8 (or 17.4%) students who correctly determined the equations for extended-response item A. This suggests that students relied on arithmetic procedures to determine their solutions, instead of their own algebraic models. Twelve of the fourteen students attempted to solve these problems using the incorrect equations discussed in the previous section, C= 63d for Hertz and C=75d for Enterprise. The remaining two students omitted the insurance fees and simply calculated the cost of the rental.

B) What is the total cost of a full one-week rental at each of the rental car companies?

Solutions → Enterprise: C=20(7)+55=$195 Hertz: C= 23(7)+40=$201

Figure 5.6 Extended-Response Task Item B

For extended-response item C, Figure 5.7, there were 15 (or 32.6%) students who were able to calculate and compare the number of days for each company and provide an accurate response for this item. 19 (or 41.3%) students were able to determine the correct answer of Enterprise, but did not have calculations to support their answer choice. Twelve (or 26.1%) students were not able to calculate or determine the accurate solution to this item. Two of the students were able to determine the number of days for one company, but not the other. Of those 15 students accurately calculating the number of days for each company, nine (or 60%) of those students determined their answers by solving the equations. The most common method used by students for attempting to solve this particular item was a listing method. The majority of students (63% or 29 students) chose to use systematic substitution to list out multiple answer possibilities based on the equations. This demonstrates the reliance students have on their arithmetic skills and their inability to recognize and utilize the language of algebra. Other methods of solving attempted by students were proportions, simple division, random guessing, and some students decided to use their answers in item B to justify a response in item C.

C) Martin has only allowed for \$350 in his budget for car rental expenses. Which company will give him the greatest number of days for \$350?

Solutions → *<u>Enterprise:</u> $20d+55=350$ → $20d=295$ → $d=14.75\approx 14$ days

<u>Hertz</u>: $23d+40=350$ → $23d=310$ → $d=13.48\approx 13$ days

Figure 5.7 Extended-Response Task Item C

For extended-response task item D, Figure 5.8, only 13 (or 28.3%) of students were able to correctly answer this question and give mathematical support for their solution. Ten (or 21.7%) of students were able to give a correct solution of 5, but did not provide sufficient mathematical support. The remaining 23 (or 50%) students were unable to determine the answer for this item. Similar to item C, the most common approach taken by students in their attempt to

solve this problem was a listing method using systematic substitution. It was also noted that, amongst students who did not correctly answer this item, students seemed to randomly guess values with no mathematical support.

D) How many days, d, will it take for the total cost, C, to the be the same at each company?

Solutions → Enterprise = Hertz   20d+55=23d+40
3d=15 where d=5 days

Figure 5.8 Extended Response Task Item D

For extended-response Task Item E, Figure 5.9, students answers were judged based on the justification provided for their choice company. 18 (or 39.1%) out of the 46 students assessed were able to provide sound justification as to which company would be more appropriate and for what length of time. These students showed a deeper understanding of the relationship that exists within the quantities (total cost and number of days) given in the problem. The remaining 28 (or 60.9%) students were not able to recognize the prevailing relationship between the two companies and their costs. Eight (or 17.4%) of these students provided justifications that were specific to work performed in items B through D. This was a sound attempt to support their answer choice, but they did not realize that the total costs calculated in these sections were specific to that particular situation. The remaining 20 (or 43.5%) of these students provided very weak support for their answers, such as "Hertz (or Enterprise) because it's cheaper" or "Enterprise because its $3 less per day." There particular answers reflect only a surface level understanding of the relationship between the equations they constructed and the values they calculated.

E) Based on your answers in items B through D, which rental car company would you recommend?

Solutions → a) Hertz for shorter trips (5 days or less) this company would be cheaper <u>OR</u>
b) Enterprise for longer trips (5 days or more) this company would be cheaper.

Figure 5.9 Extended-Response Task Item E

5.7 Student Survey

In the survey (Appendix C), the first section (Appendix C1) asked students to provide feedback on how they perceived their abilities to perform tasks that allowed them to: a) analyze and solve word problems, b) solve problems involving all forms of rational numbers, and c) solve equations in different forms. Sixty students participated in this section of the survey, which contained eleven questions. Question 1 through 5 focused on key features to solving word problems, questions 6 through 8 focused on operations with rational numbers, and questions 9 through 11 focused on solving equations. Students selected from the following choices for each question: strongly disagree, disagree, neutral, agree, or strongly agree. Table 5.10 displays the survey questions and the number of students selecting each choice for each question.

Questions 1 through 5 give insight into students' perceived abilities to decode and solve basic word problems. The survey shows that 73% of students agree that they can comprehend what they read, 82% of the students agree that they have sufficient mathematics vocabulary, 80% of the students agree they are competent in detecting mathematical keywords, and 85% agree they can select the appropriate operations necessary to solve mathematical word problems. This shows that these students have strong confidences in their abilities to solve to successfully solve word problems. However, 62% of the students surveyed did not agree that they used solutions to check the reasonableness of their answers. This suggests that the majority of students trust their arithmetic procedures and do not employ procedures to verify their solutions.

Table 5.10 Summary of Student Survey (Part 1)

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
1. I can comprehend information I read in word problems.	0	2	14	31	13
2. I understand the math vocabulary used in word problems.	0	0	12	20	29
3. I can identify "keywords" within word problems.	0	1	9	19	29
4. I can select the appropriate operation(s) (+, -, x, ÷) to solve word problems.	0	0	10	20	31
5. After solving problems, I use my answer(s) to check their reasonableness.	0	9	28	13	10
6. I can easily and quickly solve problems that include positive and negative numbers.	2	3	20	24	11
7. I can easily and quickly solve problems that include fractions and mixed numbers.	1	11	27	16	14
8. I can easily and quickly solve problems that include decimals.	2	3	15	19	21
9. I can easily and quickly solve one-step equations. (i.e. $x - 9 = -5$)	0	2	15	13	30
10. I can easily and quickly solve two-step equations. (i.e. $-2x + 4 = 28$)	2	4	16	20	17
11. I can easily and quickly solve multi-step equations. (i.e. $3(x - 7) + 7x = -12$)	3	6	24	17	10

Questions 6 through 8 give insight into students' perceived strengths in solving numerical problems that involve rational numbers (fractions, decimals, and integers). The survey shows that 58% of the students surveyed agree they can quickly and easily solve problems containing integers, 50% of the students agree they can quickly and easily solve problems containing fractions, and 67% of the students agree they can quickly and easily solve problems containing decimals. These percentages show students are more comfortable performing operations with decimals than fractions or integers. These lower percentages show students are less comfortable performing arithmetic operations with rational numbers than solving word problems.

Questions 9 through 11 give insight into students' perceived abilities to solve algebraic equations. The survey shows that 72% of the students surveyed agree they can quickly and easily solve one-step equations, 62% of the students agree they can quickly and easily solve two-step equations, and 45% of the students agree they can quickly and easily solve more intricate multi-step equations. These percentages show that students are more comfortable and confident in solving single-step equations than multi-step equations. This also highlights the difficulty students perceive in solving equations that contain multiple skills interlaced in the same equation, such as the distributive property, combining like terms, and rational numbers.

In the survey, the second section (Appendix C2) asked students to provide feedback on how they perceived their abilities to translate two real-world mathematical word problems into equations and to demonstrate those abilities. Sixty-one students participated in this section of the survey. Each word problem contained three survey questions and students selected from the following choices for each question: disagree, neutral, and agree. Table 5.11 displays the word problem for the one-step equation, the survey questions, and the number of students selecting each choice.

Table 5.11 Summary of Survey Results (Part 2a)

Scenario 1: Anthony has 19 pencils. He receives more pencils from Kathryn as gifts. Anthony now has 52 pencils. How many pencils did he receive from Kathryn?			
	Agree	**Neutral**	**Disagree**
1. I can determine the solution to this problem by using basic operations (+, -, x, ÷).	58	2	1
2. I can translate the following scenario into an equation.	45	10	4
3. I can determine the solution to this problem by solving the modeled equation.	43	10	6

The table shows that 58 (or 95.1%) students were confident in their ability to solve the word problem using basic arithmetic operations. Further analysis shows 48 (or 78.7%) students, including one neutral student and the one student who disagreed, were able to solve this problem using basic mathematics. Those students who were not successful in solving this problem made errors in subtraction, simply stated the operation to be performed, or wrote an equation. There were 45 (or 73.8%) students who were confident in their ability to translate the scenario into an equation. The results show 34 (or 55.7%) students, including two students who were neutral and two students who disagreed, were able to successfully model the word problem with a one-step algebraic equation. This illustrates that students were more successful solving the word problem then modeling with an equation. There were 43 (or 70.5%) who were confident in their ability to solve the equation to achieve the same solution calculated in survey question one. The results show that only 19 (or 31.1%) students, including two students who were neutral, were able to demonstrate solving the equation using inverse operations. Those students not illustrating this chose to simply write the answer they had previously calculated or illustrate the same solving technique used in survey question one.

Table 5.12 displays the word problem for the two-step equation, the survey questions, and the number of students selecting each choice. The table shows that 52 (or 85.2%) students were confident in their ability to solve the word problem using basic arithmetic operations.

Table 5.12 Summary of Survey Results (Part 2b)

Scenario 2: Kate has a savings account that contains $230. She saves $25 per month from her paycheck. How many months will it take for her account balance to reach $455?			
	Agree	**Neutral**	**Disagree**
1. I can determine the solution to this problem by using basic operations (+, -, x, ÷).	52	4	4
2. I can translate the following scenario into an equation.	36	15	7
3. I can determine the solution to this problem by solving the modeled equation.	34	12	11

Further analysis shows 32 (or 52.5%) students, including one neutral student and the one student who disagreed, were able to solve this problem using basic mathematics. Those students who were not successful in solving this problem did not do so because they wrote an equation for this question, simply wrote the operations they would use to solve, or only performed one of the two required operations. There were 36 (or 59%) students who were confident in their ability to translate the scenario into an equation. The results show 28 (or 45.9%) students, including three students who were neutral and two students who disagreed, were able to successfully model the word problem with a two-step algebraic equation. This illustrates that students were only slightly more successful solving the word problem then modeling with an equation. Those students who attempted to write the equation, but were unsuccessful, listed their equations as an arithmetic expression that resulted in the solution. There were 34 (or 55.7%) who were confident in their ability to solve the two-step equation, using inverse operations, to achieve the same solution calculated in survey question one. The results show 23 (or 37.7%) students, including

two students who were neutral and three students who disagreed, were able to demonstrate solving the equation using inverse operations. Those students who attempted to illustrate the solving technique, but were unsuccessful, repeated their arithmetic expressions. These percentages reinforce the point we have made about the difficulty students have with two-step equations as compared to one-step equations. This suggests that students have difficulty expressing relationships involving multiple operations.

CHAPTER 6: SUMMARY AND CONCLUSIONS

The overall objective of this study was to examine the disparity that seems to exist in students' abilities to solve equations, to solve word problems, and to write equations from word problems. Over the course of fourteen weeks, students enrolled in an advanced seventh grade mathematics course were given a series of algebra assignments, algebra tasks, and surveys that focused on students' abilities to write and solve algebraic equations. The data show that:

1. Students can translate verbal descriptions of simple operations on quantities into expressions involving variables and arithmetic operations.
2. Students can solve simple word problems using number sense or arithmetic skills.
3. Students can solve simple equations.
4. Students are far less competent in translating word problems into equation form than they are in solving simple word problems or equations.

When analyzing students' abilities to solve equations and solve word problems, the following stood out:

1. It was not clear if students were stronger at solving one-step equations or solving one-step word problems.
2. It was clear that students were stronger at solving two-step equations than solving two-step word problems.
3. It was clear that students were stronger at solving one-step equations than two-step equations.

Although students demonstrated strengths in some aspects of solving, there were consistent flaws noted in their abilities to solve. When students were required to solve direct equations, they showed difficulties in solving problems that included division and operations with fractions and

decimals. This demonstrates a weakness in operations that involve rational numbers and draws attention to the need for a strong arithmetic foundation of rational numbers prior to working with algebraic equations (Rotman, 1991). Students relied very heavily on elementary arithmetic procedures (i.e. guess and check, systematic substitution) to solve equations instead of using inverse operations. (Solving equations by applying inverse operations is a solution method that seeks to isolate the variable by performing arithmetic operations that reverse the effects of the operations present in the equation.) This supports the notion that students will rely on primitive processes before using sophisticated procedures, such as inverse operations (Herscovics, 1994). Students' are accurate in judging their own ability to solve equations or solve word problems. Student surveys showed that students were not very confident in their abilities to solve problems with fractions, decimals, or integers. The surveys also show that students felt more confident in their abilities to solve one-step equations than two-step equations. This shows that students in this study were aware and could acknowledge their own strengths and weakness in solving.

When analyzing students' abilities to write algebraic equations from word problems, the following stood out:

1. It was clear that most students involved in this study had some of the prerequisite knowledge necessary for modeling with algebraic equations.
2. It was clear that students were stronger at modeling equations for one-step word problems than two-step word problems.
3. It was clear that students were able to identify the important information provided by word problems, but were unable to arithmetically or algebraically relate the information to unknown quantities.

Although students demonstrated strengths in certain aspects of modeling with equations, there were limitations in their ability to translate the information from the word problems. Students showed a persistent confusion about how to use the operations retrieved from word problems. For one-step word problems, students were accurate in identifying the operation suggested by the problem, but used those operations to solve the problem rather than write the equation. For two-step word problems, the students exhibited the same confusion and also showed difficulty in determining the second operation indicated by the problems. This supports the idea that students are focusing on computational procedures in word problems instead of the conceptual frameworks (Thompson, 1994). Another common occurrence was students writing equations as strings of arithmetic operations. Instead of showing how quantities operate with the variable, students illustrated the equation as a set of numbers and operations that allow you determine the value of the unknown quantity. This is consistent with the literature in that students tend to work around the variable and have an inability to operate on or with the variable (Herscovics, 1994).

Overall, students showed an inability to translate information presented in words into algebraic equations. Students treated the unknown quantity and variable as two separate entities and they did not understand the connection between the variable and the quantity it represented. Students also had difficulty relating the important details of the problem to the unknown quantity, both arithmetically and algebraically. Student surveys show that students feel very confident in their abilities to work with word problems. Students feel comfortable being able to comprehend the information in the word problems, identify the keywords within word problems, understand the mathematical vocabulary, and select the appropriate operations for each word problem. For word problems in which equations were required, student surveys showed that

students were more confident in their ability to solve the word problem, but not as confident in their ability to write and solve the equation.

To improve students' abilities to solve equations, solve word problems, and write algebraic equations, it is my conclusion that teachers should provide instruction that directly and appropriately addresses the common student-held misconceptions highlighted in this study. When providing instruction on solving equations, teachers may improve student understanding and increase student success by discussing the meaning and purpose of equations, the meaning of the equals sign within equations, and including equations that contain: all forms of rational numbers, numbers of varying magnitudes, variables that occur more than once, and variables that occur on both sides of the equations. When providing instruction on translating words in algebraic equations, teachers may improve student understanding and increase student success by creating a conceptual understanding of the nature and purpose of variables, creating opportunities for students to use writing to describe how known information relates to unknown information, supporting students reliance on arithmetic by writing equations arithmetically prior to writing them algebraically, and providing a connection between arithmetic equations and algebraic equations. Teachers should also consult the science education literature on techniques for correcting student misconceptions and improving student learning. See for example http://www.apa.org/education/k12/misconceptions.aspx.

I would suggest that future investigations provide a narrowed focus on either the difficulty students have in solving equations or the difficulty students have with writing equations. This study broadly addressed solving equations and writing equations, which proved to be a daunting task that resulted in numerous findings. Narrowing the focus will allow the misconceptions and difficulties to be more precisely understood and addressed. Future studies

should seek to test interventions and strategies for improving students' conceptual and procedural understanding for solving equations and representing situations algebraically.

REFERENCES

"Algebra." *Wikipedia*. Wikimedia Foundation, 06 Nov. 2012. Web. 11 June 2013. <http://en.wikipedia.org/wiki/Algebra>

Baroody, Arthur J., and Ginsburg, Herbert P. "The Effects of Instruction on Children's Understanding of the "Equals" Sign." *The Elementary School Journal* 84.2 (1983): 198.

Booth, Julie L., and Kenneth R. Koedinger. *Key Misconceptions in Algebraic Problem Solving*. Carnegie Mellon University, 2007.

Capraro, Mary M., and Heather Joffrion. "Algebraic Equations: Can Middle School Students Meaningfully Translate From Words to Mathematical Symbols." *Reading Psychology* 27 (2006): 147-164.

Cocking, R. R. and Chipman, S. "Conceptual Issues Related to Mathematics Achievement of Language in Minority Children." *Linguistic and Cultural Influences on Learning Mathematics.* (1988): 17-46.

Devlin, Keith. "What Is Algebra?" Web log post. *ProfKeithDevlin*. Wordpress, 20 Nov. 2011. Web. 9 June 2013. <http://profkeithdevlin.org/2011/11/20/what-is-algebra/>.

Expressions and Equations (Grades 6-8). *Progressions for the Common Core State Standards in Mathematics.* WordPress, 2011.

"Grade 6 Mathematics Unit 8: Patterns and Algebra." *2012-2013 and 2013-2014 Transitional Comprehensive Curriculum*. Baton Rouge: LADOE, 2012. 1-21.

"Grade 7 Mathematics Unit 3: Expressions and Equations." *2012-2013 and 2013-2014 Transitional Comprehensive Curriculum*. Baton Rouge: LADOE, 2012. 1-21.

Herscovics, Nicolas. "Cognitive Obstacles Encountered in the Learning of Algebra." *Research Issues in the Learning and Teaching of Algebra. (1989): 60-86.*

Herscovics, Nicolas, and Linchevski, Liora. "A Cognitive Gap Between Arithmetic and Algebra." *Educational Studies in Mathematics* 27.1 (1994): 59-78.

Kieran, C. "Cognitive Processes Involved in Learning School Algebra." Ed. P. Nesher and J. Kilpatrick. *Mathematics and Cognition* (1990): 96-112. Cambridge University Press.

Knuth, E.J., Stephens, A. C., McNeil, N. M., and Alibali, M. W. "Does Understanding the Equal Sign Matter? Evidence from Solving Equations." *Journal for Research in Mathematics Education, 37.* (2006): 297-312.

Koedinger, K. and Nathan, M. "The Real Story Behind Story Problems: Effects of Representations on Quantitative Reasoning." *The Journal of the Learning Sciences,* 13. (2004): 129-164.

Linchevski, Liora and Williams, Julian. "Using Intuition from Everyday Life in 'Filling' the Gap in Children's Extension of Their Number Concept to Include the Negative Numbers." *Educational Studies in Mathematics*, Vol. 39, No. 1/3, Teaching and Learning Mathematics in Context (1999): 131-147.

Lott, J. W., ed. 2000. Algebra: A Gate? A Barrier? A Mystery! *Mathematics Education Dialogues* 3 (2): 1-12.

MacGregor, Mollie E. *Making Sense of Algebra: Cognitive Processes Influencing Comprehension*. Geelong, Vic.: Deakin University, 1991. Print. Language and Mathematics Learning.

MacGregor, Mollie, and Kaye. Stacey. "Cognitive Models Underlying Students' Formulation of Simple Linear Equations." *Journal for Research in Mathematics Education* 24.3 (1993): 217-32.

"Mathematics » Grade 6 » Expressions & Equations." *Common Core State Standards Initiative*. Web. 29 May 2013. <http://www.corestandards.org/Math/Content/6/EE>.

"Mathematics » Grade 7 » Expressions & Equations." *Common Core State Standards Initiative*. Web. 30 May 2013. <http://www.corestandards.org/Math/Content/7/EE>.

"Mathematics » Introduction » How to Read the Grade Level Standards." *Common Core State Standards Initiative*. Web. 30 May 2013. <http://www.corestandards.org/Math/Content/introduction>.

Mestre, J. P. "The Role of Language Comprehension in Mathematics and Problem-Solving." *Linguistic and Cultural Influences on Learning Mathematics* (1988): 201-220.

"Mission Statement." *Common Core State Standards Initiative*. Web. 29 May 2013. <http://www.corestandards.org/>.

Moses, Robert P., Mieko Kamii, Susan Swap, and Jeffrey Howard. "The Algebra Project: Organizing in the Spirit of Ella." *Harvard Educational Review* 59.4 (1989): 423-444.

Operational and Algebraic Thinking (Grades K-5). *Progressions for the Common Core State Standards in Mathematics.* WordPress, 2011.

Rotman, J. W. Arithmetic: Prerequisite to Algebra? *Annual Convention of the American Mathematical Association of Two-Year Colleges,* 1991.

“Solving Equations." *Middle School Math: Course 3*. Ed. Glenn Worthman, Tessa Henry, and Kristi Smith. Austin: Holt, Rinehart, and Winston, 2005. 13-22.

Thompson, A. G., Phillip, R., Thompson, P.W., and Boyd, B. “Calculational and Conceptual Orientations in Teaching Mathematics.” *Professional Development of Teachers of Mathematics* (1994): 79-92.

"Transitional Curriculum." *Academic Curriculum*. Louisiana Department of Education. 3 June 2013. <http://www.louisianabelieves.com/resources/library/academic-curriculum>.

APPENDIX A: ALGEBRA ASSIGNMENTS

<u>WRITING EXPRESSIONS</u> (ASSIGNMENT 1)

1) Write an expression for the quotient of ***m*** and 223
2) Write an expression for 17 decreased by ***c***
3) Write an expression for 305 reduced by ***t***
4) Write an expression for ***u*** more than 22
5) Write an expression for the product of ***a*** and 179
6) Write an expression for 6 increased by ***y***
7) Write an expression for 324 plus the product of ***h*** and 174
8) Write an expression for 136 subtracted from the quantity 270 times ***s***
9) Write an expression for 45 less than triple ***j***
10) Write an expression for twice the quantity ***f*** less 83
11) Henry picked ***p*** peaches. Ruth picked 6 times as many peaches as Henry. Write an expression that shows how many peaches Ruth picked.
12) There are 8 swimmers in the swimming club. The club bought ***c*** swimming caps to split among its members. Write an expression that shows how many swimming caps each swimmer will get.
13) Andrea had 52 coins. Then she found ***p*** more coins in a drawer. Write an expression that shows how many coins Andrea has now.
14) Kasey had ***c*** caramels. Then, Kasey's sister took 85 of the caramels. Write an expression that shows the number of caramels Kasey's has left.
15) Jenny earns $30 a day working part time at a supermarket. Write an algebraic expression to represent the amount of money she will earn in ***d*** days.
16) A small company has $1000 to distribute among its employees as a bonus. Write an expression that shows how much money each employee, ***e***, will receive.
17) Six buses were filled with ***s*** number of students and there were 7 students who had to ride with their parents. Write an expression to represent the total number of students traveling on the field trip.
18) At a school supply store, Martin purchased a magazine for $5 and 4 erases for ***x*** dollars. Write an expression to represent the total amount Martin spent on school supplies.
19) Nadia owned ***h*** comic books. She sold one-third of these and bought 11 new comic books. Write an expression to represent the total number of comic books has remaining.
20) Allen has ***k*** dollars in his savings account. In February, made a deposit that tripled the balance and in March, he withdrew $750 from his savings account. Write an expression to model the amount of money Allen has left in his savings account.

APPENDIX A: ALGEBRA ASSIGNMENTS

ONE-STEP EQUATIONS (ASSIGNMENT 2)

1) A customer pays 50 dollars for a coffee maker after a discount of 20 dollars. Write and solve an equation to determine the original price, p, of the coffee maker.

2) For a recycling project, 4 students each collected the same number of plastic bottles. They collected 72 bottles in all. Write and solve an equation to determine the number of plastic bottles, b, collected by each student.

3) The stuffed animals at the toy store are split among 4 shelves. There are 12 stuffed animals on each shelf. Write and solve an equation to determine the number of stuffed animals, a, there are in all.

4) Andrea bought a pack of gum. She gave 46 pieces of gum to her friend Leila and had 47 pieces left. Write and solve an equation to determine the number of pieces of gum, g, Andrea had originally.

5) Malik bought some buttons at the craft store. He used 4 of them for an art project and had 56 left. Write and solve an equation to determine the total number of buttons, b, Malik bought at the craft store.

6) Manuel ordered 6 pizzas for a party. There were 54 slices of pizza in all. Write and solve an equation to determine the number of slices, p, in each pizza.

7) Lisa found 10 daisies near a lake. Ellen found some daisies as well. When they put all of their daisies together, they had 21 daisies in all. Write and solve an equation to determine how many daisies, d, Ellen found.

8) Manny picked some oranges and divided them evenly among 7 of his friends. Each friend received 5 apples. Write and solve an equation to determine how many apples, a, Manny picked.

9) A toy company is going to use 14 boxes to ship some toys across the country. The company needs to ship 98 toys. Write and solve an equation to determine how many toys, t, the company should put in each box.

10) Benny bought some model cars at a garage sale. He gave 26 cars to his cousin and had 24 cars left. Write and solve an equation to determine how many cars, c, Benny bought at the garage sale.

APPENDIX A: ALGEBRA ASSIGNMENTS

TWO-STEP EQUATIONS (ASSIGNMENT 3)

1) Jennifer had \$24 to spend on seven pencils. After buying the pencils, she had \$10. Write and solve an equation to determine the cost, p, of each pencil that Jennifer bought.

2) Takira bought a tube of lip-gloss for \$4 and three bracelets. She spent a total of \$13. Write and solve and equation that can be used to determine the cost, b, of each bracelet.

3) Tyron is paid a weekly salary of \$520. He is paid an additional \$21 for every hour of overtime he works. This week his total pay (including regular salary and overtime pay) was \$604. Write and solve an equation that can be used to determine the number of hours, h, Joe worked in overtime this week.

4) The cost of a family membership at the YMCA is \$76 per month plus a one-time joining fee of \$50. The Scott family spent a total of \$430. Write and solve an equation that can be used to determine the number of months, m, the family will workout at the YMCA.

5) UPS charges a \$2.50 service fee to ship packages. They also charge \$0.75 for each pound the package weighs. Bryce paid \$7.75 to ship a package to his grandmother. Write and solve an equation that can be used to determine the weight, w, of Bryce's package.

6) Verizon charges \$49 per month and \$0.02 per kilobyte of data (Internet) used. Kamal's phone bill for the month of January was \$59. Write and solve an equation that can be used to determine the number of kilobytes, k, he used during the month of January.

APPENDIX A: ALGEBRA ASSIGNMENTS

SOLVING EQUATIONS (ASSIGNMENT 4)

$p - 20 = 50$	**$4b = 72$**
$\frac{a}{12} = 4$	**$g - 46 = 47$**
$b - 4 = 56$	**$6p = 54$**
$d + 10 = 21$	**$\frac{a}{7} = 5$**
$14t = 98$	**$c - 26 = 24$**
$7x + 10 = 24$	**$3m + 4 = 13$**
$21w + 520 = 604$	**$76f + 50 = 430$**
$0.75g + 2.50 = 7.75$	**$0.02a + 49 = 59$**

APPENDIX B: ALGEBRA TASKS

Writing Equations (Task 1)

Directions: Using your notes, list the recommended steps for writing equations. Apply these steps to the word problems given.

Scenario 1: The statue of liberty is 151 feet tall, but **with the pedestal and foundation** it is 305 feet tall. Write and solve an equation that can be used to find the height of the pedestal and foundation, *h*.

Steps for Writing Equations	Apply to Word Problem
1.	
2.	
3.	
4.	

APPENDIX B: ALGEBRA TASKS

<u>Writing Equations</u> (Task 2)

<u>Directions:</u> Using your notes, list the recommended steps for writing equations. Apply these steps to the word problems given.

<u>Scenario 2</u>: Kate swam 18 laps at practice. This is three-fourths of the number of laps that Paco swam at practice. Write and solve an equation that can be used to find the number of laps, p, that Paco swam.

Steps for Writing Equations	Apply to Word Problem
1.	
2.	
3.	
4.	

APPENDIX B: ALGEBRA TASKS

<u>Writing Equations</u> (Task 3)

<u>Directions:</u> Using your notes, list the recommended steps for writing equations. Apply these steps to the word problems given.

<u>*Scenario 3*</u>: After a tailor cut an 8.35 inch-long piece of material from a roll of fabric, the remaining portion of fabric was 42.15 inches long. Write and solve an equation that can be used to find the length of the fabric, f, before the tailor cut it.

Steps for Writing Equations	Apply to Word Problem
1.	
2.	
3.	
4.	

APPENDIX B: ALGEBRA TASKS

Writing Equations (Task 4)

Directions: Using the listed steps for writing equations, create and solve an equation for the following problem

Scenario 4: It costs $17 to attend a football banquet with Coach Les Miles. Anyone who ***attends the banquet*** can purchase raffle tickets for an autographed football for $4 each. How many raffle tickets can you purchase if you have $45 to spend?

Steps for Writing Equations	**Apply to Word Problem**
1. Identify the unknown and label it with a variable.	
2. Identify the important details.	
3. Establish a mathematical (arithmetic) relationship between the variable and the details	
4. Write and solve an equation.	

APPENDIX B: ALGEBRA TASKS

Writing and Applying Equations (Task 5)

Car Rentals

Martin is planning a trip to visit the Grand Canyon in Arizona. He needs to rent a car once he arrives there. He has received information from two rental car companies, Enterprise and Hertz. Using the following details, answer questions A-D. SHOW WORK for each question.

Enterprise Rental: \$55 for insurance plus \$20 for each day.
Hertz Rental: \$40 for insurance plus \$23 for each day.

A) Write an equation for each company that can be used to determine the total cost, *C*, for the car rental after *d* number of days.

B) What is the total cost of a full one-week rental at each of the rental car companies?

C) Martin has only allowed for \$350 in his budget for car rentals. Which company will give him the greatest number of days for \$350?

D) How many days, d, will it take for the total cost, C, to be the same at each company?

E) Based on your answers in questions B through D, which rental car company would you recommend? **Please provide an explanation to justify your choice.**

APPENDIX C: STUDENT SURVEY

<u>Survey Instructions:</u> This section of the survey will provide feedback to your instructor about your <u>*perceived ability*</u> to: a) analyze and solve word problems, b) solve problems involving all forms of rational numbers, and c) solve equations with various levels of difficulty. Please answer each question honestly and select only one answer per question. <u>Mark your response with an **X.**</u>

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
1. I can comprehend information I read in word problems.					
2. I understand the math vocabulary used in word problems.					
3. I can identify "keywords" within word problems.					
4. I can select the <u>appropriate</u> operation(s) (+, -, x, ÷) to solve word problems.					
5. After solving problems, I use my answer(s) to check their reasonableness.					
6. I can <u>easily</u> and <u>quickly</u> solve problems that include positive and negative numbers.					
7. I can <u>easily</u> and <u>quickly</u> solve problems that include fractions and mixed numbers.					
8. I can <u>easily</u> and <u>quickly</u> solve problems that include decimals.					
9. I can <u>easily</u> and <u>quickly</u> solve one-step equations. (i.e. $x - 9 = -5$)					
10. I can <u>easily</u> and <u>quickly</u> solve two-step equations. (i.e. $-2x + 4 = 28$)					
11. I can <u>easily</u> and <u>quickly</u> solve multi-step equations. (i.e. $3(x - 7) + 7x = -12$)					

Survey Instructions: This section of this survey will provide feedback to your instructor about your *perceived ability* and *practical ability* in translating word problems into algebraic equations to determine a solution. Please answer each question honestly and to the best of your ability. Select only one answer for (agree, neutral, or disagree). Mark your response with an **X.** For each question, show your ability to do each question in the column labeled '*Show Your Work*'.

Scenario 1: Anthony has 19 pencils. He receives more pencils from Kathryn as gifts. Anthony now has 52 pencils. How many pencils did he receive from Kathryn?

	Agree	Neutral	Disagree	Show Your Work
1. I can determine the solution to this problem by using basic operations (+, -, x, ÷).				
2. I can translate the following scenario into an equation.				
3. I can determine the solution to this problem by solving the modeled equation.				

Scenario 2: Kate has a savings account that contains $230. She saves $25 per month from her paycheck. How many months will it take for her account balance to reach $455?

	Agree	Neutral	Disagree	Show Your Work
1. I can determine the solution to this problem by using basic operations (+, -, x, ÷).				
2. I can translate the following scenario into an equation.				
3. I can determine the solution to this problem by solving the modeled equation.				

APPENDIX D: IRB APPROVAL AND CONSENT FORMS

Application for Exemption from Institutional Oversight

LSU

Institutional Review Board
Dr. Robert Mathews, Chair
131 David Boyd Hall
Baton Rouge, LA 70803
P: 225.578.8692
F: 225.578.5983
irb@lsu.edu
lsu.edu/irb

Unless qualified as meeting the specific criteria for exemption from Institutional Review Board (IRB) oversight, ALL LSU research/projects using living humans as subjects, or samples, or data obtained from humans, directly or indirectly, with or without their consent, must be approved or exempted in advance by the LSU IRB. This Form helps the PI determine if a project may be exempted, and is used to request an exemption.

-- Applicant, Please fill out the application in its entirety and include the completed application as well as parts A-F, listed below, when submitting to the IRB. Once the application is completed, please submit the completed application to the IRB Office or to a member of the Human Subjects Screening Committee. Members of this committee can be found at http://research.lsu.edu/CompliancePoliciesProcedures/InstitutionalReviewBoard%28IRB%29/item24737.html

-- A Complete Application Includes All of the Following:

(A) A copy of this completed form and a copy of parts B thru F.
(B) A brief project description (adequate to evaluate risks to subjects and to explain your responses to Parts 1&2)
(C) Copies of all instruments to be used.
*If this proposal is part of a grant proposal, include a copy of the proposal and all recruitment material.
(D) The consent form that you will use in the study (see part 3 for more information.)
(E) Certificate of Completion of Human Subjects Protection Training for all personnel involved in the project, including students who are involved with testing or handling data, unless already on file with the IRB. Training link: (http://phrp.nihtraining.com/users/login.php)
(F) IRB Security of Data Agreement: (http://research.lsu.edu/files/item26774.pdf)

1) Principal Investigator: Danielle Ricks **Rank:** Graduate Student

Dept: Natural Sciences **Ph:** 225-620-5141 **E-mail:** ddricks2@gmail.com

2) Co Investigator(s): please include department, rank, phone and e-mail for each
*If student, please identify and name supervising professor in this space

IRB# E8314 LSU Proposal # ______

- ✓ Complete Application
- ✓ Human Subjects Training
- ✓ IRB Security of Data Agreement

3) Project Title: Lost In Translation: Algebraic Modeling in the Middle School Classroom

Study Exempted By:
Dr. Robert C. Mathews, Chairman
Institutional Review Board
Louisiana State University
203 B-1 David Boyd Hall
225-578-8692 | www.lsu.edu/irb
Exemption Expires: 6/6/2016

4) Proposal? (yes or no) No **If Yes, LSU Proposal Number** ______

Also, if YES, either ◯ **This application completely matches the scope of work in the grant**
OR ◯ **More IRB Applications will be filed later**

5) Subject pool (e.g. Psychology students) Children < 18

*Circle any **"vulnerable populations" to be used**: (children <18; the mentally impaired, pregnant women, the ages, other). Projects with incarcerated persons cannot be exempted.

6) PI Signature [signature] **Date** 5/27/13 (no per signatures)

** **I certify my responses are accurate and complete.** If the project scope or design is later changes, I will resubmit for review. I will obtain written approval from the Authorized Representative of all non-LSU institutions in which the study is conducted. I also understand that it is my responsibility to maintain copies of all consent forms at LSU for three years after completion of the study. If I leave LSU before that time the consent forms should be preserved in the Departmental Office.

Screening Committee Action: Exempted ✓ Not Exempted ____ Category/Paragraph 1
Signed Consent Waived?: Yes No
Reviewer Mathews **Signature** [signature] **Date** 6/7/13

Parental Permission Form

Project Title: Lost In Translation: Algebraic Modeling in Middle School

Performance Site: Westdale Middle School

Investigators: The following investigator is available for questions,

M-F (7:05am – 2:45 pm)

Danielle Ricks

7th Grade Gifted Team (Mathematics)

dricks2@ebrschools.org

Purpose of the study: The purpose of this research project is to investigate and highlight the difficulties students have when modeling real-world scenarios as algebraic equations. This study also seeks to develop effective strategies and methods for educators to apply when teaching algebraic modeling to students within real-world scenarios.

Inclusion Criteria: Students enrolled in 7th Grade GS Mathematics

Description of the study: Over a period of the school year, the investigator will scaffold problem-solving techniques through collaborative group work, class/peer discussions, writing assignments, and teacher developed tasks and surveys. This information will be used to adjust lessons to improve student understanding.

Benefits: Research shows that if students develop the ability to discuss, question, and problem solve, they will develop a deeper understanding of the mathematics

Risks: There are no known risks

Right to Refuse: Participation is voluntary. A child will become part of the study only if both child and parent agree to the child's participation. At any time, either the subject may withdraw from the study or the subject's parent may withdraw the subject from the study without penalty or loss of any benefit to which they might otherwise be entitled. Students participating will not be asked to do anything outside of normal class procedures; their work will simply not be included in the data recorded.

Privacy: Investigator may review the school records of participants in this study. Results of the study may be published, but no names or identifying information will be included for publication. Subject identity will remain confidential unless law requires disclosure.

Financial Information: There is no cost for participation in the study, nor is there any compensation to the subjects for participation.

Signatures:

The study has been discussed with me and all my questions have been answered. I may direct additional questions regarding study specifics to the investigator. If I have questions about subjects' rights or other concerns, I can contact Robert C. Mathews, Chairman, Institutional Review Board, (225) 578-8692, irb@lsu.edu, www.lsu.edu/irb. I will allow my child to participate in the study described above and acknowledge the investigator's obligation to provide me with a signed copy of this consent form.

Parent's Signature:________________________

Date:_______________

The parent/guardian has indicated to me that he/she is unable to read. I certify that I have read this consent from to the parent/guardian and explained that by completing the signature line above he/she has given permission for the child to participate in the study.

Signature of Reader:________________________

Date:_______________

Study Exempted By:
Dr. Robert C. Mathews, Chairman
Institutional Review Board
Louisiana State University
203 B-1 David Boyd Hall
225-578-8692 | www.lsu.edu/irb
Exemption Expires: 6/6/2016

Investigator: D. Ricks
Project: Lost In Translation

Child Assent Form

I, ________________________________, agree to be in a study that is focused on finding ways to help middle school math students learn how to use real-world scenarios to write algebraic equations (also referred to as algebraic modeling). As a participant in the study, I agree to have the project investigator (Ms. Ricks) refer to the following areas in a written report at the conclusion of the study:

a) My performance on formal assessments (pre- and post-tests)
b) My thoughts reflected in writing assignments
c) My performance and comments on mini-projects, teacher created tasks, and surveys
d) My comments shared in whole class and small-group/peer discussions

I understand my workload and participation in class will not increase or decrease as a participant in the study and my math class will operate in a manner I am accustomed. I can decline participation in the study at any time without being penalized.

Child's Signature: ____________________________ Age: ______

Date: _________________

Witness* ________________________________ Date: _________________

* (Witness must be present for the assent process, not just the signature by the minor.)

Study Exempted By:
Dr. Robert C. Mathews, Chairman
Institutional Review Board
Louisiana State University
203 B-1 David Boyd Hall
225-578-8692 | www.lsu.edu/irb
Exemption Expires: 6/6/2016

VITA

Danielle Ricks is a native of Baton Rouge, Louisiana and received her bachelor's degree at Louisiana State University in 2005. Thereafter, she spent several years working in fitness education and program design, which ultimately led to a venture into teaching. Currently, she teaches middle school mathematics for the East Baton Rouge Parish School System. She will receive a Master of Natural Sciences degree in August 2013 from Louisiana State University. She plans to use her knowledge to continue furthering and improving mathematics education for students in her school district.