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Interpretation of power phenomena in three phase systems with neutral using instantaneous reactive power p-q theory

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INTERPRETATION OF POWER PHENOMENA IN THREE PHASE SYSTEMS WITH NEUTRAL
USING INSTANTANEOUS REACTIVE POWER P-Q THEORY

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Master of Science in Electrical Engineering

In

Department of Electrical and Computer Engineering

By
Rahul Anil Kantharia
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ABSTRACT

The Instantaneous Reactive power (IRP) p-q theory considered as a major power theory as well as a primary fundamental for switching compensator control in three-phase systems, is analyzed in order to verify its ability to describe and identify the power properties or power phenomena in a 3pN system with asymmetrical and non-sinusoidal voltages along with unbalanced loads. This analysis is an outcome of the misinterpretations of power phenomena in three-wire circuits shown by IRP p-q theory under non-sinusoidal conditions.

Different load and supply conditions such as balanced, unbalanced and Harmonic generating loads along with asymmetrical and non-sinusoidal voltages are assumed for the analysis. The analysis takes in to account illustrations demonstrating various combinations of unfavorable supply and load features in order to realize an actual 3pN system mathematically. Depending on the supply and load conditions used, first the supply voltages and currents are calculated in Clarke coordinates. Then instantaneous active, reactive and zero sequence power defined by the IRP p-q theory in terms of these voltages and currents are calculated. These calculated powers are then analyzed for errors from fundamental electrical engineering perspective.

CHAPTER 1: INTRODUCTION

1.1 Three phase systems with neutral (3pN)

Three phase power systems with neutral (3pN) systems serve usually as the last step of energy distribution. Such systems provide supply for both three phase as well as single-phase loads such as three phase motors and residential or commercial lighting and other single-phase appliances, respectively. These power systems are of a medium and low power range. Single phase systems which supply residential or commercial single phase loads such as lighting are supplied from a distribution transformer connected in Δ/y configuration, with a grounded star point A, as shown in Fig 1.1

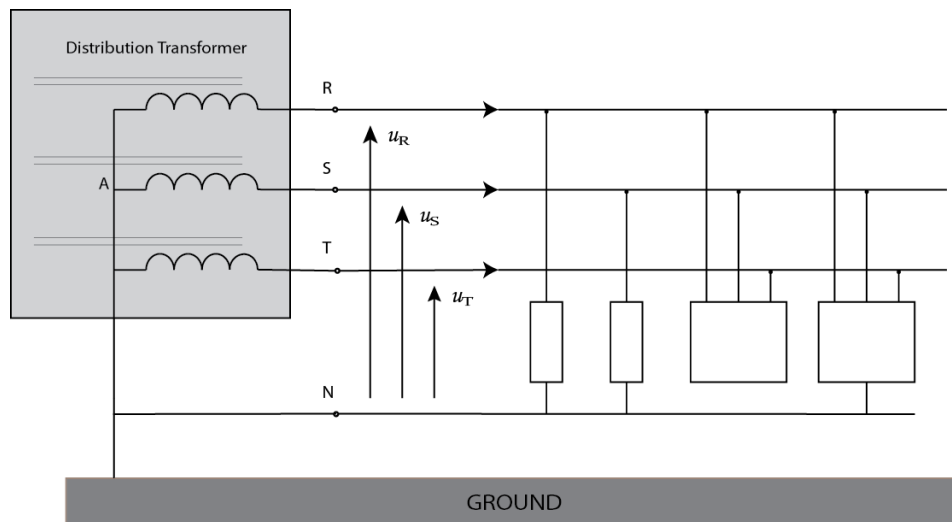


Figure 2.1: Typical 3pN system

Single-phase loads in 3pN systems are supplied by line-to-neutral voltages, whereas the three phase loads are supplied by the line-to-line voltages. These three phase loads may or may not be connected to the neutral depending on the load. The knowledge of configuration of the 3pN AC systems is essential for understanding the power phenomena in three phase electrical circuits with different kind of loads.

1.2 Unfavorable features in 3pN systems

The performance of a power system is influenced by the type of load it is supplying. Nowadays more and more single-phase non-linear loads (fluorescent bulbs, computers and TV), which often are not equally

distributed among phases, are used. Because of this unequal distribution, the 3pN system could be unbalanced. In other words, these single-phase non-linear loads draw different amount of current from individual supply lines causing unbalance in the 3pN system, which results in a large current in the neutral wire. This current is of zero sequence, which flows through the neutral wire. Power electronics in computer based devices such as rectifiers and household equipments like microwave and fluorescent bulbs are non-linear harmonic generating loads (HGLs). These loads tend to generate current harmonics and as a result distort the supply voltage. As a result, such loadings leads to low power factor, decreased efficiency and overheating of power transformers. Normally, only the presence of the reactive current has detrimental effect on transmission equipments such as lines and transformers. The 3pN systems are major source of asymmetry and harmonics and these features have an additional effect on transmission and line performance. Because voltage and current asymmetry and harmonics degrade power system's performance, these harmful features should be confined.

The confinement of these features can be performed in distribution systems at different levels. Regulating the distortion level at the customers end, imposed by the energy provider can be an option for the confinement of these features [10]. Also, development of the equipment with a low level of supply current distortion and reconfiguration of system structure of distribution systems such as transformers and lines would reduce the level of distortion and asymmetry [10]. However, harmonics reduction equipment such as resonant harmonic filters (RHF) and switching compensators, commonly known as “active power filters or power conditioners” are widely used for confining these unfavorable features. RHF are used to reduce current harmonics generated by the HGLs in the three-phase system. They operate on the principle of filtering, by creating a low impedance path for harmonic frequencies and thereby modifying the frequency properties of the system. RHF have been proven ineffective due to their resonance with supply impedance [10], however. Switching compensators, on the other hand, works on the principle of compensation, which usually is a process of reduction of harmful components of the supply current or distribution voltage by injection of the same component with an opposite sign in to the system. This

compensators not only reduce the load generated current harmonics, but also if connected in series, the distribution side generated harmonics and unbalance in the system. This section is important pertaining to the analysis and interpretations provided later in the study.

1.3 Background of two major power theories in three-phase systems

Many power theories developed, so far, for three phase systems with non-sinusoidal conditions have been in the midst of some controversy, owing to the misrepresentation of power phenomena or improper compensation scope. The subject of this thesis is related to some doubts surrounding a well-known power theory, namely the “Instantaneous Reactive power (IRP) p - q theory” developed by Akagi, Kanazawa and Nabae [6]. It is considered as a major power theory for three phase systems right now and is commonly used as a control algorithm for “switching compensators” popularly known as “active power filters” [8]. The power properties of three phase electrical loads in this theory are specified in terms of instantaneous active power p and reactive powers q . These powers are defined in terms of load voltages and currents in Clarke coordinates. This theory can be applied in three-phase systems, with or without neutral wire. The instantaneous p - q theory will be discussed in detail, later in the following chapters. Apparently, this theory has major limitations when considered as a power theory as well as a compensation control algorithm [6,8,9].

This power theory was recently challenged by Leszek S. Czarnecki for misinterpretation of power properties in three phase three-wire power systems using his own theory “Currents’ Physical Components” power theory or the “CPC power theory” in [7]. This analysis shows that the outcome of the IRP p - q theory disagrees with some common interpretations of properties in the three-phase circuits. According to the p - q theory, the instantaneous reactive current is present in the supply current in spite of an unbalanced though purely resistive load. It also shows that the values of the instantaneous powers p and q does not allow us to relate to the power phenomena in a three phase unbalanced system with sinusoidal voltage. Moreover the instantaneous identification of the power properties of three-phase power system, as suggested by the IRP p - q theory is not possible.

1.4 Thesis subject

The subject of this thesis is a study on how the IRP p-q theory identifies and describes power properties and power phenomena in 3pN systems with non-sinusoidal voltages and currents. This study involves identifying or analyzing the power properties of 3pN unbalanced systems or asymmetrically supplied systems with distorted voltages and currents, described in terms of the instantaneous reactive power p-q theory. Although, such systems have been described in terms of other theories, the scope of this thesis is limited to p-q theory. Only some elements of CPC theory for three-wire systems also are used in this thesis.

1.5 Thesis objective and approach

Since it is known that the IRP p-q theory misinterprets power phenomena in three-wire system, the objective of this thesis is a verification of whether these misinterpretations apply to 3pN unbalanced systems with non-sinusoidal voltages and currents. The approach to this research will be based on theoretical analysis of IRP p-q theory applied to 3pN unbalanced systems with non-sinusoidal voltages and currents by evaluating the results from the electrical engineering perspective.

CHAPTER 2: INSTANTANEOUS REACTIVE POWER p-q THEORY

2.1 The concept of p-q Theory

The Instantaneous Reactive power theory (IRP) p-q theory developed by Akagi, Kanazawa and Nabae in 1983, uses time domain in order to define a set of instantaneous powers [1]. These Instantaneous powers are defined in terms of instantaneous voltages and currents, which are first, transformed from phase R, S and T to $\alpha\beta 0$ coordinates by using the Clarke Transformation. This transformation produces a stationary reference frame, where coordinates α and β are orthogonal and the co-ordinate 0 corresponds to the zero sequence component [4]. However, this zero sequence coordinate differs from the zero sequence components in the symmetrical component transformation.

2.2 Instantaneous voltage, current and power definitions in Clarke coordinates

The Clarke transformation for three phase voltages and line currents, therefore are given by [1],

$$\begin{bmatrix} u_\alpha \\ u_\beta \\ u_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & \sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix}, \quad (2.1)$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix} \quad (2.2)$$

The inverse Clarke transform of these three phase voltages is,

$$\begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & 1/\sqrt{2} \\ -1/2 & \sqrt{3}/2 & 1/\sqrt{2} \\ -1/2 & -\sqrt{3}/2 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} u_\alpha \\ u_\beta \\ u_0 \end{bmatrix}. \quad (2.3)$$

Similarly inverse Clarke transform of the line currents from $\alpha\beta 0$ to RST can be calculated. According to the p-q theory, [1] the set of instantaneous powers in a three-phase system consists of the *instantaneous zero sequence power* p_0 defined as,

$$p_0 = u_0 i_0, \quad (2.4)$$

Instantaneous real power p and the *instantaneous imaginary power* q defined as,

$$p = u_\alpha i_\alpha + u_\beta i_\beta, \quad (2.5)$$

$$q = -u_\alpha i_\beta + u_\beta i_\alpha, \quad (2.6)$$

Please note that the definition for the instantaneous reactive power in their recent publication on IRP p-q theory [1] was changed as shown in eqn. (2.6) from the original in [3]. These powers can also be written in a matrix form as,

$$\begin{bmatrix} p_0 \\ p \\ q \end{bmatrix} = \begin{bmatrix} u_0 & 0 & 0 \\ 0 & u_\alpha & u_\beta \\ 0 & u_\beta & -u_\alpha \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} \quad (2.7)$$

This theory, unlike other theories does not only consider each phase of the three phase system separately but also defines them in terms of other phases. Moreover, this gives us flexibility of using the theory for three-wire systems.

2.3 Instantaneous p-q theory for three-wire systems

Since, three-wire power systems do not contain zero sequence current components, the zero sequence components i_0 and u_0 in IRP p-q theory can be considered as zero. As a result, the three-wire system can be represented in terms of reduced vector Clarke coordinates. The reduced Clarke coordinates are nothing but the representation of IRP p-q theory for three-wire systems by neglecting the zero sequence component. The reduced vectors for three phase Clarke voltages and currents are determined as,

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix}, \quad (2.8)$$

If the line voltages are measured with respect to an artificial zero, then, $u_R + u_S + u_T \equiv 0$. Since one of the three voltages is dependent on others, the supply voltages in α and β coordinates can be written in a simplified form [6],

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & 0 \\ 1/\sqrt{2} & \sqrt{2} \end{bmatrix} \begin{bmatrix} u_R \\ u_S \end{bmatrix} = \mathbf{C} \begin{bmatrix} u_R \\ u_S \end{bmatrix}, \quad (2.9)$$

Similarly, since the sum of the line currents in the three-wire systems is $i_R + i_S + i_T \equiv 0$,

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & 0 \\ 1/\sqrt{2} & \sqrt{2} \end{bmatrix} \begin{bmatrix} i_R \\ i_S \end{bmatrix} = \mathbf{C} \begin{bmatrix} i_R \\ i_S \end{bmatrix}, \quad (2.10)$$

The line currents i_R and i_S , can be calculated by taking the inverse Clarke transform of currents or voltages in $\alpha\beta$ coordinates are as follows

$$\begin{bmatrix} i_R \\ i_S \end{bmatrix} = \begin{bmatrix} \sqrt{2/3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}. \quad (2.11)$$

Therefore, the instantaneous powers in p-q theory for three-wire systems are defined as,

$$p = u_\alpha i_\alpha + u_\beta i_\beta, \quad (2.12)$$

$$q = -u_\alpha i_\beta + u_\beta i_\alpha. \quad (2.13)$$

The instantaneous active and reactive currents are defined with respect to the instantaneous powers and voltages defined above, in both α and β coordinates as given in [8],

$$i_{\alpha p} = \frac{u_{\alpha}}{u_{\alpha}^2 + u_{\beta}^2} p \quad i_{\beta p} = \frac{u_{\beta}}{u_{\alpha}^2 + u_{\beta}^2} p, \quad (2.14)$$

where $i_{\alpha p}$ and $i_{\beta p}$ are *instantaneous active currents* in α and β coordinates and

$$i_{\alpha q} = \frac{u_{\beta}}{u_{\alpha}^2 + u_{\beta}^2} p \quad i_{\beta q} = \frac{-u_{\alpha}}{u_{\alpha}^2 + u_{\beta}^2} q, \quad (2.15)$$

are *instantaneous reactive currents* in α and β coordinates. Likewise, the instantaneous active and reactive currents in phase coordinates R and S can be calculated from the currents derived in 2.21 and 2.22 as,

$$\begin{bmatrix} i_{Rp} \\ i_{Sp} \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix}, \quad \begin{bmatrix} i_{Rq} \\ i_{Sq} \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix}. \quad (2.16)$$

Unlike the definition of reactive power for 3pN systems as shown in eqn. (2.6) and for three-wire systems as shown in eqn. (2.13), the reactive power definition used in this thesis for the misinterpretations of p-q theory in three-wire systems will be used as in ref [6] and the original publication [2]. This also applies to instantaneous active and reactive currents in eqn. (2.15) and (2.16) respectively.

CHAPTER 3: MISINTERPRETATION OF POWER PHENOMENA IN IRP p - q THEORY FOR THREE-WIRE SYSTEMS

The IRP p - q theory is regarded as a mathematical fundamental for switching compensator control for years. The validity of these fundamentals in its application to the compensators has been investigated in Ref [8,9]. An evaluation of IRP p - q theory in [12], published nearly a decade after its inception, pointed out to some limitations of the p - q theory and inspired some researchers to analyze it. It is mainly done in the paper: “*Misinterpretation of p - q theory in three-wire systems*” [6] by L.S Czarnecki. Some conclusions of this paper are presented in this chapter. This chapter will serve as a base for the analysis of the p - q theory in 3pN systems.

3.1 Deficiencies of IRP p - q theory as applied to three-wire systems

Definitions of instantaneous powers p and q caused some skepticism about the theory when compared to the basic properties of a three-phase three-wire system even without any harmonic distortion. Apparently, it was found out that these power definitions of p - q powers contradict three independent features of three-wire system which is (i) permanent energy transmission to the load and associated active power, P , (ii) presence of reactive elements in the load and associated reactive power, Q , and (iii) load imbalances that causes supply current asymmetry and associated unbalanced power, D [6]. The IRP p - q theory describes sinusoidal three-wire system in terms of only two power quantities p and q and unfortunately does not take into account the unbalance in a three-wire system. Moreover, there are some doubts regarding the instantaneous identification and compensation of the reactive power of the three-phase load.

It is not important, for any theory, when considered as a fundamental for control algorithm of switching compensators that it interprets the power phenomena correctly, unless the compensation objectives are achieved. However, it is expected that it should also explain the power phenomena in a power system correctly. To have a reliable reference, contradictions to the IRP p - q theory regarding the interpretation of power phenomena are investigated using Currents’ Physical Components (CPC) power theory [7].

3.2 Comparison of instantaneous currents in p-q theory with original instantaneous current definitions in Fryze's power theory and CPC power theory

The active current definition by Fryze for single-phase systems [11] is given by,

$$i_a(t) = u(t) \frac{P}{\|u\|^2} = G_e u(t), \quad (3.1)$$

This definition can be generalized [3] as follows,

$$\mathbf{i}_a(t) = \begin{bmatrix} i_{Ra} \\ i_{Sa} \\ i_{Ta} \end{bmatrix} = \frac{P}{\|\mathbf{u}\|^2} \begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix} = G_e \mathbf{u}(t), \quad (3.2)$$

The active current is due to the permanent flow of energy, whereas the reactive current occurs due to the phase shift between the voltage and current and is given by,

$$\mathbf{i}_r(t) = \frac{-Q}{\|\mathbf{u}\|^2} \frac{d}{d(\omega t)} \mathbf{u}(t) = B_e \frac{d}{d(\omega t)} \mathbf{u}(t), \quad (3.3)$$

Where $\|\mathbf{u}\|$ is the three phase rms value of the voltage, defined as $\|\mathbf{u}\| = \sqrt{\|u_R\|^2 + \|u_S\|^2 + \|u_T\|^2}$.

The currents defined by equation (3.2) and (3.3) being instantaneous current, have nothing in common with the instantaneous currents defined in eqn. (2.14) and (2.15). However, these instantaneous current definitions for p-q theory do not relate it to the presence of the active and reactive power in the system, which will be evident in this chapter.

3.3 Misinterpretations of power phenomena for different load conditions

3.3.1 Symmetrical and sinusoidal voltage source applied to an unbalanced resistive load

Let us consider a resistive load connected as shown in the Fig. 3.1, and it is supplied from a symmetrical sinusoidal voltage through a Δ -y transformer with a turn ratio of 1:1 [6]. Let us assume, moreover, that the supply voltage is of the positive sequence and the voltage in R phase is $u_R = \sqrt{2}U \cos \omega_1 t$.

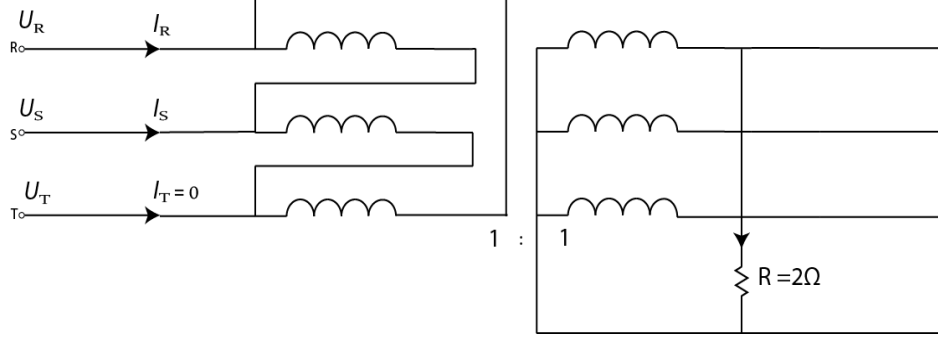


Figure 3.1: Three-wire system with an unbalanced resistive load supplied by sinusoidal and symmetrical voltage

At such assumptions, the supply voltage in the α and β coordinates is,

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \mathbf{C} \begin{bmatrix} \sqrt{2}U \cos \omega_1 t \\ \sqrt{2}U \cos(\omega_1 t - 120^\circ) \end{bmatrix} = \begin{bmatrix} \sqrt{3}U \cos \omega_1 t \\ \sqrt{3}U \sin \omega_1 t \end{bmatrix}. \quad (3.4)$$

The, line currents on the primary side are,

$$i_R = -i_S = \sqrt{2}I \cos(\omega_1 t + 30^\circ) \quad \text{and } i_T = 0. \quad (3.5)$$

Therefore, the supply currents in α and β coordinates is,

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \mathbf{C} \begin{bmatrix} i_R \\ -i_R \end{bmatrix} = \mathbf{C} \begin{bmatrix} \sqrt{2}I \cos(\omega_1 t + 30^\circ) \\ -\sqrt{2}I \cos(\omega_1 t + 30^\circ) \end{bmatrix} = \begin{bmatrix} \sqrt{3}I \cos(\omega_1 t + 30^\circ) \\ -I \cos(\omega_1 t + 30^\circ) \end{bmatrix}. \quad (3.6)$$

The instantaneous active power described in eqn. (2.19) is,

$$p = u_\alpha i_\alpha + u_\beta i_\beta = \sqrt{3}U \cos \omega_1 t [\sqrt{3}I \cos(\omega_1 t + 30^\circ)] + \sqrt{3}U \sin \omega_1 t [-I \cos(\omega_1 t + 30^\circ)], \quad (3.7)$$

$$p = \sqrt{3}UI [1 + \cos(2\omega_1 t + 60^\circ)], \quad (3.8)$$

and the instantaneous reactive power q defined by eqn. (2.20) is,

$$q = u_\alpha i_\beta - u_\beta i_\alpha = \sqrt{3}U \cos \omega_1 t [-I \cos(\omega_1 t + 30^\circ)] - \sqrt{3}U \sin \omega_1 t [\sqrt{3}I \cos(\omega_1 t + 30^\circ)], \quad (3.9)$$

$$q = -\sqrt{3}UI \sin(2\omega_1 t + 60^\circ). \quad (3.10)$$

The instantaneous active current in α and β coordinates is:

$$i_{\alpha p} = \frac{u_{\alpha}}{u_{\alpha}^2 + u_{\beta}^2} p = I \cos(\omega_1 t) [1 + \cos(2\omega_1 t + 60^\circ)], \quad (3.11)$$

$$i_{\beta p} = \frac{u_{\beta}}{u_{\alpha}^2 + u_{\beta}^2} p = I \sin(\omega_1 t) [1 + \cos(2\omega_1 t + 60^\circ)], \quad (3.12)$$

These currents in the phase coordinates are equal to,

$$\begin{aligned} \begin{bmatrix} i_{Rp} \\ i_{Sp} \end{bmatrix} &= \mathbf{C}^{-1} \begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} I \cos(\omega_1 t) [1 + \cos(2\omega_1 + 60^\circ)] \\ I \sin(\omega_1 t) [1 + \cos(2\omega_1 + 60^\circ)] \end{bmatrix} = \\ &= [1 + \cos(2\omega_1 + 60^\circ)] \sqrt{\frac{2}{3}} I \begin{bmatrix} \cos(\omega_1 t) \\ \cos(\omega_1 t - 120^\circ) \end{bmatrix}. \end{aligned} \quad (3.13)$$

The instantaneous reactive current in α and β coordinates is equal to,

$$i_{\alpha q} = \frac{-u_{\beta}}{u_{\alpha}^2 + u_{\beta}^2} q = I \sin(\omega_1 t) [\sin(2\omega_1 + 60^\circ)] \quad (3.14)$$

$$i_{\beta q} = \frac{u_{\alpha}}{u_{\alpha}^2 + u_{\beta}^2} q = -I \cos(\omega_1 t) [\sin(2\omega_1 + 60^\circ)] \quad (3.15)$$

and in phase coordinates,

$$\begin{aligned} \begin{bmatrix} i_{Rq} \\ i_{Sq} \end{bmatrix} &= \mathbf{C}^{-1} \begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} I \sin(\omega_1 t) [\sin(2\omega_1 + 60^\circ)] \\ -I \cos(\omega_1 t) [\sin(2\omega_1 + 60^\circ)] \end{bmatrix} = \\ &= [\sin(2\omega_1 + 60^\circ)] \sqrt{\frac{2}{3}} I \begin{bmatrix} \sin(\omega_1 t) \\ \sin(\omega_1 t - 120^\circ) \end{bmatrix}. \end{aligned} \quad (3.16)$$

This illustration shows that the instantaneous active and reactive current values obtained from the eqn. (3.13) and (3.16) do not fit with the common electrical engineering comprehension of reactive current. According to IRP p-q theory a reactive current can occur in purely resistive circuits. Additionally, the

calculated reactive and active currents of a linear load with sinusoidal supply voltage contain harmonic components. The instantaneous identification of the load properties based on IRP p-q theory is rather vague, as the values for reactive power at different instant of a cycle could be different. In addition, it does not have any provision for identifying the load unbalance in the system.

3.3.2 Symmetrical and sinusoidal voltage source applied to an unbalanced inductive load

Let us consider another circuit with purely reactive load as shown in Fig. 3.2 and supplied with symmetrical sinusoidal voltage source of a positive sequence [6]. The voltage assumed in R phase is $u_R = \sqrt{2}U\cos\omega_1 t$ similar to the supply used in the previous illustration. The supply voltage in α and β coordinates is same as equation (3.4). The line currents in this circuit are equal to,

$$i_R = -i_S = \sqrt{2}I\cos(\omega_1 t - 60^\circ) \text{ and } i_T = 0. \quad (3.17)$$

The supply currents in α and β coordinates are equal to,

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \mathbf{C} \begin{bmatrix} i_R \\ -i_R \end{bmatrix} = \mathbf{C} \begin{bmatrix} \sqrt{2}I\cos(\omega_1 t - 60^\circ) \\ -\sqrt{2}I\cos(\omega_1 t - 60^\circ) \end{bmatrix} = \begin{bmatrix} \sqrt{3}I\cos(\omega_1 t - 60^\circ) \\ -I\cos(\omega_1 t - 60^\circ) \end{bmatrix}. \quad (3.18)$$

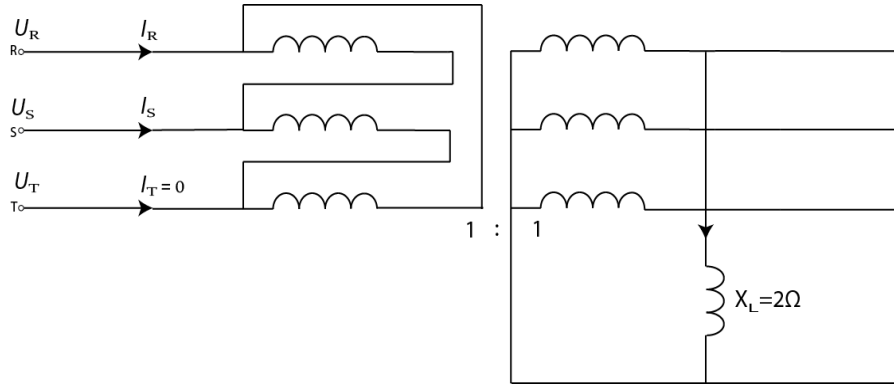


Figure 3.2: Three-wire system with an unbalanced inductive load supplied by sinusoidal and symmetrical voltage

The instantaneous active and reactive powers are,

$$p = u_\alpha i_\alpha + u_\beta i_\beta = \sqrt{3}U\cos\omega_1 t[\sqrt{3}I\cos(\omega_1 t - 60^\circ)] + \sqrt{3}U\sin\omega_1 t[-I\cos(\omega_1 t - 60^\circ)], \quad (3.19)$$

$$p = \sqrt{3}UI \cos(2\omega_1 t - 30^\circ), \quad (3.20)$$

$$q = u_\alpha i_\beta - u_\beta i_\alpha = \sqrt{3}U \cos \omega_1 t [-I \cos(\omega_1 t + 30^\circ)] - \sqrt{3}U \sin \omega_1 t [\sqrt{3}I \cos(\omega_1 t + 30^\circ)], \quad (3.21)$$

$$q = -\sqrt{3}UI [1 + \sin(2\omega_1 t - 30^\circ)]. \quad (3.22)$$

Consequently, the instantaneous active current in α and β coordinates have values,

$$\begin{aligned} i_{\alpha p} &= \frac{u_\alpha}{u_\alpha^2 + u_\beta^2} p = I \cos(\omega_1 t) \cos(2\omega_1 t + 60^\circ), \\ &= \frac{I}{2} [\cos(\omega_1 t - 30^\circ) + \cos(3\omega_1 t - 30^\circ)], \end{aligned} \quad (3.23)$$

$$\begin{aligned} i_{\beta p} &= \frac{u_\beta}{u_\alpha^2 + u_\beta^2} p = I \sin(\omega_1 t) \cos(2\omega_1 t - 30^\circ), \\ &= \frac{I}{2} [\sin(\omega_1 t - 30^\circ) - \sin(3\omega_1 t - 30^\circ)], \end{aligned} \quad (3.24)$$

and in phase coordinates it is equal to,

$$\begin{bmatrix} i_{Rp} \\ i_{Sp} \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} \frac{I}{2} [\cos(\omega_1 t - 30) + \cos(3\omega_1 t - 30^\circ)] \\ -\frac{I}{2} [\sin(\omega_1 t - 30) - \sin(3\omega_1 t - 30^\circ)] \end{bmatrix} \quad (3.25)$$

As a result,

$$\begin{aligned} i_{Rp} &= \frac{I}{\sqrt{6}} [\cos(\omega_1 t - 30) + \cos(3\omega_1 t - 30^\circ)], \\ i_{Sp} &= -\frac{I}{2\sqrt{6}} [\cos(\omega_1 t - 30) + \cos(3\omega_1 t - 30^\circ)] - \frac{I}{2\sqrt{2}} [\sin(\omega_1 t - 30) - \sin(3\omega_1 t - 30^\circ)]. \end{aligned} \quad (3.26)$$

The active power in this circuit is equal to zero; however the IRP p-q theory shows that the instantaneous active current in a purely reactive circuit is not zero. The active current in this circuit also contains a third harmonic even though the circuit has no source of distortion. Therefore, p-q theory misinterprets the actual phenomena in the system, when applied with unbalance even in sinusoidal conditions. This has been proven in section (3.3.1) and (3.3.2) [6]. Nevertheless, using the Currents' Physical Components power theory for sinusoidal conditions drafted in next two sections clarifies this ambiguity.

3.4 CPC power theory for three-wire systems with sinusoidal conditions

Any three-phase load supplied by a three-wire system, supplied by a symmetrical and sinusoidal voltage has an equivalent load in Δ structure as shown in Fig. 3.3 with $u_R = \sqrt{2}U_R \cos \omega_1 t$. This load can be described in terms of two characteristic admittances [7],

$$Y_e = G_e + jB_e = Y_{RS} + Y_{ST} + Y_{TR}, \quad (3.27)$$

which is the equivalent admittance, and the unbalanced admittance,

$$A = Ae^{j\psi} = -(Y_{ST} + \alpha Y_{TR} + \alpha^* Y_{RS}), \quad \text{where } \alpha = 120^\circ. \quad (3.28)$$

The complex three-phase supply voltage and load current are arranged in to vectors [7];

$$\begin{bmatrix} U_R \\ U_S \\ U_T \end{bmatrix} = \mathbf{U}, \quad \begin{bmatrix} U_R \\ U_T \\ U_S \end{bmatrix} = \mathbf{U}^\#, \quad (3.29)$$

$$\mathbf{i} = \begin{bmatrix} I_R \\ I_S \\ I_T \end{bmatrix} = \sqrt{2} Re \begin{bmatrix} I_R \\ I_S \\ I_T \end{bmatrix} e^{j\omega_1 t} = \sqrt{2} Re \mathbf{I} e^{j\omega_1 t}. \quad (3.30)$$

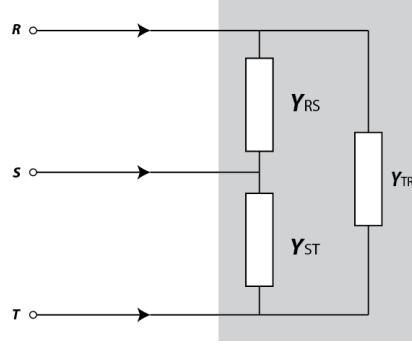


Figure 3.3: Equivalent load in Δ structure

According to the CPC theory, the load current can be decomposed in to three components,

$$\mathbf{i} = \mathbf{i}_a + \mathbf{i}_r + \mathbf{i}_u , \quad (3.31)$$

where,

$$\mathbf{i}_a = \sqrt{2} \text{Re}\{G_e \mathbf{U} e^{j\omega_1 t}\} , \quad (3.32)$$

$$\mathbf{i}_r = \sqrt{2} \text{Re}\{jB_e \mathbf{U} e^{j\omega_1 t}\} , \quad (3.33)$$

$$\mathbf{i}_u = \sqrt{2} \text{Re}\{A \mathbf{U}^\# e^{j\omega_1 t}\} . \quad (3.34)$$

These three vectors will be referred to as active, reactive and unbalanced currents. The rms values of these currents are equal to,

$$\|\mathbf{i}_a\| = G_e \|\mathbf{u}\| , \quad \|\mathbf{i}_r\| = |B_e| \|\mathbf{u}\| \text{ and } \|\mathbf{i}_u\| = A \|\mathbf{u}\|. \quad (3.35)$$

These three currents are proved to be mutually orthogonal, which means that their scalar product is equal to zero. Therefore, the square of their rms values satisfies the following relationship,

$$\|\mathbf{i}\|^2 = \|\mathbf{i}_a\|^2 + \|\mathbf{i}_r\|^2 + \|\mathbf{i}_u\|^2 . \quad (3.36)$$

Multiplying the equation (3.35) by the square of three phase rms supply voltage results in the power equation of three-phase system,

$$S^2 = P^2 + Q^2 + D^2, \quad (3.37)$$

where P is the *active power*, Q is the *reactive power* and D is the *unbalanced power*, defined as ,

$$P = \|u\| \|i_a\| = G_e \|u\|^2, \quad (3.38)$$

$$Q = \pm \|u\| \|i_r\| = -B_e \|u\|^2 \quad (3.39)$$

$$D = \|u\| \|i_u\| = A \|u\|^2 \quad (3.40)$$

The power equation and definitions (3.38), (3.39) and (3.40) emphasize the fact that three independent and distinctive physical phenomena in a linear circuit with sinusoidal and symmetrical voltage contribute to the apparent power of the system. These are (i) permanent energy transmission to the load and associated active power, P , (ii) presence of reactive elements in the load and associated reactive power, Q , and (iii) load imbalances that causes supply current asymmetry and associated unbalanced power, D . Moreover, the association of all the above three currents with a typical phenomena in this type of circuit, gives the theory its name Currents' Physical Components (CPC) power theory [7].

The CPC theory is applied to the circuit in illustration (3.3.1) in order to show the theory's effectiveness in identifying the properties of the load. Comparing the circuit in fig 3.3 with fig 3.1, the admittances of the load circuit are $Y_{RS} = \frac{1}{G_{RS}} = 0.5S$, Y_{ST} & $Y_{TR} = 0$. Therefore, the equivalent admittance is,

$$Y_e = G_e + jB_e = Y_{RS} + Y_{ST} + Y_{TR} = 0.5 S, \quad (3.41)$$

thus $G_e = 0.5S$ and $B_e = 0$, while the unbalanced admittance according to the eqn. 3.28 is,

$$A = Ae^{j\psi} = -(Y_{ST} + \alpha Y_{TR} + \alpha^* Y_{RS}) = 0.5e^{j60^\circ} S. \quad (3.42)$$

The, three-phase active and unbalanced current in the load are equal to [6],

$$\mathbf{i}_a = \begin{bmatrix} i_{Ra} \\ i_{Sa} \\ i_{Ta} \end{bmatrix} = \sqrt{2} Re \left\{ G_e \begin{bmatrix} U \\ Ue^{-j120^\circ} \\ Ue^{j120^\circ} \end{bmatrix} e^{j\omega_1 t} \right\} = 60\sqrt{2} \begin{bmatrix} \cos \omega_1 t \\ \cos(\omega_1 t - 120^\circ) \\ \cos(\omega_1 t + 120^\circ) \end{bmatrix} \text{ A}, \quad (3.43)$$

$$\mathbf{i}_u = \begin{bmatrix} i_{Ru} \\ i_{Su} \\ i_{Tu} \end{bmatrix} = \sqrt{2} Re \left\{ A \begin{bmatrix} U \\ Ue^{+j120^\circ} \\ Ue^{-j120^\circ} \end{bmatrix} e^{j\omega_1 t} \right\} = 60\sqrt{2} \begin{bmatrix} \cos(\omega_1 t + 60^\circ) \\ \cos(\omega_1 t + 180^\circ) \\ \cos(\omega_1 t - 60^\circ) \end{bmatrix} \text{ A}. \quad (3.44)$$

Since in this circuit $B_e = 0$, the reactive current will be zero. Thus, the supply line current of such a load is the sum of only active and unbalanced current, namely,

$$\mathbf{i} = \mathbf{i}_a + \mathbf{i}_u = 103.9\sqrt{2} \begin{bmatrix} \cos(\omega_1 t + 30^\circ) \\ \cos(\omega_1 t - 150^\circ) \\ 0 \end{bmatrix} \text{ A}. \quad (3.45)$$

Thus, the supply current of this resistive unbalanced circuit contains only the active and the unbalance currents, which is expected. The CPC power theory proves to identify the features of a three-phase three-wire sinusoidal system correctly.

CHAPTER 4: HARMONIC GENERATING LOADS (HGLs) AND SUPPLY DISTORTION IN TERMS OF IRP p-q THEORY FOR THREE-WIRE SYSTEMS

Definitions of instantaneous powers p and q in IRP p-q theory being are supposed to describe physical properties of the system in a mathematical way in terms of $\alpha\beta$ coordinates. However, these instantaneous powers in p-q theory, when expressed directly in terms of three-phase voltages and currents, describe the system in a more direct way. Moreover, they can be further simplified and made easier for calculations [5]. This chapter, therefore, uses these definitions of instantaneous powers in phase coordinates to analyze the performance of IRP p-q theory for harmonic generating loads (HGLs) and supply harmonics.

4.1 Instantaneous powers expressed in phase coordinates

The three-phase supply voltage and current can be arranged in vectors,

$$\mathbf{u}(t) \triangleq [u_R(t), u_S(t), u_T(t)]^T, \quad (4.1)$$

$$\mathbf{i}(t) \triangleq [i_R(t), i_S(t), i_T(t)]^T, \quad (4.2)$$

where the voltages are measured with respect to an artificial zero. Therefore in such systems,

$$u_R(t) + u_S(t) + u_T(t) = 0 \quad (4.3)$$

$$i_R(t) + i_S(t) + i_T(t) = 0 \quad (4.4)$$

The instantaneous active power p is synonymous to commonly known instantaneous power, which means the rate of energy flow, specified in terms of three phase voltages and currents as,

$$\begin{aligned} p = \bar{p} + \tilde{p} &\triangleq \frac{dW}{dt} = \mathbf{u}^T \mathbf{i} = u_R i_R + u_S i_S + u_T i_T \\ &= (u_R - u_T) i_R + (u_S - u_T) i_S = u_{RT} i_R + u_{ST} i_S \end{aligned} \quad (4.5)$$

The instantaneous reactive power q can be expressed in phase coordinates as,

$$\begin{aligned}
q &\triangleq u_\alpha i_\beta - u_\beta i_\alpha \\
&= \sqrt{\frac{3}{2}} u_R \left(\frac{1}{\sqrt{2}} i_R + \sqrt{2} i_S \right) - \left(\frac{1}{\sqrt{2}} u_R + \sqrt{2} u_S \right) \sqrt{\frac{3}{2}} i_R = \\
&= \sqrt{3} (u_R i_S - u_S i_R)
\end{aligned} \tag{4.6}$$

4.2 Instantaneous powers for resistive balanced HGLs at sinusoidal and symmetrical voltage

Harmonic generating loads cause supply current distortion and this distortion can cause supply voltage distortion. However, at a sufficiently strong supply source, this voltage distortion does not affect the system performance and can be neglected. Let us assume a resistive balanced HGL three-wire circuit supplied with symmetrical (of the positive sequence) and sinusoidal voltage as shown in Fig. 4.1 [5].

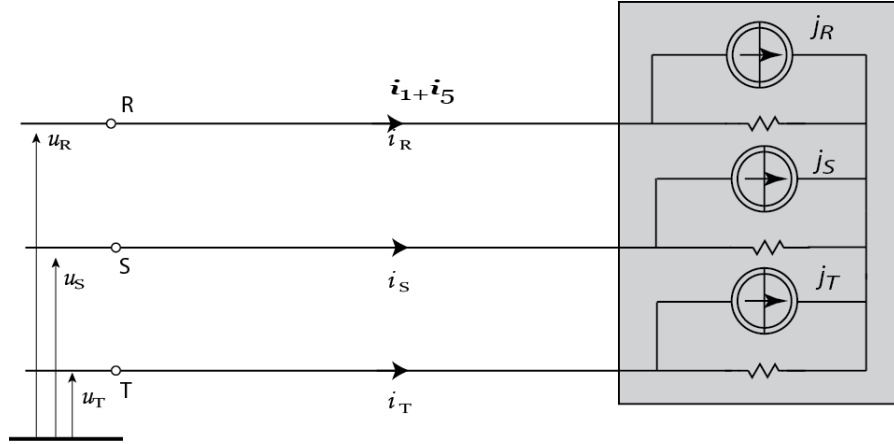


Figure 4.1: Three-wire system with a balanced resistive HGL supplied by sinusoidal and symmetrical voltage

The instantaneous power p is,

$$p = \sum_{n \in N} p_n$$

where the power of the n^{th} order harmonics is,

$$p_n = u_{RTn} i_{Rn} + u_{STn} i_{Sn}, \tag{4.7}$$

whereas the instantaneous reactive power q can be expressed as,

$$q = \sum_{n \in N} q_n = q_n \triangleq u_\alpha i_\beta - u_\beta i_\alpha = \sqrt{3}(u_R i_S - u_S i_R)$$

Assuming that the voltage contain fundamental and the current has n-th order harmonic,

$$q_n = \sqrt{3} (u_{R1} i_{Sn} - u_{S1} i_{Rn}) \quad (4.8)$$

As the supply voltage is sinusoidal and symmetrical, the line to ground voltage for R, S and T phases are,

$$\begin{aligned} u_R &= u_{R1} = \sqrt{2}U_1 \cos(\omega_1 t), \\ u_S &= u_{S1} = \sqrt{2}U_1 \cos(\omega_1 t - 120^\circ), \\ u_T &= u_{T1} = \sqrt{2}U_1 \cos(\omega_1 t + 120^\circ), \end{aligned} \quad (4.9)$$

The line-to-line voltages have values,

$$\begin{aligned} u_{RT1} &= \sqrt{6}U_1 \cos(\omega_1 t - 30^\circ), \\ u_{ST1} &= \sqrt{6}U_1 \cos(\omega_1 t - 90^\circ). \end{aligned} \quad (4.10)$$

The HGL generates 5th order harmonic which is a negative sequence harmonic hence,

$$\begin{aligned} i_R &= \sqrt{2}I_1 \cos(\omega_1 t) + \sqrt{2}I_5 \cos(5\omega_1 t + \alpha_5), \\ i_S &= \sqrt{2}I_1 \cos(\omega_1 t - 120^\circ) + \sqrt{2}I_5 \cos(5\omega_1 t + \alpha_5 + 120^\circ), \end{aligned} \quad (4.11)$$

Therefore, the instantaneous power p is,

$$\begin{aligned} p &= u_{RT1} i_{Rn} + u_{ST1} i_{Sn}, \\ &= 2\sqrt{3}U_1 \cos(\omega_1 t - 30^\circ) [I_1 \cos(\omega_1 t) + I_5 \cos(5\omega_1 t + \alpha_5)] + \end{aligned}$$

$$\begin{aligned}
& 2\sqrt{3}U_1 \cos(\omega_1 t - 90^\circ) [I_1 \cos(\omega_1 t - 120^\circ) + I_5 \cos(5\omega_1 t + \alpha_5 + 120^\circ)] \\
& = 3U_1 I_1 + 3U_1 I_5 \cos(6\omega_1 t + \alpha_5) = p_1 + p_5 = \bar{p} + \tilde{p} , \tag{4.12}
\end{aligned}$$

where the first term in equation represents the constant component of the active power of the fundamental harmonic and the second component is the oscillating component which has the sixth order harmonic frequency. These components when calculated using the p - q theory, requires a high pass or low pass filter in order to decompose.

The instantaneous reactive power q of the considered load is,

$$\begin{aligned}
q &= \sqrt{3} (u_{R1} i_{Sn} - u_{S1} i_{Rn}) = 2\sqrt{3}U_1 \{ \cos(\omega_1 t) [I_1 \cos(\omega_1 t - 120^\circ) + I_5 \cos(5\omega_1 t + \alpha_5 + 120^\circ)] \\
&\quad - \cos(\omega_1 t - 120^\circ) [I_1 \cos(\omega_1 t) + I_5 \cos(5\omega_1 t + \alpha_5)] \\
&= -3U_1 I_5 \sin(6\omega_1 t + \alpha_5). \tag{4.13}
\end{aligned}$$

This equation shows that the instantaneous reactive power is composed of only oscillating component of the sixth order harmonic frequency. The load does not have the reactive power $Q = \bar{q}$ because the load is purely resistive and balanced for the fundamental harmonic.

Now let us assume that the load shown in fig 4.1 generates 7th order current harmonic [5] instead of the 5th order current harmonic, while all the other parameters are same. The 7th order current harmonic is of a positive sequence, thus the supply currents are,

$$\begin{aligned}
i_R &= \sqrt{2}I_1 \cos(\omega_1 t) + \sqrt{2}I_5 \cos(7\omega_1 t + \alpha_7), \\
i_S &= \sqrt{2}I_1 \cos(\omega_1 t - 120^\circ) + \sqrt{2}I_7 \cos(7\omega_1 t + \alpha_7 - 120^\circ), \tag{4.14}
\end{aligned}$$

Therefore, the instantaneous powers are,

$$p = u_{RT1}i_{Rn} + u_{ST1}i_{Sn} = 3U_1I_1 + 3U_1I_7 \cos(7\omega_1t + \alpha_7) \quad (4.15)$$

$$\begin{aligned} q &= \sqrt{3} (u_{R1}i_{Sn} - u_{S1}i_{Rn}) = 2\sqrt{3}U_1\{\cos(\omega_1t) [I_1 \cos(\omega_1t - 120^\circ) + I_7 \cos(7\omega_1t + \alpha_7 - 120^\circ)] \\ &\quad - \cos(\omega_1t - 120^\circ) [I_1 \cos(\omega_1t) + I_7 \cos(7\omega_1t + \alpha_7)]\} \\ &= -3U_1I_7 \sin(6\omega_1t + \alpha_7). \end{aligned} \quad (4.16)$$

The calculation of instantaneous active power shows that the fundamental harmonic component remains the same and the load-generated harmonic does not have any effect on the oscillating components of the instantaneous powers as a whole. In other words, the frequency unexpectedly remains the same, whereas only the RMS and phase values changes. As a result, the equations (4.12), (4.13), (4.15) and (4.16) show that the oscillating instantaneous powers of the load discussed above, does not reflect the actual property of the load.

4.3 Instantaneous powers for resistive balanced load at non-sinusoidal but symmetrical voltage

Let us consider a resistive balanced three-phase load supplied from a symmetrical non-sinusoidal voltage consisting of a 5th order voltage harmonic [5].

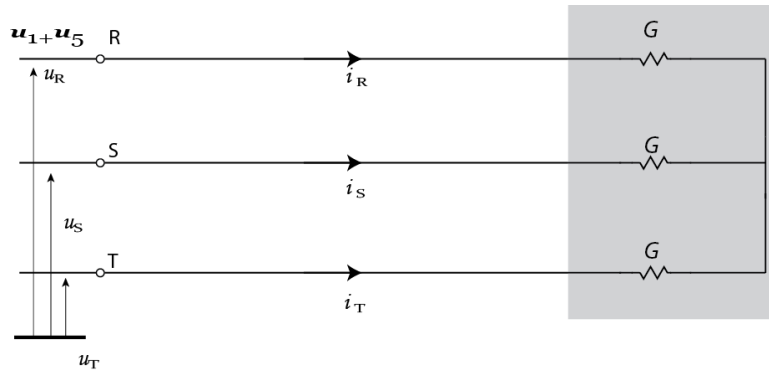


Figure 4.2: Three-wire system with a balanced resistive load supplied by non-sinusoidal but symmetrical voltage

The three phase voltages referred to an artificial zero are equal to,

$$u_R = \sqrt{2}U_1 \cos(\omega_1t) + \sqrt{2}U_5 \cos(5\omega_1t),$$

$$\begin{aligned}
u_S &= \sqrt{2}U_1 \cos(\omega_1 t - 120^\circ) + \sqrt{2}U_5 \cos(5\omega_1 t + 120^\circ), \\
u_T &= \sqrt{2}U_1 \cos(\omega_1 t + 120^\circ) + \sqrt{2}U_5 \cos(5\omega_1 t - 120^\circ),
\end{aligned} \tag{4.17}$$

The instantaneous active power p according to the traditional instantaneous power definition can be defined as,

$$\begin{aligned}
p &= \mathbf{u}^T \mathbf{i} = \mathbf{u}^T G \mathbf{u} = G[\mathbf{u}_1 + \mathbf{u}_5]^T [\mathbf{u}_1 + \mathbf{u}_5] \\
&= G\mathbf{u}_1^T \mathbf{u}_1 + \mathbf{u}_5^T \mathbf{u}_5 + G(\mathbf{u}_1^T \mathbf{u}_5 + \mathbf{u}_5^T \mathbf{u}_1).
\end{aligned} \tag{4.18}$$

The first two terms in the equation (4.18) are the constant components of the instantaneous power, whereas the third term can be expanded as,

$$\begin{aligned}
G(\mathbf{u}_1^T \mathbf{u}_5 + \mathbf{u}_5^T \mathbf{u}_1) &= G[u_{1R}u_{5R} + u_{1S}u_{5S} + u_{1T}u_{5T}] + G[u_{5R}u_{1R} + u_{5S}u_{1S} + u_{5T}u_{1T}] \\
&= 2G[u_{1R}u_{5R} + u_{1S}u_{5S} + u_{1T}u_{5T}]
\end{aligned} \tag{4.19}$$

Eventually, formula (4.19) gives,

$$\begin{aligned}
&= 4GU_1U_5\{\cos(\omega_1 t) \cos(5\omega_1 t) + [\cos(\omega_1 t - 120^\circ) \cos(5\omega_1 t + 120^\circ)] \\
&\quad + [\cos(\omega_1 t + 120^\circ) \cos(5\omega_1 t - 120^\circ)]\} \\
&= 6GU_1U_5 \cos 6\omega_1 t
\end{aligned} \tag{4.20}$$

$$p = \bar{p} + 6GU_1U_5 \cos 6\omega_1 t \tag{4.21}$$

The instantaneous reactive power q for this load is zero. Comparison of equations (4.12), (4.13) with (4.21), does not allow for differentiating between a balanced resistive load supplied by non-sinusoidal voltage and a balanced HGL at sinusoidal symmetrical voltage and shows that the theory is not capable of providing any information regarding the reactive and active power oscillating components. Hence, after

the analysis of the three-wire circuits in illustration 4.2 and 4.3 for IRP p-q theory, we can conclude that the IRP p-q theory does not allow us to properly identify the properties of load under non-sinusoidal conditions.

CHAPTER 5: ANALYSIS OF IRP p-q THEORY FOR UNBALANCED AND ASYMMETRIC THREE-PHASE SYSTEMS WITH NEUTRAL

Description of three-phase systems with neutral (3pN) in terms of IRP p-q powers is investigated in this chapter under various conditions. The definitions for 3pN systems mentioned in Chapter2 are used. Instantaneous voltages, currents in $\alpha\beta 0$ coordinate and instantaneous powers are calculated. However, before proceeding to the unbalanced and asymmetrical conditions, description of this system in terms of IRP p-q theory is presented.

5.1 Balanced resistive load with sinusoidal and symmetrical supply voltage

A three-phase system with neutral circuit with a balanced purely resistive load is shown in Fig. 5.1. The supply voltage is considered to be sinusoidal and symmetrical of positive sequence. Z_s is the internal impedance of the source.

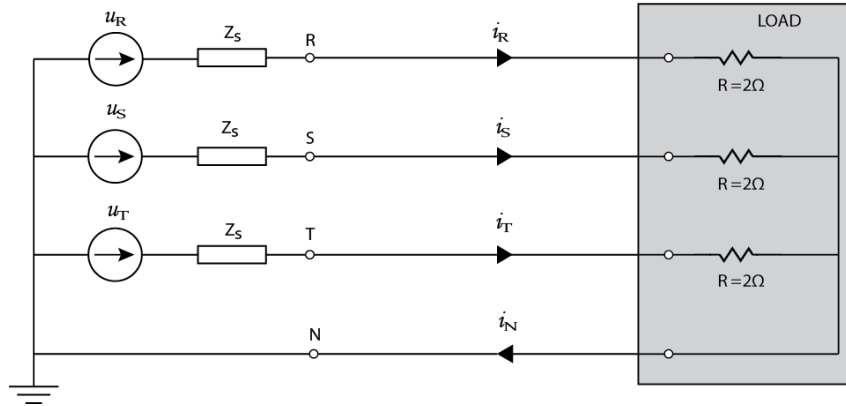


Figure 5.1: 3pN system with a balanced resistive load supplied by sinusoidal and symmetrical voltage

The phase voltages supplied with respect to neutral are given as

$$\begin{aligned} u_R &= \sqrt{2}U \cos(\omega_1 t), \\ u_S &= \sqrt{2}U \cos(\omega_1 t - 120^\circ), \\ u_T &= \sqrt{2}U \cos(\omega_1 t + 120^\circ). \end{aligned}$$

As the system is balanced and resistive, the line currents in the system can be written as

$$i_R = \sqrt{2}I \cos(\omega_1 t),$$

$$i_S = \sqrt{2}I \cos(\omega_1 t - 120^\circ),$$

$$i_T = \sqrt{2}I \cos(\omega_1 t + 120^\circ).$$

The Clarke vector [1] of the supply voltage is,

$$\begin{bmatrix} u_\alpha \\ u_\beta \\ u_0 \end{bmatrix} = \mathbf{C} \begin{bmatrix} \sqrt{2}U \cos(\omega_1 t) \\ \sqrt{2}U \cos(\omega_1 t - 120^\circ) \\ \sqrt{2}U \cos(\omega_1 t + 120^\circ) \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \sqrt{2}U \cos(\omega_1 t) \\ \sqrt{2}U \cos(\omega_1 t - 120^\circ) \\ \sqrt{2}U \cos(\omega_1 t + 120^\circ) \end{bmatrix}, \quad (5.1)$$

$$\begin{bmatrix} u_\alpha \\ u_\beta \\ u_0 \end{bmatrix} = \begin{bmatrix} \sqrt{3}U \cos(\omega_1 t) \\ \sqrt{3}U \sin(\omega_1 t) \\ 0 \end{bmatrix}, \quad (5.2)$$

similarly, the Clarke vector of three-phase current is,

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \mathbf{C} \begin{bmatrix} \sqrt{2}I \cos(\omega_1 t) \\ \sqrt{2}I \cos(\omega_1 t - 120^\circ) \\ \sqrt{2}I \cos(\omega_1 t + 120^\circ) \end{bmatrix} = \begin{bmatrix} \sqrt{3}I \cos(\omega_1 t) \\ \sqrt{3}I \sin(\omega_1 t) \\ 0 \end{bmatrix}. \quad (5.3)$$

The instantaneous active and reactive power calculated with these voltages and currents in $\alpha\beta 0$ coordinates are,

$$p = u_\alpha i_\alpha + u_\beta i_\beta = 3UI [\cos^2(\omega_1 t) + \sin^2(\omega_1 t)] = 3UI, \quad (5.4)$$

$$q = -u_\alpha i_\beta + u_\beta i_\alpha = -3UI [\cos(\omega_1 t) \sin(\omega_1 t) - \cos(\omega_1 t) \sin(\omega_1 t)] = 0, \quad (5.5)$$

and the zero sequence power is,

$$p_0 = u_0 i_0 = 0. \quad (5.6)$$

The balanced, purely resistive system supplied with sinusoidal and symmetrical voltage contains no zero sequence power and no reactive power. The system delivers only the active power to the load.

5.2 Balanced inductive load with sinusoidal and symmetrical supply voltage

Let us consider a balanced purely inductive load with supply voltages same as used in section 5.1 and as shown in Fig. 5.2.

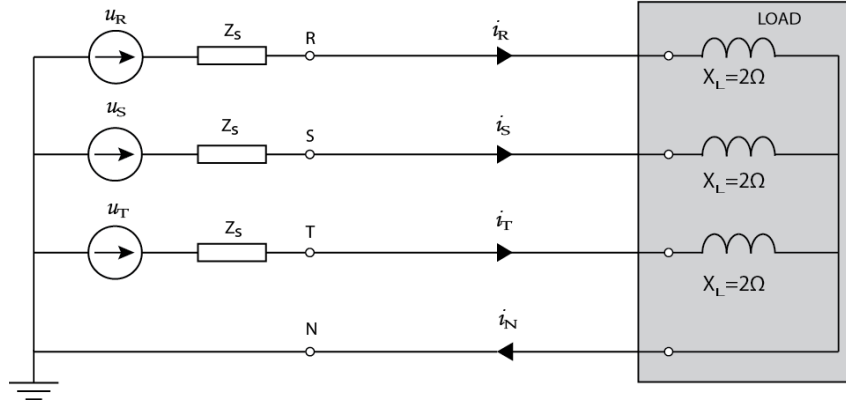


Figure 5.2: 3pN system with balanced inductive load supplied by sinusoidal and symmetrical voltage

The line currents in such a system are,

$$i_R = \sqrt{2}I \cos(\omega_1 t + 90^\circ),$$

$$i_S = \sqrt{2}I \cos(\omega_1 t - 30^\circ),$$

$$i_T = \sqrt{2}I \cos(\omega_1 t + 210^\circ).$$

Voltages and currents in Clarke coordinates are,

$$\begin{bmatrix} u_\alpha \\ u_\beta \\ u_0 \end{bmatrix} = \begin{bmatrix} \sqrt{3}U \cos(\omega_1 t) \\ \sqrt{3}U \sin(\omega_1 t) \\ 0 \end{bmatrix}, \quad (5.7)$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \sqrt{2}I \cos(\omega_1 t + 90^\circ) \\ \sqrt{2}I \cos(\omega_1 t - 30^\circ) \\ \sqrt{2}I \cos(\omega_1 t + 210^\circ) \end{bmatrix}$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} -\sqrt{2}I \sin(\omega_1 t) - \frac{\sqrt{2}}{2}I \sin(\omega_1 t) \\ \frac{3}{\sqrt{2}}I \cos(\omega_1 t) \\ 0 \end{bmatrix}. \quad (5.8)$$

The instantaneous active power is equal to,

$$\begin{aligned} p &= u_\alpha i_\alpha + u_\beta i_\beta, \\ p &= \sqrt{\frac{2}{3}} \left\{ \sqrt{3}U \cos(\omega_1 t) \left[-\sqrt{2}I \sin(\omega_1 t) - \frac{\sqrt{2}}{2}I \sin(\omega_1 t) \right] + \sqrt{3}U \sin(\omega_1 t) \left[\frac{3}{\sqrt{2}}I \cos(\omega_1 t) \right] \right\}, \\ &= \sqrt{\frac{2}{3}} UI \cos(\omega_1 t) \sin(\omega_1 t) \left[-\sqrt{6} - \frac{\sqrt{6}}{2} + \frac{3\sqrt{3}}{2} \right] = 0, \end{aligned} \quad (5.9)$$

and the instantaneous reactive power:

$$\begin{aligned} q &= -u_\alpha i_\beta + u_\beta i_\alpha, \\ q &= \sqrt{\frac{2}{3}} \left\{ -\sqrt{3}U \cos(\omega_1 t) \left[\frac{3}{\sqrt{2}}I \cos(\omega_1 t) \right] + \sqrt{3}U \sin(\omega_1 t) \left[-\sqrt{2}I \sin(\omega_1 t) - \frac{\sqrt{2}}{2}I \sin(\omega_1 t) \right] \right\}, \\ &= -3UI [\cos^2(\omega_1 t) + \sin^2(\omega_1 t)] = -3UI. \end{aligned} \quad (5.10)$$

The zero sequence power is equal to zero. The instantaneous active power p is zero as the load is purely inductive and reactive power q has a negative value. The values of instantaneous powers calculated in illustration 5.1 and 5.2 shows that the p-q theory successfully describes a 3pN system under balanced and symmetrical conditions.

5.3 Unbalanced resistive load with sinusoidal and symmetrical supply voltage

In this illustration, a 3pN system with an unbalanced resistive load, connected as shown in Fig. 5.3, is supplied by sinusoidal and symmetrical voltage. The Clarke vector of the supply voltage for this system is

same as given by eqn. (5.7). The line currents i_S and i_T are equal to zero and $i_R = \sqrt{2}I \cos(\omega_1 t)$. As a result, the line current contains the positive, negative and zero sequence current components [4]. The

Clarke vector for these line currents is,

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \sqrt{2}I \cos(\omega_1 t) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{3}}I \cos(\omega_1 t) \\ 0 \\ \sqrt{\frac{2}{3}}I \cos(\omega_1 t) \end{bmatrix}. \quad (5.11)$$

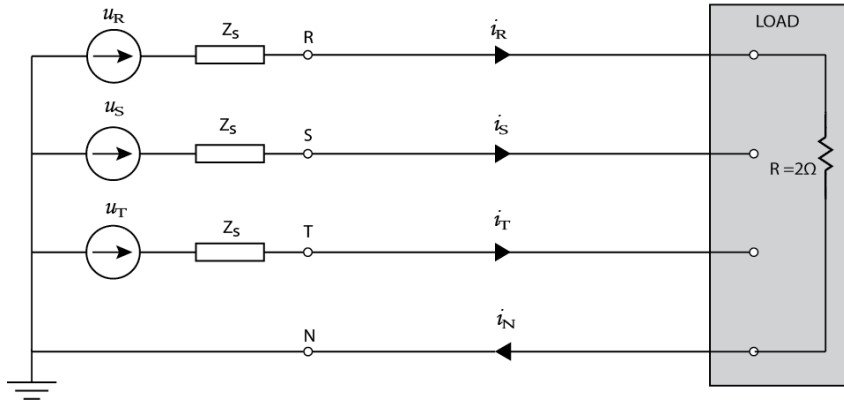


Figure 5.3: 3pN system with unbalanced resistive load supplied by sinusoidal and symmetrical voltage

The instantaneous active power is,

$$\begin{aligned} p &= \sqrt{3}U \cos(\omega_1 t) \left[\frac{2}{\sqrt{3}}I \cos(\omega_1 t) \right] + \sqrt{3}U \sin(\omega_1 t) [0], \\ &= 2UI \cos^2(\omega_1 t) = UI[1 + \cos(2\omega_1 t)], \end{aligned} \quad (5.12)$$

and the instantaneous reactive power:

$$\begin{aligned} q &= -\sqrt{3}U \cos(\omega_1 t) [0] + \sqrt{3}U \sin(\omega_1 t) \left[\frac{2}{\sqrt{3}}I \cos(\omega_1 t) \right] \\ &= 2UI \cos(\omega_1 t) \sin(\omega_1 t) = UI \sin(2\omega_1 t), \end{aligned} \quad (5.13)$$

The zero sequence power,

$$p_0 = u_0 i_0 = 0 \left[\sqrt{\frac{2}{3}} I \cos(\omega_1 t) \right] = 0. \quad (5.14)$$

Such a system, when described using IRP p-q theory, contains reactive power regardless of the fact that the load is resistive. The presence of the reactive power can be an effect of the load imbalance; however, it is difficult to comment on it so early, as the theory itself does not provide any fundamentals for separate load imbalance or asymmetry evaluations.

5.4 Unbalanced inductive load with sinusoidal and symmetrical supply voltage

Let us consider a similar system as in illustration 5.3, but with an unbalanced inductive load as shown in Fig. 5.4. According to the load structure the line currents i_S and i_T are equal to zero, whereas $i_R = \sqrt{2}I \cos(\omega_1 t + 90^\circ)$. The supply voltages are sinusoidal and symmetrical. Therefore, the line currents in Clarke coordinates are,

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \sqrt{2}I \cos(\omega_1 t + 90^\circ) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{-2}{\sqrt{3}} I \sin(\omega_1 t) \\ 0 \\ \sqrt{\frac{2}{3}} I \sin(\omega_1 t) \end{bmatrix}.$$

The instantaneous active power is,

$$\begin{aligned} p &= \sqrt{3}U \cos(\omega_1 t) \left[\frac{-2}{\sqrt{3}} I \sin(\omega_1 t) \right] + \sqrt{3}U \sin(\omega_1 t) [0], \\ &= -2UI \cos(\omega_1 t) \sin(\omega_1 t) = -UI \sin(2\omega_1 t), \end{aligned} \quad (5.15)$$

and the instantaneous reactive power:

$$q = -\sqrt{3}U \cos(\omega_1 t) [0] + \sqrt{3}U \sin(\omega_1 t) \left[\frac{-2}{\sqrt{3}} I \sin(\omega_1 t) \right],$$

$$= -2UI \sin^2(\omega_1 t) = -UI[1 - \cos(2\omega_1 t)], \quad (5.16)$$

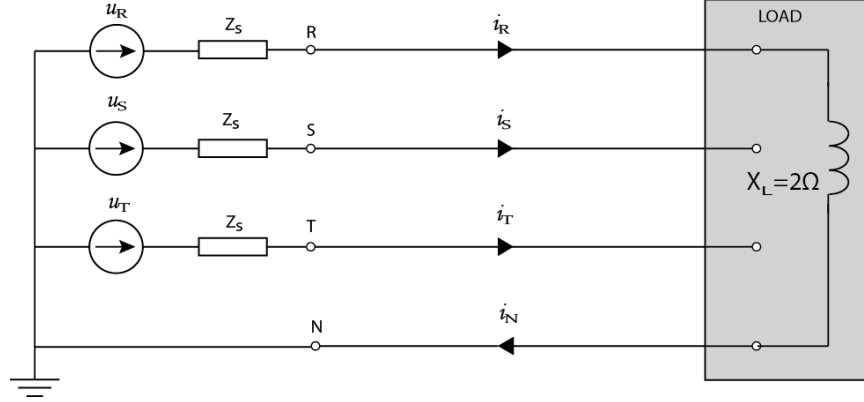


Figure 5.4: 3pN system with unbalanced inductive load supplied by sinusoidal and symmetrical voltage
The zero sequence power,

$$p_0 = u_0 i_0 = 0 \left[-\sqrt{\frac{2}{3}} I \sin(\omega_1 t) \right] = 0. \quad (5.17)$$

This system with unbalanced inductive load, according to the p-q theory has a non-zero instantaneous active power regardless of the load being unbalanced though purely inductive. Therefore, we can say that the instantaneous powers calculated in this system show similar errors when compared to the values obtained in illustration 5.3. However, the inability of the theory to represent a three-phase system cannot be single-handedly determined by the two previous illustrations. The following illustrations would be beneficial for further studies.

5.5 Balanced resistive load with sinusoidal but asymmetrical supply voltage

Unlike, all the previous illustrations, the load in the illustration considered is supplied from an asymmetrical but sinusoidal voltage source. The voltage source is as shown in Fig. 5.5 and the load is purely resistive and balanced and the source impedances are neglected. The supply voltage contains all symmetrical components, namely positive, negative and zero sequence [4] and the phase voltages in natural RST coordinates have values as $u_R = \sqrt{2}U \cos(\omega_1 t)$ and $u_S = u_T = 0$ and the line currents are

$$i_R = \sqrt{2}I \cos(\omega_1 t),$$

$$i_S = i_T = \frac{-\sqrt{2}}{2}I \cos(\omega_1 t),$$

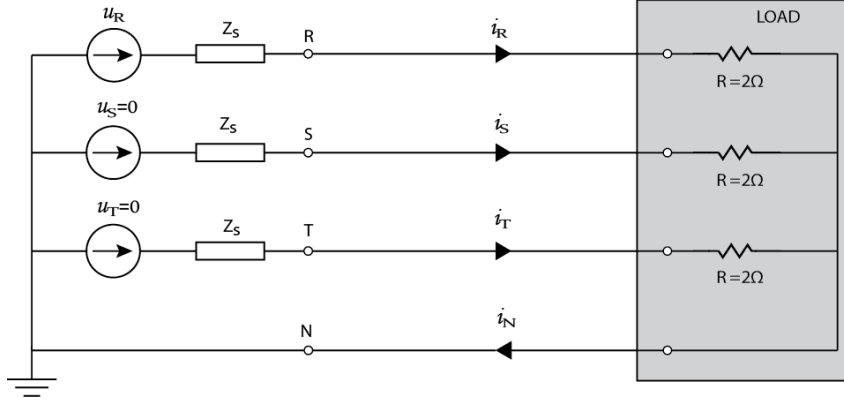


Figure 5.5: 3pN system with balanced resistive load supplied by asymmetrical but sinusoidal voltage

The line currents in Clarke coordinates are equal to,

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \mathbf{C} \begin{bmatrix} \sqrt{2}I \cos(\omega_1 t) \\ \frac{-\sqrt{2}}{2}I \cos(\omega_1 t) \\ \frac{-\sqrt{2}}{2}I \cos(\omega_1 t) \end{bmatrix} = \begin{bmatrix} \sqrt{3}I \cos(\omega_1 t) \\ 0 \\ 0 \end{bmatrix}, \quad (5.18)$$

whereas, the supply voltage in Clarke coordinates,

$$\begin{bmatrix} u_\alpha \\ u_\beta \\ u_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \sqrt{2}U \cos(\omega_1 t) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{3}}U \cos(\omega_1 t) \\ 0 \\ \sqrt{\frac{2}{3}}U \cos(\omega_1 t) \end{bmatrix}. \quad (5.19)$$

The instantaneous active and reactive powers have values,

$$\begin{aligned} p &= \left[\frac{2}{\sqrt{3}}U \cos(\omega_1 t) \right] \sqrt{3}I \cos(\omega_1 t) + [0] \\ &= 2UI \cos^2(\omega_1 t) = UI[1 + \cos(2\omega_1 t)], \end{aligned} \quad (5.20)$$

$$\begin{aligned}
q &= \left[\frac{-2}{\sqrt{3}} U \cos(\omega_1 t) \right] [0] + [0] \sqrt{3} I \cos(\omega_1 t) = \\
&= 0,
\end{aligned} \tag{5.21}$$

The zero sequence power is,

$$p_0 = u_0 i_0 = 0 \left[\sqrt{\frac{2}{3}} U \cos(\omega_1 t) \right] = 0. \tag{5.22}$$

The value of the instantaneous powers obtained in this illustration shows us that, since the load is purely resistive, it contains a non-zero active power and no reactive power. However, the zero sequence power is zero. Therefore, we can say that the IRP p-q theory reflects the power property clearly of a balanced resistive load under asymmetrical conditions.

5.6 Balanced inductive load with sinusoidal but asymmetrical supply voltage

Let us consider a balanced inductive load supplied by the same voltage source as used in illustration 5.5. The system is as shown in Fig. 5.6. The phase voltages remain the same, while the line currents in the system are

$$\begin{aligned}
i_R &= \sqrt{2} I \cos(\omega_1 t + 90^\circ), \\
i_S &= i_T = \frac{-\sqrt{2}}{2} I \cos(\omega_1 t + 90^\circ).
\end{aligned}$$

The supply voltage in $\alpha\beta 0$ coordinates is,

$$\begin{bmatrix} u_\alpha \\ u_\beta \\ u_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \sqrt{2} U \cos(\omega_1 t) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{3}} U \cos(\omega_1 t) \\ 0 \\ \sqrt{\frac{2}{3}} U \cos(\omega_1 t) \end{bmatrix}, \tag{5.23}$$

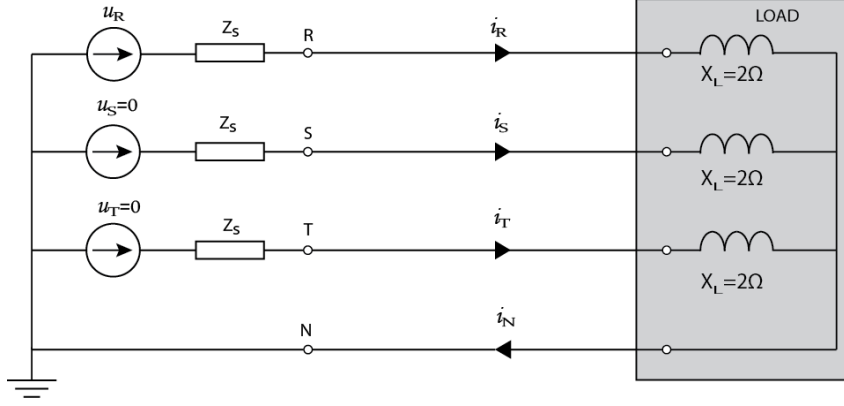


Figure 5.6: 3pN system with balanced inductive load supplied by asymmetrical but sinusoidal voltage and the line-currents in the same coordinates:

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \sqrt{2}I \cos(\omega_1 t + 90^\circ) \\ -\frac{\sqrt{2}}{2}I \cos(\omega_1 t + 90^\circ) \\ -\frac{\sqrt{2}}{2}I \cos(\omega_1 t + 90^\circ) \end{bmatrix} = \begin{bmatrix} -\sqrt{3}I \sin(\omega_1 t) \\ 0 \\ 0 \end{bmatrix}. \quad (5.24)$$

The instantaneous active power is,

$$\begin{aligned} p &= \left[\frac{2}{\sqrt{3}} U \cos(\omega_1 t) \right] [-\sqrt{3}I \sin(\omega_1 t)] + [0], \\ &= -2UI \cos(\omega_1 t) \sin(\omega_1 t) = -UI \sin(2\omega_1 t), \end{aligned} \quad (5.25)$$

and the instantaneous reactive power:

$$\begin{aligned} q &= \left[\frac{-2}{\sqrt{3}} U \cos(\omega_1 t) \right] [0] + [0] [-\sqrt{3}I \sin(\omega_1 t)] = \\ &= 0. \end{aligned} \quad (5.26)$$

The zero sequence power therefore,

$$p_0 = u_0 i_0 = \left[\sqrt{\frac{2}{3}} U \cos(\omega_1 t) \right] 0 = 0. \quad (5.27)$$

However, in this illustration it shows that an instantaneous reactive power does not occur for a balanced inductive load though supplied by an asymmetrical voltage. Moreover, it also has a negative active power. In contrast to the previous illustration, this illustration shows that the IRP p-q theory is inconsistent with its identification of power properties for different kind of loads when supplied by an asymmetrical but sinusoidal voltage

CHAPTER 6: ANALYSIS OF IRP p-q THEORY FOR NON-SINUSOIDAL THREE-PHASE SYSTEMS WITH NEUTRAL

Similar to the analysis for three-wire system in Chapter 4, this chapter also uses definitions derived in natural phase coordinates from the $\alpha\beta 0$ coordinates. These definitions are used for analyzing the IRP p-q theory under non-sinusoidal conditions.

6.1 Representation of instantaneous powers for 3pN systems in phase coordinates

The sum of supply currents in three-phase systems with a neutral is given as,

$$i_R(t) + i_s(t) + i_T(t) = i_N(t), \quad (6.1)$$

where $i_N(t)$ is the current in the neutral conductor. However, sum of supply voltages in a 3pN system measured with respect to neutral is not always,

$$u_R(t) + u_s(t) + u_T(t) = 0, \quad (6.2)$$

as this is only valid for systems supplied by symmetrical voltage (positive and negative sequence) .

Otherwise, it can be written as

$$u_R(t) + u_s(t) + u_T(t) = u_N(t) \quad (6.3)$$

Since, we also know that,

$$\begin{bmatrix} u_\alpha \\ u_\beta \\ u_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} u_R \\ u_s \\ u_T \end{bmatrix},$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} i_R \\ i_s \\ i_T \end{bmatrix},$$

the instantaneous powers can be defined in terms of natural phase coordinates. The instantaneous active power and reactive power can be defined as,

$$\begin{aligned}
 p &= u_\alpha i_\alpha + u_\beta i_\beta = \\
 &= \left[\sqrt{\frac{2}{3}} u_R - \frac{1}{\sqrt{6}} u_S - \frac{1}{\sqrt{6}} u_T \right] \left[\sqrt{\frac{2}{3}} i_R - \frac{1}{\sqrt{6}} i_S - \frac{1}{\sqrt{6}} i_T \right] + \left[\frac{1}{\sqrt{2}} u_S - \frac{1}{\sqrt{2}} u_T \right] \left[\frac{1}{\sqrt{2}} i_S - \frac{1}{\sqrt{2}} i_T \right], \\
 &= \frac{2}{3} [u_R i_R + u_S i_S + u_T i_T] - \frac{1}{3} i_S [u_R + u_T] - \frac{1}{3} i_T [u_R + u_S] - \frac{1}{3} i_R [u_S + u_T], \tag{6.4}
 \end{aligned}$$

According to eqn. (6.2), if the voltage with respect to neutral in a 3pN system is sinusoidal and symmetrical, the above eqn. can be modified as,

$$p = \frac{2}{3} [u_R i_R + u_S i_S + u_T i_T] + \frac{1}{3} u_S i_S + \frac{1}{3} u_R i_R + \frac{1}{3} u_T i_T = \tag{6.5}$$

$$= u_R i_R + u_S i_S + u_T i_T, \tag{6.6}$$

The instantaneous reactive power is,

$$\begin{aligned}
 q &= -u_\alpha i_\beta + u_\beta i_\alpha = \\
 &= \left[-\sqrt{\frac{2}{3}} u_R + \frac{1}{\sqrt{6}} u_S + \frac{1}{\sqrt{6}} u_T \right] \left[\frac{1}{\sqrt{2}} i_S - \frac{1}{\sqrt{2}} i_T \right] + \left[\frac{1}{\sqrt{2}} u_S - \frac{1}{\sqrt{2}} u_T \right] \left[\sqrt{\frac{2}{3}} i_R - \frac{1}{\sqrt{6}} i_S - \frac{1}{\sqrt{6}} i_T \right], \tag{6.7}
 \end{aligned}$$

$$q = \frac{1}{\sqrt{3}} \{u_R [i_T - i_S] + u_S [i_R - i_T] + u_T [i_S - i_R]\}, \tag{6.8}$$

or it can be written as,

$$q = \frac{1}{\sqrt{3}} \{i_T [u_R - u_S] + i_S [u_T - u_R] + i_R [u_S - u_T]\}. \tag{6.9}$$

The zero sequence power is,

$$p_0 = u_0 i_0 = \frac{1}{3} [u_R + u_S + u_T] [i_R + i_S + i_T] = \quad (6.10)$$

$$= 3u_z i_z$$

where u_z and i_z are zero sequence current and voltages respectively.

6.2 Calculations of instantaneous powers in a 3pN system with resistive balanced HGL at sinusoidal voltage

Let us consider a 3pN system as shown in Fig. 6.1, which contains a balanced resistive HGL supplied from a sinusoidal and symmetrical (positive sequence) voltage source with respect to neutral, such that the sum of the phase voltages is equal to zero. The line currents in this system are composed of fundamental as well as 5th order harmonic. The HGL generates 5th order symmetrical current harmonic.

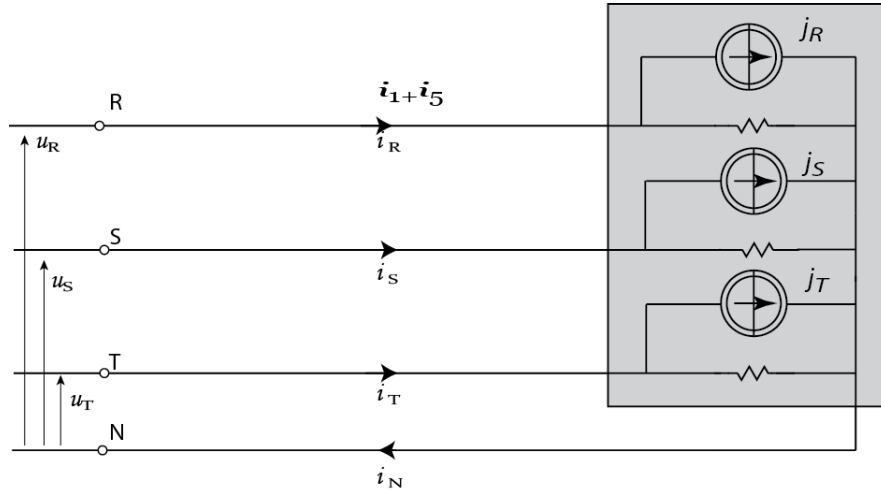


Figure 6.1: 3pN system with balanced resistive HGL supplied by non-sinusoidal but symmetrical voltage

The phase voltages in this system as

$$u_R = \sqrt{2}U_1 \cos(\omega_1 t),$$

$$u_S = \sqrt{2}U_1 \cos(\omega_1 t - 120^0),$$

$$u_T = \sqrt{2}U_1 \cos(\omega_1 t + 120^0),$$

whereas the line currents in the system can be written as,

$$i_R = \sqrt{2}I_1 \cos(\omega_1 t) + \sqrt{2}I_5 \cos(5\omega_1 t + \alpha_5),$$

$$i_S = \sqrt{2}I_1 \cos(\omega_1 t - 120^0) + \sqrt{2}I_5 \cos(5\omega_1 t + \alpha_5 + 120^0),$$

$$i_T = \sqrt{2}I_1 \cos(\omega_1 t + 120^0) + \sqrt{2}I_5 \cos(5\omega_1 t + \alpha_5 - 120^0), \quad (6.11)$$

Therefore, calculating the instantaneous active and reactive power using equations (6.6) and (6.8) we get the instantaneous active power as,

$$\begin{aligned} p &= u_R i_R + u_S i_S + u_T i_T, \\ &= \{2U_1 \cos(\omega_1 t) [I_1 \cos(\omega_1 t) + I_5 \cos(5\omega_1 t + \alpha_5)] + \\ &2U_1 \cos(\omega_1 t - 120^0) [I_1 \cos(\omega_1 t - 120^0) + I_5 \cos(5\omega_1 t + \alpha_5 + 120^0)] \\ &+ 2U_1 \cos(\omega_1 t + 120^0) [I_1 \cos(\omega_1 t + 120^0) + I_5 \cos(5\omega_1 t + \alpha_5 - 120^0)]\}, \end{aligned} \quad (6.12)$$

$$p = 3U_1 I_1 + 3U_1 I_5 \cos(6\omega_1 t + \alpha_5), \quad (6.13)$$

and the instantaneous reactive power as,

$$\begin{aligned} q &= \frac{1}{\sqrt{3}} \{u_R [i_T - i_S] + u_S [i_R - i_T] + u_T [i_S - i_R]\} \\ &= \{[2U_1 I_1 \cos(\omega_1 t) \sin(\omega_1 t) + 2U_1 I_5 \cos(\omega_1 t) \sin(5\omega_1 t + \alpha_5)] + \\ &[2U_1 I_1 \cos(\omega_1 t - 120^0) \sin(\omega_1 t + 60^0) - 2U_1 I_5 \cos(\omega_1 t - 120^0) \sin(5\omega_1 t - 60^0 + \alpha_5)] + \\ &[2U_1 I_1 \cos(\omega_1 t + 120^0) \sin(\omega_1 t - 60^0) - 2U_1 I_5 \cos(\omega_1 t + 120^0) \sin(5\omega_1 t + 60^0 + \alpha_5)]\} \end{aligned}$$

$$q = 3U_1 I_5 \sin(6\omega_1 t + \alpha_5). \quad (6.14)$$

The zero sequence power is equal to zero and these values of the instantaneous powers are identical to the equations (4.12) and (4.13) in chapter 4. It is because; this system is supplied by sinusoidal and symmetrical voltage and has non-sinusoidal though symmetrical currents. Moreover, this can be a verification of the similar condition illustrated in section 4.2.

Now, let us replace the 5th order harmonic generating load with a 7th order harmonic generating resistive balanced load. Unlike, the 5th order harmonic, the 7th order harmonic is of a positive sequence. The phase voltages remain the same, whereas the line currents for this system shown in fig 6.2 are,

$$\begin{aligned} i_R &= \sqrt{2}I_1 \cos(\omega_1 t) + \sqrt{2}I_5 \cos(7\omega_1 t + \alpha_7), \\ i_S &= \sqrt{2}I_1 \cos(\omega_1 t - 120^\circ) + \sqrt{2}I_7 \cos(7\omega_1 t + \alpha_7 - 120^\circ), \\ i_T &= \sqrt{2}I_1 \cos(\omega_1 t + 120^\circ) + \sqrt{2}I_5 \cos(7\omega_1 t + \alpha_7 + 120^\circ), \end{aligned} \quad (6.15)$$

Calculating the instantaneous active power as,

$$\begin{aligned} p &= u_R i_R + u_S i_S + u_T i_T, \\ &= \{2U_1 \cos(\omega_1 t) [I_1 \cos(\omega_1 t) + I_7 \cos(7\omega_1 t + \alpha_7)] + \\ &\quad 2U_1 \cos(\omega_1 t - 120^\circ) [I_1 \cos(\omega_1 t - 120^\circ) + I_7 \cos(7\omega_1 t + \alpha_7 - 120^\circ)] \\ &\quad + 2U_1 \cos(\omega_1 t + 120^\circ) [I_1 \cos(\omega_1 t + 120^\circ) + I_7 \cos(7\omega_1 t + \alpha_7 + 120^\circ)]\}, \end{aligned} \quad (6.16)$$

$$p = 3U_1 I_1 + 3U_1 I_7 \cos(6\omega_1 t + \alpha_7), \quad (6.17)$$

and the instantaneous reactive power,

$$\begin{aligned}
q &= \frac{1}{\sqrt{3}} \{u_R[i_T - i_S] + u_S[i_R - i_T] + u_S[i_S - i_R]\} \\
&= \{[-2U_1I_1 \cos(\omega_1 t) \sin(\omega_1 t) - 2U_1I_7 \cos(\omega_1 t) \sin(7\omega_1 t + \alpha_7)] + \\
&\quad [2U_1I_1 \cos(\omega_1 t - 120^\circ) \sin(\omega_1 t + 60^\circ) + 2U_1I_7 \cos(\omega_1 t - 120^\circ) \sin(7\omega_1 t + 60^\circ + \alpha_7)] + \\
&\quad [2U_1I_1 \cos(\omega_1 t + 120^\circ) \sin(\omega_1 t - 60^\circ) + 2U_1I_7 \cos(\omega_1 t + 120^\circ) \sin(7\omega_1 t - 60^\circ + \alpha_7)]\} \\
q &= -3U_1I_7 \sin(6\omega_1 t + \alpha_7). \tag{6.18}
\end{aligned}$$

Comparing equations (6.17), (6.18) with (6.13), (6.14) respectively shows that the change in harmonic order in the two previous illustrations does not have any effect on the harmonic order of instantaneous powers. There is only a change in magnitude.

6.3 Calculations of instantaneous powers in a 3pN system with resistive balanced load at non-sinusoidal but symmetrical voltage

A 3pN system with a balanced resistive load and supplied by a non-sinusoidal voltage is shown in Fig. 6.2. The voltage is symmetrical and it is distorted by a 5th order harmonic. The assumed phase voltages can be written as,

$$\begin{aligned}
u_R &= \sqrt{2}U_1 \cos(\omega_1 t) + \sqrt{2}U_5 \cos(5\omega_1 t), \\
u_S &= \sqrt{2}U_1 \cos(\omega_1 t - 120^\circ) + \sqrt{2}U_5 \cos(5\omega_1 t + 120^\circ), \\
u_T &= \sqrt{2}U_1 \cos(\omega_1 t + 120^\circ) + \sqrt{2}U_5 \cos(5\omega_1 t - 120^\circ), \tag{6.19}
\end{aligned}$$

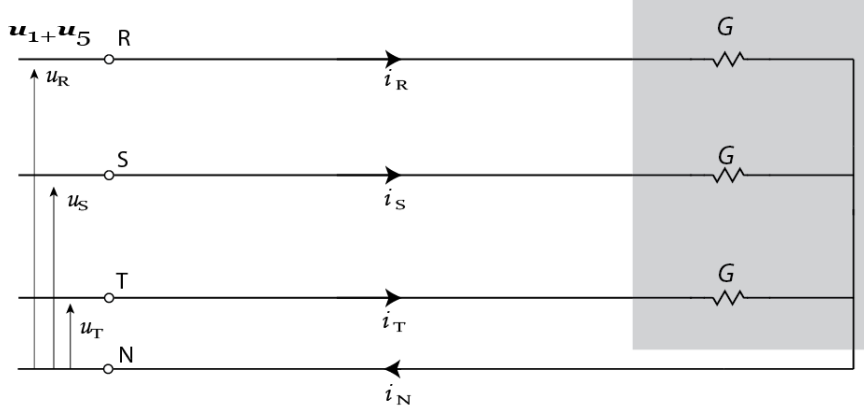


Figure 6.2: 3pN system with balanced resistive load supplied by sinusoidal and symmetrical voltage and the line currents are,

$$i_R = \sqrt{2}I_1 \cos(\omega_1 t),$$

$$i_S = \sqrt{2}I_1 \cos(\omega_1 t - 120^\circ),$$

$$i_T = \sqrt{2}I_1 \cos(\omega_1 t + 120^\circ),$$

Therefore, the instantaneous active powers in this system is,

$$\begin{aligned} p &= u_R i_R + u_S i_S + u_T i_T, \\ &= \{2I_1 \cos(\omega_1 t) [U_1 \cos(\omega_1 t) + U_5 \cos(5\omega_1 t)] + \\ &\quad 2I_1 \cos(\omega_1 t - 120^\circ) [U_1 \cos(\omega_1 t - 120^\circ) + U_5 \cos(5\omega_1 t + 120^\circ)] \\ &\quad + 2I_1 \cos(\omega_1 t + 120^\circ) [U_1 \cos(\omega_1 t + 120^\circ) + U_5 \cos(5\omega_1 t - 120^\circ)]\}, \end{aligned} \quad (6.20)$$

$$p = 3U_1 I_1 + 3U_5 I_1 \cos(6\omega_1 t), \quad (6.21)$$

and the instantaneous reactive power is,

$$q = \frac{1}{\sqrt{3}} \{u_R [i_T - i_S] + u_S [i_R - i_T] + u_T [i_S - i_R]\} =$$

$$2U_5I_1 \cos(6\omega_1t + 120^0) - 2U_5I_1 \cos(6\omega_1t - 120^0) - \\ U_5I_1 \cos(6\omega_1t + 240^0) + U_5I_1 \cos(6\omega_1t - 240^0) \quad (6.22)$$

$$q = -3U_5I_1 \sin(6\omega_1t) \quad (6.23)$$

After calculating the instantaneous powers for this 3pN system, it is clear that the IRP p-q theory cannot differentiate between a three-phase balanced resistive load supplied by a non-sinusoidal voltage and a three-phase balanced resistive HGL supplied by a sinusoidal supply voltage. This is concluded after the values of instantaneous active and reactive powers in eqn. (6.21) and (6.23) are very identical to the powers calculated in illustration 6.2 for 5th order harmonic. Moreover, the reactive power has a non-zero value for a balanced resistive load. Thus, the above two illustrations show that the IRP p-q theory does not identify the power properties correctly under non-sinusoidal conditions.

CHAPTER 7: CONCLUSION

This thesis, in general shows that the IRP p-q theory, akin to the three-wire systems, when considered as a power theory for 3pN systems, is incapable of explaining the power phenomena in electrical system. The results of the analysis for asymmetrical voltage and unbalanced load conditions in 3pN system prove that according to IRP p-q theory, an unbalanced though resistive load consists of reactive power and similarly an unbalanced inductive load has a non-zero active power. Moreover, the major deficiency of this theory would be its inconsistency with the identification of the power properties of the load under asymmetrical conditions. It is seen that the theory properly interprets the power phenomena for a balanced resistive load under asymmetry, but gives erroneous results when used for a balanced inductive load under same supply conditions. Due to this inconsistency, it is difficult to rely on such a theory for interpretation of power phenomena.

The analysis of IRP p-q theory for non-sinusoidal conditions such as distorted supply voltage and harmonic-generating loads also provides us with an evaluation of performance of the p-q theory. For an instance, it shows that the values of instantaneous powers in a 3pN system with a balanced HGL supplied by a sinusoidal and symmetrical voltage does not change with the harmonic order. In other words, the values of instantaneous powers do not change when a 5th order current harmonic generating load is replaced by a 7th order current HGL. Moreover, the main issue surrounding the IRP p-q theory under non-sinusoidal conditions is identification of source of distortion. For an example, using p-q theory does not clearly determine whether the source of distortion is from supply or load, as the instantaneous values for illustrations (6.2) and (6.3) are almost the same.

Hence, the analysis provided in this thesis proves that the misinterpretations of power properties pointed out in IRP p-q theory for three-wire systems prevail in four-wire systems too.

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