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OF A DISK IN A VISCOUS FLUID.

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HIGH-AMPLITUDE TORSIONAL OSCILLATIONS
OF A DISK IN A VISCOUS FLUID

A Dissertation

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Louisiana State University and
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in

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TABLE OF CONTENTS

	Page
I. INTRODUCTION	1
II. THEORETICAL EXPRESSIONS FOR THE ADDED MOMENT OF INERTIA AND THE DAMPING	3
III. EXPERIMENT	
IV. RESULTS AND CONCLUSIONS	8
REFERENCES	14
APPENDIX I	
CORNER PARAMETERS FOR THE OSCILLATING DISK; CONSEQUENCES FOR LIQUID HELIUM II.	AI-1
APPENDIX II	
QUANTITIES USED IN THE CALCULATION OF $\Delta I_a/I_a$ AND $\Delta D/D_1$	AII-1
APPENDIX III	
BOUNDARY VARIATIONS	AIII-1
VITA	16

LIST OF TABLES

Table	Page
I. Dimensions of the Experimental Disks	6b
II. Torsion Constants of the Quartz Rods at 23.3°C	AII-1a
III. Fluid Viscosities and Densities	AII-2a
IV. Experimental Quantities Measured	AII-2b
V. Calculated Quantities	AII-2c
VI. Equations Used to Calculate $\Delta I_a/I_a$ and $\Delta D/D_1$	AII-3a

LIST OF FIGURES

Figure	Page
1. Notation for the Experimental Disk	6a
2. Amplitude Dependence of the Relative Changes in Logarithmic Decrement D and Added Moment of Inertia I_a for Disk of Different Thickness	8a
3. Amplitude Dependence of the Relative Changes in Logarithmic Decrement D and Added Moment of Inertia I_a for Disks of Different Radius	9a
4. Empirical Correlations of $\Delta I_a/I_a$ for Low Amplitudes	9b
5. Empirical Correlations of $\Delta I_a/I_a$ at High Amplitude	10a
6. Empirical Correlation for $\Delta D/D_1$	10b
7. Period of Oscillation as a Function of Amplitude for Three Positions of the Lower Plane Boundary d .	11a
8. The Dissipative (C_D) and Inertial Parts (C_I) of the Corner Parameter vs. the Ratio of Disk Thickness t to Penetration Depth λ .	AI-2a
9. Corrected Values of Normal Fluid Density ρ_n for $C_I = 0$.	AI-3a
10. The Change in Logarithmic Decrement ΔD vs. Amplitude ϵ for Three Positions of the Lower Plane Boundary d .	AIII-1a
11. The Change in Period ΔT vs. Amplitude ϵ for Three Positions of the Cylindrical Boundary	AIII-1b
12. The Change in Logarithmic Decrement ΔD vs. Amplitude ϵ for Three Positions of the Cylindrical Boundary	AIII-1c
13. The Change in Logarithmic Decrement ΔD vs. Amplitude ϵ for Two Positions of the Cylindrical Boundary	AIII-1d

ABSTRACT

Measurements have been made of the logarithmic decrement and period of disks, of radius 3.0 to 8.5 cm, performing torsional oscillations at amplitudes in the range 0.01 to 1.6 rad in fluids of kinematic viscosity 0.29 to 3.06 centistokes. Both the logarithmic decrement and period were observed to depend on the amplitude of oscillation in qualitative agreement with a theory due to Rosenblat which predicts amplitude effects due to secondary flow. It is shown that a consequence of the theory is that the relative changes in the added moment of inertia I_a and the logarithmic decrement D should be independent of the kinematic viscosity of the fluid and the size of the disk. The observed relative changes were smaller than predicted and were dependent on the radius of the disk but were relatively insensitive to kinematic viscosity, disk thickness, or distance to the outer boundaries. The observed relative changes are given by the following empirical correlation:

$$\Delta I_a / I_a = -0.029 (a/\lambda)^{1/2} (\epsilon - 0.21)^2$$

for $0 < (a/\lambda)^{1/2} (\epsilon - 0.21)^2 < 3.0$,

$$\Delta I_a / I_a = -0.224 [\epsilon - 1.27 (\lambda/a)^{1/4}]$$

at higher amplitudes, and

$$\Delta D/D_1 = 0.0045 (a/\lambda)^{3/4} (\epsilon - 0.32)^2$$

for $0.32 < \epsilon < 1.6$ rad. a is the radius of the disk, λ is the viscous penetration depth and ϵ is the amplitude of oscillation in radians. These empirical correlations point out the existence of critical amplitudes which must be exceeded before amplitude effects occur. Below 0.2 rad the period was constant within 0.001% and below 0.3 rad the logarithmic decrement was constant within 0.3%. New corner parameters for the oscillating disk were obtained from the data in the range $0.01 < \epsilon < 0.2$ rad; these parameters show that the effect of the fluid located at the sharp corner of the disk, where the plane faces and cylindrical edge meet, is to increase the logarithmic decrement but to have no effect on the added moment of inertia. This result is used to correct the pile of disks measurements of the normal fluid density of liquid helium II.

I. INTRODUCTION

The problem of a disk oscillating in a viscous fluid has been studied extensively, from the early experimental work of Coulomb¹ and theoretical work of Stokes² to the recent calculation of corner effects by Azpeitia and Newell^{3,4} and the high precision viscosity measurements of Kestin and co-workers.^{5,6} Most of this work, however, assumed oscillations of low amplitude, so that radial flow could be neglected.

Recently, Rosenblat⁷ investigated the radial and axial secondary flows due to the torsional oscillations of an infinite plane in an incompressible viscous fluid of density ρ and kinematic viscosity ν . He solved the appropriate Navier-Stokes equations by a perturbation method in which the velocity components were expanded in power series in the amplitude ϵ . The first order solution gives the first approximation to the azimuthal velocity, from which the shearing stress and torque can be calculated. If the angular velocity of the plane is $\dot{\theta} = \epsilon\omega\sin\omega t$, then the torque on a section of radius a is given by

$$M_0 = -\pi a^4 \rho \epsilon (\nu \omega^3 / 2)^{1/2} (\sin\omega t + \cos\omega t). \quad (1)$$

The second order solution gives the first non-vanishing approximation to the radial and axial velocities, both of

which have steady and fluctuating parts. The steady parts and the fluctuating part of the axial velocity persist outside of the viscous boundary layer (thickness $O(\nu/\omega)^{1/2}$). The torque given by Eq (1) is not changed in the second order approximation. However, the third order approximation to the azimuthal velocity results in an amplitude dependent torque given by

$$M = -\pi a^4 \rho \epsilon (\nu \omega^{3/2})^{1/2} [(1 + 0.06\epsilon^2) \sin \omega t + (1 - 0.262\epsilon^2) \cos \omega t + 0.012\epsilon^2 (\sin 3\omega t + \cos 3\omega t)] \quad (2)$$

This problem has also been studied by Benney,⁸ who used a multiple scaling technique, and by Riley,⁹ who used an inner-outer expansion method. Riley showed that the next order term in the torque would be $O(\epsilon^3)$ and not $O(\epsilon^4)$ as suggested by Rosenblat. The connection between the work of Rosenblat and that of Benney has been discussed by Stuart.¹⁰

Hanold and Moszynski,¹¹ using disks of diam 2 to 3 inches oscillating with periods of 20 to 30 sec in water, have observed a radial-axial secondary flow. However, they observed no variation of logarithmic decrement with amplitude; an amplitude dependent logarithmic decrement is a consequence of Eq. (2) as shown below.

We report here measurements of the logarithmic decrement and the period of disks oscillating at amplitudes in the range $0.01 \leq \epsilon \leq 1.6$ rad in fluids of kinematic viscosity 0.29 to 3.06 centistokes. We have observed changes in both damping and period in qualitative agreement with Rosenblat's theory, but the magnitudes of the observed changes are smaller than predicted.

II. THEORETICAL EXPRESSIONS FOR THE ADDED MOMENT OF INERTIA AND THE DAMPING

In this section we derive expressions for the changes in added moment of inertia and logarithmic decrement which are implied by Rosenblat's theory. Consider a disk, of radius a and moment of inertia I , oscillating with angular frequency ω in a fluid of kinematic viscosity ν . If edge effects are neglected and the torque is given by Eq. (2), then the equation of motion is

$$I\ddot{\theta} + \gamma\dot{\theta} = M \quad (3)$$

where γ is the torsion constant. In vacuum, $M = 0$, and if the frequency is denoted by ω_0 then

$$\omega_0^2 I = \gamma. \quad (4)$$

If we require that at $t = 0$, $\theta = -\epsilon$ and $\dot{\theta} = 0$, then the zero order solution is $\theta_0 = -\epsilon \cos \omega_0 t$. The first order solution, θ_1 , is obtained for the case in which the disk is oscillating at very small amplitude in the fluid:

$$I\ddot{\theta}_1 + \gamma\dot{\theta}_1 = -\epsilon\omega_0^2 I_a (\sin \omega_0 t + \cos \omega_0 t) \quad (5)$$

where $I_a = \pi \rho a^4 (\nu/2\omega_0)^{1/2}$. The solution of Eq. (5) that satisfies the conditions $\theta_1(0) = -\epsilon$ and $\dot{\theta}_1(0) = 0$ is

$$\theta_1 = -\epsilon(1 - A\omega_0 t) \cos \omega_0 t - \epsilon A(1 + \omega_0 t) \sin \omega_0 t \quad (6)$$

where $A = I_a/2I$. Let τ_0 be the zero order period, i.e.

$\tau_0 = 2\pi/\omega_0$, and τ_1 be the first order period, which we will find from the condition $\dot{\theta}_1(\tau_1/2) = 0$. We define

$\Delta\tau_1 = \tau_1 - \tau_0$ and assume that $\Delta\tau_1 \ll \tau_0$, a condition which will be seen to be valid if $A \ll 1$. At $t = \tau_1/2$ we have

$\omega_0 \tau_1/2 = \pi[1 + (\Delta\tau_1/\tau_0)]$, $\cos(\omega_0 \tau_1/2) \approx -1$, and

$\sin(\omega_0 \tau_1/2) \approx -\pi\Delta\tau_1/\tau_0$. From the condition $\dot{\theta}_1(\tau_1/2) = 0$ we find

$$\Delta\tau_1 = \tau_0 A, \quad (7)$$

so that the period is increased. Since the added moment of inertia is given by

$$(\gamma/4\pi^2) (\tau_1^2 - \tau_0^2) \approx 2I\Delta\tau_1/\tau_0, \quad (8)$$

Eq. (7) states simply that I_a is the added moment of inertia. The angular displacement at $t = \tau_1/2$ is (to first

order in A)

$$\theta_1(\tau_1/2) \approx \epsilon(1 - \pi A) \approx \epsilon \exp(-\pi A). \quad (9)$$

Thus, the logarithmic decrement is

$$D_1 = 2\pi A = \pi I_a / I \quad (10)$$

We see from Eqs (7) and (10) that $\Delta\tau_1 \ll \tau_0$ and $D_1 \ll 1$ if $A \ll 1$.

The second order solution, θ_2 is obtained from

$$I\ddot{\theta}_2 + \gamma\theta_2 = M \quad (11)$$

where M is given by Eq (2) with $\omega = \omega_0$. The solution of Eq (11) subject to the conditions $\theta_2(0) = -\epsilon$ and $\dot{\theta}_2(0) = 0$ is

$$\begin{aligned} \theta_2 = & -\epsilon[\alpha \cos \omega_0 t + \beta \sin \omega_0 t \\ & + 0.003A\epsilon^3(\sin 3\omega_0 t + \cos 3\omega_0 t)] \end{aligned} \quad (12)$$

where

$$\alpha = 1 + 0.003A\epsilon^2 - A(1 + 0.06\epsilon^2)\omega_0 t \quad (13)$$

and

$$\beta = A[1 + 0.069\epsilon^2 + \omega_0 t(1 - 0.262\epsilon^2)]. \quad (14)$$

The change in period $\Delta\tau_2$ is obtained by defining

$\Delta\tau_2 = \tau_2 - \tau_0$ ($\Delta\tau_2 \ll \tau_0$) and requiring $\dot{\theta}_2 = 0$ at $t = \tau_2/2$.

The result is (to first order in A)

$$\Delta\tau_2 = \tau_0 A(1 - 0.262\epsilon^2) \quad (15)$$

so that the amplitude dependent terms cause a decrease in period. The logarithmic decrement is

$$D_2 = (\pi I_a / I) (1 + 0.06\epsilon^2). \quad (16)$$

From Eqs (7) and (15) the relative change in added moment

of inertia with amplitude is found to be

$$\Delta I_a / I_a = -0.262\epsilon^2; \quad (17)$$

and from Eqs. (10) and (16), the relative change of logarithmic decrement is found to be

$$\Delta D / D_1 = 0.06\epsilon^2. \quad (18)$$

The results of this approach show that the higher harmonic terms in Eq. (2) do not contribute to the changes in damping and added inertia. Equations (17) and (18) are compared with the experimental results in Section IV.

III EXPERIMENT

The experimental disks (Fig. 1 and Table I) were made of brass except disk 7 which was made of aluminum. A torsion pendulum was formed with these disks by attaching them to the bottom of a brass rod (40 cm long and 6 mm diam) which was suspended from a quartz torsion rod (80 cm long and 0.8 to 1.4 mm diam). Inertial disks, outside the bath, were attached near the top of the brass rod to reduce the damping. A mirror on the brass rod was used to reflect light from a straight-filament lamp onto a curved scale 1 m in radius and 3.4 m long. The same light beam was used to activate a photomultiplier tube located at the zero of the scale. The pulse from the photomultiplier triggered an

Fig. 1. Notation for the Experimental Disk; Radius a , Thickness t , Distance to the Cylindrical Boundary b and Distance to the Lower Plane Boundary d .

Table I. Dimensions of the Experimental Disks
(Radius a , Thickness t).

Disk	1	2	3	4	5	6	7
$a(\text{cm})$	3.008	3.011	3.013	5.097	5.079	8.550	2.999
$t(\text{cm})$	0.167	0.642	0.953	0.312	0.631	0.655	5.983

electronic timing circuit built around a Computer Measurements Corp. Model 226A counter-timer. In this way single periods of about 6 sec could be measured to within 0.05 msec at 1 rad and 0.15 msec at 0.2 rad.

The quartz torsion rods were drawn by hand and the ends were thickened to about 3 mm diam. Small stainless steel cylinders (10 mm long and 5 mm I.D.) were slipped on the ends and attached with Stycast 3020 epoxy resin. The rods were then mounted by gripping the steel cylinders. Quartz torsion rods made in this way supported over 3 kg.

The torsion constant γ (Eq. 4) was determined by measuring the period of oscillation in air for a disk of known inertia. The temperature coefficient of γ near 24°C was found to be $\gamma^{-1}(\partial\gamma/\partial T) = (12 \pm 2) \times 10^{-5}/^{\circ}\text{C}$, which is close to the value found for thin fibers by Hussey, Good, and Reynolds.¹² The torsion properties of the quartz were remarkably independent of amplitude: for deflections up to 2 rad in air, the period changed less than 0.04 msec and the logarithmic decrement changed less than 4×10^{-5} for all rods except the thickest, for which these changes were 0.4 msec and 23×10^{-5} respectively; the data in liquids were corrected for these small changes. The "nuisance decrement" due to losses in the torsion rod ranged from 0.0013 (for thickest rod) to 0.0003.

The pendulum was released from rest at an amplitude of about 2 rad, and periods and amplitudes were observed as the oscillations decayed to about 0.1 rad. Occasionally, readings were taken down to 0.01 rad. The inertial disks were chosen large enough so that up to 200 swings could be observed; the constant $A = I_a/2I$ was never more than 4×10^{-3} and consequently the logarithmic decrement was never greater than 0.03. Several runs were made for each disk and fluid, and the temperature of the room and the fluid were recorded before and after each run. The fluids used were carbon disulfide, acetone, water, and isopropyl alcohol. The period was always within 3% of 6 sec, so if the kinematic viscosity is ν , the penetration depth λ , defined as $(2\nu/\omega)^{1/2}$, ranged from 0.07 to 0.23 cm. Most of the runs were made with the disk centered in a cylindrical container of diam 24.4 cm and with a fluid depth of 20 cm; the effect of nearer boundaries is discussed below.

IV. RESULTS AND CONCLUSIONS

The observed values of $\Delta I_a/I_a$ and $\Delta D/D_1$ are compared with Eqs. (17) and (18) in Figs. 2 and 3. Figure 2 shows the results for disks 1, 2, 3 and 7 in water. Although the thickness t was varied by a factor of 36, no systematic

Fig. 2. Amplitude Dependence of the Relative Changes in Logarithmic Decrement D and Added Moment of Inertia I_a for Disks of Different Thickness. The Solid Lines are Obtained from the Theory of Rosenblat (Eqs. 17,18).

change with thickness was observed. This result was expected since the added moment of inertia and damping due to the cylindrical edge (considered as a section of an infinite surface) can be found from an exact, amplitude independent solution of the Navier-Stokes equation. We thus conclude that the observed changes $\Delta I_a/I_a$ and $\Delta D/D_1$ are due only to the action of the fluid on the plane faces of the disk. The effect produced by a variation in radius is shown in Fig. 3 where disks 1, 5 and 6 were used in water. Contrary to the theory, $\Delta I_a/I_a$ and $\Delta D/D_1$ are radius dependent. A weak dependence on kinematic viscosity was also observed and in general the magnitudes of $\Delta I_a/I_a$ and $\Delta D/D_1$ increased with increasing radius and decreased with increasing penetration depth, $\lambda = (2\nu/\omega)^{1/2}$.

Attempts to obtain empirical correlations of the results are shown in Figs. 4, 5 and 6. Figure 4 shows that the relation

$$\Delta I_a/I_a = -0.029 (a/\lambda)^{1/2} (\epsilon - 0.21)^2 \quad (19)$$

represents the data for $0 < (a/\lambda)^{1/2} (\epsilon - 0.21)^2 < 3.0$. It should be mentioned, however, that the relation $\Delta I_a/I_a = -0.02 (a/\lambda)^{1/2} \epsilon^3$ is also a reasonable fit in the same range of ϵ . For higher amplitudes $\Delta I_a/I_a$ is linear in

Fig. 3. Amplitude Dependence of the Relative Changes in Logarithmic Decrement D and Added Moment of Inertia I_a for Disks of Different Radius. The Solid Lines are Obtained from the Theory of Rosenblat (Eqs. 17, 18).

Fig. 4. Empirical Correlation of $\Delta I_a/I_a$ for Low Amplitude. See Table I for Disk Dimensions.

the amplitude; Fig. 5 shows that the data in this range is correlated by

$$\Delta I_a / I_a = -0.224[\epsilon - 1.27(\lambda/a)^{1/4}]. \quad (20)$$

Figure 6 shows that the relation

$$\Delta D/D_1 = 0.0045 (a/\lambda)^{3/4} (\epsilon - 0.32)^2 \quad (21)$$

represents the damping data reasonably well over the entire amplitude range $0.32 < \epsilon < 1.6$. $\Delta D/D_1 = 0.006 (a/\lambda)^{1/2} \epsilon^3$ also fits the data but only for $(a/\lambda)^{1/2} \epsilon^3 < 7.0$.

Our observations agree with Hanold and Moszynski's result¹¹ that the low amplitude logarithmic decrement is independent of the starting amplitude. Applying the empirical correlation Eq. (21) to Hanold and Moszynski's largest disk ($a = 3.629\text{cm}$, $\lambda = 0.25\text{cm}$) we find that $\Delta D/D_1 < 0.011$ at the maximum amplitude ($\epsilon = 0.91$). Since their logarithmic decrement observations were apparently taken only at low amplitude, it is understandable that they did not see any amplitude dependence.

For amplitudes below 0.3 rad down to the lowest amplitudes employed in this work (0.01 rad), the logarithmic decrement was constant within experimental accuracy (0.3%). Therefore, the use of the oscillating disk as a viscometer does not impose severe amplitude restrictions. For $0.01 < \epsilon < 0.2$ rad, the period was constant within 0.001%. Foch and Barriol¹³ reported that a tip Reynolds number ($R = \epsilon \omega a^2 / \nu$) of 60 had to be exceeded for the onset of

Fig. 5. Empirical Correlation of $\Delta I_a / I_a$ at High Amplitude. See Fig. 4 for Table of Symbols.

Fig. 6. Empirical Correlation for $\Delta D/D_1$ (a) at Low Amplitudes (b) Over the Entire Amplitude Range $.32 < \epsilon < 1.6$ Rad. See Fig. 4 for Table of Symbols.

observable secondary flow. Contrary to this result, Hanold and Moszynski observed a gradual development of the secondary flow pattern with increasing amplitude. Our data indicate that there are critical amplitudes (≈ 0.2 rad for the period and 0.3 rad for the damping) which must be exceeded before the torque becomes amplitude dependent. Since these critical amplitudes are independent of the radius and kinematic viscosity, they cannot be associated with a critical tip Reynolds number.

The observed changes in period and damping were found to be independent of the distance to the boundaries, at least when the boundaries were several penetration depths away from the disk. The distance b to the outer cylindrical boundary (Fig. 1) was varied from 7.32 cm to 0.82 cm, and the distance d to the lower boundary was varied from 7.4 cm to 0.90 cm. Figure 7 shows the results of period measurements for three values of d for disk No. 1 in water; similar results were obtained for the damping. Hanold and Moszynski have suggested that the difference between their observed secondary flow and that predicted by Rosenblat is due to Rosenblat's neglect of the radial pressure gradient. In a second paper, Rosenblat¹⁴ considered the case of a disk oscillating in a fluid bounded by a second parallel disk

Fig. 7. Period of Oscillation as a Function of Amplitude for Three Positions of the Lower Plane Boundary d . (Disk No. 1 in Water).

at a distance d . For the case $d/\lambda \equiv \sigma \gg 1$, he obtained a radial pressure gradient given by:

$$\frac{\partial P}{\partial r} = \frac{3\rho\epsilon^2\omega^2r}{4} \left[\frac{1}{\sigma^2} - \frac{\sqrt{2}}{\sigma^3} - \frac{2-\sqrt{2}}{6-6i\sigma} e^{2i\omega t} \right] \quad (22)$$

For the data in Fig. 7, the variation in d from 7.4 cm to 0.9 cm changes the steady part of Eq. (22) by a factor of 58. However, the maximum value of the fluctuating part is changed only by a factor of 0.8 and moreover the fluctuating part dominates the steady part. Therefore, the results shown in Fig. 7 indicate that the amplitude effects are not influenced by the steady part of the radial pressure gradient, but they leave open the question of the influence of the fluctuating part.

In summary, measurements of the period and damping of oscillating disks give results in qualitative agreement with the theory of Rosenblat; in particular, $\Delta I_a/I_a$ decreases and $\Delta D/D_1$ increases with increasing ϵ , and these changes are independent of the disk thickness and only weakly dependent on the kinematic viscosity. Contrary to the theory, however, Eqs. (19) and (21) show that critical amplitudes must be exceeded before amplitude effects occur and that both $\Delta I_a/I_a$ and $\Delta D/D_1$ are radius dependent, the strongest dependence on radius being $a^{3/4}$ in the correlation for $\Delta D/D_1$.

At high amplitude $\Delta I_a / I_a$ is linear in ϵ . Finally, both period and damping are independent of the boundary positions (for $\sigma \gg 1$ and $b/\lambda \gg 1$). The differences between the theory and experiment may be due to the finite radius of the disk and presence of a fluctuating radial pressure gradient.

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APPENDIX I

Corner Parameters for the Oscillating Disk; Consequences for Liquid Helium II.

There has been a long-standing discrepancy, amounting to 83% at 1.2°K, between the values of normal fluid viscosity η_n of liquid helium II obtained from the oscillating disk method¹⁵ and those obtained from the rotating cylinder viscometer.¹⁶ Benson and Hollis-Hallett¹⁷ have suggested that this discrepancy is due primarily to errors in the values of normal fluid density ρ_n obtained from the oscillating pile of disks, since the single disk measures the product $\eta_n \rho_n$. The purpose of this appendix is to confirm their suggestion and to point out a source of error in the pile of disks technique.

The difficulty with oscillating disk measurements is in the proper treatment of the sharp corner where the cylindrical edge and plane faces meet. Several empirical corner corrections have been devised. The most thoroughly tested is that of Kestin and co-workers,^{18,19} in which the moment M of the fluid on the disk is written as $M = C_k M_\infty$, where C_k is an empirical factor and M_∞ is the moment on the edge and faces considered as sections of infinite surfaces. Dash and Taylor¹⁵ have used

$$M - M_{\infty} = 8\pi a^3 \eta \dot{\theta} (1 + i) C_{DT} \quad (AI-1)$$

where a is the disk radius, η is the viscosity, $\dot{\theta}$ is the angular velocity and C_{DT} is an empirical parameter (≈ 0.34). Both of these approaches assume that the real and imaginary parts of the additional moment are equal. We have found, however, that two parameters are necessary: the results can be expressed as

$$M - M_{\infty} = 8\pi a^3 \eta \dot{\theta} (C_D + iC_I) \quad (AI-2)$$

where C_D is the dissipative part (≈ 0.3) and C_I is the inertial part (≈ 0). For each of the runs discussed in Section IV, values of C_I and C_D were determined from the period and damping data respectively in the very low amplitude region where both were constant. The results are shown in Fig. 8 where C_D and C_I are plotted vs. t/λ (λ is the penetration depth). The results for C_D are in good agreement with Dash and Taylor's C_{DT} (obtained from damping measurements) and both C_D and C_I agree well with the theory of Azpeitia and Newell^{3,4} shown by the dashed lines. If we assume small damping and $\lambda \gg a$, then the theory gives for the thin disk ($t' \equiv t/\lambda \sqrt{2} \ll 1$):

$$\begin{aligned} C_D &= 0.25 - 0.11t'(0.21 + \ln t') \\ &\quad - 0.12(t/a) (1.1 + \ln t') \\ C_I &= 0.11t'(1.78 + \ln t') - 0.09t/a \end{aligned} \quad (AI-3)$$

Fig. 8. The Dissipative (C_D) and Inertial Parts (C_I) of the Corner Parameter vs. the Ratio of Disk Thickness t to Penetration Depth λ .

and for the thick disk ($t' \gg 1$), $C_D = 0.30$ and $C_I = 0$. In Fig. 8 we have drawn in a reasonable interpolation between the thin and thick disk results. Both theory and experiment indicate that C_D and C_I are insensitive to disk thickness over a wide range (note, however, the C_I point at $t/\lambda = 45.5$).

From these results we conclude that $C_I \approx 0$ and, therefore, the effect of the corner is to increase the damping but not the added moment of inertia. These results produce no change in Dash and Taylor's formula for the determination of the viscosity from the damping of a single disk when the fluid density is known; but they are important in the pile of disks measurements of ρ_n since in this case the inertial part of the moment is involved.

We have recalculated ρ_n from the data of Dash and Taylor, using their Eq. (33c) with the corner parameter set equal to zero. The results are shown in Fig. 9; the discrepancy between ρ_n obtained from the pile of disks and ρ_n calculated from second sound velocity¹² is cut in half. Moreover, since the corner for a single disk has no inertial effect, it is unlikely that the fluid at the cylindrical edge of the disk pile can be represented by a uniform layer. These questions of edge and corner effects in the pile of disks can best be settled by measurements in fluids of known density.

Fig. 9. Corrected Values of Normal Fluid Density
 ρ_n for $C_I = 0$.

APPENDIX II

Quantities Used in the Calculation of

$$\Delta I_a/I_a \text{ and } \Delta D/D_1.$$

In this appendix we list the values of the quantities used to calculate the added moment of inertia I_a , the logarithmic decrement D and the relative changes in I_a and D for each run.

The total moment of inertia I can be found from Eq. (4), and the total added moment of inertia I_{at} is found from Eq. (8). Both of these equations involve the torsion constant γ . Table II lists the values of $\gamma/4\pi^2$ and the coefficient of temperature dependence of γ for each torsion rod. The temperature dependence of γ for torsion rod No. 3 is not known because this rod broke before its temperature coefficient could be determined. We have listed a few references which contain useful information on quartz.^{20,21,22}

The kinematic viscosities of isopropyl alcohol and acetone were measured with a Cannon-Fenske viscometer and their densities were measured with a Westphal balance. The measured values agreed well with those listed in the American Institute of Physics Handbook²³ and the densities and viscosities of the other fluids were taken from the handbook. This reference also gives the temperature

Table II. Torsion Constants of the
Quartz Rods at 23.3°C.

Torsion Rod	$\gamma/4\pi^2$ (g/cm ² /sec ²)	$\gamma^{-1} (\partial\gamma/\partial T)$ (°C ⁻¹)
1	130.4	11.76×10^{-5}
2	837.1	8.42×10^{-5}
3	6,866	- - - -
4	2,040	12.14×10^{-5}
5	473.5	15.32×10^{-5}

dependence of the viscosity and this was used to account for temperature variations. The fluids used in the experiment are listed in Table III along with their viscosities and densities. T_F is the temperature of the fluid.

A list of all the runs is given in Table IV with the quantities measured. These quantities include the period in air τ_0 , the low amplitude period in the fluid τ_1 , the measured values of the total added moment of inertia I_{at} , and the measured values of the total damping at low amplitude D_T . The "nuisance decrement" due to losses in the torsion rod has been accounted for, so the values of D_T in the table are for the fluid only. It should also be mentioned that in calculating the corner parameter C_I (see Appendix I), the small effect of the air on τ_0 was calculated and subtracted to give the period in vacuum.

A list of the calculated quantities is given in Table V. I_{at} is the total added moment of inertia, given by

$$I_{at} = I_a + I_E \quad (\text{AII-1})$$

$$\text{where } I_a \equiv 0.5\pi\rho\lambda a^4 \quad (\text{faces}) \quad (\text{AII-2})$$

$$\text{and } I_E \equiv \pi\rho t\lambda a^3 \quad (\text{cylindrical edge}) \quad (\text{AII-3})$$

The calculated value of the total logarithmic decrement is given by

$$D_T = D_l + D_E + D_c \quad (\text{AII-4})$$

Table III. Fluid Viscosities and Densities

No.	Fluid	$T_F(^{\circ}\text{C})$	$\eta(\text{g/cm sec})$	$\rho(\text{g/cm}^3)$
1	H_2O	23.5	.00924	.997
2	Isopropyl Alcohol	23.0	.0219	.788
3	Acetone	23.8	.00308	.781
4	CS_2	21.5	.00362	1.26

Table IV. Experimental Quantities Measured

Run	Disk (Table I)	Fluid (Table III)	Torsion Rod (Table II)	τ_1 (Sec)	τ_0 (Sec)	I_{at} (g-cm ²)	D_T
1	1	1	1	5.96428	5.95218	18.809	.01425
2	1	2	1	5.96885	5.95216	25.956	.02077
3	1	3	1	5.95817	5.95193	9.696	.00694
4	1	4	1	5.96070	5.95243	12.852	.00957
5	2	1	1	5.87473	5.85902	24.033	.01843
6	3	1	1	6.09730	6.07935	28.504	.02006
7	4	1	2	6.19004	6.17495	156.193	.01639
8	5	1	2	6.19206	6.17495	177.138	.01835
9	6	1	3	6.10702	6.09148	1301.58	.01655
10	6	3	4	5.95420	5.92745	648.53	.02940
11	7	1	5	6.01729	6.00210	86.477	.01641

Table V. Calculated Quantities

Run	I_{at} (g-cm ²)	I_a (g-cm ²)	I_E (g-cm ²)	λ (cm)	D_T	D_l	D_E	D_C
1	18.927	17.036	1.891	.133	.01428	.01153	.00136	.00138
2	25.888	23.301	2.587	.229	.02097	.01575	.00194	.00327
3	9.656	8.691	.965	.086	.00704	.00589	.00068	.00046
4	13.300	11.971	1.329	.074	.00959	.00811	.00093	.00054
5	24.049	16.860	7.189	.131	.01850	.01177	.00534	.00139
6	28.383	17.385	10.998	.135	.02024	.01126	.00760	.00137
7	160.72	143.19	17.529	.135	.01682	.01402	.00178	.00101
8	178.93	143.32	35.608	.137	.01869	.01403	.00363	.00103
9	1305.9	1132.4	173.46	.135	.01657	.01389	.00208	.00059
10	664.04	575.81	88.224	.088	.02959	.02501	.00389	.00069
11	83.147	16.663	66.484	.131	.01639	.00305	.01298	.00036

$$\text{where } D_1 \equiv \pi I_a / I \quad , \quad (\text{AII-5})$$

$$D_E \equiv (\pi I_E / I) [1 + (3\lambda / 2a)] \quad (\text{AII-6})$$

$$\text{and } D_C \equiv 4C_D \pi^2 \rho \lambda^2 a^3 / I. \quad (\text{AII-7})$$

C_D is the corner parameter defined in Appendix I; we have used Dash and Taylor's value, $C_D = 0.34$. The penetration depth λ was calculated from the values in Tables III and IV and the relation

$$\lambda = (2\nu/\omega)^{1/2} = (\eta\tau/\pi\rho)^{1/2} \quad (\text{AII-8})$$

Using these results we now list in Table VI the appropriate equations for the calculation of $\Delta I_a / I_a$ and $\Delta D / D_1$ for each run. The general equations are

$$\Delta I_a / I_a = (\gamma / 4\pi^2) (\tau^2 - \tau_1^2) / I_a \quad (\text{AII-9})$$

$$\text{and } \Delta D / D_1 = (D - D_T') / D_1 \quad (\text{AII-10})$$

where τ and D are respectively the period and logarithmic decrement at arbitrary amplitudes. I_a and D_1 are calculated values, and D_T' is the measured low amplitude logarithmic decrement including the "nuisance decrement".

Table VI. Equations Used to Calculate $\Delta I_a/I_a$ and $\Delta D/D_1$

Run	$\Delta I_a/I_a$	$\Delta D/D_1$
1	$7.6571(\tau^2 - 35.5726)$	86.702 (D-0.01467)
2	$5.5989(\tau^2 - 35.6272)$	63.492 (D-0.02120)
3	$15.011(\tau^2 - 35.4998)$	169.63 (D-0.00743)
4	$10.897(\tau^2 - 35.5299)$	123.24 (D-0.00999)
5	$7.7329(\tau^2 - 34.5124)$	84.962 (D-0.01882)
6	$7.5012(\tau^2 - 37.1771)$	88.779 (D-0.02042)
7	$5.8461(\tau^2 - 38.3166)$	71.301 (D-0.01713)
8	$5.8410(\tau^2 - 38.3416)$	71.286 (D-0.01908)
9	$6.0637(\tau^2 - 37.2957)$	71.994 (D-0.01786)
10	$3.5435(\tau^2 - 35.4525)$	39.989 (D-0.02974)
11	$28.424(\tau^2 - 36.2078)$	327.60 (D-0.01686)

APPENDIX III

Boundary Variations

In Section IV it was pointed out that variations in the position of the boundaries produce no change in either the period or the damping and we presented period data (Fig. 7) showing this result for three positions of the lower boundary. In this appendix we present all of our data for variations in both the outer cylindrical boundary and the lower plane boundary.

Figure 10 shows the change in damping ΔD as a function of ϵ for three positions of the lower plane boundary d . The period data for these runs was given in Fig. 7. This data is for disk 1 oscillating in water. Water was used in all the runs given below. Figures 11 and 12 show the change in period ΔT and change in damping ΔD respectively for three positions of the outer cylindrical boundary. In this case disk 3 was used. In order to obtain smaller values of b (the distance from the disk to the cylindrical boundary), disk 5 was also used. Figure 13 shows ΔD as a function of ϵ for disk 5 where now the smallest value of b is 0.83 cm.

All of these data indicate that the observed changes in period and damping are unaffected by boundary nearness, at least when there is no viscous interaction between the boundary and the disk.

Fig. 10. The Change in Logarithmic Decrement ΔD vs. Amplitude ϵ for Three Positions of the Lower Plane Boundary d . Disk No. 1 in Water.

Fig. 11. The Change in Period ΔT vs. Amplitude ϵ for Three Positions of the Cylindrical Boundary. Disk No. 3 in Water.

Fig. 12. The Change in Logarithmic Decrement ΔD vs. Amplitude ϵ for Three Positions of the Cylindrical Boundary. Disk No. 3 in Water.

Fig. 13. The Change in Logarithmic Decrement ΔD vs. Amplitude ϵ for Two Positions of the Cylindrical Boundary. Disk No. 5 in Water.

VITA

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