2015

On the Thermally Induced Failure of Rolling Element Bearings

Jafar Takabi

Louisiana State University and Agricultural and Mechanical College

Follow this and additional works at: https://digitalcommons.lsu.edu/gradschool_dissertations

Part of the [Mechanical Engineering Commons](#)

Recommended Citation

Takabi, Jafar, "On the Thermally Induced Failure of Rolling Element Bearings" (2015). *LSU Doctoral Dissertations*. 1451.

https://digitalcommons.lsu.edu/gradschool_dissertations/1451

This Dissertation is brought to you for free and open access by the Graduate School at LSU Digital Commons. It has been accepted for inclusion in LSU Doctoral Dissertations by an authorized graduate school editor of LSU Digital Commons. For more information, please contact gradetd@lsu.edu.

ON THE THERMALLY INDUCED FAILURE OF ROLLING ELEMENT BEARINGS

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Mechanical and Industrial Engineering

by

Jafar Takabi

B.S., Isfahan University of Technology, 2004

M.S., Amirkabir University of Technology (Tehran Polytechnic), 2007

May 2015

This dissertation is dedicated to my loving and supportive wife, Mary Evelyn White, to my beloved mother and father, Houriyeh and Mohammad, to my supportive sisters and brothers and also to my American host families, Lisi Oliver and Malcolm Richardson.

Acknowledgements

I would like to specially thank Prof. Michael M. Khonsari, the major professor and committee chairman, for his continuous support and invaluable guidance throughout this research. I would also like to thank the committee members: Prof. Shengmin Guo, Prof. Guoqiang Li, Prof. Wanjun Wang and Prof. William A. Adkins for their time and kind help.

I would also like to thank all of my colleagues in the CeRoM laboratory of Louisiana State University for their kind help and support throughout my research.

Table of Contents

Acknowledgements.....	iii
Abstract.....	viii
Chapter I: Overview.....	1
1.1. Introduction.....	1
1.2. References.....	4
Chapter II: A review study on the thermally induced seizure of bearings.....	5
2.1. Introduction.....	5
2.2. Thermal seizure of journal bearings	7
2.2.1. Operating clearance	7
2.2.2. TIS in unlubricated or grease-lubricated journal bearings	9
2.2.3. TIS in fully lubricated (hydrodynamic) journal bearings.....	16
2.3. Thermal seizure of rolling element bearings	21
2.3.1. Bearing preload	21
2.3.2. Analytical work	23
2.3.3. Experimental work	26
2.4. Conclusions.....	30
2.5. Nomenclature.....	31
2.6. References.....	33
Chapter III: Developing a comprehensive thermal model of rolling element bearings	38
3.1. Introduction.....	38
3.2. Test Rig Assembly.....	39
3.2.1. Description of Apparatus.....	39
3.2.2. Instrumentation.....	40
3.3. Modeling and simulations.....	41
3.3.1. Frictional heat generation Model	42
3.3.2. Heat Transfer Model	43
3.3.3. Thermal Expansion Model	48
3.4. Experimental tests.....	49
3.5. Results and discussion	51
3.5.1. Verification.....	51
3.5.2. Effects of rotational speed	53
3.5.3. Effects of external radial load	55
3.5.4. Effects of air heat convection.....	57

3.5.5.	Effects of oil viscosity	58
3.6.	Conclusions	59
3.7.	Nomenclature	60
3.8.	References	62

Chapter IV: Developing dynamic model of rolling element bearings and studying the influence of traction coefficient on the dynamic behavior of bearing components

4.1.	Introduction	65
4.2.	Modeling approach	67
4.2.1.	Dynamic model description	67
4.2.2.	Traction model	71
4.2.2.1.	Lubricant film thickness calculations	72
4.2.2.2.	Traction force calculations	73
4.3.	Experimental validations	76
4.4.	Results and discussion	78
4.4.1.	Assumptions about the experimental procedure	79
4.4.2.	Influence of the traction models on the cage rotational speed	79
4.4.3.	Effects of bearing load and rotational speed on the wear rate	88
4.5.	Conclusions	90
4.6.	Nomenclature	91
4.7.	References	92

Chapter V: Dynamic performance of roller bearings operating under low rotational speeds with consideration of surface roughness

5.1.	Introduction	96
5.2.	Modeling and simulations	97
5.2.1.	Dynamic model of roller bearings	97
5.2.2.	Simplified dynamic model	97
5.2.3.	Traction coefficient calculation	98
5.2.3.1.	Rigid solid–Isoviscous (R–I)	100
5.2.3.2.	Elastic solid–Isoviscous (E–I)	101
5.2.3.3.	Rigid solid–Piezoviscous (R–V)	101
5.2.3.4.	Elastic solid–Piezoviscous (E–V)	101
5.2.3.5.	Implementation of lubrication regimes’ map in the dynamic model	103
5.2.3.5.1.	An illustrative example:	104
5.3.	Results and discussion	106
5.3.1.	Simulations	106
5.3.2.	Variation of film thickness and heat generation in loading zone	107
5.3.3.	Effect of surface roughness	112
5.3.4.	Effect of radial load	113
5.3.4.1.	Effect of radial load on the wear rate	115

5.4.	Conclusions.....	116
5.5.	Nomenclature.....	117
5.6.	References:.....	118
Chapter VI: Coupling thermal and dynamic models of rolling element bearings		123
6.1.	Introduction.....	123
6.2.	Thermal failure of rolling element bearings at high operating temperatures and large radial loads	124
6.2.1.	Experimental apparatus	124
6.2.1.1.	Instrumentation.....	126
6.2.2.	Modeling	126
6.2.2.1.	Thermo-mechanical model of the deep-groove ball bearing.....	127
6.2.2.2.	Dynamic model of the deep-groove ball bearing	128
6.2.3.	Simulation approach.....	130
6.2.3.1.	Evolution of temperature of bearing elements	130
6.2.3.2.	Implementing the temperature rise in the dynamic simulations.....	131
6.2.4.	Results and discussion.....	132
6.2.4.1.	Vibration monitoring of the test bearing during the thermal failure	133
6.2.4.2.	Results of dynamic simulations.....	135
6.2.4.2.1.	Time-variation of minimum film thickness.....	135
6.2.4.2.2.	Time-variation of cage contact forces and wear rate of the cage	137
6.3.	Thermal failure of spindle bearings of high-speed machine tools	138
6.3.1.	Experimental apparatus	139
6.3.2.	Modeling	140
6.3.2.1.	State-space model of the spindle bearings assembly.....	140
6.3.2.2.	Dynamic model of the spindle bearing.....	141
6.3.3.	Simulation Approach.....	142
6.3.3.1.	Evolution of the bearing elements temperature.....	142
6.3.3.2.	Implementing the temperature rise in the dynamic simulations.....	143
6.3.4.	Results and discussion.....	143
6.3.4.1.	Evolution of thermally-induced preload during thermally-induced failure	143
6.3.4.2.	Results of dynamic simulations.....	144
6.3.4.2.1.	Time-variation of minimum film thickness.....	144
6.3.4.2.2.	Time-variation of cage contact forces and wear rate of the cage	146
6.4.	Conclusions.....	147
6.5.	Nomenclature.....	148
6.6.	References.....	149
Chapter VII: Summary and Future Works.....		153
7.1.	Summary and Conclusions	153
7.2.	Recommendations for Future Works.....	154

Appendix A: Load—Deflection Relationship of ball bearings.....	156
Appendix B: Thermo-mechanical model of spindle bearings	157
Appendix C: Letters of Permission to Use Published Material	159
Vita	162

Abstract

This dissertation is devoted to the investigation of thermally induced seizure of rolling element bearings. A comprehensive thermal model of the rolling element bearings is developed which can predict the operating temperature of the bearing components in a wide range of operating conditions. The validity of this thermal model is verified by comparing the simulation results with a set of experiments. The results of simulations reveal that the rotational speed, oil viscosity and cooling rate of the housing have a significant influence on the operating temperature of the rolling bearings. To provide detailed information about all of the contact forces between the bearing components, a dynamic model of rolling element bearings is developed that can utilize different rheological models and traction curves in order to calculate the traction coefficient between the rollers and the raceways. The validity of this dynamic model is verified by comparing the simulation results with the previously published experimental results. The simulation results show that the simplified traction curves can be utilized in dynamic simulations only in operating conditions with low slide-to-roll ratios. This dynamic model is also employed to investigate the effect of surface roughness on the dynamic behavior of roller bearings operating at low rotational speeds and large radial loads. It was shown that an increase in the radial load results in a proportional increase in the wear rate and an exponential increase in the heat generation, although it does not affect the film thickness noticeably. Finally, the developed thermal and dynamic models are combined in a unified simulation approach to investigate two types of thermally induced failure in rolling element bearings. The simulations results revealed that the cage failure can occur during the thermal failure in radially-loaded rolling bearings operating at high temperatures, while a severe surface damage and disruption of the lubricant film can occur during the thermally induced failure of spindle bearings in high speed machine tools.

Chapter I: Overview

1.1. Introduction

Rolling element bearings are one of the most crucial components integrated in many rotating machineries. Their function is to provide a low-frictional rotary motion between a shaft and other mechanical components. Thus, performance of rolling element bearings plays a key role in the overall efficiency of the rotating machineries, and their failure could result in tremendous costs associated with the system shutdown. Failure of rolling element bearings can occur due to a diverse range of conditions and damages including: fatigue failure, wear and mechanical damage, corrosion damage, lubricant deficiency, etc.

Recent advancements in the field of tribology pertaining to bearing design, the development of superior bearing materials and related manufacturing processes have greatly improved bearing performance and life expectancy. Nevertheless, the industry still experiences different types of bearing failure, many of which are difficult to predict and often occur rapidly with little warning. An important class of such a failure is the so-called thermally induced seizure (TIS) of rolling element bearings which is a premature and detrimental bearing failure. It occurs at high speeds or high temperature applications such as gas turbine engines and high speed machine tools. In view of the scarcity and complexity of the predictive models for thermal failure of roller bearings, the development of a comprehensive approach to address this type of bearing failure is still a challenging task in the field of tribology. This dissertation provides the necessary steps to be taken in a novel, comprehensive approach to analyze the thermally induced seizure of rolling elements bearings in a wide range of operating conditions.

In the first step, an extensive literature review study is carried out in order to survey the state-of-the-art modeling of thermally induced failure of the bearings. This is documented in Chapter II which presents significant contributions made by the research community to investigate this mode of bearing failure in both journal and rolling element bearings. In spite of the different geometries and operating conditions of journal and rolling element bearings, the root cause of the thermally induced failure of these bearings is similar and can provide insight into the nature of TIS if studied collectively. There are different sources of the friction inside the bearing that result in heat generation and temperature rise of the bearing components. This frictional heat should be naturally or forcibly removed from the bearing assembly for the stable and safe operation of the bearing. If the balance between the heat generation and heat dissipation is disrupted, depending on the bearing type and the operating conditions, different types of damages occur such as thermoelastic instability (TEI), scuffing, thermal runaway and thermally induced seizure (TIS).

The focus of the present study is on the TIS of rolling element bearings in which the rotational speed and operating temperature play key roles.

Having conducted the literature review, it became apparent that the first task is to develop a comprehensive thermal model that can calculate the operating temperature of the bearings components in different operating conditions. Chapter III presents a comprehensive thermal analysis of ball bearings with provision for frictional heat generation, heat transfer processes and thermal expansion of bearing components. The evolution of temperature with time in a deep-groove ball bearing in an oil-bath lubrication system is investigated both experimentally and analytically. The complexities associated with the thermal analysis of the rolling bearings are due to the fact that the temperature has a substantial impact on multiple parameters involved in the thermal analysis. The temperature of the bearing components has a significant effect on the lubricant's viscous properties which, in turn, determines the film thickness of the lubricant between the contact surfaces. This film thickness is also affected by the dimensional change of the bearing components due to the operating temperature. On the other hand, the lubricant's viscous properties and the rotational speed can considerably alter the frictional heat generation inside the bearing assembly, which results in the temperature change of the bearing components. This complex coupling between the bearing temperature, the viscous properties and the frictional heat generation shows the extreme importance of the operating temperature of the bearing components as the fundamental variable of any thermal model of the rolling element bearings.

Thermal analysis of a bearing assembly, developed in Chapter III, cannot solely provide detailed information about some of the critical bearing-performance parameters such as film thickness and contact forces between the bearing components and the cage. Therefore in the next step, the developed thermal analysis should be coupled with a comprehensive analytical model of the rolling element bearings. There are generally two types of analytical models for rolling element bearings: quasi-static models and dynamic models. Considering the fact that the cage failure is reported in the literature as a possible outcome of TIS of some rolling bearings, dynamic models appear ideal to be coupled with the thermal model of the rolling bearings. Presented in the Chapter IV is a fully dynamic model of the cylindrical roller bearings that can utilize different rheological models and traction curves in order to calculate the time variation of the dynamic forces and moments between the bearings components and the cage.

It is well-known that the traction coefficient is the most influential parameter of any dynamic model of rolling element bearings. Traction coefficient is strongly influenced by the operating temperature and pressure. Thus, the bearing geometry, sliding velocities and rolling velocities of the bearing elements all play a role on the traction characteristics. Available in the literature are different Newtonian and Non-Newtonian rheological models [1-5] that can be used to calculate the traction coefficient in elastohydrodynamic contacts between the rollers and the raceways. However, a number of references [6-8] have used simplified traction curves in the dynamic

models due to the lack of experimental results for different types of lubricants. The traction coefficient is a function of only the slide-to-roll ratio in these simplified traction curves. Chapter IV investigates the operating conditions in which employing these simplified traction curves can lead to satisfactory results. Comparison of the simulation results to the previously published experimental measurements [3, 9, 10] reveal that the simplified traction curves can be used only for the operating conditions with low sliding velocities at the roller/raceway interfaces.

Having conducted the previous dynamic simulations, it became clear that further research is needed to study the behavior of roller bearings with provision for surface roughness. This is necessary to answer these fundamental questions: First, how can the traction coefficient be calculated for a roller passing through the loading and unloading zones of a radially-loaded rolling bearing, since different regimes of lubrication exist between the roller and the raceways as the roller travels in the orbital direction. This is, in fact, due to the variation of the contact pressure and its corresponding effect on the viscosity and deformation as the roller passes through the loading zone. Second, what is the influence of surface roughness on the dynamic performance of rolling element bearings. This is particularly important if the speed is low or the applied load is high. Chapter V presents the dynamic behavior of radially-loaded roller bearings operating at high loads and low speeds with provision of different surface roughness values. The simulations take into account different lubrication regimes (Rigid solid-Isoviscous, Elastic solid-Isoviscous, Rigid solid-Piezoviscous and Elastic solid-Piezoviscous) in the loaded and unloaded sections of the bearing. The simulation results provide a detailed understanding of the variation of the film thickness, wear rate and heat generation between the roller and the raceways as the roller travels in the orbital direction. The results of dynamic simulations carried out in Chapters IV and V are useful in selecting an appropriate traction curve for the dynamic simulations of thermally induced failure of rolling element bearings carried out in Chapter VI.

Finally In the last step, the developed thermal and dynamic models of the rolling element bearings are combined in a unified framework in order to provide a detailed analysis of thermally induced seizure of rolling element bearings. Chapter VI presents a comprehensive experimental and analytical investigation of two types of thermal failure of rolling element bearings. The first type is concerned with the thermal failure of rolling bearings that can occur at high rotational speeds and large radial loads, and the second type deals with spindle bearings of high-speed machine tools. Developed in this chapter is a simulation technique that considers the thermal expansion of the bearing elements in the dynamic simulations of the rolling bearings during the thermal failure.

Chapter VII summarizes the main findings of the present study which explain the role of critical parameters and complex mechanisms involved in the thermal failure of rolling element bearings. It is hoped that these results can be used in the bearing design and also in the predictive models

and monitoring systems. At the end, a series of recommendations are given for the possible future research based on the findings of the current study.

1.2. References

- [1] Bair S, Winer WO. Rheological model for elastohydrodynamic contacts based on primary laboratory data. *Journal of Lubrication Technology-Transactions of the Asme*. 1979;101:258-265.
- [2] Bair S, Winer WO. The high-pressure high shear-stress rheology of liquid lubricants. *Journal of Tribology-Transactions of the Asme*. 1992;114:1-13.
- [3] Gupta PK, Cheng HS, Forster NH. Viscoelastic effects in mil-l-7808-type lubricant .1. Analytical formulation. *Tribology Transactions*. 1992;35:269-274.
- [4] Johnson KL, Tevaarwerk JL. Shear behavior of elastohydrodynamic oil films. *Proceedings of the Royal Society of London Series a-Mathematical Physical and Engineering Sciences*. 1977;356:215-236.
- [5] Masjedi M, Khonsari MM. Theoretical and experimental investigation of traction coefficient in line-contact ehl of rough surfaces. *Tribology International*. 2014;70:179-189.
- [6] Ashtekar A, Sadeghi F. A new approach for including cage flexibility in dynamic bearing models by using combined explicit finite and discrete element methods. *Journal of Tribology-Transactions of the Asme*. 2012;134:041502.
- [7] Gupta PK. *Advanced dynamics of rolling elements*. Springer-Verlag, New York. 1984.
- [8] Weinzapfel N, Sadeghi F. A discrete element approach for modeling cage flexibility in ball bearing dynamics simulations. *Journal of Tribology-Transactions of the Asme*. 2009;131:021102.
- [9] Forster NH, Schrand JB, Gupta PK. Viscoelastic effects in mil-l-7808-type lubricant .2. Experimental-data correlations. *Tribology Transactions*. 1992;35:275-280.
- [10] Gupta PK. Viscoelastic effects in mil-l-7808-type lubricant .3. Model implementation in bearing dynamics computer code. *Tribology Transactions*. 1992;35:724-730.

Chapter II: A review study on the thermally induced seizure of bearings

2.1. Introduction

It is hard to imagine a machine that does not utilize a bearing of some sort, and it is of no surprise that bearings are one considered to be the key component of almost all rotating machinery. Bearings are designed to provide load-carrying capacity that supports the rotation or the sliding motion of one solid body relative to another, typically, stationary body.

Radially-loaded bearings can be broadly grouped into two categories: journal bearings and rolling element bearings; see Figure 2.1. In spite of different operational mechanisms and geometrical configurations, both journal bearings and rolling bearings are susceptible to a peculiar form of failure known as seizure whose root cause is thermal effects. When seizure occurs, the bearing cannot support motion and the system totally shuts down.

The key to the safe operation of bearings is a design with an effective thermal management: one that ensures that the heat generation within the system is in balance with heat dissipation. Different types of damage are likely to occur if this balance is disrupted. One class of failure, for example, is a localized surface damage known as thermoelastic instability (TEI) that occurs due to unstable thermal growth that manifests itself in the form of macroscopic hot spots or dark patches on the surface that can be viewed with the naked eye [1-6]. The occurrence of TEI has been widely reported in automatic brakes [7-9], clutches [10, 11] and mechanical seals [12-18]. The underlying cause of TEI is intensive localized frictional heating brought about by concentrated temperatures that create rather large local thermal expansion with significant contact pressure and stress that further aggravates the frictional heating. A succinct history of the early developments on TEI is provided by Burton [19] and with additional details in his book [20].

Scuffing is another type of related damage mode of localized form. It occurs between sliding surfaces that exhibit “welding” without evidence of localized melting. There is typically significant wear associated with scuffing failure. The interested reader is referred to Peterson and Winer [21] and an authoritative review by Dyson [22]. This type of failure is also thought to be caused by abrasive particles that enters a lubrication system and bridges itself across the clearance gap and causes local damage [23-27].

The literature also contains valuable information on the so-called thermal runaway which if not controlled eventually lead to bearing seizure. For example, recent investigations on air-lubricated foil bearings concentrate on thermal runaway caused by high bearing preload that reduces the

available clearance to the extent of becoming insufficient with the consequence of increasing frictional heat [28-30].

Having briefly described the different types of thermal failures, we now turn our attention fully to bearing failures with thermally-induced seizure (TIS), as its root cause. TIS can occur in many different operating conditions such as:

- During the start-up period of a journal bearing that have been out of service for a relatively long period of time (e.g., compressor of air conditioning units);
- Due to preload change in rolling bearings (e.g., spindle bearings of high speed machine tools);
- During interruption in the lubricant supply due to contamination or clogging of the filter;
- During temporary interruption in the lubricant supply due to the maneuvering actions in aircrafts;
- Due to harsh environmental conditions (e.g., placement of the bearings near elevated temperatures); and
- Due to the severity of operating speed or external load.

The occurrence of TIS, regardless of the loading condition or the lubrication state of the bearing, can be analyzed by examining the balance between the frictional heat generation inside the bearing and the dissipation of heat to the surroundings through the housing and/or the removal of heat via the lubricating fluid. In the case of massive housings, if the housing accommodation is less than the thermal expansion of the shaft and rotating elements, a catastrophic thermal seizure of the bearing may occur rapidly. This typically manifests itself as the loss of operating clearance in the journal bearings and as the sharp increase of thermally induced preload in the rolling element bearings. Burton [31, 32] was probably the first researcher to utilize this simple concept in order to develop analytical models for thermal seizure of journal bearings as well as rolling bearings.

Wang [33] published a literature review of the studies on the TIS of dry and lubricated conformal contacts along with a summary of the studies on the TIS in non-conformal contacts. She explained that the loss of clearance in journal bearings can occur due to differential thermal expansions of the shaft and the bushing or due to the build-up of wear debris. Also noted was that the variation of the surface geometry, surface roughness and chemical composition of contact surfaces have a considerable effect on the occurrence of TIS in journal bearings.

Figure 2.1. Schematic of (a) journal bearings (b) rolling element bearings under a radial load

In this paper, we provide a review of the-state-of-the-art research in modeling and predicting thermal failure in both journal bearings and rolling element bearings. Section 2, is devoted to the survey of important papers pertaining to TIS in lubricated and unlubricated journal bearings. In Section 3 we provide a review of noteworthy analytical and experimental investigations on TIS in rolling element bearings, and conclude the paper with a summary in Section 4.

2.2. Thermal seizure of journal bearings

2.2.1. Operating clearance

Operating clearance between the rotating shaft and the bushing (or sleeve) is one of most the crucial design parameters in journal bearings [34]. A bearing with too large of a clearance can experience whirl instability and if too small, it can suffer from overheating. A bearing must maintain its designed clearance by balancing the internal heat generation and dissipation from the system to the surroundings. If the rate of heat generation exceeds the dissipation capacity, then TIS sets in and bearing loses its operating clearance. TIS can occur very rapidly in journal bearings when the rotating shaft is in dry contact with the sleeve. References [34-36] provide detailed governing equations and associated solution methodologies, which we describe later in this paper.

Table 2.1 provides a guideline to help distinguish the different characteristics of the published studies on the thermal seizure of journal bearings. It should be mentioned that “Unloaded” models consider concentric shaft and bushing in the analysis which can represent the operating condition of journal bearings in many industrial applications such as vertical pumps, mixers, and the like. The “Loaded” models consider the operation of an eccentric shaft within the clearance

space of a bushing, i.e. what is commonly experienced in a radially loaded journal bearing. Furthermore, these loading conditions can determine whether the governing equations should be developed and solved in one dimension (1D models for unloaded bearings) or two dimensions (2D models for loaded bearings).

Table. 2.1. Summary of the studies on the thermal seizure of journal bearings

Reference	Lubrication state	Loading condition	Developed mathematical model	Investigated parameters
[31]	Grease lubricated	Unloaded	1D, Infinitely long journal bearings	Thickness of housing, Internal and external cooling
[37]	Grease lubricated	Loaded	2D, Infinitely long journal bearings	Housing thickness
[38]	Unlubricated, Lubricated	Loaded, Unloaded	1D, 2D, Infinitely long journal bearings	Bearing size, Load, Rotational speed
[39]	Grease lubricated	Unloaded	1D, Infinitely long journal bearings	Seizure-time, Load, Speed, Material properties
[40]	Grease lubricated	Loaded	1D, 2D, Short and infinitely long journal bearings	Seizure-time, Bearing size, Misalignment
[41, 42]	Unlubricated, Lubricated	Loaded	2D, Infinitely long journal bearings	Bearing size, Speed, Internal and external cooling, Housing flexibility
[43]	Fully lubricated	Unloaded	Lumped mass, Short and infinitely long journal bearings	Seizure map, Bearing size, Rotational speed, Material of shaft, Lubricant viscosity
[44]	Unlubricated,	Loaded	2D, Infinitely long journal bearings	Temperature-dependent material properties, Initial clearance, Load, Speed
[33]	Unlubricated, Lubricated	Loaded, Unloaded	1D, 2D, Short and infinitely long journal bearings	Literature review

Table. 2.1. Summary of the studies on the thermal seizure of journal bearings [cont'd]

Reference	Lubrication state	Loading condition	Developed mathematical model	Investigated parameters
[45]	Fully Lubricated	Unloaded	1D, Short journal bearings	Variation of rotational speed and clearance, Thermal conductivity and expansion of shaft and bushing
[46]	Fully Lubricated	Unloaded	1D, Infinitely long journal bearing	Speed, Oil viscosity, Internal cooling, Material properties of the shaft and the bushing
[47]	Unlubricated, Lubricated	Loaded	2D, Short journal bearings	Seizure-time, Load, Speed, Bearing size, Material properties of the shaft and the bushing
[48]	Unlubricated, Grease Lubricated	Loaded	2D, Infinitely long journal bearing	Oscillatory motion, Load, Clearance, Friction coefficient, Oscillation parameters, Convective heat transfer and direction of variable load, Solid and hollow shaft

2.2.2. TIS in unlubricated or grease-lubricated journal bearings

Burton [31] presented a simple mathematical model in order to study the effect of cooling and housing thickness on the TIS of unloaded journal bearings. This is a one-dimensional (1D) model which utilizes the equations of thermal expansion only for the bushing in an infinitely long journal bearing. It is assumed that the heat input from the bearing clearance into the bushing and the shaft is constant. Burton [31] showed that if external cooling of the housing is the only manner in which the bearing is cooled, the relative expansion or encroachment in the bearing can monotonically increase and result in bearing seizure. Accordingly, Burton [31] presented the following criterion for the “safe operation” of journal bearings:

$$\frac{\alpha f V_s^2 r_{bi}}{k c_i} \left[\frac{\phi(r_{bo} - r_{bi})}{c_i} + 0.5 \right] \leq 0.25 \quad (2.1)$$

$$\text{with the encroachment factor } \phi = \frac{8k\left\{1+h_{bi}\left[\frac{(r_{bo}-r_{bi})}{k}+\frac{1}{h_{bo}}\right]\right\}}{r_{bi}\alpha q_s(r_{bo}-r_{bi})} \quad (2.2)$$

where α and k are the thermal expansion coefficient and thermal conductivity of the bushing/housing, respectively. f , V_s and q_s are, respectively, the friction coefficient, the sliding speed, and the heat flow per unit of surface area. The inner and outer radius of the bushing/housing and initial clearance are represented by r_{bi} and r_{bo} and c_i , respectively. In the encroachment factor ϕ in Equation 2.2, h_{bi} and h_{bo} represent the effective heat transfer convection coefficient inside and outside of the bushing/housing, respectively. Burton [31] indicated that the effect of the housing thickness on the operating clearance is significant for the bearings with solid shafts, while it has a negligible effect in case of thin-walled shafts.

Heckmann and Burton [37] subsequently used a similar approach in order to investigate the TIS behavior in the loaded journal bearings. This model is based on the thermal expansion of only the bushing in steady-state conditions. The effect of loading is implemented in the model by considering the heat input and the contact pressure over the loaded zone at the inner surface of the bushing. Based on this model, they presented the following formula to calculate the “operating clearance” (c) in loaded journal bearings:

$$c = \left(\frac{2nW_l}{\pi E}\right) \left(\frac{V_s}{V_s^*}\right) \left[1.87 - \left(\frac{0.0707}{2\left(\frac{r_{bo}}{r_{bi}} - 1\right)^{0.25}}\right) \left(\frac{25}{\left(\frac{r_{bo}}{r_{bi}} - 1\right)^2}\right)\right] \quad (2.3)$$

$$V_s^* = \frac{2k}{\alpha E f r_{bi}} \quad (2.4)$$

where W_l , E and n are the load per unit length, modulus of elasticity and the heat partitioning factor, respectively. V_s is the sliding speed and V_s^* is the critical sliding speed calculated from Equation 2.4. They showed that in order to avoid seizure, the value of the quantities inside the bracket in Equation 2.3 should be zero or negative. This means that TIS in loaded journal bearings occurs if the ratio of the outer radius to the inner radius of the bushing is greater than a specific value ($\frac{r_{bo}}{r_{bi}} > 1.715$).

Bishop and Ettles [38] developed a more advanced mathematical model than that of the original Burton’s model [31] in which they considered the thermal expansion of the shaft along with the thermal expansion of the bushing in unlubricated, unloaded journal bearings with non-metallic liners. They assumed that there is no heat dissipation from the housing and that all of the frictional heating is dissipated by the shaft and the lubricant (thermoplastic bushing). The outer surface of the bushing is also assumed to be rigidly constrained. Bishop and Ettles [38] carried out a set of simulations to investigate the effect of the bearing size, load and rotational speed on the TIS of unloaded dry journal bearings where a friction coefficient of 0.05 was considered for

dry journal bearings. They assumed that thermal seizure takes place either when the interface temperature reaches 320 oC or when the interface pressure reaches 120 MPa. They showed that the thermal seizure occurs in a few seconds after the start-up of dry journal bearing with thermoplastic bushing. They also reported that the “seizure-time” varies with the square root of the shaft size in the case of loaded dry journal bearings.

The next important contribution is the work of Dufrane and Kannel [39] who presented an analytical model in order to predict the seizure-time in unloaded journal bearings operating with insufficient lubrication. This thermal seizure may occur due to the dry contact in three different operating conditions: at start-up, due to the clogged lubrication system or as a result of sudden increase of the load. This mathematical model is similar to the Bishop and Ettles model [38] while it considers different rates of heat dissipation through the shaft and the housing. By neglecting the heat dissipation from the housing (e.g. thick-walled housings), Dufrane and Kannel [39] presented the following linear equation for “seizure-time”:

$$\delta \approx - \frac{4\alpha n V_s f P (1 + \vartheta)}{\pi \rho C_p} t \quad (2.5)$$

where δ is the net diametral growth of bushing relative to the shaft. $P = \frac{W_1}{D}$ is the bearing pressure while D is the shaft diameter. ϑ , C_p , ρ and t are the Poisson’s ratio, specific heat, density of the bearing material and time, respectively. They assumed that the thermal seizure takes place when the bearing clearance is completely lost (when δ reaches to the initial diametral clearance).

Khonsari and Kim [49] pointed out a problematic implication of Equation 2.5 which suggests that all bearings are susceptible to seizure regardless of their clearance size. This is illustrated in Figure 2.2 which shows a linear relationship between the clearance loss and the time to seizure. Therefore, this equation implies that no matter what the initial clearance ratio is, the bearing will eventually seize due to thermal effects. To gain further insight into this problem, Khonsari and Kim [49] developed a finite element model of the bearing-shaft assembly and reported that the variation of the clearance loss with time turns out to be non-linear, see Figure 2.2. Thus, Equation 2.5 is valid only for short time periods associated with the rapid transients when the heat dissipation from the housing can be ignored [9]. In fact, Dufrane and Kannel [39] experiments with a grease-lubricated journal bearing, composed of a carbon steel shaft and an alloy bronze bushing, with three different clearance values, revealed usefulness of this analysis. In these applications, thermal seizure occurred in less than 20 seconds and the predictions of Equation 2.5 agreed fairly well with the measurements. Dufrane and Kannel [39] also examined the application of this model for the main crankshaft bearing of internal combustion engines, as well as the bearings of rotary pumps where the model prediction agreed roughly well with the measurements. Based on the experimental results, they concluded that “for many applications the thermal growth of the bushing can be ignored because of the constraint by the housing”.

Figure 2.2. Variation of clearance loss as a function of time based on the models of Dufrane and Kannel [39] and Khonsari and Kim [40, 49] for a grease lubricated journal bearing operating under 4400 N and 250 rpm. [40]

Inspired by the work of Dufrane and Kannel [39], Khonsari and Kim [40] further investigated the seizure-time in finite loaded journal bearings. Similarly, by neglecting the thermal expansion of the bushing (thick-walled housing), they solved the governing equations for transient thermoelastic behavior of the shaft by finite element (FE) methods. Two models were considered in the analyses: one-dimensional and two dimensional models. Assumed in the one-dimensional model is a constant temperature distribution in the axial and circumferential directions of the shaft, while the two-dimensional model incorporated heat-input area along a small portion of the axial direction of the shaft to mimic what a bearing with a misaligned shaft would experience. Unlike the linear behavior of the seizure-time reported by Dufrane and Kannel [39], the simulation results showed that the clearance loss increases rapidly with time but eventually flattens out. These linear and nonlinear trends of the seizure-time are shown in Figure 2.2 where it illustrates the results of application of these two models [39, 40] on a journal bearings operating under a radial load of 4400 N and rotational speed of 250 rpm. Khonsari and Kim [40] also showed that the seizure-time in misaligned bearings is much shorter than the aligned bearings due to the local heating and higher temperature rise in misaligned bearings. They compared their simulation results with the experimental results carried out by Dufrane and Kannel [39] and showed that their model can provide a better prediction for the seizure-time. Note, that a major shortcoming of these models is the limitation of the analysis to radial expansion of the shaft only. Consideration of the expansion of the bushing is important because if it is constrained in a rigid housing, the bushing can only expand inward toward the already expanding shaft and contribute to the severity of seizure.

Hazlett and Khonsari [41, 42] developed a 2D FEM in order to investigate TIS in infinitely-long loaded journal bearings where the thermal expansion of the bushing was considered along with the thermal expansion of the shaft. This model was shown to be capable of predicting the behavior of thermal seizure in dry and lubricated journal bearings. They introduced a seizure criterion stating that the seizure is imminent when the frictional torque exceeds the torque supplied to the shaft. This is illustrated in Figure 2.3a for a specific bearing. It shows the occurrence of a sharp increase in the frictional torque a few seconds before the clearance totally closes up and seizure occurs. Comparison of this figure with Figure 2.2 shows that a seizure criterion based on the frictional torque provides a more realistic prediction for the onset of the seizure than the criterion based on the clearance loss.

Figure 2.3. (a) Time-variation of frictional torque and (b) Bushing ovalization before the ultimate seizure in a grease lubricated journal bearing operating under 4400 N and 250 rpm [41, 42]

Hazlett and Khonsari [41, 42] further described a new scenario for thermal seizure of journal bearings where they showed that the seizure may occur before the onset of the full contact

between the shaft and the bushing. This is shown in Figure 2.3b which shows an “ovalization” of the bushing due to the non-symmetrical heating. This ovalization confirms the practical evidence of the wear patterns seen on the bushing of seized journal bearings. Furthermore, the results of a sensitivity study on the different bearing parameters and operating conditions showed that a thinner housing, shorter bushing, lower contact resistance and improved internal cooling all tend to decrease the likelihood of the seizure, while the effect of housing cooling is negligible. The simulation results also revealed a nonlinear relation between the seizure-time and the rotational speed in such that the higher rotational speed significantly decreases the seizure-time.

Wang et al. [44] investigated the burn-off failure between the cone of a jammed rolling bearing and the railroad axle by means of a 2D finite element model similar to the Hazlett and Khonsari model [41, 42]. They utilized temperature-dependent material properties for the axle and the cone in their simulations. The coefficient of friction was also considered to be temperature-dependent over a range of temperatures up to 1000 oC. A set of simulations were carried out for a range of initial radial clearances, which led to similar conclusions to the simulation results of Hazlett and Khonsari [41, 42].

Krithivasan and Khonsari [47, 50] utilized and improved the 2D finite element model presented by Hazlett and Khonsari [41, 42] in order to investigate TIS occurring in loaded journal bearings during the startup period when friction is dominated by dry contact and also during the transient flow disturbance in an otherwise normal operating conditions. A high coefficient of friction between contact area of the shaft and the bushing was assumed to simulate the condition of start-up seizure. In this case, they reported that the TIS is initiated by the ovalization of the bushing along with the uniform outward expansion of the shaft. They also performed an extensive set of parametric simulations over the range of parameters shown in Table 2.2 and presented following empirical relationship based on the simulation results to calculate the dimensionless seizure-time ($\overline{t_{ss}}$):

$$\overline{t_{ss}} = 90.494 \varepsilon^{-1.2478} \lambda^{-1.6494} \quad \text{for} \quad 500 < \frac{r_s}{c_i} < 1000 \quad (2.6)$$

$$\overline{t_{ss}} = 1.825 \varepsilon^{-1.2478} \lambda^{-1.02718} \quad \text{for} \quad 1000 < \frac{r_s}{c_i} < 5100$$

$$\overline{t_{ss}} = \frac{t_{ss} k}{r_s^2} \quad \varepsilon = \frac{fW\alpha\omega}{k} \quad \lambda = \frac{r_e}{L} \quad r_e = \frac{r_s r_{bi}}{r_{bi} - r_s} = \frac{r_s(r_s + c_i)}{c_i} \quad (2.7)$$

where t_{ss} is the seizure-time during the start-up and λ is the modified aspect ratio. r_s is the shaft radius and L is the length of the bearing. W and ω are the normal load and the angular velocity of the shaft, respectively. It should be noted that the boundary conditions used in this analysis allows some flexibility for the outward expansion of the bushing and the seizure time will be appreciably lower if the bushing was completely constrained. This empirical formula is valid for the operating conditions close to the ranges given in Table 2.2. It is worthwhile to point out,

however, that the authors showed that this empirical formula was capable of predicting close results when compared to those reported by of Hazlett and Khonsari [41, 42], Wang et al. [44] and Bishop and Ettles [38].

Table 2.2. Range of parameters used in the statistical analysis

Rotational speed (rpm)	Shaft radius (m)	(L/D)	Friction coefficient	Seizure-time (s)
150 - 2000	0.010 – 0.0375	0.5 - 2	0.1 – 0.3	0 – 120

The second type of TIS investigated by Krithivasan and Khonsari [47] was the study of the transient flow disturbance in journal bearings leading to failure. This thermal seizure is triggered by an interruption in the oil supply and was modeled by increasing the friction coefficient and decreasing the convective heat transfer coefficient. They reported the same ovalization of the busing in this type of TIS similar to the TIS during the start-up with the only difference being the time taken to seize. They presented the following empirical relationship based on the simulation results to calculate the dimensionless seizure-time ($\overline{t_{sp}}$).

$$\overline{t_{sp}} = (\varepsilon^{0.5489} (0.0065 \lambda + 9.314))^{-1} \quad (2.8)$$

$$\overline{t_{sp}} = \frac{t_{sp}}{t_{ref}} \quad \varepsilon = \frac{fW\alpha\omega}{k} \quad \lambda = \frac{r_e}{L} \quad r_e = \frac{r_s r_{bi}}{r_{bi} - r_s} = \frac{r_s(r_s + c_i)}{c_i} \quad (2.9)$$

where t_{sp} is the seizure-time when flow disturbance occurs. t_{ref} is the transition time for the bearing to go from fully lubricated conditions to boundary lubrication condition. The validity of this empirical formula for the case of thermal seizure during the transient flow disturbance was examined against the simulation results obtained from the developed finite element model.

The majority of the published papers on the TIS are focused on the unidirectional rotating journal bearings while there are many applications that the journal bearing operates in an oscillatory fashion. Wen and Khonsari [48, 51] presented an analytical method to investigate the TIS in infinitely-long loaded journal bearings with oscillatory motion. This method combines the transfer matrix with a 2D FE model where an iterative transient solution is used in the thermomechanical coupling (TMC) process in order to automatically determine the heat partitioning factor between the shaft and the bushing. Wen and Khonsari [48, 51] investigated the effect of wide range of operating parameters for different cases of oscillating shaft, oscillating bushing and variable load direction. They reported a different ovalization process during the TIS than the one reported in references [41, 42, 47]. It was shown that the bushing ovalization occurs only for the bushing of the journals rotating unidirectionally. In the case of oscillating shaft, both the shaft and the bushing ovalized during the TIS. This is illustrated in Figures 2.4 and 2.5 where the variation of the temperature and the clearance are plotted at cycles 300, 379 and 391, respectively.

Figure 2.4. Temperature rise at (a) cycle 300, (b) cycle 379 and (c) cycle 391 when seizure occurs. ($W = 4400$ N, $c_i = 0.025$ mm, $f = 0.15$, $freq = 10$ Hz, $h_{bo} = 80$ W/m²K and 30 rad angular oscillation amplitude with solid oscillating shaft) [48, 51]

Figure 2.5. Variation of (a) clearance along the contact surface, (b) profile of the inner surface of the bushing, and (c) profile of the outer surface of the shaft at cycles 300, 379 and 391.

($W = 4400$ N, $c_i = 0.025$ mm, $f = 0.15$, $freq = 10$ Hz, $h_{bo} = 80$ W/m²K and 30 rad angular oscillation amplitude with solid oscillating shaft) [48, 51]

Wen and Khonsari also reported that an increase in the friction coefficient, applied load, amplitude and frequency of the oscillatory motion will reduce the seizure-time significantly due to the increase in the heat generation. It was shown that the external cooling of the bushing can prolong the seizure-time significantly in the case of oscillating bushing while it has a negligible effect in the case of oscillating shaft. Another interesting finding reported is that the seizure-time is shorter for a hollow shaft than a solid shaft exposed to the same convective cooling rate on the inside and around the perimeter. Based on the simulation results, Wen and Khonsari [48, 51] concluded that improving the lubrication, enhancing internal and external cooling, increasing the designated clearance and lowering the oscillation frequency can prolong the seizure-time significantly.

2.2.3. TIS in fully lubricated (hydrodynamic) journal bearings

As discussed in the Section 2.2, the simple mathematical model presented by Burton [31] is generally applicable for grease lubricated bearings. Nevertheless, it is also representative of the

heavy-loaded hydrodynamically-lubricated journal bearings where the viscous heat dissipation around the bearing is small compared to the loaded zone. Burton [31] reported that if the shaft or the lubricant are the only source of cooling of the bearing and the exterior of the bearing is insulated, the operating clearance of the lubricated bearing may drop transiently depending on the operating conditions, but it may continue to work for a long time. In this case, Burton [31] presented following criterion for the “safe operation” of the journal bearing:

$$\frac{\alpha f V_s^2 r_{bi}}{k c_i} \left[\frac{\psi(r_{bo} - r_{bi})}{2 c_i} + \frac{\psi}{2 \psi_\infty} \right] \leq 0.30 \quad (2.10)$$

$$\text{with encroachment factor} \quad \psi = \frac{\delta_c k}{\alpha q_s} \left[\frac{2}{(r_{bo} - r_{bi})} + \frac{h_{bi}}{k} \right] \frac{1}{r_{bi}} \quad (2.11)$$

where δ_c is the clearance reduction due to relative thermal expansion. The encroachment factor ψ is defined by Equation 2.11 and ψ_∞ is the value of ψ when $\frac{r_{bo}}{r_{bi}} \rightarrow \infty$. If this criterion is not met, then the bearing can become unstable due to small disturbances in operating conditions and eventually seize in a short time.

Bishop and Ettles [38] investigated TIS in loaded, lubricated journal bearings with non-metallic liners. Their model assumes that the outer surface of the bushing is insulated and rigidly constrained. They carried out a set of simulations to investigate the effect of the bearing size, load and rotational speed on the TIS of loaded, lubricated journal bearings where a friction coefficient of 0.005 was considered for hydrodynamic lubrication. It was assumed that the thermal seizure takes place either when the interface temperature reaches 320 oC or when the interface pressure reaches 120 MPa. Based on the results of a set of simulations for a wide range of loads, speeds and initial clearances, Bishop and Ettles [38] proposed an easy-to-use seizure number ($\frac{P V_s}{C_i}$) which should not be exceeded for the thermally-safe operation of hydrodynamic journal bearings. Here, $P = \frac{W_l}{D}$ is the interface pressure in which D is the shaft diameter.

A simple analytical model was presented by Pascovici et al. [43] to predict the thermal seizure in fully-lubricated concentric (unloaded) journal bearings. Considering the entire bearing as a lumped-system, they presented a closed-form analytical solution for the energy balance equations and developed the following criteria to predict the onset of TIS in unloaded hydrodynamic journal bearings:

$$\frac{\mu_0 \alpha r_s^4}{h_{bo} r_{bo}} \left(\frac{\omega}{c_i} \right)^2 < 1 \quad \text{For infinitely long journal bearings} \quad (2.12)$$

$$\frac{\mu_0 \alpha L r_s^2}{h_s} \left(\frac{\omega}{c_i} \right)^2 < 1 \quad \text{For submerged journal bearings} \quad (2.13)$$

where μ_0 is the initial lubricant velocity and L is the bearing length. h_s and h_{bo} are, respectively, the heat convection coefficient from the shaft and from the bushing/housing. According to these

criteria, in order to avoid TIS a shorter bushing, less viscous lubricant and a shaft material with smaller thermal expansion coefficient must be chosen. They also reported that among the involved parameters, the shaft speed and the radius-to-clearance ratio have the most important effect on TIS. Based on this model, they provided a generalized “seizure map” to predict the onset of TIS in short and long hydrodynamic journal bearings, see Figure 2.6. The dimensionless seizure-time (τ^*) and parameter Γ are defined in Figure 2.6 where A , A_f , H and M are the available area for convective heat transfer, friction area, overall convective heat transfer and system thermal capacity, respectively. It was also indicated that the thermal behavior of finite and infinitely long journal bearings is almost identical at high speeds and more discrepancy is expected between them at lower rotational speeds. This unique solution is best suited for bearings with a small Biot number ($\frac{h_{bo}L_c}{k} < 0.1$).

Figure 2.6. Seizure map for fully lubricated unloaded journal bearings [43]

Jang et al. [45] presented a comprehensive theoretical analysis of TIS in unloaded hydrodynamic journal bearings. This analysis is based on the coupled numerical solution of governing equations for transient fluid velocity, heat transfer and thermomechanical expansion in the radial direction. They verified the simulation results with the experiments with two different oils and initial radial clearances on a custom-designed apparatus in which the outward expansion of the bushing was constrained in a submerged lubricated journal bearing. It was shown that the shaft speed decreases significantly with time until the seizure occurs, while the frictional torque remains almost constant until it rises rapidly right before the complete loss of the clearance, see Figures 2.7a and 2.7b. Therefore, TIS may be avoided by monitoring the motor’s torque so that the operation can be halted prior to complete loss of clearance. The results of a series of parametric studies also revealed that a shaft with low thermal conductivity and low thermal expansion

coefficient along with a bushing with a high thermal conductivity can extend the seizure-time considerably.

(a)

(b)

Figure 2.7. Time-variation of (a) shaft speed and (b) frictional torque before the ultimate seizure in a non-rate-controlled fully lubricated journal bearing [45]

(a)

(b)

Figure 2.8. Time-variation of (a) operating clearance and (b) frictional torque before the ultimate seizure in a rate-controlled fully lubricated journal bearing [46]

In a separate study, Jang et al. [46] utilized the former 1D model developed in the previous study [45] to calculate the seizure-time in unloaded lubricated journal bearings while they assumed that the shaft speed remains constant during the entire operation (a rate-controlled system). It was shown that the clearance decreases linearly with time, but the rate of this decrease significantly increases right before the ultimate seizure, see Figure 2.8a. The trend reveals that the behavior of the frictional torque is nonlinear: there is an initial drop followed by a relatively constant value which rises rapidly right before the ultimate seizure, see Figure 2.8b. The viscosity distribution

and velocity gradient were identified as the parameters causing this nonlinear behavior. Jang et al. [46] carried out an extensive parametric study and showed that enhancing the convective heat transfer along with using a low-viscosity index (VI) lubricant can help to extend the seizure-time. It was also shown that the seizure-time reduces by increasing the rotational speed in an exponential manner.

The thermal failure of hydrodynamic journal bearings also might occur due to the presence of the rigid particulate contamination in the lubricant. This mode of thermal failure, also known as thermally-induced scuffing failure, was investigated in a paper by Khonsari et al. [24] where they presented a model to evaluate the flash temperature between a fixed rigid particle embedded into the bearing liner while in contact with the rotating shaft. It was assumed that scuffing occurs if the flash temperature exceeds a critical value. Subsequently, based on the developed model, they presented following useful equations in order to estimate an appropriate filter size.

$$Pe_D \leq Pe_{Dcr} \quad (2.14)$$

$$Pe_{Dcr} = \chi + 0.13425 \frac{\chi^2 K^2}{\sqrt{\beta}} + \sqrt{\left(\chi + 0.13425 \frac{\chi^2 K^2}{\sqrt{\beta}}\right)^2 + \chi^2 (K^2 - 1)} \quad (2.15)$$

$$\chi = \frac{5.236 \bar{T}_{fcr} \sqrt{\beta}}{\left\{ \beta \cos^{-1} \sqrt{1 - \frac{1}{\beta}} - \sqrt{\beta - 1} + 2 \bar{\tau}_1 (\beta - \sqrt{\beta(\beta - 1)}) \right\} K}$$

$$K = \frac{k_1}{k_p} \quad Pe_D = \frac{UD_p}{\epsilon_1} \quad \bar{T}_f = \frac{T_f k_1}{\sigma_1 \epsilon_1} \quad \bar{\tau}_1 = \frac{\tau_1}{\epsilon_1} \quad (2.16)$$

where Pe_D is Peclet number based on the particle diameter (D_p) and Pe_{Dcr} is the critical Peclet number. β is the hardness ratio and σ is the hardness. U , T_f , ϵ and τ are, respectively, linear velocity of slider, average flash temperature, thermal diffusivity and yield shear stress. It should be mentioned that the subscripts 1 refers to harder material while subscript p refers to particle in Equations 2.14-2.16. The authors showed that these relationships can be used to determine the critical particle size that could cause failure and accordingly choose an appropriate filter size.

2.3. Thermal seizure of rolling element bearings

2.3.1. Bearing preload

The number of published papers on the thermal seizure of rolling bearings is much less than that of the journal bearings due to the inherent complications associated with the kinematics and geometry of the rolling bearings. The problem associated with the thermal seizure of the rolling bearings is the sharp increase in the thermally induced preload due to the excessive heat generation inside the bearings. Several studies in the literature [52-62] have reported this type of thermal seizure in the spindle bearings of high speed CNC machine tools where the thermal

expansion of the bearing is greatly confined by the lower thermal expansion of the massive housing. Bearing preload is the combination of the initial preload and thermally induced preload. The initial preload is normally constant and typically set by lock nuts in spindle bearings to maintain the bearing stiffness required in high speed machining. The initial preload can also help to increase the fatigue life and decrease the bearing noises. Thermally induced preload are the induced contact loads between the bearing elements due to the uneven thermal expansion of the bearing elements during the operation. Table 2.3 provides a summary of the published papers on the thermal seizure of rolling element bearings.

Table. 2.3. Summary of the studies on the thermal seizure of rolling bearings

Reference	Type of rolling element bearing	Operating conditions	Developed model	Investigated parameters
[32]	Grease lubricated angular contact ball bearings	Under thrust loads and at high rotational speeds	Static model using thermal resistances	Expansion and accommodation curves, Thickness of housing
[52]	Spindle bearings of high speed machine tools (Ball and roller bearings)	Under thrust and radial loads and at high rotational speeds	First-order ODE based on the heat conservation and thermal expansion equation	Effect of oil viscosity, External forces, Friction of the seal on the temperature evolution of the rolling elements
[53, 54]	Oil-circulated pair of thrust ball bearings	Preloaded under thrust load and at high rotational speeds	Linear first-order ODE based on experimental results	Effect of rotational speed, Oil flow-rate, Initial preload and bearing kinematics on the transient behavior of the thermal preload and the frictional torque
[58-62]	A pair of oil lubricated back-to-back angular contact ball bearings	Preloaded under thrust load and at high rotational speeds	Experimental investigation	Effect of rotational speed, Oil flow rate, Type of oil, Housing design and housing cooling on the variation of the thermal preload and the frictional torque

Table. 2.3. Summary of the studies on the thermal seizure of rolling bearings [cont'd]

Reference	Type of rolling element bearing	Operating conditions	Developed model	Investigated parameters
[63]	A pair of oil lubricated back-to-back angular contact ball bearings	Preloaded under thrust load and at high rotational speeds	Experimental investigation	Effect of rotational speed, Lubricant flow rate and initial preload on the thermally induced preload
[64]	A pair of back-to-back angular contact ball bearings	Preloaded under thrust load and at high rotational speeds	Experimental investigation	Effect of cooling of rotating shaft on the thermally induced preload
[58-62]	Grease lubricated double-row cylindrical roller bearings, Oil circulated back-to-back angular contact ball bearings	Preloaded and at high rotational speeds	State-observer technique to estimate the thermally-induced preload indirectly from the outer ring temperature and spindle speed	Effect of rotational preload, Rotational speed, Lubrication and cooling rate on the thermally induced preload and frictional torque of the bearing

2.3.2. Analytical work

The work presented by Burton and Staph [32] is probably the first mathematical model for thermally activated seizure of lightly lubricated (or grease lubricated) angular contact ball bearings operating under thrust loads and at high rotational speeds (100,000 RPM). This static model is based on a simple heat transfer model of the bearing assembly using the thermal resistance analogy. They presented a novel approach by treating the thermal behavior of the rolling bearing in terms of the expansion and accommodation curves which can serve as a useful tool to predict the thermal instability of the bearing; see Figure 2.9. This approach introduces the thermal expansion of the shaft/inner race and rolling elements as destabilizing factors while the thermal expansion of the housing/outer race is introduced as stabilizing factor (or bearing accommodation to expansion). They introduced the following criterion for the stability of rolling bearings:

$$\left[\frac{d\gamma}{d(F/F_0)} \right]_{\text{expansion}} \leq \left[\frac{d\gamma}{d(F/F_0)} \right]_{\text{accommodation}} \quad (2.17)$$

where γ is the radial expansion or accommodation. F is the contact load between the ball and the race and F_0 is the nominal contact load on the ball at design contact angle. According to Equation 2.17 if the rate of clearance change with respect to the loading for accommodation curve is larger than that of the expansion curve, the rolling bearing is in the stable condition. This is also shown in Figure 2.9 where three expansion curves (dashed lines) are plotted along with the accommodation curve (solid line). As Figure 2.9 shows, the bearing is in the stable operating condition for very thin-wall housings ($\frac{R_{bo}-R_{bi}}{L} = 0$), since the accommodation curve is larger than the expansion curve after the first intersection point. As the wall thickness of the housing increases ($\frac{R_{bo}-R_{bi}}{L} = \frac{1}{2}$) the expansion curve shifts to the right and there would be a second intersection point beyond which the bearing becomes unstable. Finally, if the wall thickness of the housing is very large ($\frac{R_{bo}-R_{bi}}{L} = 1$), the expansion curve can shift too far to the right so that there is no longer any intersection with the accommodation curve and the bearing operation would become unstable. Although Burton and Staph's model [32] can predict the thermal instability in steady-state conditions, it is unable to predict the thermal instability during the warm-up process or transient conditions. Another drawback of this model is the simplifying assumptions made in the heat transfer analysis and thermal expansion model which limits the application of this model for different bearing assemblies.

Figure 2.9. Thermal expansion functions for three housing thicknesses. Lower intersections indicate stable operating conditions. Absence of intersection indicates unstable operation [32].

Sud et al. [53, 54] presented a simple model to predict the variation of thermally induced preload with time. This model is a linear first-order differential equation developed based on the results of a set of experiments carried out on two custom-designed test rigs built to measure the growth

of thermally induced preload in a thrust bearing and an angular contact ball bearing. Since this model is solely based on the nature of the experimental results, its validity is very limited to the specific test rigs and investigated operating conditions and cannot be used for different bearing assemblies.

Privot [52] presented following first-order differential equations based on the heat conservation equation and thermal expansion of only the rolling element. One can solve these differential equations to calculate the temperature evolution of the rolling element with respect to time:

$$\frac{1}{2} f R_i \omega \left(\frac{\alpha R_b \Delta T}{2.13 \times 10^{-8} \sqrt{\sum \rho}} + F_I^{2/3} \right)^{3/2} = h_{bc} S_b \Delta T + \frac{d(\Delta T)}{dt} m_b C_p \quad \text{for ball bearings} \quad (2.18)$$

$$\frac{1}{2} f R_i \omega \left(\frac{\alpha R_b \Delta T L_b^{0.8}}{1.53 \times 10^{-10}} + F_I^{0.9} \right)^{1.11} = h_{bc} S_b \Delta T + \frac{d(\Delta T)}{dt} m_b C_p \quad \text{for roller bearings} \quad (2.19)$$

where R_i is the radius of inner race and R_b, L_b, S_b and m_b are, respectively, the radius, length, area of the outer surface and the mass of rolling element. h_{bc} is the convection heat transfer coefficient from the roller and F_I is the initial preload on one of the rolling elements. t is time and T is the temperature. ΔT is the temperature increase of the roller and $\sum \rho$ is the curvature sum of the roller bearing. Based on these differential equations, he developed a very useful block diagram for the thermal behavior of the rolling bearings. He used this block diagram for a comprehensive study of the stabilizing and destabilizing parameters involved in the thermal seizure of the rolling bearings. He concluded that the stability margin is not large except in the case of bearings lubricated with large oil flow. The slightest change in the oil viscosity, external forces, friction of the seal and many other parameters can results in a thermal instability. Subsequently he proposed a special design for an anti-seizure spindle bearing featuring a very thin outer race that is not pressed into the housing bore. Although Privot's model can predict the transient thermal behavior of the rolling bearings as well as the steady-state behavior, the broad assumptions used in the model limit the validity of some of the conclusions for general applications. Among these assumptions is neglecting the thermal expansion of the shaft and the housing which has been experimentally proven to play a key role in the thermal seizure of the rolling bearings [53, 55, 56, 60, 63].

Tu and Stein [58-62] presented one of the first comprehensive mathematical models for predicting the thermal behavior of rolling element bearings in transient as well as steady-state condition. The temperature distribution between the bearing elements, the shaft and the housing is calculated form a comprehensive thermal analysis of the bearing assembly. Then, thermally induced preload was estimated —after calculating the associated thermal expansions— using the Hertzian load-deflection formula. Tu and Stein [58-62] compared this estimated thermal preload with the experimental measurements and used it as the feedback to improve the accuracy in the calculation of the frictional heat generation inside the bearing. They utilized this model to develop a low-cost, state-observer technique that can be used to estimate the thermally-induced

preload indirectly from the easy-to-measure parameters such as the outer ring temperature and the spindle speed. The drawback of this model is the need for the special instruments of preload measurement which is utilized in order to calculate and tune up some of the parameters such as the contact resistances, thermal constriction resistances and frictional heat generation.

2.3.3. Experimental work

Carmichael and Davies [55-57] conducted a set of experiments and measured the variation of the preload and frictional torque in a pair of preloaded back-to-back angular contact ball bearings utilized in an idealized main spindle assembly. They investigated the effect of rotational speed, oil flow rate, type of oil, housing design and housing cooling on the variation of the thermally induced preload and the frictional torque of the bearing. The experimental results showed that an increase in the shaft speed increases the thermal preload and the frictional torque significantly; see Figure 2.10. It is also shown that the effect of the oil flow rate on the thermal behavior of the bearing is dependent on the type of the oil. Generally, greater oil flow rate decreases the thermal preload and the frictional torque due to the cooling effect, but this decrease in the thermal preload is less effective when dealing with lubricants with higher viscosities. Carmichael and Davies [55-57] reported much larger thermal preloads and frictional torques in the case of the water cooling of housing compared to the air cooling. This is due to the larger housing accommodation in case of air cooling compared to the water cooling of the housing. It is also reported that in the case of air cooling the thermal preload increases momentarily up to a maximum value and then it drops gradually towards a steady-state value. The variation of thermal preload in the case of water cooling is found to be different as it increases continuously until it reaches a maximum steady-state value; see Figure 2.10.

Sud et al. [53, 54] carried out a set of experiments on a custom-designed apparatus and measured the transient behavior of the thermal preload and the frictional torque of a preloaded pair of thrust ball bearings while the housing was cooled by water circulation. They investigated the effect of rotational speed, oil flow rate, initial preload and bearing kinematics on the transient thermal behavior of the bearing assembly. The influence of the bearing kinematics was investigated by using flat plates and grooved plates as the race ways of the thrust roller bearings utilized in the experiments. For both types of the raceways, they reported that the rotational speed had a significant influence on the thermal preload similar to the Carmichael and Davies results [55-57], while the frictional torque did not change considerably with the rotational speed. Sud et al. [53, 54] identified a critical rotational speed in which the thermally stable bearing becomes unstable.

Figure 2.10. Variation of (b) frictional torque with time for various shaft speeds at a lubricant supply of 2.5 gal/h with and without cooling water [55-57].

They concluded that this critical speed is dependent upon the thermoelastic response of the system and the rate of heat input into the bearing assembly. This behavior is shown in Figure 2.11 where it is shown that the bearing becomes unstable at rotational speeds larger than 800 rpm. Based on the experimental results, they presented the following simple model in form of a prismatic bar in order to calculate the heat input at any dimensionless time ($\frac{t}{t_c}$):

$$Q_0[1 + t_c \frac{d\tau}{dt} (\frac{t}{t_c})] \quad (2.20)$$

where t_c is a time constant and $\frac{d\tau}{dt}$ is rate of change of equivalent torque. Q_0 is an initial rate of heat input. One can solve this differential equation to determine the overall thermal expansion at any time ($\frac{t}{t_c}$). It was also shown that, in the case of flat races, lubrication lowers the thermal preload compared to the dry bearing, while the non-lubricated bearing with grooved races is thermally unstable.

(a)

(b)

Figure 2.11. Variation of (a) axial load and (b) frictional torque for different rotational speeds under 750 lbf initial preload and lubricant flow rate of 0.25 lb/hr [53, 54].

Lacey et al. [63] conducted an extensive experimental study on a pair of preloaded back-to-back angular contact ball bearings to investigate the effect of rotational speed, lubricant flow rate and initial preload on the thermally induced preload. They reported similar results to the Carmichael and Davies results [55-57] about the influence of oil flow rate on the thermal preload. They also showed that at lower speeds, the thermal preload increases gradually and reaches to a maximum steady-state value while at the higher speeds, the thermal preload reaches to a higher steady-state value after dropping from a transient peak; see Figure 2.12. This shows a nonlinear relation between the thermal preload and the rotational speed. Subsequently, in order to overcome the problem of thermal instability in spindle bearings, Lacey et al. [63] proposed a hydraulic preload system that can adjust and maintain an optimum preload in the bearing in low speeds as well as high speeds. This mechanism was built and proved its efficiency in the spindle bearings rotating up to 1×10^6 DN numbers.

Figure 2.12. Nonlinear relationship between thermal preload and rotational speed [63].

Chun [64] investigated the effect of cooling of rotating shaft in a rolling bearing assembly. He showed that increasing the cooling rate of the shaft decreases the thermal preload and results in a stable thermal behavior of the rolling bearing. This is in agreement with the analytical results that present the thermal expansion of the shaft and the rolling elements as the destabilizing factors and the thermal expansion of the housing as the stabilizing factor.

As explained in Section 3.2, Tu and Stein [58-62] developed a low-cost monitoring technique that can be used to estimate the thermally-induced preload inside the bearing from easy-to-measure parameters of outer ring temperature and spindle speed. They investigated the applicability of this technique to estimate the variation of the thermal preload in a radially-preloaded grease-lubricated double-row cylindrical roller bearing used in high-precision spindle machine tools. It was shown that this monitoring technique can successfully estimate the thermal preload in thermally-stable low speeds as well as thermally-unstable high speeds. The application and accuracy of this monitoring technique was also examined for the experimental

results reported by Lacey et al. [63] where the variation of the thermal preload was successfully estimated for a wide range of rotational speeds and initial preloads.

Although the existing literature reviewed in this paper provide very useful information about the thermal failure of rolling element bearings, the need remains for development of a comprehensive analytical model that can shed light on all the phenomena involved in the thermal failure of rolling bearings in different applications and under different operating conditions. Also, in addition to the unbounded increase of the thermally induced preload, there are experimental evidences [65] that cage failure may occur due to the overheating of rolling bearings. This is in fact due to the complex and coupled phenomena involved in the thermal failure of the rolling bearings in different applications. Perhaps the lack of an accurate and comprehensive model— one that can calculate the heat generation inside the bearing assembly in a wide range of application— is still the biggest challenge in modeling of thermal failure of roller bearings. What complicates the analyses even more is the need for proper consideration of the contact forces and moments between the rolling elements, the raceways and the cage whose behavior are functions of the rheological behavior of the lubricant and the operating temperature of bearing elements. To this end, additional difficulties arise when one considers the need to assess the influence of some of the hard-to-measure thermal parameters such as contact resistances between the raceways, the shaft and the housing and constriction resistances between the rolling elements and the raceways. To the best knowledge of the authors, none of the presented models in the literature for thermal failure of the rolling element bearings provide comprehensive and detailed information about some of the critical parameters of the rolling bearing performance such as film thickness and interaction forces and moments between the bearing components. It is believed that a comprehensive analysis of thermal failure of the rolling bearings is also needed to provide a clear and detailed analysis of these critical bearing parameters during the thermal failure of bearings under different operating conditions.

2.4. Conclusions

In spite of different and complicated processes involved in the TIS of bearings at different operating conditions, it is possible to draw following conclusions about TIS of bearings:

- Seizure can occur in loaded, unloaded, lubricated, unlubricated bearings.
- Rotational speed plays the key role in the TIS of bearings. The likelihood of TIS increases exponentially at higher rotational speeds.
- Increasing the lubricant flow rate and cooling rate of the shaft along with decreasing the cooling rate of the housing can significantly reduce the likelihood of TIS in bearings.
- The thickness of the housing has the same effect similar to the external cooling on the TIS of bearings.

- The clearance loss varies nonlinearly with time in journal bearings during the TIS.
- Journal bearings with hollow shaft are more susceptible to seizure than the journal bearings with solid shaft working at the same operating conditions.
- The detailed analysis of TIS in journal bearings showed the ovalization of the bushing before the ultimate seizure in unidirectional journal bearings. This ovalization occurs for both of the shaft and the bushing for the case of oscillating journal bearings.
- Advanced comprehensive analytical models can be developed in order to analyze the thermal failure of rolling bearings. This calls for realistic formulation of the rheological behavior of the lubricant and the heat generation inside the rolling bearings. Such detailed models can provide appropriate information about the film thickness, traction coefficient and all of the interaction forces and moments between the bearing elements.
- It appears that monitoring of the frictional torque of the bearings is the easiest and least expensive method that can be used to predict the TIS in bearings.

2.5. Nomenclature

A	Total available area for convective heat transfer, $A_s + A_b$
A_b	Outside surface area of the bushing
A_f	Friction area
A_s	Surface area of the shaft
C_p	Specific heat
c	Operating clearance of journal bearing
c_i	Initial clearance of journal bearing
D	Shaft diameter
D_p	particle diameter
E	Modulus of elasticity
F	Contact load between the ball and the raceway
F_0	Nominal contact load on the ball at design contact angle
F_l	Initial preload on one of the rolling elements
f	Friction coefficient
H	Overall convective heat transfer, $\frac{A_s h_s + A_b h_{bo}}{A_s + A_b}$
h_{bc}	Heat convection coefficient from the roller
h_{bi}	Heat convection coefficient from inside the bushing/housing
h_{bo}	Heat convection coefficient from outside of the bushing/housing
h_s	Heat convection coefficient from shaft
k	Thermal conductivity
L	Length of the bushing
L_b	Length of rolling element

L_c	Characteristic length ($\frac{volume}{area}$)
M	System thermal capacity
m_b	Mass of rolling element of rolling element
n	Heat portioning factor
P	Bearing pressure
Pe_D	Peclet number based on the particle diameter (D_p)
Pe_{Dcr}	Critical Peclet number
Q_0	Initial rate of heat input
q_a	Heat flow per unit of surface area
R_b	Radius of rolling element
R_i	Radius of inner raceway
r_{bi}	Inner radius of bushing
r_{bo}	Outer radius of bushing
r_s	Shaft radius
S_b	Outer surface of rolling element
T	Temperature
T_f	Average flash temperature
t	Time
t_c	Time constant
t_{ref}	transition time for bearing to go from fully lubricated to boundary lubrication condition
t_{sp}	Seizure-time when flow disturbance occurs
$\overline{t_{sp}}$	Dimensionless seizure-time when flow disturbance occurs
t_{ss}	Seizure-time during the start-up
$\overline{t_{ss}}$	Dimensionless seizure-time during the start-up
U	Linear velocity of slider
V_s	Sliding speed
V_s^*	Critical sliding speed
W	Radial load on the bearing
W_l	Load per unit length
α	Thermal expansion coefficient
β	Hardness ratio
γ	Radial expansion or accommodation of roller bearing
ρ	Density
δ	Net diametral growth of bushing relative to the shaft
ϑ	Poisson's ratio
ϵ	Thermal diffusivity
τ	Yield shear stress
σ	Hardness
δ_c	clearance reduction due to relative thermal expansion
ψ	Encroachment factor for lubricated bearing
ϕ	Encroachment factor for unlubricated bearing
μ_0	Initial lubricant velocity
ω	Rotational speed
λ	Modified aspect ratio

τ^*	Dimensionless seizure-time
ΔT	Temperature difference
$\sum \rho$	Curvature sum

2.6. References

- [1] Barber JR. Thermoelastic instabilities in sliding of conforming solids. Proceedings of the Royal Society of London Series a-Mathematical and Physical Sciences. 1969;312:381-394.
- [2] Banerjee BN, Burton RA. Instability for parallel sliding of solid-surfaces separated by a viscous-liquid film. Journal of Lubrication Technology-Transactions of the Asme. 1976;98:157-166.
- [3] Jang JY, Khonsari M. Thermoelastic instability in mechanical systems with provision for surface roughness. In: Hetnarski R, editor. Encyclopedia of thermal stresses: Springer Netherlands; 2014. p. 5699-5714.
- [4] Jang JY, Khonsari MM. Thermoelastic instability including surface roughness effects. Journal of Tribology-Transactions of the Asme. 1999;121:648-654.
- [5] Jang JY, Khonsari MM. Thermoelastic instability with consideration of surface roughness and hydrodynamic lubrication. Journal of Tribology-Transactions of the Asme. 2000;122:725-732.
- [6] Jang JY, Khonsari MM. A generalized thermoelastic instability analysis. P R Soc A. 2003;459:309-329.
- [7] Barber JR. Elasticity. Kluwer Academic, Boston. 1992:200-201.
- [8] Lee K, Barber JR. Frictionally excited thermoelastic instability in automotive disk brakes. Journal of Tribology-Transactions of the Asme. 1993;115:607-614.
- [9] Lee K, Barber JR. The effect of shear tractions on frictionally-excited thermoelastic instability. Wear. 1993;160:237-242.
- [10] Jang JY, Khonsari MM. On the formation of hot spots in wet clutch systems. Journal of Tribology-Transactions of the Asme. 2002;124:336-345.
- [11] Zagrodzki P. Analysis of thermomechanical phenomena in multidisk clutches and brakes. Wear. 1990;140:291-308.
- [12] Jang JY, Khonsari MM. Thermoelastic instability of two-conductor friction system including surface roughness. J Appl Mech-T Asme. 2004;71:57-68.
- [13] Lebeck AO. Effect of ring deflection and heat-transfer on the thermoelastic instability of rotating face seals. Wear. 1980;59:121-133.
- [14] Lebeck AO. Principles and design of mechanical face seals. Wiley, New York. 1991.

- [15] Netzel JP. Observations of thermoelastic instability in mechanical face seals. *Wear*. 1980;59:135-148.
- [16] Peng ZC, Khonsari MM, Pascovici MD. On the thermoelastic instability of a thin-film-lubricated sliding contact: A closed-form solution. *Proceedings of the Institution of Mechanical Engineers Part J-Journal of Engineering Tribology*. 2003;217:197-204.
- [17] Burton RA, Nerlikar V, Kilapart.Sr. Thermoelastic instability in a seal-like configuration. *Wear*. 1973;24:177-188.
- [18] Kilaparti R, Burton RA. Moving hot-spot configuration for a seal-like geometry, with frictional heating, expansion and wear. *Asle Transactions*. 1977;20:64-70.
- [19] Burton RA. Thermal deformation in frictionally heated contact. *Wear*. 1980;59:1-20.
- [20] Burton RA. *Heat, bearings and lubrication*. Springer, New York. 2000.
- [21] Winer WO, Peterson MB. *Diagnosis of wear mechanisms*,. Amer Soc of Mech Engineers, New York. 1980:283-311.
- [22] Dyson A. Scuffing - a review. *Tribology*. 1975;8:77-87 and 117-121.
- [23] Khonsari MM, Wang SH. On the role of particulate contamination in scuffing failure. *Wear*. 1990;137:51-62.
- [24] Khonsari MM, Pascovici MD, Kucinski BV. On the scuffing failure of hydrodynamic bearings in the presence of an abrasive contaminant. *Journal of Tribology-Transactions of the Asme*. 1999;121:90-96.
- [25] Pascovici MD, Khonsari MM. Scuffing failure of hydrodynamic bearings due to an abrasive contaminant partially penetrated in the bearing over-layer. *Journal of Tribology-Transactions of the Asme*. 2001;123:430-433.
- [26] Hirst W. Scuffing and its prevention. *Chartered Mechanical Engineer*. 1974;21:88-92.
- [27] Lee SC. Scuffing modeling and experiments for heavily loaded elastohydrodynamic lubrication contacts. PhD Dissertation, Northwestern University. 1989.
- [28] Peng ZC, Khonsari MM. A thermohydrodynamic analysis of foil journal bearings. *Journal of Tribology-Transactions of the Asme*. 2006;128:534-541.
- [29] DellaCorte C, Radil KC, Valco MJ, Heshmat H. Performance and durability of high temperature foil air bearings for oil-free turbomachinery. NASA/TM-2000-200187. 1999.
- [30] Radil KC, Howard SA, Dykas B. The role of radial clearance on the performance of foil air bearings. NASA/TM—2002-211705 2002.

- [31] Burton RA. Thermal aspects of bearing seizure. *Wear*. 1965;8:157-172.
- [32] Burton RA, Staph HE. Thermally activated seizure of angular contact bearings. *Asle Transactions*. 1967;10:408-415.
- [33] Wang QA. Seizure failure of journal-bearing conformal contacts. *Wear*. 1997;210:8-16.
- [34] Khonsari MM, Booser ER. *Applied tribology-bearing design and lubrication*. John Wiley & Sons Ltd, Chichester, UK. 2008.
- [35] Burton RA. *Heat, bearings, and lubrication: Engineering analysis of thermally coupled shear flows and elastic solid boundaries* Springer; 2000 edition (January 1, 2000). 1999.
- [36] Pinkus O. Thermal aspects of fluid film tribology. *Amer Society of Mechanical* (February 1990). 1990.
- [37] Heckmann SR, Burton RA. Seizure of eccentrically loaded journal bearings and sealing rings. *Journal of Lubrication Technology-Transactions of the Asme*. 1979;101:419-423.
- [38] Bishop JL, Ettles CMM. The seizure of journal bearings by thermoelastic mechanisms. *Wear*. 1982;79:37-52.
- [39] Dufrane KF, Kannel JW. Thermally induced seizures of journal bearings. *Journal of Tribology-Transactions of the Asme*. 1989;111:288-292.
- [40] Khonsari MM, Kim HJ. On thermally induced seizure in journal bearings. *Journal of Tribology-Transactions of the Asme*. 1989;111:661-667.
- [41] Hazlett TL, Khonsari MM. Finite-element model of journal bearings undergoing rapid thermally induced seizure. *Tribology International*. 1992;25:177-182.
- [42] Hazlett TL, Khonsari MM. Thermoelastic behavior of journal bearings undergoing seizure. *Tribology International*. 1992;25:183-187.
- [43] Pascovici MD, Khonsari MM, Jang JY. On the modeling of a thermomechanical seizure. *Journal of Tribology-Transactions of the Asme*. 1995;117:744-747.
- [44] Wang H, Conry TF, Cusano C. Effects of cone/axle rubbing due to roller bearing seizure on the thermomechanical behavior of a railroad axle. *Journal of Tribology-Transactions of the Asme*. 1996;118:311-319.
- [45] Jang JY, Khonsari MM, Pascovici MD. Thermohydrodynamic seizure: Experimental and theoretical analysis. *Journal of Tribology-Transactions of the Asme*. 1998;120:8-15.
- [46] Jang JY, Khonsari MM, Pascovici MD. Modeling aspects of a rate-controlled seizure in an unloaded journal bearing. *Tribology Transactions*. 1998;41:481-488.

- [47] Krithivasan R, Khonsari MM. Thermally induced seizure in journal bearings during startup and transient flow disturbance. *Journal of Tribology-Transactions of the Asme*. 2003;125:833-841.
- [48] Wen J, Khonsari MM. Thermomechanical coupling in oscillatory systems with application to journal bearing seizure. *Journal of Tribology-Transactions of the Asme*. 2009;131.
- [49] Khonsari MM, Kim HJ. Discussion: "Thermally induced seizures of journal bearings" (dufrane, k. F., and kannel, j. W., 1989, *asme j. Tribol.*, 111, pp. 288–292). *Journal of Tribology*. 1989;111:292-292.
- [50] Khonsari MM, Booser ER. Design bearings that don't seize. *Machine Design*. Jan. 8, 2004;82-83.
- [51] Wen J, Khonsari MM, Hua DY. Three-dimensional heat transfer analysis of pin-bushing system with oscillatory motion: Theory and experiment. *Journal of Tribology-Transactions of the Asme*. 2011;133.
- [52] Pruvot FC. High speed bearings for machine tool spindles. *Annals of the CIRP*. 1980;29:293-297.
- [53] Sud ON, Davies PB, Halling J. The thermal behaviour of rolling bearing assemblies subjected to preload. *Wear*. 1974;27:237-249.
- [54] Sud ON. The transient thermal behavior of ball thrust race assemblies. PhD Thesis, University of Salford. 1971.
- [55] Carmicha GDT, Davies PB. The influence of transient thermal distortion on the behavior of an assembly containing two angular contact ball bearings. *Proc Instn Mech Engrs*. 1970;184:26-32.
- [56] Carmicha GDT, Davies PB. Factors which affect transient behavior of preloaded ball-bearing assenblies. *Asle Transactions*. 1972;15:1-7.
- [57] Carmicha GDT. The effects of thermal distortion in a main spindle assembly. PhD Thesis, University of Salford. 1969.
- [58] Tu JF. On-line preload monitoring for high-speed anti-friction spindle bearings using robust state observers PhD dissertation, Department of Mechanical Engineering, The University of Michigan, Ann Arbor, MI. 1991.
- [59] Tu JF. Thermoelastic instability monitoring for preventing spindle bearing seizure. *Tribology Transactions*. 1995;38:11-18.
- [60] Tu JF, Stein JL. Online preload monitoring for antifricition spindle beatings of high-speed machine-tools. *Journal of Dynamic Systems Measurement and Control-Transactions of the Asme*. 1995;117:43-53.

- [61] Tu JF, Stein JL. Active thermal preload regulation for machine tool spindles with rolling element bearings. *Journal of Manufacturing Science and Engineering-Transactions of the Asme*. 1996;118:499-505.
- [62] Tu JF, Stein JL. Model error compensation for observer design. *Int J Control*. 1998;69:329-345.
- [63] Lacey SJ, Wardle FP, Poon SY. High-speed bearings for cnc machine-tool spindles. *Cme-Charter Mech Eng*. 1983;30:51-56.
- [64] Chun, Jiang. Experimental investigation of water cooling inside the spindle bearing system. *Ji Chuang/Machine Tools*. 1988;12:15-19.
- [65] TA H. Rolling bearing analysis. New York: Wiley. 2001;4th ed.

Chapter III: Developing a comprehensive thermal model of rolling element bearings *

3.1. Introduction

The operating temperature plays a key role in the overall performance of a bearing system. Affected by the bearing temperature are many critical parameters, such as the lubricant viscosity, load—carrying capacity, load distribution and power loss. In contrast to the thermal analysis of journal bearings which has been investigated by many researchers [1-12], the thermal analysis of the roller bearings has not been addressed thoroughly because of their inherent geometric and dynamic complexity. Hence, only a few experimental and theoretical investigations are available to assist the designers.

The literature on the thermal analysis of rolling bearings can broadly be categorized in two groups. The first category focuses on developing a model to estimate the temperature of roller bearing components. One of the pioneering works on prediction of the bearing temperature is presented by Harris [13]. Applying the basic laws of heat transfer to rolling bearing assembly and using the lumped-mass assumption, he estimated the steady-state temperature of the bearing components. Later, Winer et al. [14] successfully improved the Harris's model by solving a set of governing equations for the heat transfer in a complete assembly of a tapered roller bearing including the housing. This model is able to estimate the temperature of the bearing elements during the transient as well as the steady-state condition.

The second category includes the investigation of rolling bearing failure or seizure due to excessive heat generation inside the bearing. This kind of thermo-elastic seizure can occur in spindle bearings of high speed machines and often leads to premature and catastrophic bearing failure. This typically results because of an imbalance between the frictional heat generation inside the bearing and heat dissipation to the surroundings. Consequently, the bearing components undergo excessive thermal expansion resulting in sudden increase in the contact loads. One of the early theoretical model for the thermal failure of roller bearings was developed by Burton et al. [15] for the case of dry or lightly-lubricated angular contact ball bearings where consideration was given to the relative expansion of bearing components, the bearing accommodation, as well as the changes in the contact angle. They presented a novel idea for comparing the combination of geometric and elastic accommodations with the nominal clearance wherein the expansion of inner ring and shaft was treated as a de-stabilizing factor and that of the housing as a stabilizing factor. Also assumed in the work of Burton et al. [15] is that for a bearing without internal cooling, seizure can be determined according to the steady-state thermal

* Reprinted by permission of *Tribology International* (See Appendix C)

behavior instead of the transient behavior. However, the static nature of their model renders it less amenable for evaluating the transient behavior of the bearing before it reaches the steady-state phase. A simple linear first-order differential equation for estimating thermally-induced preload during the transient phase was proposed by Sud et al. [16] based on the experimental preload change. Considering the conservation of thermal energy only for the rolling elements, Pruvot [17] proposed another simple first-order differential equation to estimate the temperature of the rolling elements of spindle tapered roller bearings in high speed machine tools. Moreover, he developed a more complete model by considering thermal energy balance and variable temperatures for the spindle and housing. It is noteworthy to mention that Pruvot employed concepts from control theory to model the thermal behavior of roller bearing by the virtue of block diagrams to clarify the overall heat transfer process, and studied the effect of many parameters in his mathematical model. However, this model does not consider the heat conduction between the bearing components and thus suffers from the lack of generality in treating heat transfer problems in roller bearings. Tu and Stein [18] presented one of the most comprehensive thermal analysis of the roller bearings. They proposed a dynamic mathematical model of spindle system in high speed machine tools for use in an online preload monitoring system. Their model uses a range of readily measurable system quantities, such as housing temperature and spindle speed, as a feedback to estimate the transient thermally-induced preload in a spindle bearing. This approach was also used for state-space observer design [19].

The present study is aimed at developing a general model applicable to a wide range of practical applications. To this end, a new model is proposed and simulations are carried out to investigate the thermal behavior of the bearing systems. The methodology proposed in the present work is amenable to roller bearing systems with different geometries and complex boundary conditions. The proposed model is used to predict the evolution of the transient temperatures in the bearing components. Appropriate formulas and solution methodology are proposed to estimate the value of the critical parameters for a roller bearing with oil-bath lubrication, such as induced thermal preload, contact resistances and convective heat transfer coefficients inside and outside of the bearing assembly. To validate the model, experiments are carried out on a bearing test rig for different values of rotational speed and external radial loads. Further simulations are carried out in order to investigate the importance of different parameters on the thermal response of the ball bearing assembly.

3.2. Test Rig Assembly

3.2.1. Description of Apparatus

To investigate the thermal behavior of a roller bearing, a test rig was designed and constructed. The rig is designed for installing a 75-mm (2.953 in) bore bearing to operate at variable speeds up to 5600 RPM; see Figure 3.1. It is capable of testing different bearing materials such as stainless steel balls in a brass cage as well as ceramic balls in a polyamide cage. Heavy duty

roller bearings with DN number (the mean diameter of the bearing in millimeter times the rotational speed of the shaft in RPM) as high as 574,000 can be safely tested on this test rig. Equipped with a variety of sensors such as temperature, torque, vibration and acoustic signals, this test rig can be used for failure prognosis in roller bearing systems.

Figure 3.1 shows a schematic view of the test rig. The speed of motor could be set independently by a variable frequency drive. Installed between the motor and the shaft are two couplings. The coupling closer to the support bearing is of torque limiting type whereas the other one is a standard coupling. To protect the motor, VFD, and instrumentation, the torque limiting coupling was chosen to act as a safety mechanism at the failure. This coupling is connected to the support bearings and the test bearing by means of a 76.2 mm (3 inch) diameter shaft. The two support bearings are selected to be double-row self-aligning ball bearings (SKF 2315 K C3) due to their low coefficient of rolling friction and self-compensation for angular misalignments. Selected as the test bearing is a single-row deep groove ball bearing (SKF 6315/C3). Oil-bath lubrication system is used for the bearings in the test rig. However, the test bearing can accommodate other types of lubrication methods. Not displayed in this figure are the pneumatic system and the instrumentation equipment. The pneumatic system is separately designed and constructed to apply the external radial force on the test bearing. A maximum ram pressure of 160 Psi can achieved using this pneumatic system.

Figure 3.1. A Schematic of the Test Rig

3.2.2. Instrumentation

Instrumentation includes a torque sensor (Interface T8 ECO Rotary Torque Transducer) between the shaft and the driving motor to measure the frictional torque of the bearing system. The torque-meter is calibrated and can measure the frictional torque of the system up to 15 N-m.

Attached to the outer surface of the housing in the test and support-bearings are two K-type thermocouples. Two additional K-type thermocouples are mounted on the outer raceway of the support bearings. Another K-type thermocouple is placed inside the oil-bath of the housing of test bearing to measure the oil temperature. It is noted that only the temperature in the test bearing is of interest and the temperatures in the support bearing are used only for checking results and comparison purposes. A data acquisition system is also employed to collect and transfer the data to a computer for further analysis. A termination block with a built-in cold-junction compensation was used to measure the thermocouples output. The output from the data acquisition system is continuously monitored and analyzed using the LabView™ software.

3.3. Modeling and simulations

Due to the multiplicity of parameters involved, a well-ordered block diagram, similar to the proposed preload model by Tu and Stein [18], is developed for analyzing the heat transfer processes and facilitating the parametric study of the bearing system.

Figure 3.2. block diagram of the entire model

Figure 3.2 depicts the block diagram constructed to investigate the thermal behavior of the rolling element bearing system. According to Tu and Stein [18] the entire process of the thermal analysis can be divided into four inter-connected stages, namely frictional heat generation, heat transfer, thermal expansion and the corresponding induced preload. The total frictional heat generation within a roller bearing can be calculated either analytically or experimentally in the first stage by considering the key parameters, such as external load, initial and induced preload, lubrication type, rotational speed, friction coefficient and the appropriate geometrical parameters. Analyzed in the second stage are all the heat transfer processes including heat conduction within and between the bearing components as well as free and forced convection from the surrounding environment. Once the temperature field is available, the oil viscosity and dimensional changes of the bearing components are calculated in the third stage. The last stage of the analysis involves the calculation of the bearing preload based on the Hertzian contact theory and the evaluation of the clearance change as a result of thermal expansion. The analysis is, then, repeated from the first stage with the bearing preload and viscosity as additional input to update the frictional heat generation used in the first stage. This procedure is continued iteratively until the steady-state condition is attained, i.e. the temperature changes in the successive cycles become negligible. It is to be mentioned that the heat radiation is neglected owing to the relatively low surface temperatures in the present study.

3.3.1. Frictional heat generation Model

The total frictional torque in a roller bearing is a measure of the power loss in contacting surfaces of bearing components as well as the power loss due to the viscous drag. The analytical estimation of the frictional torque is burdened with the multiplicity of factors, such as friction between the rolling elements, the raceways and the cage, and the change in the lubricant viscosity due to the rise in temperature, and hence is accompanied by uncertainties [20]. An empirical relationship proposed by Palmgren [13, 21] is used in the present study which has been found to yield realistic results [19]. It is noteworthy to mention that these formulas assume that an elasto-hydrodynamic film exists between the contacting surfaces inside the rolling bearing.

The total frictional torque in a roller bearing is given by Palmgren [13, 21]

$$M = M_l + M_v + M_f \quad (3.1)$$

where M_l , M_v and M_f are, respectively, the frictional torques due to the applied load, viscous effect and that due to the flanges. However, the roller end-flange friction torque (M_f) is not considered for the case of ball bearings. Stein and Tu (1994) [18] modified this equation to consider the effect of induced thermal preload:

$$M_v = 0.8 f_v (\mu_v n)^{\frac{2}{3}} (r_{in} + r_b)^3 \quad (3.2)$$

$$M_l = \mu_l f_l (F_n + F_t) (r_{in} + r_b) \quad (3.3)$$

where f_v and μ_l denote the rolling friction coefficient and the sliding friction coefficient, respectively. F_n and F_t are the normal contact force and the thermally-induced preload, respectively. The inner ring and the ball radii are r_{in} and r_b , respectively, and μ_v is the lubricant viscosity given in Centistokes(cSt). The rotational speed is denoted by n in revolutions per minute, and f_l is a coefficient related to the rolling direction which is dependent upon bearing design as specified in the manufacturer's catalog.

The total frictional heat generation is:

$$Q_f = \frac{2\pi n}{60} M \quad (3.4)$$

It is noted that this formula is valid if the level of oil inside the housing of the bearing is according to the value recommended by the bearing manufacturer, and appropriate equations should be employed in the case of different oil levels [22]. This can influence the accuracy of the frictional torque prediction for different oil levels. It is necessary to emphasize that these equations are intended to be used under normal operating condition of a roller bearing and that any major deviations such as misalignment of the shaft or insufficient lubrication is expected to dramatically change the resulting frictional torque. Accordingly, in the present study, the estimated frictional torque is modified using correction factors obtained by means of torque sensor output.

3.3.2. Heat Transfer Model

In the present study, the lumped-mass assumption is considered to be suitable for the rollers and bearing raceways since their Biot number is small enough ($Biot < 0.1$). Furthermore, in many applications the mass of the rollers and the raceways are small compared to the housing and the shaft. Therefore, this assumption is not expected to produce any considerable error in the predictions. Unlike many studies available in literature that consider either a lumped-mass assumption [15, 18] or a full thermal network [14] for the entire model, the present study adopts a composite approach. In this approach, the entire model is discretized into a set of isothermal elements represented by thermal nodes (nodes 1 through 17); see Figure 3.3. A network of thermal resistance elements is used to represent the housing and the shaft whereas the balls and raceways are modeled as lumped thermal masses connected to the network by means of contact resistances. Considering the energy balance equation for each node, one can write the governing equations for the thermal analysis of the roller bearing assembly.

The general statement of energy balance is:

$$\dot{E}_{in} + \dot{E}_g = \dot{E}_{out} + \dot{E}_{sto} \quad (3.5)$$

where $\dot{E}_{in}$ and $\dot{E}_{out}$ represent the inlet and outlet energy rate, respectively. $\dot{E}_g$ is the energy generation rate and $\dot{E}_{sto}$ is the rate of the energy storage inside the control volume.

Figure 3.3 shows the entire thermal network of the test bearing consisting of 17 thermal nodes for the bearing assembly with 9 nodes for the shaft, 4 nodes for the housing and a total of 3 nodes for the ball, the inner and outer raceways. It should be mentioned that only one node (node 17) is considered for the lubricant in the oil bath system. Clearly, in the presence of lubricant circulation appropriate fluid equations considering inlet and outlet flow of the oil should be considered. As discussed in Section 5, the number of nodes implemented in the thermal network is found to be sufficient for prediction of bearing temperature. Owing to the symmetry of the model with respect to the shaft centerline and the vertical section, only a quarter of the actual model is used in this study. Also assumed in this model, is that the entire inner surface of the housing has the same temperature since it is in contact with the oil.

Figure 3.3. Thermal resistance network of the bearing assembly

One of the advantages of this model compared to other models [15, 17, 18, 23], is that it does not require heat partitioning factors owing to the continuity of the thermal model. In fact, some studies assume a simple heat partitioning factor (e.g., 0.5) without considering the appropriate boundary conditions with questionable accuracy.

In this model, the mass of the cage is added to that of the balls and both are presented as one node in the thermal resistance network. The energy balance equation for this node is

$$\frac{T_{in} - T_b}{R_{e1}} + \frac{T_{out} - T_b}{R_{e2}} + \frac{T_{oil} - T_b}{R_{c.oil}} + Q_f = m_b C_b \frac{\partial T_b}{\partial t} \quad (3.6)$$

where Q_f is the total frictional heat generation calculated in Section 3.1. T_{in} , T_{out} , T_b and T_{oil} are the temperature of inner race, outer race, ball and oil, respectively. $R_{c.oil}$ is the convective resistance due to lubricant in contact with the balls, and can be estimated using the formula presented in table 3.1 with the corresponding Nusselt number given by [13]

$$\overline{Nu} = \frac{h_v D}{k_a} = 0.33 Re_D^{0.5} Pr^{0.4} \quad (3.7)$$

where Re_D is the Reynolds number for the rotating ball and h_v is the convection coefficient relevant to the lubrication system within the ball bearing.

The thermal constriction resistances associated with the contact between the ball and the inner and outer raceways is calculated based on the following formula presented in reference [24] based on the analytical model of Yovanovich [25, 26]

$$R_c = \frac{\Psi(a/b)}{4ka}, \quad \Psi(a/b) = \frac{2}{\pi} K\left(e, \frac{\pi}{2}\right) \quad (3.8)$$

where a and b represent the semi-major and semi-minor axes of the elliptical contact area, respectively. k is the thermal conductivity and $K\left(e, \frac{\pi}{2}\right)$ is the complete elliptic integral of the first kind, where e is the eccentricity of the contact ellipse. The equivalent thermal constriction resistances can be calculated by the following equations

$$R_{ei} = \frac{R_{ci}}{n_b}, \quad R_{eo} = \frac{R_{co}}{n_b} \quad (3.9)$$

where R_{ei} and R_{eo} are the equivalent thermal constriction resistances associated with the contact between the ball and the inner and outer raceways, respectively. The number of loaded rollers forming thermal resistances in parallel configuration is denoted by n_b . (Figure 3.4)

Figure 3.4. thermal constriction resistances

Similar to Equation 3.2, the energy balance equations for the inner and outer raceways are written considering the thermal resistance network shown in Figures 3.3 – 3.4.

$$\frac{T_7 - T_{in}}{R_{cont.i}} + \frac{T_b - T_{in}}{R_{e1}} + \frac{T_{oil} - T_{in}}{R_{c.oil}} = m_{in} C_{in} \frac{\partial T_{in}}{\partial t} \quad (3.10)$$

$$\frac{T_b - T_{out}}{R_{e2}} + \frac{T_{13} - T_{out}}{R_{cont.o}} + \frac{T_{oil} - T_{out}}{R_{c.oil}} = m_{out} C_{out} \frac{\partial T_{out}}{\partial t} \quad (3.11)$$

where T_7 and T_{13} are the temperature of the nodes on the surface of the shaft and the inner surface of the housing, respectively; see Figure 3.3. It is noted that $R_{cont.i}$ is the thermal contact resistance between the inner raceway and the shaft, and $R_{cont.o}$ is the thermal contact resistance between the outer raceway and the housing. Analytical estimation of these contact resistances is a complicated task on the account that they depend on the interference between the bearing, the shaft and the housing which, in turn, is subject to variation as the temperature and therefore the dimension of the bearing components change during the test. The values used for $R_{cont.i}$ and $R_{cont.o}$ in the present study are estimated 0.02 WK-1 and 0.1 WK-1 respectively, which are obtained according to the experimental results in the literature [27-30]. The simulations were run with different values of contact resistances within a typical range, but only to ascertain the negligible effect of these parameters on the final resulting temperatures.

In order to write the energy balance equation for the nodes in contact with air, one needs to estimate the convective heat transfer coefficient on the outer surfaces of the bearing assembly. The forced convection coefficient for the rotating shaft can be estimated by Becker's equation [31]:

$$\overline{Nu} = 0.119 Re_r^{2/3} \quad 800 \leq Re_r \leq 100000 \quad (3.12)$$

where $\overline{Nu}$ is the Nusselt number and Re_r is the Reynolds number for the rotating shaft. Therefore, The convective heat transfer coefficient h , calculated by following formula

$$\overline{Nu} = \frac{h D_{sh}}{k_a} \quad (3.13)$$

Where D_{sh} is the shaft diameter and k_a is the thermal conductivity of the air at the appropriate temperature. The convective heat transfer coefficient on the housing surface is considered to be a more critical parameter in view of the relatively larger surface of the housing comparing to the surface of the rotating shaft. Due to the complex geometry of the housing experimental measurement is preferred over the analytical approach to estimate the convection coefficient for the housing. To this end, an experimental approach proposed by Meneghetti [32] and also used by others [33] is adopted. According to the detailed theoretical analysis presented in [32] series of tests are carried out until the temperature of the housing as well as the other bearing components reaches the steady state. Subsequently, the tests are stopped by shutting down the motor with the temperatures still being monitored. According to the theoretical analysis presented in detail elsewhere [32], the rate of the temperature drop at the onset of the interruption of the test can be used to estimate the convection coefficient. Figure 3.5 depicts the application

of this approach on a set of experiments whereas the cooling condition after stopping the motor is the same. This figure clearly shows that the calculated air convection coefficient for these experiments is almost constant and equals $15 \text{ Wm}^{-2}\text{K}^{-1}$.

Figure 3.5. Estimation of the convection coefficient for the housing by shutting down the motor after the temperatures reach the steady state condition

Finally, to calculate the remaining the conductive and convective resistances for the shaft and the housing, the following formulas are used.

Table 3.1. Conductive and convective resistances formulas for the shaft and housing

$R_{2-1}, R_{3-2}, R_{5-4}, R_{6-5}, R_{8-7}, R_{9-8}$	$\left(\ln \frac{R_2}{R_1}\right) / 2\pi kL$; for the radial conduction in the shaft
$R_{4-1}, R_{5-2}, R_{6-3}, R_{7-4}, R_{8-5}, R_{9-6}, R_{13-14}, R_{14-15}, R_{13-16}, R_{16-15}$	$\frac{\Delta L}{AK}$; for the axial conduction
$R_{oil_convection}, R_{air_convection}$:	$\frac{1}{hA}$; for the convection

Using the following parameters:

$$\bar{T} = \frac{T - T_a}{T_a}, \quad \bar{t} = \frac{t}{mc_p R} \quad (3.14)$$

where c_p is the specific heat of the material and T_a is the ambient temperature. The dimensionless form of Equations 3.6, 3.10 and 3.11 can be shown as

$$\frac{R_{c,oil}(\bar{T}_{in} - \bar{T}_b)}{R_{e1}} + \frac{R_{c,oil}(\bar{T}_{out} - \bar{T}_b)}{R_{e2}} + \frac{R_{c,oil}(\bar{T}_{oil} - \bar{T}_b)}{R_{c,oil}} + \frac{R_{c,oil} \cdot Q_f}{T_a} = \frac{\partial \bar{T}_b}{\partial \bar{t}} \quad (3.15)$$

$$\frac{R_{c.oil}(\bar{T}_7 - \bar{T}_{in})}{R_{cont.i}} + \frac{R_{c.oil}(\bar{T}_b - \bar{T}_{in})}{R_{e1}} + (\bar{T}_{oil} - \bar{T}_{in}) = \frac{\partial \bar{T}_{in}}{\partial \bar{t}}$$

$$\frac{R_{c.oil}(\bar{T}_b - \bar{T}_{out})}{R_{e2}} + \frac{R_{c.oil}(\bar{T}_{13} - \bar{T}_{out})}{R_{cont.o}} + (\bar{T}_{oil} - \bar{T}_{out}) = \frac{\partial \bar{T}_{out}}{\partial \bar{t}}$$

where the dimensionless frictional heat generation is presented by $\frac{R_{c.oil}Q_f}{T_a}$.

Using the thermal balance equations for all of the thermal nodes in the bearing assembly, one can write a set of 17 differential equations, similar to set of Equations 3.15, including 17 unknown temperatures. For this purpose, a computer code is developed in MATLAB™ to solve the system of differential equation using the ODE45 function (an explicit Rung-Kutta method).

Since the viscosity of lubricant drops with temperature rise, it should be updated each iteration according to the following formula

$$\mu_v = \mu_{vi} e^{-\beta(T_{oil} - T_i)} \quad (3.16)$$

where μ_{vi} is the initial viscosity at initial temperature T_i , and β is the viscosity-temperature coefficient of the lubricant.[12, 34]

3.3.3. Thermal Expansion Model

Bearing preload is normally considered as a combination of initial preload and thermally-induced preload. The initial preload is composed of the internal contact forces between the rolling bearing components during the installation of the bearing on the shaft and housing, including lock nut loads and interference fitting loads. The initial preload plays a key role to reach an appropriate stiffness for the bearing assembly which can improve the fatigue life of the bearing. The thermally-induced preload is associated with uneven expansion of the bearing components because of the different operating temperatures due to heat generation inside the bearing. During the operation as the temperature of the bearing assembly increases, components of the bearing experience different expansion rates due to differences in their temperature and geometry. As a result, the initial interference at the bearing installation is affected leading to changes of the thermally-induced preload. Neither the initial preload nor the thermally-induced preload are easily measurable because of the major difficulties for installing sensors inside the rotating bearings. Nevertheless, presented in the literature [18, 35, 36] are several indirect techniques to estimate the bearing preloads by measuring the induced strain of the shaft or the housing in contact with the bearing.

The thermally-induced preload can affect the frictional torque as well as the heat generation rate according to Equation 3.3. Catastrophic failure can occur due to rapid increase of the thermally-induced preload, if the heat generation rate inside the bearing assembly exceeds the rate of heat dissipation from the bearing.

It is worthwhile to note that the formulas presented here are suitable for the test rig and deep-groove ball bearing used in this study. For other bearing configurations, appropriate formulas must be used.

The radial thermal expansion of the inner ring ε_{in} and the balls ε_b are assumed to be linear and calculated as follows

$$\varepsilon_b = \alpha_b r_b T_b \quad (3.17)$$

$$\varepsilon_{in} = \alpha_{in} r_{in} T_{in} \quad (3.18)$$

where α_b and α_{in} are the linear expansion coefficients of the corresponding material. The radial expansion of outer ring is estimated by an analytical formula given by Timoshenko [37] and defines as [18, 19]

$$\varepsilon_{out} = \frac{\alpha_o (1 + \vartheta) \cdot r_{out}}{3 (r_{out} + r_h)} [T_{out}(2r_{out} + r_h) + T_h(2r_h + r_{out})] \quad (3.19)$$

Hence, the total radial interference change becomes

$$\varepsilon_t = \varepsilon_b + \frac{(\varepsilon_{in} - \varepsilon_{out})}{2} \quad (3.20)$$

Next, the thermally-induced preload is estimated by the Hertzian contact theory according to the following equation

$$F_t = k_t \varepsilon_t^n \quad (3.21)$$

where n is taken to be equal to 1.5 for ball bearings and 1.11 for roller bearings and k_t is a coefficient for the radial elastic contact of ball bearing and can be calculated using formulas given in Appendix A [13].

It is worth mentioning that the initial deformation of the bearing due to the installation procedure is not taken into account for calculation of the total interference, on the account that a small amount of interference fit was used for installation of the bearing on the shaft. However, the effect of initial interference can be readily considered using the formulas given in [18, 19].

3.4. Experimental tests

As mentioned earlier in Section 2, experiments are performed using a specially designed test rig. The test bearing used in the experiments is a single-row, deep-groove ball bearing (SKF 6315/C3) with a ball diameter of 26.9875 mm. The housing type used for the test bearing is SAF 315. All tests are conducted in a laboratory at the controlled environmental temperature of 25-27 °C and relative humidity of 20–25% .

In each experiment, first, a specific external load is applied to the bearing in the radial direction using the previously mentioned pneumatic system. Next, the electric motor is turned on and run at a desired constant speed. The acceleration time for the motor to reach the test speed is approximately 15-20 seconds. Measured during the test are the temperature of the outer raceway and the outer surface of the housing as well as the frictional torque of the bearing system. These measurements are recorded on a computer at intervals of 0.1 second by means of a data acquisition board (NI PCIe-6321) and in conjunction with the LabViewTM software. All of the tests are continued until the steady state condition is attained. Minimum of three hours was required for the temperature measurements to become steady. Tests are stopped upon attainment of the steady state by shutting down the motor. However, measurement of the temperature continues until the system cools down to the ambient temperature in the laboratory.

The experiments are purposefully designed to investigate the effect of two important test parameters, namely the speed and the external load. Using a wide range of external loads and rotational speeds of the shaft, each time one test parameter is kept constant and separate tests are carried out for different values of the other parameters. The ranges of the external radial loads and the rotational speeds used in this study are presented in table 3.2.

Table 3.2. The range of the test parameters.

External Radial Load (N)	Rotational Speed (RPM)
1678	0
1397	500
840	1500
0	2000
	2500
	3000

It is noteworthy to mention that the value of maximum Hertzian stress depends not only on the external radial loads but also on the rotational speed due to effect of centrifugal forces. Table 3.3 shows the maximum Hertzian pressure in the bearing only for the case of 2000 RPM rotational speed under different loads.

Table 3.3. Maximum Hertzian stresses for the condition of 2000 RPM under different radial loads

External Radial Load (N)	Max. Hertzian Stress (MPa)
1678	1033.5
1397	962
840	775

3.5. Results and discussion

The results of the experiments are presented and compared with the predictions based on the mathematical model introduced earlier.

3.5.1. Verification

Predictions based on the model described are shown in Figures 3.6-3.7. Figure 3.6 shows the comparison of the measured and predicted housing temperature, revealing reasonably close agreement. Maximum of 2 degree discrepancy for the steady state condition is considered acceptable owing to their negligible effect on the dimensional change of the ball bearing components. These factors are dependent on the accuracy of several parameters such as contact resistances, forced air convections and frictional heat generation inside the ball bearing.

It is worthwhile to note that the temperature of the outer raceway subject the other loads and rotational speeds are found to follow the same trend, but they are not shown here for brevity. The comparison of the experimental and theoretical result for the oil temperature in a oil-bath lubrication system is shown in Figure 3.7. This figure shows the validity of the model for predicting the oil temperature in the transient phase as well as the steady state condition. It should be noted that the comparison between the experimental and mathematical model is only possible for the housing, oil and the outer raceway with this test rig. The temperature of the balls and inner raceway are predicted, but not measured.

Figure 3.6. Comparison of the predicted housing temperature to the measured value by thermocouples for the test condition of 2000 rpm and 1678 N radial load

One can also readily deduce from Figures 3.6-3.7 that the number of thermal nodes used in the model is adequate for the analysis of this bearing assembly. Subsequently, the proposed mathematical model can be used to predict the effect of other parameters.

Figure 3.7. Comparison of predicted oil temperature to the measured value by thermocouples for the test condition of 2000 rpm and 1678 N radial load

Shown in Figure 3.8 is the predicted thermally-induced preload in the ball bearing based on the Equation 3.21 for the test condition of 2000 RPM and 1678 N radial load. The thermally-induced preload increases rapidly during the first 400 seconds of operation, and then gradually decreases to zero after 2250 seconds. This behavior is dependent upon the bearing configuration and operating conditions. At the beginning of the test, a large portion of the frictional heat generation inside the roller bearing transfers to balls or rollers, and due to small mass of rollers compared to other parts, the temperature of the balls become much greater that of the inner raceway and outer raceway. As time goes by, more of the frictional heat dissipates throughout the housing and the shaft, and the temperature difference between rollers and the outer ring and the housing becomes smaller. Thus, the interference between bearing components changes. Finally after 2250 seconds, there is no induced thermal load in the ball bearing, because the expansion of housing is bigger than the total expansion of the balls and the inner raceway. It is noteworthy to mention that the trend presented in Figure 3.8 is due to the heavy duty bearing used in this study, but depending to the imposed load and boundary condition, the induced thermal preload can drop down to a non-zero value after the sharp rise at the beginning of the test; See Figure 3.14. This behavior is consistent with the results reported by Stein and Tu [18] in which they show the chance of

thermal seizure of roller bearings in high speed applications. As Stein and Tu [18] showed this stable induced preload could give a significant increase to the frictional heat generation of the roller bearing in a harsh condition and lead to an imbalance between heat generation inside bearing and heat dissipation from the bearing which finally results in a catastrophic failure.

Figure 3.8. Prediction of thermally-induced preload for the test condition of 2000 rpm and 1678 N radial load ($h_a = 15 \text{ W/m}^2 \cdot \text{K}$)

3.5.2. Effects of rotational speed

Figure 3.9 shows the non-dimensionalized frictional heat generation measured during a set of experiments with constant load of 1678 N at different speeds. Shown in a small frame at the top of Figure 3.9 is the motor's acceleration time which during this period, where the frictional torque rapidly increases from zero to its maximum value. As seen, the frictional heat generation decreases rapidly at the beginning of the test and continues to drop slowly until it reaches a steady-state value. According to the trend observed in Figure 3.9, it can be readily deduced that at the initial stage of the test there is an imbalance between the rate of heat generation inside and the rate of heat dissipation from the bearing. However, as the test continues these two energy rates come to a state of equilibrium, i.e. the steady state.

Figure 3.9. Frictional heat generation calculated based on the torque measurements for the constant radial load of 1678 N.

Figure 3.10. Temperature evolution of the outer raceway measured by thermocouples for a set of experiments with constant external radial load of 1678 N

Figure 3.10 shows the temperature evolution of the outer raceway of the test bearing during a set of experiments in which the applied radial load is kept constant (1678 N) while the shaft speed is changed from 1000 RPM to 2500 RPM. As seen the temperature of the outer raceway increases from the ambient temperature exponentially until it reaches the steady state temperature approximately after 3 hours. Referring to this figure, the steady state temperature of the outer raceway increases by almost 7 °C per for every 500 RPM increment of the rotational speed of the shaft. It is useful to note that the evolution of the housing temperature is qualitatively similar to that of the outer raceway, not shown here for brevity.

3.5.3. Effects of external radial load

Shown in Figure 3.11 is the frictional heat generation rate at different values of the external load and constant rotational speed of 2500 RPM. For brevity, the results are plotted for only one rotational speed. Comparing Figures 3.9 and 3.11, one can deduce that the effect of rotational speed on the value of the frictional torque in oil-bath systems is more pronounced compared to the external load. This effect can be attributed to the more dominant contribution of the viscous friction in the total frictional torque of the bearing.

Figure 3.11. Frictional heat generation calculated based on the torque measurements for the constant rotational speed of 2500 rpm.

Figure 3.12 shows the temperature of the outer raceway for a set of experiments in which the shaft speed is constant (2500 RPM) while the external radial load varies from zero to 1678 N. Except the no-load case, one can deduce that the temperature difference is almost 2 degree for the external loads of 839 N and 1678 N. This small temperature rise is due to the heavy-duty design of the ball bearing and the small imposed load compared to the static load rating of the

test bearing which is equal to 76.5 KN. Comparison of Figures 3.10 and 3.12 signify the importance of the rotational speed on the thermal analysis of a roller bearing, which is known to be the primary cause of roller bearing failures in high speed applications under low loads.

Figure 3.12. Temperature evolution of the outer raceway measured by thermocouples for a set of experiments with constant speed of 2500 rpm

Figure 3.13. Temperature evolution of the outer raceway for different heat convection coefficients with constant external radial load of 1678 N and 2000 rpm

3.5.4. Effects of air heat convection

Figure 3.13 shows the effect of convection coefficient of air on the temperature evolution of the outer ring. According to Figure 3.13, the temperature of bearing components needs more time to reach the steady state condition when the heat convection coefficient is reduced. An increase in the convection coefficient results in more heat dissipation from the housing, and therefore the bearing temperature reaches a lower steady state value in a shorter time.

Shown in Figure 3.14 is the effect of air convection coefficient on the thermally-induced preload in the ball bearing for the test condition of 2000 RPM and 1678 N radial load. Although the effect of cooling on the temperature of the bearing components is found to be profound in Figure 3.13, Figure 3.14 shows a slight impact of the air convection on the maximum value of the induced thermal preload. It can be deduced that the cooling of housing would lead to a longer time of the induced preload and shorter time of the transient temperature evolution, and it can result in higher induced preload at the steady-state phase.

Figure 3.14. Prediction of thermally-induced Preload for different air convection coefficient with constant external radial load of 1678 N and 2000 rpm

It should be mentioned that the presented trend of the thermally-induced preload is in agreement with the experimental results presented by Camichael and Davies [38]. They reported an excessively high thermally-induced preload inside the roller bearing for the case of water cooling of the housing which can lead to the thermal failure of the roller bearing. Conversely, Chun [39] showed that the cooling of the shaft can considerably reduces the thermally-induced preload in the bearing.

3.5.5. Effects of oil viscosity

The effect of lubricant viscosity is shown in Figure 3.15 where the temperature of outer raceway and other bearing components rise considerably along with increasing oil viscosity in the oil-bath bearing assembly. This trend could be interpreted based on Equation 3.2 which reveals the viscous frictional torque as the more significant part of the frictional torque in an oil-bath bearing system. Consequently, the temperature differences for different viscosities in Figure 3.15 are at the same order of the temperature differences due to different rotational speeds presented earlier in Figure 3.10.

Figure 3.16 shows a significant effect of the oil viscosity on the thermally-induced preload which highlights the role of viscous frictional torque on the total frictional heat generation inside the bearing assembly. The different trends shown in Figures 3.14 and 3.16 can be attributed to the higher temperature and temperature gradient of the bearing components in Figure 3.16 which results into a considerable difference of the induced thermal preload.

It can be concluded from the simulations that the rotational speed, viscosity, boundary conditions and constriction resistances have a significant impact on the behavior of the thermally-induced preload and thermal instability of the bearing assemblies.

Figure 3.15. Temperature evolution of the outer raceway for different oils with constant external radial load of 1678 N and 2000 rpm

Figure 3.16. Prediction of thermally-induced Preload for different oils with constant external radial load of 1678 N and 2000 rpm

3.6. Conclusions

A comprehensive mathematical model to analyze the thermal behavior of a ball bearing is presented. Considered in conjunction with the transient heat transfer model is the dimensional change of the bearing components due to unequal temperature rise of the bearing components. The model provides a practical tool for evaluation of the transient and steady state characteristics of rolling element bearings owing to the lumped assumption of the bearing components and the finite number of thermal nodes in the heat transfer model of the entire system. The employed modeling technique makes it applicable for a wide range of ball bearing assemblies using the appropriate presented methods to estimate the critical parameters, such as convection coefficients and contact resistances. However, for a given bearing system in practice one needs to consider the appropriate material properties, boundary conditions etc, while the generality of the approach remains the same.

Also presented is a new experimental approach to estimate the heat convection coefficient from the housing, which is difficult to predict analytically and has a major effect on the thermal behavior of the bearing assembly. It is shown that the complicated cooling effect of the lubricant in the oil-bath bearing assembly can also be resolved by considering the presented formula. The validity of the mathematical model is demonstrated by means of sets of experiments on a special

test rig. The thermal response of a deep groove ball bearing with oil-bath lubrication under different external radial loads is studied for various rotational speeds. The capability of the mathematical model to estimate the thermally-induced preload in the bearing makes it a valuable method which can be used for further investigation of the thermal seizure failure of the ball bearings. Although the significant effect of high rotational speeds on the temperature or thermal failure of the ball bearings has been already shown in literature, additional simulations reveal that the oil viscosity or viscous frictional torque and the heat convection coefficient have the same order of importance on the thermal response of the bearing assembly and their change have a significant effect on the temperature of the bearing components. The presented results reveal the rotational speed, oil viscosity and thermal boundary conditions as the critical parameters which should be considered to avoid the thermal failure of the ball bearings. Although high rotational speed experiments are not possible on the current test rig set up, it is believed that the proposed modeling technique is applicable for thermal analysis of high speed bearings, since dynamic effects such as centrifugal loads and gyroscopic moments have been considered in the analysis. However cage stability issues might arise in very high speed ranges which then require further analysis.

3.7. Nomenclature

a	Semi-major axis of the contact area (m)
b	Semi-minor axis of the contact area (m)
C_b	Specific heat of ball or roller (J/kg.K)
C_{in}	Specific heat of inner ring (J/kg.K)
C_{out}	Specific heat of outer ring (J/kg.K)
d_m	Bearing pitch diameter (m)
D	Ball diameter (m)
e	Eccentricity of contact ellipse $e = \left[1 - \left(\frac{b}{a}\right)^2\right]^2$
$\dot{E}_{in}$	Inlet energy (W)
$\dot{E}_g$	Energy generation (W)
$\dot{E}_{out}$	Outlet energy (W)
$\dot{E}_{sto}$	Stored energy (W)
f_i	$\frac{r_{ic}}{D}$
f_o	$\frac{r_{oc}}{D}$
f_l	Coefficient related to loading direction
f_v	Rolling coefficient of friction
F_n	Contact force induced by external loading (N)
F_t	Thermally-induced preload (N)
h	Air convection coefficient (W/m ² .K)
h_v	Convection coefficient related to lubrication system (W/m ² .K)
k	Thermal conductivity (W/m.K)

k_a	Thermal conductivity of air (W/m.K)
k_t, k_n	Bearing stiffness coefficient
m_b	Mass of ball or roller (kg)
m_{in}	Mass of inner ring (kg)
m_{out}	Mass of outer ring (kg)
M	Total frictional torque (N.m)
M_l	Frictional torque due to load (N.m)
M_v	Viscous frictional torque (N.m)
M_f	Frictional torque of flanges (N.m)
n	Rotational speed (rad/s)
nb	Number of balls in contacts with rings
$\overline{Nu}$	Nusselt number
Pr	Prandtl Number
Q_f	Total frictional heat generation (W)
r_b	Radius of ball or roller(m)
r_{in}	Radius of inner ring (m)
r_{ic}	Inner raceway groove curvature radius (m)
r_h	Radius of housing (m)
r_{out}	Radius of outer ring (m)
r_{oc}	outer raceway groove curvature radius (m)
R_{ci}	Thermal constriction resistance between inner raceway and ball (K/W)
R_{co}	Thermal constriction resistance between outter raceway and ball (K/W)
R_{ei}	Equivalent thermal constriction resistance related to inner ring contact (K/W)
R_{eo}	Equivalent thermal constriction resistance related to outer ring contact (K/W)
R_c	Constriction resistances between ball and inner and outer rings (K/W)
$R_{c.oil}$	Convective thermal resistance associated with oil (K/W)
$R_{cont.i}$	Contact resistance between inner ring and shaft (K/W)
$R_{cont.o}$	Contact resistance between outer ring and housing (K/W)
Re_r	Reynolds number
t	Time (s)
T_b	Temperature of ball or roller (K)
T_h	Temperature of housing (K)
T_a	Ambient temperature (K)
T_i	Initial temperature (K)
T_{in}	Temperature of inner ring (K)
T_{oil}	Temperature of oil (K)
T_{out}	Temperature of outer ring (K)
α_b	Linear expansion coefficient of ball or roller (m/m.K)
α_{in}	Linear expansion coefficient of inner ring (m/m.K)
α_o	Linear expansion coefficient of outer ring (m/m.K)
β	Temperature –viscosity coefficient
ε_b	Thermal expansion of ball or roller (m)
ε_{in}	Thermal expansion of inner ring (m)

ε_t	Resultant change of interference fit (m)
ε_{out}	Thermal expansion of outer ring (m)
μ_l	Friction coefficient due to loading
μ_v	Lubricant viscosity (Pa.s)
μ_{vi}	Initial viscosity (Pa.s)
ϑ	Poisson ratio
γ	$\gamma = \frac{D \cdot \cos \theta}{d_m}$
ρ	Curvature (1/m)
δ	Contact deformation (m)
δ^*	Dimensionless contact deformation
θ	Contact angle
$F(\rho)$	Curvature difference
$\sum \rho$	Curvature sum (1/m)
$K\left(e, \frac{\pi}{2}\right)$	Complete elliptic integral of the first kind

3.8. References

- [1] Fillon M, Frene J, Khonsari MM, Wang SH. A finite volume analysis of the thermohydrodynamic performance of finite journal bearings - Discussion. J Tribol-T Asme. 1990;112:565-.
- [2] Keogh PS, Khonsari MM. Influence of inlet conditions on the thermohydrodynamic state of a fully circumferentially grooved journal bearing. J Tribol-T Asme. 2001;123:525-32.
- [3] Khonsari MM, Jang JY, Fillon M. On the generalization of thermohydrodynamic analyses for journal bearings. J Tribol-T Asme. 1996;118:571-9.
- [4] Jang JY, Khonsari MM, Pascovici MD. Thermohydrodynamic seizure: Experimental and theoretical analysis. J Tribol-T Asme. 1998;120:8-15.
- [5] Khonsari MM, Kim HJ. On Thermally Induced Seizure in Journal Bearings. J Tribol-T Asme. 1989;111:661-7.
- [6] Pascovici MD, Khonsari MM, Jang JY. On the modeling of a thermomechanical seizure. J Tribol-T Asme. 1995;117:744-7.
- [7] Wang QA. Seizure failure of journal-bearing conformal contacts. Wear. 1997;210:8-16.
- [8] Khonsari MM. A review of thermal effects in hydrodynamic bearings. 1. Slider and thrust - bearings. Asle Trans. 1987;30:19-25.
- [9] Khonsari MM. A review of thermal effects in hydrodynamic bearings. 2. Journal bearings. Asle Trans. 1987;30:26-33.

- [10] Wen J, Khonsari MM, Hua DY. Three-Dimensional Heat Transfer Analysis of Pin-Bushing System With Oscillatory Motion: Theory and Experiment. *J Tribol-T Asme*. 2011;133.
- [11] Cristea AF, Pascovici M, Fillon M. Clearance and lubricant selection for avoiding seizure in a circumferential groove journal bearing based on a lumped model analysis. *Mecanique & Industries*. 2011;12:399-408.
- [12] Jang JY, Khonsari MM, Pascovici MD. Modeling aspects of a rate-controlled seizure in an unloaded journal bearing. *Tribol T*. 1998;41:481-8.
- [13] Harris TA. Rolling bearing analysis. 4th ed. New York: Wiley; 2001.
- [14] Winer WO, Bair S, Gecim B. Thermal-Resistance of a Tapered Roller Bearing. *Asle Trans*. 1986;29:539-47.
- [15] Burton RA, Staph HE. Thermally Activated Seizure of Angular Contact Bearings. *Asle Trans*. 1967;10:408-&.
- [16] Sud ON, Davies PB, Halling J. Thermal-Behavior of Rolling Bearing Assemblies Subjected to Preload. *Wear*. 1974;27:237-49.
- [17] Pruvot FC. High Speed Bearings for Machine Tool Spindles. *Annals of the CIRP*. 1980;29:4.
- [18] Stein JL, Tu JF. A State-Space Model for Monitoring Thermally-Induced Preload in Antifriction Spindle Bearings of High-Speed Machine-Tools. *J Dyn Syst-T Asme*. 1994;116:372-86.
- [19] Tu JF, Stein JL. Model error compensation for observer design. *Int J Control*. 1998;69:329-45.
- [20] Harris TA, Barnsby RM. Tribological performance prediction of aircraft gas turbine mainshaft ball bearings. *Tribol T*. 1998;41:60-8.
- [21] Palmgren A, Ruley B, SKF Industries inc. Ball and roller bearing engineering. Philadelphia,: SKF Industries, inc.; 1945.
- [22] SKF General Catalogue. 2003.
- [23] Amiri M, Khonsari MM, Brahmeshwarkar S. On the Relationship Between Wear and Thermal Response in Sliding Systems. *Tribol Lett*. 2010;38:147-54.
- [24] Nakajima K. Thermal contact resistance between balls and rings of a bearing under axial, radial, and combined loads. *Journal of Thermophysics and Heat Transfer*. 1995;9:88-95.
- [25] Yovanovich MM. Thermal constriction resistance of contacts on a half-space: Integral formulation. *AIAA*. 1975:75-708.

- [26] Yovanovich MM, Schneider GE. Thermal constriction resistance due to a circular annular contact. AIAA. 1976:76-142.
- [27] Salerno LJ, Kittel P. Thermal conductance of augmented pressed metallic contacts at liquid Helium temperatures. Cryogenics. 1993;33.
- [28] Mian MN, Al-Astrabadi FR, O'Callaghan PW, Probert SD. Thermal resistance of pressed contacts between steel surfaces: Influence of oxide films. Mechanical Engineering Science. 1979;21.
- [29] J. SL, P. K. Thermal contact conductance. NASA Technical Memorandum 1997;110429.
- [30] Yovanovich MM. Four decades of research on thermal contact, gap, and joint resistance in microelectronics. IEEE Trans Compon Packaging Technol. 2005;28:182-206.
- [31] Ozerdem B. Measurement of convective heat transfer coefficient for a horizontal cylinder rotating in quiescent air. Int Commun Heat Mass. 2000;27:389-95.
- [32] Meneghetti G. Analysis of the fatigue strength of a stainless steel based on the energy dissipation. Int J Fatigue. 2007;29:81-94.
- [33] Naderi M, Khonsari MM. Real-time fatigue life monitoring based on thermodynamic entropy. Struct Health Monit. 2011;10:189-97.
- [34] Khonsari MM, Booser ER. Applied tribology : bearing design and lubrication. 2nd ed. Chichester, England ; Hoboken, NJ: Wiley; 2008.
- [35] Eschmann P, Hasbargen L, Brändlein J. Ball and roller bearings : theory, design, and application. 2nd ed. New York: Wiley; 1985.
- [36] Carmicha.Gd, Davies PB. Measurement of thermally induced preloads in bearings. Strain. 1970;6:162-5.
- [37] Timoshenko S, Goodier JN. Theory of elasticity. 3d ed. New York,; McGraw-Hill; 1969.
- [38] Carmicha.Gd, Davies PB. Factors which affect transient behavior of preloaded ball-bearing assenblies. Asle Trans. 1972;15:1-&.
- [39] Chun, Jiang. Experimental investigation of water cooling inside the spindle bearing system. Ji Chuang/Machine Tolls. 1988;12:15-9.

Chapter IV: Developing dynamic model of rolling element bearings and studying the influence of traction coefficient on the dynamic behavior of bearing components *

4.1. Introduction

Historically, analytical models for simulating the performance of rolling element bearings are divided into two categories: quasi-static and dynamic models. Developed first by Jones [1] and extended by Harris [2] are the quasi-static models based on the equilibrium of static forces and moments acting on the rolling elements, whereas the dynamic models are based on the integration of the differential equations of motions for rolling elements with respect to time.

Although the well-known quasi-static models are widely used to estimate the overall load distribution as well as bearing stiffness and fatigue life, their applicability to analyze the cage motion and the dynamic instabilities of rolling elements are quite limited due to many simplifying assumptions and constraints. In contrast, dynamic models provide a real-time simulation of the rolling elements motions and thus enable the designer to specify any desired time-varying load as well as lubricant rheology. Since the dynamic models constitute a set of time-dependent ordinary differential equations that describe the motion of the rolling elements as well as the races and the cage, the treatment of these equations tends to demands a considerable computational time.

A report of one of the first dynamic bearing models is given by Walters [3] in which six degrees of freedom (DOF) for the ball-separator motion and four DOF for the ball motion are employed in conjunction with lubrication effects. Yet some assumptions inherent in the quasi-static models remained in the model. Kannel [4] presented a simplified analytical model by considering the in-plane motions of the cage in an angular contact bearing. Implemented in Kannel's model is the elastohydrodynamic (EHL) lubrication regime with the assumption of no gross slip at the ball-races contact. To the authors' best of knowledge, Gupta [5-8] presented the first fully dynamic model for ball and cylindrical roller bearings considering six DOF of motion for all the rollers, the races and the cage. Gupta's model solves the general translational and rotational motion of the bearing elements using the Newton's law and the Euler's equations. Included in Gupta's model are the cage-roller and the cage-race interactions in addition to the EHL lubrication regime as well as the provision for churning and drag losses. Based on this model, Gupta developed a commercial software called ADORE which was upgraded and advanced thereafter [9].

* Reprinted by permission of *Tribology Transactions* (See Appendix C)

Owing to the cumbersome calculations of fully dynamic model, some researchers have presented simplified dynamic models in order to make analytical solutions more cost effective and to make parametric design studies practical. For example, Meeks [10-12] presented a simplified dynamic model by considering six DOF for the cage with the consideration of inelastic contact of cage-roller and cage-race interactions, while a quasi-static model was used to calculate the contact load and stress between the ball and raceways in lieu of a time-variant solution. Also available in the literature [13-17] are several simplified dynamic models based on Gupta's model in which the effect of cage flexibility and race-cage and roller-cage pocket clearance on the cage instability and bearing performance are investigated. Discussions on some of the key developments in the rolling element bearing modeling techniques over the last several decades and an in-depth comparison between different dynamic and quasi-static models are given by Gupta [9].

It is well understood that the lubricant rheology plays a prominent role in the overall performance as well as the dynamic behavior of the rolling element bearings where the traction coefficient between the bearing elements determines the orbital acceleration, the velocity of the rolling elements and the resultant normal and traction forces between the rollers, races and the cage [18]. It is also well established that the traction coefficient in EHL contacts is not only a function of the slip velocity but also depends on the contact pressure, the temperature, the rolling velocity and the rheological properties of the lubricant. The combination of these factors renders the development of a predicting model difficult. Additional complexities arise because of the coupling of the slip velocity with the shear heating between the contacting surfaces. These effects can alter the lubricant rheological properties, which in turn can change the rotational acceleration of the bearing elements. This can directly affect the resultant collisions between the rollers and the cage. Therefore, a realistic traction model for the lubricant is vital in order to perform a realistic dynamic analysis of the rolling element bearings. Presented in the literature and applicable to the dynamic models are the Newtonian and viscoelastic models to estimate the lubricant behavior in EHL contacts. The former assumes that the lubricant behavior remains Newtonian in the contact area regardless of the magnitude of the contact pressure. In the case of very high contact pressures, the Newtonian fluid model yields unrealistic traction coefficients while the viscoelastic model tends to be more satisfactory [18, 19]. A realistic dynamic model should employ the correct traction coefficient associated with each contact between the rollers, the races and the cage owing to the existence of different contact loads and sliding velocities within a bearing. However, presented in the literature are several studies [15, 17, 20] in which simplified traction curves—independent of the contact pressure and temperature—are employed in exponential or bi-linear forms for all of the contacts in order to accomplish a parametric study with reduced computational time.

The current investigation is aimed at providing information to the researchers as to the applicability of the rheological models necessary to obtain realistic results. The main objective of the present study is to examine the utility of simplified traction curves as compared to the more comprehensive viscoelastic rheological models (discussed in Section 2.2.) to calculate the traction coefficient used in prediction of the dynamic behavior of cylindrical roller bearings. Considering the lack of viscoelastic models for many lubricants, results of the present study can be used to predict the errors associated with using simplified traction curves in dynamic models for different operating conditions of a roller bearing assembly. Developed in the present study is a simplified dynamic model based on the work of Gupta [22] in order to simulate the dynamic motion of components of a cylindrical roller bearing subjected to a constant radial load. Extensive simulations are carried out for a wide range of radial loads and speeds to investigate the effect of simplified traction curves and viscoelastic models on the rotational velocity of the cage and the numerical results are compared with the experimental results presented by Harris [21]. Since the cage velocity and the roller orbital velocity have a significant impact on the magnitude of the contact load between the roller and the cage, employing an inappropriate rheological model or traction curve in the dynamic model might produce misleading results for the cage wear rate. Therefore, simulations are carried out using the viscoelastic rheological models and results are presented to investigate the effect of different loads and rotational speeds on the average wear rate of the cage, which can be used to study the bearing life and the dynamic instability of rolling element bearings.

4.2. Modeling approach

4.2.1. Dynamic model description

This section briefly describes the fundamental concepts of the dynamic model employed in the computer program developed in the present study to simulate the dynamic behavior of rolling element bearings. Next, the simplified assumptions made for the simulations in the present study are discussed. More thorough description is available in Ref. [22].

Generally, in a fully dynamic model all of the bearing elements have six degrees of freedom (DOF). Three translational coordinates define the location of the center of the mass and three rotational coordinates define the angular orientation of the element with respect to the center of the mass of the element. Referring to Figure 4.1, the interaction variable, δ , between the bearing elements is the fundamental parameter of the dynamic modeling technique. For example, the normal deflection, δ , between the roller and the inner race is described by

$$\delta = (R_{ir} + R_r) - |r_{rb}^{ia}| \quad (4.1)$$

where R_{ir} is the radius of curvature of the inner raceway along the rolling direction and R_r is the radius of the roller. $|r_{rb}^{ia}|$ is the length of the relative position vector of the center of the mass of the roller with respect to the inner raceway center expressed in the roller azimuth frame (ia), and

can be calculated by an appropriate rotation of vector r_{rb}^i around the Z_i axis of the fixed inertial frame. Vector r_{rb}^i is equal to

$$\vec{r}_{rb}^i = \vec{r}_b^i - \vec{r}_r^i \quad (4.2)$$

where r_{rb}^i is the relative position vector of the center of the mass of the roller with respect to the inner raceway center defined in the fixed inertial frame. The vectors $\vec{r}_b^i$ and $\vec{r}_r^i$ are the position vectors of the mass center of the roller and inner race in the fixed inertial frame, respectively; see Figure 4.1.

Referring to Figure 4.1, the position and translational motion of the center of the mass of each element are normally defined in an inertial Cartesian or cylindrical coordinate system (C. S.) fixed in the space (X_i, Y_i, Z_i) while the orientation and rotational motion of each element are calculated in a body-fixed coordinate frame, (x_1, y_1, z_1), for the roller number 1 and (X_r, Y_r, Z_r) for the inner race way in Figure 4.1, coincident with the principal triad of the element. The present study considers an inertial frame located at the center of mass of the outer raceway, owing to the fixed position of the outer raceway in space. The external load is applied to the inner raceway, and the rollers and the cage are considered to be solid bodies with six DOFs. The cage is assumed to be a solid, single-piece with the details discussed in Section 3.

Figure 4.1. Basic coordinate frames to calculate the interaction (δ) between the roller and the inner raceway

Figure 4.2 shows a simplified flowchart of the dynamic model for any two elements, named A and B, within the rolling element bearing. Calculated from the quasi-static models are the predefined position and velocity vectors of the elements where the classical raceway-control hypothesis [2] is employed to estimate the initial angular velocity of the rolling elements. The program proceeds to calculate the deflection of the two elements in contact using the relative

position vector of the elements (see vector r_{rb}^i in Figure 4.1). Subsequently, an appropriate load-deflection relation is employed in order to calculate the normal contact force between the elements. In the present study Palmgren's empirical load-deflection formula (Equation 4.3) for the line contact is used owing to the simple radial cylindrical bearing studied here [23].

$$\delta = 0.39 \left[\frac{4(1 - \nu_A^2)}{E_A} + \frac{4(1 - \nu_B^2)}{E_B} \right]^{0.9} \frac{Q^{0.9}}{l^{0.8}} \quad (4.3)$$

where subscripts A and B refer to bodies A and B in contact. The parameters δ , Q and l are, respectively, the normal displacement, the normal contact force and the effective length of the line contact which is equal to the total length of the roller minus the radii of the rounded edges. The modulus of elasticity and the Poisson's ratios of the materials in contact are denoted by E and ν , respectively.

Figure 4.2. Simplified flowchart of the dynamic model for two elements within the roller bearing [9, 22]

Next, the tangential traction forces in the contact area are determined by considering the computed normal contact load and the slide-to-roll ratio along with an appropriate traction model. Determination of the traction coefficient, calculated from the traction model, plays the key role in the calculation of the total forces and moments acting on the element. This is discussed in depth in Section 2.2. The total force and moment acting on each element are determined by adding the appropriate churning and drag forces, discussed in Section 3, to the previously calculated normal and traction contact loads between each bearing element and its surrounding elements. These total forces are expressed in the directions of the inertial coordinate frame where the Newton's laws of motion are used to calculate the translational acceleration of each element.

$$m\ddot{x} = F_x \quad (4.4a)$$

$$m\ddot{r} - mr\dot{\theta}^2 = F_r \quad (4.4b)$$

$$mr\ddot{\theta} + 2m\dot{r}\dot{\theta} = F_\theta \quad (4.4c)$$

where x , r and θ are the axial, radial and orbital coordinates of the cylindrical inertial frame, respectively. F_x , F_r and F_θ are the components of the total force acting on the element, defined in the cylindrical inertial frame. Computed from the total loads and the geometry of the elements are the total moments acting on each element due to the contact with other elements expressed in the body-fixed coordinate frame of each element. The classical Euler equations of motions are employed to calculate the rotational acceleration of each element

$$I_1\dot{\omega}_1 - (I_2 - I_3)\omega_2\omega_3 = M_1 \quad (4.5a)$$

$$I_2\dot{\omega}_2 - (I_3 - I_1)\omega_1\omega_3 = M_2 \quad (4.5b)$$

$$I_3\dot{\omega}_3 - (I_1 - I_2)\omega_2\omega_1 = M_3 \quad (4.5c)$$

where the subscripts of 1-3 represents the three axes of the body-fixed coordinate frame and I , M and ω are, respectively, the principal moment of inertia, total moment and the angular velocity of the element defined in the body-fixed coordinate frame. The translational and rotational acceleration vectors of the bearing elements are calculated once the position and velocity vectors of the elements are known. Then, using an integration algorithm with an appropriate time step size, the position and velocity vectors of the bearing elements are predicted in the next time step. This procedure is continued until the program reaches the predefined final time. The present study employs a fourth-order Runge-Kutta-Fehlberg integration formula which uses an adaptive step size control by estimating the local truncation error for efficient computations with acceptable accuracy at the same time.

Employed for the simulations in the present study is a simplified dynamic model wherein two assumptions are made to decrease the computational time while maintaining the acceptable

accuracy of the results. First, the equilibrium constraint equations in the radial and axial directions are employed to remove the effect of the free vibration of the rollers between the races which results in the high frequency motions of the rollers. In this fashion, the equations of the translational motion of the rollers are reduced to only one equation in the orbital direction. It is also assumed that the rollers and the cage are allowed to rotate about their primary axes. One can find the details of these constraint equations in references [3,22]. Another assumption made in the present study compared to the fully dynamic model of Gupta is about neglecting the radius of the contact curvature in the calculation of the tractive forces. This assumption has been used in the literature and it is believed to have a negligible effect on the final results.

4.2.2. Traction model

This section explains the simple traction curves, called “simplified traction curves” in the present study, and two viscoelastic rheological models frequently used in the modeling of rolling element bearings. Shown in Figure 4.3 is a typical traction curve of a lubricant in EHL contacts, along with its associated simplified traction curve. The simplified traction curve in this study is calculated by fitting a straight line to the values of typical traction curve up to the maximum traction coefficient. Beyond this point the traction coefficient is assumed to remain constant for higher sliding velocities. Both of these traction curves characterize traction at a particular set of operating conditions with constant rolling speed, contact pressure and temperature. The traction coefficient, however, may vary with a change in any one of these parameters. Thus, the determination of the different lubricants behavior under a wide range of pressures and temperatures, as experienced in the roller-raceway contact of rolling element bearings, becomes a very complex and challenging task.

Figure 4.3. A typical traction curve and its associated simplified traction curve

The existing literature is rich with methodologies and results for the prediction of film thickness and traction force in an EHL contact using the Newtonian fluid assumptions. Nevertheless, calculation of the traction forces in the severe conditions of EHL contacts is still a complex task due to the severity of operating conditions in the contact involving high pressure, high shear rate and shear heating of the lubricant. Presented in the literature are several hypotheses [18, 19, 24, 25] to predict the non-linear relationship between the shear stress and shear strain rate in EHL contacts by considering viscoelasticity, limiting shear stress, shear thinning and shear heating. The present study employs the general form of viscoelastic model that was first introduced by Johnson and Tevaarwerk [19] and Bair and Winer [18, 26]. These models are often used in the literature [22, 24, 27-29] for realistic characterization of the traction force in the EHL contacts. See detailed explanation in Section 2.2.2.

4.2.2.1. Lubricant film thickness calculations

Computations of the traction coefficient require one to first determine the EHL film thickness by considering the lubricant properties calculated at the operating conditions associated with the inlet of the contact area. Then the computed film thickness can be used with a traction model in order to estimate the traction force in the contact area.

Employed in the present study is Dowson et al. [30] formula to calculate the isothermal minimum film thickness (h_0) and Central film thickness (h_c) at the center of the contact area. In dimensionless form the minimum film thickness is:

$$H_0 = \frac{2.65 \bar{U}^{0.7} G_h^{0.54}}{W^{0.13}} \quad H_c = \frac{4}{3} H_0 \quad (4.6)$$

where the dimensionless parameters $\bar{U}$, G_h , H_c , H_0 and W are defined by the following equations:

$$\bar{U} = \frac{\mu_0 U}{2E'R_m} \quad , \quad G_h = \lambda E' \quad , \quad W = \frac{Q}{lE'R_m} \quad , \quad H_c = \frac{h_c}{R_m} \quad , \quad H_0 = \frac{h_0}{R_m} \quad (4.7)$$

where R_m , l and E' are the equivalent radius, effective contact length and effective elastic modulus, respectively. μ_0 is the reference viscosity of the lubricant at the contact inlet, and λ is the pressure-viscosity coefficient of the lubricant.

Cheng et al. [31] and Wilson and Sheu [32] investigated the change in the film thickness owing to the thermal effects and showed that the film thickness drops considerably at high values of slide-to-roll ratios. The following so-called “thermal reduction factor” developed by Hsu and Lee [33] for line contact is used to account for the drop in the film thickness:

$$\Phi = \frac{1}{1 + 0.0766 G G_h^{0.687} W^{0.447} L^{0.527} e^{0.875S}} \quad , \quad L = \left(-\frac{\partial \mu}{\partial T}\right) \frac{U^2}{K_f} \quad (4.8)$$

where U and S are rolling velocity and slide-to-roll ratio, respectively. K_f and μ are, respectively, thermal conductivity and viscosity of the lubricant, while T represents the temperature in Kelvin.

4.2.2.2. Traction force calculations

Johnson and Tevaarwerk [19] and Bair and Winer [18, 26] presented different non-Newtonian rheological models in order to model the behavior of the lubricant viscosity under very high contact pressures experienced in EHL contacts. The general form of these non-Newtonian rheological models, or viscoelastic models, is given in Equation 4.9 and includes separate elastic and viscous terms.

$$\dot{\gamma} = \frac{1}{G} \frac{\partial \tau}{\partial t} + \frac{\tau_0}{\mu} f\left(\frac{\tau}{\tau_0}\right) \quad (4.9)$$

where τ , τ_0 , $\dot{\gamma}$ and G are the shear stress, the limiting shear stress or Eyring stress, the shear strain rate and the shear modulus of the lubricant, respectively. The effect of elastic term, i.e. the first term in RHS of Equation 4.9, becomes significant under high pressures and low strain rates, while the viscous term, i.e. the second term in RHS of Equation 4.9, is the prominent factor in the case of high sliding velocities. Johnson and Tevaarwerk [19] introduced a (sinh) function for f , while Bair and Winer [18, 26] presented a (atanh) function of f . As it is explained in [27, 28], care should be exercised in applying parameter τ_0 in Equation 4.9. Parameter τ_0 represents the Eyring stress in the case of using (sinh) function while it should be replaced by the limiting shear stress when employing (atanh) function. The limiting shear stress is higher than the Eyring stress and is equal to the ultimate shear stress beyond which the lubricant flows plastically regardless of the shear strain.

A dimensionless form of the Equation 4.9 is shown in Equation 4.10a along with the associated non-dimensional parameters presented in Equation 4.10b where $\frac{\partial}{\partial t}$ obtained by replacing $U \frac{\partial}{\partial y}$ as explained by Johnson et al. [19].

$$\frac{\partial \bar{\tau}}{\partial \bar{y}} = \dot{\bar{\gamma}} - \frac{f(\bar{\tau})}{D} \quad (4.10a)$$

$$\dot{\bar{\gamma}} = \frac{U_s}{h} \quad \bar{\tau} = \frac{\tau}{\tau_0} \quad \bar{y} = \frac{y}{b} \quad \dot{\bar{\gamma}} = \frac{U_s G b}{h U \tau_0} \quad D = \frac{\mu U}{G b} \quad (4.10b)$$

where b and U_s are the semi-minor axis of the contact area and the sliding velocity of the contact surfaces, respectively. D is the so-called Deborah number, and $\dot{\bar{\gamma}}$ is the dimensionless strain rate. y represents the direction along the minor axis of the contact area. Although viscoelastic models yield realistic results, there are some limitations associated with their application in the analytical treatment. Shear modulus, effective viscosity and critical shear stress are three important parameters in viscoelastic models that are difficult to measure for a wide range of temperatures, pressures and speeds. These parameters are not readily available for all types of lubricants in different operating conditions.

Employed in the present study is an approach for estimating the traction coefficient using the viscoelastic models that requires less numerical complexities and lower computational time for

dynamic analyses. This approach was first presented by Gupta et al. [27, 28, 34] to study the viscoelastic characteristics of MIL-L-7808-type lubricant. They showed that the related traction forces can be directly derived from Equation 4.9, if the parameters μ , G and τ are replaced by the effective average viscosity (μ_e), effective shear modulus (G_e) and the effective critical shear stress (τ_{oe}). In order to calculate the effective parameters of MIL-L-7808-type lubricant, they carried out a set of experimental measurements using a high-pressure viscometer and a rolling-disk traction machine for a wide range of pressure and temperature. It is assumed in this method that the effective values of the viscosity, shear modulus and critical shear stress are functions of the operating temperature (T) and Hertzian contact pressure (p) :

$$\mu_e = \mu_o \exp\{(\lambda_1 p + \lambda_2 p^2) + (\beta_1 \Delta T + \beta_2 \Delta T^2) + (\gamma_1 p \Delta T + \gamma_2 p^2 \Delta T^2)\},$$

$$\Delta T = \left(\frac{1}{T} - \frac{1}{T_0}\right) \quad (4.11)$$

$$G_e = \bar{G} \exp\left\{\alpha_G p + \beta_G \left(\frac{1}{T} - \frac{1}{T_0}\right)\right\} \quad (4.12)$$

$$\tau_{oe} = \bar{\tau}_o \exp\left\{\alpha_\tau p + \beta_\tau \left(\frac{1}{T} - \frac{1}{T_0}\right)\right\} \quad (4.13)$$

where μ_o is the reference viscosity and λ , β and γ are, respectively, pressure, temperature and pressure-temperature coefficients of viscosity obtained from measuring the viscosity of MIL-L-7808-type lubricant on a high-pressure viscometer. $\bar{G}$, $\bar{\tau}_o$, α_G , β_G , α_τ and β_τ are the coefficients associated with the pressure and temperature for the shear modulus and critical shear stress derived by curve-fitting the predictions and the measured traction data from a rolling-disk traction machine for a wide range of pressure and temperature experiments. The details of this approach are given in references [27, 28, 34].

Table 4.1. Properties of the MIL-L-7808-type lubricant [27]

Viscosity coefficients		Shear modulus and critical shear stress coefficients	
T_0	300 K	T_0	300 K
μ_o	0.030391 Pa.s	$\bar{G}$	8.00×10^6 Pa
λ_1	1.2925×10^{-8} 1/Pa	α_G	3.35×10^{-9} 1/Pa
λ_2	-2.4538×10^{-18} 1/Pa ²	β_G	3.00×10^3 K
β_1	6.7295×10^2 K	$\bar{\tau}_o$	1.50×10^6 Pa
β_2	-2.2642×10^6 (K) ²	α_τ	2.45×10^{-9} 1/Pa
γ_1	9.3937×10^{-6} (K/Pa)	β_τ	1.20×10^3 K
γ_2	2.9840×10^{-12} (K/Pa) ²		

Table 4.1 shows the values of these coefficients for the MIL-L-7808 lubricant for the case of using $f = \tanh$; see Equations 4.9 and 4.10a. The use of these coefficients is found to provide a good correlation between the measured and the predicted traction coefficients [27]. Finally, the traction forces are calculated by assuming constant values of the effective viscosity, effective shear modulus and effective critical shear stress over the entire contact area in order to shorten the computational time of the dynamic model.

Figure 4.4. Traction coefficient values of MIL-L-7808 lubricant with Johnson et al. model (sinh) [27]

Figure 4.5. Traction coefficient values of MIL-L-7808 lubricant with Bair and Winer model (atanh) [27] and their associated simplified traction curves

The traction coefficient of MIL-L-7808-type lubricant calculated from the parameters of Table 4.1 and viscoelastic models are plotted in Figures 4.4 and 4.5 versus different slide-to-roll ratios of contact surfaces. It is noteworthy to mention that these figures depict just a few of the viscoelastic model predictions for the experiments carried out in Reference [27], and the related operating conditions are shown in the legend of these figures. The values of the effective viscosity, effective shear modulus and effective critical shear stress associated with these operating conditions are calculated based on the Equations 4.9-4.13 and the approach presented in references [27, 28, 34]. As shown in Figures 4.4 and 4.5, at low slide-to-roll ratios, at a fixed temperature and rolling speed, the lubricant behavior is mostly elastic and the traction coefficient increases with the contact pressure. As the slide-to-roll ratio increases, the viscous term becomes more prominent and the value of traction coefficient reaches to a constant value. The slight difference in the slope of the viscoelastic traction curve at high sliding velocities, depicted in Figure 4.4 compared to Figure 4.5, is due to the different definition of the parameter τ_0 according to the Johnson [19] or Bair [18, 26] viscoelastic models.

Shown in Figure 4.5 are also two simplified traction curves of MIL-L-7808-type lubricant as a function of slide-to-roll ratio, and independent of the value of contact pressure and temperature. These simplified traction curves are generally used in the literature for the purpose of parametric studies and are used in the present study to investigate the accuracy of their results in comparison with the experimental results as well as the results of the viscoelastic rheological models.

4.3. Experimental validations

The results of the dynamic model developed in the present study are compared with the experimental investigations of Harris [21]* pertaining to skidding in high speed roller bearings. There, he presented an analytical quasi-static model along with the semi-empirical elasto-hydrodynamic formulations. Skidding is generally caused by the gross sliding between the contact surfaces of the rollers and raceways where insufficient lubrication between the contact surfaces can considerably increase the surface shear stresses and finally lead to a surface damage called smearing. Skidding is usually seen in high speed applications under low loads and can be minimized by increasing the external load; However, this remedy tends to reduce the fatigue life of the bearing [35].

Harris measured the cage speed, or the orbital speed of the rollers, in a single-row, cylindrical-roller bearing with a pitch diameter of 0.18288 m and 6.34999×10^{-5} m diametral clearance. The bearing had 36 cylindrical rollers with 0.01397 m diameter and roller length of 0.020066 m. This roller bearing is lubricated by a MIL-L-7808-type lubricant in an oil bath lubrication system.

* Experimental measurements are carried out by Richard C. Malott of Allison Division of General Motors Corporations.

Harris carried out a set of experiments with different inner raceway rotational speeds under a wide range of radial loads and compared the result of his analytical model with the experimental results; see Figure 4.6. He assumed that the bearing is rigidly supported on a solid shaft in a solid housing and the frictional drag forces due to the raceways are ignored since the bearing under investigation has no guide flange on the outer raceway. Several semi-empirical relationships are employed in his quasi-static model in order to calculate the traction and drag forces resulting from the MIL-L-7808-type lubricant used in the experiments [21].

Figure 4.6. Cage Rotational Speed under different radial loads and inner raceway rotational speeds with diester type lubricant MIL-L-7808 [21]

The viscous drag forces and churning moments on the cage and rollers due to the presence of lubricant in the bearing cavity can have a significant impact on the rotational motion of the rolling elements, especially in high speed applications. In oil-bath lubrication, the size and geometry of the reservoir oil along with the oil-level in the bearing cavity have a substantial effect on the magnitude of the churning moments and drag forces. However, due to the lack of the required data about the oil-level and bearing cavity in reference [21], in this study we calculate the drag forces using the same semi-empirical relationships used by Harris in the related experiments:

$$\bar{Q}_{\text{drag},i} = 18.4(1 - \zeta)G_h^{-0.3}\bar{U}_i^{0.7} \quad (4.14a)$$

$$\bar{Q}_{\text{drag,o}} = 18.4(1 + \zeta)G_h^{-0.3}\bar{U}_o^{0.7} \quad (4.14b)$$

where subscripts of i and o are related to inner raceway and outer raceway contacts, respectively, and the dimensionless parameter ζ is the ratio of roller diameter to pitch diameter of the bearing. It is assumed that the churning moments for the related experiments are negligible due to the low rotational speed studied in the present study. Also, employed in the experiments and calculations is the synthetic oil MIL-L-7808 with the properties shown in the Table 4.1.

Several additional assumptions are made in the present study about the cage dimensions and material properties. Since the investigated cylindrical bearing is operating at low rotational speeds, it is assumed that the cage is a solid single-piece cage with cylindrical-shaped pockets that are guided by the cylindrical rollers. A clearance of 5×10^{-4} m is assumed between the rollers and the cage pockets. It is noteworthy to mention that the roller-cage pocket friction coefficient and the roller-cage pocket clearance can have a significant effect on the cage angular velocity specially in high speed rotational speeds, as shown in references [15,20,22] when employing a simplified traction curve.

The classical formulation of line contact is employed to estimate the contact load between the cage and the rollers. Subsequently, the resulting wear values are estimated based on these contact forces. In order to calculate time-averaged wear-volume rate of the cage ($\dot{W}_r$), the classical Archard wear formula is employed in the present study [20]:

$$\dot{W}_r = \frac{1}{t} \int_0^t \frac{k_w Q u_s}{H_v} dt \quad (4.15)$$

where k_w and H are the wear coefficient and hardness of the cage material. They are assumed to be 1×10^{-5} and 1×10^9 , respectively. Q , t and u_s are the contact load, time and sliding velocity, respectively. The contact load Q is calculated based on the Hertzian line-contact theory due to the assumptions made about the geometry of the cage. It should be mentioned that the hydrodynamic load between the roller and the cage pocket are estimated from the hydrodynamic solution of the lubricant film between the roller and the cage pocket.

4.4. Results and discussion

As explained in Section 2, the quasi-static solution is the initial conditions for the dynamic solution. Therefore, the dynamic models need considerable amount of computational time in order to reach the final or steady solution from the initial conditions. The computational time depends on the difference between the initial conditions (quasi-static solution) and the final steady solution. Thus, the dynamic model should be run for constant values of bearing load and rotational speed in order to calculate the rotational velocity of the cage. Table 4.2 present the loads and speeds considered in the simulations.

Table 4.2. values of operating conditions for each dynamic simulation

External Radial Load (N):	557 (125 lb)	1112 (250 lb)	1668 (375 lb)	2224 (500 lb)	6673 (1500 lb)	11121 (2500 lb)
Rotational Speed (RPM):	2000	3500	5000	6500		

Under the conditions simulated, the application of both of the viscoelastic rheological models presented by Johnson and Tevaarwerk [19] and Bair and Winer [18, 26] leads to very close results for the cage speed, therefore the results are shown for one of them (Bair and Winer) in Figures 4.7-4.10. Moreover, using the traction curves I and II also yield very close results and only one of them (Model I) is presented in these figures.

4.4.1. Assumptions about the experimental procedure

To determine the traction coefficient it is necessary to first evaluate the lubricant operating temperature with a realistic viscoelastic rheological model. This, in turn, requires the knowledge of thermal boundary conditions of the system of the interest. However, given that the experimental results by Harris [21] do not explicitly report the boundary conditions, one needs to employ a trial-and-error procedure to estimate the pertinent operating temperature for the lubricant. To this end, first, an assumption is made regarding the boundary conditions that closely match the experimental environment including the ambient temperature of $T_a = 300$ K and convection heat transfer coefficient of $h_a = 15$ for the housing. The forced convection coefficient of the shaft is also calculated by means of appropriate empirical correlations [36, 37]. Next, an initial value is assumed for the lubricant operating temperature and the dynamic model is used to obtain the rotational speed of the cage. Through comparison with the experimentally measured cage speeds [21], the value of the lubricant operating temperature is updated and the new cage speed is obtained by employing the dynamic model. This trial-and-error procedure is continued until the predicted cage speed matches the experimentally reported value, and the appropriate corresponding lubricant operating temperature is obtained in this fashion. This trial-and-error procedure is carried out for four cases of low sliding velocities. A total of four shaft rotation speeds of 2000, 3500, 5000 and 6500 RPM are considered, all subjected to a constant radial load of 11121 N (2500 lb). The corresponding predicted lubricant operating temperatures for these speeds are: 330, 360, 390 and 420 K. In order to obtain the lubricant operating temperature under lower radial loads, the measurements reported in an earlier study [36] are used. According to this earlier study, 10 K temperature drop is expected for every 4448 N (1000 lb) decrease in the radial load [36]. In this fashion the lubricant operating temperatures are estimated for each rotational speed and for the range of radial loads used in this study. It must be noted that the observation in [36] are deemed applicable to the present study inasmuch as they are obtained for a bearing system with a similar oil-bath lubrication and bearing dimensions.

4.4.2. Influence of the traction models on the cage rotational speed

Depicted in Figures 4.7 and 4.8 are the comparison of the cage rotational speed between the experimental measurements and results of the different rolling element bearing modeling

approaches. Referring to Figure 4.7, the evolution of the cage speed is predicted for the bearing subjected to radial load of 6673 N (1500 lb) and four different inner race rotational speeds of 2000, 3500, 5000 and 6500 RPM. Figure 4.8 illustrates the comparison for the same rotational speeds but with a considerably lower radial load of 557 N (125 lb). Figure 4.7 clearly shows that the dynamic model with an appropriate viscoelastic rheological model, can predict a more realistic cage speed compared to the quasi-static model, even for the case of high radial loads where the error is negligible. One can readily deduce from Figure 4.7 that a dynamic model may result in larger errors in case of employing a simplified traction curve instead of an appropriate viscoelastic model. However, for the case of low rotational speed and high bearing load in which the sliding velocity is low, application of a simplified traction curve leads to the lower error compared to Harris's model. Comparing Figures 4.7(a) to (d) reveals that the discrepancy between the experimental results and the results of dynamic models, regardless of the applied traction model, tends to become greater as the rotational speed and associated sliding velocity increases.

The simulations are performed on a computer equipped with 2.53 GHz processor and 16 GB of memory. The computation time for the simulations varies from 4 to 6 hours when using viscoelastic rheological models, depending on the difference between the initial conditions and the final steady result. The required simulation time is about 3 hours in case of employing simplified traction curves. The initial adaptive step size of integration for most of the simulations is about 1×10^{-4} while it is lower by one order for the higher rotational speeds. Numerical experiments show that the error tolerance of 1×10^{-3} leads to acceptable results while the accuracy of the final results does not change appreciably by reducing this error tolerance.

It is also noteworthy to mention that the initial conditions (quasi-static solution) of dynamic models using viscoelastic rheological models and simplified traction curves are the same and the slight difference between the starting points of dynamic models, shown in Figures 4.7 and 4.8, is simply because of the curve fitting errors during post processing of results.

According to the trends seen in Figures 4.7(a)-(d), it can be observed that as the speed increases the running simulation time takes longer to reach the final or steady solution, thus requiring more computations. This increase in the simulation time is due to the constant error tolerance in all of the simulations and the fact that the difference between the final steady solution and the initial conditions increases owing to the larger sliding velocities in higher rotational speeds.

(a)

(b)

Figure 4.7. Cage rotational speed under 6673 N (1500 lb) radial loads and (a) 2000 RPM
(b) 3500 RPM

Figure 4.7. Cage rotational speed under 6673 N (1500 lb) radial loads and (c) 5000 RPM (d) 6500 RPM [cont'd]

Figure 4.8. Cage rotational speed under 557 N (125 lb) radial loads and (a) 2000 RPM (b) 3500 RPM

Figure 4.8. Cage rotational speed under 557 N (125 lb) radial loads and (c) 5000 RPM (d) 6500 RPM [cont'd]

Shown in Figure 4.8 is the comparison between the experiment, the quasi-static model by Harris and two dynamic models with different traction models under 557 N (125 lb) radial load. In contrast to Figure 4.7, it is seen that the discrepancy between Harris's model and the experimental results is greater for the bearing under 557 N (125 lb) radial load and different rotational speeds while the dynamic model with the viscoelastic rheological model predicts much more realistic results. Although the errors associated with all models become greater as the rotational speed increases, the dynamic model with the viscoelastic model still gives a better estimation. The most likely reason for the deviation at higher speeds is the uncertainty in the applied operating temperature of the lubricant in the viscoelastic model. Seen in Figures 4.8(a)-(d) is the same trend of increased simulation time with speed as it is observed in Figure 4.7, although the simulation time of dynamic models in Figure 4.8 is longer than the required time in Figure 4.7. This increase in computational cost can be attributed to the higher sliding velocities in case of lower radial load in Figure 4.8 compared to Figure 4.7.

It is interesting to note that, regardless of the applied load and the rotational speed, the time required to reach the final steady solution for the dynamic model with a simplified traction curve is nearly constant, about 0.5 second for the bearing configuration studied. Nevertheless, the simulation time for the dynamic model with a viscoelastic model depends on the applied load, drag force and the rotational speed, such that it rises as the rotational speed increases or the applied load decreases. In fact, one should expect longer computational times associated with the dynamic models using the viscoelastic behavior owing to the numerical solution of the differential equation of the viscoelastic rheological models.

Figure 4.9 presents the results of running a set of simulations with the viscoelastic rheological model for all of the operating conditions presented in Table 4.2. The cage speed presented in Figure 4.9 is calculated by analyzing the final steady speed of the cage under the constant load and the constant rotational speed associated with each set of data in Table 4.2. Also presented in Figures 4.10(a) and (b) is the results of the dynamic model considering the simplified traction curves I and II, shown in Figure 4.5, under the same operating conditions specified in Figure 4.9. The final steady values shown in Figures 4.9-4.10 are calculated from the curve-fitted results shown in Figures 4.7-4.8 by considering a constant admissible error in the magnitude of the cage angular velocity. It should be mentioned that the results presented in Figure 4.9 are due to the assumed lubricant temperature and experiment procedure discussed in section 4.1. The reverse trend of cage speed at very low loads, under 1112 N (250 lb), is probably due to thermo-elastic effects. As it is mentioned by Harris [21], this phenomenon is likely because of the radial expansion of the bearing elements and resultant induced thermal preload under these operating conditions.

Comparison of the general trend of solutions in Figures 4.6, 4.9 and 4.10 shows that a dynamic model with an appropriate viscoelastic model can provide a more realistic prediction of the

dynamic motion of bearing elements better than other bearing modeling techniques. Figure 4.9 demonstrates the superior capability of the dynamic model with viscoelastic rheological model to simulate the real dynamic motion of bearing elements under the high external load wherein the sliding velocity is low. As the load is reduced, the error associated with dynamic model using viscoelastic models increases due to the fact that the exact operating temperature of the lubricant in the experiments is not provided in reference [21]. It should be noted that the larger errors in higher rotational speeds are probably due to some assumptions like negligible churning moments considered in the simulations owing to the lack of experimental data.

Comparing Figures 4.6 and 4.10 shows that the results of the dynamic model with simplified traction curves are in close agreement with the results of Harris's model under the very high radial loads, but as the radial load goes down, the results of model by Harris decreases exponentially while the results of the dynamic model with simplified traction curves remains almost constant. Comparison of Figures 4.10(a) and (b) also shows a slight discrepancy between the results obtained by employing two simplified traction curves presented in Figures 4.4 and 4.5. Although it may seem that the results of the two traction curves I, II are identical according to Figures 4.10(a) and (b), a detailed comparison of the results reveals that the cage speeds obtained by traction curve I are slightly higher than those obtained by traction curve II. For example, The cage speed for the operating condition of 6673 N (1500 lb) and 2000 RPM is equal to 910 RPM in case of employing traction curve I, while it is equal to 905 RPM in case of using traction curve II. This is, in fact, due to the higher traction coefficients that are used in traction curve I.

Figure 4.9. Comparison between the experimental results and the results of dynamic model with

Figure 4.10. Comparison between the experimental results and the results of dynamic model with the (a) simplified traction curve II (b) simplified traction curve I

4.4.3. Effects of bearing load and rotational speed on the wear rate

By applying Equation 4.15 and the material coefficients mentioned in section 3, it is possible to estimate the time-averaged wear rate of the cage. The wear values presented in the present study are only for the purpose of parametric study. As mentioned in section 1, the dynamic modeling technique is the only approach available in the literature to simulate and estimate cage interactions and associated dynamic effects with a sufficient accuracy. The wear rates presented in Figures 4.11-4.13 are the output of the developed dynamic model in this study along with the viscoelastic rheological model, and they represent the total wear rate of the cage calculated by adding the wear rates of all of the cage pockets. The same set of simulations employed in the previous section to calculate the cage rotational speed are carried out in order to evaluate the effect of bearing load and rotational speed on the cage wear rate. . It is noteworthy to mention that the assumed value of the wear coefficient in Section 3 is for the case of dry contact, and choosing other wear coefficients may alter only the magnitude of the cage wear rates while the general trends shown in Figures 4.11-4.13 will not change.

Figure 4.11 shows the time-averaged wear rate of the cage under the constant radial load of 557 N (125 lb) and different rotational speeds. It is seen that the time-averaged wear rate initially starts from zero and rapidly rises to a maximum value after which it gradually drops and approaches a fixed value. This behavior is simply because of the definition of the wear rate equation in Equation 4.15 where the time-averaged wear rate is a function of the magnitude of the contact load and the time duration of the contact. Therefore, the sudden increase of the wear rate followed by a gradual reduction to a constant value is due to the transient behavior of the rollers-cage contacts, owing to the existence of a clearance between the rollers and the cage in the initial conditions of the dynamic simulation. The steady wear rate is an indicator of the stable motion of the cage and it can be used to predict the total wear of the bearing elements through the bearing life. It is noteworthy to mention that an increasing wear rate shows the instability of the cage motion and have been studied in references [22, 34]. It is easily deduced from Figure 4.11 that the wear rate of the cage increases with the rotational speed owing to the higher sliding velocities. It should be mentioned that the results for other radial loads less than 2224 N (500 lb) have the similar trend of Figure 4.11 with different magnitudes of the cage wear rate. This is, in fact, due to the lower cage velocities in lower radial loads in case of using viscoelastic rheological models in the dynamic simulations, as shown in Figure 4.9.

Shown in Figure 4.12 is the time-averaged wear rate of the cage for the case of constant rotational speed and different radial loads. It shows that the cage wear rate increases in the case of lower external radial loads. This behavior, in fact, shows that the magnitude of the contact loads between the cage and the rollers directly depend on the magnitude of the sliding velocities in the contact areas between the races and the rollers. The higher sliding velocity results in the larger contact load between the roller and the cage due to their higher velocities during the

contact. Therefore, the cage wear rate increases under the low radial loads because of the larger contact loads between the cage and the rollers in the studied bearing setup.

Figure 4.11. Estimated time-averaged wear rate of the cage in under 557 N (125 lb) radial load

Figure 4.12. Estimated time-averaged wear rate of the cage for the constant 5000 RPM rotational speed of the inner race

Figure 4.13 Estimated time-averaged wear rate of the cage for different radial loads and inner raceway rotational speeds using the dynamic model with viscoelastic rheological model

In order to gain more insight into the effect of the bearing load and the rotational speed on the wear rate of the investigated cylindrical bearing, a set of simulations are carried out for the operating conditions presented in Table 4.2, and the steady-state wear rate values associated with each set of data are presented in Figure 4.13. Comparison of the trend of the wear rates in Figure 4.13 reveals that the wear rate of the cage increases as the load decreases or the rotational speed increases due to the higher sliding velocities under high speeds and low loads. This behavior is in agreement with the trend of the change in cage speed with regards to the bearing load and the rotational speed.

4.5. Conclusions

The present study investigates the influence of different traction models available in the literature on the dynamic behavior of a cylindrical roller bearing under radial load. A simplified dynamic model is developed for the investigated roller bearing assembly. Considered in conjunction with two simplified traction curves are two appropriate viscoelastic models of MIL-L-7808-type lubricant based on the non-Newtonian rheological models presented by Johnson and Tevaarwerk [19] and Bair and Winer [18, 26]. The comparison of the experimental and simulation results indicates that the dynamic model with a viscoelastic rheological model can provide a much more realistic prediction of the dynamic motion of the bearing elements compared to other models, so long as correct operating temperature and accurate lubricant properties are employed. The

presented results show that an increase of the radial load and a decrease of the rotational speed result in lower sliding velocities and more stable dynamic motion of the bearing elements. It is also shown that the cage wear rate rises by increasing the rotational speed and decreasing the bearing load.

An in-depth analysis of the simulation results reveal that the application of viscoelastic rheological model in the dynamic model demands longer computational time compared to employing the simplified traction curve owing to the numerical solution of the differential equation associated with the viscoelastic rheological model. It is also shown that the computational time of the viscoelastic dynamic model rises with higher rotational speeds and lower loads due to higher sliding velocities. Furthermore, the comparison of the results indicates that the application of simplified traction curves in the dynamic model can be justified only for the cases of low sliding velocities. A comprehensive and more accurate solution can be derived if the effects of the thermal expansion and the thermal preload are considered in the dynamic model, especially in high temperature operating conditions.

4.6. Nomenclature

b	Semi-minor width of contact area (m)
D	Deborah Number, $\mu U / Gb$
d_m	Bearing pitch diameter (m)
E	Elastic modulus (Pa)
E'	$[0.5[(1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2]]^{-1}$
f	Race curvature factor
f_c	Frequency of contact
F	Force (N)
G_h	$\lambda E'$
G	Shear modulus (Pa)
$\bar{G}$	Reference shear modulus (Pa)
h_c	Lubricant film thickness at the center of the contact (m)
H_c	h_c / R_m
H_r	Hardness of cage material (Pa)
I	Mass moment of inertia (kg.m^2)
K_f	Thermal conductivity (W/m.K)
K_w	Wear coefficient
l	Effective roller length (m)
L	Thermal loading parameter
m	Mass (kg)
M	Moment (N.m)
P	Pressure (Pa)
Q	Contact load (N)
W	$Q / lE'R_m$
$\bar{Q}_{drag}$	Drag force (N)

$\bar{R}$	Radius of the contact curvature
R_r	Roller radius (m)
R_{ir}	Race curvature radius (m)
R_m	Equivalent radius (m) , $R_1 R_2 / (R_1 + R_2)$
S	Slide to roll ratio, $2 (U_1 - U_2) / (U_1 + U_2)$
$\dot{\gamma}$	Shear strain rate (1/s)
$\dot{\bar{\gamma}}$	$U_s G b / h U \tau_o$
t	Time (s)
T	Temperature (K)
T_0	Reference temperature (K)
U	Rolling velocity (m/s), $(U_1 + U_2) / 2$
$\bar{U}$	$\mu_o U / 2 E' R_m$
U_s	Sliding velocity (m/s)
W_r	Time-averaged wear rate (m ³)
y	Axis along the minor axis of the contact area (m)
$\bar{y}$	y / b
α_G	Pressure-shear modulus coefficient (1/Pa)
α_τ	Pressure-critical shear stress coefficient (1/Pa)
β	Temperature-viscosity coefficient (1/K)
β_G	Temperature-shear modulus coefficient (K)
β_τ	Temperature-critical shear stress coefficient (K)
δ	Load deflection (m)
λ	Pressure-viscosity coefficient (1/Pa)
ζ	D_r / d_m
μ	Viscosity (Pa.s)
μ_o	Reference viscosity (Pa.s)
τ	Shear stress (Pa)
$\bar{\tau}$	τ / τ_o
τ_o	Limiting shear stress or Eyring stress (Pa)
$\bar{\tau}_o$	Reference critical shear stress (Pa)
ΔT	$T - T_0$
Φ	Thermal reduction factor
ν	Poisson ratio
ω	Rotational velocity (Rad/s)
$\dot{\omega}$	Rotational acceleration (Rad/s ²)

4.7. References

- [1] Jones AB. A general theory for elastically constrained ball and radial roller bearings. Transactions of the ASME Series D, Journal of Basic Engineering. 1960;82:309-320.
- [2] TA H. Rolling bearing analysis. New York: Wiley. 2001;4th ed.
- [3] Walters CT. Dynamics of ball bearings. Journal of Lubrication Technology. 1971;93:1-10.

- [4] Kannel JW, Bupara SS. Simplified model of cage motion in angular contact bearings operating in ehd lubrication regime. Journal of Lubrication Technology-Transactions of the Asme. 1978;100:395-403.
- [5] Gupta PK. Dynamics of rolling-element bearings .1. Cylindrical roller bearing analysis. Journal of Lubrication Technology-Transactions of the Asme. 1979;101:293-304.
- [6] Gupta PK. Dynamics of rolling-element bearings .3. Ball-bearing analysis. Journal of Lubrication Technology-Transactions of the Asme. 1979;101:312-318.
- [7] Gupta PK. Dynamics of rolling-element bearings .4. Ball-bearing results. Journal of Lubrication Technology-Transactions of the Asme. 1979;101:319-326.
- [8] Gupta PK. Dynamics of rolling-element bearings .2. Cylindrical roller bearing results. Journal of Lubrication Technology-Transactions of the Asme. 1979;101:305-311.
- [9] Gupta PK. Current status of and future innovations in rolling bearing modeling. Tribology Transactions. 2011;54:394-403.
- [10] Meeks CR. The dynamics of ball separators in ball-bearings .2. Results of optimization study. Asle Transactions. 1985;28:288-295.
- [11] Meeks CR, Ng KO. The dynamics of ball separators in ball-bearings .1. Analysis. Asle Transactions. 1985;28:277-287.
- [12] Meeks CR, Tran L. Ball bearing dynamic analysis using computer methods .1. Analysis. Journal of Tribology-Transactions of the Asme. 1996;118:52-58.
- [13] Ashtekar A, Sadeghi F. A new approach for including cage flexibility in dynamic bearing models by using combined explicit finite and discrete element methods. J Tribol-T Asme. 2012;134.
- [14] Ashtekar A, Sadeghi F, Stacke LE. Surface defects effects on bearing dynamics. P I Mech Eng J-J Eng. 2010;224:25-35.
- [15] Ghaisas N, Wassgren CR, Sadeghi FA. Cage instabilities in cylindrical roller bearings. Journal of Tribology-Transactions of the Asme. 2004;126:681-689.
- [16] Saheta V. Dynamics of rolling element bearings using discrete element method. 2001.
- [17] Weinzapfel N, Sadeghi F. A discrete element approach for modeling cage flexibility in ball bearing dynamics simulations. Journal of Tribology-Transactions of the Asme. 2009;131.
- [18] Bair S, Winer WO. Rheological model for elastohydrodynamic contacts based on primary laboratory data. Journal of Lubrication Technology-Transactions of the Asme. 1979;101:258-265.

- [19] Johnson KL, Tevaarwerk JL. Shear behavior of elastohydrodynamic oil films. Proceedings of the Royal Society of London Series a-Mathematical Physical and Engineering Sciences. 1977;356:215-236.
- [20] Gupta PK. Frictional instabilities in ball-bearings. Tribology Transactions. 1988;31:258-268.
- [21] Harris TA. An analytical method to predict skidding in high speed roller bearings. Asle Transactions. 1966;9:229-241.
- [22] Gupta PK. Advanced dynamics of rolling elements. Springer-Verlag, New York. 1984.
- [23] Palmgren A. Ball and roller bearing engineering, 3rd edition. SKF Industries Inc. 1959.
- [24] Evans CR, Johnson KL. Regimes of traction in elastohydrodynamic lubrication. Proceedings of the Institution of Mechanical Engineers Part C-Journal of Mechanical Engineering Science. 1986;200:313-324.
- [25] Bair S, Winer WO. The high-pressure high shear-stress rheology of liquid lubricants. Journal of Tribology-Transactions of the Asme. 1992;114:1-13.
- [26] Gecim B, Winer WO. A film thickness analysis for line contacts under pure rolling conditions with a non-newtonian rheological model. Journal of Lubrication Technology-Transactions of the Asme. 1981;103:305-318.
- [27] Forster NH, Schrand JB, Gupta PK. Viscoelastic effects in mil-l-7808-type lubricant .2. Experimental-data correlations. Tribology Transactions. 1992;35:275-280.
- [28] Gupta PK, Cheng HS, Forster NH. Viscoelastic effects in mil-l-7808-type lubricant .1. Analytical formulation. Tribology Transactions. 1992;35:269-274.
- [29] Olver AV, Spikes HA. Prediction of traction in elastohydrodynamic lubrication. Proceedings of the Institution of Mechanical Engineers Part J-Journal of Engineering Tribology. 2001;215:310-310.
- [30] Dowson D, Higginson G. The effect of material properties on the lubrication of elastic rollers. Journal of Mechanical Engineering Science. 1960:188-194.
- [31] Cheng HS. A refined solution to thermal-elastohydrodynamic lubrication of rolling and sliding cylinders. Asle Transactions. 1965;8:397-410.
- [32] Wilson WRD, Sheu S. Effect of inlet shear heating due to sliding on elastohydrodynamic film thickness. Journal of Lubrication Technology-Transactions of the Asme. 1983;105:187-188.
- [33] Hsu CH, Lee RT. An efficient algorithm for thermal elastohydrodynamic lubrication under rolling/sliding line contacts. Journal of Tribology-Transactions of the Asme. 1994;116:762-769.
- [34] Gupta PK. Viscoelastic effects in mil-l-7808-type lubricant .3. Model implementation in bearing dynamics computer code. Tribology Transactions. 1992;35:724-730.

- [35] TA H. Rolling bearing analysis. New york, Wiley. 2001;4th ed.:1043-1070.
- [36] Takabi J, Khonsari MM. Experimental testing and thermal analysis of ball bearings. Tribology International. 2013;60:93-103.
- [37] Ozerdem B. Measurement of convective heat transfer coefficient for a horizontal cylinder rotating in quiescent air. Int Commun Heat Mass. 2000;27:389-395.

Chapter V: Dynamic performance of roller bearings operating under low rotational speeds with consideration of surface roughness *

5.1. Introduction

The key parameters that govern the performance of rolling element bearings are the film thickness and traction coefficient; lubricant film protects the surface and traction coefficient determines the power loss. Over the past three decades many rheological models have been proposed to estimate the lubricant film thickness and traction coefficient [1-8]. Although it is possible to calculate the film thickness in different lubrication regimes with a good accuracy, reliable prediction of the traction coefficient under severe contact pressures in rolling bearings still remains to be a challenging task.

Traction coefficient is also a highly influential parameter in dynamic modeling of rolling bearings, where elastohydrodynamic lubrication (EHL) prevails. It is a function of the temperature and pressure as well as the sliding and rolling velocities of the bodies in contact. Reported in the dynamic models of several studies [9-13] are the use of simplified traction curves that vary as a function of only the sliding velocity. A more accurate method of calculating the traction coefficient of Newtonian fluids is presented by Gupta [14]. Later, several studies utilized more advanced non-Newtonian models [8, 15-24] in dynamic analysis of rolling bearings [25-27]. Extension of the results with consideration of different rheological relationships is presented in reference [28] that gives the state of the art in rolling bearing modeling. Recently, Takabi and Khonsari [29] employed a non-Newtonian model implemented in a dynamic model of a cylindrical roller bearing and compared the accuracy of the results with a simplified traction curve. They showed that the simplified traction curves may be used only in the case of low sliding velocities. At high sliding velocities, considerable errors can occur in the final results of dynamic models.

A further review of available models in the open literature [9, 10, 12-14, 27, 28, 30-32] reveals that the influence of surface roughness is neglected in the calculation of traction coefficient in the existing dynamic models. The effect of surface roughness becomes important when the film thickness is small and does not fully separate the contact surfaces. This is typically the case at high operating temperature or low rotational speed and in heavily-loaded bearings. Accordingly, in the present paper we study the effect of surface roughness on the dynamic performance of roller bearings in low rotational speeds and under large contact loads. For this purpose, a simplified dynamic model of cylindrical roller bearings is developed which employs the results

* Reprinted by permission of *Tribology International* (See Appendix C)

of a recent study [33] to calculate the traction coefficient with provision of surface roughness that concomitantly affects the film thickness, heat generation and wear rate.

5.2. Modeling and simulations

5.2.1. Dynamic model of roller bearings

Rolling element bearing analytical models can be broadly classified as either quasi-static or dynamic models. The widely-used quasi-static models are based on the equilibrium of static forces and moments acting on each rolling element, whereas in dynamic models the integration of the differential equations of motions with respect to time is considered [30, 31, 34-39]. Unlike the quasi-static models, dynamic models can provide a real-time simulation of the motion of the bearing elements under any operating condition.

The first fully dynamic model of the rolling element bearings was presented by Gupta [40-43] where six degrees of freedom (DOF) were considered for all of the bearing elements including rollers, cage and race-ways. Based on the Gupta's model the translational and rotational motions of each bearing element are predicted by solving the Newton and Euler equations of motions, respectively. Presented in the literature are also several simplified dynamic models [9, 10, 12, 13, 32, 44] to reduce the required computational time and to make parametric design studies. Reference [28] presents a thorough review and the current state-of-the-art of dynamic models of rolling element bearings.

Developed for the present study is a dynamic model of cylindrical roller bearings similar to Gupta's model. The basic concept and details of this dynamic model is presented in references [14, 29]. The next section presents the simplified assumptions made for the dynamic model of the present study and their effects on the final results. Then, the approach developed in the current work to calculate the traction forces in different regimes of lubrication is presented in Section 2.2.

5.2.2. Simplified dynamic model

In the general form of a fully dynamic model, rollers can freely move between the races, and this free motion of the rollers between the races leads to a high frequency response in the results. The frequency (c_f) of this vibratory motion of the rollers can be calculated by employing the Hertzian load-deflection formula for the elastic contact between the rollers and the raceways [14]:

$$c_f = \frac{1}{2\pi} \sqrt{\frac{nQ}{\delta m}} \quad (5.1)$$

where δ and Q are the normal displacement and the normal contact force between the roller and the raceway, respectively. m is the mass of the roller, and n is equal to 1.11 for the case of line contacts. This high frequency motion can be filtered out by an equilibrium constraint for the

special cases, such as the current study, where the position of the inner raceway and the outer raceway are fixed with respect to one another [14]. Thus, the radial and axial position of the loaded rollers between the races is fixed for any orbital position and can be calculated from the quasi-static solution associated with any orbital position of the rollers; see Figure 5.1. Considering the fact that the radial load and the rotational speed are kept constant in each simulation, the present study employs the equilibrium constraint in the radial direction for the loaded rollers in order to remove the high frequency motions, thus increasing the computational speed. This increase in the computational speed occurs because the Newton equations of motion are reduced to just orbital direction for each roller. The details of these constraint equations can be found in reference [14].

Figure 5.1. Positions of rollers and races in a bearing under a constant unidirectional radial load

It is also assumed that the rollers and the cage are allowed to rotate only about their primary axes. Therefore, the Euler equations reduce to only one equation for each roller and the cage, i.e. rotational motion about their primary axis of rotation. It is believed that these assumptions about the direction of rotation of the cage and the rollers have negligible effects on the final results of the present study because of the assumed geometry of the cage discussed in Section 3.1. Another assumption made in the present study, compared to the fully dynamic models, is about neglecting the radius of the contact curvature in the calculation of the traction forces. This assumption has been widely used in the literature and is also believed to have a negligible effect on the results.

5.2.3. Traction coefficient calculation

Available literature contains different approaches for calculating the traction forces in the dynamic models of rolling bearings, but their application is restricted to a limited range of

operating conditions. This limited range is due to the significant effect of contact pressure on the lubricant viscosity and the deformation of the contact surfaces. For example, in the case of radially-loaded cylindrical roller bearings appropriate formulas should be used for the different regimes of lubrication as the roller passes through the loading zone; see Figure 5.1. This is, in fact, due to the variation of contact pressure between the rollers and the raceways in different locations within the loading zone. Therefore, in order to simulate a realistic dynamic behavior of the bearing elements, it is important to figure out the appropriate lubrication regime for each contact as the rollers travel in orbital direction through the loading zone.

In this study, a map of lubrication regimes, presented by Winer and Cheng [45] is utilized to find the appropriate lubrication regime for each roller-race contact; see Figure 5.2. This map is based on the different studies available in the literature [46] about different lubrication regimes in line contacts. Figure 5.2 depicts four different lubrication regimes that may exist in the lubricated line contacts [45, 47]. As shown, the elastohydrodynamic lubrication (EHL, Elastic solid-Piezoviscous)—which is widely used for the roller bearings in the literature—covers only a small region on the map of the lubrication regimes. Thus, depending on the operating condition, other regimes of lubrication may exist in the lubricated line contacts. Winer and Cheng [45] introduced two dimensionless variables g_v (dimensionless viscosity parameter) and g_E (dimensionless elasticity parameter) in order to determine the governing lubrication regime of a line contact. These parameters are defined as follows.

Figure 5.2. Lubrication regimes of line contacts [47]

$$g_v = \left(\frac{\alpha^2 w_l^3}{\mu_0 R^2 u} \right)^{1/2} = \left(\frac{G^2 W^3}{U} \right)^{1/2} \quad (5.2)$$

$$g_E = \left(\frac{w_l^2}{\mu_0 E' R u} \right)^{1/2} = \left(\frac{W^2}{U} \right)^{1/2} \quad (5.3)$$

$$G = \alpha E' \quad U = \frac{\mu_0 u}{E' R} \quad W = \frac{w_l}{E' R} \quad (5.4)$$

where α is pressure-viscosity coefficient, and μ_0 is viscosity of the lubricant at the line-contact inlet. R and E' are, respectively, the equivalent contact radius and the effective modulus of elasticity. w_l is the load per unit contact length and u is the average surface velocity or rolling speed. Equation 5.4 also presents the definition of dimensionless parameters G , W and U . Using Equations 5.2 to 5.4, one can easily find the dimensionless viscosity and elasticity parameters in order to determine the governing lubrication regime in a line contact under a given load and speed. Once the lubrication regime is determined, one can proceed to predict the film thickness using an appropriate formula associated with these lubrication regimes. The following sections briefly introduce these lubrication regimes along with the formulas that have been employed in the current investigation to calculate the film thickness and traction coefficient with provision for surface roughness.

5.2.3.1. Rigid solid–Isoviscous (R–I)

The effect of contact pressure on the lubricant viscosity and the elastic deformation is negligible in this regime. Therefore, the dimensionless film thickness (h') is considered constant [45, 47]:

$$h' = 4.9 \quad (5.5)$$

where this dimensionless film thickness is defined as:

$$h' = \frac{h_{\min} w_l}{\mu_0 R u} \quad (5.6)$$

where $h_{\min}$ is the minimum film thickness of the lubricant in the (R–I) lubrication regime. In order to calculate the traction force in this regime, the following equation was used because the viscosity is constant in this regime due to the negligible effect of contact pressure on the lubricant viscosity.

$$\tau = \mu \frac{u_s}{h} \quad (5.7)$$

where u_s is the slip velocity and μ is the lubricant viscosity which is equal to the inlet viscosity μ_0 in this regime. h is the central film thickness which is equal to $h_{\min}$ for this regime since the elastic deformation of contacting bodies is neglected.

5.2.3.2. Elastic solid–Isoviscous (E–I)

Depending on the type of the lubricant and the solid materials, a lubrication regime can be classified as “soft EHL” in situations where the contact pressure has a significant effect on the elastic deformation, but it does not appreciably alter the lubricant viscosity. The dimensionless film thickness in this regime can be calculated using

$$h' = 0.3 g_E^{0.8} \quad (5.8)$$

where the dimensionless film thickness (h') is defined by Equation 5.6. [45, 47]

5.2.3.3. Rigid solid–Piezoviscous (R–V)

In this lubrication regime, it is believed that the effect of contact pressure on the elastic deformation of the contacting bodies can be neglected akin to the rigid solid–isoviscous regime. Nonetheless, the contact pressure has a significant effect on the lubricant viscosity and should be considered in the analysis. The dimensionless film thickness is a function of only the dimensionless viscosity parameter [45, 47]:

$$h' = 1.66 g_v^{2/3} \quad (5.9)$$

Here it is assumed that the Equation 5.7 can be used to calculate the traction forces taking into consideration the effect of contact pressure on the viscosity (μ). Employed in the present study for this purpose is the Roelands [6, 48] pressure-viscosity relationship, which is widely used in the literature:

$$\frac{\mu}{\mu_0} = \exp\{ (\ln \mu_0 + 9.67) \times [-1 + (1 + 5.1 \times 10^{-9} p_h)^{z_1}] \} \quad (5.10)$$

$$z_1 = \frac{\alpha}{5.1 \times 10^{-9} (\ln \mu_0 + 9.67)} \quad (5.11)$$

where p_h and μ_0 are hydrodynamic pressure and viscosity at atmospheric pressure respectively. z_1 is the viscosity–pressure index that can be calculated from Equation 5.11. It is noteworthy to mention that Roelands equation is best suited for applications with moderate contact pressure, while Tait’s state equation should be used in high contact pressures.

5.2.3.4. Elastic solid–Piezoviscous (E–V)

This regime is widely studied in the literature where it is also called elastohydrodynamic (EHL) or hard EHL regime. This is the regime where the effect of the contact pressure on the lubricant viscosity and the elastic deformation of the contact surfaces cannot be neglected due to the high contact pressure (typically between 0.5 to 3 GPa). Winer and Cheng [45] presented the following formula for the dimensionless film thickness based on the dimensionless parameters g_E and g_v :

$$h' = 2.65 g_v^{0.5} g_E^{0.06} \quad (5.12)$$

which provides the same result for the minimum film thickness ($h_{\min}$) calculated from the well-known Dowson and Higginson's formula (Equation 5.13) [3] if the dimensionless parameters are replaced by their appropriate definitions.

$$\frac{h_{\min}}{R} = 2.65 \frac{G^{0.54} U^{0.7}}{W^{0.13}} \quad (5.13)$$

Note that Equations 5.12-5.13 are developed for the smooth contact surfaces and must be modified for rough surfaces.

Presented in the literature are several studies [2, 49, 50] in which the roughness of the contact surfaces are considered by different means in the elastohydrodynamic analysis. This study employs the results of recent studies presented by Masjedi and Khonsari [33, 48] that consider the effect of the surface roughness on the film thickness and traction coefficient in EHL regime of line contacts. They presented the following formulas (Equations 5.14-5.16) to calculate the dimensionless parameters of central (H_c) and minimum ($H_{\min}$) film thickness and the asperity load ratio (L_a) [48]:

$$H_c = \frac{h_c}{R} = 2.691 W^{-0.135} U^{0.705} G^{0.556} \times (1 + 0.2 \bar{\sigma}^{1.222} V^{-0.223} W^{-0.229} U^{-0.748} G^{-0.842}) \quad (5.14)$$

$$H_{\min} = \frac{h_{\min}}{R} = 1.652 W^{-0.077} U^{0.716} G^{0.695} \times (1 + 0.026 \bar{\sigma}^{1.120} V^{0.185} W^{-0.312} U^{-0.809} G^{-0.977}) \quad (5.15)$$

$$L_a = 0.005 W^{-0.408} U^{-0.088} G^{0.103} \times \{ \ln(1 + 4470 \bar{\sigma}^{6.015} V^{1.168} W^{0.485} U^{-3.741} G^{-2.898}) \} \quad (5.16)$$

where h_c and $h_{\min}$ are the central and minimum film thickness in the line-contact EHL. The following equations also define the dimensionless parameters (V , $\bar{\sigma}$) used in Equations 5.14-5.16:

$$V = \frac{H}{E'} \quad \bar{\sigma} = \frac{\sigma}{R} \quad (5.17)$$

where H is the hardness of the surface and σ is the equivalent surface roughness which is equal to $\sqrt{\sigma_1^2 + \sigma_2^2}$. These formulas present the film thickness and asperity load ratio for the case of fully-flooded, steady-state and isothermal operating conditions.

Masjedi and Khonsari [33] also presented the following formula for corresponding traction coefficient in a line-contact EHL with provision of surface roughness. This equation is developed by curve-fitting the results of a set of simulations where they solved the Reynolds, energy and asperity micro-contact equations in a quantitative EHL analysis, by considering the free volume

theory and limiting shear stress of the lubricant. This equation is developed for specific type of lubricant (SAE-30), which is studied in the current investigation.

$$f = f_c \left(\frac{L_a}{100} \right) + \left(1 - \frac{L_a}{100} \right) \times \left\{ \Lambda \left(1 - e^{-\left(0.507 S \bar{W}^{2.487} \bar{U}^{0.229} \Lambda^{-1} \left(1 - \frac{L_a}{100} \right)^{4.438} \right)} \right) - 0.058 \bar{W}^{-0.326} (\bar{U}\bar{K})^{0.067} S^{0.507} \right\} \quad (5.18)$$

where f and f_c are, respectively, the traction coefficient and asperity friction coefficient. Λ is the limiting shear stress coefficient and S is the slide-to-roll ratio. Equations 5.19 also present the formulas for the rest of the dimensionless parameters in Equation 5.18.

$$\bar{W} = \frac{W}{2 \times 10^{-5}} \quad \bar{U} = \frac{U}{3 \times 10^{-12}} \quad \bar{K} = \frac{K}{4 \times 10^8} \quad K = \frac{E'R}{\sqrt{k\mu_0 T_0}} \quad (5.19)$$

where k is the thermal conductivity of the lubricant.

5.2.3.5. Implementation of lubrication regimes' map in the dynamic model

It should be noted that employing of the formulas presented in Sections 2.2.1-2.2.4 in the dynamic model of rolling bearing is not a straightforward procedure, due to the limited range of input parameters associated with different lubrication regimes. This section provides the details of the approach developed in the present study to calculate film thickness and the traction coefficient for different lubrication regimes.

Although Equations 5.14 and 5.15 can be used to calculate the film thickness in EHL line-contact with provision of surface roughness for different types of liquid lubricants, Equation 5.18 is developed and valid for a specific lubricant (SAE 30). Therefore, some of the lubricant properties are kept constant in the simulations. The limiting shear stress (Λ) and pressure-viscosity coefficient of SAE 30 are considered to be 0.091 and 2.5×10^{-8} , respectively. Consequently the dimensionless parameter G is equal to 5700 considering common alloy steel (SAE 52100) as the bearing material which results in the dimensionless parameter $E' = 2.28 \times 10^{11}$. The dimensionless hardness parameter V is also considered to be 0.01 for this bearing material. The range of input parameters in curve-fitting of Equation 5.18 is also limited to the range of parameters summarized in Table 5.1 [33].

Table 5.1. Range of curve fit for input parameters [33]

Parameter	W	U	$\bar{\sigma}$	S	K
min	2×10^{-5}	3×10^{-12}	0	0	4×10^8
max	3×10^{-4}	3×10^{-11}	4×10^{-5}	0.20	4×10^9

As shown in Figure 5.2, the boundaries (shaded region) between the different lubrication regimes are not clearly defined. According to Winer and Cheng [45], if g_v and g_E fall on these boundaries, the film thickness formula from either region near the boundaries can be used with a small error. In the present study, however, we implement a modified map of lubrication regimes shown in Figure 5.3. The new boundaries (solid lines) in this figure are different from the boundaries in Figure 5.2 (dashed lines) due to the limited input range of the parameters in Equation 5.18. The lower boundaries of the EHL region in Figure 5.3 are the same as Figure 5.2 and the two vertical and horizontal lines, crossing at the lowest boundary point of EHL region, define clear boundaries between the (R-I), (R-V) and (E-I) lubrication regimes.

Figure 5.3. Modified regimes of lubrication for line contacts

5.2.3.5.1. An illustrative example:

Consider two identical steel rollers in a rolling-disk traction machine with the radius of 50.8 mm and the width of 10 mm. These rollers are lubricated by SAE 30 with the thermal conductivity of 0.145 W/mK and inlet temperature of $T_0 = 293$ K. The linear velocity of the roller surfaces in contact is 0.21 m/s and 0.19 m/s. The surface roughness (R_q) of each roller is 0.18 μm and their Vickers hardness is 2.35 GPa. Assumed that the asperity friction coefficient, associated with this surface roughness, is $f_c = 0.12$. Considering these parameters and the applied load, one can easily calculate the dimensionless parameters g_E and g_v from Equations 5.2-5.4, in order to determine the governing lubrication regime. Next, appropriate formulas in Sections 2.2.1-2.2.4 can be employed to calculate the film thickness and the traction coefficient between the rollers. This procedure is utilized for this example while the normal contact force between the rollers is increased from 0 to 4000 N and the results are presented in Figures 5.4 and 5.5.

Figure 5.4 demonstrate the variation of the minimum film thickness for a wide range of contact loads between the rollers in this example. As the contact load increases from very small values, three different lubrication regimes are identified. At small contact loads, the lubrication regime is rigid solid–isoviscous and the film thickness is calculated from Equation 5.5, due to the negligible effect of contact pressure on the viscosity and deformation. As the load increases the film thickness drops drastically until the rigid solid–piezoviscous lubrication regime is reached and the minimum film thickness is calculated from Equation 5.9. The minimum film thickness remains almost constant in this regime until the lubrication regime changes to elastic solid–piezoviscous when the contact load exceeds 1000 N. Afterward, according to Equation 5.15, the minimum film thickness within the EHL regime gradually decreases with increasing the load. The maximum contact pressure reaches to 751 MPa under the 4000 N contact load. Note that the rather abrupt changes of film thickness in Figure 5.4 is because of the defined solid-line boundaries between different lubrication regimes shown in Figure 5.3. The influence of these transitions on the dynamic performance can be neglected since they occur in a very small region at the corners of the loading zone and in a very short time [45]. The shaded area in Figure 5.4 presents a range of acceptable solutions in the case of choosing other boundaries within the shaded region of Figure 5.2.

Figure 5.4. Minimum film thickness of lubricant in different lubrication regimes of a line-contact between two steel rollers lubricated by SAE 30

Shown in Figure 5.5 is the variation of traction coefficient from the small to large contact loads between the rollers in the example. The traction coefficient is very small in the rigid solid–

isoviscous regime, but it gradually rises with increasing the load. The value of the traction coefficient in the rigid solid–piezoviscous regime increases with a higher slope due to the effect of higher contact pressures on the lubricant viscosity. This sharp increase in the traction coefficient continues as the EHL regime begins to govern the lubrication regime. The traction coefficient in EHL regime, determined using Equation 5.18, gradually reaches to a constant value in very high contact loads. This is, in fact, due to the limiting shear stress (Λ) in Equation 5.18, which is the ultimate shear stress beyond which the lubricant flows plastically regardless of the shear strain [8, 15, 40, 51]. The traction coefficient is predicted to plateau at 0.075 for radial loads higher than 3000 N. This figure can provide a better understanding of how the traction coefficient between the roller and the raceways varies as the roller travels through the loading zone in the orbital direction.

Figure 5.5. Traction coefficient in different lubrication regimes of line-contact

5.3. Results and discussion

5.3.1. Simulations

The results presented in this section pertain to a set of simulations wherein the lubrication map is incorporated into the model to study the dynamic behavior of roller bearings. For this purpose a cylindrical roller bearing (SKF 2360EC) is chosen with the bore diameter of 0.300 m and outer diameter of 0.620 m. This bearing has 13 rollers with the diameter of 0.09485 m and a normal diametral clearance. It is assumed that the cage is solid single-piece with cylindrical-shaped pockets guided by the cylindrical rollers, with a clearance of 1×10^{-3} m between the rollers and the cage pockets. This roller bearing is assumed to be lubricated by an oil pick-up ring lubrication system with negligible drag forces and churning moments.

The effect of surface roughness is expected to be appreciable in the case of low rotational speeds under high contact loads, as it is shown in references [33, 48]. In order to investigate the effect of surface roughness on the performance of the roller bearings, three sets of simulations are carried out at two different operating conditions in each set. The operating conditions are the same for each set and only the surface roughness (R_q) of the rollers and the raceways are different (smooth, $0.18\ \mu\text{m}$ and $0.36\ \mu\text{m}$). The rotational speed of the bearing is equal to 50 rpm for both operating conditions in each set of simulations while two different radial loads (150 kN and 300 kN) are considered. The simulations are performed on a computer equipped with a 2.53 GHz processor and 16 GB memory. The computation time typically takes about 6 hours.

5.3.2. Variation of film thickness and heat generation in loading zone

The results presented in the following sections are based on the orbital position of the rollers shown in Figure 5.6a. It is assumed that the external radial load is applied on the inner ring and in the Z-direction shown in Figure 5.6a. The numbering of 13 rollers in the roller bearing begins from the most heavily loaded roller at orbital position $\theta = 0$ and it continues in a counter clockwise direction. It should be noted that the presented results in this section are for the steady state condition, when the transient behavior in the motion of the bearing elements is diminished.

Figure 5.6b shows the distribution of the contact load on the roller bearing under constant radial loads of 150 kN and 300 kN. This figure demonstrates the variation of the contact load between the roller and the raceway, as the roller travels in the orbital direction. It should be mentioned that the contact loads within the loading zone and between roller/inner race and roller/outer race are very close in low rotational speed of bearing due to the small centrifugal forces. The distribution of the contact loads for the case of 150 kN and 300 kN are, respectively, about 40 and 50 degree angles from the z-axes ($\theta = 0$), although only 3 rollers are loaded at each moment in both cases. The results in this section are for these ranges of orbital position of the rollers due to the negligible effect of surface roughness outside of the loading zone.

Shown in Figures 5.7a and 5.7b are the variation of the minimum film thickness between the roller/inner race and roller/outer race, respectively. The film thickness shown in these figures is the result of simulation for the case of $0.18\ \mu\text{m}$ surface roughness and operating condition of 50 rpm and 150 kN. As these figures show, the governing lubrication regime in the loading zone is mostly hard EHL and it changes to (R–V) and (R–I) lubrication regimes in the vicinity of the loading zones' end points. Presented insets in Figures 5.7a and 5.7b show the variation of the film thickness in EHL regime more precisely, both of which are similar to the trend shown in Figure 5.4. It is noteworthy to mention that the high values of the film thickness at the endpoints of the loading zone, shown in Figures 5.7a and 5.7b, are due to the curve-fitting limits of Equations 5.2-5.4 associated with the rigid solid–isoviscous lubrication regime. Nevertheless, these large film thicknesses at the endpoints of the loading zone have a negligible effect on the dynamic behavior of the bearing elements and final results.

(a) Assumed positive direction of orbital position of rollers in roller bearing

(b) Distribution of the loading zone on the roller bearing under 150 kN and 300 kN radial loads

Figure 5.6. Definition of the orbital position and loading zone in simulation results

As shown in Figure 5.7a, outside the loading zone, the minimum film thickness between the inner race and the roller is considered to be nil in the dynamic model, although there is a lubrication film between the roller and inner race outside the loading zone. This is simply because in this zone the normal contact force between the roller and the inner race is nil and has a negligible effect on the dynamic behavior of the bearing elements. On the other hand, as shown in Figure 5.7b, the minimum film thickness between the roller and the outer race outside of the loading zone is non-zero since the centrifugal forces are present between the roller and the outer race.

(a) Film thickness variation between roller and inner race

(b) Film thickness variation between roller and outer race

Figure 5.7. Film thickness variation for operating condition of 50 rpm and 150 kN and $R_q = 0.18 \mu\text{m}$ surface roughness

(c) Film thickness variation in hard EHL regime

Figure 5.7. Film thickness variation for operating condition of 50 rpm and 150 kN and $R_q = 0.18 \mu\text{m}$ surface roughness [cont'd]

The comparison of minimum film thickness between the roller and the raceways in the EHL regime is shown in Figure 5.7c. Although the contact load between the roller and the inner race is almost equal to the contact load between the roller and the outer race, the thickness of the lubricant film between the roller and the outer race is larger than the film thickness between the roller and the inner race. This is, in fact, due to the different values of the equivalent contact radius (R) for roller/inner race and roller/outer race contacts, and also small centrifugal forces in low rotational speeds. The larger film thickness between the roller and outer race also shows that the effect of surface roughness between the roller and the inner race is more significant than that of the roller and the outer race.

Figure 5.8 presents the comparison between the viscous heat generation at the roller/inner race and roller/outer race interfaces for the same operating condition of Figure 5.7. The viscous heat generation is calculated during the simulations based on the following equation:

$$\dot{Q}_{\text{gen}} = F_s \cdot u_s \quad (5.20)$$

where F_s is the traction force, and u_s is the sliding velocity between the contact surfaces. It should be noted that the effect of asperity-to-asperity contact is considered in the amount of heat generation by using Equation 5.18 in the calculation of traction force, F_s . The effect of the surface roughness on the traction force F_s is considered only for EHL regime in the simulations due to the negligible effect of surface roughness in other lubrication regimes.

As Figure 5.8 shows, the rate of heat generation between the roller/inner race is much higher than that between the roller/outer race. This phenomenon is mostly due to the higher sliding

velocities between the roller/inner race compared to the roller/outer race. This higher heat generation at the roller/inner race interface has a significant importance in the thermal analysis of heavily-loaded roller bearings rotating at low speeds. Presented in the literature are several studies [52-54] assuming that the generated heat inside the bearing assembly dissipates equally through the inner race and outer race. Figure 5.8 clearly shows that this assumption is not applicable for the thermal analysis of roller bearings operating at low rotational speeds under large radial loads [55, 56].

Also shown in Figure 5.8 is the variation of the sliding velocity between the roller/inner race as the roller travels in the orbital direction. The amount of heat generation at the most heavily loaded position ($\theta = 0$) is minute due to low sliding velocity, and it increases appreciably as the roller travels towards the middle of the loading zone, where it reaches to a maximum value. Then, it drops with the same rate until it reaches to zero at the end of the contact zone. This variation of heat generation can be explained by considering Equation 5.20 along with the variation of the sliding velocity shown in Figure 5.8. The first increase is due to the fact that the sliding velocity increases as the roller travels from ($\theta = 0$) towards the middle of the loading zone, while the traction coefficient does not change considerably since hard EHL regime governs the lubrication behavior in this region. Although the normal contact force decreases in this range, the rate of change in the traction force (traction coefficient $\times$ normal contact load) is small compared to the increasing rate of sliding velocities which results in the rise of heat generation. However, the traction coefficient drops sharply after the middle of the loading zone since the lubrication regimes changes to (R-V) and (R-I) lubrication regimes in low contact pressures; see Figure 5.5. This sudden drop in the traction coefficient is more profound compared to the increase in the sliding velocities in a way that the multiplication of these two parameters (traction force $\times$ sliding velocity) decreases after the maximum point towards the end of the loading zone.

Figure 5.8. Variation of viscous heat generation and sliding velocity for operating condition of 50 rpm and 150 kN and $R_q = 0.18 \mu\text{m}$ surface roughness

The results presented in this section are for the case of low rotational speed where the contact forces between the rollers and the cage are mostly hydrodynamic and small compared to roller/raceways contact forces. It is also assumed that the churning moments and drag forces are negligible and do not have a significant effect on the sliding velocities of the rollers. However, at higher rotational speeds the amount of heat generation between the rollers and raceways can be much greater due to substantial effects of the lubricant drag forces and direct contact forces between the rollers and the cage. In that case, the roller/cage contact forces and lubricant drag forces can alter the sliding velocities of contact surfaces significantly, which results in much higher viscous heat generation between the rollers and raceways [14, 29].

5.3.3. Effect of surface roughness

Figure 5.9 demonstrates the effect of different surface roughness on the film thickness and viscous heat generation between the rollers and the races. The results shown in Figure 5.9 are for the operating condition of 50 rpm and 150 kN and three different roughness (R_q) values of contact surfaces.

Figure 5.9b demonstrates the comparison between the amount of heat generation between the roller and the races for the smooth and two rough surfaces. A surface roughness of $R_q = 0.01 \mu\text{m}$ is assumed in the simulations for the smooth surfaces. The amount of heat generation increases with increasing surface roughness. This behavior can be attributed to the effect of asperity load ratio (L_a) on the traction coefficient; see Equation 5.18. The asperity load ratio increases significantly as the surface roughness increases, and this will result in a larger traction coefficient in rough contact surfaces, as shown in reference [33]. Subsequently, this larger traction coefficient results in a more appreciable viscous heat generation according to Equation 5.20. Note that the film thickness increases at higher surface roughness in spite of the increase of the heat generation; see Figure 5.9a. This is, in fact, due to the contribution of the load carried by the asperities as well as the influence of asperities on the lubricant flow, as shown in reference [48]. It should be mentioned that the effect of heat generation and temperature rise at the roller/race contact is considered in the derivation of Equation 5.18 where the energy equation is solved coupled with the Reynolds and asperity micro-contact equations [33].

Also, it is interesting to note that as the surface roughness increases from the $0.18 \mu\text{m}$ to $0.36 \mu\text{m}$, heat generation increases about 10%. The increase in heat generation in the case of a surface roughness higher than $0.36 \mu\text{m}$ is expected to be much more since the asperity load ratio significantly increases in higher surface roughness values. Nonetheless, the highest surface roughness chosen for the simulations in the present study is $0.36 \mu\text{m}$ because the surface roughness of most of the commercial roller bearings is generally smaller than $0.36 \mu\text{m}$.

(a) Film thickness variation in hard EHL regime

(b) Heat generation variation in loading zone

Figure 5.9. Effect of different surface roughness (R_q) on (a) film thickness and (b) heat generation for operating condition of 50 rpm and 150 kN

5.3.4. Effect of radial load

Presented in Figure 5.10 is the comparison of film thickness and heat generation for the two operating conditions of 150 kN and 300 kN both rotating at the same speed of 50 rpm. The surface roughness $R_q = 0.36 \mu\text{m}$ in both operating conditions. As shown in Figure 5.10a, the film thickness decreases when the radial load on the bearing increases [48]. As this figure shows the governing lubrication regime is mixed EHL even under large radial load of 300 kN which results in a maximum contact pressure of 1.03 GPa at zero orbital position. It should be mentioned that the results presented in this section are derived by assuming a value of 293 K for the temperature of the lubricant at the inlet zone. It is believed that this assumption has a negligible effect on the

trend of the results presented in this section. However, a comprehensive thermal analysis of the bearing assembly for these operating conditions can provide a better prediction of the lubricant temperature at the inlet zone which leads to more accurate results.

(a) Film thickness variation in hard EHL regime

(b) Heat generation variation in the loading zone

Figure 5.10. Effect of different radial loads on (a) film thickness (b) heat generation for rotational speed of 50 rpm and $R_q = 0.36 \mu\text{m}$ surface roughness

Figure 5.10b shows a significant rise in the heat generation between the rollers and the race ways by increasing the contact load. It is interesting to note that increasing the radial load by 2 times, raises the heat generation by about 10 fold. This can be explained by considering the results presented in reference [33] where it is shown that the traction coefficient increases with increasing the contact loads at low rotational speeds. This larger traction coefficient accompanied by higher sliding velocities result in a significant rise of viscous heat generation under larger radial loads. It should be noted that the effect of heat generation and temperature rise at the roller/race contact is considered in Equation 5.18 which determines the traction coefficient. The significant rise in heat generation under larger radial loads is due to the low rotational speed simulated in the present study. It is shown in reference [33] that at higher slide-to-roll ratios, heat generation at the contact interface reduces the lubricant viscosity which results in lower traction coefficients. Consequently, the traction force and the viscous heat generation between the rollers and races decrease at higher rotational speeds.

5.3.4.1. Effect of radial load on the wear rate

It is well-known that the reduction of the film thickness under large loads and at low speeds can shift the lubrication regime into the mixed-lubrication, in which the contact surfaces are not fully separated by the lubricant and there is a partial contact between the asperities of the contact surfaces [6, 57]. These asperity contacts may result in a considerable wear of the contact surfaces depending on the number of the asperity contacts. Therefore, it is important to study the wear rate of the investigated bearing under radial loads of 150 kN and 300 kN since mixed-lubrication regime ($\frac{h_{min}}{\sigma} < 3$) is governed in a large portion of the loading zone. It is assumed in this study that the surface roughness does not change considerably due to the wear, because the simulations pertain to the steady-state condition, after ending the running-in process. This study employs an approach presented in reference [58] in order to predict the wear in mixed lubricated contacts. The wear rate between the roller and the raceway ($\dot{W}_r$) is calculated based on a modified expression of the well-known Archard wear formula:

$$\dot{W}_r = k_w \varphi \left(\frac{L_a}{100} \right) \left(\frac{Qu_s}{H} \right) \quad (5.21)$$

where Q , H and u_s are, respectively, normal contact load, hardness of the material and sliding velocity between the contact surfaces. k_w is the dry wear coefficient which is assumed to be 1.26×10^{-4} for the general case of steel on steel contact [59]. L_a is the asperity load ratio which is calculated from Equation 5.16. φ is the fractional film defect coefficient which is calculated based on the following formula [58, 60]:

$$\varphi = 1 - \exp \left\{ - \left[\frac{a_x}{u_s t_0} \exp \left(- \frac{E_a}{R_g T_s} \right) \right] \right\} \quad (5.22)$$

where a_x , t_0 and E_a are properties of the lubricant and are equal to 3×10^{-10} m, 3×10^{-12} s and 49×10^3 J.mole⁻¹, respectively [58, 61]. R_g is the gas constant (8.31 J.mole⁻¹.K⁻¹) and

T_s is the absolute temperature of the contact surface that varies in the orbital direction. Employed in the current study is the thermal analysis method of reference [62] which results in absolute surface temperatures of 295 K and 297 K at the orbital position zero for the radial loads of 150 kN and 300 kN, respectively. This relatively small increase in the contact temperature is due to the low sliding velocities shown in Figure 5.8.

Figure 5.11 shows that the increase in the wear rate is proportional with the increase of the radial load. This is, in fact, due to the significant effect of the asperity load ratio (L_a) on the wear rate as defined by Equation 5.21. Comparison of Figures 5.6b and 5.11 also reveals a similar variation of the contact load and the wear rate in the loading zone. The wear rate and contact load are maximum at the orbital position of zero and they gradually decrease towards the edges of the loading zone. These results also show the importance of the roughness and the radial load on the life of the bearings operating at low rotational speeds and large radial loads.

Figure 5.11. Effect of different radial loads on the wear rate for rotational speed of 50 rpm and $R_q = 0.36 \mu m$ surface roughness

5.4. Conclusions

A simplified dynamic model of cylindrical roller bearings is developed to study the effect of surface roughness on the performance of roller bearings operating at low rotational speeds subjected to large radial loads. Employed in this dynamic model is the results of a recent study [33] to calculate the traction coefficient in different lubrication regimes that exist at these operating conditions of roller bearings. The effect of different surface roughness and radial loads is studied on the variation of three critical parameters of the roller bearing performance: lubricant film thickness, wear rate and rate of heat generation between the rollers and the races. The results of dynamic simulations provide useful insight into the variation of these parameters as the rollers

travels in the orbital direction. It is shown that the film thickness and the rate of heat generation between the rollers and the race ways increase at greater surface roughness. The simulation results also reveal that, in the case of very large radial loads, wear rate increases proportionally to the increase in the radial load. Nonetheless, the rate of heat generation between the rollers and the raceways increases significantly and out of proportion with the increase in the radial load. This shows the importance of both surface roughness and contact load together on the wear rate and heat generation between the rollers and the raceways. These factors should be carefully investigated in the thermo-mechanical analysis of roller bearings operating at low speeds and under large loads.

5.5. Nomenclature

a_x	Diameter of area associated with an adsorb molecule, m
c_f	Frequency of this elastic contact, Hz
E	Elastic modulus, Pa
E_a	Heat of adsorption of the lubricant on surface, kJ/mole
E'	Effective modulus of elasticity, $[0.5[(1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2]]^{-1}$
F_s	Traction force, N
f	Traction coefficient
f_c	Asperity friction coefficient
g_E	Dimensionless elasticity parameter, $\left(\frac{w_l^2}{\mu_0 E' R u}\right)^{1/2}$
g_v	Dimensionless viscosity parameter, $\left(\frac{\alpha^2 w_l^3}{\mu_0 R^2 u}\right)^{1/2}$
G	Dimensionless material number, $\alpha E'$
h	Film thickness, m
h_{min}	Minimum film thickness, m
h'	Dimensionless film thickness, $\frac{h_{min} w_l}{\mu_0 R u}$
H	Vickers hardness, Pa
H_c	Dimensionless central film thickness, $\frac{h_c}{R}$
H_{min}	Dimensionless minimum film thickness, $\frac{h_{min}}{R}$
k	Thermal conductivity of lubricant, W/m.K
k_w	Original Archard wear coefficient
K	Dimensionless thermal number, $\frac{E' R}{\sqrt{k \mu_0 T_0}}$
l	Contact length, m
L_a	Asperity load ratio (percentage)
m	Mass, kg
n	Load-deflection exponent, line-contact, 1.11
p_h	hydrodynamic pressure, Pa
Q	Contact load, N
$\dot{Q}_{gen}$	Rate of heat generation, W

R	Equivalent contact radius, $(1/R_1 \pm 1/R_2)^{-1}$, m
R_g	Gas constant. J/mole.K
S	Slide-to-roll ratio, u_s/u
t_0	Fundamental time of vibration of the molecule in the adsorb state, s
T_o	Inlet temperature, K
T_s	Absolute temperature of the surface, K
u	Rolling velocity, m/s
u_s	Sliding velocity, m/s
U	Dimensionless velocity number, $\frac{\mu_0 u}{E' R}$
V	Dimensionless hardness number, $\frac{H}{E'}$
w_l	Load per contact length, N/m
W	Dimensionless load number, $\frac{w_l}{E' R}$
$\dot{W}_r$	Wear rate, $\frac{m^3}{s}$
z_1	$\alpha/5.1 \times 10^{-9}(\ln \mu_0 + 9.67)$
α	Pressure-viscosity coefficient, m^2/N
φ	Fractional film defect coefficient
δ	Load deflection, m
ν	Poisson ratio
μ	Lubricant viscosity, Pa.s
μ_0	Inlet viscosity, Pa.s
σ	Standard deviation of the surface heights, m
$\bar{\sigma}$	Dimensionless surface roughness number, $\frac{\sigma}{R}$
τ	Shear stress, Pa
Λ	Limiting shear stress coefficient

5.6. References:

- [1] Jang JY, Khonsari MM. Elastohydrodynamic line-contact of compressible shear thinning fluids with consideration of the surface roughness. Journal of Tribology-Transactions of the Asme. 2010;132:034501.
- [2] Johnson KL, Greenwood Ja, Poon SY. Simple theory of asperity contact in elastohydrodynamic lubrication. Wear. 1972;19:91-108.
- [3] Dowson D, Higginson GR. Elasto-hydrodynamic lubrication. Pergamon Press, London. 1977.
- [4] Dowson D, Toyoda S. Central film thickness formula for elastohydrodynamic line contacts. Proceedings of the 5th Leeds-Lyon Symposium on Tribology, London. 1978:60-65.
- [5] Hamrock BJ. Fundamentals of fluid film lubrication. McGraw-Hill, New York. 1994.

- [6] Khonsari MM, Booser ER. Applied tribology-bearing design and lubrication. John Wiley & Sons Ltd, Chichester, UK. 2008.
- [7] Moes H. Optimum similarity analysis with applications to elastohydrodynamic lubrication. *Wear*. 1992;159:57-66.
- [8] Bair S, Winer WO. Rheological model for elastohydrodynamic contacts based on primary laboratory data. *Journal of Lubrication Technology-Transactions of the Asme*. 1979;101:258-265.
- [9] Ashtekar A, Sadeghi F. A new approach for including cage flexibility in dynamic bearing models by using combined explicit finite and discrete element methods. *Journal of Tribology-Transactions of the Asme*. 2012;134:041502.
- [10] Ghaisas N, Wassgren CR, Sadeghi FA. Cage instabilities in cylindrical roller bearings. *Journal of Tribology-Transactions of the Asme*. 2004;126:681-689.
- [11] Gupta PK. Frictional instabilities in ball-bearings. *Tribology Transactions*. 1988;31:258-268.
- [12] Saheta V. Dynamics of rolling element bearings using discrete element method. Master's Thesis, Purdue University, West Lafayette, IN. 2001.
- [13] Weinzapfel N, Sadeghi F. A discrete element approach for modeling cage flexibility in ball bearing dynamics simulations. *Journal of Tribology-Transactions of the Asme*. 2009;131:021102.
- [14] Gupta PK. Advanced dynamics of rolling elements. Springer-Verlag, New York. 1984.
- [15] Bair S, Winer WO. The high-pressure high shear-stress rheology of liquid lubricants. *Journal of Tribology-Transactions of the Asme*. 1992;114:1-13.
- [16] Johnson KL, Tevaarwerk JL. Shear behavior of elastohydrodynamic oil films. *Proceedings of the Royal Society of London Series a-Mathematical Physical and Engineering Sciences*. 1977;356:215-236.
- [17] Liu Y, Wang QJ, Bair S, Vergne P. A quantitative solution for the full shear-thinning ehl point contact problem including traction. *Tribology Letters*. 2007;28:171-181.
- [18] Jang JY, Khonsari MM, Bair S. On the elastohydrodynamic analysis of shear-thinning fluids. *P R Soc A*. 2007;463:3271-3290.
- [19] Khonsari MM, Hua DY. Thermal elastohydrodynamic analysis using a generalized non-newtonian formulation with application to bair-winer constitutive equation. *Journal of Tribology-Transactions of the Asme*. 1994;116:37-46.

- [20] Kumar P, Khonsari MM. Traction in ehl line contacts using free-volume pressure-viscosity relationship with thermal and shear-thinning effects. *Journal of Tribology-Transactions of the Asme*. 2009;131:011503.
- [21] Bair S, Khonsari MM. Generalized reynolds equations for line contact with double-newtonian shear-thinning. *Tribology Letters*. 2005;18:513-520.
- [22] Bair S, Khonsari M. An ehd inlet zone analysis incorporating the second newtonian. *Journal of Tribology-Transactions of the Asme*. 1996;118:341-343.
- [23] Bair S, Khonsari M, Winer WO. High-pressure rheology of lubricants and limitations of the reynolds equation. *Tribology International*. 1998;31:573-586.
- [24] Kumar P, Khonsari MM. On the role of lubricant rheology and piezo-viscous properties in line and point contact ehl. *Tribology International*. 2009;42:1522-1530.
- [25] Forster NH, Schrand JB, Gupta PK. Viscoelastic effects in mil-l-7808-type lubricant .2. Experimental-data correlations. *Tribology Transactions*. 1992;35:275-280.
- [26] Gupta PK. Viscoelastic effects in mil-l-7808-type lubricant .3. Model implementation in bearing dynamics computer code. *Tribology Transactions*. 1992;35:724-730.
- [27] Gupta PK, Cheng HS, Forster NH. Viscoelastic effects in mil-l-7808-type lubricant .1. Analytical formulation. *Tribology Transactions*. 1992;35:269-274.
- [28] Gupta PK. Current status of and future innovations in rolling bearing modeling. *Tribology Transactions*. 2011;54:394-403.
- [29] Takabi J, Khonsari MM. On the influence of traction coefficient on the cage angular velocity in roller bearings. *Tribology Transactions*. 2014;57:793-805.
- [30] Kannel JW, Bupara SS. Simplified model of cage motion in angular contact bearings operating in ehd lubrication regime. *Journal of Lubrication Technology-Transactions of the Asme*. 1978;100:395-403.
- [31] Kingsbury E, Walker R. Motions of an unstable retainer in an instrument ball-bearing. *Journal of Tribology-Transactions of the Asme*. 1994;116:202-208.
- [32] Meeks CR, Tran L. Ball bearing dynamic analysis using computer methods .1. Analysis. *Journal of Tribology-Transactions of the Asme*. 1996;118:52-58.
- [33] Masjedi M, Khonsari MM. Theoretical and experimental investigation of traction coefficient in line-contact ehl of rough surfaces. *Tribology International*. 2014;70:179-189.
- [34] Walters CT. Dynamics of ball bearings. *Journal of Lubrication Technology*. 1971;93:1-10.

- [35] Kleckner RJ, Pirvics J, Castelli V. High-speed cylindrical rolling element bearing analysis cybean - analytic formulation. Journal of Lubrication Technology-Transactions of the Asme. 1980;102:380-390.
- [36] Meeks CR, Ng KO. The dynamics of ball separators in ball-bearings .1. Analysis. Asle Transactions. 1985;28:277-287.
- [37] Poplawsk JV. Slip and cage forces in a high-speed roller bearing. Journal of Lubrication Technology-Transactions of the Asme. 1972;94:143-150.
- [38] Poplawsk JV. Computer code cobra. J V Poplawski & Associates: Bethlehem, PA. 1989.
- [39] Sague JE. Computer code prebes. J E Sague Associates: Philadelphia. 1987.
- [40] Gupta PK. Dynamics of rolling-element bearings .1. Cylindrical roller bearing analysis. Journal of Lubrication Technology-Transactions of the Asme. 1979;101:293-304.
- [41] Gupta PK. Dynamics of rolling-element bearings .3. Ball-bearing analysis. Journal of Lubrication Technology-Transactions of the Asme. 1979;101:312-318.
- [42] Gupta PK. Dynamics of rolling-element bearings .4. Ball-bearing results. Journal of Lubrication Technology-Transactions of the Asme. 1979;101:319-326.
- [43] Gupta PK. Dynamics of rolling-element bearings .2. Cylindrical roller bearing results. Journal of Lubrication Technology-Transactions of the Asme. 1979;101:305-311.
- [44] Meeks CR. The dynamics of ball separators in ball-bearings .2. Results of optimization study. Asle Transactions. 1985;28:288-295.
- [45] Winer WO, Cheng HS. Film thickness, contact stress and surface temperatures. Wear Control Handbook. 1980:81-141.
- [46] Johnson KL. Regimes of elastohydrodynamic lubrication. Journal of mechanical engineering science. 1970;12:9-16.
- [47] Khonsari MM, KUMAR P. Lubrication regimes - line contacts. Encyclopedia of Tribology. 2013:2113-2115.
- [48] Masjedi M, Khonsari MM. Film thickness and asperity load formulas for line-contact elastohydrodynamic lubrication with provision for surface roughness. Journal of Tribology-Transactions of the Asme. 2012;134:011503.
- [49] Patir N, Cheng HS. Application of average flow model to lubrication between rough sliding surfaces. Journal of Lubrication Technology-Transactions of the Asme. 1979;101:220-230.
- [50] Akbarzadeh S, Khonsari MM. Performance of spur gears considering surface roughness and shear thinning lubricant. Journal of Tribology-Transactions of the Asme. 2008;130:021503.

- [51] Bair S. Elastohydrodynamic film forming with shear thinning liquids. *Journal of Tribology-Transactions of the Asme*. 1998;120:173-178.
- [52] Burton RA, Staph HE. Thermally activated seizure of angular contact bearings. *Asle Transactions*. 1967;10:408-415.
- [53] Pruvot FC. High speed bearings for machine tool spindles. *Annals of the CIRP*. 1980;29:293-297.
- [54] Stein JL, Tu JF. A state-space model for monitoring thermally-induced preload in antifriction spindle bearings of high-speed machine-tools. *Journal of Dynamic Systems Measurement and Control-Transactions of the Asme*. 1994;116:372-386.
- [55] Takabi J, Khonsari MM. Experimental testing and thermal analysis of ball bearings. *Tribology International*. 2013;60:93-103.
- [56] DAMIAN I, PALEU V. Power loss computation in angular contact ball bearings. *Mechanical Testing and Diagnosis*. 2013;3:18-24.
- [57] Jang JY, Khonsari MM, Maki R. Three-dimensional thermohydrodynamic analysis of a wet clutch with consideration of grooved friction surfaces. *Journal of Tribology-Transactions of the Asme*. 2011;133:011703.
- [58] Beheshti A, Khonsari MM. An engineering approach for the prediction of wear in mixed lubricated contacts. *Wear*. 2013;308:121-131.
- [59] Rabinowicz E. *Friction and wear of materials*. Second ed Wiley, New York; Chichester. 1995.
- [60] Akbarzadeh S, Khonsari MM. On the prediction of running-in behavior in mixed-lubrication line contact. *Journal of Tribology-Transactions of the Asme*. 2010;132:032102.
- [61] Wu SF, Cheng HS. A sliding wear model for partial-ehl contacts. *Journal of Tribology-Transactions of the Asme*. 1991;113:134-141.
- [62] Akbarzadeh S, Khonsari MM. Thermoelastohydrodynamic analysis of spur gears with consideration of surface roughness. *Tribology Letters*. 2008;32:129-141.

Chapter VI: Coupling thermal and dynamic models of rolling element bearings

6.1. Introduction

The useful life of the rolling bearings has been greatly improved in the recent years due to the advancements in the field of tribology, bearing design, bearing material and related manufacturing processes. However, thermally induced failure of the rolling bearings still presents a great danger to the industrial machineries. This type of bearing failure is a premature and detrimental type of bearing malfunction which can occur at high temperatures or high-speed applications such as gas turbine engines or high-speed machine tools. This failure may occur rapidly and long before reaching the designed life of the rolling element bearing.

The useful life of the rolling bearings has greatly improved in the recent years due to the advancements in the field of tribology, bearing design, bearing materials and related manufacturing processes. However, thermally-induced failure in rolling element bearings still presents a great danger to the industrial machineries. This type of bearing failure is a premature and detrimental type of bearing malfunction which can occur at high temperatures or high-speed applications such as gas turbine engines or high-speed machine tools. This type of failure may occur rapidly and long before reaching the designed life of the rolling element bearing.

The general mechanism of thermal failure of rolling bearings can be explained in a straightforward manner. Viscous dissipation in the rolling element bearings raises the temperature of the lubricant and the bearing components during the normal operating conditions. For the bearing to run safely, this frictional heat should be naturally or forcibly removed from the bearing assembly until a balance is reached between the rate of heat generation and heat dissipation rate from the bearing assembly. However, this equilibrium can become unstable if the rate of heat dissipation from the bearing assembly is less than the rate of frictional heat generation. As a result, the temperature can reach a dangerous level where a rapid thermal failure can occurs.

The literature contains reports on a number of studies [1-8] where different types of damages have been investigated as a result of thermally-induced failure of rolling bearings. For example, references [1-7] report that an excessive thermally-induced preload between the rolling elements and the raceways can lead to a rapid thermal failure, while in some other studies [8] the thermal failure of rolling bearings is reported to yield cage failure or cage-instability. This is in fact due to the complex and coupled phenomena involved in the thermal failure of the rolling bearings in different applications. Most of these complexities are due to, but not limited to, the contact forces

and moments between the rolling elements, the raceways and the cage which are the functions of the rheological behavior of the lubricant and the operating temperature of bearing elements. Additional complexities arise due to the uncertainty in the influence of some of the hard-to-measure thermal parameters, such as contact resistances between the raceways, the shaft and the housing and constriction resistances between the rolling elements and the raceways. To the best of present authors' knowledge, none of the available models in the literature for thermal failure of the rolling bearings provides comprehensive and detailed information about some of the critical parameters of the rolling bearing performance including, for example, film thickness and interaction forces and moments between the bearing components.

In this paper, the results of dynamic simulations for two types of thermally-induced failure of rolling bearings are compared to gain insight into the underlying mechanisms for different types of damages resulting from these thermal failures. For each type, a set of dynamic simulations of the rolling bearing is carried out in which the thermal expansion of the bearing elements is calculated from the results of an appropriate thermo-mechanical model of the bearing assembly. In Sec. 2, the thermal failure of a grease-lubricated, deep-groove ball bearing operating at high temperatures and under large radial loads is investigated both analytically and experimentally. Experiments are carried out on a custom-designed test rig where the thermal failure starts with the signs of instability in the cage motion that progresses and leads to an ultimate seizure of the ball bearing. In Sec. 3, the thermal failure of a grease-lubricated cylindrical roller bearing, usually used as the spindle bearing of high-speed machine tools, is investigated where the thermally-induced preload increases sharply and results in a rapid bearing seizure. The results of the current study can provide useful information about the critical parameters and complicated processes involved in the thermal failure of rolling element bearings which can be used for performance prediction, design consideration and use in health-monitoring systems.

6.2. Thermal failure of rolling element bearings at high operating temperatures and large radial loads

The first type of thermal failure of rolling bearings, as mentioned in Sec. 1, manifest itself as a dynamic instability of the cage motion leading to an ultimate bearing seizure due to the cage failure. This type of thermal failure usually occurs at high operating temperatures and excessive contact loads. The following sections provide the details of the experimental apparatus and the simulation approach developed to investigate this type of thermal failure.

6.2.1. Experimental apparatus

The experimental apparatus (SpectraQuest Bearing Prognostics Simulator) shown in Figure 6.1a is capable of imposing the radial loads up to 20 kN on different types of rolling element bearings with 25 mm bore diameter and 52 mm outer diameter. The main shaft is supported on two rows of robust heavy-duty rolling bearings, and is connected to an AC motor by means of a coupling. This AC motor can provide rotational speeds up to 3600 rpm which provides safe testing

capability of rolling bearings with DN numbers (bore diameter of the bearing in millimeter times the rotational speed of shaft in rpm) as high as 90,000. A grease-lubricated, deep groove ball bearing (SKF 6205/C3) is selected as the test bearing in the current investigation, while the support bearings are selected to be two heavy-duty rolling bearings (KOYO 6310/C3, NACHI NU210E). The deep groove ball bearing is packed with commercial LGWA 2 SKF grease while the support bearings are lubricated with SAE 30 in an oil-bath lubrication system.

(a)

(b)

Figure 6.1. (a) Test rig used in the current investigation of thermal failure of rolling bearings (b) Schematic of the frictional torque measurement system

As Figure 6.1a shows the hydraulic system is consisted of an oil reservoir, a ball valve, a manual hydraulic pump, a pressure gauge and a two-way hydraulic cylinder. In order to minimize the

misalignment between the rotating shaft and the hydraulic cylinder, a pulling external force is applied to the test bearing by means of a custom-designed rod-end bearing which fastens the hydraulic cylinder to the aluminum housing of the test bearing. In order to shorten the time at which the thermal failure occurs, the outer surface of the rotating shaft and housings of the support bearings are thermally insulated using a thermal insulation foam. This insulation results in a higher thermal expansion rate of the shaft and consequently larger thermally-induced preload inside the test bearing.

6.2.1.1. Instrumentation

A variable frequency drive (VFD) provides complete control on the rotational speed of the AC motor while it produces the necessary data to monitor the power consumption of the system; see Figure 6.1a. A tachometer installed on the motor frame measures and records any changes in the rotational speed of the shaft during the experiments. The temperature of the housing and outer ring of the test bearing are measured by means of two J-type thermocouples, while an infrared camera is utilized in order to measure the temperature of the rotating shaft at a point close to the test bearing. It should be noted that these temperatures are recorded only for the comparison with the simulation results. Five accelerometers are also utilized in the experiments to monitor the vibrational behavior of the apparatus. Two accelerometers are mounted on the side and top of the housing, perpendicular to the centerline of the test bearing. Three other accelerometers are mounted on the top of the housing of support bearings and AC motor.

Shown in Figure 6.1b is the frictional torque measurement system connected to the outer raceway of the test bearing by means of two steel and aluminum housings. The transducer design of this system can directly measure the frictional torque of the test bearing and minimize the torque associated with the bending forces and the supporting structure. The frictional torque is captured by a sampling rate of 20 kHz and recorded in a computer by means of a torque output bridge and a high-frequency data acquisition board. The frictional torque measurement system is calibrated prior to each test by means of an accurate torque wrench. The basic sensitivity of this system is 125 lb-in/volt at $\Delta V=3.00$ volts, while the maximum measurable torque is equal to 278 lb-in.

6.2.2. Modeling

To investigate thermally-induced failure, an appropriately formulated analytical model is needed wherein the thermal expansion/contraction of the bearing elements are taken into account. For this purpose, first, the evolution of the operating temperature of the bearing elements is predicted. Then, this temperature evolution of the bearing elements is used to calculate the thermal expansion of the bearing components in the dynamic simulations of the rolling bearing during the thermal failure.

6.2.2.1. Thermo-mechanical model of the deep-groove ball bearing

To properly estimate the operating temperature of the bearing elements during the thermal failure, a block diagram of the thermo-mechanical processes involved in the thermal analysis of the investigated test bearing is developed; see Figure 6.2. This model consists of four interconnected stages: heat generation, heat dissipation, dimensional changes and thermally-induced preload. These are discussed as follows.

First, frictional heat generation due to the lubricant viscosity and internal contact forces and moments inside the bearing assembly are determined. In order to calculate the total frictional torque (M_t) of the ball bearing, the empirical formulas of Palmgren [8, 9] is used:

$$M_t = M_l + M_v \quad M_l = f_l F_r d_p \quad M_v = 0.8 f_v (v_o n)^{2/3} d_p^3 \quad (6.1)$$

where M_l and M_v are the frictional torque due to the load and the lubricant, respectively. n is the rotational speed in rpm and v_o is the viscosity in centistokes. f_l and f_v are the coefficients dependent on the type of the bearing and lubrication system. d_p is the pitch diameter of the bearing and F_r is the radial load imposed on the bearing. The dimensions of the ball bearing used in this study as the test bearing are given in Table 6.1.

Figure 6.2. Four basic stages of the thermo-mechanical model [10, 11]

In the second stage, the distribution of the frictional heat throughout the bearing assembly is modeled using appropriate empirical formulas and heat transfer equations in order to predict the operating temperature of the bearing components. The temperature distribution throughout the bearing assembly is calculated by considering a network of thermal resistances which includes adequate number of thermal nodes. Next, the energy balance equation is applied for each node along with the consideration of the appropriate empirical formulas to estimate the heat transfer parameters such as the heat convection coefficient and thermal constriction resistances [10].

Then, the final set of energy balance equations are solved numerically. References [10, 11] provide more details about the heat transfer equations and appropriate empirical formulas. It should be noted that this thermal resistance network is unique for each bearing assembly and should appropriately take into consideration the effect of different geometries of the housing and different boundary conditions.

In the third stage, the thermal expansion of the bearing elements due to the different operating temperatures is calculated by means of elasticity formulas and simple equations for linear thermal expansion. The radial thermal expansion of the ball (ϵ_b), inner race (ϵ_{in}) and outer race (ϵ_{out}) is calculated based on the following equations [10, 11]:

$$\begin{aligned}\epsilon_b &= \alpha_b \cdot r_b \cdot T_b & \epsilon_{in} &= \alpha_{in} \cdot r_{in} \cdot T_{in} \\ \epsilon_{out} &= \frac{\alpha_o}{3} \cdot \frac{(1 + \nu) \cdot r_{out}}{r_{out} + r_h} \cdot [T_{out} \cdot (2r_{out} + r_h) + T_h \cdot (2r_h + r_{out})]\end{aligned}\tag{6.2}$$

where subscripts b, in, out and h respectively, refers to ball, inner race, outer race and the housing. T, α and r are the temperature, thermal expansion coefficient and radius respectively.

The induced thermal preload (F_{th}) between the bearing elements, due to the uneven thermal expansion of the bearing elements, is determined in the fourth stage using the Hertzian theory:

$$F_{th} = k_t \cdot \epsilon_t^{z_1}\tag{6.3}$$

where k_t is the bearing stiffness. z_1 is equal to 1.5 for ball bearings and 1.11 for roller bearings [8]. ϵ_t is the total radial interference change calculated from the following equation:

$$\epsilon_t = \epsilon_b + \frac{(\epsilon_{in} - \epsilon_{out})}{2}\tag{6.4}$$

6.2.2.2. Dynamic model of the deep-groove ball bearing

A dynamic model of the deep-groove ball bearing is developed in the present study to simulate all of the dynamic interactions between the bearing elements. In order to reduce the computational time, it is assumed that the cage is allowed to rotate only about its primary axis, which is parallel to the centerline of the ball bearing, while the balls are allowed to rotate about any arbitrary axis. Equilibrium constraint equation in the radial direction is also employed to remove the effects of free vibration of the balls between the raceways. This equilibrium constraint equation reduces the three equations of translational motion of the ball into one equation of motion in the orbital direction which result in considerably shorter computational time. The basic concept and details of this dynamic model and constraint equation are given in references [12, 13].

In order to calculate the normal contact force (Q) from the normal deflection (δ) between the ball and the raceway, the following equation is utilized in the present study which is derived from the Hertzian theory for steel bodies [8]:

$$Q = \left(\frac{\delta}{2.79 \times 10^{-4} \delta^* \Sigma \rho^{1/3}} \right)^{3/2} \quad (6.5)$$

where δ^* and $\Sigma \rho$ are the dimensionless contact deformation and the curvature sum of point contact.

The radius of the contact curvature ($\bar{R}$) in the calculation of the traction forces should also be considered due to the large external loads on the ball bearing. Figure 6.3 shows the contact between the ball and the outer raceway. The position of each point (p) within the contact area with respect to the ball center (B), shown by (r_{pb}) in Figure 6.3, is calculated using the following equation [12]:

$$r_{pb} = \left\{ \begin{array}{c} ax \\ by \\ \sqrt{\bar{R}^2 - (ax)^2} - \sqrt{\bar{R}^2 - a^2} + \sqrt{\frac{d^2}{4} - a^2} \end{array} \right\} \quad \bar{R} = \frac{2fd}{2f+1} \quad (6.6)$$

where a and b are the semi-width of the contact area associated with x and y axes of the contact-area coordinate system, respectively. d and f are the ball diameter and the race curvature factor, respectively. It is also assumed that the variation of the traction forces along the y -axis of the contact ellipse is negligible. This assumption is widely used in the literature for ball bearings and has a negligible effect on the final traction forces and moments due to the large ellipticity parameter ($k = \frac{a}{b}$) of the contact ellipse [12-14].

Figure 6.3. schematic of curvature of contact area pointed by $\bar{R}$

6.2.3. Simulation approach

In order to simulate the dynamic motion of bearing elements during thermal failure, the change in the thermal expansion of the bearing elements should be appropriately implemented in the dynamic simulations. For this purpose, the operating temperature of the bearing components is calculated from the thermo-mechanical model explained in Sec. 2.2.1. Then, a special approach is utilized to consider the thermal expansion of the bearing components, calculated from Equation 6.2, in the dynamic simulations.

6.2.3.1. Evolution of temperature of bearing elements

Figure 6.4 compares the experimental results with the analytical results of the thermo-mechanical analysis of the test bearing for two operating conditions. The radial loads imposed are 5 kN and 10 kN while the rotational speed is 3600 rpm. These radial loads are selected to demonstrate two different operating conditions where stable and unstable dynamic motion of the cage is observed during the experiments. Presented in Sec. 2.4.1 are the results of the vibration monitoring of the ball bearing during the experiments. According to these results, the thermal instability of the ball bearing manifest itself as an unstable cage-motion when the bearing is under 10 kN radial load. On the other hand, no sign of bearing failure or unstable cage-motion is observed for an extended time of the experiment in the case of 5 kN radial load.

Figure 6.4. Temperature change of bearing elements of investigated ball bearing rotating at 3600 rpm and under 50 kN and 100 kN

As shown in Figure 6.4, the experimental measurements and the analytical predictions are in good agreement for these two operating conditions. It should be noted that the comparison between the simulation and the experimental results are shown only for the outer ring/housing temperature since the instrumentation of the test rig provides only the temperature of the outer ring/housing and the shaft. The temperatures of the inner race and the ball, shown in Figure 6.4, are the output of the developed thermo-mechanical model and will be used in Sec. 2.3.2 to calculate the dimensional change of the bearing elements in the dynamic simulations.

As Figure 6.4 shows, there is no sensible fluctuation in the temperature of the components of the ball bearing rotating at 3600 rpm and 10 kN radial load. This is due to the fact that the transient instability in the cage motion occurs in less than a second which has a negligible effect on the frictional heat generation inside the bearing. According to Figure 6.4, the evolution of the temperature of the bearing elements in the case of unstable cage-motion (10 kN) is very similar to the stable operation of the bearing rotating under 5kN radial load. The only difference is the high operating temperatures of the bearing elements in the case of 10 kN radial load compared to 5 kN.

6.2.3.2. Implementing the temperature rise in the dynamic simulations

As shown in Sec. 2.4.1, the first sign of the cage instability of the investigated ball bearing rotating at 1400 rpm and 10 kN occurs approximately at 1700 s which means that the shaft rotates about 39,667 times before the first cage instability occurs. Therefore, to simulate the dynamic behavior of the bearing elements, simulation of the dynamic forces and moments between the bearing elements are required in all of the 39,667 revolutions of the shaft. The dynamic simulations are carried out on a computer equipped with 2.53 GHz processor and 16 GB of memory. It requires one hour of computational time to simulate the dynamic behavior of the bearing components for each 20 rotations of the shaft. Considering the extensive computational time required for the dynamic simulations, it is obvious that a full simulation for all of the revolutions of the shaft is not practical. Therefore, in the present study the 2000 seconds duration is divided into 4 equal sections and the dynamic simulations are run at specific moments in time: at 1 s, 500 s, 1000 s, 1500 s and 2000 s. The thermal expansion of the bearing elements for each dynamic simulation is calculated from the temperature change of the bearing elements shown in Figure 6.4.

It is well-known that the traction coefficient is the most influential parameter of any dynamic model of rolling bearings and if not realistically characterized, it can produce considerable errors in the final results. Presented in the literature [12, 15-21] are different proposed Newtonian and Non-Newtonian rheological models to calculate the traction coefficient in elastohydrodynamic (EHL) contacts between the rollers and the raceways. Consideration must be given to the operating temperature, pressure, bearing geometry, sliding velocities and rolling velocities of the bearing elements. To reduce the computational time, several studies [14, 22, 23] have employed

simplified traction curves in which the traction coefficient is a function only of the sliding velocity of the contact surfaces. Takabi and Khonsari [13] investigated the influence of employing these simplified traction curves compared to visco-elastic rheological models, and concluded that these simplified traction curves can be used in the dynamic modeling of rolling bearings in case of low sliding velocities or low slide-to-roll ratios.

Figure 6.5. (a) simplified traction curves used in dynamic simulations (b) Schematic view showing radial displacement of inner raceway under radial loads

Figure 6.5b demonstrates a schematic of the eccentric position of the inner and outer raceways of the investigated ball bearing under the radial loads. Since the sliding velocities are high in the loading zone, the application of a simplified traction curve is not appropriate in the present study. Moreover, the variation of the traction coefficient with the high temperatures, experienced in this study, should be somehow considered in the dynamic simulations. Unfortunately, rheological data and traction behavior for the particular grease used in the experiments are unavailable. Therefore, a series of simplified traction curves, shown in Figure 6.5a, are utilized to estimate the traction coefficient in each dynamic simulation carried out at specific moments of 1 s, 500 s, 1000 s, 1500 s and 2000 s. It should be mentioned that the variation of traction coefficient in these set of simplified traction curves is estimated based on the experimentally measured traction curves reported in references [24, 25] for a different type of grease.

6.2.4. Results and discussion

Presented in the following sections are the results of the experiments and simulations carried out in the present study. Table 6.1 provides the dimensions of the ball bearing used in the present study as the test bearing. It should be noted that there was no indication of cage instability in the recorded temperatures shown in Figure 6.4, while the accelerometers captured an unstable motion of the cage when the ball bearing was rotating at 3600 rpm and 10 kN radial load.

Table 6.1. Dimensions of the investigated ball bearing

Bore diameter	0.025 m
Outer diameter	0.052 m
Number of the balls (N)	9
Ball diameter (d)	0.0079380 m
Pitch diameter (d_p)	0.0385 m
Inner race curvature factor ($f \cdot d$)	0.5240 m
Outer race curvature factor ($f \cdot d$)	0.5240 m

6.2.4.1. Vibration monitoring of the test bearing during the thermal failure

By monitoring vibration of the test bearing, it is possible to obtain the necessary information to capture the transient phenomenon of unstable motion of the cage during the experiments. For this purpose two accelerometers are mounted on the top and side of the housing of test bearing and record the vibrations of the test bearing during the experiments with a sampling rate of 100 kHz.

Figure 6.6. Time-frequency waterfall of vibration signal

Figure 6.6 presents a waterfall plot of the spectrum analysis of the recorded vibrations in the time-frequency domain during the experiment. As this waterfall plot clearly indicates, the first sign of the cage instability is observed about 1700 s after beginning of the test. This unstable motion of the cage manifests itself as a transient but a large magnitude acceleration occurring at

the frequency of 900 Hz. It should be mentioned that the operation is halted after 2000 s in order to prevent severe damage to the equipment in the case of failure of the investigated ball bearing. This transient vibration was observed at the same operating temperature of the bearing components in three sets of experiments carried out with 10 KN radial load. In one of the experiments the machine continued to work longer than 2000 seconds in which the cage instability progressed in time and the bearing ultimately seized in about 2 hours.

The frequency of this high acceleration is different from the signature frequencies seen in the classical vibrational analysis of the rolling bearings. These signature frequencies of the bearing failure are independent of the time and are equal to the specific frequencies resulting from the surface damage of the bearing elements. These frequencies are calculated based on the following formulas [8]:

$$f_{bpfi} = \frac{N}{2} (f_o - f_i) \left(1 + \frac{d}{d_p} \cos(\theta)\right) \quad (6.7)$$

$$f_{bpfo} = \frac{N}{2} (f_o - f_i) \left(1 - \frac{d}{d_p} \cos(\theta)\right) \quad (6.8)$$

$$f_{bsf} = \frac{d_p}{d} \frac{(f_o - f_i)}{2} \left(1 - \left(\frac{d}{d_p} \cos(\theta)\right)^2\right) \quad (6.9)$$

$$f_c = \frac{f_i \left(1 - \frac{d}{d_p} \cos(\theta)\right)}{2} + \frac{f_o \left(1 + \frac{d}{d_p} \cos(\theta)\right)}{2} \quad (6.10)$$

where f_{bpfi} and f_{bpfo} are the ball pass frequency on the inner race and outer race, respectively. f_{bsf} is the ball spin frequency and f_c is the cage frequency. f_i and f_o are also rotational speed of the inner race and the outer race, respectively. N , d_p and θ are, respectively, the number of the balls, the pitch diameter of the bearing and the contact angle. These frequencies are equal to the following values for the investigated ball bearing rotating at 3600 rpm (60 Hz):

$$f_{bpfi} = 760.2 \text{ Hz} , f_{bpfo} = 439.3 \text{ Hz} , f_{bsf} = 125.1 \text{ Hz} , f_c = 23.8 \text{ Hz} \quad (6.11)$$

Figure 6.6 shows that neither of these signature frequencies nor their harmonics are observed in the time-frequency domain and the observed transient frequency of 900 Hz, shown in Figure 6.6, is also not close to any of these frequencies. This confirms that this transient frequency is associated with the unstable motion of the cage.

This high acceleration is accompanied with an audible noise and is also captured with the torque measurement system installed on the test bearing. Figure 6.7 shows a spike occurring at about 1740 s in the steady-state value of the frictional torque of the test bearing. This spike has the

same transient nature akin to the vibration signal and is associated with the unstable motion of the cage.

Figure 6.7. Frictional torque of investigated ball bearing

6.2.4.2. Results of dynamic simulations

A set of dynamic simulations is carried out using the traction curves shown in Figure 6.5a to investigate the dynamic behavior of the bearing elements during the experiments. The thermal expansion of each component is calculated based on the operating temperatures shown in Figure 6.4. The results of these dynamic simulations for the operating condition of 3600 rpm and 10 kN radial load are presented in Figures 6.8-6.9. It is also assumed in the present study that the surface roughness of the balls and the raceways is equal to $0.05 \mu\text{m}$ which results in a combined surface roughness ($\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$) value of $0.07 \mu\text{m}$.

6.2.4.2.1. Time-variation of minimum film thickness

Figure 6.8 demonstrates the variation of minimum film thickness between the ball, the inner and the outer raceways for the most heavily loaded position in the loading zone shown in Figure 6.5b. This minimum film thickness ($h_{\min}$) is calculated based on the widely-used formula presented by Hamrock and Dowson [26] for EHL point-contacts:

$$\frac{h_{\min}}{R} = 3.63 U^{0.68} G^{0.49} W^{-0.073} (1 - e^{-0.68k}) \quad (6.12)$$

$$W = \frac{Q}{E'R^2}, \quad U = \frac{\mu_0 u}{E'R}, \quad G = \zeta E', \quad k = \frac{a}{b} \quad (6.13)$$

where the dimensionless parameters of load, speed and material are, respectively, presented by W , U and G , and are defined by Equation 6.13. R and E' are equivalent contact radius and effective modulus of elasticity, respectively. k is the ellipticity parameter of the contact area with the semi-major width of a and the semi-minor width of b . Q is the contact load and u is the

average velocity or rolling speed of contact surfaces. ζ is the pressure-viscosity coefficient of the lubricant and μ_0 is the viscosity of the lubricant at the inlet zone of contact surfaces.

The viscosity of the lubricant at the inlet zone (μ_0) should be updated for each simulation since the operating temperature of the lubricant significantly changes during the experiment. In order to calculate the viscosity appropriately for different oil temperatures, the current investigation uses the thermal properties of the base oil of the grease (LGWA 2 SKF) used in the experiments. Temperature of the lubricant at the inlet zone of the balls/raceways contacts is assumed to be the averaged-temperature of the contact surfaces which can be calculated from Figure 6.4. Then, the viscosity of the lubricant at the inlet zone at any operating temperature (T) is calculated by the Vogel [27] formula:

$$\mu_0 = \mu_a e^{-\beta \cdot (T - T_a)} \quad (6.14)$$

where β and μ_a are, respectively, the temperature-viscosity coefficient and the viscosity of the lubricant at reference temperature of $T_a = 40$ C.

Figure 6.8. Time-variation of minimum film thickness for operating condition of 3600 rpm and 10 kN radial load on the investigated ball bearing

As Figure 6.8 shows, the minimum film thickness drops rapidly after the beginning of the test. This drastic drop is due to the combined effect of the large induced thermal preload and sudden increase of the oil temperature at the inlet zone of the contact surfaces. After 200 s, the induced thermal preload drops quickly and the rate of increase in oil temperature drops concomitantly; see Figure 6.4. Therefore, the minimum film thickness decreases gradually after 200 s. This gradual decrease in the minimum film thickness continues up to 2000 s when the operation is halted. This trend is the same for the film thickness between the ball and both of inner and outer

raceways, where the film thickness between the ball and the outer race is larger than the inner race.

The shaded area in Figure 6.8 presents the area of the boundary lubrication regime for the investigated ball bearing. Presented in the literature are several studies [28] where the lubrication regime is considered to be in the boundary lubrication regime if the minimum film thickness is lower than the combined surface roughness of the contact surfaces ($\frac{h_{min}}{\sigma} < 1$). Referring to Figure 6.8, the minimum film thickness remains above the boundary lubrication regime (shaded area) during the experiments with 10 kN radial load. The minimum film thickness is larger than the boundary lubrication limit especially at the time of 1720 s when the experimental results show an unstable motion of the cage. Therefore, it is concluded that the contact surfaces between the ball and the raceways are not severely damaged during this type of thermal failure.

According to Figure 6.8, the roller/outer race film thickness is greater than the roller/inner race film thickness. This can be attributed to two factors. First, the equivalent contact radius (R) of the roller/outer race is larger than the roller/inner race and this results in a thicker film there; see Equations 6.12-6.13. Second, the operating temperature of the lubricant at the roller/outer race is lower than the roller/inner race contact because it is assumed to be the mean of temperatures of the contact surfaces at the inlet zone, see Figure 6.4.

6.2.4.2.2. Time-variation of cage contact forces and wear rate of the cage

Figure 6.9 presents the time-variation of the interactions between the balls and the cage as the operating temperature of the bearing elements rise. As the temperature increases, the traction coefficient decreases and the thermal expansion of the balls and the shaft increases. The higher sliding velocities due to the larger thermal expansions along with the lower traction coefficients results in larger frictional moments between the balls and the raceways, which lead to larger contact forces between the balls and the cage. Examination of Figure 6.9 reveals that the magnitude of the normal contact forces between the balls and the cage may reach to high values above 100 N, in which they can cause a permanent deformation of the cage depending on the material properties and strength of the cage. This permanent deformation of the cage can cause excessive vibrations and heat generation leading to a catastrophic seizure of the bearing in a short time.

The rapid increase in the cage wear rate, shown in Figure 6.9, can be explained by considering the Archard's wear equation used in the simulation to calculate the time-averaged wear volume rate ($\dot{W}_r$) of the cage:

$$\dot{W}_r = \frac{1}{t} \int_0^t \frac{k_w Q u_s}{H_v} dt \quad (6.15)$$

where k_w and H_v are the wear coefficient and hardness of the cage material. These parameters are assumed to be $k_w = 1 \times 10^{-5}$ and $H_v = 1 \times 10^9$, respectively. Q , t and u_s are, respectively, the contact load, time and sliding velocity. Figure 6.9 shows that the balls/cage contact forces increase almost linearly and in proportion with the decrease of the traction coefficient, while the time-averaged wear rate increases exponentially. This can be attributed to the increase of the number of the ball/cage contact forces and the duration of these contact forces as the traction coefficient decreases. Therefore, the combined effect of the larger contact loads and the longer time of the contacts between the balls and the cage lead to a rapid increase of the cage wear rate based on Equation 6.15.

Figure 6.9. Time-variation of balls/cage contact forces and time-averaged wear-rate of cage for operating condition of 3600 rpm and 10 kN radial load on the investigated ball bearing

These results are in agreement with the experimental results where the ultimate thermal failure of the investigated ball bearing under 10 kN occurred after about 2 hours. After this thermal failure, the failed bearing disassembled. Careful examination of the bearing elements revealed a permanent oval deformation of the cage as the main reason of the bearing seizure. Considerable amount of wear was also observed on the inner surface of the cage pockets where the balls are in contact with the cage.

6.3. Thermal failure of spindle bearings of high-speed machine tools

The second type of thermal failure of rolling bearings is likely to occur in the spindle bearings of high-speed machine tools (CNC) where the high temperature gradient throughout the bearing assembly can induce large thermal preloads between the rollers and the raceways. This type of thermal seizure of the rolling bearings has been investigated in the literature both experimentally and analytically [1-7, 11, 29-36]. The results of these studies introduce the thermal expansion of the inner ring and the rolling elements as de-stabilizing factors, while the thermal expansion of

the outer ring and the housing are stabilizing factors. Unlike the first type of thermal failure of rolling bearings, this type of thermal failure occurs suddenly in a short time without any indication of instability in the cage motion beforehand. The following sections provide the details of the experiments, carried out by Tu and Stein [11, 33-36], and the simulation approach developed in the present study to investigate this type of thermal failure of rolling bearings.

6.3.1. Experimental apparatus

Shown in Figure 6.10 is the schematic of the test rig utilized by Tu and Stein [6, 11, 33] to investigate the thermal failure of a grease-lubricated roller bearing in a high-speed lathe spindle. This high-precision spindle is supported on one end by a set of angular-contact ball bearings (SKF 234422B), and a double-row cylindrical roller bearing (NN SKF 3022 K/W33) on the other end. Owing to the special design of this spindle, it is assumed that the angular-contact ball bearings take all of the axial loads on the spindle, while the cylindrical roller bearing takes all of the radial loads on the spindle. A system of V-belt and pulley is utilized to connect the spindle to an AC-motor which is able to rotate the spindle at rotational speeds up to 4000 rpm. This cylindrical roller bearing is packed with synthetic grease (Mobilith SHC 15).

Figure 6.10. Schematic of the spindle bearings assembly of the investigated high-speed machine tools [6, 11]

As shown in Figure 6.10, Tu and Stein [6, 11, 33] installed two Promess sensors between the housing and the outer ring of both rolling bearings. These Promess sensors along with a Wheatstone bridge and a data acquisition board provide an indirect measurement of the induced thermal preload inside the rolling bearings. They also measured the temperature of the housing, outer ring and inner wall of the spindle by means of the thermocouples shown in Figure 6.10. It should be noted that the temperature of the spindle and the housing are not required for the proposed state-space model, presented by Tu and Stein [6, 11, 33] and discussed in Sec. 3.2.1,

and are recorded in the experiments just for the comparison with the analytical results. The rotational speed and power consumption of the AC-motor are also monitored and sent to a computer by means of the acquisition board.

6.3.2. Modeling

A modeling technique similar to the modeling approach described in Sec. 2 is employed in this section in order to investigate thermally-induced failure of spindle bearings of high speed machine tools. However, the state-space model presented by Tu and Stein [6, 11, 33] is utilized for the thermo-mechanical model due to the lack of the necessary data about the dimensions and geometry of the apparatus and also imposed boundary conditions during the experiments.

6.3.2.1. State-space model of the spindle bearings assembly

Tu and Stein [6, 11, 33] developed a dynamic mathematical model and a state observer technique that can be used to estimate the thermally-induced preload inside the bearing using easy-to-measure parameters such as the outer ring temperature and the spindle speed. State observation is an estimation technique in control theory to reconstruct hard-to-measure state variables of a dynamic system, i.e. the thermally-induced preload in the present study, using the input and output measured feedbacks. The state-space model proposed by Tu and Stein [6, 11, 33] is based on a thermo-mechanical model of the spindle bearings assembly shown in Figure 6.11. This thermo-mechanical model is similar to the thermo-mechanical model shown in Figure 6.2, while it utilizes the resultant induced preload (F_{th}) in a feedback loop to adjust the frictional heat generation at the first step. More details of this thermo-mechanical model are given in references [11]. Appendix B describes the final state-space model developed by Tu and Stein [6, 11, 33] for the investigated cylindrical roller bearing utilized in the experimental apparatus discussed in Sec. 3.1.

Figure 6.11. Four basic stages of the thermo-mechanical model [6, 11]

6.3.2.2. Dynamic model of the spindle bearing

Developed for purpose of the present study is a simplified dynamic model of the investigated cylindrical roller bearing wherein several assumptions are made in order to reduce the computational time. It is assumed that the rollers and the cage are allowed to rotate only about their primary axis which is parallel to the centerline of the roller bearing. Equilibrium constraint equation in the radial direction is also employed to remove the effect of free vibration of the rollers between the raceways on the final results by reducing the three equations of translational motion of the rollers into one equation of motion in the orbital direction. The effect of the radius of contact curvature is also neglected in the calculation of traction forces between the rollers and the raceways. This assumption is widely used in the literature for line contacts and is believed to have a negligible effect on the final results of the current investigation. The basic concept and the details of this dynamic model are given in references [12, 13]. It should also be mentioned that a single-piece cage is considered in the simulations where a 5×10^{-4} m diametral clearance is assumed between the rollers and the cage pockets.

As mentioned in Sec. 2.3.2, the simplified traction curves can be used in the dynamic modeling of rolling bearings in the case of low slide-to-roll ratios [13]. Figure 6.12a Shows the simplified traction curve used in the dynamic simulations of the present study due to the lack of experimental results for the type of grease used in the experiments carried out in references [11, 33-35]. This simplified traction curve is estimated based on the experimentally measured traction coefficient for a different type of grease reported in references [24, 25]. It is believed that employing this traction curve results in negligible error on the dynamic behavior of the bearing elements due to the low slide-to-roll ratios between the rollers and the raceways. This low slide-to-roll ratio is in fact due to the concentric position of the inner raceway and the outer raceway in the dynamic model which results in low sliding velocities between the rollers and the raceways; see Figure 6.12b. This concentric inner and outer raceway is also due to the interference fitting between the investigated bearing, the shaft and the housing which results in an even distribution of the contact loads on the rollers at any orbital position.

Figure 6.12. (a) Simplified traction curve used in dynamic model. (b) Schematic view of concentric inner and outer raceways

6.3.3. Simulation Approach

Here a simulation approach similar to the one explained in Sec. 2.3 is utilized to consider the thermal expansion of the bearing components in the dynamic simulations of the investigated roller bearing rotating at 1400 rpm during the thermal failure.

6.3.3.1. Evolution of the bearing elements temperature

The mathematical model presented in Equation B.1-B.6 are employed in the current investigation to estimate the operating temperature of the bearing elements and the thermally-induced preload in different operating conditions of the investigated cylindrical roller bearing described in Sec. 3.1.

Figure 6.13 shows the temperature of the bearing elements calculated in the current study from the mathematical model presented in Equation B.1-B.6 for spindle speeds of 900 and 1400 rpm. These simulated temperatures are close to the experimental results measured by Tu and Stein [6, 11, 33] in the experiments. Figure 6.13 shows a rapid and sudden increase in the temperature of the roller as the thermal instability begins after about 500 s for the rotational speed of 1400 rpm. This evolution of the temperature is completely different in the case of 900 rpm where the bearing is operating in a thermally stable condition. The results shown in Figure 6.13 are used in dynamic simulation of the investigated roller bearing, discussed in Sec. 3.4.2, in order to calculate the dimensional change of the bearing elements during the thermal failure.

Figure 6.13. Simulated temperature rise of bearing elements at rotational speeds of 900 and 1400 rpm

Comparison of the evolution of temperature of the bearing elements shown in Figures 6.4 and 6.13 reveals that different mechanisms are involved in the first and second type of thermal failure

of the rolling bearings. Although the steady state temperature of the bearing elements shown in Figure 6.13 is an indicator of stable operation of the rolling bearing, this is not the case in Figure 6.4 where the thermal instability can be observed in a rolling bearing operating at high steady-state temperatures and under large radial loads. The operating temperature of the bearing elements in the second type of thermal failure of rolling bearings is also much lower than the first type examined in this study.

6.3.3.2. Implementing the temperature rise in the dynamic simulations

As shown in Figures 6.13 and 6.14, the thermal failure of the roller bearing rotating at 1400 rpm occurs approximately in 900 s that means the shaft spins about 21,000 times before the thermal failure occurs. In order to simulate the dynamic behavior of the bearing elements, the dynamic model should calculate all of the interaction forces and moments between the bearing elements in all of the 21,000 revolutions of the shaft. Undoubtedly, running the dynamic simulation for all of the revolutions of the shaft during this type of thermal failure is not practical. Therefore, in the present study the whole 900 seconds is divided into 9 sections and the simulations are run at specific moments in time: at 100 s, 200 s, ... 900 s. The thermal expansion of the bearing elements in each of these dynamic simulations is calculated from the temperature rise of the bearing elements shown in Figure 6.13.

6.3.4. Results and discussion

The following sections present the experimental results carried out by Tu and Stein [6, 11, 33] and the simulation results carried out in the present study, respectively. Table 6.2 shows the dimensions of the cylindrical roller bearing used in the dynamic simulations.

Table 6.2. Dimensions of the investigated ball bearing

Bore diameter	0.112 m
Outer diameter	0.170 m
Number of the rollers (N)	26 (each row)
Roller diameter (d)	0.014 m
Pitch diameter (d_p)	0.1410 m

6.3.4.1. Evolution of thermally-induced preload during thermally-induced failure

Figure 6.14 presents the comparison between the simulated bearing preload and the experimental results measured by Tu and Stein [6, 11, 33] for the spindle speeds of 350, 900 and 1400 rpm. Examination of the prediction results in Figure 6.14 shows that the proposed mathematical model, presented by Equations B.1-B.6, is able to estimate the evolution of bearing preload with an acceptable accuracy in low rotational speed as well as high rotational speed where the thermal instability of the investigated bearing occurs. At 1400 rpm, this thermal instability manifests itself in the form of a sudden increase in the magnitude of the bearing preload after about 500 s. In order to prevent any damage to the AC-motor and the apparatus [6, 11], the apparatus is halted when the preload reaches to 7000 N.

Figure 6.14. Simulated preload vs measured preload for different operating conditions [6, 11]

6.3.4.2. Results of dynamic simulations

A set of dynamic simulations is carried out using the traction curves shown in Figure 6.12a to investigate the dynamic behavior of the bearing elements during the experiments. The thermal expansion of the bearing components is calculated based on the operating temperatures shown in Figure 6.13. The results of these dynamic simulations for the case of thermal failure (1400 rpm) are presented in Figures 6.15-6.16. It is also assumed that the surface roughness of the rollers and the raceways are equal to $0.05 \mu\text{m}$ which results in a combined surface roughness ($\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$) value of $0.07 \mu\text{m}$.

6.3.4.2.1. Time-variation of minimum film thickness

Figure 6.15 shows the variation of minimum film thickness of the lubricant between the rollers and the raceways during the thermal failure (1400 rpm). This minimum film thickness ($h_{\min}$) is calculated based on the widely-used formula presented by Dowson and Higginson [37] for EHL line-contacts:

$$\frac{h_{\min}}{R} = 2.65 \frac{G^{0.54} U^{0.7}}{W^{0.13}} \quad (6.16)$$

$$W = \frac{W_1}{E'R} \quad (6.17)$$

where W_1 is the load per unit of contact length. The definition of the rest of the parameters is similar to the Equations 6.12-6.13. An approach similar to the simulation approach explained in Sec. 2.4.2.1 is also utilized to account for the influence of the temperature rise on the inlet viscosity based on the Equation 6.14.

Figure 6.15. Time-variation of minimum film thickness for the case of thermal failure of the investigated roller bearing (1400 rpm)

The variation of minimum film thickness between the roller and the inner-and-outer raceways, shown in Figure 6.15, could be explained by considering the results shown in Figures 6.13 and 6.14. At the beginning of the experiment, the contact surfaces are completely separated from each other and there is no metal-to-metal contact between the roller and the raceways. However, the film thickness drops suddenly and the lubrication regime shifts to the mixed lubrication regime just before 100 s of the bearing operation. This sudden drop in the film thickness can be attributed to the sudden increase of the contact load and the operating temperature of the bearing elements shown in Figures 6.13 and 6.14, respectively. The sharp increase of the contact load is due to the high temperature gradient across the bearing assembly at the beginning of the experiment. The sudden temperature rise of the lubricant also decreases the lubricant viscosity drastically which results in the sudden drop of the film thickness before 100 s.

The minimum film thickness increases slightly between 100 s and 400 s. The slight increase of the film thickness between 100 s and 400 s can be attributed to the coupled effect of the contact loads and the temperature of bearing elements shown in Figures 6.13 and 6.14. In this period, although the contact load reduces slightly, the operating temperature of the lubricant remains almost constant; see Figure 6.13. Therefore, considering Equation 6.16, the minimum film thickness increases minuscule in this time period.

As shown in Figure 6.15, the minimum film thickness decreases gradually after 400 s until the operation is halted. The reduction rate of film thickness in this phase is much lower than the first

phase (1000 s) since the rate of increase in the temperature of the lubricant is much lower than the initial phase. The shaded area in Figure 6.15 presents the area of the boundary lubrication regime ($\frac{h_{min}}{\sigma} < 1$) for the investigated roller bearing [28]. Figure 6.15 shows that the lubrication regime falls into the boundary lubrication regime after 800 s and 1000 s for the roller/inner and roller/outer raceway contacts, respectively. This means that metal-to-metal contact occurs between asperities of the contact surfaces after these points and severe wear and damage of bearing elements will occur if the experiment is continued. This is completely different from the first type of thermal failure, discussed in Sec. 2.4.2.1 where the minimum film thickness does not drop into the boundary lubrication regime when the bearing shows the first sign of the cage instability. This is in fact due to the different trends of the operating temperature of the bearing elements and loading conditions in these two types of thermal failure of rolling bearings.

According to the results shown in Figure 6.15, the roller/outer race film thickness is greater than the roller/inner race film thickness. This can be attributed to two factors. First, the equivalent contact radius (R) of the roller/outer race is larger than the roller/inner race which results in higher film thickness, based on Equations 6.16-6.17. Second, the operating temperature of the lubricant at roller/outer race is slightly lower than the roller/inner race contact due to the averaged techniques used in the present study to estimate the oil temperature.

6.3.4.2.2. Time-variation of cage contact forces and wear rate of the cage

Figure 6.16 shows the time variation of two important parameters in the dynamic motion of the cage in the case of thermal failure of the investigated roller bearing before shutting down the operation. This figure shows that the magnitude of the largest normal contact forces between the rollers and the cage pockets are almost constant up to 600 s and there is a gradual decrease afterwards. This gradual, minor decrease of these contact forces can be attributed to the lower sliding velocity at the roller/race contact surfaces as the contact load increases. The lower sliding velocity results in lower traction forces and consequently smaller roller/cage contact forces. The same trend is seen for the steady-state value of the timed-averaged wear rate of the cage during the experiment. The small reduction of the cage wear rate after 600 s is due to the smaller roller/cage contact forces at this time period. It should be mentioned that Archard's wear formula, presented in Equation 6.15, is employed in the dynamic simulations in order to calculate the time-averaged wear rate of the cage during the thermal failure.

Figure 6.16 also shows that the dynamic motion of the cage is stable in the case of second type of thermal failure and there is no significant contact forces between the rollers and the cage. This behavior is clearly different from the first type of thermal failure discussed in Sec. 2.4.2.2 where the cage failure occurs due to the very large contact forces between the rollers and the cage. Note also that the magnitude of roller/cage contact forces and cage wear rates, shown in Figure 6.16, are considerably less than most of bearing applications where there is external forces on the rolling bearings. This is in fact due to small slide-to-roll ratios of contact surfaces at the

roller/raceway contacts of the investigated cylindrical bearing with concentric inner and outer raceways.

Figure 6.16. Time-variation of rollers/cage contact forces and averaged wear-rate of cage for the case of thermal failure of the investigated roller bearing (1400 rpm)

6.4. Conclusions

The current study presents a comprehensive and detailed analysis of thermal failure of rolling element bearings to gain insight into the complex phenomena involved in this type of bearing failure. For this purpose, two types of thermal failure of rolling bearings with different physical mechanisms are investigated both analytically and experimentally.

For the first type, the thermal failure of a deep groove ball bearing rotating under large radial loads and at high operating temperatures is investigated comprehensively. The experimental results show that this type of thermal failure starts with the signs of instability of the cage motion progresses and eventually leads to an ultimate bearing seizure due to the cage failure. It is also shown that a frictional torque measurement system or online monitoring of power consumption of the motor can capture this transient unstable motion of the cage. Nevertheless, there is no indication of the cage instability in the measured temperatures. The results of a set of simulations reveal that the main reason for this unstable motion of the cage is the very high operating temperature of the bearing elements along with the high sliding velocities between the balls and the raceways which result in large balls/cage contact forces and severe wear rate of the cage. The simulation results also show that the minimum film thickness between the ball and the raceways does not fall into the boundary lubrication regime in this type of thermal failure of rolling bearings.

For the second type, the thermal failure of a cylindrical roller bearing used in high-speed machine tools is investigated extensively. The experimental results [6, 11, 33] show that a catastrophic thermal failure of spindle bearings operating at high rotational speeds can occur suddenly with no distinctive change in the vibrational behavior of the bearing assembly. The results of a set of simulations reveal that this sudden thermal seizure of the bearing is due to the sharp increase of the thermally-induced preload inside the bearing as a result of high temperature gradient across the bearing assembly. The simulation results also show that the minimum film thickness drops into the boundary lubrication regime where severe wear and surface damage occurs at the roller/race interface in a short time. The dynamic simulations confirm that there is no cage failure in this type of thermal failure because the magnitude of the rollers/cage contact forces and consequently wear rate of the cage are very low during this type of thermal failure.

6.5. Nomenclature

a	Semi-major width of contact area (m)
b	Semi-minor width of contact area (m)
d	Roller/ball diameter (m)
d_p	Pitch diameter of bearing (m)
E	Elastic modulus (Pa)
E'	Effective modulus of elasticity $[0.5[(1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2]]^{-1}$
F_{ip}	Initial preload (N)
F_r	Radial load on the bearing (N)
F_{th}	Thermally-induced preload (N)
f	Race curvature factor
f_{bpfi}	Ball pass frequency, inner ring (Hz)
f_{bpfo}	Ball pass frequency, outer ring (Hz)
f_{bsf}	Ball spin frequency (Hz)
f	Race curvature factor
f_c	Cage frequency (Hz)
f_i	Inner ring rotational speed (Hz)
f_o	Outer ring rotational speed (Hz)
$f(u_1, z)$	Frictional heat function vector
G	Dimensionless material number $[\zeta E']$
H_v	Hardness (Pa)
h_{min}	Minimum film thickness (m)
k	Ellipticity parameter $[a/b]$
k_t	Bearing stiffness coefficient
k_w	Wear coefficient
M_t	Total frictional torque (Nm)
M_l	Frictional torque due to load (Nm)
M_v	Viscous frictional torque (Nm)

N	Number of rollers/balls
n	Rotational speed (rpm)
Q	Contact load (N)
R	Equivalent contact radius (m)
$\bar{R}$	Radius of contact curvature (m)
r_b	Radius of roller (m)
r_{in}	Radius of inner ring (m)
r_h	Radius of housing (m)
r_{out}	Radius of outer ring (m)
T	Temperature (C)
T_a	Reference temperature (C)
T_h	Temperature of housing (C)
T_{oh}	Characteristic temperature of outer ring/housing (C)
T_{inn}	Temperature of inner ring (C)
T_{out}	Temperature of outer ring (C)
T_r	Temperature of roller (C)
t	Time (s)
$u(t)$	Input vector of state-space model
U	Dimensionless velocity number $[\frac{\mu_0 u}{E'R}]$
u	Rolling velocity (m/s)
u_s	Sliding velocity (m/s)
W	Dimensionless load number $[\frac{W_l}{E'R}]$
W_l	Load per contact length (N/m)
$\dot{W}_r$	Time-averaged wear rate (m ³ /s)
$x(t)$	State vector
x	Axis along the major axis of contact area (m)
y	Axis along the minor axis of contact area (m)
α	Linear thermal expansion coefficient (m/mK)
ζ	Pressure-viscosity coefficient (m ² /N)
β	Temperature-viscosity coefficient
μ_0	Inlet viscosity (Pa.s)
μ_a	Viscosity at reference temperature (Pa.s)
σ	Combined surface roughness, (m)
θ	Contact angle (rad)
ν	Poisson ratio
ν_o	Viscosity (centistokes)

6.6. References

[1] Burton RA, Staph HE. Thermally activated seizure of angular contact bearings. *Asle Transactions*. 1967;10:408-415.

- [2] Carmichael GDT, Davies PB. The influence of transient thermal distortion on the behavior of an assembly containing two angular contact ball bearings. *Proc Instn Mech Engrs.* 1970;184:26-32.
- [3] Carmichael GDT, Davies PB. Factors which affect transient behavior of preloaded ball-bearing assemblies. *Asle Transactions.* 1972;15:1-7.
- [4] Chun, Jiang. Experimental investigation of water cooling inside the spindle bearing system. *Ji Chuang/Machine Tools.* 1988;12:15-19.
- [5] Lacey SJ, Wardle FP, Poon SY. High-speed bearings for cnc machine-tool spindles. *Cme-Charter Mech Eng.* 1983;30:51-56.
- [6] Stein JL, Tu JF. A state-space model for monitoring thermally-induced preload in antifriction spindle bearings of high-speed machine-tools. *Journal of Dynamic Systems Measurement and Control-Transactions of the Asme.* 1994;116:372-386.
- [7] Sud ON, Davies PB, Halling J. Thermal-behavior of rolling bearing assemblies subjected to preload. *Wear.* 1974;27:237-249.
- [8] TA H. Rolling bearing analysis. New York: Wiley. 2001;4th ed.
- [9] Palmgren A. Ball and roller bearing engineering, 3rd edition. SKF Industries Inc. 1959.
- [10] Takabi J, Khonsari MM. Experimental testing and thermal analysis of ball bearings. *Tribology International.* 2013;60:93-103.
- [11] Tu JF. On-line preload monitoring for high-speed anti-friction spindle bearings using robust state observers PhD dissertation, Department of Mechanical Engineering, The University of Michigan, Ann Arbor, MI. 1991.
- [12] Gupta PK. Advanced dynamics of rolling elements. Springer-Verlag, New York. 1984.
- [13] Takabi J, Khonsari MM. On the influence of traction coefficient on the cage angular velocity in roller bearings. *Tribology Transactions.* 2014;57:793-805.
- [14] Saheta V. Dynamics of rolling element bearings using discrete element method. Master's Thesis, Purdue University, West Lafayette, IN. 2001.
- [15] Bair S, Winer WO. Rheological model for elastohydrodynamic contacts based on primary laboratory data. *Journal of Lubrication Technology-Transactions of the Asme.* 1979;101:258-265.
- [16] Gupta PK, Cheng HS, Forster NH. Viscoelastic effects in mil-l-7808-type lubricant .1. Analytical formulation. *Tribology Transactions.* 1992;35:269-274.
- [17] Bair S, Winer WO. The high-pressure high shear-stress rheology of liquid lubricants. *Journal of Tribology-Transactions of the Asme.* 1992;114:1-13.

- [18] Johnson KL, Tevaarwerk JL. Shear behavior of elastohydrodynamic oil films. Proceedings of the Royal Society of London Series a-Mathematical Physical and Engineering Sciences. 1977;356:215-236.
- [19] Masjedi M, Khonsari MM. Film thickness and asperity load formulas for line-contact elastohydrodynamic lubrication with provision for surface roughness. Journal of Tribology-Transactions of the Asme. 2012;134:011503.
- [20] Masjedi M, Khonsari MM. Theoretical and experimental investigation of traction coefficient in line-contact ehl of rough surfaces. Tribology International. 2014;70:179-189.
- [21] Masjedi M, Khonsari MM. On the effect of surface roughness in point-contact ehl: Formulas for film thickness and asperity load. Tribology International. 2015;82:228-244.
- [22] Ashtekar A, Sadeghi F. A new approach for including cage flexibility in dynamic bearing models by using combined explicit finite and discrete element methods. Journal of Tribology-Transactions of the Asme. 2012;134:041502.
- [23] Weinzapfel N, Sadeghi F. A discrete element approach for modeling cage flexibility in ball bearing dynamics simulations. Journal of Tribology-Transactions of the Asme. 2009;131:021102.
- [24] Wang YS, Yang BY. An investigation into grease behavior in thermal ehl circular contacts. Tribology Transactions. 2006;49:449-453.
- [25] Wang YS, Li P, Cao JW, Li Y. The elastohydrodynamic traction of a grease lgt2. Journal of Tribology-Transactions of the Asme. 2013;135.
- [26] Hamrock BJ, Dowson D. Isothermal elastohydrodynamic lubrication of point contacts .3. Fully flooded results. Journal of Lubrication Technology-Transactions of the Asme. 1977;99:264-276.
- [27] Khonsari MM, Booser ER. Applied tribology-bearing design and lubrication. John Wiley & Sons Ltd, Chichester, UK. 2008.
- [28] Hamrock BJ. Fundamentals of fluid film lubrication. McGraw-Hill, New York. 1994.
- [29] Carmichael GDT. The effects of thermal distortion in a main spindle assembly. PhD Thesis, University of Salford. 1969.
- [30] Pruvot FC. High speed bearings for machine tool spindles. Annals of the CIRP. 1980;29:293-297.
- [31] Sud ON. The transient thermal behavior of ball thrust race assemblies. PhD Thesis, University of Salford. 1971.
- [32] Sud ON, Davies PB, Halling J. The thermal behaviour of rolling bearing assemblies subjected to preload. Wear. 1974;27:237-249.

- [33] Tu JF. Thermoelastic instability monitoring for preventing spindle bearing seizure. Tribology Transactions. 1995;38:11-18.
- [34] Tu JF, Stein JL. Online preload monitoring for antifriction spindle bearings of high-speed machine-tools. Journal of Dynamic Systems Measurement and Control-Transactions of the Asme. 1995;117:43-53.
- [35] Tu JF, Stein JL. Active thermal preload regulation for machine tool spindles with rolling element bearings. Journal of Manufacturing Science and Engineering-Transactions of the Asme. 1996;118:499-505.
- [36] Tu JF, Stein JL. Model error compensation for observer design. Int J Control. 1998;69:329-345.
- [37] Dowson D, Higginson GR. Elasto-hydrodynamic lubrication. Pergamon Press, London. 1977.

Chapter VII: Summary and Future Works

7.1. Summary and Conclusions

The objective of this dissertation is to perform a comprehensive analysis of thermally induced failure of rolling element bearings. The previous chapters present the necessary steps that were taken to reach to this goal. These steps include the three main tasks of developing a comprehensive thermal model, developing a fully dynamic model of rolling element bearings and finally combining these models in a unified simulation approach.

First, a comprehensive mathematical model was presented to analyze the thermal behavior of a ball bearing. Considered in conjunction with the transient heat transfer model was the dimensional change of the bearing components due to unequal temperature rise of the bearing components. Experiments were performed for different speeds and loads to validate the model. The predicted temperatures under different loads and speeds were found to be in close agreement with those measured experimentally. Further simulations showed that the rotational speed, oil viscosity and heat convection coefficient are the most influential parameters in the thermal behavior of the bearing components. The simulation results revealed that the high rotational speed, oil viscosity and housing cooling rate lead to the larger temperature gradient and thermally-induced preload in ball bearings. However this thermal model is unable to provide detail information about the dynamic contact forces and moments between the bearing components, and therefore it needs to be combined with an analytical model of rolling element bearings.

Next, a dynamic model of rolling element bearings was developed which can calculate all of the contact forces and moments between the rolling elements, the raceways and the cage. Given that the traction coefficient is a crucial parameter of any dynamic model of rolling bearings, this dynamic model was used to further investigate the influence of different rheological models on the dynamic performance of a radially-loaded cylindrical roller bearing. Comparison of the simulation results to those of published experimental measurements revealed that the dynamic model with an appropriate viscoelastic rheological model is capable of realistically predicting the dynamic response of rolling elements under various operating conditions. However, in the case of low sliding velocities employing a simplified traction curve may lead to acceptable results. The dynamic model was further extended to study the cage wear rate, and the results indicate that the high rotational speeds and low radial loads can increase the cage wear rate due to high sliding velocities between the rollers and the races. However these dynamic simulations are not able to provide any information about the effect of surface roughness on the heat generation and dynamic behavior of rolling element bearings.

The developed dynamic model was employed along with the results of recent studies to investigate the effect of different surface roughness and lubrication regimes on the dynamic behavior of radially-loaded roller bearings operating at high loads and low speeds. The simulation results provided a detailed understanding of the variation of the film thickness, wear rate and heat generation between the rollers and the raceways as the rollers travel in the orbital direction. Simulation results revealed that an increase in the radial load results in a proportional increase in the wear rate and an exponential increase in the heat generation, although it does not affect the film thickness noticeably.

Finally, the developed thermal and dynamic models of the rolling element bearings were coupled in a unified simulation approach in order to investigate the thermally induced seizure of rolling element bearings. For this purpose, two different types of thermal failure of rolling element bearings were investigated and compared both experimentally and analytically. The first type is concerned with the thermal failure of rolling bearings that can occur at high rotational speeds and large radial loads, and the second type deals with spindle bearings of high-speed machine tools. The results of dynamic simulations for the first type of thermal failure showed that the unstable motion of the cage can lead to an ultimate bearing seizure because of the cage failure due to the large rollers/cage contact forces and high wear rate of the cage. However, no considerable surface damage and wear at the roller/raceways interface is expected. On the contrary, for the second type of thermal failure of rolling element bearings, the simulation results revealed that the minimum film thickness at the roller/raceways interface drops into the mixed lubrication regime resulting in severe surface damage and wear at the roller/raceways contact surfaces. This sudden thermal seizure of the spindle bearings is due to the rapid rise of the thermally induced preload inside the bearing assembly while there is no sign of cage instability during the thermal failure.

7.2 Recommendations for Future Works

Based on the results presented in this study, the following recommendations are made for the possible future research.

- The developed dynamic model of rolling element bearings can be extended by considering deformable models for the cage. By implementing these models in the dynamic simulations it would be possible to calculate the deformation of the cage due to the large contact forces between the rollers and the cage which occur in the first type of thermally induced failure of rolling element bearings.
- More simulations can be carried out in order to investigate the influence of different clearance values between the roller and the cage pocket on the dynamic behavior and thermally induced failure of rolling element bearings. It would also be possible to

investigate the effect of different geometrical shapes of the cage pockets. The results of these simulations would provide valuable information for the bearing manufacturers to design an anti-seizure bearing for spindle bearings of high speed machine tools.

- The developed dynamic model can be extended by implementing different non-Newtonian rheological models available in the literature. However, the application of these non-Newtonian models is very limited since they require very hard-to-measure parameters which are not available in the literature currently.
- The simulation results presented in Chapter VI can be extended to be used in the design of appropriate monitoring systems that can send warning signals to the operator to halt the operation before thermally induced failure occurs.

Appendix A: Load—Deflection Relationship of ball bearings

Presented in the Harris's book[13] is the following formulas to calculate the coefficient of the elastic contact k_n , based on the Hertzian contact theory

$$F = k_n \delta^n \quad (\text{A.1})$$

where F is the normal contact load, δ is the contact deformation and $n = 1.5$ for ball bearings and $n = 1.11$ for roller bearings. The coefficient of elastic contact k_n for the steel ball-steel raceway point contact can be calculated by the following formula

$$k_p = 2.15 \times 10^{11} \sum \rho^{-1/2} (\delta^*)^{-3/2} \quad (\text{A.2})$$

where δ^* is the dimensionless contact deformation which is function of the curvature difference $F(\rho)$ and can be calculated based on appropriate equations as well as tables and graphs presented in reference [13]. The curvature sum $\sum \rho$ and curvature difference $F(\rho)$ are defined based on the following formulas for the inner raceway and outer raceway contacts

$$\sum \rho_i = \frac{1}{D} \left(4 - \frac{1}{f_i} + \frac{2\gamma}{1-\gamma} \right) \quad (\text{A.3})$$

$$F(\rho)_i = \frac{\frac{1}{f_i} + \frac{2\gamma}{1-\gamma}}{4 - \frac{1}{f_i} + \frac{2\gamma}{1-\gamma}} \quad (\text{A.4})$$

$$\sum \rho_o = \frac{1}{D} \left(4 - \frac{1}{f_o} + \frac{2\gamma}{1-\gamma} \right) \quad (\text{A.5})$$

$$F(\rho)_o = \frac{\frac{1}{f_o} - \frac{2\gamma}{1-\gamma}}{4 - \frac{1}{f_o} - \frac{2\gamma}{1-\gamma}} \quad (\text{A.6})$$

$$\gamma = \frac{D \cdot \cos \theta}{d_m} \quad , \quad f_i = \frac{r_{ic}}{D} \quad , \quad f_o = \frac{r_{oc}}{D} \quad (\text{A.7})$$

Appendix B: Thermo-mechanical model of spindle bearings

The final state-space model, based on the thermo-mechanical model of the cylindrical roller bearing of the specific apparatus shown in Figure 6.10, was presented by Tu and Stein [6, 11, 30] and is given by Equations B.1-B.2.6:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -3.7572 \times 10^{-5} & 2.9820 \times 10^{-6} & 0 \\ 0 & -0.0325 & 0.0104 \\ 0 & 2.4829 \times 10^{-6} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1.4068 \times 10^{-5} \\ 0.0080 \\ 2.495 \times 10^{-5} \end{bmatrix} f(u_1, z) + \begin{bmatrix} b_1 \\ 0.0731 \\ 0 \end{bmatrix} u_2 \quad (\text{B.1})$$

$$a_{33} = \begin{cases} -1.7236 \times 10^{-4} - 6.7165 \times 10^{-5} x_3 & x_3 < 5 \\ -4.6103 \times 10^{-4} & x_3 \geq 5 \end{cases} \quad (\text{B.2})$$

$$b_1 = \begin{cases} -0.7009 \times 10^{-4} - 0.7933 \times 10^{-4} u_2 & u_2 < 5 \\ -1.9832 \times 10^{-4} & u_2 \geq 5 \end{cases} \quad (\text{B.3})$$

$$f(u_1, z) = (0.001u_1^{2/3} + 1.152 \times 10^{-4}z)u_1 \quad (\text{B.4})$$

$$z = F_{th} = 3.303 \times 10^9 \times (7.020 \times 10^{-8} x_2 + 2.707 \times 10^{-9} F_{ip}^{0.9} + 3.896 \times 10^{-7} x_3 - 1.037 \times 10^{-6} x_1)^{1.11} \quad (\text{B.5})$$

$$u_1 = n \quad , \quad u_2 = T_{out} \quad (\text{B.6a})$$

$$x_1 = T_{oh} \quad , \quad x_2 = T_r \quad , \quad x_3 = T_{inn} \quad (\text{B.6b})$$

$$T_{oh} = (2r_{out} + r_h)T_{out} + (2r_h + r_{out})T_h \quad (\text{B.6c})$$

where x_1 , x_2 and x_3 are the state variables representing temperature of housing/outer race (T_{oh}), roller (T_r) and inner race (T_{inn}), respectively. u_1 and u_2 are the control variables representing, respectively, the rotational speed of the bearing (n) and temperature of the outer ring (T_{out}). It should be noted that T_{oh} is the average temperature of the housing (T_h) and outer race (T_{out}) which is defined by Equation B.6c, where the radius of the housing and outer race are shown by r_h and r_{out} , respectively. According to Equation B.5, the thermally induced preload (F_{th}) in the bearing is calculated from the thermal expansion of the bearing elements, which are calculated

from bearing elements temperatures (x_1 , x_2 , x_3), and also the initial preload (F_{ip}) which is assumed to be constant and equal to 810 N for the investigated experimental setup.

Appendix C: Letters of Permission to Use Published Material

The Permissions from Springer and Elsevier publishing companies are presented in the following pages:

[Home](#) [Create Account](#) [Help](#) [Li](#)

Taylor & Francis
Taylor & Francis Group

Tribology Transactions

Title: On the Influence of Traction Coefficient on the Cage Angular Velocity in Roller Bearings

Author: Jafar Takabi, M. M. Khonsari

Publication: Tribology Transactions

Publisher: Taylor & Francis

Date: Sep 3, 2014

Copyright © 2014 Taylor & Francis

LOGIN

If you're a **copyright.com** user, you can login to RightsLink using your copyright.com credentials. Already a **RightsLink** user or want to [learn more?](#)

Thesis/Dissertation Reuse Request

Taylor & Francis is pleased to offer reuses of its content for a thesis or dissertation free of charge contingent on resubmission of permission request if work is published.

[BACK](#)[CLOSE WINDOW](#)

Copyright © 2015 [Copyright Clearance Center, Inc.](#) All Rights Reserved. [Privacy statement.](#) [Terms and Conditions.](#) Comments? We would like to hear from you. E-mail us at customercare@copyright.com

ELSEVIER LICENSE TERMS AND CONDITIONS

Mar

This is a License Agreement between Jafar Takabi ("You") and Elsevier ("Elsevier") provided by Copyright Clearance Center (CCC). The license consists of your order details, the terms and conditions provided by Elsevier, and the payment terms and conditions provided by CCC.

All payments must be made in full to CCC. For payment instructions, please see information listed at the bottom of this form.

Supplier	Elsevier Limited The Boulevard, Langford Lane Kidlington, Oxford, OX5 1GB, UK
Registered Company Number	1982084
Customer name	Jafar Takabi
Customer address	Louisiana State University BATON ROUGE, LA 70803
License number	3581670532524
License date	Mar 03, 2015
Licensed content publisher	Elsevier
Licensed content publication	Tribology International
Licensed content title	Experimental testing and thermal analysis of ball bearings
Licensed content author	Jafar Takabi, M.M. Khonsari
Licensed content date	April 2013
Licensed content volume number	60
Licensed content issue number	n/a
Number of pages	11
Start Page	93
End Page	103
Type of Use	reuse in a thesis/dissertation
Portion	full article
Format	both print and electronic
Are you the author of this Elsevier article?	Yes
Will you be translating?	No
Title of your thesis/dissertation	On the Thermally Induced Failure of Rolling Element Bearings
Expected completion date	May 2015
Estimated size (number of pages)	170
Elsevier VAT number	GB 494 6272 12
Price	0.00 USD
VAT/Local Sales Tax	0.00 USD / 0.00 GBP
Total	0.00 USD
Terms and Conditions	

INTRODUCTION

1. The publisher for this copyrighted material is Elsevier. By clicking "accept" in connection with completing this licensing transaction, you agree that the following terms and conditions apply to this transaction (along with the Billing and Payment terms and conditions established by Copyright Clearance Center, Inc. ("CCC"), at the time that you opened your Rightslink account that are available at any time at <http://myaccount.copyright.com>).

ELSEVIER LICENSE TERMS AND CONDITIONS

Mar 03, 2015

This is a License Agreement between Jafar Takabi ("You") and Elsevier ("Elsevier") provided by Copyright Clearance Center ("CCC"). The license consists of your order details, the terms and conditions provided by Elsevier, and the payment terms and conditions.

All payments must be made in full to CCC. For payment instructions, please see information listed at the bottom of this form.

Supplier	Elsevier Limited The Boulevard, Langford Lane Kidlington, Oxford, OX5 1GB, UK
Registered Company Number	1982084
Customer name	Jafar Takabi
Customer address	Louisiana State University BATON ROUGE, LA 70803
License number	3581670615715
License date	Mar 03, 2015
Licensed content publisher	Elsevier
Licensed content publication	Tribology International
Licensed content title	On the dynamic performance of roller bearings operating under low rotational speeds with consideration of surface roughness
Licensed content author	None
Licensed content date	June 2015
Licensed content volume number	86
Licensed content issue number	n/a
Number of pages	10
Start Page	62
End Page	71
Type of Use	reuse in a thesis/dissertation
Intended publisher of new work	other
Portion	full article
Format	both print and electronic
Are you the author of this Elsevier article?	Yes
Will you be translating?	No
Title of your thesis/dissertation	On the Thermally Induced Failure of Rolling Element Bearings
Expected completion date	May 2015
Estimated size (number of pages)	170
Elsevier VAT number	GB 494 6272 12
Price	0.00 USD
VAT/Local Sales Tax	0.00 USD / 0.00 GBP
Total	0.00 USD

Vita

Jafar Takabi was born in Sabzevar, Iran in 1982. He received his diploma in mathematics and physics from the high school of National Organization for Development of Exceptional Talents of Iran (NODET). He received his BS in Mechanical Engineering from Isfahan University of Technology in 2004. He received his MS in Mechanical Engineering from Amirkabir University of Technology (Tehran Polytechnic) in 2007. He worked as a researcher at the New Technology Research Centre (NTRC) in Amirkabir University of Technology from 2005 to 2006. He also worked as a Stationary Equipment Engineer for Sazeh Consultant and Construction Co., a highly prestigious oil and gas company in Iran, from 2006 to 2009. At 2009 he decided to pursue his graduate studies and joined Center for Rotating Machinery (CeRoM), Louisiana State University, Baton Rouge, LA, USA as a Ph.D candidate. He has been a research assistant in CeRoM lab under the supervision of Prof. Michael M. Khonsari, Dow Chemical Endowed Chair and Professor of Mechanical Engineering. Jafar Takabi has published 4 scientific articles in internationally peer-reviewed journals in the field of tribology and 3 in international conferences in the field of Manufacturing Engineering. He is currently working as a Senior Mechanical Engineer for Chicago Bridge & Iron Company (CB&I), Baton Rouge.

Jafar Takabi is expected to receive his Doctor of Philosophy degree at the 2015 spring commencement.