Measurement of Continuum Breakdown Using a Disc Spin-Down Experiment in Low Pressure Air

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MEASUREMENT OF CONTINUUM BREAKDOWN USING A DISC SPIN-DOWN EXPERIMENT IN LOW PRESSURE AIR

A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Doctor of Philosophy

in

The Department of Mechanical and Industrial Engineering

by

Tathagata Acharya
B.E., University of Pune, 2004
M.S., Louisiana State University, 2008
May 2014
To my parents,

Prabir and Shanta Acharya
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Abstract

As flow becomes rarefied, a quantity called as the tangential momentum accommodation coefficient (TMAC) becomes important because it is a measure of the momentum transport from a gas molecule to a surface. Very few experimental measurements of continuum breakdown in boundary layer flows exist. All experimental measurements of the TMAC in macro-scale boundary layer flows have been done in the continuum slip and the transition flow regimes. Moreover the experimental apparatus used by previous researchers cannot accommodate for materials that are planar by nature such as those used in the field of aerospace and microfabrication.

The objectives of this research include the experimental measurement of continuum breakdown in a macro-scale boundary layer flow, development of a test facility such that TMAC may be measured in the free molecular flow regime, and the measurement of TMAC for various gases versus material interactions.

An experimental facility is built which consists of a disc spin down experiment in various gas pressures from atmospheric pressure through the free molecular flow regime. The real time deceleration torque is measured during the disc spin-down in each ambient pressure. The deceleration torque is non-dimensionalized suitably and is plotted against Reynolds number. The non-dimensional curves show self-similarity and therefore continuity in the viscous flow regime. Self-similarity breaks down when the viscous forces are no longer dominant and therefore it is a measure of continuum breakdown. This is also confirmed through the departure of the CFD and the semi-analytic von Karman curves from the experimental curves.

A differential scavenging system is designed and incorporated into the apparatus and it facilitates measurements in the free molecular flow regime. TMAC for interactions between
several gases and certain aerospace materials are measured in the free molecular flow regime. While most measurements show TMAC values of 0.7 or above in the different gases, the values are consistently low in carbon dioxide. The results are of significant impact for future space missions to Mars because the Martian atmosphere contains carbon dioxide predominantly and a lower TMAC suggests lower atmospheric drag on space vehicles.
Chapter 1 Introduction

1.1 Rarefied gas dynamics

Researchers since the past century have been involved with studying flow physics associated with continuum and rarefied flow regimes. The flow physics gradually varies from continuum through the free molecular flow regime. The Navier Stokes equations govern the flow physics in the continuum regime. In the transition regime, the Navier Stokes equations may still be used, but with slip boundary conditions. In the free molecular flow regime, the Navier Stokes equations cannot be used at all. Flow regimes are characterized by a non-dimensional term called as the Knudsen (Kn) number, which is given by equation (1.1).

\[ Kn = \frac{\lambda}{L}. \]  

(1.1)

In the above equation, \( \lambda \) is the mean free path between two successive collisions and \( L \) is the characteristic length scale. At continuum, the value of the Knudsen number is very small and is of the order of 0.00010. The value increases as flow becomes rarefied. In the free molecular flow regime, the value of the Knudsen number is of the order of 1 or more (Gombosi, 1994).

1.1.1 Mean free path

The concept of mean free path was introduced by Rudolf Clausius in 1858. A single species gas composed of hard molecules is considered, with all molecules having the same diameter, \( d \). As shown in Figure 1.1, a given molecule undergoes a collision whenever the distance between the centers of this molecule and any other molecule approaches the diameter, \( d \). The free path is the distance traveled by a molecule between two successive collisions.
A very simplified situation is considered, where a molecule moves with a uniform molecular speed $C$. At this time all the other molecules are standing still. The actual zig-zag path of the molecule and its sphere of influence is illustrated in Figure 1.2. The moving molecule is indicated by the black circle, while the ones with which it collides are the white circles.

It may be imagined that the path of the molecule is straightened as shown in Figure 1.3. Here the centers of the target molecules retain their position relative to the path of the moving molecule prior to their collision.

As shown in Figure 1.3, the centers of all the molecules lie within the straight cylindrical volume swept out by the sphere of influence. Since the molecule is traveling at the uniform speed $C$, the volume swept out per unit time will be $\pi d^2 C$, where $d$ is the diameter of the
molecule. If \( n \) denotes the number of molecules per unit volume of gas, the number of centers lying within the cylindrical volume is \( \pi d^2 n C \). It will be noted that each of these centers correspond to a collision. Therefore the product will also represent the number of collisions, \( \Theta \). It will be given by the equation (1.2).

\[
\Theta = \pi d^2 C n, \tag{1.2}
\]

where \( \pi d^2 C \) is the volume swept by the colliding molecule per unit time, while \( n \) is the number of target molecules per unit volume, and \( \Theta \) is the number of collisions per unit time. Also, the distance traveled by the colliding molecule per unit time is \( C \). Thus the mean free path may be given by the equation (1.3).

\[
\lambda = \frac{C}{\Theta} = \frac{1}{\pi d^2 n}. \tag{1.3}
\]

Equation (1.3) accurately gives the mean free path for a particle moving with a speed infinitely greater than the other particles. This is approximately the case for a free electron with a very small mass moving in a gas of normal molecular mass. For greater accuracy the derivation is modified to account for the motion of the target molecules. The revised equation for the mean free path will be as follows (Vincenti, and Kruger, 1965).

\[
\lambda = \frac{1}{\sqrt{2\pi d^2 n}} = \frac{m}{\sqrt{2\pi d^2 \rho_g}}, \tag{1.4}
\]

where \( \rho_g \) is the gas density, \( m \) is the mass of a gas molecule and is given by:

\[
\rho_g = mn. \tag{1.5}
\]
1.1.2 Flow physics in various regimes

As mentioned earlier, the relative magnitude of mean free path to the length scale of a domain determines the flow regime. In continuum the mean free path $\lambda$ must be much smaller than the length scale $L$. Here the length scale may be given by the equation (1.6) (Gombosi, 1994).

$$L = \left| \frac{G}{\nabla G} \right|,$$  \hspace{1cm} (1.6)

where $G$ is the fastest varying macroscopic quantity within the mean free path. Physically this will mean that in continuum, all macroscopic properties are small within the mean free path. A gradient local length scale has been used to define local Knudsen numbers in a flow, which is given by the equation (1.7) (Boyd, Chen, and Chandler, 1995).

$$\left( Kn \right)_{GLL} = \frac{\lambda}{\frac{Q}{dQ/dl}},$$  \hspace{1cm} (1.7)

where $\left( Kn \right)_{GLL}$ is known as the gradient length local Knudsen number, $Q$ is the flow property and $l$ is some distance between two points in the flow field. It has been shown that the flow regimes change from continuum through the free molecular flow regime with the increase in Knudsen number.

The variation of flow regimes with increase in Knudsen number is as shown in Figure 1.4 (Gad-El-Hak, 1999).
As may be seen from Figure 1.4, when Knudsen number is less than 0.001, the flow regime is in continuum. In this regime, Navier-Stokes equations with no-slip boundary conditions may be used to solve fluid-flow problems. Between Knudsen numbers 0.001 and 0.01, continuum or Navier Stokes equations may still hold good, but slip boundary conditions may need to be applied. The flow regime between Knudsen numbers 0.01 and 1 is called as the transition flow regime. In this regime, the continuum gradually breaks down and specialized methods such as Direct Simulation Monte Carlo techniques may be used to solve the Boltzmann equation. Beyond Knudsen number 1, the flow is termed as free molecular and the gas molecules behave as independent bodies and the mean free path between two successive collisions may exceed the gradient length scale by several orders of magnitude.

In the free molecular flow regime, the drag associated may now be strictly a function of the interaction between the gas molecules and a solid wall, since very few interactions between individual molecules take place. In this regime, the momentum transfer from a particular gas molecule to a surface becomes a function of a parameter known as the tangential momentum accommodation coefficient (Gombosi, 1994).
Figures 1.5 and 1.6 show two extreme models and describe the interaction between a gas molecule and a surface in the free molecular flow regime. The two models show the two extreme cases of momentum transfer from a gas molecule to the wall. The two different types of interactions between the gas molecule and the corresponding solid surface are called as specular and diffuse reflection and are as shown below.

In the specular reflection model, the interaction is treated as a perfectly elastic collision, with no momentum being transferred from a gas molecule to the surface. In this case, the gas molecules act as bouncing balls and the tangential momentum accommodation coefficient $\sigma_t$ associated with such an interaction is 0. In the other extreme case, the interaction is termed as diffuse. The gas molecule in this case hits the surface, gets absorbed and is then re-emitted in a random direction by the surface with the velocity of the gas molecule being set by the wall temperature.

It has been observed that the tangential momentum accommodation coefficient associated with the interaction of gas molecules and real surfaces are closer to the diffuse reflection model (Gombosi, 1994). More details and quantitative treatment of the gas versus surface interaction will follow in the later part of this chapter. The following few sections will address the various
flow regimes in tubes. In order to understand the transition from the continuum flow regime the discussion starts with one of the canonical flows – ‘Poiseuille.’

1.1.2.1 Poiseuille flow in a tube

At ambient pressure and temperature, the molecular mean free path, $\lambda$, is negligibly small as compared to the associated length scale. For flow through a long tube, the characteristic length scale is the radius of the tube. It will also be assumed that the flow velocity is not too large and consequently the flow is laminar. The steady state motion is hence controlled by viscosity. The flow problem is axially symmetric around the longitudinal axis of the tube. Therefore the bulk velocity of the gas, $u$, is a function of the radial distance from the axis of the tube. The flow may be subdivided into co-axial cylindrical shells of thickness $dr$ around the $z$ axis. No slip conditions at the boundary suggest that the velocity of the gas at the wall is zero. The velocity $u(r)$ increases from zero at the wall to the maximum at the axis. Now, a cylinder with radius $r$ and a finite length $\Delta z$ is considered. Figure 1.7 shows the control volume and is analyzed as follows.

![Figure 1.7: Poiseuille flow in a tube of finite length](image)

In this case, there are two different forces acting against each other. A pressure difference is trying to accelerate the fluid between the bases of the cylinder. A viscous force is acting in the
opposite direction, and is trying to resist the flow. The governing equation of motion is as follows:

\[ \pi r^2 \Delta p - (2\pi r \Delta z) \times \left( \mu \frac{du}{dr} \right) = 0. \]  
\[ (1.8) \]

The cylinder with the finite length is treated as a solid body and therefore the pressure gradient force accelerates all the gas inside the cylinder. Equation (1.8) could be re-arranged to obtain the equation:

\[ \frac{du}{dr} = \frac{r \Delta p}{2\mu \Delta z}. \]  
\[ (1.9) \]

Equation (1.9) could be solved with a boundary condition \( u(a) = 0 \), where \( a \) is the radius of the cylinder. The following result is obtained.

\[ u(r) = -\frac{a^2 - r^2}{4\mu} \frac{\Delta p}{\Delta z}. \]  
\[ (1.10) \]

It will also be noted that in this case \( \lambda << a \). Re-iterating, the mean free path is much smaller than the length scale of the problem which is the radius of the cylinder. The negative sign in the above equation suggests that the gas will flow towards the lower pressure region. This result may be used to calculate the total flux of molecules in the tube, and is as follows (Gombosi, 1994):

\[ \Phi_{\text{total}} = n \int_0^a 2\pi ru(r)dr = -\frac{2\pi n \Delta p}{4\mu \Delta z} \frac{a^2 - r^2}{2} dr = -\frac{\pi a^4}{8\mu} \frac{\Delta p}{\Delta z}. \]  
\[ (1.11) \]
In equation (1.11), $\Phi_{total}$ is the total flux of molecules per unit volume, $n$ is the particle number density, $a$ is the radius of the cylinder, $\Delta z$ is the length of the cylinder, $\mu$ is the dynamic viscosity of the gas, and $\Delta p$ is the pressure difference between the bases of the cylinder. Thus the total mass flux will be given by the equation (1.12).

$$\Phi_{\text{mass}} = nm \int_0^a 2\pi r u(r) dr = -\frac{2\pi n m}{4\mu} \int_0^a \frac{\Delta p}{\Delta z} r(a^2 - r^2) dr = -\frac{\pi a^4}{8\mu} mn \frac{\Delta p}{\Delta z} = -\frac{\pi a^4}{8\mu} \rho g \frac{\Delta p}{\Delta z}. \quad (1.12)$$

1.1.2.2 Slip flow in a tube

In slip flow, the shortest characteristic length of the problem is comparable to the mean free path. In order to understand this situation, it will be assumed that the mean free path, $\lambda$, is smaller than the length scale, which is the radius of the tube. However the mean free path is not negligibly small as compared to the length scale. In this condition a modified Poiseuille formula is obtained with ‘slippage’ of the gas being introduced at the walls of the tube where the velocity of the gas molecule on the wall is different from the wall velocity as shown in Figure 1.8.

The two planes are separated by a distance, $h$. While the plane at the top moves with a speed of $u_o$, the plane at the bottom is stationary. It is assumed that the mean free path of the molecular collisions is somewhat smaller than the distance between the two parallel planes. This
means that after colliding with a wall, each molecule undergoes several molecular collisions before it collides with the other wall. However, the molecule does not undergo many molecular collisions as in the case of viscous flow. In this case, it is assumed that a molecule collides with the moving wall at \( z = h \). The \( y \) components of the average velocity of molecules just before and just after the collision with the upper wall are \( q_1 \) and \( q_2 \) respectively. As mentioned before in section 1.3, there are two different types of collisions associated with such interactions between the gas molecules and the wall. The first type is the elastic or specular collision. In this case the tangential velocity of the colliding molecule is conserved and therefore \( q_2 = q_1 \). The other type of collision is called as diffuse reflection. In this case, a molecule is temporarily absorbed and then re-emitted. The average tangential velocity of diffusely reflected particles is the same as the velocity of the wall. Therefore, for these particles \( q_2 = u_o \). The relative importance of the two types of reflections is characterized by the tangential momentum accommodation coefficient, \( \sigma_t \). This is defined as the fraction of molecules that undergo diffuse reflection. Thus the average tangential velocity just after the collision could be expressed as a function of the tangential momentum accommodation coefficient. The equation is as follows:

\[
q_2 = \sigma u_o + (1 - \sigma)q_1. \tag{1.13}
\]

The average gas tangential velocity at the upper wall, \( u_h \), is the mean of the average pre and post collision velocities. The derivation for the average tangential velocity is as follows:

\[
u_h = \frac{q_1 + q_2}{2} = \frac{1}{2}(q_1 + \sigma u_o + (1 - \sigma)q_1). \tag{1.14}
\]

This leads to the following result for the average gas tangential velocity:
\[ u_h = \frac{\sigma}{2} u_o + \frac{2 - \sigma}{2} q_1, \quad (1.15) \]

where \( u_o \) is a known quantity. However \( u_h \) and \( q_1 \) are still unknown. The second equation relating the two unknown parameters could be obtained using a method called as the mean free path method. According to the mean free path method, \( q_1 \) will reflect the average tangential flow velocity at a distance of \( 2\lambda/3 \) from the wall, where the last collision has occurred. Hence, expanding the tangential velocity function to the first order, equation (1.16) is obtained.

\[ q_1 = u_h - \frac{2}{3} \lambda \left[ \frac{du}{dz} \right]_{z=h}. \quad (1.16) \]

Equation (1.17) may be deduced from equations 1.15 and 1.16.

\[ u_h = u_o - \frac{2 - \sigma}{\sigma} \frac{2}{3} \lambda \left[ \frac{du}{dz} \right]_{z=h}. \quad (1.17) \]

The slip velocity is defined as the difference between the velocity of the wall, \( u_o \) and the average gas velocity at the wall, \( u_h \). Hence the equation for the slip velocity is as follows.

\[ u_{slip} = \frac{2 - \sigma}{\sigma} \frac{2}{3} \lambda \left[ \frac{du}{dz} \right]_{z=h}. \quad (1.18) \]

There are two limiting cases which may be considered. In the event of perfect slip (\( \sigma = 0 \)), the slip velocity is mathematically undefined. However, in the other case, in the event of diffuse reflection (\( \sigma = 1 \)), the slip velocity is given by the equation (1.19).

\[ u_{slip} = \frac{2}{3} \lambda \left[ \frac{du}{dz} \right]_{z=h}. \quad (1.19) \]
In the most practical cases, the accommodation coefficient is always between 0 and 1 and hence a finite slip velocity is obtained. If the above result is applied for a low density gas in a long circular tube, the problem will be very similar to Poiseuille flow, but with slippage introduced on the walls. It will be recalled that for Poiseuille flow, velocity may be related to the pressure gradient by equation (1.9).

\[
\frac{du}{dr} = \frac{r}{2\mu} \frac{\Delta p}{\Delta z}.
\]  \hspace{1cm} (1.9)

Using equations 1.9 and 1.18, and since gas flows in the direction of lower pressure, the equation for slip velocity is as follows:

\[
u_{\text{slip}} = -\frac{1}{3} \frac{a}{\mu} \frac{\Delta p}{\Delta z}.
\]  \hspace{1cm} (1.20)

In the above equation, the coordinate \( z \) denotes the distance along the longitudinal axis of the tube. The cross sectional velocity profile may hence be evaluated by solving the differential equation given by equation 1.9, and applying the boundary condition, \( u(a) = u_{\text{slip}} \). Thus the velocity profile obtained is as follows:

\[
u(r) = u_{\text{slip}} - \frac{a^2 - r^2}{4\mu} \frac{\Delta p}{\Delta z} = -\left( a^2 + \frac{4}{3} \lambda a - r^2 \right) \frac{1}{4\mu} \frac{\Delta p}{\Delta z}.
\]  \hspace{1cm} (1.21)

It will be noted that for slip flow, the correction term, \( 4\lambda a/3 \) is proportional to the mean free path. The slip flow cross sectional velocity profile may be used to calculate the modified Poiseuille formula for the total flux in the tube (Gombosi, 1994).
The mass flux may hence be obtained by multiplying the above equation by molecular mass \( m \), and is as shown below.

\[
\Phi_{\text{mass}} = \int_0^a u(r)2\pi r\,dr = -\frac{\pi a^4}{8\mu} \rho g \frac{\Delta p}{\Delta z} \left(1 + \frac{8}{3} \frac{\lambda}{a} \right).
\]  

\[ (1.23) \]

1.1.2.3 Free molecular flow in a tube

In this case, flow inside a very long tube of circular cross section is considered. The radius of the tube is \( a \) units. In this case, the mean free path between two successive collisions is much less than the radius, \( a \) of the tube. Hence Knudsen number \( Kn \) is much larger than 1 and the basic assumptions for viscous flow are not valid anymore. It is assumed that the number of intermolecular collisions is negligible as compared to the number of collisions between the gas molecules and the wall. In other words, when \( Kn >> 1 \), a gas molecule collides many times with the wall before it encounters another molecule. The flow of gas is determined almost entirely by wall-molecule interactions and is considered unaffected by intermolecular collisions. Thus in such a situation, the flow problem is reduced to the determination of effects of molecular collisions with the wall.

As in the case of slip flow, gas-surface interactions in free molecular flow is also classified into specular and diffuse reflection. In the case of specular reflection, the collision between the gas molecules and the wall is elastic. The tangential velocity of the gas molecule is conserved. When reflection is diffused, the gas molecule is temporarily absorbed and then re-
emitted diffusely. Real experiments have shown that reflection is closer to being diffusive than specular. In order to determine the escape flux a simple model as shown in Figure 1.9 may be considered where the tangential momentum accommodation coefficient is assumed as 1.

![Figure 1.9: Diffuse reflection – free molecular flow in a tube](image)

In the model shown in Figure 1.9, the molecules may be envisioned as particles escaping from a hypothetical equilibrium gas layer on the other side of the surface, assumed to be in thermal equilibrium with the wall. In this given model, the total flux in the tube will be given by the equation (1.24) (Gombosi, 1994).

\[
\Phi_{max} = \frac{4\pi \ a^4}{3} \ \frac{8kT}{\pi m} \ \frac{d \rho_s}{dz}.
\]  

(1.24)

In the above equation, \( k \) is the Boltzmann constant, \( T \) is the temperature inside the long time, \( m \) is the mass of gas molecule, \( a \) is the radius of the tube, and \( \lambda \) is the mean free path between two successive collisions. \( n(z) \) is the gas concentration between the two ends of the tube. The concentration is a slowly varying function of the longitudinal coordinate \( z \). Thus, in this case the characteristic length scale of \( n(z) \) is much larger than the radius of the tube, \( a \). The following section will deal with the details of free molecular aerodynamics and the associated momentum accommodation for the cases when reflection is not completely diffuse, where the value of tangential momentum accommodation coefficient is less than 1.
1.1.2.4 Free molecular aerodynamics

In the free molecular flow regime, Knudsen number is of the order of 1 or more. In such flows occurring at very low gas densities, the interaction between the gas molecules is neglected and only the interaction between the gas molecules and the solid surface is taken into account. As shown before, the mean free path is inversely proportional to the density of gas and may be calculated using the equation (1.4) (Vincenti, and Kruger, 1965).

\[
\lambda = \frac{1}{\sqrt{2}\pi m d^2} = \frac{m}{\sqrt{2} \pi \rho_g d^2} = \frac{kT}{\sqrt{2} \pi P d^2}.
\]  

(1.4)

In the above equation, \(m\) is the mass of gas molecule, and \(d\) is the collision diameter of the molecule. For air, \(m\) is approximately \(48 \times 10^{-27}\) kg and \(d\) is \(4.2 \times 10^{-10}\) m. \(\rho_g\) is the gas density, \(P\) is the gas pressure, \(T\) is the gas temperature, and \(k\) is the Boltzmann constant. For air under room temperature, the mean free path is approximately \(5.0 \times 10^{-8}\) m. It may also be noted that above approximately 150 km, the condition \(Kn >> 1\) is satisfied. In the h~150 km, the interaction is free molecular but the number density is still high enough to make macroscopic averages meaningful. In the free molecular regime, the molecules that hit the surface are reflected and travel very far before colliding with other molecules. Thus, the interaction of the reflected particles with the incident stream may be neglected. Some of the basic assumptions of free molecular flow are as follows:

a. In the free molecular flow regime, Knudsen number is of the order of unity or more. In other words, the mean free path of the gas molecules is larger than the characteristic length dimension of the body which is assumed to be in a gas flow of infinite extent.
b. The molecules that hit the surface of the body are reflected or re-emitted, and usually travel very far from the body before colliding with other molecules. The reflected molecules therefore ‘forget’ their previous interaction with the body before they hit again. Also, the effect of reflected molecules on the particle streams incident on the body is considered negligible. In other words, the incident flow is assumed to be completely undisturbed by the presence of the body.

As an immediate consequence of the basic assumptions, no shock wave is expected to form in the free molecular flow regime. The boundary layer will be very thick and diffuse and will have no effect on the incident flow. From the practical point of view of the main assumptions, it follows that in the free molecular flow regime, the incident and the reflected or re-emitted molecules may be treated separately. The incoming gas particles are assumed to be in local thermodynamic equilibrium characterized by Maxwell-Boltzmann velocity distribution. The Maxwell-Boltzmann velocity distribution shows the velocity distribution of a gas in equilibrium and it is given by equation (1.25) (Gombosi, 1994).

\[
f(v) = \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left( -\frac{mv^2}{2kT} \right).
\]  

(1.25)

When \( Kn >> 1 \), there is only a small probability that one of the scattered molecules will interact with the incoming beam in such a way that it will be scattered back to the body. There is an even smaller probability that it will strike the body before it reaches equilibrium with the local beam after many collisions. The assumption that the reflected particles play only a negligible role in the formation of incident particles may not be true for some special cases. For example in the case of concave surfaces, the reflected elements may directly reach another surface element.
without colliding with other molecules first. The analysis of free molecular flows around concave surfaces is much complicated and proper incident streams must be taken into account. Hence, in the following section, free molecular flow around convex surfaces will be addressed.

1.1.3 Transfer of tangential momentum and the momentum accommodation

The determination of the flux of mass, momentum, or energy carried by the gas molecules reflected or re-emitted from the surface of the body requires a specification of the interaction between the impinging particles and the surface. For detailed description of the interaction, it is important to determine the velocity distribution function of the re-emitted particles. However, in the free molecular flow regime, such detailed knowledge is not necessary and it is sufficient to know certain average parameters characterizing the interaction. As has been discussed before for slip flows, there are two different types of interactions between a solid surface and the gas molecules. In specular reflection between the gas molecules and a surface, the gas molecules hit a surface and bounce back like ‘bouncing balls.’ This interaction is assumed to be elastic. In diffuse reflection the gas molecules hit a surface and are then re-emitted in random directions. The direction of the reflected particles is randomly distributed in $2\pi$ solid angle, and the velocity distribution is Maxwellian. The two different modes of interaction between the gas molecules and a solid surface provide insight to the transfer of mass, momentum and translational energy. However, for the purpose of this research work, only momentum transfer and hence tangential momentum accommodation coefficient will be discussed. In the section on slip flow, it was assumed that a fraction $\sigma_t$ of the incident particles is reflected diffusely, while the rest is reflected specularly. The diffusely reflected fraction contributes to the tangential momentum transfer between the molecules and the wall. The remaining fraction, $(1-\sigma_t)$ is reflected specularly with a reversal of the normal momentum component. This fraction
contributes only to the normal component of the momentum transfer to the wall. Based on some experimental evidence, the situation is more complicated in free molecular flow and a single parameter $\sigma_t$ may not be sufficient to characterize both the normal and tangential momentum transfer in an adequate manner. In order to adequately specify the tangential component of the momentum accommodation coefficient, equation (1.26) is used (Gombosi, 1994).

\[
\sigma_t = \frac{\Phi_i^{(mv)}}{\Phi_r^{(mv)} - \Phi_w^{(mv)}} .
\]  

(1.26)

In equation 1.25, $\sigma_t$ is the tangential momentum accommodation coefficient for a given gas versus surface interaction. $\Phi_i^{(mv)}$ is the magnitude of the incident tangential momentum flux and $\Phi_r^{(mv)}$ is the magnitude of the reflected tangential momentum flux. In order to calculate incident tangential momentum from the dilute gas to the surface of a solid body, it is assumed that the incident molecules are temporarily trapped by the surface and they lose their tangential momentum on contact. The incident flux of the tangential momentum is given by equation (1.27).

\[
\Phi_i^{(mv)} = mn_i \left( \frac{\beta_t}{\pi} \right)^{1/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dv_1 dv_2 \left[ v_i \exp(-\beta_t c_1^2) \exp(-\beta_t c_2^2) \right] \int_0^\infty dv_3 \exp(-\beta_t c_3^2), \]  

(1.27)

where $v_i$ is the projection of the particle velocity to the $(x_1, x_2)$ plane which is tangential to the surface element. The integrals over $v_1$ and $v_2$ may be readily evaluated and the equation (1.28) for incident tangential momentum flux is obtained.

\[
\Phi_i^{(mv)} = mn_i \left( \frac{\beta_t}{\pi} \right)^{1/2} u_i \int_{-\infty}^{\infty} dc_3 v_3 \exp(-\beta_t c_3^2). \]  

(1.28)
The incident tangential momentum flux is related to the incident mass flux by the equation (1.29):

$$
\Phi_i^{(mv)} = u_i \Phi_i^{(m)}. \quad (1.29)
$$

The incident mass flux, $\Phi_i^{(m)}$, is given by equation (1.30).

$$
\Phi_i^{(m)} = \frac{1}{4} mn_i \bar{v}_i \left( \exp\left(-s_3^2\right) + \sqrt{\pi} s_3 \left[ 1 + \text{erf}(s_3) \right] \right), \quad (1.30)
$$

where $m$ is the molecular mass of the incident gas, $n_i$ is the number of molecules incident on the surface, and $s_3$ is a dimensionless velocity term given by equation (1.31).

$$
s_3 = \beta_i^{1/2} u_3 = u_3 \sqrt{\frac{m}{2kT_i}}. \quad (1.31)
$$

Here $u_3$ is the random particle velocity. Thus the following equation is derived for the incident tangential momentum flux.

$$
\Phi_i^{(mv)} = p_i \left\{ s_i \left[ \frac{\exp(-s_3^2)}{\sqrt{\pi}} + s_i s_3 \left[ 1 + \text{erf}(s_3) \right] \right] \right\}, \quad (1.32)
$$

where $p_i$ is the incident gas pressure, and $s_i$ is a non-dimensionless tangential gas velocity given by equation (1.33):

$$
s_i = u_i \sqrt{\frac{m}{2kT_i}}, \quad (1.33)
$$

where $k$ is the Boltzmann constant and $T_i$ is the gas temperature. In the case of diffuse reflection, the reflected molecules are re-emitted isotropically. Therefore the reflected flux of tangential
momentum must vanish. Thus it follows that \( \Phi_{w}^{(mv)} = 0 \). This is also true because the directional distribution is symmetric around a normal vector of the surface element. Hence, equation 1.26 may be re-written as the following:

\[
\sigma_{i} = \frac{\Phi_{i}^{(mv)} - \Phi_{r}^{(mv)}}{\Phi_{i}^{(mv)}}.
\]  \hspace{1cm} (1.34)

In the case of specular reflection, the incident tangential momentum flux \( \Phi_{i}^{(mv)} \) is equal to the reflected tangential momentum flux \( \Phi_{r}^{(mv)} \) and it follows that the tangential momentum accommodation coefficient is zero. In the case of diffuse reflection, the reflected tangential momentum flux \( \Phi_{r}^{(mv)} \) is zero. Hence in the case of diffuse reflection, tangential momentum accommodation coefficient is 1.

Also the shearing stress acting on the surface is the difference of the incident and reflected fluxes of tangential momentum. Thus the equation of shear stress associated with a surface versus gas interaction in free molecular flow is given by the following equation for the case of diffuse reflection when \( \sigma_{i} = 1 \) (Gombosi, 1994).

\[
\tau_{i} = p_{i}s_{i}\left[ \frac{\exp\left(-\frac{s_{i}^{2}}{\pi}\right)}{\sqrt{\pi}} + s_{i}\left[1 + \text{erf}\left(s_{i}\right)\right] \right].
\]  \hspace{1cm} (1.35)

It will be noted that in equation (1.34), shear stress associated with the gas-surface interaction is no longer a function of dynamic viscosity as in the continuum regime. However, equation 1.34 describes the condition of diffuse reflection when the entire gas molecules incident on a surface are absorbed by the surface and then re-emitted in random directions with the velocity being set by wall temperature.
This situation corresponds to a tangential momentum accommodation coefficient of 1. However, in the most practical cases, it has been shown by researchers that the values of tangential momentum accommodation coefficients for most gas-surface interactions are between 0 and 1. Thus the equation for shear stress in free molecular flow is revised as follows (Gombosi, 1994):

\[ \tau_i = \sigma_i p_i s_i \left( \frac{\exp(-s_i^2)}{\sqrt{\pi}} + s_i \left[ 1 + \text{erf}(s_i) \right] \right), \]  

(1.36)

where \( \sigma_i \) is the momentum accommodation coefficient. The following section will detail some of the earlier work by researchers associated with the measurement of tangential momentum accommodation coefficient.

1.2 Previous work in rarefied gas dynamics

Since the past century, researchers have been involved with the identification of continuum breakdown and measurement of momentum accommodation for gas-surface interactions in free molecular flow regime.

1.2.1 Application of Knudsen number to viscous flows

The Knudsen number was named after Martin Knudsen and was universally accepted as a parameter for determining the flow regime. Several researchers in the twentieth century became associated with the science of rarefied gas flows. H.S. Tsien in 1946 was one of the earliest researchers associated with the field of gas dynamics who defined a new variable known as Tsien’s parameter, which was given by equation (1.37):
\[ T_{si} = M_\infty / \sqrt{Re_\infty} . \]  

(1.37)

Here \( M_\infty \) is the Mach number of a free stream and \( Re_\infty \) is the corresponding Reynolds number (Tsien, 1946). Tsien had shown that for hypersonic velocities, the flow was rarefied when \( T_{si} \) was approximately greater than 1. When \( T_{si} \) was between 0.01 and 1, Tsien termed the flow as slip flow. Tsien’s work stated that the first failure in continuum theory would occur at gas-solid interfaces where the empirical conditions of continuity of tangential velocity should give rise to slip and temperature-jump boundary conditions. It was important to know that for small yet significant mean free paths, if it made sense to solve Navier-Stokes equations subjecting them to slip boundary conditions. The other question was whether the influence of slip boundary conditions became significant only at a degree of rarefaction where Navier Stokes equations failed completely. Professor Harold Grad’s studies of the continuum theory as an approximation to the Boltzmann equation warned that in principle the Navier Stokes equations with slip boundary conditions lacked consistency; however its trial in certain cases yielded satisfactory results. A principal result of Grad’s studies was that Navier Stokes equations were asymptotically equivalent to the Boltzmann equation as the degree of rarefaction was decreased. The exceptions to this law were found in the thin Knudsen layers next to the solid walls, and in strong shock waves (Grad, 1963).

In 1961, Cheng considered the hypersonic flow around a blunt body and introduced Cheng’s parameter which is as follows (Cheng, 1961).

\[ C_{\ast} = \frac{C^* M_\infty^2}{Re_\infty} , \]  

(1.38)
where \( C^* = \mu'T_*/\mu_*T^* \) and \( T^* = (T_s + T_w)/2 \). \( T_s \) is the temperature immediately behind the normal bow shock and \( T_w \) is the body wall temperature. \( \mu_* \) is the viscosity of the free stream. Cheng’s parameter was hence known as a modified Tsien’s parameter and was used as an identification of continuum breakdown in hypersonic flows.

Later in 1977, Bird proposed that for a high speed expanding flow, the onset of non-equilibrium conditions was governed by a local ‘break-down’ parameter, \( P \). He defined the parameter in both unsteady and steady flow using equations (1.39 and 1.40) (Bird, 1977).

\[
P \propto \frac{\nabla \rho}{\rho} = \frac{\Lambda}{L_\rho}, \quad \text{for steady flow} \tag{1.39}
\]

\[
\text{And } \quad P = \frac{D \ln \left( \frac{\rho}{\rho_\infty} \right)}{Dt}, \quad \text{for unsteady flow} \tag{1.40}
\]

In equation (1.39) \( U \) is the flow speed, \( L_\rho \) is the flow length derived from density gradients and \( \Lambda = U\tau \) is a local mean free path. \( \rho_\infty \) is the free stream gas density.

Several researchers performed numerical simulations to predict continuum breakdown. Boyd (Boyd, 2003), in his work in 2003 showed that continuum breakdown in gas expansions and shock waves proceeds through different physical mechanisms and was best predicted by different types of breakdown parameters. It was explained that for a gas expansion, as the gas expanded rapidly from an orifice, the temperature and the density decreased along the jet axis and hence decreased the molecular collision rate. This behavior lead to freezing of the internal energy modes in which the temperatures of the vibrational and rotational modes leveled out to
values higher than the translational temperature. The translational temperature was considered in terms of one mode in the direction parallel to the jet axis and two modes perpendicular to the jet axis. At large distances from the jet source, the population of molecules close to the axis had very small thermal velocities in the perpendicular direction; otherwise they would drift away from the axis. Thus in the absence of collisions, the translational temperature of the perpendicular modes continued to decrease with distance away from the source. It was also shown that the parallel translational mode did not experience the geometric effect and the temperature only continued to decrease while there were collisions to provide mixing between the perpendicular and parallel modes. The parallel translational energy mode froze at a temperature that was higher than that corresponding to the perpendicular mode. The mentioned phenomena were measured experimentally by Cattolica at al. (Cattolica et al., 1974) and computed using DSMC by Bird (Bird, 1994) in free jet expansions. For shock waves, it was explained that in the frame of reference of a stationary shock wave, it involved the mixing of the velocity distribution functions of two very different equilibrium populations of gas which happened at a very small length scale. Thus very different physical processes governed the continuum breakdown in shock waves. Non-equilibrium velocity distribution functions in the middle of a hypersonic shock wave have been measured experimentally and using DSMC by Pham-Van-Diep et al. (Pham-Van-Diep, 1989).

The effects of continuum breakdown have also been studies in subsonic flows. Martin and Boyd studied momentum and heat transfer in a laminar boundary layer with slip flow over a flat plate (Martin, and Boyd, 2006). Flow was simulated in the laminar boundary layer using a slip boundary condition and a scaling based on the boundary layer thickness. The slip condition changed the boundary layer structure from a self-similar profile to a two-dimensional structure.
While the slip conditions generally led to a decrease in overall drag, two-dimensional effects caused local increases in skin friction. With increases in the slip parameter in the boundary layer, the velocity increased and the wall shear stress decreased.

Aziz expanded these results to boundary layer flow over a flat plate with slip flow and constant heat flux surface condition (Aziz, 2010). Martin and Boyd also studied the laminar boundary layer flow over a wedge with slip boundary conditions (Martin, and Boyd, 2010). The results showed decreased skin friction, boundary layer thickness, velocity thickness, and momentum thickness because of the introduction of slip on the boundaries. Laminar slip boundary layer flows over a circular cylinder has also been studied (Seo, and Song, 2012). It has been shown that with the increase in slip length, the drag coefficient decreases since the frictional component of drag is reduced. Slip effects on mixed convective flow and heat transfer from a vertical plate has been studied (Cao, and Baker, 2009). The results were presented for the effect of non-continuum upon the slip velocity, temperature jump, wall shear stress, and boundary layer thickness in both gases and liquids. In another work, Cao and Baker modeled natural convection over a vertical isothermal plate using first order momentum and thermal discontinues on the wall (Cao, and Baker, 2009 #2). It was shown that the presence of nonequilibrium at the wall resulted in a nonsimilar boundary layer problem. Nonsimilar velocity and temperature distributions within the boundary layer were obtained. Results showed effects of nonequilibrium on wall slip velocity, wall shear stress, and boundary layer thickness for both gaseous and liquid flows.

Several previous researchers have simulated continuum breakdown in viscous hypersonic flows. Particle and continuum methods of a high altitude extreme Mach number reentry flow have been studied (Ozawa et al., 2010). Zhong, Ozawa, and Levin, compared high-
altitude hypersonic wake flows of slender and blunt bodies using the Direct Simulation Monte Carlo (DSMC) method. Near-wake flows were characterized by features of low density, low Reynolds number, high temperature, and thermal nonequilibrium. For the two geometries, significant differences in the near-wake flow were observed in the spatial distribution of gas temperatures, the degree of chemical dissociation, and the sensitivity of recirculation length to freestream number density (Zhong, Ozawa, and Levin, 2008). Lofthouse, Boyd and Wright studied the effects of continuum breakdown on hypersonic aerodynamics (Lofthouse, Scalabrin, and Boyd, 2008). Holman and Boyd investigated the effects of continuum breakdown on the surface properties of a hypersonic sphere (Holman, and Boyd, 2009). They observed that as the global Knudsen number was increased, the amount of continuum breakdown in the flow and in the surface was increased. The surfaces properties were affected as there was an increase in the differences between CFD and DSMC. When the Mach number was increased, the amount of continuum breakdown observed in the flow was increased, but the gradient length local Knudsen number remained approximately constant.

1.2.2. Measurement of momentum accommodation

The experimental measurement of tangential momentum accommodation coefficient has been done by very few researchers since the past century. Some of the significant work are as follows:

1.2.2.1 Millikan’s rotating cylinder experiment

In 1923, Millikan performed the rotating cylinder experiment for the measurement of slip coefficient between a surface and gas molecules. This experiment followed the famous oil drop where he noticed that the experiments by which the value of the electronic charge was
determined by the droplet method gave consistent results only when this law was modified by a factor \((1 + A*\frac{l}{a})\), where \(l\) was the mean free path and \(a\) was the radius of the droplet. As shown in Figure 1.10, the experiment involved co-axial cylinders where the inner cylinder was rotated and the torque on the outer cylinder was measured (Millikan, 1923).

As shown in Figure 1.10, the inner torsionally suspended cylinder had radius \(a\), while the outer cylinder had radius \(b\). The length of the inner cylinder was \(L\). The angular velocity of the layer of gas at any radius between \(a\), and \(b\) was \(\omega\). It was shown that at lower gas pressures, the ‘no slip’ boundary condition was not valid and slippage was necessary at the boundaries. The slip coefficient associated was derived and was given by the following equation.

\[
\xi = \left( \frac{\mu}{\mu_a} - 1 \right) \frac{1}{K}. \tag{1.41}
\]

\(\mu_a\) was the apparent viscosity of the gas when ‘no slip’ boundary conditions were applied. \(\mu\) was the actual viscosity of the gas when slip boundary conditions were applied. \(K\) was a dimensionless number based on the dimensions of the two cylinders:

\[
K = \frac{2(a^3 + b^3)}{ab(b^2 - a^2)}. \tag{1.42}
\]
It will be noted that since many materials used in the aerospace industry are available as planar materials with very limited ability to put them into non-planar geometries, the methods that require co-axial cylinders are difficult to implement.

1.2.2.2 Gabis et al.’s magnetically levitated sphere

Gabis et al. used a spinning rotor gauge to measure the torque that was induced on a suspended spherical rotor by gas molecules. Spheres of diameters, 2 mm, 2.25 mm, and 2.75 mm were used in the system shown in Figure 1.11. The measurement of spin-down time of the levitated spheres was done. The time for the spheres’ rotation to slow down was measured. The torque and momentum accommodation in transition flow regime was extracted (Gabis, Loyalka, and Storvick, 1996).

![Figure 1.11: Gabis et al.’s experimental setup](image)

Figure 1.11: Gabis et al.’s experimental setup
One shortcoming of this method was that it required the material to be magnetic and formed into a sphere and hence would not be useful for many aerospace or microfabrication related materials.

1.2.2.3 Arkilic et al.’s gaseous slip flow in long micro-channels

Arkilic et al., in their work in 1997, performed an analytic and experimental investigation into gaseous flow with slight rarefaction through long micro-channels. They performed a 2D analysis of the Navier-Stokes with a first order slip velocity boundary condition. The effect of slip upon the pressure distribution was also derived by them and it was obtained that the slip velocity would lead directly to the wall normal migration of mass. They fabricated a two wafer system that included the channel wafer and a capping wafer.

The channels were 7500µm long, 52µm wide and 1.33 µm deep. Helium at 300 K was used for analysis within the micro-channels. A Knudsen Number of 0.155 was reported at the outlet of the micro-channel. They mentioned that although the outlet Knudsen Number was within the transitional flow regime, the results of the slip model with full tangential momentum accommodation coefficient seemed to fit the experimental data nicely (Arkilic, Schmidt, and Breuer, 1996). It will be noted here that in this system the flow can range from the continuum regime to the free-molecular flow regime over the length of the channel. In such situations, the determination of local shear stress becomes extremely complicated.

As a result, it may be concluded that the measurement of pressure drop through a micro-channel was not an ideal method for the measurement of momentum accommodation coefficient.
1.2.2.4 Torque on a stationary disc near a rotating disc – Chambers et al.

Chambers et al., in their work in 1992, measured the torque on a stationary disc placed near a rotating disc. The velocity of the gas molecules reflected from the rotating disc would depend on the angular velocity of the rotating disc, and the momentum accommodation coefficient of the rotating disc. As these molecules collided with a nearby stationary disc, a torque was induced that depended on these quantities, the ambient conditions and the momentum accommodation coefficient of the stationary disc (Chambers, Chew, and Troup, 1992). This method had two weaknesses. First, it assumed that the momentum accommodation on both the discs were identical. Second, because it relied on the torque created by the particles impinging from the rotating disc, instead of measuring the torque on the stationary disc, the sensitivity of the method was reduced.

1.3 Applications of rarefied gas dynamics

The science of rarefied gas dynamics finds its applications in several fields of engineering. Some of the significant applications are into upper atmosphere aerodynamics, micro-technology, and the vacuum systems. The following sub-sections detail some of these applications.

1.3.1 Upper atmosphere aerodynamics

The knowledge of the flow physics associated with rarefied gases is important in the field of upper atmosphere aerodynamics. In the upper atmosphere, flow becomes rarefied due to increase in the mean free path between two successive collisions between air molecules.
The Knudsen number increases with the increase in mean free path. In the case of re-entry vehicles there are large variations of flow around the vehicles leading to some regions being in continuum while others in free molecular flow regimes.

An example is a hypersonic flow over a blunt re-entry vehicle. In the fore-body region, the free-stream gas is compressed and heated by a bow shock wave. For free stream conditions near peak heating and peak dynamic pressure, the flow is continuum in this region. In the wake of the vehicle, the flow expands and goes through the transition flow regime and may become free-molecular (Boyd, Chen, and Chandler, 1995).

A technology termed as ‘aero-braking’ may be employed by vehicles that uses atmospheric drag to land successfully on a planet (Walberg 1985). Using this technology, a vehicle arriving at a planet will skim through the upper atmosphere of the planet to lose momentum and drop into the orbit. Much of the vehicle’s time will be spent in rarefied flow regimes under these conditions. Thus the knowledge of rarefied flow physics becomes important for such technologies to be successfully employed.

1.3.2 Micro-technology

In the micro-scale, flow characteristics may be rarefied because of the small length scale. Under ambient conditions, while the mean free path between two successive collisions of gas molecules remains the same, the length scale associated vastly diminishes in the micro-regime. As a result a relatively large Knudsen number may be obtained and the physics of flow may differ from continuum.

The knowledge of flow physics is critical for the success of micro-technology. For example, in micro-devices such as micro-resonators vibrating in any medium other than high
vacuum, it has been seen that the dissipation to the surrounding fluid is a major loss mechanism. Such damping is critical to the performance of the resonators and has been determined in the free molecular flow regime (Martin et al., 2008).

Micro technology finds its usage in many applications. Micro-machines have had their impact in the fields of medicine, biology, optics, mechanical and electrical engineering. In various bio-medical applications, fluid transport is required in drug delivery and in chemical and DNA analyses (Ho, and Tai, 1998).

Micro-devices such as micro-mixers, micro-pumps, and micro-valves have been associated with such applications (Zhang, Xing, and Li, 2007). Micro-electromechanical devices came into being as early as the mid 1960s and significant commercial production of such devices started in the 1980s. In the 1990s, micro-accelerometers were being used as triggers for air bags in cars. Volume production of micro-fluidic devices began in this decade.

Vibrating cantilevers, bridge and paddle micro-structures have been used in mass sensing of hazardous compounds (Lavrik, Sepanik, and Datskos, 2004), atomic force microscopy (Martin, Fathy, and Houston, 2008), radio-frequency devices (Nguyen, 1999), data storage (Vettiger et al., 2002), pressure measurement (Bianco et al., 2006), temperature measurement (Roy et al., 1991), and optical micro-electromechanical system (Toshiyoshi, and Fujita, 1996).

Researchers have also studied rarefied flows through long micro-channels and have developed a mass flow measurement technique for micro-channel flow (Arkilic, Schmidt, and Breuer, 1997). In the micro-channel the flow associated may be rarefied because of large Knudsen numbers associated with the small length scale. Flow in a microchannel may hence vary from continuum through the free molecular flow regime.
1.3.3 Vacuum systems

In vacuum systems, the density of gas is much lesser than under atmospheric conditions. Hence the mean free path between two successive collisions of gas molecules would be larger than the mean free path under ambient conditions. An example of such vacuum systems is chemical vapor deposition (CVD), which is used to produce high purity, high performance solid materials. This process is used in the chemical industry to produce thin films. During CVD the vapor is exposed to one or more volatile materials which react or decompose on the substrate surface to produce the desired deposit. Rarefied gas transport in a localized region over the CVD feature have been simulated by researchers to study the evolution of the film profile which provides dynamic step-coverage performance and micro-structural detail of the growing film (Coronell, and Jensen, 1994).

1.4 Motivation for research

The failure of continuum equations in gas dynamics under certain physical conditions is the reason for the development of the field of rarefied gas dynamics. It has been widely accepted that the continuum equations become invalid at large Knudsen numbers. Physically a large Knudsen number translates to the lack of space to accommodate sufficient number of collisions in a gas to maintain equilibrium velocity distribution function given by Maxwell-Boltzmann velocity distribution function. At large global Knudsen numbers, the entire gas flow system may be modeled using non-continuum kinetic methods such as Direct Simulation Monte Carlo (DSMC). However, such kinetic methods are computationally expensive. It was observed that although many researchers over the decades were involved with numerical predictions of continuum break-down, very few were exclusively involved with experimentation.
This research has three objectives. The first objective is to perform an experimental measurement of continuum breakdown. Next, the tangential momentum accommodation coefficient for interactions between air and different materials is measured using the uniquely developed experimental apparatus that allows TMAC measurements in the free molecular flow regime for boundary layer type flows. Finally it is shown that the experimental facility is capable of TMAC measurements associated with other gases such as carbon dioxide and argon.

In this work a thin disc is levitated in vacuum, rotated and is allowed to spin down. The spin down time is extracted from the experimental data and the corresponding deceleration torque is measured. The torque is non-dimensionalized suitably and is plotted against Reynolds number from atmospheric pressure through the free molecular flow regime and the continuum breakdown is measured. The tangential momentum accommodation coefficient (TMAC) in the free molecular flow regime is extracted using an analytic equation derived for the system in the free molecular flow regime. The details of the conceptual design of experimental facility and corresponding results will follow in the subsequent chapters.
Chapter 2 Design of a Disc Spin-down Experiment

2.1 The disc spin-down experiment

The first objective of this research is to measure continuum breakdown. For this purpose, a disc spin-down experiment is planned in various ambient air pressures. The disc is mounted on a shaft, rotated, and is spun down at different ambient pressures of air as shown in Figure 2.1. As the disc spins down, the deceleration torque on the disc surface is measured. The deceleration torque in the continuum flow regime is due to the viscous losses associated with the rotation of the disc. As flow becomes rarefied, the viscous losses reduce. In the free molecular flow regime, the deceleration torque is no longer due to the viscous losses.

A 3.5 inch diameter disc is initially used for the experiments. The thickness of the disc is 0.5 mm. The disc is mounted on a 0.25 inch solid steel shaft. Chapter 3 of this manuscript will show the results with the 3.5 inch diameter disc. In that chapter it will also be shown that the 3.5 inch diameter disc is inadequate towards accomplishment of the research objectives and
therefore an 8.25 inch diameter disc is used to achieve the desired objectives. The next section discusses the rotating disc problem in the various flow regimes.

2.2 Rotating disc in the viscous flow regime

The deceleration torque on the rotating disc in the viscous flow regime is studied using an established semi-analytical model such as the von Karman Viscous Pump, and using computational fluid dynamics.

2.2.1 von Karman Viscous Pump

The von Karman Viscous Pump model assumes that a large disc rotates at a constant angular velocity $\omega$ about an axis beneath a Newtonian viscous fluid which is otherwise at rest. The viscous drag of the rotating disc sets up a swirling flow towards the disc. Because of radial symmetry, the velocity components $v_r$, $v_\theta$, and $v_z$ and the pressure $p$ are independent of $\theta$. Hence $v_r$, $v_\theta$, $v_z$, and $p$ are solved as functions of $r$ and $z$. The governing equations are as follows:

$$\frac{1}{r} \frac{\partial}{\partial r} (rv_r) + \frac{\partial}{\partial z} (v_z) = 0,$$

(2.1)

$$v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\mu}{\rho} \left( \frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} + \frac{\partial^2 v_r}{\partial z^2} - \frac{v_r}{r^2} \right),$$

(2.2)

$$v_r \frac{\partial v_\theta}{\partial r} + v_z \frac{\partial v_\theta}{\partial z} = \frac{1}{r} v_r v_\theta = \frac{\mu}{\rho} \left( \frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial v_\theta}{\partial r} + \frac{\partial^2 v_\theta}{\partial z^2} - \frac{v_\theta}{r^2} \right),$$

(2.3)

$$v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\mu}{\rho} \left( \frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2} \right).$$

(2.4)

The boundary conditions are given as:
At $z = 0$: \( v_r = v_z = 0, \) \hspace{1cm} (2.5)

\( v_0 = r\omega, \) \hspace{1cm} (2.6)

\( p = \text{constant} = 0, \) \hspace{1cm} (2.7)

At $z \to \infty$: \( v_r = v_0 = \partial v_z / \partial z = 0, \) \hspace{1cm} (2.8)

von Karman in his paper showed the solution to this problem (Kármán 1921). A similarity solution was found assuming that \( v_r/r, v_\theta/r, v_z, \) and \( p \) are functions of \( z \) only. Since the only parameters in the problem were \( \omega \) and \( v \), a dimensionless variable \( z^* \) was defined by the following equation:

\[ z^* = \frac{z \sqrt{\omega / v}}{\sqrt{\omega}}. \] \hspace{1cm} (2.9)

The new dimensionless variables \( F, G, H, \) and \( P \) were proposed as follows:

\[ v_r = r\omega F(z^*), \] \hspace{1cm} (2.10)

\[ v_\theta = r\omega G(z^*), \] \hspace{1cm} (2.11)

\[ v_z = \sqrt{\omega} v H(z^*), \] \hspace{1cm} (2.12)

\[ p = \rho \omega v P(z^*). \] \hspace{1cm} (2.13)

As a result, equations 2.1 through 2.4 may be reduced to the following equations:

\[ H' = -2F, \] \hspace{1cm} (2.14)

\[ F'' = -G^2 + F^2 + F'H, \] \hspace{1cm} (2.15)
Equations (2.5) through (2.9) are used to write the non-dimensional boundary conditions:

\[ F(z^* = 0) = H(z^* = 0) = P(z^* = 0) = 0, \]
\[ G(z^* = 0) = 1; F(z^* = \infty) = G(z^* = \infty) = 0. \]

In the above set of equations, equation (2.17) is an uncoupled equation. Therefore equations (2.14), (2.15), and (2.16) may be solved before solving for \( P \) in equation (2.17).

Accurate numerical solutions to the set of equations (2.14) through (2.19) were obtained by Roger and Lance in 1960 (Rogers and Lance 1960). A Runge-Kutta subroutine was used to solve the equations.

Initial conditions were chosen in a way such that \( F \) and \( G \) vanished for large values of \( z^* \).

The correct initial conditions were chosen as:

\[ F'(z^* = 0) = 0.5102, \]
\[ G'(z^* = 0) = -0.6159. \]

The circumferential wall shear stress on the disc is given by:

\[ \tau_{z\theta} = \mu \left. \frac{\partial u_\theta}{\partial z} \right|_{z=0} = \rho \nu G'_{0} \frac{\mu}{\rho} \omega^3, \]

where \( G'_{0} = G'(0) = -0.6159 \).

The torque on a rotating disc will be given by:
\[ T = 2 \int_{r_i}^{r} r \rho \gamma dA. \]  \hspace{1cm} (2.24)

Using equations (2.22), (2.23), and (2.24), the torque will be given by:

\[ T = G_o \left(r_o^4 - r_i^4\right)\pi \omega \frac{3}{2} \mu \frac{1}{2} \rho \frac{1}{2}. \]  \hspace{1cm} (2.25)

where \( G_o = -0.6159 \), \( r_o \) is the radius of the disc, \( r_i \) is the radius of the shaft, \( \omega \) is the angular velocity of the disc, \( \mu \) is the dynamic viscosity, and \( \rho \) is the density of the fluid. Therefore equation 2.25 may be used to predict the torque associated with a disc in the viscous flow regime. Researchers have shown that the von Karman Viscous pump model matches with experimental results between \( Re = 4000 \) and \( Re = 3.2 \times 10^5 \). Flow instabilities were encountered above \( Re = 3.2 \times 10^5 \). The Reynolds number was calculated based on the radius of the disc (Theodorsen and Regier 1944).

### 2.2.2 Computational fluid dynamics (CFD)

Computational fluid dynamics (CFD) simulations are performed with both the 3.5 inch diameter disc and with the 8.25 inch diameter disc. CFD is performed in order to obtain an acceptable shape and size of the test chamber and to anticipate the pressure at which the free molecular flow regime may be encountered. The torque on the disc rotating with various angular velocities and at various ambient pressures of air is calculated.

#### 2.2.2.1 CFD with the 3.5 inch diameter disc

An acceptable shape and size of the test facility is one that will not show signs of wall effects on the disc surface. Gambit 2.2 is used as a modeling and meshing tool while Fluent 6.2 is used for the simulations. Steady state simulations are performed for the disc rotating at various
speeds between 1000 radians per second and 10 radians per second. Wall shear stress contour profiles are plotted on the surface of the disc in order to investigate for any wall effects. The first set of simulations is performed for the disc rotating inside a tetrahedral chamber of size 28 inches x 28 inches x 28 inches. Since the disc is axisymmetric, only a quarter of the disc face is simulated. The shear stress contour profile on the surface of the disc, from the simulation inside the tetrahedral chamber is shown in Figure 2.2. The contours of shear stress on the surface of the disc are non uniform and are distorted on the edges. The distortion in the wall shear stress contour profile suggests wall effects due to the inadequate shape of the test chamber.

![Shear stress unit: N/m²](image)

Figure 2.2: Shear stress contours on the disc rotating at 1000 rad/s – tetrahedral chamber

For subsequent simulations, a cylindrical chamber of diameter 28 inches by length 28 inches is chosen.

#### 2.2.2.1 Modeling and meshing technique

Since the disc is axisymmetric, it is sufficient to model only a quarter of the geometry with the disc rotating inside. A large volume is created that contains both the test chamber geometry and the disc at the center. A second fluid volume is created by extruding from the face of the disc. This volume is split from the large volume. The ‘splitting’ operation helps continue
the mesh from the disc geometry to the test chamber wall. Hexagonal elements are used for meshing the simulation geometry. Adaptive meshing technique is used with the mesh being finer near the surface of the disc. Total number of elements is 3920 and the total number of nodes is 4950. ‘Symmetry’ boundary condition is applied on the central plane of the disc. Periodic boundary conditions are applied on the perpendicular planes in both the smaller volume and the larger volume. Wall boundary conditions are applied to all the other planes. The disc is simulated for its rotations in air at various pressures from atmospheric through the slip flow regime where Knudsen Number is of the order of 0.01 and the meshed geometry is shown in Figure 2.3.

![Figure 2.3: Quarter model of the cylindrical test chamber with the 3.5 inch diameter rotating disc](image)

**2.2.2.1.2 Simulation results – 3.5 inch diameter disc**

Simulations are performed with the disc rotating with various angular velocities ranging from 10 rad/s through 700 rad/s at ambient pressures from atmospheric pressure through 0.001 atmospheres. The Reynolds Number associated with the flow with the disc rotating at 700 rad/s in atmospheric pressure is about 378730. Three dimensional laminar steady state simulations are performed for the disc rotating at 700 rad/s inside the cylindrical test chamber. Pressure based solver with implicit formulation is used. Pressure discretization is standard while momentum
discretization is first order upwind. The under relaxation factors for pressure, density, body forces and momentum were 0.30, 1.0, 1.0, and 0.70 respectively. Pressure-velocity coupling is done using ‘Simple Algorithm’ scheme. Wall shear stress contours on the surface of the disc are investigated for any wall effects. Figures 2.4 through 2.7 show the wall shear stress contour profile for the disc rotating in atmospheric pressure at various speeds.

Figure 2.4: Wall shear stress (Pa) at 1 atm.,
700 rad/s

Figure 2.5: Wall shear stress (Pa) at 1 atm.,
500 rad/s

Figure 2.6: Wall shear stress (Pa) at 1 atm.,
200 rad/s

Figure 2.7: Wall shear stress (Pa) at 1 atm.,
100 rad/s

The wall shear stress contour profiles suggest that the wall effects on the surface of the disc rotating inside the test chamber of diameter 28 inches by length 28 inches are negligible.
The size of the test chamber is increased to 56 inches by 56 inches and then to 100 inches by 100 inches. There are no significant changes on the wall shear stress contour profile. Thus the cylindrical test chamber with a diameter of 28 inches by a length of 28 inches is determined to be an acceptable shape and size of the test chamber.

Computational fluid dynamics is also used to determine the gas pressure corresponding to a Knudsen Number of $1 \times 10^{-2}$, which may be assumed as the limit of continuum-slip flow regime. Free molecular flow regime is assumed to be present at two orders of magnitude below this limit. Although no slip boundary conditions are applied in this flow regime for the purpose of simulations, only an error of about 5% is expected. More simulations are performed with the disc rotating at various speeds, from 10 radians per second to 700 radians per second, and at various pressures from $1 \times 10^{-1}$ atm through $1 \times 10^{-3}$ atm. Figures 2.8 through 2.10 show wall shear stress contour profiles of the disc rotating at 700 radians/s at various gas pressures between atmospheric pressure and $1 \times 10^{-3}$ atmospheres.

Figure 2.8: Wall shear stress (Pa) at 0.1 atm., 700 rad/s
Figure 2.9: Wall shear stress (Pa) at 0.01 atm., 700 rad/s
Knudsen Number is the ratio of the mean free path to the gradient length scale where $\lambda$ is the mean free path between two successive collisions of gas molecules. $L$ is the gradient length scale (Vincenti, and Kruger, 1965).

$$\lambda = \frac{m}{\sqrt{2\pi\rho_g d^2}}. \quad (1.4)$$

For air, the molecular mass $m$ is $48.1 \times 10^{-27}$ kg, and collision diameter $d$ is $4.19 \times 10^{-10}$ m (Bird, 1994). The gradient length scale could be represented by the following equation (Oran, Oh, and Cybyk, 1998). Also for boundary layer type flows this length scale is equivalent to the boundary layer thickness.

$$L = \frac{V_{mag}}{dV_{mag}/dz}. \quad (2.26)$$

In equation 1.4, the mean free path $\lambda$ varies inversely as the gas density $\rho_g$. The gradient length scale in equation 2.41 may be calculated by knowing the velocity magnitude and the velocity gradient at each of the gas pressures. Figure 2.11 shows the plot of Knudsen number versus pressure for the 3.5 inch diameter disc.
Figure 2.11: Knudsen Number as a function of gas (air) pressure – 3.5 inch disc

The gas pressure corresponding to the slip limit is evaluated as $1 \times 10^{-4}$ atmospheres approximately. Hence, the free molecular flow regime is anticipated at a gas pressure that is two orders of magnitude lower than this pressure. Hence the free molecular flow domain is expected at an air pressure of $1 \times 10^{-6}$ atmospheres, which is equivalent to $1 \times 10^{-1}$ Pa or $7.6 \times 10^{-4}$ Torr.

Summarizing, a cylindrical test chamber of diameter 28 inches by length 28 inches is sufficient for the disc spin-down experiments in order to minimize wall effects on the surface of the disc. It is also learnt that a vacuum pressure of $7.6 \times 10^{-4}$ Torr may be necessary inside the test chamber to achieve free molecular flow.

**2.2.2.2 CFD with the 8.25 inch diameter disc**

The experimental results with the 3.5 inch diameter disc have been shown in chapter 3. Also it will be shown that the 3.5 inch diameter disc is inadequate towards the achievement of the desired results. Therefore a larger disc of diameter 8.25 inches is used for the experiments. Following are the computational fluid dynamics results with the 8.25 inch diameter disc.
2.2.2.2.1 Modeling and meshing technique – 8.25 inch diameter disc

Similar to the smaller disc, only a quarter of the 8.25 inch diameter disc is modeled since the disc is axisymmetric in nature. The geometry is created by integrating three pieces – the disc and the shaft with the extrusion of the disc through the wall of the chamber, a section immediately above the disc, and the rest of the chamber much above the disc.

The section with the disc is composed of hexagonal elements with 2506416 nodes. The total number of elements in this piece is 2437500. The section immediately above the disc has 975000 hexagonal elements with 1017456 nodes. The third piece is further away from the disc and for this section an adaptive mesh has been used. In this section the hexagonal elements increase in size with a growth ratio of 1.07 through the first 15 rows. Thereafter the same mesh is continued through the rest of the volume. The total number of elements used in the entire volume is 3481416. Figure 2.12 shows the meshed geometry with the 8.25 inch diameter disc inside the cylindrical chamber.

Figure 2.12: Quarter model of the cylindrical test chamber with the 8.25 inch diameter disc
2.2.2.2 Simulation results – 8.25 inch diameter disc

For the large disc the wall shear stress contours are plotted to investigate for wall effects. The simulations are performed from 50 rad/s through 300 rad/s in steps of 50 and at 345.6 rad/s. Figure 2.13 shows the wall shear stress contours on the surface of the 8.25 inch diameter disc rotating at 345.6 rad/s in atmospheric pressure.

![Wall shear stress contours on the 8.25 inch diameter disc at 1 atm – 345.6 rad/s](image)

Figure 2.13: Wall shear stress contours on the 8.25 inch diameter disc at 1 atm – 345.6 rad/s

The uniform wall shear stress contours on the surface of the disc shows that the wall effects on the surface of the disc are still negligible. This shows that the cylindrical chamber with a diameter of 28 inches and length 28 inches is an acceptable shape and size for the 8.25 inch diameter disc as well. The definition of Knudsen number is used to obtain the plot of Knudsen number versus pressure for the 8.25 inch diameter disc. The pressure corresponding to the continuum slip flow limit is obtained through CFD simulations. A Knudsen number of 0.01 is assumed at the continuum slip flow limit.

Figure 2.14 shows the Knudsen number versus the ambient pressure for the 8.25 inch diameter disc.
These results show that with the large disc the continuum slip flow limit is obtained at an ambient chamber pressure of approximately 100 Pa. This suggests that the free molecular flow regime may be encountered at a chamber pressure of 1 Pa or less. Figure 2.15 is the plot of Knudsen number versus angular velocity.
Figure 2.15 shows that the Knudsen number does not change significantly with angular velocity. The gradient length scale is equivalent to the boundary layer thickness which reduces with increase in angular velocity of the disc and is shown in Figure 2.16.

![Gradient length scale versus angular velocity](image)

**Figure 2.16: Gradient length scale versus angular velocity**

Figure 2.17 shows that the gradient length scale only slightly reduces along the radius of the disc.

![Gradient length scale versus radial distance](image)

**Figure 2.17: Gradient length scale versus radial distance**
In Figure 2.17 the rotational speed of 300 rad/s is chosen. The steep decrease in the gradient length scale between $r = 0$ and $r = 0.003175$ m is because of the presence of the shaft on which the disc is mounted. However the length scale over the surface of the disc does not change significantly.

Figure 2.18 is the plot of Knudsen number versus the radial length over the disc. The plot shows that Knudsen number does not change significantly over the surface of the disc at various ambient pressures. Therefore the flow regime does not change over the surface of the disc as it rotates at a given rotational speed in an ambient air pressure.

![Figure 2.18: Variation of the Knudsen number over the surface of the disc](image)

### 2.2.3 Non-dimensionalization of the deceleration torque during disc spin-down

The computational fluid dynamics results show that a cylindrical chamber of diameter 28 inches by length 28 inches is acceptable for the spin down experiments with both the 3.5 inch diameter disc and the 8.25 inch diameter disc. It also shows that for the 8.25 inch diameter disc the continuum slip flow limit may be encountered at an ambient pressure of 100 Pa inside the
test chamber. Therefore the free molecular flow regime may be encountered at an ambient pressure of 1 Pa or less. Both the CFD results and the von Karman Viscous Pump analysis help predict the deceleration torque on the 8.25 inch diameter disc in the viscous flow regime. In order to measure continuum breakdown a scale is chosen such that the non-dimensional curves are self similar only in the continuum flow regime. Therefore torque is non-dimensionalized in using the equation (2.27):

$$T^* = \frac{T}{\mu \omega D^3},$$

(2.27)

where $T^*$ is the non-dimensional torque, $\mu$ is the dynamic viscosity, $\omega$ is the angular velocity, and $D$ is the disc diameter. The scale $\mu \omega D^3$ has the effect of dynamic viscosity of the gas. This scale will work in the viscous flow regime while it will gradually fail when viscosity is no longer dominant.

### 2.2.4 Comparison between von Karman and the CFD results

The deceleration torque on the 8.25 inch diameter disc is non-dimensionalized using the scale shown and is plotted against Reynolds number. The torque given by the von Karman Viscous Pump model is given by equation (2.25) which is as follows:

$$T = G_o \left( r_o^4 - r_i^4 \right) \pi \omega^{3/2} \mu^{1/2} \rho^{1/2},$$

(2.25)

where $G_o = 0.6159$, $r_o =$ radius of the disc $= 0.105$ m, $r_i =$ radius of the shaft $= 0.003175$ m, $\mu$ is the dynamic viscosity, $\omega$ is the angular velocity, and $\rho$ is the gas density. The torque is non-dimensionalized using the scale shown and is as follows:
\[ T^* = 0.02534 \mu^{-\frac{1}{3}} \rho^{\frac{1}{3}} \omega^{\frac{1}{3}}. \] (2.28)

Equation (2.28) may be re-written as:

\[ T^* = \frac{0.02534}{D} \sqrt{\text{Re}}, \] (2.29)

where \( D \) = diameter of the disc. Therefore for a 0.21 m diameter disc, equation (2.29) reduces to:

\[ T^* = 0.05536 \sqrt{\text{Re}}. \] (2.30)

The non-dimensional curves obtained using CFD and von Karman Viscous Pump are superimposed on a plot against Reynolds number and is shown in Figure 2.19. The plot shows a good agreement between CFD and von Karman Viscous Pump results through a Reynolds number of approximately 200 after which the CFD curve levels out.

![Figure 2.19: Comparison between von Karman and CFD results](image)
2.3 Rotating disc in the free molecular flow regime

The earlier section shows how the disc spin down experiment may be modeled in the continuum flow regime using computational fluid dynamics (CFD) and using the von Karman Viscous Pump model. Also a scale is developed to non-dimensionalize torque in the viscous flow regime. In the free molecular flow regime it is assumed that viscosity is no longer the dominant quantity. Therefore an analytic expression for the deceleration torque on the disc in the free molecular flow regime for the disc spin-down experiment is derived and is as follows.

As shown in chapter 1, the shear stress \( \tau \) on the surface of the disc in the free molecular flow regime is given by equation (1.35):

\[
\tau = \sigma_i P_i s_t \left[ \frac{\exp(-s_3^2)}{\sqrt{\pi}} + s_3 [1 + \text{erf}(s_3)] \right],
\]

(1.35)

\[
s_3 = \sin(\alpha) \sqrt{\frac{m}{2kT_i}} u,
\]

(2.31)

\[
s_t = \cos(\alpha) \sqrt{\frac{m}{2kT_i}} u,
\]

(2.32)

where \( \sigma_i \) is the TMAC for a specific gas versus surface interaction, \( P_i \) is the ambient pressure, \( s_3 \) and \( s_t \) are normalized velocities based on thermal velocity \( c \) and incident angle \( \alpha \), \( m \) is the mass of gas molecule, \( k \) is the Boltzmann Constant, \( T_i \) is the ambient temperature, and \( u \) is the tangential velocity associated with the rotation of the disc. The thermal velocity \( c \) is represented by the equation (2.33):

\[
c = \sqrt{\frac{2kT_i}{m}}.
\]

(2.33)
When the incident angle is 0, the term \( s_t \) disappears, and the shear stress on the surface of the disc is obtained:

\[
\tau = \frac{\sigma_t P_i s_t}{\sqrt{\pi}}.
\]  

(2.34)

Replacing \( s_t \), equation (2.34) can be re-written as:

\[
\tau = \frac{\sigma_t P_i u}{\sqrt{\pi}} \sqrt{\frac{m}{2kT_i}} = \frac{\sigma_t P_i r \omega}{\sqrt{\pi}} \sqrt{\frac{m}{2kT_i}},
\]  

(2.35)

where the tangential velocity \( u \) is the product of \( r \) and \( \omega \), \( r \) is the radius of the disc and \( \omega \) is the angular velocity of the rotational disc. Shear stress can be used to obtain the torque on the disc using equation (2.36).

\[
T = \int \tau dA \times r = \int_\rho_0^{\rho_1} 2\sigma_t P_i \omega \sqrt{\frac{m\pi}{2kT_i}} r^3 dr.
\]  

(2.36)

Therefore the free molecular torque on the rotating disc-shaft system is derived as:

\[
T = \sigma_t P_i \omega \sqrt{\frac{m\pi}{2kT_i}} \left( r_o^4 - r_i^4 \right).
\]  

(2.37)

The deceleration torque is a product of the disc moment of inertia and the angular deceleration and hence equation (2.38) evolves:

\[
-I \frac{d\omega}{dt} = \sigma_t P_i \omega \sqrt{\frac{m\pi}{2kT_i}} \left( r_o^4 - r_i^4 \right),
\]  

(2.38)

where \( I \) is the moment of inertia, \( \omega \) is the angular velocity, \( r_o \) is the radius of the disc, and \( r_i \) is the radius of the shaft. The moment of inertia is given by the following:

\[
I = \frac{1}{2} M \left( r_o^2 - r_i^2 \right),
\]  

(2.39)
where $M$ is the mass of the rotating disc. The differential equation of angular velocity as a function of time may be solved to obtain TMAC in the free molecular flow regime. The decelerating torque associated with the disc will also include a contribution from the bearing. Assuming the bearing torque, $B$ to be a linear function of angular velocity, $\omega$, it will follow:

$$B = B_o \omega, \quad (2.40)$$

$$-l \frac{d\omega}{dt} = \sigma_i P_i \frac{m \pi}{2kT_i} \left( r_o^4 - r_i^4 \right) \omega + B_o \omega \quad , \quad (2.41)$$

Equation (2.41) may be restructured as:

$$\frac{d\omega}{dt} = -\frac{\sigma_i P_i}{l} \frac{m \pi}{2kT_i} \left( r_o^4 - r_i^4 \right) \omega - \frac{B_o \omega}{l} \quad . \quad (2.42)$$

Here the angular velocity $\omega$ is a function of time, $t$, $B_o$ is a constant associated with the bearing, $P_i$ is the ambient pressure, $r_o$ is the radius of the spinning disc, $r_i$ is the radius of the shaft on which the disc is mounted, $m$ is the mass of gas molecule, $k$ is the Boltzmann constant, $T_i$ is the ambient temperature, and $\sigma_t$ is the TMAC. Equation (2.42) is a first order ordinary differential equation. The solution to the differential equation is as follows:

$$\omega = \omega_o \exp\left[ -\left( \frac{\sigma_i P_i}{l} \frac{m \pi}{2kT_i} \left( r_o^4 - r_i^4 \right) + \frac{B_o}{l} \right) t \right] . \quad (2.43)$$

In equation (2.43), $\omega_o$ is the rotational speed at which the spin down starts. In the free molecular flow regime the angular velocity is only a function of the TMAC since the other quantities are constants. TMAC can be evaluated using the derived solution of the spin-down
differential equation. Also assuming a TMAC value of 1, an estimate for the maximum acceptable bearing torque is calculated as $10^{-7}$ Nm.

### 2.4 Summary

In this chapter computational fluid dynamics (CFD) simulations suggest that a cylindrical chamber with diameter 28 inches by length 28 inches is acceptable for the disc spin down experiment. Simulations also suggest that the free molecular flow regime may be encountered at an ambient pressure of 1 Pa or less inside the chamber. A scale is derived such that the non-dimensional torque curves are self similar in the viscous flow regime. The non-dimensionalized torque curves using CFD simulations and analytical von Karman model show good agreement through a Reynolds number of 200 approximately. Also an analytic equation for the free molecular torque on the disc is derived as a function of the tangential momentum accommodation coefficient.
Chapter 3 Initial Experiments

A disc spin-down experiment is performed in various ambient pressures from atmospheric pressure through the free molecular flow regime. The disc is 3.5 inches (0.09 m) in diameter and 0.5 mm thick. As shown in Figure 3.1, the disc is mounted on a 0.25 inch diameter (0.006 m) solid steel shaft which is held between frictionless bearings. The following sections detail the components used in the system.

![Diagram of the spin-down experiment with bearings](image)

Figure 3.1: The spin-down experiment with bearings

3.1 System integration

The initial experiments are performed using the 3.5 inch diameter disc. The computational fluid dynamics simulations shown in chapter 2 suggest that a cylindrical chamber of diameter 28 inches by length 28 inches is an acceptable shape and size of the test chamber. The analysis also shows that the free molecular flow regime may be encountered at a chamber pressure of 1 Pa or less. In order to facilitate frictionless rotation of the disc, an air bearing fixture is used. A motor clutch system is used to spin-up the disc mounted on the shaft and to disengage it for spin-down. A stainless steel vacuum chamber of diameter 30 inches by length 30 inches (0.76 m x 0.76 m) is used. As shown in Figure 3.2, the chamber is connected to a rotary vane pump and a turbomolecular pump. The data acquisition system is constructed by connecting...
a laser sensor/amplifier system to a computer through National Instruments DAQ hardware and the software interface is done using National Instruments LabView. Following is the block diagram of the initial experimental apparatus.

![Block Diagram of Initial Experimental Apparatus](image)

**Figure 3.2: The initial experimental apparatus**

### 3.1.1 The vacuum chamber

The vacuum chamber is 0.76 m in diameter and 0.76 m in length. The chamber has one door on its flat face. There is a glass window at the center of the door which can be used for monitoring the disc spin down experiment. The chamber is connected to the vacuum pumps which are explained in the following section. Figure 3.3 shows a picture of the vacuum chamber connected to the turbo-molecular vacuum pump at one end and with the instrumentation for measuring real time pressure and temperature inside the chamber.
The details of the ports on the vacuum chamber are as follows:

a. Vacuum port: The vacuum port has an outer diameter of 0.038 m, has a NW25 flange and a manual valve for controlling pressure inside the vacuum chamber.

b. Vacuum gauge port: Two vacuum gauge ports, with outer diameter of 0.025 m are located on the top surface of the vacuum chamber. One port has an ion vacuum gauge installed with the digital read-out, while the other one has a convection vacuum gauge installed. While the convection gauge can measure the pressure inside the chamber from atmospheric through a vacuum pressure of $1 \times 10^{-3}$ Torr (0.13 Pa), the ion gauge is capable of measuring pressures from $10^{-3}$ Torr through $10^{-8}$ Torr ($1.33 \times 10^{-6}$ Pa).

c. Vent port: This is a 0.038 m outer diameter port with NW25 flange and a manual valve. This is used while restoring the chamber back to atmospheric conditions.
d. Electrical feed-through: A NW25 port is used for a 20 pin, 700 V, 50 A per pin electrical feed-through. The feed-through is required to provide power to a motor-clutch system used for spin up and spin down of the disc.

e. Temperature reader feed-through: A NW25 port is used for a temperature reader. The temperature reader is connected via an RTD cable and is compression fitted. The temperature controller has the following features:

• 1/16 DIN panel size
• Thermocouple, RTD, milli-ampere, milli-volts and voltage inputs
• Output # 1- Relay, voltage pulse, current or linear voltage
• Output # 2- Relay or voltage pulse for control or alarm output
• RS 485 communications port

f. Thermocouple feed-through: A separate NW16 port for 2 pin type K thermocouple feed-through.

g. Drain plug: A 0.025 m outer diameter port with NW25 flange is connected to a drain plug used for increasing the chamber vacuum pressure to any higher pressure below atmospheric with greater control.

h. Gate valve port: An ISO 160 port is available for a gate valve, required for the turbo-molecular pump

i. View port: An ISO 200, 0.2 m viewing port is constructed on the stainless steel door.
j. Air inlet and scavenging ports: Four ¼ inch NPT full coupling ports are constructed. Two of these are used for air bearing stage operations while the other two are kept for unforeseen requirements and are plugged presently.

3.1.2 The vacuum pumps

Two vacuum pumps are used to reduce the chamber pressure from atmospheric through high vacuum. A rotary vane pump is capable of reducing the chamber pressure from atmospheric to as low as 13 Pa approximately, while a turbomolecular pump can reduce the chamber pressure to as low as 0.0001 Pa.

A rotary vane pump, Laco model W2V40 is used for roughing purposes. This pump is a two staged, oil sealed rotary vane pump with a capacity of 400 liters per minute. This has NW25 inlet and outlet ports. It works on a single phase motor with 120 volts and 60 Hz, and has an ON/OFF switch. This pump also has a clear sump molecular sieve trap mounted near the pump inlet port. An oil mist eliminator is connected to the pump exhaust. A clear 0.025 m diameter PVC hose with NW25 flanges, assorted clamps and C-rings are used to connect this pump to the vacuum port. This pump is used for reducing the pressure in the chamber from atmospheric through 0.10 Torr (13 Pa).

A turbomolecular pump, Edwards Next 300D is used to reduce the chamber pressure from 0.10 Torr (13 Pa) through $1 \times 10^{-6}$ Torr. This pump has a pumping speed of 300 liters per second. This pump has a nominal rotational speed of 60,000 rpm. The checklists related to the operation of the vacuum chamber and the pumps are provided in appendix 1. The following section explains the air bearing stage used.
3.1.3 The air bearing fixture

Frictionless rotation of the disc is desired during its spin-down. For this purpose, bearings are required that allow smooth frictionless rotation of the shaft. At the start, magnetic bearings are considered. However, since magnetic bearing are not off-the-shelf items and because of the complexities associated with their integration into the experimental system, the author eventually chose the air bearing as an alternative. An air bearing stage for the purpose is designed and manufactured by Guidance Dynamics Corporation, Simi Valley, California.

The air bearing stage has two $6.4 \times 10^{-3}$ m air bushings, which levitated the $6.4 \times 10^{-3}$ m steel shaft between them. The shaft has an $8.9 \times 10^{-2}$ m disc mounted on it. The disc rotates integrally with the shaft. The air bushings are designed for a maximum radial load of 12 N. A maximum air inlet pressure of 410 kPa is recommended for the bushings operating under maximum radial load. The air gap at maximum recommended load is $3.8 \times 10^{-6}$ μm. The bushing inside diameter is $6.4 \times 10^{-3}$ m, outside diameter is $1.5 \times 10^{-2}$ m, and the length is $3.2 \times 10^{-2}$ m. The mass of each bushing is 11 grams. These air bushings are standard parts bought from NEWWAY air bearings and are incorporated into the air bearing stage. Following is a detailed drawing of the $6.4 \times 10^{-3}$ m air bushing.

Figure 3.4 shows the air bushings have a pressure port through which air enters the bushing. The O-rings act as vacuum seals and create resistance in the path of leakage. The gap between two O-rings on either side may potentially be leakage paths through which air may escape into a given system. The air from these grooves needs to be scavenged out of the system. Therefore the air bearing stage is constructed in a way so as to have one air inlet connection and
two air scavenging connections to each bushing. The standard $6.4 \times 10^3$ m air bushings are housed inside aluminum housings.

Figure 3.4: 0.25” NEWWAY air bushing used for the spin-down experiment

Figure 3.5 shows the picture of the fabricated air bearing stage for the disc spin down experiment.

The details of the components of the air bearing stage are as follows:
a. Air bushing housing: The two air bushing housings are 0.036 m in length and 0.026 m in diameter. These have a through hole of diameter 0.065 m. Each bushing has an air inlet connection and two air scavenging connections.

b. Motor-clutch system: A vacuum compatible motor is chosen for the spinning up of the rotating shaft. The motor is 0.12 m long, with a maximum rated speed of 11,500 rpm and the maximum rated torque of 0.085 N-m. The speed of the motor can be controlled by a potentiometer kept outside of the vacuum chamber. The electrical feed-through is used for connecting the cables to the motor-clutch system. A solenoid for motor engagement with a bias spring together constitutes the clutch system for engagement and disengagement of the motor shaft with the spin-shaft.

c. Polyurethane hoses: These hoses are used for air inlet and air scavenging. These have an outer diameter of 0.0032 m and an inner diameter of 0.0016 m. The air inlet hose is used for pumping in air at about 410 KPa to the air inlet port on each bushing. The two scavenging ports are intended to be connected to a dedicated vacuum pump.

d. Base plate: The base plate supports the air bearing stage assembly.

Earlier CFD simulations were done for an ideal situation where the disc was mounted on a long shaft which was being held by frictionless bearings that were far apart from each other. An exactly similar arrangement was not available for experiments due to manufacturing constraints. However the air bearing fixture facilitates frictionless rotation inside the vacuum chamber. The frictional torque associated with the bearing may be measured using the spin-down experiment of the shaft by itself and can be subtracted from the cumulative torque.
3.1.4 The data acquisition system

The data acquisition system is used for logging in real time experimental data on to a computer for further post processing. The goal of this experiment is to record real time instantaneous rotational speed of the disc during spin down.

3.1.4.1 The laser sensor-amplifier system

A sensor is used for identifying a ‘dark’ mark on the surface of the rotating disc. A digital laser sensor with an amplifier is used for recording real time rotational speed as many times the dark mark on the disc passes through the laser sensor. The sensor has an adjustable focus and offers a maximum detection range of 1.2 m. The sensor-amplifier combination logs in data if the dark mark stays under the laser beam for at-least 80 µs and the time lag between the first and the second count is at-least 80 µs. It is evaluated that in order for the disc rotational speed to be read, a mark with a diameter of about 2 mm is sufficient. Figure 3.6 shows the location of the mark and the beam spot on the disc surface as the disc rotates.

![Diagram of the laser sensor-amplifier system](image-url)

Figure 3.6: Calculation of the size of the dark mark on the disc
R = radius of the disc = 0.044 m
r = radius of the dark mark to be evaluated
\( r_1 \) = adjustable radius of the beam spot = 0.4 mm = 4 x 10^{-4} m
\( 2\theta = \omega t \)
\( \omega \) = rotational speed of the disc at the start of spin-down
\( t \) = minimum time that the spot needs to spend under the beam

\[ \tan \theta = \frac{r - r_1}{R - r}. \] (3.1)

Thus \( \theta \) can be evaluated from the initial rotational speed \( \omega \), and from the time the mark spends under the laser beam. R and \( r_1 \) are known. Hence r is the only unknown which is evaluated. Figure 3.7 shows the plot of minimum size of the mark on the disc versus the angular velocity of the disc. Four different disc diameters of 3.5 inches, 6 inches, 6.5 inches, and 8.25 inches are used for this analysis.

Figure 3.7: Minimum required radius of the mark versus disc angular velocity
Figure 3.7 shows that if a large disc of diameter 8.25 inches is used for the spin-down experiments, the mark on the surface of the disc needs to be at least 4.5 mm in diameter if the disc is rotated at 1000 rad/s. Similarly for a 3.5 inch diameter disc rotating at 1000 rad/s, a mark with a diameter of 2 mm is sufficient. The three other disc diameters are simulated for the mark size because it will be shown by the end of this chapter that the 3.5 inch diameter disc is inadequate towards obtaining the research objectives.

3.1.4.2 Data logging hardware

The sensor-amplifier is connected to a National Instruments Data Acquisition Controller Board (DAQmx-9174) with NI module 9411 through which signals are read on to the computer. NI 9411 is a module designed for six channels, 500 ns differential/single ended digital inputs. Figure 3.8 shows the circuitry for real time measurement of rotational speed.

![Figure 3.8: Block diagram of the circuitry for real time measurement of rotational speed](image)

Each channel of the NI 9411 module is compatible with 5 to 24 V signals. NI 9411 is a correlated digital module and can perform correlated measurements, triggering and synchronization when installed in an NI Compact DAQ chassis. The digital input from the
amplifier is read and logged on to the computer and the driver for the same is written using NI software, LabVIEW 8.6.

3.1.4.3 LabView Programming

A LabVIEW code is written in order to have the sensor-amplifier system interact with the computer. A significant step for the creation of the code is making the counter input channel to frequency. The ‘edge’ parameter is used to determine if the counter will begin measuring on a rising or a falling edge. The ‘read VI’ is called to return the next frequency measurement. Figure 3.9 shows the LabView block diagram for real time rotational speed measurement.

Figure 3.9: LabView block diagram for real time measurement of rotational speed

A 10 second timeout is set so that an error is returned if a period is not returned in the specified time limit. The test facility combines the four systems namely the vacuum chamber, vacuum pumps, the air bearing stage, and the data acquisition system. Successful operation depends on precise system integration which involves effective air inlet and scavenging to and
from the two air bushings, calibrating the sensor/amplifier system and writing the driver using LabVIEW 8.6.

3.2 Experimental procedure

As mentioned, the DC motor is used to rotate the shaft held between the bearings. The electromagnetic clutch lets it disengage from the rotating shaft at the start of the spin down procedure. As the disc spins down, a laser beam focuses on to a dark mark on the disc face. A count is generated each time the mark passes through the beam and at each instant the number of counts is communicated to the computer via the data acquisition system. From the logged in data, the viscous torque on the disc is measured using the following steps.

\[ \omega = 2\pi n, \]  

(3.2)

\[ \alpha = \frac{\Delta \omega}{\Delta t}, \]  

(3.3)

\[ I = \frac{1}{2} M \left( r_o^2 - r_i^2 \right), \]  

(3.4)

\[ T = I \alpha. \]  

(3.5)

In the above equations, \( n \) is the number of disc rotations each second. \( \omega \) is the instantaneous angular velocity, \( \alpha \) is the instantaneous angular deceleration, \( M \) is the mass of the disc, \( r_o \) is the radius of the disc, \( r_i \) is the radius of the shaft on which the disc is mounted, \( I \) is the moment of inertia of the disc, and \( T \) is the instantaneous decelerating torque associated with disc rotation. Hence, using equations 3.2 through 3.5 the instantaneous deceleration torque is measured and is plotted as a function of the instantaneous angular velocity over the duration of spin down.
3.3 Shaft and bearing torque

In order to measure the contribution of the bearing and the shaft torque, experiments are performed without the disc, but with the shaft being spun down at various gas pressures. The laser beam sensor is focused on to a dark mark on the curved surface of the shaft. As many times as the mark passed through the beam, a count is generated and is logged on to the computer using the designed data acquisition system. Various shaft spin-down experiments are performed from atmospheric pressure through a vacuum pressure of 10 Torr (1300 Pa). Following are the spin-down curves of the shaft in a representative high pressure and a low pressure.

In Figures 3.10 and 3.11, the grey curve shows the raw data. A polynomial curve-fit is used to obtain the smoothened spin-down data. The goodness of fit is 0.996.

![Figure 3.10: Shaft spin-down in an ambient pressure of 703 Torr](image)

\[ y = -0.0043x^3 + 0.2896x^2 - 7.776x + 74.849 \]

\[ R^2 = 0.9959 \]
Figure 3.11: Shaft spin-down in an ambient pressure of 10 Torr

For all the spin-down data from atmospheric pressure through an ambient pressure of 10 Torr, the deceleration torque is obtained as a function of the shaft angular velocity. It is observed that all the torque curves at different pressures collapse on one another as shown in Figure 3.12.

Figure 3.12: Shaft torque versus angular velocity
3.4 Preliminary experimental results

The experiments are performed using 3.5 inch diameter discs of three different materials. During the experiments it is observed that the existing experimental apparatus shown in Figure 3.2 is inadequate to achieve the research objectives as the chamber pressure cannot be reduced below 1300 Pa (10 Torr). The issues associated are explained in the final section of this chapter.

3.4.1 Aluminum disc

The first set of experiments is performed on an aluminum disc. The aluminum disc is 0.089 m in diameter and was 0.50 mm thick. The aluminum disc weighs 33 grams. Spin-down tests are performed from atmospheric pressure through 10 Torr (1.3 KPa). Figures 3.13 and 3.14 show the representative raw data and curve-fitting are shown for the aluminum disc in atmospheric pressure and in an ambient pressure of 10 Torr.

![Graph showing raw data and curve-fitting for aluminum disc]

Figure 3.13: Aluminum disc raw data in atmospheric pressure
Figure 3.14: Aluminum disc raw data in 10 Torr

Figure 3.15 shows the smoothened spin-down curves for the 3.5 inch diameter aluminum disc from atmospheric pressure through 10 Torr.

Figure 3.15: Rotational speed versus time for the aluminum disc
The instantaneous rotational speed is used to calculate the instantaneous deceleration torque on the surface of the disc using equations (3.2) through (3.5). The deceleration torque is plotted as a function of instantaneous angular velocity.

Figure 3.16 shows the plot of instantaneous deceleration torque on the surface of the disc versus the disc angular velocity for the aluminum disc. It is observed that the deceleration torque is a linear function of angular velocity.

The tests are performed at various pressures from atmospheric (100 kPa), through an air pressure of 10 Torr (1.3 kPa). Tests are performed in steps of 100 Torr from atmospheric pressure through 100 Torr (13 kPa), and then at 50 Torr (6.7 kPa) and 10 Torr (1.3 kPa). At higher pressures, the curves have a steeper gradient than the curves at lower pressures. Also the linearity of the curves becomes more apparent at low pressures.
3.4.2 Acrylic disc

The acrylic disc is 0.089 m in diameter, 0.5 mm in thickness, and weighs 23 grams. The moment of inertia of the disc is $2.26 \times 10^{-5}$ Kg-m$^2$. For the acrylic disc, the tests are performed in steps of 100 Torr (13 KPa) from atmospheric pressure (100 KPa) through 100 Torr (13 KPa), and then in steps of 10 Torr from 100 Torr through 10 Torr (1.3 KPa).

Figure 3.17 shows the raw data and curve-fitting for the aluminum disc in atmospheric pressure.

![Graph showing the raw data and curve fitting for the acrylic disc in atmospheric pressure. The goodness of fit is 0.999.](image)

**Figure 3.17: Acrylic disc raw data in atmospheric pressure**

A third order polynomial curve fit is used to obtain the smoothened form of the raw data. The goodness of fit is 0.99.

Figure 3.18 shows the raw data and the curve fitting in 10 Torr (1300 Pa) which is a representative low pressure.
Figure 3.18: Acrylic disc raw data in 10 Torr

Figure 3.19 shows the smoothened raw data for the acrylic disc from atmospheric pressure through 10 Torr.

Figure 3.19: Rotational speed versus time for the acrylic disc

The raw data is post-processed to obtain the disc deceleration torque as functions of angular velocity and is shown in Figure 3.20.
As in the case of the aluminum disc, torque is a linear function of angular velocity. Similar to the aluminum disc, the gradient of the torque curves reduce with reduction in pressure.

### 3.4.3 Steel disc

Another set of experiments is performed with a steel disc having the same diameter of 0.089 m and the same thickness of 0.50 mm. The following figure shows the plot of rotations per second versus time for the steel disc. The steel disc weighs 38 grams. The moment of inertia of the steel disc is $3.73 \times 10^5 \text{ Kg-m}^2$.

Figure 3.21 shows the spin-down raw data for the steel disc in atmospheric pressure. A fourth order polynomial fit is used with a goodness of fit of 0.99.
Figure 3.21: Steel disc raw data in atmospheric pressure

Figure 3.22 shows the steel disc raw data in 10 Torr. Similar to the spin-down data in atmospheric pressure, a fourth order polynomial fit is used with a goodness of fit of 0.99.

Figure 3.22: Steel disc raw data in 10 Torr
The smoothened raw data in all ambient pressures from atmospheric pressure through 10 Torr are obtained and is shown in Figure 3.23.

Figure 3.23: Rotational speed versus time for the steel disc

Figure 3.24 shows the plot of torque versus disc angular velocity for the steel disc:

Figure 3.24: Instantaneous deceleration torque versus angular velocity – steel disc
The tests on the steel disc are performed at various air pressures from atmospheric pressure through a vacuum pressure of 100 Torr (13 KPa) in steps of 100 Torr. Below 100 Torr, the tests were performed at 50 Torr (6.7 KPa) and 10 Torr (1.3 KPa). At with the other discs, torque appears to be a linear function of angular velocity.

In chapter 2 a governing equation for the disc torque in the free molecular flow regime is derived which is as follows:

\[
T = \sigma_t P_i \omega \sqrt{\frac{m \pi}{2kT_i}} \left( r_2^4 - r_1^4 \right)
\]

(2.37)

It is observed that this equation does not fit into the torque data at the lowest data point of 1.3 KPa. Hence the free molecular flow regime is not reached as yet.

### 3.4.4 Non-dimensionalization of torque

Equation (2.27) is used to non-dimensionalize torque and it is plotted against Reynolds number:

\[
T^* = \frac{T}{\mu \omega D^3}
\]

(2.27)

The non-dimensional curves are expected to be self similar in the viscous flow regime. In continuum the disc torque is due to the viscous losses. As flow becomes rarefied, the effect of viscosity reduces and it becomes negligible in the free molecular flow regime. A viscous scale is will show self similarity, and therefore continuity in the viscous regime. Also the viscous scale will fail as continuum breaks down. Figure 3.25, Figure 3.26, and Figure 3.27 are the plots of non-dimensional torque versus Reynolds number for the aluminum, acrylic, and steel discs respectively.
Figure 3.25: Non-dimensional torque versus Reynolds number – aluminum disc

Figure 3.26: Non-dimensional torque versus Reynolds number – acrylic disc
Figure 3.27: Non-dimensional torque versus Reynolds number – steel disc

Figure 3.28 shows the plot for the aluminum, acrylic, and the steel disc superimposed.

Figure 3.28: Combined plot of non-dimensional torque versus Reynolds number
The above results show that the non-dimensional curves from atmospheric pressure through 1.3 kPa collapse on each other for all the different materials. This shows self-similarity of the non-dimensional curves within the range of pressures. However, these results do not convey any information about the torque characteristics at lower pressures. Therefore, continuum breakdown cannot be measured from the present set of results.

3.5 Summary

The results obtained in this chapter do not provide any information about continuum breakdown because experiments at lower pressures cannot be performed. There are two main problems that need to be addressed which are as follows:

a. High shaft torque: As shown in Figure 3.8 the shaft torque is of the order of $10^5$ N-m. Based on some calculations it is revealed that the disc torque in the free molecular flow regime will be of the order of $10^6$ N-m using the existing 3.5 inch diameter disc. Therefore, in order to have a larger disc torque as compared to the shaft torque, a larger disc of 8.25 inch diameter is chosen.

b. Inability to experiment at pressures lower than 1.3 kPa: The ambient pressure inside the test chamber cannot be reduced below 1.3 kPa with the existing set of vacuum pumps while the air bearing is powered on. It is observed that the leakage from the two air bushings into the vacuum chamber is too large for the only rotary vane pump to scavenge. Also, since the chamber pressure cannot be reduced to a pressure of 100 Pa or less, the turbomolecular pump cannot be switched on.

These issues will be addressed in the following chapter, and a modified experimental apparatus will be introduced in order to achieve the research objectives.
Chapter 4 Experimental Measurement of Continuum Breakdown

The two major problems associated with the apparatus used for initial experiments are – (a) high shaft torque from the air bearing fixture, and (b) inability to perform experiments below 1.3 kPa. Following measures are taken to fix these issues.

4.1 Experiments with a larger disc

One of the major problems with the existing apparatus is the large shaft torque compared to the deceleration torque on the 3.5 inch diameter disc in the free molecular flow regime. Assuming a tangential momentum accommodation of 1 the disc free molecular torque is atleast an order of magnitude lower than the shaft torque as shown below in Figure 4.1.

![Figure 4.1: Shaft torque and free molecular torque on the 3.5 inch diameter disc](image)

This suggests that in the free molecular flow regime the uncertainties associated with the measurement of disc torque will be extremely high. Therefore the present design of the air bearing fixture is inadequate for experimentation in the free molecular flow regime. By replacing
the 3.5 inch diameter disc by an 8.25 inch diameter disc, the disc torque in the free molecular flow regime can be increased to the order of $10^{-4}$ N-m. This value is approximately two orders of magnitude above the shaft torque. The modified air bearing fixture will allow more accurate measurement of torque during the spin-down of an 8.25 inch disc mounted on the 0.25 inch shaft.

4.2 Improved air-scavenging system

The ability of the apparatus to pull vacuum inside the chamber is completely dependent on the rotary vane pump and the turbomolecular pump attached to the chamber. However the turbomolecular pump only works when the pressure inside the vacuum chamber reduces to 300 Pa approximately. Since this pressure is never reached in the chamber, the turbomolecular pump is unused as yet. As a result, with the existing experimental apparatus a lowest pressure of only 1.3 kPa has been achieved inside the vacuum chamber.

Lower pressures inside the vacuum chamber are achieved by introducing additional air scavenging points on each air bushing and connecting them to separate dedicated vacuum pumps. A tortuous path is created between the first pair of scavenging ports and the second pair of scavenging ports, and between the second pair of scavenging ports and the annular gap between the shaft and the air bushings. The tortuous paths add resistance to the path of the air leakage and restrict air from escaping into the vacuum chamber.

The tortuous paths are created by making grooves on the shaft. The scavenging ports closest to the air inlet port are termed as the pre-tortuous path scavenging ports. These ports are connected to a large single phase rotary vane pump outside the chamber. The bulk of the air otherwise going into the vacuum chamber is scavenged from these ports. The ports further away
are the post-tortuous path scavenging ports. These are connected to a smaller rotary vane pump outside the vacuum chamber. The schematic of the experimental facility is shown in Figure 4.2.

![Figure 4.2: The modified experimental apparatus](image)

The location of the air bushings is elevated to accommodate for an 8.25 inch diameter disc. There is a clearance of 2 inches below the disc from the base of the fixture.

In addition to the rotary vane pump and the turbomolecular pump attached to the vacuum chamber, a high strength rotary vane pump is connected to the pre-tortuous path scavenging ports. For this purpose a single phase pump with a rated capacity of 30 m$^3$/hr is used. On the post tortuous path scavenging port, a dedicated rotary vane pump with a capacity of 3 m$^3$/hr is used.
Figure 4.3 shows the details of the modified air bushing assembly.

The red lines indicate the air inlet into the air bushing. The bright green lines adjacent to the inlet show the pre-tortuous path scavenging points. Bulk of the air is scavenged from these locations. The residual air encounters the tortuous path grooves and is scavenged at the post-tortuous path scavenging points shown by the dull green lines.

4.3 Shaft torque

With the modified fixture the shaft torque is measured to be of the order of $10^{-7}$ N-m which is at least two orders of magnitude below the anticipated free molecular disc torque. Also as before the shaft torque remained constant at all ambient pressures. Figure 4.4 shows the shaft torque as a function of angular velocity with the modified experimental apparatus.
Figure 4.4: Shaft torque versus angular velocity with the modified air bearing

Figure 4.5 shows a comparison between the shaft torques between the old fixture without the differential scavenging system and the new fixture with the differential scavenging system.

Figure 4.5: Comparison of the shaft torques between the old and the new systems
The reduction in the shaft torque may be due to the improved air scavenging system. As a result of the improved scavenging system the requirement of volume flow rate of air at the bearing inlet has reduced by 70% approximately. The results from the CFD simulations suggest that for the spin-down experiments the continuum slip flow limit exists around 100 Pa. Therefore the free molecular flow regime may be encountered at 1 Pa or less.

4.4 Experimental results

The spin-down experiments are performed with three aluminum discs of thickness 0.5 mm. The aluminum discs are 0.15 m (6 inches), 0.17 m (6.5 inches), and 0.21 m (8.25 inches) in diameter. The masses of the three discs are 47.37 grams, 29.35 grams, and 25.07 grams respectively. The real time rotational frequencies of the discs are logged into a computer. The raw spin-down data is curve fitted to obtain the smoothened spin-down curve. Figure 4.6 shows a representative case of the 0.15 m diameter disc spin-down in atmospheric pressure.

![Figure 4.6: Spin-down raw data and curve-fitting with the 0.15 m diameter disc in 1 atm.](image-url)
Figures 4.7, 4.8, and 4.9 show the rotational frequencies of the three discs as functions of time.

Figure 4.7: Rotational frequency versus time for the 0.15 m diameter disc

Figure 4.8: Rotational frequency versus time for the 0.17 m diameter disc

For each data point from atmospheric pressure through 228 Pa, the three rotary vane pumps shown in Figure 4.2 are used. The turbomolecular pump is switched on only when the rotary vane pumps reduce the ambient pressure inside the vacuum chamber to 228 Pa or less.
The turbo-molecular pump is used to operate at lower pressures and brings the ambient air pressure in the chamber down to less than 1 Pa. It is observed that at the lowest pressures the rotational frequency reduces very slowly over time.

![Figure 4.9: Rotational frequency versus time for the 0.21 m diameter disc](image)

The gradient of the torque curves gradually reduces with reduction in air pressure. Figures 4.10, 4.11, and 4.12 show the plots of disc torque versus angular velocity for 0.15 m, 0.17 m, and 0.21 m disc respectively. From the raw data, the plots for instantaneous deceleration torque versus angular velocity are generated for the three different sized aluminum discs using equations (3.2) through (3.5) shown in chapter 3 which are as follows.

\[ \omega = 2\pi n \]  
\[ \alpha = \frac{\Delta \omega}{\Delta t} \]  
\[ I = \frac{1}{2} M \left( r_o^2 - r_i^2 \right) \]  
\[ T = I\alpha \]
Figure 4.10: Torque versus angular velocity for the 0.15 m diameter disc

Figure 4.11: Torque versus angular velocity for 0.17 m diameter disc
It will be noted here that the torque in the viscous flow regime may not be a linear function of angular velocity while it is a linear function in the free molecular flow regime. This is why a linear interpolation may lead to a residual torque. The instantaneous deceleration torque obtained as a function of angular velocity at different air pressures is non-dimensionalized using equation (2.27) shown in chapter 2, which is as follows:

$$T^* = \frac{T}{\mu \omega D^3}.$$ \hspace{1cm} (2.27)

Therefore the non-dimensional torque for all the three diameters are evaluated and plotted against Reynolds number. As shown in Figure 4.13, non-dimensional curves for all the three discs between Reynolds numbers $10^3$ and $10^6$ overlap on one another. At the three lowest Reynolds numbers the non-dimensional torque does not change with Reynolds number.
The non-dimensional experimental results are superimposed with the plot shown by Figure 2.17. Figure 2.17 shows a good agreement between the non-dimensional CFD simulation results and the analytically obtained non-dimensional torque using the von Karman Viscous Pump model. The torque obtained from the CFD simulations are between atmospheric pressure and $1 \times 10^3$ atmospheres. The free-molecular flow torque given by equation (2.25) is calculated at the three lowest air pressures, non-dimensionalized and superimposed on this plot. Figure 4.13 shows the superimposed experimental, numerical, and analytical non-dimensionalized von Karman and free molecular torque as function of Reynolds number over the range of air pressures. It is seen that the von Karman and the CFD results overlap with the high Reynolds number experimental results between Reynolds numbers $10^5$ and approximately 300. This suggests self similarity and therefore continuity in the viscous flow regime. Continuum breaks down below a Reynolds number of approximately 300 where the experimental curves lose self similarity and the CFD results do not overlap with the experimental results anymore.

![Figure 4.13: Non-dimensionalized torque versus Reynolds number for all discs superimposed](image)

Figure 4.13: Non-dimensionalized torque versus Reynolds number for all discs superimposed
The experimental curves between Reynolds numbers 10 and 300 do not overlap with the continuum CFD, von Karman or the analytical free molecular flow results. This suggests that they are in the transition flow regime. The free molecular torque equation given by equation (2.25) is fitted to the data points at the lowest pressures. The tangential momentum accommodation coefficient (TMAC) for air-aluminum interaction is computed to be 0.74. This value is consistent for all three disc sizes. The free molecular flow regime is obtained at air pressures of 0.71 Pa, 0.89 Pa, and 1 Pa. The transition flow regime is encountered at air pressures between 1 and 100 Pa. The continuum flow regime exists at an ambient pressure of approximately 100 Pa or more inside the test chamber. These results are consistent with the values estimated using continuum CFD.

Figure 4.14 is a detailed plot of torque versus angular velocity in the transition flow regime.

Figure 4.14: Torque versus angular velocity in the transition flow regime
4.5 Experimental uncertainty analysis

The experimental uncertainty analysis is done for the measurement of torque. For this purpose equation (2.27) is considered which is as follows:

\[ T = \frac{1}{2} M (r_o^2 - r_i^2) \alpha, \]  

(2.27)

where M is the mass of the disc.

The general equation of experimental uncertainty is used which is given by:

\[ \varepsilon T = \sqrt{\left( \frac{\partial T}{\partial M} \varepsilon M \right)^2 + \left( \frac{\partial T}{\partial \alpha} \varepsilon \alpha \right)^2 + \left( \frac{\partial T}{\partial R} \varepsilon R \right)^2 + \left( \frac{\partial T}{\partial r} \varepsilon r_i \right)^2}. \]  

(4.1)

The total uncertainty in the measurement of torque is obtained at \(4.5 \times 10^{-7}\) N-m. This is approximately two orders of magnitude below the disc torque in the free molecular flow regime for all the three discs. The individual contributions to the total uncertainty from the measurements of mass, angular deceleration and the disc and shaft radii are given in Table 4.1.

Table 4.1: Contributions to experimental uncertainty

<table>
<thead>
<tr>
<th>Measured quantity</th>
<th>Uncertainty in measurement</th>
<th>Contribution to uncertainty (N-m)</th>
<th>Percentage contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>(5.0 \times 10^{-6}) kg</td>
<td>(1.78 \times 10^{-7})</td>
<td>15.55 %</td>
</tr>
<tr>
<td>(\frac{d\omega}{dt})</td>
<td>(1.0 \times 10^{-3}) rad/s^2</td>
<td>(2.6 \times 10^{-7})</td>
<td>33.18 %</td>
</tr>
<tr>
<td>(r_o)</td>
<td>0.01 mm</td>
<td>(3.23 \times 10^{-7})</td>
<td>51.21 %</td>
</tr>
<tr>
<td>(r_i)</td>
<td>0.01 mm</td>
<td>(9.78 \times 10^{-9})</td>
<td>0.06 %</td>
</tr>
</tbody>
</table>
Chapter 5 Measurement of Tangential Momentum Accommodation Coefficient (TMAC)

5.1 TMAC measurements in the free molecular flow regime

For the present work, the TMAC, $\sigma_t$, is measured in the free molecular flow regime. Chapter 2 of the dissertation shows the derivation of equation (2.37) to measure the free molecular flow torque for the experimental system:

$$T_{FM} = \sigma_t P_i \omega \sqrt{\frac{m \pi}{2kT_i}} (r_o^4 - r_i^4),$$

where $T_{FM}$ is the free molecular disc torque, $P_i$ is the ambient pressure, $\omega$ is the disc angular velocity, $m$ is the mass of the gas molecule, $k$ is the Boltzmann constant, $T_i$ is the ambient temperature, $r_o$ is the radius of the disc, and $r_i$ is the radius of the shaft on which the disc is mounted. For air, $m$ is $48.1 \times 10^{-27}$ kg.

In the free molecular flow regime TMAC for a particular gas versus material interaction may be obtained by knowing the free molecular pressure and the ambient temperature at a given instantaneous angular velocity since all other quantities are constants. Also the equation shows that the free molecular flow torque is a linear function of the tangential momentum accommodation coefficient. This indicates that the free molecular torque in the disc spin-down system is highly sensitive to the tangential momentum accommodation coefficient. Therefore precise measurements of TMAC are possible through the measurements of free molecular torque. Figure 5.1 is the plot of the disc deceleration torque versus TMAC in the free molecular flow regime for the present experimental system.
As part of this project, the TMAC for interactions between different gases and materials are measured in the free molecular flow regime. The materials used for measurement are aluminum, kapton, carbon fibre, titanium, and copper. Figures 5.2, 5.3, 5.4, 5.5, and 5.6 are the pictures of the aluminum, kapton, carbon fibre, titanium, and copper discs used in experiments.
Figure 5.2 shows the aluminum disc, 0.15m in diameter and having a mass of 25.07 grams. Another 0.21m diameter aluminum disc is covered by DuPont’s kapton and it has a mass of 51.8 grams and is shown in Figure 5.3. Figure 5.4 shows a 0.21m carbon fibre disc is used which has a mass of 39.2 grams. The titanium disc shown in Figure 5.5 has a mass of 80.17
grams while the copper disc shown in Figure 5.6 has a mass of 155.9 grams. All the discs are 0.5 mm thick.

The surface roughness of these discs is measured. Table 5.1 shows the arithmetic average of the absolute values ($R_a$) on each of the discs.

Table 5.1: Measured roughness of the discs used in experiments
(Courtesy: Ali Beheshti, Department of Mechanical and Industrial Engineering, LSU)

<table>
<thead>
<tr>
<th>Disc material</th>
<th>Ra (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>0.3</td>
</tr>
<tr>
<td>Kapton</td>
<td>0.086</td>
</tr>
<tr>
<td>Carbon fibre</td>
<td>2.7</td>
</tr>
<tr>
<td>Titanium</td>
<td>0.3</td>
</tr>
<tr>
<td>Copper</td>
<td>0.2</td>
</tr>
</tbody>
</table>

5.1.1 TMAC measurements in air and air like gases

The TMACs for materials such as aluminum, kapton, and carbon fibre are measured in air, nitrox36 (64% nitrogen and 36% oxygen), and pure nitrogen. These discs are rotated in the different gases and are allowed to spin down in the free molecular flow regime. The TMAC is measured using the deceleration torque:

$$ T = \sigma_t \rho \omega \left[ \frac{m \pi}{2kT_i} \left( r_o^4 - r_i^4 \right) \right], $$

(2.37)
The molecular masses of air, nitrox36, and nitrogen are $48.1 \times 10^{-27}$ kg, $46.51 \times 10^{-27}$ kg, and $48.89 \times 10^{-27}$ kg respectively. Previously simulations had suggested that the free molecular flow regime in air may be encountered at ambient pressures of 1 Pa or less.

In order to perform the experiments in the free molecular flow regime, the pressure in the test chamber is reduced to 0.01 Pa without powering the air bearings on. When the chamber pressure reaches 0.01 Pa, the gases are supplied to the air bearings and the pressure in the chamber is increased to 0.7 Pa approximately as the gas enters the chamber from the air bearings. The experiment is conducted when the steady state conditions inside the chamber are reached.

Next, the shaft torque is measured as a function of angular velocity in the free molecular flow regime when the air bushings are powered on by nitrogen, air, and nitrox36. The shaft torque is later subtracted from the total torque to obtain the disc torque in the free molecular flow regime.

The shaft torque in all the cases is always less than $10^{-6}$ N-m, which is atleast two orders of magnitude lesser than the disc torque in the free molecular flow regime. Also the experimental uncertainty analysis shown in chapter 3 suggests that the maximum experimental uncertainty in the measurement of the disc torque is $4.5 \times 10^{-7}$ N-m. This suggests that the shaft torques of the order of $4 \times 10^{-7}$ N-m or less are insignificant since the shaft torque is subtracted from the total torque to obtain the disc torque.

Figure 5.7 shows the plot of shaft torque versus angular velocity for the steel shaft spin-down in air, nitrogen, and nitrox36, which is a mixture of 64% nitrogen and 36% oxygen. Although all shaft torques are of the same order of magnitude, the largest shaft torque is obtained in nitrox36.
The disc torque is obtained by subtracting the shaft torque from the total torque. Equation (2.25) is used to obtain the tangential momentum accommodation coefficient (TMAC) from the measured disc torque:

\[
T_{FM} = \sigma_r \rho_i \omega \sqrt{\frac{m \pi}{2kT_i}} \left( r_o^4 - r_i^4 \right)
\]  

(2.25)

Table 5.2 shows the TMAC for aluminum, kapton, and carbon fibre in air and air-like gases.

Table 5.2: TMAC for aluminum, kapton, and carbon fibre in nitrogen, air, and nitrox36

<table>
<thead>
<tr>
<th>Materials</th>
<th>Nitrogen</th>
<th>Air</th>
<th>Nitrox36</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>0.77 ± 0.02</td>
<td>0.74 ± 0.03</td>
<td>0.69 ± 0.03</td>
</tr>
<tr>
<td>Kapton</td>
<td>0.73 ± 0.03</td>
<td>0.71 ± 0.02</td>
<td>0.68 ± 0.02</td>
</tr>
<tr>
<td>Carbon fibre</td>
<td>0.91 ± 0.02</td>
<td>0.90 ± 0.02</td>
<td>0.86 ± 0.02</td>
</tr>
</tbody>
</table>
It is observed that the highest TMACs are obtained for pure nitrogen and TMAC reduces with increase in the percentage of oxygen. Also carbon fibre has a consistently higher TMAC than aluminum and kapton. The smaller TMAC for aluminum and kapton may be due to smoother surfaces. Figure 5.8 shows a plot of the variation in TMAC with increasing percentage of nitrogen in the air-like gases.

Figure 5.8: Variation in TMAC with increasing percentage of nitrogen

The differences between the TMAC values with nitrogen and air are in accordance with the TMAC value of nitrogen and air versus smooth silicon shown by Chew, where TMAC with nitrogen was approximately 5% larger than TMAC with air (Chew 2009). It is also observed that TMAC reduces with increasing percentage of oxygen.

These results suggest that the prediction of TMAC may be possible from individual gas molecules. However more experiments are required to be done with higher percentages of oxygen. Gases with higher percentages of oxygen were not used in the present experimental
apparatus because it was hazardous to use high percentages of oxygen in the air bushings. Any spark from the electric motor could have caused fire hazards.

The TMAC for titanium and copper in air are also measured. The TMAC for the titanium and the copper discs in air are $0.77 \pm 0.03$ and $0.58 \pm 0.07$. The heavy copper disc requires greater volume flow rate of air into the air bushings. The larger error bar is due to the slightly inadequate air supply in the air bushings in the free molecular flow regime with the heavy copper disc. The TMAC for aluminum in nitrox32, a mixture of 32% oxygen and 68% nitrogen is measured as $0.68 \pm 0.03$.

5.1.2 TMAC measurements in carbon dioxide and argon

TMAC measurements are done for materials such as aluminum, kapton, and carbon fibre versus carbon dioxide and argon in the free molecular flow regime. The TMAC results in carbon dioxide are important for successful landing of space vehicles on Mars since the Martian atmosphere is primarily composed of carbon dioxide (Leighton and Murray 1966).

On the other hand argon as an inert gas is used in semiconductor processing to which chemically reacting gases needed for chemical vapor deposition are added (Chu, Qin et al. 1996). Since these are associated with rarefied flow conditions it is important to learn about the TMAC for argon-substrate interactions.

The shaft torques in argon and carbon dioxide are measured and is shown in Figure 5.9. It is observed that the shaft torque in these gases is still of the order of $10^7$ N-m. However the shaft torque in argon is slightly higher than in carbon dioxide. This is because of the fact that the shaft torque is governed by the viscous forces of the gas in the annular region between the shaft and the gas-bushings and the viscosity of argon is higher than the viscosity of carbon dioxide.
Figure 5.9: Shaft torque versus angular velocity in argon and carbon dioxide

Table 5.3 shows the TMAC measurements for the interactions of aluminum, kapton, and carbon fibre with argon and carbon dioxide.

Table 5.3: TMAC for aluminum, kapton, and carbon fibre in argon and carbon dioxide

<table>
<thead>
<tr>
<th>Materials</th>
<th>Argon</th>
<th>Carbon dioxide</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>0.76 ± 0.02</td>
<td>0.42 ± 0.01</td>
</tr>
<tr>
<td>Kapton</td>
<td>0.60 ± 0.03</td>
<td>0.57 ± 0.02</td>
</tr>
<tr>
<td>Carbon Fibre</td>
<td>0.74 ± 0.02</td>
<td>0.35 ± 0.03</td>
</tr>
</tbody>
</table>

The TMAC for argon versus aluminum is in reasonable accordance with Selden et al.’s result (Selden, Gimelshein et al. 2009). However, it is observed that the TMAC values involving carbon dioxide are consistently small. Since the current calculations of atmospheric drag on space vehicles in the Martian atmosphere assume TMAC values of unity, values much lesser may suggest towards capabilities of incorporating larger payload in these vehicles. However
more analysis on the interactions between carbon dioxide and aerospace materials are required for such conclusions. The results presently obtained may encourage researchers to investigate into the chemistry of interactions between carbon dioxide and the surfaces.

5.1.3 Variation of TMAC with molar mass

The variation of the tangential momentum accommodation coefficient for the interactions between all the different gases and the aluminum disc is studied. Table 5.4 shows the TMAC for the interactions between aluminum and the various gases.

Table 5.4: TMAC for aluminum versus all gases

<table>
<thead>
<tr>
<th>Gas</th>
<th>TMAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrogen</td>
<td>0.77</td>
</tr>
<tr>
<td>Air</td>
<td>0.74</td>
</tr>
<tr>
<td>Nitrox36</td>
<td>0.69</td>
</tr>
<tr>
<td>Nitrox32</td>
<td>0.68</td>
</tr>
<tr>
<td>Argon</td>
<td>0.76</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Nitrogen is the lightest gas among the different gases used. The largest TMAC of 0.77 is measured in nitrogen. The TMAC value reduces to 0.74 for air which is slightly lighter than nitrogen. The value reduces to 0.69 in Nitrox36 and 0.68 in Nitrox32 with the increasing molar mass of the gas. The smallest TMAC is measured in carbon dioxide which is also the heaviest gas used. The molar mass of carbon dioxide is 44 grams and the TMAC measured in carbon dioxide is as low as 0.42. Argon appears to be the outlier since it is almost as heavy as carbon
dioxide and yet the TMAC measures as high as 0.76. However Argon is the only monatomic inert gas used among all the gases. Figure 5.10 shows the plot of TMAC versus molar mass for the different gases versus aluminum.

To conclude future researchers may want to perform more experiments with gases having the intermediate molar masses and measure the TMAC values. While most gases yielded a TMAC value of approximately 0.7 or above, low values were consistently obtained with carbon dioxide. Investigators may perform molecular dynamic simulations to study the interactions between the carbon dioxide molecules and the different materials.

To summarize, in this section the TMAC measurements for various materials versus gases are shown in the free molecular flow regime. Thus far the previous researchers have been able to measure TMAC for gas-surface interactions in macro-scale boundary layer flows in the continuum slip flow regime or the transition flow regime only. Section 5.2 details some of the significant work done by previous researchers. Sensitivity analysis on their apparatus will show
that the torque in the free molecular flow regime is the more sensitive to TMAC than in the continuum regime.

5.2 Previous TMAC measurements

In 1923 R. A. Millikan noticed that the experiments by which the value of the electronic charge was determined by the droplet method gave consistent results only when the hydrodynamic law was modified by a factor \((1 + A \frac{\lambda}{a})\):

\[
F = \frac{6 \pi \mu v}{\left(1 + A \frac{\lambda}{a}\right)}, \quad (5.1)
\]

where \(\lambda\) was the mean free path and \(a\) was the radius of the droplet.

The hydrodynamic theory gave a first approximation of the correction factor as \((1 + \xi/a)\), where \(\xi\) was the coefficient of slip. Kinetic theory gave the value of \(A\) as 0.7004 when all the gas molecules were diffusely reflected from the surface.

Millikan derived an expression where the fraction of diffusely reflected gas molecules was \(\sigma_t\). Hence the fraction of gas molecules that were specularly reflected was \((1 - \sigma_t)\). The correction factor was given by equation (5.2):

\[
C = \left[1 + 0.7004 \left(\frac{2/\sigma_t - 1}{\sigma_t}\right) \frac{\lambda}{a}\right], \quad (5.2)
\]

where \(C\) is the correction factor. The fraction of diffusely reflected particles is \(f\) and it is equivalent to the tangential momentum accommodation coefficient (TMAC). Millikan performed a co-axial cylinder experiment to measure TMAC. In this experiment the outer cylinder was rotated and the torsional moment on the inner cylinder was measured.
The TMAC values were measured by Millikan on various materials in gases such as air, carbon dioxide, helium, and hydrogen. Table 5.5 shows the measured TMAC values by Millikan (Millikan, 1923).

Table 5.5: TMAC values measured by R. A. Millikan

<table>
<thead>
<tr>
<th>Gas versus surface</th>
<th>TMAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air – machined brass</td>
<td>1</td>
</tr>
<tr>
<td>Carbon dioxide – machined brass</td>
<td>1</td>
</tr>
<tr>
<td>Air – mercury</td>
<td>1</td>
</tr>
<tr>
<td>Air – oil</td>
<td>0.895</td>
</tr>
<tr>
<td>Carbon dioxide – oil</td>
<td>0.92</td>
</tr>
<tr>
<td>Hydrogen – oil</td>
<td>0.925</td>
</tr>
<tr>
<td>Air – glass</td>
<td>0.89</td>
</tr>
<tr>
<td>Helium – oil</td>
<td>0.874</td>
</tr>
<tr>
<td>Air – fresh shellac</td>
<td>0.79</td>
</tr>
</tbody>
</table>

Gabis, Loyalka, and Storvick measured TMAC for the interactions of stainless steel spheres with various gases such as Argon, Helium, and Krypton using a spinning rotor gauge. Their experiments were performed in the slip and the transition flow regimes. They magnetically levitated the rotating sphere and measured the deceleration during its spin-down. Figures 5.11 and 5.12 show the TMAC results in different gases as a function of inverse Knudsen number.

Their results are presented as two plots of TMAC versus inverse Knudsen number, $a$:

$$a = Kn_r^{-1}$$  \hspace{1cm} (5.3)
Knudsen number was calculated based on the radius of the levitated spheres.

Their results were compared with those from Thomas and Lord at $Kn=0.5$. It was shown that TMAC varied between 0.8 and 1. Gabis et al. mentioned that their measured and the predicted torque in the rarefied flow regime were only in qualitative agreement and the sphere motion was influenced by wall induced effects. Their measurements of TMAC varied with Knudsen number and they concluded that additional data in the free molecular flow regime were required to fully indicate the variability of TMAC with Knudsen number (Gabis et al., 1996).

Arkilic, Schmidt, and Breuer measured TMAC in silicon micromachined channels using the slip-flow solutions to the Navier Stokes equations. Their channels had a native oxide on prime silicon. The nominal height of the channel was 1.33 $\mu$m, the width of the channel was 52.25 $\mu$m, and the length of the channel was 7500 $\mu$m. Measurements were done with two different gases – argon, and nitrogen. The reported TMAC values were $0.80 \pm 0.01$, and $0.88 \pm 0.01$ respectively. The problem associated with Arkilic’s measurements was that the flow varied.
from continuum through the rarefied flow regimes within the long microchannel reducing the accuracy of measurements (Arkilic et al., 1997).

In a review article, Agarwal and Prabhu mentioned that the TMAC values for monatomic gases was about 0.93 and was almost constant with Knudsen number and this value could be employed for the most commonly available surface materials (Colin, Lalonde et al. 2004). They remarked that for nonmonatomic gases TMAC decreased with an increase in Knudsen number (Agrawal and Prabhu 2008).

5.3 Sensitivity analysis of TMAC measurements in the continuum regime

All previous TMAC measurements in macro-scale boundary layer type flows were performed in the continuum slip flow regime or the transition flow regime. The TMAC measurements in the transition flow regime are associated with issues owing to the breakdown in the viscosity law which suggests that the measurements in the continuum slip flow regime may be more accurate. Therefore for the purpose of sensitivity analysis, work from Millikan in 1923 and from Gabis and his co-workers in 1996 are chosen.

5.3.1 Sensitivity analysis of Millikan’s experiment

Millikan performed the coaxial cylinder experiment where he rotated the outer cylinder and measured the torque on the inner cylinder and measured the tangential momentum accommodation coefficient in the continuum slip flow regime. In this section the torque on the inner cylinder in Millikan’s experiment is re-derived using full scale Navier Stokes equations in the continuum slip flow regime. The continuity equation in the cylindrical coordinate system is as follows:
\[
\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho u_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho u_\theta) + \frac{\partial}{\partial z} (\rho u_z) = 0, \quad (5.4)
\]

where \( \rho \) is the gas density, \( u_r, u_\theta, \) and \( u_z \) are the velocities in the \( r, \theta, \) and \( z \) directions respectively. The momentum equations in the \( r, \theta, \) and \( z \) directions are as follows:

\[
\begin{align*}
\rho \left( \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z \frac{\partial u_r}{\partial z} \right) &= \rho g_r - \frac{\partial \rho}{\partial r} + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial (r u_r)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} - 2 \frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2} \right], \\
\rho \left( \frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + u_z \frac{\partial u_\theta}{\partial z} \right) &= \rho g_\theta - \frac{\partial \rho}{\partial \theta} + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_\theta}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + 2 \frac{\partial u_r}{\partial r} \frac{u_\theta}{r^2} + \frac{\partial^2 u_\theta}{\partial z^2} \right], \\
\rho \left( \frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} \right) &= \rho g_z - \frac{\partial \rho}{\partial z} + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right].
\end{align*}
\]

(5.5)

\[
\begin{align*}
\frac{\partial u}{\partial t} &= 0, \text{ because the flow conditions are steady state,} \\
u_z &= 0, \frac{\partial}{\partial \theta} = 0, \frac{\partial}{\partial z} = 0, \text{ fully developed conditions.} \\
\rho g &= 0, \text{ ignoring body forces.}
\end{align*}
\]

(5.8)

(5.9)

(5.10)
Since $u_\theta$ needs to be evaluated for the calculation of shear stress on the surface of the inner cylinder and hence the torque, equation (4.4) is considered and is reduced to the following:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_\theta}{\partial r} \right) - \frac{u_\theta}{r^2} = 0$$

(5.11)

or

$$\frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} (u_\theta r) \right] = 0$$

(5.12)

therefore

$$u_\theta = \frac{C_1 r}{2} + \frac{C_2}{r}$$

(5.13)

In order to evaluate the constants $C_1$ and $C_2$, slip boundary conditions are set on the walls of the inner cylinder and the outer cylinder:

at $r = r_1$;

$$u_\theta = \left( \frac{2 - \sigma_t}{\sigma_t} \right) \lambda \frac{\partial u_\theta}{\partial r} \bigg|_{r=r_1}$$

(5.14)

at $r = r_2$;

$$u_\theta = r_2 \omega_2 - \left( \frac{2 - \sigma_t}{\sigma_t} \right) \lambda \frac{\partial u_\theta}{\partial r} \bigg|_{r=r_2}$$

(5.15)

where $\sigma_t$ is the tangential momentum accommodation coefficient, $r_1$ and $r_2$ are the radii of the inner and the outer cylinders respectively, and $\omega_2$ is the rotational speed of the outer cylinder.

Applying the boundary conditions shown by equations (5.14) and (5.15) to equation (5.13), the following expression for the velocity $u_\theta$ is obtained:

$$u_\theta = \left[ \frac{r_2^3 \omega_2 \sigma_t \left( (2 - \sigma_t) \lambda + r_1 \sigma_t \right)}{r_2^2 - r_1^2 \left( (2 - \sigma_t)^2 \lambda^2 + r_1 r_2 \sigma_t^2 \right)} + \left( \frac{r_1^3 + r_2^3}{r_1^2 + r_2^2} \lambda \sigma_t (2 - \sigma_t) (r_1 + r_2) \right) \right] r + $$

$$\left[ \frac{r_2^3 r_1^2 \omega_2 \sigma_t \left( (2 - \sigma_t) \lambda - r_1 \sigma_t \right)}{r_2^2 - r_1^2 \left( (2 - \sigma_t)^2 \lambda^2 + r_1 r_2 \sigma_t^2 \right)} + \left( \frac{r_1^3 + r_2^3}{r_1^2 + r_2^2} \lambda \sigma_t (2 - \sigma_t) (r_1 + r_2) \right) \right] \right)$$

(5.16)
Since shear stress in the cylindrical coordinate system is given by:

$$\tau_{r,\theta} = \mu r \frac{d}{dr} \left( \frac{u_{\theta}}{r} \right),$$  \hspace{1cm} (5.17)

the equation for the shear stress on the inner cylinder may be given by:

$$\tau_{r,\theta}\bigg|_{r=r_1} = \frac{2\mu_2^3 \sigma_i (r_1 \sigma_i - (2 - \sigma_i) \lambda)}{(r_2^2 - r_1^2) \left[ (2 - \sigma_i)^2 \lambda^2 + r_1^2 \sigma_i^2 \right] + \left[ r_1^2 + r_2^2 \right] \lambda \sigma_i (2 - \sigma_i) (r_1 + r_2) }.$$  \hspace{1cm} (5.18)

The torque on the inner cylinder may be given by:

$$T_{\text{Millikan}} = \tau_{r,\theta}\bigg|_{r=r_1} AL$$  \hspace{1cm} (5.19)

Therefore the equation for torque on the inner cylinder of Millikan’s coaxial cylinder experiment is as follows:

$$T_{\text{Millikan}} = \frac{4\pi \mu L r_1^2 \sigma_i (r_1 \sigma_i - (2 - \sigma_i) \lambda)}{(r_2^2 - r_1^2) \left[ (2 - \sigma_i)^2 \lambda^2 + r_1^2 \sigma_i^2 \right] + \left[ r_1^2 + r_2^2 \right] \lambda \sigma_i (2 - \sigma_i) (r_1 + r_2) },$$  \hspace{1cm} (5.20)

where $\sigma_i$ is the tangential momentum accommodation coefficient (TMAC), and $\lambda$ is the mean free path.

In order to see how the torque on the inner cylinder varied with tangential momentum accommodation coefficient in Millikan’s coaxial cylinder experiment, it is necessary to assume the values for some of the parameters in the above equation. Since the experiment was performed in the continuum slip flow regime, it is assumed that the Knudsen number for the flow was 0.01. Some of the other assumptions for this analysis are as follows:

Gap size between the two cylinders = 5 mm = 0.005 m
Radius of the inner cylinder = $r_i = 50 \text{ mm} = 0.05 \text{ m}$

Radius of the outer cylinder = $r_o = 55 \text{ mm} = 0.055$

Rotational velocity of the outer cylinder = $100 \text{ rad/s}$

Height of the coaxial cylinders = $L = 1 \text{ m}$

The mean free path for these conditions is $= \lambda = 5 \times 10^{-5} \text{ m}$

Figure 5.13 shows the variation in the measured torque on the inner cylinder as a function of the tangential momentum accommodation coefficient.

![Torque vs TMAC](image)

Figure 5.13: Torque versus TMAC for Millikan’s coaxial cylinder experiment

It is observed that torque in the continuum slip flow regime the torque on the inner cylinder does not change much for TMAC values of greater than 0.5.

Figure 5.14 shows the rate of change of torque with TMAC as a function of TMAC.
Figure 5.14: Change in torque with TMAC versus TMAC for Millikan’s experiment

It is observed that the torque on the inner cylinder in the continuum slip flow regime is a weak function of TMAC at TMAC values larger than 0.5. This suggests that although this experiment may be able to measure TMAC values of less than 0.5, the measurements of TMAC values greater than 0.5 may be associated with large experimental uncertainties.

5.3.2 Sensitivity analysis of Gabis et al.’s experiment

Gabis, Loyalka, and Storvick measured the deceleration torque on a magnetically levitated sphere in various gases (Gabis et al., 1996). An analytical solution for the torque in the continuum slip flow regime was derived by Williams and Loyalka (Williams and Loyalka 1991).

\[
\tilde{T}(a, \sigma) = \frac{8\pi \tilde{a} \mu \omega}{1 + \left( \frac{3 c_m}{a} \right)},
\]

(5.21)

where \( \tilde{T} \) is the deceleration torque on the sphere, \( \tilde{a} \) is the sphere radius, \( \mu \) is the dynamic viscosity of the gas, \( \omega \) is the instantaneous angular velocity of the sphere, \( a \) is the inverse
Knudsen number based on the radius of the sphere, and \( c_m \) is a slip coefficient given by equation (5.22):

\[
c_m = \frac{5\pi}{16} \left(1 + \frac{\sigma_i (3.77 - \pi)}{2\pi} \right) \left(2 - \sigma_i \right) \frac{\sigma_i}{\sigma_f}.
\]  

(5.22)

For their experiments, Gabis et al. used Argon as the gas in the slip flow regime. Also the range of inverse Knudsen number used was: 10 < \( a \) < 100. Knudsen number is calculated using the following equation:

\[
Kn = \frac{\lambda_\mu}{\bar{a}},
\]  

(5.23)

where \( \lambda_\mu \) is the molecular mean free path and is given by equation (5.24):

\[
\lambda_\mu = \frac{8}{5\sqrt{\pi}} \left( \frac{\mu}{p} \right) \left( \frac{2kT_o}{m} \right)^{\frac{1}{2}},
\]  

(5.24)

where \( \mu \) is the dynamic viscosity of the gas, \( p \) is the ambient pressure, \( k \) is the Boltzmann constant, \( m \) is the mass of gas molecule, and \( T_0 \) is the ambient temperature.

The values of the constants in equations (5.21) and (5.22) are as follows:

\[
\bar{a} = 2mm = 0.002m
\]

\[
\mu = 0.0000212 \text{ Pa} - \text{s}
\]

\[
\omega = 1000 \text{ rad} / \text{s}
\]

\[
m = 66.4 \times 10^{-27} \text{ kg} \text{ (molecular mass of argon)}
\]

\[
p = 100 \text{ Pa}
\]
\[ T_0 = 298 \text{ K} \]

In order to study the sensitivity of the measured deceleration torque on the TMAC, the plot of deceleration torque as a function of TMAC is obtained in the slip flow regime. Figure 5.15 shows the plot of sphere spin-down torque versus tangential momentum accommodation coefficient. The plot shows that the deceleration torque hardly changes with any change in TMAC beyond a value of 0.7, where torque is a weak function of TMAC.

Figure 5.15: Torque versus TMAC for Gabis et al.’s magnetic sphere experiment

The derivative of torque as a function TMAC is plotted against TMAC in Figure 5.16. The derivative is small for TMAC values larger than 0.7. Also it will be noted that for most materials in gases the TMAC values measure 0.7 or larger.
Figure 5.16: Change in torque with TMAC versus TMAC for Gabis et al.’s experiment

To summarize, it has been shown that the measurement uncertainties associated with Millikan’s experimental apparatus are large for TMAC values of 0.5 or above. Similar uncertainties are associated with Gabis et al.’s apparatus for TMAC values larger than 0.7. It has been shown that in the continuum slip flow regime the torque associated with Millikan’s inner cylinder and Gabis et al.’s magnetically levitated sphere were weak functions of TMAC.

This problem is addressed in the present experimental apparatus where TMAC is measured in the free molecular flow regime where torque is a strong function of TMAC. Therefore uncertainties associated with the measurement of TMAC in the present apparatus are much lesser than in the apparatus used by the researchers previously.
Chapter 6 Conclusions and Future Work

6.1 Conclusions

There were three different objectives of this research. The first objective was to measure the continuum breakdown in boundary layer type flows. The second objective was to show that the experimental apparatus was capable of measuring tangential momentum accommodation coefficient (TMAC) for gas versus surface interactions in the free molecular flow regime. The final objective was to measure TMAC between certain gases and some aerospace materials.

Chapter 4 of the manuscript shows continuum breakdown in a boundary layer type flow around a rotating disc in low pressure air. This is shown by Figure 4.13 which is as follows:

![Figure 4.13: Non-dimensional torque versus Reynolds number for all the three discs](image)

Figure 4.13 shows the non-dimensional experimental curves, the computational fluid dynamics (CFD) curve, the von Karman Viscous Pump curve, and the analytically obtained free
molecular flow curves for the discs with three different diameters. Continuum breakdown may be shown by the departure of the CFD curve from the experimental results near a Reynolds number of approximately 300. Calculations show that continuum breakdown occurs around an ambient pressure of approximately 100 Pa.

Chapter 5 shows that this experimental facility is capable of TMAC measurements in boundary layer type flows in the free molecular flow regime. This chapter also shows why TMAC measurements in the free molecular flow regime are far more accurate than corresponding measurements in the continuum slip flow regime. This experimental apparatus is the first instance of such measurements in the free molecular flow regime in macro-scale boundary layer type flows.

Finally chapter 5 also details some TMAC measurements between certain aerospace materials and some gases. The gases used are nitrogen, air, nitrox36 (36% oxygen), nitrox32 (32% oxygen), argon, and carbon dioxide. For all the gases the free molecular flow regime is encountered around an ambient pressure of 1 Pa or less. The TMAC for interactions between air and metals such as aluminum, titanium, and copper are measured as approximately 0.74, 0.77, and 0.58 respectively while the TMAC between air and non-metals such as kapton and carbon fibre are measured as approximately 0.71 and 0.90 respectively. Air is replaced with nitrogen and the TMAC for the interactions between nitrogen and aluminum, kapton, and carbon fibre are measured as approximately 0.78, 0.73, and 0.92 respectively. Nitrox36 is used and the TMAC for nitrox36 versus aluminum, kapton, and carbon fibre are measured as approximately 0.69, 0.68, and 0.86 respectively. The TMAC for argon versus aluminum, kapton, and carbon fibre are measured as approximately 0.76, 0.60, and 0.74 respectively while the TMAC for
carbon dioxide versus aluminum, kapton, and carbon fibre are measured as approximately 0.42, 0.57, and 0.35 respectively.

It is observed that the TMAC values for most materials in the gases other than carbon dioxide are approximately 0.7 or more. The TMAC results in carbon dioxide are consistently small. Figure 5.10 in chapter 5 shows the plot of TMAC versus the molar masses of the different gases used:

![Figure 5.10: TMAC versus molar mass for different gases versus aluminum](image)

This plot shows that the TMAC generally reduces with increase in molar mass of the gas with argon being the outlier. Also the TMAC values are generally in the proximity of 0.7 with the exception of carbon dioxide. The TMAC value for the interaction between argon and aluminum of 0.76 is in reasonable agreement with Selden et al.’s measurement (Selden et al., Phys. of Fluids, 2009). It will be noted that argon is the only monatomic inert gas used in these experiments. The TMAC values for the interactions between carbon dioxide and materials such as aluminum, kapton, and carbon fibre are particularly low in the free molecular flow regime.
The measurements of TMAC of carbon dioxide on certain aerospace materials are of particular interest to The National Aeronautics and Space Administration (NASA) because they require those for their 2020 Mars Mission Plan and a part of this research has been funded by NASA Langley and Louisiana Space Grant Consortium. The measurements of TMAC associated with carbon dioxide are the first real estimates and the TMAC value earlier used for drag calculations was 1. A lower than unity value of TMAC suggests that the atmospheric drag on the surface of a space vehicle may be over-predicted and will leave more room for scientific payload during future missions.

Summarizing, no experimental measurements for continuum breakdown in macro-scale boundary layer type flows were done before this research work. All previous researchers measured TMAC on materials in the continuum slip and the transition flow regimes. It is shown that for the highest accuracy, TMAC measurements should be done in the free molecular flow regime. The disc spin-down experiment is unique because it provides the rarefied gas dynamics community with TMAC measurements in the free molecular flow regime for macro-scale boundary layer flows for the first time.

Finally none of the earlier experimental apparatus used by researchers could accommodate for materials that were planar by nature. Since materials used in the field of microfabrication will generally require the test surfaces to be planar, the experimental apparatus used by previous researchers cannot be used to measure TMAC on most materials used in microfabrication.
6.2 Future work

Some of the specific areas where this research may be expanded are as follows:

a. Molecular dynamic simulations: The experimental results showed much lower values of TMAC for interactions between carbon dioxide and aluminum, kapton, and carbon fibre. Molecular dynamic simulations may provide researchers with more insight into the chemistry of interactions.

b. TMAC measurements for monatomic inert gases: Agrawal and Prabhu in their review paper mentioned that in non-monatomic gases the TMAC may reduce with increased degree of rarefaction (Agrawal and Prabhu, JVST A, 2008). Their conclusions were based on a work by Colin et al., where they measured TMAC for interactions between helium and silicon and between nitrogen and silicon in a microchannel (Colin et al., 2004). It was shown that the TMAC for helium-silicon interactions remained constant at 0.93 within a Knudsen number range of 0.029-0.22 while TMAC varied between 0.93 and 1 for nitrogen-silicon interactions within a Knudsen number range of 0.005-0.03. In the present research the only monatomic gas used is argon. Therefore it will be interesting to see how the TMAC values measure for different inert gas-material interactions.

c. TMAC measurements with varying temperature: At present all TMAC measurements have been done in ambient temperatures. It will be interesting to see if TMAC changes with increasing temperatures. For this purpose a heating mechanism may be created by which the disc temperature can be varied and the corresponding TMAC can be measured.

d. Hardware modifications: In the present apparatus, the air bearing fixture is the heart of the facility. Although it facilitates near frictionless rotation of the disc, it requires additional instrumentation to scavenge the air that will otherwise leak into the vacuum chamber. If the
air bearing fixture can be replaced by an active magnetic bearing fixture then the complexities associated with the scavenging may be eliminated.

Also the present experimental apparatus with the air bearing fixture may not be suitable for measurements in combustible gases such as hydrogen and methane. Such measurements with the air bearing fixture will required these gases to be pumped into the air bushings following the same procedure. Since these gases have to be pumped into the bushings at gauge pressure of higher than 1 atmosphere, there are possibilities of fire hazards with the electrical motor being operated. Perhaps by the use of a magnetic bearing fixture such combustible gases can be used for the experiments. TMAC measurements in hydrogen and methane may help model the gas-surface interactions in gas giant planets such as Jupiter and Saturn.

e. TMAC measurements with gas mixtures: Additional experiments with gas mixtures may provide researchers with valuable data that may help establish mixing laws. Gas mixtures such as those between two different inert gases may be useful towards establishing such laws.
References


Appendix

1. Checklist to power on the experimental facility

   a. Switch on the main power source.
   b. Check if all valves (the vacuum valve, the vent valve and the fore-line valve) are closed.
   c. Make sure that the vacuum gauge (IGM 402 Hornet) is switched on.
   d. Make sure that the chamber door is closed and bolted.
   e. Make sure that the turbo-pump controller is switched off. At this time, the turbo-pump switch should be in ‘STOP’ position.
   f. Switch on the roughing pump (Laco W2V40).
   g. Immediately open the fore-line valve. The fore-line hose connects the high vacuum pump exit to the chamber via two T-flanges and the vacuum hose in series. At this time the ‘gate valve’ should be closed.
   h. Open the vacuum valve on the chamber.
   i. Watch the chamber and the fore-line pressure reduce together to 100 Torr.
   j. When the chamber pressure reduces to 100 Torr, switch on the second rotary vane pump (Pfeiffer Hena 25). This pump will be connected to the pre-tortuous path scavenging port.

   Note that at this time the air bearing fixture is not powered on.
   k. Watch the two rotary vane pumps bring the chamber pressure down to about 3 Torr.
   l. Switch on the second rotary vane pump (Pfeiffer Duo 20). This pump will be connected to the post-tortuous path scavenging port.
   m. The three rotary vane pumps bring down the chamber pressure down to 0.5 Torr approximately.
n. At this time switch on the turbo-pump controller. The turbo pump switch should still be in the ‘STOP’ position.

o. Open the gate valve.

p. Turn the turbo-pump controller knob to the ‘START’ position.

q. Close the vacuum valve but keep the gate valve and the fore-line valve open.

r. The chamber pressure starts reducing and will be reduced to a maximum lowest pressure of 0.00001 Torr.

s. When the lowest pressure is reached, switch on the air bearing fixture. When the air bearing fixture is perfectly calibrated, it will levitate the disc-shaft system with a gauge air pressure of 15 psi.

2 Checklist for switching off the test facility after completion of experiments

a. Make sure that the fore-line valve is open and the vacuum valve is closed

b. Close the gate valve. By doing so, the chamber is disconnected from the turbo-molecular pump.

c. Open the vent valve. By doing so, air starts going into the vacuum chamber and gradually the chamber is restored back to atmospheric pressure.

d. Turn the turbo-molecular pump knob on the controller to ‘STOP.’ By doing so, the rotor inside the pump starts spinning down. However since this takes a while to stop rotating completely, the ‘gate valve’ at this time should remain closed

e. Watch out for the ‘yellow light’ on the control panel. Close the fore-line valve when the light turns off.

f. Switch off the roughing pump

g. Open the chamber door
h. Note the reduction in intensity humming noise from the turbo-molecular pump to make sure that the rotors have completely stopped rotating

i. Keeping the door open, open the vacuum valve. The cavity inside the valve is restored back to atmosphere

j. Open the fore-line valve. The fore-line hose is restored back to atmosphere

k. Switch off the turbo-molecular pump controller

l. Open the gate valve for once and then close it again

m. Close all the valves

n. Switch off power
Vita

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