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Fixed Point Sets of Transformation Groups on Infinite Product Spaces.

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FIXED POINT SETS OF TRANSFORMATION GROUPS ON
INFINITE PRODUCT SPACES.**

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**FIXED POINT SETS OF TRANSFORMATION GROUPS
ON INFINITE PRODUCT SPACES**

A Dissertation

**Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy**

in

The Department of Mathematics

**by
James Edward West
B.S., Louisiana State University, 1964
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ABSTRACT

By X_0 is meant the countably infinite Cartesian product of a topological space X with itself. A topological space X is said to have the reflective isotopy property if the homeomorphism T of $X \times X$ onto itself defined by the formula $T(x_1, x_2) = (x_2, x_1)$ is isotopic to the identity. A topological group G is said to be freely representable as a transformation group on the space X if there exists an action of G on X as a transformation group such that no element of G except the identity has a fixed point. If for each closed subset F of X there exists an action of G on X as a transformation group such that F is the set of fixed points of each element of G except the identity, then G is said to be fully freely representable on X , or fully free with respect to X .

Theorem 2.1: Let X be a topological space with the reflective isotopy property, and let G be a topological group. Assume that for some topological space Y , X is homeomorphic to Y_0 and Y^n is perfectly normal for all positive integers n . In order that G be fully free with respect to X it is necessary and sufficient that G be freely representable on X . This generalizes a theorem of V. L. Klee, Jr. [7]. Corollaries to the proof of Theorem 2.1 are the following three theorems which are included without proof:

Theorem 2.2: If X , Y , and G are as in Theorem 2.1 and if Z is a topological space such that $Z \times Y^n$ is perfectly normal for each positive integer n , then G is fully free with respect to $Z \times X$ if it is freely representable on X .

Theorem 2.3: Let X be a topological space, and assume that for some space Y , X is homeomorphic to Y_0 . Let Y^n be binormal for each positive integer n . Then in order that for each topological group G , any two actions of G on X as a transformation group be isotopically equivalent, it is necessary and sufficient that X have the reflective isotopy property.

Theorem 2.4: Any topological group which is freely representable on $s (= \prod_{i=1}^{\infty} (-1,1)_i)$ is fully free with respect to s . The statement "two actions of a topological group G on a space X as a transformation group are isotopically equivalent" means that there exists a continuous function $H : G \times X \times [0, 1] \rightarrow X$ such that for each $t \in [0,1]$, $H \upharpoonright G \times X \times \{t\}$ is an action of G on X , and $H \upharpoonright G \times X \times \{0\}$ and $H \upharpoonright G \times X \times \{1\}$ are the two actions above. Theorem 2.3 is a generalization of a theorem of Raymond Wong [11].

• Chapter III is devoted to the special case in which X is a separable, infinite-dimensional Frechet space. Theorem 2.1 is sharpened to Theorem 3.1: A topological group is fully free with respect to each separable, infinite-dimensional Frechet space if and only if it can act effectively on some separable, infinite-dimensional Frechet space as a transformation group. From this follows Corollary 3.1: A compact group is fully free with respect

to all separable, infinite-dimensional Frechet spaces if and only if it is metrizable.

CHAPTER 1: INTRODUCTION AND NOTATION

Introduction

The study of Topological Algebra has greatly benefited from the technique of representing abstract structures in terms of homeomorphisms of a topological space onto itself. This often results in the discovery of much information concerning the space used for the representation as well as information concerning the object represented. Even very simple algebraic objects such as cyclic groups yield considerable topological information this way, for in 1941 P. A. Smith [10] proved that if p is a prime and X is a compact (or locally compact and finite-dimensional) Hausdorff space which is acyclic in mod p homology, then any homeomorphism of X onto itself of period p has a non-empty fixed point set which must also be acyclic mod p .

Analogous topological information about infinite-dimensional linear topological spaces is not so readily available, however, for in 1956 V. L. Klee, Jr. [7] proved that any compact subset of a separable, infinite-dimensional Hilbert space can be made the fixed point set of a homeomorphism of any period (greater than one, of course).

The object of this paper is to investigate further the possibilities suggested by the above work of Klee and to extend

it to wider classes of groups, spaces, and sets. We obtain two generalizations by techniques which are based on the structure of infinite product spaces and are entirely different from those of Klee.

In order to shorten and clarify the statements of the principal results, we first make some definitions. Almost all non-standard definitions used in this paper will be made here. A topological group, G , is defined to be a topological space with a group structure in which inversion is continuous on G and multiplication is a continuous function from $G \times G$ to G . No separation properties are required.

For any topological space X , the group of all homeomorphisms of X onto itself will be denoted by $G(X)$. By "transformation group" we mean a triple (G, X, α) where G is a topological group, X is a topological space, and α is a continuous function from $G \times X$ to X such that $g \mapsto \alpha|_{\{g\} \times X}$ defines a homomorphism of G into $G(X)$. For simplicity, we write $\alpha|_{\{g\} \times X}$ as $\alpha(g)$. This definition is non-standard in that usually it is required that X be a Hausdorff space. (See, for example, Montgomery and Zippin, Topological Transformation Groups [9], p. 40). The Hausdorff property is dropped here because the construction in Chapter II does not require it. The function α is said to be an action of G on X . The action is said to be effective if $g \mapsto \alpha(g)$ is a monomorphism of G into $G(X)$ and free if $(g, x) \in G \times X$ and $\alpha(g, x) = x$ imply that g is the identity of G .

Let a topological group G be said to be effectively representable as a transformation group on X , or effective with respect to X , if there exists a transformation group (G, X, α) where α is an effective action. If α can additionally be required to be a free action, let G be said to be freely representable as a transformation group on X , or free with respect to X . If for each closed subset F of X there is an action α_F such that 1) (G, X, α_F) is a transformation group and 2) F is the set of fixed points of $\alpha(g)$ for each $g \in G$ except the identity, then let G be said to be fully freely representable as a transformation group on X , or fully free with respect to X .

Let X be a topological space. If for any two disjoint closed subsets, H and K , of X there exist two disjoint open subsets, U and V , of X containing H and K , respectively, then X is called normal. If $X \times [0, 1]$ is normal, then X is said to be binormal. If X is normal and every closed subset of X is a G_δ -set, then X is called perfectly normal.

We now state the two principal results of this paper after making one more definition. Let a space X be said to have the reflective isotopy property if the homeomorphism T of $X \times X$ onto itself defined by the formula $T(x_1, x_2) = (x_2, x_1)$ is isotopic to the identity map of $X \times X$ onto itself. (For any topological space Y , an isotopy between two elements, g and h , of $G(Y)$ is a continuous function H from $Y \times [0, 1]$ onto Y such that 1) $H|Y \times \{t\} \in G(Y)$ for each $t \in [0, 1]$, 2) $H|Y \times \{0\} = g$, and 3) $H|Y \times \{1\} = h$.) The first result

is Theorem 2.1: Let X be a topological space with the reflective isotopy property, and let G be a topological group. Suppose that for some topological space Y , X is the countably infinite Cartesian product of Y with itself and that Y^n is perfectly normal for each positive integer n . Then in order that G be fully free with respect to X it is necessary and sufficient that G be free with respect to X . The second main result is Theorem 3.1: In order that a topological group be fully free with respect to each separable, infinite-dimensional Frechet space, it is necessary and sufficient that it be effective with respect to some separable, infinite-dimensional Frechet space. From this theorem we obtain the following corollary: A compact topological group G is fully free with respect to each separable, infinite-dimensional space if and only if G is metrizable. This corollary clearly demonstrates that Theorem 3.1 is a direct generalization of Klee's theorem mentioned above.

Several other theorems are included without proof. These are corollaries of the main results or can be proved by minor modifications of the constructions of the proofs of these two theorems.

Notational Conventions:

The positive integers will always be denoted by N ; the closed interval $[-1,1]$, by I ; and the open interval $(-1,1)$ by I^0 . If X is a space, X^n will always denote $\prod_{i=1}^n X_i$, where $X_i = X$ for $i = 1, \dots, n$. The space $\prod_{i \in N} X_i$, where $X_i = X$ for all

$i \in N$, will generally be denoted by X_i , where the subscript is not to be confused with an indexed copy of X because all indices will be elements of N .

If X is a product space of the form $X = Y_1 \times Y_2 \times \dots \times Y_m$ or $X = Y_1 \times Y_2 \times \dots$ for some spaces Y_i , $i = 1, \dots, m$ or $i \in N$, then the projection function of X onto the n^{th} factor space, Y_n , will be denoted by π_n , and the projection function of X onto the product of the first n factor spaces will be denoted by τ_n . An element x of X will often be written as a string of coordinates $x = (x_1, x_2, \dots, x_m)$ or $x = (x_1, x_2, \dots)$ in which case x_i is to be understood as $\pi_i(x)$.

The symbol ' $\sim$ ' will always mean 'is homeomorphic to'; the letter 'e' will always denote the identity element of whatever group is under discussion, and if K is a subset of a space, X , then $\text{cl.}(K)$ will always mean the closure of K in X .

The letter 's' will denote the space l_0^0 , both for simplicity and because this is the symbol commonly used to refer to the linear space R_0 , where R is the real numbers. In general, we are not dealing with the linear structure of s and so identify R_0 with l_0^0 , as these two products are clearly homeomorphic to each other.

CHAPTER 11

GENERAL THEOREMS ABOUT INFINITE PRODUCT SPACES

The purpose of this chapter is to prove the first result stated in the introduction and to note some of its immediate corollaries.

Theorem 2.1: Let X be a topological space with the reflective isotopy property, and let G be a topological group. Suppose that for some topological space Y , X is homeomorphic to Y_0 and Y^n is perfectly normal for each $n \in \mathbb{N}$. Then in order that G be fully free with respect to X it is necessary and sufficient that G be free with respect to X .

Proof of Theorem 2.1:

Free representability is necessary for fully free representability, for $\emptyset$ is a closed subset of X and $\alpha_\emptyset$ is the requisite free action of G on X as a transformation group.

To demonstrate the sufficiency of free representability, let F be a closed subset of X . We first construct a sequence of "annular" closed sets, $\{K_i\}_{i \in \mathbb{N}}$, such that $X - F = \bigcup_{i \in \mathbb{N}} K_i$. The K_i 's will be the closures of pairwise disjoint open sets, each of which has a boundary composed of two disjoint closed sets (an "F-ward" boundary and an "outward" boundary in some sense). Finally, $K_i \cap K_{i+1}$ will be precisely the intersection of the "F-ward" boundary of K_i and the "outward" boundary of K_{i+1} .

If $|i - j| > 1$, then $K_i \cap K_j$ will be empty.

Step 1: Construction of $\{K_i\}_{i \in \mathbb{N}}$.

Throughout the proof we shall identify X with Y_0 in order to simplify the formulas used in the construction of the final action of G on X .

Let $\{U_i\}_{i \in \mathbb{N}}$ be a countable collection of open subsets of X satisfying the following conditions:

- 1.) $U_1 = X$,
- 2.) $F = \bigcap_{i \in \mathbb{N}} U_i$,
- 3.) $U_i = \tau_i^{-1}(\tau_i(U_i))$ for each $i \in \mathbb{N}$, and
- 4.) cl. $(U_{i+1}) \subset U_i$ for each $i \in \mathbb{N}$.

Such a collection may be constructed inductively as follows:

For each $i \in \mathbb{N}$, let $\{V_{ij}\}_{j \in \mathbb{N}}$ be a collection of open subsets of Y^i such that cl. $(\tau_i(F)) = \bigcap_{j \in \mathbb{N}} V_{ij}$. The existence of

$\{V_{ij}\}_{j \in \mathbb{N}}$ is guaranteed by the perfect normality of Y^i . Let

$\tilde{V}_{ij} = \tau_i^{-1}(V_{ij})$. Let $U_1 = X$. Now suppose that for $0 < i < n$

a set U_i has been chosen in such a manner that U_i satisfies

(3) above and $\{U_i\}_{1 \leq i \leq n-2}$ satisfies (4) above. By the normality of Y^n , there exists an open set, W , of Y^n such that

cl. $(\tau_n(F)) \subset \tau_n((\bigcap_{1 \leq i \leq n-1} U_i) \cap (\bigcap_{1 \leq i, j \leq n} \tilde{V}_{ij}))$. Define

$U_n = \tau_n^{-1}(W)$. By induction, there exists a countably infinite

collection of such U_i 's satisfying (1), (3), and (4) together

with the additional property that $U_n \subset \tilde{V}_{ij}$ whenever $n \geq i, j$.

This is sufficient to guarantee that $F = \bigcap_{i \in \mathbb{N}} U_i$ as follows:

For each $i \in \mathbb{N}$, F is clearly contained in U_i . On the other

hand, if $x \in X - F$, then there exists a basic neighborhood, 0 , of x which misses F . Therefore for some $i \in N$, $\tau_i(0) \cap \tau_i(F) = \tau_i(0) \cap \text{cl.}(\tau_i(F)) = \emptyset$. Since $\tau_i(x) \notin \text{cl.}(\tau_i(F))$, there is a $j \in N$ for which $\tau_i(x) \notin V_{ij}$. As $U_{i+j} \subset \tilde{V}_{ij}$, we have $x \notin U_{i+j}$ and $F \supset \bigcap_{i \in N} U_i$. Thus $F = \bigcap_{i \in N} U_i$.

Let $K_i = \text{cl.}(U_i - U_{i+1})$ for each $i \in N$. Note that $K_i = \tau_{i+1}^{-1}(\tau_{i+1}(K_i))$ and hence $K_i \sim \tau_{i+1}(K_i) \times X$. Also, as $X \sim X \times X$ (for example, let $\eta: X \times X \rightarrow X$ be defined by $\eta(x_1, x_2) = (\pi_1(x_1), \pi_1(x_2), \pi_2(x_1), \pi_2(x_2), \dots)$), we have $K_i \sim \tau_{i+1}(K_i) \times (X \times X)$.

We shall define a free action of G on K_i for each $i \in N$ so that the actions converge to the trivial action as i becomes large. These actions will be defined so as to agree on the "interfaces" between the K_i 's. Once this is done, all that will remain will be to "piece the actions together".

Step 2: Definitions of the necessary functions with which to describe the actions on the K_i 's.

a) By Urysohn's lemma, let for each $i \in N$, $\bar{\varphi}_i: \tau_{i+1}(K_i) \rightarrow [0, 1]$ such that $\bar{\varphi}_i \mid \text{cl.}(\tau_{i+1}(U_i)) - \tau_{i+1}(U_i) = 0$, $\bar{\varphi}_i \mid \text{cl.}(\tau_{i+1}(U_{i+1})) - \tau_{i+1}(U_{i+1}) = 1$, and $\bar{\varphi}_i$ is continuous.

Let $\varphi_i = \bar{\varphi}_i \circ \tau_{i+1}$. Then $\varphi_i: K_i \rightarrow [0, 1]$, depends only on the first $n + 1$ coordinates, and separates the "F-ward" and "outward" boundaries of K_i .

b) Let $v = X \times X \times [0, 1] \rightarrow X \times X$ be an isotopy such that $v(x_1, x_2, 0) = (x_1, x_2)$ and $v(x_1, x_2, 1) = (x_2, x_1)$.

- c) Let $\sigma : G(X) \rightarrow G(X)$ be the "shift" homomorphism, i.e., for $h \in G(X)$ and $x = (x_1, x_2, \dots) \in X$, $\sigma(h)(x) = (x_1, \pi_1 h(x_2, x_3, \dots), \pi_2 h(x_2, x_3, \dots), \dots)$. Note that σ is a monomorphism. Let σ^i denote the monomorphism of $G(X)$ into itself obtained by applying σ i times.
- d) Let ω be the monomorphism of $G(X)$ into itself defined by the formula $\omega(h) = \eta(h \circ \pi_1) \eta^{-1}$, where η is as defined earlier and $h \circ \pi_1 : X \times X \rightarrow X \times X$ is defined by the formula $h \circ \pi_1 (x_1, x_2) = (h(x_1), x_2)$.
- e) For each $i \in \mathbb{N}$, let $\xi_i : K_i \rightarrow K_i$ be the homeomorphism defined by the formula $\xi_i(x) = \sigma^{i+1}(\eta \vee (\eta^{-1}(\cdot), \varphi_i(x))) (x)$.
- f) Let $\alpha : G \times X \rightarrow X$ be a free action of G on X as a transformation group guaranteed by the hypothesis that G is free with respect to X .

Step 3: Definition of the action of G on X .

Let $\beta : G \times X \rightarrow X$ be the function defined by the following formula: for $(g, x) \in G \times X$,

$$\beta(g, x) = \begin{cases} \xi_i^{-1}(\sigma^{i+1} \omega \alpha(g)) \xi_i(x), & \text{if } x \in K_i \\ x, & \text{if } x \in F \end{cases}.$$

The rest of the proof is devoted to the demonstration that β is indeed an action of G on X and that if $g \neq e$ then $\beta(g)$ has F as its fixed point set.

- a) β is well-defined, for if $g \in G$ and $x \in K_i \cap K_{i+1}$, then $\varphi_i(x) = 1$ and thus $\xi_i(x) = \sigma^{i+1}(\eta \vee (\eta^{-1}(\cdot), \varphi_i(x))) (x) = \sigma^{i+1}(\eta \vee (\eta^{-1}(\cdot), 1)) (x) = (x_1, \dots, x_{i+1}, \pi_1(\eta \vee (\eta^{-1}(\cdot), 1)(x_{i+j+1})_{j \in \mathbb{N}}), \pi_2(\cdot), \dots)$

$$= (x_1, \dots, x_{i+1}, \pi_1(\eta((x_{i+2j+1})_{j \in \mathbb{N}}, (x_{i+2j})_{j \in \mathbb{N}})), \pi_2(\quad), \dots), \text{ i.e.,}$$

$$\pi_{i+2j} \xi_i(x) = x_{i+2j+1} \text{ and } \pi_{i+2j+1} \xi_i(x) = x_{i+2j}.$$

Now as $\tau_{i+1} \xi_i(x) = \tau_{i+1}(x)$, we have $\xi_i: K_i \rightarrow K_i$. Also,

$$\tau_{i+1}((\sigma^{i+1} \omega \alpha(g)) \xi_i(x)) = \tau_{i+1} \xi_i(x) \text{ implies that } (\sigma^{i+1} \omega \alpha(g)) \xi_i: K_i \rightarrow K_i$$

and $\varphi_i((\sigma^{i+1} \omega \alpha(g)) \xi_i(x)) = \varphi_i(x) = 1$, so ξ_i^{-1} merely interchanges the $(i+2j)^{\text{th}}$ coordinate and the $(i+2j+1)^{\text{st}}$ coordinate of $(\sigma^{i+1} \omega \alpha(g)) \xi_i(x)$

for each $j \geq 1$. Now $(\sigma^{i+1} \omega \alpha(g)) \xi_i(x) =$

$$= (x_1, \dots, x_{i+1}, \pi_1(\alpha(g, (x_{i+2j+1})_{j \in \mathbb{N}})), x_{i+2}, \pi_2(\alpha(g, (x_{i+2j+1})_{j \in \mathbb{N}})), \dots).$$

Combining these gives us $\xi_i^{-1}(\sigma^{i+1} \omega \alpha(g)) \xi_i(x) =$

$$= (x_1, \dots, x_{i+1}, x_{i+2}, \pi_1(\alpha(g, (x_{i+2j+1})_{j \in \mathbb{N}})), x_{i+4}, \pi_2(\alpha(g, (x_{i+2j+1})_{j \in \mathbb{N}})), \dots).$$

On the other hand, $\varphi_{i+1}(x) = 0$, so $\xi_{i+1}(x) = x$ and $\xi_{i+1}^{-1}(\sigma^{i+2} \omega \alpha(g)) \xi_{i+1}(x) =$

$$= (\sigma^{i+2} \omega \alpha(g))(x) = (x_1, \dots, x_{i+2}, \pi_1(\alpha(g, (x_{i+2j+1})_{j \in \mathbb{N}})), x_{i+4}, \pi_2(\quad), \dots) =$$

$$= \xi_i^{-1}(\sigma^{i+1} \omega \alpha(g)) \xi_i(x).$$

b) $\beta(g) \mid K_i \in G(K_i)$ for each $i \in \mathbb{N}$, because $\sigma^{i+1} \omega \alpha(g) \in G(K_i)$

and ξ_i is a continuous one-to-one function of K_i onto itself with

a continuous inverse $\xi_i^{-1}(x) = (\sigma^{i+1}(\eta_V^{-1}(\eta^{-1}(\cdot), \varphi_i(x))))(x)$, and

K_i is an invariant set under $\sigma^{i+1} \omega \alpha(g)$.

c) Together, (a) and (b) imply that for $g \in G$, $\beta(g) \mid \bigcup_{i \in \mathbb{N}} K_i \in G(\bigcup_{i \in \mathbb{N}} K_i)$.

However, this set is X -F and is open, so it suffices to prove that

for each $g \in G$, $\beta(g)$ is continuous on F in order to demonstrate

that $\beta(g) \in G(X)$. This is easy, for if $x \in F$ and U is an open

subset of X containing x , then there is a basic neighborhood V

containing x and contained in U . Now there exists an $i \in \mathbb{N}$ making

$$\tau_i^{-1}(\tau_i^{-1}V) = V \text{ Now for } n \geq i, \alpha(g): V \cap U_n \rightarrow V \cap U_n \text{ by the}$$

properties of the shift homomorphism. Therefore $\beta(g)$ is continuous on F and thus on X . But $\beta(g^{-1}) = (\beta(g))^{-1}$ is also continuous on X by the same proof, so $\beta(g) \in G(X)$.

d) To show that β is an action of G on X as a transformation group, there remains only to show that β is a continuous function from $G \times X$ to X , for it is clear that $g \mapsto \beta(g)$ is a homomorphism of G into $G(X)$. On $G \times F$, β is continuous, because in the proof given in (c) above, $\alpha(g) : V \cap U_n \rightarrow V \cap U_n$ for all $g \in G$. To show that β is continuous on $G \times \bigcup_{i \in \mathbb{N}} K_i$, it suffices to show continuity on each $G \times K_i$, as no more than two $G \times K_i$'s intersect. Now in order to show that β is continuous on $G \times K_i$, it is sufficient to show that $(g, x) \mapsto \xi_i \beta(g, x) \xi_i^{-1}$ is continuous because $\xi_i \in G(K_i)$ for each $i \in \mathbb{N}$. This is easy, however, for α is continuous, ω merely restricts the action of α to the odd coordinates, and σ^{i+1} merely shifts the action of $\omega \alpha(g)$ to coordinates of index greater than $i+1$, so $\xi_i \beta \xi_i^{-1} = \sigma^{i+1} \omega \alpha$ is continuous on $G \times K_i$. Therefore β is an action of G on X as a transformation group.

e) By construction, F is contained in the fixed point set of $\beta(g)$ for all $g \in G$, but it is also true by the introduction of α that if $g \neq e$, then F is the set of fixed points of $\beta(g)$ as follows: If $x \in K_i$ for some $i \in \mathbb{N}$, then $\beta(g, x) = \xi_i^{-1}(\sigma^{i+1} \omega \alpha(g)) \xi_i(x)$ and $\beta(g)$ moves x if and only if $\sigma^{i+1} \omega \alpha(g)$ moves $\xi_i(x)$. But if $g \neq e$, then $\alpha(g)$ moves every point of X and the introduction of ω and σ^{i+1} do not change this property.

Thus we have constructed a continuous map $\beta : G \times X \rightarrow X$ which defines an action of G on X as a transformation group (G, X, β) where if $g \neq e$ and $g \in G$ then F is the set of fixed points of $\beta(g)$.

Hence G is fully free with respect to X and the theorem is proved.

It should be noted here that if the hypothesis that " Y is perfectly normal for each $n \in N$ " is weakened to " Y is normal for each $n \in N$ ", then by the same proof, we obtain the following modification of the theorem: Let X be a topological space with the reflective isotopy property, and let G be a topological group. Suppose that for some topological space Y , $X = Y_0$ and Y^n is normal for all $n \in N$. Let C be the set of all closed subsets F of X such that $\text{cl.}(\tau_n(F))$ is a G_δ -set in Y^n for each n . Then in order that G be free with respect to X it is necessary and sufficient that for each $F \in C$, there exist an action α_F of G on X as a transformation group with the property that for each $g \in G$ such that $g \neq e$, $\alpha_F(g, x) = x$ if and only if $x \in F$.

IMMEDIATE CONSEQUENCES:

The next statements follow immediately from theorem 2.1 as corollaries or from minor changes in the proof.

Theorem 2.2: If X , Y , and G are as in theorem 2.1 and if Z is a topological space such that $Z \times Y^n$ is perfectly normal for each $n \in N$, then G is fully free with respect to $Z \times X$ if it is free with respect to X .

Let G be a topological group and X be a topological space. Let two actions, α and β , of G on X as a transformation group be called isotopically equivalent if there exists a continuous function $H : G \times X \times I \rightarrow X$ such that 1) $H(g, x, 0) = \alpha(g, x)$, 2) $H(g, x, 1) = \beta(g, x)$, and 3) for each $t \in [0, 1]$, $H \mid G \times X \times \{t\}$ is an action of G on X as a transformation group.

Theorem 2.3: Let X be a topological space and assume that for some topological space Y , X is homeomorphic to Y_0 . Let Y^n be binormal for each $n \in \mathbb{N}$. Then in order that for each topological group G , any two actions of G on X as a transformation group be isotopically equivalent, it is necessary and sufficient that X have the reflective isotopy property.

This theorem is a generalization of a theorem of Raymond Wong [11] which appears in his dissertation. Stated in our terminology, it is as follows: "Let X be a separable metric space. Then in order that every homeomorphism of X_0 onto itself be isotopic to the identity, it is necessary and sufficient that the homeomorphism g defined on X_0 by $g(x_1, x_2, x_3, x_4, \dots) = (x_2, x_1, x_3, x_4, \dots)$ be isotopic to the identity."

Theorem 2.4: Any topological group which is freely representable on $s (= \prod_{i \in \mathbb{N}} I_i^0)$ is fully free with respect to s .

In the proof of this theorem, one may replace the reflective isotopy ν used in the proof of theorem 2.1 by an isotopy between the identity of $G(s)$ and the homeomorphism g defined by the formula $g(x_1, x_2, x_3, x_4, \dots) = (x_2, -x_1, x_4, -x_3, \dots)$. Note that by a similar modification of the proof of theorem 2.3 we may prove

that s has the reflective isotopy property, which has already been established by Wong [11] as a corollary to his isotopy theorem mentioned above.

CHAPTER III:

SEPARABLE, INFINITE-DIMENSIONAL FRECHET SPACES

In this chapter we specialize to the class of separable, infinite-dimensional Frechet spaces. A Frechet space is a locally convex, complete, metrizable, linear, topological space. Recent papers of R. D. Anderson [2], C. Bessaga and A. Pełczyński [4], and M. I. Kadec [6] have together established that all such spaces are homeomorphic. We use s as a topological model to prove the following theorem:

Theorem 3.1: In order that a topological group be fully free with respect to every separable, infinite-dimensional Frechet space it is necessary and sufficient that it be effectively representable with respect to some separable, infinite-dimensional Frechet space.

This theorem follows readily from Theorem 2.4 once we have established the following strictly topological theorem about s :

Theorem 3.2: Let s_0 be the countably infinite Cartesian product of s with itself, and for $q \in s_0$, let x_i be the i^{th} coordinate of q for each $i \in \mathbb{N}$. Let $T = \{q \in s_0 \mid \{x_i\} \text{ is dense in } s\}$. Then T is homeomorphic to s .

The proof of this theorem is not self-contained, but uses the theorem of R. D. Anderson [1, cor. 5.5] which states "Let s be regarded as the product of lines and as space, and let $\{K_i\} \ i > 0$ be any collection of closed sets in s such that for each i ,

K_i is bounded above (or below) in infinitely many directions. Then

$s = \bigcup_{i \geq 0} K_i$ is homeomorphic to s .¹¹

Proof of Theorem 3.2: In this proof it will be convenient to regard s and s_0 as distinct "factorizations" of the same space as follows: Let $\alpha_1, \alpha_2, \dots$ be an infinite collection of pairwise disjoint infinite subsets of N such that $N = \bigcup_{i \in N} \alpha_i$. For $i \in N$, let $s_i = \prod_{j \in \alpha_i} I_j^0$. Let $s = \prod_{j \in N} I_j^0$, and let $s_0 = \prod_{i \in N} s_i$. When we wish to express an element q of this space in the " s " factorization, we shall write its coordinate string as $q = (y_1, y_2, \dots)$, and when we wish to express q in the " s_0 " factorization, we shall write $q = (x_1, x_2, \dots)$. Thus when we are working with the " s " factorization, $\pi_i(q) = y_i$, and $\tau_i(q) = (y_1, \dots, y_i)$. When we are working in the " s_0 " factorization, $\pi_i(q) = x_i$, and $\tau_i(q) = (x_1, \dots, x_i)$.

For $i \in N$, let $\varphi_i: s_i \rightarrow s$ be the homeomorphism which assigns to each sequence of coordinates in s_i the same sequence of coordinates in s , i.e., if $x_i \in s_i$, then $\varphi_i(x_{i_1}, x_{i_2}, \dots) = (y_1, y_2, \dots)$, where $y_j = x_{i_j}$ for each $j \in N$.

Let $\{v_i\}_{i \in N}$ be a countable dense subset of s . For each i , $j \in N$ let $H_{ij} = \{q \mid q \in s_0, \rho(\varphi_k(x_k), v_i) \geq 1/j \text{ for each } k \in N\}$ where ρ is a metric for s . (Examples are 1)

$$\rho(q_1, q_2) = \left[\sum_{i \in N} 2^{-i} |\pi_i q_1 - \pi_i q_2|^2 \right]^{1/2} \text{ and 2)}$$

$$\rho(q_1, q_2) = \sum_{i \in N} 2^{-i} |\pi_i q_1 - \pi_i q_2| \text{ .) Now each } H_{ij} \text{ is closed,}$$

$$\text{and } s_0 - T = \bigcup_{i, j \in N} H_{ij} \text{ .}$$

For each triple $(i, j, k) \in N \times N \times N$, let U_{ijk} be a basic product neighborhood of $\varphi_k^{-1}(v_i)$ which is contained in $\varphi_k^{-1}(\{q \mid q \in s, \rho(q, v_i) < 1/j\})$. Let $\psi : N \rightarrow N \times N$ be a function such that for each $(i, j) \in N \times N$, $\psi^{-1}(i, j)$ is infinite. For each $k \in N$, let v_k be a homeomorphism of s_k onto itself so that there is an $l_k \in \alpha_k$ and an ϵ_k such that $0 < \epsilon_k < 1$ and $\pi_{l_k} v_k(s_k - U_{ijk})$ is contained in $(\epsilon_k^{-1}, 1 - \epsilon_k)$, where $(i, j) = \psi(k)$. This can be done simply because U_{ijk} is a basic product neighborhood. Let $v : s_0 \rightarrow s_0$ be defined by the formula $v(q) = (v_1(x_1), v_2(x_2), \dots)$ for each $q \in s_0$. Then $v \in G(s_0)$.

Now consider the "s" factorization. For each H_{ij} there exist infinitely many l_k 's $\in N$ and ϵ_k 's $\in (0, 1)$ such that $\pi_{l_k} v(H_{ij})$ is contained in $(\epsilon_k^{-1}, 1 - \epsilon_k)$. This satisfies the hypotheses of Anderson's theorem quoted above, so $s \sim s_0 \sim s_0 - \bigcup_{i, j \in N} H_{ij} = T$.

This finishes the proof of theorem 3.2.

Proof of theorem 3.1: Let G be any topological group which is effectively representable as a transformation group on s . Let α be an action of G on s so that (G, s, α) is an effective transformation group. Let the topological group $\prod_{i \in N} G_i$, where $G_i = G$ for all $i \in N$, act on s_0 by the natural action, β , defined by $\beta((g_1, \dots), (x_1, x_2, \dots)) = (\alpha(g_1, x_1), \alpha(g_2, x_2), \dots)$. Let $\delta : G \rightarrow \prod_{i \in N} G_i$ be the diagonal map, i.e., $\delta(g) = (g, g, g, \dots)$ for each $g \in G$. Finally, let η be any homeomorphism of s onto T . Let $\zeta : G \times s \rightarrow s$ be defined by the formula $\zeta(g, q) = \eta^{-1}(\beta(\delta(g), \eta(q)))$.

To see that (G, s, ζ) is a free transformation group, we must verify that $\zeta|_{\{g\} \times s} \in G(s)$, that $g \mapsto \zeta|_{\{g\} \times s} = \zeta(g)$ is a monomorphism of G into $G(s)$, that ζ is continuous, and that $\zeta(g, q) = q$ for some $(g, q) \in G \times s$ implies that $g = e$. For each $g \in G$, $\zeta(g)$ is an element of $G(s)$ by the following argument: Given $q \in s$, $\zeta(g)(q) = \eta^{-1}(\beta\delta(g))\eta(q)$. Now $\beta\delta : g \mapsto G(s_0)$ and η is a homeomorphism of s onto T , so to see that $\zeta(g) \in G(s)$ it suffices to prove that $\beta\delta(g)$ leaves T invariant. This is true, however, because if $p \in T$, then $\{x_i\}_{i \in \mathbb{N}}$ is dense in s ($x_i = \pi_i(p)$) and since α is an action $\{\alpha(g, x_i)\}_{i \in \mathbb{N}} = \{\pi_i(\beta\delta(g))(p)\}_{i \in \mathbb{N}}$ is also dense in s and $\beta\delta(g) : T \rightarrow T$ for each $g \in G$. Hence $\beta\delta(g)(T) = T$ as G is a group and $\beta\delta(g^{-1}) = (\beta\delta(g))^{-1}$. The function $g \mapsto \zeta(g)$ for $g \in G$ is a homomorphism, for δ and β are homomorphisms; it is a monomorphism for the same reason. For $g \in G$, $g \neq e$, $\zeta(g)$ has no fixed points because if $q \in s$ then $\eta(q) \in T$ and $\{\pi_i\eta(q)\}_{i \in \mathbb{N}}$ is dense in s . Since α is an effective action of G on s , $\alpha(g)$ must move some point, and because s is a Hausdorff space, $\alpha(g)$ must move some $\pi_i\eta(q)$. Therefore $\eta^{-1}(\beta\delta(g))\eta(q) \neq q$. Finally, ζ is continuous because each of the following functions is continuous and ζ is the composition of them: $G \times s \xrightarrow{1 \times \eta} G \times T \xrightarrow{\delta \times 1} \prod_{i \in \mathbb{N}} G_i \times T \xrightarrow{\beta} T \xrightarrow{\eta^{-1}} s$. This shows that a topological group which is effective with respect to s is free with respect to s . By theorem 2.4, G is fully free with respect to s , and by the above mentioned results of Anderson, of Bessaga and Pełczyński, and of Kadec, the full statement of the theorem follows.

Corollary 3.1: A compact topological group is fully free with respect to each separable infinite-dimensional Frechet space if and only if it is metrizable.

Proof of Corollary: In order that a topological group be metrizable, it is necessary and sufficient that it be a Hausdorff space and that it satisfy the first axiom of countability. It is a simple matter to show that a topological group G acting freely on a first countable Hausdorff space must be both first countable and Hausdorff. Thus a topological group which is compact is fully free with respect to each separable, infinite-dimensional Frechet space only if it is metrizable. On the other hand, if G is a compact metric group, then G can be embedded isomorphically in a countably infinite product of general linear groups of finite-dimensional Euclidean spaces. Each of these groups acts naturally on $\mathbb{R}^n$ and hence on $\prod_{i=1}^n \mathbb{R}^n$ (for some $n \in \mathbb{N}$) as an effective transformation group. By taking the product action of the product of these groups on the product of these open n -cubes in the same way as β was defined in the proof of theorem 3.1, we induce an effective action of G on S and apply theorem 3.1 to finish the proof.

A natural question arises at this point. Since any compact metric group G can act on any separable, infinite-dimensional Frechet space X as a transformation group in such a manner that any given closed subset of X is the fixed point set of each element of G but the identity, what can be said about the possible invariant sets of X under the action of G ? A partial

answer to this question has been pointed out to the author by R. D. Anderson, combining a theorem of J. de Groot [5; see also 3, Ch. 4] and one of Klee [8] with the three papers mentioned at the beginning of the chapter.

Theorem 3.3: Let (G, X, α) be a transformation group, where G is a compact group and X is a compact metric space. Let Y be a separable, infinite-dimensional Frechet space, and let $\varphi : X \rightarrow Y$ be an embedding of X in Y . Then there exists a transformation group (G, Y, β) , such that for any $(g, x) \in G \times X$, $\beta(g, \varphi(x)) = \varphi(\alpha(g, x))$, i.e., any action of a compact group G on a compact subset of a separable, infinite-dimensional Frechet space Y can be extended to an action of G on Y .

The proof of this theorem is immediate from the above results. De Groot proved that there exists an embedding, ψ , of X in a separable, infinite-dimensional Hilbert space H and a continuous homomorphism, ξ , of G into $G(H)$ under the topology of pointwise convergence on $G(H)$ such that $\psi\alpha(g, x) = \xi(g)(\psi(x))$. The kernel of ξ is the set of all $g \in G$ such that $\alpha(g, x) = x$ for each $x \in X$. Klee's theorem says that any homeomorphism from one compact subset of H onto another can be extended to a homeomorphism of H onto itself. Let η be such an extension of $\varphi\psi^{-1}$ and let $\beta : G \times H \rightarrow H$ be defined by $\beta(g, q) = \eta(\xi(g))\eta^{-1}(q)$, for each $(g, q) \in G \times H$. Then β is an action of G on H , and for $(g, x) \in G \times X$, $\beta(g, \varphi(x)) = \eta(\xi(g))\eta^{-1}\varphi(x) = \eta(\xi(g))\psi(x) = \eta\psi(\alpha(g, x)) = \varphi(\alpha(g, x))$. This and the fact that all separable, infinite-dimensional Frechet spaces are homeomorphic yield the result.

CHAPTER IV

SOME THEOREMS ABOUT $G(X)$

Perhaps the most natural group of homeomorphisms of a space X to consider is $G(X)$. In this chapter, we prove that if $G(X)$ is made into a topological space by giving it the compact-open topology or the topology of pointwise convergence, then the constructions of the proofs of theorems 2.1, 3.1, and 3.2 yield embeddings of $G(X)$ into itself as well as homomorphisms. We also prove that if $G(s)$ is given the topology of uniform convergence then the constructions yield an embedding of the topological group $G(s)$ into itself.

Theorem 4.1: Let X be a topological space with the reflective isotopy property. Suppose $X = Y_0$ for some topological space Y and that Y^n is perfectly normal for all $n \in \mathbb{N}$. Let $G(X)$ be made into a topological group by giving it the discrete topology; let $F \neq \emptyset$ be any closed subset of X , and let α_F be the action constructed in theorem 2.1. If $\tilde{G}(X)$ is the topological space obtained by giving $G(X)$ the compact-open topology, then $g \mapsto \alpha_F(g)$ is an embedding of $\tilde{G}(X)$ into itself.

Proof of theorem 4.1: Let all terminology used in the proof of theorem 2.1 be assumed except that " β " is to be replaced by " α_F ", and " $\alpha(g, x)$ ", by " $g(x)$ ". Thus

$$\alpha_F(g, x) = \begin{cases} \xi_i^{-1} (\sigma^{i+1} \omega(g)) \xi_i(x) & , \text{ if } x \in K_i \\ x & , \text{ if } x \in F \end{cases} ,$$

where $\xi_i(x) = \sigma^{i+1}(\eta \vee (\eta^{-1}(\cdot), \varphi_i(x))) (x)$.

First, we note that ω and σ are embeddings of $\widetilde{G}(X)$ into itself, for ω merely restricts the action of the elements of $G(X)$ to the odd coordinates, and σ merely shifts the action of $G(X)$ one index higher in coordinate.

Now, because $\xi_i \in G(K_i)$ for each $i \in N$ and because $\sigma^{i+1}\omega : G(X) \rightarrow G(K_i)$ for each $i \in N$, we have that $g \mapsto \xi_i^{-1}(\sigma^{i+1}\omega(g))\xi_i$ defines an embedding of $\widetilde{G}(X)$ into $\widetilde{G}(K_i)$ for each non-empty K_i .

To see that α_F is a continuous function from $\widetilde{G}(X)$ into itself, we show that any subbasic neighborhood of $\alpha_F(g)$ has an open inverse for each $g \in \widetilde{G}(X)$. Let C be any compact set of X , and let U

be any open set. Let $g \in \widetilde{G}(X)$ and assume $\alpha_F(g) : C \rightarrow U$. As $\alpha_F(g)(C)$ is compact, there is a finite cover of it by basic open product neighborhoods contained in U . Let $V_1, \dots, V_n$ be such a cover. Now each $V_i, i=1, \dots, n$, is of the form

$V_{i,1} \times V_{i,2} \times \dots \times V_{i,m(i)} \times \prod_{j > m(i)} Y_j$. If $i_0 = \max\{m(i) \mid i=1, \dots, n\}$,

then for $x \in F \bigcup_{j \in N} (K_{i_0+j})$, $\alpha_F(h, x) \in V_i$ if and only if $x \in V_i$,

so the problem is reduced to the case of the first i_0 of the

K_i 's. Now for each $j \in N$, K_j is closed, so $C \cap K_j$ is compact,

and $V_i \cap K_j$ is relatively open for $i = 1, \dots, n$. Because

$g \mapsto \xi_i^{-1}(\sigma^{i+1}\omega(g))\xi_i$ is an embedding of $\widetilde{G}(X)$ into $\widetilde{G}(K_i)$

for each non-empty K_i , there exist open sets $\{W_{ij}\}_{\substack{1 \leq i \leq n \\ 1 \leq j \leq i_0}}$

in $\widetilde{G}(X)$ containing g such that $\alpha_F(h, x) \in V_i \cap K_j$ whenever

$h \in W_{ij}$ and $x \in C \cap K_j$. Then $W = \bigcap_{\substack{1 \leq i \leq n \\ 1 \leq j \leq i_0}} W_{ij}$ is an open set

containing $-g$, and $\alpha_F : W \times C \rightarrow U$. Therefore $g \mapsto \alpha_F(g)$ is a continuous function. Now $\alpha_F(g) \rightarrow g$ is continuous because $g \mapsto \xi_i^{-1}(\sigma^{i+1}\omega(g))\xi_i$ is an embedding into $\tilde{G}(K_i)$ for each non-empty K_i . This finishes the proof of theorem 4.1.

It is easy to see that theorem 4.1 remains true if the phrase "compact-open topology" is replaced by "topology of pointwise convergence".

Corollary 4.1: If X is a separable, infinite-dimensional Frechet space, then the action constructed in the proofs of theorems 2.4, 3.1, and 3.2 define an embedding of $\tilde{G}(X)$ into itself, where $\tilde{G}(X)$ is the topological space obtained by giving $G(X)$ the compact-open topology, and $F \neq X$.

Proof of corollary 4.1: Here it suffices to demonstrate that the function $\mu\beta\delta : \tilde{G}(s) \rightarrow \tilde{G}(T)$ is an embedding, where $\tilde{G}(T)$ is the space obtained by giving $G(T)$ the compact-open topology, β and δ are from the proof of theorem 3.1, and μ denotes the restriction to T . This, however, is clear.

Theorem 4.2: Let $G(s)$ be made a topological group under the topology of uniform convergence defined by either of the metrics mentioned in Chapter III. Then the constructions in the proofs of theorem 2.4 and 3.1 yield embeddings of $G(s)$ into itself, if $F \neq X$.

Proof of theorem 4.2: First, the isotopy used to replace the reflective isotopy and each φ_i in the proof of theorem 2.4 can be made uniformly continuous by requiring them to have natural extensions to the Hilbert Cube, $\prod_{i \in \mathbb{N}} I_i$, with s embedded in the

obvious way. This is enough to guarantee that ξ_i is a uniform isomorphism of K_i for each $i \in \mathbb{N}$. Also, ω and σ are embeddings of $G(s)$ into itself. Therefore, $g \mapsto \xi_i^{-1}(\sigma^{i+1}(\omega(g)))\xi_i$ is an embedding of $G(s)$ into $G(K_i)$ for each i such that K_i is non-empty. From an argument similar to the one in the proof of theorem 4.1, this is sufficient to demonstrate that the new action constructed by the techniques of Chapter II is an embedding of $G(s)$ into itself. We use the same approach to the proof of theorem 3.1, i.e., we embed s in $\prod_{i \in \mathbb{N}} I_i$ in the natural way and note that the homeomorphism ν in the proof of theorem 3.2 can be required to have a natural extension to $\prod_{i \in \mathbb{N}} I_i$. Now the entire proof of Corollary 5.5 in Anderson's paper [1] is done in the Hilbert Cube, so the homeomorphism η can be specified to be a uniform isomorphism. To finish the proof it suffices to show that the function from $G(s)$ into itself defined by $g \mapsto \beta \delta(g)$, where s_0 is considered as a "refactorization" of s , is an embedding, but this is precisely the same as showing that the injection function from a metric space onto the diagonal of its countably infinite Cartesian product is an embedding, which is well known.

Corollary 4.2: Any separable, infinite-dimensional Frechet space X admits a bounded metric ρ such that if $G(X)$ is given the topology of uniform convergence under ρ , then the constructions of the proofs of theorems 2.4 and 3.1 induce embeddings of $G(X)$ into itself, if $F \neq X$.

Proof of Corollary 4.2: Let μ be any homeomorphism of X onto s . Let ρ be the metric for X induced by μ from the metric on s used in the proof of theorem 4.2. Then μ is an isometry and theorem 4.2 applies.

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