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A comparative study of geometry curricula

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A COMPARATIVE STUDY OF GEOMETRY CURRICULA

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ABSTRACT

In the United States, geometry has long been offered to high school students in the tenth grade. Attempts have been made in recent years to expand the role of geometry across grades Pre-K through twelve. However, based on the latest TIMSS results, although students in the United States made gains in most content areas, they still struggle with geometric concepts compared to their counterparts in other nations of the world, primarily those in certain Asian countries like Singapore and China. We argue that the structure of the curriculum and the instructional strategies used in these countries may lead to more progressive reform strategies for the United States curriculum. These strategies may provide the catalyst to push our students back to the head of the class when assessed locally, nationally, and internationally.

CHAPTER 1 – OVERVIEW OF GEOMETRY

What is Geometry?

According to Webster's Dictionary, geometry is *a branch of mathematics that deals with the measurement, properties, and relationships of points, lines, angles, surfaces, and solid; broadly**:** the study of properties of given elements that remain invariant under specified transformations*. H. M. S. Coxeter described geometry as "…*the most elementary of the sciences that enable man, by purely intellectual processes, to make predictions (based on observation) about physical world. The power of geometry, in the sense of accuracy and utility of these deductions, is impressive, and has been a powerful motivation for the study of logic in geometry."* Geometry can also be seen as the mathematics of logic and reasoning. According to the average citizen, geometry is the mathematics of shapes and their characteristics. More commonly, Geometry is the math class taken in the 10^{th} grade, usually between Algebra I and Algebra II. This sequence of courses has been in place since the late nineteenth century.

Based on the previously mentioned definitions, geometry's scope should not be limited to simply being a course taught in high schools, one that is more often than not a requirement for high school graduation. In the United States, the study of geometry has been restricted to a sophomore year course in high school that addresses specifically triangles, quadrilaterals and circles and the properties of each.

Many students approach geometry with hesitation because they lack the foundation to be successful in the course as it is designed. Successfully introducing geometry to students should be done over the course of years. Students as early as Prekindergarten should have an introduction to geometric concepts and ideas. Young children are able to build knowledge of shapes, analyze them, combine shapes and create with shapes. (Sarama and Clements, 2006) The National Council of Teachers of Mathematics in its publication *Principles and Standards for*

School Mathematics (NCTM, 2000) notes that geometry offers students a chance to develop reasoning and justification skills. It points also to the fact that through modeling and spatial visualization, students become better problem solvers. The document goes on to say that students can use geometric representations to make sense of other areas of mathematics, and should therefore be integrated when possible.

What has occurred too often is that labels have been associated with high school math classes, such as geometry; many educators are hesitant to introduce geometric concepts in the early grades. Students in elementary school are exposed to shapes and the concepts of perimeter and area, but the depth of knowledge is lacking. The study of geometry should serve as a scaffold over the course of a student's educational experience; that is to say students should be exposed to geometry in a manner that new concepts are built upon by previously taught concepts. Glenda Lappan, past-president of NCTM, said in the NCTM Bulletin from December 1999, "We must build a geometry strand that engages our students in this interesting and important area of mathematics throughout their school experience, from pre-K through grade 12." Only then can students be ensured to fully grasp geometry in a true sense of its meaning. Restricting geometry to one year robs students of its beauty and the importance of its study to the real world.

Why is it important to study geometry? Geometry provides its students with opportunities to "see" mathematics. Many topics in mathematics are made clearer through geometric representations. Many students are visual learners; ideas become more relevant through spatial visualization. Geometry enhances spatial understanding and the concepts related to space. Research has associated students' ability to perform geometric tasks to their competency in critically reasoning through problems. (Johnston-Wilder and Mason, 2005) Geometry has also been linked to efficiency in applications involving proportional reasoning. (Slovin, 2000) How then do we encourage a course of study in Geometry that is first and

foremost practical and whose inception begins in Prekindergarten? What was the original intent of those early practitioners who first developed the concepts we know as geometric in nature? It should follow that concepts that lend themselves to being easily investigated through tactile measures should be introduced early, as students are encouraged to use their senses to discover mathematical concepts. Geometric thinking can be developed as early as in infancy. Some of the first things children recognize are basic shapes. Children learn to develop relationships between objects. As they get older, they learn how to interpret and relate information about two- and three- dimensional representations.

A Brief History of Geometry

The study of geometry can be traced back to early cavemen who drew pictures of obtuse triangles in the Indus Valley (now part of India and Pakistan) and to Babylonian civilization. Early practitioners of Geometry developed properties associated with length, angles, area, and volume that were applicable to enhancing particular crafts, such as construction. The civilizations of ancient Egypt, Babylon, India, Pakistan, and Greece have all made significant contributions. Geometry as a practical science was developed based on man's intuition and his relationship with nature. Many basic geometric constructions mimic things we find in nature. Nature has served as an inspiration to everything geometric.

The early Egyptians had a practical approach to geometry. They used geometry to help solve life's problems. The geometry of early Egypt was influenced by their need for practicality to tackle problems associated with construction and economics. The reasons that the Egyptians did not develop a more formal look into geometric concepts are unclear. The Greeks used the concepts of geometry that they learned from the Egyptians to develop geometry into the deductive science we believe was the foundation for Euclid's *Elements*. The *Elements* was the backbone for geometry textbooks for more than 2000 years. Because of the formality of Euclid's

Elements, geometry has often been viewed as something impractical and restricted to being driven by proof.

Leading up to the time Euclid wrote the *Elements*, other notable Greeks made contributions to geometry. Thales of Miletus was a Greek philosopher. He is credited with the saying "Space is the greatest thing, as it contains all things." Thales was familiar with properties of similar and right triangles and used applications of these properties to solve problems. The theorem attributed to him provides insight to the relationships that exists when two lines that intersect are intersected by a pair of parallel lines. Figure 1.1 is an illustration of Thales' theorem. Given that segment PQ and segment PR intersect at point P, and that segments ST and QR intersect segments PQ and PR and are parallel to each other, the ratios of the segments formed are equal.

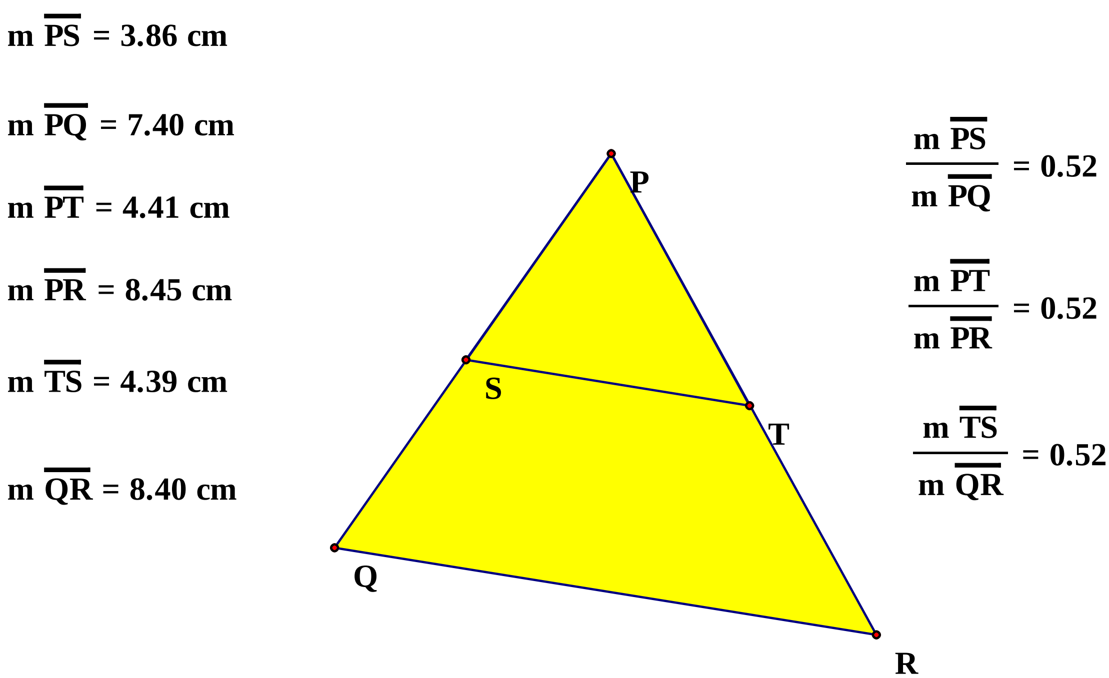

Figure 1.1 – Thales' theorem

Pythagoras of Samos was a Greek mathematician most often attributed with the Pythagorean Theorem. Pythagoras was the founder of the school of thought known as

Pythagoreanism. His followers, known as “Pythagoreans” were mathematicians and philosophers who laid the foundation for geometry as a set of principles and axioms that predated Euclid by some 200 years and led to the writing of the *Elements*. The Pythagoreans are most commonly known by the Pythagorean Theorem, which was known by earlier civilizations, such as the Babylonians but proven by the Greeks. The theorem says that the sum of the areas of the two squares on the two legs of a right triangle is equal to the area of the square on the hypotenuse. Euclid proved the theorem by deduction in proposition 47 of book 1 of the *Elements.*

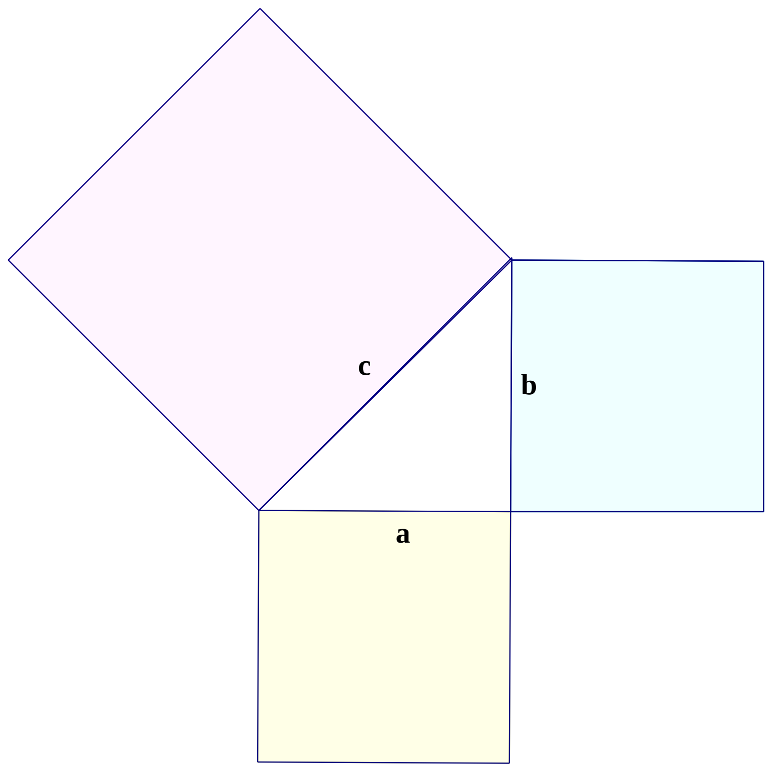

Figure 1.2 Illustration of Pythagorean Theorem

Euclid of Alexandria was a Greek mathematician who wrote the *Elements* around 300 BC. Although others had previously discovered many of the concepts in Geometry, Euclid provided a comprehensive presentation of using deductive reasoning in simplifying what was becoming a more complicated science. The *Elements* provide a logical development of

geometric concepts and other areas of mathematics. The *Elements* begins with definitions, postulates, and *common notions*, statements (propositions) logically deduced from the postulates.

Table 1.1 - Overview of Euclid's Elements

Book	Definitions	Postulates/Axioms (Common Notions)	Propositions
I Plane Geometry	23	10	48
II "Book of Geometrical Algebra" Plane Geometry	2		14
III Circles Plane Geometry	11		37
IV Constructions Plane Geometry	7		16
V Proportions of Magnitude	18		25
VI Proportions of Geometry	4		33
VI Number Theory	22		39
VIII Proportions in Number Theory and Geometric Sequences			27
IX Number Theory			36
X Irrational Magnitudes	16		115
XI Three-Dimensional Geometry	28		39
XII Volumes of Solids			18
XIII Platonic Solids			18

The series of 13 books follows a logical sequence; each set of propositions can be proven using information from prior books. Each book develops a specific topic, presenting it as a series of propositions and proofs.

Books I – VI, X, and XI of the *Elements* begin with a comprehensive list of definitions.

These definitions provide a basis for concepts discussed throughout the *Elements*. There is a logical progression in the way the definitions are listed, from basic definitions, such as the definitions for point, line, and surface, to more significant definitions that are built on the previous ones.

The definitions in the *Elements* in Book I are followed by 5 postulates. Book I is the only book of the *Elements* containing postulates. The postulates are statements that are accepted without a burden of proof. The first three postulates are illustrated through use of constructions. The five postulates of Book I are as follows:

- Between any two points a line can be drawn; Euclid defined a point as that of which there is no part and a line as a length without breadth. He went on to further state that the extremities of lines are points. (Fitzpatrick, 2008)
- Lines can be extended.
- Given two points, a circle can be constructed; one of the points is considered the circle's center and the second point is the other endpoint of the radius.
- All right angles are congruent, and;
- If two lines intersected by the transversal are parallel, then the corresponding angles of a transversal are congruent.

Subsequent to the postulates are 5 axioms called Common Notions, which refer specifically to magnitudes such as straight lines.

Finally, the *Elements* are completed with propositions. The propositions are statements that are accompanied by proofs of the statements. Each proof can be developed using a prior definition, postulate, common notion, or a previous proven proposition.

Euclid's *Elements* is one of the oldest texts dedicated to the study of mathematics. It is the most universally read textbook written. In the medieval period, when the *quadrivium* was part of the curriculum for university students, knowledge of Euclid's text was a requirement. We still see pervasive influences of Euclid in modern geometry. For example, because of the sequence in Euclid's text, plane geometry is more often than not studied in advance of solid geometry. Because of this emphasis placed on plane geometry, however, many students are not

afforded the opportunity to study solid geometry.

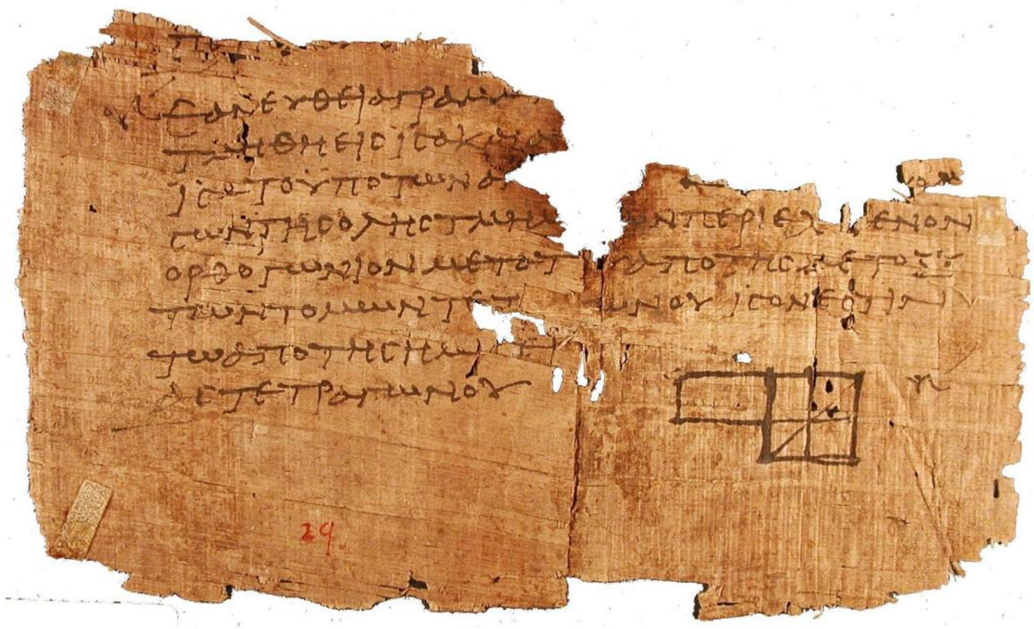

Figure 1.3 - Euclid's Elements on papyrus

Beyond Euclid, many other mathematicians have played an important role in the development of geometry. Archimedes of Syracuse was a Greek mathematician along the order of Euclid. He is noted for determining how to find the volume of a sphere, stating that its volume is two-thirds of a circumscribed cylinder. He also found a method to calculate the surface area of a sphere, based also on the circumscribed cylinder. Archimedes used the "method of exhaustion" to approximate the value of pi.

In 1637, French mathematician Rene' Descartes introduced what is known as analytic geometry. Analytic geometry is an area of geometry devoted to studying points with respect to a coordinate system. The approach to solving geometric problems in analytic geometry is algebraic in nature. It primarily uses equations and formulas to find things such as slope and distance. Analytic geometry is most often introduced in an algebra course, and further studied in geometry and beyond.

Other areas of mathematics were influenced by geometry. In the 17^{th} century, major

advancements were made in the area of calculus. It is debatable who discovered modern calculus, Isaac Newton or Gottfried Liebniz. James Gregory using Euclidean methods in 1668 had proved the Fundamental Theorem of Calculus. Both Newton and Liebniz formalized the proof after Gregory. Newton is noted for applying calculus to physics and Liebniz is credited for developing the notation currently used.

The study of topology arose in the 18th century. Topology is the mathematical study of the properties that are preserved through deformations, twistings, and stretchings of objects. (http://mathworld.wolfram.com/Topology.html) Leonhard Euler's paper '*The Seven Bridges of Konigsberg"* is considered the foundation for what is now known as topology.

Until the 19th century, most geometry was Euclidean. However, since then the idea of geometry has gone through many developments. Non-Euclidean geometry provides alternatives to Euclid's Parallel Postulate, hyperbolic and elliptic geometries. The Parallel Postulate states that given a line and a point not on the line, there is only one line passing through the point parallel to the given line. Hyperbolic geometry proposes that there are at least two lines passing through the point that will no intersect the given line. Elliptic geometry is based on the premise that given a line and a point not on the line, there are no lines parallel to the given line.

CHAPTER 2 - THE AMERICAN GEOMETRY CURRICULUM

The study of geometry in the United States started with Euclid. As a result of its comprehensive nature, *Elements* was the model for Geometry textbooks used at the advent of teaching Geometry in the United States. The *Elements* provided the foundation for the Geometry curriculum, as we know it, in the United States. Due to its logical characteristics, geometry was approached from the standpoint of using definitions and postulates to reach conclusions. In the nineteenth century, when formal geometry appeared in classrooms across the United States, students' understanding of concepts and ideas was attained through reasoning. Students based all what they knew about Geometry on given facts; students were not asked to reach conclusions through investigative approaches.

The *Elements* is responsible for the scope of geometry that is commonly studied today. Geometry was introduced in American classrooms in the mid 1800's; the curriculum was designed specifically around the *Elements*. Euclid's text became the catalyst for studying geometry, providing the scope and sequence of what needed to be studied.

Euclid's *Elements* was the major geometry text studied in Europe. Between 1482 and 1900, 1100 editions of the text were published. In 1570, Henry Billingsley published the first English translation of Euclid, *The elements of geometrie of the most ancient philosopher Euclide of Megara*.

In the United States, geometry was taught exclusively in colleges and universities until the middle nineteenth century. Harvard College, founded in 1636, first offered geometry as a course of study in the third year of a three-year program of study. In 1655, when Harvard became a four-year institution, geometry was still taught in the last year. By 1733, both Harvard and Yale were offering geometry as a senior-year course, and Euclid was used as the text. In 1744, Yale began to offer geometry as a second-year course; Harvard did not adopt this position

until 1787. By 1818, Harvard began offering geometry as a first year course.

Prior to 1844, geometry was not a requirement for college admission; Harvard made elementary geometry a requirement in this year. Yale followed suit in 1855, requiring two books from John Playfair's *Elements of Geometry*. By 1887, students applying for admission needed knowledge of plane geometry. Harvard and Yale became the benchmark for admission requirements for other college and universities across the country.

In 1794, Adrien Marie Legendre wrote *Eléments de Géométrie*. This text was viewed as an alternative to Euclid's. Legendre set about to improve Euclid's text by rearranging and simplifying the propositions. First applauded in Europe, Legendre's text was introduced in the United States in 1817 by Claude Crozet at West Point. Translated into English by Charles Davies in 1819, Legendre's simplified text made it possible for the first real change in how geometry was approached. Although Euclid had been the preeminent text, Legendre offered the most successful attempt to supplant Euclid in geometry course of study. By the time John Farrar of Harvard University translated Legendre into English in 1891, this text had become an important fixture in the realm of geometry.

Geometry did find a place at schools other than colleges and universities prior to 1844. According to author Calvin Olin Davis in *Public Secondary Education,* academies such as Phillips Exeter in New Hampshire taught geometry as part of their course of study. (http://books.google.com/books?id=SIgWAAAAIAAJ&pg=PR3) Grammar schools in colonial states taught geometry in tandem with navigation and surveying as they prepared students for excelling at particular trades. Boston Latin School, a college preparatory school, offered geometry from 1815-1828 in years four and five of a five-year program. However, what we now know as high school did not appear in the United States until 1821. By 1860, only 40 high schools existed in the country. The first such school, Boston English High School, offered

geometry during the 2nd year of a 3-year course of study. The British, who were staunch proponents of Euclid's text, heavily influenced the curriculum in the United States. This was due, in part, to the colonial status. French influence came later in the form of Legendre's text, and served as an alternative to Euclid. However, it was not until 1890 when a compromise text was introduced. The 42nd edition of Charles Davies' translation of Legendre, *Elements of geometry and trigonometry, from the works of A. M. Legendre,* (http://books.google.com/books?id=b4YAAAAAMAAJ&dq=charles%20davies%201871%20geometry&pg=PR1) served as a marriage of two views on geometry. Although Davies, in his preface, applauded Legendre for his treatment of elementary geometry, he also felt that the fact that Legendre used diagrams to describe the propositions of Euclid was regrettable. In his text, Davies combined the approaches of both Euclid and Legendre to the propositions in an attempt to lessen the difficulty a novice learner may have in comprehending the propositions without lessening their importance.

During the latter part of the nineteenth century, the first attempt at real reform in the geometry curriculum began to take shape in the United States. In 1892, The National Education Association, cognizant that the educational system in the United States was in need of repair, formed the Committee of Ten. The Committee of Ten consisted of ten educators from across the country, led by Charles William Eliot, president of Harvard University. One of the tasks assigned to the committee was to review the mathematics curriculum in the United States. In the nineteenth century, there were many schools of thought as applied to mathematics education, especially the place of geometry in the curriculum. The committee recommended in its final report that "the study of demonstrative geometry should begin at the end of the first year's study of algebra, and be carried on by the side of algebra for the next two years, occupying about two hours and a half a week." (Report of the Committee of Ten, 1892,) The committee also

recommended that plane and solid geometry could be mastered if concrete geometry was well taught. They also suggested that a student's first introduction to geometry should begin in kindergarten, and that by age ten students should be exposed to concrete geometry for at least one hour per week for three years. The committee's final recommendation held that geometry was essential to the mathematics curriculum and that it alone was essential to the development of a student's ability to reason and solves problems.

At the turn of the twentieth century, although many attempts were made to revise geometry curricula nationwide, many schools still used English translations of Euclid's text as their primary geometry textbook. High schools in the country suggested that all students take algebra and geometry, even though the rates of failure for both were extensive. In England, physicist John Perry began a reform movement to change the way geometry was taught in schools. Perry believed that geometry could be taught effectively sans proof. E. H. Moore, based on his support of Perry's views, suggested that students be allowed to explore problems, thus using analytical skills and applying these skills to solving mathematical problems. Moore believed that schools should integrate math and science and that mathematics classrooms should be lab-based. He also felt that there was a need for an "ambitious program of educational reform for secondary schools and colleges." (Final Report of the National Committee of Fifteen on Geometry Syllabus, 1912) The catalyst for Moore's call for change was a curriculum that at that time was heavy on algorithm but disconnected from the real world. The idea that geometry could be taught using concrete approaches appealed to him. In this sense, geometry could be introduced to younger children without fear of failure. Making geometry more concrete would also allow the ideas of geometry to be introduced to children in earlier grades. As part of his reform message, Moore believed a key component was teacher training in content knowledge and the advent of junior high schools and junior colleges.

In 1908, The American Federation of Teachers of Mathematical and Natural Sciences and the NEA commissioned the National Committee of Fifteen on the Geometry Syllabus. The committee published its Provisional Report in 1911. The members of the committee were distinguished educators from the nation's colleges and universities and secondary schools. The committee's task was to consider the role of geometry in the school curriculum. It produced a report that ultimately recommended a geometry syllabus. In determining how the syllabus would be devised, the committee reviewed how geometry was taught in Europe, looked at geometry textbooks and their structure, identified resources to help in the teaching of geometry, and stated the importance of a geometry curriculum that was a balance between abstract and concrete ideas.

The Committee of Fifteen recommended that in the syllabus, a list of axioms, definitions, postulates and symbols be included. In regards to the propositions, the committee felt that educators should not dismiss formal proof, but felt in any "well-regulated course in geometry" nearly 100 of the theorems required formal proof. However, the committee also felt that it was not necessary to attempt to study the list of theorems in their entirety or in any particular order. Some of the theorems could be restated in more general terms for the benefit of the students and those central to the study of geometry should be emphasized. They also suggested that algebra be taught in ninth grade, geometry in tenth grade, and a combination of both in grade eleven, which is still tradition in most high schools today.

In addition to the syllabus recommendations, the committee addressed the need for geometry to be introduced informally to students in elementary grades. The committee offered the opinion that special courses for students with vocational goals were not necessary; it decided that certain theorems could be ignored or highlighted to meet the needs of the students. It also stressed the importance of using scale drawing and modeling to demonstrate measurement.

Overall, the Committee of Fifteen recommended that a successful geometry course of

study required a marriage of abstract and concrete exercises, pedagogical changes to sustain how students learned geometry, textbooks that were successfully aligned to the committee's recommendations, and college entrance examination items that modeled the geometry being taught in secondary schools. The committee believed that the study of geometry was essential to developing reasoning skills for students.

Prior to the Committee of Fifteen's report, geometry was considered key to a student's development of logical reasoning skills. The course served as a training ground for students to become more adept at reasoning not only in geometry, but to carry the reasoning skills with them in to other areas of study with the same thoroughness needed to address the formality of the geometry course. Subsequent to the report, geometry in the United States in the 20th century also served as an avenue for students to be prepared for the workplace. The concept that the study of geometry would provide students with tools for success in the workplace arose out of the committee's desire to include teaching of applications in geometry. Geometry was also viewed as a means to expose students to the work of mathematicians. Students taking geometry would be engaged in making and proving conjectures and modeling the concepts that allow students to become problem solvers. Finally, students enrolled in geometry courses can use geometric ideas to model problems while intuitively reasoning through them.

Throughout the 20th century, little changed about teaching geometry in American classrooms. In grades K-8, geometry was integrated into a curriculum that covered a variety of strands. In the primary grades, students learned the names of basic shapes, and how these shapes related to things around them. Later, they were exposed to perimeter and area of polygons, and surface area and volume of solids. Students were introduced to properties of planar figures, symmetry and proportionality. As students moved on to high school, it was still regarded as the course most frequently offered to students in the tenth grade. However, by mid-century, only

one-third of tenth grade students taking mathematics were enrolled in a geometry class. (Kliebard, 2004) The geometry course was Euclidean in nature, and students were asked to use deductive reasoning to solve problems. In the latter half of the century, mathematics educators began to view geometry from a more investigative approach. Students were now being introduced to methods of induction and students used conjecture as a means to explore geometry. Dynamic software systems like *Geometer's Sketchpad* became available for classroom use.

As the century progressed, and the number of students in high schools across the country increased, a shift in the mathematics requirements in the country began to take place. In 1983, the National Commission on Excellence in Education released *A Nation at Risk: The Imperative for Educational Reform.* The report findings were startling. In 1980, 35 states only required one high school mathematics class for graduation. Students in high school taking a mathematics class most often were choosing between Algebra I and General Mathematics. The secondary curriculum was referred to as "homogenized" and "cafeteria style". As a result of the report, reforms in our educational system began to take place. In most states, at least three mathematics classes were required for high school graduation. The commission recommended that high school mathematics contain both algebraic and geometric content. In 1994, the National Center for Education Statistics (NCES) released *High School Students Ten Years After "A Nation at Risk"*. The report showed that the number of students taking high school geometry increased from 48.4% in 1982 to 70.4% in 1992.

The presence of geometry in the American school curriculum remains strong. Educators are aware of the importance of exposing students to concepts that will allow them to develop critical thinking and reasoning skills. Often, geometry is the avenue to which these skills are reached. However, American students still struggle with geometric concepts compared to students internationally.

CHAPTER 3 – STUDENT ACHIEVEMENT ON ASSESSMENTS

The National Assessment of Educational Progress (NAEP) in mathematics provides an indication of how well students at grades four, eight and twelve perform compared to established benchmarks. NAEP is developed based on the 2005 mathematics framework designed by the National Assessment Governing Board. The mathematics framework is two-dimensional; the assessment focuses on content and cognition. The 2007 NAEP mathematics assessment contained the following content strands:

- Number properties and operations
- Measurement
- Geometry
- Data Analysis and Probability
- Algebra

The cognitive dimension of the NAEP assessment addresses the complexity of the assessment items. Items are categorized as *low, moderate,* or *high*. This dimension also includes aspects of reasoning, procedural skills, conceptual knowledge, and problem solving. All NAEP test booklets administered are uniform, so it is a good indicator of student achievement on a national level.

NAEP was first administered as a state mathematics assessment in 1990. Overall mathematics scores at grade eight increased from 263 that year to 281 in 2007. Gains for scores for the geometry domain were identical to the composite score gains, but the geometry scores were slightly below the composite. In 1990, the average score for geometry was 260; in 2007 it rose to 278.

In 1964, the International Association for the Evaluation of Educational Achievement (IEA) commissioned the First International Mathematics Study and 12 countries, including the United States, participated. The purpose of the study was to examine math achievement in the participating countries and look at how curriculum and instructional methods affected student

performance. The study showed that students exposed to mathematics reform performed better than their counterparts. (FIMS, 1964) It also showed that students exposed to inquiry-based learning in mathematics outperformed their peers. For the United States, at grade 8, students performed worse overall than 10 of 11 countries. On a 70-item mathematics test, the mean number of items correct for students in the United States was 17.8, compared to 32.3 for the highest scoring country, Israel.

As a result of the FIMS, the Second International Mathematics Study (SIMS) was commissioned in 1981-82. This time, 24 countries participated in the study. The purpose of the study was to investigate curricula and instructional practices, and analyze results of the student assessment. The goal of the study was to provide each of the 24 countries a snapshot of how their mathematics education programs compared to the other participating countries.

Students were assessed in several content areas, including geometry. The geometry assessment consisted of 39 items. Fifteen countries scored better than the United States on the eighth grade assessment in geometry. The US mean percent of items correct was 37.8, compared to 57.6 percent correct for Japan.

In 1980, The National Council of Teachers of Mathematics introduced their *Agenda for Action* that highlighted the need to review the direction that mathematics education was taking on a national level. One of the recommendations of the agenda was that "…from the earliest years, the basic mathematical tools should be acquired within the framework of usage and application…" In other words, the agenda pointed to a need for a developmental sequence for learning goals from Pre-Kindergarten through grade 12. The need for effective sequencing is nowhere more important than in the study of geometry.

In a report of the 2007 Trends in International Mathematics and Science Study (TIMSS) released in December 2008, the United States showed marked gains in the overall scale score as

opposed to other countries around the world. When the TIMSS study was first introduced in 1995, the United States had a scale score in grade eight of 492; in 2007 the scale score was 16 points higher at 508. This score was 8 points higher than the overall TIMSS average scale score of 500. Of the 48 countries participating in 2007 at the eighth grade level, the United States scored better than 37 countries and had scores that were not significantly different than 5 other countries, Hungary, England, Russian Federation, Lithuania, and Czech Republic. Only 5 countries fared significantly better in the overall scale score than the United States: Chinese Taipei, The Republic of Korea, Singapore, Hong Kong SAR, and Japan.

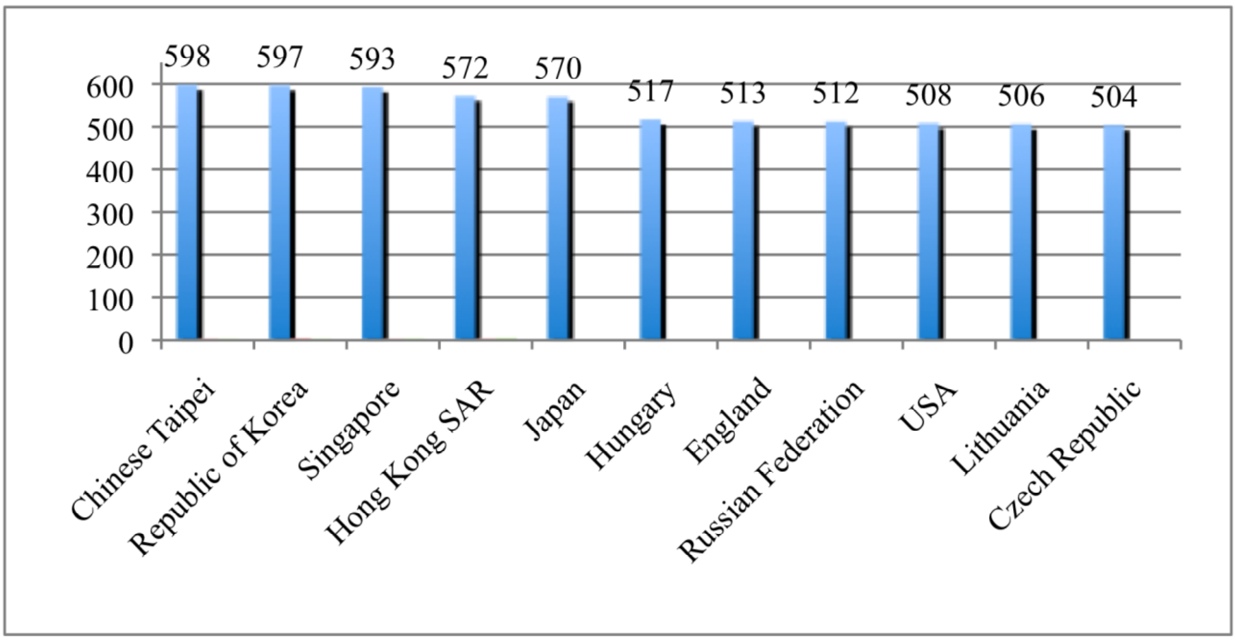

Figure 3.1 – TIMSS 2007 Grade Eight Mathematics Scale Scores

Compared to the TIMSS scale score average of 500, the United States fared well at grade eight in two of the four content domains covered on the 2007 TIMSS, *number* and *data and chance*. In *algebra*, there was no significant difference between the United States scale score and the TIMSS average scale score. However, in the content domain of *geometry*, the United States scored significantly below the TIMSS scale score average by 20 scale score points. For this content domain, 14 countries scored better than the United States, including the nine

countries that scored better than the scale score average, Chinese Taipei, Republic of Korea, Singapore, Japan, Hong Kong SAR, England, Russian Federation, Hungary, and Lithuania. As usual, many factors could contribute to the performance of the United States in the content domain of *geometry*. The study looks closely at scale score as they relate to gender, race, and poverty. The study, however, does not offer any conjecture regarding the United States' dismal showing in the *geometry* content domain. What is also not addressed is the structure of the geometry curriculum used in the United States as compared to other countries. When American students were administered the TIMSS assessment in 2007, how much geometry had they been exposed to? How does this compare with the level of exposure students in the countries that fared better than the United States in this content domain? It is difficult to pinpoint an exact reason or reasons for students performing poorly in the *geometry* domain. It is possible however to compare the different curricula based on the position and scheduling of geometry in each curriculum and the manner in which the geometry is presented.

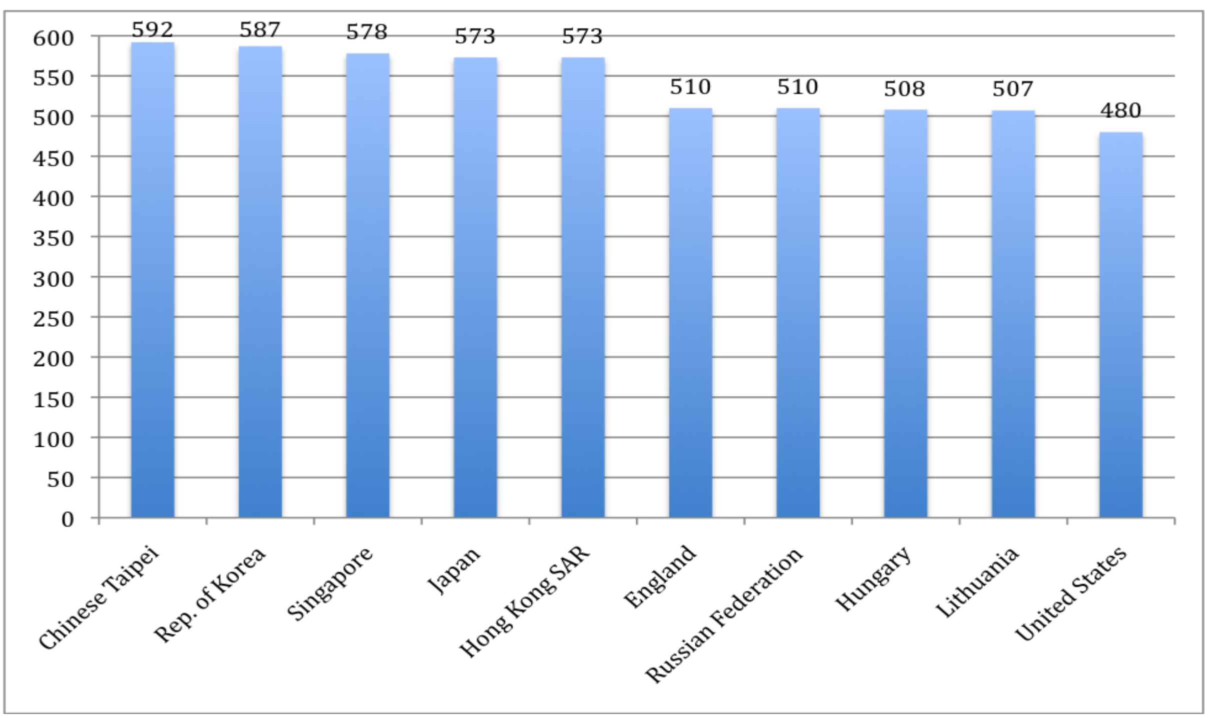

Figure 3.2 – TIMSS 2007 Grade Eight Geometry Content Domain Scale Scores

The *geometry* content domain for TIMSS covers twenty-two percent of the grade 8 assessment covered geometry. Students in grade eight were expected to be able to analyze two- and three- dimensional figures, use and apply the Pythagorean Theorem, and complete measurement tasks competently. Also encompassed in the *geometry* content domain were coordinate geometry, spatial visualization and transformations.

The *geometry* content domain of TIMSS covered three specific topics: shapes, measurement, and location and movement. For each topic, a set of benchmarks is addressed. Students at grade eight are also asked to be able to construct geometric figures, analyze compound figures, interpret views of solids and solve problems using properties of congruence and similarity.

In 2000, 32 countries, including the United States, participated in the first Program for International Student Assessment (PISA) of 15-year olds in Reading, Mathematics and Science Literacy. PISA was sponsored by the Organization for Economic Cooperation and Development (OECD). The PISA assessment was designed to tests students' abilities to solve problems that had real-world context. Mathematics literacy was defined in terms of three contexts: content, process, and situation. Students taking the assessment were judged on how well they could use processes to solve problems they might encounter in real life. In mathematics literacy, American students average score was 493, compared to the OECD average of 500. Students from seventeen of the 32 participating countries fared better on average than American students. Table 3.1 lists the countries that fared better overall than the United States in the mathematics literacy category. Statistically, the top eight countries had scores that were overall significantly better than the United States. When PISA was offered again in 2003, over a quarter of a million students from 41 countries took the assessment. Sixty-one percent of the American students taking the test were in tenth grade and thirty-one percent were enrolled in geometry. The

mathematics literacy assessments contained problems from four domains: *space and shape, change and relationships, quantity*, and *uncertainty*. Data was available for 39 of the 41 countries. Once again, the United States fared poorly compared with its international partners. The results from PISA 2003 show that students in the United States had an average score that fell 10 points to 483, compared to the OECD average of 500. Twenty-three countries scored better overall than the United States. Students in the United States scored worst in the content domain *space and shape*, with an average score of 472.

Table 3.1 Countries with higher PISA 2000 scores than United States

Japan	Switzerland	Iceland
Rep. of Korea	United Kingdom	Sweden
New Zealand	Belgium	Ireland
Finland	France	Norway
Australia	Austria	Czech Republic
Canada	Denmark	

The third PISA assessment took place in 2006. Students from fifty-seven countries participated. For a third consecutive time, the United States' mathematics literacy average score was below the OECD average. The PISA 2006 overall average was 474, compared to the OECD average 498. Internationally, 34 countries scored better than the United States. Twenty-five of the 34 countries had scores that were statistically significantly different than the United States. Students in the United States at the 90th percentile had an average score of 593; the OECD average score at the 90th percentile was 615.

Based on what we know is covered on international assessments, how we present topics in mathematics, and more specifically geometry, to our students is important. Countries that consistently outperform the United States have mandated national curricula. The absence of a required national curriculum in this country poses some serious questions for educators to

consider. We should examine how diverse mathematics instruction is. As individual states and local districts around the nation interpret the standards suggested by the NCTM and other organizations in various ways, students may be exposed to varying degrees of geometry instruction. We must also consider that the NCTM standards are recommendations. NCTM being a professional organization, means states have no obligation to follow their suggested guidelines

Table 3.2 Countries with higher PISA 2003 scores than United States

Finland	New Zealand	Ireland
Rep. of Korea	Czech Republic	Slovak Republic
Netherlands	Iceland	Norway
Japan	Denmark	Luxembourg
Canada	France	Poland
Belgium	Sweden	Hungary
Switzerland	Austria	Spain
Australia	Germany	

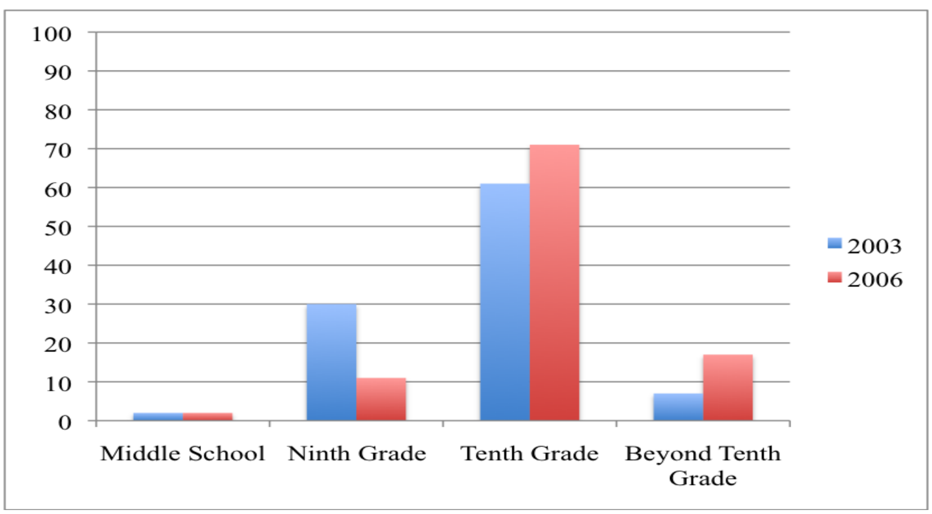

Figure 3.3 - Percentage of American Students Taking PISA by Grade.

What is necessary for successful completion of secondary geometry has become subjective. *A Nation at Risk* was released more than a quarter of century ago. We find ourselves as a nation still at risk when it comes to mathematics education.

CHAPTER 4 – SURVEY OF GEOMETRY TEXTBOOKS

As a result of the Committee of Fifteen report, it was recommended that geometry textbooks be patterned on the geometry syllabus. Over the course of the past 100 years since the committee's reports, secondary geometry textbooks in the United States have remained fairly consistent regarding their content. Many publishers have tried to ensure that their texts are balanced between abstract concepts and concrete applications. Although the text of Euclid has long since been replaced, his work still exerts a strong influence on the structure of geometry textbooks. The geometry textbook has also often been determinative of the course content and structure. Many classroom teachers feel uncomfortable attempting to teach without the textbook at hand. For some, it provides a pacing guide that takes students through a school year. The sequencing of the tenth grade course has long been dependent upon how the text is ordered.

The Advent of American Geometry Textbooks

Most early geometry books used in the United States were translations of Euclid's *Elements* or Legendre's text. One of the first high school geometry textbooks written by an American author (not a translation) was *Elements of Geometry: with Practical Applications to Mensuration* (1858) by Benjamin Greenleaf. It is also one of the first texts to include a problem set at the end of the book. Greenleaf's text contained 14 chapters and was 320 pages in length. Greenleaf used Legendre's text as a model for his, and sought to improve on the geometry textbooks of the day. In the preface, Greenleaf states that the exercises were an attempt to "test the thoroughness of the scholar's geometrical knowledge." (Greenleaf, 1858)

In 1870, William Chauvenet wrote *A Treatise on Elementary Geometry* much like Greenleaf's textbook. At the end of Chauvenet's book was a series of exercises arranged by chapter. There were no solutions available; the exercises were derived from *Traité de Géométrie Élémentaire* by Èugene Rouché and Charles de Comberousse (1874).

Most geometry textbooks of the past century and a half were arranged by chapters or books, each addressing a specific overarching theme. George Wentworth and David Eugene Smith offered *Plane and Solid Geometry*. The text comprised 9 books with exercises at the end of each book. The text was 480 pages in length.

As curriculum changes were made in the beginning of the twentieth century, the look of geometry textbooks changed dramatically. In the middle of the twentieth century, an average geometry textbook covered about 500 pages of material. By the end of the century, the number of pages in a typical American geometry book increased to more than 800 pages and the pages became larger and more complex individually. This was due in part to the increasing number of topics included in curricula and the desire for the textbooks to include more illustrations and examples.

Curriculum Reform and Its Effect on Textbook Design

Since 1980, when the NCTM first published its *An Agenda for Action*, the direction of school mathematics in this country has been driven by its call for diverse instructional strategies. In the twentieth century, the textbook was the driving force for instruction in America's classrooms. The textbook was considered the best representation of the curriculum and was seen as the link between the curriculum and classroom instruction. The prevailing use of textbooks in the classroom influenced how mathematics content was delivered and received. Teachers were more likely than not to present material that was included in a textbook; if the textbook did not include specific mathematics topics, classroom teachers did often not regard those topics. In most classrooms in the United States the sequence of what was taught paralleled how it appeared in the textbook. Even as reform in the curriculum began to take shape, the textbook was still the primary source modeling curriculum revision; many textbook publishers used state curricula to design "state-specific" textbooks that incorporated standards and benchmarks as established by

the state.

In 1989, NCTM presented the first of its publications on curriculum, assessment, and standards. *Curriculum and Evaluation Standards for School Mathematics* provided educators with a guideline for the development and implementation of the mathematics curriculum. In 1991, NCTM published *Professional Standards for Teaching Mathematics* as a companion guide to the *Curriculum and Evaluation Standards*. In tandem, these documents provided mathematics educators with a framework for school mathematics reform through the end of the 20th century. Many publishers used these documents to guide the design of textbooks that modeled key concepts of the framework as established by NCTM. With the advent of NCTM's *Principles and Standards for School Mathematics (PSSM,* 2000), many textbook providers attempted to design books that incorporated all of the principles and standards set forth by NCTM.

The question still remains: are textbooks good representations of what the curriculum is requiring students to know? We will use the geometry standards of the NCTM as a model and will examine a series of books intended for high school geometry to see how well they line up with the principles and standards of NCTM.

Textbook Survey

The following geometry textbooks were included in the survey:

- Scott-Foresman *University of Chicago School Mathematics Project Geometry* (1997)
- Glencoe *Geometry* (1999)
- Everyday Learning *Connected Geometry* (2000)
- McDougal Littell *Geometry* (2007)
- Key Curriculum Press *Discovering Geometry* (2008)
- Pearson *Center for Math Education Project Geometry* (2009)

In the survey, each textbook was examined for content as it related to the NCTM standards for geometry. We also attempted to correlate themes from Euclid's *Elements*: triangle congruence, parallel lines, parallelograms, area, and the Pythagorean Theorem. The textbooks

were compared based overarching topics covered, number of textbook pages devoted to topics, types of exercises, and types of assessments.

Note that the NCTM geometry standards were never meant to apply specifically to a high school geometry course, but to the geometry that students should be exposed to over the course of their entire school experience. However, textbook publishers have referred to the standards as the basis for their course textbook design. For example, Glencoe refers specifically to the 1989 NCTM standards and the 1995 NCTM Assessment Standards as a model for its textbook, Glencoe *Geometry*. Glencoe states that the textbook "implements the shift from geometry as a course in proof to geometry as a representation of the world around us." According to its publisher, Key Curriculum's *Discovering Geometry: An Investigative Approach* "exemplifies the *Principles and Standards for School Mathematics* set forth by the National Council of Teachers of Mathematics (NCTM) in 2000."

Ultimately, our survey seeks to reveal how closely related textbooks are to the standards and to each other. If the textbooks used NCTM Standards as a guide, there should be no noticeable difference in what content is available or how content is presented. Each of the texts should follow the standards in a manner that does not distinguish one text from the other. We are not suggesting that the texts be identical; we are merely stating that the presentation of topics should be consistent from text to text. Students using different textbooks should not be exposed to conflicting content. If on the other hand, the books lack depth of content, or if content is poorly translated by classroom teachers, this may explain why students in the United States are weak in the geometry content strand. If students across the country are exposed to varying degrees of geometry, this inconsistency only adds to the problems this country is facing as we attempt to provide our students with the knowledge and skills they need to be successful.

Table 4.1 – NCTM Geometry Standards – Grades 9-12

Instructional programs from prekindergarten through grade 12 should enable all students to—	In grades 9–12 all students should—
Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships	• Analyze properties and determine attributes of two- and three dimensional objects; • Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them; • Establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others; • Use trigonometric relationships to determine lengths and angle measures.
Specify locations and describe spatial relationships using coordinate geometry and other representational systems	• Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations; • Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.
Apply transformations and use symmetry to analyze mathematical situations	• Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices; • Use various representations to help understand the effects of simple transformations and their compositions.
Use visualization, spatial reasoning, and geometric modeling to solve problems	• Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools; • Visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections; • Use vertex-edge graphs to model and solve problems; • Use geometric models to gain insights into, and answer questions in, other areas of mathematics; • Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas

Table 4.2 – Triangle Congruence

Triangle Congruence (SAS, SSS, ASA, AAS) and Isosceles Triangles			
Textbook	Number of Sections/Pages devoted to topic	Number of Exercises/% of total	Types of Assessment
Scott-Foresman *The University of Chicago School Mathematics Project Geometry*	5 sections/19 pages	81 divided into three sets • Covering the Reading (40, 49.4%) • Applying the Math (38, 46.9%) • Exploration (3, 3.7%)	Paper-Pencil Quizzes and Tests Performance Tests Portfolio Cooperative Group Self-Assessment Projects

Glencoe *Geometry*	6 sections/20 pages	232 divided into five sets
		• Communicating Mathematics (24, 10.3%)
		• Guided Practice (55, 23.7%)
		• Practice A (20, 8.6%)
		• Practice B (89, 38.4%)
		• Practice C (44, 18.7%)
		    ○ Critical Thinking (6, 2.6%)
		    ○ Applications/Problem Solving (15, 6.5%)
			Paper-Pencil Quizzes and Tests
			• Multiple Choice
			• Short Answer
			Performance Assessment
			Cooperative Learning
			Project
			Modeling
			Journal Writing
			Portfolio
			Self-Evaluation

Everyday Learning *Connected Geometry*	1 section with activities embedded in the section/5 pages	25 divided into two sets • On Your Own (15, 60.0%) • Take It Further (10, 40.0%)	Lesson Specific Quizzes Mid-Unit Examinations End-of-Unit Examinations Journal Writing
McDougal Littell *Geometry*	7 sections/25 pages	258 divided into four sets • Guided Practice (50, 19.4%) • Practice and Application (185, 71.7%) • Test Preparation (13, 5.0%) • Challenge (10, 3.9%)	Paper-Pencil Quizzes and Tests Performance Tests Portfolio Cooperative Group Self-Assessment Projects
Key Curriculum Press *Discovering Geometry*	6 sections with investigations/16 pages	106 with five labeled problem types • Applications (1, 0.9%) • Connections (1, 0.9%) • Constructions (6, 5.7%) • Developing Proof (30, 28.3%) • Mini Investigations (0, 0.0%)	Paper-Pencil Quizzes and Tests Constructive Assessment Geometer's Sketchpad Investigations Projects and Explorations
Pearson *Center for Math Education Project Geometry*	1 section/3 pages	17 exercises divided into three sets • Check Your Understanding (7, 41.2%) • On Your Own (9, 52.9%) • Maintain Your Skills (1, 5.9%)	Paper-Pencil Mid-chapter and Chapter Tests Investigations Projects Mathematical Reflections

Table 4.3 – Parallel Lines

Parallel Lines (Parallel Postulate and Converse)			
Textbook	Number of Sections/Pages devoted to topic	Number of Exercises/% of total	Types of Assessment
Scott-Foresman *The University of Chicago School Mathematics Project Geometry*	3 sections/9 pages	63 divided into three sets • Covering the Reading (32, 50.8%) • Applying the Math (31, 49.2%) • Exploration (0, 0.0%)	Paper-Pencil Quizzes and Tests Performance Tests Portfolio Cooperative Group Self-Assessment Projects

Glencoe *Geometry*	5 sections/16 pages	230 divided into five sets
		• Communicating Mathematics (27, 11.7%)
		• Guided Practice (47, 20.4%)
		• Practice A (45, 19.6%)
		• Practice B (81, 35.2%)
		• Practice C (35, 15.2%)
		  ○ Critical Thinking (7, 3.0%)
		  ○ Applications/Problem Solving (14, 6.1%)
			Paper-Pencil Quizzes and Tests
			• Multiple Choice
			• Short Answer
			Performance Assessment
			Cooperative Learning
			Project
			Modeling
			Journal Writing
			Portfolio
			Self-Evaluation

Everyday Learning *Connected Geometry*	1 section with activities embedded in the section/3 pages	6 divided into two sets • On Your Own (4, 66.7%) • Take It Further (2, 33.3%)	Lesson Specific Quizzes Mid-Unit Examinations End-of-Unit Examinations Journal Writing
McDougal Littell *Geometry*	7 sections/21 pages	289 divided into four sets • Guided Practice (58, 20.1%) • Practice and Application (217, 75.1%) • Test Preparation (11, 3.8%) • Challenge (13, 4.5%)	Paper-Pencil Quizzes and Tests Performance Tests Portfolio Cooperative Group Self-Assessment Projects
Key Curriculum Press *Discovering Geometry*	2 sections with investigations/5 pages	29 with five labeled problem types • Applications (4, 3.2%) • Connections (0, 0.0%) • Constructions (12, 9.6%) • Developing Proof (37, 29.6%) • Mini-Investigations (3, 10.3%)	Paper-Pencil Quizzes and Tests Constructive Assessment Geometer's Sketchpad Investigations Projects and Explorations
Pearson *Center for Math Education Project Geometry*	2 sections/6 pages	27 exercises divided into 3 sets • Check Your Understanding (11, 40.7%) • On Your Own (13, 48.1%) • Maintain Your Skills (3, 11.1%)	Paper-Pencil Mid-chapter and Chapter Tests Investigations Projects Mathematical Reflections

Table 4.4 – Parallelograms

Parallelograms			
Textbook	Number of Sections/Pages devoted to topic	Number of Exercises/% of total	Types of Assessment
Scott-Foresman *The University of Chicago School Mathematics Project Geometry*	7 sections/25 pages	110 divided into three sets • Covering the Reading (49, 44.5%) • Applying the Math (57, 51.8%) • Exploration (4, 3.6%)	Paper-Pencil Quizzes and Tests Performance Tests Portfolio Cooperative Group Self-Assessment Projects
Glencoe *Geometry*	5 sections/17 pages	232 divided into five sets • Communicating Mathematics (24, 10.3%) • Guided Practice (55, 23.7%) • Practice A (20, 8.6%) • Practice B (89, 38.4%) • Practice C (44, 18.7%) ○ Critical Thinking (6, 2.6%) ○ Applications/Problem Solving (15, 6.5%)	Paper-Pencil Quizzes and Tests • Multiple Choice • Short Answer Performance Assessment Cooperative Learning Project Modeling Journal Writing Portfolio Self-Evaluation
Everyday Learning *Connected Geometry*	1 section with activities embedded in the section/4 pages	13 divided into two sets • On Your Own (12, 92.3%) • Take It Further (1, 7.7%)	Lesson Specific Quizzes Mid-Unit Examinations End-of-Unit Examinations Journal Writing
McDougal Littell *Geometry*	7 sectons/25 pages	396 divided into four sets • Guided Practice (78, 19.7%) • Practice and Application (294, 74.2%) • Test Preparation (12, 3.0%) • Challenge (12, 3.0%)	Paper-Pencil Quizzes and Tests Performance Tests Portfolio Cooperative Group Self-Assessment Projects
Key Curriculum Press *Discovering Geometry*	7 sections with investigations/21 pages	125 with five labeled problem types • Applications (4, 3.2%) • Connections (3, 2.4%) • Constructions (12, 9.6%) • Developing Proof (37, 29.6%) • Mini Investigations (0, 0.0%)	Paper-Pencil Quizzes and Tests Constructive Assessment Geometer's Sketchpad Investigations Projects and Explorations
Pearson *Center for Math Education Project Geometry*	4 sections/9 pages	94 divided into three sets • Check Your Understanding (35, 37.2%) • On Your Own (49, 52.1%) • Maintain Your Skills (10, 10.6%)	Paper-Pencil Mid-chapter and Chapter Tests Investigations Projects Mathematical Reflections

Table 4.5 – Area

Area (triangles, parallelograms, circles)			
Textbook	Number of Sections/Pages devoted to topic	Number of Exercises/% of total	Types of Assessment
Scott-Foresman *The University of Chicago School Mathematics Project Geometry*	5 sections/17 pages	97 divided into three sets • Covering the Reading (42, 43.3%) • Applying the Math (49, 50.5%) • Exploration (6, 6.2%)	Paper-Pencil Quizzes and Tests Performance Tests Portfolio Cooperative Group Self-Assessment Projects

Glencoe *Geometry*	7 sections/25 pages	232 divided into five sets
		• Communicating Mathematics (24, 10.3%)
		• Guided Practice (55, 23.7%)
		• Practice A (20, 8.6%)
		• Practice B (89, 38.4%)
		• Practice C (44, 18.7%)
		  ○ Critical Thinking (6, 2.6%)
		  ○ Applications/Problem Solving (15, 6.5%)
			Paper-Pencil Quizzes and Tests
			• Multiple Choice
			• Short Answer
			Performance Assessment
			Cooperative Learning
			Project
			Modeling
			Journal Writing
			Portfolio
			Self-Evaluation

Everyday Learning *Connected Geometry*	1 section with activities embedded in the section/7 pages	21 divided into two sets • On Your Own (11, 52.4%) • Take It Further (10, 47.6%)	Lesson Specific Quizzes Mid-Unit Examinations End-of-Unit Examinations Journal Writing
McDougal Littell *Geometry*	6 sections/19 pages	258 divided into four sets • Guided Practice (50, 19.4%) • Practice and Application (185, 71.7%) • Test Preparation (13, 5.0%) • Challenge (10, 3.9%)	Paper-Pencil Quizzes and Tests Performance Tests Portfolio Cooperative Group Self-Assessment Projects
Key Curriculum Press *Discovering Geometry*	6 sections with investigations/12 pages	122 with five labeled problem types • Applications (17, 13.9%) • Connections (2, 1.6%) • Constructions (2, 1.6%) • Developing Proof (0, 0.0%) • Mini Investigations (1, 0.8%)	Paper-Pencil Quizzes and Tests Constructive Assessment Geometer's Sketchpad Investigations Projects and Explorations
Pearson *Center for Math Education Project Geometry*	6 sections/18 pages	77 divided into three sets • Check Your Understanding (28, 36.4%) • On Your Own (32, 41.6%) • Maintain Your Skills (17, 22.1%)	Paper-Pencil Mid-chapter and Chapter Tests Investigations Projects Mathematical Reflections

Table 4.6 – Pythagorean Theorem

Pythagorean theorem			
Textbook	Number of Sections/Pages devoted to topic	Number of Exercises/% of total	Formal Assessment Alternative Assessment
Scott-Foresman *The University of Chicago School Mathematics Project Geometry*	1 section/5 pages	25 divided into three sets • Covering the Reading (17, 68.0%) • Applying the Math (7, 28.0%) • Exploration (1, 4.0%)	Paper-Pencil Quizzes and Tests Performance Tests Portfolio Cooperative Group Self-Assessment Projects

Glencoe *Geometry*	1 sections/5 pages	232 divided into five sets
		• Communicating Mathematics (6, 13.3%)
		• Guided Practice (8, 17.8%)
		• Practice A (6, 13.3%)
		• Practice B (15, 33.3.4%)
		• Practice C (10, 22.2%)
		  ○ Critical Thinking (1, 2.2%)
		  ○ Applications/Problem Solving (3, 6.7%)
			Paper-Pencil Quizzes and Tests
			• Multiple Choice
			• Short Answer
			Performance Assessment
			Cooperative Learning
			Project
			Modeling
			Journal Writing
			Portfolio
			Self-Evaluation

Everyday Learning *Connected Geometry*	1 section with activities embedded in the section/7 pages	32 divided into two sets • On Your Own (24, 75.0%) • Take It Further (8, 25.0%)	Lesson Specific Quizzes Mid-Unit Examinations End-of-Unit Examinations Journal Writing
McDougal Littell *Geometry*	2 sections/6 pages	84 divided into four sets • Guided Practice (16, 19.0%) • Practice and Application (62, 73.8%) • Test Preparation (3, 3.6%) • Challenge (3, 3.6%)	Paper-Pencil Quizzes and Tests Performance Tests Portfolio Cooperative Group Self-Assessment Projects
Key Curriculum Press *Discovering Geometry*	6 sections/15 pages	107 with five labeled problem types • Applications (2, 1.9%) • Connections (1, 0.9%) • Constructions (0, 0.0%) • Developing Proof (6, 5.6%) • Mini Investigations (2, 1.9%)	Paper-Pencil Quizzes and Tests Constructive Assessment Geometer's Sketchpad Investigations Projects and Explorations
Pearson *Center for Math Education Project Geometry*	3 sections/6 pages	31 divided into three sets • Check Your Understanding (6, 19.4%) • On Your Own (22, 71.0%) • Maintain Your Skills (3, 9.7%)	Paper-Pencil Mid-chapter and Chapter Tests Investigations Projects Mathematical Reflections

Based on the survey, the textbooks examined offered educators and students a variety of presentations. The University of Chicago School Mathematics Project (UCSMP), Glencoe, and McDougal Littell texts were the most traditional in arrangement. Each textbook was arranged in chapters, and the chapters were divided into lessons specific to the chapter topic. The lessons contained several examples and were followed by sets of exercises. In the exercise sets, the problems considered applicable to the real world or challenging were often located at the end. The percentage of problems of this sort was relatively low compared to the traditional practice problems. There was little difference evident between these texts and the typical geometry textbook used in the United States.

The UCSMP text covered 14 chapters, with each chapter divided into between 7 and 9 sections (115 sections total). All chapters had at least one in-class activity preceding a lesson that served as an introduction to the lesson. Each chapter ended with suggested projects, summary and vocabulary, progress self-test, and a chapter review.

The Glencoe text contained 13 chapters and 79 total sections, with each chapter varying in length between 5 to 8 lessons. In all of the chapters there were mini-lessons used to illustrate concepts through modeling or technology. Each chapter contained self-tests that addressed the first half of the material covered in the chapter. The chapters ended with chapter highlights that concentrated on vocabulary, chapter study guide and assessment exercises, and suggested alternative assessments.

McDougal Littell's text had 12 chapters, each divided into 6 or 7 lessons (80 total lessons). There were two quizzes in the chapters with six lessons; the chapters with seven lessons contained three quizzes. Each chapter also contained concept and technology activities relative to the lessons they preceded. The chapters end with a summary, chapter review, chapter test, standardized test, and a chapter project.

Comparatively, the other texts were much less traditional. The average number of pages devoted to the 5 themes surveyed varied greatly for each textbook. The more traditional books averaged from 15-22 pages to cover triangle congruence, parallel lines, quadrilaterals and area. The less traditional texts averaged from 5-12 pages to cover the same topics. The Key Curriculum text had a slightly different arrangement. Although the thirteen chapters were divided into specific lessons, each lesson consisted of student investigations, many of which could be done in small cooperative learning groups. The Key Curriculum text also had an introductory "Chapter 0", which exposed students to geometry in nature and art, symmetry in nature, and practice using geometry tools.

CME Project Geometry contained eight chapters, with each chapter divided in to specific lessons. What set this book apart from the more traditional texts were the number of pages dedicated to each topic and the subsequent number of exercises. Contrary to other texts, this book had relatively few exercises for students to complete.

The least traditional of the books surveyed was the *Connected Geometry* text. This book was divided into six units, with each unit dedicated to a theme. The units were divided into lessons relative to the theme. Each lesson contained cooperative learning activities for students to complete. There are no examples of problems in the text; students are required to develop their own examples to use to help them work on the lesson exercises.

The textbook survey reveals the variety in approaches to geometry that exists in the United States. Each textbook author/publisher has the autonomy to represent a set of state standards as they see fit. Since there is no national curriculum to follow, and states curricula vary, students across the country see geometry in many different lights. Throw in the current climate of state accountability tests, and textbooks are being tailored to meet the needs of specific states based on their curriculum guidelines. We see that the playing field may not be

level. In the transient world we live in, students should have the luxury of agreement between one text and another based on its content. The National Mathematics Advisory Panel issued its final report on the state of mathematics education in 2008. Chapter 8 of the report addresses the issues associated with textbooks in the United States. The committee made three recommendations regarding mathematics textbooks:

- Textbooks should be mathematically accurate;
- Textbooks should be more compact and coherent, and;
- States and districts should strive for greater agreement regarding topics covered in mathematics classrooms. (Math Advisory Panel Report, 2008)

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CHAPTER 5 – THE SINGAPORE CURRICULUM

For the better part of the 20^{th} century, the United States was recognized as being a leader in the world of mathematics. The number of mathematics specialists who practiced in the country, as well as its stand as a leader in the areas of engineering, science, and finance, all contributed to its status. However, because of a stagnant educational system, the role of the United States as a leader has diminished. In the final report of the National Mathematics Advisory Panel, the manner in which mathematics education was delivered to students was considered something that was "broken and must be fixed". The final report of the *Third International Mathematics and Science Study* (1995) found that the country of Singapore held the top spot on the international test, and has remained in the top five countries on the subsequent tests. As a matter of record, Asian countries have held the top five spots since 1995 on international assessments. On the 1999 *Trends in Mathematics and Science Study* assessment, 46 percent of the students from Singapore who took the test were in the top 10 percent of all test-takers; nine percent of students from the United States were in this group. Students from Singapore in the bottom quartile outperformed two-thirds of students from the United States on the same assessment. Singapore continued to outpace the United States on TIMSS assessments in 2003 and 2007. Figure 5.1 illustrates the comparisons between grade eight students in the 90^{th} and 10 percentiles on the study in 2007.

Why do results of studies show students in Singapore consistently outperforming students in this country? The common thread that exists between the high achieving countries on the international assessments is a national curriculum. Each country is dedicated to presenting a uniform mathematics curriculum to all of its students, primarily in grades K-8, and uses textbooks modeled specifically after that curriculum. Singapore is one of these countries. In recent years, a push has been on to infuse our curriculum and model our textbooks with lessons

based on the apparent successes that have occurred in mathematics education in Singapore.

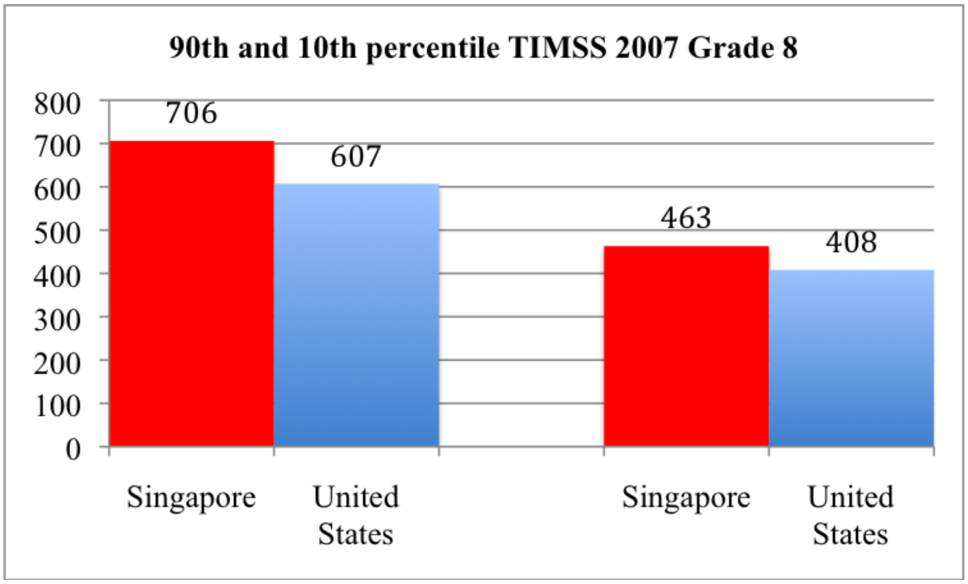

Figure 5.1 – 90th and 10th percentile comparisons, TIMSS 2007

Effects of TIMSS Results

Students in Singapore did not participate in the First or Second International Mathematics Study. After the Singapore Mathematics Curriculum Framework was introduced, Singapore participated in the Third International Mathematics and Science Study. For grade eight, Singapore finished first among all nations participating in TIMSS. This trend continued with TIMSS in 1999 and 2003. In 2007, Singapore did not finish first, but did finish as one of the top five countries in performance on the eighth grade mathematics test. Singapore's scores on all four assessments were significantly better than the United States.

Many schools in the United States have decided to use a version of Singapore's mathematics curriculum as theirs. The "Singapore Math Method" has increased in popularity in this country and internationally. In 2000, the North Middlesex Regional School District in Massachusetts implemented Singapore Math as an alternative to their state mathematics curriculum. Pilot programs were implemented in Baltimore, Maryland City Schools, Montgomery County, Maryland Public Schools, and Paterson Public School No. 2 in Paterson,

New Jersey to replace existing math programs. The state of California, in 2007, approved allocation of funds for teachers in grades one through five to be able to purchase textbooks based on the Singapore mathematics curriculum. Textbook publisher Houghton Mifflin Harcourt has recently released *Math in Focus*, a textbook series for grades K-5 based on the Singapore curriculum. The call for a revised curriculum, and dismal international test scores, seem to be the catalyst for many in the United States to use the Singapore curriculum as a model.

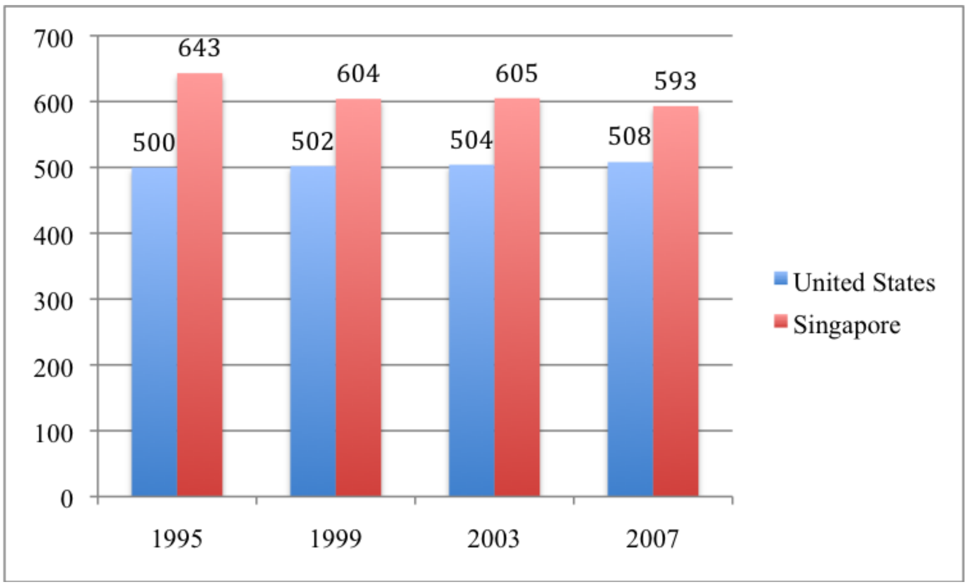

Figure 5.2 – Comparison of TIMSS Scores 1995-2007

A Brief Overview of Singapore's Educational System

After World War II, Singapore was still under British rule, but immersed in a curriculum that was primarily Chinese. The educational system went through many different phases based on the outside influences affecting it: British, Chinese, Malaysian, and Indian. Singapore struggled with developing an educational system that could meet the needs of its diverse population. Because of the changes in its governmental structure, curriculum changed as well based on the ruling entity. After several shifts in status, including joining with Malaysia for a short time in 1963, Singapore became an independent nation in 1965.

Much has been said about the educational system in Singapore. It is directed by the

country's Ministry of Education. The national curriculum in place provides a guide for instruction in both primary and secondary schools. The curriculum also includes an assessment piece for all students. Schooling in Singapore is divided into two distinct groups. Primary school covers the first six years of a student's education; it is required for all students in Singapore. The foundation level occurs from grades one through four. In mathematics, students in these grades focus heavily on literacy and numeracy. Instructional time is dedicated primarily to English, the student's natural language (Chinese, Malay, or Tamil), and mathematics. Once students have completed grade four, they are assessed to decide where they should be placed in grade five, the beginning of the orientation level. Students performing well on the assessment are place in what is referred to as EM1/EM2. In EM1/EM2, students continue their studies from the foundation level and are also studying science. Students who don't fare as well on the assessment are place in EM3. They are exposed to the same studies as students in EM1/EM2, but are afforded extra time and are given remediation.

When students in Singapore complete grade 6, they take an assessment to determine which secondary *stream* they will enroll in. The special/express stream is an advanced curriculum, akin to college preparatory. The non-academic subjects focus on life skills. These can be classes on civic and moral education to physical education. Project work is a part of the curriculum, but is not considered an exam subject. The content-based curriculum comprises languages, mathematics, science, humanities, and the arts. Students completing the special/express stream take the Joint Cambridge University and Singapore *O-level* college entrance examination. The normal stream is less advanced. In the normal academic stream, students take 6-8 courses in preparation for the Singapore-Cambridge General Certificate of Education (GCE) examination. English, mathematics, the mother tongue, science, and humanities are required. The normal technical stream offers students 5-7 courses, of which

English, mathematics, the mother tongue, and computer applications are mandatory.

The Singapore Ministry of Education has outlined what it expects it students to know and be able to do once they complete primary, secondary, and post-secondary. The basis for these outcomes is the belief that the purpose of education is to develop the whole person.

Singapore Mathematics Curriculum

The Singapore mathematics curriculum is based on a framework called the *Pentagon model*. This model was first introduced in 1990. Based on a dismal showing in the Second International Science Study, and coupled with an international movement to reform math and science education, changes took place in Singapore' curriculum. In the center of the model is problem solving; surrounding problem solving on the five sides of the pentagon are concepts, processes, metacognition, attitudes, and skills.

The Singapore mathematics syllabi are designed to ensure that a common curriculum is addressed nationwide. In each document, a detailed outline is provided to guide teachers through its implementation. The syllabi are geared to provide students with a well-rounded mathematics education, one that prepares them ultimately for post-secondary school and beyond. Emphasis is placed on numeracy, reasoning, and critical thinking and problem solving. (Singapore Ministry of Education Primary Syllabus, 2007) The goals for both primary and secondary mathematics education are clearly defined in the syllabi. They include ensuring that all students obtain skills and concepts that prepare them for continuous learning and those students can use these skills and apply them to solve problems. It is also important that students enjoy mathematics and be able to reason and communicate mathematics effectively. Finally, students should see connections between mathematics and other subject areas and be able to use mathematics tools as they learn and apply mathematics.

The Primary Syllabus covers foundation grades one through four and orientation grades

five and six. Specific mathematics content is provided for each grade level. The mathematics curriculum content is integrated and spirals in successive years. The depth of the mathematics content covered increases from one level to the next.

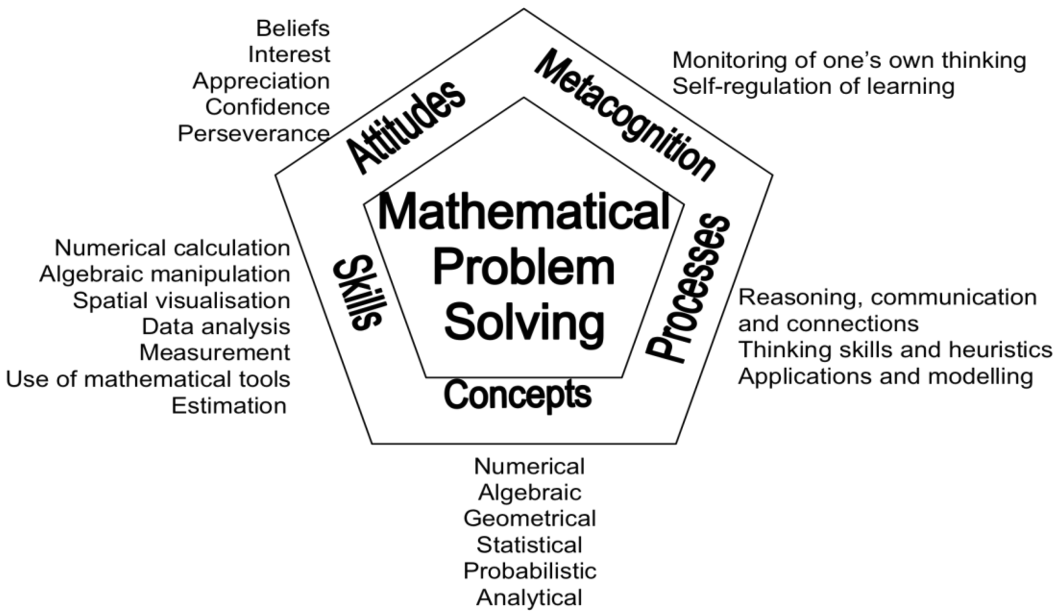

Figure 5.3 – Singapore Pentagon Model

Calculator use is encouraged and students are introduced to them in Primary 5. The use of calculators is seen as a balance between computation and problem solving and as a means to assist students who may be experiencing subject difficulty. The Secondary Math Syllabus is arranged similarly to the primary. It covers secondary years one through four and is designed to prepare students for the *O-level* college preparatory or *N-level* academic and technical tests. The syllabus addresses topics specific to the tests.

Comparisons

It has been mentioned that the math curriculum in Singapore is being adapted to meet the needs of an increasing number of school districts in the United States. Singapore's success on international tests prompted the attention they have received in recent years. NCTM has made

bold moves in this country over the past two decades in an attempt to steer mathematics education the right way.

Table 5.1 – Singapore Primary Mathematics Syllabus Topics

<table>
<tr><th>Primary 1</th><th>Primary 2</th><th>Primary 3</th><th>Primary 4</th><th>Primary 5</th><th>Primary 6</th></tr>
<tr><td rowspan="2">Whole Numbers
Measurement
Geometry
Data Analysis</td><td rowspan="2">Whole Numbers
Fractions
Measurement
Geometry
Data Analysis</td><td rowspan="2">Whole Numbers
Fractions
Measurement
Geometry
Data Analysis</td><td rowspan="2">Whole Numbers
Fractions
Decimals
Measurement
Geometry
Data Analysis</td><td>EM1/EM2:
Whole Numbers
Fractions
Decimals
Percentage
Ratio
Measurement
Geometry
Data Analysis</td><td>EM1/EM2
Fractions
Percentage
Ratio
Speed
Measurement
Geometry
Data Analysis
Algebra</td></tr>
<tr><td>EM3:
Whole Numbers
Fractions
Decimals
Measurement
Geometry
Data Analysis</td><td>EM3:
Fractions
Decimals
Percentage
Measurement
Geometry
Data Analysis</td></tr>
</table>

Table 5.2 – Singapore Secondary Mathematics Syllabus Topics

Secondary One	Secondary Two	Secondary Three-Four
Number and Algebra • Numbers and the Four Operations • Ratio, Rate, and Proportion • Percentage • Speed • Algebraic Representation and Formulas • Algebraic Manipulation • Functions and Graphs • Solutions of Equations and Inequalities Geometry and Measurement • Angles, Triangles, and Polygons • Mensuration Statistics and Probability • Data Handling	Number and Algebra • Ratio, Rate, and Proportion • Algebraic Manipulation • Functions and Graphs • Solutions of Equations • Set Language and Notation Geometry and Measurement • Congruence and Similarity • Pythagoras' Theorem • Mensuration Statistics and Probability • Data Analysis • Probability	Number and Algebra • Numbers and the Four Operations • Functions and Graphs • Solutions of Equations and Inequalities • Applications of Mathematics in practical situations • Matrices Geometry and Measurement • Congruence and Similarity • Properties of Circles • Trigonometry • Mensuration • Coordinate Geometry • Vectors in Two-Dimensions Statistics and Probability • Data Analysis • Probability

Every state has standards based on the NCTM standards. Yet, we still find that our curriculum models tend to include a large number of topics that teachers don't have time to effectively cover.

There are vast differences between the United States and Singapore. In 2006, there were a projected 55 million students in more than 1600 school districts in grades K-12 in the United States; Singapore's K-12 enrollment in 2006 was about half a million. Singapore is a small nation with a fairly homogeneous population; there are three ethnic groups in the country, with three-fourths of its population being Chinese.

Because of its size it could be inferred that it may be less difficult for Singapore's Ministry of Education to successfully implement its mathematics program nationally. However, many nations globally offer national curricula. The United States Department of Education, created in 1980, does not mandate curriculum for the country. Its purpose as a governmental agency is to establish policy regarding financial aid, collect data and research on American schools, bring attention to issues regarding education, and ensure equity in education. States are left with the responsibility to create curricula that, successfully implemented, lead to students' academic successes.   Most states, however, provide school districts with general curricula with no clear implementation guidelines. Teachers in the same district could interpret the curriculum differently. Teachers in Singapore universally are given notes for implementation of curriculum and specific assessment strategies to use.

Geometry in the United States is still considered a course as opposed to a strand. Our curriculum, similar to most countries, is fairly integrated for grades K-8. Students are exposed to a variety of topics that fit in to each of the strand definitions. However, we find that students in these grades have little exposure to geometry because of the vast exposure to number concepts. In most states, each curriculum grade band through grade twelve incorporates all of the strands

as prescribed by NCTM. Our high stakes tests are built based on an integrated set of strands. However, we still offer specific courses to students in high school. Many colleges and universities in this country still require the traditional menu of math courses for admission: Algebra I, Geometry, and Algebra II. Some states offer an integrated math series for high school students, but these classes do not meet the requirements for post-secondary admission. In Louisiana, all students who entered high school during 2008-2009 were required to enroll in at least two years of the Louisiana Core 4 curriculum, the required curriculum for the state's colleges and universities.

Singapore has approached primary and secondary mathematics education differently. Each year, students are exposed to topics that are part of an integrated curriculum; there is, for example, no tenth grade Geometry as a stand-alone class. The spiraling effect of the curriculum allows students to build on ideas based on prerequisite knowledge. Because there is differentiation in the curriculum, all students are not taking the same classes. Students are placed in a program of study based on their unique abilities, but with the same goal in mind, success after secondary school. This may mean university or technical school. Students in Singapore are not all taking the same secondary exit examination. Their exams are based solely on the syllabus they have been exposed to and the prescribed path they are embarking upon.

The textbooks used in Singapore also vary greatly compared to those used in the United States. Because the focus is placed on the curriculum being integrated, the textbooks are written is the same manner. There are no books dedicated to specific subjects like Algebra I or Geometry; each text is written with the mathematics syllabus as a guide. In 2009, there were only four textbook series available for adoption in the primary grades and six textbook series available for adoption in Singapore for the secondary schools. For the last textbook adoption in Louisiana (2004-2005), there were eight series options for textbooks for the elementary grades,

ten series options for middle school, and seven publishers offering sixty-seven different book options for high school.

In an effort to expose students to a more rigorous curriculum, we have lost sight of the effectiveness of our curriculum. In the state of Louisiana, students have a variety of courses they can combine in high school that will meet basic graduation requirements. Yet our students are still lacking what they need to be successful mathematics students. The integrated approach used in countries like Singapore seems to be a model we need to investigate further.

CHAPTER 6 - CURRICULUM REVISION IN THE 21ST CENTURY

The curriculum in the United States continues to be wider than it is deep. Students are expected to master a number of concepts by the end of each grade. In Louisiana, for example, the mathematics curriculum consists of six strands. Each strand is divided into benchmarks and for each benchmark there are a set of specific *grade level expectations*, mathematics skills that students should know and be able to do. There are forty-eight grade level expectations for eighth grade students to master. In an average school year, students need to cover these topics at an average of 1 topic per 3.75 days. This does not take in to account instructional time missed for tests, field trips, school programs, etc. There needs to be a shift in how the curriculum is presented in order for American students to rise to the challenge of their international counterparts, especially those in Asian countries like Singapore.

How then do we go about making the change? Reviewing any mathematics curriculum as a whole is a daunting task. There are at least five strands to consider and the many associated benchmarks and skills. The first logical step would appear to be making the curriculum we use more compact and integrated. Our traditions have prevented us from adopting radical changes in what we teach. How do we decide what to throw out and what to keep? It is important to maintain the integrity of the curriculum while simplifying it. Many middle school curricula for grades six through eight are repetitive, inundated with skills applying to solving problems with fractions, decimals, and percents. This is the way we have approached middle school mathematics for so long; it has been difficult for us to make amends. NCTM offered an alternative in *PSSM*; they noted American students' struggles with geometric concepts on state, national and international tests. Their suggestion was to increase the number of geometric concepts taught in the middle grades. Students however are still expected to master the number operations and algebraic skills often associated with middle school mathematics. It is imperative

that concepts previously restricted to the middle grades be moved down to the elementary grades. Note the Singapore geometry syllabus for Primary Four. Students are exposed to properties of rectangles and squares and tessellations at this level. Most students in the United States do not address these topics until middle grades. If our focus is to enhance middle school mathematics with more geometry, we need to make sure our elementary school mathematics programs provide students with curriculum models whose focus is on number sense and number operations. Let's move fractions, decimals and percents to the elementary school campus.

Trying to ensure that the needs of all students are met poses a challenge as well. The Singapore curriculum is differentiated through its syllabus from Primary Five. Most curricula used in the United States address differentiation, but only by means of strategies that can be used for implementation. Equity is one of the key principles established by NCTM. What exactly does it mean though for our curriculum to be equitable? Equity provides each student with the opportunity to learn and succeed. According to NCTM, equity requires high expectations for all students (PSSM, 2000). In Singapore, the differing needs of students are accomplished through curricula that respond to identify needs and programs that support the curricula. It is impossible to ensure equity in education as long as the curricula nationwide vary so greatly.

Common, Coherent, Challenging

True curriculum revision takes time. The problem that we have encountered is that we have lost patience. Because our students are continuously lagging behind, we have become anxious for change. In our anxiety, we have missed opportunities to affect curriculum revisions that lead to a common, coherent and challenging model. The question begs to be asked, what does it mean for a curriculum to be common, coherent, and challenging? Because of the culture we currently exist in with states having sole autonomy over individual curricula, how do we go about developing a common national curriculum that meets the needs of all of our students?

States for so long have had absolute control over its educational practices. Many state boards of education may be hesitant about embracing a common national curriculum. No Child Left Behind (NCLB) addressed commonality in the sense of assessment and accountability; curriculum revision was tied solely to standards of accountability.

A common national curriculum does not mean it has to be federally controlled. The curriculum should be common in the sense that students across the country are expected to learn and know the same set of ideas at the end of a particular grade. Thus it must be practical. As mentioned before, Singapore has a common national curriculum, but it is a relatively small country compared to the United States. However, even in Singapore, local school districts still make decisions regarding issues like textbook adoption and instructional strategies. NCTM continuously works on revising its set of standards, but it, being a professional organization, can only suggest the direction the country needs to go in regarding school mathematics. But it is clear; we must adopt a set of standards as a nation that in its essence is common from state to state. NCTM has recommended a set of guidelines to ensure commonality across the nation:

- A curriculum is more than a collection of activities: It must be coherent, focused on important mathematics, and well articulated across the grades.
- Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. Learning mathematics with understanding is essential.
- If a voluntary national mathematics curriculum is developed, the topics studied in that curriculum must be taught and learned in an equitable manner in a setting that ensures that problem solving, reasoning, connections, communication, and conceptual understanding are all developed simultaneously along with procedural fluency.
- As stated above, a potential national curriculum must include important mathematics. Content should include the following key content areas.
    - Number and Operations with Procedural Fluency
    - Algebra
    - Geometry and Measurement
    - Data Analysis, Statistics and Probability

Recently, the Carnegie-IAS Commission on Mathematics and Science Education released the

report *The Opportunity Equation: Transforming Mathematics and Science Education for Citizenship and the Global Economy*. (www.opportunityeducation.org). The report states that the current state of the mathematics curriculum is in need of revision. It recommends that a common set of national standards be developed that are clear, concise, and more challenging. It also notes that schools need to be redesigned in order to effectively reach all students.

In 2009, 49 states and territories, including the District of Columbia, Puerto Rico, and the Virgin Islands, have agreed to be part of the Common Core State Standards Initiative, sponsored by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO). Participants in the initiative have made commitments to develop common standards in mathematics education from grades K-12. The standards will be based on research and compared closely to international standards. They will also be developed with college and career readiness in mind and be rigorous.

Local, state, and federal government must partner with leading mathematics educators to seriously examine the state of mathematics education. The old traditions of local rule must be set aside in order for us to successfully develop a common national mathematics curriculum that is practical, integrated and differentiated to meet the needs of our students.

As we attempt to develop a common national curriculum, we must ensure that the curriculum is also coherent. Coherence in a curriculum demands that students understand how the mathematics they are currently exposed to relate to what they have previously experienced. Coherence requires that the sequence of topics is sensible. In the United States, the number of topics that students are expected to master in any given school year is many. We also find that students are exposed to varying degrees of the same topics over and over again. Table 6.1 compares the geometry topics covered in the Louisiana middle school mathematics geometry benchmarks, the suggested NCTM geometry standards for middle grades, and the Singapore

Secondary Two syllabus. The Louisiana benchmarks and NCTM standards are not grade-specific. They are assigned to grade bands. The NCTM standards clearly state that these are things students in grades 6-8 should be able to do. Even though Louisiana has grade-level expectations, they are coupled with the grade band benchmarks. The Singapore curriculum, however, provides specific grade expectations that are built upon based on the previous grade. The hierarchy of learning is evident; the repeat performances we often see in middle schools around the country are not present. If we are to ensure student success, we need to provide curriculum that is scaffold in a way that makes sense and provides students with the best chance for academic success. Students need to know how what they are learning relates to what they have learned in the past and how it relates to their own experiences. Mathematics should not be a jumble of topics that appear randomly drawn from a hat. Educators need to provide students with a defined foundation for school mathematics.

A coherent curriculum is also one not solely based on the textbook. The number of topics teachers attempt to cover during a school year is often a direct reflection of the size of the textbook. More does not always reflect better; textbooks in Singapore are comparatively thin to those in the United States, but are content-rich. In an effort to model curriculum, our textbooks are jammed-pack with information that is lightly glossed over by classroom teachers. The real meat of the mathematics is often missed. As we struggle to develop coherent curricula, our textbooks need to model depth, not breadth.

A coherent curriculum is one whose spotlight is directed and aimed at students being skillful in key areas of mathematics, thereby preparing them to progress from one level to the next in a rational manner.

Clearly, our mathematics curriculum should offer challenges to our students. Challenging mathematics does not equate to more problems. It is also not representative of

contrived "real-world" problems. Our students should be exposed to curriculum that requires them to think about the mathematics and use critical thinking to help them become problem solvers.

Table 6.1 – Comparison of Grade Eight Geometry Standards

	Louisiana	NCTM	Singapore
Grade 8	**Strand G: Geometry** **Standard:** In problem-solving investigations, students demonstrate an understanding of geometric concepts and applications involving one-, two-, and three-dimensional geometry, and justify their findings. • G-1-M using estimation skills to describe, order, and compare geometric measures • G-2-M identifying, describing, comparing, constructing, and classifying geometric figures and concepts • G-3-M making predictions regarding transformations of geometric figures (for example, make predictions regarding translations, reflections, and rotations of common figures) • G-4-M constructing two- and three-dimensional models • G-5-M making and testing conjectures about geometric shapes and their properties • G-6-M demonstrating an understanding of the coordinate system (for example, locate points, identify coordinates, and graph points in a coordinate plane to represent real-world situations) • G-7-M demonstrating the connection of geometry to the other strands and to real-life situations (for example, applications of the Pythagorean Theorem)	Characteristics and Properties of 2- and 3-dimensional shapes • Describe, classify, understand relationships among types of objects • Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects Developing Mathematical Arguments Coordinate geometry • Properties of geometric shapes on the coordinate plane Transformations and Symmetry • Flips, turns, slides, and scaling • Congruence, similarity, and line or rotational symmetry of objects Visualization, spatial reasoning, and geometric modeling	Congruence and Similarity Pythagorean theorem Mensuration • Volume and surface area of pyramids, cones, and spheres

This ability to solve problems will translate across curricula and beyond secondary school.

When describing mathematics curriculum that is challenging, the word “rigor” often comes up. What does it mean for mathematics to be rigorous? Rigor is often associated with difficult; is a rigorous curriculum one that is difficult? Rigor should imply a curriculum that allows students to acquire skills and apply these skills in problem-solving situations. There is a misconception that a rigorous curriculum is one that has lots of activities. The activities in a curriculum that exhibits rigor should address the content in a way that students can think and communicate mathematically and critically. A challenging curriculum is content-rich.

One of the issues of mathematics curricula is that the prerequisites for rigor are not satisfied in previous grades. Curriculum revision requires that rigor be present at all grade levels. It also means that this curriculum is not reserved for the best and the brightest. All students should be afforded the opportunity to experience a deep and challenging curriculum. The aim of the curriculum is to foster a student’s ability to understand how to address complex problems. Students may be required to investigate solutions prior to being presented specific content. A curriculum that gets students to think on a regular basis is one that is rigorous. Students become more active in the learning process. If exposed to a challenging curriculum, they rely less on the classroom teacher and more on the reasoning skills they have developed. Our textbooks contain page after page of examples for students to follow, and then problem after problem that are modeled after those examples. Very few textbooks require students to develop concepts on their own. As curriculum is revised, the textbook needs to go through a makeover as well. As a tool of learning, it should enhance what the curriculum is trying to accomplish, not compete with it.

Finally, there needs to be a concerted effort to ensure that classroom teachers receive adequate training regarding the mathematics curriculum. Professional development in recent

years has become prescriptive; school districts are using professional development as a means to cure the ills of the system, for example, classroom management. Teacher training should be content-rich, pedagogically sound, and research-based. The Singapore Ministry of Education has as its vision a "well-qualified, competent and committed teaching force dedicated to continuous learning and excellent practice". It is committed to providing teachers with constant professional development and educational resources to ensure that its curriculum is accurately delivered. School districts in the United States should follow this pattern. Unless the facilitators of the curriculum are adequately prepared, it will not serve its purpose effectively.

SUMMARY

The latest TIMSS data paints a bleak picture for mathematics education in the United States. Our students are still not competing at a level of competency as students in other countries like Singapore. Geometry especially has been an area of marked weakness for our students. The importance of geometry in the curriculum is well documented. From the days of Euclid, geometry has played an important role in education. Students exposed to geometry can walk away from it with a keen sense of reasoning and using deductive and inductive skills to solve problems. Historically, we have relegated geometry to a one-year course in the tenth grade of high school. Elementary and middle schools curricula have in the last century included some topics of geometry in an attempt to model an integrated math. However, these curricula are still heavy with topics related to number sense and number operations. Some curricula have become overloaded as a result of the need to expose students to a variety of math topics.

Singapore's Ministry of Education has developed a national curriculum that appears to work. It is fully integrated and differentiated, without students missing out on key concepts. It is spiraled to provide students with foundations for success from year to year. It is designed to give students the best opportunity to achieve. It also is dedicated to making sure students experience post-secondary success. Evidence of Singapore's reform in mathematics education is its continual success on assessments like TIMSS. Students from Singapore outscored their counterparts from the United States in all areas of the assessment, especially in the geometry content domain.

Revising the mathematics curriculum has happened consistently since *A Nation at Risk* was released. We are well aware of our shortcomings; they continue to be highlighted on the international stage. Our hope is that now we can truly work on developing a curriculum in this country that has common goals, is coherent and focused, and that challenges our body of

learners. Recent developments have suggested that as a nation we are ready to accept the challenge. In 2006, NCTM published *Curriculum Focal Points for Pre-Kindergarten through Grade 8 Mathematics: A Quest for Coherence.* Its purpose was to identify the key topics and ideas that need to be addressed in mathematics by grade level, and eliminate the laundry list of expectations we see in many states' curricula. It is also designed to provide those in the educational community with a comprehensive program of study that will lead to students' successes as problem solvers, in and out of the mathematics classroom. This document is a first step in changing the mathematics landscape in this country. We cannot afford to continue to lag behind other countries. In a changing world with changing requirements for the workplace, our students need a curriculum that not only provides them with what they need academically, but one that will help them transfer their abilities to solve problems to experiences outside of the classroom.

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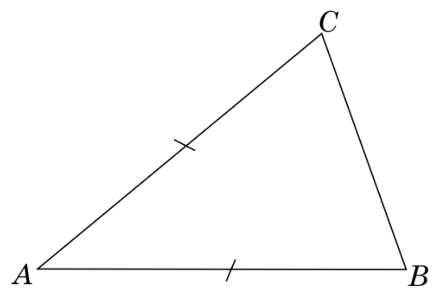
APPENDIX A: RELEASED TIMSS GEOMETRY ITEMS 2003



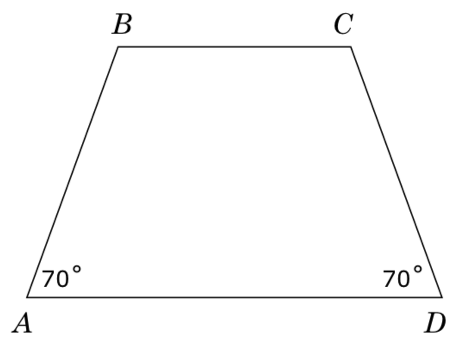
The triangle ABC has $AB = AC$.

Draw a line to divide triangle ABC into two congruent triangles.

Item Number: M032403

Figure A.1 TIMSS 2003 Released Item M032403

ABCD is a trapezoid.



Another trapezoid, *GHIJ* (not shown), is congruent (the same size and shape) to *ABCD*. Angles *G* and *J* each measure 70°. Which of these could be true?

Ⓐ $GH = AB$

Ⓑ Angle *H* is a right angle.

Ⓒ All sides of *GHIJ* are the same length.

Ⓓ The perimeter of *GHIJ* is 3 times the perimeter of *ABCD*.

Ⓔ The area of *GHIJ* is less than the area of *ABCD*.

Item Number: M012015

Figure A.2 TIMSS 2003 Released Item M012015

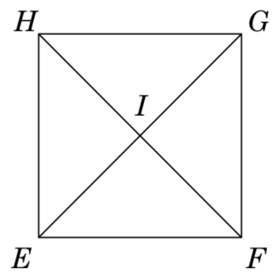
In square $EFGH$, which of these is FALSE?

Ⓐ ΔEIF and ΔEIH are congruent.

Ⓑ ΔGHI and ΔGHF are congruent.

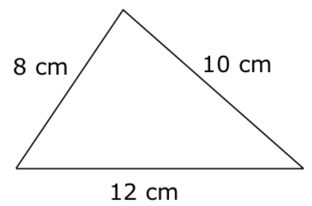
Ⓒ ΔEFH and ΔEGH are congruent.

Ⓓ ΔEIF and ΔGIH are congruent.

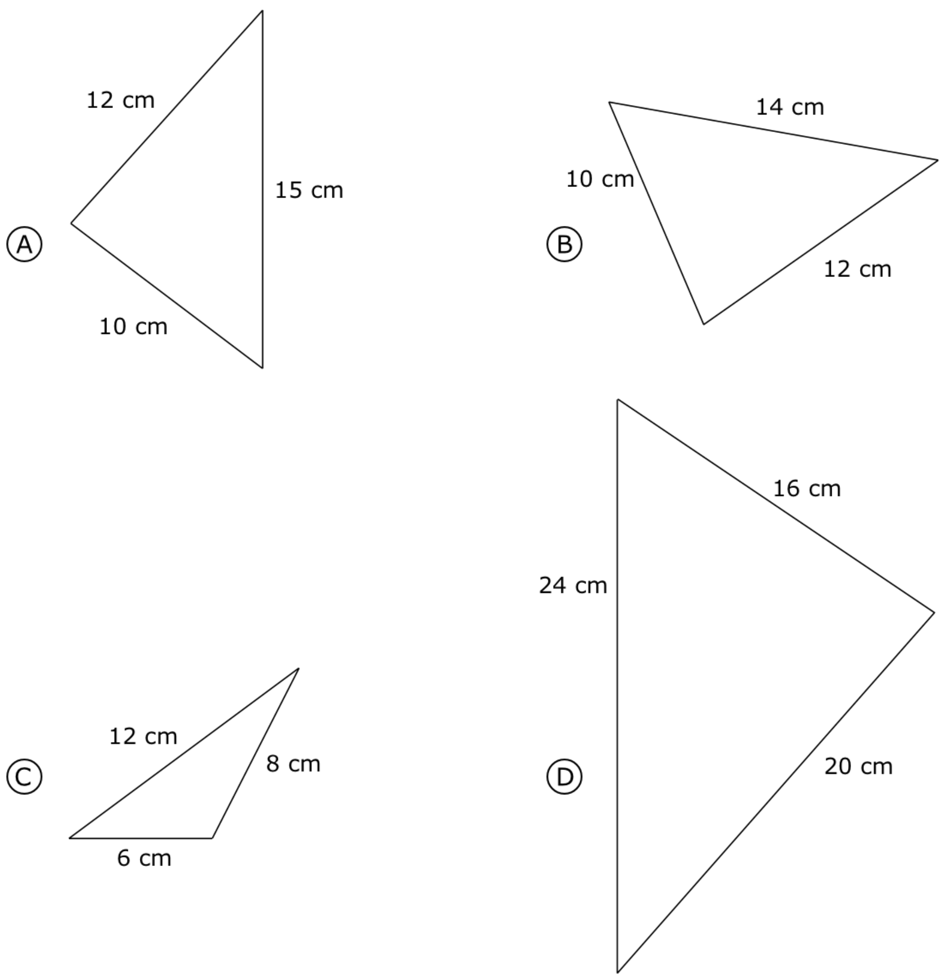


Item Number: M012005

Figure A.3 TIMSS 2003 Released Item M012005



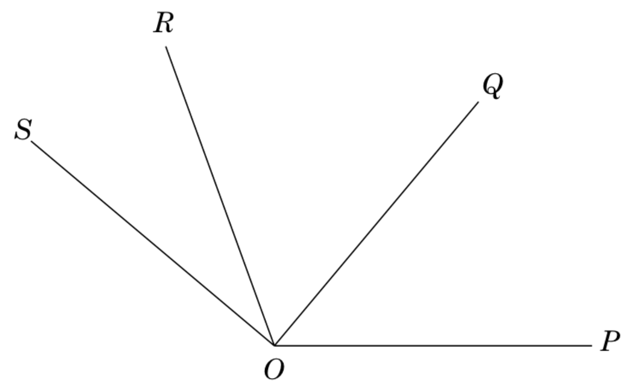
Which of the following triangles is similar to the triangle shown above?



Item Number: M032261

Figure A.4 TIMSS 2003 Released Item M032261

In the figure, the measure of $\angle POR$ is 110°, the measure of $\angle QOS$ is 90°, and the measure of $\angle POS$ is 140°.



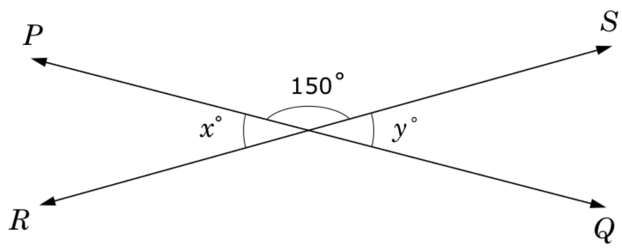
What is the measure of $\angle QOR$?

Answer: ________________

Item Number: M022202

Figure A.5 TIMSS 2003 Released Item M022202

In the figure, PQ and RS are intersecting straight lines.



What is the value of $x + y$?

Ⓐ 15

Ⓑ 30

Ⓒ 60

Ⓓ 180

Ⓔ 300

Item Number: M012039

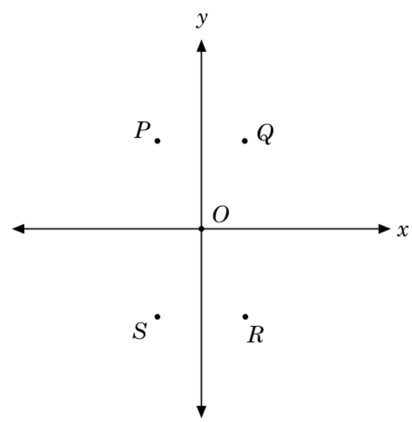
Figure A.6 TIMSS 2003 Released Item M012039

A straight line passes through the points (2,3) and (4,7). Which of these points is also on the line?

- (A) (0,2)
- (B) (1,2)
- (C) (2,4)
- (D) (3,5)
- (E) (4,5)

Item Number: M022016

Figure A.7 TIMSS 2003 Released Item M022016



In the coordinate plane above, which point could have coordinates (2,–4)?

Ⓐ P

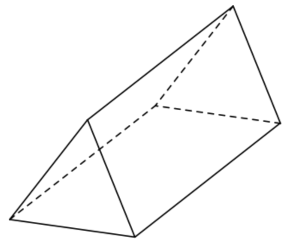
Ⓑ Q

Ⓒ R

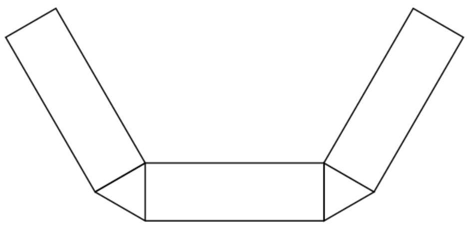
Ⓓ S

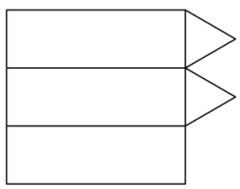
Item Number: M032588

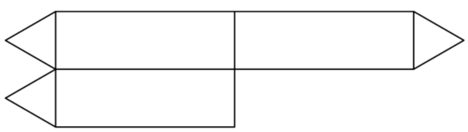
Figure A.8 TIMSS 2003 Released Item M032588

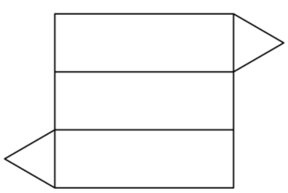


Which of these could be folded to make a shape like the 3-D figure above?

Ⓐ 

Ⓑ 

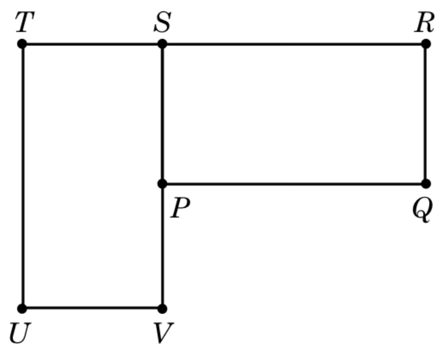
Ⓒ 

Ⓓ 

Item Number: M032489

Figure A.9 TIMSS 2003 Released Item M032489

Rectangle $PQRS$ can be rotated (turned) onto rectangle $UVST$.



What point is the center of rotation?

(A) P

(B) R

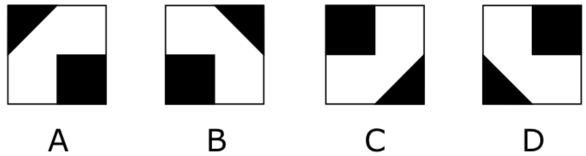
(C) S

(D) T

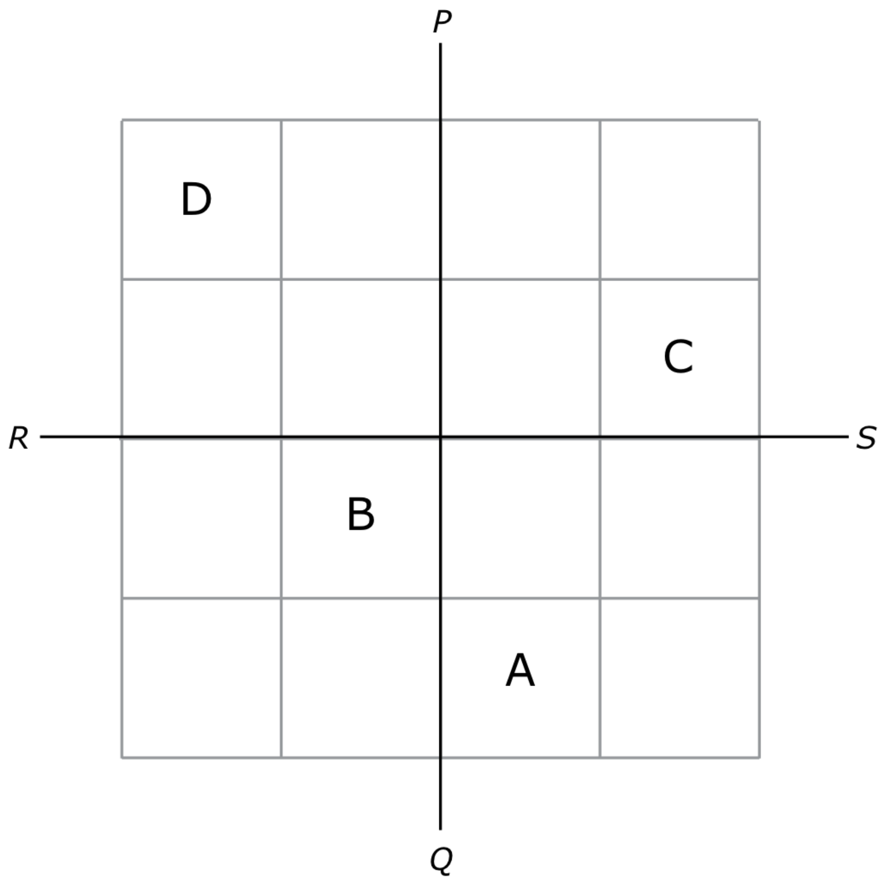
(E) V

Item Number: M022154

Figure A.10 TIMSS 2003 Released Item M022154



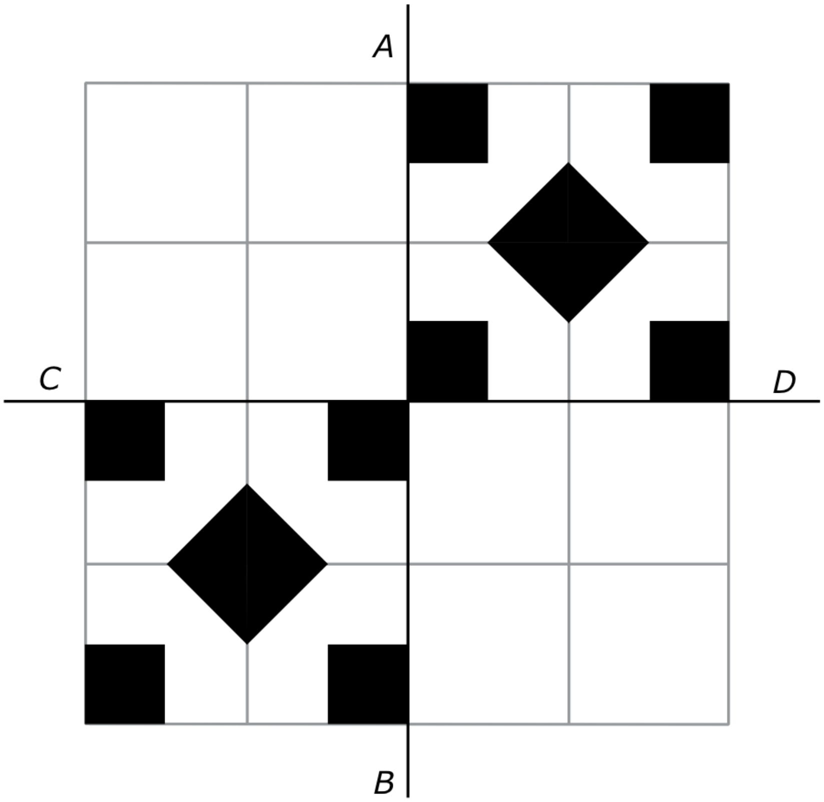
Continue to identify the tiles as shown above. On the grid below, write the letters A, B, C, or D to make a symmetrical pattern where PQ and RS would be lines of symmetry. Arrange the tiles to make a pattern.



Item Number: M032745

Figure A.11 TIMSS 2003 Released Item M032745

There are several ways of arranging the tiles so that they form patterns. The grid below has been shaded to show how tiles can be placed on some of the squares. The pattern can be continued so that AB and CD are lines of symmetry.

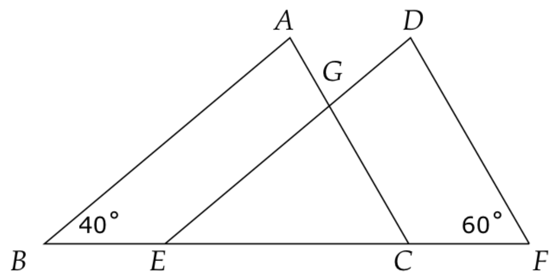


Shade in all the remaining squares on the grid so that the resulting pattern is symmetrical about line AB, and also is symmetrical about line CD.

Item Number: M032743

Figure A.12 TIMSS 2003 Released Item M032743

In this figure, triangles ABC and DEF are congruent with $BC = EF$.



What is the measure of angle EGC?

Ⓐ 20°

Ⓑ 40°

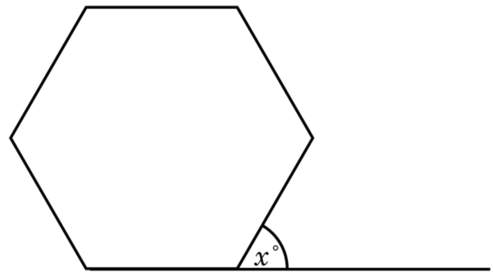
Ⓒ 60°

Ⓓ 80°

Ⓔ 100°

Item Number: M012026

Figure A.13 TIMSS 2003 Released Item M012026

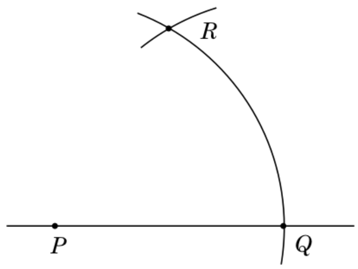


The figure above is a regular hexagon. What is the value of x?

Answer: ________________

Item Number: M032693

Figure A.14 TIMSS 2003 Released Item M032693



In the figure above, an arc of a circle with center P has been drawn to cut the line at Q. Then an arc with the same radius and center Q was drawn to cut the first arc at R. What would be the size of angle PRQ?

Ⓐ 30º

Ⓑ 45º

Ⓒ 60º

Ⓓ 75º

Item Number: M032689

Figure A.15 TIMSS 2003 Released Item M032689

APPENDIX B: TIMSS 2003 GEOMETRY ITEM COMPARISONS

Table B.1: Percent Correct on Geometry Items

Item Number	Singapore Percent Correct	United States Percent Correct	International Average
M032403	88	54	52
M012015	81	69	60
M012005	72	55	56
M032261	75	46	42
M022202	58	22	28
M012039	84	47	50
M022016	56	51	35
M032588	77	72	56
M032489	94	76	68
M022154	64	49	47
M032745	11	6	4
M032743	77	67	49
M012026	79	36	45
M032693	62	20	33
M032689	49	34	46

APPENDIX C: NAEP 2007 RELEASED GEOMETRY ITEMS

Question Number	Complexity	Calculator (Y/N)	Key
4	Low	Y	A

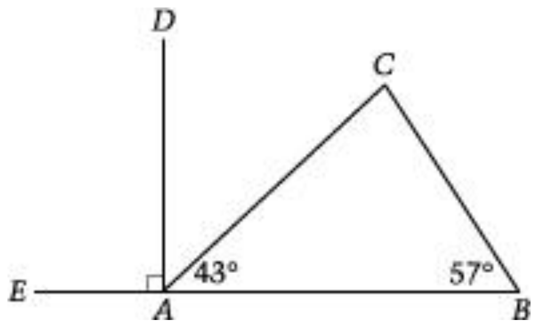


Figure C.1 - Determine measure of angle in triangle

In the figure above, what is the measure of angle DAC?

A. 47º
B. 57º
C. 80º
D. 90º
E. 13º

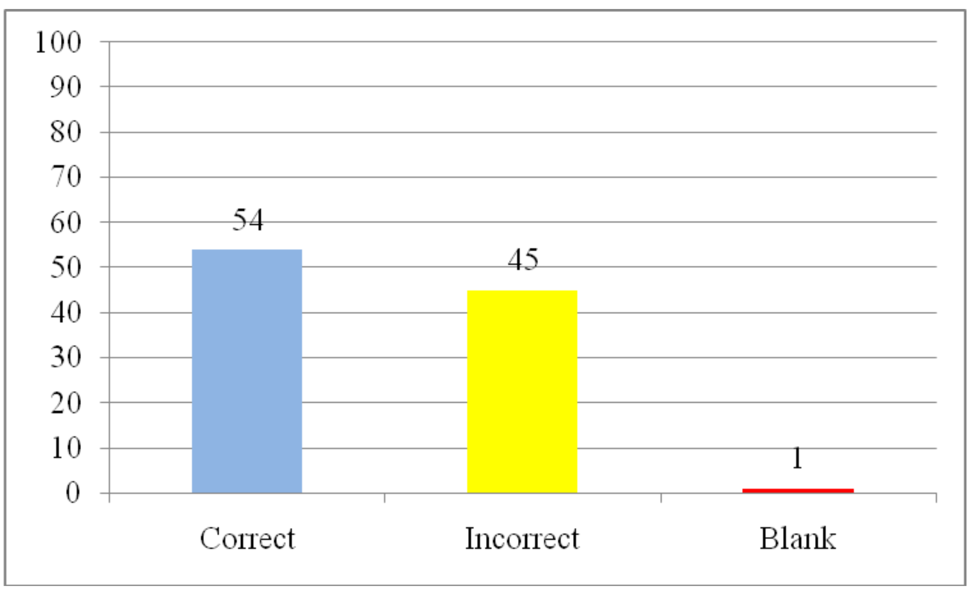


Figure C.2 – Percentage of student responses – item 4

Question Number	Complexity	Calculator (Y/N)	Key
6	Low	Y	E

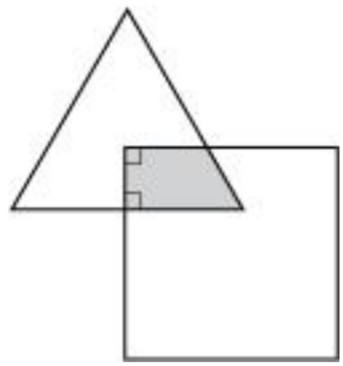


Figure C.3 - Recognize shape formed by overlapping figures

In the figure above, the intersection of the triangle and the square forms the shaded region. What is the shape of this region?

A. An equilateral triangle
B. A rectangle
C. A square
D. A rhombus
E. A trapezoid

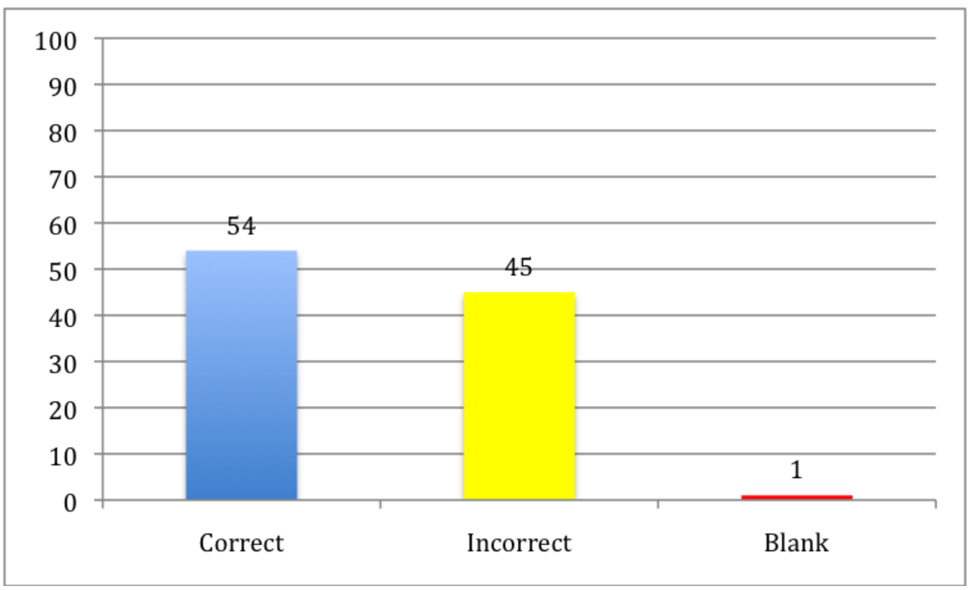


Figure C.4 – Percentage of student responses – item 6

Question Number	Complexity	Calculator (Y/N)	Key
11	Moderate	Y	Draw rectangle with dimensions 5 in. by 10 in.

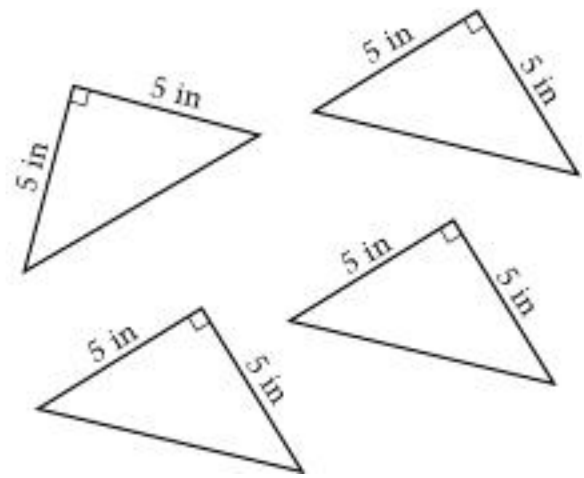


Figure C.5 - Assemble given shapes and determine total area

Make a drawing in the space below to show how the four triangles shown above could fit together without overlapping to make a rectangle that is not a square. Show the dimensions of the rectangle on your drawing. What is the area of this rectangle?

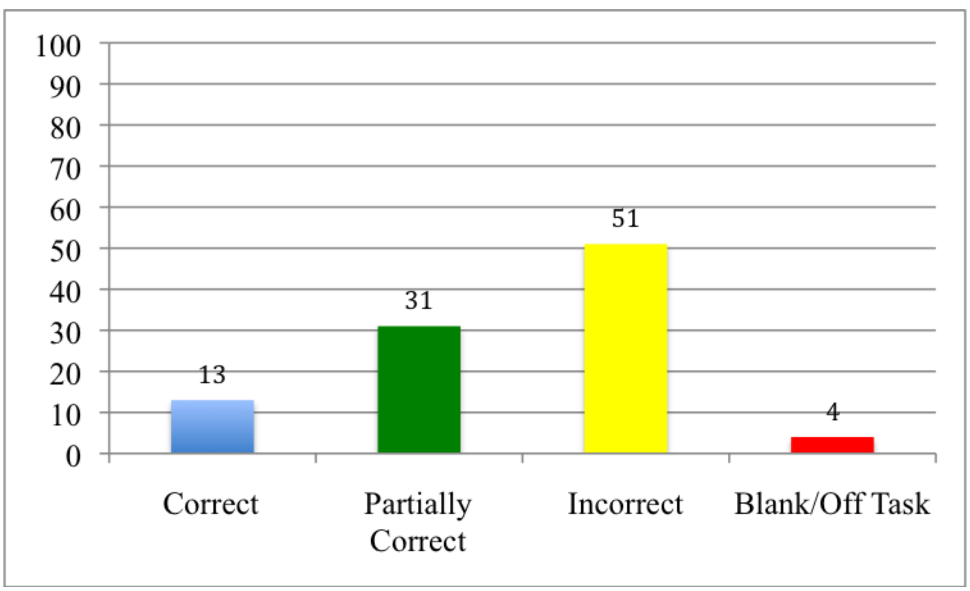


Figure C.6 – Percentage of student responses – item 11

Question Number	Complexity	Calculator (Y/N)	Key
16	Moderate	Y	x = 20, with appropriate work shown

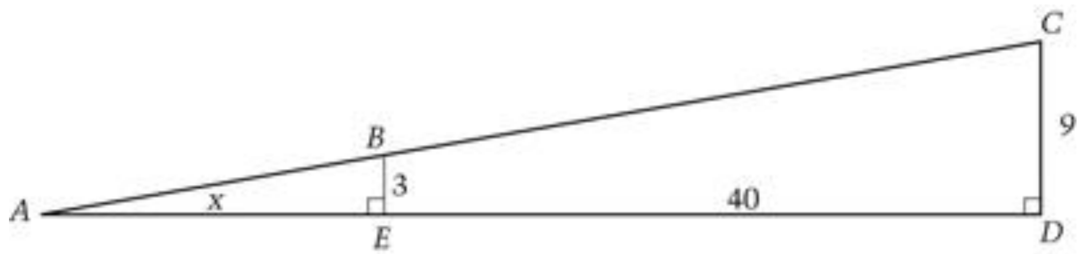


Figure C.7 - Use similarity of right triangles to solve problems

The figure above shows two right angles. The length of *AE* is *x* and the length of *DE* is 40.

Show all of the steps that lead to finding the value of *x*. Your last step should give the value of *x*.

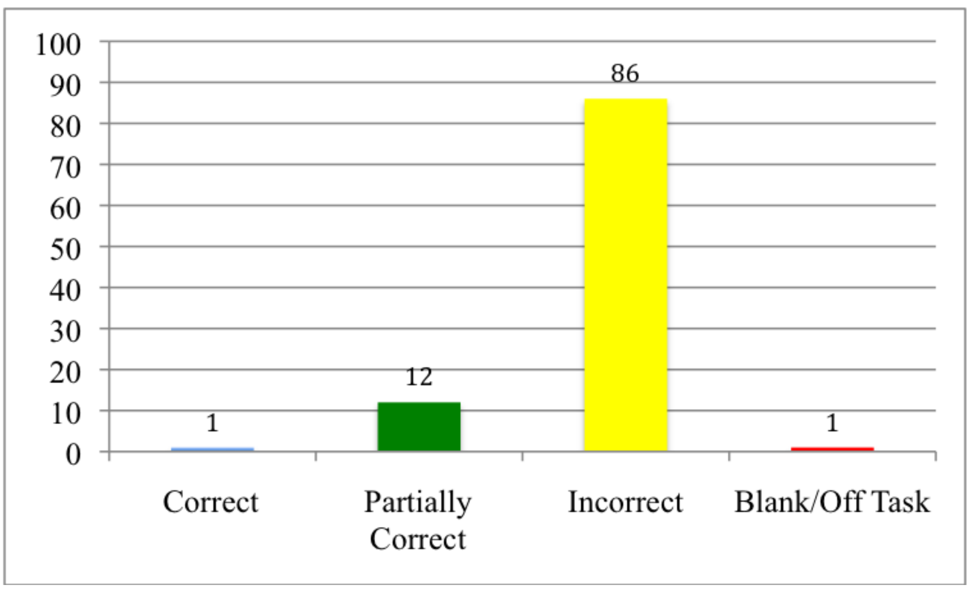


Figure C.8 – Percentage of student responses – item 16

APPENDIX D: STATISTICAL MODERATION

Statistical moderation was used to adjust the TIMSS 2007 grade 8 scores for the United States to NAEP scores with the same mean and standard deviation as TIMSS.

TIMSS level = *A* + *B*(NAEP level), where

A = TIMSS mean score – *B*(mean NAEP score), and *B* = TIMSS standard deviation/NAEP standard deviation.

For the 2007 assessments, the mean NAEP score for the United States was 281, and the mean TIMSS score was 508. The NAEP standard deviation was 36.07, and the TIMSS standard deviation was 76.74.

Therefore,

$$B = \frac{\sigma TIMSS}{\sigma NAEP} = \frac{76.74}{36.07} \approx 2.12752$$

$$A = \mu TIMSS - B(\mu NAEP) = 508.45 - 2.12752(281.35) = -90.13$$

TIMSS level = $-90.13 + 2.12752$(NAEP level)

Using this comparison, we can adjust the TIMSS scores to NAEP achievement levels. The NAEP achievement levels for 2007 were as follows:

- *Basic* – 262
- *Proficient* – 299
- *Advanced* – 333

Using the model above, the corresponding levels for TIMSS are:

- *Basic* – 467
- *Proficient* – 546
- *Advanced* – 618

Based on this model, the United States score of 508 on TIMSS 2007 would fall in the *Basic* achievement level. Singapore's score of 593 would be considered *Proficient.*

VITA

Robyn Williams Carlin was born in Chelsea, Massachusetts, the daughter of Robert Williams and the late Myrtle Washington Williams. She teaches high school mathematics at the Iberville Math, Science, and Arts Academy in Iberville Parish, Louisiana. She received her Bachelor of Science degree in mathematics in 1983 from Southern University and A&M College in Baton Rouge, Louisiana. She previously taught secondary mathematics in Baton Rouge and in Washington, District of Columbia. She was the 2001 Louisiana High School Teacher of the Year. She is married to Winton Burnell Carlin, Jr. and is the mother of one daughter, Nicole and grandmother of three, Marcus, Jordan, and Tyler.