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Radio-Frequency Size Effect Measurements in Cadmium.

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IN CADMIUM.

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**RADIO-FREQUENCY SIZE EFFECT
MEASUREMENTS IN CADMIUM**

A Dissertation

**Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy**

in

The Department of Physics and Astronomy

by

**Ray Christensen Jones
B.S., Louisiana State University, 1963
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ABSTRACT

The Fermi surface of cadmium is determined from calipers obtained through radio-frequency size effect measurements. The 6 MHz to 8 MHz rf measurements were performed at 1.2°K on high purity monocrystals of cadmium approximately 0.5mm in thickness. These measurements were made by observing the discontinuity in the surface impedance occurring when the carrier orbit diameters span the sample thickness. The Fermi surface was calipered using samples whose normals to the surfaces were parallel to one of the three principle symmetry directions: $[0001]$, $[10\bar{1}0]$, and $[11\bar{2}0]$ with magnetic field rotations in the plane of the sample. Calipers are assigned to definite orbits on the first and second band hole surfaces and on both sheets of the third band electron surfaces.

The third band electron "lens" is found to be smooth and round with a major diameter of $1.560 \text{ \AA}^{-1}$ and varying less than 1% in the basal plane. The minor diameter has a caliper of $0.556 \text{ \AA}^{-1}$. Calipers are given for the first band hole surface except near its pointed ends. The second band hole surface is discussed in detail with the assignment of twelve series of calipers to definite orbits on this sheet of the Fermi surface. Both the first and second bands of holes are found to have considerably smaller cross sections than predicted by the single orthogonalized plane wave construction. The detailed shape of the third band electron

"star" is not determined due to the reinterant nature of the orbits on this sheet of the Fermi surface. No calipers were observed which could be assigned to the fourth band electron "cigars". The present results are in good agreement with the ultrasonic data only for the "lens". The magnetic fields used in the investigation were less than 1 kG so no magnetic breakdown effects were observed.

A theoretical calculation of the line shapes encountered in the radio-frequency size effect is presented in an appendix by solving the Boltzmann equation in conjunction with Maxwell's equations of electrodynamics. The calipers obtained in the experiment are tabulated at the end of the paper.

1. INTRODUCTION

In the last ten years or so rapid progress has been made in developing experimental techniques for obtaining information about the electronic structure of metals. The results of the investigations of several metals show, at least within experimental accuracy, that there is a constant energy surface, the Fermi surface, which at $T = 0^{\circ}\text{K}$ separates the occupied from the unoccupied electron states in k -space. From all of the experimental methods that have been tried four techniques have evolved which give the most direct information about the size and shape of the Fermi surfaces of metals.

Of these four techniques, perhaps the most widely used has been the de Haas-van Alphen effect. In observing the de Haas-van Alphen effect one measures the influence of quantized electronic states in a magnetic field upon the various transport properties or on the magnetic susceptibility of the metals as these states pass through the Fermi level with increasing magnetic field. Values of the extremal areas of the various sheets of the Fermi surface are deduced from the periodic variation of these properties as the magnetic field is increased. The technique suffers from the difficulty of trying to separate the many different periods that may be observed in more complicated metals and from the fact that only the area and not the shape of a particular section of the Fermi surface can be obtained. The salient virtue of the technique, however, is that it does not

require as long a mean free path of the carriers as do the other techniques.

The rate of change of area with respect to energy evaluated at the Fermi energy can be obtained through cyclotron resonance measurements. Once both the area and its derivative with respect to energy are known, in principle, the Fermi surface can be constructed, but in practice this is a very difficult procedure requiring the fitting of data to assumed models. The cyclotron resonance experiments suffer from the rather stringent conditions placed on the preparation of samples and from the fact that the frequency of scattering of the carriers must be much less than the frequency of the radiation applied to observe the resonance.

A much more direct method of determining the size and shape of the Fermi surface is to caliper it across its extremal dimensions. There are two techniques available for doing this. The first effect to be investigated was the oscillatory change in the attenuation of ultrasound propagating in a sample subjected to an increasing magnetic field applied perpendicular to the direction of propagation of the sound wave. The oscillatory behavior occurs when diameters of extremal orbits of carriers on the Fermi surface become equal to multiples of the wavelength of the ultrasound. In order to observe this geometric resonance the mean free path of the carriers must be long compared to the acoustic wavelength and the gross acoustic attenuation must be small so that the small oscillatory changes can be observed. As in both the de Haas-van Alphen and cyclotron resonance techniques the information that is available from the geometric

resonance data depends upon ones ability to separate many periodic oscillations superimposed upon one another.

The remaining technique for determining extremal dimensions of the Fermi surface is the radio-frequency size effect (RFSE) first reported by Gantmakher¹ and the application of this technique to cadmium is the subject of the remainder of this paper. Experimentally, the RFSE consists of observing singularities in the surface impedance of a thin single crystalline slab placed in a magnetic field which is imposed parallel to the faces of the sample. At magnetic field strengths such that an extremal cyclotron orbit of a carrier spans the sample thickness a discontinuity in the surface impedance of the sample is observed. The magnetic field strength at which the discontinuity occurs for a given sample thickness is directly proportional to an extremal dimension of the Fermi surface. Since there is only a single discontinuity corresponding to each extremal dimension the data is easy to analyze; however, the technique is limited to very pure metals in which the mean free path can be made several times the thickness of the samples.

The Fermi surface of cadmium has been studied by each of the above mentioned techniques. Several de Haas-van Alphen measurements have been performed, the most recent of which were by Grassie² and Tsui and Stark.³ Cyclotron resonance measurements have been performed by Galt et al.⁴ and by Shaw et al.⁵ Rather incomplete calipers of the Fermi surface have been obtained from the geometric resonance data of Gibbons and Falicov,⁶ Daniel and Mackinnon,⁷ and Chang.⁸ Only as a result of the present investigation have extensive calipers for field rotations in the basal plane become available.⁹

The results of all of the above experimental investigations have established that the Fermi surface of cadmium differs from the free electron surface obtained from the single orthogonalized plane wave (OPW) construction of Harrison¹⁰ in certain important aspects. The experimental dimensions of the surface are considerably smaller than predicted by the single OPW construction, and degeneracies at certain points in the Brillouin zone are removed by the effects of finite spin-orbit coupling. At this time there is no published detailed calculation of the electronic band structure for cadmium from which the Fermi surface can be deduced. There is, however, a detailed calculation using the pseudopotential method with spin-orbit coupling added in progress in this laboratory which will use the results of this experiment to obtain parameters necessary for the calculation.

In the sections which follow, first a discussion of the free electron Fermi surface for cadmium is given followed by a general discussion of the RFSE and the details of the present experimental technique. The last two sections present the data and its interpretation. In Appendix I the phenomenological theory of Kaner and Fal'ko for the line shape for the RFSE is explained, and Appendix II contains a detailed listing of the calipers obtained for each direction investigated.

II. THEORY

A. The Fermi Surface of Cadmium

Cadmium is a divalent metal (atomic number 48) which forms a hexagonal close packed lattice of lattice parameters $a = 2.968 \text{ \AA}$ and $c = 5.520 \text{ \AA}$.¹¹ The Brillouin zone, its dimensions, and the radius of the free electron Fermi sphere (k_F) for cadmium are shown in Fig. 1. Experimental work on several polyvalent metals has shown that in order to obtain an experimental picture of the Fermi surface of a metal it is necessary to start from some approximate theoretical model which can be suitably modified to agree with the experimental information. For this reason, the single OPW construction of Harrison was performed and the results are shown in Figs. 2 and 3. The first band contains only two small pockets of holes ("caps") about the point H in the Brillouin zone. The first band has been separated by spin-orbit splitting from the main region of holes existing in the second band. The second band, usually called the "monster", consists of a surface which is multiply-connected in the KH direction. The third band (electrons) contains one central pocket around Γ ("lens"), and three sheets around L generated by two intersecting discs ("stars"). The latter are in fact connected at points in the AL line to pockets in the fourth band ("cigars") which are entirely contained within the intersection of the two discs which comprise the stars. The free electron construction without spin-orbit coupling predicts that the

CADMIUM

$$\begin{aligned}\overline{\Gamma A} &= 0.568 \text{ \AA}^{-1} \\ \overline{\Gamma M} &= 1.223 \text{ \AA}^{-1} \\ \overline{\Gamma K} &= 1.413 \text{ \AA}^{-1} \\ k_F &= 1.407 \text{ \AA}^{-1}\end{aligned}$$

Fig. 1

(a)

(b)

Fig. 2

Fig. 3

second band monster is connected in the direction of the MK line but is shown pinched off in this direction here because of lack of evidence of a connected orbit in the de Haas-van Alphen and magneto-acoustic experiments by several workers.^{12,13}

B. The Radio-Frequency Size Effect

In the presence of an applied magnetic field, $\underline{H}$, the carriers in a metal move in reciprocal space on the Fermi surface in planes perpendicular to the direction of $\underline{H}$. The projections of the real space orbits that result from this motion onto a plane perpendicular to $\underline{H}$ have the same shape as the orbits in reciprocal space but are rotated 90° about $\underline{H}$. The relation between a dimension $\underline{k}$ in reciprocal space and the corresponding dimension $\underline{r}$ in real space is given as

$$\hbar \underline{k} = (e/c) \underline{H} \times \underline{r} \quad ,$$

where $\hbar$ is Planck's constant divided by two pi, e is the electronic charge and c is the velocity of light. Thus if a real space dimension of an electron orbit is given at a known magnetic field, the corresponding orbit dimension in reciprocal space can be determined.

If a thin flat single crystal of metal is placed in a coil (see Fig. 4) carrying a radio frequency current of angular frequency ω , then there is induced in the metal an antisymmetric electromagnetic field near the surfaces which extends into the metal to a depth δ (the skin depth). If the frequency ω is high enough that $\delta \ll d$ (the sample thickness), and yet low enough that $\omega \ll \Omega = eH/m^*c$,

$$k_c = \frac{e}{\hbar c} H d = 0.015194 H d$$

H in gauss

d in mm

k_c in $\text{\AA}^{-1}$

Fig. 4

where m^* is the effective mass of the carriers and Ω is the cyclotron frequency of the carriers, then the carriers experience a nearly static antisymmetric electric field near the surfaces of the sample. That is, the electrons complete many cyclotron orbits and are scattered before the field changes appreciably. At magnetic field strengths such that carriers on the Fermi surface having orbits with reciprocal space extremal dimensions in the direction of $\underline{n} \times \underline{H}$, where $\underline{n}$ is the unit normal to the sample surfaces, span the sample thickness, a discontinuity in the surface impedance occurs. The discontinuity arises from the relatively large increase in the number of carriers whose real space orbits pass through the skin layer on both of the sample faces. At lower magnetic field strengths, the carriers strike the surface and cannot complete an entire orbit. The details of this process are given in Appendix I.

Experimentally the magnetic field strength H is measured at the point of discontinuity in the surface impedance of a sample of known thickness, d , then an extremal diameter of the Fermi surface, k_c , can be calibrated using the relation

$$k_c = (e/\hbar c) H d = 0.015194 H d \quad ,$$

where k_c is in $\text{\AA}^{-1}$, H is in gauss, and d is in millimeters.

III. EXPERIMENT

A. Sample Preparation

The samples used in this experiment were cut from a zone-refined bar of cadmium obtained from Cominco Products Incorporated (quoted purity: 99.9999%). A portion of this bar was etched in 30% fuming nitric acid and 70% glacial acetic acid for one minute so that the boundaries between the crystals could be observed. A large crystal was then selected and cut from the bar by spark erosion using a tungsten wire as the cutting tool. The resulting monocrystal was oriented by Laue back reflection X-ray photographs and a slab approximately 3 mm thick was cut from the crystal. Slabs cut in this manner had normals to their surfaces nearly parallel to one of the three principal symmetry directions ($[0001]$, $[10\bar{1}0]$, $[11\bar{2}0]$).

Since the opposite faces of the finished samples needed to be as parallel as possible, a technique was developed to obtain this parallelism without depending on alignment by X rays. The spark cutter is equipped with a planing wheel which can be used to obtain flat surfaces. This planing wheel was first checked for flatness in a lathe using a precision indicator. It was then mounted on the servo arm of the spark cutter and not moved for the duration of the sample preparation. The arm on which the goniometer was mounted in the spark cutter was also fixed in position with a mark placed

on the arm so that the goniometer could be removed and replaced in exactly the same position. An aluminum block of slightly larger area than the cadmium slab was glued (using silver paint) to the goniometer table to be used as a mount for the final planing of the sample. Aluminum was used because it is itself readily planed. The goniometer was set in the $(0^{\circ}, 0^{\circ})$ position which made the table approximately parallel to the planing wheel, and the upper surface of the aluminum block was planed flat which resulted in a mounting surface parallel to the planing wheel. The previously cut slab of cadmium was glued to the surface of the aluminum block, and the desired principal axis was aligned parallel to the known direction of an X-ray beam with as great an accuracy as possible ($\pm 1^{\circ}$). The goniometer was again mounted in the spark cutter and one side of the cadmium was planed flat. Following this, the specimen was removed from the aluminum block and turned over so that the flat surfaces of the aluminum and the flat aligned surface of the cadmium were placed together. Care was taken when gluing the cadmium back to the aluminum to insure that no silver paint ran between the two flat surfaces causing a misalignment. The goniometer was then returned to the $(0^{\circ}, 0^{\circ})$ position and the specimen was planed to the desired thickness.

Samples obtained with the above procedure were typically 20mm by 30mm by 0.5mm. These slabs were then dipped in a solution of 75% fuming nitric acid and 25% water at 0°C . This etch removes the thin layer of spark damage on the surface, but because of the large surface to volume ratio of these slabs it was necessary to etch only for short periods since the acid around the sample quickly came to a boil.

The polishing time was usually about one second and then the slab was quickly dipped in water. The surfaces obtained in this manner were shiney, but not mirror-like.

Samples of 5mm by 5mm were then cut by spark erosion from the central portion of the slab. The edges of the large slab were discarded because of possible rounding of the corners during the polishing procedure. The orientation of these finished samples were again checked and the normals to the faces were always found to be within 1° of arc of the desired crystallographic axis.

Since one of the two experimental parameters is the thickness of the sample, it must be determined as accurately as possible. The most satisfactory procedure for this measurement was found to be measurement of the larger dimensions of the sample with a traveling microscope so that the surface area of the parallel faces could be computed, then weigh the sample to determine its mass. From the known density of cadmium the thickness, d , could be determined using the relation

$$d = \frac{(\text{mass})}{(\text{density})(\text{area})}$$

This gives the average thickness very accurately at room temperature, but the thickness at 1°K must be known since all of the magnetic field measurements were performed at that temperature. The thermal expansion data of Grüneisen and Goens¹⁶ gives the following dimensional change in cadmium from 1° to 293°K :

$$\begin{array}{ll} \parallel [0001] & 1.38\% , \\ \perp [0001] & 0.40\% , \end{array}$$

and these corrections were applied to determine d for the calculations.

The resistivity ratio was measured on a larger sample and was found to be $\rho_{300^{\circ}\text{K}}/\rho_{4.2^{\circ}\text{K}} = 20800$. The amplitude of the size effect peaks became about 20 times as intense by reducing the temperature from 4.2°K to 2°K at which point no further increase in amplitude was observed.

B. Experimental Apparatus

The sample was placed on a thin piece of mica which supported the sample during the experiment and a flat coil of No. 45 copper wire was wound around both the sample and the mica support. Care was exercised to have the sample occupy as much of the cross sectional area of the coil as possible. This coil forms part of the resonant circuit of a marginal oscillator¹⁷ operating from 6 MHz to 8 MHz. This range of frequencies was selected because it satisfied the conditions for the RFSE, $\delta \ll d$ and $\omega \ll \Omega$. The coil and sample were immersed in liquid He^4 inside a conventional double dewar system, (see Fig. 5), and the temperature was reduced to 1.2°K by pumping.

The magnetic field was supplied by a small iron core magnet which could be rotated in the horizontal plane of the sample. The angle the magnetic field formed with one of the crystallographic axes of the sample could be set within $\pm 0.1^{\circ}$ and could be determined to within 0.5° from the symmetry of the data. During a run the angle of the magnet was rotated 30° on either side of the axis of the rf coil, thus it was necessary to remove the sample and rewind the coil to cover the full 90° between axes for two of the sample orientations. A transistorized power supply was constructed for the magnet which

Fig. 5

had the capability of sweeping the magnet current through zero and applying a small current in the reverse direction so that the residual field of approximately 30 gauss could be canceled to obtain low fields. A Hall probe, which was calibrated in place with an NMR gaussmeter, was used to measure the magnetic field. The Hall voltage was measured with a Leeds and Northrup K-2 potentiometer, and the Hall probe control current was supplied by a Hewlett Packard current regulated supply. During a sweep of the magnetic field markers were placed on the recording to calibrate each trace.

A pair of modulation coils was wound on plastic forms and mounted on the iron pole pieces of the magnet to provide an ac amplitude modulation of the applied magnetic field. A frequency of 40 Hz was selected for the amplitude modulation since at this frequency the skin depth in cadmium is large enough to allow penetration of the modulation into the entire sample.

The grid leak detector built into the second stage of the marginal oscillator is sensitive to any change in rf power absorption, and thus to the real part of the surface impedance of the sample in the tank coil. An increase in the sample impedance leads to a positive voltage at the output of the detector while a decrease produces a negative output. Thus, the output of the detector of the oscillator was a 40 Hz sine wave whose amplitude was proportional to $\partial R / \partial H$, where R is the real part of the surface impedance. The output of the oscillator was amplified by a tuned amplifier (see Fig. 5) and then fed into a phase sensitive detector. The dc output of the phase detector was applied to the Y axis of an XY recorder whose X axis was swept proportional to the applied magnetic field. The resulting plot was $\partial R / \partial H$ vs H .

IV. EXPERIMENTAL RESULTS

A. Data

The data for $\underline{n} \parallel [0001]$ were recorded by rotating the magnet in one degree steps over a 40° range which covered both axes $[10\bar{1}0]$ and $[11\bar{2}0]$. In Fig. 6 some representative recorder traces are shown. Figure (6e) shows the rapidly changing surface impedance at low magnetic field strengths. This rapid change obscures the much smaller size effect peaks in this region. Figure (6a) shows a size effect peak superimposed on the large slope at low fields. Figures (6b) and (6c) show the primary and secondary "lens" peaks, respectively. The secondary peak arises at magnetic fields such that two orbit diameters span the sample thickness. The secondary peak here is about 200 times weaker than the primary peak and is also inverted. With the magnetic field in the basal plane the "lens" orbits are elliptical and intersect at points of maximum curvature (see Fig. 4), thus the low intensity of the secondary peak is expected from this consideration. Figure (6d) is a recorder trace in the medium field range where most of the peaks occur.

Figure 7 shows a recorder trace for $\underline{n} \parallel [10\bar{1}0]$. The magnetic field is rotated from $[0001]$ to $[11\bar{2}0]$ in 1° and 2° steps. There are generally fewer size effect peaks observed for this orientation than for the other two. The double field "lens" peak is shown on this curve along with the primary "lens" peak and two other peaks.

Fig. 6

Fig. 7

The "lens" peaks were observed to as high as fourth order. Their polarity seemed to invert for each successive occurrence although the line shape gets more complex and smaller in amplitude for the higher harmonics and their polarity is not easily determined.

A typical recorder trace obtained for $\underline{n} \parallel [11\bar{2}0]$ is shown in Fig. 8. A large number of peaks occurred with considerable amplitude in this orientation. The primary and secondary "lens" peaks are shown along with several other peaks. The "lens" peak was again observed to as high as fourth order. No peak other than the "lens" was seen to exhibit the multiple field behavior.

B. Determination of the Critical Magnetic Fields and the Calipers

There are several line shapes evident in Figs. 6, 7, and 8, most of which are considerably more complex than those predicted in Appendix I. It is not always evident which of the several points in the lines to choose as the critical magnetic field. Fukumoto and Strandberg¹⁴ determined the shift in magnetic field of various points in the line versus ω , the applied frequency. They found that the lowest field extremum in the curve $\partial R / \partial H$ vs H had a smaller ω dependence than any of the other points in the line and thus concluded that this was the point to choose as the critical magnetic field. The author came to the same conclusion by noting the change in line shapes due to changing the orientation of the coil relative to the sample axes and to the magnetic field. The low field extremum had the only value of magnetic field that corresponded to an extremum for both coil orientations.

Fig. 8

The critical magnetic fields were then converted into calipers by using the formulas in Appendix II. The numerical caliper values are tabulated there for reference. The experimental caliper values are plotted vs the direction of the applied magnetic field in Figs. 9, 10, 11, 12, 13, 14, and 15. The data have been grouped into series by inspection of the recorder traces. Most of the individual series have been labeled with a Roman letter for reference.

Fig. 9

Fig. 10

Fig. II

Fig. 12

Fig. 13

Fig. 14

Fig. 15

V. DATA ANALYSIS AND CALIPER ASSIGNMENT

In order to clarify the discussion below, two definitions will be stated here. An "extremal orbit" is defined as an orbit on the Fermi surface in k-space the length of whose projection on a plane perpendicular to $\underline{n}$ is an extremum with respect to k_H (the component k in the direction of $\underline{H}$). An "extremal caliper" is defined as a vector quantity in k-space which is in the direction of $\pm \underline{n} \times \underline{H}$ and has a magnitude equal to the length of the line segment formed by projecting an extremal orbit (in k-space) onto a plane perpendicular to $\underline{n}$. Thus an extremal caliper has a fixed magnitude and direction, but is free to be translated in k-space.

The radio-frequency size effect obtains "extremal calipers" on the Fermi surface. Simply the term "caliper" will be used hereafter.

The individual sheets of the Fermi surface and the assignment of the calipers to them are discussed in turn below. This chapter is ended with a discussion of the unassigned data.

A. Third Band Electron "Lens"

There was no difficulty in assigning particular calipers to the third band electron surface called the "lens". The "lens" produced the most intense line observed for each orientation. With $\underline{n} \parallel [0001]$ the "lens" was seen to have a circular cross section in

the IMK plane with diameter $k_c = 1.560 \text{ \AA}^{-1}$ * which is plotted in Fig. 9. The "lens" calipers obtained from the $n \parallel [10\bar{1}0]$ data lie in the IAHK plane and are given in Fig. 10, while the calipers obtained from the $n \parallel [11\bar{2}0]$ data lie in the IALM plane and are plotted in Fig. 11. The arcs in Figs. 10 and 11 outline the single OPW cross sections of the "lens". The data are seen to agree quite well with the single OPW predictions except near the edge of the "lens" where considerable rounding occurs. A summary of some of the most recent "lens" calipers is given in Table I. The diameters reported in this investigation are in reasonable agreement with the available ultrasonic data. The "lens", however, is found to be quite smooth with no bump at the center as reported by Gibbons and Falicov,⁶ and no dip as observed by Chang.⁸

The cross sectional areas of the "lens" could be computed from the formula for the area of an ellipse, $S = \pi k_1 k_2 / 4$, where k_1 and k_2 are the major and minor axes of the ellipse. From the data obtained with $n \parallel [0001]$ the circular cross section in the IMK plane has an area of $1.911 \text{ \AA}^{-1}$, and from the major diameters obtained with $n \parallel [10\bar{1}0]$ and $n \parallel [11\bar{2}0]$ the area is $1.861 \text{ \AA}^{-2}$, both of which compare reasonably well with the value of $1.874 \text{ \AA}^{-2}$ obtained by Grassie.² The area of the elliptical cross section in the IAHK plane can be obtained with $n \parallel [10\bar{1}0]$, and the IALM cross section

*The previously published value of $k_c = 1.5825 \text{ \AA}^{-1}$ ⁹ is in error due to incorrect selection of the critical magnetic field. Subsequent investigation by the author and Fukumoto and Strandberg¹⁴ indicated the correct point to choose.

TABLE 1. Comparison of caliper values for
"lens" obtained by different sources.

Source						
$\underline{n}, \underline{g}^a \parallel :$	$[11\bar{2}0]$		$[10\bar{1}0]$		$[0001]$	
$\underline{H} \parallel :$	$[0001]$	$[10\bar{1}0]$	$[0001]$	$[11\bar{2}0]$	$[10\bar{1}0]$	$[11\bar{2}0]$
Deaton ^b		0.550	1.556	0.550		
Daniel and Mackinnon ^c	1.458	0.54	1.428	0.498		
Gibbons ^d and Falicov ^d	1.560	0.550		0.540		
Chang ^e	1.510	0.542	1.516	0.540		
Present Investigation	1.563	0.559	1.516 ^f	0.553	1.560	1.560
Free Electron Values	1.658	0.540	1.658	0.540	1.658	1.658

^aUltrasonic attenuation obtains the same calipers as the RFSE with $\underline{g}$, the direction of sound propagation, in the same direction as $\underline{n}$.

^bSee Ref. 15.

^cSee Ref. 7.

^dSee Ref. 6.

^eSee Ref. 8.

^fThe line shape became rather complex as $\underline{H}$ neared $[0001]$, thus this result is subject to a larger error than the other values in this row. The same diameter is calipered with $\underline{n} \parallel [0001]$ and $\underline{H} \parallel [10\bar{1}0]$ and is considered more accurate.

can be computed using the $\underline{n} \parallel [11\bar{2}0]$ data. The results are $0.658 \text{ \AA}^{-2}$ and $0.686 \text{ \AA}^{-2}$, respectively, which are in fair agreement with Grassie's result of $0.608 \text{ \AA}^{-2}$.²

B. The Second Band Hole Surface

The second band hole surface ("monster") consists of two trifoliate surfaces per Brillouin zone which are connected in the $[0001]$ direction and which are mirror reflections of each other in the ΓAHK and ΓALM planes (see Figs. 1 and 3). The calipers obtained with $\underline{n}$ parallel to one of the three principal directions ($[0001]$, $[10\bar{1}0]$, or $[11\bar{2}0]$) on these two surfaces are degenerate, and all the calipers for a given magnetic field direction are obtained by considering only one of the trifoliate surfaces (see Figs. 16, 17, and 18). In Fig. 16 the large solid line cloverleaf represents the ΓMK cross section ("waist") of the "monster". The shaded portion represents the AHL cross section ("neck"), and the dashed lines indicate the extremities of the cross sections intermediate to the ΓMK and AHL planes.

Because it has no inversion symmetry across the $(11\bar{2}0)$ plane the "monster" has no central calipers for $\underline{n} \parallel [0001]$ or $\underline{n} \parallel [10\bar{1}0]$, i.e., it has no series of calipers that have a common fixed point on a symmetry point or line in the Brillouin zone. Thus while the direction of a caliper is known to be that of $\underline{n} \times \underline{H}$, there is no known point, line, or plane on which to fix the position of the caliper. This greatly complicates the determination of the Fermi surface from the extremal caliper data. Figure 16 was determined by a method of self consistent caliper assignment which shall be described below.

Fig. 16

Fig. 17

Fig. 18

A surface is first drawn similar to the single OPW Fermi surface. The extremal dimensions of this surface are measured for various directions and compared with the experimental data. The figure is then modified in an attempt to obtain a closer fit to the data and its extremal dimensions are again measured. This process is continued until a surface is obtained that yield the experimental calipers. This process is more accurate when several calipers have overlapping ranges on the Fermi surface, i.e., when two or more different calipers have one common end point.

The cross sectional areas of the "monster" in the AHL and IMK planes are known from de Haas-van Alphen data³ to be $0.060 \text{ \AA}^{-2}$ and $0.579 \text{ \AA}^{-2}$, respectively. This greatly facilitated the analysis because the cross sectional area of the "monster neck" in the AHL plane combined with the calipers obtained in the two directions $[10\bar{1}0]$ and $[11\bar{2}0]$ on the "neck", and the fact that the "neck" must contain the first zone "cap" determined the shape of the "neck" in this plane rather accurately.

Open orbits existing on the second band hole surface with H in the (0001) plane have their extremities perpendicular to $[0001]$ in the AHL and IMK planes. Thus open orbits generated on this surface will caliper the difference in extent of the surface in these two planes. These orbits have a near sinesoid shape similar to the outline of the second band surface shown in Fig. 17.

The projection of the "monster" in the plane determined by two adjacent HK lines is shown in Fig. 17 along with the cross section of the "cap" in that plane which is enclosed by the "monster".

Calipers lying in this plane are obtained with $\underline{n} \parallel [10\bar{1}0]$. As previously mentioned, the calipers for the "monster" in this plane are non-central, thus the method of self consistent caliper assignments must be employed.

The calipers obtained with $\underline{n} \parallel [11\bar{2}0]$ for the "monster" and the "cap" are central in the sense that there exists a symmetry line in the Brillouin zone through which all of the calipers in a given series have their midpoints. These lines are either the ΓK or the AH lines (these lines are normal to the page in Fig. 18). Since these calipers are not fixed to a single point on these lines the calipers obtained do not all lie in the same plane. Figure 18 was obtained by plotting in polar form the half calipers perpendicular to the AH or the ΓK lines. The figure that results is a shadowgram, or projection of the extremal size, of the second band hole surface on a plane perpendicular to the ΓK and AH lines.

Caliper series labeled A, B, C, D, E, M, P, Q, T, W_1 , X, and Y were assigned to definite orbits on the "monster" surface. Representative calipers of each of these except X are shown by lines labeled with small letters in Figs. 16, 17, and 18. The angles to which these representative calipers correspond are shown by the arrows in Figs. 12, 13, 14, and 15. Series A and B were assigned as minimum and maximum open orbits around the arms of the "monster". With $\underline{n} \parallel [0001]$ there exist two distinct D orbits for $\underline{H}$ within 7° of $[10\bar{1}0]$ which have the same caliper values. This interpretation is supported by a more intense D line in this range.

After constructing the cross sections of the second zone from calipers obtained by series A, B, C, and D together with the known cross sectional areas of the IMK and AHL cross sections, it was found that series E corresponds to both the expected angular dependence and size of the type caliper indicated by ee' in Fig. 16. For field directions approximately $\pm 10^\circ$ from the basal plane, closed orbits lying in the concave portion of the second zone ("valley") can exist. These orbits have a maximum extent only by virtue of the fact that they increase in size to a point where they spill out of this concave portion of the surface. This orbit was also observed for $\underline{n} \parallel [11\bar{2}0]$ with $\underline{H}$ within 10° of $[10\bar{1}0]$ (series X). In this case the "valley" was calipered in directions which were nearly parallel to $[0001]$ with one end of the caliper being fixed at the rim of the "valley" near the AHL plane.

Inspection of the series P and Q in Fig. 13 lead naturally to the assignment of P to the "waist" orbit and Q to the extended waist orbit (see Fig. 17). Assignment of series M to the "neck" orbit then followed. Similarly, assignment of W_1 and Y in Fig. 15 to the "waist" orbit and the extended "waist" orbit for $\underline{n} \parallel [11\bar{2}0]$ was immediate (see Fig. 18). Series T (Fig. 14) was easily seen to be due to the minimal "neck" caliper (see Fig. 18).

The solid curves in Figs. 17 and 18 have been determined by at least one series of calipers. The tips on the "monster arms" were determined from Fig. 16 and the dashed lines represent interpolated points.

The calipers in the region of overlap in Figs. 16, 17, and 18 were found to agree within experimental accuracy.

C. The First Band Hole Surface

The first band hole surface, "cap", consists of two sections per Brillouin zone centered at the point H which have three fold symmetry about $[0001]$. This surface has the same symmetry properties as the second band hole surface discussed previously; thus, all the calipers for this surface can be obtained by considering only one of the two sections.

In Fig. 19 the AHL cross section of the "cap" is shown as the solid figure within the second band "neck" (dashed figure). The Γ AHK cross section, the $(10\bar{1}0)$ plane, is shown within the second band cross section in Fig. 17. The same remarks stated above for calipers obtained with $\underline{n} \parallel [11\bar{2}0]$ on the second band hole surface are also valid for the first zone hole surface. Thus the extremal calipers obtained for the "cap" with $\underline{n} \parallel [11\bar{2}0]$ do not all lie in a plane, and the outline in Fig. 18 is not a true cross section of the "cap", but a shadowgram.

The caliper series labeled F, L, and S have been assigned to the first band hole surface. The representative calipers shown in Figs. 17, 18, and 19 are for the angles indicated by the arrows in Figs. 9, 12, and 14. The assignments of L and S to the "cap" are quite natural considering the assignment of M and T to the "neck" of the second band (see Figs. 12, 14, 17, and 18). The assignment of F arose from the assignment of L since the same caliper is obtained

Fig. 19

with $\underline{H} \parallel [10\bar{1}0]$ for $\underline{n} \parallel [0001]$ and with $\underline{H} \parallel [0001]$ for $\underline{n} \parallel [10\bar{1}0]$. The AHL cross section shown in Fig. 19 and the Γ AHK cross section, Fig. 17, were obtained using the previously described self consistent method with the known areas of $0.063 \text{ \AA}^{-2}$ for the AHL cross section and $0.105 \text{ \AA}^{-2}$ for the Γ AHK cross section as determined from the de Haas-van Alphen data.³ The dashed portion of the Γ AHK cross section shown in Fig. 17 was extrapolated to give the correct areas. No similar extrapolation was possible for the section shown in Fig. 18 because here the calipers do not trace out the maximum area orbit on the "cap" obtained with $\underline{H} \parallel [11\bar{2}0]$.

D. The Third Band "Stars"

The third band "stars" consist of three non-equivalent electron pockets per Brillouin zone centered at the points L. The "stars" have only two fold symmetry about each of the LH, LM, and LA lines and the three non-equivalent "stars" are rotated 120° from each other in the (0001) plane. Thus to assign calipers to the "stars" all three pockets must be considered with $\underline{n} \parallel [0001]$, and two must be considered both with $\underline{n} \parallel [10\bar{1}0]$ and with $\underline{n} \parallel [11\bar{2}0]$. This implies that with $\underline{n} \parallel [0001]$ one would expect to obtain three calipers for the stars in the 30° range of rotation of $\underline{H}$ between $[10\bar{1}0]$ and $[11\bar{2}0]$ which degenerate into two calipers with $\underline{H}$ along either of the axes. If these three series were connected end to end on a polar plot then the projection of the "stars" onto the (0001) plane would be obtained. With $\underline{n} \parallel [10\bar{1}0]$ the calipers would trace the two projections of the stars onto the $(10\bar{1}0)$ plane. The

calipers obtained with $\underline{n} \parallel [11\bar{2}0]$ should trace the AHL cross section in addition to the cross section formed by a plane rotated 60° along the LM line from the Γ ALM plane and passing through the point L.

Series G, N, R, and U have been assigned to the "stars". Series G was assigned as the projection of the star onto the (0001) plane. This series could be followed only 56° instead of the 90° expected. This is because the lines produced by these calipers are of low intensity and they became obscured by the more intense lines as they moved to higher magnetic fields. Series N was assigned as the projection of the "stars" onto the (10 $\bar{1}$ 0) plane. Only one instead of the expected two calipers were observed and series N deviated from what was expected from the single OPW model as $\underline{H}$ was rotated away from [0001] - the values of N increasing instead of decreasing. Series R has been assigned to the AHL cross section and U has been assigned to the cross section rotated 60° from the Γ ALM plane. The calipers R and U agree reasonably well with the single OPW values.

The diameter of the "star" along the HL line was slightly smaller than that predicted by the single OPW construction. The projection of the "star" onto the (0001) plane was considerably smaller than predicted for directions parallel to the AL line. No calipers were observed which would give the maximum size of the "stars" in the [0001] direction.

E. Unassigned Data

Series H, I, J, K, V, W_2 , and W_3 have not been assigned to particular orbits. The peaks obtained for series H, V, W_2 , and

W_3 were of moderate intensity while series I, J, and K were of low intensity relative to the other peaks observed (see Figs. 6 and 8).

The series H remained an almost constant value less than the minimal open orbit A. W_2 and W_3 have the same appearance as the "waist" orbit W_1 but have smaller values for each orientation of H in the $(11\bar{2}0)$ plane.

VI. CONCLUSIONS

The third band "lens" was found to be round and smooth with the principle diameters given in Table I. The separation in k-space between the "lenses" in adjacent Brillouin zones has been calculated and is presented in Table II.

The second band hole surface ("monster") is much smaller near the AHL plane than predicted by the single OPW method, and is somewhat smaller in the Γ MK plane than predicted. Series W_2 and W_3 are similar to the "waist" series W_1 . It would seem that calipers W_2 and W_3 might indicate the presence of openings in the bottom of the "valley" of the "monster" near the HK line. However, this is not believed to be the case because no similar series were obtained for $\underline{n} \parallel [10\bar{1}0]$, and the existence of the "valley" orbit X places stringent requirements on the positions of any openings that might occur. The most convincing argument against their existence, however, is the fact that no position for the openings could be found which yielded the calipers W_2 and W_3 . In particular, the value of W_3 is larger than one half of the value of W_1 for each direction of $\underline{H}$ which should not be the case if W_3 arose from orbits passing through an opening. It is then concluded that the second band hole surface is continuous along the HK line, and the existence of series W_2 and W_3 cannot be explained on the basis of an opening.

TABLE II. Separation of the various sheets of
the Fermi surface in k-space.

Position of Separation	Separation ($\text{\AA}^{-1}$)
Between:	
First band holes and second band holes in AHL plane	
along AH line	0.004
along HL line	0.014
Second band holes and third band electrons ("lens") in Γ MK plane	
along Γ K line	0.063
Arms of the second band hole surfaces in the Γ MK plane	
parallel to MK line	0.127
Second band holes and third band electrons ("stars")	
along HL	0.251
Third band "lenses" in adjacent Brillouin zones	
along Γ A line	0.580

The separation between the "monster arms" is calculated from Fig. 16 and presented in Table II. The separation between the second band hole surface and the third band electron surface along the HL line is calculated from information in Figs. 1, 12, 16, and 19.

The first band hole surface was found to be much smaller near the AHL plane than predicted by the single OPW calculation. There are very small separations between the first and second band hole surfaces in this plane, and they are presented in Table II. This very small separation is responsible for the magnetic breakdown that has been observed in several experiments.²⁻⁴ The extent of the first band hole surface along the HK line is implied by a combination of the present data with the area data of Tsui and Stark³ (see Fig. 17).

The third band electron "star" is found to deviate significantly from the single OPW surface. Not enough data is available on this surface to assign to it a definite shape.

No k_c values could be assigned to the fourth band electrons ("cigars"). The small radius of curvature expected on the ends of the "cigar" would make the line extremely weak, and the small dimensions in the other directions would place the peak at very low magnetic fields where it would be difficult to detect due to rapidly changing surface impedance which is always encountered at low magnetic fields. Thus the fact that no size effect peaks are observed which can be attributed to the fourth band "cigars" by no means precludes their existence.

APPENDIX I

Calculation of the Line Shape of the Radio-Frequency Size Effect in Metals

In this section line shapes for the size effect peaks are calculated under rather idealized conditions. To do this the method of Kaner and Fal'ko¹⁸ is followed and expanded upon where necessary. The bulk of the mathematical manipulation will be left to the reader, but a description of each step involved in developing the theory is given.

To construct a theory of the size effect line shape it is necessary to solve Maxwell's equations simultaneously with the kinetic equation (Boltzmann's equation) for the electron distribution in the plate. The coordinate system used here is chosen as follows: The x axis coincides with the direction of the constant magnetic field $\underline{H}$ (the vector $\underline{H}$ is parallel to the surface of the plate), and the z axis is parallel to the inward normal to the surface of the plate $z = 0$. All quantities are assumed to have a time dependence of $\exp(-i\omega t)$, where ω is the frequency of the applied electromagnetic field. Maxwell's two curl equations are combined by eliminating $\underline{H}$. The displacement current $\frac{\partial \underline{D}}{\partial t}$ is neglected because ω is assumed to be much smaller than the conductivity of the metal σ . This yields the following

equations:

$$\frac{\partial^2 E_\alpha(z)}{\partial z^2} = -i \frac{4\pi\omega}{c^2} j_\alpha(z), \quad (\alpha = x, y) \quad (1)$$

The Boltzmann equation for f , the equilibrium addition to the Fermi distribution function $f_0(\epsilon)$, is obtained by considering the following equation:

$$\left. \frac{\partial f}{\partial t} \right|_{\text{scattering}} + \left. \frac{\partial f}{\partial t} \right|_{\text{fields}} + \left. \frac{\partial f}{\partial t} \right|_{\text{diffusion}} = \frac{\partial f}{\partial t}.$$

f is assumed to have time dependence $\exp(-i\omega t)$, thus

$\partial f / \partial t = -i\omega f$. If the concept of a relaxation time is introduced, then $\partial f / \partial t |_{\text{scattering}} = -\nu f$,¹⁹ where ν is the frequency of scattering. The term due to the interaction with the electromagnetic fields is known to be¹⁹:

$$\left. \frac{\partial f}{\partial t} \right|_{\text{fields}} = -\Omega \frac{\partial f}{\partial \tau} + e E_\alpha v_\alpha \frac{\partial f_0}{\partial \epsilon},$$

where v , p , and ϵ are the electron velocity, momentum, and energy, respectively; $\Omega = eH/mc$ is the cyclotron frequency, which depends on ϵ and p_x ; $2\pi m(\epsilon, p_x) = \partial S(\epsilon, p_x) / \partial \epsilon$, where $S(\epsilon, p_x)$ is the area of the intersection of the equal-energy surface with the plane $p_x = \text{constant}$; and τ is the dimensionless time of motion of the electron along the orbit in p -space, $\tau = \Omega t$.

The term due to diffusion is simply¹⁹:

$$\left. \frac{\partial f}{\partial t} \right|_{\text{diffusion}} = - v_z \frac{\partial f}{\partial z} .$$

By rearranging terms, the following is obtained for the Boltzmann equation:

$$v_z \frac{\partial f}{\partial z} + \Omega \frac{\partial f}{\partial \tau} + (v - i\omega)f = e E_\alpha v_\alpha \frac{\partial f_0}{\partial \epsilon} . \quad (2)$$

The current density j_α is given by

$$j_\alpha = - \frac{2e}{h^3} \int v_\alpha f d^3p . \quad (3)$$

The field component E_z (the Hall field) has been neglected in the above discussion. It has been shown²⁰ that E_z leads only to a small correction in Eqs. (1) - (3). This is due to the fact that the main contribution of the current density is made by electrons moving almost parallel to the surface of the metal. It can therefore be assumed that $E_z = 0$, and $j_z = 0$.

The function f is set equal to zero outside the plate, i.e.

$$f(z, \underline{v}) = 0 , \quad \begin{cases} z < 0 \\ z > d \end{cases} , \quad (4)$$

where d is the thickness of the plate.

The conditions for diffuse reflection of the electrons from the surface of the plate are given as:

$$f(0, \underline{v})|_{\underline{v}_z > 0} = 0 ; \quad f(d, \underline{v})|_{\underline{v}_z < 0} = 0 . \quad (5)$$

To solve Eq. (3) for f , the Laplace transformation is applied:

$$F(p, \tau) \equiv \int_0^d f(z, \tau) e^{-pz} dz$$

and

$$G(p, \tau) \equiv \int_0^d \frac{e}{\Omega} \frac{\partial f_0}{\partial \epsilon} \underline{v} \cdot \underline{E}(z) e^{-pz} dz .$$

Equation (3) is multiplied through by $\exp(-pz)dz/\Omega$ and integrated from zero to d yielding:

$$\frac{\partial F}{\partial \tau} + \left(\gamma + \frac{pv_z}{\Omega}\right)F = G(p, \tau) + \frac{v_z}{\Omega} [f(0, \tau) - f(d, \tau)e^{-pd}] , \quad (6)$$

where $\gamma = (\nu - i\omega)/\Omega$. This is solved to yield:

$$F(p, \tau) = \int_{-\infty}^{\tau} d\tau_1 \exp\left[\int_{\tau_1}^{\tau} \left(\gamma + \frac{pv_z}{\Omega}\right) d\tau_2\right] \\ \times \left\{ G(p, \tau_1) + \frac{v_z(\tau_1)}{\Omega} [f(0, \tau_1) - f(d, \tau_1) e^{-pd}] \right\} . \quad (7)$$

The inverse transformation is applied

$$f(z, \tau) \equiv \frac{1}{2\pi i} \int_{s-i\infty}^{s+i\infty} F(p, \tau) e^{pz} dp$$

$$\begin{aligned}
f(z, \tau) \equiv & \int_{-\infty}^{\tau} d\tau_1 \exp\left(\int_{\tau}^{\tau_1} \gamma d\tau_2\right) \left\{ g\left(z + \frac{1}{\Omega} \int_{\tau}^{\tau_1} v_z d\tau_2, \tau_1\right) \right. \\
& + \frac{v_z(\tau_1)}{\Omega} \left[f(0, \tau_1) \delta\left(z + \frac{1}{\Omega} \int_{\tau}^{\tau_1} v_z d\tau_2\right) \right. \\
& \left. \left. - f(d, \tau_1) \delta\left(z - d + \frac{1}{\Omega} \int_{\tau}^{\tau_1} v_z d\tau_2\right) \right] \right\} .
\end{aligned} \tag{8}$$

The delta functions arise from

$$\delta(z) \exp(-zs) = \frac{1}{2\pi i} \int_{s-i\infty}^{s+i\infty} \exp(\lambda z) d\lambda$$

with $s = 0$.

It follows from the boundary conditions (5) that

$$f(0, \tau) = -\text{sgn}[v_z] f(0, \tau) , \text{ and } f(d, \tau) = \text{sgn}[v_z] f(d, \tau) , \tag{9}$$

where

$$\text{sgn}(x) \equiv \begin{cases} 1 & (x > 0) \\ -1 & (x < 0) \end{cases} .$$

Defining the function $u(z, \tau)$ as:

$$u(z, \tau) \equiv \exp\left(\int_0^{\tau} \gamma d\tau_2\right) f(z, \tau) , \tag{10}$$

and using (9), Eq. (8) becomes

$$\begin{aligned}
 & u(z, \tau) + \int_{-\infty}^{\tau} d\tau_1 \, u\left(z + \frac{1}{\Omega} \int_{\tau}^{\tau_1} v_z d\tau_2, \tau_1\right) \\
 & \times \frac{|v_z(\tau_1)|}{\Omega} \left\{ \delta\left(z + \frac{1}{\Omega} \int_{\tau}^{\tau_1} v_z d\tau_2\right) \right. \\
 & \left. + \delta\left(z - d + \frac{1}{\Omega} \int_{\tau}^{\tau_1} v_z d\tau_2\right) \right\} \\
 & = \int_{-\infty}^{\tau} \exp\left(\int_0^{\tau_1} \gamma d\tau_2\right) g\left(z + \frac{1}{\Omega} \int_{\tau}^{\tau_1} v_z d\tau_2, \tau_1\right) d\tau_1 .
 \end{aligned}$$

Making use of the delta functions, this reduces to

$$\begin{aligned}
 & u(z, \tau) + \sum_s u\left(z + \frac{1}{\Omega} \int_{\tau}^{\lambda_s(z, \tau)} v_z d\tau_2, \lambda_s(z, \tau)\right) \\
 & = \int_{-\infty}^{\tau} d\tau_1 \exp\left(\int_0^{\tau_1} \gamma d\tau_2\right) g\left(z + \frac{1}{\Omega} \int_{\tau}^{\tau_1} v_z d\tau_2, \tau_1\right) , \quad (11)
 \end{aligned}$$

where $\lambda_s(z, \tau)$ are defined as the roots of one of the equations,

$$z + \frac{1}{\Omega} \int_{\tau}^{\lambda_s(z, \tau)} v_z d\tau_2 = 0, d ,$$

such that $\tau \geq \lambda_1 > \lambda_2 > \dots > -\infty$. λ_1 then is the instant of the last collision of the electron with the boundaries. For electrons that do not collide with the boundaries $\lambda_1 = -\infty$.

By direct substitution it is easy to show that the solution to Eq. (11) is

$$u(z, \tau) = \int_{\lambda_1(z, \tau)}^{\tau} d\tau_1 \exp\left(\int_0^{\tau_1} \gamma d\tau_2\right) g\left(z + \frac{1}{\Omega} \int_{\tau}^{\tau_1} v_z d\tau_2, \tau_1\right), \quad (13)$$

where it is noted that

$$\lambda_1\left(z + \frac{1}{\Omega} \int_{\tau}^{\lambda_s} v_z d\tau_2, \lambda_s\right) = \lambda_{s+1}(z, \tau). \quad (14)$$

The solution to the Boltzmann equation (2) is then obtained for the boundary conditions (5) by using (10)

$$f = \frac{e}{\Omega} \frac{\partial f_0}{\partial \epsilon} \int_{\lambda_1(z, \tau)}^{\tau} d\tau' v_{\alpha}(\tau') E_{\alpha}\left(z + \frac{1}{\Omega} \int_{\tau}^{\tau'} v_z(\tau'') d\tau''\right) \times \exp\left[\frac{\nu - i\omega}{\Omega} (\tau' - \tau)\right]. \quad (15)$$

The solution (15) has a rather simple physical meaning. Electrons that do not collide with the boundaries of the metal are described by the distribution function of the unbounded metal. The presence of the boundary affects the distribution of the electrons whose trajectories cross at least one of the boundaries during each revolution. Because of the diffuse nature of the reflection, these electrons are "ineffective" in contributing to the current density in the metal. The electrons that do not collide with the boundary can pass through the skin layer Ω/ν times ($\Omega \gg \nu$). It is these electrons that

determine the current density in the metal.

The region in phase space occupied by the effective electrons can be obtained in the following manner. The unperturbed equations of motion of the electrons are:

$$\dot{\underline{p}} = - \frac{e}{c} \underline{v} \times \underline{H} . \quad (16)$$

The dot denotes the first derivative with respect to time. The component of motion normal to the surface is determined by

$$\dot{z} = \frac{c}{eH} \dot{p}_y , \quad (17)$$

which has a solution

$$z(\tau) = - \frac{c}{eH} p_y(\tau) + z_0 . \quad (18)$$

z_0 depends only on p_x and ϵ . The condition that the electron orbit be contained completely in the plate is

$$z_{\max} - z_{\min} < d , \quad (19)$$

where

$$z_{\max} - z_{\min} = \frac{c}{eH} (p_{y\max} - p_{y\min}) \equiv D , \quad (20)$$

(maximum and minimum with respect to τ).

For closed orbits (the only kind considered here) the motion of the electron in the yz plane is bound. $D(\epsilon, p_x)$ is the maximum dimension of the orbit along the z axis. The electrons that do not collide with the boundaries are those for which

$$z_{\min} > 0, \quad z_{\max} < d, \quad (21)$$

that is, from (21) and (18)

$$\frac{c}{eH} p_{y\max} < z_0 < d + \frac{c}{eH} p_{y\min}. \quad (22)$$

Solving (18) for z_0 and substituting into the above

$$z_1(\tau, p_x) \equiv \frac{c}{eH} (p_{y\max} - p_y) < z < d - \frac{c}{eH} (p_y - p_{y\min}) \equiv z_2(\tau, p_x). \quad (23)$$

The usual experimental arrangement is to place the metal plate in a flat coil and impose the radio-frequency field antisymmetrically on the two faces. It is simpler theoretically to consider a unilateral excitation, and this is the problem considered hereafter.

The radio-frequency size effect is concerned with the properties of the reflected wave from the metal determined by the surface impedance tensor $Z_{\alpha\beta}$ defined as

$$Z_{\alpha\beta} \equiv \frac{4\pi i\omega}{c^2} \frac{E_\alpha(0)}{E'_\beta(0)}. \quad (24)$$

$E_\alpha(0)$ and $E'_\beta(0)$ are the tangential components of the electric

field and their normal derivatives on the surface $z = 0$, respectively.

To find $Z_{\alpha\beta}$ Maxwell's equations (1) must be solved. For convenience they shall be solved in the Fourier representation. $E_\alpha(z)$ is then formally continued outside the plate as follows:

$$E_\alpha(z) = 0 \quad (z > d), \quad E_\alpha(-z) = E_\alpha(z) \quad . \quad (25)$$

The Fourier components are defined as

$$\begin{aligned} \xi_\alpha(k) &= 2 \int_0^d dz E_\alpha(z) \cos kz \\ E_\alpha(z) &= \pi^{-1} \int_0^\infty dk \xi_\alpha(k) \cos kz \quad . \end{aligned} \quad (26)$$

By multiplying (1) by $2 \cos kz dz$ and integrating from zero to d , the following is obtained:

$$\begin{aligned} k^2 \xi_\alpha(k) + 2E'_\alpha(0) - 2E'_\alpha(d) \cos kd - 2kE_\alpha(d) \sin kd \\ = 4\pi i \omega c^{-2} j_\alpha(k) \quad . \end{aligned} \quad (27)$$

Assuming that the z dependence of $E_\alpha(z)$ is of the form $\exp(i\omega z/c)$, it follows that $E'(d) = i\omega c^{-1} E_\alpha(d)$, and therefore the third term on the left side of Eq. (27) is $kc/\omega \sim c/\omega\delta$ times smaller than the fourth term. The term $2kE_\alpha(d) \sin kd$ describes the change in the electromagnetic field at the surface $z = 0$ due to reflection from the surface $z = d$. This term must be considered

if anomalous penetration of the field takes place. If the thickness d of the sample is considerably greater than the depth of penetration δ (the usual experimental condition), then this term may be neglected. In this case the influence of the finite sample thickness is manifest only in the manner in which the Fourier component of the current density $j_\alpha(k)$ is changed. If the sample thickness is much greater than the characteristic electron orbit dimension D_0 , the field distribution and surface impedance are very nearly those of a semi-infinite metal.

The Fourier component of the current density $j_y(k)$ can now be calculated. The size effect takes place by "cutoff" of the electrons with extremal diameter D . In the simplest case of a singly-connected and convex Fermi surface, the extremum of the function $D(p_x) = D_0$ is attained on the central section $p_x = 0$. The current produced by the electrons with $D(p_x) \approx D_0$ is along the y axis.

Since the frequency ν is of the order of 10^9 sec^{-1} , and the applied radio-frequency ω is of the order of 10^7 sec^{-1} , then:

$$\omega \ll \nu.$$

Thus $\gamma \rightarrow \nu/\Omega \ll 1$.

If Eq. (15) is substituted into Eq. (3) and E_y is expressed in the Fourier representation similar to Eq. (26), then the following results:

$$\begin{aligned}
j_y(k) &= \frac{2e^2}{\pi h^3} \int_{p_{x \min}}^{p_{x \max}} \frac{m}{\Omega} dp_x \theta(d-D) \int_0^{2\pi} d\tau v_y(\tau, p_x) \\
&\times \int_{z_1(\tau, p_x)}^{z_2(\tau, p_x)} dz \cos kz \int_{-\infty}^{\tau} d\tau' v_y(\tau', p_x) \exp[\gamma(\tau' - \tau)] \\
&\times \int_0^{\infty} dk' \xi_y(k') \cos k'[z + \frac{c}{eH}(p_y(\tau, p_x) - p_y(\tau', p_x))] \quad (28)
\end{aligned}$$

where

$$\theta(x) = \begin{cases} 1 & (x > 0) \\ 0 & (x < 0) \end{cases}.$$

By the use of $\theta(d-D)$ and by setting the limits of integration on z by Eq. (23), only the electrons that do not collide with the boundaries contribute to $j_y(k)$.

The integral with respect to z can be performed in elementary fashion. To simplify the calculation of the integrals with respect to τ and τ' , it shall be assumed that the projections of the electron orbits on the yz plane are circles. Then

$$v_y(\tau) = v_{\perp}(p_x) \sin \tau, \quad p_y = m(p_x) v_y(\tau, p_x). \quad (29)$$

In the limiting case as $\gamma \rightarrow 0$, Eq. (28) becomes

$$\begin{aligned}
j_y(k) &= \frac{4e^2}{h^3 v} \int_0^{\infty} dk' \frac{\xi_y(k')}{k-k'} \int_{p_{x \min}}^{p_{x \max}} dp_x m v^2 \theta(d-D) \\
&\times J_1(kR) J_1(k'R) \cdot [\sin(k-k')(d-R) - \sin(k-k')R], \quad (30)
\end{aligned}$$

where $J_n(\lambda) = \frac{1}{\pi} \int_0^\pi \cos(n\phi - \lambda \sin \phi) d\phi$ is a Bessel function and $2R = D(p_x)$.

Equation (27) can be solved for $\xi_y(k)$ by neglecting the aforementioned terms to yield:

$$\xi_y(k) = \frac{-2E'_y(0)}{k^2 - i4\pi\omega c - 2\sigma_{yy}(k)}, \quad (31)$$

where $\sigma_{\alpha\beta}(k) \equiv j_\alpha(k)/\xi_\beta(k)$.

The surface impedance Z as defined in Eq. (24) can be written as:

$$Z \equiv R + iX = \frac{4i\omega}{c^2 E'_y(0)} \int_0^\infty dk \xi_y(k) \quad (32)$$

because $E_y(0) = \frac{1}{\pi} \int_0^\infty dk \xi_y(k) \cos k0$.

It is not the surface impedance that is measured in the radio-frequency size effect, but rather the first derivative of Z with respect to the static magnetic field H . Since the line width is small ($\Delta H/H \sim \delta/d$), and $k_c = \frac{e}{\hbar c} Hd$ (k_c is constant for a given line), the derivative of Z with respect to H is the same (within a constant factor) as the derivative with respect to d , the thickness of the plate. Therefore,

$$Z' \equiv \frac{\partial Z}{\partial d} = \frac{4i\omega}{c^2 E'_y(0)} \int_0^\infty dk \frac{\partial \xi_y(k)}{\partial d}. \quad (33)$$

By differentiating the simplified Eq. (27) with respect to d , and using expression (30) for $j_y(k)$, the following result is obtained:

$$(k^2 - i \frac{4\pi\omega}{c^2} \sigma_{yy}(k)) \frac{\partial \xi_y(k)}{\partial d} = \frac{4\pi i \omega}{c^2} \frac{4e^2}{h^3 v}$$

$$\times \int_{p_{x \min}}^{p_{x \max}} dp_x m v^2 \theta(d - D)$$

$$\times \int_0^\infty dk' \xi_y(k') \cos[(k-k')(d-R)] J_1(kR) J_1(k'R) . \quad (34)$$

By solving Eq. (34) for $\partial \xi_y(k)/\partial d$, substituting this into Eq. (33), and using Eq. (31), one obtains, assuming even symmetry in p_x :

$$Z' = \left(\frac{4\omega}{c^2}\right)^2 \frac{4\pi e^2}{h^3 v [E'_y(0)]^2} \int_0^{p_{x \max}} dp_x m v^2 \theta(d-D) B^2 , \quad (35)$$

where

$$B = 2 \int_0^\infty dk \xi_y(k) \sin k(d-R) J_1(kR) . \quad (36)$$

Near a radio-frequency size effect line, $R \sim \frac{d}{2}$, thus $d-R \sim R$. In the region of small k where $J_1(kR)$ differs markedly from its asymptotic form the factor $\sin k(d-R)$ is small. Therefore the integral is not significantly altered by replacing $J_1(kR)$ by its asymptotic form

$$J_1(kR) \rightarrow \sqrt{\frac{2}{\pi k R}} \sin(kR - \frac{\pi}{4}) .$$

Using the trigonometric identity for the product of two sine functions, B becomes:

$$B = -\left(\frac{2}{\pi R}\right)^{1/2} \int_0^{\infty} \frac{dk}{k^{1/2}} \xi_y(k) \left\{ \sin[k(d-D) - \frac{\pi}{4}] + \sin(kd + \frac{\pi}{4}) \right\} \quad (37)$$

The second term in curly brackets is a rapidly oscillating term, whereas the first term changes more slowly over the limits of the line ($|d-D| \sim 1/k$). The second term will then be neglected. This yields an expression for the surface impedance of:

$$Z'(d-D_0) = \left(\frac{4\omega}{c^2}\right)^2 \frac{8\pi e^2}{h^3 v [E'_y(0)]^2} \int_0^{p_x \max} dp_x \frac{mv^2}{R} \times \theta(d-D) \psi^2(d-D) \quad (38)$$

where

$$\psi(q) = -\frac{1}{\sqrt{\pi}} \int_0^{\infty} \frac{dk}{k^{1/2}} \xi_y(k) \sin[kq - \frac{\pi}{4}] \quad (39)$$

If $\xi_y(k)$ is converted back to the coordinate representation in Eq. (39), then

$$\psi(q) = -\frac{2}{\sqrt{\pi}} \int_0^d dz E_y(z) \int_0^{\infty} \frac{dk}{k^{1/2}} \cos kz \sin(kq - \frac{\pi}{4}) \quad (40)$$

The integral over k can be computed by writing:

$$\begin{aligned} \cos kz \sin(kq - \frac{\pi}{4}) &= \frac{1}{2\sqrt{2}} [\sin k(z+q) - \sin k(z-q) \\ &\quad - \cos k(z+q) - \cos k(z-q)] . \end{aligned}$$

The integral over k is then

$$\int_0^{\infty} \frac{dk}{k^{1/2}} \cos kz \sin(kq - \frac{\pi}{4}) = \begin{cases} -\frac{\sqrt{\pi}}{2} \frac{1}{\sqrt{z-q}} & (z > q) \\ 0 & (z < q) \end{cases} \quad (41)$$

Thus

$$\psi(q) = \int_q^d dz \frac{E_y(z)}{(z-q)^{1/2}} . \quad (42)$$

Expressions (38), (39), and (31) contain the solution to the line shape of the radio-frequency size effect. The integrations cannot be carried through in general form, thus some simplified field distribution must be assumed. The electric field will be assumed to be of the form:

$$E_y(z) = E_y'(0) k_{\text{eff}}^{-1} \exp(-k_{\text{eff}} z) , \quad (43)$$

where the effective damping decrement k_{eff} is given by:

$$-i \frac{4\pi\omega}{c^2} \sigma(k_{\text{eff}}) = k_{\text{eff}}^2 , \quad \text{or} \quad k_{\text{eff}} = \delta^{-1} \exp(-\frac{\pi i}{6}) . \quad (44)$$

Substitution of Eq. (43) into Eq. (39) yields:

$$\psi^2(d-D) = \pi [E'_y(0)]^2 k_{\text{eff}}^{-3} \exp[-2k_{\text{eff}}(d-D)] , \quad (45)$$

where the approximation

$$\int_0^D \frac{\exp(-k_{\text{eff}} u)}{u^{1/2}} du \approx \int_0^\infty \frac{\exp(-k_{\text{eff}} u)}{u^{1/2}} du$$

has been made.

Size Effect for Maximum Diameter

The function $D(p_x)$ will be expanded in powers of p_x about the point $p_x = 0$ where $D(p_x)$ is a maximum as follows:

$$D(p_x) = D_{\text{max}} + \frac{1}{2} D'' \Big|_{p_x=0} p_x^2 , \quad (46)$$

$(\partial D / \partial p_x \Big|_{p_x=0} = 0)$. Because $D(0)$ is a maximum, $D'' \Big|_{p_x=0}$ is negative.

By substituting Eqs. (45), and (46) into Eq. (38) and assuming that $p_{x \text{ max}}$ is large, one obtains for the strong field side of the line ($D_{\text{max}} < d$):

$$Z^1(d-D_{\text{max}}) = C \exp[-2k_{\text{eff}}(d-D_{\text{max}})] , \quad (47)$$

where

$$C = \left(\frac{8\pi\omega}{c^2}\right)^2 \frac{\sqrt{\pi} e^2}{h^3 v k_{\text{eff}}^{7/2}} \left(\frac{mv^2}{R |D''|^{1/2}}\right)_{p_x=0} . \quad (48)$$

On the weak field side of the line ($d < D_{\max}$) the step function $\theta(d-D)$ effectively changes the limits of integration over p_x to

$$\sqrt{\frac{2(d-D_{\max})}{D''|_{p_x=0}}} \equiv \mu < p_x < p_{x \max} .$$

This is done by subtracting the integration from zero to μ from Eq. (47) in the following manner:

$$Z'(d-D_{\max}) = C \exp[-2k_{\text{eff}}(d-D_{\max})] \times \{1 - \Phi(\sqrt{2k_{\text{eff}}(D_{\max}-d)})\} , \quad (49)$$

where

$$\Phi(z) = 2\pi^{-1/2} \int_0^z dq \exp(-q^2) . \quad (50)$$

Size Effect for Minimal Diameter

In this case $D''|_{p_x=0}$ is positive. On the strong field side of the line ($d > D_{\min}$) the step function $\theta(d-D)$ limits the integration of p_x from zero to $[2(d-D_{\min})/D''|_{p_x=0}]^{1/2}$, thus

$$Z'(d-D_{\min}) = Cw(\sqrt{2k_{\text{eff}}(d-D_{\min})}) , \quad (51)$$

where

$$w(z) = 2\pi^{-1/2} \int_0^z \exp(q^2 - z^2) dq \quad . \quad (52)$$

On the weak field side ($d < D_{\min}$), $Z' = 0$ because all electrons collide with the boundary.

The real and imaginary parts of Z' are plotted in Fig. 20. It will be noted that the line for a maximum caliper is sharper on the high field side, while the line for a minimum caliper is sharper on the low field side. This suggests a method for differentiating between a maximum and a minimum caliper for circular orbits.

(a)

(b)

Fig. 20

APPENDIX II

Experimental Calipers for Cadmium

Obtained Using the Radio-Frequency Size Effect

The three samples used to obtain the calipers had a room temperature thickness of:

$\underline{n} \parallel [0001]$	0.401 mm ,
$\underline{n} \parallel [10\bar{1}0]$	0.499 mm ,
$\underline{n} \parallel [11\bar{2}0]$	0.5395 mm .

After correction was made for the thermal contraction, the calipers could be obtained using the formulas:

$\underline{n} \parallel [0001]$	$k_c = 0.00600H (\text{\AA}^{-1})$,
$\underline{n} \parallel [10\bar{1}0]$	$k_c = 0.007552H (\text{\AA}^{-1})$,
$\underline{n} \parallel [11\bar{2}0]$	$k_c = 0.008164H (\text{\AA}^{-1})$.

Tables III, IV, V, VI, VII, VIII, IX, and X contain the caliper values obtained in the experiment with the exception of the constant $k_c = 1.560 \text{\AA}^{-1}$ caliper for the "lens" with $\underline{n} \parallel [0001]$.

TABLE III. Caliper values obtained for $n \parallel [0001]$. The magnetic field was rotated in the (0001) plane.

Angular Direction of Magnetic Field	k_c ($\text{\AA}^{-1}$)			
	(A)		(B)	(C)
$[11\bar{2}0]$	0.540		0.564	0.327
1°	0.529	0.544	0.556	0.327
2°	0.525	0.551		0.329
3°	0.521	0.557		0.330
4°	0.515	0.565	0.545	0.329
5°	0.507	0.575	0.540	0.332
6°	0.503	0.584	0.536	0.330
7°	0.501	0.590	0.533	0.333
8°	0.498	0.600	0.525	0.333
9°	0.498	0.611	0.522	0.339
10°	0.491	0.615		0.341
11°	0.489	0.635	0.515	0.341
12°	0.485	0.647	0.513	0.342
13°	0.485	0.660	0.507	0.348
14°	0.482	0.674	0.506	0.353
15°	0.482	0.689	0.503	0.356
16°	0.480	0.707	0.500	0.360
17°	0.479	0.720	0.500	0.366
18°	0.479		0.493	0.371
19°	0.477		0.494	0.375
20°	0.477		0.489	0.381
21°	0.477		0.488	0.381
22°	0.476		0.489	0.392
23°	0.476			0.408
24°	0.476			0.417
25°	0.476			0.432
26°	0.474			
27°	0.474			
28°				
29°	0.476			
$[10\bar{1}0]$	0.476			

TABLE IV. Caliper values obtained for $n \parallel [0001]$. The magnetic field was rotated in the (0001) plane.

Angular Direction of Magnetic Field	k_c ($\text{\AA}^{-1}$)			
	(D)	(E)	(F)	(G)
$[11\bar{2}0]$		0.287	0.300	0.176
1°		0.288	0.300	
2°		0.289	0.304	0.177
3°		0.290	0.304	0.175
4°		0.290	0.306	
5°		0.291	0.300	0.176
6°		0.293	0.300	0.262
7°		0.296	0.302	0.178
8°		0.295		0.175
9°		0.299		
10°		0.299		0.177
11°	0.774	0.303		0.242
12°	0.773	0.307		0.179
13°	0.761	0.309	0.297	0.176
14°	0.750	0.314	0.296	
15°	0.741	0.317	0.299	0.175
16°	0.741	0.323	0.294	0.226
17°	0.731	0.326	0.293	0.182
18°	0.728	0.330	0.294	0.183
19°	0.725	0.336	0.293	0.222
20°	0.719	0.344	0.294	0.185
21°	0.716	0.350	0.294	0.214
22°	0.711		0.294	0.187
23°	0.708		0.297	0.210
24°	0.704		0.293	0.187
25°	0.702		0.292	0.190
26°	0.700		0.291	0.209
27°	0.699			0.191
28°	0.699			0.193
29°	0.699			0.206
$[10\bar{1}0]$	0.702			0.198
				0.196

TABLE V. Caliper values obtained for $n \parallel [0001]$. The magnetic field was rotated in the (0001) plane.

Angular Direction of Magnetic Field	k_c ($\text{\AA}^{-1}$)			
	(H)	(I)	(J)	(K)
$[11\bar{2}0]$	0.453	0.314	0.360	
1°	0.443	0.462	0.313	0.360
2°	0.435	0.470	0.313	0.359
3°	0.434	0.477	0.314	0.359
4°	0.429	0.483	0.315	0.355
5°	0.425	0.491	0.314	
6°	0.423	0.503	0.314	
7°	0.419	0.510	0.314	
8°	0.414	0.522	0.314	
9°	0.414	0.528	0.314	
10°	0.407	0.534	0.320	
11°	0.404	0.546	0.323	
12°	0.401	0.552	0.325	
13°	0.401	0.566	0.330	
14°	0.399	0.582	0.336	
15°	0.398	0.597	0.341	
16°	0.396	0.614	0.344	
17°	0.396	0.626	0.350	
18°	0.395	0.644	0.356	
19°	0.392	0.659	0.363	
20°	0.389	0.672	0.371	
21°	0.388		0.377	
22°	0.387			0.505
23°	0.386			0.515
24°	0.384			0.523
25°	0.384			0.540
26°	0.383			0.555
27°	0.384			0.576
28°	0.384			0.596
29°	0.384			
$[10\bar{1}0]$	0.384			

TABLE VI. "Lens" caliper values obtained for $\eta \parallel [10\bar{1}0]$. The magnetic field was rotated in the $(10\bar{1}0)$ plane.

Angular Direction of Magnetic Field	k_c ($\text{\AA}^{-1}$)	Angular Direction of Magnetic Field	k_c ($\text{\AA}^{-1}$)
[0001]	1.516	42°	0.744
1°	1.514	44°	0.729
2°	1.503	46°	0.706
3°	1.495	48°	0.693
4°	1.482	50°	0.680
5°	1.460	52°	0.663
6°	1.435	54°	0.653
7°	1.405	56°	0.638
8°	1.382	58°	0.631
9°	1.352	60°	0.623
10°	1.322	62°	0.608
12°	1.269	64°	0.604
14°	1.220	66°	0.603
16°	1.171	68°	0.591
18°	1.123	70°	0.583
20°	1.076	72°	0.574
22°	1.038	74°	0.572
24°	0.999	76°	0.566
26°	0.963	78°	0.565
28°	0.927	80°	0.561
30°	0.891	82°	0.557
32°	0.865	84°	0.555
34°	0.838	86°	0.553
36°	0.812	88°	0.553
38°	0.785	[11 $\bar{2}$ 0]	0.553
40°	0.767		

TABLE VII. Caliper values obtained for $\underline{n} \parallel [10\bar{1}0]$. The magnetic field was rotated in the $(10\bar{1}0)$ plane.

Angular Direction of Magnetic Field	k_c ($\text{\AA}^{-1}$)			
	(L)	(M)	(N)	(P)
[0001]	0.268	0.310	0.532	
2°	0.266	0.308	0.531	
4°	0.264	0.308	0.531	
6°	0.261	0.308	0.532	
8°	0.261	0.310	0.531	
10°	0.262	0.312	0.532	
12°	0.261	0.313	0.536	
14°	0.261	0.313	0.536	
16°	0.261		0.540	0.925
18°	0.262		0.544	0.903
20°	0.264		0.544	0.880
22°	0.262		0.549	0.869
24°	0.266		0.555	0.861
26°	0.270		0.561	
28°	0.268		0.565	0.846
30°	0.274		0.572	0.833
32°	0.274		0.578	0.823
34°	0.281		0.583	0.826
36°	0.287		0.593	
38°			0.604	0.819
40°			0.610	0.818
42°	0.291		0.623	0.819
44°				0.819

TABLE VII continued

Angular Direction of Magnetic Field	k_c ($\text{\AA}^{-1}$)		
	(L)	(P)	(Q)
46°	0.306	0.821	
48°		0.827	
50°	0.319	0.836	
52°		0.840	1.643
54°	0.336	0.853	1.641
56°		0.863	1.646
58°	0.346	0.876	1.650
60°		0.892	1.652
62°	0.355	0.908	1.661
64°		0.933	1.673
66°	0.374	0.955	1.690
68°		0.982	1.711
70°	0.400	1.012	1.737
72°		1.050	1.771
74°	0.431		1.813
76°			1.862
78°	0.476		1.918
80°			1.994

TABLE VIII. "Lens" caliper values obtained for $n \parallel [11\bar{2}0]$.The magnetic field was rotated in the $(11\bar{2}0)$ plane.

Angular Direction of Magnetic Field	k_c ($\text{\AA}^{-1}$)	Angular Direction of Magnetic Field	k_c ($\text{\AA}^{-1}$)
[0001]	1.563	42°	0.747
1°	1.556	44°	0.733
2°	1.560	46°	0.708
3°	1.542	48°	0.688
4°	1.517	50°	0.674
5°	1.493	52°	0.657
6°	1.460	54°	0.657
7°	1.442	56°	0.638
8°	1.411	58°	0.631
9°	1.383	60°	0.618
10°	1.356	62°	0.610
12°	1.289	64°	0.605
14°	1.226	66°	0.594
16°	1.172	68°	0.592
18°	1.125	70°	0.583
20°	1.086	72°	0.574
22°	1.038	74°	0.578
24°	1.003	76°	0.574
26°	0.967	78°	0.567
28°	0.929	80°	0.565
30°	0.894	82°	0.555
32°	0.870	84°	0.559
34°	0.847	86°	0.557
36°	0.815	88°	0.555
38°	0.792	[10 $\bar{1}$ 0]	0.559
40°	0.772		

TABLE IX. Caliper values obtained for $\underline{n} \parallel [11\bar{2}0]$. The magnetic field was rotated in the $(11\bar{2}0)$ plane.

Angular Direction of Magnetic Field	k_c ($\text{\AA}^{-1}$)				
	(T)	(Y)	(W ₁)	(W ₂)	(W ₃)
[0001]	0.347				
2°	0.351				
4°	0.347				
6°	0.353				
8°	0.357				
10°	0.365		1.104	0.925	
12°	0.367		1.063	0.886	
14°	0.374	1.963	1.027	0.867	0.704
16°	0.380		1.002	0.837	0.686
18°	0.390		0.982	0.823	0.672
20°	0.400	1.820	0.959	0.810	0.665
22°	0.412		0.935	0.798	0.645
24°	0.429		0.925	0.784	0.645
26°	0.443		0.912	0.774	0.635
28°	0.461		0.898	0.765	0.625
30°	0.480	1.715		0.759	0.621
32°	0.506			0.751	0.621
34°	0.535			0.749	0.616
36°	0.569			0.743	0.616
38°	0.614		0.861	0.737	
40°	0.678		0.861	0.743	0.616
42°			0.867		0.616
44°		1.670	0.867		0.623

TABLE IX continued

Angular Direction of Magnetic Field		$k_c \text{ (Å}^{-1}\text{)}$		
	(x)	(w ₁)	(w ₂)	(w ₃)
46°		0.870		0.629
48°		0.874		0.618
50°		0.880	0.761	0.641
52°		0.890	0.770	0.651
54°		0.902	0.784	
56°		0.914	0.792	
58°		0.923	0.804	0.682
60°		0.939	0.821	0.706
62°		0.963	0.841	0.727
64°		0.992	0.870	0.761
66°		1.018	0.894	0.790
68°		1.057	0.937	0.839
70°		1.092	0.978	0.884
72°			1.039	0.965
74°			1.108	
76°			1.225	
78°				
80°	0.804			
81°	0.816			
82°	0.847			
83°	0.880			
84°	0.912			
85°	0.933			
86°	0.980			
87°	1.023			

TABLE X. Caliper values obtained for $\underline{n} \parallel [11\bar{2}0]$. The magnetic field was rotated in the $(11\bar{2}0)$ plane.

Angular Direction of Magnetic Field	k_c ($\text{\AA}^{-1}$)			
	(R)	(S)	(U)	(V)
[0001]	0.180	0.294		
2°	0.180	0.296		
4°	0.180	0.298		
6°	0.180	0.298		
8°	0.180	0.302		
10°	0.182	0.302		
12°	0.182	0.304		
14°	0.184	0.308		
16°	0.184	0.310		
18°	0.186	0.314		
20°	0.188	0.320		
22°	0.190	0.320		
24°	0.192	0.325		
26°	0.196	0.327		0.351
28°	0.198	0.327		0.359
30°	0.200	0.327		0.369
32°	0.204	0.327	0.369	0.384
34°	0.210	0.331	0.376	0.388
36°	0.208	0.331	0.382	0.406
38°	0.218	0.337	0.390	0.420

TABLE X continued

Angular Direction of Magnetic Field	k_c ($\text{\AA}^{-1}$)			
	(R)	(S)	(U)	(V)
40°	0.222	0.339	0.400	0.439
42°	0.229	0.343	0.414	0.463
44°	0.237	0.347	0.429	0.490
46°	0.245	0.351	0.441	0.523
48°	0.253	0.357	0.461	0.574
50°	0.263	0.364	0.486	0.641
52°	0.280	0.366	0.510	
54°	0.282	0.380	0.545	
56°	0.292	0.382	0.584	
58°	0.310	0.391		
60°	0.335	0.398		
62°		0.410		
64°		0.424		
66°		0.434		
68°		0.449		
70°		0.463		
72°		0.480		
74°		0.496		
76°		0.523		
78°		0.549		

REFERENCES

1. V. F. Gantmakher, Zh. Eksperim. i Teor. Fiz. 42, 1416 (1962) [English transl.: Soviet Phys.—JETP 15, 982 (1962)]; Zh. Eksperim. i Teor. Fiz. 43, 345 (1962) [English transl.: Soviet Phys.—JETP 16, 247 (1962)]; Zh. Eksperim. i Teor. Fiz. 44, 811 (1963) [English transl.: Soviet Phys.—JETP 17, 549 (1963)]; V. F. Gantmakher and E. A. Kaner, Zh. Eksperim. i Teor. Fiz. 45, 1430 (1963) [English transl.: Soviet Phys.—JETP 18, 988 (1964)]; V. F. Gantmakher and I. P. Krylov, Zh. Eksperim. i Teor. Fiz. 49, 1054 (1965) [English transl.: Soviet Phys.—JETP 22, 734 (1966)]; I. P. Krylov and V. F. Gantmakher, Zh. Eksperim. i Teor. Fiz. 51, 740 (1966) [English transl.: Soviet Phys.—JETP 24, 492 (1967)].
2. A. D. C. Grassie, Phil. Mag. 9, 847 (1964).
3. D. C. Tsui and R. W. Stark, Phys. Rev. Letters 16, 19 (1966).
4. J. K. Galt, F. R. Merritt and J. N. Klauder, Phys. Rev. 139, A823 (1965).
5. M. P. Shaw, T. G. Eck, and D. A. Zych, Phys. Rev. 142, 407 (1966).
6. D. F. Gibbons and L. M. Falicov, Phil. Mag. 8, 117 (1963).
7. M. R. Daniel and L. Mackinnon, Phil. Mag. 8, 537 (1963).
8. F. H. S. Chang, PhD dissertation, The University of Texas, 1966 (unpublished).
9. R. G. Goodrich and R. C. Jones, Phys. Rev. 156, 745 (1967).

10. W. A. Harrison, Phys. Rev. 118, 1190 (1960).
11. J. G. Anderson and W. F. Love, Bull. Am. Phys. Soc. 8, 258 (1963).
12. W. A. Harrison, Phys. Rev. 126, 497 (1962).
13. T. G. Berlincourt, Phys. Rev. 94, 1172 (1954).
14. A. Fukumoto and M. W. P. Strandberg, Phys. Letters 23, 200 (1966).
15. B. C. Deaton, PhD dissertation, The University of Texas, 1962 (unpublished).
16. E. Grüneisen and E. Goens, Z. Phys. 29, 141 (1924).
17. W. D. Knight, Rev. Sci. Instr. 32, 95 (1961).
18. E. A. Kaner and V. L. Fal'ko, Zh. Eksperim. i Teor. Fiz. 51, 586 (1966) [English transl.: Soviet Phys.—JETP 24, 392 (1967)].
19. J. M. Ziman, Principles of the Theory of Solids (Cambridge University Press, Cambridge, England, 1964), p. 179.
20. M. Ya. Azbel' and E. A. Kaner, Zh. Eksperim. i Teor. Fiz. 32, 837 (1957) [English transl.: Soviet Phys.—JETP 5, 685 (1957)].

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