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Cyclotron Resonance in Thallium.

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CYCLOTRON RESONANCE IN THALLIUM

A Dissertation

Submitted to the Graduate Faculty of the
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requirements for the degree of
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ABSTRACT

The results of cyclotron resonance measurements in high purity samples of thallium are reported. The measurements were performed at a microwave frequency of 34 GHz in the Azbel-Kaner geometry with the effective mass values being calculated from the periodicity of the subharmonic resonance series observed. The anisotropy of the effective masses in all three principal planes was measured and the various mass branches are assigned to orbits occurring on both parts of the third zone, the fourth zone and the fifth zone sheets of the Fermi surface.

Effective mass values are assigned to both portions of the third band. The data for the third zone "crown" agrees well with the known Fermi surface with a value of $m^*/m_0 = .65 \pm .02$ for $\underline{H}$ in the basal plane and $m^*/m_0 = .85$ for $\underline{H}$ along $[0001]$ in the $(11\bar{2}0)$ plane. No accurate description of the smaller section of the third zone exists, but mass values are assigned to this section from the free electron description of this portion of the Fermi surface and the angular dependence of the mass values. Mass values are assigned to several different orbits on the fourth zone, with resonances due to central orbits on the arms dominating the cyclotron resonance. The mass value observed for $\underline{H}$ along $[10\bar{1}0]$ in the basal plane is $m^*/m_0 = 0.5$ for this central orbit. The existence of the three "posts" in the corner of the honeycomb

network of the fourth zone complicates the assignment of orbits on other sections of the fourth zone. Mass values are also assigned to the fifth zone, with the values observed in the basal plane being nearly isotropic.

Considerable effects due to magnetic breakdown are observed and a discussion of these effects is given. The existence of breakdown complicates the resonances and prevents the resolution of some orbits that are expected.

A theoretical treatment of cyclotron resonance is presented by solving the Boltzmann transport equation in conjunction with Maxwell's equations of electrodynamics. Results are obtained for both the longitudinal and transverse case of cyclotron resonance. A description of the computer program used as an aid in analyzing the data is contained at the end of the paper.

1. INTRODUCTION

This paper reports the results of a cyclotron resonance experiment in the Azbel'-Kaner¹ geometry on high purity single crystals of thallium. The band structure and Fermi surface (FS) of Tl has been shown from theoretical considerations to bear a close resemblance to the free-electron model with the departures from this model being due to effects of the finite crystal potentials and spin-orbit coupling.² Prior experimental studies³⁻⁸ of the FS of thallium have shown general agreement with this hypothesis. The present measurements were performed to supplement the previous knowledge of the FS of Tl by providing information about the anisotropy of the effective mass of the carriers in the different energy bands as a function of crystallographic directions.

Magnetoresistance measurements^{3,4} have provided qualitative information about the topology of the thallium FS and about the energy gaps in the band structure. Details of the structure of the FS have been reported from studies of the magnetoacoustic effect^{5,6,7} and the de Haas-van Alphen effect.⁸ A theoretical study of the FS of thallium was performed by Soven² who used a relativistic modification of the orthogonalized-plane-wave method to obtain the energy band structure. The results of these experiments and the energy band calculation show that the FS of thallium consists of six nonequivalent sheets spread throughout

the third through sixth zone. The two sheets in the third zone form simple closed surfaces bounding unoccupied regions of the Brillouin zone. The larger of these has its center of symmetry at the point A of the Brillouin zone and has roughly the shape of a hexagonal "crown" as shown in Fig. 1. The smaller portion of the third zone sheet consists of six elongated surfaces centered at the point M and extending along the line MK in the Brillouin zone as shown in Fig. 2. The fourth zone sheet (Fig. 3) is a honeycomb-like network spanning the rectangular faces of the Brillouin zone and bounding occupied regions of momentum space. The fifth zone (Fig. 4) and sixth zone consist of pockets of electrons with symmetry around $\underline{H}$ and extended in the HK direction.

Since its prediction by Azbel' and Kaner (AK),¹ cyclotron resonance has been used extensively in the investigation of the FS of metals. In this type of experiment a static magnetic field $\underline{H}$ is applied parallel to a plane surface of a metal single crystal. An rf electric field is then applied either parallel or perpendicular to $\underline{H}$ in the plane surface of the sample depending on the direction of the velocity of the carriers to be observed. The measurements are performed under extreme anomalous skin effect⁹ conditions where the mean free path and orbit radii of the carriers are much larger than the skin depth. The static magnetic field causes the carriers to move in helical orbits passing through the skin depth where they may absorb energy from the rf field several times. This motion is similar to that of an electron in a cyclotron with a single dee with resonance occurring when the rf frequency is a

Fig. 1

Fig. 2

Fig. 3

Fig. 4

multiple of the cyclotron frequency $\omega_c = eH/m^*c$, where m^* is the effective mass of the electron. If the magnetic field is not applied parallel to the surface of the sample or if the surface is rough, the resonant series is attenuated or under some conditions splitting of the series will occur.¹⁰

The results of the several calculations (see Appendix A) of the surface impedance of a metal subjected to the above conditions^{1,11-13} show that both the absorptive or real part and the dispersive or imaginary part of the surface impedance are periodic in $1/H$ and that this period is related to the effective mass m^* of the carriers as follows

$$m^* = e^2 \omega_c \Delta(1/H) \quad , \quad (1)$$

where $\Delta(1/H)$ is the period of the oscillating surface impedance, ω is the angular rf frequency, e is the electronic charge, and c is the speed of light. A knowledge of the effective mass of a carrier yields the rate of change of the size of a given section of the FS with carrier energy E since

$$m^* = \frac{\hbar^2}{2\pi} \left(\frac{\partial A}{\partial E} \right)_{E_f} \quad , \quad (2)$$

where A is the cross-sectional area normal to the applied magnetic field of the relevant section of the FS, $\hbar$ is Planck's constant divided by 2π , and E_f is the Fermi energy.

In the following sections the results of the measurement of effective masses as a function of crystallographic orientation in the three principal planes of thallium are presented. Where possible, these mass values are assigned to orbits on a section of the FS. Utilization of Soven's calculations² and the available de Haas-van Alphen⁸ and geometric resonance⁵⁻⁷ data are made in the assignment of these orbits. In Appendix A, a theoretical calculation of the surface impedance of a metal is performed by solving the Boltzmann transport equation in conjunction with Maxwell's equations. Appendix B describes in detail the computer program used to analyze the experimental data.

II. EXPERIMENT

The samples used in this experiment were prepared from a zone-refined bar of thallium metal obtained from Cominco Products Incorporated with a quoted impurity content of 1 ppm. The strain-anneal technique of crystal growth was adopted to produce single crystals.¹⁴ A bar of thallium two inches long was cooled to 77°K and placed in a vise while still at that temperature. The strain was induced by thermal expansion of the bar during warming to 300°K. After the bar reached room temperature it was placed in an oven and annealed for fifteen days at 493°K which is 10°K below the phase transition from the hcp phase to the bcc phase for thallium. After this annealing process the bar contained three large crystals from which two oriented samples for each principal plane were cut by spark erosion into rectangular parallelepipeds approximately 11 mm by 11 mm by 3 mm. Orientation of these samples to within $\pm 1^\circ$ was obtained by use of Laue back diffraction x-ray techniques. After orienting and cutting, one surface of each of the samples was spark planed and the samples were stored in glycerin to prevent oxidation. Samples prepared in this manner gave a resistivity ratio $R_{300^\circ\text{K}}/R_{4.2^\circ\text{K}}$ of 4000.

The surface upon which the measurements were performed was made flat and smooth by a chemical polishing technique in which the planed surface of the crystal was lightly lapped on a smooth

glass surface submersed in a solution of 4 parts glacial acetic acid to 1 part 30% hydrogen peroxide. When a satisfactory polish was achieved, the crystal was immediately flooded with anhydrous glycerin to prevent oxidation. With a thin film of glycerin remaining on the crystal, it was attached to the sample holder and placed in the dewar assembly. The dewar was then evacuated and pre-cooling procedures were begun. Attempts at placing the crystal in liquid nitrogen immediately after polishing to prevent oxidation proved unsatisfactory because the polishing solution remaining on the surface would freeze and strain the surface.

An attempt was made to electropolish the samples using a technique originally developed for use on rare earth metals.* This method did not yield as smooth a surface as the chemical lapping method described above. In order to prevent oxidation of the chemically polished surfaces the polishing of the crystals was attempted in an inert atmosphere. The glove box that was used was equipped with a pump that circulated the helium gas from the glove box through heated copper filings and a charcoal trap operating at liquid nitrogen temperatures. An inert atmosphere was achieved in this manner after the circulating pump was in operation for several hours. However, when the chemical polishing was started, the circulating pump could not remove all of the acetic acid vapors, and consequently the atmosphere inside the glove box

*See R. G. Goodrich, PhD Thesis, University of California, Riverside and references cited therein.

contained a considerable percentage of acetic acid. The small screws used on the sample holder and the fragility of the sample caused the attaching of the crystal to the sample holder and the transferring of it to the dewar assembly from the glove box to be a problem.

The measurements were performed at 1.2°K and at a microwave frequency of approximately 34 GHz. The derivative with respect to H of the surface impedance of the samples vs $1/H$ was recorded by a standard reflection spectrometer utilizing field modulation and phase sensitive detection. A block diagram of the experimental apparatus appears in Fig. 5. Magnetic fields up to 13.5 kG were provided by a 12 inch low impedance iron magnet system equipped with a $1/H$ sweep unit.

The sample formed one wall of a rectangular resonant cavity operating in the TE_{103} mode. The design of a sample holder for cyclotron resonance experiments requires the following degrees of freedom: (1) orientation of the magnetic field with respect to the crystal axes in the plane of the sample surface, (2) small-angle tipping of the magnetic field with respect to the sample surface, and (3) the polarization of the rf current with respect to the magnetic field. Two different designs were used, one allowing the sample to be rotated in a horizontal plane while the other sample holder rotated the sample in a vertical plane. Each design allowed for the variation of the angle between the magnetic field and the crystallographic axes by a gear arrangement while keeping the rf polarization constant with respect to the magnetic field. With the sample placed horizontally, the magnet could be

MICROWAVE CIRCUIT

Fig. 5

rotated about a vertical axis to align $\underline{H}$ either parallel or perpendicular to the rf current. With this design only small angle field tipping studies ($\pm 2^\circ$) could be performed by adjustment of the leveling screws on the magnet. With the sample placed vertically, field tipping studies could be performed at any desired angle by rotating the magnet about its vertical axis. However, for this design, $\underline{H}$ could only be aligned perpendicular to the rf current.

The analysis of the data was based upon the periodicity in $1/H$ of the subharmonic resonances. Recorder traces of the derivative of the real part of the surface impedance dR/dH versus $1/H$ were obtained from the experiments. In order to obtain sufficient resolution of the different mass series separate overlapping curves were taken at low fields and high fields. Inverse magnetic field values were assigned to each well resolved resonance maximum, and all values from a given orientation of crystal axis with respect to the steady magnetic field were punched onto cards for analyzing by an IBM 1620 computer. (See Appendix B for a description and listing of the computer program.) The computer was programmed to pick out all possible mass series consisting of three or more points regardless of the amplitude of the individual resonances. Series with some subharmonics missing were accepted by the program. For a given orientation, true mass series were then chosen by consulting the amplitude dependence of the recorder tracings and referring to tracings at adjacent orientations. For each mass series so obtained, the computer was also programmed to perform a least squares fit to a straight line to obtain its slope,

intercept and standard deviation, and from the calculated slope the effective mass was obtained. In addition, the phase shift or axis intercept for infinite field was calculated. The effective mass values with error flags determined by the standard deviation from the least squares fit (when the error was larger than the symbol) were then plotted vs angle by a Calcomp Plotter. Thus, the error flags shown in Figs. 7, 8, 9, and 10 refer only to the accuracy with which the points fit a straight line. Other sources of experimental error such as frequency measurement, magnet calibration, etc. cause an uncertainty of from $\pm 2\%$ to $\pm 5\%$ for each data point.

III. RESULTS AND DISCUSSION

Effective mass values as a function of crystallographic directions were investigated for the three principal planes with the rf current polarized both perpendicular and parallel to $\underline{H}$. The effect of small angle tipping of the magnetic field out of the surface of the samples was investigated for several orientations of the magnetic field in the crystals. For the orientations where the field tipping was investigated, the only effect was an attenuation of the signal to noise ratio as the field was tipped out of the surface up to $\pm 2^\circ$. Peak splitting of the mass series was not observed as in previous investigations¹⁰ of field tipping on other metals with a less complicated FS. The present behavior has been found previously in metals having complicated Fermi surfaces where only a small fraction of the carriers are contributing to the resonant part of the surface impedance.¹⁵

A. (0001) Plane

The signals for the (0001) plane are the strongest observed with many subharmonics occurring for most of the mass series found. This plane was the only one for which the signal to noise ratio was sufficiently large to enable the data taken with the rf current parallel to the applied field to be analyzed. The stronger signals for this orientation may be in part due to the fact that a

better polish could be obtained for this surface than for surfaces parallel to the other principal planes. An example of the resonance data for this plane is shown in Fig. 6. In Fig. 7 the measured cyclotron effective masses as a function of magnetic field orientation are displayed for j_{rf} perpendicular to H while Fig. 8 gives the measured effective masses for j_{rf} parallel to H .

The series of mass values labeled a and c in Fig. 7 are assigned to orbits centered at the point L on the "arms" of the fourth zone (Fig. 3) hexagonal mesh. The orbit that occurs for H along $[10\bar{1}0]$ is shown by the equivalent orbits lying in Sections $C'-C'$ and $C-C$ in Fig. 3. The resonances labeled A_i in Fig. 6 are due to this orbit and are seen to dominate the resonance effect. When the field is rotated away from $[10\bar{1}0]$, two nonequivalent orbits centered on L are expected. One of these becomes the orbit denoted by Sec. $A'-A'$ when H is along $[11\bar{2}0]$ and gives rise to mass branch a in Fig. 7. The second set of nonequivalent orbits is rotated towards the MLH plane and no longer exists when the field is directed 11° from $[11\bar{2}0]$. These orbits give rise to branch c shown on Fig. 7. The assignment of mass branch a and c to these orbits is strengthened by comparing the angular dependence of the masses with that expected from a cylinder with its axis along LH as denoted by the solid lines in Fig. 7. This curve only has one adjustable parameter (m^*/m_0 for H along $[10\bar{1}0]$) and the fit is accurate within experimental error for the measured mass values. Priestley⁸ also obtained a reasonable fit to the areas expected for a cylinder from his DHVA data arising from orbits on this portion of the fourth zone.

Fig. 7

Fig. 8

For the field directed along $[10\bar{1}0]$ the orbits in Sec. C-C and C'-C' have velocity components both perpendicular and parallel to $\underline{H}$. As the field is rotated toward $[11\bar{2}0]$ the velocity components parallel to $\underline{H}$ increase in magnitude for the extremal orbits being rotated toward the MLH plane while they decrease for the orbits being rotated toward Sec. A'-A'. The data for $\mathcal{L}_{rf}$ parallel to $\underline{H}$ (Fig. 8) demonstrates this effect. Branch c is observed over the same angular range as it is for $\mathcal{L}_{rf}$ perpendicular to $\underline{H}$, but branch a becomes unobservable for $\underline{H}$ rotated a few degrees from $[10\bar{1}0]$.

Since the details of the shape of the arms of the fourth zone determine the magnitude of the velocity components parallel to $\mathcal{L}_{rf}$, the cylindrical nature of the arms has important effects on the interpretation of data arising from other orbits on this part of the FS. For example, an orbit such as the one shown by Sec. P-P in Fig. 3 for $\underline{H}$ along $[11\bar{2}0]$ should have a large component of velocity along $\underline{H}$. Therefore, it is reasonable to assign the mass series p for $\mathcal{L}_{rf} \parallel \underline{H}$ in Fig. 8 to such an orbit. This mass series occurs only over a small angular range and has the magnitude and angular dependence expected from this type orbit. Similarly, the mass series q and r in Fig. 8 are tentatively assigned to the orbits shown by Sec. Q-Q and Sec. R-R in Fig. 3. The uncertainty in the size and shape of the "posts" near the point H of the Brillouin zone complicates the assignment of the exact position of such orbits, since extremal mass orbits for this series exist only over a few degrees before the posts cut them off. There are three disconnected segments of this mass branch p, q, r vs field

orientation corresponding to three arrangements of extremal orbits between the posts. Assignments of some of these mass series to orbits on the third zone crown may be possible. However, the shape of the crown causes extremal orbits to have smaller velocity components in the direction of $\underline{H}$ and these mass series are detected only for $\underline{j}_{rf}$ parallel to $\underline{H}$.

The mass series labeled e in Figs. 7 and 8 are assigned to orbits on the fourth zone "lower arms." This orbit is shown as Sec. E-E in Fig. 3 for $\underline{H}$ along $[11\bar{2}0]$ with similar orbits occurring as $\underline{H}$ is rotated away from $[11\bar{2}0]$. This orbit remains near the fourth zone posts in a position such that the mass remains a minimum. The signals from this orbit disappear as $\underline{H}$ is rotated more than 15° away from $[11\bar{2}0]$ as expected.

The series of mass values labeled j in Fig. 7 are assigned to orbits on the hexagonal "cookie" of the third zone of the FS (Fig. 1). The orbit that occurs with $\underline{H}$ along $[10\bar{1}0]$ is shown by Sec. J-J in Fig. 1 and the resonances due to this orbit are labeled as J_1 in Fig. 6. Central orbits centered on the point A of the Brillouin zone are expected as the field is rotated toward $[11\bar{2}0]$ where the orbit denoted by Sec. G-G in Fig. 1 occurs for $\underline{H}$ along $[11\bar{2}0]$. Resonances associated with these orbits are considerably weaker than the resonances associated with orbits on the "upper arms" of the fourth zone.

The mass series labeled b in Fig. 7 is tentatively assigned to orbits centered at M on the third zone "dog bone" (Fig. 2). The orbit that occurs for $\underline{H}$ along $[11\bar{2}0]$ is shown as Sec. B-B

in Fig. 2, while the orbit lying in Sec. B'-B' occurs for $\underline{H}$ along $[10\bar{1}0]$. The angular dependence of this mass series makes this assignment plausible, but the mass values appear rather large for this small portion of the FS. However, the magnitude of m^* as given in Eq. (2) depends upon the curvature of $E(k)$ vs k and not the actual size of the FS. Estimates from the $E(k)$ vs k curves given by Soven² show that m^* for this section would be in reasonable agreement with the mass series b. This branch does not occur for $\underline{j}_{rf}$ parallel to $\underline{H}$, making the assignment more plausible since no resonances would be expected from such an orbit in the parallel field configuration. These resonances become weaker near $[10\bar{1}0]$ with the first and second harmonic being difficult to resolve, as may be verified by noting the resonances labeled B₁ in Fig. 6 which occur for $\underline{H}$ along $[10\bar{1}0]$. This effect is probably due to mixing with the other resonances.

The series of masses labeled d in Fig. 7 are weak resonances with at most three or four subharmonics occurring. These mass values are tentatively assigned to orbits on the fifth zone (Fig. 4) pocket of electrons. Soven's calculations indicate that equivalent orbits should be expected for the fifth zone for all field orientations in the basal plane. The assignment is strengthened by noting that Coon, Grenier, and Reynolds⁷ reported constant calipers within experimental error for the fifth zone in their geometric resonance data. It may be noted that the four mass values not assigned to other orbits occurring with $\underline{H}$ within 3° of $[10\bar{1}0]$ probably do not belong to the fifth zone mass

series, but to an orbit on the fourth zone such as Sec. R-R of Fig. 3. No assignment for the mass series f in Fig. 8 could be made.

B. $(11\bar{2}0)$ Plane

The effective mass values obtained for field rotations in the $(11\bar{2}0)$ plane are displayed in Fig. 9. The signal strength and number of resonances is appreciably less for this plane than for the (0001) plane and, therefore, fewer mass branches were obtained. Considerable effects due to magnetic breakdown were observed for this plane, especially for H parallel to $[0001]$. The effect of magnetic breakdown on the data will be discussed later.

The series of masses labeled a (open squares) in Fig. 9 is assigned to orbits centered at the point L of the fourth zone hexagonal mesh. These resonances are strong for H along $[10\bar{1}0]$ with the signal strength due to this orbit decreasing as H is rotated toward $[0001]$ and becoming unobservable for H applied 32.5° from $[10\bar{1}0]$. This mass series corresponds to the same fourth band cylinder with its axis along the line LH as was discussed for the (0001) plane. The expected mass values for the cylinder for these orientations are shown by the solid line in Fig. 9.

The mass branches labeled j and f in Fig. 9 are assigned to orbits on the third zone crown. The series j is assigned to the central orbit centered at the point A of the Brillouin zone while f is assigned to non-central orbits on the "shadow"

Fig. 9

of the crown. An example of the "shadow" orbit is shown by Sec. F-F in Fig. 1. The resonances giving rise to series j near $[10\bar{1}0]$ are weak and difficult to resolve. When $\underline{H}$ is rotated to within 22.5° of $[0001]$ the resonances producing the j series become very strong and dominate the resonance effect at low fields. An example of this orbit for $\underline{H}$ applied along $[0001]$ is shown by Sec. E-E in Fig. 1. For sufficiently high magnetic field strengths, magnetic breakdown occurs between the third and fourth zone across the points of double degeneracy along the line AL. Initial effects due to magnetic breakdown are observed between 2-5 kG.

The mass branch labeled b (the mass values connected by the dashed line in Fig. 9) is assigned to orbits centered at the point M on the "dog bone" of the third zone. The resonances producing this branch are weak and can be resolved only over a narrow angular region.

The branch labeled d (shown by the solid squares in Fig. 9) is observed for field rotations out to 27.5° from $[10\bar{1}0]$ and is tentatively assigned to orbits on the fifth zone. The phase shift for these mass values is large (approximately $n_0 = \frac{1}{2}$) while the phase shifts for mass values assigned to the fifth zone in the (0001) plane data are less than $n_0 = 0.1$. The resonances giving rise to series a together with these resonances give a reasonable fit to a straight line, and were at first thought to be an effect of mass doubling¹⁰ due to surface roughness or a slight tilt of $\underline{H}$ out of the plane of the surface. However, as $\underline{H}$ is rotated in this plane, the peaks in the different branches shift with respect

to one another and the amplitudes of the peaks are in no way related. No explanation for this large phase shift can be proposed at this time, although it is pointed out that for this orientation the electrons on the fifth zone portion of the FS have approximately one-half of their orbits lying within the skin depth since the elongated edge of the pockets (Fig. 4) is parallel to the surface of the sample. In the (0001) data the fifth zone electrons have only a small portion of their orbits within the skin depth.

c. $(10\bar{1}0)$ Plane

The effective mass values for field rotations in the $(10\bar{1}0)$ plane are displayed in Fig. 10. The data for H applied near $[11\bar{2}0]$ are of low signal to noise ratio for this plane, and therefore, the effective masses for this region show considerable scatter.

The mass branch labeled a (open squares) in Fig. 10 agrees with the mass obtained for the fourth zone arms when H is along $[11\bar{2}0]$. In general, this branch follows the curve for the same cylinder as noted previously with its axes along the LH line, although the scatter is large. These resonances are very weak with only a few subharmonics occurring. The second branch d (solid squares) found in this angular region may be assigned to orbits on the fifth zone. Similar phase shifts as discussed for the $(11\bar{2}0)$ plane data were also found for this mass branch. The branch d' between 15° and 35° may be the extension of the fifth zone branch at these angles. The resonances corresponding to this branch become very strong at angles between 20° and 25° from $[0001]$,

Fig. 10

but they are the only ones observed in this angular region.

The remaining mass branch on Fig. 10 labeled as n is thought to be due to a different orbit than the one causing branch d' . Although it is very unlikely that a mass extremal should exist on the fourth zone "posts", these posts may be the only part of the FS giving such low mass values for this range of orientation; furthermore, these posts are leaning 32° from the $[0001]$ direction at which angle the mass series n passes near a minimum value and the corresponding resonances are strongest. The splitting of these resonances at some angles are perhaps due to orbits on different posts if this assignment is valid, but most probably are due to surface roughness or a slight tilt of H out of the surface of the sample.

Another assignment which may be considered for this branch is the orbit indicated by the dotted oval in Fig. 3. As was pointed out by Priestley⁸ the symmetry of the Fermi surface about the point H is $\bar{6}_m2$, thus H is not an inversion center making the location of an extremal mass orbit such as this one difficult. This assignment is weakened by the fact that the magnitude of the observed masses is smaller than might be expected for this orbit.

The two branches with $m^*/m_0 \gtrsim 1$ which occur when H is near $[0001]$ on Fig. 10 are evidently due to orbits around the third zone crown and the fourth zone network. Neither of the mass values for H along $[0001]$ correspond to the central orbit crown mass obtained in the $(11\bar{2}0)$ plane data, but the angles over which these branches are observed are consistent with assigning branch f to the non-central "shadow" orbit on the crown as shown by Sec. F-F on Fig. 1

and branch g to the orbit around the central section of the fourth zone as shown in Sec. G-G of Fig. 3. The resonant series giving the masses on branch f become very strong for H near [0001] and dominate the surface impedance for these angles with as many as 25 subharmonics being obtained.

D. Effect of magnetic Breakdown

The recorder traces displayed in Fig. 11 show effects of magnetic breakdown similar to those reported by Moore¹⁶ in cyclotron resonance measurements on gallium. The a series dominates at fields below 2 kG, while at higher fields the first through third harmonics are not detectable and the fourth and fifth harmonics are reduced in signal strength. Moore¹⁶ predicts that magnetic breakdown produces a decrease in signal amplitude and a broadening of a given resonance peak due to an apparent $\omega_c \tau_{app} = \omega_c \left(\frac{1}{\tau} + \frac{1}{\tau'} \right)^{-1}$ where τ' is an effective relaxation time due to magnetic breakdown.

For small breakdown probabilities $\omega_c \tau'$ can be approximated by $\omega_c \tau' \approx 1/rP$ where r is the number of equivalent Bragg reflection (or breakdown) points on the orbit and P is the breakdown probability. Blount¹⁷ and Pippard¹⁸ have shown that the probability of an electron tunneling from one piece of the FS to another rather than undergoing normal Bragg reflection is

$$P = \exp(-H_0/H) \quad , \quad (3)$$

where H_0 is the characteristic breakdown field at which the probability of breakdown is equal to 1/e of its zero field value. An

Fig. 11

estimate of H_0 may be made by assuming that the broadenings of the peaks is due only to a decrease in $\omega_c \tau_{app}$.^{*} Referring to the previous equations, it follows that

$$H_0 = (1/H_m - 1/H_n)^{-1} \ln(nd_n/md_m) \quad , \quad (4)$$

where m and n represent subharmonic numbers where broadening occurs and d_m and d_n denote the increase in width of the m^{th} and n^{th} harmonic, respectively, where these differences are taken with respect to an average of the widths of those peaks unaffected by breakdown. In this manner a value of $H_0 \approx 12 \pm 3$ kG is obtained for a field direction approximately 2° from $[0001]$.

A more general effect of the rather low fields at which magnetic breakdown occurs is shown in the absence of any mass branches due to the third zone crown and the very weak signals for the fourth zone upper arms in the data for the $(10\bar{1}0)$ plane. For field rotations in this plane the central orbits on the crown lie in the skin depth at their point of contact along the AL line. The cylindrical shape of the fourth zone upper arms allows non-central orbits removed from the point of contact but having the same mass as the central orbit to be observed. The de-phasing that occurs at this breakdown point apparently causes much more serious effects on the resonance conditions than when the same orbits contact the

^{*}This is to say that for constant τ , the width of a subharmonic resonance is directly proportional to H . Hence, for $1/H$ sweeps, the width of the resonances should be constant.

skin depth at a point removed from the breakdown point. The central crown orbit is observed in the (0001) plane and $(11\bar{2}0)$ plane data for field orientations where breakdown is occurring, with only the decrease in amplitude of the first few harmonics as mentioned above.

APPENDIX A

In this section, the surface impedance of a metal is calculated for the conditions under which Azbel'-Kaner type cyclotron resonance is observed. The surface impedance is calculated by solving the Boltzmann transport equation in conjunction with Maxwell's equations. A quadratic dispersion law is assumed throughout as well as the existence of an isotropic relaxation time τ for the carriers. A boundary condition of specular reflection at the surface shall be assumed, but differences due to diffuse scattering shall be pointed out. The surface impedance is calculated for a microwave field polarized both perpendicular and parallel to a static magnetic field H_0 applied parallel to a plane surface of the metal.

This presentation will hopefully give the reader a full knowledge of how the surface impedance is calculated and the mathematics involved. However, the bulk of the algebra and mathematical manipulations are omitted.

A requirement for cyclotron resonance to be observed is that the electron complete several orbits before it is scattered. This is equivalent to requiring that $\omega\tau \gg 1$. The values of mean free path and relaxation times of interest here correspond to the extreme anomalous skin effect region where the microwave field is rapidly damped inside a metal. Under these conditions the mean free path of the electrons is much larger than the penetration

depth of the rf field and therefore the electric field varies considerably during an electron mean free path. Hence, the current at a point within the metal does not depend upon the electric field at that point alone, but also on its value within a sphere centered at the point under consideration and of a radius equal to the mean free path. A theoretical treatment of the anomalous skin effect has been given in great detail by Reuter and Sondheimer.⁹ Their work gives the relation of the current density $\underline{j}$ to the electric field $\underline{E}$ as obtained from a solution of the Boltzmann transport equation for the conduction electrons in the metal. This relation, in conjunction with Maxwell's equations, gives an integro-differential equation governing the variation of the electric field inside the sample. The solution of this equation permits one to obtain the power absorbed for a variety of experimental conditions.

Azbel' and Kaner extended the results of Reuter and Sondheimer to the case where a metal is subjected both to a microwave electric field polarized in the plane of the sample and a constant magnetic field H_0 parallel to the sample surface. Azbel' and Kaner¹ predicted a resonance behavior of the surface impedance of the metal under these conditions. They considered the boundary condition of diffuse scattering of electrons at the surface which resulted in considerable mathematical difficulties. Rodriguez¹¹ and Mattis and Dresselhaus¹² approached the problem in the same manner, but for simplicity assumed specular reflection as the boundary condition, even though diffuse scattering has been demonstrated to be closer to reality. The justification of the use of specular reflection

is not only that it simplifies the mathematics but that the essential features of the phenomenon are satisfactorily described. Reuter and Sondheimer⁹ have shown that, when $H_0 = 0$, the surface impedance, if the scattering is specular, differs from the result obtained in the case of diffuse scattering only by a factor of 8/9. Therefore, specular reflection shall be assumed for purposes of illustration here with the differences due to use of diffuse scattering being noted. Recently, a theoretical treatment of the problem was done by Hartmann and Luttinger¹³ for arbitrary boundary conditions using an integral form of the Boltzmann transport equation. The rapidly varying factor in their theoretical expression for the surface impedance agrees with that of Azbel' and Kaner. The use of only this factor for typical $\omega\tau$ has shown that the resonant frequencies vary by only 1/10 of a percent from the resonant frequencies obtained from the entire expression.

Consider a semi-infinite sample of metal with a plane surface. Take a system of Cartesian coordinates x, y, z with the xz plane in the plane of the sample and the y axis normal to this plane and pointing into the metal (refer to Fig. 12). The external field H_0 will be taken as parallel to the z axis with the rf field polarized in the xz plane.

The complete set of equations governing the motion of the electrons consists of Maxwell's inhomogeneous equations:

$$\Delta x \underline{H} = \frac{4\pi}{c} \underline{J} \quad \Delta x \underline{E} = - \frac{1}{c} \frac{\partial \underline{H}}{\partial t} \quad (1)$$

and the Boltzmann transport equation for the electron distribution function f :

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \nabla_r f + \frac{e}{\hbar} [\underline{E} + \frac{1}{c} \underline{v} \times \underline{H}] \cdot \nabla_k f + (f - f_0) / \tau = 0, \quad (2)$$

where $\underline{v} = \hbar \underline{k} / m$ is the velocity of the electrons, e and m their charge and mass, and $\underline{k}$ their wave vector. The electromagnetic fields referred to in the above equations are the total fields. The function f_0 is the Fermi distribution function. The collision of the electrons with the lattice imperfections and the thermal phonons are taken into account by the assumption of the existence of a relaxation time τ , which is a function of electron energy alone.

A solution of equation (2) shall be assumed to be of the form

$$f = f_0 + f_1 \quad (3)$$

where the deviation f_1 of the electron distribution function from its equilibrium function f_0 is proportional to the microwave electric field $\underline{E}$, and has the same time periodicity as $\underline{E}$.^{*} The force exerted on the electrons by the microwave magnetic field shall

^{*} If diffuse scattering boundary conditions were assumed, this would mean that an electron would lose all memory of its past trajectory when it struck the surface. This is equivalent to requiring

$$f_1|_{z=0, v_t > 0} = 0$$

be neglected in equation (2) since it is v_F/c times smaller than the force exerted by the electric field $\underline{E}$. Here, v_F denotes the Fermi velocity of the electrons, which is on the order of 10^8 cm/sec for thallium. The term $\underline{E} \cdot \nabla_k f_1$ in equation (2) shall also be neglected since it is of the order E^2 . With these assumptions, equation (2) reduces to

$$(1/\tau + i\omega)f_1 + v_y \frac{\partial f}{\partial y} + \frac{e}{\hbar} \underline{E} \cdot \nabla_k f_0 + \frac{e}{\hbar c} \underline{v} \times \underline{H}_0 \cdot \nabla_k f = 0 \quad (4)$$

The velocity of the electrons is defined as

$$\underline{v} = 1/\hbar \nabla_k \epsilon \quad , \quad (5)$$

where ϵ is the energy of the electrons. Since f_0 is a function of ϵ only, we may write

$$\nabla_k f_0 = \nabla_k \epsilon \frac{\partial f_0}{\partial \epsilon} = \hbar \underline{v}_k \frac{\partial f_0}{\partial \epsilon} \quad (6)$$

The function $\frac{\partial f_0}{\partial \epsilon}$ has an appreciable value only in an energy range of a few KT about the Fermi level, and since the experiments are performed well below the Fermi temperature, a good approximation is that

$$\frac{\partial f_0}{\partial \epsilon} \approx \delta(\epsilon - \epsilon_F) \quad , \quad (7)$$

*To solve the Boltzmann transport equation, it becomes more convenient to introduce a function

$$g = f_1(z, v) - f_1(z, -v) \quad .$$

The reason for diffuse scattering being more difficult to solve is the fact that f_1 depends on z and v for this case, while for specular reflection f_1 is a function of energy alone.

where ϵ_F is the Fermi energy and δ is the Dirac δ function. Using the properties of Dirac δ functions, we may write

$$\delta(\epsilon - \epsilon_F) = \delta\left(\frac{mv^2}{2} - \frac{mv_F^2}{2}\right) = \frac{1}{mv_F} \delta(v - v_F) \quad (8)$$

Using (6), the term involving f_0 in the triple product of equation (4) vanishes identically. Transforming to spherical coordinates as shown in Fig. (12), the Boltzmann transport equation reduces to

$$(1 + i\omega\tau)f_1 + \omega_c \tau \frac{\partial f_1}{\partial \varphi} + v\tau \sin\theta \sin\varphi \frac{\partial f_1}{\partial y} = - \frac{e\tau}{m} \delta(v-v_F) \underline{E} \cdot \underline{n} \quad (9)$$

where $\underline{n}$ is a unit vector in the direction of $\underline{v}$ and $\omega_c = eH_0/mc$. Equation (9) is a linear partial differential equation with the independent variables y and φ , which θ appears only as a parameter. It becomes convenient in solving for f_1 to introduce the Fourier transforms of $\underline{E}$ and f_1 , and therefore we define

$$\underline{E}(s) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} \underline{E}(y) \exp(isy) dy \quad (10)$$

$$\Phi(s) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} f_1(y) \exp(isy) dy \quad (11)$$

With these substitutions, Eq. (9) becomes

$$(1 + i\omega\tau - isv\tau \sin\theta \sin\varphi)\Phi + \omega_c \tau \frac{\partial \Phi}{\partial \varphi} = - \frac{e\tau}{m} \delta(v-v_F) \underline{E} \cdot \underline{n} \quad (12)$$

Equation (12) is now in the form of a linear differential equation as below:

$$\frac{dg}{dx} = -\alpha(x) g + f(x) \quad . \quad (13)$$

A solution for this differential equation may be found in the following manner. A solution for the homogeneous part ($f(x)=0$) is

$$g = A \exp(-\alpha'(x)) \quad (14)$$

where A is some constant of integration and α' is the primitive or indefinite integral of α . To obtain the inhomogeneous solution to (13), we allow A to become a function of x . By taking the derivative of g with respect to x , and requiring the solution to be unique, we determine that

$$A = \int_x^\infty f(x') \alpha'(x') dx' \quad . \quad (15)$$

Hence the solution to the differential equation (13) is

$$g(x) = \int_x^\infty f(x') \exp(\alpha'(x') - \alpha'(x)) dx' \quad . \quad (16)$$

By making the proper substitutions from equation (12) into equation (16), the solution to the Boltzmann transport equation reduces to

$$\Phi = \frac{e}{m\omega_c} \delta(v-v_F) \int_\varphi^\infty dp' \mathcal{E}(s) \cdot \mathcal{U}(\theta, \varphi') \times \\ \exp[\gamma(\varphi' - \varphi) + i a \gamma \sin \theta (\cos \varphi' - \cos \varphi)] \quad , \quad (17)$$

where

$$\gamma = (1 + i\omega\tau)/\omega_0\tau \quad (18)$$

$$a = sv\tau/(1 + i\omega\tau) \quad (19)$$

Note that the function Φ is single valued with respect to φ , i.e. $\Phi(\varphi + 2\pi) = \Phi(\varphi)$. The infinite limits in equation (17) may be changed to an integration over a single period by noting that

$$\Phi = \int_{\varphi}^{\varphi+2\pi} d\varphi' f(\varphi') + \int_{\varphi+2\pi}^{\infty} d\varphi' f(\varphi') \quad , \quad (20)$$

where $f(\varphi')$ represents the integrand of equation (17). By noting that the latter term in equation (20) is $\exp(2\pi\gamma) \times \Phi(\varphi+2\pi)$, and recalling that Φ is single-valued, we may write equation (17) as

$$\begin{aligned} \Phi = \frac{e}{m\omega_c} \delta(v-v_F) [1 - \exp(2\pi\gamma)]^{-1} \int_{\varphi}^{\varphi+2\pi} d\varphi' \mathcal{L}(s) \cdot \mathcal{N}(\theta, \varphi') \\ \times \exp[\gamma(\varphi' - \varphi) + i a \gamma \sin\theta (\cos\varphi' - \cos\varphi)] \quad . \end{aligned} \quad (21)$$

This integral solution for the Fourier transform of the variable electron distribution function shall be used in the calculations.

To calculate the surface impedance, it is necessary to calculate the current density. The current density is given by

$$\mathcal{J} = \frac{e}{4\pi} \int f_1 \mathcal{V} d^3k \quad , \quad (22)$$

which is equivalent to

$$\underline{j}(s) = \frac{e}{4\pi^3} \left(\frac{m}{\hbar}\right)^3 \int \Phi_1 \underline{v} d^3v \quad . \quad (23)$$

By substituting (21) into (23), we obtain

$$\begin{aligned} \underline{j}(s) = & \frac{3}{4} (Ne^2 / \pi m \omega_c) [\exp(2\pi\gamma) - 1]^{-1} \int_0^\pi d\theta \sin\theta \\ & \times \int_0^{2\pi} d\varphi \underline{n}(\theta, \varphi) \int_\varphi^{\varphi+2\pi} d\varphi' \underline{\xi}(s) \cdot \underline{n}(\theta, \varphi') \\ & \times \exp[\gamma(\varphi' - \varphi) + i\alpha\gamma \sin\theta (\cos\varphi' - \cos\varphi)] \quad , \quad (24) \end{aligned}$$

where N is the number of conduction electrons per unit volume.

The integration over dv was performed in arriving at (24) as follows

$$\int_0^\infty v^3 \delta(v - v_F) d^3v = v_F^3 = \frac{3\hbar^3}{8\pi m^3} N \quad . \quad (25)$$

Equation (24) defines a conductivity tensor which relates the components of $\underline{j}(s)$ to those of $\underline{\xi}(s)$. The components of the current density shall be evaluated for the x , y , z components by noting that

$$\underline{\xi}(s) \cdot \underline{n}(\theta, \varphi') = \sin\theta \cos\varphi' \xi_x + \sin\theta \sin\varphi' \xi_y + \cos\theta \xi_z \quad (26)$$

$$\underline{n}(\theta, \varphi) = \sin\theta \cos\varphi \hat{i} + \sin\theta \sin\varphi \hat{j} + \cos\theta \hat{k} \quad . \quad (27)$$

The change of variable

$$\alpha = \frac{1}{2}(\varphi' - \varphi) \quad \beta = \frac{1}{2}(\varphi' + \varphi) \quad (28)$$

along with (26) and (27) allows one to write (24) as

$$\begin{aligned} \mathcal{J}(s) = & \frac{3}{4} (Ne^2 / \pi m \omega_c) [\exp(2\pi\gamma) - 1]^{-1} \int_0^\pi d\theta \sin\theta \\ & \times \int_0^\pi d\alpha \int_\alpha^{\alpha+2\pi} d\beta [\sin\theta \cos(\beta-\alpha) \hat{i} + \sin\theta \sin(\beta-\alpha) \hat{j} + \cos\theta \hat{k}] \\ & \times [\xi_x \sin\theta \cos(\alpha+\beta) + \xi_y \sin\theta \sin(\alpha+\beta) + \xi_z \cos\theta] \\ & \times \exp[2\gamma\alpha - 2iay \sin\theta \sin\alpha \sin\beta] \quad . \end{aligned} \quad (29)$$

This expression may be expressed in a more convenient form by the introduction of the following functions:

$$\xi_{\pm} = \xi_x \pm \xi_y \quad \xi_{\pm}^* = \xi_y \pm \xi_x \quad . \quad (30)$$

Using these functions along with some trigometric identities, we may write

$$\begin{aligned} \mathcal{J}(s) = & \frac{3}{4} (Ne^2 / \pi m \omega_c) [\exp(2\pi\gamma) - 1]^{-1} \int_0^\pi d\theta \sin\theta \int_0^\pi d\alpha \exp(2\gamma\alpha) \\ & \times \int_\alpha^{\alpha+2\pi} d\beta \{ [\xi_- \cos 2\beta + \xi_+ \cos 2\alpha + \xi_+^* \sin 2\beta + \xi_-^* \sin 2\alpha] \sin^2\theta \\ & + \cos\theta \hat{k} [\xi_x \sin\theta \cos(\alpha+\beta) + \xi_y \sin\theta \sin(\alpha+\beta) + \xi_z \cos\theta] \} \\ & \times \exp(-2iay \sin\theta \sin\alpha \sin\beta) \quad . \end{aligned} \quad (31)$$

In order to perform the integration over $d\beta$, the exponential of the sine function must be expressed as a series expansion of Bessel Functions:

$$e^{iz \sin \theta} = J_0(z) + 2 \sum_{n=1}^{\infty} [J_{2n}(z) \cos 2n\theta + i J_{2n-1}(z) \sin(2n-1)\theta] . \quad (32)$$

With this expansion, the integral over $d\beta$ results in an integral of sine and cosine functions. These are orthogonal functions with the following properties:

$$\int_0^{2\pi} \sin(nx+a) \sin kx \, dx = \int_0^{2\pi} \cos(nx+a) \cos kx \, dx = 2\pi \cos a \, \delta_{n,k} \quad (33)$$

$$\int_0^{2\pi} \sin(nx+a) \cos kx \, dx = - \int_0^{2\pi} \sin nx \cos(kx+a) \, dx = 2\pi \sin a \, \delta_{n,k} , \quad (34)$$

where $\delta_{n,k}$ is one if n is equal to k and zero otherwise.

Performing the integral over $d\beta$, equation (31) reduces to

$$\begin{aligned} j(s) = & \frac{3}{2} (Ne^2 / m\omega_c) [\exp(2\pi\gamma) - 1]^{-1} \int_0^{\pi} d\theta \sin \theta \int_0^{\pi} d\alpha \exp(2\gamma\alpha) \\ & \times \{ [2\xi_- J_2(b) + \xi_+ \cos 2\alpha J_0(b) + \xi_-^* \sin 2\alpha J_0(b)] \sin^2 \theta \\ & + [\xi_y \sin \theta \cos \alpha J_1(b) + \xi_z \cos \theta J_0(b)] \cos \theta \hat{k} \} , \quad (35) \end{aligned}$$

where

$$b = 2a\gamma \sin \theta \sin \alpha . \quad (36)$$

Referring to the properties of Bessel Functions, the n^{th} order Bessel Function may be written as an infinite series:

$$J_n(b) = \sum_{r=0}^{\infty} \frac{(-1)^r (b/2)^{n+2r}}{r! (n+r)!} . \quad (37a)$$

Looking ahead, it may be noticed that the z_y component of $\underline{j}$ vanishes since the integral over $d\theta$ for this term results in a sine function which vanishes when evaluated at the limits. The remaining integrals result (in some cases after an integration by parts) in an integral of the form

$$\begin{aligned} \int e^{ax} \sin^n x \, dx &= \frac{1}{a^2 + n^2} [(a \sin x - n \cos x) e^{ax} \sin^{n-1} x \\ &+ n(n-1) \int e^{ax} \sin^{n-2} x \, dx] . \end{aligned} \quad (37b)$$

When the limits of integration from (35) are used to evaluate the first term of (37b), this term vanishes due to the $\sin^{n-1} x$ term. Hence, after performing r such integrals, we may write

$$\int_0^{\pi} d\alpha \exp(2\gamma\alpha) \sin^{2r} \alpha = \frac{2r(2r-1)}{(2\gamma)^2 + (2r)^2} \cdot \frac{(2r-2)(2r-3)}{(2\gamma)^2 + (2r-2)^2} \cdots \frac{1}{2\gamma} [\exp(2\pi\gamma) - 1] \quad (38a)$$

$$= \frac{(2r)!}{(2\gamma)^{2r+1}} [\exp(2\pi\gamma) - 1] \prod_{v=0}^r \left[1 + \frac{\gamma^2}{\gamma^2} \right]^{-1} , \quad (38b)$$

where the symbol $\prod_{i=m}^n$ denotes the produce of terms with index i

ranging from m to n . Making the proper substitutions, we may reduce (35) to

$$\begin{aligned}
 j(s) = & \frac{3}{2} \frac{\sigma}{1+i\omega\tau} \sum_{r=0}^{\infty} \frac{(-1)^r (2r)!}{(r!)^2 2^{2r+1}} \int_0^{\pi} d\theta \{ \sin^{2r+3} \theta \\
 & \times [\xi_+ + (\frac{2a^2 \gamma^2 \sin^2 \theta \xi_-}{(r+2)(r+1)} + \frac{\xi_-^*}{2\gamma(r+1)} - 2\xi_+) \frac{(2r+2)(2r+1)}{(2\gamma)^2 + (2r+2)^2}] \\
 & + \sin^{2r+1} \theta \cos^2 \theta \xi_z \hat{k} \} \prod_{v=0}^r [1 + \frac{v^2}{\gamma^2}]^{-1}, \quad (39)
 \end{aligned}$$

where $\sigma = Ne^2\tau/m$ is the classical electrical conductivity. The remaining integration over $d\theta$ may be made by use of the following integration formulas:

$$\int \cos^m x \sin^n x = \frac{\cos^{m-1} x \sin^{n+1} x}{m+n} + \frac{m-1}{m+n} \int \cos^{m-2} x \sin^n x dx \quad (40)$$

$$\int_0^{\pi} \sin^n x dx = \sqrt{\pi} \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{n}{2}+1)}, \quad (41)$$

where Γ is the Gamma function. The properties of the Gamma function are

$$\Gamma(n) = (n-1)! \quad (42)$$

$$\Gamma(n+\frac{1}{2}) = \frac{(2n)!}{2^{2n} n!} \sqrt{\pi} \quad (43)$$

The current density may be evaluated with the use of equations (40) - (43). Writing the current density in its tensor form, we have

$$j_x = \sigma_{xx} \xi_x + \sigma_{xy} \xi_y \quad (44)$$

$$j_y = \sigma_{yx} \xi_x + \sigma_{yy} \xi_y \quad (45)$$

$$j_z = \sigma_{zz} \xi_z \quad (46)$$

where the components of the tensor are

$$\sigma_{xx} = \frac{3\sigma}{1+i\omega\tau} \sum_{r=0}^{\infty} \frac{(-1)^r a^{2r}}{(2r+1)(2r+3)} \left\{ 1 - \frac{2r(r+1)^2}{\gamma^2} \right\} \times \prod_{n=0}^{r+1} [1 + (n\gamma^{-1})^2]^{-1} \quad (47)$$

$$\sigma_{yy} = \frac{3\sigma}{1+i\omega\tau} \sum_{r=0}^{\infty} \frac{(-1)^r a^{2r}}{2r+3} \prod_{n=0}^{r+1} [1 + (n\gamma^{-1})^2]^{-1} \quad (48)$$

$$\sigma_{xy} = -\sigma_{yx} = -\frac{3\sigma}{1+i\omega\tau} \sum_{r=0}^{\infty} \frac{(-1)^r a^{2r} (r+1)}{2r+3} \prod_{n=0}^{r+1} [1 + (n\gamma^{-1})^2]^{-1} \quad (49)$$

$$\sigma_{zz} = \frac{3\sigma}{1+i\omega\tau} \sum_{r=0}^{\infty} \frac{(-1)^r a^{2r}}{(2r+1)(2r+3)} \prod_{n=0}^r [1 + (n\gamma^{-1})^2]^{-1} \quad (50)$$

Note that $j(s) = j(-s)$ in the above expressions. The equations (47) - (50) are useful for the numerical computations of the coefficients of the conductivity tensor for large values of H_0 (i.e., $|\gamma| \ll 1$) and for any value of a , and for $|a| \ll 1$ and any value of H_0 in the range in which $\omega_c \tau \sim 1$. For example

if ξ is along the z axis, there is only a component of j along z . For high fields where $\omega_c \tau \gg 1$, only the $\gamma = 0$ term is important in the series expansion of σ_{zz} . The current density reduces to

$$j_z = (\sigma_0 / 1 + i\omega\tau) \xi_z, \quad (51)$$

which is just Ohm's law. For small a , we have $sv_F \tau \ll |1 + i\omega\tau|$ which is roughly equivalent to $\frac{\lambda}{\delta} \ll |1 + i\omega\tau|$ where λ is the mean free path and δ is the skin depth. Resonance is unlikely to occur in this region. The current density reduces to Ohm's law for the leading term in (50) while the remaining terms are proportional to a^{2r} , $r = 1, 2, \dots$.

For conditions under which resonance is observed, it becomes convenient to express the components of the conductivity tensor as a power series in a^{-1} . The method to obtain this power series is outlined by Rodriguez¹⁹ in an appendix to his paper on ultrasonic attenuation. The expansion involves taking the complex integral over a certain contour. The exact procedure may be found in Rodriguez's paper while we shall only quote the result of the expansion:

$$\begin{aligned} \sigma_{xx} = & \frac{\sigma}{1 + i\omega\tau} \left[\frac{3\pi}{4a} \coth(\pi\gamma) - \frac{3}{a^2} + \frac{3\pi}{4a^3} \coth(\pi\gamma) \left(1 + \frac{3}{4\gamma^2} \right) \right. \\ & \left. - 3 \sum_{r=0}^{\infty} \frac{(-1)^r a^{-2r-4}}{(2r+1)(2r+3)} \left\{ 1 + \frac{2(r+1)^2(r+2)}{\gamma^2} \right\} \prod_{n=0}^r \{1 + (n\gamma^{-1})^2\} \right] \quad (52) \end{aligned}$$

$$\sigma_{xy} = -\sigma_{yx} = \frac{\sigma}{1+i\omega\tau} \left[-\frac{3\pi}{4a^3} \frac{\coth(\pi\gamma)}{\gamma} + \frac{3}{\gamma} \sum_{r=0}^{\infty} \frac{(-1)^r a^{-2r-4} (r+1)}{2r+1} \prod_{n=0}^r \{1 + (n\gamma^{-1})^2\} \right] \quad (53)$$

$$\sigma_{yy} = \frac{\sigma}{1+i\omega\tau} \left[-\frac{3\pi}{2a^3} \coth(\pi\gamma) + \frac{3}{a^2} + 3 \sum_{r=0}^{\infty} \frac{(-1)^r a^{-2r-4}}{2r+1} \prod_{n=0}^r \{1 + (n\gamma^{-1})^2\} \right] \quad (54)$$

$$\sigma_{zz} = \frac{\sigma}{1+i\omega\tau} \left[\frac{3\pi}{4a} \coth(\pi\gamma) + \frac{3\pi}{4a^3} \left(1 + \frac{1}{4\gamma^2}\right) \coth(\pi\gamma) + 3 \sum_{r=0}^{\infty} \frac{(-1)^r a^{-2r-2}}{(2r+1)(2r-1)} \prod_{n=0}^r \{1 + (n\gamma^{-1})^2\} \right] \quad (55)$$

This completes the calculation of the current density tensor. The remaining equation at our disposal is a result of Maxwell's equations:

$$\frac{\partial^2 \underline{E}}{\partial y^2} + (\omega^2/c^2) \underline{E} = (4\pi i\omega/c^2) \underline{j} \quad (56)$$

The boundary condition imposed on the fields because of our choice of specular reflection is

$$\underline{E}(-z) = \underline{E}(z) \quad \underline{E}_{-s} = \underline{E}_s \quad j_{-s}(-\omega_c) = j_s(\omega_c) \quad (57)$$

With these conditions, the problem may now be considered in an infinite medium.

Taking the Fourier transform of (56), we note that $\underline{E}(y)$ has a cusp at $y = 0$, because there the field is damped in both the direction of the positive and negative y axis. This also follows from (57) since

$$E'(+0) = -E'(-0) \quad , \quad (58)$$

where the prime denotes derivative with respect to y , and the arguments $+0$ and -0 indicate that the limits are to be evaluated for positive and negative values of y , respectively. Hence the Fourier transform of (56) is

$$-\sqrt{2/\pi} \underline{E}'(+0) - s^2 \underline{\xi}(s) + (\omega/c)^2 \underline{\xi}(s) = (4\pi i \omega / c^2) \underline{j}(s) \quad . \quad (59)$$

The surface impedance is defined as

$$Z_z = \frac{4\pi}{c} \frac{E_z(0)}{H_x(0)} \quad (60)$$

$$Z_x = -\frac{4\pi}{c} \frac{E_x(0)}{H_z(0)} \quad (61)$$

for the two cases of longitudinal and transverse cyclotron resonance, respectively. Here $\underline{H}$ is the microwave magnetic field, so we may write

$$Z_z = -\frac{4\pi i \omega}{c^2} \frac{E_z(+0)}{E'_z(+0)} \quad (62)$$

$$Z_x = - \frac{4\pi i \omega}{c^2} \frac{E_x(+0)}{E'_x(+0)} \quad (63)$$

If we consider the longitudinal case with $\underline{E}$ parallel to the z axis, the only nonvanishing component of $\underline{j}$ is in the z direction. Hence from (58), we find that

$$\xi_z(s) = - \sqrt{2/\pi} E'_z(+0) [s^2 - (\omega/c)^2 + (4\pi i \omega/c^2) \sigma_{zz}(s)]^{-1} . \quad (64)$$

Taking the inverse transform of (64), we have

$$E_z(y) = -(2/\pi) E'_z(+0) \int_0^\infty [s^2 - (\omega/c)^2 + (4\pi i \omega/c^2) \sigma_{zz}(s)]^{-1} \cos(sy) ds . \quad (65)$$

The term $(\omega/c)^2$ results from the displacement current and can be neglected as argued by Reuter and Sondheimer.⁹ The conductivity component σ_{zz} may be found by recalling that we are in the extreme anomalous skin effect region ($l \gg \delta$) and a is of the order of $1/\delta$. Hence only the first term in the asymptotic expansion (54) need be taken into account.

$$\sigma_{zz} = \frac{3\pi\sigma}{4a(1+i\omega\tau)} \coth(\pi\gamma) \quad (66)$$

Substituting (64) and (65) into (61), we find

$$Z_z = \left(\frac{8i\omega}{c^2}\right) \int_0^\infty [s^3 + (3\pi^2 i \omega \sigma / c^2 l) \coth(\pi\gamma)]^{-1} s ds . \quad (67)$$

The term containing $1/\omega\tau$ was neglected since we have required

$\omega\tau$ be large. Upon integration (67) yields

$$z_z = \frac{8}{9} \left(\frac{\sqrt{3} \pi \omega^2}{c^4 \sigma} \right)^{1/3} (1 + i\sqrt{3}) \tanh^{1/3}(\pi\gamma) \quad . \quad (68)$$

We now consider the case when the microwave field is polarized perpendicular to $\underline{H}_0$ along the x axis. A field E_y in the direction of the normal to the surface of the sample is present due to the buildup of a space charge by the resonant electrons. This current may be found as follows by using some properties of electrodynamics:

$$(\nabla \cdot \underline{E})_y = \frac{\partial E_y}{\partial y} = 4\pi\rho \quad (69)$$

$$(\nabla \cdot \underline{J})_y + \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial y} (J_y + \frac{i\omega}{4\pi} \xi_y) = 0 \quad . \quad (70)$$

Taking the Fourier transform of (70), we require

$$j_y(s) = -(i\omega/4\pi) \xi_y(s) \quad . \quad (71)$$

Using (45), we find that

$$\xi_y(s) = [\sigma_{xy}/(\sigma_{yy} + i\omega/4\pi)] \xi_x(s) \quad (72)$$

so that with (44), we deduce that

$$j_x(s) = \sigma_x \xi_x(s) \quad (73)$$

where

$$\sigma_x = \sigma_{xx} + \frac{\sigma_{xy}^2}{\sigma_{yy} + i\omega/4\pi} \quad (74)$$

Hence the surface impedance is calculated exactly as for the longitudinal case with σ_x replacing σ_{zz} . As before only the term containing a^{-1} in the expansion of σ_x is of importance.

$$\sigma_x = \frac{3\pi\sigma}{4a(1 + i\omega\tau)} \quad (75)$$

Therefore in this limit $\sigma_x = \sigma_{zz}$ and $Z_x = Z_z$.

The real part of the surface impedance is proportional to the power absorbed by the metal. Hence neglecting the constants for the moment, we may calculate the real component of Z which we shall write as

$$Z = e^{i\pi/3} \left[\frac{e^{a+ib} - e^{-a-ib}}{e^{a+ib} + e^{-a-ib}} \right]^{1/3} \quad (76)$$

where

$$a = \frac{2\pi}{\omega_c \tau} \quad b = \frac{2\pi\omega}{\omega_c} \quad (77)$$

By multiplying top and bottom of (76) by the complex conjugate of the denominator, we have

$$Z = e^{i\pi/3} \left[\frac{\sinh 2a + i \sin 2b}{\cosh 2a + \cos 2b} \right]^{1/3} \quad (78)$$

The real part R of Z is defined as

$$R = \frac{Z + Z^*}{2} \quad (79)$$

The solution for R in this case may be found by cubing R :

$$8R^3 = Z^3 + Z^{*3} + 6ZZ^*R \quad (80)$$

Substituting from (78), we find

$$R^3 - \frac{3}{4} \frac{[\sinh^2(2a) + \sin^2(2b)]^{1/3}}{[\cosh(2a) + \cos(2b)]^{2/3}} R + \frac{1}{4} \frac{\sinh(2a)}{\cosh(2a) + \cos(2b)} = 0. \quad (81)$$

This is in the form of a general cubic equation. The form of the solution depends upon the "discriminate"

$$\frac{p^2}{4} + \frac{q^3}{27} \quad , \quad (82)$$

where p is the constant term and q is the coefficient of the first order term. If (82) is larger than zero there will be one real and two conjugate imaginary roots. If (82) is zero there will be three real roots of which at least two are equal. If (82) is negative, there will be three real and unequal roots.

For our case, substituting from (81) into (82), we find that

$$\frac{p^2}{4} + \frac{q^3}{27} = \frac{1}{64} \frac{-\sin^2 2b}{[\cosh 2a + \cos 2b]^2} < 0 \quad (83)$$

Hence there are three real and unequal roots.

$$2\sqrt{-a/3} \cos \phi/3 \quad (84)$$

$$2\sqrt{-a/3} \cos (\phi/3 + 120^\circ) \quad (85)$$

$$2\sqrt{-a/3} \cos (\phi/3 + 240^\circ) \quad (86)$$

where

$$\phi = \cos^{-1} \left[-\frac{b}{2} / \sqrt{(-a^3/27)} \right] . \quad (87)$$

For our case, the angle ϕ is

$$\phi = \cos^{-1} [-\sinh 2a / \sqrt{(\sinh^2 2a + \sin^2 2b)}] . \quad (88)$$

This is equivalent to

$$\phi = \tan^{-1} (\sin 2b / \sinh 2a) . \quad (89)$$

Therefore the final solution for R is

$$R = \cos \left(\frac{\phi + \pi}{3} \right) \frac{[\sinh^2 2a + \sin^2 2b]^{1/6}}{[\cosh^2 a - \sin^2 b]^{1/3}} \quad (90)$$

where ϕ is given by (89). By choosing a value for the parameter

$\omega\tau$ (which is equal to a/b), the real part of the surface impedance is found to oscillate if $\omega\tau > 1$, with these oscillations being periodic in $1/H$.

This completes our calculation of the surface impedance. The results (equations (68) and (90)) show the same periodic behavior as predicted by Azbel' and Kaner, with only the line shapes being different. The periodic behavior of the experimental results agrees with these theoretical predictions, but the line shapes of the resonances, in general, do not agree as well with the theoretical predictions.

APPENDIX B

Description of the SPS Program

The computer program used in analyzing the data was written by use of the Symbolic Programming System (SPS). An explanation of the operation of the program follows with a listing of the program and a flow diagram appearing at the end of the appendix. Hopefully, this appendix will explain the operation of the program requiring only a small knowledge of computers on the part of the reader. The role of each statement in the program would require some knowledge of SPS programming, and is unnecessary for an understanding of the operation of the program.

A considerable portion of the program is to insure that all numbers used in mathematical operations were of the same order fields and that these numbers were all floating point or all fixed point numbers. The smallest possible fields consistent with the accuracy desired were used, thus conserving storage space and speeding up execution of the program.

As directed by statements 8-23, the program discriminates between five different types of input cards by checking the first character of each card. The input cards contain either a data point ($1/H$ value), crystallographic orientation, microwave frequency, or one of two type parameters used in the calculations.

If an input card does not have an acceptable character in the first position, a statement (501) is printed on the typewriter and the computer then halts. The input cards will be accepted in any order, but safeguards are added to the program whereby the computer types out a statement if the input cards are out of intended order.

An input card that contains an integer in the first position is interpreted as giving the microwave frequency in multiples of giga-hertz. If an integer is the first character, control of the program is transferred from statement 23 to statement 111. Each time a value of frequency is read into the computer, a statement (498) is typed out to this effect to insure that the frequency was not changed by mistake. Statements 119-123 allow the computer to calculate the constant $e/\omega c m_0$ from the frequency value.

A data input card has a decimal point in the first position with the 1/H value following in units of inverse kilogauss. The control of the computer is transferred to statements 60-110. This section of the program is used to determine the range in which mass values are sought, or equivalently, the possible subharmonics that a given 1/H value may represent. Two parameters, referred to as DX1 and DX2, must be set by the programmer which give the limits in which mass values are sought. These "DX" cards are read in on a separate input card with the following format:

Lb.xxxE-xxb.xxxE-xx

where the first value is DX1 and the second DX2. The data points

are divided by integers, beginning with one and increasing by integral steps. If the resultant value (referred to hereafter as $1/nH$, where n denotes the integer) falls in the range

$$DX1 < 1/nH < DX2$$

then $1/nH$ and n are stored such that their storage space, in theory, is known. Once $1/nH$ is less than $DX2$, the computer branches to read another card. A running total of the number of $1/nH$ values stored for a given crystallographic orientation is kept in order to obtain the storage space of each value.

An input card containing a Y in the first position gives the angle of crystallographic orientation. This card is used to denote that all data points from the previous angle have been read. Control of the program is transferred to statement 137 where the value of angle read in is stored, and the value that was stored is transferred to a position to be punched onto an output statement. Then if at least two data points are stored, the program branches to statement 149, where "ordering" occurs as described below. If for any reason, two "angle" cards are read in succession, a statement (500) is typed out and the computer halts. This prevents the computer from doing the same calculations for a set of data points twice in a row. Also, precautions are taken to insure that the last group of data points are included by use of the last card indicator.

In the "ordering" phase of the program, all of the $1/nH$ values are put into descending order. Care is taken to be sure that, when the storage location of a $1/nH$ value is changed, the storage location of n is also changed so that their relative storage location is known. This part of the program is the most time consuming part for as many as 200 numbers must be ordered. The program then branches to "group", if a parameter, referred to as CX, is defined.

The parameter CX appears on an input card in the format CXb.xxxE-xx. This parameter sets the error which is acceptable in grouping $1/nH$ values to form a period. The value chosen for this parameter is very critical and determines the effectiveness of the program. If CX is too small, several "true" periods may be neglected. If CX is too large, too many "false" periods are formed, and these having a large number of subharmonics.

In the "grouping" phase of the program, sets of three or more $1/nH$ values are sought where the first $1/nH$ value is less than CX larger than the second, the second is less than CX larger than the third, etc. Selected in this manner, a set of $1/nH$ values are used to form a period from which an effective mass value will be calculated.

The statements 215-329 are used to calculate various quantities from the points in a selected period. It might be mentioned that a Fortran II program was written to do this portion of the program for cards fitting the correct format. The part of the SPS program meant to pick out periods has been completed at this point in the program.

A least square straight line fit for the subharmonic number versus the $1/H$ field value for the points in a series is calculated. The slope of this line, the intercepts for both the subharmonic and $1/H$ axis, and the standard deviation of these points from a straight line is calculated. From the slope of the line, the effective mass of the set of points is calculated. An error for this value of effective mass is given by multiplying the standard deviation by $e/\omega c m_0$ in order to put it in units of mass.

Statements 323-437 are output statements giving the format of the information to be punched out. The information read out is the angle, effective mass value, standard deviation, number of subharmonics in a period, intercepts, and the slope of the line. In addition, for each point in a period, the $1/H$ value, subharmonic number n , and $1/nH$ are punched out.

The remainder of the statements define constants or variables and their field length. Included in these are the output format statements and the messages for typeout on the typewriter.

There are certain shortcomings of this program for which no solution could be found. In the choice of the points in a mass series, no restriction whatsoever is placed on the subharmonic numbers; i.e., points in a mass series may have the same subharmonic number or certain subharmonic numbers may be omitted. Checking to see if any subharmonic numbers are equal would be trivial, but a reliable way of determining which value to discard could not be found. Likewise, no formula to discard mass series due to missing subharmonics could be found, since some subharmonics

could not be resolved for "true" mass series. Another disadvantage of the program is that no knowledge of the amplitude dependence of the peaks is included, which is certainly necessary for the assignment of the peaks to mass series.

Once the programmer has the output of the program, "true" mass series must be chosen as described in the main body of the text. Hence the problem of analyzing the data has resulted in discriminating between "true" and "false" mass series.

Listing of SPS Program

1.	BEGIN	RACDINPUT	47.	TD	DX2-7, INPUT+24
2.		WACDINPUT	48.	TD	DX2-6, INPUT+26
3.		TFM N,0,9	49.	TD	DX2-5, INPUT+28
4.		TFM NN,0,9	50.	TD	DX2-1, INPUT+34
5.		TFM TDS,5,10	51.	TD	DX2, INPUT+36
6.		SF F2-11	52.	SF	DX2-7
7.	READ	BLC LASTCD	53.	SF	DX2-1
8.		RACDINPUT	54.	SF	DX2
9.		TF TDSF, TDS	55.	B	READ
10.		TD TDS, INPUT-1	56.	DARR	RCTY
11.		SF TDS-1	57.		WATYNOFREQ
12.		CM TDS,0,10	58.		H
13.		BNE *+36	59.	B	READ
14.		BD DATA, INPUT	60.	DATA	TD H-7, INPUT+2
15.		B PAT	61.		TD H-6, INPUT+4
16.		CM TDS,6,10	62.		TD H-5, INPUT+6
17.		BE ANGLE	63.		TD H-4, INPUT+8
18.		CM TDS,4,10	64.		TD H-1, INPUT+14
19.		BE CXCD	65.		TD H, INPUT+16
20.		CM TDS,5,10	66.		SF H-7
21.		BE DXCD	67.		SF H-1
22.		CM TDS,7,10	68.		SF H
23.		BE FREQP	69.		BNF READ, DX1-7
24.	PAT	RCTY	70.		TFM Q,1,9
25.		WATYSCREW	71.	DIVIDE	TFL QF, ZERO
26.		H	72.		BD TEN, Q-1
27.		B READ	73.		TD QF-7, Q
28.	LASTCD	CM TDS,0,10	74.		SF QF-7
29.		BNE READ+12	75.		TFM QF,1,10
30.		BNF READ+12, OH+1	76.		B *+60
31.		TF N, NN	77.	TEN	TD QF-7, Q-1
32.		B ORDER	78.		TD QF-6, Q
33.		CM TDS,4,10	79.		SF QF-7
34.		BNE READ+12	80.		TFM QF,2,10
35.		BNF READ+12, OH-7	81.		TFL HH, H
36.		BNF READ+12, OH+1	82.		FDIVHH, QF
37.		TF N, NN	83.		TFL NUM1, HH
38.		B GROUP	84.		FSUBNUM1, DX2
39.	DXCD	TD DX1-7, INPUT+6	85.		BNF ADD, NUM1-2
40.		TD DX1-6, INPUT+8	86.		TFL NUM1, HH
41.		TD DX1-5, INPUT+10	87.		FSUBNUM1, DX1
42.		TD DX1-1, INPUT+16	88.		BNF *+36, NUM1-2
43.		TD DX1, INPUT+18	89.		TF NN, N
44.		SF DX1-7	90.		B READ
45.		SF DX1-1	91.		MM N,2,9
46.		SF DX1	92.		SF 95

93.	TF	N1,99	143.	RCTY
94.	MM	N,8,9	144.	WATYTWANG
95.	SF	95	145.	H
96.	TF	N2,99	146.	BNF READ,OH-7
97.	A	N1, TABLE	147.	BNF READ,OH+1
98.	A	N2, TABLE+5	148.	TF N, NN
99.	TF	N3, N1	149.	ORDER TF NM1, N
100.	SM	N3, 1, 7	150.	SM NM1, 1, 9
101.	TD	N1, Q, 6	151.	MM NM1, 2, 9
102.	TD	N3, Q-1, 6	152.	SF 95
103.	SF	N3, , 6	153.	TF N1, 99
104.	TFL	N2, HH, 6	154.	A N1, TABLE
105.	AM	N, 1, 9	155.	MM N, 2, 9
106.	ADD	AM Q, 1, 9	156.	SF 95
107.	CM	Q, 20, 9	157.	TF N2, 99
108.	BNE	DIVIDE	158.	A N2, TABLE
109.	TF	NN, N	159.	TF N4, TABLE
110.	B	READ	160.	TF N3, TABLE+5
111.	FREQP	RCTY	161.	BARE TF N5, N3
112.		WATYNUFREQ	162.	AM N5, 8, 7
113.	TD	F1-7, INPUT	163.	TF N6, N4
114.	TD	F1-6, INPUT+2	164.	AM N6, 2, 7
115.	TD	F1-5, INPUT+6	165.	GO TFL A1, N3, 11
116.	TD	F1-4, INPUT+8	166.	FSUBA1, N5, 11
117.	TD	F1-3, INPUT+10	167.	BNF CYCLE, A1-2
118.	SF	F1-7	168.	TFL B1, N3, 11
119.	TFM	F1, 2, 10	169.	TFL N3, N5, 611
120.	TFL	FREQ, F1	170.	TFL N5, B1, 6
121.	FMULFREQ	TWOPI	171.	TF A2, N4, 11
122.	TFL	FCONS, CONS	172.	TF N4, N6, 611
123.	FDIVFCONS	FREQ	173.	TF N6, A2, 6
124.	B	READ	174.	CYCLE AM N5, 8, 7
125.	CXCD	TD CX-7, INPUT+8	175.	AM N6, 2, 7
126.		TD CX-6, INPUT+10	176.	C N6, N2
127.		TD CX-5, INPUT+12	177.	BNE GO
128.		TD CX-1, INPUT+18	178.	AM N3, 8, 7
129.		TD CX, INPUT+20	179.	AM N4, 2, 7
130.	SF	CX-7	180.	C N4, N1
131.	SF	CX-1	181.	BNE BARF
132.	SF	CX	182.	BNF READ, CX-7
133.	BNF	READ, OH-7	183.	GROUP TFM P, 0, 9
134.	BNF	READ, OH+1	184.	TF NMIN1, N
135.	TF	N, NN	185.	SM NMIN1, 1, 9
136.	B	GROUP	186.	TF NMIN2, N
137.	ANGLE	BLC READ+12	187.	SM NMIN2, 2, 9
138.	TF	OUT1+26, F2	188.	NEWPR TFM NG, 1, 10
139.	SF	INPUT+1	189.	COMPARMM P, 8, 9
140.	TF	F2, INPUT+12	190.	SF 95
141.	C	TDSF, TDS	191.	TF N1, 99
142.	BNE	*+48	192.	A N1, TABLE+5

193.	TF	N2,N1	243.	FADDSUMY,A1
194.	AM	N2,8,7	244.	FMULA1,A2F
195.	TFL	A1,N1,11	245.	FADDSUMXY,A1
196.	FSUBA1	N2,11	246.	FMULA2F,A2F
197.	FSUBA1	CX	247.	FADDSUMX2,A2F
198.	BNF	CHECK,A1-2	248.	AM Q,1,9
199.	AM	P,1,9	249.	C Q,NG
200.	AM	NG,1,10	250.	BNE FIGURE
201.	C	P,NMIN1	251.	WACDBLANK
202.	BNE	COMPAR	252.	TFL FNG,ZERO
203.	CHECK CM	NG,1,10	253.	BD FAP,NG-1
204.	BE	*+36	254.	TD FNG-7,NG
205.	CM	NG,2,10	255.	SF FNG-7
206.	BNE	COMPUT	256.	TFM FNG,1,10
207.	C	P,NMIN2	257.	B *+36
208.	BE	*+60	258.	FAP TF FNG-6,NG
209.	C	P,NMIN1	259.	TFM FNG,2,10
210.	BE	*+36	260.	TFL SLOPE,FNG
211.	AM	P,1,9	261.	FMULSLOPE,SUMXY
212.	B	NEWPR	262.	TFL A1,SUMX
213.	TFM	N,0,9	263.	FMULA1,SUMY
214.	B	READ	264.	FSUBSLOPE,A1
215.	COMPUTTFL	SUMX,ZERO	265.	TFL A1,FNG
216.	BNF	DARR,FCONS-7	266.	FMULA1,SUMX2
217.	TFL	SUMY,ZERO	267.	TFL B1,SUMX
218.	TFL	SUMXY,ZERO	268.	FMULB1,SUMX
219.	TFL	SUMX2,ZERO	269.	FSUBA1,B1
220.	TFM	Q,0,9	270.	FDIVSLOPE,A1
221.	FIGURETF	PMQ,P	271.	TFL AINT,SUMY
222.	S	PMQ,Q	272.	TFL A1,SLOPE
223.	MM	PMQ,2,9	273.	FMULA1,SUMX
224.	SF	95	274.	FSUBAINT,A1
225.	TF	N1,99	275.	FDIVAJNT,FNG
226.	A	N1,TABLE	276.	SF SLOPE-2
227.	TFL	A2F,ZERO	277.	TFL XINT,AINT
228.	TF	A2,N1,11	278.	FDIVXINT,SLOPE
229.	BD	RASP,A2-1	279.	CF SLOPE-2
230.	TD	A2F-7,A2	280.	TFL EFM,FCONS
231.	SF	A2F-7	281.	FDIVEFM,SLOPE
232.	TFM	A2F,1,10	282.	TFM Q,0,9
233.	B	*+36	283.	TFL SIG2,ZERO
234.	RASP TF	A2F-6,A2	284.	RATS TF PMQ,P
235.	TFM	A2F,2,10	285.	S PMQ,Q
236.	MM	PMQ,8,9	286.	MM PMQ,2,9
237.	SF	95	287.	SF 95
238.	TF	N2,99	288.	TF N1,99
239.	A	N2,TABLE+5	289.	A N1,TABLE
240.	TFL	A1,N2,11	290.	TF A2,N1,11
241.	FMULA1	A2F	291.	TFL A2F,ZERO
242.	FADDSUMX	A2F	292.	BD BFTSPL,A2-1

293.	TD	A2F-7,A2	343.	TDM	OUT3+31,7
294.	SF	A2F-7	344.	BD	**+36,NG-1
295.	TFM	A2F,1,10	345.	TDM	OUT3+32,0
296.	B	**+36	346.	TDM	OUT3+31,0
297.	BFTSPLTF	A2F-6,A2	347.	WACDOUT1	
298.	TFM	A2F,2,10	348.	TD	OUT5+4,AINT-7
299.	MM	PMQ,8,9	349.	TD	OUT5+6,AINT-6
300.	SF	95	350.	TD	OUT5+8,AINT-5
301.	TF	N2,99	351.	TD	OUT5+10,AINT-4
302.	A	N2, TABLE+5	352.	TD	OUT5+16,AINT-1
303.	TFL	A1,A2F	353.	TD	OUT5+18,AINT
304.	FMULA1,	SLOPE	354.	TDM	OUT5+13,0
305.	FADDA1,	AINT	355.	BNF	**+24,AINT
306.	TFL	C1,N2,11	356.	TDM	OUT5+13,2
307.	FMULC1,	A2F	357.	TDM	OUT5-1,0
308.	TFL	NUM2,ONE	358.	BNF	**+24,AINT-2
309.	FDIVNUM2,	C1	359.	TDM	OUT5-1,2
310.	TFL	B1,ONE	360.	TD	OUT6+4,SLOPE-7
311.	FDIVB1,	A1	361.	TD	OUT6+6,SLOPE-6
312.	FSUBB1,	NUM2	362.	TD	OUT6+8,SLOPE-5
313.	FMULB1,	B1	363.	TD	OUT6+10,SLOPE-4
314.	FADDSIG2,	B1	364.	TD	OUT6+16,SLOPE-1
315.	AM	Q,1,9	365.	TD	OUT6+18,SLOPE
316.	C	Q,NG	366.	TDM	OUT6+13,0
317.	BNE	RATS	367.	BNF	**+24,SLOPE
318.	TFL	C1,FNG	368.	TDM	OUT6+13,2
319.	FSUBC1,	ONE	369.	TDM	OUT6-1,0
320.	FDIVSIG2,	C1	370.	BNF	**+24,SLOPE-2
321.	FSQRSD,	SIG2	371.	TDM	OUT6-1,2
322.	FMULSD,	FCONS	372.	TD	OUT4+18,XINT-7
323.	TD	OUT2+4,EFM-7	373.	TD	OUT4+20,XINT-6
324.	TD	OUT2+6,EFM-6	374.	TD	OUT4+22,XINT-5
325.	TD	OUT2+8,EFM-5	375.	TD	OUT4+24,XINT-4
326.	TD	OUT2+10,EFM-4	376.	TD	OUT4+30,XINT-1
327.	TD	OUT2+16,EFM-1	377.	TD	OUT4+32,XINT
328.	TD	OUT2+18,EFM	378.	TDM	OUT4+27,0
329.	TDM	OUT2+13,0	379.	BNF	**+24,XINT
330.	BNF	**+24,EFM	380.	TDM	OUT4+27,2
331.	TDM	OUT2+13,2	381.	TDM	OUT4+13,0
332.	TD	OUT3+4,SD-7	382.	BNF	**+24,XINT-2
333.	TD	OUT3+6,SD-6	383.	TDM	OUT4+13,2
334.	TD	OUT3+8,SD-5	384.	WACDOUT4	
335.	TD	OUT3+10,SD-4	385.	TFM	Q,0,9
336.	TD	OUT3+16,SD-1	386.	BRAP	TF
337.	TD	OUT3+18,SD	387.	S	PMQ,Q
338.	TDM	OUT3+13,0	388.	MM	PMQ,8,9
339.	BNF	**+24,SD	389.	SF	95
340.	TDM	OUT3+13,2	390.	TF	N1,99
341.	TD	OUT3+34,NG	391.	A	N1, TABLE+5
342.	TD	OUT3+32,NG-1	392.	MM	PMQ,2,9

393.		SF	95	443.	NG	DS	2
394.		TF	N2,99	444.	TDSF	DS	2
395.		A	N2, TABLE	445.	A2	DS	2
396.		TFL	A1, N1, 11	446.	TDS	DS	2
397.		TF	A2, N2, 11	447.	PMQ	DS	3
398.		TFL	A2F, ZERO	448.	NN	DS	3
399.		BD	BONK, A2-1	449.	DD	DS	3
400.		TD	A2F-7, A2	450.	NM1	DS	3
401.		SF	A2F-7	451.	R	DS	3
402.		TFM	A2F, 1, 10	452.	Q	DS	3
403.		B	*+36	453.	P	DS	3
404.	BONK	TF	A2F-6, A2	454.	N	DS	3
405.		TFM	A2F, 2, 10	455.	NMIN2	DS	3
406.		FMULA	A2F, A1	456.	NMIN1	DS	3
407.		TD	OUT7+6, A2F-7	457.	N4	DS	5
408.		TD	OUT7+8, A2F-6	458.	N3	DS	5
409.		TD	OUT7+10, A2F-5	459.	N2	DS	5
410.		TD	OUT7+12, A2F-4	460.	N1	DS	5
411.		TD	OUT7+18, A2F-1	461.	N5	DS	5
412.		TD	OUT7+20, A2F	462.	N6	DS	5
413.		TD	OUT7+30, A2-1	463.	FREQ	DS	8
414.		TD	OUT7+32, A2	464.	EFM	DS	8
415.		TDM	OUT7+29, 7	465.	FNG	DS	8
416.		BD	*+36, A2-1	466.	SD	DS	8
417.		TDM	OUT7+29, 0	467.	SIG2	DS	8
418.		TDM	OUT7+30, 0	468.	FCONS	DS	8
419.		TD	OUT7+42, A1-7	469.	F1	DS	8
420.		TD	OUT7+44, A1-6	470.	XINT	DS	8
421.		TD	OUT7+46, A1-5	471.	A2F	DS	8
422.		TD	OUT7+48, A1-4	472.	AIN	DS	8
423.		TD	OUT7+54, A1-1	473.	SLOPE	DS	8
424.		TD	OUT7+56, A1	474.	SUMX2	DS	8
425.		WACDOUT	7	475.	SUMXY	DS	8
426.		AM	Q, 1, 9	476.	SUMY	DS	8
427.		C	Q, NG	477.	SUMX	DS	8
428.		BNE	BRAP	478.	C1	DS	8
429.		AM	P, 1, 9	479.	B1	DS	8
430.		C	P, N	480.	A1	DS	8
431.		BE	*+60	481.	HH	DS	8
432.		TF	DD, N	482.	QF	DS	8
433.		SM	DD, 1, 9	483.	NUM2	DS	8
434.		C	P, DD	484.	NUM1	DS	8
435.		BNE	NEWPR	485.	DX2	DS	8
436.		TFM	N, 0, 9	486.	DX1	DS	8
437.		B	READ	487.	H	DS	8
438.	M	DSB	2, 1000	488.	CX	DS	8
439.	OH	DSB	8, 1000	489.	F2	DS	12
440.	TABLE	DSA	M, OH	490.	DC		6,000000
441.	INPUT	DAS	80	491.	ZERO	DC	2,-99
442.	BLANK	DAS	80	492.		DC	6,628319

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493. TWOP1 DC 2,02
494.      DC 6,175880
495. CONS DC 2,00
496.      DC 6,100000
497. ONE  DC 2,01
498. NUFREQDAC 37,A NEW VALUE OF FREQUENCY WAS READ IN'
499. NOFREQDAC 35,THERE IS NO VALUE FOR FREQ DEFINED'
500. TWOANGDAC 40,TWO ANGLE CARDS WERE READ CONSECUTIVELY'
501. SCREW DAC 21,INPUT DATA INCORRECT'
502. OUT1 DAC 21,ANGLE = 000.0 M* = ,
503. OUT2 DAC 19, .0000E 00 +OR- = ,
504. OUT3 DAC 18, .0000E 00 N = 00,
505.      DAS 22
506. OUT4 DAC 28,XINT = .0000E 00 YINT = ,
507. OUT5 DAC 27, .0000E 00(1/G) SLOPE = ,
508. OUT6 DAC 15, .0000E 00(1/G),
509.      DAS 10 -
510. OUT7 DAC 29, .0000E-00 00 .0000E-00,
511.      DAS 51
512.      DENDBEGIN

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Fig. 12

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VITA

Wilbur Lynn Dahlquist was born September 2, 1942, in DeQuincy, Louisiana. His secondary education was completed in DeQuincy, Louisiana and in September 1960 he enrolled in McNeese State College. He graduated Magna Cum Laude with a Bachelor of Science degree in physics from McNeese in August 1963. In January 1961, he was married to the former Charline Day of DeQuincy, Louisiana. In September 1963, he enrolled for a graduate curriculum in physics at Louisiana State University, Baton Rouge, Louisiana and is a candidate for the degree of Doctor of Philosophy. He has held a N.A.S.A. Pre-doctoral Fellowship for the years 1964-1967 and is a member of the American Physical Society.

EXAMINATION AND THESIS REPORT

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Major Field: Physics

Title of Thesis: Cyclotron Resonance in Thallium

Approved:

R. G. Goodrich

Major Professor and Chairman

Max Goodrich

Dean of the Graduate School

EXAMINING COMMITTEE:

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Date of Examination:

July 14, 1967