11-1-2012

Structural thermal noise in gram-scale mirror oscillators

Abraham R. Neben
LIGO, Massachusetts Institute of Technology

Timothy P. Bodiya
LIGO, Massachusetts Institute of Technology

Christopher Wipf
LIGO, Massachusetts Institute of Technology

Eric Oelker
LIGO, Massachusetts Institute of Technology

Thomas Corbitt
LIGO, Massachusetts Institute of Technology

See next page for additional authors

Recommended Citation

Neben, A., Bodiya, T., Wipf, C., Oelker, E., Corbitt, T., & Mavalvala, N. (2012). Structural thermal noise in gram-scale mirror oscillators. *New Journal of Physics, 14* https://doi.org/10.1088/1367-2630/14/11/115008

Authors

Abraham R. Neben, Timothy P. Bodiya, Christopher Wipf, Eric Oelker, Thomas Corbitt, and Nergis Mavalvala

New Journal of Physics
The open access journal at the forefront of physics

Deutsche Physikalische Gesellschaft DPG IOP Institute of Physics

Structural thermal noise in gram-scale mirror oscillators

To cite this article: Abraham R Neben *et al* 2012 *New J. Phys.* **14** 115008

New Journal of Physics
The open access journal for physics

Structural thermal noise in gram-scale mirror oscillators

Abraham R Neben[1], **Timothy P Bodiya**[1], **Christopher Wipf**[1], **Eric Oelker**[1], **Thomas Corbitt**[1,2] **and Nergis Mavalvala**[1,3]
[1] LIGO Laboratory, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
[2] Department of Physics, Louisiana State University, Baton Rouge, LA, USA
E-mail: nergis@ligo.mit.edu

New Journal of Physics **14** (2012) 115008 (10pp)
Received 13 July 2012
Published 12 November 2012
Online at http://www.njp.org/
doi:10.1088/1367-2630/14/11/115008

Abstract. The thermal noise associated with mechanical dissipation is a ubiquitous limitation to the sensitivity of precision experiments ranging from frequency stabilization to gravitational wave interferometry. We report on the thermal noise limits to the performance of 1 gm mirror oscillators that are part of a cavity optomechanics experiment to observe quantum radiation pressure noise. Thermal noise limits the observed cavity displacement spectrum from 80 Hz to 5 kHz. We present a calculation of the thermal noise, based on finite element analysis of the dissipation due to structural damping, and find it to be in excellent agreement with the experimental result. We conclude with the predicted thermal noise for an improved oscillator design, which should be capable of revealing the noise that arises from quantum backaction in this system.

---

[3] Author to whom any correspondence should be addressed.

New Journal of Physics **14** (2012) 115008
1367-2630/12/115008+10$33.00

Contents

1. Introduction

All mechanical systems exhibit thermally driven fluctuations. Callen and Greene give a general description of thermal noise, the fluctuation dissipation theorem, which relates the level of thermal noise at each frequency to the amount of damping at that frequency [1]. Such damping is well modeled with a complex spring constant $k \rightarrow k(1 + \mathrm{i}\phi(f))$, where $\phi(f)$ is known as the loss angle. When dissipation arises from the familiar velocity-dependent damping force due to, for instance, air resistance, it results in *viscous* damping, with $\phi(f) \propto f$. However, Saulson [2] proposed that in high vacuum with appropriate isolation, the damping in a mirror-suspension system (such as those used in gravitational wave detectors) would likely be *structural*: that is, dominated by internal friction in the suspension and mirror materials, with $\phi(f)$ constant over the frequency band of interest. While the displacement noise due to either form of damping is identical near a mechanical resonance, away from resonance the two forms look quite different, with the amplitude spectral density of the structural form *steeper* by a factor of $f^{-1/2}$. The thermal noise spectrum of mechanical resonance has been well studied, but in fact it is the off-resonant behavior that reveals the underlying dissipation mechanism.

This thermal noise due to mechanical dissipation provides a fundamental limitation to many precision measurement experiments, such as gravitational wave detectors [3], frequency stabilization [4, 5], opto and electro-mechanics [6] and tests of deviations from Newtonian gravity via torsion pendulum. Most measurements of thermal noise exhibit non-structural damping [6, 7]. The exceptions to date include measurements at very low frequencies, below 10 Hz, in reference cavities [4, 5] and in torsion pendulums used for testing deviations from Newtonian gravity [8].

Here we report on an interferometric displacement measurement limited by *off-resonance* suspension thermal noise in the broad frequency band 80–5000 Hz, achieving a maximum displacement sensitivity of $2 \times 10^{-17}\,\mathrm{m}\,\sqrt{\mathrm{Hz}}^{-1}$ at 5 kHz. Our measurement over that band matches thermal noise predictions made from a finite element model of the mirror and its suspension. Our experiment is uniquely sensitive to structural thermal noise at frequencies below a few kHz due to the mm-scale optical beam diameter, which reduces the impact of coating thermal noise and thermoelastic substrate noise [9]. This is in strong contrast to previous thermal noise measurements, where much smaller beam sizes (<0.1 mm) were used in order to magnify the effect of thermal noise [10–12]. Moreover, in the 80–200 Hz band our thermal noise is primarily due to *above*-resonance thermal noise. Our results should, therefore, be of interest both to those in the cavity optomechanics community working toward quantum noise limited experiments [6] and those in the gravitational wave community contending with structural thermal noise in the design of mirror suspensions for interferometric gravitational wave detectors [3].

2. Theory

The fluctuation dissipation theorem provides a unified way of understanding thermal noise. It relates the power spectral density S_x of fluctuations in a generalized coordinate x to its dissipation into a coupled thermal reservoir at temperature T as:

$$S_x(f) = \frac{k_{\mathrm{B}}T}{\pi^2 f^2}|\Re[Y(f)]|, \tag{1}$$

where k_{B} is Boltzmann's constant, and $Y(f)$ is the generalized mechanical admittance at frequency f. The admittance acquires a real part when the system has some loss. It is computed from the complex velocity response $v(f)$ of the coordinate to an oscillating driving force $F(f)$:

$$Y(f) = v(f)/F(f) = \mathrm{i}(2\pi f)x(f)/F(f). \tag{2}$$

For systems with a single degree of freedom, the admittance can be calculated exactly from its equation of motion, and then equation (1) yields the exact thermal noise spectrum. For multi-mode or continuous systems, one may often obtain a good approximation to the measured thermal noise by summing the thermal noise power spectral density due to each mode, but only if the normal modes are reasonably orthogonal (a condition dependent on the modal Qs and frequencies) and sufficiently many modes are included in the sum. This approximation may be written as:

$$S_x(f) \approx \sum_{\mathrm{mode}\,n} \frac{4k_{\mathrm{B}}T}{8\pi^3 m_n f}\left(\frac{f_n^2\phi_n}{(f^2-f_n^2)^2+f_n^4\phi_n^2}\right), \tag{3}$$

where m_n is the modal mass of the nth mode with frequency f_n and loss angle ϕ_n. Typically at least the lowest 100 modes must be included to obtain good estimates for the mirror substrate thermal noise in a gravitational wave interferometer [13].

The extent to which each normal mode couples to the generalized coordinate x of interest is quantified by its change Δx_n under maximal expansion of that mode, via its modal mass $m_n = U/(\frac{1}{2}\omega_n^2\Delta x_n^2)$, where U is the stored energy in the mode. For a system made of several materials, the loss angle ϕ_n of mode n is the sum over the loss angles θ_i of the materials weighted by $\alpha_{n,i}$, the fraction of energy stored in that material at maximal expansion:

$$\phi_n = 1/Q_n = \sum_{\mathrm{material}\ i} \theta_i\alpha_{n,i}. \tag{4}$$

Based on the fluctuation dissipation theorem, Levin derived a 'direct approach' to calculate the exact thermal noise spectrum of mirror position as measured by an incident laser beam, as in an interferometer [14]. One calculates the mechanical admittance by applying a generalized force F, oscillating at frequency f, that drives only the generalized coordinate x. As measured by a

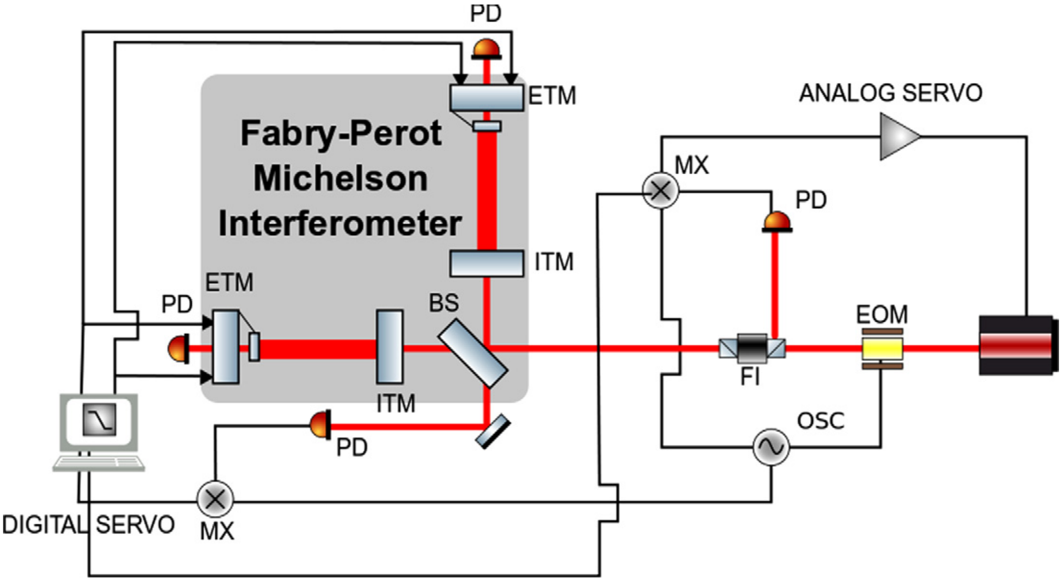

Figure 1. Schematic layout of the experiment. Several watts of 1064 nm laser power are incident on a Michelson interferometer that has Fabry–Perot cavities in each arm. The beam splitter (BS) and input mirrors (ITM) are 250 gm mirrors, while the end mirrors (ETM) are 1 gm mirrors suspended as pendulums. An electro-optic modulator (EOM) driven by a crystal oscillator (OSC) applies phase modulation sidebands on the laser light to generate Pound–Drever–Hall error signals for the arm cavities. Photodetectors (PD) at the different output ports are demodulated by mixers (MX) to derive error signals for the arm cavities and for the Michelson interferometer. Control signals generated using hybrid digital and analogue servo loops are fed back to the interferometer mirror positions and to the laser frequency.

laser, the mirror position is $x(t) = \int \mathrm{d}^2\vec{r}\, w(\vec{r}) y(\vec{r}, t)$, where $w(r)$ is the intensity weight of the beam at radius r, and $y(\vec{r}, t)$ is the deformation of the surface at position $\vec{r}$ in the $\hat{x}$-direction at time t. For a Gaussian beam, we have $w(r) = \mathrm{e}^{-(r/r_0)^2}/(\pi r_0^2)$, where r_0 is the radius at which the intensity has dropped to $1/e^2$. After finding the admittance, the power spectral density of the thermal noise is computed with equation (1). This formulation is particularly amenable to numerical calculation with finite element simulations.

In our analysis, we use Levin's direct approach to calculate predicted thermal noise levels, but the normal mode method provides a simple way to interpret those results qualitatively. For instance, thermal noise due to a particular mode often dominates in a particular frequency band, and knowledge of which mode is the culprit may suggest a route to mitigating it. Further, in a multi-material system, the thermal noise from the *lossiest* (highest ϕ) material may often completely overwhelm the thermal noise from lower loss components (see equation (4)) over certain frequency bands, even if the amount of lossy material is minuscule in comparison. We also observed that computing thermal noise via the normal modes approach with finite element models is about an order of magnitude faster than the Levin approach, and with the inclusion of the lowest ~100 modes, agreed at the ~30% level.

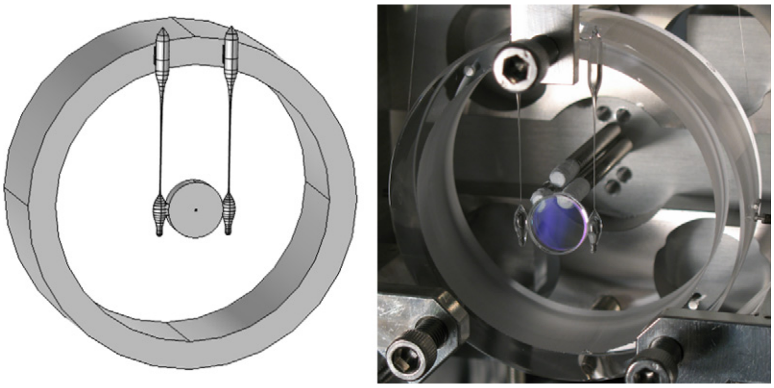

Figure 2. Left panel: COMSOL model of a 1 gm mirror suspended by fused silica fibers, showing the fibers that taper in the middle and are thicker at the ends where the epoxy bonding is done. The 1 gm mirror pendulum is attached to a fused silica ring that is itself suspended as a pendulum. This double pendulum system provides greater isolation from seismic noise, but has little impact on the accuracy of the model due to the much greater mass of the outer ring. Right panel: photograph of the 1 gm mirror suspension.

Table 1. Material properties and their assumed values.

Property	Fused silica	Vac-Seal epoxy
Density	$2.20\,\mathrm{g\,cm^{-3}}$	$1.01\,\mathrm{g\,cm^{-3}}$
Young's modulus	73.1 GPa	2.75 GPa[a]
Poisson's ratio	0.17	0.25[b]
Loss angle	3×10^{-7} [c]	0.035[d]

[a] Per manufacturer specification for comparable epoxies: http://www.masterbond.com

[b] The predicted thermal noise level depends very weakly on the Poisson ratio. The expected value is between 0 and 0.5, here we use the average.

[c] All fused silica loss occurs in the fibers where surface losses dominate. With minimum diameter ~150 μm, Gretarsson and Harry find this value [16].

[d] See discussion in text.

3. Experiment

A schematic of the experimental setup is shown in figure 1. The optomechanical system consists of two Fabry–Perot cavities with movable mirrors, embedded in the arms of a Michelson interferometer. The Michelson enables a sensitive readout of the differential displacement of the two cavities. A 1 gm mirror oscillator forms the end mirror of each cavity. The primary goal of the experiment is to observe the quantum backaction of the differential displacement measurement on the 1 gm mirrors. Details of the experimental design can be found in [15].

Central to the present work are the 1 gm mirror oscillators, which are suspended from fused silica fibers, as shown in figure 2. The fibers are bonded to the mirror with Vac-Seal epoxy (Tra-Con), see table 1. Each fiber is 40 mm in length, and tapers from a maximum diameter of 3 mm to a minimum of ~150 μm. A thicker region, called the 'ear', is located at the bottom, and is

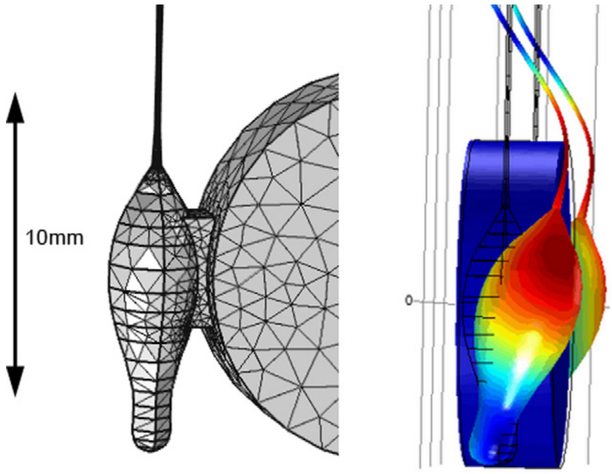

Figure 3. Left panel: 1 gm mirror suspension shown with the COMSOL mesh. The model was generated from high-resolution photographs of the mirror and ears. Right panel: image of the displacement of the twisting ear mode, with blue being least displaced and red being most displaced.

about 10 mm long and 1.5 mm in diameter. The shape of the ear was informed by the need for a steep spatial gradient at the transition between the small diameter fiber and the large diameter ear, which allows for better decoupling of ear and mirror motion. See figure 3, for a close-up of the ear region.

The purpose of the tapering fibers is to minimize the thermal noise of the fundamental mode of oscillation (which occurs at about 10 Hz), by ensuring that it stores energy mostly in the pristine fused silica near the middle of the fiber, and not in the lossy epoxy at either end. Quality factors of 10^6 or greater are measured for the 10 Hz mode, suggesting that the thermal noise of this mode is far better than what is required to observe the quantum backaction. However, another thermal noise floor limits the measured displacement noise spectrum in the 80–5000 Hz band, as shown in figure 4. In the next section, we present the results of finite element simulations confirming that the ear-mirror epoxy bond is the origin of the observed thermal noise.

4. Thermal noise analysis

The finite element method is an approach to solving for continuum dynamics in various physical scenarios by discretizing the geometry. We perform finite element simulations with the COMSOL Multiphysics package, using its structural mechanics module. With that software we model the three-dimensional geometry and then implement the Levin approach [14]. We apply a Gaussian load to the mirror face, and solve for the steady-state mirror response to the load oscillating at a set of frequencies $\{f\}$, yielding the complex admittance $Y(\{f\})$ directly. See table 1 for the material parameters used in the simulations. Using experimental data for different quantities of glue we infer the loss angle, $\phi_{\mathrm{epoxy}} = 0.035$, of the epoxy from the amplitude spectra density. This value results in excellent agreement between our experimental data and the output from the model. We take this result as confirmation that our displacement noise floor is dominated in the 80–5000 Hz band by structural thermal noise. The 80–200 Hz band is in the

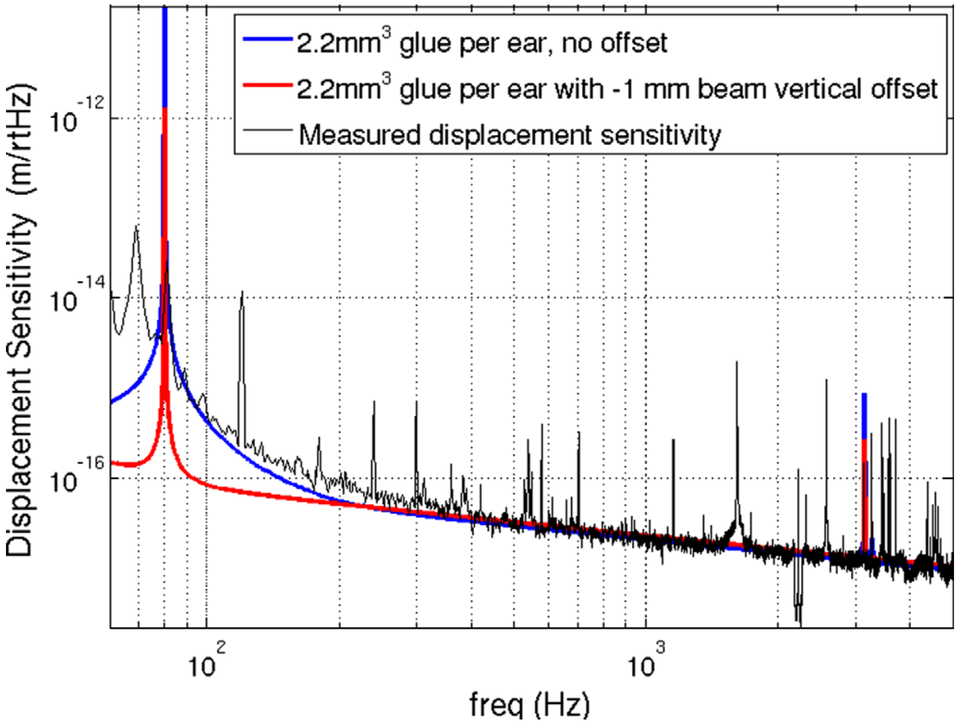

Figure 4. Thermal noise calculated from COMSOL model with normal mode method, along with the measured sensitivity, and the predicted sensitivity with a 0 and -1 mm vertical beam offset from the mirror center. Non-uniform optical losses over the mirror surface prevented us from exploring the full space of beam offsets. Experimental upgrades underway should allow us to explore a wider range of beam offsets in the future.

above-resonance 'wing' of the 80 Hz pitch mode. If all the pitch modal energy were in the very low loss fused silica, then we would expect to measure $Q = 1/\phi_{\mathrm{fusedsilica}} \sim 3 \times 10^6$. However, we measure only $\sim 5 \times 10^5$, indicating both that pitch thermal noise is setting our noise floor in that band, and that epoxy loss is dominating that noise. In the 200–5000 Hz band the spectrum shows the characteristic $f^{-1/2}$ below-resonance thermal noise spectrum of a structurally damped oscillator, which is well reproduced by simulations. The modes responsible for this noise are the bending modes of the ear–mirror joint, with resonant frequencies starting around 30 kHz, outside the band of our measurement. See figure 3, for an example of one such ear–mirror bending mode.

5. Thermal noise mitigation

Various schemes have been presented to reduce the level of thermal noise in an interferometer [17–20], relying either on carefully tuned passive cancelations or multiple measurements to disentangle the thermal noise from displacement signal. Our modeling results suggest several simpler geometric ways to lower our noise floor.

In the 80–200 Hz band, the measured spectrum is dominated by the thermal noise from the pitch mode of the oscillator. Introducing a vertical offset of the incident laser beam [14],

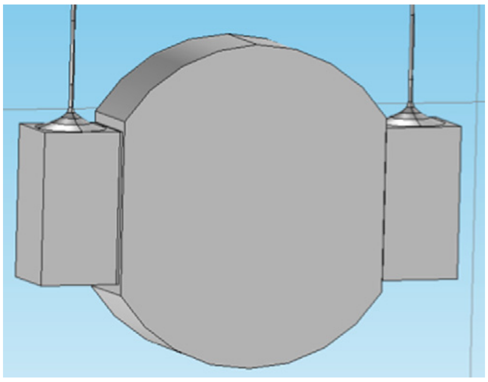

Figure 5. COMSOL model of chopped mirror suspended with square ears.

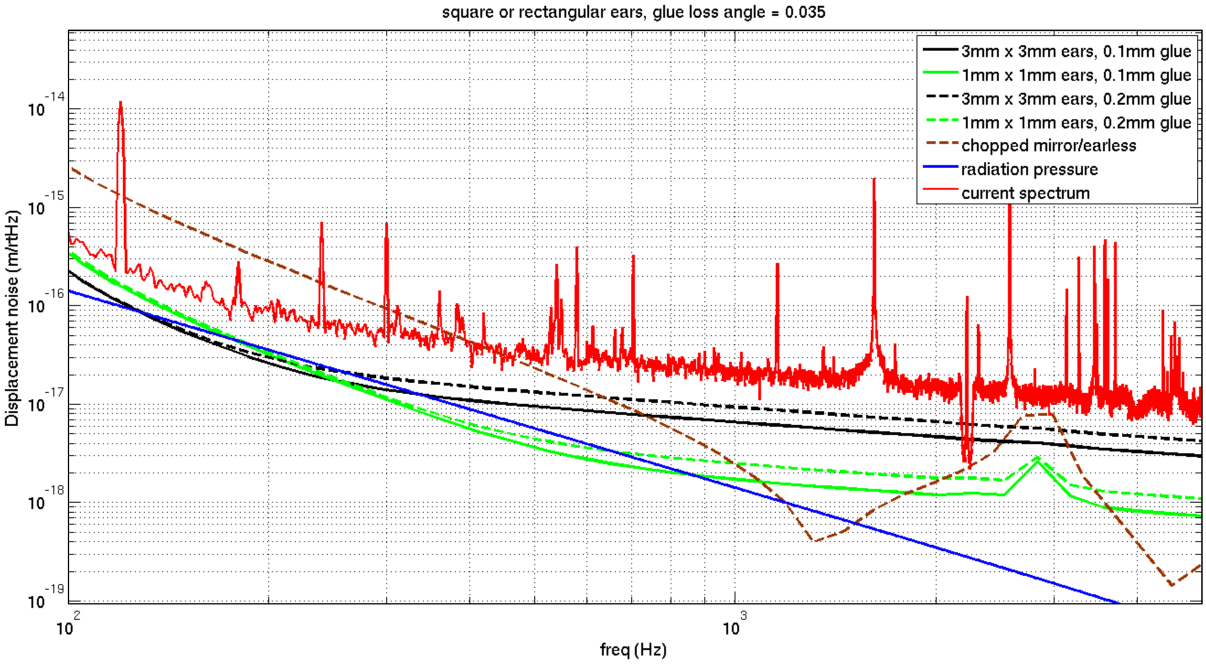

Figure 6. Predicted thermal noise for various alternative ear geometries from COMSOL simulations.

shifting it ≈1 mm from the center of the mirror, places the beam near the node of the pitch mode. This offset should decrease the sensed pitch thermal noise (by increasing the modal mass), as depicted in figure 4.

Above 200 Hz, the thermal noise of the mirror–ear bending modes takes over. As the loss angle ϕ_{epoxy} is five orders of magnitude larger than $\phi_{\text{fusedsilica}}$ (for ∼150 μm fibers [16]), it is the epoxy that dominates both the pitch and the mirror–ear bending thermal noise. To minimize the effect of this thermal noise in our measurement band, we studied alternative mirror–ear geometries to maximize the strength of the ear–mirror bond per amount of epoxy. By maximizing this quantity, we redistribute the thermal noise of the mirror–ear modes to higher frequencies, out of the measurement band. This quantity of merit is optimized in the case of plane-on-plane contact, and we propose a design that incorporates ears with square cross-section, and 'chopped' mirrors with flattened regions on the circular edge. Figure 5 shows a model of this square-ears/chopped-mirror geometry, and figure 6 shows the thermal noise for variations on that geometry predicted from our modeling. We expect these square ears to be the most expedient path forward, and are pursuing this design for the next iteration of the end mirror suspension. This chopped geometry is also compatible with other adhesives such as silicate bonding [21].

6. Conclusion

We have experimentally demonstrated and theoretically analyzed the structural thermal noise-limited performance of a 1 gm mirror oscillator, finding excellent agreement between the model and the measurement. We have also proposed some avenues to reduce the thermal noise floor in this experiment. The observation of off-resonance thermal noise has importance for a broad class of experiments in cavity optomechanics and gravitational wave interferometry. Accurate modeling of these complex structures depends on detailed knowledge of their geometry and material properties, but lets us better understand and optimize their mechanical design. The optimizations we have presented should permit our experiment to enter the quantum backaction-limited regime.

Acknowledgments

We thank our colleagues at the LIGO Laboratory, especially Rich Mittleman and Myron McInnis for experimental support. This work is supported by NSF grant PHY-1068772 and in part by the Department of Energy Office of Science Graduate Fellowship Program (DOE SCGF), made possible in part by the American Recovery and Reinvestment Act of 2009, administered by ORISE-ORAU under contract no. DE-AC05-06OR23100. E Oelker gratefully acknowledges support from the DOE Office of Basic Sciences. N Mavalvala gratefully acknowledges support from the MacArthur Foundation.

References

[1] Callen H B and Greene R F 1952 On a theorem of irreversible thermodynamics *Phys. Rev.* **86** 702–10

[2] Saulson P R 1990 Thermal noise in mechanical experiments *Phys. Rev.* D **42** 2437–45

[3] Scientific LIGO Collaboration 2009 LIGO: the laser interferometer gravitational-wave observatory *Rep. Prog. Phys.* **72** 076901

[4] Numata K, Kemery A and Camp J 2004 Thermal-noise limit in the frequency stabilization of lasers with rigid cavities *Phys. Rev. Lett.* **93** 250602

[5] Ludlow A D, Huang X, Notcutt M, Zanon-Willette T, Foreman S M, Boyd M M, Blatt S and Ye J 2007 Compact, thermal-noise-limited optical cavity for diode laser stabilization at 1×10^{-15} *Opt. Lett.* **32** 641–3

[6] Kippenberg T J and Vahala K J 2008 Cavity optomechanics: back-action at the mesoscale *Science* **321** 1172–6

[7] Starr J B 1990 Squeeze-film damping in solid-state accelerometers *Solid-State Sensor and Actuator Workshop 1990, 4th Technical Digest, IEEE* pp 44–7

[8] Adelberger E G, Heckel B R and Nelson A E 2003 Tests of the gravitational inverse-square law *Annu. Rev. Nucl. Part. Sci.* **53** 77–121

[9] Harry G M *et al* 2002 Thermal noise in interferometric gravitational wave detectors due to dielectric optical coatings *Class. Quantum Grav.* **19** 897

[10] Numata K 2003 Direct measurement of mirror thermal noise *PhD Thesis* University of Tokyo

[11] Numata K, Ando M, Yamamoto K, Otsuka S and Tsubono K 2003 Wide-band direct measurement of thermal fluctuations in an interferometer *Phys. Rev. Lett.* **91** 260602

[12] Black E D, Villar A and Libbrecht K G 2004 Thermoelastic-damping noise from sapphire mirrors in a fundamental-noise-limited interferometer *Phys. Rev. Lett.* **93** 241101

[13] Gillespie A and Raab F 1995 Thermally excited vibrations of the mirrors of laser interferometer gravitational-wave detectors *Phys. Rev.* D **52** 577–85

[14] Levin Yu 1998 Internal thermal noise in the LIGO test masses: a direct approach *Phys. Rev.* D **57** 659–63

[15] Corbitt T, Chen Y, Khalili F, Ottaway D, Vyatchanin S, Whitcomb S and Mavalvala N 2006 Squeezed-state source using radiation-pressure-induced rigidity *Phys. Rev.* A **73** 023801

[16] Gretarsson A M and Harry G M 1999 Dissipation of mechanical energy in fused silica fibers *Rev. Sci. Instrum.* **70** 4081–7

[17] Kimble H J, Lev B L and Ye J 2008 Optical interferometers with reduced sensitivity to thermal noise *Phys. Rev. Lett.* **101** 260602

[18] Zhao Y, Wilson D J, Ni K-K and Kimble H J 2012 Suppression of extraneous thermal noise in cavity optomechanics *Opt. Express* **20** 3586–612

[19] Khalili Ya F 2005 Reducing the mirrors coating noise in laser gravitational-wave antennae by means of double mirrors *Phys. Lett.* A **334** 67–72

[20] Mours B, Tournefier E and Vinet J-Y 2006 Thermal noise reduction in interferometric gravitational wave antennas: using high order TEM modes *Class. Quantum Grav.* **23** 5777

[21] Sneddon P H, Bull S, Cagnoli G, Crooks D R M, Elliffe E J, Faller J E, Fejer M M, Hough J and Rowan S 2003 The intrinsic mechanical loss factor of hydroxy-catalysis bonds for use in the mirror suspensions of gravitational wave detectors *Class. Quantum Grav.* **20** 5025