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The Magnetoacoustic Effect in Thallium.

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THE MAGNETOACOUSTIC EFFECT IN THALLIUM

A Dissertation

Submitted to the Graduate Faculty of the
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in partial fulfillment of the
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in

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by
Julian Barham Coon
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ABSTRACT

Geometric resonances in 99.9999⁰/₀ purity thallium metal have been observed with longitudinal sound waves propagated along the three major axes at frequencies up to 162 Mc. The calipers of both sections of the third band hole surface and those assigned to the fourth band electron surfaces are in general agreement with the band calculation of Soven, even though some data associated with the fourth band appears to agree well with the nearly-free electron theory. Evidence of open orbits in the $[10\bar{1}0]$ direction has been seen which is in agreement with the predicted degeneracy of the third and fourth bands along AL. No open orbits were seen in the $[0001]$ direction, establishing further evidence for the non-connectivity of the fourth band electron surface in the c-direction.

The theory of Pippard for the absorption of energy from a sound wave due to interactions with electrons in metals is discussed and expanded in the appendices. Results are obtained for longitudinal waves when $\omega\tau < 1$ for both $q\ell \gg 1$ and $q\ell \ll 1$ in the absence of a magnetic field. The theory is also presented for the attenuation of longitudinal sound waves in the presence of a transverse magnetic field under the conditions $q\ell \gg 1$, $\omega_c\tau \gg 1$, $\omega\tau < 1$. The oscillatory behavior of the attenuation coefficient corresponding to geometric resonances is discussed.

1. INTRODUCTION

In recent years, rapid progress has been made in the experimental determination of the Fermi surface in metals. The success of methods such as the de Haas van Alphen effect, cyclotron resonance, and the magnetoacoustic effect has to a large extent been due to the advances in the theory of the behavior of electrons in metals. Harrison¹ presented the nearly-free electron Fermi surfaces for a large group of metals in 1960. The results were modified for the hcp metals when Cohen and Falicov² showed that due to spin orbit coupling, the degeneracy along the AHL plane of the hexagonal zone was removed. Later Cohen and Falicov³ demonstrated that magnetic breakdown should alter the topology of the nearly-free electron Fermi surface, and Blount⁴ predicted that these effects should begin to occur at magnetic fields determined by $\hbar\omega_c \approx E_g^2/E_F$.

The usefulness of the theory has been well confirmed by experiment in a group of the divalent as well as some of the higher valency metals. Thallium, which has the principal sections of the Fermi surface centered in the AHL plane, is a logical choice for a meaningful extension of the comparison between experiment and theory.

The electronic properties of thallium were first investigated by Alekseevskii and Gaidukov,⁵ who measured the transverse magnetoresistance in fairly high magnetic fields. Mackintosh, Spanel, and Young⁶ performed the same type of measurements, and their interpretation was consistent with the nearly-free electron picture. Ultrasonic attenuation measurements have been performed by Rayne⁷ and by Eckstein, Ketterson, and

Priestley⁸ (hereafter referred to as EKP), and Priestley⁹ has also measured the de Haas van Alphen effect. Soven^{10,11} performed a relativistic O.P.W. calculation for the Fermi surface of thallium, which along with the nearly-free electron results, will be compared to the present results. In most of the instances where the present results overlap those of EKP and Rayne, the assignments of the calipers to the different portions of the Fermi surface are in general agreement.

The present experiment uses the method of geometric resonances and to a lesser extent, open orbit resonances to study the major features of the low magnetic field Fermi surface of thallium. Each of three single crystals of high purity (99.9999%) hcp thallium metal were prepared so that sound waves could be propagated along one of the three major crystallographic axes in the presence of a transverse magnetic field. The attenuation coefficient was recorded by the method discussed in the section on experimental procedures. The resulting curves were analyzed and the values of the calipers are presented in the form of polar plots and tables. The interpretation of the data is discussed in the section dealing with experimental results, and is in acceptable agreement with the predictions of Soven. Effects were observed which are probably related to magnetic breakdown occurring at or near the points of degeneracy of the third and fourth bands in the AHL plane, and are discussed in detail. Evidence of open orbits in the $[10\bar{1}0]$ direction was seen, but since the line shape of the open orbit resonances could not be studied, no definite information concerning the open orbit was obtained. The absence of resonances corresponding to open orbits in the c-direction is interpreted as evidence that the fourth band electron surface does not contact the Γ MK plane near the

corners of the Brillouin zone, contrary to the predictions of the nearly-free electron theory.

The theory of acoustic attenuation in metals is discussed in Appendices I and II. The development follows that of Pippard, wherein the physical processes by which the electrons absorb energy from the passing sound wave are discussed using the hard Fermi surface approximation.

An approach for a general solution of the attenuation coefficient in the absence of a magnetic field under the restriction $\omega\tau < 1$ is presented. A particular solution is obtained for purely longitudinal sound waves, and is evaluated in the free-electron approximation. Both the case of $q\ell \gg 1$ and $q\ell \ll 1$ are discussed. The solutions are in agreement with the results of other investigators.

The physical considerations used in obtaining a solution for the attenuation coefficient in the absence of a magnetic field are then applied to the case of attenuation in the presence of a transverse magnetic field. The solution is carried out in detail for purely longitudinal waves propagated in a direction perpendicular to the applied magnetic field.

The assumption of $q\ell \gg 1$ is made and results obtained for the oscillatory geometric resonances. The periodicity of the oscillatory component in the attenuation coefficient in the reciprocal of the magnetic field is discussed as applied to the present experiment. The results are in agreement with both Pippard and Cohen, Harrison, and Harrison in the limit $\omega\tau < 1$ and $\omega_c\tau \gg 1$.

II. EXPERIMENT

A single transducer pulse-echo technique, similar to that described by Morse,¹² was employed during the course of this investigation. The pulsed-oscillator, receiver, and cathode ray tube assembly was a self-contained unit, the Sperry Products¹³ Attenuation Comparator, style 56A001. The video output from the comparator was fed to a pulse echo selector-demodulator unit designed by Kamm and Bohm.¹⁴ The only modification was the omission of the final stage logarithmic converter. It was found more convenient to deliver the output of the demodulator unit to a Hewlett Packard¹³ Model 410 C voltmeter. This served the twofold purpose of offering a direct visual monitoring of the integrated pulse height, while furnishing the additional amplification necessary for input to the Moseley¹³ 60B Logarithmic Converter. The final signal, which was now proportional to the attenuation coefficient, was recorded on the Y-axis of a Moseley¹³ Model 2-D X-Y recorder with strip chart attachment. (A block diagram of the equipment is shown in Fig. 1.)

The magnetic field was provided by a Varian¹³ twelve inch magnet system and through use of the Fieldial regulation system, the magnetic field could be determined to within 0.5%.

The samples used in the experiment were prepared from a zone-refined bar of thallium metal obtained from Cominco Products Incorporated¹³ (quoted purity: 99.9999%). There was only one single crystal in the bar as received that was of suitable size for an ultrasonic specimen. It was spark machined into a rectangular parallelepiped approximately 14mm X 13mm X 5mm, with the normal to the large faces within $\pm 1^\circ$ of the [0001] axis.

To obtain the other samples the strain-anneal technique of crystal growth was adopted wherein a two inch length of the thallium bar was subjected to a longitudinal compression resulting in about a one percent deformation in length. The sample was then etched to remove the oxide coating and sealed in a Pyrex glass tube under vacuum. Next the sample was placed in an oven and annealed at 220°C for ten days. The annealing temperature was chosen to be 10°C below the temperature of the transition from the body centered cubic to the hexagonal close-packed phase.¹⁵ As a result of the annealing, four or five large single crystals were produced, and two were selected for use. These crystals were spark machined into rectangular parallelepipeds $13\text{mm} \times 13\text{mm} \times 3\text{mm}$ and $15\text{mm} \times 16\text{mm} \times 3\text{mm}$, the normal to the large faces of the former being along the $[11\bar{2}0]$ axis, while the normal to the large faces of the latter was along the $[10\bar{1}0]$ axis. Both orientations were within $\pm 1^{\circ}$, again checked by Laue X-ray photographs. Final surface preparation was achieved by spark planing. It was found that smooth, parallel faces could be obtained in this manner. The faces of one of the samples were polished by using dilute nitric acid as an acid polishing reagent. This procedure, although improving the appearance of the sample somewhat, did not yield a mirror-like surface, nor did it have any significant effect on the ultrasonic signal.

It should be mentioned that one of the crystals showed evidence of being slightly mosaic. However, this structure was of small extent compared to the bulk of the sample, and the orientations differed by less than 1° . No effects of the mosaic structure were observable in the data.

All three samples had mean free paths in excess of 2.0 mm at 1.2°K, indicating that the strain-anneal technique could produce samples as good as those grown from the melt. The mean free path was calculated by the usual relation

$$l = (n + 1/2) \pi \lambda \quad , \quad (1)$$

where λ was the wave length of sound at 15 Mc/sec, and n is the number of oscillations observed in the attenuation coefficient. The three samples had $n = 6, 5, \text{ and } 5$, respectively.

Since thallium oxidizes readily when exposed to the atmosphere, it was found necessary to store the samples in glycerine. Two days prior to each run, the sample was removed from the glycerine and cleansed with anhydrous methyl alcohol. A Valpey Corporation¹³ X-cut gold-plated quartz transducer, 1/4" in diameter with 15 Mc/sec fundamental, was immediately bonded to the sample using a Dow Corning¹³ 200 silicone oil, the viscosity of which was 2,000,000 centistokes. The sample was placed in the sample holder with a spring-loaded contact button applying light pressure to the transducer. The arrangement was then placed under vacuum for two days to allow the high viscosity bonding agent to be squeezed flat under the transducer. This technique was found to be entirely satisfactory, giving a strong echo pattern down to liquid helium temperatures with no evidence of faulty bonding.

The data was taken at 1.2°K to reduce the scattering of the electrons by thermal phonons thereby enhancing the oscillatory component of the attenuation curve.

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III. THEORY AND DATA ANALYSIS

The general theory of magnetoacoustic attenuation has been given by Cohen, Harrison, and Harrison¹⁶ and by Pippard.¹⁷ The principal result of their work, in the case of geometric resonances, is that the attenuation coefficient is periodic in the reciprocal of the magnetic field, and that this period is related to the k space caliper of the electron orbit. The method of Pippard is discussed in detail in Appendices I and II for the case of purely longitudinal sound waves in the presence of a transverse magnetic field, and for the readers' benefit, the notation in the two appendices is chosen to coincide with that of Pippard wherever possible.

In the geometry where $\hat{q}$, the wave vector of a longitudinal sound wave, is perpendicular to the applied magnetic field, the relation may be simply expressed as

$$k_D = \frac{e\lambda}{ch \Delta(1/H)} \quad , \quad (2)$$

where k_D is twice the radial caliper of the Fermi surface in the direction $\hat{q} \times \hat{H}$; λ is the wavelength of sound and $\Delta(1/H)$ is the period of the oscillations in reciprocal field. It is generally assumed that the value of k_D measured by the geometric resonance technique is associated with an extremal value of k assumed on the orbit. However, it is well recognized that this need not always be the case, and this point has been discussed by several authors.^{11,16,17} During the analysis of this experiment the assumption of measuring the extremal value of k will be made with few exceptions since this assumption gives very reasonable results. The exceptions will be discussed in the section on experimental results.

Theoretical considerations show that the third and fourth bands of thallium should be degenerate along the line AL as shown in Fig. 2. Soven predicts that the magnetic fields required to cause breakdown between the third and fourth bands in directions slightly different from $[10\bar{1}0]$ should be within the range of fields studied in this experiment. Hence, both open orbits and extended orbits should be possible with the magnetic field in the $[11\bar{2}0]$ direction, and further, if $\hat{q}$ is in the $[0001]$ direction, open orbit resonances^{18,19} should be observed in the attenuation coefficient.

The values of the magnetic field for which the open orbit resonances should occur are given by

$$H_n = \frac{ch k_o}{e\lambda n} \quad , \quad (3)$$

where k_o is the Brillouin zone extension along the orbit and n is an integer.

The analysis of the data was based primarily upon interpretation of graphs of the relation

$$1/H = an + c \quad , \quad (4)$$

where a and c are to be determined by the straight line fit, and n is an integer. Interpretation was complicated, however, by the appearance of the second harmonics of the strong oscillations at fields typically above 1200 gauss, and a considerable amount of harmonic mixing was also noted. Fortunately, it was found that by plotting the data using Eq. (4) for magnetic fields less than those above which the data exhibited high harmonic content, the periods could be determined with precisions varying from $\pm 2\%$ to $\pm 5\%$. Analysis of the data in the region of

high harmonic content could then be accomplished by assigning the remaining peaks to extensions of the lines plotted for the lower field region. These lines were then corrected to give a best fit for the overall data. In this manner, a self-consistent type of interpretation was possible, giving precisions ranging from $\pm 1\%$ to $\pm 5\%$.

Another aid in the interpretation of the data was the $n = 0$ phase shift, denoted by the c in Eq. (4). Although it was not possible to determine c for all of the oscillations, especially those whose relative amplitude had become small at higher fields, it was ascertainable for most of the shorter period, and therefore higher field, oscillations. Knowledge of the phase shift greatly facilitated the analysis in the high harmonic content regions.

IV. EXPERIMENTAL RESULTS

A. Geometric Resonances

The results for $\hat{q}$ parallel to $[0001]$ are shown in Table I, each column corresponding to a possible assignment, and the data is displayed in Fig. 3. The entries in column (1) are assigned to the third band hole portion of the Fermi surface which is centered on A, and is typically referred to as the "cookie" or "crown". The orbits traversed on the crown which give rise to dominant effects in the magnetoacoustic attenuation appear to be central orbits, i.e., those in a plane passing through A and perpendicular to the magnetic field. An example of this type of orbit is the path C-C shown in Fig. 4, and the corresponding caliper in the $[11\bar{2}0]$ direction of $1.83 \times 10^8 \text{ cm}^{-1}$ is in good agreement with Rayne, EKP, and Soven. As the magnetic field is rotated toward the $[11\bar{2}0]$ axis, it is observed that electrons on central crown orbits

TABLE 1. Caliper values for $\hat{q}$ in the [0001] direction. The magnetic field was rotated in the (0001) plane. Error limits presented refer only to uncertainties in the determination of the periods.

Angular Direction of Caliper	$k_D \times 10^{-8} \text{ cm}^{-1}$				
	(1)	(2)	(3)	(4)	(5)
3°	$1.79 \pm .02^b$	$.72 \pm .02^a$		$1.40 \pm .05$	$.18 \pm .02$
$0^\circ \equiv [11\bar{2}0]$	$1.83 \pm .02^b$	$.72 \pm .02^a$			$.14 \pm .02$
3°	$1.79 \pm .02^b$	$.72 \pm .02^a$		$1.40 \pm .05$	$.13 \pm .02$
6°	$1.72 \pm .02$	$.73 \pm .02$		$1.40 \pm .02^c$	$.16 \pm .02$
9°	$1.68 \pm .02$	$.66 \pm .02$		$1.25 \pm .02^c$	$.15 \pm .02$
12°	$1.61 \pm .02$	$.61 \pm .02$		$1.24 \pm .02^c$	$.15 \pm .02$
15°	$1.60 \pm .02^b$	$.64 \pm .03$	$.54 \pm .02$	$1.79 \pm .05^d$	$.16 \pm .02$
18°	$1.58 \pm .02^b$	$.60 \pm .03$	$.51 \pm .02^a$		$.13 \pm .02$
21°	$1.56 \pm .02^b$	$.63 \pm .02$	$.53 \pm .02^a$		$.13 \pm .02$
24°	$1.55 \pm .02$	$.65 \pm .03$	$.54 \pm .02^c$		$.14 \pm .02$
27°	$1.56 \pm .02$	$.61 \pm .02$	$.53 \pm .02^c$		$.13 \pm .02$
$30^\circ \equiv [10\bar{1}0]$	$1.56 \pm .02$	$.61 \pm .02$	$.52 \pm .02^c$		$.13 \pm .02$
33°	$1.54 \pm .02$	$.59 \pm .02$	$.51 \pm .02^c$		$.15 \pm .02$

a Oscillations corresponding to this caliper are dominant at low magnetic fields.

b Oscillations corresponding to this caliper are dominant at high magnetic fields.

c Oscillations corresponding to this caliper are dominant over the range of magnetic fields investigated.

d Weak period-not due to same portion of Fermi surface as the other entries in this column.

participate less in the magnetoacoustic attenuation. (See footnotes in Table I.)

When the magnetic field is brought to within 15° of the $[11\bar{2}0]$ axis, it is assumed non-central crown orbits are responsible for the observed calipers. In particular, the caliper of $1.56 \times 10^8 \text{ cm}^{-1}$ corresponding to the $[10\bar{1}0]$ direction is interpreted as due to the orbit B-B, Fig. 4. Note that the caliper due to the B-B orbit should be approximately the $[10\bar{1}0]$ projection of the $[11\bar{2}0]$ caliper obtained from the C-C orbit, thereby causing the ratio $k_{[10\bar{1}0]} / k_{[11\bar{2}0]}$ to have an approximate value of $\cos 30^\circ$. The observed value of the ratio is .85, which is within experimental error of the theoretical value of .866. It should be pointed out, however, that the $[10\bar{1}0]$ caliper observed by Rayne and EKP is in close agreement with the expected G-G caliper, suggesting that indeed a central orbit may be observed.

The calipers in column (2) are assigned to the fourth band network, the $[10\bar{1}0]$ caliper being due to the orbit shown by Sec. A'-A', Fig. 5. The calipers proceeding from $[10\bar{1}0]$ to $[11\bar{2}0]$ are expected to come from similar orbits centered on L, occurring between the Sec. A'-A' and the Sec. C'-C' as the field is rotated in the basal plane. Again the interpretation is strengthened by noting that the ratio of the $[10\bar{1}0]$ caliper to the $[11\bar{2}0]$ caliper in column (2) is .85, i.e., of the order of $\cos 30^\circ$. The broken lines in Fig. 3 outline the free electron caliper expected for those orbits and are seen to be in good agreement with the experimental points.

The calipers in column (3), Table I are assigned to a cut such as B'-B' and are seen to be in agreement with Soven. It is expected that this orbit should become unstable as the magnetic field is rotated from

$[11\bar{2}0]$ toward $[10\bar{1}0]$, and the present results indicate that the orbit indeed does disappear around 15° from the $[11\bar{2}0]$ direction.

The calipers in column (4) are assigned to non-central crown orbits similar to D-D which will be discussed in the section dealing with results for $\hat{q}$ parallel to the $[11\bar{2}0]$ axis. It is interesting to note that these orbits dominate the attenuation effects in directions 6° to 12° from $[11\bar{2}0]$, while becoming very weak as the magnetic field was made parallel to the $[11\bar{2}0]$ axis.

The assignment of column (5) could be made to the fifth band which consists of small pockets of electrons approximately in the shape of an ellipsoid of revolution centered on H and elongated along HK. While the orbits involved in calipering the fifth band with the magnetic field in the basal plane are stationary only over a small portion of the pocket, the condition of $\hat{q} \cdot \hat{v}$ small, which leads to large attenuation effects, is fulfilled over most of the surface. Another equally reasonable assignment would be the small pockets of third band holes centered on M. These surfaces would produce stationary orbits with the magnetic field in the basal plane, but should have a large curvature in the direction of $\hat{q}$. The periods corresponding to column (5) were determined from no more than three oscillations, giving rise to the large uncertainties reported, and therefore the values of the calipers cannot be used to differentiate between the possible assignments.

The highest convenient working frequency for $\hat{q}$ parallel to $[10\bar{1}0]$ was 106 Mc/sec, with two echoes visible over most of the range of magnetic fields studied. As in the case with $\hat{q}$ parallel to $[0001]$, harmonic content in high field data was quite large, but could be analyzed satisfactorily by the methods discussed in the data evaluation section.

TABLE II. Caliper values for $\hat{q}$ in the $[10\bar{1}0]$ direction. The magnetic field was rotated in the $(10\bar{1}0)$ plane. Error limits presented refer only to uncertainties in the determination of the periods.

Angular Direction of Caliper	$k_D \times 10^{-8} \text{ cm}^{-1}$				
	(1)	(2)	(3)	(4)	(5)
$90^\circ \equiv [11\bar{2}0]$	$1.75 \pm .02^b$	$.57 \pm .02$		$1.95 \pm .02$	$.77 \pm .02^a$
85°	$1.78 \pm .02^c$			$2.07 \pm .02$	$.35 \pm .05^d$
80°	$1.89 \pm .02$	$.55 \pm .02$		$2.38 \pm .02^c$	
75°	$1.92 \pm .02^b$	$.53 \pm .02^a$			
70°	$1.91 \pm .02^b$	$.56 \pm .02^a$			$2.22 \pm .02^d$
65°	$1.75 \pm .02^b$	$.55 \pm .02^a$			$.11 \pm .02$
60°	$1.41 \pm .02^b$	$.57 \pm .02^a$			$.11 \pm .02$
55°	$1.08 \pm .02^b$	$.53 \pm .02$			$.09 \pm .02$
50°	$.85 \pm .02^c$	$.47 \pm .02$			$.11 \pm .02$
45°	$.76 \pm .02^c$		$.86 \pm .02$		$.11 \pm .02$
40°	$.70 \pm .02^b$		$.80 \pm .02^a$		$.09 \pm .02$
35°	$.66 \pm .02^b$		$.73 \pm .02^a$		$.08 \pm .02$
30°	$.63 \pm .02^b$		$.72 \pm .02^a$		$.08 \pm .01$
25°	$.58 \pm .02^b$		$.69 \pm .02^a$		$.08 \pm .01$
20°	$.58 \pm .02^b$		$.68 \pm .02^a$		$.08 \pm .01$
15°	$.56 \pm .02^b$		$.67 \pm .02^a$		$.08 \pm .01$
10°	$.53 \pm .02^b$		$.65 \pm .02^a$		$.08 \pm .01$
5°	$.53 \pm .02^c$				$.08 \pm .01$
$0^\circ \equiv [0001]$	$.52 \pm .02^c$				$.08 \pm .01$
-5°	$.52 \pm .02^c$				$.09 \pm .01$

a Oscillations corresponding to this caliper are dominant at low magnetic fields.

b Oscillations corresponding to this caliper are dominant at high magnetic fields.

c Oscillations corresponding to this caliper are dominant over the range of magnetic fields studied.

d Weak oscillations - caliper unassigned.

- Assigned To Third Band Holes
- ▲ Assigned To Fourth Band Electrons
- Unassigned

The caliper values for $\hat{q}$ parallel to $[10\bar{1}0]$ and the magnetic field in the $(10\bar{1}0)$ plane are presented in Table II, and displayed in polar form in Fig. 6. The first column in Table II reports calipers that are interpreted as due to orbits around third band crown, and further it was seen that the crown orbits produced the largest effects in the magneto-acoustic attenuation. A plot of the data in column (1) along with the nearly-free electron surface is shown in Fig. 7. The $[11\bar{2}0]$ caliper of $1.75 \times 10^8 \text{ cm}^{-1}$ is seen to be approximately 3% smaller than is expected from theory, and will be discussed in the following section.

It is tempting to interpret the calipers in column (1) observed between approximately 10° and 20° from the $[11\bar{2}0]$ direction as due to a real bulge in the $AHK\Gamma$ section of the principal third band hole surface. However, by comparing the area of this hypothetical section to that predicted by Soven and observed by Priestley, it is found to be approximately 17% too large. A more reasonable assumption would be to attribute the large calipers observed over this narrow range of angles to the bumps occurring on the top edge of the crown as shown in Fig. 4. The assignment is strengthened by noting that the curvature of the orbit in the region of the bumps is smaller than the curvature of the orbit as it passes through the $AHK\Gamma$ plane. The result would be a large contribution to the attenuation due to that portion of the orbit in the region of the bumps.

The $[0001]$ caliper for the crown is measured from a simple orbit such as G-G, Fig. 4, and its magnitude is in good agreement with theory and other experiments.

Column (2) is tentatively assigned to the small pocket of third band holes, centered on M. This surface appears to be deformed from the

free electron picture, and was observed only for $\hat{q}$ in the $[10\bar{1}0]$ direction. The experimental $[11\bar{2}0]$ caliper of $.57 \times 10^8 \text{ cm}^{-1}$ is in fair agreement with Soven's prediction of $.60 \times 10^8 \text{ cm}^{-1}$.

Column (3) is assigned to a non-central set of orbits on the crown probably of the type B-B. This experiment indicates that the orbits become unstable as the magnetic field is tilted toward $[11\bar{2}0]$, and no caliper for this type of orbit is seen for the field along the $[11\bar{2}0]$ axis.

The assignment of column (4), Table II, to a particular portion of the Fermi surface is quite difficult if the predictions of either Soven or the free electron theory are valid for regions near the corner of the zone. The calculations of Soven show that the fourth band is certainly deformed from the nearly-free electron predictions around H, and its shape still unknown from experiment. It is thus tempting to assign the data of column (4) to hole orbits inside the fourth band similar to $D'-D'$. A firm assignment of these points awaits further experiment.

The calipers in column (5), except for the 3 first entries, may be assigned entirely to the fifth band, indicating less elongation than expected theoretically. An equally reasonable assignment would be to attribute the calipers in the $[0001]$ and nearby directions to the small pocket of third band holes centered on M, while leaving the remaining calipers as due to the fifth band. Additional data would be necessary to confirm either assignment.

With $\hat{q}$ parallel to the $[11\bar{2}0]$ axis, the attenuation was found to be much larger than for the other two directions, thus limiting the frequency to 75 Mc/sec, and only one strong echo could be seen over the range of

TABLE III. Caliper values for $\hat{q}$ in the $[11\bar{2}0]$ direction. The magnetic field was rotated in the $(11\bar{2}0)$ plane. Error limits presented refer only to uncertainties in the determination of the periods.

Angular Direction of Caliper	$k_D \times 10^{-8} \text{ cm}^{-1}$				
	(1)	(2)	(3)	(4)	(5)
$90^\circ = [10\bar{1}0]$	$1.37 \pm .02^c$		$2.15 \pm .05^d$		$1.59 \pm .02$
85°				$1.56 \pm .02^c$	$1.71 \pm .02$
80°	$1.45 \pm .02^b$			$1.69 \pm .02$	$1.88 \pm .02^a$
75°	$1.43 \pm .02^c$			$1.82 \pm .02$	
70°	$1.41 \pm .02^c$		$1.79 \pm .02$		
65°	$1.42 \pm .02^c$		$1.83 \pm .03$	$.57 \pm .02$	
60°	$1.41 \pm .02^c$			$.56 \pm .02$	
55°	$1.13 \pm .02^c$			$.56 \pm .02$	
50°	$.96 \pm .02^c$			$.58 \pm .02$	
45°	$.84 \pm .02^c$			$.57 \pm .02$	
40°	$.75 \pm .02$			$.59 \pm .02$	
35°	$.63 \pm .02^c$	$.77 \pm .02$		$.63 \pm .02^a$	
30°	$.56 \pm .02^a$	$.83 \pm .02^b$			
25°	$.59 \pm .03^a$	$.83 \pm .02^b$			
20°	$.53 \pm .02$	$.75 \pm .02^c$			
15°	$.53 \pm .02$	$.74 \pm .02^c$			
10°	$.52 \pm .02$	$.71 \pm .02^c$			
5°	$.51 \pm .02$	$.71 \pm .02^c$			
$0^\circ = [0001]$		$.67 \pm .02$		$.82 \pm .02$	

a Oscillations corresponding to this caliper are dominant at low magnetic fields.

b Oscillations corresponding to this caliper are dominant at high magnetic fields.

c Oscillations corresponding to this caliper are dominant over the range of magnetic fields investigated.

d This caliper not assigned with other calipers in this column.

e This caliper can be assigned to both columns (1) and (4)

- Assigned To Third Band Holes
- ▲ Assigned To Fourth Band Electrons
- " " " " "

fields studied. Even at this lower frequency, the oscillations seen were sufficient in number to allow accurate identification of the periods.

The data for $\hat{q}$ along $[11\bar{2}0]$ is presented in Table III and is plotted in polar form in Fig. 8. The calipers entered in column (1) are assigned to the crown, and the orbits traversed should correspond to central orbits. These calipers are in reasonable agreement with the free electron theory as may be seen in Fig. 9. The caliper measured in the $[10\bar{1}0]$ direction shows a large deviation from both the free electron Fermi surface and that predicted by Soven, the disagreement with the latter being approximately 5%.

Another unusual observation in connection with the anomalous $[10\bar{1}0]$ caliper is the behavior of the data as the magnetic field is brought into alignment with the c-axis. The data for the magnetic field 5° from the $[0001]$ direction shows a strong set of oscillations that have been assigned to either the crown or the fourth band, and these oscillations continue to be strong until the magnetic field is within 1° of the symmetry direction. As the field is aligned exactly along the $[0001]$ axis, the strong set of oscillations disappear, and in fact, at 75 Mc almost no oscillatory effect is seen until the field reaches 600 gauss. At this point one oscillation of small amplitude appears, followed by a series of oscillations, the first of which is very large in amplitude almost as if an absorption edge had been reached. The data for 45 Mc was also examined for this direction, and a similar, although less pronounced, effect occurred at almost the same value of the sound wave frequency divided by the magnetic field. For both frequencies, the period determined from the high field oscillations

present corresponded to the reported caliper of $1.37 \times 10^8 \text{ cm}^{-1}$.

A complete explanation of this anomalous result cannot be advanced at the present time, nonetheless, there are points of interest which may be discussed. When the field is exactly in the c-direction, the central orbits E-E and D'-D' are in contact as they pass through the six points of degeneracy in the AHL plane. Therefore, an electron originally on one of the orbits is able to pass to the other with a finite probability during the course of its lifetime. The question then arises are the separate orbits able to exist in a coherent manner, and are the electrons on these orbits able to participate strongly in the attenuation.

In the present experiment neither the $[11\bar{2}0]$ or the $[10\bar{1}0]$ caliper predicted from Soven's calculations corresponding to the central orbit E-E is observed; however, in both cases a set of weak oscillations are seen which lead to smaller calipers than those expected. This may be interpreted as evidence that the orbits in the AHL plane do not participate strongly in the attenuation, and that the observed effects are due to either orbits slightly displaced from the AHL plane or even orbits such as F-F which are shifted far off the AHL plane. The fact that electrons on the central E-E type of orbit would have an appreciable component of velocity in the direction of the magnetic field in the region where the orbit passes through the AL line would strengthen the assumption that the most stable orbit is probably shifted away from the AHL plane.

Should the field be tilted away from the c-direction, the central orbits would again become simple orbits.

The present calipers obtained for the crown central orbit with the field off the c-axis extrapolate to a $[10\bar{1}0]$ caliper of the crown of approximately $1.47 \times 10^8 \text{ cm}^{-1}$ which is in better agreement with the results of both Soven and EKP; the latter reports calipers which could be produced by a stable central crown orbit for the field along $[0001]$.

With the magnetic field in the $[10\bar{1}0]$ direction, oscillations corresponding to the $[0001]$ caliper of the crown are not sufficiently well resolved to be analyzed. However, the calipers for directions slightly different from $[0001]$ extrapolate to a caliper of $.50 \times 10^8 \text{ cm}^{-1}$ for the crown along $A\Gamma$ which agrees well with the value in Table II and with the theoretical prediction of Soven of $.49 \times 10^8 \text{ cm}^{-1}$.

Column (2) is assigned to a set of non-central orbits on the third band crown, possibly the same type of orbit for which the calipers are reported in column (4) of Table I. In both cases these orbits appear as the magnetic field is brought toward the $[10\bar{1}0]$ direction, but are also strongly diminished as the field is made parallel to $[10\bar{1}0]$. The caliper is not seen at all with $\hat{q}$ parallel to $[0001]$, probably because the oscillations are much weaker than the other two observed, and that is probably due to the fact that the corresponding orbit passes near the degeneracy points in the direction being calipered.

The first entry in column (3) is approximately equal to the zone dimension in the $[10\bar{1}0]$ direction and is probably due to extended orbits on the fourth band hexagonal network which occur when there is a slight misalignment of the sample. Oscillations for this period were seen at magnetic fields up to 5000 gauss at which point the attenuation became too large to extend the measurements to higher fields. The remaining entries in column (3) may be assigned to non-central calipers of the crown.

Column (4) is assigned to the fourth band electron surface in the following manner. The first entries would be caused by hole orbits inside the fourth band, $D'-D'$ for example, and the calipers obtained between 25° and 55° could be interpreted as due to electron orbits of the type $E'-E'$ around the fourth band surface planes passing through HL. In strong support of this assignment is the matching of the two groups of data points when reduced to a polar plot centered on L in the $ALM\Gamma$ plane shown in Fig. 10. Even though the orbit $E'-E'$ appears to be strongly distorted, it is able to contribute to the attenuation over the range of angles reported since the calipered portion of the orbit, that portion where the orbit passes through the $ALM\Gamma$ plane, satisfies the condition that $\hat{q} \cdot \hat{v}$ be small. The fact that the calipers first appear at 25° from the $[11\bar{2}0]$ axis is somewhat surprising, although the $E'-E'$ type of orbit can exist even at this small angle.

The $[0001]$ caliper of $.82 \times 10^8 \text{ cm}^{-1}$ most probably comes from the $C'-C'$ orbit, even so, a $E'-E'$ type of orbit is not excluded. In either case it would determine the $[0001]$ caliper at L of the fourth band where it agrees well with Soven's prediction.²⁰

The entries in column (5) could arise from the $D'-D'$ hole type orbits produced on the fourth band network. They would correspond to a non-central caliper approximately complementary to the $B'-B'$ section.

B. Open Orbits

Open orbits in thallium metal were first observed by Alekseevskii and Gaidukov in the transverse magnetoresistance with the magnetic field in the basal plane. Mackintosh et al. found the open orbits and explained them in terms of magnetic breakdown consistent with the free electron

model. Rayne reported open orbits in the c -direction which also agreed with the free electron model. More recently EKP have observed open orbit resonances due to open orbits along AL but have failed to see any indication of the open orbits in the c -direction. A search for open orbit resonances was carried out during the present experiment with particular interest in investigating the connectivity of the fourth band in the $[0001]$ direction.

The c -axis open orbit resonance should occur for either of the two configurations $\hat{H}$ parallel to $[11\bar{2}0]$, $\hat{q}$ parallel to $[10\bar{1}0]$ or $\hat{H}$ parallel to $[10\bar{1}0]$, $\hat{q}$ parallel to $[11\bar{2}0]$. In this experiment both geometries were studied and no evidence of open orbit resonances of the type seen by Gavenda in cadmium was seen. This is in agreement with the predictions of Soven and the observations of EKP.

For the magnetic field along $[11\bar{2}0]$ and $\hat{q}$ along $[0001]$, EKP reported open orbit resonances corresponding to the open orbit parallel to the line AL along which the third and fourth bands are degenerate. Evidence of this resonance was seen in this experiment, but as the crystal used was fairly thick, the attenuation was too high to study the actual line shape for the magnetic fields at which the proposed resonances were observed. Thus no conclusive evidence of the open orbit may be reported.

A similar search for open orbits parallel to $[11\bar{2}0]$ of the type discussed by Mackintosh yielded negative results, probably due to the fact that our observations were restricted to magnetic fields well below those required for breakdown.

For the geometry $\hat{q}$ parallel to $[11\bar{2}0]$ and the magnetic field parallel to the $[0001]$ axis, extended orbits were observed which are

present on the fourth band network whenever the magnetic field is slightly misaligned. The reported caliper of $2.15 \times 10^8 \text{ cm}^{-1}$ is compatible with this interpretation.

V. CONCLUSIONS

The results of the present experiment are in acceptable agreement with the relativistic band calculation performed by Soven. Calipers corresponding to both central and non-central orbits on the principal section of the third band hole surface were observed, and for certain orientations the non-central orbits appeared to contribute more strongly to the magnetoacoustic attenuation than the central orbits. Possible effects due to the degeneracy of the third and fourth bands along the line AL were seen for the magnetic field in both the $[11\bar{2}0]$ and the $[0001]$ direction. A set of weak oscillations reported in column (2), Table II, have been tentatively assigned to the small section of third band holes.

Both electron-like and hole-like orbits on the fourth band electron surface are reported and the associated calipers are consistent with Soven's model. The shape of the fourth band near the corner of the zone is still in question as is evidenced by the $[11\bar{2}0]$ direction data reported in Table II. Extended orbits with the magnetic field near the c-direction were observed and attributed to the fourth band. No conclusive evidence concerning the connectivity of the fourth band surface in the c-direction was obtained, although the absence of open orbit resonances in the data suggests the surface is not connected.

Further experiments are in progress in this laboratory investigating the Fermi surface of thallium, including the study of the transport effects and cyclotron resonance, the results of the latter tentatively confirming the existence of stationary non-central crown orbits.

APPENDIX I

CALCULATION OF THE ATTENUATION OF SOUND WAVES IN METALS

A. Attenuation in the Absence of a Magnetic Field

In the following problem, the method of Pippard¹⁷ to determine the attenuation of a sound wave due to interactions with conduction electrons will be studied and expanded upon when clarity demands. The presentation will hopefully be simple enough to allow ease in reading, but the bulk of the algebraic manipulations will be left to the reader. Emphasis will be placed on the physical processes involved rather than the handling of the rather cumbersome mathematical expressions which will arise. Results will also be taken from the doctoral dissertation of Raoul B. Weil,²¹ University of California, Riverside, California, since he has previously clarified some of the initial arguments of Pippard. The coordinate system used is shown in Fig. 11.

The basis for the calculation of ultrasonic attenuation in a metal in the absence of external magnetic fields may be outlined as follows. Assume that the sonic frequencies are low enough ($f < 1\text{KMc}$) so that the electrons are able to follow the motion of the lattice instantaneously, i.e., that the plasma frequency is greater than the frequency of the sound wave, thereby forbidding space charge effects. Also under this condition no transverse electric currents are created. These two assumptions require that the electronic current relative to the lattice must vanish.

The method is now to analyze the effects of the sound wave on the electrons by looking at the resulting deformations of the Fermi surface;

those which tend to hold the electrons on the equilibrium surface, which is deformed to follow the ionic motion, and those which tend to cause the electrons to leave the surface, thereby relaxing back to the deformed equilibrium surface and giving up energy. The effect of the electric fields created by the lattice are calculated from the assumption that the total current must be equal to zero since the electrons are able to follow the motion of the lattice. The calculated electric fields, along with the displacement forces, are then used to calculate the energy dissipated by the electron, which is proportional to the attenuation of the sound wave.

It will be convenient to develop relations for the components of the strain tensor which will be useful during the following discussion. To do this one may proceed as follows:

Consider a sound wave propagated in the x direction with wave vector $q_x = q$ and angular velocity ω . The particle displacements ξ_j will be of the form

$$\xi_j = \xi_j \exp [-i(qx - \omega t)] \quad . \quad (1)$$

The components of the strain tensor are defined as

$$\epsilon_{ij} \equiv \left(\frac{\partial \xi_i}{\partial x_j} \right) \quad , \quad (2)$$

which differs from the customary symmetric definition, but is more convenient in the analysis of this problem. Thus

$$\begin{aligned} \epsilon_{xx} &= -iq \xi_x & \epsilon_{yy} &= \epsilon_{zz} = 0 & \epsilon_{yz} &= \epsilon_{zy} = 0 \\ \epsilon_{yx} &= -iq \xi_y & \epsilon_{xy} &= \epsilon_{xz} = 0 & \epsilon_{zx} &= -iq \xi_z \end{aligned} \quad (3)$$

The dilation Θ is

$$\Theta = \sum_i \epsilon_{ii} = -iq \xi_x \quad . \quad (4)$$

If the particle velocity is $\hat{u}$ and the velocity of sound is c_s , then

$$u_x = \frac{\partial \xi_x}{\partial t} = +i\omega \xi_x$$

$$u_y = +i\omega \xi_y \quad ; \quad u_z = +i\omega \xi_z \quad . \quad (5)$$

Thus the tensor ϵ_{ij} may be written as a vector and related to $\hat{u}$ by

$$\hat{u} = -c_s \hat{\epsilon} \quad . \quad (6)$$

since $c_s = \omega/q$.

Now consider the effect of static strains on the Fermi surface from which the equilibrium position of the surface due to deformations may be calculated. If the strain is small, it may be assumed that the deformation of the Fermi surface is also small and is proportional to the strain. Thus if Δk_n is the deformation of the Fermi surface normal to the surface, a vector deformation coefficient $\hat{K}$ may be defined such that

$$\Delta k_n = \hat{K} \cdot \tilde{\epsilon} \quad . \quad (7)$$

$\hat{K}$ will be taken positive for shifts to regions of higher energy. By symmetry considerations it is expected that

$$\hat{K}(\mathbf{k}) = \hat{K}(-\mathbf{k}) \quad . \quad (8)$$

Higher symmetries may be found for particular lattice structures, however, only the inversion symmetry requirement will be assumed.

An equation of continuity may be written for the conservation of the number of occupied states in reciprocal space. This may be done by realizing that when a deformation occurs, the number of states passing through the surface must be equal to the number of states contained in the volume in reciprocal space given by

$$\delta V_{rs} = - \Theta V_{rs} \quad (9)$$

Thus if ρ is the density of states at the Fermi level then

$$\int \rho(\Delta k_n) dS = \rho \delta V_{rs} \quad (10)$$

where it will be assumed for this integral and those to follow that integration over dS implies integration over the closed Fermi surface.

If ρ is constant

$$\int (\Theta K_x) dS + \int \Theta k_x \cos \varphi dS = 0 \quad (11)$$

since $\delta k_y = 0 = \delta k_z$, and $\delta k_x = - \epsilon_{xx} k_x = - \Theta k_x$. Thus

$$\int (K_x + k_x \cos \varphi) dS = 0 \quad , \quad (12)$$

where φ is shown in Fig. 11 and is the angle between $\hat{v}$ and the x axis. The integral $\int K_y dS = 0$ since the corresponding component of $\hat{\epsilon}$, i.e., e_{yx} represents a shear which cannot be associated with a change in volume. The integral

$$\int k_y \cos \varphi dS \quad (13)$$

vanishes identically. Similarly for the z components. Hence

$$\int (\hat{K} + \hat{k} \cos \varphi) dS = 0 \quad . \quad (14)$$

The energy variation along the Fermi surface is given by

$$\delta W = \left(\frac{\partial W}{\partial k_n} \right) \Delta k_n = \hbar v \hat{K} \cdot \hat{\epsilon} \quad (15)$$

and if the strain is periodic then deformations of the type $\hat{\epsilon} e^{-iqx}$ occur in real space, causing δW on the Fermi surface to be of the form

$$\delta W = \hbar v \hat{K} \cdot \hat{\epsilon} e^{-iqx} \quad . \quad (16)$$

Thus the electron may be regarded as moving in a force field F_x given by

$$F_x = -iq \hbar v \hat{K} \cdot \hat{\epsilon} e^{-iqx} \quad (17)$$

and it is F_x which maintains the electrons in equilibrium on the deformed Fermi surface. Pippard states that F_x has its origin partly in electrostatic fields resulting from a minute charge imbalance and partly from an interaction between the electron and the non-uniformly strained lattice.

There are two effects present other than the electric field due to the lattice which cause the electron to leave the Fermi surface during the passage of the sound wave. The first is due to the fact that different parts of the lattice are in motion relative to one another which causes the electron to change its momentum value as it moves through the lattice. To calculate the effect of this interaction

assume the observer is fixed relative to a point in the lattice which will be called the origin for convenience. If the lattice were undistorted, and electron wave front of vector k_i passed through the origin, the next wave front would be located at a point r_i in the direction of propagation of the electron wave front, where $r_i = 2\pi/k_i$.

Now consider a lattice distorted by the sound wave. If, as is assumed, the electron follows the motion of the lattice adiabatically thereby changing its momentum, the following relation must be satisfied for the wave fronts

$$(\hat{k} + \delta\hat{k}) \cdot (\hat{r} + \delta\hat{r}) = 2\pi \quad . \quad (18)$$

Upon neglecting second order terms, it is found that

$$\delta\hat{k} \cdot \hat{r} = -\hat{k} \cdot \delta\hat{r} \quad (19)$$

but $\delta\hat{r} = \tilde{\epsilon} \cdot \hat{r}$, hence

$$\delta\hat{k} \cdot \hat{r} = -\hat{k} \cdot (\tilde{\epsilon} \cdot \hat{r}) \quad \text{or} \quad \delta\hat{k} = -\hat{k} \cdot \tilde{\epsilon} \quad . \quad (20)$$

For the sound wave propagated only in the x direction we have seen that $\tilde{\epsilon} = \hat{\epsilon}$, thus the results may be simplified to yield

$$\delta k_x = -\hat{\epsilon} \cdot \hat{k} \quad , \quad \delta k_y = 0 = \delta k_z \quad . \quad (21)$$

The value of $\hat{k}$ may now be calculated, and if second order terms are again neglected

$$\dot{k}_x = -\hat{\epsilon} \cdot \hat{k} \quad , \quad \dot{k}_y = 0 = \dot{k}_z \quad . \quad (22)$$

The force, f_1 , is related to $\dot{k}$ by the usual relation $\hbar\dot{k} = f_1$ so that the effects on the electron may be replaced by a fictitious force in the

stationary lattice according to the relation

$$f_1 = - \hbar \hat{\epsilon} \cdot \hat{k} = i \hbar q \hat{u} \cdot \hat{k} \quad (23)$$

The component of this force in the direction of the electronic velocity is the portion of f_1 of interest, thus redefine f_1 by

$$f_1 = i \hbar q \hat{u} \cdot \hat{k} \cos \varphi \quad (24)$$

Now consider the effect of the motion of the wave traveling through the metal. The electron on the Fermi surface experiences the force F_x which tends to keep the electron on the Fermi surface if the strain is static. However, since the wave travels past a point in the metal with velocity c_s , the force F_x does work to remove the electron from the Fermi surface by an amount $F_x c_s \delta t$ during time δt . To find the fictitious force, f_2 , which may be considered as performing the same amount of work on the electron of velocity v , traveling in the stationary lattice, one equates

$$F_x c_s \delta t = f_2 v \delta t \quad (25)$$

It is understood that f_2 is the component of the fictitious force along the electron velocity. Hence,

$$f_2 = - i q \hbar c_s \hat{K} \cdot \hat{\epsilon} \quad (26)$$

which may be rewritten more conveniently as

$$f_2 = i q \hbar \hat{K} \cdot \hat{u} \quad (27)$$

The sum of the two fictitious forces tending to remove the electrons from the equilibrium position of the deformed Fermi surface is

$$f = f_1 + f_2 = iq\hbar \hat{D} \cdot \hat{u} \quad (28)$$

where $\hat{D} = \hat{K} + \hat{k} \cos \varphi$.

In addition to f_1 , the electrons are acted upon by a real force due to the electric field, $\hat{E}$, associated with the wave. In a metal at frequencies less than approximately 1 KMc, the electrons are able to follow the motion of the lattice ions well enough to neutralize any current due to the motion of the ions. This may be stated differently, i.e., the electric fields created by the sound wave moving through the lattice are equal and opposite to the electric field related to the electronic current by

$$E_j = \rho_{ij} J_i \quad (29)$$

The quantity ρ_{ij} is the usual resistivity tensor. Therefore, if the electronic current due to f can be calculated, the electronic fields due to the lattice motion can be calculated in a self-consistent manner.

To calculate the electronic current, the following procedure is adopted. If a group of electrons gain energy in excess of the Fermi energy by an amount ΔW , the surface is displaced outwards by an amount $\Delta k = \frac{\Delta W}{\hbar v}$. The volume of the displaced surface $dV = (\Delta k) dS = \frac{\Delta W}{\hbar v} dS$. The volume multiplied by the density of states per unit volume of metal gives the number of electrons per unit volume of the metal moving with velocity v . Thus the electronic current $\hat{J}_e$ may be written as

$$\hat{J}_e = \frac{e}{4\pi^3 \hbar} \int \frac{v \Delta W}{v} dS, \quad (30)$$

where the integral is taken over the Fermi surface.

The quantity ΔW may be calculated from knowledge of the effective force f acting on the electrons parallel to their motion, taking into account the total history of the electron, by the expression

$$\Delta W(o) = \int_{-\infty}^0 v f \exp \left(- \int_t^0 \frac{dt}{\tau} \right) dt \quad . \quad (31)$$

In zero magnetic field, v and τ are constant, thus

$$\Delta W(o) = v \int_{-\infty}^0 f e^{t/\tau} dt \quad . \quad (32)$$

The force f has a time dependence of the form

$$f(t) = f e^{i(\omega - \hat{q} \cdot \hat{v}) t} \quad , \quad (33)$$

and since q is in the x direction,

$$f(t) = f e^{i(\omega - qv \cos \varphi) t} \quad . \quad (34)$$

Therefore

$$\Delta W = f v \tau (1 + i\omega\tau - iqv\tau \cos \varphi)^{-1} \quad . \quad (35)$$

The current $\hat{J}_e$ may now be expressed as

$$J_e = \frac{e}{4\pi^3 n} \int \frac{f \hat{v} \tau dS}{(1 + i\omega\tau - iq\ell \cos \varphi)} \quad , \quad (36)$$

where $\ell = v\tau$, the mean free path of the electron.

The electric field, $\hat{E}$, due to the lattice may now be expressed as

$$E_j = \frac{-ieq}{4\pi^3} \rho_{ij} \int \frac{v_i \tau (\hat{D} \cdot \hat{u}) dS}{1 + i\omega\tau - iq\ell \cos \varphi} \quad . \quad (37)$$

At this point Pippard makes the approximation that $\omega\tau$ is neglected in comparison with $(1 - iq\ell \cos \varphi)$. The assumption is easily justified for the case $q\ell \gg 1$, but is on less firm theoretical grounds when either the mean free path of the electrons becomes small or the sonic wavelength becomes too long. However, for the present experiment where $q\ell \gtrsim 30$, the approximation is justified.

Thus if $\hat{\ell} = \hat{v}\tau$ and $\hat{a} = q\ell$, then

$$E_j = \frac{-ieq}{4\pi^3} \rho_{ij} \int \frac{\tau v_i (\hat{D} \cdot \hat{u})}{(1 - ia \cos \varphi)} dS \quad (38)$$

Using Eq. (14), the above expression reduces to

$$E_j = \frac{e}{4\pi^3} \rho_{ij} \int a_i \frac{(\hat{D} \cdot \hat{u}) a \cos \varphi}{(1 + a^2 \cos^2 \varphi)} dS ,$$

where the integral

$$\int \frac{\tau v_i (\hat{D} \cdot \hat{u}) dS}{(1 + a^2 \cos^2 \varphi)}$$

is seen to vanish by virtue of the central symmetry of $\hat{D}$.

The total force, F , parallel to the motion of the electron may now be written as

$$F = f_1 + f_2 + e \left(\frac{\hat{E} \cdot \hat{a}}{a} \right)$$

$$F = iq\hbar(\hat{D} \cdot \hat{u}) + \frac{e^2}{4\pi^3 a} \rho_{ij} a_j \int \frac{a_i (\hat{D} \cdot \hat{u}) a \cos \varphi dS}{(1 + a^2 \cos^2 \varphi)} \quad (39)$$

The total energy shift, ΔW , of the electrons from the Fermi surface due to the sound wave may now be calculated by Eq. (32) with the substitution of F for f . The number of states excited above the Fermi level is related to ΔW by

$$\Delta N = \frac{\Delta W}{4\pi^3 \hbar v} dS \quad (40)$$

and the average excess energy of these electrons above the equilibrium Fermi energy is $\Delta W/2$. If the relaxation time is τ , then the average rate of energy dissipated by the electron system per unit volume is

$$\dot{Q} = \frac{1}{8\pi^3 \hbar} \int \frac{|\Delta W|^2}{\ell} dS \quad (41)$$

To find the attenuation coefficient α , $\dot{Q}$ is divided by the energy density per unit time due to the sound wave. Therefore,

$$\alpha = \frac{\dot{Q}}{(1/2 M u^2 c_s)} \quad (42)$$

where M is the density of the metal.

In terms of ΔW , the attenuation coefficient becomes

$$\alpha = (4\pi^3 \hbar M c_s^2)^{-1} \int \frac{|\Delta W|^2}{\ell} dS \quad (43)$$

The value of ΔW is found by substitution of Eq. (39) into Eq. (32) which yields, upon restoring the time dependence of F

$$\Delta W = v \int_{-\infty}^0 F e^{(1/\tau + i\omega - iqv \cos \theta)t} dt \quad (44)$$

or

$$\Delta W = \frac{v F \tau}{(1 - |a \cos \varphi|)}$$

if $\omega \tau$ is again neglected.

Thus,

$$|\Delta W|^2 = \frac{\ell^2 |F|^2}{1 + a^2 \cos^2 \varphi} \quad (45)$$

Therefore

$$\alpha = (4\pi^3 \hbar m c_s u^2)^{-1} \int \frac{|F|^2 \ell dS}{1 + a^2 \cos^2 \varphi} \quad (46)$$

To calculate $|F|^2$ the assumption will be made that ρ_{ij} is real since $\omega \tau$ is small. The value $|F|^2$ in this approximation is

$$|F|^2 = q^2 \hbar^2 u^2 + \left(\frac{e^2}{4\pi^3 a}\right)^2 \rho_{ij} \rho_{kl} a_j a_\ell l_i l_k u^2, \quad (47)$$

where $\mathcal{Q} = \frac{\hat{\mathbf{D}} \cdot \hat{\mathbf{u}}}{u}$ and

$$l_i = \int a_i \frac{\mathcal{Q} a \cos \varphi}{1 + a^2 \cos^2 \varphi} dS$$

Therefore the expression for α becomes

$$\alpha = \frac{\hbar q}{4\pi^3 m c_s} \left\{ \int \frac{\mathcal{Q}^2 a dS}{1 + a^2 \cos^2 \varphi} + \left(\frac{e^2}{4\pi^3 \hbar q}\right)^2 \rho_{ij} \rho_{kl} l_i l_k A_{jl} \right\} \quad (48)$$

where

$$A_{jl} = \int \frac{a_j a_\ell dS}{a(1 + a^2 \cos^2 \varphi)}$$

To simplify some of the expressions obtained, it is instructive to calculate the value of the conductivity tensor σ_{ij} defined by

$$J_j = \sigma_{ij} E_i \quad (49)$$

where σ_{ij} has the property

$$\sigma_{ij} \rho_{ik} = \delta_{jk}$$

Consider an electric field of the form $\hat{E}(t) = \hat{E} e^{i\omega t}$. The current due to $\hat{E}(t)$ may be calculated in the usual manner, again neglecting ωr with respect to $(1 - i a \cos \varphi)$, by using as the force in Eq.(32) the value $e \hat{E} \cdot \hat{a}/a$ to obtain ΔW , which may be substituted in Eq. (30) to obtain J_j

$$J_j = \frac{e^2 E_i}{4\pi^3 n q} \int \frac{a_i a_j dS}{a(1 - i a \cos \varphi)} = \frac{e^2 E_i}{4\pi^3 n q} \int \frac{a_i a_j dS}{a(1 + a^2 \cos^2 \varphi)} \quad (50)$$

Thus

$$\sigma_{ij} = \frac{e^2}{4\pi^3 n q} A_{ij} \quad (51)$$

and

$$\rho_{ij} \rho_{kl} \rho_{ik} A_{jl} = \frac{4\pi^3 n q}{e^2} \rho_{ij} \rho_{kl} \rho_{ik} \sigma_{jl} = \frac{4\pi^3 n q}{e^2} \rho_{ij} \rho_{il} \quad (52)$$

using the Kronecker delta property of the ρ 's and σ 's.

Thus if B_{ij} is defined by

$$B_{ij} = \frac{4\pi^3 n q}{e^2} \rho_{ij} \quad (53)$$

then

$$\alpha = \frac{\hbar q}{4\pi^3 M c_s} \left\{ \int \frac{\mathcal{D}^2 a dS}{1 + a^2 \cos^2 \varphi} + B_{ij} l_i l_j \right\} \quad (54)$$

This is the general result for a sound wave of arbitrary polarization propagated in the x direction with wave vector q . It is instructive to evaluate the expression for α for a simple but interesting case, that of a purely longitudinal wave. For a longitudinal wave propagated in the x direction

$$u_x = u \quad u_y = 0 = u_z \quad (55)$$

Therefore

$$\mathcal{D} = \frac{\hat{D} \cdot \hat{u}}{u} = D_x = K_x + k_x \cos \varphi \quad (56)$$

and noting that $a_x = a \cos \varphi$, it is seen that

$$I_x = \int \frac{\mathcal{D}^2 a^2 \cos^2 \varphi}{1 + a^2 \cos^2 \varphi} dS = \int \mathcal{D}^2 dS - \int \frac{\mathcal{D}^2 dS}{1 + a^2 \cos^2 \varphi} \quad (57)$$

But according to Eq. (1) the first integral on the right vanishes, hence

$$I_x = - \int \frac{\mathcal{D}^2 dS}{1 + a^2 \cos^2 \varphi} \quad (58)$$

$I_y = 0 = I_z$ due to symmetry, leaving the only B_{ij} to be determined as B_{xx} .

For the case of purely longitudinal or purely transverse sound waves, it may be easily verified that $\sigma_{ij} = 0$, $i \neq j$. This implies ρ_{ij}

is also diagonal with the result that

$$B_{xx} = (A_{xx})^{-1} = \int \frac{a \cos^2 \varphi}{1 + a^2 \cos^2 \varphi} dS \quad (59)$$

with $a_x = a \cos \varphi$.

The general result for longitudinal waves is therefore

$$\alpha = \frac{\hbar q}{4\pi^3 N c_s} \left\{ \int \frac{\mathcal{D}^2 dS}{1 + a^2 \cos^2 \varphi} + \left[\int \frac{\mathcal{D} dS}{1 + a^2 \cos^2 \varphi} \right]^2 / \int \frac{a \cos^2 \varphi dS}{1 + a^2 \cos^2 \varphi} \right\} \quad (60)$$

If the general result is evaluated for the free electron model of a metal, the value of K_x is easily determined by the following procedure

$$E_F \propto n^{2/3} \propto (\text{Vol})^{-2/3}, \quad (61)$$

thus

$$\frac{\delta E}{E_F} = \frac{2\delta k}{k_F} = -2/3 \frac{\delta V}{V}, \quad (62)$$

but

$$\frac{\delta V}{V} \Theta = \epsilon_{xx} \quad (63)$$

and K_x is defined by $\delta k = \epsilon_{xx} K_x$.

According to Eq. (62)

$$\delta k = -1/3 \epsilon_{xx} k_F \quad (64)$$

so that K_x may be identified as $-1/3 k_F$ for the free electron model, which is a constant over Fermi surface and may therefore be removed from under the integral sign.

The evaluation of the integrals is straightforward and only the result need be reported which is

$$\alpha = \frac{nm}{Mv_s \tau} \left\{ \frac{a^2 \tan^{-1} a}{3(a - \tan^{-1} a)} - 1 \right\} \quad (65)$$

It is interesting to note that for $a = ql \gg 1$,

$$\alpha \rightarrow \frac{nm}{Mv_s \tau} \left(\frac{\pi}{6} \right) ql \quad , \quad (66)$$

and the attenuation is directly proportional to q , whereas for $a \ll 1$

$$\alpha \rightarrow \frac{nm}{Mv_s \tau} \left\{ \frac{a^2(a - \frac{a^3}{3!} + \dots)}{3[a - (a - \frac{a^3}{3!} + \dots)]} - 1 \right\} \quad (67)$$

$$\alpha \rightarrow \frac{nm}{Mv_s \tau} \frac{q^2 l^2}{3}$$

and the attenuation is now proportional to the square of the sonic wave vector.

B. Attenuation in a Transverse Magnetic Field

The attenuation of sound waves in a metal in the presence of a transverse magnetic field may be treated in a manner very similar to that of the preceding section since the same effects are responsible for the attenuation. The main difference caused by the introduction of the magnetic field is that the electrons are now considered to move around the Fermi surface.

Suppose the magnetic field is in the z direction. The k -space trajectory of an electron in the absence of collisions is along the intersection of a surface of constant energy by planes of constant k_z . Onsager²² has shown that the projections of the paths in real space on the $z = 0$ plane are related to the paths in k -space by a rotation of $\pi/2$ about the z -axis and scaled by a factor of $\hbar c/eH$. (Pippard uses a system of units where $c = 1$.)

It will be assumed throughout the course of the following discussion that the portions of the Fermi surface under consideration are closed surfaces, and will be viewed as seen in the repeated zone scheme. Since it is the usual procedure to treat almost filled bands of electrons as hole bands, the theory will be presented in a form applicable to both kinds of carriers. It should be recalled at this point that closed surfaces of holes surround states of higher energy, while closed electron surfaces surround states of lower energy. It will be assumed that the magnetic field is directed along the z -axis into the plane of the paper such that electrons traverse orbits in a

clockwise direction while hole type orbits move in a counter-clockwise sense.

Again, as in the preceding section, the procedure is to calculate the energy shift of the electrons due to a fictitious force $\hat{F}(\hat{k}, t)$ which is parallel to the motion of the electrons and is of the form

$$F(\hat{k}, t) = F \exp [i(\omega t - qx)] \quad . \quad (68)$$

Now consider an electron on the plane $x = 0$ at time $t = 0$ and at a point s on its k -space orbit. The equation of motion governing the motion of the electron is

$$\hbar \frac{d}{dt} \hat{k}(s) = \frac{-e}{c} \hat{v}(s) \times \hat{H} \quad . \quad (69)$$

Thus if at time t' the electron now finds itself at the point s' , then the k_y coordinate has changed from $k_y(s)$ to $k'_y(s')$. Thus from Eq. (69) it may be seen that the new x coordinate is given by $\hbar(k_y - k_{y'})/eH$. The wavelike variation of $F(\hat{k}, t)$ may therefore be written as

$$\exp[i(\omega t - \frac{q\hbar}{eH} (k_y - k_{y'}))] \quad . \quad (70)$$

The expression for the energy shift due to the force $F(\hat{k}, t)$ is now seen to be given by

$$\Delta W(0) = \int_{-\infty}^0 v F \exp[-\frac{iq\hbar}{eH} (k_y - k_{y'}) + i\omega t' - \int_{t'}^0 \frac{dt''}{\tau}] dt' \quad . \quad (71)$$

At this point the time dependent term $\exp(i\omega t')$ is neglected since the inclusion of this term results in an expression of the

form

$$\int_{t'}^0 (1 + i\omega\tau) \frac{dt''}{\tau} \quad (72)$$

and for the sonic frequencies under consideration, $\omega\tau \ll 1$.

The Lorentz force acting on the electron is $\frac{evH}{c} \sin\theta$, thus

$$\dot{s} = (evH / \hbar c) \sin\theta, \quad (73)$$

where the same sign is possible for both electron and hole orbits since s is measured along the direction of motion. Therefore the product vdt' appearing in the integral for ΔW reduces to

$$vdt' = (ch / eH) \operatorname{cosec}\theta' ds' \quad (74)$$

Similarly the integral involving the relaxation time becomes

$$-\int_{t'}^0 \frac{dt''}{\tau} = + \int_s^{s'} \frac{ch}{\tau veH \sin\theta''} ds'' \quad (75)$$

Defining β and λ by

$$\beta \equiv chq/eH \quad (76)$$

$$\lambda \equiv \beta/a \sin \theta$$

where again $a = qv\tau = q\ell$, the expression for ΔW reduces to

$$\Delta W(0) = \frac{ch}{eH} \exp(-i\beta k_y) \int_{-\infty}^s F \operatorname{cosec}\theta' \exp(i\beta k_y') + \int_s^{s'} \lambda ds'' ds' \quad (77)$$

the integrals having been converted from time integrals to line integrals along the path of the electrons in k -space.

The integrals in ΔW may be reduced further if one recalls that the paths involved are closed paths, and that $F \operatorname{cosec}\theta' \exp(-i\beta k_y')$

and $\lambda(s'')$ are periodic. Thus the integral is of the general form

$$I(s) = \int_{-\infty}^s f(s') \exp \left(\int_s^{s'} \lambda ds'' \right) ds' \quad (78)$$

where $f(s')$ and $\lambda(s'')$ are periodic functions. If s_0 designates the perimeter of the k-space orbit, then

$$I(s) = \int_{s-s_0}^s f(s') \exp \left(\int_s^{s'} \lambda ds'' \right) ds' + \int_{s-2s_0}^{s-s_0} f(s') \exp \left(\int_s^{s'} \lambda ds'' \right) ds' + \dots \quad (79)$$

By a change of the variable of integration in each integral of the sum to bring the limits of integration back to $s-s_0 \rightarrow s$, it may be seen that

$$\begin{aligned} I(s) = & \int_{s-s_0}^s f(s') \exp \left(\int_s^{s'} \lambda ds'' \right) ds' + \int_{s-s_0}^s f(s'-s_0) \exp \left(\int_s^{s'} \lambda ds'' \right) ds' \\ & + \int_{s-s_0}^s f(s'-2s_0) \exp \left(\int_s^{s'} \lambda ds'' \right) ds' \quad . \quad (80) \end{aligned}$$

The periodicity of $f(s')$ implies that $f(s'-ns_0) = f(s')$, thus

$$I(s) = \int_{s-s_0}^s ds' f(s') \left\{ \exp \int_s^{s'} \lambda ds'' + \exp \int_s^{s'-s_0} \lambda ds'' + \exp \int_s^{s'-2s_0} \lambda ds'' \right\} \quad (81)$$

the integral in the exponential may be written as

$$\int_s^{s'-ns_0} \lambda ds'' = \int_s^{s'} \lambda ds'' - \int_{s'-ns_0}^{s'} \lambda ds'' = \int_s^{s'} \lambda ds'' - n \oint \lambda ds'' \quad , \quad (82)$$

since $\lambda(s'')$ is periodic. Therefore

$$I(s) = \frac{1}{1 - \exp(-\oint \lambda ds'')} \int_{s-s_0}^s f(s') \exp \left(\int_s^{s'} \lambda ds'' \right) ds' \quad . \quad (83)$$

The result may be applied to the integral for ΔW yielding

$$\Delta W(0) = \frac{\hbar c}{eH} \frac{\exp(-i\beta ky)}{1 - e^{-\mu}} \int_{s-s_0}^s F \operatorname{cosec} \theta' \exp(i\beta ky') + \int_s^{s'} \lambda ds'' ds' \quad (84)$$

where $\mu = \frac{2\pi}{\omega_c \tau}$ has been written for $\oint \lambda ds''$ which is the probability of collision during one revolution of the orbit. It is understood that τ is the mean relaxation time, the average being taken on the orbit around the Fermi surface; and ω_c is the cyclotron frequency defined as

$$\omega_c = \frac{eH}{m_c} \quad (85)$$

where m_c is the cyclotron mass defined in the usual manner.

The current density $\hat{J}$ is given by

$$J_i = \frac{e}{4\pi^3 \hbar} \int \frac{v_i \Delta W}{v} dS \quad (86)$$

The surface integral may be broken up and evaluated by

$$\int dS = \int dk_z \oint \frac{ds}{\sin \theta} \quad , \quad (87)$$

hence

$$\begin{aligned} J_x &= \frac{e}{4\pi^3 \hbar} \int dk_z \oint \Delta W \cos \phi \, ds \quad , \\ J_y &= \frac{e}{4\pi^3 \hbar} \int dk_z \oint \Delta W \sin \phi \, ds \quad , \\ J_z &= \frac{e}{4\pi^3 \hbar} \int dk_z \oint \Delta W \cot \theta \, ds \quad . \end{aligned} \quad (88)$$

The force F , which must be substituted in the expression for ΔW to allow the calculation of $\hat{J}$, is made up of three parts as discussed in

the preceding section. For an electron moving on a closed k-space orbit, they may be written as $F = f_1 + f_2 + f_3$, where $f_1 = i\hbar \hat{\mathbf{K}} \cdot \hat{\mathbf{u}}$, and is due to deformation, $f_2 = i\hbar \hat{\mathbf{k}} \cdot \hat{\mathbf{u}} \sin\theta \cos\phi$, and is due to relative velocity, $f_3 = \frac{e \hat{\mathbf{E}} \cdot \hat{\mathbf{v}}}{v}$, and is due to the electric fields.

At this point, it becomes convenient to specialize to the case of a purely longitudinal sound wave propagated in the x direction. This is the configuration of the present experiment, and is the case (l,k) in the notation of Pippard, which will be adopted in this discussion. The force F assumes the form

$$F = i\hbar u (K_x + k_x \sin\theta \cos\phi) + eE_x \sin\theta \cos\phi + eE_y \sin\theta \sin\phi$$

Substitution in ΔW , and then into the equations for $\hat{\mathbf{J}}$, results in

$$J_x = \frac{c}{4\pi^3 H} \int \frac{dk_z}{1-e^{-\mu}} \oint \exp(-i\beta k_y) \cos\phi G(s, s_0) ds, \quad (89)$$

$$J_y = \frac{c}{4\pi^3 H} \int \frac{dk_z}{1-e^{-\mu}} \oint \exp(-i\beta k_y) \sin\phi G(s, s_0) ds,$$

and

$$J_z = \frac{c}{4\pi^3 H} \int \frac{dk_z}{1-e^{-\mu}} \oint \exp(-i\beta k_y) \cot\theta G(s, s_0) ds \quad (90)$$

where

$$G(s, s_0) = \int_{s-s_0}^s \exp(i\beta k_y) + \int_s^{s'} \lambda ds'' \{ i\hbar u K_x \operatorname{cosec}\theta' + (i\hbar u k_x + eE_x) \cos\phi' + eE_y \sin\phi' \} ds' \quad (91)$$

The integrals involved in the calculation of $\hat{\mathbf{J}}$ are simplified in Appendix II, hence it is possible at this point to write down the

following expressions for the current density.

$$\begin{aligned}
 J_x &= \frac{ce}{\pi H \beta^2} \left\{ \frac{Hu}{c} (B_{13} + B_{35}) + E_x (B_{07} - B_{33}) + E_y B_{13} \right\} \\
 J_y &= \frac{ce}{\pi H \beta^2} \left\{ \frac{Hu}{c} (B_{11} + B_{15}) - E_x B_{13} + E_y B_{11} \right\} \\
 J_z &= 0
 \end{aligned} \tag{92}$$

In the preceding expressions, the following notation is adopted

$$\begin{aligned}
 I_0 &= \frac{1}{2\pi} \oint \lambda ds = (\omega_c \tau)^{-1} \\
 I_n &= \frac{1}{2\pi} \oint \eta_n e^{i\beta k_y} ds; \quad n = 1, 2, \dots, 6
 \end{aligned} \tag{93}$$

where in the present paper, to conform with Pippard,

$$\begin{aligned}
 \eta_1 &= -i\beta \sin\phi \text{ or } \beta^2 k_z \cos\phi, \\
 \eta_3 &= \lambda
 \end{aligned} \tag{94}$$

$$\eta_5 = \beta^2 k_x \operatorname{cosec}\theta$$

and $I_7 = \frac{1-e^{-\mu}}{2\pi}$.

The B_{mn} are defined by

$$B_{mn} = \int \frac{I_m I_n}{1-e^{-\mu}} dk_z \tag{95}$$

It should be noted that I_1 is dependent upon whether the orbit is electron-like or hole-like, whereas I_3 , I_5 , and I_7 are not.

In defining the η_n , the property of inversion symmetry is used, i.e., for each orbit in the plane k_z , there exists a corresponding orbit

in the plane $-k_z$ which is of the same shape as the original orbit, but rotated through 180° about the z axis. Note that η_n is real if its sign is unchanged by inversion, whereas η_n is imaginary if inversion does change the sign. It may be seen that this causes the $B_{m,n}$ to be real.

The present results are in agreement with those of Pippard at this point, and may be compared by approximating $\mu \ll 1$ and noting that in this approximation

$$\begin{aligned}
 1 - e^{-\mu} &\approx \frac{2\pi}{\omega_c \tau} \\
 I_7 &\rightarrow I_0 \\
 B_{mn} &\rightarrow \frac{eH}{2\pi c} I_{mn}
 \end{aligned} \tag{96}$$

where I_{mn} is defined by Pippard. The η_n are defined so as to agree with Pippard.

For a metal at sonic frequencies less than approximately 1 KMc/sec, the total current should vanish. It is therefore possible to determine E_x and E_y from Eqs. (92), setting $J_x = 0 = J_y$.

$$\begin{aligned}
 E_x &= \left\{ \frac{(B_{15}B_{13} - B_{11}B_{35})}{B_{11}(B_{07} - B_{33}) + B_{13}^2} \right\} \frac{uH}{c} \equiv Y_1 \frac{uH}{c} \\
 E_y &= - \left\{ 1 + \frac{(B_{07} - B_{33})B_{15} + B_{13}B_{35}}{B_{11}(B_{07} - B_{33}) + B_{13}^2} \right\} \frac{uH}{c} \equiv Y_2 \frac{uH}{c} .
 \end{aligned} \tag{97}$$

Substitution in the equation for F gives

$$F = i\hbar u (K_x + k_x \sin\theta \cos\phi) + \frac{euH}{c} Y_1 \sin\theta \cos\phi - \frac{euH}{c} (1 + Y_2) \sin\theta \sin\phi . \tag{98}$$

If the expression for F is substituted in Eq.(77) for ΔW , and the

integrations are performed, it may be seen that

$$\Delta W(0) = \frac{2\pi i \hbar u}{\beta} \left\{ (I_5 - Y_1 I_3 - Y_2 I_1) \frac{\exp(-i\beta k_y)}{1-e^{-\mu}} + \frac{Y_1}{2\pi} \right\} \quad (99)$$

where the integrals are evaluated according to Appendix II.

To calculate the attenuation coefficient, α_{ik} , the value of $|\Delta W|^2$ must be known, and may be written as

$$|\Delta W|^2 = \frac{4\pi^2 \hbar^2 u^2}{\beta^2} \left\{ \frac{|I_5 - Y_1 I_3 - Y_2 I_1|^2}{(1-e^{-\mu})^2} + \frac{2Y_1}{2\pi(1-e^{-\mu})} \mathcal{R}[(I_5 - Y_1 I_3 - Y_2 I_1) \exp(-i\beta k_y)] + \left(\frac{Y_1}{2\pi}\right)^2 \right\} \quad (100)$$

since Y_1 is real. $\mathcal{R}$ denotes the operation of taking the real part.

Converting the surface integral in Eq.(43) for α gives

$$\alpha_{ik} = \frac{eH}{4\pi^2 \hbar^2 M c_s u^2} \int dk_z \oint |\Delta W|^2 \lambda ds \quad (101)$$

where $\hbar c / eH \lambda \sin \theta$ has been substituted for ℓ .

When Eq.(100) for $|\Delta W|^2$ is substituted in Eq.(101) for α , and the path integrals are performed, the result is

$$\alpha_{ik} = \frac{eH}{\pi M c_s \beta^2 c} \int dk_z \left\{ \frac{\mu |I_5 - I_3 Y_1 - I_1 Y_2|^2}{(1-e^{-\mu})^2} + \frac{2Y_1}{(1-e^{-\mu})} [I_3 I_5 - I_3 I_3 Y_1 - I_1 I_3 Y_2] + \frac{\mu Y_1^2}{4\pi^2} \right\} \quad (102)$$

The integration of the first and third terms is straight forward since only λ varies over the path. The second term may be integrated

around the path recalling that for a complex function and a real path, the integral of the real part is equal to the real part of the integral.

To perform the integrations over k_z , the following procedure is adopted. In the first term it is approximated that $\mu/1-e^{-\mu}$ is constant over the integration and may be removed from beneath the integral sign. The second term integrates immediately recalling that the B_{mn} are real. To integrate the third term, multiply by $2\pi l_\gamma/1-e^{-\mu}$. The resulting expression for α_{ik} is

$$\alpha_{ik} = \frac{eH}{\pi M c_s \beta^2 c} \left\{ \frac{\mu}{1-e^{-\mu}} [B_{55} + Y_1^2 B_{33} + Y_2^2 B_{11} - 2Y_1 B_{35} - 2Y_2 B_{15} + 2Y_1 Y_2 B_{13}] + 2Y_1 [B_{35} - Y_1 B_{33} - Y_2 B_{13}] + Y_1^2 B_{07} \right\} \quad (103)$$

(The difference in sign shown in the present work as compared to that of Pippard in the $2Y_1 B_{35}$ term is probably a typographical error in the original article.)

A meaningful simplification can be accomplished at this point if the term

$$\frac{\mu}{1-e^{-\mu}} \approx 1, \quad \text{i.e., } \mu = \frac{2\pi}{\omega_c \tau} \ll 1 \quad (104)$$

where it occurs as the coefficient of the leading term in Eq. (103).

The approximation is valid over the range of fields studied in the present experiment, and physically corresponds to the electron being able to complete at least one orbit before being scattered. The factor of $1-e^{-\mu}$ will be retained unapproximated in the B_{mn} . With the approximation, α_{ik} becomes

$$\alpha_{ik} = \frac{eH}{\pi M c_s \beta^2 c} \{ B_{55} - 2B_{15} Y_2 + B_{11} Y_2^2 + Y_1^2 (B_{07} - B_{33}) \} \quad (105)$$

The term $B_{11}Y_2^2 + Y_1^2(B_{07} - B_{33})$ may be simplified to give

$$\frac{(B_{07} - B_{33})B_{15}^2 + B_{11}(B_{35})^2}{B_{11}(B_{07} - B_{33}) + B_{13}^2}, \quad (106)$$

hence

$$\alpha_{ik} = \frac{eH}{\pi M c_s \beta^2 c} \left\{ \frac{B_{07}(B_{11}B_{55} - B_{15}^2)}{B_{11}(B_{07} - B_{33}) + B_{13}^2} + \frac{B_{11}(B_{35}^2 - B_{33}B_{55}) + B_{13}^2B_{55} + B_{33}B_{15}^2 - 2B_{13}B_{15}B_{35}}{B_{11}(B_{07} - B_{33}) + B_{13}^2} \right\}. \quad (107)$$

Equation(107) reduces to Pippards result when the approximation that $1 - e^{-\mu} \simeq \mu$ is made in the B_{mn} .

The above expression for α_{ik} may be applied to an arbitrary Fermi surface and will be valid for both electron and hole bands. However, the major feature of interest, i.e., the oscillatory effect seen in the geometric resonance region of the magnetoacoustic attenuation, may be elucidated by application to a particularly simple model of the Fermi surface; that of the free electron model. In this model K_x may be evaluated as shown in the preceding section, and along with ω_c , τ , and l , will take the same value over all of the Fermi surface. Thus it may be seen that

$$\eta_5 = \beta K_x a \eta_3 \quad (108)$$

which allows one to write

$$B_{55} = \beta K_x a B_{35} = (\beta K_x a)^2 B_{33} \quad (109)$$

and

$$B_{15} = \beta K_x \text{ a } B_{13} \quad (110)$$

where $K_x = -1/3 k_o$, k_o being defined as the radius of the free-electron Fermi sphere. It may now be seen that the entire second term in Eq.(107) for α_{ik} vanishes, and defining A_{mn} by

$$B_{mn} = (1-e^{-\mu})^{-1} A_{mn}, \quad (111)$$

α_{ik} is given by

$$\alpha_{ik} = \frac{eHk_o^2 q^2 \ell^2}{9\pi M c_s c (1-e^{-\mu})} \left\{ \frac{A_{07}(A_{11}A_{33} - A_{13}^2)}{A_{11}(A_{07} - A_{33}) + A_{13}^2} \right\}. \quad (112)$$

If the coefficient A_{07} in the numerator is factored and evaluated, the remaining term may be simplified algebraically. Thus with

$$A_{07} = \frac{k_o (1-e^{-\mu})}{\pi \omega_c \tau} \quad (113)$$

and $k_o^3 = 3\pi^2 n$; where n is the free electron density, then

$$\alpha_{ik} = \frac{nmq^2 \ell^2}{3Mv_s \tau} \left\{ \frac{A_{07}A_{11}}{A_{11}(A_{07} - A_{33}) + A_{13}^2} - 1 \right\}. \quad (114)$$

The remaining integrals may be evaluated in the free-electron approximation in a manner similar to the following example. Recall

$$I_1 = \frac{1}{2\pi} \oint -i\beta \sin\phi \exp(i\beta k_y) ds \quad (115)$$

With $Bk_0 \equiv X$, then

$$I_1 = \frac{-iX \sin \theta}{2\pi} \int_0^{2\pi} \exp(iX \sin \theta \sin \phi) \sin \phi d\phi \quad . \quad (116)$$

Recalling the expansion of the exponential in Bessel functions

$$\exp(ir \sin \phi) = \sum_{n=-\infty}^{\infty} J_n(r) \exp(in\phi) \quad , \quad (117)$$

and making the substitution $\psi = \phi - \pi$, one finds

$$I_1 = \frac{-iX \sin \theta}{2\pi} \sum_{n=-\infty}^{\infty} (-1)^{n+1} J_n(X \sin \theta) \int_{-\pi}^{\pi} \exp(in\psi) \sin \psi d\psi \quad . \quad (118)$$

The orthonormality properties of the sine and cosine functions, along with the relation

$$J_0'(z) = -J_1(z) = J_{-1}(z) \quad (119)$$

result in

$$I_1 = -X \sin \theta J_0'(X \sin \theta) \quad , \quad (120)$$

where the prime denotes differentiation with respect to the argument.

Similarly for I_3 , it is found that

$$I_3 = \frac{1}{2\pi} \oint \lambda \exp(i\beta k_y) ds = (X/a) J_0(X \sin \theta) \quad . \quad (121)$$

Note that $X/a = (\omega_c \tau)^{-1}$.

The integrals A_{mn} become

$$A_{07} = \frac{k_0 X}{a\pi} (1 - e^{-\mu}) \quad , \quad (122a)$$

$$A_{11} = qk_o X^2 \int_0^{\pi/2} [J_o'(X \sin \theta)]^2 \sin^3 \theta d\theta, \quad (122b)$$

$$A_{33} = \frac{2k_o X^2}{a^2} \int_0^{\pi/2} [J_o(X \sin \theta)]^2 \sin \theta d\theta, \quad (122c)$$

$$A_{13} = \frac{-2k_o X^2}{a} \int_0^{\pi/2} J_o(X \sin \theta) J_o'(X \sin \theta) \sin^2 \theta d\theta. \quad (122d)$$

It may be seen that

$$A_{11} = 2k_o X^2 s_o(X), \quad (123a)$$

$$A_{33} = \frac{2k_o X^2}{a^2} g_o(X), \quad (123b)$$

and

$$A_{13} = \frac{-k_o X^2}{a} g_o'(X), \quad (123c)$$

where $s_o(X)$, $g_o(X)$, and $g_o'(X) \equiv \frac{d}{dX} g_o(X)$ are the same functions introduced by Cohen, Harrison, and Harrison, and are defined in Eqs. (122b), (122c), and (122d).

The attenuation coefficient becomes

$$\alpha_{ik} = \frac{nmq^2 \ell^2}{3Mc_s \tau} \left\{ \frac{1}{\left[1 - \frac{2\pi X}{a(1-e^{-\mu})} g_o \right] + \frac{\pi X}{2a(1-e^{-\mu})} \frac{(g_o')^2}{s_o}} - 1 \right\}. \quad (124)$$

In the limit $\mu \ll 1$, the result of Pippard is again obtained

$$\alpha_{ik} \xrightarrow{\mu \ll 1} \frac{nmq^2 \ell^2}{3Mc_s \tau} \left\{ \frac{1}{(1-g_o) + \frac{(g_o')^2}{4s_o}} - 1 \right\}. \quad (125)$$

Cohen, Harrison, and Harrison have numerically evaluated the expression in brackets and have presented the results in their paper published in 1960. The main result is that α_{ik} is oscillatory, and the period is related to k_y by

$$k_y^c = \frac{e\lambda}{ch \Delta(1/H)} \quad , \quad (126)$$

where $\Delta(1/H)$ is the period in reciprocal field k_y^c is the y direction caliper of the electron orbit in k -space.

The oscillatory part of α_{ik} comes predominantly from the $g_o'(X)$ term, and the maxima and minima occur when $g_o'(X)$ vanishes. The oscillations are thus due to the oscillations in $J_o(2X)$ as may be seen as follows.

Recall that

$$g_o(X) = \int_0^{\pi/2} J_o^2(X \sin \theta) \sin \theta \, d\theta = \frac{1}{X} \sum_{m=0}^{\infty} J_{2m+1}^2(2X) \quad . \quad (127)$$

Therefore taking the derivative with respect to X of $g_o(X)$, and using the relation

$$2J_n' = J_{n-1} - J_{n+1} \quad (128)$$

it is seen that

$$g_o'(X) = \frac{1}{X} [J_o(2X) - g_o(X)] \quad . \quad (129)$$

Since the oscillatory part of g_o is much smaller in magnitude than that of J_o , then the oscillatory effect in α_{ik} will be dominated by the zeroes of $J_o(2X)$.

APPENDIX II

The purpose of this appendix is to illustrate the procedure for reduction of the integrals occurring in Appendix I. In general, it will be shown that for l large, the double integrals can be reduced to products of single integrals, which certainly facilitates the evaluation of the attenuation coefficient.

First consider integrals of the type

$$L = \oint \exp(-i\beta k_y) \cos\phi \, ds \int_{s-s_0}^s X(s') \exp(i\beta k_y' + \int_s^{s'} \lambda ds'') ds' , \quad (1)$$

which, for example, appear in the integral expression for J_x . Note that

$$\frac{d}{ds} \{ \exp(-i\beta k_y) \} = +i\beta \cos\phi \exp(-i\beta k_y) \quad (2)$$

taking into consideration the positive direction of s . Thus L may be integrated once by parts, giving

$$L = \frac{+i}{\beta} \oint \exp(-i\beta k_y) \frac{d}{ds} \left\{ \int_{s-s_0}^s X(s') \exp(i\beta k_y' + \int_s^{s'} \lambda ds'') ds' \right\} ds , \quad (3)$$

since the initial term in the integration contains as a product the closed line integral of a perfect differential of a well behaved periodic function.

Through the use of Leibnitz rule, the differentiation may be performed resulting in

$$L = \frac{+i}{\beta} \oint X(s) ds$$

$$\frac{-i}{\beta} \oint X(s-s_0) \left(\exp \int_s^{s-s_0} \lambda ds'' \right) ds \quad (4)$$

$$\frac{+i}{\beta} \oint \exp(-i\beta k_y) ds \int_{s-s_0}^s X(s') \exp(i\beta k_y' + \int_s^{s'} \lambda ds'') [-\lambda(s)] ds'$$

since $k_y(s) = k_y(s-s_0)$. Also since $X(s-s_0) = X(s)$ and

$$\int_s^{s-s_0} \lambda ds'' = - \oint \lambda ds'' \quad (5)$$

then

$$L = \frac{1}{\beta} (1-e^{-\mu}) \oint X(s) ds - \frac{1}{\beta} \oint \lambda \exp(-i\beta k_y) ds \oint X(s') \exp(i\beta k_y') ds' \quad (6)$$

where the exponential damping term is neglected in the last integral.

The approximation is valid unless it causes the second integral to

vanish. This occurs when $X(s') = A \cos \phi'$. In this case both the first

and last integrals vanish, thereby requiring a further integration by parts for the second integral before the damping term is neglected.

This is done as follows. Consider the second integral in Eq. (6) with the exponential restored, i.e.,

$$L = \frac{-i}{\beta} \oint \lambda \exp(-i\beta k_y) ds \int_{s-s_0}^s A \cos \phi' \exp(i\beta k_y' + \int_s^{s'} \lambda ds'') ds' \quad (7)$$

But

$$\int_{s-s_0}^s \cos\phi' \exp(i\beta k_Y' + \int_s^{s'} \lambda ds'') ds' = \frac{-1}{i\beta} \int_{s-s_0}^s \frac{d}{ds'} [\exp(i\beta k_Y')] \exp(\int_s^{s'} \lambda ds'') ds'. \quad (8)$$

Integration by parts of the right hand side of Eq. (8) gives

$$\frac{-1}{i\beta} \{ \exp(i\beta k_Y) (1-e^{-\mu}) \} + \frac{-1}{i\beta} \int_{s-s_0}^s \lambda(s') \exp(i\beta k_Y' + \int_s^{s'} \lambda ds'') ds'. \quad (9)$$

At this point the damping term may again be neglected, and by substitution of Eq. (9) in Eq. (7), it is seen that

$$L = \frac{A}{\beta^2} \{ \mu(1-e^{-\mu}) - \oint \lambda \exp(-i\beta k_Y) ds \oint \lambda' \exp(i\beta k_Y') ds' \} \quad (10)$$

For integrals of the type

$$M = \oint \exp(-i\beta k_Y) \sin\phi \, ds \int_{s-s_0}^s X(s') \exp(i\beta k_Y' + \int_s^{s'} \lambda \, ds'') ds', \quad (11)$$

which typically appear in J_Y , the damping term may be neglected immediately, yielding

$$M = \oint \exp(-i\beta k_Y) \sin\phi \, ds \oint X(s') \exp(i\beta k_Y') ds', \quad (12)$$

unless, of course, $X(s') = A \cos\phi'$. In this case integration by parts of the s' integral gives

$$M = \frac{-iA}{\beta} \oint \sin\phi \exp(-i\beta k_Y) ds \oint \lambda \exp(i\beta k_Y') ds'. \quad (13)$$

Integrals of the type

$$N = \oint \exp(-i\beta k_Y) \cot\theta \, ds \int_{s-s_0}^s X(s') \exp(i\beta k_Y' + \int_s^{s'} \lambda ds'') ds' \quad (14)$$

may be simplified by neglecting the damping term unless $X(s') = k_z \sin \theta' \cos \phi'$, in which case partial integration is required. However, for the case under consideration, it suffices to write

$$N = \oint \exp(-i\beta k_y) \cot \theta \, ds \oint X(s') \exp(i\beta k_y') ds' \quad (15)$$

For extension to cases other than longitudinal sound waves the reader is referred to the source article by Pippard.

It is now possible to evaluate some of the integrals in J_x , J_y , and J_z . Consider

$$J_x = \frac{c}{4\pi^3 H} \int \frac{dk_z}{1-e^{-\mu}} I_x \quad , \quad (16)$$

where

$$I_x = \oint \exp(-i\beta k_y) \cos \phi \, G(s, s_0) ds \quad (17)$$

The integration of I_x is broken up into parts for ease in handling.

The first part involves

$$I_{x1} = \oint \exp(-i\beta k_y) \cos \phi \, ds \int_{s-s_0}^s i q \hbar u K_x' \operatorname{cosec} \theta' \exp(i\beta k_y' + \int_s^{s'} \lambda ds'') ds' \quad (18)$$

which may be evaluated using Eq. (6) to give

$$I_{x1} = \frac{(2\pi)^2 q \hbar u}{\beta^3} I_3 I_5 \quad (19)$$

Similarly, the term involving $eE_y \sin \phi'$ gives

$$I_{x4} = \frac{(2\pi)^2 e E_y}{\beta^2} I_1 I_3 \quad (20)$$

The next integral is I_{x2} defined by

$$I_{x2} = \oint \exp(-i\beta k_y) \cos\phi \, ds \int_{s=s_0}^s i q \hbar u k'_x \cos\phi' \exp(i\beta k'_y + \int_s^{s'} \lambda ds'') ds' . \quad (21)$$

Application of Eq. (6) gives

$$I_{x2} = \frac{q \hbar u}{\beta} \oint \lambda \exp(-i\beta k_y) ds \oint k'_x \cos\phi' \exp(i\beta k'_y) ds' . \quad (22)$$

The second integral in Eq. (22) may be integrated by parts to give

$$I_{x2} = \frac{(2\pi)^2 q \hbar u}{\beta^3} I_1 I_3 . \quad (23)$$

The integral

$$I_{x3} = \oint \exp(-i\beta k_y) \cos\phi \, ds \int_{s=s_0}^s e E_x \cos\phi' \exp(i\beta k'_y + \int_s^{s'} \lambda ds'') ds' , \quad (24)$$

is simplified through the use of Eq. (10) which yields

$$I_{x3} = \frac{(2\pi)^2 e E_x}{\beta^2} (I_0 I_7 - I_3 I_3) . \quad (25)$$

Since $I_x = I_{x1} + I_{x2} + I_{x3} + I_{x4}$, J_x may be calculated to give

$$J_x = \frac{ec}{H\beta^2 \pi} \left\{ \frac{\hbar u}{c} (B_{13} + B_{35}) + E_x (B_{07} - B_{33}) + E_y B_{13} \right\} \quad (26)$$

The y component of the current density is

$$J_y = \frac{c}{4\pi^3 H} \int \frac{dk_z}{1-e^{-\mu}} I_y , \quad (27)$$

where

$$I_y = \oint \exp(-i\beta k_y) \sin\phi \, G(s, s_0) ds . \quad (28)$$

I_y may be evaluated using Eqs. (12) and (13) to give

$$I_y = \frac{(2\pi)^2 e}{\beta^2} \left\{ \frac{Hu}{c} (I_{15} + I_{11}) - E_x I_{13} + E_y I_{11} \right\} , \quad (29)$$

and substitution of Eq. (29) into Eq. (27) gives for J_y

$$J_y = \frac{ec}{\pi\beta^2 H} \left\{ \frac{Hu}{c} (B_{11} + B_{15}) - E_x B_{13} + E_y B_{11} \right\} . \quad (30)$$

By using Eq. (15) for the integral N , which occurs in J_z , and performing the integral over k_z , it may be seen that J_z vanishes.

REFERENCES

1. W. A. Harrison, Phys. Rev. 118, 1182 (1960).
2. M. H. Cohen and L. M. Falicov, Phys. Rev. 130, 92 (1962).
3. M. H. Cohen and L. M. Falicov, Phys. Rev. Letters 7, 231 (1961).
4. E. I. Blount, Phys. Rev. 126, 1636 (1962).
5. N. E. Alekseevskii and Yu. P. Gaidukov, Zh. Eksperim. i. Teor. Fiz. 43, 2094 (1962) [English transl.: Soviet Phys.—JETP 16, 1481 (1963)].
6. A. R. Mackintosh, L. E. Spanel, and R. C. Young, Phys. Rev. Letters 10, 434 (1963).
7. J. A. Rayne, Phys. Rev. 131, 653 (1963).
8. Y. Eckstein, J. B. Ketterson, and M. G. Priestley (to be published).
9. M. G. Priestley (to be published).
10. P. Soven, Phys. Rev. 137, A1706 (1965).
11. P. Soven, Phys. Rev. 137, A1717 (1965).
12. R. W. Morse, Progress in Cryogenics (Academic Press Inc., New York, 1959), edited by K. Mendelssohn, Vol. 1, p. 219.
13. Sperry Products, Danbury, Connecticut; Hewlett-Packard Co., Palo Alto, California; F. L. Moseley Co., Pasadena, California; Varian Associates, Palo Alto, California; Cominco Products Inc., Spokane, Washington; Valpey Corporation, Holliston, Massachusetts; Dow Corning Corporation, Midland, Michigan.
14. G. N. Kamm and H. V. Bohm, Rev. Sci. Instr. 33, 957 (1962).

15. G. A. Saunders and A. W. Lawson, Phys. Rev. 135, A1161 (1964).
16. M. H. Cohen, M. J. Harrison, and W. A. Harrison, Phys. Rev. 117, 937 (1960).
17. A. B. Pippard, Proc. Roy. Soc. (London), A257, 165 (1960).
18. A. A. Galkin, E. A. Kaner, and A. P. Korolyuk, Zh. Eksperim. i Teor. Fiz. 39, 1517 (1960) [English transl.: Soviet Phys — JETP 12, 1055 (1961)].
19. J. D. Gavenda and B. C. Deaton, Phys. Rev. Letters 8, 208 (1962).
20. As measured from Fig. 7 of reference 8.
21. Raoul B. Weil, Ph.D. Dissertation, University of California, Riverside, California, 1965 (unpublished).
22. L. Onsager, Phil. Mag. 46, 1104 (1952).

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